Computer algebra independent integration tests

7-Inverse-hyperbolic-functions/7.1-Inverse-hyperbolic-sine/7.1.4-f-x-^m-d+e-x^2-^p-a+b-arcsinh-c-x-^n

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July 28, 2021 Compiled on July 28, 2021 at 3:43am

Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [663]. This is test number [187].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (663)	% 0.00 (0)
Mathematica	% 100.00 (663)	% 0.00 (0)
Maple	% 75.11 (498)	% 24.89 (165)
Maxima	% 39.06 (259)	% 60.94 (404)
Fricas	% 36.35 (241)	% 63.65 (422)
Sympy	% 27.90 (185)	% 72.10 (478)
Giac	% 14.33 (95)	% 85.67 (568)
Mupad	% 20.06 (133)	% 79.94 (530)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

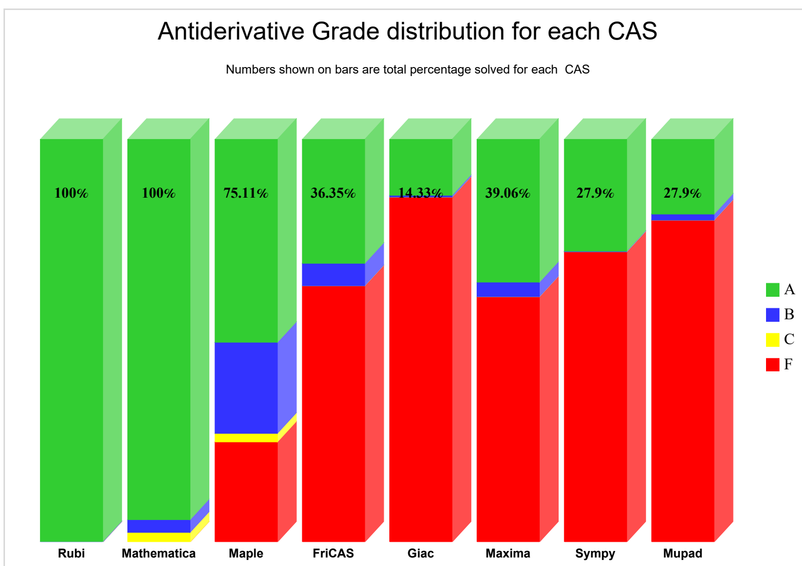
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

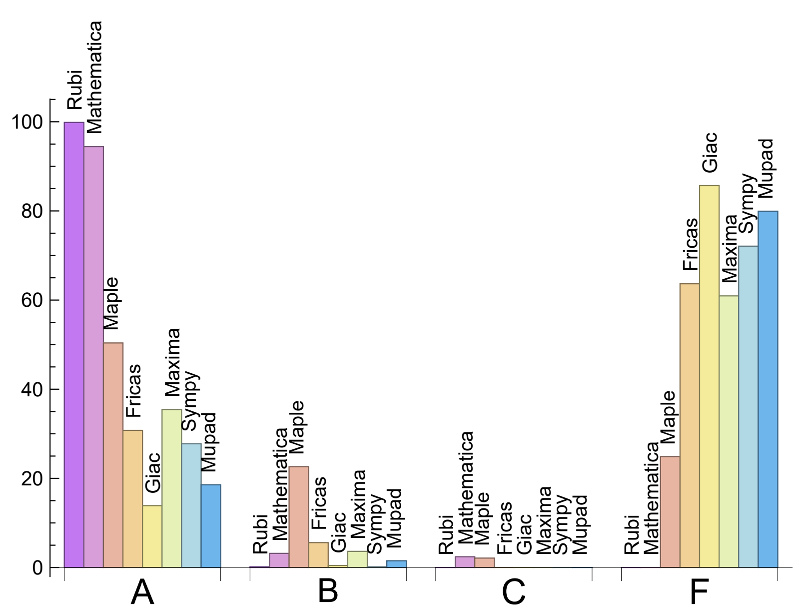
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.85	0.15	0.00	0.00
Mathematica	94.42	3.17	2.41	0.00
Maple	50.38	22.62	2.11	24.89
Maxima	35.44	3.62	0.00	60.94
Fricas	30.77	5.58	0.00	63.65
Sympy	27.75	0.15	0.00	72.10
Giac	13.88	0.45	0.00	85.67
Mupad	18.55	1.51	0.00	79.94

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	165	81.82 %	0.00 %	18.18 %
Maxima	404	79.46 %	1.98 %	18.56 %
Fricas	422	84.83 %	0.00 %	15.17 %
Sympy	478	80.33 %	19.67 %	0.00 %
Giac	568	48.77 %	0.18 %	51.06 %
Mupad	530	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

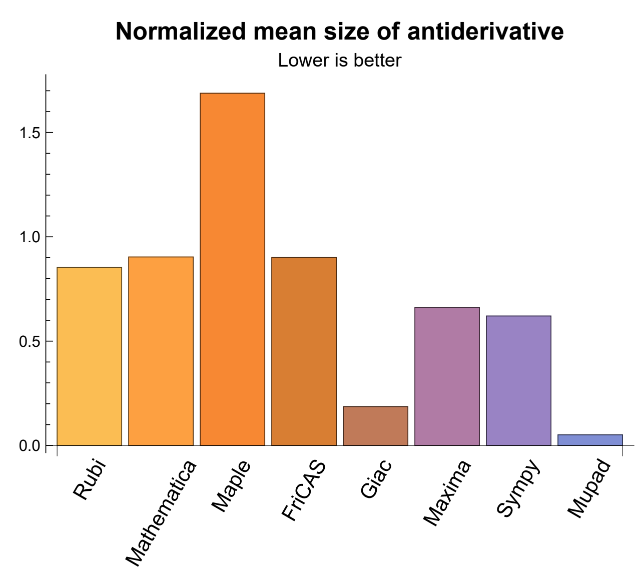
1.3 Performance

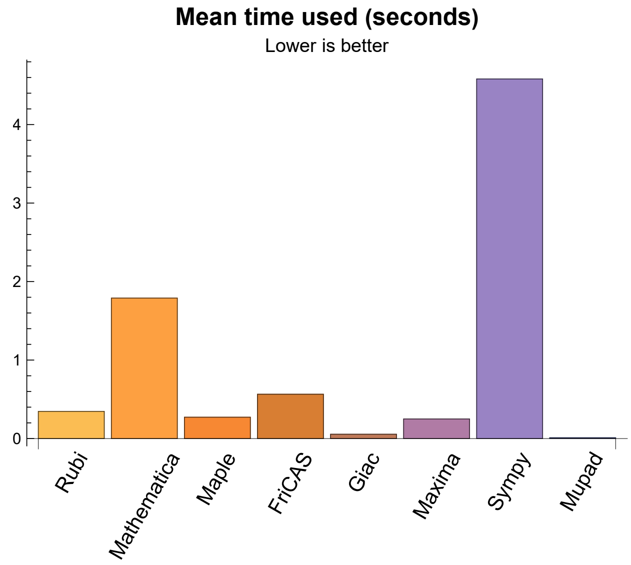
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.35	201.41	0.85	174.00	1.00
Mathematica	1.79	225.28	0.90	143.00	0.91
Maple	0.27	353.52	1.69	189.50	1.40
Maxima	0.25	108.47	0.66	18.00	0.71
Fricas	0.56	120.32	0.90	57.00	0.91
Sympy	4.58	94.89	0.62	0.00	0.00
Giac	0.06	8.65	0.19	0.00	0.00
Mupad	0.01	0.27	0.05	-1.00	-0.03

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{188, 189, 190, 321, 322, 323, 324, 325, 326, 327, 341, 353, 354, 360, 361, 362, 363, 368, 369, 370, 371, 376, 377, 378, 379, 386, 387, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 409, 410, 415, 416, 417, 418, 423, 424, 425, 426, 431, 432, 433, 434, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 466, 471, 475, 476, 480, 484, 488, 489, 493, 499, 500, 505, 506, 510, 511, 515, 516, 520, 521, 525, 526, 527, 532, 533, 622, 623, 627, 628, 632, 633, 636, 637, 641, 642, 645, 646, 647, 648, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {6, 8, 15, 17, 24, 26, 203, 205, 212, 214, 221, 223, 263, 265, 271, 273, 279, 281, 297, 299}

Mathematica $\{6, 8, 15, 17, 24, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 48, 50, 51, 52, 53, 54, 59, 61, 67, 69, 75, 77, 86, 88, 96, 98, 107, 109, 117, 119, 124, 132, 134, 140, 151, 153, 161, 163, 172, 174, 182, 184, 203, 204, 205, 206, 212, 213, 214, 215, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 245, 247, 248, 249, 250, 251, 256, 257, 262, 263, 264, 265, 270, 271, 272, 273, 278, 279, 280, 281, 287, 288, 289, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 331, 332, 333, 338, 339, 340, 347, 349, 462, 463, 464, 465, 467, 468, 469, 470, 472, 473, 477, 481, 485, 486, 490, 494, 495, 496, 497, 501, 502, 507, 508, 545, 551, 564, 565, 574, 575, 580, 581, 586, 587, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 611, 612, 617, 629, 630, 631, 634, 635, 638, 639, 640, 643, 644, 649, 650, 651\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
```

```
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

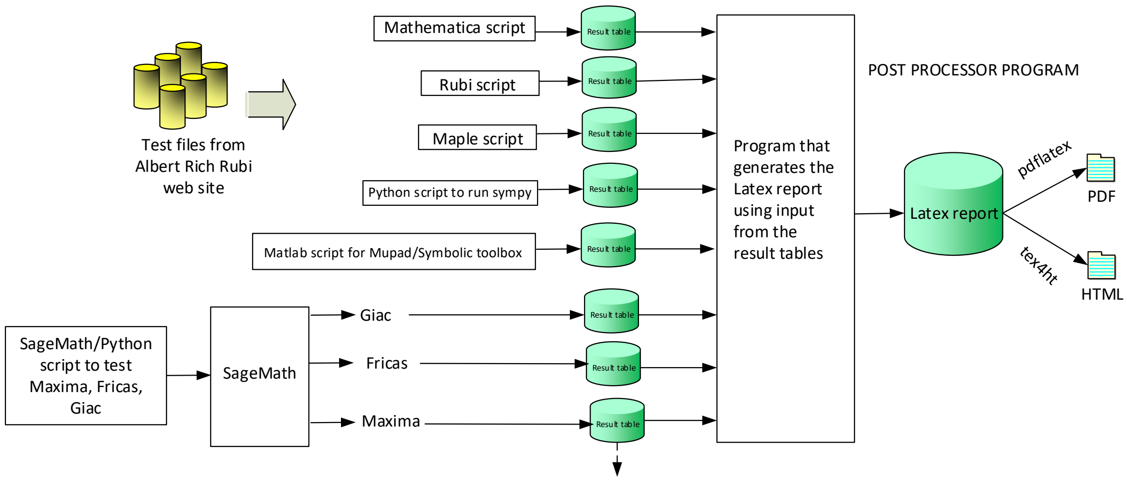
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"

    The following field present only in Rubi and Mathematica Tables

13. integer. 1 if result was verified or 0 if not verified.

    The following fields present only in Rubi Tables

14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,..}" which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

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May 11, 2021

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663 }

B grade: { 73 }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 34, 36, 37, 39, 40, 41, 46, 47, 48, 49, 50, 51, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 227, 229, 231, 234, 236, 237, 238, 240, 242, 243, 244, 245, 246, 247, 249, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 582, 583, 584, 585, 588, 589, 590, 592, 593, 595, 596, 597, 599, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663 }

B grade: { 31, 33, 35, 42, 44, 228, 230, 233, 241, 331, 551, 564, 580, 581, 586, 587, 591, 594, 598, 600, 601 }

C grade: { 38, 43, 45, 52, 54, 226, 232, 235, 239, 248, 250, 348, 612, 649, 650, 651 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49, 50, 52, 54, 55, 56, 59, 61, 63, 64, 66, 67, 69, 71, 72, 74, 75, 77, 79, 80, 81, 82, 83, 84, 86, 88, 96, 98, 107, 109, 112, 113, 114, 115, 116, 117, 119, 124, 126, 129, 132, 134, 137, 139, 140, 142, 144, 150, 151, 153, 158, 161, 162, 163, 172, 174, 177, 178, 179, 180, 181, 182, 184, 188, 189, 190, 198, 199, 200, 201, 202, 204, 206, 207, 208, 209, 210, 211, 213, 215, 216, 217, 218, 219, 220, 222, 224, 226, 228, 252, 282, 283, 284, 285, 286, 287, 288, 289, 300, 302, 305, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 404, 405, 406, 407, 408, 409, 410, 415, 416, 417, 418, 423, 424, 425, 426, 431, 432, 433, 434, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 458, 459, 460, 461, 474, 475, 476, 479, 480, 483, 484, 487, 488, 489, 492, 493, 498, 499, 500, 504, 505, 506, 509, 510, 511, 515, 516, 520, 521, 525, 526, 527, 531, 532, 533, 606, 607, 608, 609, 610, 615, 616, 618, 619, 620, 621, 622, 623, 625, 626, 627, 628, 632, 633, 636, 637, 641, 642, 645, 646, 647, 648, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663 }

B grade: { 42, 51, 53, 57, 58, 60, 62, 65, 68, 70, 73, 76, 78, 85, 87, 89, 91, 93, 95, 97, 99, 100, 102, 104, 106, 108, 110, 111, 118, 120, 121, 122, 123, 125, 127, 128, 130, 131, 133, 135, 136, 138, 141, 143, 145, 146, 147, 148, 149, 152, 154, 156, 160, 164, 165, 167, 169, 171, 173, 175, 176, 183, 203, 205, 212, 214, 221, 223, 230, 232, 235, 237, 239, 241, 244, 246, 248, 250, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 301, 303, 304, 307, 309, 310, 311, 312, 313, 314, 315, 317, 319, 411, 412, 413, 414, 419, 420, 421, 422, 427, 428, 429, 430, 435, 436, 437, 438, 439, 613, 614, 624 }

C grade: { 90, 92, 94, 101, 103, 105, 155, 157, 159, 166, 168, 170, 611, 612 }

F grade: { 185, 186, 187, 191, 192, 193, 194, 195, 196, 197, 225, 227, 229, 231, 233, 234, 236, 238, 240, 242, 243, 245, 247, 249, 251, 306, 308, 316, 318, 331, 332, 333, 401, 402, 403, 455, 456, 457, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 477, 478, 481, 482, 485, 486, 490, 491, 494, 495, 496, 497, 501, 502, 503, 507, 508, 512, 513, 514, 517, 518, 519, 522, 523, 524, 528, 529, 530, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 617, 629, 630, 631, 634, 635, 638, 639, 640, 643, 644, 649, 650, 651 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 7, 9, 11, 12, 14, 16, 18, 25, 27, 55, 57, 63, 65, 71, 73, 79, 80, 82, 84, 85, 89, 92, 95, 97, 103, 106, 110, 111, 112, 113, 114, 115, 116, 118, 120, 122, 127, 128, 130, 136, 138, 144, 145, 147, 149, 150, 152, 154, 157, 160, 162, 168, 169, 171, 175, 176, 177, 178, 179, 180, 181, 183, 188, 189, 190, 198, 200, 258, 260, 266, 268, 274, 276, 283, 285, 286, 290, 292, 294, 295, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 337, 341, 343, 345, 346, 353, 354, 360, 361, 362, 363, 368, 369, 370, 371, 376, 377, 378, 379, 385, 386, 387, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 409, 410, 415, 416, 417, 418, 423, 424, 425, 426, 431, 432, 433, 434, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 466, 471, 475, 476, 480, 484, 488, 489, 493, 499, 500, 505, 506, 510, 511, 515, 516, 520, 521, 525, 526, 527, 531, 532, 533, 539, 555, 556, 557, 561, 562, 563, 566, 567, 568, 569, 591, 606, 607, 608, 609, 610, 613, 614, 615, 616, 622, 623, 627, 628, 632, 633, 636, 637, 641, 642, 645, 646, 648, 653, 655, 656, 657, 658, 659, 660, 661, 662, 663 }

B grade: { 10, 13, 19, 20, 21, 22, 23, 62, 87, 104, 199, 201, 202, 207, 208, 209, 210, 211, 216, 217, 218, 219, 220, 255 }

C grade: { }

F grade: { 6, 8, 15, 17, 24, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 56, 58, 59, 60, 61, 64, 66, 67, 68, 69, 70, 72, 74, 75, 76, 77, 78, 81, 83, 86, 88, 90, 91, 93, 94, 96, 98, 99, 100, 101, 102, 105, 107, 108, 109, 117, 119, 121, 123, 124, 125, 126, 129, 131, 132, 133, 134, 135, 137, 139, 140, 141, 142, 143, 146, 148, 151, 153, 155, 156, 158, 159, 161, 163, 164, 165, 166, 167, 170, 172, 173, 174, 182, 184, 185, 186, 187, 191, 192, 193, 194, 195, 196, 197, 203, 204, 205, 206, 212, 213, 214, 215, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 256, 257, 259, 261, 262, 263, 264, 265, 267, 269, 270, 271, 272, 273, 275, 277, 278, 279, 280, 281, 282, 284, 287, 288, 289, 291, 293, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 331, 332, 333, 334, 335, 336, 338, 339, 340, 342, 344, 347, 348, 349, 350, 351, 352, 355, 356, 357, 358, 359, 364, 365, 366, 367, 372, 373, 374, 375, 380, 381, 382, 383, 384, 388, 389, 390, 391, 392, 406, 407, 408, 411, 412, 413, 414, 419, 420, 421, 422, 427, 428, 429, 430, 435, 436, 437, 438, 439, 462, 463, 464, 465, 467, 468, 469, 470, 472, 473, 474, 477, 478, 479, 481, 482, 483, 485, 486, 487, 490, 491, 492, 494, 495, 496, 497, 498, 501, 502, 503, 504, 507, 508, 509, 512, 513, 514, 517, 518, 519, 522, 523, 524, 528, 529, 530, 534, 535, 536, 537, 538, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 558, 559, 560, 564, 565, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 611, 612, 617, 618, 619, 620, 621, 624, 625, 626, 629, 630, 631, 634, 635, 638, 639, 640, 643, 644, 647, 649, 650, 651, 652, 654 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 19, 20, 21, 22, 23, 25, 27, 40, 47, 49, 55, 63, 79, 80, 82, 90, 101, 112, 113, 114, 115, 118, 120, 122, 128, 130, 136, 138, 144, 145, 147, 149, 154, 155, 157, 159, 166, 168, 170, 177, 178, 179, 180, 183, 188, 189, 190, 198, 199, 200, 201, 202, 207, 208, 209, 210, 211, 216, 217, 218, 219, 220, 244, 258, 260, 266, 268, 274, 276, 282, 283, 284, 285, 290, 292, 294, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 341, 342, 343, 344, 345, 353, 354, 360, 361, 362, 363, 368, 369, 370, 371, 376, 377, 378, 379, 386, 387, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 409, 410, 415, 416, 417, 418, 423, 424, 425, 426, 431, 432, 433, 434, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 487, 492, 504, 509, 515, 516, 520, 521, 525, 526, 527, 532, 533, 606, 607, 608, 609, 610, 613, 614, 615, 622, 623, 627, 628, 647, 648, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663 }

B grade: { 7, 9, 16, 18, 57, 62, 65, 71, 73, 84, 87, 89, 92, 94, 103, 105, 116, 127, 152, 181, 237, 246, 286, 346, 385, 461, 531, 539, 556, 557, 561, 566, 567, 616, 649, 650, 651 }

C grade: { }

F grade: { 6, 8, 15, 17, 24, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 48, 50, 51, 52, 53, 54, 56, 58, 59, 60, 61, 64, 66, 67, 68, 69, 70, 72, 74, 75, 76, 77, 78, 81, 83, 85, 86, 88, 91, 93, 95, 96, 97, 98, 99, 100, 102, 104, 106, 107, 108, 109, 110, 111, 117, 119, 121, 123, 124, 125, 126, 129, 131, 132, 133, 134, 135, 137, 139, 140, 141, 142, 143, 146, 148, 150, 151, 153, 156, 158, 160, 161, 162, 163, 164, 165, 167, 169, 171, 172, 173, 174, 175, 176, 182, 184, 185, 186, 187, 191, 192, 193, 194, 195, 196, 197, 203, 204, 205, 206, 212, 213, 214, 215, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 245, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 259, 261, 262, 263, 264, 265, 267, 269, 270, 271, 272, 273, 275, 277, 278, 279, 280, 281, 287, 288, 289, 291, 293, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 347, 348, 349, 350, 351, 352, 355, 356, 357, 358, 359, 364, 365, 366, 367, 372, 373, 374, 375, 380, 381, 382, 383, 384, 388, 389, 390, 391, 392, 406, 407, 408, 411, 412, 413, 414, 419, 420, 421, 422, 427, 428, 429, 430, 435, 436, 437, 438, 439, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 488, 489, 490, 491, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 505, 506, 507, 508, 510, 511, 512, 513, 514, 517, 518, 519, 522, 523, 524, 528, 529, 530, 534, 535, 536, 537, 538, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 558, 559, 560, 562, 563, 564, 565, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 611, 612, 617, 618, 619, 620, 621, 624, 625, 626, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 19, 20, 21, 22, 23, 55, 57, 65, 66, 80, 81, 82, 83, 84, 85, 112, 113, 114, 115, 116, 177, 178, 179, 180, 181, 188, 189, 198, 199, 200, 201, 202, 207, 208, 209, 210, 211, 216, 217, 218, 219, 220, 253, 255, 282, 283, 284, 285, 286, 323, 324, 325, 327, 328, 329, 330, 341, 342, 343, 344, 345, 346, 353, 354, 360, 361, 362, 363, 368, 369, 370, 371, 376, 377, 378, 379, 385, 386, 387, 393, 394, 395, 396, 397, 398, 399, 400, 402, 403, 404, 405, 409, 410, 415, 416, 417, 418, 423, 424, 425, 426, 431, 432, 433, 434, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 456, 457, 458, 459, 460, 461, 466, 471, 475, 476, 480, 488, 489, 493, 499, 500, 505, 515, 516, 520, 527, 531, 532, 533, 606, 607, 608, 609, 610, 613, 614, 615, 616, 622, 627, 632, 633, 636, 641, 642, 645, 647, 648, 652, 653, 654, 656, 657, 658, 659, 660, 661, 662, 663 }

B grade: { 60 }

C grade: { }

F grade: { 6, 7, 8, 9, 15, 16, 17, 18, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 56, 58, 59, 61, 62, 63, 64, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 182, 183, 184, 185, 186, 187, 190, 191, 192, 193, 194, 195, 196, 197, 203, 204, 205, 206, 212, 213, 214, 215, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 254, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 326, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 347, 348, 349, 350, 351, 352, 355, 356, 357, 358, 359, 364, 365, 366, 367, 372, 373, 374, 375, 380, 381, 382, 383, 384, 388, 389, 390, 391, 392, 401, 406, 407, 408, 411, 412, 413, 414, 419, 420, 421, 422, 427, 428, 429, 430, 435, 436, 437, 438, 439, 455, 462, 463, 464, 465, 467, 468, 469, 470, 472, 473, 474, 477, 478, 479, 481, 482, 483, 484, 485, 486, 487, 490, 491, 492, 494, 495, 496, 497, 498, 501, 502, 503, 504, 506, 507, 508, 509, 510, 511, 512, 513, 514, 517, 518, 519, 521, 522, 523, 524, 525, 526, 528, 529, 530, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 611, 612, 617, 618, 619, 620, 621, 623, 624, 625, 626, 628, 629, 630, 631, 634, 635, 637, 638, 639, 640, 643, 644, 646, 649, 650, 651, 655 }

2.1.7 Giac

A grade: { 111, 115, 176, 180, 188, 189, 190, 285, 324, 325, 326, 327, 341, 345, 353, 354, 361, 363, 369, 371, 377, 379, 386, 387, 395, 396, 398, 400, 404, 405, 409, 410, 416, 418, 424, 426, 432, 434, 442, 444, 446, 448, 450, 452, 454, 458, 459, 460, 475, 476, 480, 484, 488, 489, 493, 499, 500, 505, 506, 510, 511, 527, 531, 532, 533, 610, 622, 623, 627, 628, 632, 633, 636, 637, 641, 642, 645, 646, 647, 648, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663 }

B grade: { 118, 183, 616 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 113, 114, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 178, 179, 181, 182, 184, 185, 186, 187, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 342, 343, 344, 346, 347, 348, 349, 350, 351, 352, 355, 356, 357, 358, 359, 360, 362, 364, 365, 366, 367, 368, 370, 372, 373, 374, 375, 376, 378, 380, 381, 382, 383, 384, 385, 388, 389, 390, 391, 392, 393, 394, 397, 399, 401, 402, 403, 406, 407, 408, 411, 412, 413, 414, 415, 417, 419, 420, 421, 422, 423, 425, 427, 428, 429, 430, 431, 433, 435, 436, 437, 438, 439, 440, 441, 443, 445, 447, 449, 451, 453, 455, 456, 457, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 477, 478, 479, 481, 482, 483, 485, 486, 487, 490, 491, 492, 494, 495, 496, 497, 498, 501, 502, 503, 504, 507, 508, 509, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 528, 529, 530, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 611, 612, 613, 614, 615, 617, 618, 619, 620, 621, 624, 625, 626, 629, 630, 631, 634, 635, 638, 639, 640, 643, 644, 649, 650, 651 }

2.1.8 Mupad

A grade: { 188, 189, 190, 321, 322, 323, 324, 325, 326, 327, 341, 353, 354, 360, 361, 362, 363, 368, 369, 370, 371, 376, 377, 378, 379, 386, 387, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 409, 410, 415, 416, 417, 418, 423, 424, 425, 426, 431, 432, 433, 434, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 466, 471, 475, 476, 480, 484, 488, 489, 493, 499, 500, 505, 506, 510, 511, 515, 516, 520, 521, 525, 526, 527, 532, 533, 622, 623, 627, 628, 632, 633, 636, 637, 641, 642, 645, 646, 647, 648, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663 }

B grade: { 116, 181, 286, 346, 385, 393, 440, 461, 531, 610 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 342, 343, 344, 345, 347, 348, 349, 350, 351, 352, 355, 356, 357, 358, 359, 364, 365, 366, 367, 372, 373, 374, 375, 380, 381, 382, 383, 384, 388, 389, 390, 391, 392, 406, 407, 408, 411, 412, 413, 414, 419, 420, 421, 422, 427, 428, 429, 430, 435, 436, 437, 438, 439, 462, 463, 464, 465, 467, 468, 469, 470, 472, 473, 474, 477, 478, 479, 481, 482, 483, 485, 486, 487, 490, 491, 492, 494, 495, 496, 497, 498, 501, 502, 503, 504, 507, 508, 509, 512, 513, 514, 517, 518, 519, 522, 523, 524, 528, 529, 530, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 624, 625, 626, 629, 630, 631, 634, 635, 638, 639, 640, 643, 644, 649, 650, 651 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	87	124	184	113	151	0	-1
normalized size	1	1.00	0.70	1.00	1.48	0.91	1.22	0.00	-0.01
time (sec)	N/A	0.119	0.136	0.037	0.361	0.549	5.684	0.000	0.000

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	88	113	166	109	138	0	-1
normalized size	1	1.00	0.73	0.94	1.38	0.91	1.15	0.00	-0.01
time (sec)	N/A	0.098	0.051	0.012	0.360	0.497	3.758	0.000	0.000

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	78	105	145	103	126	0	-1
normalized size	1	1.00	0.76	1.03	1.42	1.01	1.24	0.00	-0.01
time (sec)	N/A	0.102	0.076	0.010	0.442	0.597	2.020	0.000	0.000

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	77	94	127	98	117	0	-1
normalized size	1	1.00	0.89	1.08	1.46	1.13	1.34	0.00	-0.01
time (sec)	N/A	0.041	0.050	0.008	0.530	0.560	1.157	0.000	0.000

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	86	76	97	83	90	0	-1
normalized size	1	1.00	1.15	1.01	1.29	1.11	1.20	0.00	-0.01
time (sec)	N/A	0.060	0.042	0.007	0.482	0.691	0.521	0.000	0.000

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	113	162	0	0	0	0	-1
normalized size	1	1.00	1.02	1.46	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.057	0.272	0.000	0.635	0.000	0.000	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	74	69	64	156	0	0	-1
normalized size	1	1.00	1.12	1.05	0.97	2.36	0.00	0.00	-0.02
time (sec)	N/A	0.081	0.025	0.010	0.513	0.587	0.000	0.000	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	111	175	0	0	0	0	-1
normalized size	1	1.00	0.87	1.37	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.054	0.241	0.000	0.643	0.000	0.000	0.000

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	93	87	91	169	0	0	-1
normalized size	1	1.00	1.16	1.09	1.14	2.11	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.031	0.014	0.339	0.584	0.000	0.000	0.000

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	119	167	319	165	230	0	-1
normalized size	1	1.00	0.66	0.92	1.76	0.91	1.27	0.00	-0.01
time (sec)	N/A	0.208	0.093	0.014	0.570	0.716	16.104	0.000	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	115	156	292	161	218	0	-1
normalized size	1	1.00	0.64	0.87	1.62	0.89	1.21	0.00	-0.01
time (sec)	N/A	0.175	0.078	0.011	0.623	0.579	10.996	0.000	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	111	148	261	153	202	0	-1
normalized size	1	1.00	0.71	0.94	1.66	0.97	1.29	0.00	-0.01
time (sec)	N/A	0.169	0.080	0.008	0.549	0.647	6.138	0.000	0.000

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	104	137	234	149	190	0	-1
normalized size	1	1.00	0.87	1.14	1.95	1.24	1.58	0.00	-0.01
time (sec)	N/A	0.065	0.131	0.005	0.478	0.867	4.162	0.000	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	95	119	194	133	165	0	-1
normalized size	1	1.00	0.74	0.93	1.52	1.04	1.29	0.00	-0.01
time (sec)	N/A	0.102	0.113	0.007	0.325	0.518	2.259	0.000	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	173	231	0	0	0	0	-1
normalized size	1	1.00	1.01	1.34	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.203	0.183	0.165	0.000	0.509	0.000	0.000	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	124	114	143	228	0	0	-1
normalized size	1	1.00	1.03	0.95	1.19	1.90	0.00	0.00	-0.01
time (sec)	N/A	0.157	0.116	0.011	0.474	0.508	0.000	0.000	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	143	262	0	0	0	0	-1
normalized size	1	1.00	0.76	1.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.211	0.324	0.320	0.000	0.600	0.000	0.000	0.000

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	133	114	137	243	0	0	-1
normalized size	1	1.00	1.06	0.90	1.09	1.93	0.00	0.00	-0.01
time (sec)	N/A	0.159	0.115	0.013	0.450	0.707	0.000	0.000	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	143	206	465	201	289	0	-1
normalized size	1	1.00	0.63	0.91	2.06	0.89	1.28	0.00	-0.00
time (sec)	N/A	0.281	0.114	0.020	0.487	0.614	39.153	0.000	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	139	195	429	197	280	0	-1
normalized size	1	1.00	0.70	0.98	2.16	0.99	1.41	0.00	-0.01
time (sec)	N/A	0.169	0.105	0.017	0.397	0.539	28.240	0.000	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	135	187	388	189	265	0	-1
normalized size	1	1.00	0.67	0.93	1.92	0.94	1.31	0.00	-0.00
time (sec)	N/A	0.249	0.098	0.009	0.642	0.612	16.746	0.000	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	128	176	352	185	253	0	-1
normalized size	1	1.00	0.88	1.21	2.43	1.28	1.74	0.00	-0.01
time (sec)	N/A	0.070	0.162	0.007	0.469	0.632	11.422	0.000	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	119	156	301	169	221	0	-1
normalized size	1	1.00	0.70	0.92	1.77	0.99	1.30	0.00	-0.01
time (sec)	N/A	0.161	0.153	0.007	0.555	0.610	6.235	0.000	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	189	284	0	0	0	0	-1
normalized size	1	1.00	0.86	1.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.284	0.125	0.198	0.000	0.632	0.000	0.000	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	163	151	231	276	0	0	-1
normalized size	1	1.00	1.02	0.94	1.44	1.72	0.00	0.00	-0.01
time (sec)	N/A	0.221	0.149	0.012	0.484	0.672	0.000	0.000	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	184	313	0	0	0	0	-1
normalized size	1	1.00	0.74	1.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.303	0.411	0.386	0.000	0.439	0.000	0.000	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	171	155	208	289	0	0	-1
normalized size	1	1.00	0.98	0.89	1.20	1.66	0.00	0.00	-0.01
time (sec)	N/A	0.256	0.150	0.013	0.453	0.550	0.000	0.000	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	170	266	0	0	0	0	-1
normalized size	1	1.00	1.09	1.71	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.241	0.282	0.135	0.000	0.542	0.000	0.000	0.000

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	181	161	0	0	0	0	-1
normalized size	1	1.00	1.34	1.19	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.195	0.188	0.125	0.000	0.502	0.000	0.000	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	121	215	0	0	0	0	-1
normalized size	1	1.00	1.12	1.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.140	0.151	0.013	0.000	0.578	0.000	0.000	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	167	98	0	0	0	0	-1
normalized size	1	1.00	2.29	1.34	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.063	0.046	0.000	0.590	0.000	0.000	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	135	171	0	0	0	0	-1
normalized size	1	1.00	1.93	2.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.084	0.006	0.000	0.652	0.000	0.000	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	207	74	0	0	0	0	-1
normalized size	1	1.00	3.39	1.21	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.117	0.081	0.066	0.000	0.762	0.000	0.000	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	182	202	0	0	0	0	-1
normalized size	1	1.00	1.80	2.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.150	0.137	0.015	0.000	0.542	0.000	0.000	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	240	266	0	0	0	0	-1
normalized size	1	1.00	2.12	2.35	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.198	0.237	0.126	0.000	0.545	0.000	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	247	261	0	0	0	0	-1
normalized size	1	1.00	1.58	1.67	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.247	0.191	0.019	0.000	0.682	0.000	0.000	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	268	285	0	0	0	0	-1
normalized size	1	1.00	1.57	1.67	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.241	0.334	0.021	0.000	0.534	0.000	0.000	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	241	206	0	0	0	0	-1
normalized size	1	1.00	1.66	1.42	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.191	0.215	0.213	0.000	0.534	0.000	0.000	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	221	240	0	0	0	0	-1
normalized size	1	1.00	1.74	1.89	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.134	0.258	0.013	0.000	0.567	0.000	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	74	61	0	65	0	0	-1
normalized size	1	1.00	1.35	1.11	0.00	1.18	0.00	0.00	-0.02
time (sec)	N/A	0.050	0.061	0.005	0.000	0.577	0.000	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	216	234	0	0	0	0	-1
normalized size	1	1.00	1.74	1.89	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.107	0.011	0.000	0.731	0.000	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	234	283	0	0	0	0	-1
normalized size	1	1.00	2.13	2.57	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.177	0.415	0.120	0.000	0.557	0.000	0.000	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	253	267	0	0	0	0	-1
normalized size	1	1.00	1.51	1.59	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.187	0.549	0.020	0.000	0.681	0.000	0.000	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	326	311	0	0	0	0	-1
normalized size	1	1.00	2.23	2.13	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.260	0.448	0.145	0.000	0.649	0.000	0.000	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	239	264	311	332	0	0	0	0	-1
normalized size	1	1.10	1.30	1.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.309	0.577	0.020	0.000	0.524	0.000	0.000	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	341	313	0	0	0	0	-1
normalized size	1	1.00	1.83	1.68	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.233	0.680	0.016	0.000	0.518	0.000	0.000	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	79	108	0	99	0	0	-1
normalized size	1	1.00	0.81	1.11	0.00	1.02	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.142	0.015	0.000	0.540	0.000	0.000	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	340	307	0	0	0	0	-1
normalized size	1	1.00	1.85	1.67	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.176	0.247	0.014	0.000	0.607	0.000	0.000	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	56	76	0	98	0	0	-1
normalized size	1	1.00	0.70	0.95	0.00	1.22	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.072	0.009	0.000	0.676	0.000	0.000	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	341	295	0	0	0	0	-1
normalized size	1	1.00	1.92	1.66	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.141	0.152	0.012	0.000	0.595	0.000	0.000	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	289	451	0	0	0	0	-1
normalized size	1	1.00	1.82	2.84	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.252	0.591	0.197	0.000	0.624	0.000	0.000	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	298	357	0	0	0	0	-1
normalized size	1	1.00	1.34	1.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.238	1.053	0.020	0.000	0.624	0.000	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	353	575	0	0	0	0	-1
normalized size	1	1.00	1.52	2.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.344	0.830	0.261	0.000	0.511	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	295	345	380	425	0	0	0	0	-1
normalized size	1	1.17	1.29	1.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.367	0.929	0.020	0.000	0.696	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	111	106	164	134	158	221	0	-1
normalized size	1	1.02	0.97	1.50	1.23	1.45	2.03	0.00	-0.01
time (sec)	N/A	0.121	0.200	0.152	0.410	0.560	15.677	0.000	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	181	79	170	0	0	0	0	-1
normalized size	1	1.52	0.66	1.43	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.198	0.170	0.067	0.000	0.651	0.000	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	105	63	108	73	127	141	0	-1
normalized size	1	1.72	1.03	1.77	1.20	2.08	2.31	0.00	-0.02
time (sec)	N/A	0.068	0.120	0.059	0.502	0.687	2.204	0.000	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	111	69	112	0	0	0	0	-1
normalized size	1	1.66	1.03	1.67	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.141	0.057	0.000	0.512	0.000	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	89	177	131	171	0	0	0	0	-1
normalized size	1	1.99	1.47	1.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.192	0.197	0.215	0.000	0.609	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	B	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	105	75	155	0	0	110	0	-1
normalized size	1	1.72	1.23	2.54	0.00	0.00	1.80	0.00	-0.02
time (sec)	N/A	0.108	0.167	0.165	0.000	0.672	3.008	0.000	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	113	201	185	243	0	0	0	0	-1
normalized size	1	1.78	1.64	2.15	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.195	3.321	0.300	0.000	0.660	0.000	0.000	0.000

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	106	78	501	133	217	0	0	-1
normalized size	1	1.71	1.26	8.08	2.15	3.50	0.00	0.00	-0.02
time (sec)	N/A	0.090	0.143	0.306	0.604	0.766	0.000	0.000	0.000

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	127	100	195	145	199	0	0	-1
normalized size	1	1.02	0.80	1.56	1.16	1.59	0.00	0.00	-0.01
time (sec)	N/A	0.143	0.169	0.111	0.491	0.617	0.000	0.000	0.000

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	254	154	240	0	0	0	0	-1
normalized size	1	1.54	0.93	1.45	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.321	0.343	0.113	0.000	0.708	0.000	0.000	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	146	72	139	85	167	221	0	-1
normalized size	1	1.90	0.94	1.81	1.10	2.17	2.87	0.00	-0.01
time (sec)	N/A	0.087	0.125	0.065	0.381	0.550	82.232	0.000	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	180	111	170	0	0	185	0	-1
normalized size	1	1.62	1.00	1.53	0.00	0.00	1.67	0.00	-0.01
time (sec)	N/A	0.112	0.237	0.063	0.000	0.634	38.263	0.000	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	134	249	180	227	0	0	0	0	-1
normalized size	1	1.86	1.34	1.69	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.304	0.371	0.271	0.000	0.579	0.000	0.000	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	177	122	222	0	0	0	0	-1
normalized size	1	1.64	1.13	2.06	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.168	0.296	0.224	0.000	0.714	0.000	0.000	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	155	270	292	295	0	0	0	0	-1
normalized size	1	1.74	1.88	1.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.302	1.659	0.523	0.000	0.514	0.000	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	184	125	622	0	0	0	0	-1
normalized size	1	1.60	1.09	5.41	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.219	0.233	0.290	0.000	0.671	0.000	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	143	108	226	156	263	0	0	-1
normalized size	1	1.01	0.77	1.60	1.11	1.87	0.00	0.00	-0.01
time (sec)	N/A	0.153	0.203	0.115	0.519	0.615	0.000	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	337	196	301	0	0	0	0	-1
normalized size	1	1.58	0.92	1.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.458	0.606	0.109	0.000	0.749	0.000	0.000	0.000

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	B	A	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	193	80	170	96	225	0	0	-1
normalized size	1	2.08	0.86	1.83	1.03	2.42	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.152	0.072	0.367	0.610	0.000	0.000	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	254	153	228	0	0	0	0	-1
normalized size	1	1.54	0.93	1.38	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.165	0.384	0.069	0.000	0.789	0.000	0.000	0.000

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	179	329	257	284	0	0	0	0	-1
normalized size	1	1.84	1.44	1.59	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.430	0.363	0.357	0.000	0.736	0.000	0.000	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	257	168	283	0	0	0	0	-1
normalized size	1	1.64	1.07	1.80	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.236	0.421	0.294	0.000	0.688	0.000	0.000	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	205	355	349	356	0	0	0	0	-1
normalized size	1	1.73	1.70	1.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.431	1.954	0.623	0.000	0.664	0.000	0.000	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	266	179	692	0	0	0	0	-1
normalized size	1	1.60	1.08	4.17	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.294	0.377	0.351	0.000	0.549	0.000	0.000	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	28	26	28	40	0	0	-1
normalized size	1	1.00	0.88	0.81	0.88	1.25	0.00	0.00	-0.03
time (sec)	N/A	0.030	0.013	0.037	0.613	0.681	0.000	0.000	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	215	108	193	174	161	182	0	-1
normalized size	1	1.44	0.72	1.30	1.17	1.08	1.22	0.00	-0.01
time (sec)	N/A	0.257	0.181	0.122	0.442	0.633	10.091	0.000	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	170	111	188	0	0	185	0	-1
normalized size	1	1.35	0.88	1.49	0.00	0.00	1.47	0.00	-0.01
time (sec)	N/A	0.226	0.233	0.107	0.000	0.665	9.015	0.000	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	142	82	133	117	132	122	0	-1
normalized size	1	1.45	0.84	1.36	1.19	1.35	1.24	0.00	-0.01
time (sec)	N/A	0.157	0.140	0.106	0.383	0.659	3.943	0.000	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	97	69	125	0	0	92	0	-1
normalized size	1	1.29	0.92	1.67	0.00	0.00	1.23	0.00	-0.01
time (sec)	N/A	0.121	0.179	0.068	0.000	0.563	4.147	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	64	49	72	55	96	60	0	-1
normalized size	1	1.52	1.17	1.71	1.31	2.29	1.43	0.00	-0.02
time (sec)	N/A	0.065	0.080	0.065	0.506	0.602	2.439	0.000	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	53	28	0	85	0	-1
normalized size	1	1.00	1.00	2.12	1.12	0.00	3.40	0.00	-0.04
time (sec)	N/A	0.030	0.016	0.054	0.500	0.660	2.523	0.000	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	96	72	0	0	0	0	-1
normalized size	1	1.00	1.71	1.29	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.119	0.155	0.045	0.000	0.489	0.000	0.000	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	63	42	84	101	132	0	0	-1
normalized size	1	1.54	1.02	2.05	2.46	3.22	0.00	0.00	-0.02
time (sec)	N/A	0.088	0.111	0.091	0.462	0.627	0.000	0.000	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	115	137	185	225	0	0	0	0	-1
normalized size	1	1.19	1.61	1.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.213	2.595	0.187	0.000	0.605	0.000	0.000	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	141	99	372	121	222	0	0	-1
normalized size	1	1.45	1.02	3.84	1.25	2.29	0.00	0.00	-0.01
time (sec)	N/A	0.182	0.158	0.118	0.453	0.583	0.000	0.000	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	140	131	224	0	196	0	0	-1
normalized size	1	1.02	0.96	1.64	0.00	1.43	0.00	0.00	-0.01
time (sec)	N/A	0.172	0.216	0.319	0.000	0.681	0.000	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	181	147	269	0	0	0	0	-1
normalized size	1	1.38	1.12	2.05	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.257	0.356	0.327	0.000	0.705	0.000	0.000	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	88	87	158	119	165	0	0	-1
normalized size	1	1.02	1.01	1.84	1.38	1.92	0.00	0.00	-0.01
time (sec)	N/A	0.142	0.158	0.343	0.593	0.661	0.000	0.000	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	105	78	196	0	0	0	0	-1
normalized size	1	1.31	0.98	2.45	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.140	0.291	0.158	0.000	0.462	0.000	0.000	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	70	52	103	0	127	0	0	-1
normalized size	1	1.56	1.16	2.29	0.00	2.82	0.00	0.00	-0.02
time (sec)	N/A	0.072	0.112	0.092	0.000	0.835	0.000	0.000	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	76	66	132	58	0	0	0	-1
normalized size	1	1.49	1.29	2.59	1.14	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.039	0.099	0.082	0.552	0.826	0.000	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	94	119	143	156	0	0	0	0	-1
normalized size	1	1.27	1.52	1.66	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.222	0.318	0.229	0.000	0.624	0.000	0.000	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	95	69	180	119	0	0	0	-1
normalized size	1	1.02	0.74	1.94	1.28	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.138	0.152	0.148	0.555	0.779	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	162	212	269	234	0	0	0	0	-1
normalized size	1	1.31	1.66	1.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.350	3.830	0.304	0.000	0.507	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	156	127	601	0	0	0	0	-1
normalized size	1	1.02	0.83	3.93	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.175	0.200	0.324	0.000	0.898	0.000	0.000	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	256	202	970	0	0	0	0	-1
normalized size	1	1.33	1.05	5.05	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.425	0.474	0.510	0.000	0.548	0.000	0.000	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	149	132	231	0	218	0	0	-1
normalized size	1	1.02	0.90	1.58	0.00	1.49	0.00	0.00	-0.01
time (sec)	N/A	0.181	0.215	0.484	0.000	0.892	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	178	166	897	0	0	0	0	-1
normalized size	1	1.28	1.19	6.45	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.283	0.363	0.355	0.000	0.699	0.000	0.000	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	107	93	175	138	187	0	0	-1
normalized size	1	1.02	0.89	1.67	1.31	1.78	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.183	0.516	0.609	0.605	0.000	0.000	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	119	88	707	137	0	0	0	-1
normalized size	1	1.49	1.10	8.84	1.71	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.170	0.297	0.527	0.685	0.000	0.000	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	114	72	124	0	165	0	0	-1
normalized size	1	1.52	0.96	1.65	0.00	2.20	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.136	0.108	0.000	0.626	0.000	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	147	100	618	126	0	0	0	-1
normalized size	1	1.36	0.93	5.72	1.17	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.143	0.115	0.488	0.745	0.000	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	187	209	220	0	0	0	0	-1
normalized size	1	1.26	1.41	1.49	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.339	0.766	0.261	0.000	0.613	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	153	123	778	0	0	0	0	-1
normalized size	1	1.02	0.82	5.19	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.176	0.202	0.293	0.000	0.934	0.000	0.000	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	247	325	331	314	0	0	0	0	-1
normalized size	1	1.32	1.34	1.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.475	6.309	0.340	0.000	0.664	0.000	0.000	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	212	142	1153	236	0	0	0	-1
normalized size	1	1.02	0.68	5.54	1.13	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.240	0.228	0.254	0.521	0.994	0.000	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	121	363	143	0	0	124	-1
normalized size	1	1.00	0.60	1.82	0.72	0.00	0.00	0.62	-0.00
time (sec)	N/A	0.123	0.180	0.219	0.411	0.587	0.000	0.394	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	63	74	83	83	82	0	-1
normalized size	1	1.00	0.73	0.86	0.97	0.97	0.95	0.00	-0.01
time (sec)	N/A	0.155	0.051	0.076	0.367	0.523	2.294	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	48	82	59	55	65	0	-1
normalized size	1	1.00	0.69	1.17	0.84	0.79	0.93	0.00	-0.01
time (sec)	N/A	0.111	0.041	0.046	0.323	0.658	1.202	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	42	40	55	62	42	0	-1
normalized size	1	1.00	0.86	0.82	1.12	1.27	0.86	0.00	-0.02
time (sec)	N/A	0.103	0.040	0.041	0.326	0.632	0.760	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	47	26	38	24	38	-1
normalized size	1	1.00	1.00	1.68	0.93	1.36	0.86	1.36	-0.04
time (sec)	N/A	0.046	0.028	0.040	0.356	0.539	0.443	0.413	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	23	10	0	11
normalized size	1	1.00	1.00	0.92	0.85	1.77	0.77	0.00	0.85
time (sec)	N/A	0.022	0.006	0.005	0.348	0.656	0.340	0.000	0.106

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	57	42	0	0	0	0	-1
normalized size	1	1.00	1.68	1.24	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.092	0.102	0.056	0.000	0.642	0.000	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	56	25	39	0	71	-1
normalized size	1	1.00	1.07	2.07	0.93	1.44	0.00	2.63	-0.04
time (sec)	N/A	0.064	0.036	0.082	0.341	0.433	0.000	0.332	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	126	150	0	0	0	0	-1
normalized size	1	1.00	1.58	1.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.660	0.092	0.000	0.614	0.000	0.000	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	120	578	134	158	0	0	-1
normalized size	1	1.00	0.69	3.30	0.77	0.90	0.00	0.00	-0.01
time (sec)	N/A	0.161	0.111	0.262	0.405	0.456	0.000	0.000	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	129	320	0	0	0	0	-1
normalized size	1	1.00	0.71	1.77	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.194	0.810	0.227	0.000	0.557	0.000	0.000	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	92	321	73	127	0	0	-1
normalized size	1	1.00	0.88	3.06	0.70	1.21	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.091	0.138	0.433	0.482	0.000	0.000	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	120	222	0	0	0	0	-1
normalized size	1	1.00	1.08	2.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.451	0.139	0.000	0.593	0.000	0.000	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	168	331	0	0	0	0	-1
normalized size	1	1.00	0.95	1.87	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.195	0.453	0.182	0.000	0.668	0.000	0.000	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	129	263	0	0	0	0	-1
normalized size	1	1.00	1.23	2.50	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.324	0.175	0.000	0.659	0.000	0.000	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	223	377	0	0	0	0	-1
normalized size	1	1.00	1.11	1.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.195	2.927	0.239	0.000	0.707	0.000	0.000	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	131	946	133	217	0	0	-1
normalized size	1	1.00	1.24	8.92	1.25	2.05	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.242	0.257	0.533	0.733	0.000	0.000	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	130	872	145	199	0	0	-1
normalized size	1	1.00	0.60	4.02	0.67	0.92	0.00	0.00	-0.00
time (sec)	N/A	0.177	0.162	0.273	0.395	0.670	0.000	0.000	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	251	421	0	0	0	0	-1
normalized size	1	1.00	0.99	1.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.310	0.725	0.296	0.000	0.527	0.000	0.000	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	102	559	85	167	0	0	-1
normalized size	1	1.00	0.70	3.83	0.58	1.14	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.131	0.168	0.442	0.697	0.000	0.000	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	200	318	0	0	0	0	-1
normalized size	1	1.00	1.11	1.77	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.805	0.168	0.000	0.485	0.000	0.000	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	248	428	0	0	0	0	-1
normalized size	1	1.00	1.00	1.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.303	0.772	0.207	0.000	0.684	0.000	0.000	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	200	392	0	0	0	0	-1
normalized size	1	1.00	1.13	2.21	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.170	0.789	0.208	0.000	0.670	0.000	0.000	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	352	472	0	0	0	0	-1
normalized size	1	1.00	1.30	1.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.310	4.463	0.270	0.000	0.645	0.000	0.000	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	214	1107	0	0	0	0	-1
normalized size	1	1.00	1.16	6.02	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.231	0.766	0.260	0.000	0.569	0.000	0.000	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	140	996	156	263	0	0	-1
normalized size	1	1.00	0.53	3.74	0.59	0.99	0.00	0.00	-0.00
time (sec)	N/A	0.192	0.191	0.279	0.444	0.783	0.000	0.000	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	388	537	0	0	0	0	-1
normalized size	1	1.00	1.15	1.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.466	0.937	0.338	0.000	0.580	0.000	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	112	863	96	225	0	0	-1
normalized size	1	1.00	0.58	4.47	0.50	1.17	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.152	0.206	0.371	0.659	0.000	0.000	0.000

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	317	427	0	0	0	0	-1
normalized size	1	1.00	1.25	1.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.160	0.717	0.209	0.000	0.625	0.000	0.000	0.000

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	361	540	0	0	0	0	-1
normalized size	1	1.00	1.10	1.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.439	1.265	0.248	0.000	0.626	0.000	0.000	0.000

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	270	506	0	0	0	0	-1
normalized size	1	1.00	1.05	1.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.238	1.469	0.241	0.000	0.532	0.000	0.000	0.000

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	467	588	0	0	0	0	-1
normalized size	1	1.00	1.32	1.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.447	6.745	0.302	0.000	0.612	0.000	0.000	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	287	1316	0	0	0	0	-1
normalized size	1	1.00	1.08	4.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.300	0.971	0.303	0.000	0.528	0.000	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	28	26	28	40	0	0	-1
normalized size	1	1.00	0.88	0.81	0.88	1.25	0.00	0.00	-0.03
time (sec)	N/A	0.029	0.013	0.000	0.472	0.600	0.000	0.000	0.000

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	119	625	174	161	0	0	-1
normalized size	1	1.00	0.55	2.91	0.81	0.75	0.00	0.00	-0.00
time (sec)	N/A	0.264	0.168	0.291	0.379	0.540	0.000	0.000	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	151	347	0	0	0	0	-1
normalized size	1	1.00	0.79	1.81	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.252	0.616	0.337	0.000	0.596	0.000	0.000	0.000

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	93	358	117	132	0	0	-1
normalized size	1	1.00	0.65	2.52	0.82	0.93	0.00	0.00	-0.01
time (sec)	N/A	0.158	0.130	0.234	0.361	0.552	0.000	0.000	0.000

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	121	247	0	0	0	0	-1
normalized size	1	1.00	1.02	2.08	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.146	0.729	0.250	0.000	0.768	0.000	0.000	0.000

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	74	148	55	96	0	0	-1
normalized size	1	1.00	1.16	2.31	0.86	1.50	0.00	0.00	-0.02
time (sec)	N/A	0.061	0.104	0.117	0.410	0.421	0.000	0.000	0.000

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	48	77	28	0	0	0	-1
normalized size	1	1.00	1.02	1.64	0.60	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.057	0.051	0.047	0.379	0.612	0.000	0.000	0.000

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	129	234	0	0	0	0	-1
normalized size	1	1.00	1.06	1.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.188	0.333	0.162	0.000	0.627	0.000	0.000	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	67	183	101	132	0	0	-1
normalized size	1	1.00	1.06	2.90	1.60	2.10	0.00	0.00	-0.02
time (sec)	N/A	0.089	0.163	0.171	0.344	0.583	0.000	0.000	0.000

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	229	380	0	0	0	0	-1
normalized size	1	1.00	1.13	1.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.292	3.018	0.326	0.000	0.622	0.000	0.000	0.000

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	135	791	121	222	0	0	-1
normalized size	1	1.00	0.96	5.61	0.86	1.57	0.00	0.00	-0.01
time (sec)	N/A	0.184	0.204	0.277	0.354	0.708	0.000	0.000	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	220	148	362	0	197	0	0	-1
normalized size	1	1.04	0.70	1.71	0.00	0.93	0.00	0.00	-0.00
time (sec)	N/A	0.294	0.266	0.293	0.000	0.614	0.000	0.000	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	161	366	0	0	0	0	-1
normalized size	1	1.00	0.78	1.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.282	0.468	0.333	0.000	0.463	0.000	0.000	0.000

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	141	143	260	119	166	0	0	-1
normalized size	1	1.04	1.05	1.91	0.88	1.22	0.00	0.00	-0.01
time (sec)	N/A	0.182	0.238	0.225	0.487	0.758	0.000	0.000	0.000

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	146	232	0	0	0	0	-1
normalized size	1	1.00	1.12	1.78	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.168	0.201	0.251	0.000	0.597	0.000	0.000	0.000

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	82	164	0	128	0	0	-1
normalized size	1	1.00	1.17	2.34	0.00	1.83	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.162	0.140	0.000	0.628	0.000	0.000	0.000

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	100	143	58	0	0	0	-1
normalized size	1	1.00	1.32	1.88	0.76	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.172	0.111	0.347	0.652	0.000	0.000	0.000

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	231	274	0	0	0	0	-1
normalized size	1	1.00	1.19	1.41	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.303	0.850	0.181	0.000	0.672	0.000	0.000	0.000

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	163	205	119	0	0	0	-1
normalized size	1	1.00	1.14	1.43	0.83	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.154	0.284	0.171	0.413	0.792	0.000	0.000	0.000

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	381	389	0	0	0	0	-1
normalized size	1	1.00	1.33	1.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.431	6.374	0.295	0.000	0.577	0.000	0.000	0.000

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	216	965	0	0	0	0	-1
normalized size	1	1.00	0.95	4.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.289	0.316	0.314	0.000	0.902	0.000	0.000	0.000

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	222	1607	0	0	0	0	-1
normalized size	1	1.00	0.79	5.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.435	1.004	0.475	0.000	0.598	0.000	0.000	0.000

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	225	154	394	0	219	0	0	-1
normalized size	1	1.07	0.73	1.88	0.00	1.04	0.00	0.00	-0.00
time (sec)	N/A	0.309	0.271	0.283	0.000	0.707	0.000	0.000	0.000

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	191	1430	0	0	0	0	-1
normalized size	1	1.00	0.94	7.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.296	0.531	0.374	0.000	0.604	0.000	0.000	0.000

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	149	151	262	138	188	0	0	-1
normalized size	1	1.03	1.05	1.82	0.96	1.31	0.00	0.00	-0.01
time (sec)	N/A	0.182	0.254	0.227	0.589	0.635	0.000	0.000	0.000

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	118	1153	137	0	0	0	-1
normalized size	1	1.00	0.99	9.69	1.15	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.125	0.199	0.277	0.390	0.573	0.000	0.000	0.000

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	130	198	0	166	0	0	-1
normalized size	1	1.00	1.14	1.74	0.00	1.46	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.188	0.145	0.000	0.690	0.000	0.000	0.000

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	143	1005	126	0	0	0	-1
normalized size	1	1.00	0.97	6.84	0.86	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.197	0.153	0.357	0.816	0.000	0.000	0.000

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	247	364	0	0	0	0	-1
normalized size	1	1.00	0.94	1.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.415	1.200	0.200	0.000	0.511	0.000	0.000	0.000

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	227	1257	0	0	0	0	-1
normalized size	1	1.00	1.06	5.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.213	0.312	0.207	0.000	0.983	0.000	0.000	0.000

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	400	400	437	546	0	0	0	0	-1
normalized size	1	1.00	1.09	1.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.560	6.560	0.306	0.000	0.613	0.000	0.000	0.000

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	267	1790	236	0	0	0	-1
normalized size	1	1.00	0.90	6.03	0.79	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.372	0.361	0.290	0.439	1.104	0.000	0.000	0.000

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	121	363	143	0	0	124	-1
normalized size	1	1.00	0.60	1.82	0.72	0.00	0.00	0.62	-0.00
time (sec)	N/A	0.112	0.147	0.002	0.422	0.645	0.000	0.681	0.000

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	63	74	83	83	82	0	-1
normalized size	1	1.00	0.73	0.86	0.97	0.97	0.95	0.00	-0.01
time (sec)	N/A	0.146	0.051	0.002	0.416	0.588	2.267	0.000	0.000

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	48	82	59	55	65	0	-1
normalized size	1	1.00	0.69	1.17	0.84	0.79	0.93	0.00	-0.01
time (sec)	N/A	0.103	0.035	0.001	0.401	0.710	1.203	0.000	0.000

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	42	40	55	62	42	0	-1
normalized size	1	1.00	0.86	0.82	1.12	1.27	0.86	0.00	-0.02
time (sec)	N/A	0.081	0.037	0.000	0.538	0.522	0.754	0.000	0.000

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	47	26	38	24	38	-1
normalized size	1	1.00	1.00	1.68	0.93	1.36	0.86	1.36	-0.04
time (sec)	N/A	0.042	0.028	0.000	0.412	0.502	0.429	0.515	0.000

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	23	10	0	11
normalized size	1	1.00	1.00	0.92	0.85	1.77	0.77	0.00	0.85
time (sec)	N/A	0.020	0.006	0.001	0.401	0.650	0.339	0.000	0.096

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	57	42	0	0	0	0	-1
normalized size	1	1.00	1.68	1.24	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.085	0.044	0.000	0.000	0.559	0.000	0.000	0.000

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	56	25	39	0	71	-1
normalized size	1	1.00	1.07	2.07	0.93	1.44	0.00	2.63	-0.04
time (sec)	N/A	0.059	0.039	0.000	0.469	0.583	0.000	0.442	0.000

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	126	150	0	0	0	0	-1
normalized size	1	1.00	1.58	1.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.144	0.027	0.000	0.000	0.528	0.000	0.000	0.000

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	257	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.172	0.489	180.000	0.000	0.628	0.000	0.000	0.000

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	188	0	0	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.297	0.056	180.000	0.000	0.731	0.000	0.000	0.000

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	118	0	0	0	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.130	0.045	180.000	0.000	0.474	0.000	0.000	0.000

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.071	3.812	0.480	0.000	0.616	0.000	0.000	0.000

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	116	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.158	5.633	0.548	0.000	0.559	0.000	0.000	0.000

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	207	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.253	5.960	0.607	0.000	0.659	0.000	0.000	0.000

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	618	618	332	0	0	0	0	0	-1
normalized size	1	1.00	0.54	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.556	1.282	1.644	0.000	0.636	0.000	0.000	0.000

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	390	390	233	0	0	0	0	0	-1
normalized size	1	1.00	0.60	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.330	0.520	1.419	0.000	0.535	0.000	0.000	0.000

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	179	0	0	0	0	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.203	0.069	1.339	0.000	0.555	0.000	0.000	0.000

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	129	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.192	0.054	0.390	0.000	0.580	0.000	0.000	0.000

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	206	0	0	0	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.331	0.231	0.534	0.000	0.507	0.000	0.000	0.000

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	286	0	0	0	0	0	-1
normalized size	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.471	0.422	0.539	0.000	0.662	0.000	0.000	0.000

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	97	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.034	0.303	0.000	0.699	0.000	0.000	0.000

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	201	330	441	260	388	0	-1
normalized size	1	1.00	0.71	1.17	1.56	0.92	1.37	0.00	-0.00
time (sec)	N/A	0.477	0.291	0.059	0.541	0.446	11.400	0.000	0.000

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	186	267	442	240	332	0	-1
normalized size	1	1.00	0.94	1.35	2.23	1.21	1.68	0.00	-0.01
time (sec)	N/A	0.568	0.266	0.044	0.470	0.535	7.858	0.000	0.000

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	177	248	346	225	313	0	-1
normalized size	1	1.00	0.86	1.20	1.68	1.09	1.52	0.00	-0.00
time (sec)	N/A	0.342	0.248	0.042	0.439	0.666	5.212	0.000	0.000

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	155	191	347	204	269	0	-1
normalized size	1	1.00	1.15	1.41	2.57	1.51	1.99	0.00	-0.01
time (sec)	N/A	0.134	0.261	0.038	0.365	0.754	3.332	0.000	0.000

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	135	166	230	178	224	0	-1
normalized size	1	1.00	1.08	1.33	1.84	1.42	1.79	0.00	-0.01
time (sec)	N/A	0.144	0.206	0.049	0.722	0.625	1.453	0.000	0.000

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	166	165	209	425	0	0	0	0	-1
normalized size	1	0.99	1.26	2.56	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.246	0.387	0.168	0.000	0.617	0.000	0.000	0.000

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	192	252	0	0	0	0	-1
normalized size	1	1.00	1.47	1.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.318	0.433	0.181	0.000	0.725	0.000	0.000	0.000

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	180	179	212	515	0	0	0	0	-1
normalized size	1	0.99	1.18	2.86	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.310	0.378	0.371	0.000	0.567	0.000	0.000	0.000

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	245	278	0	0	0	0	-1
normalized size	1	1.00	1.55	1.76	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.393	0.798	0.360	0.000	0.666	0.000	0.000	0.000

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	386	251	428	760	368	563	0	-1
normalized size	1	1.00	0.65	1.11	1.97	0.95	1.46	0.00	-0.00
time (sec)	N/A	0.739	0.407	0.057	0.453	0.682	30.819	0.000	0.000

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	237	344	762	348	515	0	-1
normalized size	1	1.00	0.80	1.16	2.57	1.18	1.74	0.00	-0.00
time (sec)	N/A	1.037	0.364	0.048	0.613	0.588	22.949	0.000	0.000

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	227	346	619	327	483	0	-1
normalized size	1	1.00	0.75	1.14	2.04	1.08	1.59	0.00	-0.00
time (sec)	N/A	0.594	0.404	0.046	0.377	0.669	12.702	0.000	0.000

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	208	268	621	307	430	0	-1
normalized size	1	1.00	1.02	1.31	3.04	1.50	2.11	0.00	-0.00
time (sec)	N/A	0.206	0.500	0.039	0.545	0.563	8.816	0.000	0.000

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	191	264	457	278	389	0	-1
normalized size	1	1.00	0.89	1.23	2.14	1.30	1.82	0.00	-0.00
time (sec)	N/A	0.256	0.406	0.051	0.541	0.623	4.766	0.000	0.000

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	257	256	323	586	0	0	0	0	-1
normalized size	1	1.00	1.26	2.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.441	0.396	0.233	0.000	0.622	0.000	0.000	0.000

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	306	400	0	0	0	0	-1
normalized size	1	1.00	1.34	1.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.523	1.170	0.221	0.000	0.578	0.000	0.000	0.000

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	305	719	0	0	0	0	-1
normalized size	1	1.00	1.12	2.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.501	0.882	0.482	0.000	0.538	0.000	0.000	0.000

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	357	408	0	0	0	0	-1
normalized size	1	1.00	1.44	1.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.707	0.912	0.418	0.000	0.724	0.000	0.000	0.000

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	465	465	299	520	1109	444	702	0	-1
normalized size	1	1.00	0.64	1.12	2.38	0.95	1.51	0.00	-0.00
time (sec)	N/A	1.058	0.477	0.099	0.465	0.596	71.195	0.000	0.000

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	285	415	1112	424	654	0	-1
normalized size	1	1.00	0.76	1.10	2.96	1.13	1.74	0.00	-0.00
time (sec)	N/A	1.657	0.457	0.089	0.603	0.653	55.916	0.000	0.000

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	382	275	438	922	403	626	0	-1
normalized size	1	1.00	0.72	1.15	2.41	1.05	1.64	0.00	-0.00
time (sec)	N/A	0.861	0.510	0.054	0.449	0.576	32.873	0.000	0.000

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	256	339	925	383	573	0	-1
normalized size	1	1.00	0.98	1.30	3.54	1.47	2.20	0.00	-0.00
time (sec)	N/A	0.257	0.665	0.046	0.453	0.537	24.281	0.000	0.000

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	239	354	712	354	524	0	-1
normalized size	1	1.00	0.82	1.22	2.45	1.22	1.80	0.00	-0.00
time (sec)	N/A	0.403	0.562	0.056	0.468	0.731	13.357	0.000	0.000

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	337	336	416	706	0	0	0	0	-1
normalized size	1	1.00	1.23	2.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.684	0.693	0.301	0.000	0.646	0.000	0.000	0.000

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	466	516	0	0	0	0	-1
normalized size	1	1.00	1.52	1.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.739	1.427	0.305	0.000	0.695	0.000	0.000	0.000

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	354	355	459	838	0	0	0	0	-1
normalized size	1	1.00	1.30	2.37	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.750	1.202	0.800	0.000	0.681	0.000	0.000	0.000

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	461	528	0	0	0	0	-1
normalized size	1	1.00	1.41	1.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.021	1.099	0.667	0.000	0.559	0.000	0.000	0.000

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	365	0	0	0	0	0	-1
normalized size	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.553	1.324	0.341	0.000	0.515	0.000	0.000	0.000

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	279	380	0	0	0	0	-1
normalized size	1	1.00	1.40	1.91	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.408	0.455	0.197	0.000	0.654	0.000	0.000	0.000

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	293	0	0	0	0	0	-1
normalized size	1	1.00	1.48	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.293	0.594	0.154	0.000	0.678	0.000	0.000	0.000

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	281	223	0	0	0	0	-1
normalized size	1	1.00	2.68	2.12	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.184	0.226	0.067	0.000	0.588	0.000	0.000	0.000

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	274	0	0	0	0	0	-1
normalized size	1	1.00	1.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.242	0.148	0.000	0.648	0.000	0.000	0.000

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	400	354	0	0	0	0	-1
normalized size	1	1.00	3.45	3.05	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.208	0.360	0.109	0.000	0.586	0.000	0.000	0.000

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	363	0	0	0	0	0	-1
normalized size	1	1.00	1.78	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.335	1.064	0.313	0.000	0.646	0.000	0.000	0.000

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	419	719	0	0	0	0	-1
normalized size	1	1.00	2.16	3.71	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.388	1.051	0.213	0.000	0.560	0.000	0.000	0.000

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	602	0	0	0	0	0	-1
normalized size	1	1.00	2.03	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.639	7.960	0.363	0.000	0.641	0.000	0.000	0.000

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	482	0	0	0	0	0	-1
normalized size	1	1.00	1.73	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.531	2.229	0.368	0.000	0.611	0.000	0.000	0.000

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	320	499	0	0	0	0	-1
normalized size	1	1.00	1.50	2.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.405	1.099	0.337	0.000	0.693	0.000	0.000	0.000

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	385	0	0	0	0	0	-1
normalized size	1	1.00	1.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.298	1.864	0.236	0.000	0.694	0.000	0.000	0.000

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	145	222	0	185	0	0	-1
normalized size	1	1.00	1.71	2.61	0.00	2.18	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.220	0.105	0.000	0.610	0.000	0.000	0.000

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	403	0	0	0	0	0	-1
normalized size	1	1.00	1.92	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.245	1.417	0.119	0.000	0.562	0.000	0.000	0.000

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	428	724	0	0	0	0	-1
normalized size	1	1.00	2.22	3.75	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.340	1.857	0.192	0.000	0.730	0.000	0.000	0.000

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	549	0	0	0	0	0	-1
normalized size	1	1.00	1.91	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.544	7.211	0.338	0.000	0.668	0.000	0.000	0.000

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	594	799	0	0	0	0	-1
normalized size	1	1.00	2.35	3.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.581	0.910	0.233	0.000	0.563	0.000	0.000	0.000

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	401	401	764	0	0	0	0	0	-1
normalized size	1	1.00	1.91	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.963	9.037	0.464	0.000	0.623	0.000	0.000	0.000

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	552	0	0	0	0	0	-1
normalized size	1	1.00	1.72	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.545	3.397	0.541	0.000	0.639	0.000	0.000	0.000

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	186	523	0	283	0	0	-1
normalized size	1	1.00	1.11	3.13	0.00	1.69	0.00	0.00	-0.01
time (sec)	N/A	0.337	0.398	0.375	0.000	0.433	0.000	0.000	0.000

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	550	0	0	0	0	0	-1
normalized size	1	1.00	1.73	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.417	2.512	0.403	0.000	0.657	0.000	0.000	0.000

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	152	432	0	273	0	0	-1
normalized size	1	1.00	1.05	2.98	0.00	1.88	0.00	0.00	-0.01
time (sec)	N/A	0.146	0.224	0.151	0.000	0.575	0.000	0.000	0.000

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	546	0	0	0	0	0	-1
normalized size	1	1.00	1.77	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.364	2.336	0.210	0.000	0.602	0.000	0.000	0.000

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	560	1129	0	0	0	0	-1
normalized size	1	1.00	2.04	4.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.507	3.777	0.337	0.000	0.651	0.000	0.000	0.000

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	716	0	0	0	0	0	-1
normalized size	1	1.00	1.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.767	7.697	0.489	0.000	0.614	0.000	0.000	0.000

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	759	1436	0	0	0	0	-1
normalized size	1	1.00	1.99	3.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.803	9.463	0.426	0.000	0.587	0.000	0.000	0.000

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	529	529	937	0	0	0	0	0	-1
normalized size	1	1.00	1.77	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.307	10.295	0.617	0.000	0.509	0.000	0.000	0.000

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	420	284	486	0	0	0	0	-1
normalized size	1	1.40	0.95	1.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.382	0.931	0.134	0.000	0.593	0.000	0.000	0.000

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	294	202	350	0	0	405	0	-1
normalized size	1	1.40	0.96	1.67	0.00	0.00	1.93	0.00	-0.00
time (sec)	N/A	0.227	0.569	0.118	0.000	0.757	95.262	0.000	0.000

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	184	124	213	0	0	0	0	-1
normalized size	1	1.51	1.02	1.75	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.353	0.104	0.000	0.554	0.000	0.000	0.000

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	72	47	0	88	0	-1
normalized size	1	1.00	1.00	2.88	1.88	0.00	3.52	0.00	-0.04
time (sec)	N/A	0.050	0.024	0.084	0.356	0.522	2.809	0.000	0.000

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	104	179	153	306	0	0	0	0	-1
normalized size	1	1.72	1.47	2.94	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.177	0.386	0.193	0.000	0.563	0.000	0.000	0.000

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	204	292	293	1730	0	0	0	0	-1
normalized size	1	1.43	1.44	8.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.282	0.630	0.362	0.000	0.695	0.000	0.000	0.000

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	358	358	222	1162	302	316	0	0	-1
normalized size	1	1.00	0.62	3.25	0.84	0.88	0.00	0.00	-0.00
time (sec)	N/A	0.479	0.274	0.412	0.457	0.729	0.000	0.000	0.000

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	207	701	0	0	0	0	-1
normalized size	1	1.00	0.71	2.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.372	1.324	0.350	0.000	0.637	0.000	0.000	0.000

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	166	657	183	249	0	0	-1
normalized size	1	1.00	0.92	3.65	1.02	1.38	0.00	0.00	-0.01
time (sec)	N/A	0.152	0.272	0.230	0.426	0.671	0.000	0.000	0.000

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	200	482	0	0	0	0	-1
normalized size	1	1.00	1.09	2.62	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.120	1.068	0.214	0.000	0.675	0.000	0.000	0.000

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	352	823	0	0	0	0	-1
normalized size	1	1.00	1.04	2.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.343	1.166	0.337	0.000	0.465	0.000	0.000	0.000

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	232	625	0	0	0	0	-1
normalized size	1	1.00	1.11	2.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.255	1.166	0.321	0.000	0.688	0.000	0.000	0.000

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	358	358	446	870	0	0	0	0	-1
normalized size	1	1.00	1.25	2.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.383	4.883	0.452	0.000	0.708	0.000	0.000	0.000

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	240	2557	0	0	0	0	-1
normalized size	1	1.00	0.82	8.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.285	0.801	0.486	0.000	0.535	0.000	0.000	0.000

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	482	482	251	1766	346	402	0	0	-1
normalized size	1	1.00	0.52	3.66	0.72	0.83	0.00	0.00	-0.00
time (sec)	N/A	0.811	0.385	0.540	0.438	0.673	0.000	0.000	0.000

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	405	405	508	934	0	0	0	0	-1
normalized size	1	1.00	1.25	2.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.664	1.191	0.486	0.000	0.671	0.000	0.000	0.000

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	198	1149	230	332	0	0	-1
normalized size	1	1.00	0.74	4.30	0.86	1.24	0.00	0.00	-0.00
time (sec)	N/A	0.219	0.364	0.294	0.388	0.515	0.000	0.000	0.000

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	329	709	0	0	0	0	-1
normalized size	1	1.00	1.12	2.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.246	1.910	0.276	0.000	0.787	0.000	0.000	0.000

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	498	498	520	1053	0	0	0	0	-1
normalized size	1	1.00	1.04	2.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.595	2.286	0.374	0.000	0.605	0.000	0.000	0.000

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	398	398	369	954	0	0	0	0	-1
normalized size	1	1.00	0.93	2.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.422	2.969	0.361	0.000	0.689	0.000	0.000	0.000

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	541	541	771	1131	0	0	0	0	-1
normalized size	1	1.00	1.43	2.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.654	7.878	0.485	0.000	0.674	0.000	0.000	0.000

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	458	2796	0	0	0	0	-1
normalized size	1	1.00	1.21	7.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.584	1.415	0.464	0.000	0.547	0.000	0.000	0.000

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	625	625	277	2014	390	525	0	0	-1
normalized size	1	1.00	0.44	3.22	0.62	0.84	0.00	0.00	-0.00
time (sec)	N/A	1.238	0.458	0.492	0.417	0.615	0.000	0.000	0.000

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	536	536	619	1204	0	0	0	0	-1
normalized size	1	1.00	1.15	2.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.047	2.063	0.578	0.000	0.599	0.000	0.000	0.000

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	366	366	224	1773	274	446	0	0	-1
normalized size	1	1.00	0.61	4.84	0.75	1.22	0.00	0.00	-0.00
time (sec)	N/A	0.294	0.426	0.406	0.490	0.627	0.000	0.000	0.000

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	420	420	499	966	0	0	0	0	-1
normalized size	1	1.00	1.19	2.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.404	1.517	0.367	0.000	0.685	0.000	0.000	0.000

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	635	635	710	1321	0	0	0	0	-1
normalized size	1	1.00	1.12	2.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.908	4.453	0.460	0.000	0.621	0.000	0.000	0.000

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	530	530	550	1223	0	0	0	0	-1
normalized size	1	1.00	1.04	2.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.613	2.176	0.446	0.000	0.761	0.000	0.000	0.000

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	687	687	990	1404	0	0	0	0	-1
normalized size	1	1.00	1.44	2.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.988	8.023	0.571	0.000	0.615	0.000	0.000	0.000

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	561	561	616	3311	0	0	0	0	-1
normalized size	1	1.00	1.10	5.90	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.851	2.356	0.551	0.000	0.715	0.000	0.000	0.000

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	98	125	0	131	146	0	-1
normalized size	1	1.00	0.64	0.82	0.00	0.86	0.95	0.00	-0.01
time (sec)	N/A	0.290	0.077	0.096	0.000	0.637	3.685	0.000	0.000

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	79	113	101	98	121	0	-1
normalized size	1	1.00	0.65	0.93	0.83	0.80	0.99	0.00	-0.01
time (sec)	N/A	0.215	0.062	0.091	0.464	0.450	2.173	0.000	0.000

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	72	69	0	102	78	0	-1
normalized size	1	1.00	0.83	0.79	0.00	1.17	0.90	0.00	-0.01
time (sec)	N/A	0.154	0.050	0.082	0.000	0.529	1.237	0.000	0.000

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	48	64	48	70	49	74	-1
normalized size	1	1.00	0.92	1.23	0.92	1.35	0.94	1.42	-0.02
time (sec)	N/A	0.078	0.033	0.075	0.519	0.474	0.723	0.377	0.000

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	23	10	0	11
normalized size	1	1.00	1.00	0.92	0.85	1.77	0.77	0.00	0.85
time (sec)	N/A	0.034	0.007	0.005	0.461	0.438	0.395	0.000	0.093

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	100	144	0	0	0	0	-1
normalized size	1	1.00	1.47	2.12	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.149	0.113	0.089	0.000	0.454	0.000	0.000	0.000

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	65	132	0	0	0	0	-1
normalized size	1	1.00	0.98	2.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.162	0.330	0.148	0.000	0.447	0.000	0.000	0.000

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	188	233	0	0	0	0	-1
normalized size	1	1.00	1.39	1.73	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.262	1.201	0.224	0.000	0.431	0.000	0.000	0.000

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	383	383	230	1227	353	319	0	0	-1
normalized size	1	1.00	0.60	3.20	0.92	0.83	0.00	0.00	-0.00
time (sec)	N/A	0.554	0.367	0.496	0.518	0.499	0.000	0.000	0.000

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	268	760	0	0	0	0	-1
normalized size	1	1.00	0.83	2.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.479	0.824	0.536	0.000	0.522	0.000	0.000	0.000

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	176	706	243	254	0	0	-1
normalized size	1	1.00	0.66	2.66	0.92	0.96	0.00	0.00	-0.00
time (sec)	N/A	0.330	0.279	0.375	0.522	0.456	0.000	0.000	0.000

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	198	530	0	0	0	0	-1
normalized size	1	1.00	0.97	2.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.275	0.891	0.399	0.000	0.459	0.000	0.000	0.000

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	127	296	125	179	0	0	-1
normalized size	1	1.00	0.92	2.14	0.91	1.30	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.280	0.186	0.513	0.458	0.000	0.000	0.000

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	62	120	47	0	0	0	-1
normalized size	1	1.00	1.32	2.55	1.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.095	0.133	0.068	0.407	0.451	0.000	0.000	0.000

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	266	564	0	0	0	0	-1
normalized size	1	1.00	1.19	2.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.342	0.866	0.296	0.000	0.489	0.000	0.000	0.000

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	168	526	0	0	0	0	-1
normalized size	1	1.00	1.01	3.15	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.227	0.458	0.311	0.000	0.499	0.000	0.000	0.000

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	455	901	0	0	0	0	-1
normalized size	1	1.00	1.26	2.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.570	5.474	0.482	0.000	0.419	0.000	0.000	0.000

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	278	2147	0	0	0	0	-1
normalized size	1	1.00	0.93	7.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.427	0.647	0.487	0.000	0.488	0.000	0.000	0.000

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	515	515	427	933	0	0	0	0	-1
normalized size	1	1.00	0.83	1.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.793	0.633	0.515	0.000	0.440	0.000	0.000	0.000

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	400	400	288	816	0	0	0	0	-1
normalized size	1	1.00	0.72	2.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.665	1.905	0.543	0.000	0.495	0.000	0.000	0.000

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	383	383	318	703	0	0	0	0	-1
normalized size	1	1.00	0.83	1.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.455	0.468	0.402	0.000	0.534	0.000	0.000	0.000

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	215	478	0	0	0	0	-1
normalized size	1	1.00	0.92	2.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.383	1.010	0.409	0.000	0.461	0.000	0.000	0.000

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	217	446	0	0	0	0	-1
normalized size	1	1.00	1.15	2.37	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.187	0.459	0.217	0.000	0.459	0.000	0.000	0.000

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	152	343	0	0	0	0	-1
normalized size	1	1.00	0.85	1.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.184	0.439	0.183	0.000	0.629	0.000	0.000	0.000

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	568	0	0	0	0	0	-1
normalized size	1	1.00	1.38	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.595	1.545	0.401	0.000	0.508	0.000	0.000	0.000

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	296	660	0	0	0	0	-1
normalized size	1	1.00	0.97	2.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.464	0.926	0.308	0.000	0.543	0.000	0.000	0.000

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	573	573	884	0	0	0	0	0	-1
normalized size	1	1.00	1.54	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.926	7.534	0.543	0.000	0.495	0.000	0.000	0.000

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	452	452	438	2609	0	0	0	0	-1
normalized size	1	1.00	0.97	5.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.841	0.888	0.503	0.000	0.506	0.000	0.000	0.000

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	512	512	333	1040	0	0	0	0	-1
normalized size	1	1.00	0.65	2.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.881	1.770	0.497	0.000	0.474	0.000	0.000	0.000

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	398	398	359	3705	0	0	0	0	-1
normalized size	1	1.00	0.90	9.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.761	1.252	0.555	0.000	0.490	0.000	0.000	0.000

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	301	705	0	0	0	0	-1
normalized size	1	1.00	0.98	2.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.505	1.108	0.390	0.000	0.476	0.000	0.000	0.000

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	280	3112	0	0	0	0	-1
normalized size	1	1.00	0.90	9.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.364	0.962	0.465	0.000	0.746	0.000	0.000	0.000

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	254	591	0	0	0	0	-1
normalized size	1	1.00	0.94	2.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.217	1.016	0.254	0.000	0.479	0.000	0.000	0.000

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	236	2729	0	0	0	0	-1
normalized size	1	1.00	0.81	9.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.294	1.268	0.246	0.000	0.474	0.000	0.000	0.000

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	518	518	547	0	0	0	0	0	-1
normalized size	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.863	3.808	0.395	0.000	0.449	0.000	0.000	0.000

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	421	421	408	3517	0	0	0	0	-1
normalized size	1	1.00	0.97	8.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.655	1.884	0.421	0.000	0.496	0.000	0.000	0.000

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	687	687	983	0	0	0	0	0	-1
normalized size	1	1.00	1.43	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.259	8.019	0.622	0.000	0.583	0.000	0.000	0.000

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	506	506	417	4955	0	0	0	0	-1
normalized size	1	1.00	0.82	9.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.094	3.051	0.532	0.000	0.725	0.000	0.000	0.000

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	366	366	178	570	0	0	0	0	-1
normalized size	1	1.00	0.49	1.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.353	0.797	0.253	0.000	0.429	0.000	0.000	0.000

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	936	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.152	2.803	1.868	0.000	0.441	0.000	0.000	0.000

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	488	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.151	0.431	1.424	0.000	0.539	0.000	0.000	0.000

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	199	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.293	1.219	0.000	0.453	0.000	0.000	0.000

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.142	2.860	0.352	0.000	0.485	0.000	0.000	0.000

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.160	4.351	0.614	0.000	0.464	0.000	0.000	0.000

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.159	4.469	0.569	0.000	0.554	0.000	0.000	0.000

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.092	0.499	0.280	0.000	0.516	0.000	0.000	0.000

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	169	270	276	248	355	0	-1
normalized size	1	1.00	0.47	0.75	0.77	0.69	0.99	0.00	-0.00
time (sec)	N/A	0.728	0.263	0.114	0.323	0.485	18.410	0.000	0.000

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	137	200	210	204	262	0	-1
normalized size	1	1.00	0.52	0.75	0.79	0.77	0.99	0.00	-0.00
time (sec)	N/A	0.421	0.150	0.063	0.319	0.549	6.634	0.000	0.000

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	99	128	124	140	150	0	-1
normalized size	1	1.00	0.65	0.84	0.81	0.92	0.98	0.00	-0.01
time (sec)	N/A	0.217	0.082	0.055	0.562	0.583	2.084	0.000	0.000

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	454	0	0	0	0	0	-1
normalized size	1	1.00	2.61	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.130	0.227	0.086	0.000	0.434	0.000	0.000	0.000

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	568	0	0	0	0	0	-1
normalized size	1	1.00	1.93	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.306	2.601	0.209	0.000	0.441	0.000	0.000	0.000

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	409	409	654	0	0	0	0	0	-1
normalized size	1	1.00	1.60	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.517	5.472	0.343	0.000	0.446	0.000	0.000	0.000

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	509	509	177	802	0	0	0	0	-1
normalized size	1	1.00	0.35	1.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.595	0.695	0.255	0.000	0.491	0.000	0.000	0.000

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	136	484	0	0	0	0	-1
normalized size	1	1.00	0.39	1.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.348	0.289	0.195	0.000	0.567	0.000	0.000	0.000

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	86	231	0	0	0	0	-1
normalized size	1	1.00	0.42	1.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.176	0.150	0.164	0.000	0.715	0.000	0.000	0.000

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	39	14	0	0	0	-1
normalized size	1	1.00	1.00	0.98	0.35	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.078	0.037	0.043	0.367	0.533	0.000	0.000	0.000

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	133	262	0	0	0	0	-1
normalized size	1	1.00	0.61	1.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.188	0.263	0.187	0.000	0.523	0.000	0.000	0.000

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	195	550	0	0	0	0	-1
normalized size	1	1.00	0.54	1.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.329	0.546	0.273	0.000	0.477	0.000	0.000	0.000

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	515	515	297	888	0	0	0	0	-1
normalized size	1	1.00	0.58	1.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.526	0.660	0.349	0.000	0.521	0.000	0.000	0.000

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.088	0.494	0.270	0.000	0.615	0.000	0.000	0.000

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	121	156	0	166	185	0	-1
normalized size	1	1.00	0.65	0.83	0.00	0.89	0.99	0.00	-0.01
time (sec)	N/A	0.498	0.078	0.120	0.000	0.465	6.470	0.000	0.000

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	98	164	127	128	148	0	-1
normalized size	1	1.00	0.64	1.07	0.83	0.84	0.97	0.00	-0.01
time (sec)	N/A	0.338	0.069	0.098	0.406	0.522	3.573	0.000	0.000

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	83	84	0	128	100	0	-1
normalized size	1	1.00	0.79	0.80	0.00	1.22	0.95	0.00	-0.01
time (sec)	N/A	0.225	0.065	0.088	0.000	0.527	2.197	0.000	0.000

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	58	90	61	92	61	101	-1
normalized size	1	1.00	0.91	1.41	0.95	1.44	0.95	1.58	-0.02
time (sec)	N/A	0.110	0.032	0.080	0.321	0.487	1.188	0.423	0.000

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	23	10	0	11
normalized size	1	1.00	1.00	0.92	0.85	1.77	0.77	0.00	0.85
time (sec)	N/A	0.031	0.006	0.005	0.350	0.424	0.682	0.000	0.099

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	146	197	0	0	0	0	-1
normalized size	1	1.00	1.43	1.93	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.162	0.137	0.097	0.000	0.569	0.000	0.000	0.000

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	97	187	0	0	0	0	-1
normalized size	1	1.00	1.10	2.12	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.189	0.210	0.178	0.000	0.447	0.000	0.000	0.000

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	304	377	0	0	0	0	-1
normalized size	1	1.00	1.45	1.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.363	4.621	0.265	0.000	0.537	0.000	0.000	0.000

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	43	42	0	0	0	0	-1
normalized size	1	1.00	0.64	0.63	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.111	0.080	0.000	0.597	0.000	0.000	0.000

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	34	33	0	0	0	0	-1
normalized size	1	1.00	0.68	0.66	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.100	0.075	0.042	0.000	0.787	0.000	0.000	0.000

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	23	22	0	0	0	0	-1
normalized size	1	1.00	0.79	0.76	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.071	0.015	0.041	0.000	0.400	0.000	0.000	0.000

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.026	0.359	0.086	0.000	0.577	0.000	0.000	0.000

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.029	1.611	0.202	0.000	0.490	0.000	0.000	0.000

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	152	199	0	0	0	0	-1
normalized size	1	1.00	0.74	0.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.508	0.360	0.514	0.000	0.717	0.000	0.000	0.000

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	179	135	178	0	0	0	0	-1
normalized size	1	0.98	0.74	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.532	0.289	0.353	0.000	0.545	0.000	0.000	0.000

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	65	79	0	0	0	0	-1
normalized size	1	1.00	0.79	0.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.273	0.175	0.206	0.000	0.440	0.000	0.000	0.000

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	117	91	118	0	0	0	0	-1
normalized size	1	0.97	0.75	0.98	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.313	0.201	0.196	0.000	0.580	0.000	0.000	0.000

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	63	79	0	0	0	0	-1
normalized size	1	1.00	0.77	0.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.183	0.129	0.154	0.000	0.496	0.000	0.000	0.000

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.426	1.956	0.279	0.000	0.597	0.000	0.000	0.000

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	45	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.300	1.148	0.290	0.000	0.531	0.000	0.000	0.000

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.126	4.733	0.631	0.000	0.528	0.000	0.000	0.000

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.125	0.828	0.722	0.000	0.523	0.000	0.000	0.000

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	241	179	238	0	0	0	0	-1
normalized size	1	0.98	0.73	0.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.616	0.657	0.449	0.000	0.471	0.000	0.000	0.000

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	152	199	0	0	0	0	-1
normalized size	1	1.00	0.74	0.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.495	0.476	0.422	0.000	0.424	0.000	0.000	0.000

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	179	136	178	0	0	0	0	-1
normalized size	1	0.98	0.74	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.417	0.445	0.278	0.000	0.420	0.000	0.000	0.000

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	109	139	0	0	0	0	-1
normalized size	1	1.00	0.76	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.287	0.253	0.313	0.000	0.520	0.000	0.000	0.000

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.853	1.987	0.372	0.000	0.469	0.000	0.000	0.000

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.604	1.437	0.310	0.000	0.460	0.000	0.000	0.000

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.145	4.686	0.829	0.000	0.434	0.000	0.000	0.000

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.141	0.856	0.589	0.000	0.434	0.000	0.000	0.000

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	241	180	238	0	0	0	0	-1
normalized size	1	0.98	0.73	0.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.577	0.991	0.536	0.000	0.509	0.000	0.000	0.000

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	197	259	0	0	0	0	-1
normalized size	1	1.00	0.74	0.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.619	0.883	0.604	0.000	0.488	0.000	0.000	0.000

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	241	180	238	0	0	0	0	-1
normalized size	1	0.98	0.73	0.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.491	0.805	0.423	0.000	0.455	0.000	0.000	0.000

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	153	199	0	0	0	0	-1
normalized size	1	1.00	0.74	0.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.359	0.554	0.509	0.000	0.443	0.000	0.000	0.000

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	195	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	1.280	2.008	0.461	0.000	0.449	0.000	0.000	0.000

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	159	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.967	1.373	0.375	0.000	0.622	0.000	0.000	0.000

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.140	4.677	0.954	0.000	0.482	0.000	0.000	0.000

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.141	0.933	0.689	0.000	0.532	0.000	0.000	0.000

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	31	36	0	0	0	0	-1
normalized size	1	1.00	0.76	0.88	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.161	0.076	0.108	0.000	0.467	0.000	0.000	0.000

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	23	0	0	0	0	-1
normalized size	1	1.00	0.81	0.85	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.155	0.081	0.095	0.000	0.430	0.000	0.000	0.000

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	24	0	0	0	0	-1
normalized size	1	1.00	0.81	0.89	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.145	0.069	0.077	0.000	0.582	0.000	0.000	0.000

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	24	0	0	0	0	-1
normalized size	1	1.00	0.81	0.89	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.144	0.024	0.002	0.000	0.442	0.000	0.000	0.000

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	0	0	0	0	-1
normalized size	1	1.00	1.00	1.11	0.00	0.00	0.00	0.00	-0.11
time (sec)	N/A	0.080	0.056	0.062	0.000	0.447	0.000	0.000	0.000

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	21	7	0	9
normalized size	1	1.00	1.00	1.11	1.00	2.33	0.78	0.00	1.00
time (sec)	N/A	0.037	0.017	0.006	0.419	0.454	0.481	0.000	0.104

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.100	1.117	0.115	0.000	0.522	0.000	0.000	0.000

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.098	0.748	0.210	0.000	0.430	0.000	0.000	0.000

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	179	136	178	0	0	0	0	-1
normalized size	1	0.98	0.74	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.459	0.338	0.329	0.000	0.442	0.000	0.000	0.000

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	109	139	0	0	0	0	-1
normalized size	1	1.00	0.76	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.403	0.221	0.395	0.000	0.446	0.000	0.000	0.000

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	117	92	118	0	0	0	0	-1
normalized size	1	0.97	0.76	0.98	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.386	0.213	0.227	0.000	0.453	0.000	0.000	0.000

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	65	79	0	0	0	0	-1
normalized size	1	1.00	0.79	0.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.292	0.181	0.196	0.000	0.577	0.000	0.000	0.000

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	50	46	58	0	0	0	0	-1
normalized size	1	0.93	0.85	1.07	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.185	0.112	0.077	0.000	0.415	0.000	0.000	0.000

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	16	28	26	0	16
normalized size	1	1.00	1.00	1.06	1.00	1.75	1.62	0.00	1.00
time (sec)	N/A	0.050	0.025	0.006	0.413	0.400	1.506	0.000	0.137

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.126	1.759	0.089	0.000	0.445	0.000	0.000	0.000

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.133	1.204	0.173	0.000	0.417	0.000	0.000	0.000

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.145	2.112	0.313	0.000	0.438	0.000	0.000	0.000

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.102	1.077	0.217	0.000	0.406	0.000	0.000	0.000

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.053	0.060	0.141	0.000	0.444	0.000	0.000	0.000

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.138	1.499	0.497	0.000	0.477	0.000	0.000	0.000

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.141	1.121	0.263	0.000	0.427	0.000	0.000	0.000

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.132	0.915	180.000	0.000	0.429	0.000	0.000	0.000

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.131	0.516	180.000	0.000	0.433	0.000	0.000	0.000

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.114	0.116	180.000	0.000	0.422	0.000	0.000	0.000

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.124	0.342	0.156	0.000	0.505	0.000	0.000	0.000

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.132	0.615	0.498	0.000	0.615	0.000	0.000	0.000

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	82	106	0	0	0	0	-1
normalized size	1	1.00	0.87	1.13	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.184	0.500	0.127	0.000	0.494	0.000	0.000	0.000

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	69	84	0	0	0	0	-1
normalized size	1	1.00	0.90	1.09	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.170	0.396	0.072	0.000	0.467	0.000	0.000	0.000

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	60	0	0	0	0	-1
normalized size	1	1.00	1.00	1.11	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.135	0.173	0.064	0.000	0.462	0.000	0.000	0.000

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.102	1.532	0.096	0.000	0.452	0.000	0.000	0.000

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.101	3.771	0.225	0.000	0.437	0.000	0.000	0.000

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	209	175	633	0	0	0	0	-1
normalized size	1	0.98	0.82	2.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.711	0.674	0.501	0.000	0.558	0.000	0.000	0.000

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	82	248	0	0	0	0	-1
normalized size	1	1.00	0.88	2.67	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.562	0.322	0.278	0.000	0.608	0.000	0.000	0.000

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	198	126	364	0	0	0	0	-1
normalized size	1	1.33	0.85	2.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.412	0.377	0.271	0.000	0.536	0.000	0.000	0.000

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	73	192	0	0	0	0	-1
normalized size	1	1.00	0.86	2.26	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.181	0.172	0.264	0.000	0.510	0.000	0.000	0.000

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.229	10.865	0.413	0.000	0.498	0.000	0.000	0.000

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.154	2.825	0.350	0.000	0.526	0.000	0.000	0.000

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.127	19.088	0.752	0.000	0.438	0.000	0.000	0.000

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.131	3.675	0.848	0.000	0.473	0.000	0.000	0.000

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	273	399	958	0	0	0	0	-1
normalized size	1	0.99	1.44	3.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.006	0.816	0.588	0.000	0.455	0.000	0.000	0.000

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	306	704	0	0	0	0	-1
normalized size	1	1.00	1.40	3.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.716	0.737	0.597	0.000	0.435	0.000	0.000	0.000

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	209	295	633	0	0	0	0	-1
normalized size	1	0.98	1.38	2.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.732	0.609	0.404	0.000	0.489	0.000	0.000	0.000

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	122	420	0	0	0	0	-1
normalized size	1	1.00	0.82	2.82	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.318	0.473	0.408	0.000	0.438	0.000	0.000	0.000

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	175	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.437	7.700	0.457	0.000	0.594	0.000	0.000	0.000

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.252	4.310	0.510	0.000	0.449	0.000	0.000	0.000

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.139	14.507	1.287	0.000	0.588	0.000	0.000	0.000

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.199	2.777	0.773	0.000	0.452	0.000	0.000	0.000

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	273	408	1070	0	0	0	0	-1
normalized size	1	0.99	1.47	3.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.215	1.376	0.739	0.000	0.481	0.000	0.000	0.000

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	413	1044	0	0	0	0	-1
normalized size	1	1.00	1.47	3.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.072	1.175	0.886	0.000	0.496	0.000	0.000	0.000

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	271	404	958	0	0	0	0	-1
normalized size	1	0.99	1.47	3.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.970	1.017	0.615	0.000	0.612	0.000	0.000	0.000

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	311	704	0	0	0	0	-1
normalized size	1	1.00	1.44	3.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.446	0.720	0.713	0.000	0.503	0.000	0.000	0.000

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	233	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.568	9.518	0.617	0.000	0.445	0.000	0.000	0.000

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.318	3.422	0.495	0.000	0.442	0.000	0.000	0.000

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.144	14.574	1.433	0.000	0.465	0.000	0.000	0.000

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.144	3.083	1.240	0.000	0.442	0.000	0.000	0.000

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	200	158	633	0	0	0	0	-1
normalized size	1	0.98	0.77	3.10	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.480	0.341	0.425	0.000	0.447	0.000	0.000	0.000

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	117	420	0	0	0	0	-1
normalized size	1	1.00	0.83	2.98	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.395	0.286	0.474	0.000	0.449	0.000	0.000	0.000

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	138	113	364	0	0	0	0	-1
normalized size	1	0.97	0.80	2.56	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.367	0.270	0.290	0.000	0.501	0.000	0.000	0.000

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	70	192	0	0	0	0	-1
normalized size	1	1.00	0.89	2.43	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.258	0.141	0.221	0.000	0.497	0.000	0.000	0.000

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	60	151	0	0	0	0	-1
normalized size	1	1.00	0.82	2.07	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.161	0.124	0.084	0.000	0.558	0.000	0.000	0.000

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	30	36	0	18
normalized size	1	1.00	1.00	1.06	1.00	1.67	2.00	0.00	1.00
time (sec)	N/A	0.045	0.012	0.007	0.359	0.544	3.060	0.000	0.139

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	47	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.147	5.442	0.152	0.000	0.480	0.000	0.000	0.000

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	47	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.148	1.249	0.126	0.000	0.466	0.000	0.000	0.000

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.137	8.095	0.702	0.000	0.465	0.000	0.000	0.000

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.201	3.603	0.358	0.000	0.463	0.000	0.000	0.000

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.093	5.362	0.251	0.000	0.434	0.000	0.000	0.000

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.111	2.438	0.144	0.000	0.469	0.000	0.000	0.000

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.132	7.827	0.553	0.000	0.492	0.000	0.000	0.000

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.130	20.005	0.321	0.000	0.493	0.000	0.000	0.000

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.137	14.365	1.000	0.000	0.560	0.000	0.000	0.000

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.136	5.874	0.826	0.000	0.483	0.000	0.000	0.000

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.092	11.057	0.495	0.000	0.455	0.000	0.000	0.000

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.110	2.859	0.252	0.000	0.507	0.000	0.000	0.000

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.131	12.779	1.015	0.000	0.456	0.000	0.000	0.000

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.132	12.088	1.190	0.000	0.613	0.000	0.000	0.000

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.128	1.025	180.000	0.000	0.569	0.000	0.000	0.000

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.128	0.567	180.000	0.000	0.459	0.000	0.000	0.000

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.113	0.126	180.000	0.000	0.479	0.000	0.000	0.000

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.152	0.358	0.201	0.000	0.499	0.000	0.000	0.000

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.140	0.667	0.533	0.000	0.431	0.000	0.000	0.000

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.140	0.903	0.720	0.000	0.490	0.000	0.000	0.000

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	23	12	0	11
normalized size	1	1.00	1.00	0.92	0.85	1.77	0.92	0.00	0.85
time (sec)	N/A	0.035	0.009	0.013	0.642	0.419	0.986	0.000	0.094

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	232	0	0	0	0	0	-1
normalized size	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.308	0.922	180.000	0.000	0.000	0.000	0.000	0.000

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	436	0	0	0	0	0	-1
normalized size	1	1.00	1.30	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.379	0.745	180.000	0.000	0.000	0.000	0.000	0.000

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	227	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.767	0.463	180.000	0.000	0.000	0.000	0.000	0.000

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	295	0	0	0	0	0	-1
normalized size	1	1.00	1.29	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.525	0.926	180.000	0.000	0.000	0.000	0.000	0.000

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	F(-2)	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	175	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.796	4.435	180.000	0.000	0.000	0.000	0.000	0.000

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	474	474	462	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.527	1.011	180.000	0.000	0.000	0.000	0.000	0.000

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	457	457	577	0	0	0	0	0	-1
normalized size	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.780	1.637	180.000	0.000	0.000	0.000	0.000	0.000

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	358	358	351	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.330	0.706	180.000	0.000	0.000	0.000	0.000	0.000

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	440	0	0	0	0	0	-1
normalized size	1	1.00	1.27	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.729	2.085	180.000	0.000	0.000	0.000	0.000	0.000

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	F(-2)	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	375	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.441	3.309	180.000	0.000	0.000	0.000	0.000	0.000

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	142	0	0	0	0	0	-1
normalized size	1	1.00	0.45	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.408	0.194	0.388	0.000	0.000	0.000	0.000	0.000

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	104	0	0	0	0	0	-1
normalized size	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.168	0.101	0.355	0.000	0.000	0.000	0.000	0.000

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	36	0	0	0	0	-1
normalized size	1	1.00	1.00	0.86	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.077	0.038	0.057	0.000	0.000	0.000	0.000	0.000

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.616	0.419	0.000	0.000	0.000	0.000	0.000

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	179	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.199	1.417	0.467	0.000	0.000	0.000	0.000	0.000

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	186	0	0	0	0	0	-1
normalized size	1	1.00	0.41	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.565	0.317	0.320	0.000	0.000	0.000	0.000	0.000

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	126	0	0	0	0	0	-1
normalized size	1	1.00	0.46	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.273	0.272	0.344	0.000	0.000	0.000	0.000	0.000

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	36	0	0	0	0	-1
normalized size	1	1.00	1.00	0.86	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.074	0.042	0.056	0.000	0.000	0.000	0.000	0.000

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.662	0.383	0.000	0.000	0.000	0.000	0.000

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	514	514	201	0	0	0	0	0	-1
normalized size	1	1.00	0.39	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.765	0.345	0.317	0.000	0.000	0.000	0.000	0.000

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	135	0	0	0	0	0	-1
normalized size	1	1.00	0.45	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.301	0.293	0.338	0.000	0.000	0.000	0.000	0.000

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	36	0	0	0	0	-1
normalized size	1	1.00	1.00	0.86	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.072	0.037	0.055	0.000	0.000	0.000	0.000	0.000

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.645	0.378	0.000	0.000	0.000	0.000	0.000

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	156	0	0	0	0	0	-1
normalized size	1	1.00	0.50	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.360	0.177	0.375	0.000	0.000	0.000	0.000	0.000

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	110	0	0	0	0	0	-1
normalized size	1	1.00	0.62	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.089	0.361	0.000	0.000	0.000	0.000	0.000

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	34	0	20	0	0	-1
normalized size	1	1.00	1.00	0.87	0.00	0.51	0.00	0.00	-0.03
time (sec)	N/A	0.062	0.027	0.065	0.000	0.442	0.000	0.000	0.000

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.350	0.460	0.000	0.000	0.000	0.000	0.000

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	173	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.158	0.925	0.452	0.000	0.000	0.000	0.000	0.000

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	433	433	210	0	0	0	0	0	-1
normalized size	1	1.00	0.48	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.548	0.301	0.316	0.000	0.000	0.000	0.000	0.000

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	133	0	0	0	0	0	-1
normalized size	1	1.00	0.51	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.276	0.247	0.346	0.000	0.000	0.000	0.000	0.000

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	34	0	20	0	0	-1
normalized size	1	1.00	1.00	0.87	0.00	0.51	0.00	0.00	-0.03
time (sec)	N/A	0.064	0.028	0.057	0.000	0.483	0.000	0.000	0.000

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.379	0.372	0.000	0.000	0.000	0.000	0.000

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	34	0	0	0	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.080	0.064	0.163	0.000	0.000	0.000	0.000	0.000

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	197	0	0	0	0	0	-1
normalized size	1	1.00	0.50	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.308	0.311	0.338	0.000	0.000	0.000	0.000	0.000

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	141	0	0	0	0	0	-1
normalized size	1	1.00	0.53	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.227	0.176	0.333	0.000	0.000	0.000	0.000	0.000

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	101	0	0	0	0	0	-1
normalized size	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.164	0.098	0.365	0.000	0.000	0.000	0.000	0.000

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	36	0	0	0	0	-1
normalized size	1	1.00	1.00	0.90	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.072	0.039	0.056	0.000	0.000	0.000	0.000	0.000

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.042	0.738	0.390	0.000	0.000	0.000	0.000	0.000

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.042	1.664	0.456	0.000	0.000	0.000	0.000	0.000

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	399	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.302	1.066	0.315	0.000	0.000	0.000	0.000	0.000

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	225	0	0	0	0	0	-1
normalized size	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.208	0.373	0.321	0.000	0.000	0.000	0.000	0.000

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	115	0	0	0	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.188	0.349	0.000	0.000	0.000	0.000	0.000

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	36	0	57	0	0	-1
normalized size	1	1.00	1.00	0.90	0.00	1.42	0.00	0.00	-0.02
time (sec)	N/A	0.071	0.036	0.056	0.000	0.430	0.000	0.000	0.000

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.741	0.388	0.000	0.000	0.000	0.000	0.000

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	1.544	0.459	0.000	0.000	0.000	0.000	0.000

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	262	0	0	0	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.377	0.402	0.325	0.000	0.000	0.000	0.000	0.000

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	122	0	0	0	0	0	-1
normalized size	1	1.00	0.67	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.155	0.346	0.000	0.000	0.000	0.000	0.000

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	36	0	57	0	0	-1
normalized size	1	1.00	1.00	0.86	0.00	1.36	0.00	0.00	-0.02
time (sec)	N/A	0.070	0.033	0.059	0.000	0.574	0.000	0.000	0.000

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.729	0.401	0.000	0.000	0.000	0.000	0.000

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	1.552	0.457	0.000	0.000	0.000	0.000	0.000

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	170	0	0	0	0	0	-1
normalized size	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.454	0.994	180.000	0.000	0.581	0.000	0.000	0.000

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	229	0	0	0	0	0	-1
normalized size	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.473	0.950	0.179	0.000	0.477	0.000	0.000	0.000

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	160	0	0	0	0	0	-1
normalized size	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.298	0.605	0.217	0.000	0.587	0.000	0.000	0.000

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	199	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.142	0.220	0.273	0.000	0.525	0.000	0.000	0.000

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.221	0.269	0.000	0.510	0.000	0.000	0.000

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	616	616	429	0	0	0	0	0	-1
normalized size	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.833	3.191	180.000	0.000	0.566	0.000	0.000	0.000

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	542	542	390	0	0	0	0	0	-1
normalized size	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.631	1.791	0.151	0.000	0.495	0.000	0.000	0.000

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	420	420	287	0	0	0	0	0	-1
normalized size	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.417	1.431	0.178	0.000	0.483	0.000	0.000	0.000

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	390	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.153	0.246	0.234	0.000	0.500	0.000	0.000	0.000

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	273	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.154	0.642	0.262	0.000	0.552	0.000	0.000	0.000

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	816	816	667	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.975	6.944	180.000	0.000	0.549	0.000	0.000	0.000

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	745	745	685	0	0	0	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.785	2.956	0.152	0.000	0.469	0.000	0.000	0.000

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	632	632	529	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.600	5.368	0.177	0.000	0.544	0.000	0.000	0.000

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	756	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.155	0.271	0.230	0.000	0.494	0.000	0.000	0.000

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	455	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.156	0.651	0.260	0.000	0.547	0.000	0.000	0.000

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.104	0.515	0.162	0.000	0.521	0.000	0.000	0.000

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	100	0	0	0	0	0	-1
normalized size	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.253	0.196	180.000	0.000	0.467	0.000	0.000	0.000

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	86	0	0	0	0	0	-1
normalized size	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.193	0.209	180.000	0.000	0.564	0.000	0.000	0.000

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	43	0	0	0	0	0	-1
normalized size	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.117	0.075	0.129	0.000	0.459	0.000	0.000	0.000

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	17	83	34	29	33
normalized size	1	1.00	1.00	1.06	1.00	4.88	2.00	1.71	1.94
time (sec)	N/A	0.038	0.009	0.006	0.387	0.461	0.866	0.884	0.285

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.102	5.934	0.148	0.000	0.495	0.000	0.000	0.000

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.102	1.931	0.234	0.000	0.511	0.000	0.000	0.000

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	361	0	0	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.597	1.481	0.401	0.000	0.560	0.000	0.000	0.000

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	273	0	0	0	0	0	-1
normalized size	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.343	1.576	0.285	0.000	0.484	0.000	0.000	0.000

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	233	0	0	0	0	0	-1
normalized size	1	1.00	1.59	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.196	0.602	0.285	0.000	0.462	0.000	0.000	0.000

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	227	0	0	0	0	0	-1
normalized size	1	1.00	1.44	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.298	0.610	0.365	0.000	0.516	0.000	0.000	0.000

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	283	0	0	0	0	0	-1
normalized size	1	1.00	1.56	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.401	1.516	0.336	0.000	0.486	0.000	0.000	0.000

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	141	0	219	565	0	0	-1
normalized size	1	1.00	0.75	0.00	1.17	3.02	0.00	0.00	-0.01
time (sec)	N/A	0.306	0.499	0.311	0.599	0.635	0.000	0.000	0.000

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	459	683	0	0	0	0	0	-1
normalized size	1	1.00	1.49	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.441	1.764	0.276	0.000	0.605	0.000	0.000	0.000

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	352	0	0	0	0	0	-1
normalized size	1	1.00	1.43	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.252	0.903	0.315	0.000	0.521	0.000	0.000	0.000

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	273	0	0	0	0	0	-1
normalized size	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.341	1.589	0.278	0.000	0.603	0.000	0.000	0.000

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	344	0	0	0	0	0	-1
normalized size	1	1.00	1.29	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.465	1.083	0.327	0.000	0.508	0.000	0.000	0.000

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	514	0	0	0	0	0	-1
normalized size	1	1.00	1.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.474	2.383	0.330	0.000	0.542	0.000	0.000	0.000

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	706	0	0	0	0	0	-1
normalized size	1	1.00	1.94	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.384	5.841	0.351	0.000	0.492	0.000	0.000	0.000

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	481	0	0	0	0	0	-1
normalized size	1	1.00	1.40	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.304	1.175	0.293	0.000	0.530	0.000	0.000	0.000

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	459	683	0	0	0	0	0	-1
normalized size	1	1.00	1.49	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.426	1.667	0.299	0.000	0.549	0.000	0.000	0.000

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	565	0	0	0	0	0	-1
normalized size	1	1.00	1.36	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.571	1.233	0.295	0.000	0.682	0.000	0.000	0.000

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	465	0	0	0	0	0	-1
normalized size	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.628	1.714	0.337	0.000	0.482	0.000	0.000	0.000

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	518	518	779	0	0	0	0	0	-1
normalized size	1	1.00	1.50	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.416	3.967	0.328	0.000	0.565	0.000	0.000	0.000

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	472	472	1005	0	0	0	0	0	-1
normalized size	1	1.00	2.13	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.444	7.429	0.332	0.000	0.629	0.000	0.000	0.000

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	465	0	0	0	0	0	-1
normalized size	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.629	1.678	0.349	0.000	0.782	0.000	0.000	0.000

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	344	0	0	0	0	0	-1
normalized size	1	1.00	1.29	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.466	1.025	0.343	0.000	0.621	0.000	0.000	0.000

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	227	0	0	0	0	0	-1
normalized size	1	1.00	1.44	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.290	0.571	0.340	0.000	0.756	0.000	0.000	0.000

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	113	0	32	0	0	0	-1
normalized size	1	1.00	1.92	0.00	0.54	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.170	0.450	0.308	0.508	0.617	0.000	0.000	0.000

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	113	0	98	445	0	0	-1
normalized size	1	1.00	1.02	0.00	0.88	4.01	0.00	0.00	-0.01
time (sec)	N/A	0.241	0.380	0.339	0.744	0.526	0.000	0.000	0.000

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	143	0	233	593	0	0	-1
normalized size	1	1.00	0.48	0.00	0.79	2.01	0.00	0.00	-0.00
time (sec)	N/A	0.337	0.505	0.334	0.711	0.749	0.000	0.000	0.000

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	517	517	781	0	0	0	0	0	-1
normalized size	1	1.00	1.51	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.422	4.019	0.333	0.000	0.578	0.000	0.000	0.000

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	515	0	0	0	0	0	-1
normalized size	1	1.00	1.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.478	2.431	0.333	0.000	0.802	0.000	0.000	0.000

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	285	0	0	0	0	0	-1
normalized size	1	1.00	1.58	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.400	1.275	0.333	0.000	0.623	0.000	0.000	0.000

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	94	0	98	445	0	0	-1
normalized size	1	1.00	0.84	0.00	0.88	3.97	0.00	0.00	-0.01
time (sec)	N/A	0.249	0.414	0.337	0.652	0.649	0.000	0.000	0.000

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	118	0	82	0	0	0	-1
normalized size	1	1.00	1.15	0.00	0.80	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.205	0.453	0.323	0.584	0.877	0.000	0.000	0.000

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	201	0	237	0	0	0	-1
normalized size	1	1.00	0.71	0.00	0.84	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.306	0.636	0.332	0.516	1.081	0.000	0.000	0.000

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	470	470	1331	0	0	0	0	0	-1
normalized size	1	1.00	2.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.439	8.753	0.366	0.000	0.523	0.000	0.000	0.000

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	706	0	0	0	0	0	-1
normalized size	1	1.00	1.95	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.392	5.641	0.353	0.000	0.744	0.000	0.000	0.000

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	131	0	220	565	0	0	-1
normalized size	1	1.00	0.71	0.00	1.19	3.05	0.00	0.00	-0.01
time (sec)	N/A	0.306	0.430	0.337	0.626	0.692	0.000	0.000	0.000

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	139	0	232	593	0	0	-1
normalized size	1	1.00	0.47	0.00	0.79	2.02	0.00	0.00	-0.00
time (sec)	N/A	0.333	0.465	0.336	0.639	0.786	0.000	0.000	0.000

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	202	0	237	0	0	0	-1
normalized size	1	1.00	0.72	0.00	0.84	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.299	0.604	0.332	0.620	1.112	0.000	0.000	0.000

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	193	0	159	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.78	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.245	0.572	0.339	0.530	0.839	0.000	0.000	0.000

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	680	680	890	0	0	0	0	0	-1
normalized size	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.108	2.410	0.294	0.000	0.614	0.000	0.000	0.000

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	508	508	705	0	0	0	0	0	-1
normalized size	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.634	1.843	0.296	0.000	0.646	0.000	0.000	0.000

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	352	0	0	0	0	0	-1
normalized size	1	1.00	1.44	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.353	1.021	0.294	0.000	0.726	0.000	0.000	0.000

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	315	0	0	0	0	0	-1
normalized size	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.508	1.170	0.339	0.000	0.656	0.000	0.000	0.000

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	544	544	594	0	0	0	0	0	-1
normalized size	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.005	4.244	0.282	0.000	0.624	0.000	0.000	0.000

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	518	518	783	0	0	0	0	0	-1
normalized size	1	1.00	1.51	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.149	8.179	0.347	0.000	0.770	0.000	0.000	0.000

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	774	774	1084	0	0	0	0	0	-1
normalized size	1	1.00	1.40	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.858	3.179	0.295	0.000	0.676	0.000	0.000	0.000

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	524	0	0	0	0	0	-1
normalized size	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.482	1.738	0.295	0.000	0.528	0.000	0.000	0.000

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	508	508	705	0	0	0	0	0	-1
normalized size	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.649	1.831	0.300	0.000	0.524	0.000	0.000	0.000

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	436	436	532	0	0	0	0	0	-1
normalized size	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.633	2.263	0.351	0.000	0.522	0.000	0.000	0.000

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	752	752	1546	0	0	0	0	0	-1
normalized size	1	1.00	2.06	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.128	9.103	0.319	0.000	0.551	0.000	0.000	0.000

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-1)	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	580	580	1609	0	0	0	0	0	-1
normalized size	1	1.00	2.77	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.204	10.243	0.280	0.000	0.748	0.000	0.000	0.000

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	548	548	735	0	0	0	0	0	-1
normalized size	1	1.00	1.34	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.612	2.343	0.301	0.000	0.583	0.000	0.000	0.000

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	774	774	1084	0	0	0	0	0	-1
normalized size	1	1.00	1.40	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.833	3.188	0.305	0.000	0.644	0.000	0.000	0.000

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	680	680	890	0	0	0	0	0	-1
normalized size	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.063	2.428	0.301	0.000	0.547	0.000	0.000	0.000

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	615	615	723	0	0	0	0	0	-1
normalized size	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.781	3.619	0.346	0.000	0.563	0.000	0.000	0.000

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	972	972	2492	0	0	0	0	0	-1
normalized size	1	1.00	2.56	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.364	11.831	0.276	0.000	0.734	0.000	0.000	0.000

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-1)	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	790	790	2622	0	0	0	0	0	-1
normalized size	1	1.00	3.32	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.365	13.525	0.278	0.000	0.683	0.000	0.000	0.000

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	615	615	723	0	0	0	0	0	-1
normalized size	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.785	3.484	0.354	0.000	0.659	0.000	0.000	0.000

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	436	436	529	0	0	0	0	0	-1
normalized size	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.669	2.210	0.352	0.000	0.564	0.000	0.000	0.000

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	315	0	0	0	0	0	-1
normalized size	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.511	1.150	0.356	0.000	0.671	0.000	0.000	0.000

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	168	0	53	0	0	0	-1
normalized size	1	1.00	2.85	0.00	0.90	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.296	0.854	0.338	0.588	0.543	0.000	0.000	0.000

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	464	464	508	0	0	0	0	0	-1
normalized size	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.753	2.263	0.366	0.000	0.676	0.000	0.000	0.000

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	942	942	524	0	0	0	0	0	-1
normalized size	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.332	7.137	0.352	0.000	0.571	0.000	0.000	0.000

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	972	972	2143	0	0	0	0	0	-1
normalized size	1	1.00	2.20	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.370	14.703	0.295	0.000	0.520	0.000	0.000	0.000

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	752	752	1346	0	0	0	0	0	-1
normalized size	1	1.00	1.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.133	11.194	0.287	0.000	0.494	0.000	0.000	0.000

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	544	544	530	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.988	5.531	0.301	0.000	0.681	0.000	0.000	0.000

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	464	464	511	0	0	0	0	0	-1
normalized size	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.704	2.277	0.362	0.000	0.470	0.000	0.000	0.000

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	488	0	0	0	0	0	-1
normalized size	1	1.00	2.18	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.423	1.362	0.344	0.000	0.521	0.000	0.000	0.000

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	743	743	754	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.909	8.341	0.347	0.000	0.521	0.000	0.000	0.000

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-1)	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	794	794	2552	0	0	0	0	0	-1
normalized size	1	1.00	3.21	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.396	13.908	0.287	0.000	0.816	0.000	0.000	0.000

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-1)	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	584	584	1617	0	0	0	0	0	-1
normalized size	1	1.00	2.77	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.205	10.217	0.291	0.000	0.616	0.000	0.000	0.000

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	522	522	788	0	0	0	0	0	-1
normalized size	1	1.00	1.51	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.155	8.237	0.351	0.000	0.475	0.000	0.000	0.000

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	942	942	528	0	0	0	0	0	-1
normalized size	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.301	7.053	0.346	0.000	0.681	0.000	0.000	0.000

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	743	743	757	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.897	8.331	0.355	0.000	0.835	0.000	0.000	0.000

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	386	386	642	0	0	0	0	0	-1
normalized size	1	1.00	1.66	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.531	8.157	0.350	0.000	0.697	0.000	0.000	0.000

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	260	451	415	333	593	0	-1
normalized size	1	1.00	0.83	1.45	1.33	1.07	1.90	0.00	-0.00
time (sec)	N/A	0.351	0.355	0.020	0.412	0.613	18.003	0.000	0.000

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	187	316	287	241	389	0	-1
normalized size	1	1.00	0.85	1.43	1.30	1.09	1.76	0.00	-0.00
time (sec)	N/A	0.261	0.282	0.006	0.356	0.534	6.649	0.000	0.000

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	125	204	180	163	240	0	-1
normalized size	1	1.00	0.85	1.39	1.22	1.11	1.63	0.00	-0.01
time (sec)	N/A	0.143	0.168	0.005	0.446	0.632	2.318	0.000	0.000

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	71	109	91	94	109	0	-1
normalized size	1	1.00	0.88	1.35	1.12	1.16	1.35	0.00	-0.01
time (sec)	N/A	0.070	0.069	0.005	0.331	0.553	0.556	0.000	0.000

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	31	30	43	26	41	28
normalized size	1	1.00	1.00	1.03	1.00	1.43	0.87	1.37	0.93
time (sec)	N/A	0.014	0.008	0.007	0.319	0.505	0.140	0.253	0.244

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	485	485	434	224	0	0	0	0	-1
normalized size	1	1.00	0.89	0.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.832	0.439	0.666	0.000	0.572	0.000	0.000	0.000

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-2)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	707	707	622	1745	0	0	0	0	-1
normalized size	1	1.00	0.88	2.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.075	1.722	1.165	0.000	0.903	0.000	0.000	0.000

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	559	559	443	1166	684	586	989	0	-1
normalized size	1	1.00	0.79	2.09	1.22	1.05	1.77	0.00	-0.00
time (sec)	N/A	0.966	0.608	0.162	0.527	0.568	13.828	0.000	0.000

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	289	620	429	380	595	0	-1
normalized size	1	1.00	0.88	1.88	1.30	1.16	1.81	0.00	-0.00
time (sec)	N/A	0.582	0.429	0.105	0.504	1.016	4.919	0.000	0.000

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	164	271	218	209	279	0	-1
normalized size	1	1.00	1.07	1.77	1.42	1.37	1.82	0.00	-0.01
time (sec)	N/A	0.277	0.224	0.089	0.436	0.972	1.373	0.000	0.000

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	74	72	72	96	82	111	-1
normalized size	1	1.00	1.61	1.57	1.57	2.09	1.78	2.41	-0.02
time (sec)	N/A	0.064	0.061	0.050	0.605	0.609	0.273	0.520	0.000

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	739	739	985	0	0	0	0	0	-1
normalized size	1	1.00	1.33	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.315	0.693	0.409	0.000	0.570	0.000	0.000	0.000

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	670	658	444	654	0	0	0	0	-1
normalized size	1	0.98	0.66	0.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.344	1.026	0.474	0.000	0.435	0.000	0.000	0.000

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	380	253	380	0	0	0	0	-1
normalized size	1	0.98	0.65	0.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.786	0.554	0.292	0.000	0.653	0.000	0.000	0.000

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	176	126	178	0	0	0	0	-1
normalized size	1	0.98	0.70	0.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.367	0.247	0.201	0.000	0.546	0.000	0.000	0.000

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	45	56	0	0	0	0	-1
normalized size	1	1.00	0.83	1.04	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.070	0.020	0.033	0.000	0.739	0.000	0.000	0.000

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.040	0.716	0.270	0.000	1.044	0.000	0.000	0.000

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.040	3.482	0.500	0.000	0.461	0.000	0.000	0.000

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	495	483	356	1036	0	0	0	0	-1
normalized size	1	0.98	0.72	2.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.875	2.077	0.398	0.000	0.603	0.000	0.000	0.000

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	239	190	438	0	0	0	0	-1
normalized size	1	0.97	0.77	1.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.476	0.913	0.250	0.000	0.523	0.000	0.000	0.000

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	81	71	118	0	0	0	0	-1
normalized size	1	0.95	0.84	1.39	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.188	0.145	0.047	0.000	0.480	0.000	0.000	0.000

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.038	11.895	0.268	0.000	0.534	0.000	0.000	0.000

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.037	28.651	0.556	0.000	0.492	0.000	0.000	0.000

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	672	672	535	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.879	6.395	180.000	0.000	0.000	0.000	0.000	0.000

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	319	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.932	2.898	180.000	0.000	0.000	0.000	0.000	0.000

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	101	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.260	0.199	0.060	0.000	0.000	0.000	0.000	0.000

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.056	7.552	0.401	0.000	0.000	0.000	0.000	0.000

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.054	16.826	0.934	0.000	0.000	0.000	0.000	0.000

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	427	427	770	0	0	0	0	0	-1
normalized size	1	1.00	1.80	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.264	4.836	180.000	0.000	0.000	0.000	0.000	0.000

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	251	0	0	0	0	0	-1
normalized size	1	1.00	1.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.251	0.636	0.066	0.000	0.000	0.000	0.000	0.000

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.065	2.889	0.394	0.000	0.000	0.000	0.000	0.000

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.065	9.447	0.947	0.000	0.000	0.000	0.000	0.000

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	608	608	530	0	0	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.195	1.233	180.000	0.000	0.000	0.000	0.000	0.000

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	218	0	0	0	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.572	0.693	180.000	0.000	0.000	0.000	0.000	0.000

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	101	0	0	0	0	0	-1
normalized size	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.100	0.062	0.000	0.000	0.000	0.000	0.000

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.060	0.139	0.397	0.000	0.000	0.000	0.000	0.000

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.057	0.267	0.907	0.000	0.000	0.000	0.000	0.000

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	303	0	0	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.691	1.519	180.000	0.000	0.000	0.000	0.000	0.000

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	137	0	0	0	0	0	-1
normalized size	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.251	0.232	0.059	0.000	0.000	0.000	0.000	0.000

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.067	0.152	0.393	0.000	0.000	0.000	0.000	0.000

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.065	0.278	0.923	0.000	0.000	0.000	0.000	0.000

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.023	5.265	0.464	0.000	0.462	0.000	0.000	0.000

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.025	3.925	0.437	0.000	0.421	0.000	0.000	0.000

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	75	0	0	326	0	0	-1
normalized size	1	1.00	1.07	0.00	0.00	4.66	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.116	0.349	0.000	0.542	0.000	0.000	0.000

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	139	0	0	738	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	5.05	0.00	0.00	-0.01
time (sec)	N/A	0.169	0.339	0.378	0.000	0.825	0.000	0.000	0.000

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	191	0	0	1354	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	5.96	0.00	0.00	-0.00
time (sec)	N/A	0.820	0.423	0.381	0.000	0.848	0.000	0.000	0.000

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.042	14.400	0.265	0.000	0.536	0.000	0.000	0.000

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.045	11.413	0.320	0.000	0.677	0.000	0.000	0.000

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.049	3.129	0.284	0.000	0.596	0.000	0.000	0.000

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.048	5.904	0.290	0.000	0.753	0.000	0.000	0.000

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.047	1.209	0.217	0.000	0.494	0.000	0.000	0.000

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.050	1.080	0.254	0.000	0.657	0.000	0.000	0.000

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.054	1.533	0.234	0.000	0.524	0.000	0.000	0.000

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.053	3.719	0.249	0.000	0.803	0.000	0.000	0.000

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.044	4.670	0.232	0.000	0.428	0.000	0.000	0.000

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.047	6.453	0.260	0.000	0.541	0.000	0.000	0.000

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.051	16.768	0.244	0.000	0.471	0.000	0.000	0.000

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.050	29.126	0.256	0.000	0.499	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [250] had the largest ratio of [.7308]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	5	1.00	22	0.227
2	A	6	6	1.00	22	0.273
3	A	5	5	1.00	22	0.227
4	A	4	3	1.00	20	0.150
5	A	5	4	1.00	19	0.210
6	A	8	8	1.00	22	0.364
7	A	6	7	1.00	22	0.318
8	A	8	8	1.00	22	0.364
9	A	6	7	1.00	22	0.318
10	A	6	6	1.00	24	0.250
11	A	7	8	1.00	24	0.333
12	A	5	5	1.00	24	0.208
13	A	5	3	1.00	22	0.136
14	A	5	5	1.00	21	0.238
15	A	12	8	1.00	24	0.333
16	A	7	7	1.00	24	0.292
17	A	12	10	1.00	24	0.417
18	A	7	8	1.00	24	0.333
19	A	5	5	1.00	24	0.208
20	A	8	7	1.00	24	0.292
21	A	5	5	1.00	24	0.208
22	A	6	3	1.00	22	0.136
23	A	5	5	1.00	21	0.238
24	A	17	8	1.00	24	0.333
25	A	7	7	1.00	24	0.292
26	A	17	10	1.00	24	0.417
27	A	8	8	1.00	24	0.333
28	A	12	8	1.00	24	0.333
29	A	8	8	1.00	24	0.333
30	A	8	6	1.00	24	0.250
31	A	5	5	1.00	22	0.227
32	A	6	4	1.00	21	0.190
33	A	7	5	1.00	24	0.208
34	A	10	8	1.00	24	0.333
35	A	9	7	1.00	24	0.292

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
36	A	15	9	1.00	24	0.375
37	A	12	9	1.00	24	0.375
38	A	8	8	1.00	24	0.333
39	A	8	6	1.00	24	0.250
40	A	2	2	1.00	22	0.091
41	A	8	6	1.00	21	0.286
42	A	9	7	1.00	24	0.292
43	A	13	11	1.00	24	0.458
44	A	12	9	1.00	24	0.375
45	A	19	11	1.10	24	0.458
46	A	12	8	1.00	24	0.333
47	A	4	3	1.00	24	0.125
48	A	10	7	1.00	24	0.292
49	A	3	3	1.00	22	0.136
50	A	10	6	1.00	21	0.286
51	A	12	8	1.00	24	0.333
52	A	16	11	1.00	24	0.458
53	A	16	10	1.00	24	0.417
54	A	23	11	1.17	24	0.458
55	A	3	4	1.02	26	0.154
56	A	5	4	1.52	26	0.154
57	A	2	1	1.72	24	0.042
58	A	3	3	1.66	23	0.130
59	A	8	6	1.99	26	0.231
60	A	3	3	1.72	26	0.115
61	A	8	6	1.78	26	0.231
62	A	3	2	1.71	26	0.077
63	A	4	5	1.02	26	0.192
64	A	8	6	1.54	26	0.231
65	A	3	2	1.90	24	0.083
66	A	6	5	1.62	23	0.217
67	A	10	7	1.86	26	0.269
68	A	6	5	1.64	26	0.192
69	A	11	8	1.74	26	0.308
70	A	6	5	1.60	26	0.192
71	A	4	5	1.01	26	0.192

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
72	A	12	8	1.58	26	0.308
73	B	3	2	2.08	24	0.083
74	A	8	6	1.54	23	0.261
75	A	13	8	1.84	26	0.308
76	A	10	8	1.64	26	0.308
77	A	13	9	1.73	26	0.346
78	A	10	7	1.60	26	0.269
79	A	3	3	1.00	12	0.250
80	A	6	4	1.44	26	0.154
81	A	5	3	1.35	26	0.115
82	A	4	4	1.45	26	0.154
83	A	3	3	1.29	26	0.115
84	A	2	2	1.52	24	0.083
85	A	1	1	1.00	23	0.043
86	A	6	4	1.00	26	0.154
87	A	2	2	1.54	26	0.077
88	A	8	6	1.19	26	0.231
89	A	4	4	1.45	26	0.154
90	A	4	5	1.02	26	0.192
91	A	7	6	1.38	26	0.231
92	A	3	5	1.02	26	0.192
93	A	3	3	1.31	26	0.115
94	A	2	2	1.56	24	0.083
95	A	2	2	1.49	23	0.087
96	A	8	6	1.27	26	0.231
97	A	4	5	1.02	26	0.192
98	A	11	8	1.31	26	0.308
99	A	5	6	1.02	26	0.231
100	A	11	6	1.33	26	0.231
101	A	5	7	1.02	26	0.269
102	A	7	5	1.28	26	0.192
103	A	4	6	1.02	26	0.231
104	A	4	3	1.49	26	0.115
105	A	3	3	1.52	24	0.125
106	A	4	4	1.36	23	0.174
107	A	11	7	1.26	26	0.269

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
108	A	5	7	1.02	26	0.269
109	A	15	10	1.32	26	0.385
110	A	5	7	1.02	26	0.269
111	A	6	4	1.00	19	0.210
112	A	5	3	1.00	21	0.143
113	A	4	4	1.00	21	0.190
114	A	3	3	1.00	21	0.143
115	A	2	2	1.00	19	0.105
116	A	1	1	1.00	18	0.056
117	A	6	4	1.00	21	0.190
118	A	2	2	1.00	21	0.095
119	A	8	6	1.00	21	0.286
120	A	6	4	1.00	26	0.154
121	A	5	4	1.00	26	0.154
122	A	2	1	1.00	24	0.042
123	A	3	3	1.00	23	0.130
124	A	8	6	1.00	26	0.231
125	A	3	3	1.00	26	0.115
126	A	8	6	1.00	26	0.231
127	A	3	2	1.00	26	0.077
128	A	7	5	1.00	26	0.192
129	A	8	6	1.00	26	0.231
130	A	3	2	1.00	24	0.083
131	A	6	5	1.00	23	0.217
132	A	10	7	1.00	26	0.269
133	A	6	5	1.00	26	0.192
134	A	11	8	1.00	26	0.308
135	A	6	5	1.00	26	0.192
136	A	7	5	1.00	26	0.192
137	A	12	8	1.00	26	0.308
138	A	3	2	1.00	24	0.083
139	A	8	6	1.00	23	0.261
140	A	13	8	1.00	26	0.308
141	A	10	8	1.00	26	0.308
142	A	13	9	1.00	26	0.346
143	A	10	7	1.00	26	0.269

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
144	A	3	3	1.00	12	0.250
145	A	6	4	1.00	26	0.154
146	A	6	4	1.00	26	0.154
147	A	4	4	1.00	26	0.154
148	A	4	4	1.00	26	0.154
149	A	2	2	1.00	24	0.083
150	A	2	2	1.00	23	0.087
151	A	7	5	1.00	26	0.192
152	A	2	2	1.00	26	0.077
153	A	9	7	1.00	26	0.269
154	A	4	4	1.00	26	0.154
155	A	8	7	1.04	26	0.269
156	A	8	7	1.00	26	0.269
157	A	5	5	1.04	26	0.192
158	A	4	4	1.00	26	0.154
159	A	2	2	1.00	24	0.083
160	A	2	2	1.00	23	0.087
161	A	9	7	1.00	26	0.269
162	A	7	7	1.00	26	0.269
163	A	12	9	1.00	26	0.346
164	A	11	8	1.00	26	0.308
165	A	12	7	1.00	26	0.269
166	A	9	6	1.07	26	0.231
167	A	8	6	1.00	26	0.231
168	A	5	4	1.03	26	0.154
169	A	4	3	1.00	26	0.115
170	A	3	3	1.00	24	0.125
171	A	4	4	1.00	23	0.174
172	A	12	8	1.00	26	0.308
173	A	8	7	1.00	26	0.269
174	A	16	11	1.00	26	0.423
175	A	12	7	1.00	26	0.269
176	A	6	4	1.00	19	0.210
177	A	5	3	1.00	21	0.143
178	A	4	4	1.00	21	0.190
179	A	3	3	1.00	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
180	A	2	2	1.00	19	0.105
181	A	1	1	1.00	18	0.056
182	A	6	4	1.00	21	0.190
183	A	2	2	1.00	21	0.095
184	A	8	6	1.00	21	0.286
185	A	6	7	1.00	24	0.292
186	A	5	6	1.00	24	0.250
187	A	4	5	1.00	22	0.227
188	A	0	0	0.00	0	0.000
189	A	0	0	0.00	0	0.000
190	A	0	0	0.00	0	0.000
191	A	9	6	1.00	26	0.231
192	A	6	5	1.00	26	0.192
193	A	3	3	1.00	26	0.115
194	A	2	2	1.00	26	0.077
195	A	4	4	1.00	26	0.154
196	A	6	4	1.00	26	0.154
197	A	1	1	1.00	21	0.048
198	A	11	10	1.00	24	0.417
199	A	14	6	1.00	24	0.250
200	A	9	10	1.00	24	0.417
201	A	7	6	1.00	22	0.273
202	A	6	4	1.00	21	0.190
203	A	10	10	0.99	24	0.417
204	A	12	9	1.00	24	0.375
205	A	10	10	0.99	24	0.417
206	A	16	8	1.00	24	0.333
207	A	16	11	1.00	26	0.423
208	A	25	7	1.00	26	0.269
209	A	14	11	1.00	26	0.423
210	A	9	7	1.00	24	0.292
211	A	10	5	1.00	23	0.217
212	A	17	12	1.00	26	0.462
213	A	17	11	1.00	26	0.423
214	A	17	12	1.00	26	0.462
215	A	24	10	1.00	26	0.385

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
216	A	21	11	1.00	26	0.423
217	A	40	9	1.00	26	0.346
218	A	19	11	1.00	26	0.423
219	A	11	7	1.00	24	0.292
220	A	14	5	1.00	23	0.217
221	A	26	13	1.00	26	0.500
222	A	24	12	1.00	26	0.462
223	A	28	15	1.00	26	0.577
224	A	31	12	1.00	26	0.462
225	A	16	10	1.00	26	0.385
226	A	10	10	1.00	26	0.385
227	A	11	8	1.00	26	0.308
228	A	6	6	1.00	24	0.250
229	A	8	5	1.00	23	0.217
230	A	9	6	1.00	26	0.231
231	A	15	10	1.00	26	0.385
232	A	12	9	1.00	26	0.346
233	A	24	11	1.00	26	0.423
234	A	15	14	1.00	26	0.538
235	A	10	9	1.00	26	0.346
236	A	11	8	1.00	26	0.308
237	A	3	3	1.00	24	0.125
238	A	11	8	1.00	23	0.348
239	A	12	9	1.00	26	0.346
240	A	20	14	1.00	26	0.538
241	A	17	15	1.00	26	0.577
242	A	32	15	1.00	26	0.577
243	A	16	13	1.00	26	0.500
244	A	8	6	1.00	26	0.231
245	A	15	10	1.00	26	0.385
246	A	5	5	1.00	24	0.208
247	A	15	9	1.00	23	0.391
248	A	17	11	1.00	26	0.423
249	A	27	15	1.00	26	0.577
250	A	23	19	1.00	26	0.731
251	A	43	17	1.00	26	0.654

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
252	A	16	8	1.40	25	0.320
253	A	10	8	1.40	25	0.320
254	A	5	5	1.51	25	0.200
255	A	1	1	1.00	25	0.040
256	A	6	6	1.72	25	0.240
257	A	9	9	1.43	25	0.360
258	A	14	8	1.00	28	0.286
259	A	10	6	1.00	28	0.214
260	A	5	4	1.00	26	0.154
261	A	5	5	1.00	25	0.200
262	A	12	8	1.00	28	0.286
263	A	7	7	1.00	28	0.250
264	A	13	10	1.00	28	0.357
265	A	9	9	1.00	28	0.321
266	A	20	14	1.00	28	0.500
267	A	17	11	1.00	28	0.393
268	A	6	6	1.00	26	0.231
269	A	10	8	1.00	25	0.320
270	A	17	12	1.00	28	0.429
271	A	14	13	1.00	28	0.464
272	A	18	15	1.00	28	0.536
273	A	16	11	1.00	28	0.393
274	A	27	18	1.00	28	0.643
275	A	25	14	1.00	28	0.500
276	A	6	6	1.00	26	0.231
277	A	16	8	1.00	25	0.320
278	A	23	16	1.00	28	0.571
279	A	23	15	1.00	28	0.536
280	A	25	20	1.00	28	0.714
281	A	27	15	1.00	28	0.536
282	A	10	5	1.00	23	0.217
283	A	8	7	1.00	23	0.304
284	A	5	5	1.00	23	0.217
285	A	3	3	1.00	21	0.143
286	A	1	1	1.00	20	0.050
287	A	8	5	1.00	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
288	A	6	6	1.00	23	0.261
289	A	13	10	1.00	23	0.435
290	A	14	7	1.00	28	0.250
291	A	11	6	1.00	28	0.214
292	A	9	7	1.00	28	0.250
293	A	6	6	1.00	28	0.214
294	A	4	3	1.00	26	0.115
295	A	2	2	1.00	25	0.080
296	A	9	6	1.00	28	0.214
297	A	6	6	1.00	28	0.214
298	A	14	11	1.00	28	0.393
299	A	9	9	1.00	28	0.321
300	A	22	13	1.00	28	0.464
301	A	15	13	1.00	28	0.464
302	A	13	9	1.00	28	0.321
303	A	8	8	1.00	28	0.286
304	A	7	5	1.00	26	0.192
305	A	6	6	1.00	25	0.240
306	A	16	11	1.00	28	0.393
307	A	14	10	1.00	28	0.357
308	A	27	15	1.00	28	0.536
309	A	24	11	1.00	28	0.393
310	A	26	11	1.00	28	0.393
311	A	17	10	1.00	28	0.357
312	A	16	7	1.00	28	0.250
313	A	9	9	1.00	28	0.321
314	A	9	7	1.00	26	0.269
315	A	9	9	1.00	25	0.360
316	A	25	13	1.00	28	0.464
317	A	19	14	1.00	28	0.500
318	A	39	18	1.00	28	0.643
319	A	32	15	1.00	28	0.536
320	A	13	10	1.00	21	0.476
321	A	0	0	0.00	0	0.000
322	A	0	0	0.00	0	0.000
323	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
324	A	0	0	0.00	0	0.000
325	A	0	0	0.00	0	0.000
326	A	0	0	0.00	0	0.000
327	A	0	0	0.00	0	0.000
328	A	24	13	1.00	19	0.684
329	A	17	11	1.00	19	0.579
330	A	10	7	1.00	17	0.412
331	A	10	6	1.00	19	0.316
332	A	18	10	1.00	19	0.526
333	A	28	11	1.00	19	0.579
334	A	24	9	1.00	21	0.429
335	A	14	8	1.00	21	0.381
336	A	6	5	1.00	21	0.238
337	A	2	2	1.00	21	0.095
338	A	7	7	1.00	21	0.333
339	A	11	10	1.00	21	0.476
340	A	17	11	1.00	21	0.524
341	A	0	0	0.00	0	0.000
342	A	13	4	1.00	23	0.174
343	A	10	6	1.00	23	0.261
344	A	6	4	1.00	23	0.174
345	A	4	3	1.00	21	0.143
346	A	1	1	1.00	20	0.050
347	A	10	6	1.00	23	0.261
348	A	7	7	1.00	23	0.304
349	A	18	10	1.00	23	0.435
350	A	7	3	1.00	19	0.158
351	A	6	3	1.00	19	0.158
352	A	5	3	1.00	17	0.176
353	A	0	0	0.00	0	0.000
354	A	0	0	0.00	0	0.000
355	A	12	5	1.00	27	0.185
356	A	12	5	0.98	27	0.185
357	A	6	5	1.00	27	0.185
358	A	9	5	0.97	25	0.200
359	A	6	5	1.00	24	0.208

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
360	A	0	0	0.00	0	0.000
361	A	0	0	0.00	0	0.000
362	A	0	0	0.00	0	0.000
363	A	0	0	0.00	0	0.000
364	A	15	5	0.98	27	0.185
365	A	12	5	1.00	27	0.185
366	A	12	5	0.98	25	0.200
367	A	9	5	1.00	24	0.208
368	A	0	0	0.00	0	0.000
369	A	0	0	0.00	0	0.000
370	A	0	0	0.00	0	0.000
371	A	0	0	0.00	0	0.000
372	A	15	5	0.98	27	0.185
373	A	15	5	1.00	27	0.185
374	A	15	5	0.98	25	0.200
375	A	12	5	1.00	24	0.208
376	A	0	0	0.00	0	0.000
377	A	0	0	0.00	0	0.000
378	A	0	0	0.00	0	0.000
379	A	0	0	0.00	0	0.000
380	A	5	3	1.00	23	0.130
381	A	5	3	1.00	23	0.130
382	A	4	3	1.00	23	0.130
383	A	4	3	1.00	23	0.130
384	A	2	2	1.00	21	0.095
385	A	1	1	1.00	20	0.050
386	A	0	0	0.00	0	0.000
387	A	0	0	0.00	0	0.000
388	A	12	5	0.98	27	0.185
389	A	9	5	1.00	27	0.185
390	A	9	5	0.97	27	0.185
391	A	6	5	1.00	27	0.185
392	A	4	4	0.93	25	0.160
393	A	1	1	1.00	24	0.042
394	A	0	0	0.00	0	0.000
395	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
396	A	0	0	0.00	0	0.000
397	A	0	0	0.00	0	0.000
398	A	0	0	0.00	0	0.000
399	A	0	0	0.00	0	0.000
400	A	0	0	0.00	0	0.000
401	A	0	0	0.00	0	0.000
402	A	0	0	0.00	0	0.000
403	A	0	0	0.00	0	0.000
404	A	0	0	0.00	0	0.000
405	A	0	0	0.00	0	0.000
406	A	8	4	1.00	19	0.210
407	A	7	4	1.00	19	0.210
408	A	6	4	1.00	17	0.235
409	A	0	0	0.00	0	0.000
410	A	0	0	0.00	0	0.000
411	A	22	6	0.98	27	0.222
412	A	16	7	1.00	27	0.259
413	A	14	7	1.33	25	0.280
414	A	7	7	1.00	24	0.292
415	A	0	0	0.00	0	0.000
416	A	0	0	0.00	0	0.000
417	A	0	0	0.00	0	0.000
418	A	0	0	0.00	0	0.000
419	A	28	6	0.99	27	0.222
420	A	19	6	1.00	27	0.222
421	A	22	8	0.98	25	0.320
422	A	10	6	1.00	24	0.250
423	A	0	0	0.00	0	0.000
424	A	0	0	0.00	0	0.000
425	A	0	0	0.00	0	0.000
426	A	0	0	0.00	0	0.000
427	A	34	6	0.99	27	0.222
428	A	28	6	1.00	27	0.222
429	A	28	8	0.99	25	0.320
430	A	13	6	1.00	24	0.250
431	A	0	0	0.00	0	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
432	A	0	0	0.00	0	0.000
433	A	0	0	0.00	0	0.000
434	A	0	0	0.00	0	0.000
435	A	13	6	0.98	27	0.222
436	A	10	6	1.00	27	0.222
437	A	10	6	0.97	27	0.222
438	A	7	7	1.00	27	0.259
439	A	5	5	1.00	25	0.200
440	A	1	1	1.00	24	0.042
441	A	0	0	0.00	0	0.000
442	A	0	0	0.00	0	0.000
443	A	0	0	0.00	0	0.000
444	A	0	0	0.00	0	0.000
445	A	0	0	0.00	0	0.000
446	A	0	0	0.00	0	0.000
447	A	0	0	0.00	0	0.000
448	A	0	0	0.00	0	0.000
449	A	0	0	0.00	0	0.000
450	A	0	0	0.00	0	0.000
451	A	0	0	0.00	0	0.000
452	A	0	0	0.00	0	0.000
453	A	0	0	0.00	0	0.000
454	A	0	0	0.00	0	0.000
455	A	0	0	0.00	0	0.000
456	A	0	0	0.00	0	0.000
457	A	0	0	0.00	0	0.000
458	A	0	0	0.00	0	0.000
459	A	0	0	0.00	0	0.000
460	A	0	0	0.00	0	0.000
461	A	1	1	1.00	20	0.050
462	A	27	7	1.00	26	0.269
463	A	32	7	1.00	26	0.269
464	A	17	9	1.00	24	0.375
465	A	14	7	1.00	23	0.304
466	A	0	0	0.00	0	0.000
467	A	32	7	1.00	28	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
468	A	42	7	1.00	28	0.250
469	A	32	9	1.00	26	0.346
470	A	19	7	1.00	25	0.280
471	A	0	0	0.00	0	0.000
472	A	24	11	1.00	23	0.478
473	A	10	9	1.00	23	0.391
474	A	2	2	1.00	23	0.087
475	A	0	0	0.00	0	0.000
476	A	0	0	0.00	0	0.000
477	A	26	12	1.00	23	0.522
478	A	11	9	1.00	23	0.391
479	A	2	2	1.00	23	0.087
480	A	0	0	0.00	0	0.000
481	A	39	14	1.00	23	0.609
482	A	13	11	1.00	23	0.478
483	A	2	2	1.00	23	0.087
484	A	0	0	0.00	0	0.000
485	A	24	11	1.00	22	0.500
486	A	10	9	1.00	22	0.409
487	A	2	2	1.00	22	0.091
488	A	0	0	0.00	0	0.000
489	A	0	0	0.00	0	0.000
490	A	26	12	1.00	22	0.546
491	A	11	9	1.00	22	0.409
492	A	2	2	1.00	22	0.091
493	A	0	0	0.00	0	0.000
494	A	6	5	1.00	17	0.294
495	A	19	7	1.00	23	0.304
496	A	14	7	1.00	23	0.304
497	A	9	7	1.00	23	0.304
498	A	2	2	1.00	23	0.087
499	A	0	0	0.00	0	0.000
500	A	0	0	0.00	0	0.000
501	A	19	7	1.00	23	0.304
502	A	14	7	1.00	23	0.304
503	A	9	8	1.00	23	0.348

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
504	A	2	2	1.00	23	0.087
505	A	0	0	0.00	0	0.000
506	A	0	0	0.00	0	0.000
507	A	18	10	1.00	23	0.435
508	A	7	6	1.00	23	0.261
509	A	2	2	1.00	23	0.087
510	A	0	0	0.00	0	0.000
511	A	0	0	0.00	0	0.000
512	A	7	5	1.00	28	0.179
513	A	10	5	1.00	26	0.192
514	A	7	5	1.00	25	0.200
515	A	0	0	0.00	0	0.000
516	A	0	0	0.00	0	0.000
517	A	13	5	1.00	28	0.179
518	A	13	5	1.00	26	0.192
519	A	10	5	1.00	25	0.200
520	A	0	0	0.00	0	0.000
521	A	0	0	0.00	0	0.000
522	A	16	5	1.00	28	0.179
523	A	16	5	1.00	26	0.192
524	A	13	5	1.00	25	0.200
525	A	0	0	0.00	0	0.000
526	A	0	0	0.00	0	0.000
527	A	0	0	0.00	0	0.000
528	A	9	4	1.00	23	0.174
529	A	6	4	1.00	23	0.174
530	A	4	3	1.00	21	0.143
531	A	1	1	1.00	20	0.050
532	A	0	0	0.00	0	0.000
533	A	0	0	0.00	0	0.000
534	A	13	8	1.00	35	0.229
535	A	8	6	1.00	35	0.171
536	A	4	4	1.00	35	0.114
537	A	6	5	1.00	35	0.143
538	A	8	8	1.00	35	0.229
539	A	6	6	1.00	35	0.171

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
540	A	12	9	1.00	35	0.257
541	A	7	6	1.00	35	0.171
542	A	8	6	1.00	35	0.171
543	A	9	7	1.00	35	0.200
544	A	10	10	1.00	35	0.286
545	A	9	9	1.00	35	0.257
546	A	9	7	1.00	35	0.200
547	A	12	9	1.00	35	0.257
548	A	13	8	1.00	35	0.229
549	A	13	7	1.00	35	0.200
550	A	7	9	1.00	35	0.257
551	A	10	8	1.00	35	0.229
552	A	13	7	1.00	35	0.200
553	A	9	7	1.00	35	0.200
554	A	6	5	1.00	35	0.143
555	A	2	2	1.00	35	0.057
556	A	5	6	1.00	35	0.171
557	A	8	8	1.00	35	0.229
558	A	7	9	1.00	35	0.257
559	A	10	10	1.00	35	0.286
560	A	8	8	1.00	35	0.229
561	A	5	6	1.00	35	0.171
562	A	3	3	1.00	35	0.086
563	A	8	8	1.00	35	0.229
564	A	10	8	1.00	35	0.229
565	A	9	9	1.00	35	0.257
566	A	6	6	1.00	35	0.171
567	A	8	8	1.00	35	0.229
568	A	8	8	1.00	35	0.229
569	A	5	5	1.00	35	0.143
570	A	23	13	1.00	37	0.351
571	A	13	11	1.00	37	0.297
572	A	6	6	1.00	37	0.162
573	A	8	6	1.00	37	0.162
574	A	19	13	1.00	37	0.351
575	A	20	12	1.00	37	0.324

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
576	A	19	15	1.00	37	0.405
577	A	11	9	1.00	37	0.243
578	A	13	11	1.00	37	0.297
579	A	11	9	1.00	37	0.243
580	A	23	15	1.00	37	0.405
581	A	21	13	1.00	37	0.351
582	A	17	9	1.00	37	0.243
583	A	19	15	1.00	37	0.405
584	A	23	13	1.00	37	0.351
585	A	17	10	1.00	37	0.270
586	A	28	19	1.00	37	0.514
587	A	25	16	1.00	37	0.432
588	A	17	10	1.00	37	0.270
589	A	11	9	1.00	37	0.243
590	A	8	6	1.00	37	0.162
591	A	2	2	1.00	37	0.054
592	A	16	11	1.00	37	0.297
593	A	30	18	1.00	37	0.486
594	A	28	19	1.00	37	0.514
595	A	23	15	1.00	37	0.405
596	A	19	13	1.00	37	0.351
597	A	16	11	1.00	37	0.297
598	A	7	7	1.00	37	0.189
599	A	21	14	1.00	37	0.378
600	A	25	16	1.00	37	0.432
601	A	21	13	1.00	37	0.351
602	A	20	12	1.00	37	0.324
603	A	30	18	1.00	37	0.486
604	A	21	14	1.00	37	0.378
605	A	10	10	1.00	37	0.270
606	A	5	5	1.00	18	0.278
607	A	5	5	1.00	18	0.278
608	A	5	5	1.00	18	0.278
609	A	4	3	1.00	16	0.188
610	A	3	2	1.00	8	0.250
611	A	18	6	1.00	18	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
612	A	26	9	1.00	18	0.500
613	A	26	7	1.00	20	0.350
614	A	17	7	1.00	20	0.350
615	A	10	7	1.00	18	0.389
616	A	3	3	1.00	10	0.300
617	A	22	7	1.00	20	0.350
618	A	42	7	0.98	20	0.350
619	A	27	7	0.98	20	0.350
620	A	15	7	0.98	18	0.389
621	A	4	4	1.00	10	0.400
622	A	0	0	0.00	0	0.000
623	A	0	0	0.00	0	0.000
624	A	26	7	0.98	20	0.350
625	A	15	7	0.97	18	0.389
626	A	5	5	0.95	10	0.500
627	A	0	0	0.00	0	0.000
628	A	0	0	0.00	0	0.000
629	A	42	9	1.00	22	0.409
630	A	23	9	1.00	20	0.450
631	A	7	6	1.00	12	0.500
632	A	0	0	0.00	0	0.000
633	A	0	0	0.00	0	0.000
634	A	32	12	1.00	20	0.600
635	A	8	7	1.00	12	0.583
636	A	0	0	0.00	0	0.000
637	A	0	0	0.00	0	0.000
638	A	39	8	1.00	22	0.364
639	A	21	8	1.00	20	0.400
640	A	6	5	1.00	12	0.417
641	A	0	0	0.00	0	0.000
642	A	0	0	0.00	0	0.000
643	A	21	8	1.00	20	0.400
644	A	7	6	1.00	12	0.500
645	A	0	0	0.00	0	0.000
646	A	0	0	0.00	0	0.000
647	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
648	A	0	0	0.00	0	0.000
649	A	6	7	1.00	20	0.350
650	A	7	9	1.00	20	0.450
651	A	8	10	1.00	20	0.500
652	A	0	0	0.00	0	0.000
653	A	0	0	0.00	0	0.000
654	A	0	0	0.00	0	0.000
655	A	0	0	0.00	0	0.000
656	A	0	0	0.00	0	0.000
657	A	0	0	0.00	0	0.000
658	A	0	0	0.00	0	0.000
659	A	0	0	0.00	0	0.000
660	A	0	0	0.00	0	0.000
661	A	0	0	0.00	0	0.000
662	A	0	0	0.00	0	0.000
663	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

3.1 $\int x^4 \left(d + c^2 d x^2\right) \left(a + b \sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=124

$$\frac{1}{7} c^2 d x^7 \left(a + b \sinh^{-1}(cx)\right) + \frac{1}{5} d x^5 \left(a + b \sinh^{-1}(cx)\right) - \frac{bd\left(c^2x^2+1\right)^{7/2}}{49c^5} + \frac{8bd\left(c^2x^2+1\right)^{5/2}}{175c^5} - \frac{bd\left(c^2x^2+1\right)^{3/2}}{105c^5} - \frac{2b\ldots}{}$$

```
[Out] -1/105*b*d*(c^2*x^2+1)^(3/2)/c^5+8/175*b*d*(c^2*x^2+1)^(5/2)/c^5-1/49*b*d*(
c^2*x^2+1)^(7/2)/c^5+1/5*d*x^5*(a+b*arcsinh(c*x))+1/7*c^2*d*x^7*(a+b*arcsin
h(c*x))-2/35*b*d*(c^2*x^2+1)^(1/2)/c^5
```

Rubi [A] time = 0.12, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {14, 5730, 12, 446, 77}

$$\frac{1}{7} c^2 d x^7 \left(a + b \sinh^{-1}(cx)\right) + \frac{1}{5} d x^5 \left(a + b \sinh^{-1}(cx)\right) - \frac{bd\left(c^2x^2+1\right)^{7/2}}{49c^5} + \frac{8bd\left(c^2x^2+1\right)^{5/2}}{175c^5} - \frac{bd\left(c^2x^2+1\right)^{3/2}}{105c^5} - \frac{2b\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-2*b*d*Sqrt[1 + c^2*x^2])/(35*c^5) - (b*d*(1 + c^2*x^2)^(3/2))/(105*c^5) +
 (8*b*d*(1 + c^2*x^2)^(5/2))/(175*c^5) - (b*d*(1 + c^2*x^2)^(7/2))/(49*c^5)
 + (d*x^5*(a + b*ArcSinh[c*x]))/5 + (c^2*d*x^7*(a + b*ArcSinh[c*x]))/7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\int x^4\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)dx=\frac{1}{5}dx^5\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{7}c^2dx^7\left(a+b\sinh^{-1}(cx)\right)-(bc)\int\frac{dx^5\left(7+\right.}{35\sqrt{1-}}$$

$$=\frac{1}{5}dx^5\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{7}c^2dx^7\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{35}(bcd)\int\frac{x^5\left(\right.}{\sqrt{}}$$

$$=\frac{1}{5}dx^5\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{7}c^2dx^7\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{70}(bcd)\,\mathrm{Subst}$$

$$=\frac{1}{5}dx^5\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{7}c^2dx^7\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{70}(bcd)\,\mathrm{Subst}$$

$$=-\frac{2bd\sqrt{1+c^2x^2}}{35c^5}-\frac{bd\left(1+c^2x^2\right)^{3/2}}{105c^5}+\frac{8bd\left(1+c^2x^2\right)^{5/2}}{175c^5}-\frac{bd\left(1+c^2x^2\right)}{49c^5}$$

Mathematica [A] time = 0.14, size = 87, normalized size = 0.70

$$\frac{d\left(105ax^5\left(5c^2x^2+7\right)+105bx^5\left(5c^2x^2+7\right)\sinh^{-1}(cx)-\frac{b\sqrt{c^2x^2+1}\left(75c^6x^6+57c^4x^4-76c^2x^2+152\right)}{c^5}\right)}{3675}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d*(105*a*x^5*(7 + 5*c^2*x^2) - (b*Sqrt[1 + c^2*x^2]*(152 - 76*c^2*x^2 + 57
*c^4*x^4 + 75*c^6*x^6))/c^5 + 105*b*x^5*(7 + 5*c^2*x^2)*ArcSinh[c*x]))/3675
```

fricas [A] time = 0.55, size = 113, normalized size = 0.91

$$\frac{525\,ac^7dx^7+735\,ac^5dx^5+105\left(5\,bc^7dx^7+7\,bc^5dx^5\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-\left(75\,bc^6dx^6+57\,bc^4dx^4-76\,bc^2dx^2\right.}{3675\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/3675*(525*a*c^7*d*x^7 + 735*a*c^5*d*x^5 + 105*(5*b*c^7*d*x^7 + 7*b*c^5*d*
x^5)*log(c*x + sqrt(c^2*x^2 + 1)) - (75*b*c^6*d*x^6 + 57*b*c^4*d*x^4 - 76*b
*c^2*d*x^2 + 152*b*d)*sqrt(c^2*x^2 + 1))/c^5
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.04, size = 124, normalized size = 1.00

$$\frac{da\left(\frac{1}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+db\left(\frac{\mathrm{arcsinh}(cx)c^7x^7}{7}+\frac{\mathrm{arcsinh}(cx)c^5x^5}{5}-\frac{c^6x^6\sqrt{c^2x^2+1}}{49}-\frac{19c^4x^4\sqrt{c^2x^2+1}}{1225}+\frac{76c^2x^2\sqrt{c^2x^2+1}}{3675}-\frac{152\sqrt{c^2x^2+}}{3675}\right.}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c^5*(d*a*(1/7*c^7*x^7+1/5*c^5*x^5)+d*b*(1/7*arcsinh(c*x)*c^7*x^7+1/5*arcs
inh(c*x)*c^5*x^5-1/49*c^6*x^6*(c^2*x^2+1)^(1/2)-19/1225*c^4*x^4*(c^2*x^2+1)
^(1/2)+76/3675*c^2*x^2*(c^2*x^2+1)^(1/2)-152/3675*(c^2*x^2+1)^(1/2)))
```

maxima [A] time = 0.36, size = 184, normalized size = 1.48

$$\frac{1}{7}\,ac^2dx^7+\frac{1}{5}\,adx^5+\frac{1}{245}\left(35\,x^7\,\mathrm{arsinh}\left(cx\right)-\left(\frac{5\,\sqrt{c^2x^2+1}\,x^6}{c^2}-\frac{6\,\sqrt{c^2x^2+1}\,x^4}{c^4}+\frac{8\,\sqrt{c^2x^2+1}\,x^2}{c^6}-\frac{16\,\sqrt{c^2x^2+}}{c^8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*c^2*d*x^7 + 1/5*a*d*x^5 + 1/245*(35*x^7*arcsinh(c*x) - (5*sqrt(c^2*x^
2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6
- 16*sqrt(c^2*x^2 + 1)/c^8)*c)*b*c^2*d + 1/75*(15*x^5*arcsinh(c*x) - (3*sqr
t(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/
c^6)*c)*b*d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\left(d\,c^2\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*asinh(c*x))*(d + c^2*d*x^2),x)
```

```
[Out] int(x^4*(a + b*asinh(c*x))*(d + c^2*d*x^2), x)
```

sympy [A] time = 5.68, size = 151, normalized size = 1.22

$$\begin{cases}\frac{ac^2dx^7}{7}+\frac{adx^5}{5}+\frac{bc^2dx^7\,\mathrm{asinh}\left(cx\right)}{7}-\frac{bcdx^6\sqrt{c^2x^2+1}}{49}+\frac{bdx^5\,\mathrm{asinh}\left(cx\right)}{5}-\frac{19bdx^4\sqrt{c^2x^2+1}}{1225c}+\frac{76bdx^2\sqrt{c^2x^2+1}}{3675c^3}-\frac{152bd\sqrt{c^2x^2+1}}{3675c^5} & \text{fo} \\ \frac{adx^5}{5} & \text{ot}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c**2*d*x**2+d)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**2*d*x**7/7 + a*d*x**5/5 + b*c**2*d*x**7*asinh(c*x)/7 - b*c*
d*x**6*sqrt(c**2*x**2 + 1)/49 + b*d*x**5*asinh(c*x)/5 - 19*b*d*x**4*sqrt(c*
*2*x**2 + 1)/(1225*c) + 76*b*d*x**2*sqrt(c**2*x**2 + 1)/(3675*c**3) - 152*b
*d*sqrt(c**2*x**2 + 1)/(3675*c**5), Ne(c, 0)), (a*d*x**5/5, True))
```

3.2 $\int x^3 \left(d + c^2 d x^2\right) \left(a + b \sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=120

$$\frac{1}{6}c^2dx^6\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}dx^4\left(a+b\sinh^{-1}(cx)\right)-\frac{bd\sinh^{-1}(cx)}{24c^4}-\frac{1}{36}bcdx^5\sqrt{c^2x^2+1}-\frac{bdx^3\sqrt{c^2x^2+1}}{36c}+\frac{bd}{}$$

```
[Out] -1/24*b*d*arcsinh(c*x)/c^4+1/4*d*x^4*(a+b*arcsinh(c*x))+1/6*c^2*d*x^6*(a+b*
arcsinh(c*x))+1/24*b*d*x*(c^2*x^2+1)^(1/2)/c^3-1/36*b*d*x^3*(c^2*x^2+1)^(1/
2)/c-1/36*b*c*d*x^5*(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = $\{14, 5730, 12, 459, 321, 215\}$

$$\frac{1}{6}c^2dx^6\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}dx^4\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{36}bcdx^5\sqrt{c^2x^2+1}-\frac{bdx^3\sqrt{c^2x^2+1}}{36c}+\frac{bdx\sqrt{c^2x^2+1}}{24c^3}-\frac{bd}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (b*d*x*Sqrt[1 + c^2*x^2])/(24*c^3) - (b*d*x^3*Sqrt[1 + c^2*x^2])/(36*c) - (
b*c*d*x^5*Sqrt[1 + c^2*x^2])/36 - (b*d*ArcSinh[c*x])/(24*c^4) + (d*x^4*(a +
 b*ArcSinh[c*x]))/4 + (c^2*d*x^6*(a + b*ArcSinh[c*x]))/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\int x^3 \left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)dx = \frac{1}{4}dx^4\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{6}c^2dx^6\left(a+b\sinh^{-1}(cx)\right)-(bc)\int\frac{dx^4\left(3+\right.}{12\sqrt{1-}}$$

$$=\frac{1}{4}dx^4\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{6}c^2dx^6\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{12}(bcd)\int\frac{x^4(}{\sqrt{}}$$

$$=-\frac{1}{36}bcdx^5\sqrt{1+c^2x^2}+\frac{1}{4}dx^4\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{6}c^2dx^6\left(a+b\sinh^{-1}\right.$$

$$=-\frac{bdx^3\sqrt{1+c^2x^2}}{36c}-\frac{1}{36}bcdx^5\sqrt{1+c^2x^2}+\frac{1}{4}dx^4\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{6}c$$

$$=\frac{bdx\sqrt{1+c^2x^2}}{24c^3}-\frac{bdx^3\sqrt{1+c^2x^2}}{36c}-\frac{1}{36}bcdx^5\sqrt{1+c^2x^2}+\frac{1}{4}dx^4\left(a+b\,\mathrm{s}\right.$$

$$=\frac{bdx\sqrt{1+c^2x^2}}{24c^3}-\frac{bdx^3\sqrt{1+c^2x^2}}{36c}-\frac{1}{36}bcdx^5\sqrt{1+c^2x^2}-\frac{bd\sinh^{-1}(cx)}{24c^4}$$

Mathematica [A] time = 0.05, size = 88, normalized size = 0.73

$$\frac{d\left(6ac^4x^4\left(2c^2x^2+3\right)+3b\left(4c^6x^6+6c^4x^4-1\right)\sinh^{-1}(cx)+bcx\sqrt{c^2x^2+1}\left(-2c^4x^4-2c^2x^2+3\right)\right)}{72c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d*(6*a*c^4*x^4*(3 + 2*c^2*x^2) + b*c*x*Sqrt[1 + c^2*x^2]*(3 - 2*c^2*x^2 -
2*c^4*x^4) + 3*b*(-1 + 6*c^4*x^4 + 4*c^6*x^6)*ArcSinh[c*x]))/(72*c^4)
```

fricas [A] time = 0.50, size = 109, normalized size = 0.91

$$\frac{12\,ac^6dx^6+18\,ac^4dx^4+3\left(4\,bc^6dx^6+6\,bc^4dx^4-bd\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-\left(2\,bc^5dx^5+2\,bc^3dx^3-3\,bcdx\right)\sqrt{c^2x}}{72\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/72*(12*a*c^6*d*x^6 + 18*a*c^4*d*x^4 + 3*(4*b*c^6*d*x^6 + 6*b*c^4*d*x^4 -
b*d)*log(c*x + sqrt(c^2*x^2 + 1)) - (2*b*c^5*d*x^5 + 2*b*c^3*d*x^3 - 3*b*c*
d*x)*sqrt(c^2*x^2 + 1))/c^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 113, normalized size = 0.94

$$\frac{da\left(\frac{1}{6}c^6x^6+\frac{1}{4}c^4x^4\right)+db\left(\frac{\operatorname{arcsinh}(cx)c^6x^6}{6}+\frac{\operatorname{arcsinh}(cx)c^4x^4}{4}-\frac{c^5x^5\sqrt{c^2x^2+1}}{36}-\frac{c^3x^3\sqrt{c^2x^2+1}}{36}+\frac{cx\sqrt{c^2x^2+1}}{24}-\frac{\operatorname{arcsinh}(cx)}{24}\right)}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c^4*(d*a*(1/6*c^6*x^6+1/4*c^4*x^4)+d*b*(1/6*arcsinh(c*x)*c^6*x^6+1/4*arcs
inh(c*x)*c^4*x^4-1/36*c^5*x^5*(c^2*x^2+1)^(1/2)-1/36*c^3*x^3*(c^2*x^2+1)^(1
/2)+1/24*c*x*(c^2*x^2+1)^(1/2)-1/24*arcsinh(c*x)))
```

maxima [A] time = 0.36, size = 166, normalized size = 1.38

$$\frac{1}{6}\,ac^2dx^6+\frac{1}{4}\,adx^4+\frac{1}{288}\left(48\,x^6\operatorname{arsinh}\left(cx\right)-\left(\frac{8\,\sqrt{c^2x^2+1}\,x^5}{c^2}-\frac{10\,\sqrt{c^2x^2+1}\,x^3}{c^4}+\frac{15\,\sqrt{c^2x^2+1}\,x}{c^6}-\frac{15\operatorname{arsinh}}{c^7}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*c^2*d*x^6 + 1/4*a*d*x^4 + 1/288*(48*x^6*arcsinh(c*x) - (8*sqrt(c^2*x^
2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 + 1)*x/c^6
- 15*arcsinh(c*x)/c^7)*c)*b*c^2*d + 1/32*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*
x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*b*d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\left(d\,c^2\,x^2+d\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2),x)
```

```
[Out] int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2), x)
```

sympy [A] time = 3.76, size = 138, normalized size = 1.15

$$\begin{cases} \frac{ac^2dx^6}{6}+\frac{adx^4}{4}+\frac{bc^2dx^6\operatorname{asinh}(cx)}{6}-\frac{bcdx^5\sqrt{c^2x^2+1}}{36}+\frac{bdx^4\operatorname{asinh}(cx)}{4}-\frac{bdx^3\sqrt{c^2x^2+1}}{36c}+\frac{bdx\sqrt{c^2x^2+1}}{24c^3}-\frac{bd\operatorname{asinh}(cx)}{24c^4} & \text{for}\ c \neq 0 \\ \frac{adx^4}{4} & \text{otherwi} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*d*x**2+d)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**2*d*x**6/6 + a*d*x**4/4 + b*c**2*d*x**6*asinh(c*x)/6 - b*c*
d*x**5*sqrt(c**2*x**2 + 1)/36 + b*d*x**4*asinh(c*x)/4 - b*d*x**3*sqrt(c**2*
x**2 + 1)/(36*c) + b*d*x*sqrt(c**2*x**2 + 1)/(24*c**3) - b*d*asinh(c*x)/(24
*c**4), Ne(c, 0)), (a*d*x**4/4, True))
```

3.3 $\int x^2 \left(d+c^2 d x^2\right) \left(a+b \sinh^{-1}(c x)\right) \, dx$

Optimal. Leaf size=102

$$\frac{1}{5} c^2 d x^5 \left(a+b \sinh^{-1}(c x)\right)+\frac{1}{3} d x^3 \left(a+b \sinh^{-1}(c x)\right)-\frac{b d \left(c^2 x^2+1\right)^{5/2}}{25 c^3}+\frac{b d \left(c^2 x^2+1\right)^{3/2}}{45 c^3}+\frac{2 b d \sqrt{c^2 x^2+1}}{15 c^3}$$

```
[Out] 1/45*b*d*(c^2*x^2+1)^(3/2)/c^3-1/25*b*d*(c^2*x^2+1)^(5/2)/c^3+1/3*d*x^3*(a+
b*arcsinh(c*x))+1/5*c^2*d*x^5*(a+b*arcsinh(c*x))+2/15*b*d*(c^2*x^2+1)^(1/2)
/c^3
```

Rubi [A] time = 0.10, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {14, 5730, 12, 446, 77}

$$\frac{1}{5} c^2 d x^5 \left(a+b \sinh^{-1}(c x)\right)+\frac{1}{3} d x^3 \left(a+b \sinh^{-1}(c x)\right)-\frac{b d \left(c^2 x^2+1\right)^{5/2}}{25 c^3}+\frac{b d \left(c^2 x^2+1\right)^{3/2}}{45 c^3}+\frac{2 b d \sqrt{c^2 x^2+1}}{15 c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (2*b*d*Sqrt[1 + c^2*x^2])/(15*c^3) + (b*d*(1 + c^2*x^2)^(3/2))/(45*c^3) - (
b*d*(1 + c^2*x^2)^(5/2))/(25*c^3) + (d*x^3*(a + b*ArcSinh[c*x]))/3 + (c^2*d
*x^5*(a + b*ArcSinh[c*x]))/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
```

```
GtQ[p, 0]
```

Rubi steps

$$\int x^2 \left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)dx = \frac{1}{3}dx^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}c^2dx^5\left(a+b\sinh^{-1}(cx)\right)-(bc)\int\frac{dx^3(}{15\sqrt{}}$$

$$= \frac{1}{3}dx^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}c^2dx^5\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{15}(bcd)\int$$

$$= \frac{1}{3}dx^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}c^2dx^5\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{30}(bcd)\,\text{Sub}$$

$$= \frac{1}{3}dx^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}c^2dx^5\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{30}(bcd)\,\text{Sub}$$

$$= \frac{2bd\sqrt{1+c^2x^2}}{15c^3}+\frac{bd\left(1+c^2x^2\right)^{3/2}}{45c^3}-\frac{bd\left(1+c^2x^2\right)^{5/2}}{25c^3}+\frac{1}{3}dx^3\left(a+b\,\text{s}\right.$$

Mathematica [A] time = 0.08, size = 78, normalized size = 0.76

$$\frac{1}{225}d\left(15ax^3\left(3c^2x^2+5\right)+15bx^3\left(3c^2x^2+5\right)\sinh^{-1}(cx)+\frac{b\sqrt{c^2x^2+1}\left(-9c^4x^4-13c^2x^2+26\right)}{c^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d*(15*a*x^3*(5 + 3*c^2*x^2) + (b*Sqrt[1 + c^2*x^2]*(26 - 13*c^2*x^2 - 9*c^
4*x^4))/c^3 + 15*b*x^3*(5 + 3*c^2*x^2)*ArcSinh[c*x]))/225
```

fricas [A] time = 0.60, size = 103, normalized size = 1.01

$$\frac{45\,ac^5dx^5+75\,ac^3dx^3+15\left(3\,bc^5dx^5+5\,bc^3dx^3\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-\left(9\,bc^4dx^4+13\,bc^2dx^2-26\,bd\right)\sqrt{c^2x^2}}{225\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/225*(45*a*c^5*d*x^5 + 75*a*c^3*d*x^3 + 15*(3*b*c^5*d*x^5 + 5*b*c^3*d*x^3)
*log(c*x + sqrt(c^2*x^2 + 1)) - (9*b*c^4*d*x^4 + 13*b*c^2*d*x^2 - 26*b*d)*s
qrt(c^2*x^2 + 1))/c^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 105, normalized size = 1.03

$$\frac{da\left(\frac{1}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+db\left(\frac{\operatorname{arcsinh}(cx)c^5x^5}{5}+\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}-\frac{c^4x^4\sqrt{c^2x^2+1}}{25}-\frac{13c^2x^2\sqrt{c^2x^2+1}}{225}+\frac{26\sqrt{c^2x^2+1}}{225}\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c^3*(d*a*(1/5*c^5*x^5+1/3*c^3*x^3)+d*b*(1/5*arcsinh(c*x)*c^5*x^5+1/3*arcs
inh(c*x)*c^3*x^3-1/25*c^4*x^4*(c^2*x^2+1)^(1/2)-13/225*c^2*x^2*(c^2*x^2+1)^
(1/2)+26/225*(c^2*x^2+1)^(1/2)))
```

maxima [A] time = 0.44, size = 145, normalized size = 1.42

$$\frac{1}{5}\,ac^2dx^5+\frac{1}{75}\left(15\,x^5\operatorname{arsinh}(cx)-\left(\frac{3\,\sqrt{c^2x^2+1}\,x^4}{c^2}-\frac{4\,\sqrt{c^2x^2+1}\,x^2}{c^4}+\frac{8\,\sqrt{c^2x^2+1}}{c^6}\right)c\right)bc^2d+\frac{1}{3}\,adx^3+\frac{1}{9}\left(3\,x^3\operatorname{arsi}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*c^2*d*x^5 + 1/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2
- 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*b*c^2*d + 1/3*a
*d*x^3 + 1/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^
2*x^2 + 1)/c^4))*b*d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\left(d\,c^2\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2),x)
```

```
[Out] int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2), x)
```

sympy [A] time = 2.02, size = 126, normalized size = 1.24

$$\begin{cases} \frac{ac^2dx^5}{5}+\frac{adx^3}{3}+\frac{bc^2dx^5\operatorname{asinh}(cx)}{5}-\frac{bcdx^4\sqrt{c^2x^2+1}}{25}+\frac{bdx^3\operatorname{asinh}(cx)}{3}-\frac{13bdx^2\sqrt{c^2x^2+1}}{225c}+\frac{26bd\sqrt{c^2x^2+1}}{225c^3} & \text{for}\ c\neq 0 \\ \frac{adx^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**2*d*x**5/5 + a*d*x**3/3 + b*c**2*d*x**5*asinh(c*x)/5 - b*c*
d*x**4*sqrt(c**2*x**2 + 1)/25 + b*d*x**3*asinh(c*x)/3 - 13*b*d*x**2*sqrt(c*
*2*x**2 + 1)/(225*c) + 26*b*d*sqrt(c**2*x**2 + 1)/(225*c**3), Ne(c, 0)), (a
*d*x**3/3, True))
```

3.4 $\int x\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=87

$$\frac{d\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{4c^2}-\frac{bdx\left(c^2x^2+1\right)^{3/2}}{16c}-\frac{3bdx\sqrt{c^2x^2+1}}{32c}-\frac{3bd\sinh^{-1}(cx)}{32c^2}$$

```
[Out] -1/16*b*d*x*(c^2*x^2+1)^(3/2)/c-3/32*b*d*arcsinh(c*x)/c^2+1/4*d*(c^2*x^2+1)
^2*(a+b*arcsinh(c*x))/c^2-3/32*b*d*x*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5717, 195, 215}

$$\frac{d\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{4c^2}-\frac{bdx\left(c^2x^2+1\right)^{3/2}}{16c}-\frac{3bdx\sqrt{c^2x^2+1}}{32c}-\frac{3bd\sinh^{-1}(cx)}{32c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-3*b*d*x*Sqrt[1 + c^2*x^2])/(32*c) - (b*d*x*(1 + c^2*x^2)^(3/2))/(16*c) -
(3*b*d*ArcSinh[c*x])/(32*c^2) + (d*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(4
*c^2)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\int x\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)\,dx=\frac{d\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{4c^2}-\frac{(bd)\int\left(1+c^2x^2\right)^{3/2}\,dx}{4c}$$
$$=-\frac{bdx\left(1+c^2x^2\right)^{3/2}}{16c}+\frac{d\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{4c^2}-\frac{(3bd)\int\sqrt{1+c^2}}{16c}$$
$$=-\frac{3bdx\sqrt{1+c^2x^2}}{32c}-\frac{bdx\left(1+c^2x^2\right)^{3/2}}{16c}+\frac{d\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{4c^2}$$
$$=-\frac{3bdx\sqrt{1+c^2x^2}}{32c}-\frac{bdx\left(1+c^2x^2\right)^{3/2}}{16c}-\frac{3bd\sinh^{-1}(cx)}{32c^2}+\frac{d\left(1+c^2x^2\right)^2}{}$$

Mathematica [A] time = 0.05, size = 77, normalized size = 0.89

$$\frac{d\left(cx\left(8acx\left(c^2x^2+2\right)-b\sqrt{c^2x^2+1}\left(2c^2x^2+5\right)\right)+b\left(8c^4x^4+16c^2x^2+5\right)\sinh^{-1}(cx)\right)}{32c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d*(c*x*(8*a*c*x*(2 + c^2*x^2) - b*Sqrt[1 + c^2*x^2]*(5 + 2*c^2*x^2)) + b*(
5 + 16*c^2*x^2 + 8*c^4*x^4)*ArcSinh[c*x]))/(32*c^2)
```

fricas [A] time = 0.56, size = 98, normalized size = 1.13

$$\frac{8\,ac^4dx^4+16\,ac^2dx^2+\left(8\,bc^4dx^4+16\,bc^2dx^2+5\,bd\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-\left(2\,bc^3dx^3+5\,bcdx\right)\sqrt{c^2x^2+1}}{32\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/32*(8*a*c^4*d*x^4 + 16*a*c^2*d*x^2 + (8*b*c^4*d*x^4 + 16*b*c^2*d*x^2 + 5*
b*d)*log(c*x + sqrt(c^2*x^2 + 1)) - (2*b*c^3*d*x^3 + 5*b*c*d*x)*sqrt(c^2*x^
2 + 1))/c^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 94, normalized size = 1.08

$$\frac{da\left(\frac{1}{4}c^4x^4+\frac{1}{2}c^2x^2\right)+db\left(\frac{\operatorname{arcsinh}(cx)c^4x^4}{4}+\frac{\operatorname{arcsinh}(cx)c^2x^2}{2}-\frac{c^3x^3\sqrt{c^2x^2+1}}{16}-\frac{5cx\sqrt{c^2x^2+1}}{32}+\frac{5\operatorname{arcsinh}(cx)}{32}\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c^2*(d*a*(1/4*c^4*x^4+1/2*c^2*x^2)+d*b*(1/4*arcsinh(c*x)*c^4*x^4+1/2*arcs
inh(c*x)*c^2*x^2-1/16*c^3*x^3*(c^2*x^2+1)^(1/2)-5/32*c*x*(c^2*x^2+1)^(1/2)+
5/32*arcsinh(c*x)))
```

maxima [A] time = 0.53, size = 127, normalized size = 1.46

$$\frac{1}{4}\,ac^2dx^4+\frac{1}{32}\left(8\,x^4\,\mathrm{arsinh}\,(cx)-\left(\frac{2\,\sqrt{c^2x^2+1}\,x^3}{c^2}-\frac{3\,\sqrt{c^2x^2+1}\,x}{c^4}+\frac{3\,\mathrm{arsinh}\,(cx)}{c^5}\right)c\right)bc^2d+\frac{1}{2}\,adx^2+\frac{1}{4}\left(2\,x^2\,\mathrm{ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*c^2*d*x^4 + 1/32*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^2 + 1)*x^3/c^2 -
 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*b*c^2*d + 1/2*a*d*x^2 +
 1/4*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsinh(c*x)/c^3))*
b*d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\left(d\,c^2\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2),x)
```

```
[Out] int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2), x)
```

sympy [A] time = 1.16, size = 117, normalized size = 1.34

$$\begin{cases} \frac{ac^2dx^4}{4}+\frac{adx^2}{2}+\frac{bc^2dx^4\,\mathrm{asinh}\,(cx)}{4}-\frac{bcdx^3\sqrt{c^2x^2+1}}{16}+\frac{bdx^2\,\mathrm{asinh}\,(cx)}{2}-\frac{5bdx\sqrt{c^2x^2+1}}{32c}+\frac{5bd\,\mathrm{asinh}\,(cx)}{32c^2} & \text{for}\ c\neq 0 \\ \frac{adx^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*d*x**2+d)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**2*d*x**4/4 + a*d*x**2/2 + b*c**2*d*x**4*asinh(c*x)/4 - b*c*
d*x**3*sqrt(c**2*x**2 + 1)/16 + b*d*x**2*asinh(c*x)/2 - 5*b*d*x*sqrt(c**2*x
**2 + 1)/(32*c) + 5*b*d*asinh(c*x)/(32*c**2), Ne(c, 0)), (a*d*x**2/2, True)
)
```

3.5 $\int \left(d+c^2 d x^2\right) \left(a+b \sinh^{-1}(c x)\right) \, dx$

Optimal. Leaf size=75

$$\frac{1}{3}c^2 d x^3 \left(a+b \sinh^{-1}(c x)\right)+d x \left(a+b \sinh^{-1}(c x)\right)-\frac{b d \left(c^2 x^2+1\right)^{3/2}}{9 c}-\frac{2 b d \sqrt{c^2 x^2+1}}{3 c}$$

```
[Out] -1/9*b*d*(c^2*x^2+1)^(3/2)/c+d*x*(a+b*arcsinh(c*x))+1/3*c^2*d*x^3*(a+b*arcs
inh(c*x))-2/3*b*d*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {5679, 12, 444, 43}

$$\frac{1}{3}c^2 d x^3 \left(a+b \sinh^{-1}(c x)\right)+d x \left(a+b \sinh^{-1}(c x)\right)-\frac{b d \left(c^2 x^2+1\right)^{3/2}}{9 c}-\frac{2 b d \sqrt{c^2 x^2+1}}{3 c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-2*b*d*Sqrt[1 + c^2*x^2])/(3*c) - (b*d*(1 + c^2*x^2)^(3/2))/(9*c) + d*x*(a
 + b*ArcSinh[c*x]) + (c^2*d*x^3*(a + b*ArcSinh[c*x]))/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 5679

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)\,dx = dx\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^2dx^3\left(a+b\sinh^{-1}(cx)\right)-(bc)\int \frac{dx\left(3+c^2\right.}{3\sqrt{1+c^2}}$$

$$= dx\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^2dx^3\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{3}(bcd)\int \frac{x\left(3+c\right.}{\sqrt{1+c}}$$

$$= dx\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^2dx^3\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{6}(bcd)\,\text{Subst}\Bigg(\int$$

$$= dx\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^2dx^3\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{6}(bcd)\,\text{Subst}\Bigg(\int$$

$$= -\frac{2bd\sqrt{1+c^2x^2}}{3c}-\frac{bd\left(1+c^2x^2\right)^{3/2}}{9c}+dx\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^2dx^3\Big($$

Mathematica [A] time = 0.04, size = 86, normalized size = 1.15

$$\frac{1}{3}ac^2dx^3+adx+\frac{1}{3}bc^2dx^3\sinh^{-1}(cx)-\frac{1}{9}bcdx^2\sqrt{c^2x^2+1}-\frac{7bd\sqrt{c^2x^2+1}}{9c}+bdx\sinh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] a*d*x + (a*c^2*d*x^3)/3 - (7*b*d*Sqrt[1 + c^2*x^2])/(9*c) - (b*c*d*x^2*Sqrt
[1 + c^2*x^2])/9 + b*d*x*ArcSinh[c*x] + (b*c^2*d*x^3*ArcSinh[c*x])/3
```

fricas [A] time = 0.69, size = 83, normalized size = 1.11

$$\frac{3\,ac^3dx^3+9\,acdx+3\left(bc^3dx^3+3\,bcdx\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-\left(bc^2dx^2+7\,bd\right)\sqrt{c^2x^2+1}}{9\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/9*(3*a*c^3*d*x^3 + 9*a*c*d*x + 3*(b*c^3*d*x^3 + 3*b*c*d*x)*log(c*x + sqrt
(c^2*x^2 + 1)) - (b*c^2*d*x^2 + 7*b*d)*sqrt(c^2*x^2 + 1))/c
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 76, normalized size = 1.01

$$\frac{da\left(\frac{1}{3}c^3x^3+cx\right)+db\left(\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}+\operatorname{arcsinh}(cx)\,cx-\frac{c^2x^2\sqrt{c^2x^2+1}}{9}-\frac{7\sqrt{c^2x^2+1}}{9}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c*(d*a*(1/3*c^3*x^3+c*x)+d*b*(1/3*arcsinh(c*x)*c^3*x^3+arcsinh(c*x)*c*x-1
/9*c^2*x^2*(c^2*x^2+1)^(1/2)-7/9*(c^2*x^2+1)^(1/2)))
```

maxima [A] time = 0.48, size = 97, normalized size = 1.29

$$\frac{1}{3}ac^2dx^3+\frac{1}{9}\left(3x^3\operatorname{arsinh}(cx)-c\left(\frac{\sqrt{c^2x^2+1}\,x^2}{c^2}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)bc^2d+adx+\frac{\left(cx\operatorname{arsinh}(cx)-\sqrt{c^2x^2+1}\right)bd}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*c^2*d*x^3 + 1/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 -
2*sqrt(c^2*x^2 + 1)/c^4))*b*c^2*d + a*d*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^
2 + 1))*b*d/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\operatorname{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + c^2*d*x^2),x)
```

```
[Out] int((a + b*asinh(c*x))*(d + c^2*d*x^2), x)
```

sympy [A] time = 0.52, size = 90, normalized size = 1.20

$$\begin{cases} \frac{ac^2dx^3}{3}+adx+\frac{bc^2dx^3\operatorname{asinh}(cx)}{3}-\frac{bcdx^2\sqrt{c^2x^2+1}}{9}+bdx\operatorname{asinh}(cx)-\frac{7bd\sqrt{c^2x^2+1}}{9c} & \text{for } c\neq 0 \\ adx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**2*d*x**3/3 + a*d*x + b*c**2*d*x**3*asinh(c*x)/3 - b*c*d*x**
2*sqrt(c**2*x**2 + 1)/9 + b*d*x*asinh(c*x) - 7*b*d*sqrt(c**2*x**2 + 1)/(9*c
), Ne(c, 0)), (a*d*x, True))
```

3.6 $\int \frac{(d+c^2dx^2)(a+b\sinh^{-1}(cx))}{x} dx$

Optimal. Leaf size=111

$$\frac{1}{2}d\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)+\frac{d\left(a+b\sinh^{-1}(cx)\right)^2}{2b}+d\log\left(1-e^{-2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{4}bcdx\sqrt{c^2}$$

```
[Out] -1/4*b*d*arcsinh(c*x)+1/2*d*(c^2*x^2+1)*(a+b*arcsinh(c*x))+1/2*d*(a+b*arcsi
nh(c*x))^2/b+d*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^2)-1/2*b*d
*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)-1/4*b*c*d*x*(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5726, 5659, 3716, 2190, 2279, 2391, 195, 215}

$$\frac{1}{2}bd\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)+\frac{1}{2}d\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{d\left(a+b\sinh^{-1}(cx)\right)^2}{2b}+d\log\left(1-e^{2\sinh^{-1}(cx)}\right)($$

Warning: Unable to verify antiderivative.

```
[In] Int[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] -(b*c*d*x*Sqrt[1 + c^2*x^2])/4 - (b*d*ArcSinh[c*x])/4 + (d*(1 + c^2*x^2)*(a
 + b*ArcSinh[c*x]))/2 - (d*(a + b*ArcSinh[c*x])^2)/(2*b) + d*(a + b*ArcSinh
[c*x])*Log[1 - E^(2*ArcSinh[c*x])] + (b*d*PolyLog[2, E^(2*ArcSinh[c*x])])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5726

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[((d + e*x^2)^p*(a + b*ArcSinh[c*x]))/(2*p), x] + (Dist[d
, Int[((d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]))/x, x], x] - Dist[(b*c*d^p)
/(2*p), Int[(1 + c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)}{x}\,dx = \frac{1}{2}d\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)+d\int \frac{a+b\sinh^{-1}(cx)}{x}\,dx-\frac{1}{2}(bcd)\int \sqrt{}$$

$$= -\frac{1}{4}bcdx\sqrt{1+c^2x^2}+\frac{1}{2}d\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)+d\,\text{Subst}\left(\int (a+b$$

$$= -\frac{1}{4}bcdx\sqrt{1+c^2x^2}-\frac{1}{4}bd\sinh^{-1}(cx)+\frac{1}{2}d\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)-$$

$$= -\frac{1}{4}bcdx\sqrt{1+c^2x^2}-\frac{1}{4}bd\sinh^{-1}(cx)+\frac{1}{2}d\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)-$$

$$= -\frac{1}{4}bcdx\sqrt{1+c^2x^2}-\frac{1}{4}bd\sinh^{-1}(cx)+\frac{1}{2}d\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)-$$

$$= -\frac{1}{4}bcdx\sqrt{1+c^2x^2}-\frac{1}{4}bd\sinh^{-1}(cx)+\frac{1}{2}d\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)-$$

Mathematica [A] time = 0.06, size = 113, normalized size = 1.02

$$\frac{1}{2}ac^2dx^2+ad\log(x)-\frac{1}{4}bcdx\sqrt{c^2x^2+1}+\frac{1}{2}bc^2dx^2\sinh^{-1}(cx)+\frac{1}{2}bd\text{Li}_2\left(e^{2\sinh^{-1}(cx)}\right)-\frac{1}{2}bd\sinh^{-1}(cx)^2+\frac{1}{4}bd\sinh^{-1}($$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (a*c^2*d*x^2)/2 - (b*c*d*x*Sqrt[1 + c^2*x^2])/4 + (b*d*ArcSinh[c*x])/4 + (b
*c^2*d*x^2*ArcSinh[c*x])/2 - (b*d*ArcSinh[c*x]^2)/2 + b*d*ArcSinh[c*x]*Log[
1 - E^(2*ArcSinh[c*x])] + a*d*Log[x] + (b*d*PolyLog[2, E^(2*ArcSinh[c*x])])
/2
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ac^2dx^2+ad+\left(bc^2dx^2+bd\right)\text{arsinh}\left(cx\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))/x, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.27, size = 162, normalized size = 1.46

$$\frac{d a\,c^2x^2}{2}+da\ln\left(cx\right)-\frac{db\,\mathrm{arcsinh}\left(cx\right)^2}{2}+\frac{db\,\mathrm{arcsinh}\left(cx\right)c^2x^2}{2}-\frac{bcdx\sqrt{c^2x^2+1}}{4}+\frac{bd\,\mathrm{arcsinh}\left(cx\right)}{4}+db\,\mathrm{arcsinh}\left(cx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x,x)
```

```
[Out] 1/2*d*a*c^2*x^2+d*a*ln(c*x)-1/2*d*b*arcsinh(c*x)^2+1/2*d*b*arcsinh(c*x)*c^2
*x^2-1/4*b*c*d*x*(c^2*x^2+1)^(1/2)+1/4*b*d*arcsinh(c*x)+d*b*arcsinh(c*x)*ln
(1+c*x+(c^2*x^2+1)^(1/2))+d*b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+d*b*arcsinh
(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+d*b*polylog(2,c*x+(c^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,ac^2dx^2+ad\log(x)+\int bc^2dx\log\left(cx+\sqrt{c^2x^2+1}\right)+\frac{bd\log\left(cx+\sqrt{c^2x^2+1}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/2*a*c^2*d*x^2 + a*d*log(x) + integrate(b*c^2*d*x*log(c*x + sqrt(c^2*x^2 +
 1)) + b*d*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\left(d\,c^2\,x^2+d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2))/x,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int\frac{a}{x}\,dx+\int ac^2x\,dx+\int\frac{b\,\mathrm{asinh}\left(cx\right)}{x}\,dx+\int bc^2x\,\mathrm{asinh}\left(cx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))/x,x)
```

```
[Out] d*(Integral(a/x, x) + Integral(a*c**2*x, x) + Integral(b*asinh(c*x)/x, x) +
 Integral(b*c**2*x*asinh(c*x), x))
```

3.7 $\int \frac{(d+c^2dx^2)(a+b\sinh^{-1}(cx))}{x^2}\,dx$

Optimal. Leaf size=66

$$c^2dx\left(a+b\sinh^{-1}(cx)\right)-\frac{d\left(a+b\sinh^{-1}(cx)\right)}{x}-bcd\sqrt{c^2x^2+1}-bcd\tanh^{-1}\left(\sqrt{c^2x^2+1}\right)$$

```
[Out] -d*(a+b*arcsinh(c*x))/x+c^2*d*x*(a+b*arcsinh(c*x))-b*c*d*arctanh((c^2*x^2+1
)^(1/2))-b*c*d*(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {14, 5730, 12, 446, 80, 63, 208}

$$c^2dx\left(a+b\sinh^{-1}(cx)\right)-\frac{d\left(a+b\sinh^{-1}(cx)\right)}{x}-bcd\sqrt{c^2x^2+1}-bcd\tanh^{-1}\left(\sqrt{c^2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] -(b*c*d*Sqrt[1 + c^2*x^2]) - (d*(a + b*ArcSinh[c*x]))/x + c^2*d*x*(a + b*Ar
cSinh[c*x]) - b*c*d*ArcTanh[Sqrt[1 + c^2*x^2]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)}{x^2}\,dx = -\frac{d\left(a+b\sinh^{-1}(cx)\right)}{x}+c^2dx\left(a+b\sinh^{-1}(cx)\right)-(bc)\int\frac{d\left(-1+c^2x^2\right.}{x\sqrt{1+c^2x^2}}$$

$$= -\frac{d\left(a+b\sinh^{-1}(cx)\right)}{x}+c^2dx\left(a+b\sinh^{-1}(cx)\right)-(bcd)\int\frac{-1+c^2x^2}{x\sqrt{1+c^2x^2}}$$

$$= -\frac{d\left(a+b\sinh^{-1}(cx)\right)}{x}+c^2dx\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{2}(bcd)\,\text{Subst}\left(\int \frac{-}{x}\right.$$

$$= -bcd\sqrt{1+c^2x^2}-\frac{d\left(a+b\sinh^{-1}(cx)\right)}{x}+c^2dx\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{2}(b$$

$$(bc$$

$$= -bcd\sqrt{1+c^2x^2}-\frac{d\left(a+b\sinh^{-1}(cx)\right)}{x}+c^2dx\left(a+b\sinh^{-1}(cx)\right)+ —$$

$$= -bcd\sqrt{1+c^2x^2}-\frac{d\left(a+b\sinh^{-1}(cx)\right)}{x}+c^2dx\left(a+b\sinh^{-1}(cx)\right)-bcd$$

Mathematica [A] time = 0.02, size = 74, normalized size = 1.12

$$ac^2dx-\frac{ad}{x}-bcd\sqrt{c^2x^2+1}-bcd\tanh^{-1}\left(\sqrt{c^2x^2+1}\right)+bc^2dx\sinh^{-1}(cx)-\frac{bd\sinh^{-1}(cx)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] -((a*d)/x) + a*c^2*d*x - b*c*d*Sqrt[1 + c^2*x^2] - (b*d*ArcSinh[c*x])/x + b
*c^2*d*x*ArcSinh[c*x] - b*c*d*ArcTanh[Sqrt[1 + c^2*x^2]]
```

fricas [B] time = 0.59, size = 156, normalized size = 2.36

$$\frac{ac^2dx^2-bcdx\log\left(-cx+\sqrt{c^2x^2+1}+1\right)+bcdx\log\left(-cx+\sqrt{c^2x^2+1}-1\right)-\sqrt{c^2x^2+1}\,bcdx-\left(bc^2-b\right)dx\,\text{lo}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] (a*c^2*d*x^2 - b*c*d*x*log(-c*x + sqrt(c^2*x^2 + 1) + 1) + b*c*d*x*log(-c*x
 + sqrt(c^2*x^2 + 1) - 1) - sqrt(c^2*x^2 + 1)*b*c*d*x - (b*c^2 - b)*d*x*log
(-c*x + sqrt(c^2*x^2 + 1)) - a*d + (b*c^2*d*x^2 - (b*c^2 - b)*d*x - b*d)*lo
g(c*x + sqrt(c^2*x^2 + 1)))/x
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 69, normalized size = 1.05

$$c\left(da\left(cx-\frac{1}{cx}\right)+db\left(\operatorname{arcsinh}(cx)\,cx-\frac{\operatorname{arcsinh}(cx)}{cx}-\sqrt{c^2x^2+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^2,x)
```

```
[Out] c*(d*a*(c*x-1/c/x)+d*b*(arcsinh(c*x)*c*x-arcsinh(c*x)/c/x-(c^2*x^2+1)^(1/2)
-arctanh(1/(c^2*x^2+1)^(1/2))))
```

maxima [A] time = 0.51, size = 64, normalized size = 0.97

$$ac^2dx+\left(cx\operatorname{arsinh}(cx)-\sqrt{c^2x^2+1}\right)bcd-\left(c\operatorname{arsinh}\left(\frac{1}{c|x|}\right)+\frac{\operatorname{arsinh}(cx)}{x}\right)bd-\frac{ad}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] a*c^2*d*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*b*c*d - (c*arcsinh(1/(c*
abs(x))) + arcsinh(c*x)/x)*b*d - a*d/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{(a+b\operatorname{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2))/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int ac^2\,dx+\int\frac{a}{x^2}\,dx+\int bc^2\operatorname{asinh}(cx)\,dx+\int\frac{b\operatorname{asinh}(cx)}{x^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))/x**2,x)
```

```
[Out] d*(Integral(a*c**2, x) + Integral(a/x**2, x) + Integral(b*c**2*asinh(c*x),
x) + Integral(b*asinh(c*x)/x**2, x))
```

3.8 $\int \frac{\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=128

$$-\frac{d\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{2x^2}+\frac{c^2d\left(a+b\sinh^{-1}(cx)\right)^2}{2b}+c^2d\log\left(1-e^{-2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{2}bc^2d$$

```
[Out] 1/2*b*c^2*d*arcsinh(c*x)-1/2*d*(c^2*x^2+1)*(a+b*arcsinh(c*x))/x^2+1/2*c^2*d
*(a+b*arcsinh(c*x))^2/b+c^2*d*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(c^2*x^2+1)^(1
/2))^2)-1/2*b*c^2*d*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)-1/2*b*c*d*(c^2*x
^2+1)^(1/2)/x
```

Rubi [A] time = 0.13, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5728, 277, 215, 5659, 3716, 2190, 2279, 2391}

$$\frac{1}{2}bc^2d\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)-\frac{d\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{2x^2}-\frac{c^2d\left(a+b\sinh^{-1}(cx)\right)^2}{2b}+c^2d\log\left(1-e^{2\sinh^{-1}(}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] -(b*c*d*Sqrt[1 + c^2*x^2])/(2*x) + (b*c^2*d*ArcSinh[c*x])/2 - (d*(1 + c^2*x
^2)*(a + b*ArcSinh[c*x]))/(2*x^2) - (c^2*d*(a + b*ArcSinh[c*x])^2)/(2*b) +
c^2*d*(a + b*ArcSinh[c*x])*Log[1 - E^(2*ArcSinh[c*x])] + (b*c^2*d*PolyLog[2
, E^(2*ArcSinh[c*x])])/2
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5728

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*ArcSinh[c
*x]))/(f*(m + 1)), x] + (-Dist[(b*c*d^p)/(f*(m + 1)), Int[(f*x)^(m + 1)*(1
+ c^2*x^2)^(p - 1/2), x], x] - Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2
)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && ILtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)}{x^3}\,dx = -\frac{d\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{2x^2}+\frac{1}{2}(bcd)\int\frac{\sqrt{1+c^2x^2}}{x^2}\,dx+\left(c^2d\right)\int\frac{a\ }{}$$

$$= -\frac{bcd\sqrt{1+c^2x^2}}{2x}-\frac{d\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{2x^2}+\left(c^2d\right)\text{Subst}\left(\int(a+b\right.$$

$$= -\frac{bcd\sqrt{1+c^2x^2}}{2x}+\frac{1}{2}bc^2d\sinh^{-1}(cx)-\frac{d\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{2x^2}-$$

$$= -\frac{bcd\sqrt{1+c^2x^2}}{2x}+\frac{1}{2}bc^2d\sinh^{-1}(cx)-\frac{d\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{2x^2}-$$

$$= -\frac{bcd\sqrt{1+c^2x^2}}{2x}+\frac{1}{2}bc^2d\sinh^{-1}(cx)-\frac{d\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{2x^2}-$$

$$= -\frac{bcd\sqrt{1+c^2x^2}}{2x}+\frac{1}{2}bc^2d\sinh^{-1}(cx)-\frac{d\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{2x^2}-$$

Mathematica [A] time = 0.05, size = 111, normalized size = 0.87

$$ac^2d\log(x)-\frac{ad}{2x^2}+\frac{1}{2}bc^2d\text{Li}_2\left(e^{2\sinh^{-1}(cx)}\right)-\frac{bcd\sqrt{c^2x^2+1}}{2x}-\frac{1}{2}bc^2d\sinh^{-1}(cx)^2+bc^2d\sinh^{-1}(cx)\log\left(1-e^{2\sinh^{-1}(cx)}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] -1/2*(a*d)/x^2 - (b*c*d*Sqrt[1 + c^2*x^2])/(2*x) - (b*d*ArcSinh[c*x])/(2*x^
2) - (b*c^2*d*ArcSinh[c*x]^2)/2 + b*c^2*d*ArcSinh[c*x]*Log[1 - E^(2*ArcSinh
[c*x])] + a*c^2*d*Log[x] + (b*c^2*d*PolyLog[2, E^(2*ArcSinh[c*x])])/2
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ac^2dx^2+ad+\left(bc^2dx^2+bd\right)\text{arsinh}\left(cx\right)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))/x^3, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.24, size = 175, normalized size = 1.37

$$c^2 da \ln(cx) - \frac{da}{2x^2} - \frac{c^2 db \operatorname{arcsinh}(cx)^2}{2} - \frac{bcd\sqrt{c^2x^2+1}}{2x} + \frac{c^2 db}{2} - \frac{db \operatorname{arcsinh}(cx)}{2x^2} + c^2 db \operatorname{arcsinh}(cx) \ln\left(1 + cx + \sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^3,x)
```

```
[Out] c^2*d*a*ln(c*x)-1/2*d*a/x^2-1/2*c^2*d*b*arcsinh(c*x)^2-1/2*b*c*d*(c^2*x^2+1
)^(1/2)/x+1/2*c^2*d*b-1/2*d*b*arcsinh(c*x)/x^2+c^2*d*b*arcsinh(c*x)*ln(1+c*
x+(c^2*x^2+1)^(1/2))+c^2*d*b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+c^2*d*b*arcs
inh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+c^2*d*b*polylog(2,c*x+(c^2*x^2+1)^(1/2
))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$bc^2 d \int \frac{\log\left(cx + \sqrt{c^2x^2+1}\right)}{x}\,dx + ac^2 d\log(x) - \frac{1}{2}\,bd\left(\frac{\sqrt{c^2x^2+1}\,c}{x} + \frac{\operatorname{arsinh}(cx)}{x^2}\right) - \frac{ad}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] b*c^2*d*integrate(log(c*x + sqrt(c^2*x^2 + 1))/x, x) + a*c^2*d*log(x) - 1/2
*b*d*(sqrt(c^2*x^2 + 1)*c/x + arcsinh(c*x)/x^2) - 1/2*a*d/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\operatorname{asinh}(c\,x))\left(d\,c^2\,x^2 + d\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2))/x^3,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int \frac{a}{x^3}\,dx + \int \frac{ac^2}{x}\,dx + \int \frac{b\operatorname{asinh}(cx)}{x^3}\,dx + \int \frac{bc^2\operatorname{asinh}(cx)}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))/x**3,x)
```

```
[Out] d*(Integral(a/x**3, x) + Integral(a*c**2/x, x) + Integral(b*asinh(c*x)/x**3
, x) + Integral(b*c**2*asinh(c*x)/x, x))
```

3.9 $\int \frac{(d+c^2dx^2)(a+b\sinh^{-1}(cx))}{x^4}\,dx$

Optimal. Leaf size=80

$$-\frac{c^2d\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{d\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{bcd\sqrt{c^2x^2+1}}{6x^2}-\frac{5}{6}bc^3d\tanh^{-1}\left(\sqrt{c^2x^2+1}\right)$$

```
[Out] -1/3*d*(a+b*arcsinh(c*x))/x^3-c^2*d*(a+b*arcsinh(c*x))/x-5/6*b*c^3*d*arctan
h((c^2*x^2+1)^(1/2))-1/6*b*c*d*(c^2*x^2+1)^(1/2)/x^2
```

Rubi [A] time = 0.08, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {14, 5730, 12, 446, 78, 63, 208}

$$-\frac{c^2d\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{d\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{bcd\sqrt{c^2x^2+1}}{6x^2}-\frac{5}{6}bc^3d\tanh^{-1}\left(\sqrt{c^2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] -(b*c*d*Sqrt[1 + c^2*x^2])/(6*x^2) - (d*(a + b*ArcSinh[c*x]))/(3*x^3) - (c^
2*d*(a + b*ArcSinh[c*x]))/x - (5*b*c^3*d*ArcTanh[Sqrt[1 + c^2*x^2]])/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)}{x^4}\,dx &= -\frac{d\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{c^2d\left(a+b\sinh^{-1}(cx)\right)}{x}-(bc)\int \frac{d\left(-1-3c^2x^2\right.}{3x^3\sqrt{1+c^2x^2}}\\ &= -\frac{d\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{c^2d\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{1}{3}(bcd)\int \frac{-1-3c^2x}{x^3\sqrt{1+c^2}}\\ &= -\frac{d\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{c^2d\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{1}{6}(bcd)\,\mathrm{Subst}\left(\int \frac{-}{x^2}\right.\\ &= -\frac{bcd\sqrt{1+c^2x^2}}{6x^2}-\frac{d\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{c^2d\left(a+b\sinh^{-1}(cx)\right)}{x}+\frac{1}{12}\\ &= -\frac{bcd\sqrt{1+c^2x^2}}{6x^2}-\frac{d\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{c^2d\left(a+b\sinh^{-1}(cx)\right)}{x}+\frac{1}{6}(5\\ &= -\frac{bcd\sqrt{1+c^2x^2}}{6x^2}-\frac{d\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{c^2d\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{5}{6}bc\end{aligned}$$

Mathematica [A] time = 0.03, size = 93, normalized size = 1.16

$$-\frac{ac^2d}{x}-\frac{ad}{3x^3}-\frac{bcd\sqrt{c^2x^2+1}}{6x^2}-\frac{bc^2d\sinh^{-1}(cx)}{x}-\frac{5}{6}bc^3d\tanh^{-1}\left(\sqrt{c^2x^2+1}\right)-\frac{bd\sinh^{-1}(cx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] -1/3*(a*d)/x^3 - (a*c^2*d)/x - (b*c*d*Sqrt[1 + c^2*x^2])/(6*x^2) - (b*d*Arc
Sinh[c*x])/(3*x^3) - (b*c^2*d*ArcSinh[c*x])/x - (5*b*c^3*d*ArcTanh[Sqrt[1 +
 c^2*x^2]])/6
```

fricas [B] time = 0.58, size = 169, normalized size = 2.11

$$-\frac{5\,bc^3dx^3\log\left(-cx+\sqrt{c^2x^2+1}+1\right)-5\,bc^3dx^3\log\left(-cx+\sqrt{c^2x^2+1}-1\right)+6\,ac^2dx^2-2\left(3\,bc^2+b\right)dx^3\log}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(5*b*c^3*d*x^3*log(-c*x + sqrt(c^2*x^2 + 1) + 1) - 5*b*c^3*d*x^3*log(-
c*x + sqrt(c^2*x^2 + 1) - 1) + 6*a*c^2*d*x^2 - 2*(3*b*c^2 + b)*d*x^3*log(-c
```

```
*x + sqrt(c^2*x^2 + 1)) + sqrt(c^2*x^2 + 1)*b*c*d*x + 2*a*d + 2*(3*b*c^2*d*
x^2 - (3*b*c^2 + b)*d*x^3 + b*d)*log(c*x + sqrt(c^2*x^2 + 1)))/x^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 87, normalized size = 1.09

$$c^{3}\left(da\left(-\frac{1}{cx}-\frac{1}{3c^{3}x^{3}}\right)+db\left(-\frac{\operatorname{arcsinh}(cx)}{cx}-\frac{\operatorname{arcsinh}(cx)}{3c^{3}x^{3}}-\frac{5\operatorname{arctanh}\left(\frac{1}{\sqrt{c^{2}x^{2}+1}}\right)}{6}-\frac{\sqrt{c^{2}x^{2}+1}}{6c^{2}x^{2}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^4,x)
```

```
[Out] c^3*(d*a*(-1/c/x-1/3/c^3/x^3)+d*b*(-arcsinh(c*x)/c/x-1/3*arcsinh(c*x)/c^3/x
^3-5/6*arctanh(1/(c^2*x^2+1)^(1/2))-1/6/c^2/x^2*(c^2*x^2+1)^(1/2)))
```

maxima [A] time = 0.34, size = 91, normalized size = 1.14

$$-\left(c\operatorname{arsinh}\left(\frac{1}{c|x|}\right)+\frac{\operatorname{arsinh}(cx)}{x}\right)bc^{2}d+\frac{1}{6}\left(\left(c^{2}\operatorname{arsinh}\left(\frac{1}{c|x|}\right)-\frac{\sqrt{c^{2}x^{2}+1}}{x^{2}}\right)c-\frac{2\operatorname{arsinh}(cx)}{x^{3}}\right)bd-\frac{ac^{2}d}{x}-\frac{ad}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] -(c*arcsinh(1/(c*abs(x))) + arcsinh(c*x)/x)*b*c^2*d + 1/6*((c^2*arcsinh(1/(
c*abs(x))) - sqrt(c^2*x^2 + 1)/x^2)*c - 2*arcsinh(c*x)/x^3)*b*d - a*c^2*d/x
 - 1/3*a*d/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\operatorname{asinh}(c\,x))\,\left(d\,c^{2}\,x^{2}+d\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2))/x^4,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int\frac{a}{x^{4}}\,dx+\int\frac{ac^{2}}{x^{2}}\,dx+\int\frac{b\operatorname{asinh}(cx)}{x^{4}}\,dx+\int\frac{bc^{2}\operatorname{asinh}(cx)}{x^{2}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))/x**4,x)
```

```
[Out] d*(Integral(a/x**4, x) + Integral(a*c**2/x**2, x) + Integral(b*asinh(c*x)/x
**4, x) + Integral(b*c**2*asinh(c*x)/x**2, x))
```

3.10 $\int x^4 \left(d+c^2 d x^2\right)^2 \left(a+b \sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=181

$$\frac{1}{9}c^4d^2x^9\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{7}c^2d^2x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}d^2x^5\left(a+b\sinh^{-1}(cx)\right)-\frac{bd^2\left(c^2x^2+1\right)^{9/2}}{81c^5}+\frac{10bd^2\left(\right.}{4}$$

```
[Out] -4/945*b*d^2*(c^2*x^2+1)^(3/2)/c^5-1/525*b*d^2*(c^2*x^2+1)^(5/2)/c^5+10/441
*b*d^2*(c^2*x^2+1)^(7/2)/c^5-1/81*b*d^2*(c^2*x^2+1)^(9/2)/c^5+1/5*d^2*x^5*(
a+b*arcsinh(c*x))+2/7*c^2*d^2*x^7*(a+b*arcsinh(c*x))+1/9*c^4*d^2*x^9*(a+b*a
rcsinh(c*x))-8/315*b*d^2*(c^2*x^2+1)^(1/2)/c^5
```

Rubi [A] time = 0.21, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {270, 5730, 12, 1251, 897, 1153}

$$\frac{1}{9}c^4d^2x^9\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{7}c^2d^2x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}d^2x^5\left(a+b\sinh^{-1}(cx)\right)-\frac{bd^2\left(c^2x^2+1\right)^{9/2}}{81c^5}+\frac{10bd^2\left(\right.}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-8*b*d^2*Sqrt[1 + c^2*x^2])/(315*c^5) - (4*b*d^2*(1 + c^2*x^2)^(3/2))/(945
*c^5) - (b*d^2*(1 + c^2*x^2)^(5/2))/(525*c^5) + (10*b*d^2*(1 + c^2*x^2)^(7/
2))/(441*c^5) - (b*d^2*(1 + c^2*x^2)^(9/2))/(81*c^5) + (d^2*x^5*(a + b*ArcS
inh[c*x]))/5 + (2*c^2*d^2*x^7*(a + b*ArcSinh[c*x]))/7 + (c^4*d^2*x^9*(a + b
*ArcSinh[c*x]))/9
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\int x^4\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)dx=\frac{1}{5}d^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{7}c^2d^2x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{9}c^4d^2x^9\left(a\right.$$

$$=\frac{1}{5}d^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{7}c^2d^2x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{9}c^4d^2x^9\left(a\right.$$

$$=\frac{1}{5}d^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{7}c^2d^2x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{9}c^4d^2x^9\left(a\right.$$

$$=\frac{1}{5}d^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{7}c^2d^2x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{9}c^4d^2x^9\left(a\right.$$

$$=\frac{1}{5}d^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{7}c^2d^2x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{9}c^4d^2x^9\left(a\right.$$

$$=-\frac{8bd^2\sqrt{1+c^2x^2}}{315c^5}-\frac{4bd^2\left(1+c^2x^2\right)^{3/2}}{945c^5}-\frac{bd^2\left(1+c^2x^2\right)^{5/2}}{525c^5}+\frac{10bd^2\left(1\right.}{44}$$

Mathematica [A] time = 0.09, size = 119, normalized size = 0.66

$$\frac{d^2\left(315ac^5x^5\left(35c^4x^4+90c^2x^2+63\right)+315bc^5x^5\left(35c^4x^4+90c^2x^2+63\right)\sinh^{-1}(cx)-b\sqrt{c^2x^2+1}\left(1225c^8x^8+2\right.}{99225c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*(315*a*c^5*x^5*(63 + 90*c^2*x^2 + 35*c^4*x^4) - b*Sqrt[1 + c^2*x^2]*(2
104 - 1052*c^2*x^2 + 789*c^4*x^4 + 2650*c^6*x^6 + 1225*c^8*x^8) + 315*b*c^5
*x^5*(63 + 90*c^2*x^2 + 35*c^4*x^4)*ArcSinh[c*x]))/(99225*c^5)
```

fricas [A] time = 0.72, size = 165, normalized size = 0.91

$$\frac{11025\,ac^9d^2x^9+28350\,ac^7d^2x^7+19845\,ac^5d^2x^5+315\left(35\,bc^9d^2x^9+90\,bc^7d^2x^7+63\,bc^5d^2x^5\right)\log\left(cx+\sqrt{c^2x^2+\right.}{99225\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/99225*(11025*a*c^9*d^2*x^9 + 28350*a*c^7*d^2*x^7 + 19845*a*c^5*d^2*x^5 +
315*(35*b*c^9*d^2*x^9 + 90*b*c^7*d^2*x^7 + 63*b*c^5*d^2*x^5)*log(c*x + sqrt
```

```
(c^2*x^2 + 1)) - (1225*b*c^8*d^2*x^8 + 2650*b*c^6*d^2*x^6 + 789*b*c^4*d^2*x
^4 - 1052*b*c^2*d^2*x^2 + 2104*b*d^2)*sqrt(c^2*x^2 + 1))/c^5
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 167, normalized size = 0.92

$$\frac{d^2 a\left(\frac{1}{9}c^9x^9+\frac{2}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+d^2 b\left(\frac{\mathrm{arcsinh}(cx)c^9x^9}{9}+\frac{2\,\mathrm{arcsinh}(cx)c^7x^7}{7}+\frac{\mathrm{arcsinh}(cx)c^5x^5}{5}-\frac{c^8x^8\sqrt{c^2x^2+1}}{81}-\frac{106c^6x^6\sqrt{c^2x^2+1}}{3969}\right.}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c^5*(d^2*a*(1/9*c^9*x^9+2/7*c^7*x^7+1/5*c^5*x^5)+d^2*b*(1/9*arcsinh(c*x)*
c^9*x^9+2/7*arcsinh(c*x)*c^7*x^7+1/5*arcsinh(c*x)*c^5*x^5-1/81*c^8*x^8*(c^2
*x^2+1)^(1/2)-106/3969*c^6*x^6*(c^2*x^2+1)^(1/2)-263/33075*c^4*x^4*(c^2*x^2
+1)^(1/2)+1052/99225*c^2*x^2*(c^2*x^2+1)^(1/2)-2104/99225*(c^2*x^2+1)^(1/2)
))
```

maxima [B] time = 0.57, size = 319, normalized size = 1.76

$$\frac{1}{9}ac^4d^2x^9+\frac{2}{7}ac^2d^2x^7+\frac{1}{2835}\left(315\,x^9\,\mathrm{arsinh}\,(cx)-\left(\frac{35\,\sqrt{c^2x^2+1}\,x^8}{c^2}-\frac{40\,\sqrt{c^2x^2+1}\,x^6}{c^4}+\frac{48\,\sqrt{c^2x^2+1}\,x^4}{c^6}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/9*a*c^4*d^2*x^9 + 2/7*a*c^2*d^2*x^7 + 1/2835*(315*x^9*arcsinh(c*x) - (35*
sqrt(c^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(c^2*x^2
+ 1)*x^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(c^2*x^2 + 1)/c^10)*c
)*b*c^4*d^2 + 1/5*a*d^2*x^5 + 2/245*(35*x^7*arcsinh(c*x) - (5*sqrt(c^2*x^2
+ 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 -
16*sqrt(c^2*x^2 + 1)/c^8)*c)*b*c^2*d^2 + 1/75*(15*x^5*arcsinh(c*x) - (3*sqr
t(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/
c^6)*c)*b*d^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(d\,c^2\,x^2+d\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*asinh(c*x))*(d + c^2*d*x^2)^2,x)
```

```
[Out] int(x^4*(a + b*asinh(c*x))*(d + c^2*d*x^2)^2, x)
```

sympy [A] time = 16.10, size = 230, normalized size = 1.27

$$\begin{cases} \frac{ac^4d^2x^9}{9}+\frac{2ac^2d^2x^7}{7}+\frac{ad^2x^5}{5}+\frac{bc^4d^2x^9\,\mathrm{asinh}\,(cx)}{9}-\frac{bc^3d^2x^8\sqrt{c^2x^2+1}}{81}+\frac{2bc^2d^2x^7\,\mathrm{asinh}\,(cx)}{7}-\frac{106bcd^2x^6\sqrt{c^2x^2+1}}{3969}+\frac{bd^2x^5\,\mathrm{asinh}\,(cx)}{5} \\ \frac{ad^2x^5}{5} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c**2*d*x**2+d)**2*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**2*x**9/9 + 2*a*c**2*d**2*x**7/7 + a*d**2*x**5/5 + b*c*
*4*d**2*x**9*asinh(c*x)/9 - b*c**3*d**2*x**8*sqrt(c**2*x**2 + 1)/81 + 2*b*c
**2*d**2*x**7*asinh(c*x)/7 - 106*b*c*d**2*x**6*sqrt(c**2*x**2 + 1)/3969 + b
*d**2*x**5*asinh(c*x)/5 - 263*b*d**2*x**4*sqrt(c**2*x**2 + 1)/(33075*c) + 1
052*b*d**2*x**2*sqrt(c**2*x**2 + 1)/(99225*c**3) - 2104*b*d**2*sqrt(c**2*x*
*2 + 1)/(99225*c**5), Ne(c, 0)), (a*d**2*x**5/5, True))
```

3.11 $\int x^3 \left(d + c^2 d x^2\right)^2 \left(a + b \sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=180

$$\frac{1}{8}c^4d^2x^8\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^2d^2x^6\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}d^2x^4\left(a+b\sinh^{-1}(cx)\right)-\frac{73bd^2\sinh^{-1}(cx)}{3072c^4}-\frac{43bcd^2}{}$$

```
[Out] -73/3072*b*d^2*arcsinh(c*x)/c^4+1/4*d^2*x^4*(a+b*arcsinh(c*x))+1/3*c^2*d^2*
x^6*(a+b*arcsinh(c*x))+1/8*c^4*d^2*x^8*(a+b*arcsinh(c*x))+73/3072*b*d^2*x*(
c^2*x^2+1)^(1/2)/c^3-73/4608*b*d^2*x^3*(c^2*x^2+1)^(1/2)/c-43/1152*b*c*d^2*
x^5*(c^2*x^2+1)^(1/2)-1/64*b*c^3*d^2*x^7*(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 43, 5730, 12, 1267, 459, 321, 215}

$$\frac{1}{8}c^4d^2x^8\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^2d^2x^6\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}d^2x^4\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{64}bc^3d^2x^7\sqrt{c^2x^2+1}-\frac{43b}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (73*b*d^2*x*Sqrt[1 + c^2*x^2])/(3072*c^3) - (73*b*d^2*x^3*Sqrt[1 + c^2*x^2]
)/(4608*c) - (43*b*c*d^2*x^5*Sqrt[1 + c^2*x^2])/1152 - (b*c^3*d^2*x^7*Sqrt[
1 + c^2*x^2])/64 - (73*b*d^2*ArcSinh[c*x])/(3072*c^4) + (d^2*x^4*(a + b*Arc
Sinh[c*x]))/4 + (c^2*d^2*x^6*(a + b*ArcSinh[c*x]))/3 + (c^4*d^2*x^8*(a + b*
ArcSinh[c*x]))/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1267

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\int x^3 \left(d+c^2dx^2\right)^2 \left(a+b\sinh^{-1}(cx)\right) dx = \frac{1}{4}d^2x^4\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^2d^2x^6\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{8}c^4d^2x^8\left(a\right.$$

$$= \frac{1}{4}d^2x^4\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^2d^2x^6\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{8}c^4d^2x^8\left(a\right.$$

$$= -\frac{1}{64}bc^3d^2x^7\sqrt{1+c^2x^2}+\frac{1}{4}d^2x^4\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^2d^2x^6\left(a+bs\right.$$

$$= -\frac{43bcd^2x^5\sqrt{1+c^2x^2}}{1152}-\frac{1}{64}bc^3d^2x^7\sqrt{1+c^2x^2}+\frac{1}{4}d^2x^4\left(a+b\sinh^{-1}(\right.$$

$$= -\frac{73bd^2x^3\sqrt{1+c^2x^2}}{4608c}-\frac{43bcd^2x^5\sqrt{1+c^2x^2}}{1152}-\frac{1}{64}bc^3d^2x^7\sqrt{1+c^2x^2}+$$

$$= \frac{73bd^2x\sqrt{1+c^2x^2}}{3072c^3}-\frac{73bd^2x^3\sqrt{1+c^2x^2}}{4608c}-\frac{43bcd^2x^5\sqrt{1+c^2x^2}}{1152}-\frac{1}{64}b$$

$$= \frac{73bd^2x\sqrt{1+c^2x^2}}{3072c^3}-\frac{73bd^2x^3\sqrt{1+c^2x^2}}{4608c}-\frac{43bcd^2x^5\sqrt{1+c^2x^2}}{1152}-\frac{1}{64}b$$

Mathematica [A] time = 0.08, size = 115, normalized size = 0.64

$$\frac{d^2\left(384ac^4x^4\left(3c^4x^4+8c^2x^2+6\right)+3b\left(384c^8x^8+1024c^6x^6+768c^4x^4-73\right)\sinh^{-1}(cx)-bcx\sqrt{c^2x^2+1}\left(144c^6x\right.\right.}{9216c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*(384*a*c^4*x^4*(6 + 8*c^2*x^2 + 3*c^4*x^4) - b*c*x*Sqrt[1 + c^2*x^2]*(
-219 + 146*c^2*x^2 + 344*c^4*x^4 + 144*c^6*x^6) + 3*b*(-73 + 768*c^4*x^4 +
1024*c^6*x^6 + 384*c^8*x^8)*ArcSinh[c*x]))/(9216*c^4)
```

fricas [A] time = 0.58, size = 161, normalized size = 0.89

$$\frac{1152\,ac^8d^2x^8 + 3072\,ac^6d^2x^6 + 2304\,ac^4d^2x^4 + 3\left(384\,bc^8d^2x^8 + 1024\,bc^6d^2x^6 + 768\,bc^4d^2x^4 - 73\,bd^2\right)\log\left(cx\right.}{9216\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/9216*(1152*a*c^8*d^2*x^8 + 3072*a*c^6*d^2*x^6 + 2304*a*c^4*d^2*x^4 + 3*(3
84*b*c^8*d^2*x^8 + 1024*b*c^6*d^2*x^6 + 768*b*c^4*d^2*x^4 - 73*b*d^2)*log(c
*x + sqrt(c^2*x^2 + 1)) - (144*b*c^7*d^2*x^7 + 344*b*c^5*d^2*x^5 + 146*b*c^
3*d^2*x^3 - 219*b*c*d^2*x)*sqrt(c^2*x^2 + 1))/c^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 156, normalized size = 0.87

$$\frac{d^2a\left(\frac{1}{8}c^8x^8 + \frac{1}{3}c^6x^6 + \frac{1}{4}c^4x^4\right) + d^2b\left(\frac{\operatorname{arcsinh}(cx)c^8x^8}{8} + \frac{\operatorname{arcsinh}(cx)c^6x^6}{3} + \frac{\operatorname{arcsinh}(cx)c^4x^4}{4} - \frac{c^7x^7\sqrt{c^2x^2+1}}{64} - \frac{43c^5x^5\sqrt{c^2x^2+1}}{1152}\right.}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c^4*(d^2*a*(1/8*c^8*x^8+1/3*c^6*x^6+1/4*c^4*x^4)+d^2*b*(1/8*arcsinh(c*x)*
c^8*x^8+1/3*arcsinh(c*x)*c^6*x^6+1/4*arcsinh(c*x)*c^4*x^4-1/64*c^7*x^7*(c^2
*x^2+1)^(1/2)-43/1152*c^5*x^5*(c^2*x^2+1)^(1/2)-73/4608*c^3*x^3*(c^2*x^2+1)
^(1/2)+73/3072*c*x*(c^2*x^2+1)^(1/2)-73/3072*arcsinh(c*x)))
```

maxima [A] time = 0.62, size = 292, normalized size = 1.62

$$\frac{1}{8}\,ac^4d^2x^8+\frac{1}{3}\,ac^2d^2x^6+\frac{1}{3072}\left(384\,x^8\operatorname{arsinh}\left(cx\right)-\left(\frac{48\,\sqrt{c^2x^2+1}\,x^7}{c^2}-\frac{56\,\sqrt{c^2x^2+1}\,x^5}{c^4}+\frac{70\,\sqrt{c^2x^2+1}\,x^3}{c^6}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*c^4*d^2*x^8 + 1/3*a*c^2*d^2*x^6 + 1/3072*(384*x^8*arcsinh(c*x) - (48*
sqrt(c^2*x^2 + 1)*x^7/c^2 - 56*sqrt(c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(c^2*x^2
+ 1)*x^3/c^6 - 105*sqrt(c^2*x^2 + 1)*x/c^8 + 105*arcsinh(c*x)/c^9)*c)*b*c^4
*d^2 + 1/4*a*d^2*x^4 + 1/144*(48*x^6*arcsinh(c*x) - (8*sqrt(c^2*x^2 + 1)*x^
5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 + 1)*x/c^6 - 15*arcs
inh(c*x)/c^7)*c)*b*c^2*d^2 + 1/32*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^2 + 1
)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*b*d^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(d\,c^2\,x^2+d\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^2,x)
```

```
[Out] int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^2, x)
```

sympy [A] time = 11.00, size = 218, normalized size = 1.21

$$\begin{cases} \frac{ac^4d^2x^8}{8} + \frac{ac^2d^2x^6}{3} + \frac{ad^2x^4}{4} + \frac{bc^4d^2x^8\operatorname{asinh}(cx)}{8} - \frac{bc^3d^2x^7\sqrt{c^2x^2+1}}{64} + \frac{bc^2d^2x^6\operatorname{asinh}(cx)}{3} - \frac{43bcd^2x^5\sqrt{c^2x^2+1}}{1152} + \frac{bd^2x^4\operatorname{asinh}(cx)}{4} - \frac{73b \\ \frac{ad^2x^4}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*d*x**2+d)**2*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**2*x**8/8 + a*c**2*d**2*x**6/3 + a*d**2*x**4/4 + b*c**4
*d**2*x**8*asinh(c*x)/8 - b*c**3*d**2*x**7*sqrt(c**2*x**2 + 1)/64 + b*c**2*
d**2*x**6*asinh(c*x)/3 - 43*b*c*d**2*x**5*sqrt(c**2*x**2 + 1)/1152 + b*d**2
*x**4*asinh(c*x)/4 - 73*b*d**2*x**3*sqrt(c**2*x**2 + 1)/(4608*c) + 73*b*d**
2*x*sqrt(c**2*x**2 + 1)/(3072*c**3) - 73*b*d**2*asinh(c*x)/(3072*c**4), Ne(
c, 0)), (a*d**2*x**4/4, True))
```

3.12 $\int x^2 \left(d+c^2 d x^2\right)^2 \left(a+b \sinh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=157

$$\frac{1}{7}c^4d^2x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{5}c^2d^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}d^2x^3\left(a+b\sinh^{-1}(cx)\right)-\frac{bd^2\left(c^2x^2+1\right)^{7/2}}{49c^3}+\frac{bd^2\left(c^2x\right.}{17}$$

```
[Out] 4/315*b*d^2*(c^2*x^2+1)^(3/2)/c^3+1/175*b*d^2*(c^2*x^2+1)^(5/2)/c^3-1/49*b*
d^2*(c^2*x^2+1)^(7/2)/c^3+1/3*d^2*x^3*(a+b*arcsinh(c*x))+2/5*c^2*d^2*x^5*(a
+b*arcsinh(c*x))+1/7*c^4*d^2*x^7*(a+b*arcsinh(c*x))+8/105*b*d^2*(c^2*x^2+1)
^(1/2)/c^3
```

Rubi [A] time = 0.17, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {270, 5730, 12, 1251, 771}

$$\frac{1}{7}c^4d^2x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{5}c^2d^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}d^2x^3\left(a+b\sinh^{-1}(cx)\right)-\frac{bd^2\left(c^2x^2+1\right)^{7/2}}{49c^3}+\frac{bd^2\left(c^2x\right.}{17}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (8*b*d^2*Sqrt[1 + c^2*x^2])/(105*c^3) + (4*b*d^2*(1 + c^2*x^2)^(3/2))/(315*
c^3) + (b*d^2*(1 + c^2*x^2)^(5/2))/(175*c^3) - (b*d^2*(1 + c^2*x^2)^(7/2))/
(49*c^3) + (d^2*x^3*(a + b*ArcSinh[c*x]))/3 + (2*c^2*d^2*x^5*(a + b*ArcSinh
[c*x]))/5 + (c^4*d^2*x^7*(a + b*ArcSinh[c*x]))/7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 771

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
```

```
GtQ[p, 0]
```

Rubi steps

$$\int x^2\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)dx=\frac{1}{3}d^2x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{5}c^2d^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{7}c^4d^2x^7\left(a\right.$$

$$=\frac{1}{3}d^2x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{5}c^2d^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{7}c^4d^2x^7\left(a\right.$$

$$=\frac{1}{3}d^2x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{5}c^2d^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{7}c^4d^2x^7\left(a\right.$$

$$=\frac{1}{3}d^2x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{5}c^2d^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{7}c^4d^2x^7\left(a\right.$$

$$=\frac{8bd^2\sqrt{1+c^2x^2}}{105c^3}+\frac{4bd^2\left(1+c^2x^2\right)^{3/2}}{315c^3}+\frac{bd^2\left(1+c^2x^2\right)^{5/2}}{175c^3}-\frac{bd^2\left(1+c^2\right.}{49c^3}$$

Mathematica [A] time = 0.08, size = 111, normalized size = 0.71

$$\frac{d^2\left(105ac^3x^3\left(15c^4x^4+42c^2x^2+35\right)-b\sqrt{c^2x^2+1}\left(225c^6x^6+612c^4x^4+409c^2x^2-818\right)+105bc^3x^3\left(15c^4x^4+4\right.\right.}{11025c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*(105*a*c^3*x^3*(35 + 42*c^2*x^2 + 15*c^4*x^4) - b*Sqrt[1 + c^2*x^2]*(-
818 + 409*c^2*x^2 + 612*c^4*x^4 + 225*c^6*x^6) + 105*b*c^3*x^3*(35 + 42*c^2
*x^2 + 15*c^4*x^4)*ArcSinh[c*x]))/(11025*c^3)
```

fricas [A] time = 0.65, size = 153, normalized size = 0.97

$$\frac{1575\,ac^7d^2x^7+4410\,ac^5d^2x^5+3675\,ac^3d^2x^3+105\left(15\,bc^7d^2x^7+42\,bc^5d^2x^5+35\,bc^3d^2x^3\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}{11025\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/11025*(1575*a*c^7*d^2*x^7 + 4410*a*c^5*d^2*x^5 + 3675*a*c^3*d^2*x^3 + 105
*(15*b*c^7*d^2*x^7 + 42*b*c^5*d^2*x^5 + 35*b*c^3*d^2*x^3)*log(c*x + sqrt(c^
2*x^2 + 1)) - (225*b*c^6*d^2*x^6 + 612*b*c^4*d^2*x^4 + 409*b*c^2*d^2*x^2 -
818*b*d^2)*sqrt(c^2*x^2 + 1))/c^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 148, normalized size = 0.94

$$\frac{d^2a\left(\frac{1}{7}c^7x^7+\frac{2}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+d^2b\left(\frac{\operatorname{arcsinh}(cx)c^7x^7}{7}+\frac{2\operatorname{arcsinh}(cx)c^5x^5}{5}+\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}-\frac{c^6x^6\sqrt{c^2x^2+1}}{49}-\frac{68c^4x^4\sqrt{c^2x^2+1}}{1225}\right.}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c^3*(d^2*a*(1/7*c^7*x^7+2/5*c^5*x^5+1/3*c^3*x^3)+d^2*b*(1/7*arcsinh(c*x)*
c^7*x^7+2/5*arcsinh(c*x)*c^5*x^5+1/3*arcsinh(c*x)*c^3*x^3-1/49*c^6*x^6*(c^2
*x^2+1)^(1/2)-68/1225*c^4*x^4*(c^2*x^2+1)^(1/2)-409/11025*c^2*x^2*(c^2*x^2+
1)^(1/2)+818/11025*(c^2*x^2+1)^(1/2)))
```

maxima [A] time = 0.55, size = 261, normalized size = 1.66

$$\frac{1}{7}\,ac^4d^2x^7+\frac{2}{5}\,ac^2d^2x^5+\frac{1}{245}\left(35\,x^7\,\operatorname{arsinh}\left(cx\right)-\left(\frac{5\,\sqrt{c^2x^2+1}\,x^6}{c^2}-\frac{6\,\sqrt{c^2x^2+1}\,x^4}{c^4}+\frac{8\,\sqrt{c^2x^2+1}\,x^2}{c^6}-\frac{16\,\sqrt{c^2x^2+1}}{c^8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*c^4*d^2*x^7 + 2/5*a*c^2*d^2*x^5 + 1/245*(35*x^7*arcsinh(c*x) - (5*sqr
t(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*
x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*b*c^4*d^2 + 2/75*(15*x^5*arcsinh(c*x
) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2
*x^2 + 1)/c^6)*c)*b*c^2*d^2 + 1/3*a*d^2*x^3 + 1/9*(3*x^3*arcsinh(c*x) - c*(
sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*d^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(d\,c^2\,x^2+d\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^2,x)
```

```
[Out] int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^2, x)
```

sympy [A] time = 6.14, size = 202, normalized size = 1.29

$$\begin{cases} \frac{ac^4d^2x^7}{7}+\frac{2ac^2d^2x^5}{5}+\frac{ad^2x^3}{3}+\frac{bc^4d^2x^7\,\mathrm{asinh}\,(cx)}{7}-\frac{bc^3d^2x^6\sqrt{c^2x^2+1}}{49}+\frac{2bc^2d^2x^5\,\mathrm{asinh}\,(cx)}{5}-\frac{68bcd^2x^4\sqrt{c^2x^2+1}}{1225}+\frac{bd^2x^3\,\mathrm{asinh}\,(cx)}{3} \\ \frac{ad^2x^3}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)**2*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**2*x**7/7 + 2*a*c**2*d**2*x**5/5 + a*d**2*x**3/3 + b*c*
*4*d**2*x**7*asinh(c*x)/7 - b*c**3*d**2*x**6*sqrt(c**2*x**2 + 1)/49 + 2*b*c
**2*d**2*x**5*asinh(c*x)/5 - 68*b*c*d**2*x**4*sqrt(c**2*x**2 + 1)/1225 + b*
d**2*x**3*asinh(c*x)/3 - 409*b*d**2*x**2*sqrt(c**2*x**2 + 1)/(11025*c) + 81
8*b*d**2*sqrt(c**2*x**2 + 1)/(11025*c**3), Ne(c, 0)), (a*d**2*x**3/3, True)
)
```

3.13 $\int x \left(d+c^2dx^2\right)^2 \left(a+b\sinh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=120

$$\frac{d^2\left(c^2x^2+1\right)^3\left(a+b\sinh^{-1}(cx)\right)}{6c^2}-\frac{bd^2x\left(c^2x^2+1\right)^{5/2}}{36c}-\frac{5bd^2x\left(c^2x^2+1\right)^{3/2}}{144c}-\frac{5bd^2x\sqrt{c^2x^2+1}}{96c}-\frac{5bd^2\sinh^{-1}(cx)}{96c^2}$$

```
[Out] -5/144*b*d^2*x*(c^2*x^2+1)^(3/2)/c-1/36*b*d^2*x*(c^2*x^2+1)^(5/2)/c-5/96*b*
d^2*arcsinh(c*x)/c^2+1/6*d^2*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))/c^2-5/96*b*d^
2*x*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] time = 0.06, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {5717, 195, 215}

$$\frac{d^2\left(c^2x^2+1\right)^3\left(a+b\sinh^{-1}(cx)\right)}{6c^2}-\frac{bd^2x\left(c^2x^2+1\right)^{5/2}}{36c}-\frac{5bd^2x\left(c^2x^2+1\right)^{3/2}}{144c}-\frac{5bd^2x\sqrt{c^2x^2+1}}{96c}-\frac{5bd^2\sinh^{-1}(cx)}{96c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-5*b*d^2*x*Sqrt[1 + c^2*x^2])/(96*c) - (5*b*d^2*x*(1 + c^2*x^2)^(3/2))/(14
4*c) - (b*d^2*x*(1 + c^2*x^2)^(5/2))/(36*c) - (5*b*d^2*ArcSinh[c*x])/(96*c^
2) + (d^2*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x]))/(6*c^2)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\int x\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)\,dx = \frac{d^2\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)}{6c^2} - \frac{\left(bd^2\right)\int\left(1+c^2x^2\right)^{5/2}\,dx}{6c}$$
$$= -\frac{bd^2x\left(1+c^2x^2\right)^{5/2}}{36c} + \frac{d^2\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)}{6c^2} - \frac{\left(5bd^2\right)\int}{}$$
$$= -\frac{5bd^2x\left(1+c^2x^2\right)^{3/2}}{144c} - \frac{bd^2x\left(1+c^2x^2\right)^{5/2}}{36c} + \frac{d^2\left(1+c^2x^2\right)^3\left(a+b\sin\right.}{6c^2}$$
$$= -\frac{5bd^2x\sqrt{1+c^2x^2}}{96c} - \frac{5bd^2x\left(1+c^2x^2\right)^{3/2}}{144c} - \frac{bd^2x\left(1+c^2x^2\right)^{5/2}}{36c} + \frac{d^2\left(\right.}{}$$
$$= -\frac{5bd^2x\sqrt{1+c^2x^2}}{96c} - \frac{5bd^2x\left(1+c^2x^2\right)^{3/2}}{144c} - \frac{bd^2x\left(1+c^2x^2\right)^{5/2}}{36c} - \frac{5bd}{}$$

Mathematica [A] time = 0.13, size = 104, normalized size = 0.87

$$\frac{d^2\left(cx\left(48acx\left(c^4x^4+3c^2x^2+3\right)-b\sqrt{c^2x^2+1}\left(8c^4x^4+26c^2x^2+33\right)\right)+3b\left(16c^6x^6+48c^4x^4+48c^2x^2+11\right)s\right.}{288c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*(c*x*(48*a*c*x*(3 + 3*c^2*x^2 + c^4*x^4) - b*Sqrt[1 + c^2*x^2]*(33 + 2
6*c^2*x^2 + 8*c^4*x^4)) + 3*b*(11 + 48*c^2*x^2 + 48*c^4*x^4 + 16*c^6*x^6)*A
rcSinh[c*x]))/(288*c^2)
```

fricas [A] time = 0.87, size = 149, normalized size = 1.24

$$\frac{48\,ac^6d^2x^6+144\,ac^4d^2x^4+144\,ac^2d^2x^2+3\left(16\,bc^6d^2x^6+48\,bc^4d^2x^4+48\,bc^2d^2x^2+11\,bd^2\right)\log\left(cx+\sqrt{c^2x^2}\right.}{288\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/288*(48*a*c^6*d^2*x^6 + 144*a*c^4*d^2*x^4 + 144*a*c^2*d^2*x^2 + 3*(16*b*c
^6*d^2*x^6 + 48*b*c^4*d^2*x^4 + 48*b*c^2*d^2*x^2 + 11*b*d^2)*log(c*x + sqrt
(c^2*x^2 + 1)) - (8*b*c^5*d^2*x^5 + 26*b*c^3*d^2*x^3 + 33*b*c*d^2*x)*sqrt(c
^2*x^2 + 1))/c^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.00, size = 137, normalized size = 1.14

$$\frac{d^2a\left(\frac{1}{6}c^6x^6+\frac{1}{2}c^4x^4+\frac{1}{2}c^2x^2\right)+d^2b\left(\frac{\operatorname{arcsinh}(cx)c^6x^6}{6}+\frac{\operatorname{arcsinh}(cx)c^4x^4}{2}+\frac{\operatorname{arcsinh}(cx)c^2x^2}{2}-\frac{c^5x^5\sqrt{c^2x^2+1}}{36}-\frac{13c^3x^3\sqrt{c^2x^2+1}}{144}\right.}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c^2*(d^2*a*(1/6*c^6*x^6+1/2*c^4*x^4+1/2*c^2*x^2)+d^2*b*(1/6*arcsinh(c*x)*
c^6*x^6+1/2*arcsinh(c*x)*c^4*x^4+1/2*arcsinh(c*x)*c^2*x^2-1/36*c^5*x^5*(c^2
*x^2+1)^(1/2)-13/144*c^3*x^3*(c^2*x^2+1)^(1/2)-11/96*c*x*(c^2*x^2+1)^(1/2)+
11/96*arcsinh(c*x)))
```

maxima [B] time = 0.48, size = 234, normalized size = 1.95

$$\frac{1}{6}ac^4d^2x^6+\frac{1}{2}ac^2d^2x^4+\frac{1}{288}\left(48x^6\operatorname{arsinh}(cx)-\left(\frac{8\sqrt{c^2x^2+1}\,x^5}{c^2}-\frac{10\sqrt{c^2x^2+1}\,x^3}{c^4}+\frac{15\sqrt{c^2x^2+1}\,x}{c^6}-\frac{15\operatorname{arsinh}}{c^7}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*c^4*d^2*x^6 + 1/2*a*c^2*d^2*x^4 + 1/288*(48*x^6*arcsinh(c*x) - (8*sqr
t(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 + 1
)*x/c^6 - 15*arcsinh(c*x)/c^7)*c)*b*c^4*d^2 + 1/16*(8*x^4*arcsinh(c*x) - (2
*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5
)*c)*b*c^2*d^2 + 1/2*a*d^2*x^2 + 1/4*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2
+ 1)*x/c^2 - arcsinh(c*x)/c^3))*b*d^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(d\,c^2\,x^2+d\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2)^2,x)
```

```
[Out] int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2)^2, x)
```

sympy [A] time = 4.16, size = 190, normalized size = 1.58

$$\begin{cases} \frac{ac^4d^2x^6}{6}+\frac{ac^2d^2x^4}{2}+\frac{ad^2x^2}{2}+\frac{bc^4d^2x^6\operatorname{asinh}(cx)}{6}-\frac{bc^3d^2x^5\sqrt{c^2x^2+1}}{36}+\frac{bc^2d^2x^4\operatorname{asinh}(cx)}{2}-\frac{13bcd^2x^3\sqrt{c^2x^2+1}}{144}+\frac{bd^2x^2\operatorname{asinh}(cx)}{2}-\frac{11b}{} \\ \frac{ad^2x^2}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*d*x**2+d)**2*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**2*x**6/6 + a*c**2*d**2*x**4/2 + a*d**2*x**2/2 + b*c**4
*d**2*x**6*asinh(c*x)/6 - b*c**3*d**2*x**5*sqrt(c**2*x**2 + 1)/36 + b*c**2*
d**2*x**4*asinh(c*x)/2 - 13*b*c*d**2*x**3*sqrt(c**2*x**2 + 1)/144 + b*d**2*
x**2*asinh(c*x)/2 - 11*b*d**2*x*sqrt(c**2*x**2 + 1)/(96*c) + 11*b*d**2*asin
h(c*x)/(96*c**2), Ne(c, 0)), (a*d**2*x**2/2, True))
```

3.14 $\int \left(d+c^2dx^2\right)^2 \left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=128

$$\frac{1}{5}c^4d^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{3}c^2d^2x^3\left(a+b\sinh^{-1}(cx)\right)+d^2x\left(a+b\sinh^{-1}(cx)\right)-\frac{bd^2\left(c^2x^2+1\right)^{5/2}}{25c}-\frac{4bd^2\left(c^2x^2\right.}{45c}$$

```
[Out] -4/45*b*d^2*(c^2*x^2+1)^(3/2)/c-1/25*b*d^2*(c^2*x^2+1)^(5/2)/c+d^2*x*(a+b*a
rcsinh(c*x))+2/3*c^2*d^2*x^3*(a+b*arcsinh(c*x))+1/5*c^4*d^2*x^5*(a+b*arcsin
h(c*x))-8/15*b*d^2*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] time = 0.10, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {194, 5679, 12, 1247, 698}

$$\frac{1}{5}c^4d^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{3}c^2d^2x^3\left(a+b\sinh^{-1}(cx)\right)+d^2x\left(a+b\sinh^{-1}(cx)\right)-\frac{bd^2\left(c^2x^2+1\right)^{5/2}}{25c}-\frac{4bd^2\left(c^2x^2\right.}{45c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-8*b*d^2*Sqrt[1 + c^2*x^2])/(15*c) - (4*b*d^2*(1 + c^2*x^2)^(3/2))/(45*c)
- (b*d^2*(1 + c^2*x^2)^(5/2))/(25*c) + d^2*x*(a + b*ArcSinh[c*x]) + (2*c^2*
d^2*x^3*(a + b*ArcSinh[c*x]))/3 + (c^4*d^2*x^5*(a + b*ArcSinh[c*x]))/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 5679

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(d+c^2dx^2\right)^2 \left(a+b\sinh^{-1}(cx)\right) dx = d^2x\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{3}c^2d^2x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}c^4d^2x^5\left(a+b\sin\right.$$

$$= d^2x\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{3}c^2d^2x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}c^4d^2x^5\left(a+b\sin\right.$$

$$= d^2x\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{3}c^2d^2x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}c^4d^2x^5\left(a+b\sin\right.$$

$$= d^2x\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{3}c^2d^2x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}c^4d^2x^5\left(a+b\sin\right.$$

$$= -\frac{8bd^2\sqrt{1+c^2x^2}}{15c}-\frac{4bd^2\left(1+c^2x^2\right)^{3/2}}{45c}-\frac{bd^2\left(1+c^2x^2\right)^{5/2}}{25c}+d^2x\left(a+b\,\mathrm{si}\right.$$

Mathematica [A] time = 0.11, size = 95, normalized size = 0.74

$$\frac{d^2\left(15acx\left(3c^4x^4+10c^2x^2+15\right)-b\sqrt{c^2x^2+1}\left(9c^4x^4+38c^2x^2+149\right)+15bcx\left(3c^4x^4+10c^2x^2+15\right)\sinh^{-1}(cx)\right.}{225c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*(15*a*c*x*(15 + 10*c^2*x^2 + 3*c^4*x^4) - b*Sqrt[1 + c^2*x^2]*(149 + 3
8*c^2*x^2 + 9*c^4*x^4) + 15*b*c*x*(15 + 10*c^2*x^2 + 3*c^4*x^4)*ArcSinh[c*x
]))/(225*c)
```

fricas [A] time = 0.52, size = 133, normalized size = 1.04

$$\frac{45\,ac^5d^2x^5+150\,ac^3d^2x^3+225\,acd^2x+15\left(3\,bc^5d^2x^5+10\,bc^3d^2x^3+15\,bcd^2x\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-\left(9\,bc^4d^2x\right.}{225\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/225*(45*a*c^5*d^2*x^5 + 150*a*c^3*d^2*x^3 + 225*a*c*d^2*x + 15*(3*b*c^5*d
^2*x^5 + 10*b*c^3*d^2*x^3 + 15*b*c*d^2*x)*log(c*x + sqrt(c^2*x^2 + 1)) - (9
*b*c^4*d^2*x^4 + 38*b*c^2*d^2*x^2 + 149*b*d^2)*sqrt(c^2*x^2 + 1))/c
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 119, normalized size = 0.93

$$\frac{d^2a\left(\frac{1}{5}c^5x^5+\frac{2}{3}c^3x^3+cx\right)+d^2b\left(\frac{\operatorname{arcsinh}(cx)c^5x^5}{5}+\frac{2\operatorname{arcsinh}(cx)c^3x^3}{3}+\operatorname{arcsinh}(cx)\,cx-\frac{c^4x^4\sqrt{c^2x^2+1}}{25}-\frac{38c^2x^2\sqrt{c^2x^2+1}}{225}-\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x)

[Out] 1/c*(d^2*a*(1/5*c^5*x^5+2/3*c^3*x^3+c*x)+d^2*b*(1/5*arcsinh(c*x)*c^5*x^5+2/3*arcsinh(c*x)*c^3*x^3+arcsinh(c*x)*c*x-1/25*c^4*x^4*(c^2*x^2+1)^(1/2)-38/225*c^2*x^2*(c^2*x^2+1)^(1/2)-149/225*(c^2*x^2+1)^(1/2)))

maxima [A] time = 0.33, size = 194, normalized size = 1.52

$$\frac{1}{5}\,ac^4d^2x^5+\frac{1}{75}\left(15\,x^5\,\mathrm{arsinh}\,(cx)-\left(\frac{3\,\sqrt{c^2x^2+1}\,x^4}{c^2}-\frac{4\,\sqrt{c^2x^2+1}\,x^2}{c^4}+\frac{8\,\sqrt{c^2x^2+1}}{c^6}\right)c\right)bc^4d^2+\frac{2}{3}\,ac^2d^2x^3+\frac{2}{9}\Big(3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")

[Out] 1/5*a*c^4*d^2*x^5 + 1/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*b*c^4*d^2 + 2/3*a*c^2*d^2*x^3 + 2/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*c^2*d^2 + a*d^2*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*b*d^2/c

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\mathrm{asinh}\,(c\,x))\,\left(d\,c^2\,x^2+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*asinh(c*x))*(d + c^2*d*x^2)^2,x)

[Out] int((a + b*asinh(c*x))*(d + c^2*d*x^2)^2, x)

sympy [A] time = 2.26, size = 165, normalized size = 1.29

$$\begin{cases} \frac{ac^4d^2x^5}{5}+\frac{2ac^2d^2x^3}{3}+ad^2x+\frac{bc^4d^2x^5\,\mathrm{asinh}\,(cx)}{5}-\frac{bc^3d^2x^4\sqrt{c^2x^2+1}}{25}+\frac{2bc^2d^2x^3\,\mathrm{asinh}\,(cx)}{3}-\frac{38bcd^2x^2\sqrt{c^2x^2+1}}{225}+bd^2x\,\mathrm{asinh}\,(c \\ ad^2x \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x)),x)

[Out] Piecewise((a*c**4*d**2*x**5/5 + 2*a*c**2*d**2*x**3/3 + a*d**2*x + b*c**4*d**2*x**5*asinh(c*x)/5 - b*c**3*d**2*x**4*sqrt(c**2*x**2 + 1)/25 + 2*b*c**2*d**2*x**3*asinh(c*x)/3 - 38*b*c*d**2*x**2*sqrt(c**2*x**2 + 1)/225 + b*d**2*x*asinh(c*x) - 149*b*d**2*sqrt(c**2*x**2 + 1)/(225*c), Ne(c, 0)), (a*d**2*x, True))

3.15 $\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=172

$$\frac{1}{4}d^2\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{2}d^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)+\frac{d^2\left(a+b\sinh^{-1}(cx)\right)^2}{2b}+d^2\log\left(1-e^{-2\sinh\right.$$

```
[Out] -1/16*b*c*d^2*x*(c^2*x^2+1)^(3/2)-11/32*b*d^2*arcsinh(c*x)+1/2*d^2*(c^2*x^2
+1)*(a+b*arcsinh(c*x))+1/4*d^2*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))+1/2*d^2*(a+
b*arcsinh(c*x))^2/b+d^2*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^2
)-1/2*b*d^2*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)-11/32*b*c*d^2*x*(c^2*x^2
+1)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5726, 5659, 3716, 2190, 2279, 2391, 195, 215}

$$\frac{1}{2}bd^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)+\frac{1}{4}d^2\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{2}d^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{d^2\left(a+b\,\mathrm{si}\right.}{2}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (-11*b*c*d^2*x*Sqrt[1 + c^2*x^2])/32 - (b*c*d^2*x*(1 + c^2*x^2)^(3/2))/16 -
 (11*b*d^2*ArcSinh[c*x])/32 + (d^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/2 +
(d^2*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/4 - (d^2*(a + b*ArcSinh[c*x])^2)
/(2*b) + d^2*(a + b*ArcSinh[c*x])*Log[1 - E^(2*ArcSinh[c*x])] + (b*d^2*Poly
Log[2, E^(2*ArcSinh[c*x])])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5726

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[((d + e*x^2)^p*(a + b*ArcSinh[c*x]))/(2*p), x] + (Dist[d
, Int[((d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]))/x, x], x] - Dist[(b*c*d^p)
/(2*p), Int[(1 + c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{x}\,dx = \frac{1}{4}d^2\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)+d\int \frac{\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(c\right.}{x}$$

$$= -\frac{1}{16}bcd^2x\left(1+c^2x^2\right)^{3/2}+\frac{1}{2}d^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}d^2\left(1\right.$$

$$= -\frac{11}{32}bcd^2x\sqrt{1+c^2x^2}-\frac{1}{16}bcd^2x\left(1+c^2x^2\right)^{3/2}+\frac{1}{2}d^2\left(1+c^2x^2\right)\left(a+b\right.$$

$$= -\frac{11}{32}bcd^2x\sqrt{1+c^2x^2}-\frac{1}{16}bcd^2x\left(1+c^2x^2\right)^{3/2}-\frac{11}{32}bd^2\sinh^{-1}(cx)+\frac{1}{2}$$

$$= -\frac{11}{32}bcd^2x\sqrt{1+c^2x^2}-\frac{1}{16}bcd^2x\left(1+c^2x^2\right)^{3/2}-\frac{11}{32}bd^2\sinh^{-1}(cx)+\frac{1}{2}$$

$$= -\frac{11}{32}bcd^2x\sqrt{1+c^2x^2}-\frac{1}{16}bcd^2x\left(1+c^2x^2\right)^{3/2}-\frac{11}{32}bd^2\sinh^{-1}(cx)+\frac{1}{2}$$

$$= -\frac{11}{32}bcd^2x\sqrt{1+c^2x^2}-\frac{1}{16}bcd^2x\left(1+c^2x^2\right)^{3/2}-\frac{11}{32}bd^2\sinh^{-1}(cx)+\frac{1}{2}$$

Mathematica [A] time = 0.18, size = 173, normalized size = 1.01

$$\frac{d^2\left(-16a^2+8abc^4x^4+32abc^2x^2+b\sinh^{-1}(cx)\left(-32a+b\left(8c^4x^4+32c^2x^2+13\right)+32b\log\left(1-e^{2\sinh^{-1}(cx)}\right)\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (d^2*(-16*a^2 + 24*a*b + 32*a*b*c^2*x^2 + 8*a*b*c^4*x^4 - 13*b^2*c*x*Sqrt[1
 + c^2*x^2] - 2*b^2*c^3*x^3*Sqrt[1 + c^2*x^2] - 16*b^2*ArcSinh[c*x]^2 + 32*
```

```
a*b*Log[1 - E^(2*ArcSinh[c*x])] + b*ArcSinh[c*x]*(-32*a + b*(13 + 32*c^2*x^
2 + 8*c^4*x^4) + 32*b*Log[1 - E^(2*ArcSinh[c*x])]) + 16*b^2*PolyLog[2, E^(2
*ArcSinh[c*x])]))/(32*b)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ac^4d^2x^4 + 2\,ac^2d^2x^2 + ad^2 + \left(bc^4d^2x^4 + 2\,bc^2d^2x^2 + bd^2\right)\text{arsinh}\left(cx\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c^
2*d^2*x^2 + b*d^2)*arcsinh(c*x))/x, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.16, size = 231, normalized size = 1.34

$$\frac{d^2a\,c^4x^4}{4}+d^2a\,c^2x^2+d^2a\ln\left(cx\right)+d^2b\,\text{arcsinh}\left(cx\right)\ln\left(1-cx-\sqrt{c^2x^2+1}\right)+d^2b\,\text{arcsinh}\left(cx\right)\ln\left(1+cx+\sqrt{c^2x^2+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x,x)
```

```
[Out] 1/4*d^2*a*c^4*x^4+d^2*a*c^2*x^2+d^2*a*ln(c*x)+d^2*b*arcsinh(c*x)*ln(1-c*x-(
c^2*x^2+1)^(1/2))+d^2*b*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-1/16*d^2*b
*c^3*x^3*(c^2*x^2+1)^(1/2)-13/32*b*c*d^2*x*(c^2*x^2+1)^(1/2)+1/4*d^2*b*arcs
inh(c*x)*c^4*x^4+d^2*b*arcsinh(c*x)*c^2*x^2+13/32*b*d^2*arcsinh(c*x)+d^2*b*
polylog(2,c*x+(c^2*x^2+1)^(1/2))+d^2*b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))-1/
2*d^2*b*arcsinh(c*x)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\,ac^4d^2x^4+ac^2d^2x^2+ad^2\log(x)+\int bc^4d^2x^3\log\left(cx+\sqrt{c^2x^2+1}\right)+2\,bc^2d^2x\log\left(cx+\sqrt{c^2x^2+1}\right)+\frac{bd^2\log\left(cx+\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/4*a*c^4*d^2*x^4 + a*c^2*d^2*x^2 + a*d^2*log(x) + integrate(b*c^4*d^2*x^3*
log(c*x + sqrt(c^2*x^2 + 1)) + 2*b*c^2*d^2*x*log(c*x + sqrt(c^2*x^2 + 1)) +
 b*d^2*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(d\,c^2\,x^2+d\right)}^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^2)/x,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int \frac{a}{x}\,dx + \int 2ac^2x\,dx + \int ac^4x^3\,dx + \int \frac{b\,\mathrm{asinh}\,(cx)}{x}\,dx + \int 2bc^2x\,\mathrm{asinh}\,(cx)\,dx + \int bc^4x^3\,\mathrm{asinh}\,(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))/x,x)
```

```
[Out] d**2*(Integral(a/x, x) + Integral(2*a*c**2*x, x) + Integral(a*c**4*x**3, x)
 + Integral(b*asinh(c*x)/x, x) + Integral(2*b*c**2*x*asinh(c*x), x) + Integ
ral(b*c**4*x**3*asinh(c*x), x))
```

3.16 $\int \frac{(d+c^2dx^2)^2(a+b\sinh^{-1}(cx))}{x^2}\,dx$

Optimal. Leaf size=120

$$\frac{1}{3}c^4d^2x^3\left(a+b\sinh^{-1}(cx)\right)+2c^2d^2x\left(a+b\sinh^{-1}(cx)\right)-\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{1}{9}bcd^2\left(c^2x^2+1\right)^{3/2}-\frac{5}{3}bcd^2\sqrt{c^2x^2}$$

```
[Out] -1/9*b*c*d^2*(c^2*x^2+1)^(3/2)-d^2*(a+b*arcsinh(c*x))/x+2*c^2*d^2*x*(a+b*ar
csinh(c*x))+1/3*c^4*d^2*x^3*(a+b*arcsinh(c*x))-b*c*d^2*arctanh((c^2*x^2+1)^
(1/2))-5/3*b*c*d^2*(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {270, 5730, 12, 1251, 897, 1153, 208}

$$\frac{1}{3}c^4d^2x^3\left(a+b\sinh^{-1}(cx)\right)+2c^2d^2x\left(a+b\sinh^{-1}(cx)\right)-\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{1}{9}bcd^2\left(c^2x^2+1\right)^{3/2}-\frac{5}{3}bcd^2\sqrt{c^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] (-5*b*c*d^2*Sqrt[1 + c^2*x^2])/3 - (b*c*d^2*(1 + c^2*x^2)^(3/2))/9 - (d^2*(
a + b*ArcSinh[c*x]))/x + 2*c^2*d^2*x*(a + b*ArcSinh[c*x]) + (c^4*d^2*x^3*(a
 + b*ArcSinh[c*x]))/3 - b*c*d^2*ArcTanh[Sqrt[1 + c^2*x^2]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{x^2}\,dx = -\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{x}+2c^2d^2x\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^4d^2x^3\left(a+b\right.$$
$$= -\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{x}+2c^2d^2x\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^4d^2x^3\left(a+b\right.$$
$$= -\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{x}+2c^2d^2x\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^4d^2x^3\left(a+b\right.$$
$$= -\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{x}+2c^2d^2x\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^4d^2x^3\left(a+b\right.$$
$$= -\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{x}+2c^2d^2x\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^4d^2x^3\left(a+b\right.$$
$$= -\frac{5}{3}bcd^2\sqrt{1+c^2x^2}-\frac{1}{9}bcd^2\left(1+c^2x^2\right)^{3/2}-\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{x}+2c$$
$$= -\frac{5}{3}bcd^2\sqrt{1+c^2x^2}-\frac{1}{9}bcd^2\left(1+c^2x^2\right)^{3/2}-\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{x}+2c$$

Mathematica [A] time = 0.12, size = 124, normalized size = 1.03

$$\frac{d^2\left(3ac^4x^4+18ac^2x^2-9a-16bcx\sqrt{c^2x^2+1}-9bcx\log\left(\sqrt{c^2x^2+1}+1\right)+3b\left(c^4x^4+6c^2x^2-3\right)\sinh^{-1}(cx)-\right.}{9x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] (d^2*(-9*a + 18*a*c^2*x^2 + 3*a*c^4*x^4 - 16*b*c*x*Sqrt[1 + c^2*x^2] - b*c^
3*x^3*Sqrt[1 + c^2*x^2] + 3*b*(-3 + 6*c^2*x^2 + c^4*x^4)*ArcSinh[c*x] + 9*b
*c*x*Log[x] - 9*b*c*x*Log[1 + Sqrt[1 + c^2*x^2]]))/(9*x)
```

fricas [B] time = 0.51, size = 228, normalized size = 1.90

$$3\,ac^4d^2x^4+18\,ac^2d^2x^2-9\,bcd^2x\log\left(-cx+\sqrt{c^2x^2+1}+1\right)+9\,bcd^2x\log\left(-cx+\sqrt{c^2x^2+1}-1\right)-3\left(bc^4+6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] 1/9*(3*a*c^4*d^2*x^4 + 18*a*c^2*d^2*x^2 - 9*b*c*d^2*x*log(-c*x + sqrt(c^2*x
^2 + 1) + 1) + 9*b*c*d^2*x*log(-c*x + sqrt(c^2*x^2 + 1) - 1) - 3*(b*c^4 + 6
*b*c^2 - 3*b)*d^2*x*log(-c*x + sqrt(c^2*x^2 + 1)) - 9*a*d^2 + 3*(b*c^4*d^2*
x^4 + 6*b*c^2*d^2*x^2 - (b*c^4 + 6*b*c^2 - 3*b)*d^2*x - 3*b*d^2)*log(c*x +
sqrt(c^2*x^2 + 1)) - (b*c^3*d^2*x^3 + 16*b*c*d^2*x)*sqrt(c^2*x^2 + 1))/x
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 114, normalized size = 0.95

$$c\left(d^2a\left(\frac{c^3x^3}{3}+2cx-\frac{1}{cx}\right)+d^2b\left(\frac{\operatorname{arcsinh}(cx)\,c^3x^3}{3}+2\operatorname{arcsinh}(cx)\,cx-\frac{\operatorname{arcsinh}(cx)}{cx}-\frac{c^2x^2\sqrt{c^2x^2+1}}{9}-\frac{16\sqrt{c^2}}{9}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^2,x)
```

```
[Out] c*(d^2*a*(1/3*c^3*x^3+2*c*x-1/c/x)+d^2*b*(1/3*arcsinh(c*x)*c^3*x^3+2*arcsin
h(c*x)*c*x-arcsinh(c*x)/c/x-1/9*c^2*x^2*(c^2*x^2+1)^(1/2)-16/9*(c^2*x^2+1)^
(1/2)-arctanh(1/(c^2*x^2+1)^(1/2))))
```

maxima [A] time = 0.47, size = 143, normalized size = 1.19

$$\frac{1}{3}ac^4d^2x^3+\frac{1}{9}\left(3x^3\operatorname{arsinh}(cx)-c\left(\frac{\sqrt{c^2x^2+1}\,x^2}{c^2}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)bc^4d^2+2ac^2d^2x+2\left(cx\operatorname{arsinh}(cx)-\sqrt{c^2x^2+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*a*c^4*d^2*x^3 + 1/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2
- 2*sqrt(c^2*x^2 + 1)/c^4))*b*c^4*d^2 + 2*a*c^2*d^2*x + 2*(c*x*arcsinh(c*x)
 - sqrt(c^2*x^2 + 1))*b*c*d^2 - (c*arcsinh(1/(c*abs(x))) + arcsinh(c*x)/x)*
b*d^2 - a*d^2/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\operatorname{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^2)/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int 2ac^2\,dx+\int\frac{a}{x^2}\,dx+\int ac^4x^2\,dx+\int 2bc^2\operatorname{asinh}(cx)\,dx+\int\frac{b\operatorname{asinh}(cx)}{x^2}\,dx+\int bc^4x^2\operatorname{asinh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))/x**2,x)
```

```
[Out] d**2*(Integral(2*a*c**2, x) + Integral(a/x**2, x) + Integral(a*c**4*x**2, x
) + Integral(2*b*c**2*asinh(c*x), x) + Integral(b*asinh(c*x)/x**2, x) + Int
egral(b*c**4*x**2*asinh(c*x), x))
```

3.17 $\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=187

$$c^2d^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{d^2\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{2x^2}+\frac{c^2d^2\left(a+b\sinh^{-1}(cx)\right)^2}{b}+2c^2d^2\log\left(1-e^{-2\ldots}\right.$$

```
[Out] -1/2*b*c*d^2*(c^2*x^2+1)^(3/2)/x+1/4*b*c^2*d^2*arcsinh(c*x)+c^2*d^2*(c^2*x^
2+1)*(a+b*arcsinh(c*x))-1/2*d^2*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/x^2+c^2*d^
2*(a+b*arcsinh(c*x))^2/b+2*c^2*d^2*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(c^2*x^2+
1)^(1/2))^2)-b*c^2*d^2*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)+1/4*b*c^3*d^2
*x*(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5728, 277, 195, 215, 5726, 5659, 3716, 2190, 2279, 2391}

$$bc^2d^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)+c^2d^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{d^2\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{2x^2}-\frac{c^2d^2\left(a+b\ldots\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] (b*c^3*d^2*x*Sqrt[1 + c^2*x^2])/4 - (b*c*d^2*(1 + c^2*x^2)^(3/2))/(2*x) + (
b*c^2*d^2*ArcSinh[c*x])/4 + c^2*d^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]) - (d
^2*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(2*x^2) - (c^2*d^2*(a + b*ArcSinh[
c*x])^2)/b + 2*c^2*d^2*(a + b*ArcSinh[c*x])*Log[1 - E^(2*ArcSinh[c*x])] + b
*c^2*d^2*PolyLog[2, E^(2*ArcSinh[c*x])]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5726

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[((d + e*x^2)^p*(a + b*ArcSinh[c*x]))/(2*p), x] + (Dist[d
, Int[((d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]))/x, x], x] - Dist[(b*c*d^p)
/(2*p), Int[(1 + c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5728

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*ArcSinh[c
*x]))/(f*(m + 1)), x] + (-Dist[(b*c*d^p)/(f*(m + 1)), Int[(f*x)^(m + 1)*(1
+ c^2*x^2)^(p - 1/2), x], x] - Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2
)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && ILtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{x^3}\,dx = -\frac{d^2\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{2x^2} + \left(2c^2d\right)\int \frac{\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}\right.}{x}$$

$$= -\frac{bcd^2\left(1+c^2x^2\right)^{3/2}}{2x} + c^2d^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right) - \frac{d^2\left(1+c^2x^2\right)^2}{}$$

$$= \frac{1}{4}bc^3d^2x\sqrt{1+c^2x^2} - \frac{bcd^2\left(1+c^2x^2\right)^{3/2}}{2x} + c^2d^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(c\right.$$

$$= \frac{1}{4}bc^3d^2x\sqrt{1+c^2x^2} - \frac{bcd^2\left(1+c^2x^2\right)^{3/2}}{2x} + \frac{1}{4}bc^2d^2\sinh^{-1}(cx) + c^2d^2\left(1+\right.$$

$$= \frac{1}{4}bc^3d^2x\sqrt{1+c^2x^2} - \frac{bcd^2\left(1+c^2x^2\right)^{3/2}}{2x} + \frac{1}{4}bc^2d^2\sinh^{-1}(cx) + c^2d^2\left(1+\right.$$

$$= \frac{1}{4}bc^3d^2x\sqrt{1+c^2x^2} - \frac{bcd^2\left(1+c^2x^2\right)^{3/2}}{2x} + \frac{1}{4}bc^2d^2\sinh^{-1}(cx) + c^2d^2\left(1+\right.$$

$$= \frac{1}{4}bc^3d^2x\sqrt{1+c^2x^2} - \frac{bcd^2\left(1+c^2x^2\right)^{3/2}}{2x} + \frac{1}{4}bc^2d^2\sinh^{-1}(cx) + c^2d^2\left(1+\right.$$

Mathematica [A] time = 0.32, size = 143, normalized size = 0.76

$$\frac{1}{4}d^2\left(2c^4x^2\left(a+b\sinh^{-1}(cx)\right) + 4c^2\left(2\log\left(1-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right) + b\mathrm{Li}_2\left(e^{2\sinh^{-1}(cx)}\right)\right) - \frac{4c^2\left(a+b\,\mathrm{s}\right.}{b}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] (d^2*((-2*b*c*Sqrt[1 + c^2*x^2])/x + b*c^2*(-(c*x*Sqrt[1 + c^2*x^2]) + ArcS
inh[c*x]) - (2*(a + b*ArcSinh[c*x]))/x^2 + 2*c^4*x^2*(a + b*ArcSinh[c*x]) -
 (4*c^2*(a + b*ArcSinh[c*x])^2)/b + 4*c^2*(2*(a + b*ArcSinh[c*x])*Log[1 - E
^(2*ArcSinh[c*x])] + b*PolyLog[2, E^(2*ArcSinh[c*x])])))/4
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{ac^4d^2x^4 + 2\,ac^2d^2x^2 + ad^2 + \left(bc^4d^2x^4 + 2\,bc^2d^2x^2 + bd^2\right)\mathrm{arsinh}\left(cx\right)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c^
2*d^2*x^2 + b*d^2)*arcsinh(c*x))/x^3, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.32, size = 262, normalized size = 1.40

$$\frac{c^4d^2a\,x^2}{2}+2c^2d^2a\ln(cx)-\frac{d^2a}{2x^2}-c^2d^2b\operatorname{arcsinh}(cx)^2+\frac{c^4d^2b\operatorname{arcsinh}(cx)\,x^2}{2}-\frac{b\,c^3d^2x\sqrt{c^2x^2+1}}{4}+\frac{b\,c^2d^2\operatorname{arcsinh}(}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^3,x)
```

```
[Out] 1/2*c^4*d^2*a*x^2+2*c^2*d^2*a*ln(c*x)-1/2*d^2*a/x^2-c^2*d^2*b*arcsinh(c*x)^
2+1/2*c^4*d^2*b*arcsinh(c*x)*x^2-1/4*b*c^3*d^2*x*(c^2*x^2+1)^(1/2)+1/4*b*c^
2*d^2*arcsinh(c*x)+1/2*d^2*b*c^2-1/2*c*d^2*b/x*(c^2*x^2+1)^(1/2)-1/2*d^2*b*
arcsinh(c*x)/x^2+2*c^2*d^2*b*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+2*c^2
*d^2*b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+2*c^2*d^2*b*arcsinh(c*x)*ln(1-c*x-
(c^2*x^2+1)^(1/2))+2*c^2*d^2*b*polylog(2,c*x+(c^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,ac^4d^2x^2+2\,ac^2d^2\log(x)-\frac{1}{2}\,bd^2\left(\frac{\sqrt{c^2x^2+1}\,c}{x}+\frac{\operatorname{arsinh}(cx)}{x^2}\right)-\frac{ad^2}{2\,x^2}+\int bc^4d^2x\log\left(cx+\sqrt{c^2x^2+1}\right)+\frac{2\,bc^2d^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*c^4*d^2*x^2 + 2*a*c^2*d^2*log(x) - 1/2*b*d^2*(sqrt(c^2*x^2 + 1)*c/x +
 arcsinh(c*x)/x^2) - 1/2*a*d^2/x^2 + integrate(b*c^4*d^2*x*log(c*x + sqrt(c
^2*x^2 + 1)) + 2*b*c^2*d^2*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\operatorname{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^2)/x^3,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int\frac{a}{x^3}\,dx+\int\frac{2ac^2}{x}\,dx+\int ac^4x\,dx+\int\frac{b\operatorname{asinh}(cx)}{x^3}\,dx+\int\frac{2bc^2\operatorname{asinh}(cx)}{x}\,dx+\int bc^4x\operatorname{asinh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))/x**3,x)
```

```
[Out] d**2*(Integral(a/x**3, x) + Integral(2*a*c**2/x, x) + Integral(a*c**4*x, x)
 + Integral(b*asinh(c*x)/x**3, x) + Integral(2*b*c**2*asinh(c*x)/x, x) + In
tegral(b*c**4*x*asinh(c*x), x))
```

3.18 $\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=126

$$c^4d^2x\left(a+b\sinh^{-1}(cx)\right)-\frac{2c^2d^2\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{bcd^2\sqrt{c^2x^2+1}}{6x^2}-bc^3d^2\sqrt{c^2x^2+1}-\frac{11}{6}b$$

```
[Out] -1/3*d^2*(a+b*arcsinh(c*x))/x^3-2*c^2*d^2*(a+b*arcsinh(c*x))/x+c^4*d^2*x*(a
+b*arcsinh(c*x))-11/6*b*c^3*d^2*arctanh((c^2*x^2+1)^(1/2))-b*c^3*d^2*(c^2*x
^2+1)^(1/2)-1/6*b*c*d^2*(c^2*x^2+1)^(1/2)/x^2
```

Rubi [A] time = 0.16, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {270, 5730, 12, 1251, 897, 1157, 388, 208}

$$c^4d^2x\left(a+b\sinh^{-1}(cx)\right)-\frac{2c^2d^2\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-bc^3d^2\sqrt{c^2x^2+1}-\frac{bcd^2\sqrt{c^2x^2+1}}{6x^2}-\frac{11}{6}b$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] -(b*c^3*d^2*Sqrt[1 + c^2*x^2]) - (b*c*d^2*Sqrt[1 + c^2*x^2])/(6*x^2) - (d^2
*(a + b*ArcSinh[c*x]))/(3*x^3) - (2*c^2*d^2*(a + b*ArcSinh[c*x]))/x + c^4*d
^2*x*(a + b*ArcSinh[c*x]) - (11*b*c^3*d^2*ArcTanh[Sqrt[1 + c^2*x^2]])/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{x^4}\,dx = -\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{2c^2d^2\left(a+b\sinh^{-1}(cx)\right)}{x}+c^4d^2x\left(a+b\sinh\right.$$

$$= -\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{2c^2d^2\left(a+b\sinh^{-1}(cx)\right)}{x}+c^4d^2x\left(a+b\sinh\right.$$

$$= -\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{2c^2d^2\left(a+b\sinh^{-1}(cx)\right)}{x}+c^4d^2x\left(a+b\sinh\right.$$

$$= -\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{2c^2d^2\left(a+b\sinh^{-1}(cx)\right)}{x}+c^4d^2x\left(a+b\sinh\right.$$

$$= -\frac{bcd^2\sqrt{1+c^2x^2}}{6x^2}-\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{2c^2d^2\left(a+b\sinh^{-1}(cx)\right)}{x}$$

$$= -bc^3d^2\sqrt{1+c^2x^2}-\frac{bcd^2\sqrt{1+c^2x^2}}{6x^2}-\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{2c^2d^2\left(\right.}{}$$

$$= -bc^3d^2\sqrt{1+c^2x^2}-\frac{bcd^2\sqrt{1+c^2x^2}}{6x^2}-\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{2c^2d^2\left(\right.}{}$$

Mathematica [A] time = 0.12, size = 133, normalized size = 1.06

$$\frac{d^2\left(6ac^4x^4-12ac^2x^2-2a+11bc^3x^3\log(x)-bcx\sqrt{c^2x^2+1}+2b\left(3c^4x^4-6c^2x^2-1\right)\sinh^{-1}(cx)-6bc^3x^3\sqrt{c^2\right.}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] (d^2*(-2*a - 12*a*c^2*x^2 + 6*a*c^4*x^4 - b*c*x*Sqrt[1 + c^2*x^2] - 6*b*c^3
*x^3*Sqrt[1 + c^2*x^2] + 2*b*(-1 - 6*c^2*x^2 + 3*c^4*x^4)*ArcSinh[c*x] + 11
*b*c^3*x^3*Log[x] - 11*b*c^3*x^3*Log[1 + Sqrt[1 + c^2*x^2]]))/(6*x^3)
```

fricas [B] time = 0.71, size = 243, normalized size = 1.93

$$\frac{6\,ac^4d^2x^4 - 11\,bc^3d^2x^3\log\left(-cx + \sqrt{c^2x^2+1}\,+1\right) + 11\,bc^3d^2x^3\log\left(-cx + \sqrt{c^2x^2+1}\,-1\right) - 12\,ac^2d^2x^2 - 2\left(3\,bc^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a*c^4*d^2*x^4 - 11*b*c^3*d^2*x^3*log(-c*x + sqrt(c^2*x^2 + 1) + 1) +
 11*b*c^3*d^2*x^3*log(-c*x + sqrt(c^2*x^2 + 1) - 1) - 12*a*c^2*d^2*x^2 - 2*
(3*b*c^4 - 6*b*c^2 - b)*d^2*x^3*log(-c*x + sqrt(c^2*x^2 + 1)) - 2*a*d^2 + 2
*(3*b*c^4*d^2*x^4 - 6*b*c^2*d^2*x^2 - (3*b*c^4 - 6*b*c^2 - b)*d^2*x^3 - b*d
^2)*log(c*x + sqrt(c^2*x^2 + 1)) - (6*b*c^3*d^2*x^3 + b*c*d^2*x)*sqrt(c^2*x
^2 + 1))/x^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 114, normalized size = 0.90

$$c^3\left(d^2a\left(cx - \frac{2}{cx} - \frac{1}{3c^3x^3}\right) + d^2b\left(\operatorname{arcsinh}(cx)\,cx - \frac{2\operatorname{arcsinh}(cx)}{cx} - \frac{\operatorname{arcsinh}(cx)}{3c^3x^3} - \sqrt{c^2x^2+1} - \frac{11\operatorname{arctanh}\left(\frac{}{\sqrt{c^2}}\right.}{6}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^4,x)
```

```
[Out] c^3*(d^2*a*(c*x-2/c/x-1/3/c^3/x^3)+d^2*b*(arcsinh(c*x)*c*x-2*arcsinh(c*x)/c
/x-1/3*arcsinh(c*x)/c^3/x^3-(c^2*x^2+1)^(1/2)-11/6*arctanh(1/(c^2*x^2+1)^(1
/2))-1/6/c^2/x^2*(c^2*x^2+1)^(1/2)))
```

maxima [A] time = 0.45, size = 137, normalized size = 1.09

$$ac^4d^2x+\left(cx\operatorname{arsinh}(cx) - \sqrt{c^2x^2+1}\right)bc^3d^2-2\left(c\operatorname{arsinh}\left(\frac{1}{c|x|}\right) + \frac{\operatorname{arsinh}(cx)}{x}\right)bc^2d^2+\frac{1}{6}\left(\left(c^2\operatorname{arsinh}\left(\frac{1}{c|x|}\right) - \frac{\sqrt{c^2x}}{x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] a*c^4*d^2*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*b*c^3*d^2 - 2*(c*arcsi
nh(1/(c*abs(x))) + arcsinh(c*x)/x)*b*c^2*d^2 + 1/6*((c^2*arcsinh(1/(c*abs(x
))) - sqrt(c^2*x^2 + 1)/x^2)*c - 2*arcsinh(c*x)/x^3)*b*d^2 - 2*a*c^2*d^2/x
- 1/3*a*d^2/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))\,\left(d\,c^2\,x^2+d\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^2)/x^4,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int ac^4\,dx+\int \frac{a}{x^4}\,dx+\int \frac{2ac^2}{x^2}\,dx+\int bc^4\,\mathrm{asinh}\,(cx)\,dx+\int \frac{b\,\mathrm{asinh}\,(cx)}{x^4}\,dx+\int \frac{2bc^2\,\mathrm{asinh}\,(cx)}{x^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))/x**4,x)
```

```
[Out] d**2*(Integral(a*c**4, x) + Integral(a/x**4, x) + Integral(2*a*c**2/x**2, x
) + Integral(b*c**4*asinh(c*x), x) + Integral(b*asinh(c*x)/x**4, x) + Integ
ral(2*b*c**2*asinh(c*x)/x**2, x))
```

3.19 $\int x^4 \left(d+c^2dx^2\right)^3 \left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=226

$$\frac{1}{11}c^6d^3x^{11}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^4d^3x^9\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{7}c^2d^3x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}d^3x^5\left(a+b\sinh^{-1}(cx)\right)$$

```
[Out] -8/3465*b*d^3*(c^2*x^2+1)^(3/2)/c^5-2/1925*b*d^3*(c^2*x^2+1)^(5/2)/c^5-1/16
17*b*d^3*(c^2*x^2+1)^(7/2)/c^5+4/297*b*d^3*(c^2*x^2+1)^(9/2)/c^5-1/121*b*d^
3*(c^2*x^2+1)^(11/2)/c^5+1/5*d^3*x^5*(a+b*arcsinh(c*x))+3/7*c^2*d^3*x^7*(a+
b*arcsinh(c*x))+1/3*c^4*d^3*x^9*(a+b*arcsinh(c*x))+1/11*c^6*d^3*x^11*(a+b*a
rcsinh(c*x))-16/1155*b*d^3*(c^2*x^2+1)^(1/2)/c^5
```

Rubi [A] time = 0.28, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {270, 5730, 12, 1799, 1620}

$$\frac{1}{11}c^6d^3x^{11}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^4d^3x^9\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{7}c^2d^3x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}d^3x^5\left(a+b\sinh^{-1}(cx)\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-16*b*d^3*Sqrt[1 + c^2*x^2])/(1155*c^5) - (8*b*d^3*(1 + c^2*x^2)^(3/2))/(3
465*c^5) - (2*b*d^3*(1 + c^2*x^2)^(5/2))/(1925*c^5) - (b*d^3*(1 + c^2*x^2)^
(7/2))/(1617*c^5) + (4*b*d^3*(1 + c^2*x^2)^(9/2))/(297*c^5) - (b*d^3*(1 + c
^2*x^2)^(11/2))/(121*c^5) + (d^3*x^5*(a + b*ArcSinh[c*x]))/5 + (3*c^2*d^3*x
^7*(a + b*ArcSinh[c*x]))/7 + (c^4*d^3*x^9*(a + b*ArcSinh[c*x]))/3 + (c^6*d^
3*x^11*(a + b*ArcSinh[c*x]))/11
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
```

```
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\int x^4 \left(d+c^2dx^2\right)^3 \left(a+b\sinh^{-1}(cx)\right)\,dx = \frac{1}{5}d^3x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{7}c^2d^3x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^4d^3x^9\left(\right.$$

$$=\frac{1}{5}d^3x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{7}c^2d^3x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^4d^3x^9\left(\right.$$

$$=\frac{1}{5}d^3x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{7}c^2d^3x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^4d^3x^9\left(\right.$$

$$=\frac{1}{5}d^3x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{7}c^2d^3x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^4d^3x^9\left(\right.$$

$$=-\frac{16bd^3\sqrt{1+c^2x^2}}{1155c^5}-\frac{8bd^3\left(1+c^2x^2\right)^{3/2}}{3465c^5}-\frac{2bd^3\left(1+c^2x^2\right)^{5/2}}{1925c^5}-\frac{bd^3}{}$$

Mathematica [A] time = 0.11, size = 143, normalized size = 0.63

$$\frac{d^3\left(3465ac^5x^5\left(105c^6x^6+385c^4x^4+495c^2x^2+231\right)+3465bc^5x^5\left(105c^6x^6+385c^4x^4+495c^2x^2+231\right)\sinh^{-1}\right.}{4002075c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^3*(3465*a*c^5*x^5*(231 + 495*c^2*x^2 + 385*c^4*x^4 + 105*c^6*x^6) - b*Sq
rt[1 + c^2*x^2]*(50488 - 25244*c^2*x^2 + 18933*c^4*x^4 + 117625*c^6*x^6 + 1
11475*c^8*x^8 + 33075*c^10*x^10) + 3465*b*c^5*x^5*(231 + 495*c^2*x^2 + 385*
c^4*x^4 + 105*c^6*x^6)*ArcSinh[c*x]))/(4002075*c^5)
```

fricas [A] time = 0.61, size = 201, normalized size = 0.89

$$\frac{363825\,ac^{11}d^3x^{11}+1334025\,ac^9d^3x^9+1715175\,ac^7d^3x^7+800415\,ac^5d^3x^5+3465\left(105\,bc^{11}d^3x^{11}+385\,bc^9d^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/4002075*(363825*a*c^11*d^3*x^11 + 1334025*a*c^9*d^3*x^9 + 1715175*a*c^7*d
^3*x^7 + 800415*a*c^5*d^3*x^5 + 3465*(105*b*c^11*d^3*x^11 + 385*b*c^9*d^3*x
^9 + 495*b*c^7*d^3*x^7 + 231*b*c^5*d^3*x^5)*log(c*x + sqrt(c^2*x^2 + 1)) -
(33075*b*c^10*d^3*x^10 + 111475*b*c^8*d^3*x^8 + 117625*b*c^6*d^3*x^6 + 1893
3*b*c^4*d^3*x^4 - 25244*b*c^2*d^3*x^2 + 50488*b*d^3)*sqrt(c^2*x^2 + 1))/c^5
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.02, size = 206, normalized size = 0.91

$$\frac{d^3a\left(\frac{1}{11}c^{11}x^{11}+\frac{1}{3}c^9x^9+\frac{3}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+d^3b\left(\frac{\operatorname{arcsinh}(cx)c^{11}x^{11}}{11}+\frac{\operatorname{arcsinh}(cx)c^9x^9}{3}+\frac{3\operatorname{arcsinh}(cx)c^7x^7}{7}+\frac{\operatorname{arcsinh}(cx)c^5x^5}{5}-\frac{c}{}\right.}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c^5*(d^3*a*(1/11*c^11*x^11+1/3*c^9*x^9+3/7*c^7*x^7+1/5*c^5*x^5)+d^3*b*(1/
11*arcsinh(c*x)*c^11*x^11+1/3*arcsinh(c*x)*c^9*x^9+3/7*arcsinh(c*x)*c^7*x^7
+1/5*arcsinh(c*x)*c^5*x^5-1/121*c^10*x^10*(c^2*x^2+1)^(1/2)-91/3267*c^8*x^8
*(c^2*x^2+1)^(1/2)-4705/160083*c^6*x^6*(c^2*x^2+1)^(1/2)-6311/1334025*c^4*x
^4*(c^2*x^2+1)^(1/2)+25244/4002075*c^2*x^2*(c^2*x^2+1)^(1/2)-50488/4002075*
(c^2*x^2+1)^(1/2)))
```

maxima [B] time = 0.49, size = 465, normalized size = 2.06

$$\frac{1}{11}ac^6d^3x^{11}+\frac{1}{3}ac^4d^3x^9+\frac{3}{7}ac^2d^3x^7+\frac{1}{7623}\left(693\,x^{11}\operatorname{arsinh}(cx)-\left(\frac{63\sqrt{c^2x^2+1}\,x^{10}}{c^2}-\frac{70\sqrt{c^2x^2+1}\,x^8}{c^4}+\frac{80\sqrt{c^2x}}{c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/11*a*c^6*d^3*x^11 + 1/3*a*c^4*d^3*x^9 + 3/7*a*c^2*d^3*x^7 + 1/7623*(693*x
^11*arcsinh(c*x) - (63*sqrt(c^2*x^2 + 1)*x^10/c^2 - 70*sqrt(c^2*x^2 + 1)*x^
8/c^4 + 80*sqrt(c^2*x^2 + 1)*x^6/c^6 - 96*sqrt(c^2*x^2 + 1)*x^4/c^8 + 128*s
qrt(c^2*x^2 + 1)*x^2/c^10 - 256*sqrt(c^2*x^2 + 1)/c^12)*c)*b*c^6*d^3 + 1/94
5*(315*x^9*arcsinh(c*x) - (35*sqrt(c^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c^2*x^2 +
 1)*x^6/c^4 + 48*sqrt(c^2*x^2 + 1)*x^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x^2/c^8 +
 128*sqrt(c^2*x^2 + 1)/c^10)*c)*b*c^4*d^3 + 1/5*a*d^3*x^5 + 3/245*(35*x^7*a
rcsinh(c*x) - (5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 +
8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*b*c^2*d^3 + 1/75
*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*
x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*b*d^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(d\,c^2\,x^2+d\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*asinh(c*x))*(d + c^2*d*x^2)^3,x)
```

```
[Out] int(x^4*(a + b*asinh(c*x))*(d + c^2*d*x^2)^3, x)
```

sympy [A] time = 39.15, size = 289, normalized size = 1.28

$$\begin{cases}\frac{ac^6d^3x^{11}}{11}+\frac{ac^4d^3x^9}{3}+\frac{3ac^2d^3x^7}{7}+\frac{ad^3x^5}{5}+\frac{bc^6d^3x^{11}\operatorname{asinh}(cx)}{11}-\frac{bc^5d^3x^{10}\sqrt{c^2x^2+1}}{121}+\frac{bc^4d^3x^9\operatorname{asinh}(cx)}{3}-\frac{91bc^3d^3x^8\sqrt{c^2x^2+1}}{3267}+\frac{3bc^2d^3}{} \\ \frac{ad^3x^5}{5}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c**2*d*x**2+d)**3*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**6*d**3*x**11/11 + a*c**4*d**3*x**9/3 + 3*a*c**2*d**3*x**7/7
 + a*d**3*x**5/5 + b*c**6*d**3*x**11*asinh(c*x)/11 - b*c**5*d**3*x**10*sqrt
(c**2*x**2 + 1)/121 + b*c**4*d**3*x**9*asinh(c*x)/3 - 91*b*c**3*d**3*x**8*s
qrt(c**2*x**2 + 1)/3267 + 3*b*c**2*d**3*x**7*asinh(c*x)/7 - 4705*b*c*d**3*x
**6*sqrt(c**2*x**2 + 1)/160083 + b*d**3*x**5*asinh(c*x)/5 - 6311*b*d**3*x**
4*sqrt(c**2*x**2 + 1)/(1334025*c) + 25244*b*d**3*x**2*sqrt(c**2*x**2 + 1)/(
4002075*c**3) - 50488*b*d**3*sqrt(c**2*x**2 + 1)/(4002075*c**5), Ne(c, 0)),
 (a*d**3*x**5/5, True))
```

3.20 $\int x^3 \left(d+c^2 d x^2\right)^3 \left(a+b \sinh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=199

$$\frac{d^3 \left(c^2x^2+1\right)^5 \left(a+b\sinh^{-1}(cx)\right)}{10c^4}-\frac{d^3 \left(c^2x^2+1\right)^4 \left(a+b\sinh^{-1}(cx)\right)}{8c^4}+\frac{49bd^3\sinh^{-1}(cx)}{5120c^4}-\frac{bd^3x\left(c^2x^2+1\right)^{9/2}}{100c^3}+\frac{7bd^3}{}$$

```
[Out] 49/7680*b*d^3*x*(c^2*x^2+1)^(3/2)/c^3+49/9600*b*d^3*x*(c^2*x^2+1)^(5/2)/c^3
+7/1600*b*d^3*x*(c^2*x^2+1)^(7/2)/c^3-1/100*b*d^3*x*(c^2*x^2+1)^(9/2)/c^3+4
9/5120*b*d^3*arcsinh(c*x)/c^4-1/8*d^3*(c^2*x^2+1)^4*(a+b*arcsinh(c*x))/c^4+
1/10*d^3*(c^2*x^2+1)^5*(a+b*arcsinh(c*x))/c^4+49/5120*b*d^3*x*(c^2*x^2+1)^(
1/2)/c^3
```

Rubi [A] time = 0.17, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {266, 43, 5730, 12, 388, 195, 215}

$$\frac{d^3 \left(c^2x^2+1\right)^5 \left(a+b\sinh^{-1}(cx)\right)}{10c^4}-\frac{d^3 \left(c^2x^2+1\right)^4 \left(a+b\sinh^{-1}(cx)\right)}{8c^4}-\frac{bd^3x\left(c^2x^2+1\right)^{9/2}}{100c^3}+\frac{7bd^3x\left(c^2x^2+1\right)^{7/2}}{1600c^3}+\frac{49}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (49*b*d^3*x*Sqrt[1 + c^2*x^2])/(5120*c^3) + (49*b*d^3*x*(1 + c^2*x^2)^(3/2)
)/(7680*c^3) + (49*b*d^3*x*(1 + c^2*x^2)^(5/2))/(9600*c^3) + (7*b*d^3*x*(1
+ c^2*x^2)^(7/2))/(1600*c^3) - (b*d^3*x*(1 + c^2*x^2)^(9/2))/(100*c^3) + (4
9*b*d^3*ArcSinh[c*x])/(5120*c^4) - (d^3*(1 + c^2*x^2)^4*(a + b*ArcSinh[c*x]
))/(8*c^4) + (d^3*(1 + c^2*x^2)^5*(a + b*ArcSinh[c*x]))/(10*c^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\int x^3 \left(d+c^2dx^2\right)^3 \left(a+b\sinh^{-1}(cx)\right) dx = -\frac{d^3\left(1+c^2x^2\right)^4\left(a+b\sinh^{-1}(cx)\right)}{8c^4}+\frac{d^3\left(1+c^2x^2\right)^5\left(a+b\sinh^{-1}(c\right.}{10c^4}$$

$$= -\frac{d^3\left(1+c^2x^2\right)^4\left(a+b\sinh^{-1}(cx)\right)}{8c^4}+\frac{d^3\left(1+c^2x^2\right)^5\left(a+b\sinh^{-1}(c\right.}{10c^4}$$

$$= -\frac{bd^3x\left(1+c^2x^2\right)^{9/2}}{100c^3}-\frac{d^3\left(1+c^2x^2\right)^4\left(a+b\sinh^{-1}(cx)\right)}{8c^4}+\frac{d^3\left(1+c\right.}{}$$

$$= \frac{7bd^3x\left(1+c^2x^2\right)^{7/2}}{1600c^3}-\frac{bd^3x\left(1+c^2x^2\right)^{9/2}}{100c^3}-\frac{d^3\left(1+c^2x^2\right)^4\left(a+b\sin\right.}{8c^4}$$

$$= \frac{49bd^3x\left(1+c^2x^2\right)^{5/2}}{9600c^3}+\frac{7bd^3x\left(1+c^2x^2\right)^{7/2}}{1600c^3}-\frac{bd^3x\left(1+c^2x^2\right)^{9/2}}{100c^3}-$$

$$= \frac{49bd^3x\left(1+c^2x^2\right)^{3/2}}{7680c^3}+\frac{49bd^3x\left(1+c^2x^2\right)^{5/2}}{9600c^3}+\frac{7bd^3x\left(1+c^2x^2\right)^{7/2}}{1600c^3}$$

$$= \frac{49bd^3x\sqrt{1+c^2x^2}}{5120c^3}+\frac{49bd^3x\left(1+c^2x^2\right)^{3/2}}{7680c^3}+\frac{49bd^3x\left(1+c^2x^2\right)^{5/2}}{9600c^3}+$$

$$= \frac{49bd^3x\sqrt{1+c^2x^2}}{5120c^3}+\frac{49bd^3x\left(1+c^2x^2\right)^{3/2}}{7680c^3}+\frac{49bd^3x\left(1+c^2x^2\right)^{5/2}}{9600c^3}+$$

Mathematica [A] time = 0.11, size = 139, normalized size = 0.70

$$\frac{d^3\left(1920ac^4x^4\left(4c^6x^6+15c^4x^4+20c^2x^2+10\right)+15b\left(512c^{10}x^{10}+1920c^8x^8+2560c^6x^6+1280c^4x^4-79\right)\sinh\right.}{76800c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^3*(1920*a*c^4*x^4*(10 + 20*c^2*x^2 + 15*c^4*x^4 + 4*c^6*x^6) - b*c*x*Sqr
t[1 + c^2*x^2]*(-1185 + 790*c^2*x^2 + 3208*c^4*x^4 + 2736*c^6*x^6 + 768*c^8
*x^8) + 15*b*(-79 + 1280*c^4*x^4 + 2560*c^6*x^6 + 1920*c^8*x^8 + 512*c^10*x
^10)*ArcSinh[c*x]))/(76800*c^4)
```

fricas [A] time = 0.54, size = 197, normalized size = 0.99

$$\frac{7680\,ac^{10}d^3x^{10} + 28800\,ac^8d^3x^8 + 38400\,ac^6d^3x^6 + 19200\,ac^4d^3x^4 + 15\left(512\,bc^{10}d^3x^{10} + 1920\,bc^8d^3x^8 + 2560\,bc\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/76800*(7680*a*c^10*d^3*x^10 + 28800*a*c^8*d^3*x^8 + 38400*a*c^6*d^3*x^6 +
 19200*a*c^4*d^3*x^4 + 15*(512*b*c^10*d^3*x^10 + 1920*b*c^8*d^3*x^8 + 2560*
b*c^6*d^3*x^6 + 1280*b*c^4*d^3*x^4 - 79*b*d^3)*log(c*x + sqrt(c^2*x^2 + 1))
 - (768*b*c^9*d^3*x^9 + 2736*b*c^7*d^3*x^7 + 3208*b*c^5*d^3*x^5 + 790*b*c^3
*d^3*x^3 - 1185*b*c*d^3*x)*sqrt(c^2*x^2 + 1))/c^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.02, size = 195, normalized size = 0.98

$$\frac{d^3a\left(\frac{1}{10}c^{10}x^{10} + \frac{3}{8}c^8x^8 + \frac{1}{2}c^6x^6 + \frac{1}{4}c^4x^4\right) + d^3b\left(\frac{\operatorname{arcsinh}(cx)c^{10}x^{10}}{10} + \frac{3\operatorname{arcsinh}(cx)c^8x^8}{8} + \frac{\operatorname{arcsinh}(cx)c^6x^6}{2} + \frac{\operatorname{arcsinh}(cx)c^4x^4}{4} - \right.}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c^4*(d^3*a*(1/10*c^10*x^10+3/8*c^8*x^8+1/2*c^6*x^6+1/4*c^4*x^4)+d^3*b*(1/
10*arcsinh(c*x)*c^10*x^10+3/8*arcsinh(c*x)*c^8*x^8+1/2*arcsinh(c*x)*c^6*x^6
+1/4*arcsinh(c*x)*c^4*x^4-1/100*c^9*x^9*(c^2*x^2+1)^(1/2)-57/1600*c^7*x^7*(
c^2*x^2+1)^(1/2)-401/9600*c^5*x^5*(c^2*x^2+1)^(1/2)-79/7680*c^3*x^3*(c^2*x^
2+1)^(1/2)+79/5120*c*x*(c^2*x^2+1)^(1/2)-79/5120*arcsinh(c*x)))
```

maxima [B] time = 0.40, size = 429, normalized size = 2.16

$$\frac{1}{10}\,ac^6d^3x^{10} + \frac{3}{8}\,ac^4d^3x^8 + \frac{1}{2}\,ac^2d^3x^6 + \frac{1}{12800}\left(1280\,x^{10}\operatorname{arsinh}(cx) - \left(\frac{128\,\sqrt{c^2x^2+1}\,x^9}{c^2} - \frac{144\,\sqrt{c^2x^2+1}\,x^7}{c^4} + \frac{168}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/10*a*c^6*d^3*x^10 + 3/8*a*c^4*d^3*x^8 + 1/2*a*c^2*d^3*x^6 + 1/12800*(1280
*x^10*arcsinh(c*x) - (128*sqrt(c^2*x^2 + 1)*x^9/c^2 - 144*sqrt(c^2*x^2 + 1)
*x^7/c^4 + 168*sqrt(c^2*x^2 + 1)*x^5/c^6 - 210*sqrt(c^2*x^2 + 1)*x^3/c^8 +
315*sqrt(c^2*x^2 + 1)*x/c^10 - 315*arcsinh(c*x)/c^11)*c)*b*c^6*d^3 + 1/1024
*(384*x^8*arcsinh(c*x) - (48*sqrt(c^2*x^2 + 1)*x^7/c^2 - 56*sqrt(c^2*x^2 +
1)*x^5/c^4 + 70*sqrt(c^2*x^2 + 1)*x^3/c^6 - 105*sqrt(c^2*x^2 + 1)*x/c^8 + 1
05*arcsinh(c*x)/c^9)*c)*b*c^4*d^3 + 1/4*a*d^3*x^4 + 1/96*(48*x^6*arcsinh(c*
x) - (8*sqrt(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(
c^2*x^2 + 1)*x/c^6 - 15*arcsinh(c*x)/c^7)*c)*b*c^2*d^3 + 1/32*(8*x^4*arcsin
```

```
h(c*x) - (2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsi
nh(c*x)/c^5)*c)*b*d^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \, (a + b \operatorname{asinh}(c\,x)) \left(d\,c^2\,x^2 + d\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^3,x)
```

```
[Out] int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^3, x)
```

sympy [A] time = 28.24, size = 280, normalized size = 1.41

$$\begin{cases} \frac{ac^6d^3x^{10}}{10} + \frac{3ac^4d^3x^8}{8} + \frac{ac^2d^3x^6}{2} + \frac{ad^3x^4}{4} + \frac{bc^6d^3x^{10}\operatorname{asinh}(cx)}{10} - \frac{bc^5d^3x^9\sqrt{c^2x^2+1}}{100} + \frac{3bc^4d^3x^8\operatorname{asinh}(cx)}{8} - \frac{57bc^3d^3x^7\sqrt{c^2x^2+1}}{1600} + bc \\ \frac{ad^3x^4}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*d*x**2+d)**3*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**6*d**3*x**10/10 + 3*a*c**4*d**3*x**8/8 + a*c**2*d**3*x**6/2
 + a*d**3*x**4/4 + b*c**6*d**3*x**10*asinh(c*x)/10 - b*c**5*d**3*x**9*sqrt(
c**2*x**2 + 1)/100 + 3*b*c**4*d**3*x**8*asinh(c*x)/8 - 57*b*c**3*d**3*x**7*
sqrt(c**2*x**2 + 1)/1600 + b*c**2*d**3*x**6*asinh(c*x)/2 - 401*b*c*d**3*x**
5*sqrt(c**2*x**2 + 1)/9600 + b*d**3*x**4*asinh(c*x)/4 - 79*b*d**3*x**3*sqrt
(c**2*x**2 + 1)/(7680*c) + 79*b*d**3*x*sqrt(c**2*x**2 + 1)/(5120*c**3) - 79
*b*d**3*asinh(c*x)/(5120*c**4), Ne(c, 0)), (a*d**3*x**4/4, True))
```

3.21 $\int x^2 \left(d+c^2 d x^2\right)^3 \left(a+b \sinh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=202

$$\frac{1}{9}c^6d^3x^9\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{7}c^4d^3x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}c^2d^3x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}d^3x^3\left(a+b\sinh^{-1}(cx)\right)-$$

```
[Out] 8/945*b*d^3*(c^2*x^2+1)^(3/2)/c^3+2/525*b*d^3*(c^2*x^2+1)^(5/2)/c^3+1/441*b
*d^3*(c^2*x^2+1)^(7/2)/c^3-1/81*b*d^3*(c^2*x^2+1)^(9/2)/c^3+1/3*d^3*x^3*(a+
b*arcsinh(c*x))+3/5*c^2*d^3*x^5*(a+b*arcsinh(c*x))+3/7*c^4*d^3*x^7*(a+b*arc
sinh(c*x))+1/9*c^6*d^3*x^9*(a+b*arcsinh(c*x))+16/315*b*d^3*(c^2*x^2+1)^(1/2
)/c^3
```

Rubi [A] time = 0.25, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {270, 5730, 12, 1799, 1620}

$$\frac{1}{9}c^6d^3x^9\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{7}c^4d^3x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}c^2d^3x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}d^3x^3\left(a+b\sinh^{-1}(cx)\right)-$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (16*b*d^3*Sqrt[1 + c^2*x^2])/(315*c^3) + (8*b*d^3*(1 + c^2*x^2)^(3/2))/(945
*c^3) + (2*b*d^3*(1 + c^2*x^2)^(5/2))/(525*c^3) + (b*d^3*(1 + c^2*x^2)^(7/2
))/(441*c^3) - (b*d^3*(1 + c^2*x^2)^(9/2))/(81*c^3) + (d^3*x^3*(a + b*ArcSi
nh[c*x]))/3 + (3*c^2*d^3*x^5*(a + b*ArcSinh[c*x]))/5 + (3*c^4*d^3*x^7*(a +
b*ArcSinh[c*x]))/7 + (c^6*d^3*x^9*(a + b*ArcSinh[c*x]))/9
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
```

```
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\int x^2 \left(d+c^2dx^2\right)^3 \left(a+b\sinh^{-1}(cx)\right) dx = \frac{1}{3}d^3x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}c^2d^3x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{7}c^4d^3x^7\left(\right.$$

$$= \frac{1}{3}d^3x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}c^2d^3x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{7}c^4d^3x^7\left(\right.$$

$$= \frac{1}{3}d^3x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}c^2d^3x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{7}c^4d^3x^7\left(\right.$$

$$= \frac{1}{3}d^3x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}c^2d^3x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{7}c^4d^3x^7\left(\right.$$

$$= \frac{16bd^3\sqrt{1+c^2x^2}}{315c^3}+\frac{8bd^3\left(1+c^2x^2\right)^{3/2}}{945c^3}+\frac{2bd^3\left(1+c^2x^2\right)^{5/2}}{525c^3}+\frac{bd^3\left(\right.}{}$$

Mathematica [A] time = 0.10, size = 135, normalized size = 0.67

$$\frac{d^3\left(315ac^3x^3\left(35c^6x^6+135c^4x^4+189c^2x^2+105\right)-b\sqrt{c^2x^2+1}\left(1225c^8x^8+4675c^6x^6+6297c^4x^4+2629c^2x\right.\right.}{99225c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^3*(315*a*c^3*x^3*(105 + 189*c^2*x^2 + 135*c^4*x^4 + 35*c^6*x^6) - b*Sqrt
[1 + c^2*x^2]*(-5258 + 2629*c^2*x^2 + 6297*c^4*x^4 + 4675*c^6*x^6 + 1225*c^
8*x^8) + 315*b*c^3*x^3*(105 + 189*c^2*x^2 + 135*c^4*x^4 + 35*c^6*x^6)*ArcSi
nh[c*x]))/(99225*c^3)
```

fricas [A] time = 0.61, size = 189, normalized size = 0.94

$$\frac{11025\,ac^9d^3x^9+42525\,ac^7d^3x^7+59535\,ac^5d^3x^5+33075\,ac^3d^3x^3+315\left(35\,bc^9d^3x^9+135\,bc^7d^3x^7+189\,bc^5d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/99225*(11025*a*c^9*d^3*x^9 + 42525*a*c^7*d^3*x^7 + 59535*a*c^5*d^3*x^5 +
33075*a*c^3*d^3*x^3 + 315*(35*b*c^9*d^3*x^9 + 135*b*c^7*d^3*x^7 + 189*b*c^5
*d^3*x^5 + 105*b*c^3*d^3*x^3)*log(c*x + sqrt(c^2*x^2 + 1)) - (1225*b*c^8*d^
3*x^8 + 4675*b*c^6*d^3*x^6 + 6297*b*c^4*d^3*x^4 + 2629*b*c^2*d^3*x^2 - 5258
*b*d^3)*sqrt(c^2*x^2 + 1))/c^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 187, normalized size = 0.93

$$\frac{d^3a\left(\frac{1}{9}c^9x^9+\frac{3}{7}c^7x^7+\frac{3}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+d^3b\left(\frac{\operatorname{arcsinh}(cx)c^9x^9}{9}+\frac{3\operatorname{arcsinh}(cx)c^7x^7}{7}+\frac{3\operatorname{arcsinh}(cx)c^5x^5}{5}+\frac{\operatorname{arcsinh}(cx)c^3x^3}{3}-\frac{c^8x^8}{}\right.}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c^3*(d^3*a*(1/9*c^9*x^9+3/7*c^7*x^7+3/5*c^5*x^5+1/3*c^3*x^3)+d^3*b*(1/9*a
rcsinh(c*x)*c^9*x^9+3/7*arcsinh(c*x)*c^7*x^7+3/5*arcsinh(c*x)*c^5*x^5+1/3*a
rcsinh(c*x)*c^3*x^3-1/81*c^8*x^8*(c^2*x^2+1)^(1/2)-187/3969*c^6*x^6*(c^2*x^
2+1)^(1/2)-2099/33075*c^4*x^4*(c^2*x^2+1)^(1/2)-2629/99225*c^2*x^2*(c^2*x^2
+1)^(1/2)+5258/99225*(c^2*x^2+1)^(1/2)))
```

maxima [B] time = 0.64, size = 388, normalized size = 1.92

$$\frac{1}{9}\,ac^6d^3x^9+\frac{3}{7}\,ac^4d^3x^7+\frac{1}{2835}\left(315\,x^9\operatorname{arsinh}(cx)-\left(\frac{35\,\sqrt{c^2x^2+1}\,x^8}{c^2}-\frac{40\,\sqrt{c^2x^2+1}\,x^6}{c^4}+\frac{48\,\sqrt{c^2x^2+1}\,x^4}{c^6}-\frac{64\,\sqrt{}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/9*a*c^6*d^3*x^9 + 3/7*a*c^4*d^3*x^7 + 1/2835*(315*x^9*arcsinh(c*x) - (35*
sqrt(c^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(c^2*x^2
+ 1)*x^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(c^2*x^2 + 1)/c^10)*c
)*b*c^6*d^3 + 3/5*a*c^2*d^3*x^5 + 3/245*(35*x^7*arcsinh(c*x) - (5*sqrt(c^2*
x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^
6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*b*c^4*d^3 + 1/25*(15*x^5*arcsinh(c*x) - (3
*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 +
 1)/c^6)*c)*b*c^2*d^3 + 1/3*a*d^3*x^3 + 1/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c
^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*d^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(a+b\,\operatorname{asinh}\left(c\,x\right)\right)\,{\left(d\,c^2\,x^2+d\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^3,x)
```

```
[Out] int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^3, x)
```

sympy [A] time = 16.75, size = 265, normalized size = 1.31

$$\begin{cases}\frac{ac^6d^3x^9}{9}+\frac{3ac^4d^3x^7}{7}+\frac{3ac^2d^3x^5}{5}+\frac{ad^3x^3}{3}+\frac{bc^6d^3x^9\operatorname{asinh}(cx)}{9}-\frac{bc^5d^3x^8\sqrt{c^2x^2+1}}{81}+\frac{3bc^4d^3x^7\operatorname{asinh}(cx)}{7}-\frac{187bc^3d^3x^6\sqrt{c^2x^2+1}}{3969}+\frac{3bc^2d}{} \\ \frac{ad^3x^3}{3}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)**3*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**6*d**3*x**9/9 + 3*a*c**4*d**3*x**7/7 + 3*a*c**2*d**3*x**5/5
 + a*d**3*x**3/3 + b*c**6*d**3*x**9*asinh(c*x)/9 - b*c**5*d**3*x**8*sqrt(c*
```

```
*2*x**2 + 1)/81 + 3*b*c**4*d**3*x**7*asinh(c*x)/7 - 187*b*c**3*d**3*x**6*sq
rt(c**2*x**2 + 1)/3969 + 3*b*c**2*d**3*x**5*asinh(c*x)/5 - 2099*b*c*d**3*x*
*4*sqrt(c**2*x**2 + 1)/33075 + b*d**3*x**3*asinh(c*x)/3 - 2629*b*d**3*x**2*
sqrt(c**2*x**2 + 1)/(99225*c) + 5258*b*d**3*sqrt(c**2*x**2 + 1)/(99225*c**3
), Ne(c, 0)), (a*d**3*x**3/3, True))
```

3.22 $\int x \left(d + c^2 d x^2\right)^3 \left(a + b \sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=145

$$\frac{d^3\left(c^2x^2+1\right)^4\left(a+b\sinh^{-1}(cx)\right)}{8c^2}-\frac{bd^3x\left(c^2x^2+1\right)^{7/2}}{64c}-\frac{7bd^3x\left(c^2x^2+1\right)^{5/2}}{384c}-\frac{35bd^3x\left(c^2x^2+1\right)^{3/2}}{1536c}-\frac{35bd^3x\sqrt{c^2x^2+}}{1024c}$$

```
[Out] -35/1536*b*d^3*x*(c^2*x^2+1)^(3/2)/c-7/384*b*d^3*x*(c^2*x^2+1)^(5/2)/c-1/64
*b*d^3*x*(c^2*x^2+1)^(7/2)/c-35/1024*b*d^3*arcsinh(c*x)/c^2+1/8*d^3*(c^2*x^
2+1)^4*(a+b*arcsinh(c*x))/c^2-35/1024*b*d^3*x*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] time = 0.07, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {5717, 195, 215}

$$\frac{d^3\left(c^2x^2+1\right)^4\left(a+b\sinh^{-1}(cx)\right)}{8c^2}-\frac{bd^3x\left(c^2x^2+1\right)^{7/2}}{64c}-\frac{7bd^3x\left(c^2x^2+1\right)^{5/2}}{384c}-\frac{35bd^3x\left(c^2x^2+1\right)^{3/2}}{1536c}-\frac{35bd^3x\sqrt{c^2x^2+}}{1024c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-35*b*d^3*x*Sqrt[1 + c^2*x^2])/(1024*c) - (35*b*d^3*x*(1 + c^2*x^2)^(3/2))
/(1536*c) - (7*b*d^3*x*(1 + c^2*x^2)^(5/2))/(384*c) - (b*d^3*x*(1 + c^2*x^2
)^(7/2))/(64*c) - (35*b*d^3*ArcSinh[c*x])/(1024*c^2) + (d^3*(1 + c^2*x^2)^4
*(a + b*ArcSinh[c*x]))/(8*c^2)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\int x\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)\,dx=\frac{d^3\left(1+c^2x^2\right)^4\left(a+b\sinh^{-1}(cx)\right)}{8c^2}-\frac{\left(bd^3\right)\int\left(1+c^2x^2\right)^{7/2}\,dx}{8c}$$
$$=-\frac{bd^3x\left(1+c^2x^2\right)^{7/2}}{64c}+\frac{d^3\left(1+c^2x^2\right)^4\left(a+b\sinh^{-1}(cx)\right)}{8c^2}-\frac{\left(7bd^3\right)\int}{}$$
$$=-\frac{7bd^3x\left(1+c^2x^2\right)^{5/2}}{384c}-\frac{bd^3x\left(1+c^2x^2\right)^{7/2}}{64c}+\frac{d^3\left(1+c^2x^2\right)^4\left(a+b\sin\right.}{8c^2}$$
$$=-\frac{35bd^3x\left(1+c^2x^2\right)^{3/2}}{1536c}-\frac{7bd^3x\left(1+c^2x^2\right)^{5/2}}{384c}-\frac{bd^3x\left(1+c^2x^2\right)^{7/2}}{64c}+$$
$$=-\frac{35bd^3x\sqrt{1+c^2x^2}}{1024c}-\frac{35bd^3x\left(1+c^2x^2\right)^{3/2}}{1536c}-\frac{7bd^3x\left(1+c^2x^2\right)^{5/2}}{384c}-$$
$$=-\frac{35bd^3x\sqrt{1+c^2x^2}}{1024c}-\frac{35bd^3x\left(1+c^2x^2\right)^{3/2}}{1536c}-\frac{7bd^3x\left(1+c^2x^2\right)^{5/2}}{384c}-$$

Mathematica [A] time = 0.16, size = 128, normalized size = 0.88

$$\frac{d^3\left(cx\left(384acx\left(c^6x^6+4c^4x^4+6c^2x^2+4\right)-b\sqrt{c^2x^2+1}\left(48c^6x^6+200c^4x^4+326c^2x^2+279\right)\right)+3b\left(128c^8x^8\right.}{3072c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^3*(c*x*(384*a*c*x*(4 + 6*c^2*x^2 + 4*c^4*x^4 + c^6*x^6) - b*Sqrt[1 + c^2
*x^2]*(279 + 326*c^2*x^2 + 200*c^4*x^4 + 48*c^6*x^6)) + 3*b*(93 + 512*c^2*x
^2 + 768*c^4*x^4 + 512*c^6*x^6 + 128*c^8*x^8)*ArcSinh[c*x]))/(3072*c^2)
```

fricas [A] time = 0.63, size = 185, normalized size = 1.28

$$\frac{384\,ac^8d^3x^8+1536\,ac^6d^3x^6+2304\,ac^4d^3x^4+1536\,ac^2d^3x^2+3\left(128\,bc^8d^3x^8+512\,bc^6d^3x^6+768\,bc^4d^3x^4+5\right.}{307}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/3072*(384*a*c^8*d^3*x^8 + 1536*a*c^6*d^3*x^6 + 2304*a*c^4*d^3*x^4 + 1536*
a*c^2*d^3*x^2 + 3*(128*b*c^8*d^3*x^8 + 512*b*c^6*d^3*x^6 + 768*b*c^4*d^3*x^
4 + 512*b*c^2*d^3*x^2 + 93*b*d^3)*log(c*x + sqrt(c^2*x^2 + 1)) - (48*b*c^7*
d^3*x^7 + 200*b*c^5*d^3*x^5 + 326*b*c^3*d^3*x^3 + 279*b*c*d^3*x)*sqrt(c^2*x
^2 + 1))/c^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 176, normalized size = 1.21

$$\frac{d^3a\left(\frac{1}{8}c^8x^8+\frac{1}{2}c^6x^6+\frac{3}{4}c^4x^4+\frac{1}{2}c^2x^2\right)+d^3b\left(\frac{\operatorname{arcsinh}(cx)c^8x^8}{8}+\frac{\operatorname{arcsinh}(cx)c^6x^6}{2}+\frac{3\operatorname{arcsinh}(cx)c^4x^4}{4}+\frac{\operatorname{arcsinh}(cx)c^2x^2}{2}-\frac{c^7x^7\sqrt{c}}{6}\right.}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c^2*(d^3*a*(1/8*c^8*x^8+1/2*c^6*x^6+3/4*c^4*x^4+1/2*c^2*x^2)+d^3*b*(1/8*a
rcsinh(c*x)*c^8*x^8+1/2*arcsinh(c*x)*c^6*x^6+3/4*arcsinh(c*x)*c^4*x^4+1/2*a
rcsinh(c*x)*c^2*x^2-1/64*c^7*x^7*(c^2*x^2+1)^(1/2)-25/384*c^5*x^5*(c^2*x^2+
1)^(1/2)-163/1536*c^3*x^3*(c^2*x^2+1)^(1/2)-93/1024*c*x*(c^2*x^2+1)^(1/2)+9
3/1024*arcsinh(c*x)))
```

maxima [B] time = 0.47, size = 352, normalized size = 2.43

$$\frac{1}{8}ac^6d^3x^8+\frac{1}{2}ac^4d^3x^6+\frac{1}{3072}\left(384\,x^8\operatorname{arsinh}(cx)-\left(\frac{48\sqrt{c^2x^2+1}\,x^7}{c^2}-\frac{56\sqrt{c^2x^2+1}\,x^5}{c^4}+\frac{70\sqrt{c^2x^2+1}\,x^3}{c^6}-105\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*c^6*d^3*x^8 + 1/2*a*c^4*d^3*x^6 + 1/3072*(384*x^8*arcsinh(c*x) - (48*
sqrt(c^2*x^2 + 1)*x^7/c^2 - 56*sqrt(c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(c^2*x^2
+ 1)*x^3/c^6 - 105*sqrt(c^2*x^2 + 1)*x/c^8 + 105*arcsinh(c*x)/c^9)*c)*b*c^6
*d^3 + 3/4*a*c^2*d^3*x^4 + 1/96*(48*x^6*arcsinh(c*x) - (8*sqrt(c^2*x^2 + 1)
*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 + 1)*x/c^6 - 15*a
rcsinh(c*x)/c^7)*c)*b*c^4*d^3 + 3/32*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^2
+ 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*b*c^2*d^3
 + 1/2*a*d^3*x^2 + 1/4*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x/c^2 - a
rcsinh(c*x)/c^3))*b*d^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\left(a+b\operatorname{asinh}\left(c\,x\right)\right)\,{\left(d\,c^2\,x^2+d\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2)^3,x)
```

```
[Out] int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2)^3, x)
```

sympy [A] time = 11.42, size = 253, normalized size = 1.74

$$\begin{cases}\frac{ac^6d^3x^8}{8}+\frac{ac^4d^3x^6}{2}+\frac{3ac^2d^3x^4}{4}+\frac{ad^3x^2}{2}+\frac{bc^6d^3x^8\operatorname{asinh}(cx)}{8}-\frac{bc^5d^3x^7\sqrt{c^2x^2+1}}{64}+\frac{bc^4d^3x^6\operatorname{asinh}(cx)}{2}-\frac{25bc^3d^3x^5\sqrt{c^2x^2+1}}{384}+\frac{3bc^2d^3x^4}{} \\ \frac{ad^3x^2}{2}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*d*x**2+d)**3*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**6*d**3*x**8/8 + a*c**4*d**3*x**6/2 + 3*a*c**2*d**3*x**4/4 +
 a*d**3*x**2/2 + b*c**6*d**3*x**8*asinh(c*x)/8 - b*c**5*d**3*x**7*sqrt(c**2
*x**2 + 1)/64 + b*c**4*d**3*x**6*asinh(c*x)/2 - 25*b*c**3*d**3*x**5*sqrt(c*
*2*x**2 + 1)/384 + 3*b*c**2*d**3*x**4*asinh(c*x)/4 - 163*b*c*d**3*x**3*sqrt
(c**2*x**2 + 1)/1536 + b*d**3*x**2*asinh(c*x)/2 - 93*b*d**3*x*sqrt(c**2*x**
2 + 1)/(1024*c) + 93*b*d**3*asinh(c*x)/(1024*c**2), Ne(c, 0)), (a*d**3*x**2
/2, True))
```

3.23 $\int \left(d+c^2dx^2\right)^3 \left(a+b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=170

$$\frac{1}{7}c^6d^3x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}c^4d^3x^5\left(a+b\sinh^{-1}(cx)\right)+c^2d^3x^3\left(a+b\sinh^{-1}(cx)\right)+d^3x\left(a+b\sinh^{-1}(cx)\right)-\frac{bc}{}$$

```
[Out] -8/105*b*d^3*(c^2*x^2+1)^(3/2)/c-6/175*b*d^3*(c^2*x^2+1)^(5/2)/c-1/49*b*d^3
*(c^2*x^2+1)^(7/2)/c+d^3*x*(a+b*arcsinh(c*x))+c^2*d^3*x^3*(a+b*arcsinh(c*x)
)+3/5*c^4*d^3*x^5*(a+b*arcsinh(c*x))+1/7*c^6*d^3*x^7*(a+b*arcsinh(c*x))-16/
35*b*d^3*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] time = 0.16, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {194, 5679, 12, 1799, 1850}

$$\frac{1}{7}c^6d^3x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}c^4d^3x^5\left(a+b\sinh^{-1}(cx)\right)+c^2d^3x^3\left(a+b\sinh^{-1}(cx)\right)+d^3x\left(a+b\sinh^{-1}(cx)\right)-\frac{bc}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-16*b*d^3*Sqrt[1 + c^2*x^2])/(35*c) - (8*b*d^3*(1 + c^2*x^2)^(3/2))/(105*c
) - (6*b*d^3*(1 + c^2*x^2)^(5/2))/(175*c) - (b*d^3*(1 + c^2*x^2)^(7/2))/(49
*c) + d^3*x*(a + b*ArcSinh[c*x]) + c^2*d^3*x^3*(a + b*ArcSinh[c*x]) + (3*c^
4*d^3*x^5*(a + b*ArcSinh[c*x]))/5 + (c^6*d^3*x^7*(a + b*ArcSinh[c*x]))/7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rule 5679

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(d+c^2dx^2\right)^3 \left(a+b\sinh^{-1}(cx)\right)\,dx = d^3x\left(a+b\sinh^{-1}(cx)\right)+c^2d^3x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}c^4d^3x^5\left(a+b\sinh\right.$$
$$= d^3x\left(a+b\sinh^{-1}(cx)\right)+c^2d^3x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}c^4d^3x^5\left(a+b\sinh\right.$$
$$= d^3x\left(a+b\sinh^{-1}(cx)\right)+c^2d^3x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}c^4d^3x^5\left(a+b\sinh\right.$$
$$= d^3x\left(a+b\sinh^{-1}(cx)\right)+c^2d^3x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}c^4d^3x^5\left(a+b\sinh\right.$$
$$= -\frac{16bd^3\sqrt{1+c^2x^2}}{35c}-\frac{8bd^3\left(1+c^2x^2\right)^{3/2}}{105c}-\frac{6bd^3\left(1+c^2x^2\right)^{5/2}}{175c}-\frac{bd^3\left(1+c\right.}{49c}$$

Mathematica [A] time = 0.15, size = 119, normalized size = 0.70

$$\frac{d^3\left(105acx\left(5c^6x^6+21c^4x^4+35c^2x^2+35\right)-b\sqrt{c^2x^2+1}\left(75c^6x^6+351c^4x^4+757c^2x^2+2161\right)+105bcx\left(5c^6x^6\right.\right.}{3675c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^3*(105*a*c*x*(35 + 35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6) - b*Sqrt[1 + c^2
*x^2]*(2161 + 757*c^2*x^2 + 351*c^4*x^4 + 75*c^6*x^6) + 105*b*c*x*(35 + 35*
c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6)*ArcSinh[c*x]))/(3675*c)
```

fricas [A] time = 0.61, size = 169, normalized size = 0.99

$$\frac{525\,ac^7d^3x^7+2205\,ac^5d^3x^5+3675\,ac^3d^3x^3+3675\,acd^3x+105\left(5\,bc^7d^3x^7+21\,bc^5d^3x^5+35\,bc^3d^3x^3+35\,bcd^3x\right.}{3675\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/3675*(525*a*c^7*d^3*x^7 + 2205*a*c^5*d^3*x^5 + 3675*a*c^3*d^3*x^3 + 3675*
a*c*d^3*x + 105*(5*b*c^7*d^3*x^7 + 21*b*c^5*d^3*x^5 + 35*b*c^3*d^3*x^3 + 35
*b*c*d^3*x)*log(c*x + sqrt(c^2*x^2 + 1)) - (75*b*c^6*d^3*x^6 + 351*b*c^4*d^
3*x^4 + 757*b*c^2*d^3*x^2 + 2161*b*d^3)*sqrt(c^2*x^2 + 1))/c
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 156, normalized size = 0.92

$$\frac{d^3a\left(\frac{1}{7}c^7x^7+\frac{3}{5}c^5x^5+c^3x^3+cx\right)+d^3b\left(\frac{\operatorname{arcsinh}(cx)c^7x^7}{7}+\frac{3\operatorname{arcsinh}(cx)c^5x^5}{5}+\operatorname{arcsinh}(cx)\,c^3x^3+\operatorname{arcsinh}(cx)\,cx-\frac{c^6x^6}{\ldots}\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c*(d^3*a*(1/7*c^7*x^7+3/5*c^5*x^5+c^3*x^3+c*x)+d^3*b*(1/7*arcsinh(c*x)*c^
7*x^7+3/5*arcsinh(c*x)*c^5*x^5+arcsinh(c*x)*c^3*x^3+arcsinh(c*x)*c*x-1/49*c
^6*x^6*(c^2*x^2+1)^(1/2)-117/1225*c^4*x^4*(c^2*x^2+1)^(1/2)-757/3675*c^2*x^
2*(c^2*x^2+1)^(1/2)-2161/3675*(c^2*x^2+1)^(1/2)))
```

maxima [B] time = 0.55, size = 301, normalized size = 1.77

$$\frac{1}{7}\,ac^6d^3x^7+\frac{3}{5}\,ac^4d^3x^5+\frac{1}{245}\left(35\,x^7\,\mathrm{arsinh}\,(cx)-\left(\frac{5\,\sqrt{c^2x^2+1}\,x^6}{c^2}-\frac{6\,\sqrt{c^2x^2+1}\,x^4}{c^4}+\frac{8\,\sqrt{c^2x^2+1}\,x^2}{c^6}-\frac{16\,\sqrt{c^2x}}{c^8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*c^6*d^3*x^7 + 3/5*a*c^4*d^3*x^5 + 1/245*(35*x^7*arcsinh(c*x) - (5*sqr
t(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*
x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*b*c^6*d^3 + 1/25*(15*x^5*arcsinh(c*x
) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2
*x^2 + 1)/c^6)*c)*b*c^4*d^3 + a*c^2*d^3*x^3 + 1/3*(3*x^3*arcsinh(c*x) - c*(
sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*c^2*d^3 + a*d^3*x +
 (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*b*d^3/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\mathrm{asinh}\,(c\,x))\,\left(d\,c^2\,x^2+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + c^2*d*x^2)^3,x)
```

```
[Out] int((a + b*asinh(c*x))*(d + c^2*d*x^2)^3, x)
```

sympy [A] time = 6.23, size = 221, normalized size = 1.30

$$\begin{cases} \frac{ac^6d^3x^7}{7}+\frac{3ac^4d^3x^5}{5}+ac^2d^3x^3+ad^3x+\frac{bc^6d^3x^7\,\mathrm{asinh}\,(cx)}{7}-\frac{bc^5d^3x^6\,\sqrt{c^2x^2+1}}{49}+\frac{3bc^4d^3x^5\,\mathrm{asinh}\,(cx)}{5}-\frac{117bc^3d^3x^4\,\sqrt{c^2x^2+1}}{1225}+b \\ ad^3x \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**6*d**3*x**7/7 + 3*a*c**4*d**3*x**5/5 + a*c**2*d**3*x**3 + a
*d**3*x + b*c**6*d**3*x**7*asinh(c*x)/7 - b*c**5*d**3*x**6*sqrt(c**2*x**2 +
 1)/49 + 3*b*c**4*d**3*x**5*asinh(c*x)/5 - 117*b*c**3*d**3*x**4*sqrt(c**2*x
**2 + 1)/1225 + b*c**2*d**3*x**3*asinh(c*x) - 757*b*c*d**3*x**2*sqrt(c**2*x
**2 + 1)/3675 + b*d**3*x*asinh(c*x) - 2161*b*d**3*sqrt(c**2*x**2 + 1)/(3675
*c), Ne(c, 0)), (a*d**3*x, True))
```

3.24 $\int \frac{\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=221

$$\frac{1}{6}d^3\left(c^2x^2+1\right)^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}d^3\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{2}d^3\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)+\frac{d^3\left(a\right.}{}$$

```
[Out] -7/72*b*c*d^3*x*(c^2*x^2+1)^(3/2)-1/36*b*c*d^3*x*(c^2*x^2+1)^(5/2)-19/48*b*
d^3*arcsinh(c*x)+1/2*d^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))+1/4*d^3*(c^2*x^2+1)
^2*(a+b*arcsinh(c*x))+1/6*d^3*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))+1/2*d^3*(a+b
*arcsinh(c*x))^2/b+d^3*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^2)
-1/2*b*d^3*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)-19/48*b*c*d^3*x*(c^2*x^2+
1)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5726, 5659, 3716, 2190, 2279, 2391, 195, 215}

$$\frac{1}{2}bd^3\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)+\frac{1}{6}d^3\left(c^2x^2+1\right)^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}d^3\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{2}d^3\left(c^2x^2\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (-19*b*c*d^3*x*Sqrt[1 + c^2*x^2])/48 - (7*b*c*d^3*x*(1 + c^2*x^2)^(3/2))/72
 - (b*c*d^3*x*(1 + c^2*x^2)^(5/2))/36 - (19*b*d^3*ArcSinh[c*x])/48 + (d^3*(
1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/2 + (d^3*(1 + c^2*x^2)^2*(a + b*ArcSinh[
c*x]))/4 + (d^3*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x]))/6 - (d^3*(a + b*ArcSi
nh[c*x])^2)/(2*b) + d^3*(a + b*ArcSinh[c*x])*Log[1 - E^(2*ArcSinh[c*x])] +
(b*d^3*PolyLog[2, E^(2*ArcSinh[c*x])])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5726

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[((d + e*x^2)^p*(a + b*ArcSinh[c*x]))/(2*p), x] + (Dist[d
, Int[((d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]))/x, x], x] - Dist[(b*c*d^p)
/(2*p), Int[(1 + c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)}{x}\,dx = \frac{1}{6}d^3\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)+d\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(\right.}{x}$$

$$= -\frac{1}{36}bcd^3x\left(1+c^2x^2\right)^{5/2}+\frac{1}{4}d^3\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{6}d^3\left(1\right.$$

$$= -\frac{7}{72}bcd^3x\left(1+c^2x^2\right)^{3/2}-\frac{1}{36}bcd^3x\left(1+c^2x^2\right)^{5/2}+\frac{1}{2}d^3\left(1+c^2x^2\right)\left(a+\right.$$

$$= -\frac{19}{48}bcd^3x\sqrt{1+c^2x^2}-\frac{7}{72}bcd^3x\left(1+c^2x^2\right)^{3/2}-\frac{1}{36}bcd^3x\left(1+c^2x^2\right)^{5/2}$$

$$= -\frac{19}{48}bcd^3x\sqrt{1+c^2x^2}-\frac{7}{72}bcd^3x\left(1+c^2x^2\right)^{3/2}-\frac{1}{36}bcd^3x\left(1+c^2x^2\right)^{5/2}$$

$$= -\frac{19}{48}bcd^3x\sqrt{1+c^2x^2}-\frac{7}{72}bcd^3x\left(1+c^2x^2\right)^{3/2}-\frac{1}{36}bcd^3x\left(1+c^2x^2\right)^{5/2}$$

$$= -\frac{19}{48}bcd^3x\sqrt{1+c^2x^2}-\frac{7}{72}bcd^3x\left(1+c^2x^2\right)^{3/2}-\frac{1}{36}bcd^3x\left(1+c^2x^2\right)^{5/2}$$

$$= -\frac{19}{48}bcd^3x\sqrt{1+c^2x^2}-\frac{7}{72}bcd^3x\left(1+c^2x^2\right)^{3/2}-\frac{1}{36}bcd^3x\left(1+c^2x^2\right)^{5/2}$$

Mathematica [A] time = 0.12, size = 189, normalized size = 0.86

$$\frac{1}{144}d^3\left(3\sinh^{-1}(cx)\left(-48a+b\left(8c^6x^6+36c^4x^4+72c^2x^2+25\right)+48b\log\left(1-e^{2\sinh^{-1}(cx)}\right)\right)+24ac^6x^6+108a\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (d^3*(216*a*c^2*x^2 + 108*a*c^4*x^4 + 24*a*c^6*x^6 - 75*b*c*x*Sqrt[1 + c^2*
x^2] - 22*b*c^3*x^3*Sqrt[1 + c^2*x^2] - 4*b*c^5*x^5*Sqrt[1 + c^2*x^2] - 72*
b*ArcSinh[c*x]^2 + 144*a*Log[1 - E^(2*ArcSinh[c*x])] + 3*ArcSinh[c*x]*(-48*
a + b*(25 + 72*c^2*x^2 + 36*c^4*x^4 + 8*c^6*x^6) + 48*b*Log[1 - E^(2*ArcSin
h[c*x])]) + 72*b*PolyLog[2, E^(2*ArcSinh[c*x])]))/144
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{ac^6d^3x^6 + 3\,ac^4d^3x^4 + 3\,ac^2d^3x^2 + ad^3 + \left(bc^6d^3x^6 + 3\,bc^4d^3x^4 + 3\,bc^2d^3x^2 + bd^3\right)\mathrm{arsinh}\left(cx\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*c^6*d^3*x^6 + 3*a*c^4*d^3*x^4 + 3*a*c^2*d^3*x^2 + a*d^3 + (b*c^
6*d^3*x^6 + 3*b*c^4*d^3*x^4 + 3*b*c^2*d^3*x^2 + b*d^3)*arcsinh(c*x))/x, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.20, size = 284, normalized size = 1.29

$$\frac{d^3a\,c^6x^6}{6}+\frac{3d^3a\,c^4x^4}{4}+\frac{3d^3a\,c^2x^2}{2}+d^3a\ln\left(cx\right)-\frac{d^3b\,c^5x^5\sqrt{c^2x^2+1}}{36}-\frac{11d^3b\,c^3x^3\sqrt{c^2x^2+1}}{72}-\frac{25bc\,d^3x\sqrt{c^2x^2+1}}{48}+d^3b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x,x)
```

```
[Out] 1/6*d^3*a*c^6*x^6+3/4*d^3*a*c^4*x^4+3/2*d^3*a*c^2*x^2+d^3*a*ln(c*x)-1/36*d^
3*b*c^5*x^5*(c^2*x^2+1)^(1/2)-11/72*d^3*b*c^3*x^3*(c^2*x^2+1)^(1/2)-25/48*b
*c*d^3*x*(c^2*x^2+1)^(1/2)+d^3*b*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+d
^3*b*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+1/6*d^3*b*arcsinh(c*x)*c^6*x^
6+3/4*d^3*b*arcsinh(c*x)*c^4*x^4+3/2*d^3*b*arcsinh(c*x)*c^2*x^2+25/48*b*d^3
*arcsinh(c*x)+d^3*b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+d^3*b*polylog(2,c*x+(
c^2*x^2+1)^(1/2))-1/2*d^3*b*arcsinh(c*x)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}\,ac^6d^3x^6+\frac{3}{4}\,ac^4d^3x^4+\frac{3}{2}\,ac^2d^3x^2+ad^3\log(x)+\int bc^6d^3x^5\log\left(cx+\sqrt{c^2x^2+1}\right)+3\,bc^4d^3x^3\log\left(cx+\sqrt{c^2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/6*a*c^6*d^3*x^6 + 3/4*a*c^4*d^3*x^4 + 3/2*a*c^2*d^3*x^2 + a*d^3*log(x) +
integrate(b*c^6*d^3*x^5*log(c*x + sqrt(c^2*x^2 + 1)) + 3*b*c^4*d^3*x^3*log(
c*x + sqrt(c^2*x^2 + 1)) + 3*b*c^2*d^3*x*log(c*x + sqrt(c^2*x^2 + 1)) + b*d
^3*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{asinh}\,(c\,x))\,\left(d\,c^2\,x^2 + d\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^3)/x,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^3)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^3\left(\int \frac{a}{x}\,dx + \int 3ac^2x\,dx + \int 3ac^4x^3\,dx + \int ac^6x^5\,dx + \int \frac{b\,\mathrm{asinh}\,(cx)}{x}\,dx + \int 3bc^2x\,\mathrm{asinh}\,(cx)\,dx + \int 3bc$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))/x,x)
```

```
[Out] d**3*(Integral(a/x, x) + Integral(3*a*c**2*x, x) + Integral(3*a*c**4*x**3,
x) + Integral(a*c**6*x**5, x) + Integral(b*asinh(c*x)/x, x) + Integral(3*b*
c**2*x*asinh(c*x), x) + Integral(3*b*c**4*x**3*asinh(c*x), x) + Integral(b*
c**6*x**5*asinh(c*x), x))
```

3.25 $\int \frac{\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=160

$$\frac{1}{5}c^6d^3x^5\left(a+b\sinh^{-1}(cx)\right)+c^4d^3x^3\left(a+b\sinh^{-1}(cx)\right)+3c^2d^3x\left(a+b\sinh^{-1}(cx)\right)-\frac{d^3\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{1}{25}bcd^3$$

```
[Out] -1/5*b*c*d^3*(c^2*x^2+1)^(3/2)-1/25*b*c*d^3*(c^2*x^2+1)^(5/2)-d^3*(a+b*arcs
inh(c*x))/x+3*c^2*d^3*x*(a+b*arcsinh(c*x))+c^4*d^3*x^3*(a+b*arcsinh(c*x))+1
/5*c^6*d^3*x^5*(a+b*arcsinh(c*x))-b*c*d^3*arctanh((c^2*x^2+1)^(1/2))-11/5*b
*c*d^3*(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {270, 5730, 12, 1799, 1620, 63, 208}

$$\frac{1}{5}c^6d^3x^5\left(a+b\sinh^{-1}(cx)\right)+c^4d^3x^3\left(a+b\sinh^{-1}(cx)\right)+3c^2d^3x\left(a+b\sinh^{-1}(cx)\right)-\frac{d^3\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{1}{25}bcd^3$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] (-11*b*c*d^3*Sqrt[1 + c^2*x^2])/5 - (b*c*d^3*(1 + c^2*x^2)^(3/2))/5 - (b*c*
d^3*(1 + c^2*x^2)^(5/2))/25 - (d^3*(a + b*ArcSinh[c*x]))/x + 3*c^2*d^3*x*(a
 + b*ArcSinh[c*x]) + c^4*d^3*x^3*(a + b*ArcSinh[c*x]) + (c^6*d^3*x^5*(a + b
*ArcSinh[c*x]))/5 - b*c*d^3*ArcTanh[Sqrt[1 + c^2*x^2]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\int \frac{(d+c^2dx^2)^3 (a+b\sinh^{-1}(cx))}{x^2} dx = -\frac{d^3(a+b\sinh^{-1}(cx))}{x} + 3c^2d^3x(a+b\sinh^{-1}(cx)) + c^4d^3x^3(a+b\text{si}$$

$$= -\frac{d^3(a+b\sinh^{-1}(cx))}{x} + 3c^2d^3x(a+b\sinh^{-1}(cx)) + c^4d^3x^3(a+b\text{si}$$

$$= -\frac{d^3(a+b\sinh^{-1}(cx))}{x} + 3c^2d^3x(a+b\sinh^{-1}(cx)) + c^4d^3x^3(a+b\text{si}$$

$$= -\frac{d^3(a+b\sinh^{-1}(cx))}{x} + 3c^2d^3x(a+b\sinh^{-1}(cx)) + c^4d^3x^3(a+b\text{si}$$

$$= -\frac{11}{5}bcd^3\sqrt{1+c^2x^2} - \frac{1}{5}bcd^3(1+c^2x^2)^{3/2} - \frac{1}{25}bcd^3(1+c^2x^2)^{5/2} - \frac{d^3}{}$$

$$= -\frac{11}{5}bcd^3\sqrt{1+c^2x^2} - \frac{1}{5}bcd^3(1+c^2x^2)^{3/2} - \frac{1}{25}bcd^3(1+c^2x^2)^{5/2} - \frac{d^3}{}$$

$$= -\frac{11}{5}bcd^3\sqrt{1+c^2x^2} - \frac{1}{5}bcd^3(1+c^2x^2)^{3/2} - \frac{1}{25}bcd^3(1+c^2x^2)^{5/2} - \frac{d^3}{}$$

Mathematica [A] time = 0.15, size = 163, normalized size = 1.02

$$\frac{d^3\left(5ac^6x^6 + 25ac^4x^4 + 75ac^2x^2 - 25a - 61bcx\sqrt{c^2x^2+1} - 25bcx\log\left(\sqrt{c^2x^2+1}+1\right) - bc^5x^5\sqrt{c^2x^2+1} - 7b\right.}{25x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] (d^3*(-25*a + 75*a*c^2*x^2 + 25*a*c^4*x^4 + 5*a*c^6*x^6 - 61*b*c*x*Sqrt[1 +
 c^2*x^2] - 7*b*c^3*x^3*Sqrt[1 + c^2*x^2] - b*c^5*x^5*Sqrt[1 + c^2*x^2] + 5
*b*(-5 + 15*c^2*x^2 + 5*c^4*x^4 + c^6*x^6)*ArcSinh[c*x] + 25*b*c*x*Log[x] -
 25*b*c*x*Log[1 + Sqrt[1 + c^2*x^2]]))/(25*x)
```

fricas [A] time = 0.67, size = 276, normalized size = 1.72

$$\frac{5\,ac^6d^3x^6 + 25\,ac^4d^3x^4 + 75\,ac^2d^3x^2 - 25\,bcd^3x\log\left(-cx+\sqrt{c^2x^2+1}+1\right) + 25\,bcd^3x\log\left(-cx+\sqrt{c^2x^2+1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] 1/25*(5*a*c^6*d^3*x^6 + 25*a*c^4*d^3*x^4 + 75*a*c^2*d^3*x^2 - 25*b*c*d^3*x*
log(-c*x + sqrt(c^2*x^2 + 1) + 1) + 25*b*c*d^3*x*log(-c*x + sqrt(c^2*x^2 +
1) - 1) - 5*(b*c^6 + 5*b*c^4 + 15*b*c^2 - 5*b)*d^3*x*log(-c*x + sqrt(c^2*x^
2 + 1)) - 25*a*d^3 + 5*(b*c^6*d^3*x^6 + 5*b*c^4*d^3*x^4 + 15*b*c^2*d^3*x^2
- (b*c^6 + 5*b*c^4 + 15*b*c^2 - 5*b)*d^3*x - 5*b*d^3)*log(c*x + sqrt(c^2*x^
2 + 1)) - (b*c^5*d^3*x^5 + 7*b*c^3*d^3*x^3 + 61*b*c*d^3*x)*sqrt(c^2*x^2 + 1
))/x
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 151, normalized size = 0.94

$$c\left(d^3a\left(\frac{c^5x^5}{5}+c^3x^3+3cx-\frac{1}{cx}\right)+d^3b\left(\frac{\operatorname{arcsinh}(cx)\,c^5x^5}{5}+\operatorname{arcsinh}(cx)\,c^3x^3+3\operatorname{arcsinh}(cx)\,cx-\frac{\operatorname{arcsinh}(cx)}{cx}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^2,x)
```

```
[Out] c*(d^3*a*(1/5*c^5*x^5+c^3*x^3+3*c*x-1/c/x)+d^3*b*(1/5*arcsinh(c*x)*c^5*x^5+
arcsinh(c*x)*c^3*x^3+3*arcsinh(c*x)*c*x-arcsinh(c*x)/c/x-1/25*c^4*x^4*(c^2*
x^2+1)^(1/2)-7/25*c^2*x^2*(c^2*x^2+1)^(1/2)-61/25*(c^2*x^2+1)^(1/2)-arctanh
(1/(c^2*x^2+1)^(1/2))))
```

maxima [A] time = 0.48, size = 231, normalized size = 1.44

$$\frac{1}{5}\,ac^6d^3x^5+\frac{1}{75}\left(15\,x^5\operatorname{arsinh}(cx)-\left(\frac{3\,\sqrt{c^2x^2+1}\,x^4}{c^2}-\frac{4\,\sqrt{c^2x^2+1}\,x^2}{c^4}+\frac{8\,\sqrt{c^2x^2+1}}{c^6}\right)c\right)bc^6d^3+ac^4d^3x^3+\frac{1}{3}\left(3\,x^3\,\mathrm{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*a*c^6*d^3*x^5 + 1/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^
2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*b*c^6*d^3 + a
*c^4*d^3*x^3 + 1/3*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*s
qrt(c^2*x^2 + 1)/c^4))*b*c^4*d^3 + 3*a*c^2*d^3*x + 3*(c*x*arcsinh(c*x) - sq
rt(c^2*x^2 + 1))*b*c*d^3 - (c*arcsinh(1/(c*abs(x))) + arcsinh(c*x)/x)*b*d^3
 - a*d^3/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^3)/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^3)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^3\left(\int 3ac^2\,dx + \int \frac{a}{x^2}\,dx + \int 3ac^4x^2\,dx + \int ac^6x^4\,dx + \int 3bc^2\operatorname{asinh}(cx)\,dx + \int \frac{b\operatorname{asinh}(cx)}{x^2}\,dx + \int 3bc^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))/x**2,x)
```

```
[Out] d**3*(Integral(3*a*c**2, x) + Integral(a/x**2, x) + Integral(3*a*c**4*x**2,
 x) + Integral(a*c**6*x**4, x) + Integral(3*b*c**2*asinh(c*x), x) + Integra
l(b*asinh(c*x)/x**2, x) + Integral(3*b*c**4*x**2*asinh(c*x), x) + Integral(
b*c**6*x**4*asinh(c*x), x))
```

3.26 $\int \frac{\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=249

$$-\frac{d^3\left(c^2x^2+1\right)^3\left(a+b\sinh^{-1}(cx)\right)}{2x^2}+\frac{3}{4}c^2d^3\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{2}c^2d^3\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{}$$

```
[Out] 7/16*b*c^3*d^3*x*(c^2*x^2+1)^(3/2)-1/2*b*c*d^3*(c^2*x^2+1)^(5/2)/x-3/32*b*c
^2*d^3*arcsinh(c*x)+3/2*c^2*d^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))+3/4*c^2*d^3*
(c^2*x^2+1)^2*(a+b*arcsinh(c*x))-1/2*d^3*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))/x
^2+3/2*c^2*d^3*(a+b*arcsinh(c*x))^2/b+3*c^2*d^3*(a+b*arcsinh(c*x))*ln(1-1/(
c*x+(c^2*x^2+1)^(1/2))^2)-3/2*b*c^2*d^3*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))
^2)-3/32*b*c^3*d^3*x*(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5728, 277, 195, 215, 5726, 5659, 3716, 2190, 2279, 2391}

$$\frac{3}{2}bc^2d^3\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)-\frac{d^3\left(c^2x^2+1\right)^3\left(a+b\sinh^{-1}(cx)\right)}{2x^2}+\frac{3}{4}c^2d^3\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{2}c^2d^3\left(\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] (-3*b*c^3*d^3*x*Sqrt[1 + c^2*x^2])/32 + (7*b*c^3*d^3*x*(1 + c^2*x^2)^(3/2))
/16 - (b*c*d^3*(1 + c^2*x^2)^(5/2))/(2*x) - (3*b*c^2*d^3*ArcSinh[c*x])/32 +
 (3*c^2*d^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/2 + (3*c^2*d^3*(1 + c^2*x^2
)^2*(a + b*ArcSinh[c*x]))/4 - (d^3*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x]))/(2
*x^2) - (3*c^2*d^3*(a + b*ArcSinh[c*x])^2)/(2*b) + 3*c^2*d^3*(a + b*ArcSinh
[c*x])*Log[1 - E^(2*ArcSinh[c*x])] + (3*b*c^2*d^3*PolyLog[2, E^(2*ArcSinh[c
*x])])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5726

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[((d + e*x^2)^p*(a + b*ArcSinh[c*x]))/(2*p), x] + (Dist[d
, Int[((d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]))/x, x], x] - Dist[(b*c*d^p)
/(2*p), Int[(1 + c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5728

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*ArcSinh[c
*x]))/(f*(m + 1)), x] + (-Dist[(b*c*d^p)/(f*(m + 1)), Int[(f*x)^(m + 1)*(1
+ c^2*x^2)^(p - 1/2), x], x] - Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2
)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && ILtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int \frac{(d+c^2dx^2)^3(a+b\sinh^{-1}(cx))}{x^3}dx = -\frac{d^3(1+c^2x^2)^3(a+b\sinh^{-1}(cx))}{2x^2} + (3c^2d)\int \frac{(d+c^2dx^2)^2(a+b\sinh}{x}$$

$$= -\frac{bcd^3(1+c^2x^2)^{5/2}}{2x} + \frac{3}{4}c^2d^3(1+c^2x^2)^2(a+b\sinh^{-1}(cx)) - \frac{d^3(1+c^2x}{}$$

$$= \frac{7}{16}bc^3d^3x(1+c^2x^2)^{3/2} - \frac{bcd^3(1+c^2x^2)^{5/2}}{2x} + \frac{3}{2}c^2d^3(1+c^2x^2)(a+b\sin$$

$$= -\frac{3}{32}bc^3d^3x\sqrt{1+c^2x^2} + \frac{7}{16}bc^3d^3x(1+c^2x^2)^{3/2} - \frac{bcd^3(1+c^2x^2)^{5/2}}{2x} + \frac{3}{2}$$

$$= -\frac{3}{32}bc^3d^3x\sqrt{1+c^2x^2} + \frac{7}{16}bc^3d^3x(1+c^2x^2)^{3/2} - \frac{bcd^3(1+c^2x^2)^{5/2}}{2x} - \frac{3}{32}$$

$$= -\frac{3}{32}bc^3d^3x\sqrt{1+c^2x^2} + \frac{7}{16}bc^3d^3x(1+c^2x^2)^{3/2} - \frac{bcd^3(1+c^2x^2)^{5/2}}{2x} - \frac{3}{32}$$

$$= -\frac{3}{32}bc^3d^3x\sqrt{1+c^2x^2} + \frac{7}{16}bc^3d^3x(1+c^2x^2)^{3/2} - \frac{bcd^3(1+c^2x^2)^{5/2}}{2x} - \frac{3}{32}$$

$$= -\frac{3}{32}bc^3d^3x\sqrt{1+c^2x^2} + \frac{7}{16}bc^3d^3x(1+c^2x^2)^{3/2} - \frac{bcd^3(1+c^2x^2)^{5/2}}{2x} - \frac{3}{32}$$

Mathematica [A] time = 0.41, size = 184, normalized size = 0.74

$$\frac{1}{32}d^3\left(8c^6x^4(a+b\sinh^{-1}(cx)) + 48c^4x^2(a+b\sinh^{-1}(cx)) - \frac{48c^2(a+b\sinh^{-1}(cx))^2}{b} + 96c^2\log\left(1-e^{2\sinh^{-1}(cx)}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] (d^3*((-16*b*c*Sqrt[1 + c^2*x^2])/x - 2*b*c^5*x^3*Sqrt[1 + c^2*x^2] - 21*b*
c^2*(c*x*Sqrt[1 + c^2*x^2] - ArcSinh[c*x]) - (16*(a + b*ArcSinh[c*x]))/x^2
+ 48*c^4*x^2*(a + b*ArcSinh[c*x]) + 8*c^6*x^4*(a + b*ArcSinh[c*x]) - (48*c^
2*(a + b*ArcSinh[c*x])^2)/b + 96*c^2*(a + b*ArcSinh[c*x])*Log[1 - E^(2*ArcS
inh[c*x])] + 48*b*c^2*PolyLog[2, E^(2*ArcSinh[c*x])]))/32
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ac^6d^3x^6 + 3\,ac^4d^3x^4 + 3\,ac^2d^3x^2 + ad^3 + (bc^6d^3x^6 + 3\,bc^4d^3x^4 + 3\,bc^2d^3x^2 + bd^3)\,\text{arsinh}\,(cx)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*c^6*d^3*x^6 + 3*a*c^4*d^3*x^4 + 3*a*c^2*d^3*x^2 + a*d^3 + (b*c^
6*d^3*x^6 + 3*b*c^4*d^3*x^4 + 3*b*c^2*d^3*x^2 + b*d^3)*arcsinh(c*x))/x^3, x
)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.39, size = 313, normalized size = 1.26

$$\frac{c^6 d^3 a\, x^4}{4}+\frac{3c^4 d^3 a\, x^2}{2}+3c^2 d^3 a \ln(cx)-\frac{d^3 a}{2x^2}+\frac{d^3 b\, c^2}{2}+\frac{c^6 d^3 b \operatorname{arcsinh}(cx)\, x^4}{4}+\frac{3c^4 d^3 b \operatorname{arcsinh}(cx)\, x^2}{2}-\frac{d^3 b \operatorname{arcsinh}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^3,x)
```

```
[Out] 1/4*c^6*d^3*a*x^4+3/2*c^4*d^3*a*x^2+3*c^2*d^3*a*ln(c*x)-1/2*d^3*a/x^2+1/2*d
^3*b*c^2+1/4*c^6*d^3*b*arcsinh(c*x)*x^4+3/2*c^4*d^3*b*arcsinh(c*x)*x^2-1/2*
d^3*b*arcsinh(c*x)/x^2+21/32*b*c^2*d^3*arcsinh(c*x)-1/2*c*d^3*b/x*(c^2*x^2+
1)^(1/2)+3*c^2*d^3*b*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+3*c^2*d^3*b*a
rcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+3*c^2*d^3*b*polylog(2,c*x+(c^2*x^2+
1)^(1/2))+3*c^2*d^3*b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))-3/2*c^2*d^3*b*arcsi
nh(c*x)^2-1/16*c^5*d^3*b*x^3*(c^2*x^2+1)^(1/2)-21/32*b*c^3*d^3*x*(c^2*x^2+1
)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\, ac^6 d^3 x^4+\frac{3}{2}\, ac^4 d^3 x^2+3\, ac^2 d^3 \log(x)-\frac{1}{2}\, bd^3\left(\frac{\sqrt{c^2x^2+1}\, c}{x}+\frac{\operatorname{arsinh}(cx)}{x^2}\right)-\frac{ad^3}{2\, x^2}+\int bc^6 d^3 x^3 \log\left(cx+\sqrt{c^2x^2+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*a*c^6*d^3*x^4 + 3/2*a*c^4*d^3*x^2 + 3*a*c^2*d^3*log(x) - 1/2*b*d^3*(sqr
t(c^2*x^2 + 1)*c/x + arcsinh(c*x)/x^2) - 1/2*a*d^3/x^2 + integrate(b*c^6*d^
3*x^3*log(c*x + sqrt(c^2*x^2 + 1)) + 3*b*c^4*d^3*x*log(c*x + sqrt(c^2*x^2 +
 1)) + 3*b*c^2*d^3*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\operatorname{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^3)/x^3,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^3)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^3\left(\int \frac{a}{x^3}\,dx+\int \frac{3ac^2}{x}\,dx+\int 3ac^4x\,dx+\int ac^6x^3\,dx+\int \frac{b\operatorname{asinh}(cx)}{x^3}\,dx+\int \frac{3bc^2\operatorname{asinh}(cx)}{x}\,dx+\int 3bc^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))/x**3,x)
```

```
[Out] d**3*(Integral(a/x**3, x) + Integral(3*a*c**2/x, x) + Integral(3*a*c**4*x,
x) + Integral(a*c**6*x**3, x) + Integral(b*asinh(c*x)/x**3, x) + Integral(3
*b*c**2*asinh(c*x)/x, x) + Integral(3*b*c**4*x*asinh(c*x), x) + Integral(b*
c**6*x**3*asinh(c*x), x))
```

3.27 $\int \frac{(d+c^2dx^2)^3(a+b\sinh^{-1}(cx))}{x^4}\,dx$

Optimal. Leaf size=174

$$\frac{1}{3}c^6d^3x^3\left(a+b\sinh^{-1}(cx)\right)+3c^4d^3x\left(a+b\sinh^{-1}(cx)\right)-\frac{3c^2d^3\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{d^3\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{bcd^3\sqrt{c^2}}{6x}$$

```
[Out] -1/9*b*c^3*d^3*(c^2*x^2+1)^(3/2)-1/3*d^3*(a+b*arcsinh(c*x))/x^3-3*c^2*d^3*(
a+b*arcsinh(c*x))/x+3*c^4*d^3*x*(a+b*arcsinh(c*x))+1/3*c^6*d^3*x^3*(a+b*arc
sinh(c*x))-17/6*b*c^3*d^3*arctanh((c^2*x^2+1)^(1/2))-8/3*b*c^3*d^3*(c^2*x^2
+1)^(1/2)-1/6*b*c*d^3*(c^2*x^2+1)^(1/2)/x^2
```

Rubi [A] time = 0.26, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {270, 5730, 12, 1799, 1621, 897, 1153, 208}

$$\frac{1}{3}c^6d^3x^3\left(a+b\sinh^{-1}(cx)\right)+3c^4d^3x\left(a+b\sinh^{-1}(cx)\right)-\frac{3c^2d^3\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{d^3\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{1}{9}bc^3d^3\,($$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] (-8*b*c^3*d^3*Sqrt[1 + c^2*x^2])/3 - (b*c*d^3*Sqrt[1 + c^2*x^2])/(6*x^2) -
(b*c^3*d^3*(1 + c^2*x^2)^(3/2))/9 - (d^3*(a + b*ArcSinh[c*x]))/(3*x^3) - (3
*c^2*d^3*(a + b*ArcSinh[c*x]))/x + 3*c^4*d^3*x*(a + b*ArcSinh[c*x]) + (c^6*
d^3*x^3*(a + b*ArcSinh[c*x]))/3 - (17*b*c^3*d^3*ArcTanh[Sqrt[1 + c^2*x^2]])
/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
```

```
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[(R*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((m + 1)*(b*c
- a*d)), x] + Dist[1/((m + 1)*(b*c - a*d)), Int[(a + b*x)^(m + 1)*(c + d*x)
^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fre
eQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[m, -1] && GtQ[Expon[Px, x],
2]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)}{x^4}\,dx = -\frac{d^3\left(a+b\sinh^{-1}(cx)\right)}{3x^3} - \frac{3c^2d^3\left(a+b\sinh^{-1}(cx)\right)}{x} + 3c^4d^3x\left(a+b\sin\right.$$

$$= -\frac{d^3\left(a+b\sinh^{-1}(cx)\right)}{3x^3} - \frac{3c^2d^3\left(a+b\sinh^{-1}(cx)\right)}{x} + 3c^4d^3x\left(a+b\sin\right.$$

$$= -\frac{d^3\left(a+b\sinh^{-1}(cx)\right)}{3x^3} - \frac{3c^2d^3\left(a+b\sinh^{-1}(cx)\right)}{x} + 3c^4d^3x\left(a+b\sin\right.$$

$$= -\frac{bcd^3\sqrt{1+c^2x^2}}{6x^2} - \frac{d^3\left(a+b\sinh^{-1}(cx)\right)}{3x^3} - \frac{3c^2d^3\left(a+b\sinh^{-1}(cx)\right)}{x}$$

$$= -\frac{bcd^3\sqrt{1+c^2x^2}}{6x^2} - \frac{d^3\left(a+b\sinh^{-1}(cx)\right)}{3x^3} - \frac{3c^2d^3\left(a+b\sinh^{-1}(cx)\right)}{x}$$

$$= -\frac{bcd^3\sqrt{1+c^2x^2}}{6x^2} - \frac{d^3\left(a+b\sinh^{-1}(cx)\right)}{3x^3} - \frac{3c^2d^3\left(a+b\sinh^{-1}(cx)\right)}{x}$$

$$= -\frac{8}{3}bc^3d^3\sqrt{1+c^2x^2} - \frac{bcd^3\sqrt{1+c^2x^2}}{6x^2} - \frac{1}{9}bc^3d^3\left(1+c^2x^2\right)^{3/2} - \frac{d^3\left(a+\right.}{}$$

$$= -\frac{8}{3}bc^3d^3\sqrt{1+c^2x^2} - \frac{bcd^3\sqrt{1+c^2x^2}}{6x^2} - \frac{1}{9}bc^3d^3\left(1+c^2x^2\right)^{3/2} - \frac{d^3\left(a+\right.}{}$$

Mathematica [A] time = 0.15, size = 171, normalized size = 0.98

$$\frac{d^3\left(6ac^6x^6+54ac^4x^4-54ac^2x^2-6a+51bc^3x^3\log(x)-3bcx\sqrt{c^2x^2+1}-2bc^5x^5\sqrt{c^2x^2+1}-50bc^3x^3\sqrt{c^2x^2+1}\right.}{18x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] (d^3*(-6*a - 54*a*c^2*x^2 + 54*a*c^4*x^4 + 6*a*c^6*x^6 - 3*b*c*x*Sqrt[1 + c
^2*x^2] - 50*b*c^3*x^3*Sqrt[1 + c^2*x^2] - 2*b*c^5*x^5*Sqrt[1 + c^2*x^2] +
6*b*(-1 - 9*c^2*x^2 + 9*c^4*x^4 + c^6*x^6)*ArcSinh[c*x] + 51*b*c^3*x^3*Log[
x] - 51*b*c^3*x^3*Log[1 + Sqrt[1 + c^2*x^2]]))/(18*x^3)
```

fricas [A] time = 0.55, size = 289, normalized size = 1.66

$$6\,ac^6d^3x^6+54\,ac^4d^3x^4-51\,bc^3d^3x^3\log\left(-cx+\sqrt{c^2x^2+1}+1\right)+51\,bc^3d^3x^3\log\left(-cx+\sqrt{c^2x^2+1}-1\right)-54\,ac^2\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/18*(6*a*c^6*d^3*x^6 + 54*a*c^4*d^3*x^4 - 51*b*c^3*d^3*x^3*log(-c*x + sqrt
(c^2*x^2 + 1) + 1) + 51*b*c^3*d^3*x^3*log(-c*x + sqrt(c^2*x^2 + 1) - 1) - 5
4*a*c^2*d^3*x^2 - 6*(b*c^6 + 9*b*c^4 - 9*b*c^2 - b)*d^3*x^3*log(-c*x + sqrt
(c^2*x^2 + 1)) - 6*a*d^3 + 6*(b*c^6*d^3*x^6 + 9*b*c^4*d^3*x^4 - 9*b*c^2*d^3
*x^2 - (b*c^6 + 9*b*c^4 - 9*b*c^2 - b)*d^3*x^3 - b*d^3)*log(c*x + sqrt(c^2*
x^2 + 1)) - (2*b*c^5*d^3*x^5 + 50*b*c^3*d^3*x^3 + 3*b*c*d^3*x)*sqrt(c^2*x^2
 + 1))/x^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 155, normalized size = 0.89

$$c^3\left(d^3a\left(\frac{c^3x^3}{3}+3cx-\frac{3}{cx}-\frac{1}{3c^3x^3}\right)+d^3b\left(\frac{\operatorname{arcsinh}(cx)\,c^3x^3}{3}+3\operatorname{arcsinh}(cx)\,cx-\frac{3\operatorname{arcsinh}(cx)}{cx}-\frac{\operatorname{arcsinh}(cx)}{3c^3x^3}\ldots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^4,x)
```

```
[Out] c^3*(d^3*a*(1/3*c^3*x^3+3*c*x-3/c/x-1/3/c^3/x^3)+d^3*b*(1/3*arcsinh(c*x)*c^
3*x^3+3*arcsinh(c*x)*c*x-3*arcsinh(c*x)/c/x-1/3*arcsinh(c*x)/c^3/x^3-1/9*c^
2*x^2*(c^2*x^2+1)^(1/2)-25/9*(c^2*x^2+1)^(1/2)-17/6*arctanh(1/(c^2*x^2+1)^(
1/2))-1/6/c^2/x^2*(c^2*x^2+1)^(1/2)))
```

maxima [A] time = 0.45, size = 208, normalized size = 1.20

$$\frac{1}{3}\,ac^6d^3x^3+\frac{1}{9}\left(3\,x^3\operatorname{arsinh}(cx)-c\left(\frac{\sqrt{c^2x^2+1}\,x^2}{c^2}-\frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)bc^6d^3+3\,ac^4d^3x+3\left(cx\operatorname{arsinh}(cx)-\sqrt{c^2x^2+1}\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*a*c^6*d^3*x^3 + 1/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2
- 2*sqrt(c^2*x^2 + 1)/c^4))*b*c^6*d^3 + 3*a*c^4*d^3*x + 3*(c*x*arcsinh(c*x)
 - sqrt(c^2*x^2 + 1))*b*c^3*d^3 - 3*(c*arcsinh(1/(c*abs(x))) + arcsinh(c*x)
/x)*b*c^2*d^3 + 1/6*((c^2*arcsinh(1/(c*abs(x))) - sqrt(c^2*x^2 + 1)/x^2)*c
- 2*arcsinh(c*x)/x^3)*b*d^3 - 3*a*c^2*d^3/x - 1/3*a*d^3/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{asinh}\,(c\,x))\,\left(d\,c^2\,x^2 + d\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^3)/x^4,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^3)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^3\left(\int 3ac^4\,dx + \int \frac{a}{x^4}\,dx + \int \frac{3ac^2}{x^2}\,dx + \int ac^6x^2\,dx + \int 3bc^4\,\mathrm{asinh}\,(cx)\,dx + \int \frac{b\,\mathrm{asinh}\,(cx)}{x^4}\,dx + \int \frac{3bc^2\,\mathrm{a}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))/x**4,x)
```

```
[Out] d**3*(Integral(3*a*c**4, x) + Integral(a/x**4, x) + Integral(3*a*c**2/x**2,
 x) + Integral(a*c**6*x**2, x) + Integral(3*b*c**4*asinh(c*x), x) + Integra
l(b*asinh(c*x)/x**4, x) + Integral(3*b*c**2*asinh(c*x)/x**2, x) + Integral(
b*c**6*x**2*asinh(c*x), x))
```

3.28 $\int \frac{x^4(a+b\sinh^{-1}(cx))}{d+c^2dx^2}\,dx$

Optimal. Leaf size=156

$$\frac{2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^5d}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4d}+\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2d}-\frac{ib\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{c^5d}+\frac{ib\text{Li}_2\left(\right.}{}$$

```
[Out] -1/9*b*(c^2*x^2+1)^(3/2)/c^5/d-x*(a+b*arcsinh(c*x))/c^4/d+1/3*x^3*(a+b*arcs
inh(c*x))/c^2/d+2*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))/c^5/d-I*
b*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c^5/d+I*b*polylog(2,I*(c*x+(c^2*x^2
+1)^(1/2)))/c^5/d+4/3*b*(c^2*x^2+1)^(1/2)/c^5/d
```

Rubi [A] time = 0.24, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5767, 5693, 4180, 2279, 2391, 261, 266, 43}

$$-\frac{ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{c^5d}+\frac{ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{c^5d}+\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2d}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4d}+\frac{2\tan^{-1}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

```
[Out] (4*b*Sqrt[1 + c^2*x^2])/(3*c^5*d) - (b*(1 + c^2*x^2)^(3/2))/(9*c^5*d) - (x*
(a + b*ArcSinh[c*x]))/(c^4*d) + (x^3*(a + b*ArcSinh[c*x]))/(3*c^2*d) + (2*(
a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c^5*d) - (I*b*PolyLog[2, (-I)*
E^ArcSinh[c*x]])/(c^5*d) + (I*b*PolyLog[2, I*E^ArcSinh[c*x]])/(c^5*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5767

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(e*(m + 2*p + 1)), x] + (-Dist[(f^2*(m - 1))/(c^2*(m +
 2*p + 1)), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(c*(m + 2*p + 1)*(1 + c
^2*x^2)^FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcS
inh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2
*d] && GtQ[n, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx = \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2d} - \frac{\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx}{c^2} - \frac{b\int \frac{x^3}{\sqrt{1+c^2x^2}}\,dx}{3cd}$$

$$= -\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4d} + \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2d} + \frac{\int \frac{a+b\sinh^{-1}(cx)}{d+c^2dx^2}\,dx}{c^4} + \frac{b\int \frac{x}{\sqrt{1+c^2x^2}}}{c^3d}$$

$$= \frac{b\sqrt{1+c^2x^2}}{c^5d} - \frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4d} + \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2d} + \frac{\text{Subst}\left(\int(a+bx)\right.}{}$$

$$= \frac{4b\sqrt{1+c^2x^2}}{3c^5d} - \frac{b\left(1+c^2x^2\right)^{3/2}}{9c^5d} - \frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4d} + \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2d} +$$

$$= \frac{4b\sqrt{1+c^2x^2}}{3c^5d} - \frac{b\left(1+c^2x^2\right)^{3/2}}{9c^5d} - \frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4d} + \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2d} +$$

$$= \frac{4b\sqrt{1+c^2x^2}}{3c^5d} - \frac{b\left(1+c^2x^2\right)^{3/2}}{9c^5d} - \frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4d} + \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2d} +$$

Mathematica [A] time = 0.28, size = 170, normalized size = 1.09

$$\frac{3ac^3x^3 - 9acx + 9a\tan^{-1}(cx) + 3bc^3x^3\sinh^{-1}(cx) - bc^2x^2\sqrt{c^2x^2+1} + 11b\sqrt{c^2x^2+1} - 9ib\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right) +}{9}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

```
[Out] (-9*a*c*x + 3*a*c^3*x^3 + 11*b*Sqrt[1 + c^2*x^2] - b*c^2*x^2*Sqrt[1 + c^2*x
^2] - 9*b*c*x*ArcSinh[c*x] + 3*b*c^3*x^3*ArcSinh[c*x] + 9*a*ArcTan[c*x] + (
```

```
9*I)*b*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - (9*I)*b*ArcSinh[c*x]*Log[1
+ I*E^ArcSinh[c*x]] - (9*I)*b*PolyLog[2, (-I)*E^ArcSinh[c*x]] + (9*I)*b*Pol
yLog[2, I*E^ArcSinh[c*x]])/(9*c^5*d)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^4 \operatorname{arsinh}(cx) + ax^4}{c^2dx^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arcsinh(c*x) + a*x^4)/(c^2*d*x^2 + d), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.14, size = 266, normalized size = 1.71

$$\frac{a\,x^3}{3c^2d}-\frac{ax}{c^4d}+\frac{a\arctan(cx)}{c^5d}+\frac{b\operatorname{arcsinh}(cx)\,x^3}{3c^2d}-\frac{b\operatorname{arcsinh}(cx)\,x}{c^4d}+\frac{b\operatorname{arcsinh}(cx)\arctan(cx)}{c^5d}-\frac{b\,x^2\sqrt{c^2x^2+1}}{9c^3d}+\frac{11b\sqrt{}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x)
```

```
[Out] 1/3/c^2*a/d*x^3-1/c^4*a/d*x+1/c^5*a/d*arctan(c*x)+1/3/c^2*b/d*arcsinh(c*x)*
x^3-1/c^4*b/d*arcsinh(c*x)*x+1/c^5*b/d*arcsinh(c*x)*arctan(c*x)-1/9/c^3*b/d
*x^2*(c^2*x^2+1)^(1/2)+11/9*b*(c^2*x^2+1)^(1/2)/c^5/d+1/c^5*b/d*arctan(c*x)
*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/c^5*b/d*arctan(c*x)*ln(1-I*(1+I*c*x)
/(c^2*x^2+1)^(1/2))-I/c^5*b/d*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I/c^5*
b/d*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\,a\left(\frac{c^2x^3 - 3\,x}{c^4d} + \frac{3\,\arctan(cx)}{c^5d}\right) + b\int \frac{x^4\log\left(cx + \sqrt{c^2x^2 + 1}\right)}{c^2dx^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/3*a*((c^2*x^3 - 3*x)/(c^4*d) + 3*arctan(c*x)/(c^5*d)) + b*integrate(x^4*l
og(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\left(a+b\,\mathrm{asinh}(c\,x)\right)}{d\,c^2\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2),x)
```

```
[Out] int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^4}{c^2x^2+1}\,dx + \int \frac{bx^4 \operatorname{asinh}(cx)}{c^2x^2+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a*x**4/(c**2*x**2 + 1), x) + Integral(b*x**4*asinh(c*x)/(c**2*x**
2 + 1), x))/d
```

3.29 $\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx$

Optimal. Leaf size=135

$$\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^4d}-\frac{\log\left(e^{2\sinh^{-1}(cx)}+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c^4d}+\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{2c^2d}-\frac{b\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{2c^4d}+\frac{b\sinh}{4c}$$

```
[Out] 1/4*b*arcsinh(c*x)/c^4/d+1/2*x^2*(a+b*arcsinh(c*x))/c^2/d+1/2*(a+b*arcsinh(
c*x))^2/b/c^4/d-(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c^4/d-1/
2*b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c^4/d-1/4*b*x*(c^2*x^2+1)^(1/2)/c
^3/d
```

Rubi [A] time = 0.20, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5767, 5714, 3718, 2190, 2279, 2391, 321, 215}

$$-\frac{b\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{2c^4d}+\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{2c^2d}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^4d}-\frac{\log\left(e^{2\sinh^{-1}(cx)}+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

```
[Out] -(b*x*Sqrt[1 + c^2*x^2])/(4*c^3*d) + (b*ArcSinh[c*x])/(4*c^4*d) + (x^2*(a +
 b*ArcSinh[c*x]))/(2*c^2*d) + (a + b*ArcSinh[c*x])^2/(2*b*c^4*d) - ((a + b*
ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(c^4*d) - (b*PolyLog[2, -E^(2*Ar
cSinh[c*x])])/(2*c^4*d)
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5767

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(e*(m + 2*p + 1)), x] + (-Dist[(f^2*(m - 1))/(c^2*(m +
 2*p + 1)), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(c*(m + 2*p + 1)*(1 + c
^2*x^2)^FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcS
inh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2
*d] && GtQ[n, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^3 \left(a + b\sinh^{-1}(cx)\right)}{d + c^2dx^2}\,dx = \frac{x^2\left(a + b\sinh^{-1}(cx)\right)}{2c^2d} - \frac{\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx}{c^2} - \frac{b\int \frac{x^2}{\sqrt{1+c^2x^2}}\,dx}{2cd}$$

$$= -\frac{bx\sqrt{1+c^2x^2}}{4c^3d} + \frac{x^2\left(a + b\sinh^{-1}(cx)\right)}{2c^2d} - \frac{\text{Subst}\left(\int (a+bx)\tanh(x)\,dx, x, \sinh^{-}\right.}{c^4d}$$

$$= -\frac{bx\sqrt{1+c^2x^2}}{4c^3d} + \frac{b\sinh^{-1}(cx)}{4c^4d} + \frac{x^2\left(a + b\sinh^{-1}(cx)\right)}{2c^2d} + \frac{\left(a + b\sinh^{-1}(cx)\right)^2}{2bc^4d} -$$

$$= -\frac{bx\sqrt{1+c^2x^2}}{4c^3d} + \frac{b\sinh^{-1}(cx)}{4c^4d} + \frac{x^2\left(a + b\sinh^{-1}(cx)\right)}{2c^2d} + \frac{\left(a + b\sinh^{-1}(cx)\right)^2}{2bc^4d} -$$

$$= -\frac{bx\sqrt{1+c^2x^2}}{4c^3d} + \frac{b\sinh^{-1}(cx)}{4c^4d} + \frac{x^2\left(a + b\sinh^{-1}(cx)\right)}{2c^2d} + \frac{\left(a + b\sinh^{-1}(cx)\right)^2}{2bc^4d} -$$

$$= -\frac{bx\sqrt{1+c^2x^2}}{4c^3d} + \frac{b\sinh^{-1}(cx)}{4c^4d} + \frac{x^2\left(a + b\sinh^{-1}(cx)\right)}{2c^2d} + \frac{\left(a + b\sinh^{-1}(cx)\right)^2}{2bc^4d} -$$

Mathematica [A] time = 0.19, size = 181, normalized size = 1.34

$$-\frac{-2ac^2x^2 + 2a\log\left(c^2x^2+1\right) + 4b\text{Li}_2\left(\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right) + 4b\text{Li}_2\left(\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right) + bcx\sqrt{c^2x^2+1} - 2bc^2x^2\sinh^{-1}(cx}{4c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

```
[Out] -1/4*(-2*a*c^2*x^2 + b*c*x*Sqrt[1 + c^2*x^2] - b*ArcSinh[c*x] - 2*b*c^2*x^2
*ArcSinh[c*x] - 2*b*ArcSinh[c*x]^2 + 4*b*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[
c*x])/Sqrt[-c^2]] + 4*b*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c]
```

```
 + 2*a*Log[1 + c^2*x^2] + 4*b*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] + 4
*b*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/(c^4*d)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{bx^3 \operatorname{arsinh}(cx) + ax^3}{c^2dx^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arcsinh(c*x) + a*x^3)/(c^2*d*x^2 + d), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.12, size = 161, normalized size = 1.19

$$\frac{a\,x^2}{2c^2d} - \frac{a\ln\left(c^2x^2+1\right)}{2c^4d} + \frac{b\operatorname{arcsinh}(cx)^2}{2c^4d} + \frac{b\operatorname{arcsinh}(cx)\,x^2}{2c^2d} - \frac{bx\sqrt{c^2x^2+1}}{4c^3d} + \frac{b\operatorname{arcsinh}(cx)}{4c^4d} - \frac{b\operatorname{arcsinh}(cx)\ln\left(1+\left(\right.\right.}{c^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x)
```

```
[Out] 1/2/c^2*a/d*x^2-1/2/c^4*a/d*ln(c^2*x^2+1)+1/2/c^4*b/d*arcsinh(c*x)^2+1/2/c^
2*b/d*arcsinh(c*x)*x^2-1/4*b*x*(c^2*x^2+1)^(1/2)/c^3/d+1/4*b*arcsinh(c*x)/c
^4/d-1/c^4*b/d*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-1/2*b*polylog(2
,-(c*x+(c^2*x^2+1)^(1/2))^2)/c^4/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}a\left(\frac{x^2}{c^2d} - \frac{\log\left(c^2x^2+1\right)}{c^4d}\right) - \frac{1}{8}b\left(\frac{2\,c^2x^2 - \log\left(c^2x^2+1\right)^2 - 4\left(c^2x^2 - \log\left(c^2x^2+1\right)\right)\log\left(cx + \sqrt{c^2x^2+1}\right) - 2\log}{c^4d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/2*a*(x^2/(c^2*d) - log(c^2*x^2 + 1)/(c^4*d)) - 1/8*b*((2*c^2*x^2 - log(c^
2*x^2 + 1)^2 - 4*(c^2*x^2 - log(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))
- 2*log(c^2*x^2 + 1))/(c^4*d) - 8*integrate(-1/2*(c^2*x^2 - log(c^2*x^2 + 1
))/(c^6*d*x^3 + c^4*d*x + (c^5*d*x^2 + c^3*d)*sqrt(c^2*x^2 + 1)), x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{d\,c^2\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2),x)
```

```
[Out] int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^3}{c^2x^2+1}\,dx + \int \frac{bx^3\operatorname{asinh}(cx)}{c^2x^2+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a*x**3/(c**2*x**2 + 1), x) + Integral(b*x**3*asinh(c*x)/(c**2*x**
2 + 1), x))/d
```

3.30 $\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx$

Optimal. Leaf size=108

$$-\frac{2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^3d}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^2d}+\frac{ib\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{c^3d}-\frac{ib\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)}{c^3d}-\frac{b\sqrt{c^2x^2+}}{c^3d}$$

```
[Out] x*(a+b*arcsinh(c*x))/c^2/d-2*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2
))/c^3/d+I*b*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d-I*b*polylog(2,I*(c
*x+(c^2*x^2+1)^(1/2)))/c^3/d-b*(c^2*x^2+1)^(1/2)/c^3/d
```

Rubi [A] time = 0.14, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5767, 5693, 4180, 2279, 2391, 261}

$$\frac{ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{c^3d}-\frac{ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{c^3d}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^2d}-\frac{2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(c\right.}{c^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

```
[Out] -((b*Sqrt[1 + c^2*x^2])/(c^3*d)) + (x*(a + b*ArcSinh[c*x]))/(c^2*d) - (2*(a
 + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c^3*d) + (I*b*PolyLog[2, (-I)*E
^ArcSinh[c*x]])/(c^3*d) - (I*b*PolyLog[2, I*E^ArcSinh[c*x]])/(c^3*d)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5767

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(e*(m + 2*p + 1)), x] + (-Dist[(f^2*(m - 1))/(c^2*(m +
 2*p + 1)), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(c*(m + 2*p + 1)*(1 + c
^2*x^2)^FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcS
inh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2
*d] && GtQ[n, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx = \frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^2d} - \frac{\int \frac{a+b\sinh^{-1}(cx)}{d+c^2dx^2}\,dx}{c^2} - \frac{b\int \frac{x}{\sqrt{1+c^2x^2}}\,dx}{cd}$$

$$= -\frac{b\sqrt{1+c^2x^2}}{c^3d} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^2d} - \frac{\text{Subst}\left(\int (a+bx)\text{sech}(x)\,dx, x, \sinh^{-1}(c\right.}{c^3d}$$

$$= -\frac{b\sqrt{1+c^2x^2}}{c^3d} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^2d} - \frac{2\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{c^3d} +$$

$$= -\frac{b\sqrt{1+c^2x^2}}{c^3d} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^2d} - \frac{2\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{c^3d} +$$

$$= -\frac{b\sqrt{1+c^2x^2}}{c^3d} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^2d} - \frac{2\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{c^3d} +$$

Mathematica [A] time = 0.15, size = 121, normalized size = 1.12

$$\frac{acx - a\tan^{-1}(cx) - b\sqrt{c^2x^2+1} + ib\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right) - ib\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right) + bcx\sinh^{-1}(cx) - ib\sinh^{-1}(cx)\log}{c^3d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

```
[Out] (a*c*x - b*Sqrt[1 + c^2*x^2] + b*c*x*ArcSinh[c*x] - a*ArcTan[c*x] - I*b*Arc
Sinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + I*b*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[
c*x]] + I*b*PolyLog[2, (-I)*E^ArcSinh[c*x]] - I*b*PolyLog[2, I*E^ArcSinh[c*
x]])/(c^3*d)
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^2\,\text{arsinh}(cx) + ax^2}{c^2dx^2+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arcsinh(c*x) + a*x^2)/(c^2*d*x^2 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arsinh}(cx) + a)x^2}{c^2dx^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^2/(c^2*d*x^2 + d), x)
```

maple [A] time = 0.01, size = 215, normalized size = 1.99

$$\frac{ax}{c^2d}-\frac{a\arctan(cx)}{c^3d}-\frac{b\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)}{c^3d}+\frac{b\arctan(cx)\ln\left(1-\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)}{c^3d}+\frac{ib\,\mathrm{dilog}\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)}{c^3d}-\frac{ib\,\mathrm{dil}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x)
```

```
[Out] 1/c^2*a/d*x-1/c^3*a/d*arctan(c*x)-1/c^3*b/d*arctan(c*x)*ln(1+I*(1+I*c*x)/(c
^2*x^2+1)^(1/2))+1/c^3*b/d*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+
I/c^3*b/d*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-I/c^3*b/d*dilog(1-I*(1+I*c
*x)/(c^2*x^2+1)^(1/2))-1/c^3*b/d*arcsinh(c*x)*arctan(c*x)+1/c^2*b/d*arcsinh
(c*x)*x-b*(c^2*x^2+1)^(1/2)/c^3/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\frac{x}{c^2d}-\frac{\arctan(cx)}{c^3d}\right)+b\int\frac{x^2\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^2dx^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] a*(x/(c^2*d) - arctan(c*x)/(c^3*d)) + b*integrate(x^2*log(c*x + sqrt(c^2*x^
2 + 1))/(c^2*d*x^2 + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^2\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{d\,c^2\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2),x)
```

```
[Out] int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{ax^2}{c^2x^2+1}\,dx+\int\frac{bx^2\operatorname{asinh}(cx)}{c^2x^2+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a*x**2/(c**2*x**2 + 1), x) + Integral(b*x**2*asinh(c*x)/(c**2*x**
2 + 1), x))/d
```

3.31 $\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx$

Optimal. Leaf size=73

$$-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^2d}+\frac{\log\left(e^{2\sinh^{-1}(cx)}+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c^2d}+\frac{b\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{2c^2d}$$

```
[Out] -1/2*(a+b*arcsinh(c*x))^2/b/c^2/d+(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^
(1/2))^2)/c^2/d+1/2*b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c^2/d
```

Rubi [A] time = 0.12, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {5714, 3718, 2190, 2279, 2391}

$$\frac{b\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{2c^2d}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^2d}+\frac{\log\left(e^{2\sinh^{-1}(cx)}+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

```
[Out] -(a + b*ArcSinh[c*x])^2/(2*b*c^2*d) + ((a + b*ArcSinh[c*x])*Log[1 + E^(2*Ar
cSinh[c*x])])/(c^2*d) + (b*PolyLog[2, -E^(2*ArcSinh[c*x])])/(2*c^2*d)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx = \frac{\text{Subst}\left(\int (a+bx)\tanh(x)\,dx, x, \sinh^{-1}(cx)\right)}{c^2d}$$
$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^2d} + \frac{2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx, x, \sinh^{-1}(cx)\right)}{c^2d}$$
$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^2d} + \frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1+e^{2\sinh^{-1}(cx)}\right)}{c^2d} - \frac{b\,\text{Subst}\left(\int \log\left(1\right.\right.}{}$$
$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^2d} + \frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1+e^{2\sinh^{-1}(cx)}\right)}{c^2d} - \frac{b\,\text{Subst}\left(\int \frac{\log(1+}{x}\right.}{2}$$
$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^2d} + \frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1+e^{2\sinh^{-1}(cx)}\right)}{c^2d} + \frac{b\text{Li}_2\left(-e^{2\sinh^{-1}(cx}\right.}{2c^2d}$$

Mathematica [B] time = 0.06, size = 167, normalized size = 2.29

$$\frac{a\log\left(c^2x^2+1\right)}{2c^2d} + \frac{b\text{Li}_2\left(-\frac{\sqrt{-c^2}e^{\sinh^{-1}(cx)}}{c}\right)}{c^2d} + \frac{b\text{Li}_2\left(\frac{\sqrt{-c^2}e^{\sinh^{-1}(cx)}}{c}\right)}{c^2d} - \frac{b\sinh^{-1}(cx)^2}{2c^2d} + \frac{b\sinh^{-1}(cx)\log\left(1-\frac{\sqrt{-c^2}e^{\sinh^{-1}(cx)}}{c}\right.}{c^2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

```
[Out] -1/2*(b*ArcSinh[c*x]^2)/(c^2*d) + (b*ArcSinh[c*x]*Log[1 - (Sqrt[-c^2]*E^Arc
Sinh[c*x])/c])/(c^2*d) + (b*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x]
)/c])/(c^2*d) + (a*Log[1 + c^2*x^2])/(2*c^2*d) + (b*PolyLog[2, -((Sqrt[-c^2
]*E^ArcSinh[c*x])/c)])/(c^2*d) + (b*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/
c])/(c^2*d)
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx\,\text{arsinh}\,(cx) + ax}{c^2dx^2+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x*arcsinh(c*x) + a*x)/(c^2*d*x^2 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arsinh}\,(cx) + a)x}{c^2dx^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x/(c^2*d*x^2 + d), x)
```

maple [A] time = 0.05, size = 98, normalized size = 1.34

$$\frac{a\ln\left(c^2x^2+1\right)}{2c^2d} - \frac{b\,\text{arcsinh}\,(cx)^2}{2c^2d} + \frac{b\,\text{arcsinh}\,(cx)\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)}{c^2d} + \frac{b\,\text{polylog}\left(2,-\left(cx+\sqrt{c^2x^2+1}\right)^2\right)}{2c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x)
```

```
[Out] 1/2/c^2*a/d*ln(c^2*x^2+1)-1/2/c^2*b/d*arcsinh(c*x)^2+1/c^2*b/d*arcsinh(c*x)
*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+1/2*b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2
)/c^2/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{8}b\left(\frac{\log\left(c^2x^2+1\right)^2-4\log\left(c^2x^2+1\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^2d}+8\int\frac{\log\left(c^2x^2+1\right)}{2\left(c^4dx^3+c^2dx+\left(c^3dx^2+cd\right)\sqrt{c^2x^2+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/8*b*((log(c^2*x^2 + 1)^2 - 4*log(c^2*x^2 + 1)*log(c*x + sqrt(c^2*x^2 + 1
)))/(c^2*d) + 8*integrate(1/2*log(c^2*x^2 + 1)/(c^4*d*x^3 + c^2*d*x + (c^3*
d*x^2 + c*d)*sqrt(c^2*x^2 + 1)), x)) + 1/2*a*log(c^2*d*x^2 + d)/(c^2*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{d\,c^2\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2),x)
```

```
[Out] int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{ax}{c^2x^2+1}\,dx+\int\frac{bx\,\mathrm{asinh}\left(cx\right)}{c^2x^2+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a*x/(c**2*x**2 + 1), x) + Integral(b*x*asinh(c*x)/(c**2*x**2 + 1)
, x))/d
```

3.32 $\int \frac{a+b\sinh^{-1}(cx)}{d+c^2dx^2}\,dx$

Optimal. Leaf size=70

$$\frac{2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{cd}-\frac{ib\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{cd}+\frac{ib\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)}{cd}$$

```
[Out] 2*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))/c/d-I*b*polylog(2,-I*(c*
x+(c^2*x^2+1)^(1/2)))/c/d+I*b*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/d
```

Rubi [A] time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5693, 4180, 2279, 2391}

$$-\frac{ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{cd}+\frac{ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{cd}+\frac{2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{cd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2),x]
```

```
[Out] (2*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*d) - (I*b*PolyLog[2, (-I
)*E^ArcSinh[c*x]])/(c*d) + (I*b*PolyLog[2, I*E^ArcSinh[c*x]])/(c*d)
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{d+c^2dx^2}\,dx = \frac{\text{Subst}\left(\int(a+bx)\text{sech}(x)\,dx, x, \sinh^{-1}(cx)\right)}{cd}$$
$$= \frac{2\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{cd} - \frac{(ib)\,\text{Subst}\left(\int\log\left(1-ie^x\right)\,dx, x, \sinh^{-1}(cx)\right)}{cd}$$
$$= \frac{2\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{cd} - \frac{(ib)\,\text{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx, x, e^{\sinh^{-1}(cx)}\right)}{cd} + \frac{(ib}{}$$
$$= \frac{2\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{cd} - \frac{ib\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{cd} + \frac{ib\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)}{cd}$$

Mathematica [A] time = 0.08, size = 135, normalized size = 1.93

$$-\frac{c\left(a\sqrt{-c^2}\tan^{-1}(cx)+bc\text{Li}_2\left(\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right)-bc\text{Li}_2\left(\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right)-bc\sinh^{-1}(cx)\log\left(\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}+1\right)+bc\sinh}{\left(-c^2\right)^{3/2}d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2),x]
```

```
[Out] -((c*(a*Sqrt[-c^2]*ArcTan[c*x] - b*c*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x]
)/Sqrt[-c^2]] + b*c*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + b
*c*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - b*c*PolyLog[2, (Sqrt[-c^2]*E
^ArcSinh[c*x])/c]))/((-c^2)^(3/2)*d))
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arsinh}\,(cx)+a}{c^2dx^2+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^2*d*x^2 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arsinh}\,(cx)+a}{c^2dx^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(c^2*d*x^2 + d), x)
```

maple [A] time = 0.01, size = 171, normalized size = 2.44

$$\frac{a\arctan(cx)}{cd}+\frac{ib\,\text{dilog}\left(1-\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)}{cd}+\frac{b\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)}{cd}-\frac{b\arctan(cx)\ln\left(1-\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)}{cd}-\frac{ib\,\text{dilog}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(c^2*d*x^2+d),x)
```

```
[Out] 1/c*a/d*arctan(c*x)+I/c*b/d*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/c*b/d*
arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/c*b/d*arctan(c*x)*ln(1-I*
(1+I*c*x)/(c^2*x^2+1)^(1/2))-I/c*b/d*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))
+1/c*b/d*arcsinh(c*x)*arctan(c*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\int \frac{\log\left(cx + \sqrt{c^2x^2 + 1}\right)}{c^2dx^2 + d}\,dx + \frac{a\arctan(cx)}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d), x) + a*arctan(c*x
)/(c*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,\mathrm{asinh}(c\,x)}{d\,c^2\,x^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(d + c^2*d*x^2),x)
```

```
[Out] int((a + b*asinh(c*x))/(d + c^2*d*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^2x^2+1}\,dx + \int \frac{b\,\mathrm{asinh}(cx)}{c^2x^2+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a/(c**2*x**2 + 1), x) + Integral(b*asinh(c*x)/(c**2*x**2 + 1), x)
)/d
```

3.33 $\int \frac{a+b\sinh^{-1}(cx)}{x(d+c^2dx^2)}\,dx$

Optimal. Leaf size=61

$$-\frac{2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}-\frac{b\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{2d}+\frac{b\text{Li}_2\left(e^{2\sinh^{-1}(cx)}\right)}{2d}$$

```
[Out] -2*(a+b*arcsinh(c*x))*arctanh((c*x+(c^2*x^2+1)^(1/2))^2)/d-1/2*b*polylog(2,
-(c*x+(c^2*x^2+1)^(1/2))^2)/d+1/2*b*polylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)/d
```

Rubi [A] time = 0.12, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5720, 5461, 4182, 2279, 2391}

$$-\frac{b\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{2d}+\frac{b\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{2d}-\frac{2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)),x]
```

```
[Out] (-2*(a + b*ArcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/d - (b*PolyLog[2, -E^
(2*ArcSinh[c*x])])/(2*d) + (b*PolyLog[2, E^(2*ArcSinh[c*x])])/(2*d)
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5720

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)}\,dx = \frac{\text{Subst}\left(\int (a+bx)\text{csch}(x)\text{sech}(x)\,dx, x, \sinh^{-1}(cx)\right)}{d}$$
$$= \frac{2\,\text{Subst}\left(\int (a+bx)\text{csch}(2x)\,dx, x, \sinh^{-1}(cx)\right)}{d}$$
$$= -\frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d} - \frac{b\,\text{Subst}\left(\int \log\left(1-e^{2x}\right)dx, x, \sinh^{-1}(cx)\right)}{d}$$
$$= -\frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d} - \frac{b\,\text{Subst}\left(\int \frac{\log(1-x)}{x}dx, x, e^{2\sinh^{-1}(cx)}\right)}{2d} + \frac{b\,\text{S}}{}$$
$$= -\frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d} - \frac{b\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{2d} + \frac{b\text{Li}_2\left(e^{2\sinh^{-1}(cx)}\right)}{2d}$$

Mathematica [B] time = 0.08, size = 207, normalized size = 3.39

$$-\frac{a\log\left(c^2x^2+1\right)}{2d} - \frac{a\sinh^{-1}(cx)}{d} + \frac{a\log\left(1-e^{2\sinh^{-1}(cx)}\right)}{d} - \frac{b\text{Li}_2\left(-\frac{\sqrt{-c^2}e^{\sinh^{-1}(cx)}}{c}\right)}{d} - \frac{b\text{Li}_2\left(\frac{\sqrt{-c^2}e^{\sinh^{-1}(cx)}}{c}\right)}{d} - \frac{b\sinh^{-1}(c}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)),x]
```

```
[Out] -((a*ArcSinh[c*x])/d) - (b*ArcSinh[c*x]*Log[1 - (Sqrt[-c^2]*E^ArcSinh[c*x])
/c])/d - (b*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/d + (a*Log
[1 - E^(2*ArcSinh[c*x])])/d + (b*ArcSinh[c*x]*Log[1 - E^(2*ArcSinh[c*x])])/
d - (a*Log[1 + c^2*x^2])/(2*d) - (b*PolyLog[2, -((Sqrt[-c^2]*E^ArcSinh[c*x]
)/c)])/d - (b*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/d + (b*PolyLog[2,
E^(2*ArcSinh[c*x])])/(2*d)
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arsinh}(cx)+a}{c^2dx^3+dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^2*d*x^3 + d*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arsinh}(cx)+a}{\left(c^2dx^2+d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)*x), x)
```

maple [A] time = 0.07, size = 74, normalized size = 1.21

$$\frac{a\ln(cx)}{d} - \frac{a\ln\left(c^2x^2+1\right)}{2d} + \frac{b\,\text{dilog}\left(\frac{1}{\left(cx+\sqrt{c^2x^2+1}\right)^2}\right)}{d} - \frac{b\,\text{dilog}\left(\frac{1}{\left(cx+\sqrt{c^2x^2+1}\right)^4}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d),x)
```

```
[Out] a/d*ln(c*x)-1/2*a/d*ln(c^2*x^2+1)+b/d*dilog(1/(c*x+(c^2*x^2+1)^(1/2))^2)-1/
4*b/d*dilog(1/(c*x+(c^2*x^2+1)^(1/2))^4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}a\left(\frac{\log\left(c^2x^2+1\right)}{d}-\frac{2\log(x)}{d}\right)+b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^2dx^3+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/2*a*(log(c^2*x^2 + 1)/d - 2*log(x)/d) + b*integrate(log(c*x + sqrt(c^2*x
^2 + 1))/(c^2*d*x^3 + d*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{a+b\,\mathrm{asinh}\left(c\,x\right)}{x\,\left(d\,c^2\,x^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{a}{c^2x^3+x}\,dx+\int\frac{b\,\mathrm{asinh}\,(cx)}{c^2x^3+x}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a/(c**2*x**3 + x), x) + Integral(b*asinh(c*x)/(c**2*x**3 + x), x)
)/d
```

3.34 $\int \frac{a+b\sinh^{-1}(cx)}{x^2(d+c^2dx^2)}\,dx$

Optimal. Leaf size=101

$$-\frac{a+b\sinh^{-1}(cx)}{dx}-\frac{2c\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}-\frac{bc\tanh^{-1}\left(\sqrt{c^2x^2+1}\right)}{d}+\frac{ibc\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{d}-\frac{ibc\text{Li}}{}$$

```
[Out] (-a-b*arcsinh(c*x))/d/x-2*c*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2)
)/d-b*c*arctanh((c^2*x^2+1)^(1/2))/d+I*b*c*polylog(2,-I*(c*x+(c^2*x^2+1)^(1
/2)))/d-I*b*c*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/d
```

Rubi [A] time = 0.15, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5747, 5693, 4180, 2279, 2391, 266, 63, 208}

$$\frac{ibc\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{d}-\frac{ibc\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{d}-\frac{a+b\sinh^{-1}(cx)}{dx}-\frac{2c\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(c\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)),x]
```

```
[Out] -((a + b*ArcSinh[c*x])/(d*x)) - (2*c*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[
c*x]])/d - (b*c*ArcTanh[Sqrt[1 + c^2*x^2]])/d + (I*b*c*PolyLog[2, (-I)*E^Ar
cSinh[c*x]])/d - (I*b*c*PolyLog[2, I*E^ArcSinh[c*x]])/d
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(d+c^2dx^2\right)}\,dx = -\frac{a+b\sinh^{-1}(cx)}{dx} - c^2\int \frac{a+b\sinh^{-1}(cx)}{d+c^2dx^2}\,dx + \frac{(bc)\int \frac{1}{x\sqrt{1+c^2x^2}}\,dx}{d}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{dx} - \frac{c\,\text{Subst}\left(\int (a+bx)\text{sech}(x)\,dx, x, \sinh^{-1}(cx)\right)}{d} + \frac{(bc)\,\text{Subst}\left(\int \frac{}{x}\right.}{2}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{dx} - \frac{2c\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{d} + \frac{b\,\text{Subst}\left(\int \frac{1}{-\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx\right.}{cd}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{dx} - \frac{2c\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{d} - \frac{bc\tanh^{-1}\left(\sqrt{1+c^2x^2}\right.}{d}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{dx} - \frac{2c\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{d} - \frac{bc\tanh^{-1}\left(\sqrt{1+c^2x^2}\right.}{d}$$

Mathematica [A] time = 0.14, size = 182, normalized size = 1.80

$$-\frac{acx\tan^{-1}(cx)+a-b\sqrt{-c^2}\,x\text{Li}_2\left(\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right)+b\sqrt{-c^2}\,x\text{Li}_2\left(\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right)+bcx\tanh^{-1}\left(\sqrt{c^2x^2+1}\right)+b\sqrt{-c^2}}{dx}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)),x]
```

```
[Out] -((a + b*ArcSinh[c*x] + a*c*x*ArcTan[c*x] + b*c*x*ArcTanh[Sqrt[1 + c^2*x^2]
] + b*Sqrt[-c^2]*x*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - b*
Sqrt[-c^2]*x*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - b*Sqrt[-
c^2]*x*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] + b*Sqrt[-c^2]*x*PolyLog[2
, (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/(d*x))
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b \operatorname{arsinh}(cx) + a}{c^2dx^4 + dx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^2*d*x^4 + d*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arsinh}(cx) + a}{(c^2dx^2 + d)x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)*x^2), x)
```

maple [A] time = 0.02, size = 202, normalized size = 2.00

$$-\frac{a}{dx} - \frac{ca \arctan(cx)}{d} - \frac{b \operatorname{arcsinh}(cx)}{dx} - \frac{cb \operatorname{arcsinh}(cx) \arctan(cx)}{d} - \frac{cb \operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)}{d} - \frac{cb \arctan(cx) \ln\left(1 + \frac{i}{\sqrt{}}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d),x)
```

```
[Out] -a/d/x-c*a/d*arctan(c*x)-b/d*arcsinh(c*x)/x-c*b/d*arcsinh(c*x)*arctan(c*x)-
c*b/d*arctanh(1/(c^2*x^2+1)^(1/2))-c*b/d*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*
x^2+1)^(1/2))+c*b/d*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*c*b/d
*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-I*c*b/d*dilog(1-I*(1+I*c*x)/(c^2*x^
2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-a\left(\frac{c \arctan(cx)}{d} + \frac{1}{dx}\right) + b \int \frac{\log\left(cx + \sqrt{c^2x^2 + 1}\right)}{c^2dx^4 + dx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] -a*(c*arctan(c*x)/d + 1/(d*x)) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(
c^2*d*x^4 + d*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{asinh}(c\, x)}{x^2\, \left(d\, c^2\, x^2 + d\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^2x^4+x^2}\,dx + \int \frac{b\,\mathrm{asinh}\,(cx)}{c^2x^4+x^2}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**2/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a/(c**2*x**4 + x**2), x) + Integral(b*asinh(c*x)/(c**2*x**4 + x**
2), x))/d
```

3.35 $\int \frac{a+b\sinh^{-1}(cx)}{x^3(d+c^2dx^2)}\,dx$

Optimal. Leaf size=113

$$\frac{2c^2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}-\frac{a+b\sinh^{-1}(cx)}{2dx^2}+\frac{bc^2\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{2d}-\frac{bc^2\text{Li}_2\left(e^{2\sinh^{-1}(cx)}\right)}{2d}-\frac{bc\sqrt{c^2}}{2}$$

```
[Out] 1/2*(-a-b*arcsinh(c*x))/d/x^2+2*c^2*(a+b*arcsinh(c*x))*arctanh((c*x+(c^2*x^
2+1)^(1/2))^2)/d+1/2*b*c^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d-1/2*b*c^
2*polylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)/d-1/2*b*c*(c^2*x^2+1)^(1/2)/d/x
```

Rubi [A] time = 0.20, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {5747, 5720, 5461, 4182, 2279, 2391, 264}

$$\frac{bc^2\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{2d}-\frac{bc^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{2d}+\frac{2c^2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}-\frac{a+b\sin}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)),x]
```

```
[Out] -(b*c*Sqrt[1 + c^2*x^2])/(2*d*x) - (a + b*ArcSinh[c*x])/(2*d*x^2) + (2*c^2*
(a + b*ArcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/d + (b*c^2*PolyLog[2, -E^
(2*ArcSinh[c*x])])/(2*d) - (b*c^2*PolyLog[2, E^(2*ArcSinh[c*x])])/(2*d)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5720

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^3\left(d+c^2dx^2\right)}\,dx = -\frac{a+b\sinh^{-1}(cx)}{2dx^2} - c^2\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)}\,dx + \frac{(bc)\int \frac{1}{x^2\sqrt{1+c^2x^2}}\,dx}{2d}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2dx} - \frac{a+b\sinh^{-1}(cx)}{2dx^2} - \frac{c^2\,\text{Subst}\left(\int (a+bx)\text{csch}(x)\text{sech}(x)\,dx, x, \sinh^{-1}\right.}{d}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2dx} - \frac{a+b\sinh^{-1}(cx)}{2dx^2} - \frac{\left(2c^2\right)\text{Subst}\left(\int (a+bx)\text{csch}(2x)\,dx, x, \sinh^{-1}(c\right.}{d}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2dx} - \frac{a+b\sinh^{-1}(cx)}{2dx^2} + \frac{2c^2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d} + \frac{(b}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2dx} - \frac{a+b\sinh^{-1}(cx)}{2dx^2} + \frac{2c^2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d} + \frac{(b}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2dx} - \frac{a+b\sinh^{-1}(cx)}{2dx^2} + \frac{2c^2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d} + \frac{bc}{}$$

Mathematica [B] time = 0.24, size = 240, normalized size = 2.12

$$\frac{-c^2\left(2\log\left(1-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)+b\text{Li}_2\left(e^{2\sinh^{-1}(cx)}\right)\right)+\frac{c^2\left(a+b\sinh^{-1}(cx)\right)^2}{b}-\frac{a+b\sinh^{-1}(cx)}{x^2}+ac^2\log}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)),x]
```

```
[Out] (-((b*c*Sqrt[1 + c^2*x^2])/x) - b*c^2*ArcSinh[c*x]^2 - (a + b*ArcSinh[c*x])
/x^2 + (c^2*(a + b*ArcSinh[c*x])^2)/b + 2*b*c^2*ArcSinh[c*x]*Log[1 + (c*E^A
rcSinh[c*x])/Sqrt[-c^2]] + 2*b*c^2*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSi
nh[c*x])/c] + a*c^2*Log[1 + c^2*x^2] + 2*b*c^2*PolyLog[2, (c*E^ArcSinh[c*x]
)/Sqrt[-c^2]] + 2*b*c^2*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - c^2*(2*
(a + b*ArcSinh[c*x])*Log[1 - E^(2*ArcSinh[c*x])] + b*PolyLog[2, E^(2*ArcSin
h[c*x])]))/(2*d)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arsinh}\,(cx)+a}{c^2dx^5+dx^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^2*d*x^5 + d*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arsinh}(cx) + a}{(c^2dx^2 + d)x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)*x^3), x)
```

maple [A] time = 0.13, size = 266, normalized size = 2.35

$$-\frac{a}{2d\,x^2}-\frac{c^2 a \ln(cx)}{d}+\frac{c^2 a \ln\left(c^2x^2+1\right)}{2d}-\frac{bc\sqrt{c^2x^2+1}}{2dx}+\frac{c^2 b}{2d}-\frac{b \operatorname{arcsinh}(cx)}{2d\,x^2}+\frac{c^2 b \operatorname{arcsinh}(cx) \ln\left(1+\left(cx+\sqrt{c^2x^2}\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d),x)
```

```
[Out] -1/2*a/d/x^2-c^2*a/d*ln(c*x)+1/2*c^2*a/d*ln(c^2*x^2+1)-1/2*b*c*(c^2*x^2+1)^
(1/2)/d/x+1/2*c^2*b/d-1/2*b/d*arcsinh(c*x)/x^2+c^2*b/d*arcsinh(c*x)*ln(1+(c
*x+(c^2*x^2+1)^(1/2))^2)+1/2*b*c^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d-
c^2*b/d*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-c^2*b/d*polylog(2,-c*x-(c^
2*x^2+1)^(1/2))-c^2*b/d*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-c^2*b/d*po
lylog(2,c*x+(c^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(\frac{c^2 \log\left(c^2x^2+1\right)}{d}-\frac{2\,c^2 \log(x)}{d}-\frac{1}{dx^2}\right)a+b\int \frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^2dx^5+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/2*(c^2*log(c^2*x^2 + 1)/d - 2*c^2*log(x)/d - 1/(d*x^2))*a + b*integrate(l
og(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^5 + d*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b \operatorname{asinh}(c\,x)}{x^3\left(d\,c^2\,x^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^2x^5+x^3}\,dx+\int \frac{b \operatorname{asinh}(cx)}{c^2x^5+x^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**3/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a/(c**2*x**5 + x**3), x) + Integral(b*asinh(c*x)/(c**2*x**5 + x**
3), x))/d
```

3.36 $\int \frac{a+b\sinh^{-1}(cx)}{x^4(d+c^2dx^2)}\,dx$

Optimal. Leaf size=156

$$\frac{2c^3\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}+\frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{dx}-\frac{a+b\sinh^{-1}(cx)}{3dx^3}-\frac{ibc^3\mathrm{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{d}+\frac{ibc^3\mathrm{Li}_2}{}$$

```
[Out] 1/3*(-a-b*arcsinh(c*x))/d/x^3+c^2*(a+b*arcsinh(c*x))/d/x+2*c^3*(a+b*arcsinh
(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))/d+7/6*b*c^3*arctanh((c^2*x^2+1)^(1/2))
/d-I*b*c^3*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/d+I*b*c^3*polylog(2,I*(c*x
+(c^2*x^2+1)^(1/2)))/d-1/6*b*c*(c^2*x^2+1)^(1/2)/d/x^2
```

Rubi [A] time = 0.25, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5747, 5693, 4180, 2279, 2391, 266, 63, 208, 51}

$$-\frac{ibc^3\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{d}+\frac{ibc^3\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{d}+\frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{dx}+\frac{2c^3\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)),x]
```

```
[Out] -(b*c*Sqrt[1 + c^2*x^2])/(6*d*x^2) - (a + b*ArcSinh[c*x])/(3*d*x^3) + (c^2*
(a + b*ArcSinh[c*x]))/(d*x) + (2*c^3*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[
c*x]])/d + (7*b*c^3*ArcTanh[Sqrt[1 + c^2*x^2]])/(6*d) - (I*b*c^3*PolyLog[2,
 (-I)*E^ArcSinh[c*x]])/d + (I*b*c^3*PolyLog[2, I*E^ArcSinh[c*x]])/d
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^4\left(d+c^2dx^2\right)}\,dx = -\frac{a+b\sinh^{-1}(cx)}{3dx^3} - c^2\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(d+c^2dx^2\right)}\,dx + \frac{(bc)\int \frac{1}{x^3\sqrt{1+c^2x^2}}\,dx}{3d}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{3dx^3} + \frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{dx} + c^4\int \frac{a+b\sinh^{-1}(cx)}{d+c^2dx^2}\,dx + \frac{(bc)\,\text{Subst}\,(}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6dx^2} - \frac{a+b\sinh^{-1}(cx)}{3dx^3} + \frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{dx} + \frac{c^3\,\text{Subst}\left(\int (a+bx)\text{sec}\right.}{d}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6dx^2} - \frac{a+b\sinh^{-1}(cx)}{3dx^3} + \frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{dx} + \frac{2c^3\left(a+b\sinh^{-1}(cx)\right)\text{t}}{d}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6dx^2} - \frac{a+b\sinh^{-1}(cx)}{3dx^3} + \frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{dx} + \frac{2c^3\left(a+b\sinh^{-1}(cx)\right)\text{t}}{d}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6dx^2} - \frac{a+b\sinh^{-1}(cx)}{3dx^3} + \frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{dx} + \frac{2c^3\left(a+b\sinh^{-1}(cx)\right)\text{t}}{d}$$

Mathematica [A] time = 0.19, size = 247, normalized size = 1.58

$$6ac^3x^3\tan^{-1}(cx)+6ac^2x^2-2a+6b\left(-c^2\right)^{3/2}x^3\text{Li}_2\left(\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right)-6b\left(-c^2\right)^{3/2}x^3\text{Li}_2\left(\frac{\sqrt{-c^2}e^{\sinh^{-1}(cx)}}{c}\right)-6b\left(-c^2\right)^{3/2}x^3$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)),x]
```

```
[Out] (-2*a + 6*a*c^2*x^2 - b*c*x*Sqrt[1 + c^2*x^2] - 2*b*ArcSinh[c*x] + 6*b*c^2*
x^2*ArcSinh[c*x] + 6*a*c^3*x^3*ArcTan[c*x] + 7*b*c^3*x^3*ArcTanh[Sqrt[1 + c
^2*x^2]] - 6*b*(-c^2)^(3/2)*x^3*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x])/Sqr
t[-c^2]] + 6*b*(-c^2)^(3/2)*x^3*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[
c*x])/c] + 6*b*(-c^2)^(3/2)*x^3*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] -
 6*b*(-c^2)^(3/2)*x^3*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/(6*d*x^3)
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arsinh}\,(cx)+a}{c^2dx^6+dx^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^2*d*x^6 + d*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{b\,\text{arsinh}\,(cx)+a}{\left(c^2dx^2+d\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)*x^4), x)
```

maple [A] time = 0.02, size = 261, normalized size = 1.67

$$-\frac{a}{3d\,x^3}+\frac{c^2a}{dx}+\frac{c^3a\arctan\,(cx)}{d}-\frac{b\,\text{arcsinh}\,(cx)}{3d\,x^3}+\frac{c^2b\,\text{arcsinh}\,(cx)}{dx}+\frac{c^3b\,\text{arcsinh}\,(cx)\arctan\,(cx)}{d}-\frac{bc\sqrt{c^2x^2+1}}{6d\,x^2}+\frac{7c^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d),x)
```

```
[Out] -1/3*a/d/x^3+c^2*a/d/x+c^3*a/d*arctan(c*x)-1/3*b/d*arcsinh(c*x)/x^3+c^2*b/d
*arcsinh(c*x)/x+c^3*b/d*arcsinh(c*x)*arctan(c*x)-1/6*b*c*(c^2*x^2+1)^(1/2)/
d/x^2+7/6*c^3*b/d*arctanh(1/(c^2*x^2+1)^(1/2))+c^3*b/d*arctan(c*x)*ln(1+I*(
1+I*c*x)/(c^2*x^2+1)^(1/2))-c^3*b/d*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1
)^(1/2))-I*c^3*b/d*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*c^3*b/d*dilog(1
-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\left(\frac{3\,c^3\arctan\,(cx)}{d}+\frac{3\,c^2x^2-1}{dx^3}\right)a+b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^2dx^6+dx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/3*(3*c^3*arctan(c*x)/d + (3*c^2*x^2 - 1)/(d*x^3))*a + b*integrate(log(c*x
 + sqrt(c^2*x^2 + 1))/(c^2*d*x^6 + d*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{asinh}(c x)}{x^4 \left(d c^2 x^2 + d\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^2x^6+x^4} dx + \int \frac{b \operatorname{asinh}(cx)}{c^2x^6+x^4} dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**4/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a/(c**2*x**6 + x**4), x) + Integral(b*asinh(c*x)/(c**2*x**6 + x**
4), x))/d
```

3.37 $\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=171

$$-\frac{3\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2}+\frac{3x\left(a+b\sinh^{-1}(cx)\right)}{2c^4d^2}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(c^2x^2+1\right)}+\frac{3ib\mathrm{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{2c^5d^2}-\frac{3ib}{}$$

```
[Out] 3/2*x*(a+b*arcsinh(c*x))/c^4/d^2-1/2*x^3*(a+b*arcsinh(c*x))/c^2/d^2/(c^2*x^
2+1)-3*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))/c^5/d^2+3/2*I*b*pol
ylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c^5/d^2-3/2*I*b*polylog(2,I*(c*x+(c^2*x^
2+1)^(1/2)))/c^5/d^2+1/2*b/c^5/d^2/(c^2*x^2+1)^(1/2)-b*(c^2*x^2+1)^(1/2)/c^
5/d^2
```

Rubi [A] time = 0.24, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5751, 5767, 5693, 4180, 2279, 2391, 261, 266, 43}

$$\frac{3ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{2c^5d^2}-\frac{3ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{2c^5d^2}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(c^2x^2+1\right)}+\frac{3x\left(a+b\sinh^{-1}(cx)\right)}{2c^4d^2}-\frac{3\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

```
[Out] b/(2*c^5*d^2*Sqrt[1 + c^2*x^2]) - (b*Sqrt[1 + c^2*x^2])/(c^5*d^2) + (3*x*(a
 + b*ArcSinh[c*x]))/(2*c^4*d^2) - (x^3*(a + b*ArcSinh[c*x]))/(2*c^2*d^2*(1
+ c^2*x^2)) - (3*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c^5*d^2) + (
((3*I)/2)*b*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^5*d^2) - (((3*I)/2)*b*PolyL
og[2, I*E^ArcSinh[c*x]])/(c^5*d^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5767

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(e*(m + 2*p + 1)), x] + (-Dist[(f^2*(m - 1))/(c^2*(m +
 2*p + 1)), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(c*(m + 2*p + 1)*(1 + c
^2*x^2)^FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcS
inh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2
*d] && GtQ[n, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^2}\,dx = -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)}+\frac{b\int\frac{x^3}{\left(1+c^2x^2\right)^{3/2}}\,dx}{2cd^2}+\frac{3\int\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx}{2c^2d}$$

$$=\frac{3x\left(a+b\sinh^{-1}(cx)\right)}{2c^4d^2}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)}-\frac{(3b)\int\frac{x}{\sqrt{1+c^2x^2}}\,dx}{2c^3d^2}+\frac{b\,\mathrm{Subst}\left(\int \frac{}{(1\cdots}\right.}{4\cdots}$$

$$=-\frac{3b\sqrt{1+c^2x^2}}{2c^5d^2}+\frac{3x\left(a+b\sinh^{-1}(cx)\right)}{2c^4d^2}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)}-\frac{3\,\mathrm{Subst}\left(\int(a+\cdots\right.}{}$$

$$=\frac{b}{2c^5d^2\sqrt{1+c^2x^2}}-\frac{b\sqrt{1+c^2x^2}}{c^5d^2}+\frac{3x\left(a+b\sinh^{-1}(cx)\right)}{2c^4d^2}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)}-$$

$$=\frac{b}{2c^5d^2\sqrt{1+c^2x^2}}-\frac{b\sqrt{1+c^2x^2}}{c^5d^2}+\frac{3x\left(a+b\sinh^{-1}(cx)\right)}{2c^4d^2}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)}-$$

$$=\frac{b}{2c^5d^2\sqrt{1+c^2x^2}}-\frac{b\sqrt{1+c^2x^2}}{c^5d^2}+\frac{3x\left(a+b\sinh^{-1}(cx)\right)}{2c^4d^2}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)}-$$

Mathematica [A] time = 0.33, size = 268, normalized size = 1.57

$$2ac^3x^3-3ac^2x^2\tan^{-1}(cx)+3acx-3a\tan^{-1}(cx)+2bc^3x^3\sinh^{-1}(cx)+3ib\left(c^2x^2+1\right)\mathrm{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)-3ib\left(c^2\cdots\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

```
[Out] (3*a*c*x + 2*a*c^3*x^3 - b*Sqrt[1 + c^2*x^2] - 2*b*c^2*x^2*Sqrt[1 + c^2*x^2
] + 3*b*c*x*ArcSinh[c*x] + 2*b*c^3*x^3*ArcSinh[c*x] - 3*a*ArcTan[c*x] - 3*a
*c^2*x^2*ArcTan[c*x] - (3*I)*b*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - (3*
I)*b*c^2*x^2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + (3*I)*b*ArcSinh[c*x]*
Log[1 + I*E^ArcSinh[c*x]] + (3*I)*b*c^2*x^2*ArcSinh[c*x]*Log[1 + I*E^ArcSin
h[c*x]] + (3*I)*b*(1 + c^2*x^2)*PolyLog[2, (-I)*E^ArcSinh[c*x]] - (3*I)*b*(
1 + c^2*x^2)*PolyLog[2, I*E^ArcSinh[c*x]])/(2*c^5*d^2*(1 + c^2*x^2))
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{bx^4\,\mathrm{arsinh}\,(cx)+ax^4}{c^4d^2x^4+2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arcsinh(c*x) + a*x^4)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2),
x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.02, size = 285, normalized size = 1.67

$$\frac{ax}{c^4d^2}+\frac{ax}{2c^4d^2\left(c^2x^2+1\right)}-\frac{3a\arctan\left(cx\right)}{2c^5d^2}+\frac{b\,\mathrm{arcsinh}\left(cx\right)x}{c^4d^2}+\frac{b\,\mathrm{arcsinh}\left(cx\right)x}{2c^4d^2\left(c^2x^2+1\right)}-\frac{3b\,\mathrm{arcsinh}\left(cx\right)\arctan\left(cx\right)}{2c^5d^2}-\frac{}{c^3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x)
```

```
[Out] 1/c^4*a/d^2*x+1/2/c^4*a/d^2*x/(c^2*x^2+1)-3/2/c^5*a/d^2*arctan(c*x)+1/c^4*b
/d^2*arcsinh(c*x)*x+1/2/c^4*b/d^2*arcsinh(c*x)*x/(c^2*x^2+1)-3/2/c^5*b/d^2*
arcsinh(c*x)*arctan(c*x)-1/c^3*b/d^2*x^2/(c^2*x^2+1)^(1/2)-1/2*b/c^5/d^2/(c
^2*x^2+1)^(1/2)-3/2/c^5*b/d^2*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2
))+3/2/c^5*b/d^2*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+3/2*I/c^5*
b/d^2*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/2*I/c^5*b/d^2*dilog(1-I*(1+I
*c*x)/(c^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,a\left(\frac{x}{c^6d^2x^2+c^4d^2}+\frac{2\,x}{c^4d^2}-\frac{3\,\arctan\left(cx\right)}{c^5d^2}\right)+b\int\frac{x^4\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^4d^2x^4+2\,c^2d^2x^2+d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*(x/(c^6*d^2*x^2 + c^4*d^2) + 2*x/(c^4*d^2) - 3*arctan(c*x)/(c^5*d^2))
 + b*integrate(x^4*log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^4 + 2*c^2*d^2*x^
2 + d^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^4\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{{\left(d\,c^2\,x^2+d\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^2,x)
```

```
[Out] int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{ax^4}{c^4x^4+2c^2x^2+1}\,dx+\int\frac{bx^4\,\mathrm{asinh}\left(cx\right)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a*x**4/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b*x**4*asinh(
c*x)/(c**4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

3.38 $\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=145

$$-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^4d^2}+\frac{\log\left(e^{2\sinh^{-1}(cx)}+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c^4d^2}-\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(c^2x^2+1\right)}+\frac{b\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{2c^4d^2}+\frac{b\sin}{2}$$

```
[Out] 1/2*b*arcsinh(c*x)/c^4/d^2-1/2*x^2*(a+b*arcsinh(c*x))/c^2/d^2/(c^2*x^2+1)-1
/2*(a+b*arcsinh(c*x))^2/b/c^4/d^2+(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^
(1/2))^2)/c^4/d^2+1/2*b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c^4/d^2-1/2*b
*x/c^3/d^2/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5751, 5714, 3718, 2190, 2279, 2391, 288, 215}

$$\frac{b\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{2c^4d^2}-\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(c^2x^2+1\right)}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^4d^2}+\frac{\log\left(e^{2\sinh^{-1}(cx)}+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c^4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

```
[Out] -(b*x)/(2*c^3*d^2*Sqrt[1 + c^2*x^2]) + (b*ArcSinh[c*x])/(2*c^4*d^2) - (x^2*
(a + b*ArcSinh[c*x]))/(2*c^2*d^2*(1 + c^2*x^2)) - (a + b*ArcSinh[c*x])^2/(2
*b*c^4*d^2) + ((a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(c^4*d^2)
+ (b*PolyLog[2, -E^(2*ArcSinh[c*x])])/(2*c^4*d^2)
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rubi steps

$$\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^2}\,dx = -\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)}+\frac{b\int \frac{x^2}{\left(1+c^2x^2\right)^{3/2}}\,dx}{2cd^2}+\frac{\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx}{c^2d}$$

$$= -\frac{bx}{2c^3d^2\sqrt{1+c^2x^2}}-\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)}+\frac{\text{Subst}\left(\int (a+bx)\tanh(x)\,dx,x,\text{sir}\right.}{c^4d^2}$$

$$= -\frac{bx}{2c^3d^2\sqrt{1+c^2x^2}}+\frac{b\sinh^{-1}(cx)}{2c^4d^2}-\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^4d^2}$$

$$= -\frac{bx}{2c^3d^2\sqrt{1+c^2x^2}}+\frac{b\sinh^{-1}(cx)}{2c^4d^2}-\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^4d^2}$$

$$= -\frac{bx}{2c^3d^2\sqrt{1+c^2x^2}}+\frac{b\sinh^{-1}(cx)}{2c^4d^2}-\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^4d^2}$$

$$= -\frac{bx}{2c^3d^2\sqrt{1+c^2x^2}}+\frac{b\sinh^{-1}(cx)}{2c^4d^2}-\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^4d^2}$$

Mathematica [C] time = 0.21, size = 241, normalized size = 1.66

$$\frac{ac^2x^2\log\left(c^2x^2+1\right)+a\log\left(c^2x^2+1\right)+a+2b\left(c^2x^2+1\right)\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)+2b\left(c^2x^2+1\right)\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)-}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

```
[Out] (a - b*c*x*Sqrt[1 + c^2*x^2] + b*ArcSinh[c*x] - b*ArcSinh[c*x]^2 - b*c^2*x^
2*ArcSinh[c*x]^2 + 2*b*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + 2*b*c^2*x^2
*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + 2*b*ArcSinh[c*x]*Log[1 + I*E^ArcS
inh[c*x]] + 2*b*c^2*x^2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] + a*Log[1 +
c^2*x^2] + a*c^2*x^2*Log[1 + c^2*x^2] + 2*b*(1 + c^2*x^2)*PolyLog[2, (-I)*E
^ArcSinh[c*x]] + 2*b*(1 + c^2*x^2)*PolyLog[2, I*E^ArcSinh[c*x]])/(2*c^4*d^2
*(1 + c^2*x^2))
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^3 \operatorname{arsinh}(cx) + ax^3}{c^4d^2x^4 + 2\,c^2d^2x^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arcsinh(c*x) + a*x^3)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2),
x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.21, size = 206, normalized size = 1.42

$$\frac{a}{2c^4d^2\left(c^2x^2+1\right)}+\frac{a\ln\left(c^2x^2+1\right)}{2c^4d^2}-\frac{b\operatorname{arcsinh}(cx)^2}{2c^4d^2}-\frac{bx}{2c^3d^2\sqrt{c^2x^2+1}}+\frac{b\,x^2}{2c^2d^2\left(c^2x^2+1\right)}+\frac{b\operatorname{arcsinh}(cx)}{2c^4d^2\left(c^2x^2+1\right)}+\frac{}{2c^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x)
```

```
[Out] 1/2/c^4*a/d^2/(c^2*x^2+1)+1/2/c^4*a/d^2*ln(c^2*x^2+1)-1/2/c^4*b/d^2*arcsinh
(c*x)^2-1/2*b*x/c^3/d^2/(c^2*x^2+1)^(1/2)+1/2/c^2*b/d^2*x^2/(c^2*x^2+1)+1/2
/c^4*b/d^2*arcsinh(c*x)/(c^2*x^2+1)+1/2/c^4*b/d^2/(c^2*x^2+1)+1/c^4*b/d^2*a
rcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+1/2*b*polylog(2,-(c*x+(c^2*x^2+
1)^(1/2))^2)/c^4/d^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{8}b\left(\frac{\left(c^2x^2+1\right)\log\left(c^2x^2+1\right)^2-4\left(\left(c^2x^2+1\right)\log\left(c^2x^2+1\right)+1\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-2}{c^6d^2x^2+c^4d^2}+8\int\frac{}{2\left(c^8d^2x^5+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/8*b*(((c^2*x^2 + 1)*log(c^2*x^2 + 1)^2 - 4*((c^2*x^2 + 1)*log(c^2*x^2 +
1) + 1)*log(c*x + sqrt(c^2*x^2 + 1)) - 2)/(c^6*d^2*x^2 + c^4*d^2) + 8*integ
rate(1/2*((c^2*x^2 + 1)*log(c^2*x^2 + 1) + 1)/(c^8*d^2*x^5 + 2*c^6*d^2*x^3
+ c^4*d^2*x + (c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2)*sqrt(c^2*x^2 + 1)), x
)) + 1/2*a*(1/(c^6*d^2*x^2 + c^4*d^2) + log(c^2*x^2 + 1)/(c^4*d^2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{{\left(d\,c^2\,x^2+d\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^2,x)
```

```
[Out] int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^3}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{bx^3\,\mathrm{asinh}\,(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a*x**3/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b*x**3*asinh(
c*x)/(c**4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

3.39 $\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=127

$$\frac{\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^3d^2}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(c^2x^2+1\right)}-\frac{ib\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{2c^3d^2}+\frac{ib\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)}{2c^3d^2}-\frac{b}{2c^3d^2\sqrt{c^2x^2}}$$

```
[Out] -1/2*x*(a+b*arcsinh(c*x))/c^2/d^2/(c^2*x^2+1)+(a+b*arcsinh(c*x))*arctan(c*x
+(c^2*x^2+1)^(1/2))/c^3/d^2-1/2*I*b*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c
^3/d^2+1/2*I*b*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d^2-1/2*b/c^3/d^2/(
c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5751, 5693, 4180, 2279, 2391, 261}

$$-\frac{ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{2c^3d^2}+\frac{ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{2c^3d^2}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(c^2x^2+1\right)}+\frac{\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(c\right.}{c^3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

```
[Out] -b/(2*c^3*d^2*Sqrt[1 + c^2*x^2]) - (x*(a + b*ArcSinh[c*x]))/(2*c^2*d^2*(1 +
 c^2*x^2)) + ((a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c^3*d^2) - ((I/
2)*b*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^3*d^2) + ((I/2)*b*PolyLog[2, I*E^A
rcSinh[c*x]])/(c^3*d^2)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rubi steps

$$\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^2}\,dx = -\frac{x\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)}+\frac{b\int\frac{x}{\left(1+c^2x^2\right)^{3/2}}\,dx}{2cd^2}+\frac{\int\frac{a+b\sinh^{-1}(cx)}{d+c^2dx^2}\,dx}{2c^2d}$$

$$= -\frac{b}{2c^3d^2\sqrt{1+c^2x^2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)}+\frac{\text{Subst}\left(\int(a+bx)\text{sech}(x)\,dx,x,\sinh\right.}{2c^3d^2}$$

$$= -\frac{b}{2c^3d^2\sqrt{1+c^2x^2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)}+\frac{\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{c^3d^2}$$

$$= -\frac{b}{2c^3d^2\sqrt{1+c^2x^2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)}+\frac{\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{c^3d^2}$$

$$= -\frac{b}{2c^3d^2\sqrt{1+c^2x^2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)}+\frac{\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{c^3d^2}$$

Mathematica [A] time = 0.26, size = 221, normalized size = 1.74

$$-\frac{-ac^2x^2\tan^{-1}(cx)+acx-a\tan^{-1}(cx)+ib\left(c^2x^2+1\right)\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)-ib\left(c^2x^2+1\right)\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)+b\sqrt{c^2}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

```
[Out] -1/2*(a*c*x + b*Sqrt[1 + c^2*x^2] + b*c*x*ArcSinh[c*x] - a*ArcTan[c*x] - a*
c^2*x^2*ArcTan[c*x] - I*b*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - I*b*c^2*
x^2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + I*b*ArcSinh[c*x]*Log[1 + I*E^A
rcSinh[c*x]] + I*b*c^2*x^2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] + I*b*(1
+ c^2*x^2)*PolyLog[2, (-I)*E^ArcSinh[c*x]] - I*b*(1 + c^2*x^2)*PolyLog[2, I
*E^ArcSinh[c*x]])/(c^3*d^2*(1 + c^2*x^2))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^2\,\text{arsinh}\,(cx)+ax^2}{c^4d^2x^4+2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arcsinh(c*x) + a*x^2)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(cx) + a)x^2}{\left(c^2dx^2 + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^2/(c^2*d*x^2 + d)^2, x)
```

maple [A] time = 0.01, size = 240, normalized size = 1.89

$$-\frac{ax}{2c^2d^2\left(c^2x^2+1\right)}+\frac{a\arctan(cx)}{2c^3d^2}-\frac{b\operatorname{arcsinh}(cx)\,x}{2c^2d^2\left(c^2x^2+1\right)}+\frac{b\operatorname{arcsinh}(cx)\arctan(cx)}{2c^3d^2}+\frac{b\arctan(cx)\ln\left(1+\frac{i(icx+1)}{\sqrt{c^2x^2+1}}\right)}{2c^3d^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x)
```

```
[Out] -1/2/c^2*a/d^2*x/(c^2*x^2+1)+1/2/c^3*a/d^2*arctan(c*x)-1/2/c^2*b/d^2*arcsin
h(c*x)*x/(c^2*x^2+1)+1/2/c^3*b/d^2*arcsinh(c*x)*arctan(c*x)+1/2/c^3*b/d^2*a
rctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2/c^3*b/d^2*arctan(c*x)*ln
(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*I/c^3*b/d^2*dilog(1+I*(1+I*c*x)/(c^2*
x^2+1)^(1/2))+1/2*I/c^3*b/d^2*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*b/
c^3/d^2/(c^2*x^2+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\,a\left(\frac{x}{c^4d^2x^2+c^2d^2}-\frac{\arctan(cx)}{c^3d^2}\right)+b\int\frac{x^2\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^4d^2x^4+2\,c^2d^2x^2+d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*(x/(c^4*d^2*x^2 + c^2*d^2) - arctan(c*x)/(c^3*d^2)) + b*integrate(x^
2*log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(a+b\,\operatorname{asinh}(c\,x)\right)}{\left(d\,c^2\,x^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^2,x)
```

```
[Out] int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^2}{c^4x^4+2c^2x^2+1}\,dx+\int \frac{bx^2\operatorname{asinh}(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a*x**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b*x**2*asinh(
c*x)/(c**4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

3.40 $\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^2} dx$

Optimal. Leaf size=55

$$\frac{bx}{2cd^2\sqrt{c^2x^2+1}} - \frac{a+b\sinh^{-1}(cx)}{2c^2d^2\left(c^2x^2+1\right)}$$

```
[Out] 1/2*(-a-b*arcsinh(c*x))/c^2/d^2/(c^2*x^2+1)+1/2*b*x/c/d^2/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5717, 191}

$$\frac{bx}{2cd^2\sqrt{c^2x^2+1}} - \frac{a+b\sinh^{-1}(cx)}{2c^2d^2\left(c^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

```
[Out] (b*x)/(2*c*d^2*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])/(2*c^2*d^2*(1 + c^
2*x^2))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^2} dx &= -\frac{a+b\sinh^{-1}(cx)}{2c^2d^2\left(1+c^2x^2\right)} + \frac{b\int \frac{1}{\left(1+c^2x^2\right)^{3/2}} dx}{2cd^2}\\ &= \frac{bx}{2cd^2\sqrt{1+c^2x^2}} - \frac{a+b\sinh^{-1}(cx)}{2c^2d^2\left(1+c^2x^2\right)}\end{aligned}$$

Mathematica [A] time = 0.06, size = 74, normalized size = 1.35

$$-\frac{a}{2c^2d^2\left(c^2x^2+1\right)} + \frac{bx}{2cd^2\sqrt{c^2x^2+1}} - \frac{b\sinh^{-1}(cx)}{2c^2d^2\left(c^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

```
[Out] -1/2*a/(c^2*d^2*(1 + c^2*x^2)) + (b*x)/(2*c*d^2*Sqrt[1 + c^2*x^2]) - (b*Arc
Sinh[c*x])/(2*c^2*d^2*(1 + c^2*x^2))
```

fricas [A] time = 0.58, size = 65, normalized size = 1.18

$$\frac{ac^2x^2 + \sqrt{c^2x^2+1}\,bcx - b\log\left(cx + \sqrt{c^2x^2+1}\right)}{2\left(c^4d^2x^2 + c^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a*c^2*x^2 + sqrt(c^2*x^2 + 1)*b*c*x - b*log(c*x + sqrt(c^2*x^2 + 1)))/
(c^4*d^2*x^2 + c^2*d^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arsinh}\,(cx) + a)x}{\left(c^2dx^2 + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x/(c^2*d*x^2 + d)^2, x)
```

maple [A] time = 0.00, size = 61, normalized size = 1.11

$$\frac{-\frac{a}{2d^2\left(c^2x^2+1\right)} + \frac{b\left(-\frac{\mathrm{arcsinh}(cx)}{2\left(c^2x^2+1\right)} + \frac{cx}{2\sqrt{c^2x^2+1}}\right)}{d^2}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x)
```

```
[Out] 1/c^2*(-1/2*a/d^2/(c^2*x^2+1)+b/d^2*(-1/2/(c^2*x^2+1)*arcsinh(c*x)+1/2*c*x/
(c^2*x^2+1)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{4}b\left(\frac{2\log\left(cx + \sqrt{c^2x^2+1}\right) + 1}{c^4d^2x^2 + c^2d^2} - 4\int \frac{1}{2\left(c^6d^2x^5 + 2c^4d^2x^3 + c^2d^2x + \left(c^5d^2x^4 + 2c^3d^2x^2 + cd^2\right)\sqrt{c^2x^2+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/4*b*((2*log(c*x + sqrt(c^2*x^2 + 1)) + 1)/(c^4*d^2*x^2 + c^2*d^2) - 4*in
tegrate(1/2/(c^6*d^2*x^5 + 2*c^4*d^2*x^3 + c^2*d^2*x + (c^5*d^2*x^4 + 2*c^3
*d^2*x^2 + c*d^2)*sqrt(c^2*x^2 + 1)), x)) - 1/2*a/(c^4*d^2*x^2 + c^2*d^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\left(a + b\,\mathrm{asinh}\,(c\,x)\right)}{\left(d\,c^2\,x^2 + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^2,x)
```

```
[Out] int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^2, x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{bx\operatorname{asinh}(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a*x/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b*x*asinh(c*x)/(
c**4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

3.41 $\int \frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=124

$$\frac{x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(c^2x^2+1\right)}+\frac{\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{cd^2}+\frac{b}{2cd^2\sqrt{c^2x^2+1}}-\frac{ib\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{2cd^2}+\frac{ib\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)}{2cd^2}$$

```
[Out] 1/2*x*(a+b*arcsinh(c*x))/d^2/(c^2*x^2+1)+(a+b*arcsinh(c*x))*arctan(c*x+(c^2
*x^2+1)^(1/2))/c/d^2-1/2*I*b*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^2+1/
2*I*b*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^2+1/2*b/c/d^2/(c^2*x^2+1)^(1
/2)
```

Rubi [A] time = 0.10, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5690, 5693, 4180, 2279, 2391, 261}

$$-\frac{ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{2cd^2}+\frac{ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{2cd^2}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(c^2x^2+1\right)}+\frac{\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{cd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^2,x]
```

```
[Out] b/(2*c*d^2*Sqrt[1 + c^2*x^2]) + (x*(a + b*ArcSinh[c*x]))/(2*d^2*(1 + c^2*x^
2)) + ((a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*d^2) - ((I/2)*b*Poly
Log[2, (-I)*E^ArcSinh[c*x]])/(c*d^2) + ((I/2)*b*PolyLog[2, I*E^ArcSinh[c*x]
])/(c*d^2)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^2}\,dx &= \frac{x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(1+c^2x^2\right)} - \frac{(bc)\int \frac{x}{\left(1+c^2x^2\right)^{3/2}}\,dx}{2d^2} + \frac{\int \frac{a+b\sinh^{-1}(cx)}{d+c^2dx^2}\,dx}{2d}\\
&= \frac{b}{2cd^2\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(1+c^2x^2\right)} + \frac{\text{Subst}\left(\int (a+bx)\text{sech}(x)\,dx, x, \sinh^{-1}(cx)\right)}{2cd^2}\\
&= \frac{b}{2cd^2\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(1+c^2x^2\right)} + \frac{\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{cd^2} - \frac{(ib)\,\text{Su}}{}\\
&= \frac{b}{2cd^2\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(1+c^2x^2\right)} + \frac{\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{cd^2} - \frac{(ib)\,\text{Su}}{}\\
&= \frac{b}{2cd^2\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(1+c^2x^2\right)} + \frac{\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{cd^2} - \frac{ib\,\text{Li}_2\,(}{}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 216, normalized size = 1.74

$$ac^2x^2\tan^{-1}(cx) + acx + a\tan^{-1}(cx) - ib\left(c^2x^2+1\right)\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right) + ib\left(c^2x^2+1\right)\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right) + b\sqrt{c^2x^2+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^2,x]
```

```
[Out] (a*c*x + b*Sqrt[1 + c^2*x^2] + b*c*x*ArcSinh[c*x] + a*ArcTan[c*x] + a*c^2*x
^2*ArcTan[c*x] + I*b*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + I*b*c^2*x^2*A
rcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - I*b*ArcSinh[c*x]*Log[1 + I*E^ArcSin
h[c*x]] - I*b*c^2*x^2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] - I*b*(1 + c^2
*x^2)*PolyLog[2, (-I)*E^ArcSinh[c*x]] + I*b*(1 + c^2*x^2)*PolyLog[2, I*E^Ar
cSinh[c*x]])/(2*d^2*(c + c^3*x^2))
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arsinh}\,(cx)+a}{c^4d^2x^4+2\,c^2d^2x^2+d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\left(c^2 d x^2 + d\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(c^2*d*x^2 + d)^2, x)
```

maple [A] time = 0.01, size = 234, normalized size = 1.89

$$\frac{ax}{2d^2\left(c^2x^2+1\right)} + \frac{a \arctan(cx)}{2c\,d^2} + \frac{b \operatorname{arcsinh}(cx)\, x}{2d^2\left(c^2x^2+1\right)} + \frac{b \operatorname{arcsinh}(cx) \arctan(cx)}{2c\,d^2} + \frac{b}{2c\,d^2\sqrt{c^2x^2+1}} + \frac{b \arctan(cx) \ln}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x)
```

```
[Out] 1/2*a/d^2*x/(c^2*x^2+1)+1/2/c*a/d^2*arctan(c*x)+1/2*b/d^2*arcsinh(c*x)*x/(c
^2*x^2+1)+1/2/c*b/d^2*arcsinh(c*x)*arctan(c*x)+1/2*b/c/d^2/(c^2*x^2+1)^(1/2
)+1/2/c*b/d^2*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2/c*b/d^2*a
rctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*I/c*b/d^2*dilog(1+I*(1+I
*c*x)/(c^2*x^2+1)^(1/2))+1/2*I/c*b/d^2*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2
))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\, a\left(\frac{x}{c^2d^2x^2 + d^2} + \frac{\arctan(cx)}{cd^2}\right) + b \int \frac{\log\left(cx + \sqrt{c^2x^2+1}\right)}{c^4d^2x^4 + 2\,c^2d^2x^2 + d^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*(x/(c^2*d^2*x^2 + d^2) + arctan(c*x)/(c*d^2)) + b*integrate(log(c*x +
 sqrt(c^2*x^2 + 1))/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{asinh}(c\,x)}{\left(d\,c^2\,x^2 + d\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(d + c^2*d*x^2)^2,x)
```

```
[Out] int((a + b*asinh(c*x))/(d + c^2*d*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^4x^4+2c^2x^2+1}\, dx + \int \frac{b \operatorname{asinh}(cx)}{c^4x^4+2c^2x^2+1}\, dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b*asinh(c*x)/(c**4
*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

3.42 $\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=110

$$\frac{a+b\sinh^{-1}(cx)}{2d^2\left(c^2x^2+1\right)}-\frac{2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}-\frac{bcx}{2d^2\sqrt{c^2x^2+1}}-\frac{b\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{2d^2}+\frac{b\text{Li}_2\left(e^{2\sinh^{-1}(cx)}\right)}{2d^2}$$

```
[Out] 1/2*(a+b*arcsinh(c*x))/d^2/(c^2*x^2+1)-2*(a+b*arcsinh(c*x))*arctanh((c*x+(c
^2*x^2+1)^(1/2))^2)/d^2-1/2*b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^2+1/2
*b*polylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)/d^2-1/2*b*c*x/d^2/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {5755, 5720, 5461, 4182, 2279, 2391, 191}

$$-\frac{b\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{2d^2}+\frac{b\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{2d^2}+\frac{a+b\sinh^{-1}(cx)}{2d^2\left(c^2x^2+1\right)}-\frac{2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)^2),x]
```

```
[Out] -(b*c*x)/(2*d^2*Sqrt[1 + c^2*x^2]) + (a + b*ArcSinh[c*x])/(2*d^2*(1 + c^2*x
^2)) - (2*(a + b*ArcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/d^2 - (b*PolyLo
g[2, -E^(2*ArcSinh[c*x])])/(2*d^2) + (b*PolyLog[2, E^(2*ArcSinh[c*x])])/(2*
d^2)
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5720

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)^2}\,dx = \frac{a+b\sinh^{-1}(cx)}{2d^2\left(1+c^2x^2\right)} - \frac{(bc)\int \frac{1}{\left(1+c^2x^2\right)^{3/2}}\,dx}{2d^2} + \frac{\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)}\,dx}{d}$$

$$= -\frac{bcx}{2d^2\sqrt{1+c^2x^2}} + \frac{a+b\sinh^{-1}(cx)}{2d^2\left(1+c^2x^2\right)} + \frac{\text{Subst}\left(\int (a+bx)\text{csch}(x)\text{sech}(x)\,dx, x, \sinh^{-1}\right.}{d^2}$$

$$= -\frac{bcx}{2d^2\sqrt{1+c^2x^2}} + \frac{a+b\sinh^{-1}(cx)}{2d^2\left(1+c^2x^2\right)} + \frac{2\,\text{Subst}\left(\int (a+bx)\text{csch}(2x)\,dx, x, \sinh^{-1}(cx)\right)}{d^2}$$

$$= -\frac{bcx}{2d^2\sqrt{1+c^2x^2}} + \frac{a+b\sinh^{-1}(cx)}{2d^2\left(1+c^2x^2\right)} - \frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d^2} - \frac{b\,\text{S}}{}$$

$$= -\frac{bcx}{2d^2\sqrt{1+c^2x^2}} + \frac{a+b\sinh^{-1}(cx)}{2d^2\left(1+c^2x^2\right)} - \frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d^2} - \frac{b\,\text{S}}{}$$

$$= -\frac{bcx}{2d^2\sqrt{1+c^2x^2}} + \frac{a+b\sinh^{-1}(cx)}{2d^2\left(1+c^2x^2\right)} - \frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d^2} - \frac{b\text{L}}{}$$

Mathematica [B] time = 0.42, size = 234, normalized size = 2.13

$$-\frac{\frac{a^2}{b} - \frac{a}{c^2x^2+1} + a\log\left(c^2x^2+1\right) + 2a\sinh^{-1}(cx) - 2a\log\left(1-e^{2\sinh^{-1}(cx)}\right) + 2b\text{Li}_2\left(\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right) + 2b\text{Li}_2\left(\frac{\sqrt{-c^2}\,e^{\text{sir}}}{c}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)^2),x]
```

```
[Out] -1/2*(a^2/b - a/(1 + c^2*x^2) + (b*c*x)/Sqrt[1 + c^2*x^2] + 2*a*ArcSinh[c*x
] - (b*ArcSinh[c*x])/(1 + c^2*x^2) + 2*b*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[
c*x])/Sqrt[-c^2]] + 2*b*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c]
 - 2*a*Log[1 - E^(2*ArcSinh[c*x])] - 2*b*ArcSinh[c*x]*Log[1 - E^(2*ArcSinh[
c*x])] + a*Log[1 + c^2*x^2] + 2*b*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]]
 + 2*b*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - b*PolyLog[2, E^(2*ArcSin
h[c*x])])/d^2
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b \operatorname{arsinh}(cx) + a}{c^4d^2x^5 + 2\,c^2d^2x^3 + d^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\left(c^2dx^2 + d\right)^2 x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^2*x), x)
```

maple [B] time = 0.12, size = 283, normalized size = 2.57

$$\frac{a\ln(cx)}{d^2} + \frac{a}{2d^2\left(c^2x^2+1\right)} - \frac{a\ln\left(c^2x^2+1\right)}{2d^2} - \frac{bcx}{2d^2\sqrt{c^2x^2+1}} + \frac{b\,c^2x^2}{2d^2\left(c^2x^2+1\right)} + \frac{b \operatorname{arcsinh}(cx)}{2d^2\left(c^2x^2+1\right)} + \frac{b}{2d^2\left(c^2x^2+1\right)} - \frac{b \operatorname{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^2,x)
```

```
[Out] a/d^2*ln(c*x)+1/2*a/d^2/(c^2*x^2+1)-1/2*a/d^2*ln(c^2*x^2+1)-1/2*b*c*x/d^2/(
c^2*x^2+1)^(1/2)+1/2*b/d^2*c^2*x^2/(c^2*x^2+1)+1/2*b/d^2*arcsinh(c*x)/(c^2*
x^2+1)+1/2*b/d^2/(c^2*x^2+1)-b/d^2*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2)
)^2)-1/2*b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^2+b/d^2*arcsinh(c*x)*ln(
1+c*x+(c^2*x^2+1)^(1/2))+b/d^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+b/d^2*arcs
inh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+b/d^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,a\left(\frac{1}{c^2d^2x^2 + d^2} - \frac{\log\left(c^2x^2+1\right)}{d^2} + \frac{2\,\log(x)}{d^2}\right) + b\int \frac{\log\left(cx + \sqrt{c^2x^2+1}\right)}{c^4d^2x^5 + 2\,c^2d^2x^3 + d^2x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*(1/(c^2*d^2*x^2 + d^2) - log(c^2*x^2 + 1)/d^2 + 2*log(x)/d^2) + b*int
egrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{asinh}(c\,x)}{x\left(d\,c^2\,x^2 + d\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)^2),x)
```

```
[Out] int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^4x^5+2c^2x^3+x}\,dx + \int \frac{b\,\text{asinh}\,(cx)}{c^4x^5+2c^2x^3+x}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a/(c**4*x**5 + 2*c**2*x**3 + x), x) + Integral(b*asinh(c*x)/(c**4
*x**5 + 2*c**2*x**3 + x), x))/d**2
```

3.43 $\int \frac{a+b\sinh^{-1}(cx)}{x^2(d+c^2dx^2)^2}\,dx$

Optimal. Leaf size=168

$$-\frac{3c^2x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(c^2x^2+1\right)}-\frac{a+b\sinh^{-1}(cx)}{d^2x\left(c^2x^2+1\right)}-\frac{3c\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}-\frac{bc}{2d^2\sqrt{c^2x^2+1}}-\frac{bc\tanh^{-1}\left(\right.}{}$$

```
[Out] (-a-b*arcsinh(c*x))/d^2/x/(c^2*x^2+1)-3/2*c^2*x*(a+b*arcsinh(c*x))/d^2/(c^2
*x^2+1)-3*c*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))/d^2-b*c*arctan
h((c^2*x^2+1)^(1/2))/d^2+3/2*I*b*c*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/d^
2-3/2*I*b*c*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/d^2-1/2*b*c/d^2/(c^2*x^2+1
)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {5747, 5690, 5693, 4180, 2279, 2391, 261, 266, 51, 63, 208}

$$\frac{3ibc\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{2d^2}-\frac{3ibc\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{2d^2}-\frac{3c^2x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(c^2x^2+1\right)}-\frac{a+b\sinh^{-1}(cx)}{d^2x\left(c^2x^2+1\right)}-\frac{3c\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)^2),x]
```

```
[Out] -(b*c)/(2*d^2*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])/(d^2*x*(1 + c^2*x^2
)) - (3*c^2*x*(a + b*ArcSinh[c*x]))/(2*d^2*(1 + c^2*x^2)) - (3*c*(a + b*Arc
Sinh[c*x])*ArcTan[E^ArcSinh[c*x]])/d^2 - (b*c*ArcTanh[Sqrt[1 + c^2*x^2]])/d
^2 + (((3*I)/2)*b*c*PolyLog[2, (-I)*E^ArcSinh[c*x]])/d^2 - (((3*I)/2)*b*c*P
olyLog[2, I*E^ArcSinh[c*x]])/d^2
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(d+c^2dx^2\right)^2}\,dx = -\frac{a+b\sinh^{-1}(cx)}{d^2x\left(1+c^2x^2\right)} - \left(3c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^2}\,dx + \frac{(bc)\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}}\,dx}{d^2}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{d^2x\left(1+c^2x^2\right)} - \frac{3c^2x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(1+c^2x^2\right)} + \frac{(bc)\,\text{Subst}\left(\int \frac{1}{x\left(1+c^2x\right)^{3/2}}\,dx, x, x^2\right)}{2d^2} + \frac{(3b}{}$$

$$= -\frac{bc}{2d^2\sqrt{1+c^2x^2}} - \frac{a+b\sinh^{-1}(cx)}{d^2x\left(1+c^2x^2\right)} - \frac{3c^2x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(1+c^2x^2\right)} - \frac{(3c)\,\text{Subst}\left(\int (a+bx)\text{s}\right.}{2}$$

$$= -\frac{bc}{2d^2\sqrt{1+c^2x^2}} - \frac{a+b\sinh^{-1}(cx)}{d^2x\left(1+c^2x^2\right)} - \frac{3c^2x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(1+c^2x^2\right)} - \frac{3c\left(a+b\sinh^{-1}(cx)\right)\text{ta}}{d^2}$$

$$= -\frac{bc}{2d^2\sqrt{1+c^2x^2}} - \frac{a+b\sinh^{-1}(cx)}{d^2x\left(1+c^2x^2\right)} - \frac{3c^2x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(1+c^2x^2\right)} - \frac{3c\left(a+b\sinh^{-1}(cx)\right)\text{ta}}{d^2}$$

$$= -\frac{bc}{2d^2\sqrt{1+c^2x^2}} - \frac{a+b\sinh^{-1}(cx)}{d^2x\left(1+c^2x^2\right)} - \frac{3c^2x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(1+c^2x^2\right)} - \frac{3c\left(a+b\sinh^{-1}(cx)\right)\text{ta}}{d^2}$$

Mathematica [C] time = 0.55, size = 253, normalized size = 1.51

$$-\frac{-\frac{a}{c^2x^3+x} + 3ac\tan^{-1}(cx) + \frac{3a}{x} + \frac{bc\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};c^2x^2+1\right)}{\sqrt{c^2x^2+1}} - 3b\sqrt{-c^2}\,\text{Li}_2\left(\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right) + 3b\sqrt{-c^2}\,\text{Li}_2\left(\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right) - \frac{b\sin}{c^2}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)^2),x]
```

```
[Out] -1/2*((3*a)/x - a/(x + c^2*x^3) + (3*b*ArcSinh[c*x])/x - (b*ArcSinh[c*x])/(
x + c^2*x^3) + 3*a*c*ArcTan[c*x] + 3*b*c*ArcTanh[Sqrt[1 + c^2*x^2]] + (b*c*
Hypergeometric2F1[-1/2, 1, 1/2, 1 + c^2*x^2])/Sqrt[1 + c^2*x^2] + 3*b*Sqrt[
-c^2]*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - 3*b*Sqrt[-c^2]*
ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - 3*b*Sqrt[-c^2]*PolyLo
g[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] + 3*b*Sqrt[-c^2]*PolyLog[2, (Sqrt[-c^2]
*E^ArcSinh[c*x])/c])/d^2
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arsinh}(cx)+a}{c^4d^2x^6+2\,c^2d^2x^4+d^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arsinh}(cx)+a}{\left(c^2dx^2+d\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^2*x^2), x)
```

maple [A] time = 0.02, size = 267, normalized size = 1.59

$$-\frac{a}{d^2x}-\frac{a\,c^2x}{2d^2\left(c^2x^2+1\right)}-\frac{3ca\arctan\left(cx\right)}{2d^2}-\frac{b\,\mathrm{arcsinh}\left(cx\right)}{d^2x}-\frac{b\,\mathrm{arcsinh}\left(cx\right)c^2x}{2d^2\left(c^2x^2+1\right)}-\frac{3cb\,\mathrm{arcsinh}\left(cx\right)\arctan\left(cx\right)}{2d^2}-\frac{3cb}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^2,x)
[Out] -a/d^2/x-1/2*a/d^2*c^2*x/(c^2*x^2+1)-3/2*c*a/d^2*arctan(c*x)-b/d^2*arcsinh(
c*x)/x-1/2*b/d^2*arcsinh(c*x)*c^2*x/(c^2*x^2+1)-3/2*c*b/d^2*arcsinh(c*x)*ar
ctan(c*x)-3/2*c*b/d^2*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+3/2*c
*b/d^2*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+3/2*I*c*b/d^2*dilog(
1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/2*I*c*b/d^2*dilog(1-I*(1+I*c*x)/(c^2*x^2
+1)^(1/2))-1/2*b*c/d^2/(c^2*x^2+1)^(1/2)-c*b/d^2*arctanh(1/(c^2*x^2+1)^(1/2
))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\,a\left(\frac{3\,c^2x^2+2}{c^2d^2x^3+d^2x}+\frac{3\,c\arctan\left(cx\right)}{d^2}\right)+b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^4d^2x^6+2\,c^2d^2x^4+d^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^2,x, algorithm="maxima")
[Out] -1/2*a*((3*c^2*x^2 + 2)/(c^2*d^2*x^3 + d^2*x) + 3*c*arctan(c*x)/d^2) + b*in
tegrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{a+b\,\mathrm{asinh}\left(c\,x\right)}{x^2\,{\left(d\,c^2\,x^2+d\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)^2),x)
[Out] int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{a}{c^4x^6+2c^2x^4+x^2}\,dx+\int\frac{b\,\mathrm{asinh}\left(cx\right)}{c^4x^6+2c^2x^4+x^2}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**2/(c**2*d*x**2+d)**2,x)
[Out] (Integral(a/(c**4*x**6 + 2*c**2*x**4 + x**2), x) + Integral(b*asinh(c*x)/(c
**4*x**6 + 2*c**2*x**4 + x**2), x))/d**2
```

3.44 $\int \frac{a+b\sinh^{-1}(cx)}{x^3(d+c^2dx^2)^2}\,dx$

Optimal. Leaf size=146

$$-\frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{d^2\left(c^2x^2+1\right)}-\frac{a+b\sinh^{-1}(cx)}{2d^2x^2\left(c^2x^2+1\right)}+\frac{4c^2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}+\frac{bc^2\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{d^2}-\frac{bc^2}{}$$

```
[Out] -c^2*(a+b*arcsinh(c*x))/d^2/(c^2*x^2+1)+1/2*(-a-b*arcsinh(c*x))/d^2/x^2/(c^
2*x^2+1)+4*c^2*(a+b*arcsinh(c*x))*arctanh((c*x+(c^2*x^2+1)^(1/2))^2)/d^2+b*
c^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^2-b*c^2*polylog(2,(c*x+(c^2*x^2
+1)^(1/2))^2)/d^2-1/2*b*c/d^2/x/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5747, 5755, 5720, 5461, 4182, 2279, 2391, 191, 271}

$$\frac{bc^2\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{d^2}-\frac{bc^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{d^2}-\frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{d^2\left(c^2x^2+1\right)}-\frac{a+b\sinh^{-1}(cx)}{2d^2x^2\left(c^2x^2+1\right)}+\frac{4c^2\tanh^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)^2),x]
```

```
[Out] -(b*c)/(2*d^2*x*Sqrt[1 + c^2*x^2]) - (c^2*(a + b*ArcSinh[c*x]))/(d^2*(1 + c
^2*x^2)) - (a + b*ArcSinh[c*x])/(2*d^2*x^2*(1 + c^2*x^2)) + (4*c^2*(a + b*A
rcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/d^2 + (b*c^2*PolyLog[2, -E^(2*Arc
Sinh[c*x])])/d^2 - (b*c^2*PolyLog[2, E^(2*ArcSinh[c*x])])/d^2
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
```

```
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5720

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^3\left(d+c^2dx^2\right)^2}\,dx = -\frac{a+b\sinh^{-1}(cx)}{2d^2x^2\left(1+c^2x^2\right)} - \left(2c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)^2}\,dx + \frac{(bc)\int \frac{1}{x^2\left(1+c^2x^2\right)^{3/2}}\,dx}{2d^2}$$

$$= -\frac{bc}{2d^2x\sqrt{1+c^2x^2}} - \frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{d^2\left(1+c^2x^2\right)} - \frac{a+b\sinh^{-1}(cx)}{2d^2x^2\left(1+c^2x^2\right)} - \frac{\left(2c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)}\,dx}{d}$$

$$= -\frac{bc}{2d^2x\sqrt{1+c^2x^2}} - \frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{d^2\left(1+c^2x^2\right)} - \frac{a+b\sinh^{-1}(cx)}{2d^2x^2\left(1+c^2x^2\right)} - \frac{\left(2c^2\right)\text{Subst}\left(\int (a+bx)\text{c}\right.}{}$$

$$= -\frac{bc}{2d^2x\sqrt{1+c^2x^2}} - \frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{d^2\left(1+c^2x^2\right)} - \frac{a+b\sinh^{-1}(cx)}{2d^2x^2\left(1+c^2x^2\right)} - \frac{\left(4c^2\right)\text{Subst}\left(\int (a+bx)\text{c}\right.}{}$$

$$= -\frac{bc}{2d^2x\sqrt{1+c^2x^2}} - \frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{d^2\left(1+c^2x^2\right)} - \frac{a+b\sinh^{-1}(cx)}{2d^2x^2\left(1+c^2x^2\right)} + \frac{4c^2\left(a+b\sinh^{-1}(cx)\right)\text{ta}}{d^2}$$

$$= -\frac{bc}{2d^2x\sqrt{1+c^2x^2}} - \frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{d^2\left(1+c^2x^2\right)} - \frac{a+b\sinh^{-1}(cx)}{2d^2x^2\left(1+c^2x^2\right)} + \frac{4c^2\left(a+b\sinh^{-1}(cx)\right)\text{ta}}{d^2}$$

$$= -\frac{bc}{2d^2x\sqrt{1+c^2x^2}} - \frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{d^2\left(1+c^2x^2\right)} - \frac{a+b\sinh^{-1}(cx)}{2d^2x^2\left(1+c^2x^2\right)} + \frac{4c^2\left(a+b\sinh^{-1}(cx)\right)\text{ta}}{d^2}$$

Mathematica [B] time = 0.45, size = 326, normalized size = 2.23

$$\frac{\frac{2a^2c^2}{b} + 2ac^2\log\left(c^2x^2+1\right) + \frac{a}{c^2x^4+x^2} + 4ac^2\sinh^{-1}(cx) - 4ac^2\log\left(1-e^{2\sinh^{-1}(cx)}\right) - \frac{2a}{x^2} + 4bc^2\text{Li}_2\left(\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right) + 4}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)^2),x]
```

```
[Out] ((2*a^2*c^2)/b - (2*a)/x^2 + (b*c)/(x*Sqrt[1 + c^2*x^2]) + (2*b*c^3*x)/Sqrt
[1 + c^2*x^2] - (2*b*c*Sqrt[1 + c^2*x^2])/x + a/(x^2 + c^2*x^4) + 4*a*c^2*A
rcSinh[c*x] - (2*b*ArcSinh[c*x])/x^2 + (b*ArcSinh[c*x])/(x^2 + c^2*x^4) + 4
*b*c^2*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x])/Sqrt[-c^2]] + 4*b*c^2*ArcSin
h[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - 4*a*c^2*Log[1 - E^(2*ArcSin
h[c*x])] - 4*b*c^2*ArcSinh[c*x]*Log[1 - E^(2*ArcSinh[c*x])] + 2*a*c^2*Log[1
 + c^2*x^2] + 4*b*c^2*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] + 4*b*c^2*P
olyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - 2*b*c^2*PolyLog[2, E^(2*ArcSinh[
c*x])])/(2*d^2)
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arsinh}\,(cx)+a}{c^4d^2x^7+2\,c^2d^2x^5+d^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^4*d^2*x^7 + 2*c^2*d^2*x^5 + d^2*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arsinh}\,(cx)+a}{\left(c^2dx^2+d\right)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^2*x^3), x)
```

maple [A] time = 0.14, size = 311, normalized size = 2.13

$$-\frac{a}{2d^2x^2}-\frac{2c^2a\ln(cx)}{d^2}-\frac{c^2a}{2d^2\left(c^2x^2+1\right)}+\frac{c^2a\ln\left(c^2x^2+1\right)}{d^2}-\frac{c^2b\,\mathrm{arcsinh}(cx)}{d^2\left(c^2x^2+1\right)}-\frac{bc}{2d^2x\sqrt{c^2x^2+1}}-\frac{b\,\mathrm{arcsinh}(cx)}{2d^2x^2\left(c^2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^2,x)
```

```
[Out] -1/2*a/d^2/x^2-2*c^2*a/d^2*ln(c*x)-1/2*c^2*a/d^2/(c^2*x^2+1)+c^2*a/d^2*ln(c
^2*x^2+1)-c^2*b/d^2*arcsinh(c*x)/(c^2*x^2+1)-1/2*b*c/d^2/x/(c^2*x^2+1)^(1/2
)-1/2*b/d^2/x^2/(c^2*x^2+1)*arcsinh(c*x)+2*c^2*b/d^2*arcsinh(c*x)*ln(1+(c*x
+(c^2*x^2+1)^(1/2))^2)+b*c^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^2-2*c^
2*b/d^2*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-2*c^2*b/d^2*polylog(2,-c*x
-(c^2*x^2+1)^(1/2))-2*c^2*b/d^2*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-2*
c^2*b/d^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}a\left(\frac{2c^2\log\left(c^2x^2+1\right)}{d^2}-\frac{4c^2\log(x)}{d^2}-\frac{2c^2x^2+1}{c^2d^2x^4+d^2x^2}\right)+b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^4d^2x^7+2c^2d^2x^5+d^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*(2*c^2*log(c^2*x^2 + 1)/d^2 - 4*c^2*log(x)/d^2 - (2*c^2*x^2 + 1)/(c^2
*d^2*x^4 + d^2*x^2)) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^
7 + 2*c^2*d^2*x^5 + d^2*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{a+b\,\mathrm{asinh}(c\,x)}{x^3\left(d\,c^2\,x^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)^2),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{a}{c^4x^7+2c^2x^5+x^3}\,dx+\int\frac{b\,\mathrm{asinh}(cx)}{c^4x^7+2c^2x^5+x^3}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**3/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a/(c**4*x**7 + 2*c**2*x**5 + x**3), x) + Integral(b*asinh(c*x)/(c
**4*x**7 + 2*c**2*x**5 + x**3), x))/d**2
```

3.45 $\int \frac{a+b\sinh^{-1}(cx)}{x^4(d+c^2dx^2)^2}\,dx$

Optimal. Leaf size=239

$$\frac{5c^3\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}+\frac{5c^2\left(a+b\sinh^{-1}(cx)\right)}{3d^2x\left(c^2x^2+1\right)}-\frac{a+b\sinh^{-1}(cx)}{3d^2x^3\left(c^2x^2+1\right)}+\frac{5c^4x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(c^2x^2+1\right)}-\frac{5ib}{}$$

```
[Out] 1/3*(-a-b*arcsinh(c*x))/d^2/x^3/(c^2*x^2+1)+5/3*c^2*(a+b*arcsinh(c*x))/d^2/
x/(c^2*x^2+1)+5/2*c^4*x*(a+b*arcsinh(c*x))/d^2/(c^2*x^2+1)+5*c^3*(a+b*arcsi
nh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))/d^2+13/6*b*c^3*arctanh((c^2*x^2+1)^(
1/2))/d^2-5/2*I*b*c^3*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/d^2+5/2*I*b*c^3
*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/d^2+1/3*b*c^3/d^2/(c^2*x^2+1)^(1/2)-1
/6*b*c/d^2/x^2/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 264, normalized size of antiderivative = 1.10, number of steps used = 19, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {5747, 5690, 5693, 4180, 2279, 2391, 261, 266, 51, 63, 208}

$$-\frac{5ibc^3\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{2d^2}+\frac{5ibc^3\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{2d^2}+\frac{5c^4x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(c^2x^2+1\right)}+\frac{5c^2\left(a+b\sinh^{-1}(cx)\right)}{3d^2x\left(c^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)^2),x]
```

```
[Out] (5*b*c^3)/(6*d^2*Sqrt[1 + c^2*x^2]) + (b*c)/(3*d^2*x^2*Sqrt[1 + c^2*x^2]) -
 (b*c*Sqrt[1 + c^2*x^2])/(2*d^2*x^2) - (a + b*ArcSinh[c*x])/(3*d^2*x^3*(1 +
 c^2*x^2)) + (5*c^2*(a + b*ArcSinh[c*x]))/(3*d^2*x*(1 + c^2*x^2)) + (5*c^4*
x*(a + b*ArcSinh[c*x]))/(2*d^2*(1 + c^2*x^2)) + (5*c^3*(a + b*ArcSinh[c*x])
*ArcTan[E^ArcSinh[c*x]])/d^2 + (13*b*c^3*ArcTanh[Sqrt[1 + c^2*x^2]])/(6*d^2
) - (((5*I)/2)*b*c^3*PolyLog[2, (-I)*E^ArcSinh[c*x]])/d^2 + (((5*I)/2)*b*c^
3*PolyLog[2, I*E^ArcSinh[c*x]])/d^2
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^4\left(d+c^2dx^2\right)^2}dx = -\frac{a+b\sinh^{-1}(cx)}{3d^2x^3\left(1+c^2x^2\right)} - \frac{1}{3}\left(5c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(d+c^2dx^2\right)^2}dx + \frac{(bc)\int \frac{1}{x^3\left(1+c^2x^2\right)^{3/2}}dx}{3d^2}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{3d^2x^3\left(1+c^2x^2\right)} + \frac{5c^2\left(a+b\sinh^{-1}(cx)\right)}{3d^2x\left(1+c^2x^2\right)} + \left(5c^4\right)\int \frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^2}dx + \frac{(bc)\,\text{Subs}}{}$$

$$= \frac{bc}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{a+b\sinh^{-1}(cx)}{3d^2x^3\left(1+c^2x^2\right)} + \frac{5c^2\left(a+b\sinh^{-1}(cx)\right)}{3d^2x\left(1+c^2x^2\right)} + \frac{5c^4x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(1+c^2x^2\right)}$$

$$= \frac{5bc^3}{6d^2\sqrt{1+c^2x^2}} + \frac{bc}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{bc\sqrt{1+c^2x^2}}{2d^2x^2} - \frac{a+b\sinh^{-1}(cx)}{3d^2x^3\left(1+c^2x^2\right)} + \frac{5c^2\left(a+b\sinh\right.}{3d^2x\left(1+c^2\right.}$$

$$= \frac{5bc^3}{6d^2\sqrt{1+c^2x^2}} + \frac{bc}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{bc\sqrt{1+c^2x^2}}{2d^2x^2} - \frac{a+b\sinh^{-1}(cx)}{3d^2x^3\left(1+c^2x^2\right)} + \frac{5c^2\left(a+b\sinh\right.}{3d^2x\left(1+c^2\right.}$$

$$= \frac{5bc^3}{6d^2\sqrt{1+c^2x^2}} + \frac{bc}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{bc\sqrt{1+c^2x^2}}{2d^2x^2} - \frac{a+b\sinh^{-1}(cx)}{3d^2x^3\left(1+c^2x^2\right)} + \frac{5c^2\left(a+b\sinh\right.}{3d^2x\left(1+c^2\right.}$$

$$= \frac{5bc^3}{6d^2\sqrt{1+c^2x^2}} + \frac{bc}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{bc\sqrt{1+c^2x^2}}{2d^2x^2} - \frac{a+b\sinh^{-1}(cx)}{3d^2x^3\left(1+c^2x^2\right)} + \frac{5c^2\left(a+b\sinh\right.}{3d^2x\left(1+c^2\right.}$$

Mathematica [C] time = 0.58, size = 311, normalized size = 1.30

$$\frac{5ac^3\tan^{-1}(cx) + \frac{a}{c^2x^5+x^3} + \frac{5ac^2}{x} - \frac{5a}{3x^3} + 5b\left(-c^2\right)^{3/2}\text{Li}_2\left(\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right) - 5b\left(-c^2\right)^{3/2}\text{Li}_2\left(\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right) - \frac{5bc\sqrt{c^2x^2+1}}{6x^2} + }{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)^2),x]
```

```
[Out] ((-5*a)/(3*x^3) + (5*a*c^2)/x - (5*b*c*Sqrt[1 + c^2*x^2])/(6*x^2) + a/(x^3
+ c^2*x^5) - (5*b*ArcSinh[c*x])/(3*x^3) + (5*b*c^2*ArcSinh[c*x])/x + (b*Arc
Sinh[c*x])/(x^3 + c^2*x^5) + 5*a*c^3*ArcTan[c*x] + (35*b*c^3*ArcTanh[Sqrt[1
 + c^2*x^2]])/6 + (b*c^3*Hypergeometric2F1[-1/2, 2, 1/2, 1 + c^2*x^2])/Sqrt
[1 + c^2*x^2] - 5*b*(-c^2)^(3/2)*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x])/Sq
rt[-c^2]] + 5*b*(-c^2)^(3/2)*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x
])/c] + 5*b*(-c^2)^(3/2)*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - 5*b*(-
c^2)^(3/2)*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/(2*d^2)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arsinh}\left(cx\right) + a}{c^4d^2x^8 + 2\,c^2d^2x^6 + d^2x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^4*d^2*x^8 + 2*c^2*d^2*x^6 + d^2*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\left(c^2 dx^2 + d\right)^2 x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^2*x^4), x)
```

maple [A] time = 0.02, size = 332, normalized size = 1.39

$$-\frac{a}{3d^2x^3}+\frac{2c^2a}{d^2x}+\frac{c^4ax}{2d^2\left(c^2x^2+1\right)}+\frac{5c^3a\arctan(cx)}{2d^2}-\frac{b\operatorname{arcsinh}(cx)}{3d^2x^3}+\frac{2c^2b\operatorname{arcsinh}(cx)}{d^2x}+\frac{c^4b\operatorname{arcsinh}(cx)\,x}{2d^2\left(c^2x^2+1\right)}+\frac{5c^3 b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^2,x)
```

```
[Out] -1/3*a/d^2/x^3+2*c^2*a/d^2/x+1/2*c^4*a/d^2*x/(c^2*x^2+1)+5/2*c^3*a/d^2*arct
an(c*x)-1/3*b/d^2*arcsinh(c*x)/x^3+2*c^2*b/d^2*arcsinh(c*x)/x+1/2*c^4*b/d^2
*arcsinh(c*x)*x/(c^2*x^2+1)+5/2*c^3*b/d^2*arcsinh(c*x)*arctan(c*x)+1/3*b*c^
3/d^2/(c^2*x^2+1)^(1/2)+13/6*c^3*b/d^2*arctanh(1/(c^2*x^2+1)^(1/2))-1/6*b*c
/d^2/x^2/(c^2*x^2+1)^(1/2)+5/2*c^3*b/d^2*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*
x^2+1)^(1/2))-5/2*c^3*b/d^2*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))
-5/2*I*c^3*b/d^2*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+5/2*I*c^3*b/d^2*dil
og(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}\left(\frac{15\,c^3\arctan(cx)}{d^2}+\frac{15\,c^4x^4+10\,c^2x^2-2}{c^2d^2x^5+d^2x^3}\right)a+b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^4d^2x^8+2\,c^2d^2x^6+d^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(15*c^3*arctan(c*x)/d^2 + (15*c^4*x^4 + 10*c^2*x^2 - 2)/(c^2*d^2*x^5 +
d^2*x^3))*a + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^8 + 2*c^2
*d^2*x^6 + d^2*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{asinh}(c\,x)}{x^4 \left(d\,c^2\,x^2 + d\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)^2),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^4x^8+2c^2x^6+x^4}\,dx + \int \frac{b\operatorname{asinh}(cx)}{c^4x^8+2c^2x^6+x^4}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**4/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a/(c**4*x**8 + 2*c**2*x**6 + x**4), x) + Integral(b*asinh(c*x)/(c
**4*x**8 + 2*c**2*x**6 + x**4), x))/d**2
```

3.46 $\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=186

$$\frac{3\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4c^5d^3}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(c^2x^2+1\right)^2}-\frac{3x\left(a+b\sinh^{-1}(cx)\right)}{8c^4d^3\left(c^2x^2+1\right)}-\frac{3ib\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{8c^5d^3}+\frac{3ibL}{}$$

```
[Out] 1/12*b/c^5/d^3/(c^2*x^2+1)^(3/2)-1/4*x^3*(a+b*arcsinh(c*x))/c^2/d^3/(c^2*x^
2+1)^2-3/8*x*(a+b*arcsinh(c*x))/c^4/d^3/(c^2*x^2+1)+3/4*(a+b*arcsinh(c*x))*
arctan(c*x+(c^2*x^2+1)^(1/2))/c^5/d^3-3/8*I*b*polylog(2,-I*(c*x+(c^2*x^2+1)
^(1/2)))/c^5/d^3+3/8*I*b*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c^5/d^3-5/8*b
/c^5/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5751, 5693, 4180, 2279, 2391, 261, 266, 43}

$$-\frac{3ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{8c^5d^3}+\frac{3ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{8c^5d^3}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(c^2x^2+1\right)^2}-\frac{3x\left(a+b\sinh^{-1}(cx)\right)}{8c^4d^3\left(c^2x^2+1\right)}+\frac{3\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

```
[Out] b/(12*c^5*d^3*(1 + c^2*x^2)^(3/2)) - (5*b)/(8*c^5*d^3*Sqrt[1 + c^2*x^2]) -
(x^3*(a + b*ArcSinh[c*x]))/(4*c^2*d^3*(1 + c^2*x^2)^2) - (3*x*(a + b*ArcSin
h[c*x]))/(8*c^4*d^3*(1 + c^2*x^2)) + (3*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSi
nh[c*x]])/(4*c^5*d^3) - (((3*I)/8)*b*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^5*
d^3) + (((3*I)/8)*b*PolyLog[2, I*E^ArcSinh[c*x]])/(c^5*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rubi steps

$$\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^3}dx = -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(1+c^2x^2\right)^2}+\frac{b\int\frac{x^3}{\left(1+c^2x^2\right)^{5/2}}dx}{4cd^3}+\frac{3\int\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^2}dx}{4c^2d}$$

$$= -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(1+c^2x^2\right)^2}-\frac{3x\left(a+b\sinh^{-1}(cx)\right)}{8c^4d^3\left(1+c^2x^2\right)}+\frac{(3b)\int\frac{x}{\left(1+c^2x^2\right)^{3/2}}dx}{8c^3d^3}+\frac{b\,\mathrm{Subs}}{}$$

$$= -\frac{3b}{8c^5d^3\sqrt{1+c^2x^2}}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(1+c^2x^2\right)^2}-\frac{3x\left(a+b\sinh^{-1}(cx)\right)}{8c^4d^3\left(1+c^2x^2\right)}+\frac{3\,\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{b}{12c^5d^3\left(1+c^2x^2\right)^{3/2}}-\frac{5b}{8c^5d^3\sqrt{1+c^2x^2}}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(1+c^2x^2\right)^2}-\frac{3x\left(a+b\sinh\right.}{8c^4d^3\left(1+\right.}$$

$$= \frac{b}{12c^5d^3\left(1+c^2x^2\right)^{3/2}}-\frac{5b}{8c^5d^3\sqrt{1+c^2x^2}}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(1+c^2x^2\right)^2}-\frac{3x\left(a+b\sinh\right.}{8c^4d^3\left(1+\right.}$$

$$= \frac{b}{12c^5d^3\left(1+c^2x^2\right)^{3/2}}-\frac{5b}{8c^5d^3\sqrt{1+c^2x^2}}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(1+c^2x^2\right)^2}-\frac{3x\left(a+b\sinh\right.}{8c^4d^3\left(1+\right.}$$

Mathematica [A] time = 0.68, size = 341, normalized size = 1.83

$$-\frac{-9ac^4x^4\tan^{-1}(cx)+15ac^3x^3-18ac^2x^2\tan^{-1}(cx)+9acx-9a\tan^{-1}(cx)-9ibc^4x^4\sinh^{-1}(cx)\log\left(1-ie^{\sinh^{-1}(cx}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

```
[Out] -1/24*(9*a*c*x + 15*a*c^3*x^3 + 13*b*Sqrt[1 + c^2*x^2] + 15*b*c^2*x^2*Sqrt[
1 + c^2*x^2] + 9*b*c*x*ArcSinh[c*x] + 15*b*c^3*x^3*ArcSinh[c*x] - 9*a*ArcTa
n[c*x] - 18*a*c^2*x^2*ArcTan[c*x] - 9*a*c^4*x^4*ArcTan[c*x] - (9*I)*b*ArcSi
nh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - (18*I)*b*c^2*x^2*ArcSinh[c*x]*Log[1 - I
*E^ArcSinh[c*x]] - (9*I)*b*c^4*x^4*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] +
 (9*I)*b*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] + (18*I)*b*c^2*x^2*ArcSinh[
c*x]*Log[1 + I*E^ArcSinh[c*x]] + (9*I)*b*c^4*x^4*ArcSinh[c*x]*Log[1 + I*E^A
rcSinh[c*x]] + (9*I)*b*(1 + c^2*x^2)^2*PolyLog[2, (-I)*E^ArcSinh[c*x]] - (9
*I)*b*(1 + c^2*x^2)^2*PolyLog[2, I*E^ArcSinh[c*x]])/(c^5*d^3*(1 + c^2*x^2)^
2)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^4\,\text{arsinh}\,(cx)+ax^4}{c^6d^3x^6+3\,c^4d^3x^4+3\,c^2d^3x^2+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arcsinh(c*x) + a*x^4)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*
d^3*x^2 + d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\,\text{arsinh}\,(cx)+a)x^4}{\left(c^2dx^2+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^4/(c^2*d*x^2 + d)^3, x)
```

maple [A] time = 0.02, size = 313, normalized size = 1.68

$$-\frac{5a\,x^3}{8c^2d^3\left(c^2x^2+1\right)^2}-\frac{3ax}{8c^4d^3\left(c^2x^2+1\right)^2}+\frac{3a\arctan(cx)}{8c^5d^3}-\frac{5b\,\text{arcsinh}\,(cx)\,x^3}{8c^2d^3\left(c^2x^2+1\right)^2}-\frac{3b\,\text{arcsinh}\,(cx)\,x}{8c^4d^3\left(c^2x^2+1\right)^2}+\frac{3b\,\text{arcsinh}\,(cx)\,\text{ar}}{8c^5d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x)
```

```
[Out] -5/8/c^2*a/d^3*x^3/(c^2*x^2+1)^2-3/8/c^4*a/d^3*x/(c^2*x^2+1)^2+3/8/c^5*a/d^
3*arctan(c*x)-5/8/c^2*b/d^3*arcsinh(c*x)*x^3/(c^2*x^2+1)^2-3/8/c^4*b/d^3*ar
csinh(c*x)*x/(c^2*x^2+1)^2+3/8/c^5*b/d^3*arcsinh(c*x)*arctan(c*x)+3/8/c^5*b
/d^3*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/8/c^5*b/d^3*arctan(c
*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/8*I/c^5*b/d^3*dilog(1+I*(1+I*c*x)
/(c^2*x^2+1)^(1/2))+3/8*I/c^5*b/d^3*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-
5/8/c^3*b/d^3*x^2/(c^2*x^2+1)^(3/2)-13/24*b/c^5/d^3/(c^2*x^2+1)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{8}a\left(\frac{5c^2x^3+3x}{c^8d^3x^4+2c^6d^3x^2+c^4d^3}-\frac{3\arctan(cx)}{c^5d^3}\right)+b\int\frac{x^4\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^6d^3x^6+3c^4d^3x^4+3c^2d^3x^2+d^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*a*((5*c^2*x^3 + 3*x)/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3) - 3*arcta
n(c*x)/(c^5*d^3)) + b*integrate(x^4*log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d^3*x
^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^4\,(a+b\,\mathrm{asinh}(c\,x))}{\left(d\,c^2\,x^2+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^3,x)
```

```
[Out] int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{ax^4}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx+\int\frac{bx^4\,\mathrm{asinh}(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a*x**4/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integral
(b*x**4*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x))/d**3
```

3.47 $\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=97

$$\frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(c^2x^2+1\right)^2}-\frac{b\sinh^{-1}(cx)}{4c^4d^3}+\frac{bx^3}{12cd^3\left(c^2x^2+1\right)^{3/2}}+\frac{bx}{4c^3d^3\sqrt{c^2x^2+1}}$$

```
[Out] 1/12*b*x^3/c/d^3/(c^2*x^2+1)^(3/2)-1/4*b*arcsinh(c*x)/c^4/d^3+1/4*x^4*(a+b*
arcsinh(c*x))/d^3/(c^2*x^2+1)^2+1/4*b*x/c^3/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5723, 288, 215}

$$\frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(c^2x^2+1\right)^2}+\frac{bx^3}{12cd^3\left(c^2x^2+1\right)^{3/2}}+\frac{bx}{4c^3d^3\sqrt{c^2x^2+1}}-\frac{b\sinh^{-1}(cx)}{4c^4d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

```
[Out] (b*x^3)/(12*c*d^3*(1 + c^2*x^2)^(3/2)) + (b*x)/(4*c^3*d^3*Sqrt[1 + c^2*x^2]
) - (b*ArcSinh[c*x])/(4*c^4*d^3) + (x^4*(a + b*ArcSinh[c*x]))/(4*d^3*(1 + c
^2*x^2)^2)
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^3}\,dx = \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} - \frac{(bc)\int \frac{x^4}{\left(1+c^2x^2\right)^{5/2}}\,dx}{4d^3}$$
$$= \frac{bx^3}{12cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} - \frac{b\int \frac{x^2}{\left(1+c^2x^2\right)^{3/2}}\,dx}{4cd^3}$$
$$= \frac{bx^3}{12cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{bx}{4c^3d^3\sqrt{1+c^2x^2}} + \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} - \frac{b\int \frac{1}{\sqrt{1+c^2x^2}}\,dx}{4c^3d^3}$$
$$= \frac{bx^3}{12cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{bx}{4c^3d^3\sqrt{1+c^2x^2}} - \frac{b\sinh^{-1}(cx)}{4c^4d^3} + \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2}$$

Mathematica [A] time = 0.14, size = 79, normalized size = 0.81

$$\frac{-3a\left(2c^2x^2+1\right)+bcx\sqrt{c^2x^2+1}\left(4c^2x^2+3\right)-3\left(2bc^2x^2+b\right)\sinh^{-1}(cx)}{12c^4d^3\left(c^2x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

```
[Out] (-3*a*(1 + 2*c^2*x^2) + b*c*x*Sqrt[1 + c^2*x^2]*(3 + 4*c^2*x^2) - 3*(b + 2*
b*c^2*x^2)*ArcSinh[c*x])/(12*c^4*d^3*(1 + c^2*x^2)^2)
```

fricas [A] time = 0.54, size = 99, normalized size = 1.02

$$\frac{3\,ac^4x^4-3\left(2\,bc^2x^2+b\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(4\,bc^3x^3+3\,bcx\right)\sqrt{c^2x^2+1}}{12\left(c^8d^3x^4+2\,c^6d^3x^2+c^4d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*c^4*x^4 - 3*(2*b*c^2*x^2 + b)*log(c*x + sqrt(c^2*x^2 + 1)) + (4*b
*c^3*x^3 + 3*b*c*x)*sqrt(c^2*x^2 + 1))/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d
^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.02, size = 108, normalized size = 1.11

$$\frac{\frac{a\left(-\frac{1}{2\left(c^2x^2+1\right)}+\frac{1}{4\left(c^2x^2+1\right)^2}\right)}{d^3}+\frac{b\left(-\frac{\operatorname{arcsinh}(cx)}{2\left(c^2x^2+1\right)}+\frac{\operatorname{arcsinh}(cx)}{4\left(c^2x^2+1\right)^2}-\frac{cx}{12\left(c^2x^2+1\right)^{\frac{3}{2}}}+\frac{cx}{3\sqrt{c^2x^2+1}}\right)}{d^3}}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x)
```

```
[Out] 1/c^4*(a/d^3*(-1/2/(c^2*x^2+1)+1/4/(c^2*x^2+1)^2)+b/d^3*(-1/2/(c^2*x^2+1)*a
rcsinh(c*x)+1/4/(c^2*x^2+1)^2*arcsinh(c*x)-1/12/(c^2*x^2+1)^(3/2)*c*x+1/3*c
*x/(c^2*x^2+1)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{16}b\left(\frac{4c^2x^2+4\left(2c^2x^2+1\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+3}{c^8d^3x^4+2c^6d^3x^2+c^4d^3}-16\int\frac{2c^2x^2}{4\left(c^{10}d^3x^7+3c^8d^3x^5+3c^6d^3x^3+c^4d^3x+\left(c^9d^3\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/16*b*((4*c^2*x^2 + 4*(2*c^2*x^2 + 1)*log(c*x + sqrt(c^2*x^2 + 1)) + 3)/(
c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3) - 16*integrate(1/4*(2*c^2*x^2 + 1)/(
c^10*d^3*x^7 + 3*c^8*d^3*x^5 + 3*c^6*d^3*x^3 + c^4*d^3*x + (c^9*d^3*x^6 + 3
*c^7*d^3*x^4 + 3*c^5*d^3*x^2 + c^3*d^3)*sqrt(c^2*x^2 + 1)), x)) - 1/4*(2*c^
2*x^2 + 1)*a/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{{\left(d\,c^2\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^3,x)
```

```
[Out] int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{ax^3}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx+\int\frac{bx^3\operatorname{asinh}(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a*x**3/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integral
(b*x**3*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x))/d**3
```

3.48 $\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=184

$$\frac{\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4c^3d^3}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{8c^2d^3\left(c^2x^2+1\right)}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(c^2x^2+1\right)^2}-\frac{ib\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{8c^3d^3}+\frac{ib\text{Li}_2\left(\right.}{8}$$

```
[Out] -1/12*b/c^3/d^3/(c^2*x^2+1)^(3/2)-1/4*x*(a+b*arcsinh(c*x))/c^2/d^3/(c^2*x^2
+1)^2+1/8*x*(a+b*arcsinh(c*x))/c^2/d^3/(c^2*x^2+1)+1/4*(a+b*arcsinh(c*x))*a
rctan(c*x+(c^2*x^2+1)^(1/2))/c^3/d^3-1/8*I*b*polylog(2,-I*(c*x+(c^2*x^2+1)^
(1/2)))/c^3/d^3+1/8*I*b*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d^3+1/8*b/
c^3/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {5751, 5690, 5693, 4180, 2279, 2391, 261}

$$-\frac{ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{8c^3d^3}+\frac{ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{8c^3d^3}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{8c^2d^3\left(c^2x^2+1\right)}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(c^2x^2+1\right)^2}+\frac{\tan^{-1}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

```
[Out] -b/(12*c^3*d^3*(1 + c^2*x^2)^(3/2)) + b/(8*c^3*d^3*Sqrt[1 + c^2*x^2]) - (x*
(a + b*ArcSinh[c*x]))/(4*c^2*d^3*(1 + c^2*x^2)^2) + (x*(a + b*ArcSinh[c*x])
)/(8*c^2*d^3*(1 + c^2*x^2)) + ((a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])
/(4*c^3*d^3) - ((I/8)*b*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^3*d^3) + ((I/8)
*b*PolyLog[2, I*E^ArcSinh[c*x]])/(c^3*d^3)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rubi steps

$$\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^3}dx = -\frac{x\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(1+c^2x^2\right)^2}+\frac{b\int\frac{x}{\left(1+c^2x^2\right)^{5/2}}dx}{4cd^3}+\frac{\int\frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^2}dx}{4c^2d}$$

$$= -\frac{b}{12c^3d^3\left(1+c^2x^2\right)^{3/2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(1+c^2x^2\right)^2}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{8c^2d^3\left(1+c^2x^2\right)}-\frac{b\int\frac{x}{\left(1+c^2x^2\right)^{3/}}}{8cd^3}$$

$$= -\frac{b}{12c^3d^3\left(1+c^2x^2\right)^{3/2}}+\frac{b}{8c^3d^3\sqrt{1+c^2x^2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(1+c^2x^2\right)^2}+\frac{x\left(a+b\sinh^{-1}(\right.}{8c^2d^3\left(1+c^2x\right.}$$

$$= -\frac{b}{12c^3d^3\left(1+c^2x^2\right)^{3/2}}+\frac{b}{8c^3d^3\sqrt{1+c^2x^2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(1+c^2x^2\right)^2}+\frac{x\left(a+b\sinh^{-1}(\right.}{8c^2d^3\left(1+c^2x\right.}$$

$$= -\frac{b}{12c^3d^3\left(1+c^2x^2\right)^{3/2}}+\frac{b}{8c^3d^3\sqrt{1+c^2x^2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(1+c^2x^2\right)^2}+\frac{x\left(a+b\sinh^{-1}(\right.}{8c^2d^3\left(1+c^2x\right.}$$

$$= -\frac{b}{12c^3d^3\left(1+c^2x^2\right)^{3/2}}+\frac{b}{8c^3d^3\sqrt{1+c^2x^2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(1+c^2x^2\right)^2}+\frac{x\left(a+b\sinh^{-1}(\right.}{8c^2d^3\left(1+c^2x\right.}$$

Mathematica [A] time = 0.25, size = 340, normalized size = 1.85

$$3ac^4x^4\tan^{-1}(cx)+3ac^3x^3+6ac^2x^2\tan^{-1}(cx)-3acx+3a\tan^{-1}(cx)+3ibc^4x^4\sinh^{-1}(cx)\log\left(1-ie^{\sinh^{-1}(cx)}\right)-3$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

```
[Out] (-3*a*c*x + 3*a*c^3*x^3 + b*Sqrt[1 + c^2*x^2] + 3*b*c^2*x^2*Sqrt[1 + c^2*x^
2] - 3*b*c*x*ArcSinh[c*x] + 3*b*c^3*x^3*ArcSinh[c*x] + 3*a*ArcTan[c*x] + 6*
a*c^2*x^2*ArcTan[c*x] + 3*a*c^4*x^4*ArcTan[c*x] + (3*I)*b*ArcSinh[c*x]*Log[
1 - I*E^ArcSinh[c*x]] + (6*I)*b*c^2*x^2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*
x]] + (3*I)*b*c^4*x^4*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - (3*I)*b*ArcS
inh[c*x]*Log[1 + I*E^ArcSinh[c*x]] - (6*I)*b*c^2*x^2*ArcSinh[c*x]*Log[1 + I
*E^ArcSinh[c*x]] - (3*I)*b*c^4*x^4*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] -
 (3*I)*b*(1 + c^2*x^2)^2*PolyLog[2, (-I)*E^ArcSinh[c*x]] + (3*I)*b*(1 + c^2
*x^2)^2*PolyLog[2, I*E^ArcSinh[c*x]])/(24*c^3*d^3*(1 + c^2*x^2)^2)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^2\operatorname{arsinh}(cx) + ax^2}{c^6d^3x^6 + 3\,c^4d^3x^4 + 3\,c^2d^3x^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arcsinh(c*x) + a*x^2)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*
d^3*x^2 + d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arsinh}(cx) + a)x^2}{\left(c^2dx^2 + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^2/(c^2*d*x^2 + d)^3, x)
```

maple [A] time = 0.01, size = 307, normalized size = 1.67

$$\frac{a\,x^3}{8d^3\left(c^2x^2+1\right)^2} - \frac{ax}{8c^2d^3\left(c^2x^2+1\right)^2} + \frac{a\arctan(cx)}{8c^3d^3} + \frac{b\operatorname{arcsinh}(cx)\,x^3}{8d^3\left(c^2x^2+1\right)^2} - \frac{b\operatorname{arcsinh}(cx)\,x}{8c^2d^3\left(c^2x^2+1\right)^2} + \frac{b\operatorname{arcsinh}(cx)\operatorname{arctan}}{8c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x)
```

```
[Out] 1/8*a/d^3*x^3/(c^2*x^2+1)^2-1/8/c^2*a/d^3*x/(c^2*x^2+1)^2+1/8/c^3*a/d^3*arc
tan(c*x)+1/8*b/d^3*arcsinh(c*x)*x^3/(c^2*x^2+1)^2-1/8/c^2*b/d^3*arcsinh(c*x
)*x/(c^2*x^2+1)^2+1/8/c^3*b/d^3*arcsinh(c*x)*arctan(c*x)+1/8/c^3*b/d^3*arct
an(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/8/c^3*b/d^3*arctan(c*x)*ln(1-
I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/8*I/c^3*b/d^3*dilog(1+I*(1+I*c*x)/(c^2*x^2
+1)^(1/2))+1/8*I/c^3*b/d^3*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/8/c*b/d
^3*x^2/(c^2*x^2+1)^(3/2)+1/24*b/c^3/d^3/(c^2*x^2+1)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8}a\left(\frac{c^2x^3 - x}{c^6d^3x^4 + 2\,c^4d^3x^2 + c^2d^3} + \frac{\arctan(cx)}{c^3d^3}\right) + b\int \frac{x^2\log\left(cx + \sqrt{c^2x^2+1}\right)}{c^6d^3x^6 + 3\,c^4d^3x^4 + 3\,c^2d^3x^2 + d^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*a*((c^2*x^3 - x)/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3) + arctan(c*x)/
(c^3*d^3)) + b*integrate(x^2*log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d^3*x^6 + 3*
c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{{\left(d\,c^2\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^3,x)
```

```
[Out] int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^2}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{bx^2\operatorname{asinh}(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a*x**2/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integral
(b*x**2*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x))/d**3
```

3.49 $\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=80

$$-\frac{a+b\sinh^{-1}(cx)}{4c^2d^3\left(c^2x^2+1\right)^2}+\frac{bx}{6cd^3\sqrt{c^2x^2+1}}+\frac{bx}{12cd^3\left(c^2x^2+1\right)^{3/2}}$$

```
[Out] 1/12*b*x/c/d^3/(c^2*x^2+1)^(3/2)+1/4*(-a-b*arcsinh(c*x))/c^2/d^3/(c^2*x^2+1
)^2+1/6*b*x/c/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {5717, 192, 191}

$$-\frac{a+b\sinh^{-1}(cx)}{4c^2d^3\left(c^2x^2+1\right)^2}+\frac{bx}{6cd^3\sqrt{c^2x^2+1}}+\frac{bx}{12cd^3\left(c^2x^2+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

```
[Out] (b*x)/(12*c*d^3*(1 + c^2*x^2)^(3/2)) + (b*x)/(6*c*d^3*Sqrt[1 + c^2*x^2]) -
(a + b*ArcSinh[c*x])/(4*c^2*d^3*(1 + c^2*x^2)^2)
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^3}\,dx = -\frac{a+b\sinh^{-1}(cx)}{4c^2d^3\left(1+c^2x^2\right)^2}+\frac{b\int \frac{1}{\left(1+c^2x^2\right)^{5/2}}\,dx}{4cd^3}$$

$$= \frac{bx}{12cd^3\left(1+c^2x^2\right)^{3/2}}-\frac{a+b\sinh^{-1}(cx)}{4c^2d^3\left(1+c^2x^2\right)^2}+\frac{b\int \frac{1}{\left(1+c^2x^2\right)^{3/2}}\,dx}{6cd^3}$$

$$= \frac{bx}{12cd^3\left(1+c^2x^2\right)^{3/2}}+\frac{bx}{6cd^3\sqrt{1+c^2x^2}}-\frac{a+b\sinh^{-1}(cx)}{4c^2d^3\left(1+c^2x^2\right)^2}$$

Mathematica [A] time = 0.07, size = 56, normalized size = 0.70

$$\frac{-3a+bcx\sqrt{c^2x^2+1}\left(2c^2x^2+3\right)-3b\sinh^{-1}(cx)}{12d^3\left(c^3x^2+c\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

```
[Out] (-3*a + b*c*x*Sqrt[1 + c^2*x^2]*(3 + 2*c^2*x^2) - 3*b*ArcSinh[c*x])/(12*d^3
*(c + c^3*x^2)^2)
```

fricas [A] time = 0.68, size = 98, normalized size = 1.22

$$\frac{3\,ac^4x^4+6\,ac^2x^2-3\,b\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(2\,bc^3x^3+3\,bcx\right)\sqrt{c^2x^2+1}}{12\left(c^6d^3x^4+2\,c^4d^3x^2+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*c^4*x^4 + 6*a*c^2*x^2 - 3*b*log(c*x + sqrt(c^2*x^2 + 1)) + (2*b*c
^3*x^3 + 3*b*c*x)*sqrt(c^2*x^2 + 1))/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)x}{\left(c^2dx^2+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x/(c^2*d*x^2 + d)^3, x)
```

maple [A] time = 0.01, size = 76, normalized size = 0.95

$$\frac{-\frac{a}{4d^3\left(c^2x^2+1\right)^2}+\frac{b\left(-\frac{\mathrm{arcsinh}(cx)}{4\left(c^2x^2+1\right)^2}+\frac{cx}{12\left(c^2x^2+1\right)^{\frac{3}{2}}}+\frac{cx}{6\sqrt{c^2x^2+1}}\right)}{d^3}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x)
```

```
[Out] 1/c^2*(-1/4*a/d^3/(c^2*x^2+1)^2+b/d^3*(-1/4/(c^2*x^2+1)^2*arcsinh(c*x)+1/12
/(c^2*x^2+1)^(3/2)*c*x+1/6*c*x/(c^2*x^2+1)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{16}\,b\left(\frac{4\,\log\left(cx+\sqrt{c^2x^2+1}\right)+1}{c^6d^3x^4+2\,c^4d^3x^2+c^2d^3}-16\int\frac{1}{4\left(c^8d^3x^7+3\,c^6d^3x^5+3\,c^4d^3x^3+c^2d^3x+\left(c^7d^3x^6+3\,c^5d^3x^4+3\,c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/16*b*((4*log(c*x + sqrt(c^2*x^2 + 1)) + 1)/(c^6*d^3*x^4 + 2*c^4*d^3*x^2
+ c^2*d^3) - 16*integrate(1/4/(c^8*d^3*x^7 + 3*c^6*d^3*x^5 + 3*c^4*d^3*x^3
+ c^2*d^3*x + (c^7*d^3*x^6 + 3*c^5*d^3*x^4 + 3*c^3*d^3*x^2 + c*d^3)*sqrt(c^
2*x^2 + 1)), x)) - 1/4*a/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{{\left(d\,c^2\,x^2+d\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^3,x)
```

```
[Out] int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx+\int \frac{bx\,\mathrm{asinh}\left(cx\right)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a*x/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integral(b*
x*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x))/d**3
```

3.50 $\int \frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=178

$$\frac{3x\left(a+b\sinh^{-1}(cx)\right)}{8d^3\left(c^2x^2+1\right)}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(c^2x^2+1\right)^2}+\frac{3\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4cd^3}+\frac{3b}{8cd^3\sqrt{c^2x^2+1}}+\frac{b}{12cd^3\left(c^2x\right.}$$

```
[Out] 1/12*b/c/d^3/(c^2*x^2+1)^(3/2)+1/4*x*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^2+3
/8*x*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)+3/4*(a+b*arcsinh(c*x))*arctan(c*x+(
c^2*x^2+1)^(1/2))/c/d^3-3/8*I*b*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^3
+3/8*I*b*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^3+3/8*b/c/d^3/(c^2*x^2+1)
^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5690, 5693, 4180, 2279, 2391, 261}

$$-\frac{3ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{8cd^3}+\frac{3ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{8cd^3}+\frac{3x\left(a+b\sinh^{-1}(cx)\right)}{8d^3\left(c^2x^2+1\right)}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(c^2x^2+1\right)^2}+\frac{3\tan^{-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^3,x]
```

```
[Out] b/(12*c*d^3*(1 + c^2*x^2)^(3/2)) + (3*b)/(8*c*d^3*Sqrt[1 + c^2*x^2]) + (x*(
a + b*ArcSinh[c*x]))/(4*d^3*(1 + c^2*x^2)^2) + (3*x*(a + b*ArcSinh[c*x]))/(
8*d^3*(1 + c^2*x^2)) + (3*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(4*c
*d^3) - (((3*I)/8)*b*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*d^3) + (((3*I)/8)*
b*PolyLog[2, I*E^ArcSinh[c*x]])/(c*d^3)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^3}\,dx = \frac{x\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} - \frac{(bc)\int \frac{x}{\left(1+c^2x^2\right)^{5/2}}\,dx}{4d^3} + \frac{3\int \frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^2}\,dx}{4d}$$

$$= \frac{b}{12cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} + \frac{3x\left(a+b\sinh^{-1}(cx)\right)}{8d^3\left(1+c^2x^2\right)} - \frac{(3bc)\int \frac{x}{\left(1+c^2x^2\right)}}{8d^3}$$

$$= \frac{b}{12cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{3b}{8cd^3\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} + \frac{3x\left(a+b\sinh^{-1}(cx)\right)}{8d^3\left(1+c^2x^2\right)}$$

$$= \frac{b}{12cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{3b}{8cd^3\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} + \frac{3x\left(a+b\sinh^{-1}(cx)\right)}{8d^3\left(1+c^2x^2\right)}$$

$$= \frac{b}{12cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{3b}{8cd^3\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} + \frac{3x\left(a+b\sinh^{-1}(cx)\right)}{8d^3\left(1+c^2x^2\right)}$$

$$= \frac{b}{12cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{3b}{8cd^3\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} + \frac{3x\left(a+b\sinh^{-1}(cx)\right)}{8d^3\left(1+c^2x^2\right)}$$

Mathematica [A] time = 0.15, size = 341, normalized size = 1.92

$$9ac^4x^4\tan^{-1}(cx) + 9ac^3x^3 + 18ac^2x^2\tan^{-1}(cx) + 15acx + 9a\tan^{-1}(cx) + 9ibc^4x^4\sinh^{-1}(cx)\log\left(1-ie^{\sinh^{-1}(cx)}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^3,x]
```

```
[Out] (15*a*c*x + 9*a*c^3*x^3 + 11*b*Sqrt[1 + c^2*x^2] + 9*b*c^2*x^2*Sqrt[1 + c^2
*x^2] + 15*b*c*x*ArcSinh[c*x] + 9*b*c^3*x^3*ArcSinh[c*x] + 9*a*ArcTan[c*x]
+ 18*a*c^2*x^2*ArcTan[c*x] + 9*a*c^4*x^4*ArcTan[c*x] + (9*I)*b*ArcSinh[c*x]
*Log[1 - I*E^ArcSinh[c*x]] + (18*I)*b*c^2*x^2*ArcSinh[c*x]*Log[1 - I*E^ArcS
inh[c*x]] + (9*I)*b*c^4*x^4*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - (9*I)*
b*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] - (18*I)*b*c^2*x^2*ArcSinh[c*x]*Lo
```

```
g[1 + I*E^ArcSinh[c*x]] - (9*I)*b*c^4*x^4*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[
c*x]] - (9*I)*b*(1 + c^2*x^2)^2*PolyLog[2, (-I)*E^ArcSinh[c*x]] + (9*I)*b*(
1 + c^2*x^2)^2*PolyLog[2, I*E^ArcSinh[c*x]])/(24*c*d^3*(1 + c^2*x^2)^2)
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\operatorname{arsinh}\left(cx\right)+a}{c^6d^3x^6+3\,c^4d^3x^4+3\,c^2d^3x^2+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2
+ d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\operatorname{arsinh}\left(cx\right)+a}{\left(c^2dx^2+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(c^2*d*x^2 + d)^3, x)
```

maple [A] time = 0.01, size = 295, normalized size = 1.66

$$\frac{ax}{4d^3\left(c^2x^2+1\right)^2}+\frac{3ax}{8d^3\left(c^2x^2+1\right)}+\frac{3a\arctan\left(cx\right)}{8c\,d^3}+\frac{b\operatorname{arcsinh}\left(cx\right)x}{4d^3\left(c^2x^2+1\right)^2}+\frac{3b\operatorname{arcsinh}\left(cx\right)x}{8d^3\left(c^2x^2+1\right)}+\frac{3b\operatorname{arcsinh}\left(cx\right)\arctan\left(cx\right.}{8c\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x)
```

```
[Out] 1/4*a/d^3*x/(c^2*x^2+1)^2+3/8*a/d^3*x/(c^2*x^2+1)+3/8/c*a/d^3*arctan(c*x)+1
/4*b/d^3*arcsinh(c*x)*x/(c^2*x^2+1)^2+3/8*b/d^3*arcsinh(c*x)*x/(c^2*x^2+1)+
3/8/c*b/d^3*arcsinh(c*x)*arctan(c*x)+3/8*c*b/d^3*x^2/(c^2*x^2+1)^(3/2)+11/2
4*b/c/d^3/(c^2*x^2+1)^(3/2)+3/8/c*b/d^3*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x
^2+1)^(1/2))-3/8/c*b/d^3*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/
8*I/c*b/d^3*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+3/8*I/c*b/d^3*dilog(1-I*
(1+I*c*x)/(c^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8}\,a\left(\frac{3\,c^2x^3+5\,x}{c^4d^3x^4+2\,c^2d^3x^2+d^3}+\frac{3\,\arctan\left(cx\right)}{cd^3}\right)+b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^6d^3x^6+3\,c^4d^3x^4+3\,c^2d^3x^2+d^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*a*((3*c^2*x^3 + 5*x)/(c^4*d^3*x^4 + 2*c^2*d^3*x^2 + d^3) + 3*arctan(c*x
)/(c*d^3)) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d^3*x^6 + 3*c^4*
d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{asinh}\left(c\,x\right)}{\left(d\,c^2\,x^2+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(d + c^2*d*x^2)^3,x)
```

```
[Out] int((a + b*asinh(c*x))/(d + c^2*d*x^2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{b\operatorname{asinh}(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integral(b*as
inh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x))/d**3
```

3.51 $\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=159

$$\frac{a+b\sinh^{-1}(cx)}{2d^3\left(c^2x^2+1\right)}+\frac{a+b\sinh^{-1}(cx)}{4d^3\left(c^2x^2+1\right)^2}-\frac{2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^3}-\frac{2bcx}{3d^3\sqrt{c^2x^2+1}}-\frac{bcx}{12d^3\left(c^2x^2+1\right)^{3/2}}$$

```
[Out] -1/12*b*c*x/d^3/(c^2*x^2+1)^(3/2)+1/4*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^2+
1/2*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)-2*(a+b*arcsinh(c*x))*arctanh((c*x+(c
^2*x^2+1)^(1/2))^2)/d^3-1/2*b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^3+1/2
*b*polylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)/d^3-2/3*b*c*x/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5755, 5720, 5461, 4182, 2279, 2391, 191, 192}

$$-\frac{b\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{2d^3}+\frac{b\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{2d^3}+\frac{a+b\sinh^{-1}(cx)}{2d^3\left(c^2x^2+1\right)}+\frac{a+b\sinh^{-1}(cx)}{4d^3\left(c^2x^2+1\right)^2}-\frac{2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)^3),x]
```

```
[Out] -(b*c*x)/(12*d^3*(1 + c^2*x^2)^(3/2)) - (2*b*c*x)/(3*d^3*Sqrt[1 + c^2*x^2])
 + (a + b*ArcSinh[c*x])/(4*d^3*(1 + c^2*x^2)^2) + (a + b*ArcSinh[c*x])/(2*d
^3*(1 + c^2*x^2)) - (2*(a + b*ArcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/d^
3 - (b*PolyLog[2, -E^(2*ArcSinh[c*x])])/(2*d^3) + (b*PolyLog[2, E^(2*ArcSin
h[c*x])])/(2*d^3)
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
```

```
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5720

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)^3}\,dx = \frac{a+b\sinh^{-1}(cx)}{4d^3\left(1+c^2x^2\right)^2} - \frac{(bc)\int \frac{1}{\left(1+c^2x^2\right)^{5/2}}\,dx}{4d^3} + \frac{\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)^2}\,dx}{d}$$

$$= -\frac{bcx}{12d^3\left(1+c^2x^2\right)^{3/2}} + \frac{a+b\sinh^{-1}(cx)}{4d^3\left(1+c^2x^2\right)^2} + \frac{a+b\sinh^{-1}(cx)}{2d^3\left(1+c^2x^2\right)} - \frac{(bc)\int \frac{1}{\left(1+c^2x^2\right)^{3/2}}\,dx}{6d^3} - (\ldots$$

$$= -\frac{bcx}{12d^3\left(1+c^2x^2\right)^{3/2}} - \frac{2bcx}{3d^3\sqrt{1+c^2x^2}} + \frac{a+b\sinh^{-1}(cx)}{4d^3\left(1+c^2x^2\right)^2} + \frac{a+b\sinh^{-1}(cx)}{2d^3\left(1+c^2x^2\right)} + \frac{\text{Subs}\ldots}{}$$

$$= -\frac{bcx}{12d^3\left(1+c^2x^2\right)^{3/2}} - \frac{2bcx}{3d^3\sqrt{1+c^2x^2}} + \frac{a+b\sinh^{-1}(cx)}{4d^3\left(1+c^2x^2\right)^2} + \frac{a+b\sinh^{-1}(cx)}{2d^3\left(1+c^2x^2\right)} + \frac{2\,\text{Sub}\ldots}{}$$

$$= -\frac{bcx}{12d^3\left(1+c^2x^2\right)^{3/2}} - \frac{2bcx}{3d^3\sqrt{1+c^2x^2}} + \frac{a+b\sinh^{-1}(cx)}{4d^3\left(1+c^2x^2\right)^2} + \frac{a+b\sinh^{-1}(cx)}{2d^3\left(1+c^2x^2\right)} - \frac{2\left(a+\ldots\right.}{}$$

$$= -\frac{bcx}{12d^3\left(1+c^2x^2\right)^{3/2}} - \frac{2bcx}{3d^3\sqrt{1+c^2x^2}} + \frac{a+b\sinh^{-1}(cx)}{4d^3\left(1+c^2x^2\right)^2} + \frac{a+b\sinh^{-1}(cx)}{2d^3\left(1+c^2x^2\right)} - \frac{2\left(a+\ldots\right.}{}$$

$$= -\frac{bcx}{12d^3\left(1+c^2x^2\right)^{3/2}} - \frac{2bcx}{3d^3\sqrt{1+c^2x^2}} + \frac{a+b\sinh^{-1}(cx)}{4d^3\left(1+c^2x^2\right)^2} + \frac{a+b\sinh^{-1}(cx)}{2d^3\left(1+c^2x^2\right)} - \frac{2\left(a+\ldots\right.}{}$$

Mathematica [A] time = 0.59, size = 289, normalized size = 1.82

$$-\frac{2a^2}{b}+\frac{2a}{c^2x^2+1}+\frac{a}{\left(c^2x^2+1\right)^2}-2a\log\left(c^2x^2+1\right)-4a\sinh^{-1}(cx)+4a\log\left(1-e^{2\sinh^{-1}(cx)}\right)-4b\text{Li}_2\left(\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right)-4b\text{Li}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)^3),x]
```

```
[Out] ((-2*a^2)/b + a/(1 + c^2*x^2)^2 - (b*c*x)/(3*(1 + c^2*x^2)^(3/2)) + (2*a)/(
1 + c^2*x^2) - (8*b*c*x)/(3*Sqrt[1 + c^2*x^2]) - 4*a*ArcSinh[c*x] + (b*ArcS
inh[c*x])/(1 + c^2*x^2)^2 + (2*b*ArcSinh[c*x])/(1 + c^2*x^2) - 4*b*ArcSinh[
c*x]*Log[1 + (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - 4*b*ArcSinh[c*x]*Log[1 + (Sqr
t[-c^2]*E^ArcSinh[c*x])/c] + 4*a*Log[1 - E^(2*ArcSinh[c*x])] + 4*b*ArcSinh[
c*x]*Log[1 - E^(2*ArcSinh[c*x])] - 2*a*Log[1 + c^2*x^2] - 4*b*PolyLog[2, (c
*E^ArcSinh[c*x])/Sqrt[-c^2]] - 4*b*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c
] + 2*b*PolyLog[2, E^(2*ArcSinh[c*x])])/(4*d^3)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arsinh}\,(cx)+a}{c^6d^3x^7+3\,c^4d^3x^5+3\,c^2d^3x^3+d^3x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^6*d^3*x^7 + 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3
+ d^3*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{b\,\text{arsinh}\,(cx)+a}{\left(c^2dx^2+d\right)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^3*x), x)
```

maple [B] time = 0.20, size = 451, normalized size = 2.84

$$\frac{a\ln(cx)}{d^3}+\frac{a}{2d^3\left(c^2x^2+1\right)}+\frac{a}{4d^3\left(c^2x^2+1\right)^2}-\frac{a\ln\left(c^2x^2+1\right)}{2d^3}-\frac{2b\,c^3x^3\sqrt{c^2x^2+1}}{3d^3\left(c^4x^4+2c^2x^2+1\right)}+\frac{2b\,c^4x^4}{3d^3\left(c^4x^4+2c^2x^2+1\right)}+\frac{b}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^3,x)
```

```
[Out] a/d^3*ln(c*x)+1/2*a/d^3/(c^2*x^2+1)+1/4*a/d^3/(c^2*x^2+1)^2-1/2*a/d^3*ln(c^
2*x^2+1)-2/3*b/d^3/(c^4*x^4+2*c^2*x^2+1)*c^3*x^3*(c^2*x^2+1)^(1/2)+2/3*b/d^
3/(c^4*x^4+2*c^2*x^2+1)*c^4*x^4+1/2*b/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x
)*c^2*x^2-3/4*b/d^3/(c^4*x^4+2*c^2*x^2+1)*c*x*(c^2*x^2+1)^(1/2)+4/3*b/d^3/(
c^4*x^4+2*c^2*x^2+1)*c^2*x^2+3/4*b/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)+2
/3*b/d^3/(c^4*x^4+2*c^2*x^2+1)-b/d^3*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/
2))^2)-1/2*b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^3+b/d^3*arcsinh(c*x)*l
n(1+c*x+(c^2*x^2+1)^(1/2))+b/d^3*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+b/d^3*ar
```

```
csinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+b/d^3*polylog(2,c*x+(c^2*x^2+1)^(1/2
))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}a\left(\frac{2c^2x^2+3}{c^4d^3x^4+2c^2d^3x^2+d^3}-\frac{2\log\left(c^2x^2+1\right)}{d^3}+\frac{4\log(x)}{d^3}\right)+b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^6d^3x^7+3c^4d^3x^5+3c^2d^3x^3+d^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/4*a*((2*c^2*x^2 + 3)/(c^4*d^3*x^4 + 2*c^2*d^3*x^2 + d^3) - 2*log(c^2*x^2
+ 1)/d^3 + 4*log(x)/d^3) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d^
3*x^7 + 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 + d^3*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{a+b\,\mathrm{asinh}\left(c\,x\right)}{x\,{\left(d\,c^2\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)^3),x)
```

```
[Out] int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{a}{c^6x^7+3c^4x^5+3c^2x^3+x}\,dx+\int\frac{b\,\mathrm{asinh}\,(cx)}{c^6x^7+3c^4x^5+3c^2x^3+x}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a/(c**6*x**7 + 3*c**4*x**5 + 3*c**2*x**3 + x), x) + Integral(b*as
inh(c*x)/(c**6*x**7 + 3*c**4*x**5 + 3*c**2*x**3 + x), x))/d**3
```

3.52 $\int \frac{a+b\sinh^{-1}(cx)}{x^2(d+c^2dx^2)^3}\,dx$

Optimal. Leaf size=222

$$-\frac{15c^2x\left(a+b\sinh^{-1}(cx)\right)}{8d^3\left(c^2x^2+1\right)}-\frac{5c^2x\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(c^2x^2+1\right)^2}-\frac{a+b\sinh^{-1}(cx)}{d^3x\left(c^2x^2+1\right)^2}-\frac{15c\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4d^3}-$$

```
[Out] -1/12*b*c/d^3/(c^2*x^2+1)^(3/2)+(-a-b*arcsinh(c*x))/d^3/x/(c^2*x^2+1)^2-5/4
*c^2*x*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^2-15/8*c^2*x*(a+b*arcsinh(c*x))/d
^3/(c^2*x^2+1)-15/4*c*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))/d^3-
b*c*arctanh((c^2*x^2+1)^(1/2))/d^3+15/8*I*b*c*polylog(2,-I*(c*x+(c^2*x^2+1)
^(1/2)))/d^3-15/8*I*b*c*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/d^3-7/8*b*c/d^
3/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {5747, 5690, 5693, 4180, 2279, 2391, 261, 266, 51, 63, 208}

$$\frac{15ibc\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{8d^3}-\frac{15ibc\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{8d^3}-\frac{15c^2x\left(a+b\sinh^{-1}(cx)\right)}{8d^3\left(c^2x^2+1\right)}-\frac{5c^2x\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(c^2x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)^3),x]
```

```
[Out] -(b*c)/(12*d^3*(1 + c^2*x^2)^(3/2)) - (7*b*c)/(8*d^3*Sqrt[1 + c^2*x^2]) - (
a + b*ArcSinh[c*x])/(d^3*x*(1 + c^2*x^2)^2) - (5*c^2*x*(a + b*ArcSinh[c*x])
)/(4*d^3*(1 + c^2*x^2)^2) - (15*c^2*x*(a + b*ArcSinh[c*x]))/(8*d^3*(1 + c^2
*x^2)) - (15*c*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(4*d^3) - (b*c*
ArcTanh[Sqrt[1 + c^2*x^2]])/d^3 + (((15*I)/8)*b*c*PolyLog[2, (-I)*E^ArcSinh
[c*x]])/d^3 - (((15*I)/8)*b*c*PolyLog[2, I*E^ArcSinh[c*x]])/d^3
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(d+c^2dx^2\right)^3}\,dx = -\frac{a+b\sinh^{-1}(cx)}{d^3x\left(1+c^2x^2\right)^2} - \left(5c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^3}\,dx + \frac{(bc)\int \frac{1}{x\left(1+c^2x^2\right)^{5/2}}\,dx}{d^3}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{d^3x\left(1+c^2x^2\right)^2} - \frac{5c^2x\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} + \frac{(bc)\,\text{Subst}\left(\int \frac{1}{x\left(1+c^2x\right)^{5/2}}\,dx, x, x^2\right)}{2d^3} + \frac{(5b}{\ }$$

$$= -\frac{bc}{12d^3\left(1+c^2x^2\right)^{3/2}} - \frac{a+b\sinh^{-1}(cx)}{d^3x\left(1+c^2x^2\right)^2} - \frac{5c^2x\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} - \frac{15c^2x\left(a+b\sinh^{-1}\right.}{8d^3\left(1+c^2x^2\right)}$$

$$= -\frac{bc}{12d^3\left(1+c^2x^2\right)^{3/2}} - \frac{7bc}{8d^3\sqrt{1+c^2x^2}} - \frac{a+b\sinh^{-1}(cx)}{d^3x\left(1+c^2x^2\right)^2} - \frac{5c^2x\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} - \frac{15}{\ }$$

$$= -\frac{bc}{12d^3\left(1+c^2x^2\right)^{3/2}} - \frac{7bc}{8d^3\sqrt{1+c^2x^2}} - \frac{a+b\sinh^{-1}(cx)}{d^3x\left(1+c^2x^2\right)^2} - \frac{5c^2x\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} - \frac{15}{\ }$$

$$= -\frac{bc}{12d^3\left(1+c^2x^2\right)^{3/2}} - \frac{7bc}{8d^3\sqrt{1+c^2x^2}} - \frac{a+b\sinh^{-1}(cx)}{d^3x\left(1+c^2x^2\right)^2} - \frac{5c^2x\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} - \frac{15}{\ }$$

$$= -\frac{bc}{12d^3\left(1+c^2x^2\right)^{3/2}} - \frac{7bc}{8d^3\sqrt{1+c^2x^2}} - \frac{a+b\sinh^{-1}(cx)}{d^3x\left(1+c^2x^2\right)^2} - \frac{5c^2x\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} - \frac{15}{\ }$$

Mathematica [C] time = 1.05, size = 298, normalized size = 1.34

$$-\frac{-\frac{15\left(a+b\sinh^{-1}(cx)\right)}{c^2x^3+x} - \frac{6\left(a+b\sinh^{-1}(cx)\right)}{x\left(c^2x^2+1\right)^2} + \frac{45\left(a+b\sinh^{-1}(cx)\right)}{x} + 45ac\tan^{-1}(cx) + \frac{2bc\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};c^2x^2+1\right)}{\left(c^2x^2+1\right)^{3/2}} + \frac{15bc\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};c^2x^2\right.}{\sqrt{c^2x^2+1}}}{\ }$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)^3),x]
```

```
[Out] -1/24*((45*(a + b*ArcSinh[c*x]))/x - (6*(a + b*ArcSinh[c*x]))/(x*(1 + c^2*x
^2)^2) - (15*(a + b*ArcSinh[c*x]))/(x + c^2*x^3) + 45*a*c*ArcTan[c*x] + 45*
b*c*ArcTanh[Sqrt[1 + c^2*x^2]] + (2*b*c*Hypergeometric2F1[-3/2, 1, -1/2, 1
+ c^2*x^2])/(1 + c^2*x^2)^(3/2) + (15*b*c*Hypergeometric2F1[-1/2, 1, 1/2, 1
 + c^2*x^2])/Sqrt[1 + c^2*x^2] + 45*b*Sqrt[-c^2]*ArcSinh[c*x]*Log[1 + (c*E^
ArcSinh[c*x])/Sqrt[-c^2]] - 45*b*Sqrt[-c^2]*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2
]*E^ArcSinh[c*x])/c] - 45*b*Sqrt[-c^2]*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-
c^2]] + 45*b*Sqrt[-c^2]*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/d^3
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arsinh}\,(cx)+a}{c^6d^3x^8+3\,c^4d^3x^6+3\,c^2d^3x^4+d^3x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^6*d^3*x^8 + 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4
+ d^3*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\left(c^2dx^2 + d\right)^3 x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^3*x^2), x)
```

maple [A] time = 0.02, size = 357, normalized size = 1.61

$$-\frac{a}{d^3x} - \frac{7a\,c^4x^3}{8d^3\left(c^2x^2+1\right)^2} - \frac{9a\,c^2x}{8d^3\left(c^2x^2+1\right)^2} - \frac{15ca\arctan(cx)}{8d^3} - \frac{b\operatorname{arcsinh}(cx)}{d^3x} - \frac{7b\operatorname{arcsinh}(cx)\,c^4x^3}{8d^3\left(c^2x^2+1\right)^2} - \frac{9b\operatorname{arcsinh}(}{8d^3\left(c^2x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^3,x)
```

```
[Out] -a/d^3/x-7/8*a/d^3*c^4*x^3/(c^2*x^2+1)^2-9/8*a/d^3*c^2*x/(c^2*x^2+1)^2-15/8
*c*a/d^3*arctan(c*x)-b/d^3*arcsinh(c*x)/x-7/8*b/d^3*arcsinh(c*x)*c^4*x^3/(c
^2*x^2+1)^2-9/8*b/d^3*arcsinh(c*x)*c^2*x/(c^2*x^2+1)^2-15/8*c*b/d^3*arcsinh
(c*x)*arctan(c*x)-15/8*c*b/d^3*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/
2))+15/8*c*b/d^3*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+15/8*I*c*b
/d^3*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-15/8*I*c*b/d^3*dilog(1-I*(1+I*c
*x)/(c^2*x^2+1)^(1/2))-15/8*b/d^3*c^3*x^2/(c^2*x^2+1)^(3/2)-47/24*b*c/d^3/(
c^2*x^2+1)^(3/2)+b*c/d^3/(c^2*x^2+1)^(1/2)-c*b/d^3*arctanh(1/(c^2*x^2+1)^(1
/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{8}\,a\left(\frac{15\,c^4x^4 + 25\,c^2x^2 + 8}{c^4d^3x^5 + 2\,c^2d^3x^3 + d^3x} + \frac{15\,c\arctan(cx)}{d^3}\right) + b\int \frac{\log\left(cx + \sqrt{c^2x^2+1}\right)}{c^6d^3x^8 + 3\,c^4d^3x^6 + 3\,c^2d^3x^4 + d^3x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*a*((15*c^4*x^4 + 25*c^2*x^2 + 8)/(c^4*d^3*x^5 + 2*c^2*d^3*x^3 + d^3*x)
 + 15*c*arctan(c*x)/d^3) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d^
3*x^8 + 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4 + d^3*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\operatorname{asinh}(c\,x)}{x^2\left(d\,c^2\,x^2 + d\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)^3),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^6x^8+3c^4x^6+3c^2x^4+x^2}\, dx + \int \frac{b\operatorname{asinh}(cx)}{c^6x^8+3c^4x^6+3c^2x^4+x^2}\, dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**2/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a/(c**6*x**8 + 3*c**4*x**6 + 3*c**2*x**4 + x**2), x) + Integral(b
*asinh(c*x)/(c**6*x**8 + 3*c**4*x**6 + 3*c**2*x**4 + x**2), x))/d**3
```

3.53 $\int \frac{a+b\sinh^{-1}(cx)}{x^3(d+c^2dx^2)^3}\,dx$

Optimal. Leaf size=232

$$-\frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2d^3\left(c^2x^2+1\right)}-\frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(c^2x^2+1\right)^2}-\frac{a+b\sinh^{-1}(cx)}{2d^3x^2\left(c^2x^2+1\right)^2}+\frac{6c^2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx\right.}{d^3}$$

```
[Out] -1/2*b*c/d^3/x/(c^2*x^2+1)^(3/2)-5/12*b*c^3*x/d^3/(c^2*x^2+1)^(3/2)-3/4*c^2
*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^2+1/2*(-a-b*arcsinh(c*x))/d^3/x^2/(c^2*
x^2+1)^2-3/2*c^2*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)+6*c^2*(a+b*arcsinh(c*x)
)*arctanh((c*x+(c^2*x^2+1)^(1/2))^2)/d^3+3/2*b*c^2*polylog(2,-(c*x+(c^2*x^2
+1)^(1/2))^2)/d^3-3/2*b*c^2*polylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)/d^3+2/3*b*
c^3*x/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5747, 5755, 5720, 5461, 4182, 2279, 2391, 191, 192, 271}

$$\frac{3bc^2\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{2d^3}-\frac{3bc^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{2d^3}-\frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2d^3\left(c^2x^2+1\right)}-\frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(c^2x^2+1\right)^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)^3),x]
```

```
[Out] -(b*c)/(2*d^3*x*(1 + c^2*x^2)^(3/2)) - (5*b*c^3*x)/(12*d^3*(1 + c^2*x^2)^(3
/2)) + (2*b*c^3*x)/(3*d^3*Sqrt[1 + c^2*x^2]) - (3*c^2*(a + b*ArcSinh[c*x]))
/(4*d^3*(1 + c^2*x^2)^2) - (a + b*ArcSinh[c*x])/(2*d^3*x^2*(1 + c^2*x^2)^2)
 - (3*c^2*(a + b*ArcSinh[c*x]))/(2*d^3*(1 + c^2*x^2)) + (6*c^2*(a + b*ArcSi
nh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/d^3 + (3*b*c^2*PolyLog[2, -E^(2*ArcSi
nh[c*x])])/(2*d^3) - (3*b*c^2*PolyLog[2, E^(2*ArcSinh[c*x])])/(2*d^3)
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5720

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^3\left(d+c^2dx^2\right)^3}\,dx = -\frac{a+b\sinh^{-1}(cx)}{2d^3x^2\left(1+c^2x^2\right)^2} - \left(3c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)^3}\,dx + \frac{(bc)\int \frac{1}{x^2\left(1+c^2x^2\right)^{5/2}}\,dx}{2d^3}$$

$$= -\frac{bc}{2d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} - \frac{a+b\sinh^{-1}(cx)}{2d^3x^2\left(1+c^2x^2\right)^2} + \frac{\left(3bc^3\right)\int \frac{1}{\left(1+c^2x^2\right)}}{4d^3}$$

$$= -\frac{bc}{2d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x}{12d^3\left(1+c^2x^2\right)^{3/2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} - \frac{a+b\sinh^{-1}(cx}{2d^3x^2\left(1+c^2x^2\right)}$$

$$= -\frac{bc}{2d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x}{12d^3\left(1+c^2x^2\right)^{3/2}} + \frac{2bc^3x}{3d^3\sqrt{1+c^2x^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} -$$

$$= -\frac{bc}{2d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x}{12d^3\left(1+c^2x^2\right)^{3/2}} + \frac{2bc^3x}{3d^3\sqrt{1+c^2x^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} -$$

$$= -\frac{bc}{2d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x}{12d^3\left(1+c^2x^2\right)^{3/2}} + \frac{2bc^3x}{3d^3\sqrt{1+c^2x^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} -$$

$$= -\frac{bc}{2d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x}{12d^3\left(1+c^2x^2\right)^{3/2}} + \frac{2bc^3x}{3d^3\sqrt{1+c^2x^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} -$$

$$= -\frac{bc}{2d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x}{12d^3\left(1+c^2x^2\right)^{3/2}} + \frac{2bc^3x}{3d^3\sqrt{1+c^2x^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} -$$

Mathematica [A] time = 0.83, size = 353, normalized size = 1.52

$$\frac{-18c^2\left(2\log\left(1-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)+b\text{Li}_2\left(e^{2\sinh^{-1}(cx)}\right)\right)+\frac{3\left(a+b\sinh^{-1}(cx)\right)}{\left(c^2x^3+x\right)^2}+\frac{9\left(a+b\sinh^{-1}(cx)\right)}{c^2x^4+x^2}+\frac{18c^2(}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)^3),x]
```

```
[Out] ((-18*b*c*Sqrt[1 + c^2*x^2])/x + (9*b*c*(1 + 2*c^2*x^2))/(x*Sqrt[1 + c^2*x^
2]) + (b*c*(3 + 12*c^2*x^2 + 8*c^4*x^4))/(x*(1 + c^2*x^2)^(3/2)) - 18*b*c^2
*ArcSinh[c*x]^2 - (18*(a + b*ArcSinh[c*x]))/x^2 + (3*(a + b*ArcSinh[c*x]))/
(x + c^2*x^3)^2 + (9*(a + b*ArcSinh[c*x]))/(x^2 + c^2*x^4) + (18*c^2*(a + b
*ArcSinh[c*x])^2)/b + 36*b*c^2*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x])/Sqrt
[-c^2]] + 36*b*c^2*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + 18
*a*c^2*Log[1 + c^2*x^2] + 36*b*c^2*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]
] + 36*b*c^2*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - 18*c^2*(2*(a + b*A
rcSinh[c*x])*Log[1 - E^(2*ArcSinh[c*x])] + b*PolyLog[2, E^(2*ArcSinh[c*x])]
))/(12*d^3)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arsinh}\,(cx)+a}{c^6d^3x^9+3\,c^4d^3x^7+3\,c^2d^3x^5+d^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^6*d^3*x^9 + 3*c^4*d^3*x^7 + 3*c^2*d^3*x^5
+ d^3*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\left(c^2dx^2 + d\right)^3 x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^3*x^3), x)
```

maple [B] time = 0.26, size = 575, normalized size = 2.48

$$-\frac{a}{2d^3x^2}-\frac{3c^2a\ln(cx)}{d^3}-\frac{c^2a}{d^3\left(c^2x^2+1\right)}-\frac{c^2a}{4d^3\left(c^2x^2+1\right)^2}+\frac{3c^2a\ln\left(c^2x^2+1\right)}{2d^3}+\frac{2c^5b\,x^3\sqrt{c^2x^2+1}}{3d^3\left(c^4x^4+2c^2x^2+1\right)}-\frac{2c^6b\,x}{3d^3\left(c^4x^4+2c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^3,x)
```

```
[Out] -1/2*a/d^3/x^2-3*c^2*a/d^3*ln(c*x)-c^2*a/d^3/(c^2*x^2+1)-1/4*c^2*a/d^3/(c^2
*x^2+1)^2+3/2*c^2*a/d^3*ln(c^2*x^2+1)+2/3*c^5*b/d^3/(c^4*x^4+2*c^2*x^2+1)*x
^3*(c^2*x^2+1)^(1/2)-2/3*c^6*b/d^3/(c^4*x^4+2*c^2*x^2+1)*x^4-3/2*c^4*b/d^3/
(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)*x^2+1/4*c^3*b/d^3/(c^4*x^4+2*c^2*x^2+1)*
x*(c^2*x^2+1)^(1/2)-4/3*c^4*b/d^3/(c^4*x^4+2*c^2*x^2+1)*x^2-9/4*c^2*b/d^3/(
c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)-1/2*c*b/d^3/(c^4*x^4+2*c^2*x^2+1)/x*(c^2*
x^2+1)^(1/2)-2/3*c^2*b/d^3/(c^4*x^4+2*c^2*x^2+1)-1/2*b/d^3/(c^4*x^4+2*c^2*x
^2+1)/x^2*arcsinh(c*x)+3*c^2*b/d^3*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2)
)^2)+3/2*b*c^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^3-3*c^2*b/d^3*arcsin
h(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-3*c^2*b/d^3*polylog(2,-c*x-(c^2*x^2+1)^(
1/2))-3*c^2*b/d^3*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-3*c^2*b/d^3*poly
log(2,c*x+(c^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{4}a\left(\frac{6\,c^4x^4+9\,c^2x^2+2}{c^4d^3x^6+2\,c^2d^3x^4+d^3x^2}-\frac{6\,c^2\log\left(c^2x^2+1\right)}{d^3}+\frac{12\,c^2\log(x)}{d^3}\right)+b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^6d^3x^9+3\,c^4d^3x^7+3\,c^2d^3x^5+d^3x^3}\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/4*a*((6*c^4*x^4 + 9*c^2*x^2 + 2)/(c^4*d^3*x^6 + 2*c^2*d^3*x^4 + d^3*x^2)
 - 6*c^2*log(c^2*x^2 + 1)/d^3 + 12*c^2*log(x)/d^3) + b*integrate(log(c*x +
sqrt(c^2*x^2 + 1))/(c^6*d^3*x^9 + 3*c^4*d^3*x^7 + 3*c^2*d^3*x^5 + d^3*x^3),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\,\mathrm{asinh}\left(c\,x\right)}{x^3\,{\left(d\,c^2\,x^2 + d\right)}^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)^3),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)^3), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^6x^9+3c^4x^7+3c^2x^5+x^3}\,dx + \int \frac{b\operatorname{asinh}(cx)}{c^6x^9+3c^4x^7+3c^2x^5+x^3}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**3/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a/(c**6*x**9 + 3*c**4*x**7 + 3*c**2*x**5 + x**3), x) + Integral(b
*asinh(c*x)/(c**6*x**9 + 3*c**4*x**7 + 3*c**2*x**5 + x**3), x))/d**3
```

3.54 $\int \frac{a+b\sinh^{-1}(cx)}{x^4(d+c^2dx^2)^3}\,dx$

Optimal. Leaf size=295

$$\frac{35c^3\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4d^3}+\frac{7c^2\left(a+b\sinh^{-1}(cx)\right)}{3d^3x\left(c^2x^2+1\right)^2}-\frac{a+b\sinh^{-1}(cx)}{3d^3x^3\left(c^2x^2+1\right)^2}+\frac{35c^4x\left(a+b\sinh^{-1}(cx)\right)}{8d^3\left(c^2x^2+1\right)}+$$

```
[Out] -1/12*b*c^3/d^3/(c^2*x^2+1)^(3/2)-1/6*b*c/d^3/x^2/(c^2*x^2+1)^(3/2)+1/3*(-a
-b*arcsinh(c*x))/d^3/x^3/(c^2*x^2+1)^2+7/3*c^2*(a+b*arcsinh(c*x))/d^3/x/(c^
2*x^2+1)^2+35/12*c^4*x*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^2+35/8*c^4*x*(a+b
*arcsinh(c*x))/d^3/(c^2*x^2+1)+35/4*c^3*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*
x^2+1)^(1/2))/d^3+19/6*b*c^3*arctanh((c^2*x^2+1)^(1/2))/d^3-35/8*I*b*c^3*po
lylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/d^3+35/8*I*b*c^3*polylog(2,I*(c*x+(c^2*
x^2+1)^(1/2)))/d^3+29/24*b*c^3/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 345, normalized size of antiderivative = 1.17, number of steps used = 23, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {5747, 5690, 5693, 4180, 2279, 2391, 261, 266, 51, 63, 208}

$$-\frac{35ibc^3\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{8d^3}+\frac{35ibc^3\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{8d^3}+\frac{35c^4x\left(a+b\sinh^{-1}(cx)\right)}{8d^3\left(c^2x^2+1\right)}+\frac{35c^4x\left(a+b\sinh^{-}\right.}{12d^3\left(c^2x^2+1\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)^3),x]
```

```
[Out] (7*b*c^3)/(36*d^3*(1 + c^2*x^2)^(3/2)) + (b*c)/(9*d^3*x^2*(1 + c^2*x^2)^(3/
2)) + (49*b*c^3)/(24*d^3*Sqrt[1 + c^2*x^2]) + (5*b*c)/(9*d^3*x^2*Sqrt[1 + c
^2*x^2]) - (5*b*c*Sqrt[1 + c^2*x^2])/(6*d^3*x^2) - (a + b*ArcSinh[c*x])/(3*
d^3*x^3*(1 + c^2*x^2)^2) + (7*c^2*(a + b*ArcSinh[c*x]))/(3*d^3*x*(1 + c^2*x
^2)^2) + (35*c^4*x*(a + b*ArcSinh[c*x]))/(12*d^3*(1 + c^2*x^2)^2) + (35*c^4
*x*(a + b*ArcSinh[c*x]))/(8*d^3*(1 + c^2*x^2)) + (35*c^3*(a + b*ArcSinh[c*x
])*ArcTan[E^ArcSinh[c*x]])/(4*d^3) + (19*b*c^3*ArcTanh[Sqrt[1 + c^2*x^2]])/
(6*d^3) - (((35*I)/8)*b*c^3*PolyLog[2, (-I)*E^ArcSinh[c*x]])/d^3 + (((35*I)
/8)*b*c^3*PolyLog[2, I*E^ArcSinh[c*x]])/d^3
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
```

```
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^4\left(d+c^2dx^2\right)^3}\,dx = -\frac{a+b\sinh^{-1}(cx)}{3d^3x^3\left(1+c^2x^2\right)^2} - \frac{1}{3}\left(7c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(d+c^2dx^2\right)^3}\,dx + \frac{(bc)\int \frac{1}{x^3\left(1+c^2x^2\right)^{5/2}}\,dx}{3d^3}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{3d^3x^3\left(1+c^2x^2\right)^2} + \frac{7c^2\left(a+b\sinh^{-1}(cx)\right)}{3d^3x\left(1+c^2x^2\right)^2} + \frac{1}{3}\left(35c^4\right)\int \frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^3}\,dx + \frac{(bc)\,\mathrm{S}}{}$$

$$= \frac{bc}{9d^3x^2\left(1+c^2x^2\right)^{3/2}} - \frac{a+b\sinh^{-1}(cx)}{3d^3x^3\left(1+c^2x^2\right)^2} + \frac{7c^2\left(a+b\sinh^{-1}(cx)\right)}{3d^3x\left(1+c^2x^2\right)^2} + \frac{35c^4x\left(a+b\sinh^{-1}\right.}{12d^3\left(1+c^2x^2\right)}$$

$$= \frac{7bc^3}{36d^3\left(1+c^2x^2\right)^{3/2}} + \frac{bc}{9d^3x^2\left(1+c^2x^2\right)^{3/2}} + \frac{5bc}{9d^3x^2\sqrt{1+c^2x^2}} - \frac{a+b\sinh^{-1}(cx)}{3d^3x^3\left(1+c^2x^2\right)^2} + \frac{7c^2}{3}$$

$$= \frac{7bc^3}{36d^3\left(1+c^2x^2\right)^{3/2}} + \frac{bc}{9d^3x^2\left(1+c^2x^2\right)^{3/2}} + \frac{49bc^3}{24d^3\sqrt{1+c^2x^2}} + \frac{5bc}{9d^3x^2\sqrt{1+c^2x^2}} - \frac{5bc\sqrt{1}}{6d}$$

$$= \frac{7bc^3}{36d^3\left(1+c^2x^2\right)^{3/2}} + \frac{bc}{9d^3x^2\left(1+c^2x^2\right)^{3/2}} + \frac{49bc^3}{24d^3\sqrt{1+c^2x^2}} + \frac{5bc}{9d^3x^2\sqrt{1+c^2x^2}} - \frac{5bc\sqrt{1}}{6d}$$

$$= \frac{7bc^3}{36d^3\left(1+c^2x^2\right)^{3/2}} + \frac{bc}{9d^3x^2\left(1+c^2x^2\right)^{3/2}} + \frac{49bc^3}{24d^3\sqrt{1+c^2x^2}} + \frac{5bc}{9d^3x^2\sqrt{1+c^2x^2}} - \frac{5bc\sqrt{1}}{6d}$$

$$= \frac{7bc^3}{36d^3\left(1+c^2x^2\right)^{3/2}} + \frac{bc}{9d^3x^2\left(1+c^2x^2\right)^{3/2}} + \frac{49bc^3}{24d^3\sqrt{1+c^2x^2}} + \frac{5bc}{9d^3x^2\sqrt{1+c^2x^2}} - \frac{5bc\sqrt{1}}{6d}$$

Mathematica [C] time = 0.93, size = 380, normalized size = 1.29

$$210ac^3\tan^{-1}(cx) + \frac{42a}{c^2x^5+x^3} + \frac{12a}{x^3\left(c^2x^2+1\right)^2} + \frac{210ac^2}{x} - \frac{70a}{x^3} + 210b\left(-c^2\right)^{3/2}\mathrm{Li}_2\left(\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right) - 210b\left(-c^2\right)^{3/2}\mathrm{Li}_2\left(\frac{\sqrt{-c^2}\,e^{\sinh}}{c}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)^3),x]
```

```
[Out] ((-70*a)/x^3 + (210*a*c^2)/x + (12*a)/(x^3*(1 + c^2*x^2)^2) - (35*b*c*Sqrt[
1 + c^2*x^2])/x^2 + (42*a)/(x^3 + c^2*x^5) - (70*b*ArcSinh[c*x])/x^3 + (210
*b*c^2*ArcSinh[c*x])/x + (12*b*ArcSinh[c*x])/(x^3*(1 + c^2*x^2)^2) + (42*b*
ArcSinh[c*x])/(x^3 + c^2*x^5) + 210*a*c^3*ArcTan[c*x] + 245*b*c^3*ArcTanh[S
qrt[1 + c^2*x^2]] + (4*b*c^3*Hypergeometric2F1[-3/2, 2, -1/2, 1 + c^2*x^2])
/(1 + c^2*x^2)^(3/2) + (42*b*c^3*Hypergeometric2F1[-1/2, 2, 1/2, 1 + c^2*x^
2])/Sqrt[1 + c^2*x^2] - 210*b*(-c^2)^(3/2)*ArcSinh[c*x]*Log[1 + (c*E^ArcSin
h[c*x])/Sqrt[-c^2]] + 210*b*(-c^2)^(3/2)*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E
```

```
^ArcSinh[c*x])/c] + 210*b*(-c^2)^(3/2)*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-
c^2]] - 210*b*(-c^2)^(3/2)*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/(48*d
^3)
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arsinh}\,(cx)+a}{c^6d^3x^{10}+3\,c^4d^3x^8+3\,c^2d^3x^6+d^3x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^6*d^3*x^10 + 3*c^4*d^3*x^8 + 3*c^2*d^3*x^6
 + d^3*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arsinh}\,(cx)+a}{\left(c^2dx^2+d\right)^3x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^3*x^4), x)
```

maple [A] time = 0.02, size = 425, normalized size = 1.44

$$-\frac{a}{3d^3x^3}+\frac{3c^2a}{d^3x}+\frac{11c^6a\,x^3}{8d^3\left(c^2x^2+1\right)^2}+\frac{13c^4ax}{8d^3\left(c^2x^2+1\right)^2}+\frac{35c^3a\arctan\,(cx)}{8d^3}-\frac{b\,\text{arcsinh}\,(cx)}{3d^3x^3}+\frac{3c^2b\,\text{arcsinh}\,(cx)}{d^3x}+\frac{11c^6}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^3,x)
```

```
[Out] -1/3*a/d^3/x^3+3*c^2*a/d^3/x+11/8*c^6*a/d^3*x^3/(c^2*x^2+1)^2+13/8*c^4*a/d^
3*x/(c^2*x^2+1)^2+35/8*c^3*a/d^3*arctan(c*x)-1/3*b/d^3*arcsinh(c*x)/x^3+3*c
^2*b/d^3*arcsinh(c*x)/x+11/8*c^6*b/d^3*arcsinh(c*x)*x^3/(c^2*x^2+1)^2+13/8*
c^4*b/d^3*arcsinh(c*x)*x/(c^2*x^2+1)^2+35/8*c^3*b/d^3*arcsinh(c*x)*arctan(c
*x)+35/8*c^5*b/d^3*x^2/(c^2*x^2+1)^(3/2)+103/24*b*c^3/d^3/(c^2*x^2+1)^(3/2)
-19/6*b*c^3/d^3/(c^2*x^2+1)^(1/2)+19/6*c^3*b/d^3*arctanh(1/(c^2*x^2+1)^(1/2
))-1/6*b*c/d^3/x^2/(c^2*x^2+1)^(3/2)+35/8*c^3*b/d^3*arctan(c*x)*ln(1+I*(1+I
*c*x)/(c^2*x^2+1)^(1/2))-35/8*c^3*b/d^3*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x
^2+1)^(1/2))+35/8*I*c^3*b/d^3*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-35/8*I
*c^3*b/d^3*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{24}a\left(\frac{105\,c^3\arctan\,(cx)}{d^3}+\frac{105\,c^6x^6+175\,c^4x^4+56\,c^2x^2-8}{c^4d^3x^7+2\,c^2d^3x^5+d^3x^3}\right)+b\int \frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^6d^3x^{10}+3\,c^4d^3x^8+3\,c^2d^3x^6+d^3x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/24*a*(105*c^3*arctan(c*x)/d^3 + (105*c^6*x^6 + 175*c^4*x^4 + 56*c^2*x^2 -
 8)/(c^4*d^3*x^7 + 2*c^2*d^3*x^5 + d^3*x^3)) + b*integrate(log(c*x + sqrt(c
^2*x^2 + 1))/(c^6*d^3*x^10 + 3*c^4*d^3*x^8 + 3*c^2*d^3*x^6 + d^3*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}\left(c\,x\right)}{x^4\,{\left(d\,c^2\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)^3),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^6x^{10}+3c^4x^8+3c^2x^6+x^4}\,dx+\int \frac{b\,\mathrm{asinh}\,(cx)}{c^6x^{10}+3c^4x^8+3c^2x^6+x^4}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**4/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a/(c**6*x**10 + 3*c**4*x**8 + 3*c**2*x**6 + x**4), x) + Integral(
b*asinh(c*x)/(c**6*x**10 + 3*c**4*x**8 + 3*c**2*x**6 + x**4), x))/d**3
```

3.55 $\int x^3\sqrt{\pi + c^2\pi x^2}\,\left(a + b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=109

$$\frac{\left(\pi c^2x^2+\pi\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5\pi^2c^4}-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3\pi c^4}+\frac{2\sqrt{\pi}\,bx}{15c^3}-\frac{1}{25}\sqrt{\pi}\,bcx^5-\frac{\sqrt{\pi}\,bx^3}{45c}$$

```
[Out] -1/3*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))/c^4/Pi+1/5*(Pi*c^2*x^2+Pi)^(5
/2)*(a+b*arcsinh(c*x))/c^4/Pi^2+2/15*b*x*Pi^(1/2)/c^3-1/45*b*x^3*Pi^(1/2)/c
-1/25*b*c*x^5*Pi^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 111, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43, 5732, 12}

$$\frac{\sqrt{\pi}\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5c^4}-\frac{\sqrt{\pi}\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c^4}+\frac{2\sqrt{\pi}\,bx}{15c^3}-\frac{1}{25}\sqrt{\pi}\,bcx^5-\frac{\sqrt{\pi}\,bx^3}{45c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (2*b*Sqrt[Pi]*x)/(15*c^3) - (b*Sqrt[Pi]*x^3)/(45*c) - (b*c*Sqrt[Pi]*x^5)/25
 - (Sqrt[Pi]*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c^4) + (Sqrt[Pi]*
(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(5*c^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rubi steps

$$\int x^3\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)dx = -\frac{\sqrt{\pi}\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c^4}+\frac{\sqrt{\pi}\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}\right.}{5c^4}$$
$$= -\frac{\sqrt{\pi}\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c^4}+\frac{\sqrt{\pi}\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}\right.}{5c^4}$$
$$= \frac{2b\sqrt{\pi}x}{15c^3}-\frac{b\sqrt{\pi}x^3}{45c}-\frac{1}{25}bc\sqrt{\pi}x^5-\frac{\sqrt{\pi}\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right.}{3c^4}$$

Mathematica [A] time = 0.20, size = 106, normalized size = 0.97

$$\frac{\sqrt{\pi}\left(15a\sqrt{c^2x^2+1}\left(3c^4x^4+c^2x^2-2\right)+b\left(-9c^5x^5-5c^3x^3+30cx\right)+15b\sqrt{c^2x^2+1}\left(3c^4x^4+c^2x^2-2\right)\sinh^{-1}(c\right.}{225c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Sqrt[Pi]*(15*a*Sqrt[1 + c^2*x^2]*(-2 + c^2*x^2 + 3*c^4*x^4) + b*(30*c*x -
5*c^3*x^3 - 9*c^5*x^5) + 15*b*Sqrt[1 + c^2*x^2]*(-2 + c^2*x^2 + 3*c^4*x^4)*
ArcSinh[c*x]))/(225*c^4)
```

fricas [A] time = 0.56, size = 158, normalized size = 1.45

$$\frac{15\sqrt{\pi+\pi c^2x^2}\left(3bc^6x^6+4bc^4x^4-bc^2x^2-2b\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{\pi+\pi c^2x^2}\left(45ac^6x^6+60ac^4x^4-15a\right.}{225\left(c^6x^2+c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/225*(15*sqrt(pi + pi*c^2*x^2)*(3*b*c^6*x^6 + 4*b*c^4*x^4 - b*c^2*x^2 - 2*
b)*log(c*x + sqrt(c^2*x^2 + 1)) + sqrt(pi + pi*c^2*x^2)*(45*a*c^6*x^6 + 60*
a*c^4*x^4 - 15*a*c^2*x^2 - (9*b*c^5*x^5 + 5*b*c^3*x^3 - 30*b*c*x)*sqrt(c^2*
x^2 + 1) - 30*a))/(c^6*x^2 + c^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.15, size = 164, normalized size = 1.50

$$a\left(\frac{x^2\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}}{5\pi c^2}-\frac{2\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}}{15\pi c^4}\right)+\frac{b\sqrt{\pi}\left(45\operatorname{arcsinh}(cx)c^6x^6+60\operatorname{arcsinh}(cx)c^4x^4-9c^5x^5\sqrt{c^2x^2+1}-\right.}{225c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2),x)
```

```
[Out] a*(1/5*x^2*(Pi*c^2*x^2+Pi)^(3/2)/Pi/c^2-2/15/Pi/c^4*(Pi*c^2*x^2+Pi)^(3/2))+
1/225*b/c^4*Pi^(1/2)/(c^2*x^2+1)^(1/2)*(45*arcsinh(c*x)*c^6*x^6+60*arcsinh(
c*x)*c^4*x^4-9*c^5*x^5*(c^2*x^2+1)^(1/2)-15*arcsinh(c*x)*c^2*x^2-5*c^3*x^3*
(c^2*x^2+1)^(1/2)-30*arcsinh(c*x)+30*c*x*(c^2*x^2+1)^(1/2))
```

maxima [A] time = 0.41, size = 134, normalized size = 1.23

$$\frac{1}{15}\,b\left(\frac{3\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}x^2}{\pi c^2}-\frac{2\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}}{\pi c^4}\right)\operatorname{arsinh}(cx)+\frac{1}{15}\,a\left(\frac{3\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}x^2}{\pi c^2}-\frac{2\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}}{\pi c^4}\right)-\frac{\left(9\sqrt{\pi}\,c^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/15*b*(3*(pi + pi*c^2*x^2)^(3/2)*x^2/(pi*c^2) - 2*(pi + pi*c^2*x^2)^(3/2)/
(pi*c^4))*arcsinh(c*x) + 1/15*a*(3*(pi + pi*c^2*x^2)^(3/2)*x^2/(pi*c^2) - 2
*(pi + pi*c^2*x^2)^(3/2)/(pi*c^4)) - 1/225*(9*sqrt(pi)*c^4*x^5 + 5*sqrt(pi)
*c^2*x^3 - 30*sqrt(pi)*x)*b/c^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\sqrt{\Pi\,c^2\,x^2+\Pi}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2),x)
```

```
[Out] int(x^3*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2), x)
```

sympy [A] time = 15.68, size = 221, normalized size = 2.03

$$\begin{cases}\frac{\sqrt{\pi}\,ax^4\sqrt{c^2x^2+1}}{5}+\frac{\sqrt{\pi}\,ax^2\sqrt{c^2x^2+1}}{15c^2}-\frac{2\sqrt{\pi}\,a\sqrt{c^2x^2+1}}{15c^4}-\frac{\sqrt{\pi}\,bcx^5}{25}+\frac{\sqrt{\pi}\,bx^4\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{5}-\frac{\sqrt{\pi}\,bx^3}{45c}+\frac{\sqrt{\pi}\,bx^2\sqrt{c^2x^2+1}\operatorname{asinh}(c}{15c^2}\\ \frac{\sqrt{\pi}\,ax^4}{4}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))*(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] Piecewise((sqrt(pi)*a*x**4*sqrt(c**2*x**2 + 1)/5 + sqrt(pi)*a*x**2*sqrt(c**
2*x**2 + 1)/(15*c**2) - 2*sqrt(pi)*a*sqrt(c**2*x**2 + 1)/(15*c**4) - sqrt(p
i)*b*c*x**5/25 + sqrt(pi)*b*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/5 - sqrt(pi
)*b*x**3/(45*c) + sqrt(pi)*b*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(15*c**2)
+ 2*sqrt(pi)*b*x/(15*c**3) - 2*sqrt(pi)*b*sqrt(c**2*x**2 + 1)*asinh(c*x)/(1
5*c**4), Ne(c, 0)), (sqrt(pi)*a*x**4/4, True))
```

3.56 $\int x^2\sqrt{\pi + c^2\pi x^2} \left(a + b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=119

$$-\frac{\sqrt{\pi}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc^3}+\frac{\sqrt{\pi}\,x\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{8c^2}+\frac{1}{4}x^3\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{16}\sqrt{\pi}\,bcx^4-$$

```
[Out] -1/16*b*x^2*Pi^(1/2)/c-1/16*b*c*x^4*Pi^(1/2)-1/16*(a+b*arcsinh(c*x))^2*Pi^(
1/2)/b/c^3+1/8*x*(a+b*arcsinh(c*x))*Pi^(1/2)*(c^2*x^2+1)^(1/2)/c^2+1/4*x^3*
(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 181, normalized size of antiderivative = 1.52, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5742, 5758, 5675, 30}

$$\frac{1}{4}x^3\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)+\frac{x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{8c^2}-\frac{\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc^3\sqrt{c^2x^2+1}}-\frac{bcx^4\sqrt{\cdot}}{16\sqrt{\cdot}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(b*x^2*Sqrt[Pi + c^2*Pi*x^2])/(16*c*Sqrt[1 + c^2*x^2]) - (b*c*x^4*Sqrt[Pi
+ c^2*Pi*x^2])/(16*Sqrt[1 + c^2*x^2]) + (x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*Arc
Sinh[c*x]))/(8*c^2) + (x^3*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/4 -
(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^2)/(16*b*c^3*Sqrt[1 + c^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int x^2\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)dx = \frac{1}{4}x^3\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right) + \frac{\sqrt{\pi+c^2\pi x^2}\int\frac{x^2\left(a+b\sinh^{-1}(cx\right.}{\sqrt{1+c^2x^2}}}{4\sqrt{1+c^2x^2}}$$

$$= -\frac{bcx^4\sqrt{\pi+c^2\pi x^2}}{16\sqrt{1+c^2x^2}} + \frac{x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{8c^2} + \frac{1}{4}x^3\sqrt{\pi+}$$

$$= -\frac{bx^2\sqrt{\pi+c^2\pi x^2}}{16c\sqrt{1+c^2x^2}} - \frac{bcx^4\sqrt{\pi+c^2\pi x^2}}{16\sqrt{1+c^2x^2}} + \frac{x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh\right.}{8c^2}$$

Mathematica [A] time = 0.17, size = 79, normalized size = 0.66

$$\frac{\sqrt{\pi}\left(\sinh^{-1}(cx)\left(4b\sinh\left(4\sinh^{-1}(cx)\right)-16a\right)+16acx\sqrt{c^2x^2+1}\left(2c^2x^2+1\right)-8b\sinh^{-1}(cx)^2-b\cosh\left(4\sin\right.\right.}{128c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Sqrt[Pi]*(16*a*c*x*Sqrt[1 + c^2*x^2]*(1 + 2*c^2*x^2) - 8*b*ArcSinh[c*x]^2
- b*Cosh[4*ArcSinh[c*x]] + ArcSinh[c*x]*(-16*a + 4*b*Sinh[4*ArcSinh[c*x]]))
)/(128*c^3)
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{\pi+\pi c^2x^2}\left(bx^2\operatorname{arsinh}(cx)+ax^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*x^2*arcsinh(c*x) + a*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\pi+\pi c^2x^2}\,(b\operatorname{arsinh}(cx)+a)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)*x^2, x)
```

maple [A] time = 0.07, size = 170, normalized size = 1.43

$$\frac{ax\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}}{4\pi c^2}-\frac{ax\sqrt{\pi c^2x^2+\pi}}{8c^2}-\frac{a\pi\ln\left(\frac{\pi x c^2}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{8c^2\sqrt{\pi c^2}}+\frac{b\sqrt{\pi}\ \operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\ x^3}{4}-\frac{bc\,x^4\sqrt{\pi}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2),x)
```

```
[Out] 1/4*a*x*(Pi*c^2*x^2+Pi)^(3/2)/Pi/c^2-1/8*a/c^2*x*(Pi*c^2*x^2+Pi)^(1/2)-1/8*
a/c^2*Pi*ln(Pi*x*c^2/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1
/4*b*Pi^(1/2)*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^3-1/16*b*c*x^4*Pi^(1/2)+1/8*
```

```
b*Pi^(1/2)/c^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x-1/16*b*x^2*Pi^(1/2)/c-1/16*
b*Pi^(1/2)/c^3*arcsinh(c*x)^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\sqrt{\Pi\,c^2\,x^2+\Pi}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2),x)
```

```
[Out] int(x^2*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\sqrt{\pi}\left(\int ax^2\sqrt{c^2x^2+1}\,dx+\int bx^2\sqrt{c^2x^2+1}\,\mathrm{asinh}\left(cx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))*(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] sqrt(pi)*(Integral(a*x**2*sqrt(c**2*x**2 + 1), x) + Integral(b*x**2*sqrt(c*
*2*x**2 + 1)*asinh(c*x), x))
```

3.57 $\int x\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right)dx$

Optimal. Leaf size=61

$$\frac{\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3\pi c^2}-\frac{1}{9}\sqrt{\pi}\,bcx^3-\frac{\sqrt{\pi}\,bx}{3c}$$

```
[Out] 1/3*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))/c^2/Pi-1/3*b*x*Pi^(1/2)/c-1/9*
b*c*x^3*Pi^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 105, normalized size of antiderivative = 1.72, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {5717}

$$\frac{\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3\pi c^2}-\frac{bcx^3\sqrt{\pi c^2x^2+\pi}}{9\sqrt{c^2x^2+1}}-\frac{bx\sqrt{\pi c^2x^2+\pi}}{3c\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(b*x*Sqrt[Pi + c^2*Pi*x^2])/(3*c*Sqrt[1 + c^2*x^2]) - (b*c*x^3*Sqrt[Pi + c
^2*Pi*x^2])/(9*Sqrt[1 + c^2*x^2]) + ((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh
[c*x]))/(3*c^2*Pi)
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)dx &= \frac{\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c^2\pi}-\frac{\left(b\sqrt{\pi+c^2\pi x^2}\right)\int\left(1+c^2x^2\right)dx}{3c\sqrt{1+c^2x^2}}\\ &= -\frac{bx\sqrt{\pi+c^2\pi x^2}}{3c\sqrt{1+c^2x^2}}-\frac{bcx^3\sqrt{\pi+c^2\pi x^2}}{9\sqrt{1+c^2x^2}}+\frac{\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c^2\pi}\end{aligned}$$

Mathematica [A] time = 0.12, size = 63, normalized size = 1.03

$$\frac{\sqrt{\pi}\left(3a\left(c^2x^2+1\right)^{3/2}-bcx\left(c^2x^2+3\right)+3b\left(c^2x^2+1\right)^{3/2}\sinh^{-1}(cx)\right)}{9c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Sqrt[Pi]*(3*a*(1 + c^2*x^2)^(3/2) - b*c*x*(3 + c^2*x^2) + 3*b*(1 + c^2*x^2
)^(3/2)*ArcSinh[c*x]))/(9*c^2)
```

fricas [B] time = 0.69, size = 127, normalized size = 2.08

$$\frac{3\sqrt{\pi + \pi c^2x^2}\left(bc^4x^4 + 2\,bc^2x^2 + b\right)\log\left(cx + \sqrt{c^2x^2 + 1}\right) + \sqrt{\pi + \pi c^2x^2}\left(3\,ac^4x^4 + 6\,ac^2x^2 - \left(bc^3x^3 + 3\,bcx\right)\sqrt{c^2 \ldots}\right.}{9\left(c^4x^2 + c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*(3*sqrt(pi + pi*c^2*x^2)*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*log(c*x + sqrt(c
^2*x^2 + 1)) + sqrt(pi + pi*c^2*x^2)*(3*a*c^4*x^4 + 6*a*c^2*x^2 - (b*c^3*x^
3 + 3*b*c*x)*sqrt(c^2*x^2 + 1) + 3*a))/(c^4*x^2 + c^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.06, size = 108, normalized size = 1.77

$$\frac{a\left(\pi\,c^2x^2 + \pi\right)^{\frac{3}{2}}}{3\pi\,c^2} + \frac{b\sqrt{\pi}\,\left(3\,\mathrm{arcsinh}\,(cx)\,c^4x^4 + 6\,\mathrm{arcsinh}\,(cx)\,c^2x^2 - c^3x^3\sqrt{c^2x^2 + 1} + 3\,\mathrm{arcsinh}\,(cx) - 3cx\sqrt{c^2x^2 \ldots}\right.}{9c^2\sqrt{c^2x^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2),x)
```

```
[Out] 1/3*a/Pi/c^2*(Pi*c^2*x^2+Pi)^(3/2)+1/9*b/c^2*Pi^(1/2)/(c^2*x^2+1)^(1/2)*(3*
arcsinh(c*x)*c^4*x^4+6*arcsinh(c*x)*c^2*x^2-c^3*x^3*(c^2*x^2+1)^(1/2)+3*arc
sinh(c*x)-3*c*x*(c^2*x^2+1)^(1/2))
```

maxima [A] time = 0.50, size = 73, normalized size = 1.20

$$\frac{\left(\pi + \pi c^2x^2\right)^{\frac{3}{2}} b\,\mathrm{arsinh}\,(cx)}{3\,\pi c^2} - \frac{\left(\pi^{\frac{3}{2}}c^2x^3 + 3\,\pi^{\frac{3}{2}}x\right)b}{9\,\pi c} + \frac{\left(\pi + \pi c^2x^2\right)^{\frac{3}{2}} a}{3\,\pi c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(pi + pi*c^2*x^2)^(3/2)*b*arcsinh(c*x)/(pi*c^2) - 1/9*(pi^(3/2)*c^2*x^3
 + 3*pi^(3/2)*x)*b/(pi*c) + 1/3*(pi + pi*c^2*x^2)^(3/2)*a/(pi*c^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\left(a + b\,\mathrm{asinh}\,(c\,x)\right)\,\sqrt{\Pi\,c^2\,x^2 + \Pi}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2),x)
```

```
[Out] int(x*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2), x)
```

sympy [A] time = 2.20, size = 141, normalized size = 2.31

$$\begin{cases} \frac{\sqrt{\pi}ax^2\sqrt{c^2x^2+1}}{3} + \frac{\sqrt{\pi}a\sqrt{c^2x^2+1}}{3c^2} - \frac{\sqrt{\pi}bcx^3}{9} + \frac{\sqrt{\pi}bx^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{3} - \frac{\sqrt{\pi}bx}{3c} + \frac{\sqrt{\pi}b\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{3c^2} & \text{for } c \neq 0 \\ \frac{\sqrt{\pi}ax^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))*(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] Piecewise((sqrt(pi)*a*x**2*sqrt(c**2*x**2 + 1)/3 + sqrt(pi)*a*sqrt(c**2*x**
2 + 1)/(3*c**2) - sqrt(pi)*b*c*x**3/9 + sqrt(pi)*b*x**2*sqrt(c**2*x**2 + 1)
*asinh(c*x)/3 - sqrt(pi)*b*x/(3*c) + sqrt(pi)*b*sqrt(c**2*x**2 + 1)*asinh(c
*x)/(3*c**2), Ne(c, 0)), (sqrt(pi)*a*x**2/2, True))
```

3.58 $\int \sqrt{\pi + c^2\pi x^2} \left(a + b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=67

$$\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)+\frac{\sqrt{\pi}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc}-\frac{1}{4}\sqrt{\pi}\,bcx^2$$

```
[Out] -1/4*b*c*x^2*Pi^(1/2)+1/4*(a+b*arcsinh(c*x))^2*Pi^(1/2)/b/c+1/2*x*(a+b*arcs
inh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 111, normalized size of antiderivative = 1.66, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5682, 5675, 30}

$$\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)+\frac{\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{c^2x^2+1}}-\frac{bcx^2\sqrt{\pi c^2x^2+\pi}}{4\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(b*c*x^2*Sqrt[Pi + c^2*Pi*x^2])/(4*Sqrt[1 + c^2*x^2]) + (x*Sqrt[Pi + c^2*P
i*x^2]*(a + b*ArcSinh[c*x]))/2 + (Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]
)^2)/(4*b*c*Sqrt[1 + c^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rubi steps

$$\int \sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)dx=\frac{1}{2}x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{\sqrt{\pi+c^2\pi x^2}\int\frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}dx}{2\sqrt{1+c^2x^2}}-\frac{(b$$

$$=-\frac{bcx^2\sqrt{\pi+c^2\pi x^2}}{4\sqrt{1+c^2x^2}}+\frac{1}{2}x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{\sqrt{\pi+c^2\pi x^2}}{4bc}$$

Mathematica [A] time = 0.14, size = 69, normalized size = 1.03

$$\frac{\sqrt{\pi}\left(2\sinh^{-1}(cx)\left(2a+b\sinh\left(2\sinh^{-1}(cx)\right)\right)+4acx\sqrt{c^2x^2+1}+2b\sinh^{-1}(cx)^2-b\cosh\left(2\sinh^{-1}(cx)\right)\right)}{8c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Sqrt[Pi]*(4*a*c*x*Sqrt[1 + c^2*x^2] + 2*b*ArcSinh[c*x]^2 - b*Cosh[2*ArcSin
h[c*x]] + 2*ArcSinh[c*x]*(2*a + b*Sinh[2*ArcSinh[c*x]])))/(8*c)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{\pi + \pi c^2 x^2}\,(b\,\text{arsinh}\,(cx) + a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.06, size = 112, normalized size = 1.67

$$\frac{xa\sqrt{\pi c^2 x^2 + \pi}}{2} + \frac{a\pi \ln\left(\frac{\pi x c^2}{\sqrt{\pi c^2}} + \sqrt{\pi c^2 x^2 + \pi}\right)}{2\sqrt{\pi c^2}} + \frac{b\sqrt{\pi}\ \text{arcsinh}\,(cx)\,\sqrt{c^2x^2+1}\ x}{2} - \frac{bc\,x^2\sqrt{\pi}}{4} + \frac{b\sqrt{\pi}\ \text{arcsinh}\,(c}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2),x)
```

```
[Out] 1/2*x*a*(Pi*c^2*x^2+Pi)^(1/2)+1/2*a*Pi*ln(Pi*x*c^2/(Pi*c^2)^(1/2)+(Pi*c^2*x
^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/2*b*Pi^(1/2)*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*
x-1/4*b*c*x^2*Pi^(1/2)+1/4*b*Pi^(1/2)/c*arcsinh(c*x)^2-1/4*b*Pi^(1/2)/c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,\text{asinh}\,(c\,x))\ \sqrt{\Pi\,c^2\,x^2 + \Pi}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\sqrt{\pi}\left(\int a\sqrt{c^2x^2+1}\,dx+\int b\sqrt{c^2x^2+1}\,\operatorname{asinh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] sqrt(pi)*(Integral(a*sqrt(c**2*x**2 + 1), x) + Integral(b*sqrt(c**2*x**2 +
1)*asinh(c*x), x))
```

3.59 $\int \frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=89

$$\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)-2\sqrt{\pi}\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)-\sqrt{\pi}\,b\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)+\sqrt{\pi}\,b\text{Li}_2\left($$

```
[Out] -b*c*x*Pi^(1/2)-2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))*Pi^(1/2
)-b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*Pi^(1/2)+b*polylog(2,c*x+(c^2*x^2+1)^
(1/2))*Pi^(1/2)+(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 177, normalized size of antiderivative = 1.99, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5742, 5760, 4182, 2279, 2391, 8}

$$-\frac{b\sqrt{\pi c^2x^2+\pi}\,\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+\frac{b\sqrt{\pi c^2x^2+\pi}\,\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(c\right.$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] -((b*c*x*Sqrt[Pi + c^2*Pi*x^2])/Sqrt[1 + c^2*x^2]) + Sqrt[Pi + c^2*Pi*x^2]*
(a + b*ArcSinh[c*x]) - (2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])*ArcTan
h[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (b*Sqrt[Pi + c^2*Pi*x^2]*PolyLog[2,
-E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (b*Sqrt[Pi + c^2*Pi*x^2]*PolyLog[2, E
^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
```

```
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right)}{x}\,dx = \sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right) + \frac{\sqrt{\pi + c^2\pi x^2}\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{1+c^2x^2}}\,dx}{\sqrt{1 + c^2x^2}} - \frac{\left(bc\sqrt{\right.}{}$$

$$= -\frac{bcx\sqrt{\pi + c^2\pi x^2}}{\sqrt{1 + c^2x^2}} + \sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right) + \frac{\sqrt{\pi + c^2\pi x^2}\ \text{Sub}}{}$$

$$= -\frac{bcx\sqrt{\pi + c^2\pi x^2}}{\sqrt{1 + c^2x^2}} + \sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right) - \frac{2\sqrt{\pi + c^2\pi x^2}\left(a\right.}{}$$

$$= -\frac{bcx\sqrt{\pi + c^2\pi x^2}}{\sqrt{1 + c^2x^2}} + \sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right) - \frac{2\sqrt{\pi + c^2\pi x^2}\left(a\right.}{}$$

$$= -\frac{bcx\sqrt{\pi + c^2\pi x^2}}{\sqrt{1 + c^2x^2}} + \sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right) - \frac{2\sqrt{\pi + c^2\pi x^2}\left(a\right.}{}$$

Mathematica [A] time = 0.20, size = 131, normalized size = 1.47

$$\sqrt{\pi}\left(a\sqrt{c^2x^2+1} - a\log\left(\pi\left(\sqrt{c^2x^2+1}+1\right)\right) + a\log(x) + b\left(\sqrt{c^2x^2+1}\sinh^{-1}(cx) + \text{Li}_2\left(-e^{-\sinh^{-1}(cx)}\right) - \text{Li}_2\left(e\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] Sqrt[Pi]*(a*Sqrt[1 + c^2*x^2] + a*Log[x] - a*Log[Pi*(1 + Sqrt[1 + c^2*x^2])
] + b*(-(c*x) + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + ArcSinh[c*x]*Log[1 - E^(-A
rcSinh[c*x])] - ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + PolyLog[2, -E^(-A
rcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])]))
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\pi + \pi c^2x^2}\,(b\,\text{arsinh}\,(cx) + a)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/x, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.22, size = 171, normalized size = 1.92

$$-\sqrt{\pi}\ \operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi\,c^2x^2+\pi}}\right)a+a\sqrt{\pi\,c^2x^2+\pi}+\sqrt{\pi}\ \operatorname{arcsinh}(cx)\ln\left(1-cx-\sqrt{c^2x^2+1}\right)b+\sqrt{\pi}\ \operatorname{arcsinh}(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/x,x)
```

```
[Out] -Pi^(1/2)*arctanh(Pi^(1/2)/(Pi*c^2*x^2+Pi)^(1/2))*a+a*(Pi*c^2*x^2+Pi)^(1/2)
+Pi^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*b+Pi^(1/2)*arcsinh(c*x)*
(c^2*x^2+1)^(1/2)*b-Pi^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*b-b*c
*x*Pi^(1/2)+b*polylog(2,c*x+(c^2*x^2+1)^(1/2))*Pi^(1/2)-b*polylog(2,-c*x-(c
^2*x^2+1)^(1/2))*Pi^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\left(\sqrt{\pi}\ \operatorname{arsinh}\left(\frac{1}{c|x|}\right)-\sqrt{\pi+\pi c^2x^2}\right)a+b\int\frac{\sqrt{\pi+\pi c^2x^2}\ \log\left(cx+\sqrt{c^2x^2+1}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -(sqrt(pi)*arcsinh(1/(c*abs(x))) - sqrt(pi + pi*c^2*x^2))*a + b*integrate(s
qrt(pi + pi*c^2*x^2)*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{(a+b\operatorname{asinh}(c\,x))\ \sqrt{\Pi\,c^2\,x^2+\Pi}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2))/x,x)
```

```
[Out] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\sqrt{\pi}\left(\int\frac{a\sqrt{c^2x^2+1}}{x}\,dx+\int\frac{b\sqrt{c^2x^2+1}\ \operatorname{asinh}(cx)}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(pi*c**2*x**2+pi)**(1/2)/x,x)
```

```
[Out] sqrt(pi)*(Integral(a*sqrt(c**2*x**2 + 1)/x, x) + Integral(b*sqrt(c**2*x**2
+ 1)*asinh(c*x)/x, x))
```

3.60 $\int \frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=61

$$-\frac{\sqrt{\pi c^2x^2+\pi}\,\left(a+b\sinh^{-1}(cx)\right)}{x}+\frac{\sqrt{\pi}\,c\left(a+b\sinh^{-1}(cx)\right)^2}{2b}+\sqrt{\pi}\,bc\log(x)$$

```
[Out] 1/2*c*(a+b*arcsinh(c*x))^2*Pi^(1/2)/b+b*c*ln(x)*Pi^(1/2)-(a+b*arcsinh(c*x))
*(Pi*c^2*x^2+Pi)^(1/2)/x
```

Rubi [A] time = 0.11, antiderivative size = 105, normalized size of antiderivative = 1.72, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {5737, 29, 5675}

$$\frac{c\sqrt{\pi c^2x^2+\pi}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2b\sqrt{c^2x^2+1}}-\frac{\sqrt{\pi c^2x^2+\pi}\,\left(a+b\sinh^{-1}(cx)\right)}{x}+\frac{bc\sqrt{\pi c^2x^2+\pi}\,\log(x)}{\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] -((Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x) + (c*Sqrt[Pi + c^2*Pi*x^2
]*(a + b*ArcSinh[c*x])^2)/(2*b*Sqrt[1 + c^2*x^2]) + (b*c*Sqrt[Pi + c^2*Pi*x
^2]*Log[x])/Sqrt[1 + c^2*x^2]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5737

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 1)*S
qrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x] -
 Dist[(c^2*Sqrt[d + e*x^2])/(f^2*(m + 1)*Sqrt[1 + c^2*x^2]), Int[((f*x)^(m
+ 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{x^2}\,dx &= -\frac{\sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{x}+\frac{\left(bc\sqrt{\pi+c^2\pi x^2}\right)\int\frac{1}{x}\,dx}{\sqrt{1+c^2x^2}}+\frac{\left(c^2\sqrt{\pi+\right.}{}\\ &= -\frac{\sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{x}+\frac{c\sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2b\sqrt{1+c^2x^2}}+\end{aligned}$$

Mathematica [A] time = 0.17, size = 75, normalized size = 1.23

$$\frac{\sqrt{\pi}\left(2\sinh^{-1}(cx)\left(acx-b\sqrt{c^2x^2+1}\right)-2a\sqrt{c^2x^2+1}+2bcx\log(cx)+bcx\sinh^{-1}(cx)^2\right)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] (Sqrt[Pi]*(-2*a*Sqrt[1 + c^2*x^2] + 2*(a*c*x - b*Sqrt[1 + c^2*x^2])*ArcSinh
[c*x] + b*c*x*ArcSinh[c*x]^2 + 2*b*c*x*Log[c*x]))/(2*x)
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\pi + \pi c^2 x^2}\,(b\,\text{arsinh}\,(cx) + a)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^2,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/x^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.16, size = 155, normalized size = 2.54

$$-\frac{a\left(\pi c^2 x^2 + \pi\right)^{\frac{3}{2}}}{\pi x} + a c^2 x\sqrt{\pi c^2 x^2 + \pi} + \frac{a c^2 \pi \ln\left(\frac{\pi x c^2}{\sqrt{\pi c^2}} + \sqrt{\pi c^2 x^2 + \pi}\right)}{\sqrt{\pi c^2}} + \frac{bc\sqrt{\pi}\ \text{arcsinh}\,(cx)^2}{2} - bc\sqrt{\pi}\ \text{arcsinh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/x^2,x)
```

```
[Out] -a/Pi/x*(Pi*c^2*x^2+Pi)^(3/2)+a*c^2*x*(Pi*c^2*x^2+Pi)^(1/2)+a*c^2*Pi*ln(Pi*
x*c^2/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/2*b*c*Pi^(1/2)
*arcsinh(c*x)^2-b*c*Pi^(1/2)*arcsinh(c*x)-b*Pi^(1/2)*arcsinh(c*x)/x*(c^2*x^
2+1)^(1/2)+b*c*Pi^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(\sqrt{\pi}\,c\,\text{arsinh}\,(cx) - \frac{\sqrt{\pi + \pi c^2 x^2}}{x}\right)a + b\int \frac{\sqrt{\pi + \pi c^2 x^2}\ \log\left(cx + \sqrt{c^2 x^2 + 1}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^2,x, algorithm="maxima
")
```

```
[Out] (sqrt(pi)*c*arcsinh(c*x) - sqrt(pi + pi*c^2*x^2)/x)*a + b*integrate(sqrt(pi
 + pi*c^2*x^2)*log(c*x + sqrt(c^2*x^2 + 1))/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a + b\,\text{asinh}\,(c\,x))\ \sqrt{\Pi\,c^2\,x^2 + \Pi}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2))/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2))/x^2, x)
```

sympy [B] time = 3.01, size = 110, normalized size = 1.80

$$-\frac{\sqrt{\pi}\,ac^2x}{\sqrt{c^2x^2+1}}+\sqrt{\pi}\,ac\operatorname{asinh}(cx)-\frac{\sqrt{\pi}\,a}{x\sqrt{c^2x^2+1}}+\sqrt{\pi}\,bc\log(x)+\frac{\sqrt{\pi}\,bc\operatorname{asinh}^2(cx)}{2}-\frac{\sqrt{\pi}\,b\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(pi*c**2*x**2+pi)**(1/2)/x**2,x)
```

```
[Out] -sqrt(pi)*a*c**2*x/sqrt(c**2*x**2 + 1) + sqrt(pi)*a*c*asinh(c*x) - sqrt(pi)
*a/(x*sqrt(c**2*x**2 + 1)) + sqrt(pi)*b*c*log(x) + sqrt(pi)*b*c*asinh(c*x)*
*2/2 - sqrt(pi)*b*sqrt(c**2*x**2 + 1)*asinh(c*x)/x
```

3.61 $\int \frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=113

$$-\frac{\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{2x^2}-\sqrt{\pi}\,c^2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{2}\sqrt{\pi}\,bc^2\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)+\frac{1}{2}\sqrt{}$$

```
[Out] -1/2*b*c*Pi^(1/2)/x-c^2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))*P
i^(1/2)-1/2*b*c^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*Pi^(1/2)+1/2*b*c^2*poly
log(2,c*x+(c^2*x^2+1)^(1/2))*Pi^(1/2)-1/2*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi
)^(1/2)/x^2
```

Rubi [A] time = 0.20, antiderivative size = 201, normalized size of antiderivative = 1.78, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5737, 30, 5760, 4182, 2279, 2391}

$$-\frac{bc^2\sqrt{\pi c^2x^2+\pi}\,\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2x^2+1}}+\frac{bc^2\sqrt{\pi c^2x^2+\pi}\,\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2x^2+1}}-\frac{\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh\right.}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] -(b*c*Sqrt[Pi + c^2*Pi*x^2])/(2*x*Sqrt[1 + c^2*x^2]) - (Sqrt[Pi + c^2*Pi*x^
2]*(a + b*ArcSinh[c*x]))/(2*x^2) - (c^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSin
h[c*x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (b*c^2*Sqrt[Pi + c^2*P
i*x^2]*PolyLog[2, -E^ArcSinh[c*x]])/(2*Sqrt[1 + c^2*x^2]) + (b*c^2*Sqrt[Pi
+ c^2*Pi*x^2]*PolyLog[2, E^ArcSinh[c*x]])/(2*Sqrt[1 + c^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5737

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 1)*S
qrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x] -
```

```
 Dist[(c^2*Sqrt[d + e*x^2])/(f^2*(m + 1)*Sqrt[1 + c^2*x^2]), Int[((f*x)^(m
+ 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right)}{x^3}\,dx = -\frac{\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right)}{2x^2} + \frac{\left(bc\sqrt{\pi + c^2\pi x^2}\right)\int \frac{1}{x^2}\,dx}{2\sqrt{1 + c^2x^2}} + \frac{\left(c^2\sqrt{\pi +\right.}{}$$

$$= -\frac{bc\sqrt{\pi + c^2\pi x^2}}{2x\sqrt{1 + c^2x^2}} - \frac{\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right)}{2x^2} + \frac{\left(c^2\sqrt{\pi + c^2\pi x^2}\right)S}{}$$

$$= -\frac{bc\sqrt{\pi + c^2\pi x^2}}{2x\sqrt{1 + c^2x^2}} - \frac{\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right)}{2x^2} - \frac{c^2\sqrt{\pi + c^2\pi x^2}\,(a}{}$$

$$= -\frac{bc\sqrt{\pi + c^2\pi x^2}}{2x\sqrt{1 + c^2x^2}} - \frac{\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right)}{2x^2} - \frac{c^2\sqrt{\pi + c^2\pi x^2}\,(a}{}$$

$$= -\frac{bc\sqrt{\pi + c^2\pi x^2}}{2x\sqrt{1 + c^2x^2}} - \frac{\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right)}{2x^2} - \frac{c^2\sqrt{\pi + c^2\pi x^2}\,(a}{}$$

Mathematica [A] time = 3.32, size = 185, normalized size = 1.64

$$\frac{1}{8}\sqrt{\pi}\left(-\frac{4a\sqrt{c^2x^2+1}}{x^2} - 4ac^2\log\left(\pi\left(\sqrt{c^2x^2+1}+1\right)\right) + 4ac^2\log(x) + bc^2\left(4\text{Li}_2\left(-e^{-\sinh^{-1}(cx)}\right) - 4\text{Li}_2\left(e^{-\sinh^{-1}(cx)}\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] (Sqrt[Pi]*((-4*a*Sqrt[1 + c^2*x^2])/x^2 + 4*a*c^2*Log[x] - 4*a*c^2*Log[Pi*(
1 + Sqrt[1 + c^2*x^2])] + b*c^2*(-2*Coth[ArcSinh[c*x]/2] - ArcSinh[c*x]*Csc
h[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 4*ArcSinh
[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 4*PolyLog[2, -E^(-ArcSinh[c*x])] - 4*Pol
yLog[2, E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Tanh[A
rcSinh[c*x]/2])))/8
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\pi + \pi c^2x^2}\,(b\,\text{arsinh}\,(cx) + a)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^3,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/x^3, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.30, size = 243, normalized size = 2.15

$$-\frac{a\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}}{2\pi x^2}-\frac{a\sqrt{\pi}\ \operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)c^2}{2}+\frac{a\sqrt{\pi c^2x^2+\pi}\ c^2}{2}-\frac{b\sqrt{\pi}\ \operatorname{arcsinh}(cx)\,c^2}{2\sqrt{c^2x^2+1}}-\frac{bc\sqrt{\pi}}{2x}-\frac{b\sqrt{\pi}\ \operatorname{arcs}}{2\sqrt{c^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/x^3,x)
```

```
[Out] -1/2*a/Pi/x^2*(Pi*c^2*x^2+Pi)^(3/2)-1/2*a*Pi^(1/2)*arctanh(Pi^(1/2)/(Pi*c^2
*x^2+Pi)^(1/2))*c^2+1/2*a*(Pi*c^2*x^2+Pi)^(1/2)*c^2-1/2*b*Pi^(1/2)/(c^2*x^2
+1)^(1/2)*arcsinh(c*x)*c^2-1/2*b*c*Pi^(1/2)/x-1/2*b*Pi^(1/2)/(c^2*x^2+1)^(1
/2)/x^2*arcsinh(c*x)-1/2*b*c^2*Pi^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(
1/2))-1/2*b*c^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*Pi^(1/2)+1/2*b*c^2*Pi^(1/
2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+1/2*b*c^2*polylog(2,c*x+(c^2*x^
2+1)^(1/2))*Pi^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\left(\sqrt{\pi}\,c^2\operatorname{arsinh}\left(\frac{1}{c|x|}\right)-\sqrt{\pi+\pi c^2x^2}\,c^2+\frac{\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}}{\pi x^2}\right)a+b\int\frac{\sqrt{\pi+\pi c^2x^2}\ \log\left(cx+\sqrt{c^2x^2+1}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^3,x, algorithm="maxima
")
```

```
[Out] -1/2*(sqrt(pi)*c^2*arcsinh(1/(c*abs(x))) - sqrt(pi + pi*c^2*x^2)*c^2 + (pi
+ pi*c^2*x^2)^(3/2)/(pi*x^2))*a + b*integrate(sqrt(pi + pi*c^2*x^2)*log(c*x
 + sqrt(c^2*x^2 + 1))/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{(a+b\operatorname{asinh}(c\,x))\ \sqrt{\Pi\,c^2\,x^2+\Pi}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2))/x^3,x)
```

```
[Out] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\sqrt{\pi}\left(\int\frac{a\sqrt{c^2x^2+1}}{x^3}\,dx+\int\frac{b\sqrt{c^2x^2+1}\ \operatorname{asinh}(cx)}{x^3}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(pi*c**2*x**2+pi)**(1/2)/x**3,x)
```

```
[Out] sqrt(pi)*(Integral(a*sqrt(c**2*x**2 + 1)/x**3, x) + Integral(b*sqrt(c**2*x*
*2 + 1)*asinh(c*x)/x**3, x))
```

3.62 $\int \frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=62

$$-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3\pi x^3}+\frac{1}{3}\sqrt{\pi}\,bc^3\log(x)-\frac{\sqrt{\pi}\,bc}{6x^2}$$

```
[Out] -1/3*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))/Pi/x^3-1/6*b*c*Pi^(1/2)/x^2+1
/3*b*c^3*ln(x)*Pi^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 106, normalized size of antiderivative = 1.71, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5723, 14}

$$-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3\pi x^3}-\frac{bc\sqrt{\pi c^2x^2+\pi}}{6x^2\sqrt{c^2x^2+1}}+\frac{bc^3\sqrt{\pi c^2x^2+\pi}\log(x)}{3\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] -(b*c*Sqrt[Pi + c^2*Pi*x^2])/(6*x^2*Sqrt[1 + c^2*x^2]) - ((Pi + c^2*Pi*x^2)
^(3/2)*(a + b*ArcSinh[c*x]))/(3*Pi*x^3) + (b*c^3*Sqrt[Pi + c^2*Pi*x^2]*Log[
x])/(3*Sqrt[1 + c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{x^4}\,dx &= -\frac{\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3\pi x^3}+\frac{\left(bc\sqrt{\pi+c^2\pi x^2}\right)\int \frac{1+c^2x^2}{x^3}\,dx}{3\sqrt{1+c^2x^2}}\\ &= -\frac{\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3\pi x^3}+\frac{\left(bc\sqrt{\pi+c^2\pi x^2}\right)\int \left(\frac{1}{x^3}+\frac{c^2}{x}\right) d}{3\sqrt{1+c^2x^2}}\\ &= -\frac{bc\sqrt{\pi+c^2\pi x^2}}{6x^2\sqrt{1+c^2x^2}}-\frac{\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3\pi x^3}+\frac{bc^3\sqrt{\pi+c^2\pi}}{3\sqrt{1+}}\end{aligned}$$

Mathematica [A] time = 0.14, size = 78, normalized size = 1.26

$$\frac{1}{3}\sqrt{\pi}\,bc^3\log(x)-\frac{\sqrt{\pi}\left(2a\left(c^2x^2+1\right)^{3/2}+3bc^3x^3+2b\left(c^2x^2+1\right)^{3/2}\sinh^{-1}(cx)+bcx\right)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] -1/6*(Sqrt[Pi]*(b*c*x + 3*b*c^3*x^3 + 2*a*(1 + c^2*x^2)^(3/2) + 2*b*(1 + c^
2*x^2)^(3/2)*ArcSinh[c*x]))/x^3 + (b*c^3*Sqrt[Pi]*Log[x])/3
```

fricas [B] time = 0.77, size = 217, normalized size = 3.50

$$-\frac{2\sqrt{\pi+\pi c^2x^2}\left(bc^4x^4+2\,bc^2x^2+b\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-\sqrt{\pi}\left(bc^5x^5+bc^3x^3\right)\log\left(\frac{\pi+\pi c^2x^6+\pi c^2x^2+\pi x^4+\sqrt{\pi}\sqrt{\pi+\pi}}{c^2x^4+x^2}\right.}{6\left(c^2x^5+x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^4,x, algorithm="fricas
")
```

```
[Out] -1/6*(2*sqrt(pi + pi*c^2*x^2)*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*log(c*x + sqrt(
c^2*x^2 + 1)) - sqrt(pi)*(b*c^5*x^5 + b*c^3*x^3)*log((pi + pi*c^2*x^6 + pi*
c^2*x^2 + pi*x^4 + sqrt(pi)*sqrt(pi + pi*c^2*x^2)*sqrt(c^2*x^2 + 1)*(x^4 -
1))/(c^2*x^4 + x^2)) + sqrt(pi + pi*c^2*x^2)*(2*a*c^4*x^4 + 4*a*c^2*x^2 - (
b*c*x^3 - b*c*x)*sqrt(c^2*x^2 + 1) + 2*a))/(c^2*x^5 + x^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.31, size = 501, normalized size = 8.08

$$-\frac{a\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}}{3\pi x^3}-\frac{2b\,c^3\sqrt{\pi}\ \operatorname{arcsinh}(cx)}{3}+\frac{b\sqrt{\pi}\ x^4\operatorname{arcsinh}(cx)\,c^7}{3c^4x^4+3c^2x^2+1}-\frac{b\sqrt{\pi}\ x^3\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\ c^6}{3c^4x^4+3c^2x^2+1}+\frac{b\sqrt{}}{18c^4x^4+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/x^4,x)
```

```
[Out] -1/3*a/Pi/x^3*(Pi*c^2*x^2+Pi)^(3/2)-2/3*b*c^3*Pi^(1/2)*arcsinh(c*x)+b*Pi^(1
/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^4*arcsinh(c*x)*c^7-b*Pi^(1/2)/(3*c^4*x^4+3*c^
2*x^2+1)*x^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^6+1/6*b*Pi^(1/2)/(3*c^4*x^4+3
*c^2*x^2+1)*x^4*c^7-1/6*b*Pi^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^2*(c^2*x^2+1)*
c^5+b*Pi^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^2*arcsinh(c*x)*c^5-2*b*Pi^(1/2)/(3
*c^4*x^4+3*c^2*x^2+1)*x*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^4-1/3*b*Pi^(1/2)/(
3*c^4*x^4+3*c^2*x^2+1)*(c^2*x^2+1)*c^3+1/3*b*Pi^(1/2)/(3*c^4*x^4+3*c^2*x^2+
1)*arcsinh(c*x)*c^3-4/3*b*Pi^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/x*arcsinh(c*x)*(
c^2*x^2+1)^(1/2)*c^2-1/6*b*Pi^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/x^2*(c^2*x^2+1)
*c-1/3*b*Pi^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/x^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2
)+1/3*b*c^3*Pi^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)
```

maxima [B] time = 0.60, size = 133, normalized size = 2.15

$$-\frac{\left(\pi^{\frac{3}{2}}\left(-1\right)^{2\,\pi+2\,\pi c^2x^2}c^2\log\left(2\,\pi c^2+\frac{2\,\pi}{x^2}\right)-\pi^{\frac{3}{2}}c^2\log\left(x^2+\frac{1}{c^2}\right)+\frac{\pi\sqrt{\pi+\pi c^4x^4+2\,\pi c^2x^2}}{x^2}\right)bc}{6\,\pi}-\frac{\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}b\operatorname{arsinh}(cx)}{3\,\pi x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^4,x, algorithm="maxima
")
```

```
[Out] -1/6*(pi^(3/2)*(-1)^(2*pi + 2*pi*c^2*x^2)*c^2*log(2*pi*c^2 + 2*pi/x^2) - pi
^(3/2)*c^2*log(x^2 + 1/c^2) + pi*sqrt(pi + pi*c^4*x^4 + 2*pi*c^2*x^2)/x^2)*
b*c/pi - 1/3*(pi + pi*c^2*x^2)^(3/2)*b*arcsinh(c*x)/(pi*x^3) - 1/3*(pi + pi
*c^2*x^2)^(3/2)*a/(pi*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))\,\sqrt{\Pi\,c^2\,x^2+\Pi}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2))/x^4,x)
```

```
[Out] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(1/2))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\sqrt{\pi}\left(\int \frac{a\sqrt{c^2x^2+1}}{x^4}\,dx + \int \frac{b\sqrt{c^2x^2+1}\,\mathrm{asinh}\,(cx)}{x^4}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(pi*c**2*x**2+pi)**(1/2)/x**4,x)
```

```
[Out] sqrt(pi)*(Integral(a*sqrt(c**2*x**2 + 1)/x**4, x) + Integral(b*sqrt(c**2*x*
*2 + 1)*asinh(c*x)/x**4, x))
```

3.63 $\int x^3 \left(\pi + c^2\pi x^2\right)^{3/2} \left(a + b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=125

$$\frac{\left(\pi c^2x^2+\pi\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7\pi^2c^4}-\frac{\left(\pi c^2x^2+\pi\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5\pi c^4}-\frac{1}{49}\pi^{3/2}bc^3x^7+\frac{2\pi^{3/2}bx}{35c^3}-\frac{8}{175}\pi^{3/2}bcx^5-\frac{\pi^{3/2}}{10}$$

```
[Out] 2/35*b*Pi^(3/2)*x/c^3-1/105*b*Pi^(3/2)*x^3/c-8/175*b*c*Pi^(3/2)*x^5-1/49*b*
c^3*Pi^(3/2)*x^7-1/5*(Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))/c^4/Pi+1/7*(P
i*c^2*x^2+Pi)^(7/2)*(a+b*arcsinh(c*x))/c^4/Pi^2
```

Rubi [A] time = 0.14, antiderivative size = 127, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {266, 43, 5732, 12, 373}

$$\frac{\pi^{3/2}\left(c^2x^2+1\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7c^4}-\frac{\pi^{3/2}\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5c^4}-\frac{1}{49}\pi^{3/2}bc^3x^7+\frac{2\pi^{3/2}bx}{35c^3}-\frac{8}{175}\pi^{3/2}bcx^5-$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (2*b*Pi^(3/2)*x)/(35*c^3) - (b*Pi^(3/2)*x^3)/(105*c) - (8*b*c*Pi^(3/2)*x^5)
/175 - (b*c^3*Pi^(3/2)*x^7)/49 - (Pi^(3/2)*(1 + c^2*x^2)^(5/2)*(a + b*ArcSi
nh[c*x]))/(5*c^4) + (Pi^(3/2)*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7*
c^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rubi steps

$$\int x^3\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)dx=-\frac{\pi^{3/2}\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5c^4}+\frac{\pi^{3/2}\left(1+c^2x^2\right)^{7/2}\left(a+b\right.}{7c^4}$$

$$=-\frac{\pi^{3/2}\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5c^4}+\frac{\pi^{3/2}\left(1+c^2x^2\right)^{7/2}\left(a+b\right.}{7c^4}$$

$$=-\frac{\pi^{3/2}\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5c^4}+\frac{\pi^{3/2}\left(1+c^2x^2\right)^{7/2}\left(a+b\right.}{7c^4}$$

$$=\frac{2b\pi^{3/2}x}{35c^3}-\frac{b\pi^{3/2}x^3}{105c}-\frac{8}{175}bc\pi^{3/2}x^5-\frac{1}{49}bc^3\pi^{3/2}x^7-\frac{\pi^{3/2}\left(1+c^2x\right.}{}$$

Mathematica [A] time = 0.17, size = 100, normalized size = 0.80

$$\frac{\pi^{3/2}\left(105a\left(5c^2x^2-2\right)\left(c^2x^2+1\right)^{5/2}+105b\left(5c^2x^2-2\right)\left(c^2x^2+1\right)^{5/2}\sinh^{-1}(cx)-bcx\left(75c^6x^6+168c^4x^4+35c\right.\right.}{3675c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Pi^(3/2)*(105*a*(1 + c^2*x^2)^(5/2)*(-2 + 5*c^2*x^2) - b*c*x*(-210 + 35*c^
2*x^2 + 168*c^4*x^4 + 75*c^6*x^6) + 105*b*(1 + c^2*x^2)^(5/2)*(-2 + 5*c^2*x
^2)*ArcSinh[c*x]))/(3675*c^4)
```

fricas [A] time = 0.62, size = 199, normalized size = 1.59

$$\frac{105\sqrt{\pi+\pi c^2x^2}\left(5\pi bc^8x^8+13\pi bc^6x^6+9\pi bc^4x^4-\pi bc^2x^2-2\pi b\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{\pi+\pi c^2x^2}\left(525\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas
")
```

```
[Out] 1/3675*(105*sqrt(pi + pi*c^2*x^2)*(5*pi*b*c^8*x^8 + 13*pi*b*c^6*x^6 + 9*pi*
b*c^4*x^4 - pi*b*c^2*x^2 - 2*pi*b)*log(c*x + sqrt(c^2*x^2 + 1)) + sqrt(pi +
 pi*c^2*x^2)*(525*pi*a*c^8*x^8 + 1365*pi*a*c^6*x^6 + 945*pi*a*c^4*x^4 - 105
*pi*a*c^2*x^2 - 210*pi*a - (75*pi*b*c^7*x^7 + 168*pi*b*c^5*x^5 + 35*pi*b*c^
3*x^3 - 210*pi*b*c*x)*sqrt(c^2*x^2 + 1)))/(c^6*x^2 + c^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.11, size = 195, normalized size = 1.56

$$a\left(\frac{x^2\left(\pi c^2x^2+\pi\right)^{\frac{5}{2}}}{7\pi c^2}-\frac{2\left(\pi c^2x^2+\pi\right)^{\frac{5}{2}}}{35\pi c^4}\right)+\frac{b\pi^{\frac{3}{2}}\left(525\operatorname{arcsinh}(cx)c^8x^8+1365\operatorname{arcsinh}(cx)c^6x^6-75c^7x^7\sqrt{c^2x^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] a*(1/7*x^2*(Pi*c^2*x^2+Pi)^(5/2)/Pi/c^2-2/35/Pi/c^4*(Pi*c^2*x^2+Pi)^(5/2))+
1/3675*b/c^4*Pi^(3/2)/(c^2*x^2+1)^(1/2)*(525*arcsinh(c*x)*c^8*x^8+1365*arcs
inh(c*x)*c^6*x^6-75*c^7*x^7*(c^2*x^2+1)^(1/2)+945*arcsinh(c*x)*c^4*x^4-168*
c^5*x^5*(c^2*x^2+1)^(1/2)-105*arcsinh(c*x)*c^2*x^2-35*c^3*x^3*(c^2*x^2+1)^(
1/2)-210*arcsinh(c*x)+210*c*x*(c^2*x^2+1)^(1/2))
```

maxima [A] time = 0.49, size = 145, normalized size = 1.16

$$\frac{1}{35}\left(\frac{5\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}x^2}{\pi c^2}-\frac{2\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}}{\pi c^4}\right)b\operatorname{arsinh}(cx)+\frac{1}{35}\left(\frac{5\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}x^2}{\pi c^2}-\frac{2\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}}{\pi c^4}\right)a-\frac{\left(75\,\pi^{\frac{3}{2}}c^6x^7\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima
")
```

```
[Out] 1/35*(5*(pi + pi*c^2*x^2)^(5/2)*x^2/(pi*c^2) - 2*(pi + pi*c^2*x^2)^(5/2)/(p
i*c^4))*b*arcsinh(c*x) + 1/35*(5*(pi + pi*c^2*x^2)^(5/2)*x^2/(pi*c^2) - 2*(
pi + pi*c^2*x^2)^(5/2)/(pi*c^4))*a - 1/3675*(75*pi^(3/2)*c^6*x^7 + 168*pi^(
3/2)*c^4*x^5 + 35*pi^(3/2)*c^2*x^3 - 210*pi^(3/2)*x)*b/c^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(\Pi\,c^2\,x^2+\Pi\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2),x)
```

```
[Out] int(x^3*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

3.64 $\int x^2 \left(\pi + c^2\pi x^2\right)^{3/2} \left(a + b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=165

$$-\frac{\pi^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{32bc^3}+\frac{\pi^{3/2}x\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{16c^2}+\frac{1}{6}x^3\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{8}\pi x^3$$

```
[Out] -1/32*b*Pi^(3/2)*x^2/c-7/96*b*c*Pi^(3/2)*x^4-1/36*b*c^3*Pi^(3/2)*x^6+1/6*x^
3*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))-1/32*Pi^(3/2)*(a+b*arcsinh(c*x))
^2/b/c^3+1/16*Pi^(3/2)*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^2+1/8*Pi*x^
3*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 254, normalized size of antiderivative = 1.54, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5744, 5742, 5758, 5675, 30, 14}

$$\frac{1}{6}x^3\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{8}\pi x^3\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)+\frac{\pi x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(c\right.}{16c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(b*Pi*x^2*Sqrt[Pi + c^2*Pi*x^2])/(32*c*Sqrt[1 + c^2*x^2]) - (7*b*c*Pi*x^4*
Sqrt[Pi + c^2*Pi*x^2])/(96*Sqrt[1 + c^2*x^2]) - (b*c^3*Pi*x^6*Sqrt[Pi + c^2
*Pi*x^2])/(36*Sqrt[1 + c^2*x^2]) + (Pi*x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSi
nh[c*x]))/(16*c^2) + (Pi*x^3*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/8
+ (x^3*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/6 - (Pi*Sqrt[Pi + c^2*
Pi*x^2]*(a + b*ArcSinh[c*x])^2)/(32*b*c^3*Sqrt[1 + c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int x^2\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)dx=\frac{1}{6}x^3\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{2}\pi\int x^2\sqrt{\pi+c^2\pi x^2}\,\left(a+\right.$$

$$=\frac{1}{8}\pi x^3\sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{6}x^3\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\,\mathrm{si}\right.$$

$$=-\frac{7bc\pi x^4\sqrt{\pi+c^2\pi x^2}}{96\sqrt{1+c^2x^2}}-\frac{bc^3\pi x^6\sqrt{\pi+c^2\pi x^2}}{36\sqrt{1+c^2x^2}}+\frac{\pi x\sqrt{\pi+c^2\pi x^2}\,\left(a+\right.}{16c^2}$$

$$=-\frac{b\pi x^2\sqrt{\pi+c^2\pi x^2}}{32c\sqrt{1+c^2x^2}}-\frac{7bc\pi x^4\sqrt{\pi+c^2\pi x^2}}{96\sqrt{1+c^2x^2}}-\frac{bc^3\pi x^6\sqrt{\pi+c^2\pi x^2}}{36\sqrt{1+c^2x^2}}+$$

Mathematica [A] time = 0.34, size = 154, normalized size = 0.93

$$\frac{\pi^{3/2}\left(-12\sinh^{-1}(cx)\left(12a+3b\sinh\left(2\sinh^{-1}(cx)\right)-3b\sinh\left(4\sinh^{-1}(cx)\right)-b\sinh\left(6\sinh^{-1}(cx)\right)\right)+144acx\sqrt{c}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Pi^(3/2)*(144*a*c*x*Sqrt[1 + c^2*x^2] + 672*a*c^3*x^3*Sqrt[1 + c^2*x^2] +
384*a*c^5*x^5*Sqrt[1 + c^2*x^2] - 72*b*ArcSinh[c*x]^2 + 18*b*Cosh[2*ArcSinh
[c*x]] - 9*b*Cosh[4*ArcSinh[c*x]] - 2*b*Cosh[6*ArcSinh[c*x]] - 12*ArcSinh[c
*x]*(12*a + 3*b*Sinh[2*ArcSinh[c*x]] - 3*b*Sinh[4*ArcSinh[c*x]] - b*Sinh[6*
ArcSinh[c*x]])))/(2304*c^3)
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{\pi+\pi c^2x^2}\left(\pi ac^2x^4+\pi ax^2+\left(\pi bc^2x^4+\pi bx^2\right)\mathrm{arsinh}\left(cx\right)\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi*a*c^2*x^4 + pi*a*x^2 + (pi*b*c^2*x^4 + p
i*b*x^2)*arcsinh(c*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\pi + \pi c^2 x^2\right)^{\frac{3}{2}} (b \operatorname{arsinh}(cx) + a) x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((pi + pi*c^2*x^2)^(3/2)*(b*arcsinh(c*x) + a)*x^2, x)
```

maple [A] time = 0.11, size = 240, normalized size = 1.45

$$\frac{ax\left(\pi c^2 x^2 + \pi\right)^{\frac{5}{2}}}{6\pi c^2} - \frac{ax\left(\pi c^2 x^2 + \pi\right)^{\frac{3}{2}}}{24c^2} - \frac{a\pi x\sqrt{\pi c^2 x^2 + \pi}}{16c^2} - \frac{a\pi^2 \ln\left(\frac{\pi x c^2}{\sqrt{\pi c^2}} + \sqrt{\pi c^2 x^2 + \pi}\right)}{16c^2\sqrt{\pi c^2}} + \frac{b\pi^{\frac{3}{2}} c^2 \operatorname{arcsinh}(cx)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/6*a*x*(Pi*c^2*x^2+Pi)^(5/2)/Pi/c^2-1/24*a/c^2*x*(Pi*c^2*x^2+Pi)^(3/2)-1/1
6*a/c^2*Pi*x*(Pi*c^2*x^2+Pi)^(1/2)-1/16*a/c^2*Pi^2*ln(Pi*x*c^2/(Pi*c^2)^(1/
2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/6*b*Pi^(3/2)*c^2*arcsinh(c*x)*(c
^2*x^2+1)^(1/2)*x^5-1/36*b*c^3*Pi^(3/2)*x^6+7/24*b*Pi^(3/2)*arcsinh(c*x)*(c
^2*x^2+1)^(1/2)*x^3-7/96*b*c*Pi^(3/2)*x^4+1/16*b*Pi^(3/2)/c^2*arcsinh(c*x)*
(c^2*x^2+1)^(1/2)*x-1/32*b*Pi^(3/2)*x^2/c-1/32*b*Pi^(3/2)/c^3*arcsinh(c*x)^
2+1/72*b*Pi^(3/2)/c^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left(a + b \operatorname{asinh}(c\,x)\right) \left(\Pi\,c^2\,x^2 + \Pi\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2),x)
```

```
[Out] int(x^2*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

3.65 $\int x \left(\pi + c^2 \pi x^2\right)^{3/2} \left(a + b \sinh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=77

$$\frac{\left(\pi c^2 x^2 + \pi\right)^{5/2} \left(a + b \sinh^{-1}(cx)\right)}{5\pi c^2} - \frac{1}{25}\pi^{3/2} b c^3 x^5 - \frac{2}{15}\pi^{3/2} b c x^3 - \frac{\pi^{3/2} b x}{5c}$$

```
[Out] -1/5*b*Pi^(3/2)*x/c-2/15*b*c*Pi^(3/2)*x^3-1/25*b*c^3*Pi^(3/2)*x^5+1/5*(Pi*c
^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))/c^2/Pi
```

Rubi [A] time = 0.09, antiderivative size = 146, normalized size of antiderivative = 1.90, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {5717, 194}

$$\frac{\left(\pi c^2 x^2 + \pi\right)^{5/2} \left(a + b \sinh^{-1}(cx)\right)}{5\pi c^2} - \frac{\pi b c^3 x^5 \sqrt{\pi c^2 x^2 + \pi}}{25\sqrt{c^2 x^2 + 1}} - \frac{2\pi b c x^3 \sqrt{\pi c^2 x^2 + \pi}}{15\sqrt{c^2 x^2 + 1}} - \frac{\pi b x \sqrt{\pi c^2 x^2 + \pi}}{5c\sqrt{c^2 x^2 + 1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(b*Pi*x*Sqrt[Pi + c^2*Pi*x^2])/(5*c*Sqrt[1 + c^2*x^2]) - (2*b*c*Pi*x^3*Sqr
t[Pi + c^2*Pi*x^2])/(15*Sqrt[1 + c^2*x^2]) - (b*c^3*Pi*x^5*Sqrt[Pi + c^2*Pi
*x^2])/(25*Sqrt[1 + c^2*x^2]) + ((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x
]))/(5*c^2*Pi)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned} \int x \left(\pi + c^2 \pi x^2\right)^{3/2} \left(a + b \sinh^{-1}(cx)\right) dx &= \frac{\left(\pi + c^2 \pi x^2\right)^{5/2} \left(a + b \sinh^{-1}(cx)\right)}{5c^2 \pi} - \frac{\left(b\pi\sqrt{\pi + c^2 \pi x^2}\right) \int \left(1 + c^2 x^2\right)^2}{5c\sqrt{1 + c^2 x^2}} \\ &= \frac{\left(\pi + c^2 \pi x^2\right)^{5/2} \left(a + b \sinh^{-1}(cx)\right)}{5c^2 \pi} - \frac{\left(b\pi\sqrt{\pi + c^2 \pi x^2}\right) \int \left(1 + 2c^2 x^2 \right.}{5c\sqrt{1 + c^2 x^2}} \\ &= -\frac{b\pi x\sqrt{\pi + c^2 \pi x^2}}{5c\sqrt{1 + c^2 x^2}} - \frac{2bc\pi x^3\sqrt{\pi + c^2 \pi x^2}}{15\sqrt{1 + c^2 x^2}} - \frac{bc^3 \pi x^5\sqrt{\pi + c^2 \pi x^2}}{25\sqrt{1 + c^2 x^2}} + \end{aligned}$$

Mathematica [A] time = 0.12, size = 72, normalized size = 0.94

$$\frac{\pi^{3/2}\left(15a\left(c^2 x^2 + 1\right)^{5/2} + 15b\left(c^2 x^2 + 1\right)^{5/2} \sinh^{-1}(cx) - bcx\left(3c^4 x^4 + 10c^2 x^2 + 15\right)\right)}{75c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Pi^(3/2)*(15*a*(1 + c^2*x^2)^(5/2) - b*c*x*(15 + 10*c^2*x^2 + 3*c^4*x^4) +
 15*b*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x]))/(75*c^2)
```

fricas [B] time = 0.55, size = 167, normalized size = 2.17

$$\frac{15\sqrt{\pi+\pi c^2x^2}\left(\pi bc^6x^6+3\,\pi bc^4x^4+3\,\pi bc^2x^2+\pi b\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{\pi+\pi c^2x^2}\left(15\,\pi ac^6x^6+45\,\pi a\right.}{75\left(c^4x^2+c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/75*(15*sqrt(pi + pi*c^2*x^2)*(pi*b*c^6*x^6 + 3*pi*b*c^4*x^4 + 3*pi*b*c^2*
x^2 + pi*b)*log(c*x + sqrt(c^2*x^2 + 1)) + sqrt(pi + pi*c^2*x^2)*(15*pi*a*c
^6*x^6 + 45*pi*a*c^4*x^4 + 45*pi*a*c^2*x^2 + 15*pi*a - (3*pi*b*c^5*x^5 + 10
*pi*b*c^3*x^3 + 15*pi*b*c*x)*sqrt(c^2*x^2 + 1)))/(c^4*x^2 + c^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.06, size = 139, normalized size = 1.81

$$\frac{a\left(\pi\,c^2x^2+\pi\right)^{\frac{5}{2}}}{5\pi\,c^2}+\frac{b\,\pi^{\frac{3}{2}}\left(15\operatorname{arcsinh}(cx)\,c^6x^6+45\operatorname{arcsinh}(cx)\,c^4x^4-3c^5x^5\sqrt{c^2x^2+1}+45\operatorname{arcsinh}(cx)\,c^2x^2-\right.}{75c^2\sqrt{c^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/5*a/Pi/c^2*(Pi*c^2*x^2+Pi)^(5/2)+1/75*b/c^2*Pi^(3/2)/(c^2*x^2+1)^(1/2)*(1
5*arcsinh(c*x)*c^6*x^6+45*arcsinh(c*x)*c^4*x^4-3*c^5*x^5*(c^2*x^2+1)^(1/2)+
45*arcsinh(c*x)*c^2*x^2-10*c^3*x^3*(c^2*x^2+1)^(1/2)+15*arcsinh(c*x)-15*c*x
*(c^2*x^2+1)^(1/2))
```

maxima [A] time = 0.38, size = 85, normalized size = 1.10

$$\frac{\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}b\operatorname{arsinh}(cx)}{5\,\pi c^2}+\frac{\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}a}{5\,\pi c^2}-\frac{\left(3\,\pi^{\frac{5}{2}}c^4x^5+10\,\pi^{\frac{5}{2}}c^2x^3+15\,\pi^{\frac{5}{2}}x\right)b}{75\,\pi c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*(pi + pi*c^2*x^2)^(5/2)*b*arcsinh(c*x)/(pi*c^2) + 1/5*(pi + pi*c^2*x^2)
^(5/2)*a/(pi*c^2) - 1/75*(3*pi^(5/2)*c^4*x^5 + 10*pi^(5/2)*c^2*x^3 + 15*pi^
(5/2)*x)*b/(pi*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(\Pi\,c^2\,x^2+\Pi\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2),x)
```

```
[Out] int(x*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2), x)
```

sympy [A] time = 82.23, size = 221, normalized size = 2.87

$$\begin{cases} \frac{\pi^{\frac{3}{2}} ac^2x^4\sqrt{c^2x^2+1}}{5} + \frac{2\pi^{\frac{3}{2}} ax^2\sqrt{c^2x^2+1}}{5} + \frac{\pi^{\frac{3}{2}} a\sqrt{c^2x^2+1}}{5c^2} - \frac{\pi^{\frac{3}{2}} bc^3x^5}{25} + \frac{\pi^{\frac{3}{2}} bc^2x^4\sqrt{c^2x^2+1}\ \mathrm{asinh}\,(cx)}{5} - \frac{2\pi^{\frac{3}{2}} bcx^3}{15} + \frac{2\pi^{\frac{3}{2}} bx^2\sqrt{c^2x^2+1}\ \mathrm{asinh}\,(c \\ \frac{\pi^{\frac{3}{2}} ax^2}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((pi**(3/2)*a*c**2*x**4*sqrt(c**2*x**2 + 1)/5 + 2*pi**(3/2)*a*x**2
*sqrt(c**2*x**2 + 1)/5 + pi**(3/2)*a*sqrt(c**2*x**2 + 1)/(5*c**2) - pi**(3/
2)*b*c**3*x**5/25 + pi**(3/2)*b*c**2*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/5
- 2*pi**(3/2)*b*c*x**3/15 + 2*pi**(3/2)*b*x**2*sqrt(c**2*x**2 + 1)*asinh(c*
x)/5 - pi**(3/2)*b*x/(5*c) + pi**(3/2)*b*sqrt(c**2*x**2 + 1)*asinh(c*x)/(5*
c**2), Ne(c, 0)), (pi**(3/2)*a*x**2/2, True))
```

3.66 $\int \left(\pi + c^2\pi x^2\right)^{3/2} \left(a + b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=111

$$\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{8}\pi x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)+\frac{3\pi^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc}-\frac{1}{16}\pi^{3/2}$$

```
[Out] -5/16*b*c*Pi^(3/2)*x^2-1/16*b*c^3*Pi^(3/2)*x^4+1/4*x*(Pi*c^2*x^2+Pi)^(3/2)*
(a+b*arcsinh(c*x))+3/16*Pi^(3/2)*(a+b*arcsinh(c*x))^2/b/c+3/8*Pi*x*(a+b*arc
sinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 180, normalized size of antiderivative = 1.62, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {5684, 5682, 5675, 30, 14}

$$\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{8}\pi x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)+\frac{3\pi\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{16bc\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-5*b*c*Pi*x^2*Sqrt[Pi + c^2*Pi*x^2])/(16*Sqrt[1 + c^2*x^2]) - (b*c^3*Pi*x^
4*Sqrt[Pi + c^2*Pi*x^2])/(16*Sqrt[1 + c^2*x^2]) + (3*Pi*x*Sqrt[Pi + c^2*Pi*
x^2]*(a + b*ArcSinh[c*x]))/8 + (x*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*
x]))/4 + (3*Pi*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^2)/(16*b*c*Sqrt[1
 + c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
```

```
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rubi steps

$$\int \left(\pi + c^2\pi x^2\right)^{3/2} \left(a + b\sinh^{-1}(cx)\right) dx = \frac{1}{4}x\left(\pi + c^2\pi x^2\right)^{3/2}\left(a + b\sinh^{-1}(cx)\right) + \frac{1}{4}(3\pi)\int \sqrt{\pi + c^2\pi x^2}\,\left(a + b\,\mathrm{s}\right.$$

$$= \frac{3}{8}\pi x\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right) + \frac{1}{4}x\left(\pi + c^2\pi x^2\right)^{3/2}\left(a + b\sinh^{-1}\right.$$

$$= -\frac{5bc\pi x^2\sqrt{\pi + c^2\pi x^2}}{16\sqrt{1 + c^2x^2}} - \frac{bc^3\pi x^4\sqrt{\pi + c^2\pi x^2}}{16\sqrt{1 + c^2x^2}} + \frac{3}{8}\pi x\sqrt{\pi + c^2\pi x^2}\left(a + \right.$$

Mathematica [A] time = 0.24, size = 111, normalized size = 1.00

$$\frac{\pi^{3/2}\left(4\sinh^{-1}(cx)\left(12a + 8b\sinh\left(2\sinh^{-1}(cx)\right) + b\sinh\left(4\sinh^{-1}(cx)\right)\right) + 80acx\sqrt{c^2x^2+1} + 32ac^3x^3\sqrt{c^2x^2+1}\right.}{128c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Pi^(3/2)*(80*a*c*x*Sqrt[1 + c^2*x^2] + 32*a*c^3*x^3*Sqrt[1 + c^2*x^2] + 24
*b*ArcSinh[c*x]^2 - 16*b*Cosh[2*ArcSinh[c*x]] - b*Cosh[4*ArcSinh[c*x]] + 4*
ArcSinh[c*x]*(12*a + 8*b*Sinh[2*ArcSinh[c*x]] + b*Sinh[4*ArcSinh[c*x]])))/(
128*c)
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{\pi + \pi c^2x^2}\left(\pi ac^2x^2 + \pi a + \left(\pi bc^2x^2 + \pi b\right)\text{arsinh}\left(cx\right)\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi*a*c^2*x^2 + pi*a + (pi*b*c^2*x^2 + pi*b)
*arcsinh(c*x)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.06, size = 170, normalized size = 1.53

$$\frac{x\left(\pi c^2x^2 + \pi\right)^{\frac{3}{2}} a}{4} + \frac{3a\pi x\sqrt{\pi c^2x^2 + \pi}}{8} + \frac{3a\pi^2\ln\left(\frac{\pi x c^2}{\sqrt{\pi c^2}} + \sqrt{\pi c^2x^2 + \pi}\right)}{8\sqrt{\pi c^2}} + \frac{b\pi^{\frac{3}{2}}c^2\sqrt{c^2x^2+1}\,\text{arcsinh}\left(cx\right)x^3}{4} - \frac{bc^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/4*x*(Pi*c^2*x^2+Pi)^(3/2)*a+3/8*a*Pi*x*(Pi*c^2*x^2+Pi)^(1/2)+3/8*a*Pi^2*l
n(Pi*x*c^2/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/4*b*Pi^(3
/2)*c^2*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^3-1/16*b*c^3*Pi^(3/2)*x^4+5/8*b*Pi
^(3/2)*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x-5/16*b*c*Pi^(3/2)*x^2+3/16*b*Pi^(3/
2)/c*arcsinh(c*x)^2-1/4*b*Pi^(3/2)/c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,\mathrm{asinh}\,(c\,x))\,\left(\Pi\,c^2\,x^2 + \Pi\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2),x)
```

```
[Out] int((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2), x)
```

sympy [A] time = 38.26, size = 185, normalized size = 1.67

$$\begin{cases} \frac{\pi^{\frac{3}{2}}ac^2x^3\sqrt{c^2x^2+1}}{4} + \frac{5\pi^{\frac{3}{2}}ax\sqrt{c^2x^2+1}}{8} + \frac{3\pi^{\frac{3}{2}}a\,\mathrm{asinh}\,(cx)}{8c} - \frac{\pi^{\frac{3}{2}}bc^3x^4}{16} + \frac{\pi^{\frac{3}{2}}bc^2x^3\sqrt{c^2x^2+1}\,\mathrm{asinh}\,(cx)}{4} - \frac{5\pi^{\frac{3}{2}}bcx^2}{16} + \frac{5\pi^{\frac{3}{2}}bx\sqrt{c^2x^2+1}\,\mathrm{asi}}{8} \\ \pi^{\frac{3}{2}}ax \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((pi**(3/2)*a*c**2*x**3*sqrt(c**2*x**2 + 1)/4 + 5*pi**(3/2)*a*x*sq
rt(c**2*x**2 + 1)/8 + 3*pi**(3/2)*a*asinh(c*x)/(8*c) - pi**(3/2)*b*c**3*x**
4/16 + pi**(3/2)*b*c**2*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/4 - 5*pi**(3/2)
*b*c*x**2/16 + 5*pi**(3/2)*b*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/8 + 3*pi**(3/
2)*b*asinh(c*x)**2/(16*c), Ne(c, 0)), (pi**(3/2)*a*x, True))
```

3.67 $\int \frac{\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=134

$$\frac{1}{3}\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\pi\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)-2\pi^{3/2}\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx\right.$$

```
[Out] -4/3*b*c*Pi^(3/2)*x-1/9*b*c^3*Pi^(3/2)*x^3+1/3*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*a
rcsinh(c*x))-2*Pi^(3/2)*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))-b
*Pi^(3/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+b*Pi^(3/2)*polylog(2,c*x+(c^2*x
^2+1)^(1/2))+Pi*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 249, normalized size of antiderivative = 1.86, number of steps used = 10, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5744, 5742, 5760, 4182, 2279, 2391, 8}

$$-\frac{\pi b\sqrt{\pi c^2x^2+\pi}\,\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+\frac{\pi b\sqrt{\pi c^2x^2+\pi}\,\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+\frac{1}{3}\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh\right.$$

Antiderivative was successfully verified.

```
[In] Int[((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (-4*b*c*Pi*x*Sqrt[Pi + c^2*Pi*x^2])/(3*Sqrt[1 + c^2*x^2]) - (b*c^3*Pi*x^3*S
qrt[Pi + c^2*Pi*x^2])/(9*Sqrt[1 + c^2*x^2]) + Pi*Sqrt[Pi + c^2*Pi*x^2]*(a +
 b*ArcSinh[c*x]) + ((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/3 - (2*Pi
*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1
 + c^2*x^2] - (b*Pi*Sqrt[Pi + c^2*Pi*x^2]*PolyLog[2, -E^ArcSinh[c*x]])/Sqrt
[1 + c^2*x^2] + (b*Pi*Sqrt[Pi + c^2*Pi*x^2]*PolyLog[2, E^ArcSinh[c*x]])/Sqr
t[1 + c^2*x^2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
```

```
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(\pi + c^2\pi x^2\right)^{3/2}\left(a + b\sinh^{-1}(cx)\right)}{x}\,dx = \frac{1}{3}\left(\pi + c^2\pi x^2\right)^{3/2}\left(a + b\sinh^{-1}(cx)\right) + \pi\int \frac{\sqrt{\pi + c^2\pi x^2}\left(a + b\,\mathrm{sinh}\right.}{x}$$

$$= -\frac{bc\pi x\sqrt{\pi + c^2\pi x^2}}{3\sqrt{1 + c^2x^2}} - \frac{bc^3\pi x^3\sqrt{\pi + c^2\pi x^2}}{9\sqrt{1 + c^2x^2}} + \pi\sqrt{\pi + c^2\pi x^2}\left(a + b\,\mathrm{s}\right.$$

$$= -\frac{4bc\pi x\sqrt{\pi + c^2\pi x^2}}{3\sqrt{1 + c^2x^2}} - \frac{bc^3\pi x^3\sqrt{\pi + c^2\pi x^2}}{9\sqrt{1 + c^2x^2}} + \pi\sqrt{\pi + c^2\pi x^2}\left(a + b\right.$$

$$= -\frac{4bc\pi x\sqrt{\pi + c^2\pi x^2}}{3\sqrt{1 + c^2x^2}} - \frac{bc^3\pi x^3\sqrt{\pi + c^2\pi x^2}}{9\sqrt{1 + c^2x^2}} + \pi\sqrt{\pi + c^2\pi x^2}\left(a + b\right.$$

$$= -\frac{4bc\pi x\sqrt{\pi + c^2\pi x^2}}{3\sqrt{1 + c^2x^2}} - \frac{bc^3\pi x^3\sqrt{\pi + c^2\pi x^2}}{9\sqrt{1 + c^2x^2}} + \pi\sqrt{\pi + c^2\pi x^2}\left(a + b\right.$$

$$= -\frac{4bc\pi x\sqrt{\pi + c^2\pi x^2}}{3\sqrt{1 + c^2x^2}} - \frac{bc^3\pi x^3\sqrt{\pi + c^2\pi x^2}}{9\sqrt{1 + c^2x^2}} + \pi\sqrt{\pi + c^2\pi x^2}\left(a + b\right.$$

Mathematica [A] time = 0.37, size = 180, normalized size = 1.34

$$\frac{1}{9}\pi^{3/2}\left(3a\sqrt{c^2x^2+1}\left(c^2x^2+4\right) - 9a\log\left(\pi\left(\sqrt{c^2x^2+1}+1\right)\right) + 9a\log(x) + 9b\left(\sqrt{c^2x^2+1}\sinh^{-1}(cx) + \mathrm{Li}_2\left(-\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (Pi^(3/2)*(3*a*Sqrt[1 + c^2*x^2]*(4 + c^2*x^2) - b*(3*c*x + c^3*x^3 - 3*(1
+ c^2*x^2)^(3/2)*ArcSinh[c*x]) + 9*a*Log[x] - 9*a*Log[Pi*(1 + Sqrt[1 + c^2*
x^2])] + 9*b*(-(c*x) + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + ArcSinh[c*x]*Log[1
```

```
- E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + PolyLog[2,
 -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])])))/9
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\pi+\pi c^2x^2}\left(\pi ac^2x^2+\pi a+\left(\pi bc^2x^2+\pi b\right)\text{arsinh}\left(cx\right)\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi*a*c^2*x^2 + pi*a + (pi*b*c^2*x^2 + pi*b)
*arcsinh(c*x))/x, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.27, size = 227, normalized size = 1.69

$$\frac{\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}a}{3}-a\pi^{\frac{3}{2}}\,\text{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)+a\sqrt{\pi c^2x^2+\pi}\,\pi+\frac{b\pi^{\frac{3}{2}}\,\text{arcsinh}\left(cx\right)\sqrt{c^2x^2+1}\,x^2c^2}{3}-b\pi^{\frac{3}{2}}\,\text{arcsir}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))/x,x)
```

```
[Out] 1/3*(Pi*c^2*x^2+Pi)^(3/2)*a-a*Pi^(3/2)*arctanh(Pi^(1/2)/(Pi*c^2*x^2+Pi)^(1/
2))+a*(Pi*c^2*x^2+Pi)^(1/2)*Pi+1/3*b*Pi^(3/2)*arcsinh(c*x)*(c^2*x^2+1)^(1/2
)*x^2*c^2-b*Pi^(3/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-b*Pi^(3/2)*po
lylog(2,-c*x-(c^2*x^2+1)^(1/2))+b*Pi^(3/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1
)^(1/2))-1/9*b*c^3*Pi^(3/2)*x^3+4/3*b*Pi^(3/2)*arcsinh(c*x)*(c^2*x^2+1)^(1/
2)-4/3*b*c*Pi^(3/2)*x+b*Pi^(3/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}\left(3\pi^{\frac{3}{2}}\,\text{arsinh}\left(\frac{1}{c|x|}\right)-3\pi\sqrt{\pi+\pi c^2x^2}-\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}\right)a+b\int\frac{\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}\log\left(cx+\sqrt{c^2x^2+1}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x,x, algorithm="maxima")
```

```
[Out] -1/3*(3*pi^(3/2)*arcsinh(1/(c*abs(x))) - 3*pi*sqrt(pi + pi*c^2*x^2) - (pi +
 pi*c^2*x^2)^(3/2))*a + b*integrate((pi + pi*c^2*x^2)^(3/2)*log(c*x + sqrt(
c^2*x^2 + 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(a+b\,\text{asinh}\left(c\,x\right)\right)\left(\Pi\,c^2\,x^2+\Pi\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2))/x,x)
```

```
[Out] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\pi^{\frac{3}{2}}\left(\int \frac{a\sqrt{c^2x^2+1}}{x}\,dx + \int ac^2x\sqrt{c^2x^2+1}\,dx + \int \frac{b\sqrt{c^2x^2+1}\,\operatorname{asinh}(cx)}{x}\,dx + \int bc^2x\sqrt{c^2x^2+1}\,\operatorname{asinh}(cx)\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x))/x,x)
```

```
[Out] pi**(3/2)*(Integral(a*sqrt(c**2*x**2 + 1)/x, x) + Integral(a*c**2*x*sqrt(c*
*2*x**2 + 1), x) + Integral(b*sqrt(c**2*x**2 + 1)*asinh(c*x)/x, x) + Integr
al(b*c**2*x*sqrt(c**2*x**2 + 1)*asinh(c*x), x))
```

3.68 $\int \frac{\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=108

$$\frac{3}{2}\pi c^2x\sqrt{\pi c^2x^2+\pi}\,\left(a+b\sinh^{-1}(cx)\right)-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x}+\frac{3\pi^{3/2}c\left(a+b\sinh^{-1}(cx)\right)^2}{4b}-\frac{1}{4}\pi^{3/2}bc^3x$$

```
[Out] -1/4*b*c^3*Pi^(3/2)*x^2-(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))/x+3/4*c*Pi
^(3/2)*(a+b*arcsinh(c*x))^2/b+b*c*Pi^(3/2)*ln(x)+3/2*c^2*Pi*x*(a+b*arcsinh(
c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 177, normalized size of antiderivative = 1.64, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5739, 5682, 5675, 30, 14}

$$\frac{3}{2}\pi c^2x\sqrt{\pi c^2x^2+\pi}\,\left(a+b\sinh^{-1}(cx)\right)+\frac{3\pi c\sqrt{\pi c^2x^2+\pi}\,\left(a+b\sinh^{-1}(cx)\right)^2}{4b\sqrt{c^2x^2+1}}-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x}-$$

Antiderivative was successfully verified.

```
[In] Int[((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] -(b*c^3*Pi*x^2*Sqrt[Pi + c^2*Pi*x^2])/(4*Sqrt[1 + c^2*x^2]) + (3*c^2*Pi*x*S
qrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/2 - ((Pi + c^2*Pi*x^2)^(3/2)*(a
+ b*ArcSinh[c*x]))/x + (3*c*Pi*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^2
)/(4*b*Sqrt[1 + c^2*x^2]) + (b*c*Pi*Sqrt[Pi + c^2*Pi*x^2]*Log[x])/Sqrt[1 +
c^2*x^2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
```

```
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(\pi + c^2\pi x^2\right)^{3/2}\left(a + b\sinh^{-1}(cx)\right)}{x^2}\,dx = -\frac{\left(\pi + c^2\pi x^2\right)^{3/2}\left(a + b\sinh^{-1}(cx)\right)}{x} + \left(3c^2\pi\right)\int \sqrt{\pi + c^2\pi x^2}\,\left(a + b\right.$$

$$= \frac{3}{2}c^2\pi x\sqrt{\pi + c^2\pi x^2}\,\left(a + b\sinh^{-1}(cx)\right) - \frac{\left(\pi + c^2\pi x^2\right)^{3/2}\left(a + b\sinh\right.}{x}$$

$$= -\frac{bc^3\pi x^2\sqrt{\pi + c^2\pi x^2}}{4\sqrt{1 + c^2x^2}} + \frac{3}{2}c^2\pi x\sqrt{\pi + c^2\pi x^2}\,\left(a + b\sinh^{-1}(cx)\right) - \frac{(\pi}{}$$

Mathematica [A] time = 0.30, size = 122, normalized size = 1.13

$$\frac{\pi^{3/2}\left(2\sinh^{-1}(cx)\left(6acx - 4b\sqrt{c^2x^2+1} + bcx\sinh\left(2\sinh^{-1}(cx)\right)\right) + 4ac^2x^2\sqrt{c^2x^2+1} - 8a\sqrt{c^2x^2+1} + 8bcx\right.}{8x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] (Pi^(3/2)*(-8*a*Sqrt[1 + c^2*x^2] + 4*a*c^2*x^2*Sqrt[1 + c^2*x^2] + 6*b*c*x
*ArcSinh[c*x]^2 - b*c*x*Cosh[2*ArcSinh[c*x]] + 8*b*c*x*Log[c*x] + 2*ArcSinh
[c*x]*(6*a*c*x - 4*b*Sqrt[1 + c^2*x^2] + b*c*x*Sinh[2*ArcSinh[c*x]])))/(8*x
)
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\pi + \pi c^2x^2}\left(\pi ac^2x^2 + \pi a + \left(\pi bc^2x^2 + \pi b\right)\operatorname{arsinh}(cx)\right)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi*a*c^2*x^2 + pi*a + (pi*b*c^2*x^2 + pi*b)
*arcsinh(c*x))/x^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.22, size = 222, normalized size = 2.06

$$-\frac{a\left(\pi c^2x^2+\pi\right)^{\frac{5}{2}}}{\pi x}+a c^2x\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}+\frac{3a c^2\pi x\sqrt{\pi c^2x^2+\pi}}{2}+\frac{3a c^2\pi^2\ln\left(\frac{\pi x c^2}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{2\sqrt{\pi c^2}}+\frac{3bc\,\pi^{\frac{3}{2}}\,\mathrm{arcsir}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))/x^2,x)
```

```
[Out] -a/Pi/x*(Pi*c^2*x^2+Pi)^(5/2)+a*c^2*x*(Pi*c^2*x^2+Pi)^(3/2)+3/2*a*c^2*Pi*x*
(Pi*c^2*x^2+Pi)^(1/2)+3/2*a*c^2*Pi^2*ln(Pi*x*c^2/(Pi*c^2)^(1/2)+(Pi*c^2*x^2
+Pi)^(1/2))/(Pi*c^2)^(1/2)+3/4*b*c*Pi^(3/2)*arcsinh(c*x)^2+1/2*b*Pi^(3/2)*a
rcsinh(c*x)*(c^2*x^2+1)^(1/2)*x*c^2-1/4*b*c^3*Pi^(3/2)*x^2-b*c*Pi^(3/2)*arc
sinh(c*x)-1/8*b*Pi^(3/2)*c-b*Pi^(3/2)*arcsinh(c*x)/x*(c^2*x^2+1)^(1/2)+b*c*
Pi^(3/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))\,\left(\Pi\,c^2\,x^2+\Pi\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2))/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\pi^{\frac{3}{2}}\left(\int ac^2\sqrt{c^2x^2+1}\,dx+\int\frac{a\sqrt{c^2x^2+1}}{x^2}\,dx+\int bc^2\sqrt{c^2x^2+1}\,\mathrm{asinh}\,(cx)\,dx+\int\frac{b\sqrt{c^2x^2+1}\,\mathrm{asinh}\,(cx)}{x^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x))/x**2,x)
```

```
[Out] pi**(3/2)*(Integral(a*c**2*sqrt(c**2*x**2 + 1), x) + Integral(a*sqrt(c**2*x
**2 + 1)/x**2, x) + Integral(b*c**2*sqrt(c**2*x**2 + 1)*asinh(c*x), x) + In
tegral(b*sqrt(c**2*x**2 + 1)*asinh(c*x)/x**2, x))
```

3.69 $\int \frac{\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=155

$$\frac{3}{2}\pi c^2\sqrt{\pi c^2x^2+\pi}\,\left(a+b\sinh^{-1}(cx)\right)-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{2x^2}-3\pi^{3/2}c^2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sin\right.$$

```
[Out] -1/2*b*c*Pi^(3/2)/x-b*c^3*Pi^(3/2)*x-1/2*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh
(c*x))/x^2-3*c^2*Pi^(3/2)*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))
-3/2*b*c^2*Pi^(3/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+3/2*b*c^2*Pi^(3/2)*po
lylog(2,c*x+(c^2*x^2+1)^(1/2))+3/2*c^2*Pi*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi
)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 270, normalized size of antiderivative = 1.74, number of steps used = 11, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5739, 5742, 5760, 4182, 2279, 2391, 8, 14}

$$-\frac{3\pi bc^2\sqrt{\pi c^2x^2+\pi}\,\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2x^2+1}}+\frac{3\pi bc^2\sqrt{\pi c^2x^2+\pi}\,\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2x^2+1}}+\frac{3}{2}\pi c^2\sqrt{\pi c^2x^2+\pi}\,\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] -(b*c*Pi*Sqrt[Pi + c^2*Pi*x^2])/(2*x*Sqrt[1 + c^2*x^2]) - (b*c^3*Pi*x*Sqrt[
Pi + c^2*Pi*x^2])/Sqrt[1 + c^2*x^2] + (3*c^2*Pi*Sqrt[Pi + c^2*Pi*x^2]*(a +
b*ArcSinh[c*x]))/2 - ((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(2*x^2)
 - (3*c^2*Pi*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c
*x]])/Sqrt[1 + c^2*x^2] - (3*b*c^2*Pi*Sqrt[Pi + c^2*Pi*x^2]*PolyLog[2, -E^A
rcSinh[c*x]])/(2*Sqrt[1 + c^2*x^2]) + (3*b*c^2*Pi*Sqrt[Pi + c^2*Pi*x^2]*Pol
yLog[2, E^ArcSinh[c*x]])/(2*Sqrt[1 + c^2*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
```

```
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x^3}\,dx = -\frac{\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{2x^2}+\frac{1}{2}\left(3c^2\pi\right)\int\frac{\sqrt{\pi+c^2\pi x^2}\,\left(a+b\right.}{x}$$

$$=\frac{3}{2}c^2\pi\sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(cx)\right)-\frac{\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(c\right.}{2x^2}$$

$$=-\frac{bc\pi\sqrt{\pi+c^2\pi x^2}}{2x\sqrt{1+c^2x^2}}-\frac{bc^3\pi x\sqrt{\pi+c^2\pi x^2}}{\sqrt{1+c^2x^2}}+\frac{3}{2}c^2\pi\sqrt{\pi+c^2\pi x^2}\,\left(a+b\,\mathrm{si}\right.$$

$$=-\frac{bc\pi\sqrt{\pi+c^2\pi x^2}}{2x\sqrt{1+c^2x^2}}-\frac{bc^3\pi x\sqrt{\pi+c^2\pi x^2}}{\sqrt{1+c^2x^2}}+\frac{3}{2}c^2\pi\sqrt{\pi+c^2\pi x^2}\,\left(a+b\,\mathrm{si}\right.$$

$$=-\frac{bc\pi\sqrt{\pi+c^2\pi x^2}}{2x\sqrt{1+c^2x^2}}-\frac{bc^3\pi x\sqrt{\pi+c^2\pi x^2}}{\sqrt{1+c^2x^2}}+\frac{3}{2}c^2\pi\sqrt{\pi+c^2\pi x^2}\,\left(a+b\,\mathrm{si}\right.$$

$$=-\frac{bc\pi\sqrt{\pi+c^2\pi x^2}}{2x\sqrt{1+c^2x^2}}-\frac{bc^3\pi x\sqrt{\pi+c^2\pi x^2}}{\sqrt{1+c^2x^2}}+\frac{3}{2}c^2\pi\sqrt{\pi+c^2\pi x^2}\,\left(a+b\,\mathrm{si}\right.$$

Mathematica [A] time = 1.66, size = 292, normalized size = 1.88

$$\pi^{3/2}\left(8ac^2x^2\sqrt{c^2x^2+1}-4a\sqrt{c^2x^2+1}+12ac^2x^2\log(x)-12ac^2x^2\log\left(\pi\left(\sqrt{c^2x^2+1}+1\right)\right)-8bc^3x^3-bc^3x^3\mathrm{csch}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] (Pi^(3/2)*(-8*b*c^3*x^3 - 4*a*Sqrt[1 + c^2*x^2] + 8*a*c^2*x^2*Sqrt[1 + c^2*
x^2] + 8*b*c^2*x^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - b*c^3*x^3*Csch[ArcSinh[
c*x]/2]^2 - b*c^2*x^2*ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 + 12*b*c^2*x^2*Ar
cSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 12*b*c^2*x^2*ArcSinh[c*x]*Log[1 + E
^(-ArcSinh[c*x])] + 12*a*c^2*x^2*Log[x] - 12*a*c^2*x^2*Log[Pi*(1 + Sqrt[1 +
 c^2*x^2])] + 12*b*c^2*x^2*PolyLog[2, -E^(-ArcSinh[c*x])] - 12*b*c^2*x^2*Po
lyLog[2, E^(-ArcSinh[c*x])] + 4*b*c*x*Sinh[ArcSinh[c*x]/2]^2 - 4*b*ArcSinh[
c*x]*Sinh[ArcSinh[c*x]/2]^2))/(8*x^2)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\pi + \pi c^2x^2}\left(\pi a c^2x^2 + \pi a + \left(\pi b c^2x^2 + \pi b\right)\operatorname{arsinh}(cx)\right)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi*a*c^2*x^2 + pi*a + (pi*b*c^2*x^2 + pi*b)
*arcsinh(c*x))/x^3, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.52, size = 295, normalized size = 1.90

$$-\frac{a\left(\pi c^2x^2 + \pi\right)^{\frac{5}{2}}}{2\pi x^2} + \frac{a c^2\left(\pi c^2x^2 + \pi\right)^{\frac{3}{2}}}{2} - \frac{3a c^2\pi^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)}{2} + \frac{3a c^2\sqrt{\pi c^2x^2 + \pi}\,\pi}{2} + b\,\pi^{\frac{3}{2}}\operatorname{arcsinh}(cx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))/x^3,x)
```

```
[Out] -1/2*a/Pi/x^2*(Pi*c^2*x^2+Pi)^(5/2)+1/2*a*c^2*(Pi*c^2*x^2+Pi)^(3/2)-3/2*a*c
^2*Pi^(3/2)*arctanh(Pi^(1/2)/(Pi*c^2*x^2+Pi)^(1/2))+3/2*a*c^2*(Pi*c^2*x^2+P
i)^(1/2)*Pi+b*Pi^(3/2)*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^2-b*c^3*Pi^(3/2)*x-
1/2*b*Pi^(3/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^2-1/2*b*c*Pi^(3/2)/x-1/2*b*
Pi^(3/2)/(c^2*x^2+1)^(1/2)/x^2*arcsinh(c*x)-3/2*b*c^2*Pi^(3/2)*arcsinh(c*x)
*ln(1+c*x+(c^2*x^2+1)^(1/2))-3/2*b*c^2*Pi^(3/2)*polylog(2,-c*x-(c^2*x^2+1)^
(1/2))+3/2*b*c^2*Pi^(3/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+3/2*b*c^
2*Pi^(3/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\left(3\pi^{\frac{3}{2}}c^2\operatorname{arsinh}\left(\frac{1}{c|x|}\right) - 3\pi\sqrt{\pi + \pi c^2x^2}\,c^2 - \left(\pi + \pi c^2x^2\right)^{\frac{3}{2}}c^2 + \frac{\left(\pi + \pi c^2x^2\right)^{\frac{5}{2}}}{\pi x^2}\right)a + b\int\frac{\left(\pi + \pi c^2x^2\right)^{\frac{3}{2}}\log\left(c\right.}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="maxima
")
```

```
[Out] -1/2*(3*pi^(3/2)*c^2*arcsinh(1/(c*abs(x))) - 3*pi*sqrt(pi + pi*c^2*x^2)*c^2
 - (pi + pi*c^2*x^2)^(3/2)*c^2 + (pi + pi*c^2*x^2)^(5/2)/(pi*x^2))*a + b*in
tegrate((pi + pi*c^2*x^2)^(3/2)*log(c*x + sqrt(c^2*x^2 + 1))/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))\,\left(\Pi\,c^2\,x^2+\Pi\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2))/x^3,x)
```

```
[Out] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\pi^{\frac{3}{2}}\left(\int \frac{a\sqrt{c^2x^2+1}}{x^3}\,dx + \int \frac{ac^2\sqrt{c^2x^2+1}}{x}\,dx + \int \frac{b\sqrt{c^2x^2+1}\,\operatorname{asinh}(cx)}{x^3}\,dx + \int \frac{bc^2\sqrt{c^2x^2+1}\,\operatorname{asinh}(cx)}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x))/x**3,x)
```

```
[Out] pi**(3/2)*(Integral(a*sqrt(c**2*x**2 + 1)/x**3, x) + Integral(a*c**2*sqrt(c
**2*x**2 + 1)/x, x) + Integral(b*sqrt(c**2*x**2 + 1)*asinh(c*x)/x**3, x) +
Integral(b*c**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/x, x))
```

3.70 $\int \frac{\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=115

$$\frac{\pi^{3/2}c^3\left(a+b\sinh^{-1}(cx)\right)^2}{2b}-\frac{\pi c^2\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3x^3}+\frac{4}{3}\pi^{3/2}bc^3$$

```
[Out] -1/6*b*c*Pi^(3/2)/x^2-1/3*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))/x^3+1/2*
c^3*Pi^(3/2)*(a+b*arcsinh(c*x))^2/b+4/3*b*c^3*Pi^(3/2)*ln(x)-c^2*Pi*(a+b*ar
csinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/x
```

Rubi [A] time = 0.22, antiderivative size = 184, normalized size of antiderivative = 1.60, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5739, 5737, 29, 5675, 14}

$$\frac{\pi c^3\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)^2}{2b\sqrt{c^2x^2+1}}-\frac{\pi c^2\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-$$

Antiderivative was successfully verified.

```
[In] Int[((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] -(b*c*Pi*Sqrt[Pi + c^2*Pi*x^2])/(6*x^2*Sqrt[1 + c^2*x^2]) - (c^2*Pi*Sqrt[Pi
 + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x - ((Pi + c^2*Pi*x^2)^(3/2)*(a + b*Ar
cSinh[c*x]))/(3*x^3) + (c^3*Pi*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^2
)/(2*b*Sqrt[1 + c^2*x^2]) + (4*b*c^3*Pi*Sqrt[Pi + c^2*Pi*x^2]*Log[x])/(3*Sq
rt[1 + c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5737

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 1)*S
qrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x] -
 Dist[(c^2*Sqrt[d + e*x^2])/(f^2*(m + 1)*Sqrt[1 + c^2*x^2]), Int[((f*x)^(m
+ 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
```

```
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(\pi + c^2\pi x^2\right)^{3/2} \left(a + b\sinh^{-1}(cx)\right)}{x^4}\,dx = -\frac{\left(\pi + c^2\pi x^2\right)^{3/2} \left(a + b\sinh^{-1}(cx)\right)}{3x^3} + \left(c^2\pi\right)\int \frac{\sqrt{\pi + c^2\pi x^2}\,\left(a + b\sin\right.}{x^2}$$

$$= -\frac{c^2\pi\sqrt{\pi + c^2\pi x^2}\,\left(a + b\sinh^{-1}(cx)\right)}{x} - \frac{\left(\pi + c^2\pi x^2\right)^{3/2} \left(a + b\sinh^{-1}(c\right.}{3x^3}$$

$$= -\frac{bc\pi\sqrt{\pi + c^2\pi x^2}}{6x^2\sqrt{1 + c^2x^2}} - \frac{c^2\pi\sqrt{\pi + c^2\pi x^2}\,\left(a + b\sinh^{-1}(cx)\right)}{x} - \frac{\left(\pi + c^2\pi x^2\right.}{}$$

Mathematica [A] time = 0.23, size = 125, normalized size = 1.09

$$\frac{\pi^{3/2}\left(\sinh^{-1}(cx)\left(6ac^3x^3 - 2b\sqrt{c^2x^2+1}\,\left(4c^2x^2+1\right)\right) - 8ac^2x^2\sqrt{c^2x^2+1} - 2a\sqrt{c^2x^2+1} + 8bc^3x^3\log(cx) + 3bc^3\right.}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] (Pi^(3/2)*(-(b*c*x) - 2*a*Sqrt[1 + c^2*x^2] - 8*a*c^2*x^2*Sqrt[1 + c^2*x^2]
 + (6*a*c^3*x^3 - 2*b*Sqrt[1 + c^2*x^2]*(1 + 4*c^2*x^2))*ArcSinh[c*x] + 3*b
*c^3*x^3*ArcSinh[c*x]^2 + 8*b*c^3*x^3*Log[c*x]))/(6*x^3)
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\pi + \pi c^2x^2}\,\left(\pi ac^2x^2 + \pi a + \left(\pi bc^2x^2 + \pi b\right)\text{arsinh}\,(cx)\right)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi*a*c^2*x^2 + pi*a + (pi*b*c^2*x^2 + pi*b)
*arcsinh(c*x))/x^4, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.29, size = 622, normalized size = 5.41

$$-\frac{a\left(\pi c^{2}x^{2}+\pi\right)^{\frac{5}{2}}}{3\pi x^{3}}-\frac{2a c^{2}\left(\pi c^{2}x^{2}+\pi\right)^{\frac{5}{2}}}{3\pi x}+\frac{2a c^{4}x\left(\pi c^{2}x^{2}+\pi\right)^{\frac{3}{2}}}{3}+a c^{4}\pi x\sqrt{\pi c^{2}x^{2}+\pi}+\frac{a c^{4}\pi^{2}\ln\left(\frac{\pi x c^{2}}{\sqrt{\pi c^{2}}}+\sqrt{\pi c^{2}\cdot}\right.}{\sqrt{\pi c^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))/x^4,x)
```

```
[Out] -1/3*a/Pi/x^3*(Pi*c^2*x^2+Pi)^(5/2)-2/3*a*c^2/Pi/x*(Pi*c^2*x^2+Pi)^(5/2)+2/
3*a*c^4*x*(Pi*c^2*x^2+Pi)^(3/2)+a*c^4*Pi*x*(Pi*c^2*x^2+Pi)^(1/2)+a*c^4*Pi^2
*ln(Pi*x*c^2/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/2*b*c^3
*Pi^(3/2)*arcsinh(c*x)^2-8/3*b*c^3*Pi^(3/2)*arcsinh(c*x)+32*b*Pi^(3/2)/(24*
c^4*x^4+9*c^2*x^2+1)*x^4*arcsinh(c*x)*c^7-32*b*Pi^(3/2)/(24*c^4*x^4+9*c^2*x
^2+1)*x^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^6+8/3*b*Pi^(3/2)/(24*c^4*x^4+9*c
^2*x^2+1)*x^4*c^7-8/3*b*Pi^(3/2)/(24*c^4*x^4+9*c^2*x^2+1)*x^2*(c^2*x^2+1)*c
^5+12*b*Pi^(3/2)/(24*c^4*x^4+9*c^2*x^2+1)*x^2*arcsinh(c*x)*c^5-20*b*Pi^(3/2
)/(24*c^4*x^4+9*c^2*x^2+1)*x*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^4-4/3*b*Pi^(3
/2)/(24*c^4*x^4+9*c^2*x^2+1)*(c^2*x^2+1)*c^3+4/3*b*Pi^(3/2)/(24*c^4*x^4+9*c
^2*x^2+1)*arcsinh(c*x)*c^3-13/3*b*Pi^(3/2)/(24*c^4*x^4+9*c^2*x^2+1)/x*arcsi
nh(c*x)*(c^2*x^2+1)^(1/2)*c^2-1/6*b*Pi^(3/2)/(24*c^4*x^4+9*c^2*x^2+1)/x^2*(
c^2*x^2+1)*c-1/3*b*Pi^(3/2)/(24*c^4*x^4+9*c^2*x^2+1)/x^3*arcsinh(c*x)*(c^2*
x^2+1)^(1/2)+4/3*b*c^3*Pi^(3/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))\,\left(\Pi\,c^2\,x^2+\Pi\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2))/x^4,x)
```

```
[Out] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(3/2))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\pi^{\frac{3}{2}}\left(\int\frac{a\sqrt{c^{2}x^{2}+1}}{x^{4}}\,dx+\int\frac{ac^{2}\sqrt{c^{2}x^{2}+1}}{x^{2}}\,dx+\int\frac{b\sqrt{c^{2}x^{2}+1}\,\operatorname{asinh}{(cx)}}{x^{4}}\,dx+\int\frac{bc^{2}\sqrt{c^{2}x^{2}+1}\,\operatorname{asinh}{(cx)}}{x^{2}}\,dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x))/x**4,x)
```

```
[Out] pi**(3/2)*(Integral(a*sqrt(c**2*x**2 + 1)/x**4, x) + Integral(a*c**2*sqrt(c
**2*x**2 + 1)/x**2, x) + Integral(b*sqrt(c**2*x**2 + 1)*asinh(c*x)/x**4, x)
 + Integral(b*c**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/x**2, x))
```

3.71 $\int x^3 \left(\pi + c^2\pi x^2\right)^{5/2} \left(a + b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=141

$$\frac{\left(\pi c^2x^2+\pi\right)^{9/2}\left(a+b\sinh^{-1}(cx)\right)}{9\pi^2c^4}-\frac{\left(\pi c^2x^2+\pi\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7\pi c^4}-\frac{1}{81}\pi^{5/2}bc^5x^9-\frac{19}{441}\pi^{5/2}bc^3x^7+\frac{2\pi^{5/2}bx}{63c^3}-\frac{1}{21}$$

```
[Out] 2/63*b*Pi^(5/2)*x/c^3-1/189*b*Pi^(5/2)*x^3/c-1/21*b*c*Pi^(5/2)*x^5-19/441*b
*c^3*Pi^(5/2)*x^7-1/81*b*c^5*Pi^(5/2)*x^9-1/7*(Pi*c^2*x^2+Pi)^(7/2)*(a+b*ar
csinh(c*x))/c^4/Pi+1/9*(Pi*c^2*x^2+Pi)^(9/2)*(a+b*arcsinh(c*x))/c^4/Pi^2
```

Rubi [A] time = 0.15, antiderivative size = 143, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {266, 43, 5732, 12, 373}

$$\frac{\pi^{5/2}\left(c^2x^2+1\right)^{9/2}\left(a+b\sinh^{-1}(cx)\right)}{9c^4}-\frac{\pi^{5/2}\left(c^2x^2+1\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7c^4}-\frac{1}{81}\pi^{5/2}bc^5x^9-\frac{19}{441}\pi^{5/2}bc^3x^7+\frac{2\pi^{5/2}bx}{63c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (2*b*Pi^(5/2)*x)/(63*c^3) - (b*Pi^(5/2)*x^3)/(189*c) - (b*c*Pi^(5/2)*x^5)/2
1 - (19*b*c^3*Pi^(5/2)*x^7)/441 - (b*c^5*Pi^(5/2)*x^9)/81 - (Pi^(5/2)*(1 +
c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7*c^4) + (Pi^(5/2)*(1 + c^2*x^2)^(9/2
)*(a + b*ArcSinh[c*x]))/(9*c^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rubi steps

$$\int x^3\left(\pi+c^2\pi x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)dx=-\frac{\pi^{5/2}\left(1+c^2x^2\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7c^4}+\frac{\pi^{5/2}\left(1+c^2x^2\right)^{9/2}\left(a+b\right.}{9c^4}$$

$$=-\frac{\pi^{5/2}\left(1+c^2x^2\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7c^4}+\frac{\pi^{5/2}\left(1+c^2x^2\right)^{9/2}\left(a+b\right.}{9c^4}$$

$$=-\frac{\pi^{5/2}\left(1+c^2x^2\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7c^4}+\frac{\pi^{5/2}\left(1+c^2x^2\right)^{9/2}\left(a+b\right.}{9c^4}$$

$$=\frac{2b\pi^{5/2}x}{63c^3}-\frac{b\pi^{5/2}x^3}{189c}-\frac{1}{21}bc\pi^{5/2}x^5-\frac{19}{441}bc^3\pi^{5/2}x^7-\frac{1}{81}bc^5\pi^{5/2}x^9$$

Mathematica [A] time = 0.20, size = 108, normalized size = 0.77

$$\frac{\pi^{5/2}\left(63a\left(7c^2x^2-2\right)\left(c^2x^2+1\right)^{7/2}+63b\left(7c^2x^2-2\right)\left(c^2x^2+1\right)^{7/2}\sinh^{-1}(cx)-bcx\left(49c^8x^8+171c^6x^6+189c^4\right.\right.}{3969c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Pi^(5/2)*(63*a*(1 + c^2*x^2)^(7/2)*(-2 + 7*c^2*x^2) - b*c*x*(-126 + 21*c^2
*x^2 + 189*c^4*x^4 + 171*c^6*x^6 + 49*c^8*x^8) + 63*b*(1 + c^2*x^2)^(7/2)*(
-2 + 7*c^2*x^2)*ArcSinh[c*x]))/(3969*c^4)
```

fricas [B] time = 0.61, size = 263, normalized size = 1.87

$$\frac{63\sqrt{\pi+\pi c^2x^2}\left(7\pi^2bc^{10}x^{10}+26\pi^2bc^8x^8+34\pi^2bc^6x^6+16\pi^2bc^4x^4-\pi^2bc^2x^2-2\pi^2b\right)\log\left(cx+\sqrt{c^2x^2+1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas
")
```

```
[Out] 1/3969*(63*sqrt(pi + pi*c^2*x^2)*(7*pi^2*b*c^10*x^10 + 26*pi^2*b*c^8*x^8 +
34*pi^2*b*c^6*x^6 + 16*pi^2*b*c^4*x^4 - pi^2*b*c^2*x^2 - 2*pi^2*b)*log(c*x
+ sqrt(c^2*x^2 + 1)) + sqrt(pi + pi*c^2*x^2)*(441*pi^2*a*c^10*x^10 + 1638*p
i^2*a*c^8*x^8 + 2142*pi^2*a*c^6*x^6 + 1008*pi^2*a*c^4*x^4 - 63*pi^2*a*c^2*x
^2 - 126*pi^2*a - (49*pi^2*b*c^9*x^9 + 171*pi^2*b*c^7*x^7 + 189*pi^2*b*c^5*
x^5 + 21*pi^2*b*c^3*x^3 - 126*pi^2*b*c*x)*sqrt(c^2*x^2 + 1)))/(c^6*x^2 + c^
4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.12, size = 226, normalized size = 1.60

$$a\left(\frac{x^{2}\left(\pi c^{2}x^{2}+\pi\right)^{\frac{7}{2}}}{9\pi c^{2}}-\frac{2\left(\pi c^{2}x^{2}+\pi\right)^{\frac{7}{2}}}{63\pi c^{4}}\right)+\frac{b\pi^{\frac{5}{2}}\left(441\,\mathrm{arcsinh}\left(cx\right)c^{10}x^{10}+1638\,\mathrm{arcsinh}\left(cx\right)c^{8}x^{8}-49c^{9}x^{9}\sqrt{c^{2}x^{2}+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] a*(1/9*x^2*(Pi*c^2*x^2+Pi)^(7/2)/Pi/c^2-2/63/Pi/c^4*(Pi*c^2*x^2+Pi)^(7/2))+
1/3969*b/c^4*Pi^(5/2)/(c^2*x^2+1)^(1/2)*(441*arcsinh(c*x)*c^10*x^10+1638*ar
csinh(c*x)*c^8*x^8-49*c^9*x^9*(c^2*x^2+1)^(1/2)+2142*arcsinh(c*x)*c^6*x^6-1
71*c^7*x^7*(c^2*x^2+1)^(1/2)+1008*arcsinh(c*x)*c^4*x^4-189*c^5*x^5*(c^2*x^2
+1)^(1/2)-63*arcsinh(c*x)*c^2*x^2-21*c^3*x^3*(c^2*x^2+1)^(1/2)-126*arcsinh(
c*x)+126*c*x*(c^2*x^2+1)^(1/2))
```

maxima [A] time = 0.52, size = 156, normalized size = 1.11

$$\frac{1}{63}\left(\frac{7\left(\pi+\pi c^{2}x^{2}\right)^{\frac{7}{2}}x^{2}}{\pi c^{2}}-\frac{2\left(\pi+\pi c^{2}x^{2}\right)^{\frac{7}{2}}}{\pi c^{4}}\right)b\,\mathrm{arsinh}\left(cx\right)+\frac{1}{63}\left(\frac{7\left(\pi+\pi c^{2}x^{2}\right)^{\frac{7}{2}}x^{2}}{\pi c^{2}}-\frac{2\left(\pi+\pi c^{2}x^{2}\right)^{\frac{7}{2}}}{\pi c^{4}}\right)a-\frac{\left(49\,\pi^{\frac{5}{2}}c^{8}x^{9}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima
")
```

```
[Out] 1/63*(7*(pi + pi*c^2*x^2)^(7/2)*x^2/(pi*c^2) - 2*(pi + pi*c^2*x^2)^(7/2)/(p
i*c^4))*b*arcsinh(c*x) + 1/63*(7*(pi + pi*c^2*x^2)^(7/2)*x^2/(pi*c^2) - 2*(
pi + pi*c^2*x^2)^(7/2)/(pi*c^4))*a - 1/3969*(49*pi^(5/2)*c^8*x^9 + 171*pi^(
5/2)*c^6*x^7 + 189*pi^(5/2)*c^4*x^5 + 21*pi^(5/2)*c^2*x^3 - 126*pi^(5/2)*x)
*b/c^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(\Pi\,c^2\,x^2+\Pi\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2),x)
```

```
[Out] int(x^3*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

3.72 $\int x^2 \left(\pi + c^2\pi x^2\right)^{5/2} \left(a + b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=213

$$-\frac{5\pi^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{256bc^3}+\frac{5\pi^{5/2}x\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{128c^2}+\frac{1}{8}x^3\left(\pi c^2x^2+\pi\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{48}\pi$$

```
[Out] -5/256*b*Pi^(5/2)*x^2/c-59/768*b*c*Pi^(5/2)*x^4-17/288*b*c^3*Pi^(5/2)*x^6-1
/64*b*c^5*Pi^(5/2)*x^8+5/48*Pi*x^3*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))
+1/8*x^3*(Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))-5/256*Pi^(5/2)*(a+b*arcsi
nh(c*x))^2/b/c^3+5/128*Pi^(5/2)*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^2+
5/64*Pi^2*x^3*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 337, normalized size of antiderivative = 1.58, number of steps used = 12, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5744, 5742, 5758, 5675, 30, 14, 266, 43}

$$\frac{1}{8}x^3\left(\pi c^2x^2+\pi\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{48}\pi x^3\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{64}\pi^2x^3\sqrt{\pi c^2x^2+\pi}\left(a+b\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-5*b*Pi^2*x^2*Sqrt[Pi + c^2*Pi*x^2])/(256*c*Sqrt[1 + c^2*x^2]) - (59*b*c*P
i^2*x^4*Sqrt[Pi + c^2*Pi*x^2])/(768*Sqrt[1 + c^2*x^2]) - (17*b*c^3*Pi^2*x^6
*Sqrt[Pi + c^2*Pi*x^2])/(288*Sqrt[1 + c^2*x^2]) - (b*c^5*Pi^2*x^8*Sqrt[Pi +
 c^2*Pi*x^2])/(64*Sqrt[1 + c^2*x^2]) + (5*Pi^2*x*Sqrt[Pi + c^2*Pi*x^2]*(a +
 b*ArcSinh[c*x]))/(128*c^2) + (5*Pi^2*x^3*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcS
inh[c*x]))/64 + (5*Pi*x^3*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/48
+ (x^3*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/8 - (5*Pi^2*Sqrt[Pi +
c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^2)/(256*b*c^3*Sqrt[1 + c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int x^2 \left(\pi + c^2\pi x^2\right)^{5/2} \left(a + b\sinh^{-1}(cx)\right) dx = \frac{1}{8}x^3 \left(\pi + c^2\pi x^2\right)^{5/2} \left(a + b\sinh^{-1}(cx)\right) + \frac{1}{8}(5\pi) \int x^2 \left(\pi + c^2\pi x^2\right)^{3/2}$$

$$= \frac{5}{48}\pi x^3 \left(\pi + c^2\pi x^2\right)^{3/2} \left(a + b\sinh^{-1}(cx)\right) + \frac{1}{8}x^3 \left(\pi + c^2\pi x^2\right)^{5/2} \left(a +\right.$$

$$= \frac{5}{64}\pi^2 x^3 \sqrt{\pi + c^2\pi x^2} \left(a + b\sinh^{-1}(cx)\right) + \frac{5}{48}\pi x^3 \left(\pi + c^2\pi x^2\right)^{3/2} \left(a\right.$$

$$= -\frac{59bc\pi^2 x^4 \sqrt{\pi + c^2\pi x^2}}{768\sqrt{1 + c^2x^2}} - \frac{17bc^3\pi^2 x^6 \sqrt{\pi + c^2\pi x^2}}{288\sqrt{1 + c^2x^2}} - \frac{bc^5\pi^2 x^8 \sqrt{\pi +}}{64\sqrt{1 + c^2}}$$

$$= -\frac{5b\pi^2 x^2 \sqrt{\pi + c^2\pi x^2}}{256c\sqrt{1 + c^2x^2}} - \frac{59bc\pi^2 x^4 \sqrt{\pi + c^2\pi x^2}}{768\sqrt{1 + c^2x^2}} - \frac{17bc^3\pi^2 x^6 \sqrt{\pi + c}}{288\sqrt{1 + c^2x}}$$

Mathematica [A] time = 0.61, size = 196, normalized size = 0.92

$$\pi^{5/2}\left(-24\sinh^{-1}(cx)\left(120a + 48b\sinh\left(2\sinh^{-1}(cx)\right) - 24b\sinh\left(4\sinh^{-1}(cx)\right) - 16b\sinh\left(6\sinh^{-1}(cx)\right) - 3b\,\mathrm{si}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Pi^(5/2)*(2880*a*c*x*Sqrt[1 + c^2*x^2] + 22656*a*c^3*x^3*Sqrt[1 + c^2*x^2]
 + 26112*a*c^5*x^5*Sqrt[1 + c^2*x^2] + 9216*a*c^7*x^7*Sqrt[1 + c^2*x^2] - 1
440*b*ArcSinh[c*x]^2 + 576*b*Cosh[2*ArcSinh[c*x]] - 144*b*Cosh[4*ArcSinh[c*
x]] - 64*b*Cosh[6*ArcSinh[c*x]] - 9*b*Cosh[8*ArcSinh[c*x]] - 24*ArcSinh[c*x
]*(120*a + 48*b*Sinh[2*ArcSinh[c*x]] - 24*b*Sinh[4*ArcSinh[c*x]] - 16*b*Sin
h[6*ArcSinh[c*x]] - 3*b*Sinh[8*ArcSinh[c*x]])))/(73728*c^3)
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{\pi + \pi c^2 x^2}\left(\pi^2 a c^4 x^6 + 2\,\pi^2 a c^2 x^4 + \pi^2 a x^2 + \left(\pi^2 b c^4 x^6 + 2\,\pi^2 b c^2 x^4 + \pi^2 b x^2\right)\operatorname{arsinh}(cx)\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi^2*a*c^4*x^6 + 2*pi^2*a*c^2*x^4 + pi^2*a*
x^2 + (pi^2*b*c^4*x^6 + 2*pi^2*b*c^2*x^4 + pi^2*b*x^2)*arcsinh(c*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\pi + \pi c^2 x^2\right)^{\frac{5}{2}}(b\operatorname{arsinh}(cx) + a)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((pi + pi*c^2*x^2)^(5/2)*(b*arcsinh(c*x) + a)*x^2, x)
```

maple [A] time = 0.11, size = 301, normalized size = 1.41

$$\frac{ax\left(\pi c^2x^2+\pi\right)^{\frac{7}{2}}}{8\pi c^2}-\frac{ax\left(\pi c^2x^2+\pi\right)^{\frac{5}{2}}}{48c^2}-\frac{5a\pi x\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}}{192c^2}-\frac{5a\pi^2x\sqrt{\pi c^2x^2+\pi}}{128c^2}-\frac{5a\pi^3\ln\left(\frac{\pi x c^2}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\right.}{128c^2\sqrt{\pi c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/8*a*x*(Pi*c^2*x^2+Pi)^(7/2)/Pi/c^2-1/48*a/c^2*x*(Pi*c^2*x^2+Pi)^(5/2)-5/1
92*a/c^2*Pi*x*(Pi*c^2*x^2+Pi)^(3/2)-5/128*a/c^2*Pi^2*x*(Pi*c^2*x^2+Pi)^(1/2
)-5/128*a/c^2*Pi^3*ln(Pi*x*c^2/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^
2)^(1/2)+1/8*b*Pi^(5/2)*c^4*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^7-1/64*b*c^5*P
i^(5/2)*x^8+17/48*b*Pi^(5/2)*c^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^5-17/288*
b*c^3*Pi^(5/2)*x^6+59/192*b*Pi^(5/2)*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^3-59/
768*b*c*Pi^(5/2)*x^4+5/128*b*Pi^(5/2)/c^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x-
5/256*b*Pi^(5/2)*x^2/c-5/256*b*Pi^(5/2)/c^3*arcsinh(c*x)^2+1/72*b*Pi^(5/2)/
c^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(a + b \operatorname{asinh}(c x)\right) \left(\Pi c^2 x^2 + \Pi\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2),x)
[Out] int(x^2*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x)),x)
[Out] Timed out
```

3.73 $\int x\left(\pi+c^2\pi x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)dx$

Optimal. Leaf size=93

$$\frac{\left(\pi c^2x^2+\pi\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7\pi c^2}-\frac{1}{49}\pi^{5/2}bc^5x^7-\frac{3}{35}\pi^{5/2}bc^3x^5-\frac{1}{7}\pi^{5/2}bcx^3-\frac{\pi^{5/2}bx}{7c}$$

```
[Out] -1/7*b*Pi^(5/2)*x/c-1/7*b*c*Pi^(5/2)*x^3-3/35*b*c^3*Pi^(5/2)*x^5-1/49*b*c^5
*Pi^(5/2)*x^7+1/7*(Pi*c^2*x^2+Pi)^(7/2)*(a+b*arcsinh(c*x))/c^2/Pi
```

Rubi [B] time = 0.09, antiderivative size = 193, normalized size of antiderivative = 2.08, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5717, 194}

$$\frac{\left(\pi c^2x^2+\pi\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7\pi c^2}-\frac{\pi^2bc^5x^7\sqrt{\pi c^2x^2+\pi}}{49\sqrt{c^2x^2+1}}-\frac{3\pi^2bc^3x^5\sqrt{\pi c^2x^2+\pi}}{35\sqrt{c^2x^2+1}}-\frac{\pi^2bcx^3\sqrt{\pi c^2x^2+\pi}}{7\sqrt{c^2x^2+1}}-\frac{\pi^2bx\cdots}{7c\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(b*Pi^2*x*Sqrt[Pi + c^2*Pi*x^2])/(7*c*Sqrt[1 + c^2*x^2]) - (b*c*Pi^2*x^3*S
qrt[Pi + c^2*Pi*x^2])/(7*Sqrt[1 + c^2*x^2]) - (3*b*c^3*Pi^2*x^5*Sqrt[Pi + c
^2*Pi*x^2])/(35*Sqrt[1 + c^2*x^2]) - (b*c^5*Pi^2*x^7*Sqrt[Pi + c^2*Pi*x^2])
/(49*Sqrt[1 + c^2*x^2]) + ((Pi + c^2*Pi*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7
*c^2*Pi)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(\pi+c^2\pi x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)dx &= \frac{\left(\pi+c^2\pi x^2\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7c^2\pi}-\frac{\left(b\pi^2\sqrt{\pi+c^2\pi x^2}\right)\int\left(1+c^2x\cdots\right.}{7c\sqrt{1+c^2x^2}}\\ &= \frac{\left(\pi+c^2\pi x^2\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7c^2\pi}-\frac{\left(b\pi^2\sqrt{\pi+c^2\pi x^2}\right)\int\left(1+3c\cdots\right.}{7c\sqrt{1+c\cdots}}\\ &= -\frac{b\pi^2x\sqrt{\pi+c^2\pi x^2}}{7c\sqrt{1+c^2x^2}}-\frac{bc\pi^2x^3\sqrt{\pi+c^2\pi x^2}}{7\sqrt{1+c^2x^2}}-\frac{3bc^3\pi^2x^5\sqrt{\pi+c^2\pi x\cdots}}{35\sqrt{1+c^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.15, size = 80, normalized size = 0.86

$$\frac{\pi^{5/2}\left(35a\left(c^2x^2+1\right)^{7/2}+35b\left(c^2x^2+1\right)^{7/2}\sinh^{-1}(cx)-bcx\left(5c^6x^6+21c^4x^4+35c^2x^2+35\right)\right)}{245c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Pi^(5/2)*(35*a*(1 + c^2*x^2)^(7/2) - b*c*x*(35 + 35*c^2*x^2 + 21*c^4*x^4 +
 5*c^6*x^6) + 35*b*(1 + c^2*x^2)^(7/2)*ArcSinh[c*x]))/(245*c^2)
```

fricas [B] time = 0.61, size = 225, normalized size = 2.42

$$\frac{35\sqrt{\pi+\pi c^2x^2}\left(\pi^2bc^8x^8+4\,\pi^2bc^6x^6+6\,\pi^2bc^4x^4+4\,\pi^2bc^2x^2+\pi^2b\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{\pi+\pi c^2x^2}\left(35\,\pi\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/245*(35*sqrt(pi + pi*c^2*x^2)*(pi^2*b*c^8*x^8 + 4*pi^2*b*c^6*x^6 + 6*pi^2
*b*c^4*x^4 + 4*pi^2*b*c^2*x^2 + pi^2*b)*log(c*x + sqrt(c^2*x^2 + 1)) + sqrt
(pi + pi*c^2*x^2)*(35*pi^2*a*c^8*x^8 + 140*pi^2*a*c^6*x^6 + 210*pi^2*a*c^4*
x^4 + 140*pi^2*a*c^2*x^2 + 35*pi^2*a - (5*pi^2*b*c^7*x^7 + 21*pi^2*b*c^5*x^
5 + 35*pi^2*b*c^3*x^3 + 35*pi^2*b*c*x)*sqrt(c^2*x^2 + 1)))/(c^4*x^2 + c^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.07, size = 170, normalized size = 1.83

$$\frac{a\left(\pi\,c^2x^2+\pi\right)^{\frac{7}{2}}}{7\pi\,c^2}+\frac{b\,\pi^{\frac{5}{2}}\left(35\operatorname{arcsinh}(cx)\,c^8x^8+140\operatorname{arcsinh}(cx)\,c^6x^6-5c^7x^7\sqrt{c^2x^2+1}+210\operatorname{arcsinh}(cx)\,c^4x^4-\right.}{245c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/7*a/Pi/c^2*(Pi*c^2*x^2+Pi)^(7/2)+1/245*b/c^2*Pi^(5/2)*(35*arcsinh(c*x)*c^
8*x^8+140*arcsinh(c*x)*c^6*x^6-5*c^7*x^7*(c^2*x^2+1)^(1/2)+210*arcsinh(c*x)
*c^4*x^4-21*c^5*x^5*(c^2*x^2+1)^(1/2)+140*arcsinh(c*x)*c^2*x^2-35*c^3*x^3*(
c^2*x^2+1)^(1/2)+35*arcsinh(c*x)-35*c*x*(c^2*x^2+1)^(1/2))/(c^2*x^2+1)^(1/2
)
```

maxima [A] time = 0.37, size = 96, normalized size = 1.03

$$\frac{\left(\pi+\pi c^2x^2\right)^{\frac{7}{2}}b\operatorname{arsinh}(cx)}{7\,\pi c^2}+\frac{\left(\pi+\pi c^2x^2\right)^{\frac{7}{2}}a}{7\,\pi c^2}-\frac{\left(5\,\pi^{\frac{7}{2}}c^6x^7+21\,\pi^{\frac{7}{2}}c^4x^5+35\,\pi^{\frac{7}{2}}c^2x^3+35\,\pi^{\frac{7}{2}}x\right)b}{245\,\pi c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*(pi + pi*c^2*x^2)^(7/2)*b*arcsinh(c*x)/(pi*c^2) + 1/7*(pi + pi*c^2*x^2)
^(7/2)*a/(pi*c^2) - 1/245*(5*pi^(7/2)*c^6*x^7 + 21*pi^(7/2)*c^4*x^5 + 35*pi
^(7/2)*c^2*x^3 + 35*pi^(7/2)*x)*b/(pi*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \left(a + b \operatorname{asinh}(c x)\right) \left(\Pi c^2 x^2 + \Pi\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2),x)
[Out] int(x*(a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x)),x)
[Out] Timed out
```

3.74 $\int \left(\pi + c^2\pi x^2\right)^{5/2} \left(a + b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=165

$$\frac{1}{6}x\left(\pi c^2x^2+\pi\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{24}\pi x\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{16}\pi^2x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}\right.$$

```
[Out] -25/96*b*c*Pi^(5/2)*x^2-5/96*b*c^3*Pi^(5/2)*x^4-1/36*b*Pi^(5/2)*(c^2*x^2+1)
^3/c+5/24*Pi*x*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))+1/6*x*(Pi*c^2*x^2+P
i)^(5/2)*(a+b*arcsinh(c*x))+5/32*Pi^(5/2)*(a+b*arcsinh(c*x))^2/b/c+5/16*Pi^
2*x*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 254, normalized size of antiderivative = 1.54, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5684, 5682, 5675, 30, 14, 261}

$$\frac{1}{6}x\left(\pi c^2x^2+\pi\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{24}\pi x\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{16}\pi^2x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-25*b*c*Pi^2*x^2*Sqrt[Pi + c^2*Pi*x^2])/(96*Sqrt[1 + c^2*x^2]) - (5*b*c^3*
Pi^2*x^4*Sqrt[Pi + c^2*Pi*x^2])/(96*Sqrt[1 + c^2*x^2]) - (b*Pi^2*(1 + c^2*x
^2)^(5/2)*Sqrt[Pi + c^2*Pi*x^2])/(36*c) + (5*Pi^2*x*Sqrt[Pi + c^2*Pi*x^2]*(
a + b*ArcSinh[c*x]))/16 + (5*Pi*x*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*
x]))/24 + (x*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/6 + (5*Pi^2*Sqrt
[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^2)/(32*b*c*Sqrt[1 + c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
```

```
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rubi steps

$$\int \left(\pi + c^2\pi x^2\right)^{5/2} \left(a + b\sinh^{-1}(cx)\right) dx = \frac{1}{6}x\left(\pi + c^2\pi x^2\right)^{5/2}\left(a + b\sinh^{-1}(cx)\right) + \frac{1}{6}(5\pi)\int \left(\pi + c^2\pi x^2\right)^{3/2} \left(a\right.$$

$$= -\frac{b\pi^2\left(1 + c^2x^2\right)^{5/2}\sqrt{\pi + c^2\pi x^2}}{36c} + \frac{5}{24}\pi x\left(\pi + c^2\pi x^2\right)^{3/2}\left(a + b\sinh\right.$$

$$= -\frac{b\pi^2\left(1 + c^2x^2\right)^{5/2}\sqrt{\pi + c^2\pi x^2}}{36c} + \frac{5}{16}\pi^2 x\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-}\right.$$

$$= -\frac{25bc\pi^2x^2\sqrt{\pi + c^2\pi x^2}}{96\sqrt{1 + c^2x^2}} - \frac{5bc^3\pi^2x^4\sqrt{\pi + c^2\pi x^2}}{96\sqrt{1 + c^2x^2}} - \frac{b\pi^2\left(1 + c^2x^2\right)^{5}}{36}$$

Mathematica [A] time = 0.38, size = 153, normalized size = 0.93

$$\frac{\pi^{5/2}\left(12\sinh^{-1}(cx)\left(60a + 45b\sinh\left(2\sinh^{-1}(cx)\right) + 9b\sinh\left(4\sinh^{-1}(cx)\right) + b\sinh\left(6\sinh^{-1}(cx)\right)\right) + 1584ac\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Pi^(5/2)*(1584*a*c*x*Sqrt[1 + c^2*x^2] + 1248*a*c^3*x^3*Sqrt[1 + c^2*x^2]
+ 384*a*c^5*x^5*Sqrt[1 + c^2*x^2] + 360*b*ArcSinh[c*x]^2 - 270*b*Cosh[2*Arc
Sinh[c*x]] - 27*b*Cosh[4*ArcSinh[c*x]] - 2*b*Cosh[6*ArcSinh[c*x]] + 12*ArcS
inh[c*x]*(60*a + 45*b*Sinh[2*ArcSinh[c*x]] + 9*b*Sinh[4*ArcSinh[c*x]] + b*S
inh[6*ArcSinh[c*x]])))/(2304*c)
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{\pi + \pi c^2x^2}\left(\pi^2ac^4x^4 + 2\,\pi^2ac^2x^2 + \pi^2a + \left(\pi^2bc^4x^4 + 2\,\pi^2bc^2x^2 + \pi^2b\right)\text{arsinh}\,(cx)\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi^2*a*c^4*x^4 + 2*pi^2*a*c^2*x^2 + pi^2*a
+ (pi^2*b*c^4*x^4 + 2*pi^2*b*c^2*x^2 + pi^2*b)*arcsinh(c*x)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [**A**] time = 0.07, size = 228, normalized size = 1.38

$$\frac{x\left(\pi c^2x^2+\pi\right)^{\frac{5}{2}} a}{6}+\frac{5a\pi x\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}}{24}+\frac{5a\,\pi^2 x\sqrt{\pi c^2x^2+\pi}}{16}+\frac{5a\,\pi^3 \ln\left(\frac{\pi x c^2}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{16\sqrt{\pi c^2}}+\frac{b\,\pi^{\frac{5}{2}}c^4 \operatorname{arcsinh}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/6*x*(Pi*c^2*x^2+Pi)^(5/2)*a+5/24*a*Pi*x*(Pi*c^2*x^2+Pi)^(3/2)+5/16*a*Pi^2
*x*(Pi*c^2*x^2+Pi)^(1/2)+5/16*a*Pi^3*ln(Pi*x*c^2/(Pi*c^2)^(1/2)+(Pi*c^2*x^2
+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/6*b*Pi^(5/2)*c^4*arcsinh(c*x)*(c^2*x^2+1)^(1/2
)*x^5-1/36*b*Pi^(5/2)*c^5*x^6+13/24*b*Pi^(5/2)*c^2*arcsinh(c*x)*(c^2*x^2+1)
^(1/2)*x^3-13/96*b*c^3*Pi^(5/2)*x^4+11/16*b*Pi^(5/2)*arcsinh(c*x)*(c^2*x^2+
1)^(1/2)*x-11/32*b*c*Pi^(5/2)*x^2+5/32*b*Pi^(5/2)/c*arcsinh(c*x)^2-17/72*b*
Pi^(5/2)/c
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\mathrm{asinh}\,(c\,x))\,\left(\Pi\,c^2\,x^2+\Pi\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2),x)
```

```
[Out] int((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

3.75 $\int \frac{\left(\pi+c^2\pi x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=179

$$\frac{1}{5}\left(\pi c^2x^2+\pi\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}\pi\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\pi^2\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)$$

```
[Out] -23/15*b*c*Pi^(5/2)*x-11/45*b*c^3*Pi^(5/2)*x^3-1/25*b*c^5*Pi^(5/2)*x^5+1/3*
Pi*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))+1/5*(Pi*c^2*x^2+Pi)^(5/2)*(a+b*
arcsinh(c*x))-2*Pi^(5/2)*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))-
b*Pi^(5/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+b*Pi^(5/2)*polylog(2,c*x+(c^2*
x^2+1)^(1/2))+Pi^2*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 329, normalized size of antiderivative = 1.84, number of steps used = 13, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5744, 5742, 5760, 4182, 2279, 2391, 8, 194}

$$-\frac{\pi^2b\sqrt{\pi c^2x^2+\pi}\,\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+\frac{\pi^2b\sqrt{\pi c^2x^2+\pi}\,\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+\frac{1}{5}\left(\pi c^2x^2+\pi\right)^{5/2}\left(a+b\right.$$

Antiderivative was successfully verified.

```
[In] Int[((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (-23*b*c*Pi^2*x*Sqrt[Pi + c^2*Pi*x^2])/(15*Sqrt[1 + c^2*x^2]) - (11*b*c^3*P
i^2*x^3*Sqrt[Pi + c^2*Pi*x^2])/(45*Sqrt[1 + c^2*x^2]) - (b*c^5*Pi^2*x^5*Sqr
t[Pi + c^2*Pi*x^2])/(25*Sqrt[1 + c^2*x^2]) + Pi^2*Sqrt[Pi + c^2*Pi*x^2]*(a
+ b*ArcSinh[c*x]) + (Pi*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/3 + (
(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/5 - (2*Pi^2*Sqrt[Pi + c^2*Pi*
x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (b*P
i^2*Sqrt[Pi + c^2*Pi*x^2]*PolyLog[2, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] +
(b*Pi^2*Sqrt[Pi + c^2*Pi*x^2]*PolyLog[2, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
```

```
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(\pi + c^2\pi x^2\right)^{5/2}\left(a + b\sinh^{-1}(cx)\right)}{x}\,dx = \frac{1}{5}\left(\pi + c^2\pi x^2\right)^{5/2}\left(a + b\sinh^{-1}(cx)\right) + \pi\int \frac{\left(\pi + c^2\pi x^2\right)^{3/2}\left(a + b\sinh\right.}{x}$$

$$= \frac{1}{3}\pi\left(\pi + c^2\pi x^2\right)^{3/2}\left(a + b\sinh^{-1}(cx)\right) + \frac{1}{5}\left(\pi + c^2\pi x^2\right)^{5/2}\left(a + b\sinh^{-}\right.$$

$$= -\frac{8bc\pi^2x\sqrt{\pi + c^2\pi x^2}}{15\sqrt{1 + c^2x^2}} - \frac{11bc^3\pi^2x^3\sqrt{\pi + c^2\pi x^2}}{45\sqrt{1 + c^2x^2}} - \frac{bc^5\pi^2x^5\sqrt{\pi + c^2\pi x}}{25\sqrt{1 + c^2x^2}}$$

$$= -\frac{23bc\pi^2x\sqrt{\pi + c^2\pi x^2}}{15\sqrt{1 + c^2x^2}} - \frac{11bc^3\pi^2x^3\sqrt{\pi + c^2\pi x^2}}{45\sqrt{1 + c^2x^2}} - \frac{bc^5\pi^2x^5\sqrt{\pi + c^2\pi}}{25\sqrt{1 + c^2x^2}}$$

$$= -\frac{23bc\pi^2x\sqrt{\pi + c^2\pi x^2}}{15\sqrt{1 + c^2x^2}} - \frac{11bc^3\pi^2x^3\sqrt{\pi + c^2\pi x^2}}{45\sqrt{1 + c^2x^2}} - \frac{bc^5\pi^2x^5\sqrt{\pi + c^2\pi}}{25\sqrt{1 + c^2x^2}}$$

$$= -\frac{23bc\pi^2x\sqrt{\pi + c^2\pi x^2}}{15\sqrt{1 + c^2x^2}} - \frac{11bc^3\pi^2x^3\sqrt{\pi + c^2\pi x^2}}{45\sqrt{1 + c^2x^2}} - \frac{bc^5\pi^2x^5\sqrt{\pi + c^2\pi}}{25\sqrt{1 + c^2x^2}}$$

$$= -\frac{23bc\pi^2x\sqrt{\pi + c^2\pi x^2}}{15\sqrt{1 + c^2x^2}} - \frac{11bc^3\pi^2x^3\sqrt{\pi + c^2\pi x^2}}{45\sqrt{1 + c^2x^2}} - \frac{bc^5\pi^2x^5\sqrt{\pi + c^2\pi}}{25\sqrt{1 + c^2x^2}}$$

Mathematica [A] time = 0.36, size = 257, normalized size = 1.44

$$\frac{1}{225}\pi^{5/2}\left(165ac^2x^2\sqrt{c^2x^2+1} + 345a\sqrt{c^2x^2+1} - 225a\log\left(\pi\left(\sqrt{c^2x^2+1}+1\right)\right) + 45ac^4x^4\sqrt{c^2x^2+1} + 225a\log\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (Pi^(5/2)*(-345*b*c*x - 55*b*c^3*x^3 - 9*b*c^5*x^5 + 345*a*Sqrt[1 + c^2*x^2
] + 165*a*c^2*x^2*Sqrt[1 + c^2*x^2] + 45*a*c^4*x^4*Sqrt[1 + c^2*x^2] + 345*
b*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + 165*b*c^2*x^2*Sqrt[1 + c^2*x^2]*ArcSinh[
c*x] + 45*b*c^4*x^4*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + 225*b*ArcSinh[c*x]*Log
[1 - E^(-ArcSinh[c*x])] - 225*b*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 2
25*a*Log[x] - 225*a*Log[Pi*(1 + Sqrt[1 + c^2*x^2])] + 225*b*PolyLog[2, -E^(
-ArcSinh[c*x])] - 225*b*PolyLog[2, E^(-ArcSinh[c*x])]))/225
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\pi+\pi c^2x^2}\left(\pi^2ac^4x^4+2\,\pi^2ac^2x^2+\pi^2a+\left(\pi^2bc^4x^4+2\,\pi^2bc^2x^2+\pi^2b\right)\text{arsinh}\left(cx\right)\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi^2*a*c^4*x^4 + 2*pi^2*a*c^2*x^2 + pi^2*a
+ (pi^2*b*c^4*x^4 + 2*pi^2*b*c^2*x^2 + pi^2*b)*arcsinh(c*x))/x, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.36, size = 284, normalized size = 1.59

$$\frac{\left(\pi\,c^2x^2+\pi\right)^{\frac{5}{2}}a}{5}+\frac{a\pi\left(\pi\,c^2x^2+\pi\right)^{\frac{3}{2}}}{3}-a\,\pi^{\frac{5}{2}}\,\text{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi\,c^2x^2+\pi}}\right)+a\sqrt{\pi\,c^2x^2+\pi}\;\pi^2-b\,\pi^{\frac{5}{2}}\,\text{polylog}\left(2,-cx-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))/x,x)
```

```
[Out] 1/5*(Pi*c^2*x^2+Pi)^(5/2)*a+1/3*a*Pi*(Pi*c^2*x^2+Pi)^(3/2)-a*Pi^(5/2)*arcta
nh(Pi^(1/2)/(Pi*c^2*x^2+Pi)^(1/2))+a*(Pi*c^2*x^2+Pi)^(1/2)*Pi^2-b*Pi^(5/2)*
polylog(2,-c*x-(c^2*x^2+1)^(1/2))+b*Pi^(5/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2
))-b*Pi^(5/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+b*Pi^(5/2)*arcsinh(c
*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-1/25*b*c^5*Pi^(5/2)*x^5-11/45*b*c^3*Pi^(5/2
)*x^3+23/15*b*Pi^(5/2)*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-23/15*b*c*Pi^(5/2)*x+
1/5*b*Pi^(5/2)*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^4*c^4+11/15*b*Pi^(5/2)*arcs
inh(c*x)*(c^2*x^2+1)^(1/2)*x^2*c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{15}\left(15\,\pi^{\frac{5}{2}}\,\text{arsinh}\left(\frac{1}{c|x|}\right)-15\,\pi^2\sqrt{\pi+\pi c^2x^2}\;-5\,\pi\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}-3\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}\right)a+b\int\frac{\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x,x, algorithm="maxima")
```

```
[Out] -1/15*(15*pi^(5/2)*arcsinh(1/(c*abs(x))) - 15*pi^2*sqrt(pi + pi*c^2*x^2) -
5*pi*(pi + pi*c^2*x^2)^(3/2) - 3*(pi + pi*c^2*x^2)^(5/2))*a + b*integrate((
pi + pi*c^2*x^2)^(5/2)*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{asinh}\,(c\,x))\,\left(\Pi\,c^2\,x^2 + \Pi\right)^{5/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2))/x,x)
```

```
[Out] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\pi^{\frac{5}{2}}\left(\int \frac{a\sqrt{c^2x^2+1}}{x}\,dx + \int 2ac^2x\sqrt{c^2x^2+1}\,dx + \int ac^4x^3\sqrt{c^2x^2+1}\,dx + \int \frac{b\sqrt{c^2x^2+1}\,\mathrm{asinh}\,(cx)}{x}\,dx + \int 2bc^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x))/x,x)
```

```
[Out] pi**(5/2)*(Integral(a*sqrt(c**2*x**2 + 1)/x, x) + Integral(2*a*c**2*x*sqrt(
c**2*x**2 + 1), x) + Integral(a*c**4*x**3*sqrt(c**2*x**2 + 1), x) + Integra
l(b*sqrt(c**2*x**2 + 1)*asinh(c*x)/x, x) + Integral(2*b*c**2*x*sqrt(c**2*x*
*2 + 1)*asinh(c*x), x) + Integral(b*c**4*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x
), x))
```

3.76 $\int \frac{\left(\pi+c^2\pi x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=157

$$\frac{5}{4}\pi c^2 x\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{15}{8}\pi^2c^2x\sqrt{\pi c^2x^2+\pi}\,\left(a+b\sinh^{-1}(cx)\right)-\frac{\left(\pi c^2x^2+\pi\right)^{5/2}\left(a+b\sinh\right.}{x}$$

```
[Out] -9/16*b*c^3*Pi^(5/2)*x^2-1/16*b*c^5*Pi^(5/2)*x^4+5/4*c^2*Pi*x*(Pi*c^2*x^2+P
i)^(3/2)*(a+b*arcsinh(c*x))-(Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))/x+15/1
6*c*Pi^(5/2)*(a+b*arcsinh(c*x))^2/b+b*c*Pi^(5/2)*ln(x)+15/8*c^2*Pi^2*x*(a+b
*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 257, normalized size of antiderivative = 1.64, number of steps used = 10, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5739, 5684, 5682, 5675, 30, 14, 266, 43}

$$\frac{5}{4}\pi c^2 x\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{15}{8}\pi^2c^2x\sqrt{\pi c^2x^2+\pi}\,\left(a+b\sinh^{-1}(cx)\right)+\frac{15\pi^2c\sqrt{\pi c^2x^2+\pi}\,\left(a+b\right.}{16b\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] (-9*b*c^3*Pi^2*x^2*Sqrt[Pi + c^2*Pi*x^2])/(16*Sqrt[1 + c^2*x^2]) - (b*c^5*P
i^2*x^4*Sqrt[Pi + c^2*Pi*x^2])/(16*Sqrt[1 + c^2*x^2]) + (15*c^2*Pi^2*x*Sqrt
[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/8 + (5*c^2*Pi*x*(Pi + c^2*Pi*x^2)^(
3/2)*(a + b*ArcSinh[c*x]))/4 - ((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]
))/x + (15*c*Pi^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^2)/(16*b*Sqrt[
1 + c^2*x^2]) + (b*c*Pi^2*Sqrt[Pi + c^2*Pi*x^2]*Log[x])/Sqrt[1 + c^2*x^2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
```

```
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(\pi + c^2\pi x^2\right)^{5/2}\left(a + b\sinh^{-1}(cx)\right)}{x^2}\,dx = -\frac{\left(\pi + c^2\pi x^2\right)^{5/2}\left(a + b\sinh^{-1}(cx)\right)}{x} + \left(5c^2\pi\right)\int \left(\pi + c^2\pi x^2\right)^{3/2}\left(a + b\right.$$

$$= \frac{5}{4}c^2\pi x\left(\pi + c^2\pi x^2\right)^{3/2}\left(a + b\sinh^{-1}(cx)\right) - \frac{\left(\pi + c^2\pi x^2\right)^{5/2}\left(a + b\sinh\right.}{x}$$

$$= \frac{15}{8}c^2\pi^2 x\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right) + \frac{5}{4}c^2\pi x\left(\pi + c^2\pi x^2\right)^{3/2}\left(a +\right.$$

$$= -\frac{9bc^3\pi^2x^2\sqrt{\pi + c^2\pi x^2}}{16\sqrt{1 + c^2x^2}} - \frac{bc^5\pi^2x^4\sqrt{\pi + c^2\pi x^2}}{16\sqrt{1 + c^2x^2}} + \frac{15}{8}c^2\pi^2 x\sqrt{\pi + c^2\pi x}$$

Mathematica [A] time = 0.42, size = 168, normalized size = 1.07

$$\frac{\pi^{5/2}\left(4\sinh^{-1}(cx)\left(60acx - 32b\sqrt{c^2x^2+1} + 16bcx\sinh\left(2\sinh^{-1}(cx)\right) + bcx\sinh\left(4\sinh^{-1}(cx)\right)\right) + 144ac^2x^2\sqrt{c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

[Out] (Pi^(5/2)*(-128*a*Sqrt[1 + c^2*x^2] + 144*a*c^2*x^2*Sqrt[1 + c^2*x^2] + 32*a*c^4*x^4*Sqrt[1 + c^2*x^2] + 120*b*c*x*ArcSinh[c*x]^2 - 32*b*c*x*Cosh[2*ArcSinh[c*x]] - b*c*x*Cosh[4*ArcSinh[c*x]] + 128*b*c*x*Log[c*x] + 4*ArcSinh[c*x]*(60*a*c*x - 32*b*Sqrt[1 + c^2*x^2] + 16*b*c*x*Sinh[2*ArcSinh[c*x]] + b*c*x*Sinh[4*ArcSinh[c*x]])))/(128*x)

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\pi + \pi c^2x^2}\left(\pi^2ac^4x^4 + 2\,\pi^2ac^2x^2 + \pi^2a + \left(\pi^2bc^4x^4 + 2\,\pi^2bc^2x^2 + \pi^2b\right)\operatorname{arsinh}(cx)\right)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="fricas")

[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi^2*a*c^4*x^4 + 2*pi^2*a*c^2*x^2 + pi^2*a + (pi^2*b*c^4*x^4 + 2*pi^2*b*c^2*x^2 + pi^2*b)*arcsinh(c*x))/x^2, x)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

maple [B] time = 0.29, size = 283, normalized size = 1.80

$$-\frac{a\left(\pi\,c^2x^2 + \pi\right)^{\frac{7}{2}}}{\pi x} + a\,c^2x\left(\pi\,c^2x^2 + \pi\right)^{\frac{5}{2}} + \frac{5a\,c^2\pi x\left(\pi\,c^2x^2 + \pi\right)^{\frac{3}{2}}}{4} + \frac{15a\,c^2\pi^2x\sqrt{\pi\,c^2x^2 + \pi}}{8} + \frac{15a\,c^2\pi^3\ln\left(\frac{\pi x\,c^2}{\sqrt{\pi\,c^2}} +\right.}{8\sqrt{\pi\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))/x^2,x)

[Out] -a/Pi/x*(Pi*c^2*x^2+Pi)^(7/2)+a*c^2*x*(Pi*c^2*x^2+Pi)^(5/2)+5/4*a*c^2*Pi*x*(Pi*c^2*x^2+Pi)^(3/2)+15/8*a*c^2*Pi^2*x*(Pi*c^2*x^2+Pi)^(1/2)+15/8*a*c^2*Pi^3*ln(Pi*x*c^2/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/4*b*Pi^(5/2)*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^3*c^4+9/8*b*Pi^(5/2)*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x*c^2-33/128*b*Pi^(5/2)*c-b*Pi^(5/2)*arcsinh(c*x)/x*(c^2*x^2+1)^(1/2)+b*c*Pi^(5/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)-1/16*b*c^5*Pi^(5/2)*x^4-9/16*b*c^3*Pi^(5/2)*x^2+15/16*b*c*Pi^(5/2)*arcsinh(c*x)^2-b*c*Pi^(5/2)*arcsinh(c*x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\operatorname{asinh}(c\,x))\,\left(\Pi\,c^2\,x^2+\Pi\right)^{5/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2))/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\pi^{\frac{5}{2}}\left(\int 2ac^2\sqrt{c^2x^2+1}\,dx+\int \frac{a\sqrt{c^2x^2+1}}{x^2}\,dx+\int ac^4x^2\sqrt{c^2x^2+1}\,dx+\int 2bc^2\sqrt{c^2x^2+1}\,\operatorname{asinh}(cx)\,dx+\int \frac{b\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x))/x**2,x)
```

```
[Out] pi**(5/2)*(Integral(2*a*c**2*sqrt(c**2*x**2 + 1), x) + Integral(a*sqrt(c**2
*x**2 + 1)/x**2, x) + Integral(a*c**4*x**2*sqrt(c**2*x**2 + 1), x) + Integr
al(2*b*c**2*sqrt(c**2*x**2 + 1)*asinh(c*x), x) + Integral(b*sqrt(c**2*x**2
+ 1)*asinh(c*x)/x**2, x) + Integral(b*c**4*x**2*sqrt(c**2*x**2 + 1)*asinh(c
*x), x))
```

3.77 $\int \frac{(\pi+c^2\pi x^2)^{5/2}(a+b\sinh^{-1}(cx))}{x^3}\,dx$

Optimal. Leaf size=205

$$\frac{5}{6}\pi c^2\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{2}\pi^2c^2\sqrt{\pi c^2x^2+\pi}\,\left(a+b\sinh^{-1}(cx)\right)-\frac{\left(\pi c^2x^2+\pi\right)^{5/2}\left(a+b\sinh^{-1}(\right.}{2x^2}$$

```
[Out] -1/2*b*c*Pi^(5/2)/x-7/3*b*c^3*Pi^(5/2)*x-1/9*b*c^5*Pi^(5/2)*x^3+5/6*c^2*Pi*
(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))-1/2*(Pi*c^2*x^2+Pi)^(5/2)*(a+b*arc
sinh(c*x))/x^2-5*c^2*Pi^(5/2)*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1
/2))-5/2*b*c^2*Pi^(5/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+5/2*b*c^2*Pi^(5/2
)*polylog(2,c*x+(c^2*x^2+1)^(1/2))+5/2*c^2*Pi^2*(a+b*arcsinh(c*x))*(Pi*c^2*
x^2+Pi)^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 355, normalized size of antiderivative = 1.73, number of steps used = 13, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5739, 5744, 5742, 5760, 4182, 2279, 2391, 8, 270}

$$-\frac{5\pi^2bc^2\sqrt{\pi c^2x^2+\pi}\,\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2x^2+1}}+\frac{5\pi^2bc^2\sqrt{\pi c^2x^2+\pi}\,\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2x^2+1}}+\frac{5}{6}\pi c^2\left(\pi c^2x^2+\pi\right.$$

Antiderivative was successfully verified.

```
[In] Int[((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] -(b*c*Pi^2*Sqrt[Pi + c^2*Pi*x^2])/(2*x*Sqrt[1 + c^2*x^2]) - (7*b*c^3*Pi^2*x
*Sqrt[Pi + c^2*Pi*x^2])/(3*Sqrt[1 + c^2*x^2]) - (b*c^5*Pi^2*x^3*Sqrt[Pi + c
^2*Pi*x^2])/(9*Sqrt[1 + c^2*x^2]) + (5*c^2*Pi^2*Sqrt[Pi + c^2*Pi*x^2]*(a +
b*ArcSinh[c*x]))/2 + (5*c^2*Pi*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x])
)/6 - ((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(2*x^2) - (5*c^2*Pi^2*
Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1
+ c^2*x^2] - (5*b*c^2*Pi^2*Sqrt[Pi + c^2*Pi*x^2]*PolyLog[2, -E^ArcSinh[c*x]
])/(2*Sqrt[1 + c^2*x^2]) + (5*b*c^2*Pi^2*Sqrt[Pi + c^2*Pi*x^2]*PolyLog[2, E
^ArcSinh[c*x]])/(2*Sqrt[1 + c^2*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(\pi + c^2\pi x^2\right)^{5/2} \left(a + b \sinh^{-1}(cx)\right)}{x^3} dx = -\frac{\left(\pi + c^2\pi x^2\right)^{5/2} \left(a + b \sinh^{-1}(cx)\right)}{2x^2} + \frac{1}{2}\left(5c^2\pi\right) \int \frac{\left(\pi + c^2\pi x^2\right)^{3/2} (}{}$$

$$= \frac{5}{6}c^2\pi \left(\pi + c^2\pi x^2\right)^{3/2} \left(a + b \sinh^{-1}(cx)\right) - \frac{\left(\pi + c^2\pi x^2\right)^{5/2} \left(a + b \sin\right.}{2x^2}$$

$$= -\frac{bc\pi^2\sqrt{\pi + c^2\pi x^2}}{2x\sqrt{1 + c^2x^2}} + \frac{bc^3\pi^2 x\sqrt{\pi + c^2\pi x^2}}{6\sqrt{1 + c^2x^2}} - \frac{bc^5\pi^2x^3\sqrt{\pi + c^2\pi x^2}}{9\sqrt{1 + c^2x^2}}$$

$$= -\frac{bc\pi^2\sqrt{\pi + c^2\pi x^2}}{2x\sqrt{1 + c^2x^2}} - \frac{7bc^3\pi^2 x\sqrt{\pi + c^2\pi x^2}}{3\sqrt{1 + c^2x^2}} - \frac{bc^5\pi^2x^3\sqrt{\pi + c^2\pi x^2}}{9\sqrt{1 + c^2x^2}}$$

$$= -\frac{bc\pi^2\sqrt{\pi + c^2\pi x^2}}{2x\sqrt{1 + c^2x^2}} - \frac{7bc^3\pi^2 x\sqrt{\pi + c^2\pi x^2}}{3\sqrt{1 + c^2x^2}} - \frac{bc^5\pi^2x^3\sqrt{\pi + c^2\pi x^2}}{9\sqrt{1 + c^2x^2}}$$

$$= -\frac{bc\pi^2\sqrt{\pi + c^2\pi x^2}}{2x\sqrt{1 + c^2x^2}} - \frac{7bc^3\pi^2 x\sqrt{\pi + c^2\pi x^2}}{3\sqrt{1 + c^2x^2}} - \frac{bc^5\pi^2x^3\sqrt{\pi + c^2\pi x^2}}{9\sqrt{1 + c^2x^2}}$$

$$= -\frac{bc\pi^2\sqrt{\pi + c^2\pi x^2}}{2x\sqrt{1 + c^2x^2}} - \frac{7bc^3\pi^2 x\sqrt{\pi + c^2\pi x^2}}{3\sqrt{1 + c^2x^2}} - \frac{bc^5\pi^2x^3\sqrt{\pi + c^2\pi x^2}}{9\sqrt{1 + c^2x^2}}$$

Mathematica [A] time = 1.95, size = 349, normalized size = 1.70

$$\pi^{5/2}\left(168ac^2x^2\sqrt{c^2x^2+1} - 36a\sqrt{c^2x^2+1} + 180ac^2x^2\log(x) - 180ac^2x^2\log\left(\pi\left(\sqrt{c^2x^2+1}+1\right)\right) + 24ac^4x^4\sqrt{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] (Pi^(5/2)*(-168*b*c^3*x^3 - 8*b*c^5*x^5 - 36*a*Sqrt[1 + c^2*x^2] + 168*a*c^
2*x^2*Sqrt[1 + c^2*x^2] + 24*a*c^4*x^4*Sqrt[1 + c^2*x^2] + 168*b*c^2*x^2*Sq
rt[1 + c^2*x^2]*ArcSinh[c*x] + 24*b*c^4*x^4*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]
- 9*b*c^3*x^3*Csch[ArcSinh[c*x]/2]^2 - 9*b*c^2*x^2*ArcSinh[c*x]*Csch[ArcSin
h[c*x]/2]^2 + 180*b*c^2*x^2*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 180*b
*c^2*x^2*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 180*a*c^2*x^2*Log[x] - 1
80*a*c^2*x^2*Log[Pi*(1 + Sqrt[1 + c^2*x^2])] + 180*b*c^2*x^2*PolyLog[2, -E^
(-ArcSinh[c*x])] - 180*b*c^2*x^2*PolyLog[2, E^(-ArcSinh[c*x])] + 36*b*c*x*S
inh[ArcSinh[c*x]/2]^2 - 36*b*ArcSinh[c*x]*Sinh[ArcSinh[c*x]/2]^2))/(72*x^2)
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\pi + \pi c^2x^2}\left(\pi^2ac^4x^4 + 2\,\pi^2ac^2x^2 + \pi^2a + \left(\pi^2bc^4x^4 + 2\,\pi^2bc^2x^2 + \pi^2b\right)\operatorname{arsinh}(cx)\right)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi^2*a*c^4*x^4 + 2*pi^2*a*c^2*x^2 + pi^2*a
+ (pi^2*b*c^4*x^4 + 2*pi^2*b*c^2*x^2 + pi^2*b)*arcsinh(c*x))/x^3, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.62, size = 356, normalized size = 1.74

$$-\frac{a\left(\pi c^{2}x^{2}+\pi\right)^{\frac{7}{2}}}{2\pi x^{2}}+\frac{a c^{2}\left(\pi c^{2}x^{2}+\pi\right)^{\frac{5}{2}}}{2}+\frac{5a c^{2}\pi\left(\pi c^{2}x^{2}+\pi\right)^{\frac{3}{2}}}{6}-\frac{5a c^{2}\pi^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^{2}x^{2}+\pi}}\right)}{2}+\frac{5a c^{2}\sqrt{\pi c^{2}x^{2}+\pi}\,\pi}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))/x^3,x)
```

```
[Out] -1/2*a/Pi/x^2*(Pi*c^2*x^2+Pi)^(7/2)+1/2*a*c^2*(Pi*c^2*x^2+Pi)^(5/2)+5/6*a*c
^2*Pi*(Pi*c^2*x^2+Pi)^(3/2)-5/2*a*c^2*Pi^(5/2)*arctanh(Pi^(1/2)/(Pi*c^2*x^2
+Pi)^(1/2))+5/2*a*c^2*(Pi*c^2*x^2+Pi)^(1/2)*Pi^2+5/2*b*c^2*Pi^(5/2)*polylog
(2,c*x+(c^2*x^2+1)^(1/2))-5/2*b*c^2*Pi^(5/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/
2))-1/9*b*c^5*Pi^(5/2)*x^3-7/3*b*c^3*Pi^(5/2)*x-1/2*b*Pi^(5/2)/(c^2*x^2+1)^
(1/2)*arcsinh(c*x)*c^2+1/3*b*Pi^(5/2)*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2*c^
4-1/2*b*c*Pi^(5/2)/x-1/2*b*Pi^(5/2)/(c^2*x^2+1)^(1/2)/x^2*arcsinh(c*x)+7/3*
b*Pi^(5/2)*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^2+5/2*b*c^2*Pi^(5/2)*arcsinh(c*
x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-5/2*b*c^2*Pi^(5/2)*arcsinh(c*x)*ln(1+c*x+(c^
2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{6}\left(15\,\pi^{\frac{5}{2}}c^{2}\operatorname{arsinh}\left(\frac{1}{c|x|}\right)-15\,\pi^{2}\sqrt{\pi+\pi c^{2}x^{2}}\,c^{2}-5\,\pi\left(\pi+\pi c^{2}x^{2}\right)^{\frac{3}{2}}c^{2}-3\left(\pi+\pi c^{2}x^{2}\right)^{\frac{5}{2}}c^{2}+\frac{3\left(\pi+\pi c^{2}x^{2}\right)^{\frac{7}{2}}}{\pi x^{2}}\right)a+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="maxima
")
```

```
[Out] -1/6*(15*pi^(5/2)*c^2*arcsinh(1/(c*abs(x))) - 15*pi^2*sqrt(pi + pi*c^2*x^2)
*c^2 - 5*pi*(pi + pi*c^2*x^2)^(3/2)*c^2 - 3*(pi + pi*c^2*x^2)^(5/2)*c^2 + 3
*(pi + pi*c^2*x^2)^(7/2)/(pi*x^2))*a + b*integrate((pi + pi*c^2*x^2)^(5/2)*
log(c*x + sqrt(c^2*x^2 + 1))/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\operatorname{asinh}(c\,x))\,\left(\Pi\,c^2\,x^2+\Pi\right)^{5/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2))/x^3,x)
```

```
[Out] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\pi^{\frac{5}{2}}\left(\int\frac{a\sqrt{c^{2}x^{2}+1}}{x^{3}}\,dx+\int\frac{2ac^{2}\sqrt{c^{2}x^{2}+1}}{x}\,dx+\int ac^{4}x\sqrt{c^{2}x^{2}+1}\,dx+\int\frac{b\sqrt{c^{2}x^{2}+1}\operatorname{asinh}(cx)}{x^{3}}\,dx+\int\frac{2bc^{2}\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x))/x**3,x)
```

```
[Out] pi**(5/2)*(Integral(a*sqrt(c**2*x**2 + 1)/x**3, x) + Integral(2*a*c**2*sqrt
(c**2*x**2 + 1)/x, x) + Integral(a*c**4*x*sqrt(c**2*x**2 + 1), x) + Integra
l(b*sqrt(c**2*x**2 + 1)*asinh(c*x)/x**3, x) + Integral(2*b*c**2*sqrt(c**2*x
**2 + 1)*asinh(c*x)/x, x) + Integral(b*c**4*x*sqrt(c**2*x**2 + 1)*asinh(c*x
), x))
```

3.78 $\int \frac{\left(\pi+c^2\pi x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=166

$$\frac{5\pi^{5/2}c^3\left(a+b\sinh^{-1}(cx)\right)^2}{4b}-\frac{5\pi c^2\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3x}-\frac{\left(\pi c^2x^2+\pi\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{3x^3}+\frac{5}{2}\pi^2c^4x$$

```
[Out] -1/6*b*c*Pi^(5/2)/x^2-1/4*b*c^5*Pi^(5/2)*x^2-5/3*c^2*Pi*(Pi*c^2*x^2+Pi)^(3/
2)*(a+b*arcsinh(c*x))/x-1/3*(Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))/x^3+5/
4*c^3*Pi^(5/2)*(a+b*arcsinh(c*x))^2/b+7/3*b*c^3*Pi^(5/2)*ln(x)+5/2*c^4*Pi^2
*x*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 266, normalized size of antiderivative = 1.60, number of steps used = 10, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5739, 5682, 5675, 30, 14, 266, 43}

$$\frac{5}{2}\pi^2c^4x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)+\frac{5\pi^2c^3\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)^2}{4b\sqrt{c^2x^2+1}}-\frac{5\pi c^2\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh\right.}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] -(b*c*Pi^2*Sqrt[Pi + c^2*Pi*x^2])/(6*x^2*Sqrt[1 + c^2*x^2]) - (b*c^5*Pi^2*x
^2*Sqrt[Pi + c^2*Pi*x^2])/(4*Sqrt[1 + c^2*x^2]) + (5*c^4*Pi^2*x*Sqrt[Pi + c
^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/2 - (5*c^2*Pi*(Pi + c^2*Pi*x^2)^(3/2)*(a +
 b*ArcSinh[c*x]))/(3*x) - ((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(3
*x^3) + (5*c^3*Pi^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*Sqrt
[1 + c^2*x^2]) + (7*b*c^3*Pi^2*Sqrt[Pi + c^2*Pi*x^2]*Log[x])/(3*Sqrt[1 + c^
2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
```

```
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(\pi + c^2\pi x^2\right)^{5/2}\left(a + b\sinh^{-1}(cx)\right)}{x^4}\,dx = -\frac{\left(\pi + c^2\pi x^2\right)^{5/2}\left(a + b\sinh^{-1}(cx)\right)}{3x^3} + \frac{1}{3}\left(5c^2\pi\right)\int \frac{\left(\pi + c^2\pi x^2\right)^{3/2}\left(\right.}{x}$$

$$= -\frac{5c^2\pi\left(\pi + c^2\pi x^2\right)^{3/2}\left(a + b\sinh^{-1}(cx)\right)}{3x} - \frac{\left(\pi + c^2\pi x^2\right)^{5/2}\left(a + b\,\mathrm{si}\right.}{3x^3}$$

$$= \frac{5}{2}c^4\pi^2x\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right) - \frac{5c^2\pi\left(\pi + c^2\pi x^2\right)^{3/2}\left(a + \right.}{3x}$$

$$= -\frac{bc\pi^2\sqrt{\pi + c^2\pi x^2}}{6x^2\sqrt{1 + c^2x^2}} - \frac{bc^5\pi^2x^2\sqrt{\pi + c^2\pi x^2}}{4\sqrt{1 + c^2x^2}} + \frac{5}{2}c^4\pi^2x\sqrt{\pi + c^2\pi x^2}$$

Mathematica [A] time = 0.38, size = 179, normalized size = 1.08

$$\frac{\pi^{5/2}\left(\sinh^{-1}(cx)\left(60ac^3x^3 + 6bc^3x^3\sinh\left(2\sinh^{-1}(cx)\right) - 8b\sqrt{c^2x^2+1}\left(7c^2x^2+1\right)\right) - 56ac^2x^2\sqrt{c^2x^2+1} - 8a\right.}{24}$$

Antiderivative was successfully verified.

```
[In] Integrate[((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] (Pi^(5/2)*(-4*b*c*x - 8*a*Sqrt[1 + c^2*x^2] - 56*a*c^2*x^2*Sqrt[1 + c^2*x^2
] + 12*a*c^4*x^4*Sqrt[1 + c^2*x^2] + 30*b*c^3*x^3*ArcSinh[c*x]^2 - 3*b*c^3*
x^3*Cosh[2*ArcSinh[c*x]] + 56*b*c^3*x^3*Log[c*x] + ArcSinh[c*x]*(60*a*c^3*x
^3 - 8*b*Sqrt[1 + c^2*x^2]*(1 + 7*c^2*x^2) + 6*b*c^3*x^3*Sinh[2*ArcSinh[c*x
]])))/(24*x^3)
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{\pi + \pi c^2x^2}\left(\pi^2ac^4x^4 + 2\,\pi^2ac^2x^2 + \pi^2a + \left(\pi^2bc^4x^4 + 2\,\pi^2bc^2x^2 + \pi^2b\right)\mathrm{arsinh}\,(cx)\right)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi^2*a*c^4*x^4 + 2*pi^2*a*c^2*x^2 + pi^2*a
+ (pi^2*b*c^4*x^4 + 2*pi^2*b*c^2*x^2 + pi^2*b)*arcsinh(c*x))/x^4, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.35, size = 692, normalized size = 4.17

$$-\frac{a\left(\pi c^2x^2+\pi\right)^{\frac{7}{2}}}{3\pi x^3}-\frac{4a c^2\left(\pi c^2x^2+\pi\right)^{\frac{7}{2}}}{3\pi x}+\frac{4a c^4x\left(\pi c^2x^2+\pi\right)^{\frac{5}{2}}}{3}+\frac{5a c^4\pi x\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}}{3}+\frac{5a c^4\pi^2x\sqrt{\pi c^2x^2+\pi}}{2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))/x^4,x)
```

```
[Out] -1/3*a/Pi/x^3*(Pi*c^2*x^2+Pi)^(7/2)-4/3*a*c^2/Pi/x*(Pi*c^2*x^2+Pi)^(7/2)+4/
3*a*c^4*x*(Pi*c^2*x^2+Pi)^(5/2)+5/3*a*c^4*Pi*x*(Pi*c^2*x^2+Pi)^(3/2)+5/2*a*
c^4*Pi^2*x*(Pi*c^2*x^2+Pi)^(1/2)+5/2*a*c^4*Pi^3*ln(Pi*x*c^2/(Pi*c^2)^(1/2)+
(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)-147*b*Pi^(5/2)/(63*c^4*x^4+15*c^2*x^2
+1)*x^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^6-56*b*Pi^(5/2)/(63*c^4*x^4+15*c^2
*x^2+1)*x*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^4-22/3*b*Pi^(5/2)/(63*c^4*x^4+15
*c^2*x^2+1)/x*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^2-1/4*b*c^5*Pi^(5/2)*x^2+7/3
*b*c^3*Pi^(5/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)-1/8*b*Pi^(5/2)*c^3+49/6*b*P
i^(5/2)/(63*c^4*x^4+15*c^2*x^2+1)*x^4*c^7+5/4*b*c^3*Pi^(5/2)*arcsinh(c*x)^2
-14/3*b*c^3*Pi^(5/2)*arcsinh(c*x)+147*b*Pi^(5/2)/(63*c^4*x^4+15*c^2*x^2+1)*
x^4*arcsinh(c*x)*c^7-49/6*b*Pi^(5/2)/(63*c^4*x^4+15*c^2*x^2+1)*x^2*(c^2*x^2
+1)*c^5+35*b*Pi^(5/2)/(63*c^4*x^4+15*c^2*x^2+1)*x^2*arcsinh(c*x)*c^5-7/3*b*
Pi^(5/2)/(63*c^4*x^4+15*c^2*x^2+1)*(c^2*x^2+1)*c^3+7/3*b*Pi^(5/2)/(63*c^4*x
^4+15*c^2*x^2+1)*arcsinh(c*x)*c^3-1/6*b*Pi^(5/2)/(63*c^4*x^4+15*c^2*x^2+1)/
x^2*(c^2*x^2+1)*c-1/3*b*Pi^(5/2)/(63*c^4*x^4+15*c^2*x^2+1)/x^3*arcsinh(c*x)
*(c^2*x^2+1)^(1/2)+1/2*b*arcsinh(c*x)*Pi^(5/2)*(c^2*x^2+1)^(1/2)*x*c^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))\,\left(\Pi\,c^2\,x^2+\Pi\right)^{5/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2))/x^4,x)
```

```
[Out] int(((a + b*asinh(c*x))*(Pi + Pi*c^2*x^2)^(5/2))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\pi^{\frac{5}{2}}\left(\int ac^4\sqrt{c^2x^2+1}\,dx+\int\frac{a\sqrt{c^2x^2+1}}{x^4}\,dx+\int\frac{2ac^2\sqrt{c^2x^2+1}}{x^2}\,dx+\int bc^4\sqrt{c^2x^2+1}\,\operatorname{asinh}(cx)\,dx+\int\frac{b\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x))/x**4,x)
```

```
[Out] pi**(5/2)*(Integral(a*c**4*sqrt(c**2*x**2 + 1), x) + Integral(a*sqrt(c**2*x
**2 + 1)/x**4, x) + Integral(2*a*c**2*sqrt(c**2*x**2 + 1)/x**2, x) + Integr
al(b*c**4*sqrt(c**2*x**2 + 1)*asinh(c*x), x) + Integral(b*sqrt(c**2*x**2 +
1)*asinh(c*x)/x**4, x) + Integral(2*b*c**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/x
**2, x))
```

3.79 $\int \sqrt{1+x^2} \sinh^{-1}(x) \, dx$

Optimal. Leaf size=32

$$-\frac{x^2}{4}+\frac{1}{2}\sqrt{x^2+1}\,x\sinh^{-1}(x)+\frac{1}{4}\sinh^{-1}(x)^2$$

```
[Out] -1/4*x^2+1/4*arcsinh(x)^2+1/2*x*arcsinh(x)*(x^2+1)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5682, 5675, 30}

$$-\frac{x^2}{4}+\frac{1}{2}\sqrt{x^2+1}\,x\sinh^{-1}(x)+\frac{1}{4}\sinh^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2]*ArcSinh[x],x]
```

```
[Out] -x^2/4 + (x*Sqrt[1 + x^2]*ArcSinh[x])/2 + ArcSinh[x]^2/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1+x^2}\sinh^{-1}(x)\,dx &= \frac{1}{2}x\sqrt{1+x^2}\sinh^{-1}(x)-\frac{\int x\,dx}{2}+\frac{1}{2}\int \frac{\sinh^{-1}(x)}{\sqrt{1+x^2}}\,dx\\ &= -\frac{x^2}{4}+\frac{1}{2}x\sqrt{1+x^2}\sinh^{-1}(x)+\frac{1}{4}\sinh^{-1}(x)^2\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.88

$$\frac{1}{4}\left(-x^2+2\sqrt{x^2+1}\,x\sinh^{-1}(x)+\sinh^{-1}(x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2]*ArcSinh[x],x]
```

```
[Out] (-x^2 + 2*x*Sqrt[1 + x^2]*ArcSinh[x] + ArcSinh[x]^2)/4
```

fricas [A] time = 0.68, size = 40, normalized size = 1.25

$$\frac{1}{2}\sqrt{x^2+1}\,x\log\left(x+\sqrt{x^2+1}\right)-\frac{1}{4}\,x^2+\frac{1}{4}\,\log\left(x+\sqrt{x^2+1}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x)*(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(x^2 + 1)*x*log(x + sqrt(x^2 + 1)) - 1/4*x^2 + 1/4*log(x + sqrt(x^2
 + 1))^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^2+1}\,\operatorname{arsinh}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x)*(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 1)*arcsinh(x), x)
```

maple [A] time = 0.04, size = 26, normalized size = 0.81

$$\frac{x\operatorname{arcsinh}(x)\sqrt{x^2+1}}{2}+\frac{\operatorname{arcsinh}(x)^2}{4}-\frac{x^2}{4}-\frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(x)*(x^2+1)^(1/2),x)
```

```
[Out] 1/2*x*arcsinh(x)*(x^2+1)^(1/2)+1/4*arcsinh(x)^2-1/4*x^2-1/4
```

maxima [A] time = 0.61, size = 28, normalized size = 0.88

$$-\frac{1}{4}\,x^2+\frac{1}{2}\left(\sqrt{x^2+1}\,x+\operatorname{arsinh}(x)\right)\operatorname{arsinh}(x)-\frac{1}{4}\,\operatorname{arsinh}(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x)*(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*x^2 + 1/2*(sqrt(x^2 + 1)*x + arcsinh(x))*arcsinh(x) - 1/4*arcsinh(x)^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \operatorname{asinh}(x)\,\sqrt{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(x)*(x^2 + 1)^(1/2),x)
```

```
[Out] int(asinh(x)*(x^2 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^2+1}\,\operatorname{asinh}{\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x)*(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + 1)*asinh(x), x)
```

3.80 $\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal. Leaf size=149

$$\frac{x^4\sqrt{\pi c^2x^2+\pi}\,\left(a+b\sinh^{-1}(cx)\right)}{5\pi c^2}+\frac{8\sqrt{\pi c^2x^2+\pi}\,\left(a+b\sinh^{-1}(cx)\right)}{15\pi c^6}-\frac{4x^2\sqrt{\pi c^2x^2+\pi}\,\left(a+b\sinh^{-1}(cx)\right)}{15\pi c^4}-\frac{8bx}{15\sqrt{\pi}}$$

```
[Out] -8/15*b*x/c^5/Pi^(1/2)+4/45*b*x^3/c^3/Pi^(1/2)-1/25*b*x^5/c/Pi^(1/2)+8/15*(
a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/c^6/Pi-4/15*x^2*(a+b*arcsinh(c*x))*
(Pi*c^2*x^2+Pi)^(1/2)/c^4/Pi+1/5*x^4*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/
2)/c^2/Pi
```

Rubi [A] time = 0.26, antiderivative size = 215, normalized size of antiderivative = 1.44, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5758, 5717, 8, 30}

$$\frac{x^4\sqrt{\pi c^2x^2+\pi}\,\left(a+b\sinh^{-1}(cx)\right)}{5\pi c^2}-\frac{4x^2\sqrt{\pi c^2x^2+\pi}\,\left(a+b\sinh^{-1}(cx)\right)}{15\pi c^4}+\frac{8\sqrt{\pi c^2x^2+\pi}\,\left(a+b\sinh^{-1}(cx)\right)}{15\pi c^6}-\frac{bx^5}{25c\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] (-8*b*x*Sqrt[1 + c^2*x^2])/(15*c^5*Sqrt[Pi + c^2*Pi*x^2]) + (4*b*x^3*Sqrt[1
 + c^2*x^2])/(45*c^3*Sqrt[Pi + c^2*Pi*x^2]) - (b*x^5*Sqrt[1 + c^2*x^2])/(25
*c*Sqrt[Pi + c^2*Pi*x^2]) + (8*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/
(15*c^6*Pi) - (4*x^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(15*c^4*Pi
) + (x^4*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(5*c^2*Pi)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{x^4\sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{5c^2\pi} - \frac{4\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi+c^2\pi x^2}}dx}{5c^2} - \frac{\left(b\sqrt{1+c^2x^2}\right)\int x}{5c\sqrt{\pi+c^2\pi x^2}}$$

$$= -\frac{bx^5\sqrt{1+c^2x^2}}{25c\sqrt{\pi+c^2\pi x^2}} - \frac{4x^2\sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{15c^4\pi} + \frac{x^4\sqrt{\pi+c^2\pi x^2}\,(a+}{5c^2\pi}$$

$$= \frac{4bx^3\sqrt{1+c^2x^2}}{45c^3\sqrt{\pi+c^2\pi x^2}} - \frac{bx^5\sqrt{1+c^2x^2}}{25c\sqrt{\pi+c^2\pi x^2}} + \frac{8\sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{15c^6\pi} - \frac{4x^2}{}$$

$$= -\frac{8bx\sqrt{1+c^2x^2}}{15c^5\sqrt{\pi+c^2\pi x^2}} + \frac{4bx^3\sqrt{1+c^2x^2}}{45c^3\sqrt{\pi+c^2\pi x^2}} - \frac{bx^5\sqrt{1+c^2x^2}}{25c\sqrt{\pi+c^2\pi x^2}} + \frac{8\sqrt{\pi+c^2\pi x^2}\,(a-}{15c^6}$$

Mathematica [A] time = 0.18, size = 108, normalized size = 0.72

$$\frac{15a\sqrt{c^2x^2+1}\,\left(3c^4x^4-4c^2x^2+8\right)+b\left(-9c^5x^5+20c^3x^3-120cx\right)+15b\sqrt{c^2x^2+1}\,\left(3c^4x^4-4c^2x^2+8\right)\sinh^{-1}}{225\sqrt{\pi}\,c^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] (15*a*Sqrt[1 + c^2*x^2]*(8 - 4*c^2*x^2 + 3*c^4*x^4) + b*(-120*c*x + 20*c^3*
x^3 - 9*c^5*x^5) + 15*b*Sqrt[1 + c^2*x^2]*(8 - 4*c^2*x^2 + 3*c^4*x^4)*ArcSi
nh[c*x])/(225*c^6*Sqrt[Pi])
```

fricas [A] time = 0.63, size = 161, normalized size = 1.08

$$\frac{15\sqrt{\pi+\pi c^2x^2}\left(3\,bc^6x^6-bc^4x^4+4\,bc^2x^2+8\,b\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{\pi+\pi c^2x^2}\left(45\,ac^6x^6-15\,ac^4x^4+6\right.}{225\left(\pi c^8x^2+\pi c^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/225*(15*sqrt(pi + pi*c^2*x^2)*(3*b*c^6*x^6 - b*c^4*x^4 + 4*b*c^2*x^2 + 8*
b)*log(c*x + sqrt(c^2*x^2 + 1)) + sqrt(pi + pi*c^2*x^2)*(45*a*c^6*x^6 - 15*
a*c^4*x^4 + 60*a*c^2*x^2 - (9*b*c^5*x^5 - 20*b*c^3*x^3 + 120*b*c*x)*sqrt(c^
2*x^2 + 1) + 120*a))/(pi*c^8*x^2 + pi*c^6)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.12, size = 193, normalized size = 1.30

$$a\left(\frac{x^4\sqrt{\pi c^2x^2+\pi}}{5\pi c^2}-\frac{4\left(\frac{x^2\sqrt{\pi c^2x^2+\pi}}{3\pi c^2}-\frac{2\sqrt{\pi c^2x^2+\pi}}{3\pi c^4}\right)}{5c^2}\right)+\frac{b\left(45\operatorname{arcsinh}(cx)\,c^6x^6-15\operatorname{arcsinh}(cx)\,c^4x^4-9c^5x^5\sqrt{c^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(1/2),x)
```

```
[Out] a*(1/5*x^4/Pi/c^2*(Pi*c^2*x^2+Pi)^(1/2)-4/5/c^2*(1/3*x^2/Pi/c^2*(Pi*c^2*x^2
+Pi)^(1/2)-2/3/Pi/c^4*(Pi*c^2*x^2+Pi)^(1/2)))+1/225*b/c^6/Pi^(1/2)/(c^2*x^2
+1)^(1/2)*(45*arcsinh(c*x)*c^6*x^6-15*arcsinh(c*x)*c^4*x^4-9*c^5*x^5*(c^2*x
^2+1)^(1/2)+60*arcsinh(c*x)*c^2*x^2+20*c^3*x^3*(c^2*x^2+1)^(1/2)+120*arcsin
h(c*x)-120*c*x*(c^2*x^2+1)^(1/2))
```

maxima [A] time = 0.44, size = 174, normalized size = 1.17

$$\frac{1}{15}\left(\frac{3\sqrt{\pi+\pi c^2x^2}\,x^4}{\pi c^2}-\frac{4\sqrt{\pi+\pi c^2x^2}\,x^2}{\pi c^4}+\frac{8\sqrt{\pi+\pi c^2x^2}}{\pi c^6}\right)b\operatorname{arsinh}(cx)+\frac{1}{15}\left(\frac{3\sqrt{\pi+\pi c^2x^2}\,x^4}{\pi c^2}-\frac{4\sqrt{\pi+\pi c^2x^2}}{\pi c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/15*(3*sqrt(pi + pi*c^2*x^2)*x^4/(pi*c^2) - 4*sqrt(pi + pi*c^2*x^2)*x^2/(p
i*c^4) + 8*sqrt(pi + pi*c^2*x^2)/(pi*c^6))*b*arcsinh(c*x) + 1/15*(3*sqrt(pi
 + pi*c^2*x^2)*x^4/(pi*c^2) - 4*sqrt(pi + pi*c^2*x^2)*x^2/(pi*c^4) + 8*sqrt
(pi + pi*c^2*x^2)/(pi*c^6))*a - 1/225*(9*c^4*x^5 - 20*c^2*x^3 + 120*x)*b/(s
qrt(pi)*c^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{\sqrt{\Pi\,c^2\,x^2+\Pi}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(1/2),x)
```

```
[Out] int((x^5*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(1/2), x)
```

sympy [A] time = 10.09, size = 182, normalized size = 1.22

$$\frac{a\left(\begin{cases}\frac{x^4\sqrt{c^2x^2+1}}{5c^2}-\frac{4x^2\sqrt{c^2x^2+1}}{15c^4}+\frac{8\sqrt{c^2x^2+1}}{15c^6} & \text{for}\ c\neq 0\\ \frac{x^6}{6} & \text{otherwise}\end{cases}\right)}{\sqrt{\pi}}+\frac{b\left(\begin{cases}-\frac{x^5}{25c}+\frac{x^4\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{5c^2}+\frac{4x^3}{45c^3}-\frac{4x^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{15c^4}- \\ 0\end{cases}\right.}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] a*Piecewise((x**4*sqrt(c**2*x**2 + 1)/(5*c**2) - 4*x**2*sqrt(c**2*x**2 + 1)
/(15*c**4) + 8*sqrt(c**2*x**2 + 1)/(15*c**6), Ne(c, 0)), (x**6/6, True))/sq
rt(pi) + b*Piecewise((-x**5/(25*c) + x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/(5
*c**2) + 4*x**3/(45*c**3) - 4*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(15*c**4)
 - 8*x/(15*c**5) + 8*sqrt(c**2*x**2 + 1)*asinh(c*x)/(15*c**6), Ne(c, 0)), (
0, True))/sqrt(pi)
```

3.81 $\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal. Leaf size=126

$$\frac{3\left(a+b\sinh^{-1}(cx)\right)^2}{16\sqrt{\pi}\,bc^5}+\frac{x^3\sqrt{\pi c^2x^2+\pi}\,\left(a+b\sinh^{-1}(cx)\right)}{4\pi c^2}-\frac{3x\sqrt{\pi c^2x^2+\pi}\,\left(a+b\sinh^{-1}(cx)\right)}{8\pi c^4}+\frac{3bx^2}{16\sqrt{\pi}\,c^3}-\frac{bx^{\cdots}}{16\sqrt{\cdots}}$$

```
[Out] 3/16*b*x^2/c^3/Pi^(1/2)-1/16*b*x^4/c/Pi^(1/2)+3/16*(a+b*arcsinh(c*x))^2/b/c
^5/Pi^(1/2)-3/8*x*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/c^4/Pi+1/4*x^3*(
a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/c^2/Pi
```

Rubi [A] time = 0.23, antiderivative size = 170, normalized size of antiderivative = 1.35, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {5758, 5675, 30}

$$\frac{x^3\sqrt{\pi c^2x^2+\pi}\,\left(a+b\sinh^{-1}(cx)\right)}{4\pi c^2}-\frac{3x\sqrt{\pi c^2x^2+\pi}\,\left(a+b\sinh^{-1}(cx)\right)}{8\pi c^4}+\frac{3\left(a+b\sinh^{-1}(cx)\right)^2}{16\sqrt{\pi}\,bc^5}-\frac{bx^4\sqrt{c^2x^2+1}}{16c\sqrt{\pi c^2x^2+\pi\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] (3*b*x^2*Sqrt[1 + c^2*x^2])/(16*c^3*Sqrt[Pi + c^2*Pi*x^2]) - (b*x^4*Sqrt[1
+ c^2*x^2])/(16*c*Sqrt[Pi + c^2*Pi*x^2]) - (3*x*Sqrt[Pi + c^2*Pi*x^2]*(a +
b*ArcSinh[c*x]))/(8*c^4*Pi) + (x^3*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x
]))/(4*c^2*Pi) + (3*(a + b*ArcSinh[c*x])^2)/(16*b*c^5*Sqrt[Pi])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^4 \left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{x^3\sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{4c^2\pi} - \frac{3\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi+c^2\pi x^2}}dx}{4c^2} - \frac{\left(b\sqrt{1+c^2x^2}\right)\int x^3\,dx}{4c\sqrt{\pi+c^2\pi x^2}}$$

$$= -\frac{bx^4\sqrt{1+c^2x^2}}{16c\sqrt{\pi+c^2\pi x^2}} - \frac{3x\sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{8c^4\pi} + \frac{x^3\sqrt{\pi+c^2\pi x^2}\,\left(a+b\sin\right.}{4c^2\pi}$$

$$= \frac{3bx^2\sqrt{1+c^2x^2}}{16c^3\sqrt{\pi+c^2\pi x^2}} - \frac{bx^4\sqrt{1+c^2x^2}}{16c\sqrt{\pi+c^2\pi x^2}} - \frac{3x\sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{8c^4\pi} + \frac{x^3\sqrt{\pi}}{}$$

Mathematica [A] time = 0.23, size = 111, normalized size = 0.88

$$\frac{4\sinh^{-1}(cx)\left(12a-8b\sinh\left(2\sinh^{-1}(cx)\right)+b\sinh\left(4\sinh^{-1}(cx)\right)\right)-48acx\sqrt{c^2x^2+1}+32ac^3x^3\sqrt{c^2x^2+1}+24}{128\sqrt{\pi}\,c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] (-48*a*c*x*Sqrt[1 + c^2*x^2] + 32*a*c^3*x^3*Sqrt[1 + c^2*x^2] + 24*b*ArcSin
h[c*x]^2 + 16*b*Cosh[2*ArcSinh[c*x]] - b*Cosh[4*ArcSinh[c*x]] + 4*ArcSinh[c
*x]*(12*a - 8*b*Sinh[2*ArcSinh[c*x]] + b*Sinh[4*ArcSinh[c*x]]))/(128*c^5*Sq
rt[Pi])
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^4\,\text{arsinh}\,(cx)+ax^4}{\sqrt{\pi+\pi c^2x^2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral((b*x^4*arcsinh(c*x) + a*x^4)/sqrt(pi + pi*c^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arsinh}\,(cx)+a)x^4}{\sqrt{\pi+\pi c^2x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^4/sqrt(pi + pi*c^2*x^2), x)
```

maple [A] time = 0.11, size = 188, normalized size = 1.49

$$\frac{a\,x^3\sqrt{\pi\,c^2x^2+\pi}}{4\pi\,c^2}-\frac{3ax\sqrt{\pi\,c^2x^2+\pi}}{8c^4\pi}+\frac{3a\ln\left(\frac{\pi x\,c^2}{\sqrt{\pi\,c^2}}+\sqrt{\pi\,c^2x^2+\pi}\right)}{8c^4\sqrt{\pi\,c^2}}+\frac{b\,\text{arcsinh}\,(cx)\,\sqrt{c^2x^2+1}\;x^3}{4c^2\sqrt{\pi}}-\frac{b\,x^4}{16c\sqrt{\pi}}-\frac{3b\,\text{ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(1/2),x)
```

```
[Out] 1/4*a*x^3/Pi/c^2*(Pi*c^2*x^2+Pi)^(1/2)-3/8*a/c^4*x/Pi*(Pi*c^2*x^2+Pi)^(1/2)
+3/8*a/c^4*ln(Pi*x*c^2/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)
```

```
+1/4*b/c^2/Pi^(1/2)*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^3-1/16*b*x^4/c/Pi^(1/2
)-3/8*b/c^4/Pi^(1/2)*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x+3/16*b*x^2/c^3/Pi^(1/
2)+3/16*b/c^5/Pi^(1/2)*arcsinh(c*x)^2+3/16*b/c^5/Pi^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{\sqrt{\Pi\,c^2\,x^2+\Pi}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(1/2),x)
```

```
[Out] int((x^4*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(1/2), x)
```

sympy [A] time = 9.02, size = 185, normalized size = 1.47

$$\frac{ax^5}{4\sqrt{\pi}\sqrt{c^2x^2+1}}-\frac{ax^3}{8\sqrt{\pi}\,c^2\sqrt{c^2x^2+1}}-\frac{3ax}{8\sqrt{\pi}\,c^4\sqrt{c^2x^2+1}}+\frac{3a\,\mathrm{asinh}\,(cx)}{8\sqrt{\pi}\,c^5}+\frac{b\left(\begin{cases}-\frac{x^4}{16c}+\frac{x^3\sqrt{c^2x^2+1}\,\mathrm{asinh}\,(cx)}{4c^2}+\frac{3x^2}{16c^3}- \\ 0\end{cases}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] a*x**5/(4*sqrt(pi)*sqrt(c**2*x**2 + 1)) - a*x**3/(8*sqrt(pi)*c**2*sqrt(c**2
*x**2 + 1)) - 3*a*x/(8*sqrt(pi)*c**4*sqrt(c**2*x**2 + 1)) + 3*a*asinh(c*x)/
(8*sqrt(pi)*c**5) + b*Piecewise((-x**4/(16*c) + x**3*sqrt(c**2*x**2 + 1)*as
inh(c*x)/(4*c**2) + 3*x**2/(16*c**3) - 3*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/(
8*c**4) + 3*asinh(c*x)**2/(16*c**5), Ne(c, 0)), (0, True))/sqrt(pi)
```

3.82 $\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal. Leaf size=98

$$\frac{x^2\sqrt{\pi c^2x^2+\pi}\,\left(a+b\sinh^{-1}(cx)\right)}{3\pi c^2}-\frac{2\sqrt{\pi c^2x^2+\pi}\,\left(a+b\sinh^{-1}(cx)\right)}{3\pi c^4}+\frac{2bx}{3\sqrt{\pi}\,c^3}-\frac{bx^3}{9\sqrt{\pi}\,c}$$

```
[Out] 2/3*b*x/c^3/Pi^(1/2)-1/9*b*x^3/c/Pi^(1/2)-2/3*(a+b*arcsinh(c*x))*(Pi*c^2*x^
2+Pi)^(1/2)/c^4/Pi+1/3*x^2*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/c^2/Pi
```

Rubi [A] time = 0.16, antiderivative size = 142, normalized size of antiderivative = 1.45, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5758, 5717, 8, 30}

$$\frac{x^2\sqrt{\pi c^2x^2+\pi}\,\left(a+b\sinh^{-1}(cx)\right)}{3\pi c^2}-\frac{2\sqrt{\pi c^2x^2+\pi}\,\left(a+b\sinh^{-1}(cx)\right)}{3\pi c^4}-\frac{bx^3\sqrt{c^2x^2+1}}{9c\sqrt{\pi c^2x^2+\pi}}+\frac{2bx\sqrt{c^2x^2+1}}{3c^3\sqrt{\pi c^2x^2+\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] (2*b*x*Sqrt[1 + c^2*x^2])/(3*c^3*Sqrt[Pi + c^2*Pi*x^2]) - (b*x^3*Sqrt[1 + c
^2*x^2])/(9*c*Sqrt[Pi + c^2*Pi*x^2]) - (2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcS
inh[c*x]))/(3*c^4*Pi) + (x^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(3
*c^2*Pi)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi+c^2\pi x^2}}dx = \frac{x^2\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{3c^2\pi} - \frac{2\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi+c^2\pi x^2}}dx}{3c^2} - \frac{\left(b\sqrt{1+c^2x^2}\right)\int x^2}{3c\sqrt{\pi+c^2\pi x^2}}$$
$$= -\frac{bx^3\sqrt{1+c^2x^2}}{9c\sqrt{\pi+c^2\pi x^2}} - \frac{2\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{3c^4\pi} + \frac{x^2\sqrt{\pi+c^2\pi x^2}\left(a+b\sin\right.}{3c^2\pi}$$
$$= \frac{2bx\sqrt{1+c^2x^2}}{3c^3\sqrt{\pi+c^2\pi x^2}} - \frac{bx^3\sqrt{1+c^2x^2}}{9c\sqrt{\pi+c^2\pi x^2}} - \frac{2\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{3c^4\pi} + \frac{x^2\sqrt{\pi}}{}$$

Mathematica [A] time = 0.14, size = 82, normalized size = 0.84

$$\frac{3a\sqrt{c^2x^2+1}\left(c^2x^2-2\right)+b\left(6cx-c^3x^3\right)+3b\sqrt{c^2x^2+1}\left(c^2x^2-2\right)\sinh^{-1}(cx)}{9\sqrt{\pi}\,c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] (3*a*(-2 + c^2*x^2)*Sqrt[1 + c^2*x^2] + b*(6*c*x - c^3*x^3) + 3*b*(-2 + c^2
*x^2)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(9*c^4*Sqrt[Pi])
```

fricas [A] time = 0.66, size = 132, normalized size = 1.35

$$\frac{3\sqrt{\pi+\pi c^2x^2}\left(bc^4x^4-bc^2x^2-2\,b\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{\pi+\pi c^2x^2}\left(3\,ac^4x^4-3\,ac^2x^2-\left(bc^3x^3-6\,bcx\right)\sqrt{\right.}{9\left(\pi c^6x^2+\pi c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/9*(3*sqrt(pi + pi*c^2*x^2)*(b*c^4*x^4 - b*c^2*x^2 - 2*b)*log(c*x + sqrt(c
^2*x^2 + 1)) + sqrt(pi + pi*c^2*x^2)*(3*a*c^4*x^4 - 3*a*c^2*x^2 - (b*c^3*x^
3 - 6*b*c*x)*sqrt(c^2*x^2 + 1) - 6*a))/(pi*c^6*x^2 + pi*c^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.11, size = 133, normalized size = 1.36

$$a\left(\frac{x^2\sqrt{\pi\,c^2x^2+\pi}}{3\pi\,c^2}-\frac{2\sqrt{\pi\,c^2x^2+\pi}}{3\pi\,c^4}\right)+\frac{b\left(3\,\mathrm{arcsinh}\,(cx)\,c^4x^4-3\,\mathrm{arcsinh}\,(cx)\,c^2x^2-c^3x^3\sqrt{c^2x^2+1}-6\,\mathrm{arcsin}\right.}{9c^4\sqrt{\pi}\,\sqrt{c^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(1/2),x)
```

```
[Out] a*(1/3*x^2/Pi/c^2*(Pi*c^2*x^2+Pi)^(1/2)-2/3/Pi/c^4*(Pi*c^2*x^2+Pi)^(1/2))+1
/9*b/c^4/Pi^(1/2)/(c^2*x^2+1)^(1/2)*(3*arcsinh(c*x)*c^4*x^4-3*arcsinh(c*x)*
c^2*x^2-c^3*x^3*(c^2*x^2+1)^(1/2)-6*arcsinh(c*x)+6*c*x*(c^2*x^2+1)^(1/2))
```

maxima [A] time = 0.38, size = 117, normalized size = 1.19

$$\frac{1}{3}\,b\left(\frac{\sqrt{\pi+\pi c^2x^2}\,x^2}{\pi c^2}-\frac{2\,\sqrt{\pi+\pi c^2x^2}}{\pi c^4}\right)\operatorname{arsinh}(cx)+\frac{1}{3}\,a\left(\frac{\sqrt{\pi+\pi c^2x^2}\,x^2}{\pi c^2}-\frac{2\,\sqrt{\pi+\pi c^2x^2}}{\pi c^4}\right)-\frac{\left(c^2x^3-6\,x\right)b}{9\,\sqrt{\pi}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/3*b*(sqrt(pi + pi*c^2*x^2)*x^2/(pi*c^2) - 2*sqrt(pi + pi*c^2*x^2)/(pi*c^4
))*arcsinh(c*x) + 1/3*a*(sqrt(pi + pi*c^2*x^2)*x^2/(pi*c^2) - 2*sqrt(pi + p
i*c^2*x^2)/(pi*c^4)) - 1/9*(c^2*x^3 - 6*x)*b/(sqrt(pi)*c^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{\sqrt{\Pi\,c^2\,x^2+\Pi}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(1/2),x)
```

```
[Out] int((x^3*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(1/2), x)
```

sympy [A] time = 3.94, size = 122, normalized size = 1.24

$$\frac{a\left(\begin{cases}\frac{x^2\sqrt{c^2x^2+1}}{3c^2}-\frac{2\sqrt{c^2x^2+1}}{3c^4} & \text{for}\: c\neq 0\\ \frac{x^4}{4} & \text{otherwise}\end{cases}\right)}{\sqrt{\pi}}+\frac{b\left(\begin{cases}-\frac{x^3}{9c}+\frac{x^2\sqrt{c^2x^2+1}\,\operatorname{asinh}(cx)}{3c^2}+\frac{2x}{3c^3}-\frac{2\sqrt{c^2x^2+1}\,\operatorname{asinh}(cx)}{3c^4} & \text{for}\: c\neq 0\\ 0 & \text{otherwise}\end{cases}\right)}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] a*Piecewise((x**2*sqrt(c**2*x**2 + 1)/(3*c**2) - 2*sqrt(c**2*x**2 + 1)/(3*c
**4), Ne(c, 0)), (x**4/4, True))/sqrt(pi) + b*Piecewise((-x**3/(9*c) + x**2
*sqrt(c**2*x**2 + 1)*asinh(c*x)/(3*c**2) + 2*x/(3*c**3) - 2*sqrt(c**2*x**2
+ 1)*asinh(c*x)/(3*c**4), Ne(c, 0)), (0, True))/sqrt(pi)
```

3.83 $\int \frac{x^2(a+b\sinh^{-1}(cx))}{\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal. Leaf size=75

$$-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4\sqrt{\pi}\,bc^3}+\frac{x\sqrt{\pi c^2x^2+\pi}\,\left(a+b\sinh^{-1}(cx)\right)}{2\pi c^2}-\frac{bx^2}{4\sqrt{\pi}\,c}$$

```
[Out] -1/4*b*x^2/c/Pi^(1/2)-1/4*(a+b*arcsinh(c*x))^2/b/c^3/Pi^(1/2)+1/2*x*(a+b*ar
csinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/c^2/Pi
```

Rubi [A] time = 0.12, antiderivative size = 97, normalized size of antiderivative = 1.29, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {5758, 5675, 30}

$$\frac{x\sqrt{\pi c^2x^2+\pi}\,\left(a+b\sinh^{-1}(cx)\right)}{2\pi c^2}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4\sqrt{\pi}\,bc^3}-\frac{bx^2\sqrt{c^2x^2+1}}{4c\sqrt{\pi c^2x^2+\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] -(b*x^2*Sqrt[1 + c^2*x^2])/(4*c*Sqrt[Pi + c^2*Pi*x^2]) + (x*Sqrt[Pi + c^2*P
i*x^2]*(a + b*ArcSinh[c*x]))/(2*c^2*Pi) - (a + b*ArcSinh[c*x])^2/(4*b*c^3*S
qrt[Pi])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi+c^2\pi x^2}}\,dx&=\frac{x\sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{2c^2\pi}-\frac{\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx}{2c^2}-\frac{\left(b\sqrt{1+c^2x^2}\right)\int x\,dx}{2c\sqrt{\pi+c^2\pi x^2}}\\&=-\frac{bx^2\sqrt{1+c^2x^2}}{4c\sqrt{\pi+c^2\pi x^2}}+\frac{x\sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{2c^2\pi}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4bc^3\sqrt{\pi}}\end{aligned}$$

Mathematica [A] time = 0.18, size = 69, normalized size = 0.92

$$\frac{\sinh^{-1}(cx)\left(2b\sinh\left(2\sinh^{-1}(cx)\right)-4a\right)+4acx\sqrt{c^2x^2+1}-2b\sinh^{-1}(cx)^2-b\cosh\left(2\sinh^{-1}(cx)\right)}{8\sqrt{\pi}c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] (4*a*c*x*Sqrt[1 + c^2*x^2] - 2*b*ArcSinh[c*x]^2 - b*Cosh[2*ArcSinh[c*x]] +
ArcSinh[c*x]*(-4*a + 2*b*Sinh[2*ArcSinh[c*x]]))/(8*c^3*Sqrt[Pi])
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^2\,\text{arsinh}\,(cx)+ax^2}{\sqrt{\pi+\pi c^2x^2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral((b*x^2*arcsinh(c*x) + a*x^2)/sqrt(pi + pi*c^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arsinh}\,(cx)+a)x^2}{\sqrt{\pi+\pi c^2x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^2/sqrt(pi + pi*c^2*x^2), x)
```

maple [A] time = 0.07, size = 125, normalized size = 1.67

$$\frac{ax\sqrt{\pi c^2x^2+\pi}}{2\pi c^2}-\frac{a\ln\left(\frac{\pi x c^2}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{2c^2\sqrt{\pi c^2}}+\frac{b\,\text{arcsinh}\,(cx)\sqrt{c^2x^2+1}\,x}{2c^2\sqrt{\pi}}-\frac{bx^2}{4c\sqrt{\pi}}-\frac{b\,\text{arcsinh}\,(cx)^2}{4c^3\sqrt{\pi}}-\frac{b}{4c^3\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(1/2),x)
```

```
[Out] 1/2*a*x/Pi/c^2*(Pi*c^2*x^2+Pi)^(1/2)-1/2*a/c^2*ln(Pi*x*c^2/(Pi*c^2)^(1/2)+(
Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/2*b/c^2/Pi^(1/2)*arcsinh(c*x)*(c^2*x
^2+1)^(1/2)*x-1/4*b*x^2/c/Pi^(1/2)-1/4*b/c^3/Pi^(1/2)*arcsinh(c*x)^2-1/4*b/
c^3/Pi^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{\sqrt{\Pi\,c^2\,x^2+\Pi}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(1/2),x)
[Out] int((x^2*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(1/2), x)
```

sympy [A] time = 4.15, size = 92, normalized size = 1.23

$$\frac{ax\sqrt{c^2x^2+1}}{2\sqrt{\pi}\,c^2}-\frac{a\,\mathrm{asinh}\,(cx)}{2\sqrt{\pi}\,c^3}+\frac{b\left(\begin{cases}-\frac{x^2}{4c}+\frac{x\sqrt{c^2x^2+1}\,\mathrm{asinh}\,(cx)}{2c^2}-\frac{\mathrm{asinh}^2\,(cx)}{4c^3} & \text{for } c\neq 0\\ 0 & \text{otherwise}\end{cases}\right)}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(1/2),x)
[Out] a*x*sqrt(c**2*x**2 + 1)/(2*sqrt(pi)*c**2) - a*asinh(c*x)/(2*sqrt(pi)*c**3)
+ b*Piecewise((-x**2/(4*c) + x*sqrt(c**2*x**2 + 1)*asinh(c*x)/(2*c**2) - as
inh(c*x)**2/(4*c**3), Ne(c, 0)), (0, True))/sqrt(pi)
```

3.84 $\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal. Leaf size=42

$$\frac{\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{\pi c^2}-\frac{bx}{\sqrt{\pi}\,c}$$

```
[Out] -b*x/c/Pi^(1/2)+(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/c^2/Pi
```

Rubi [A] time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.52, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5717, 8}

$$\frac{\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{\pi c^2}-\frac{bx\sqrt{c^2x^2+1}}{c\sqrt{\pi c^2x^2+\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] -((b*x*Sqrt[1 + c^2*x^2])/(c*Sqrt[Pi + c^2*Pi*x^2])) + (Sqrt[Pi + c^2*Pi*x^
2]*(a + b*ArcSinh[c*x]))/(c^2*Pi)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi+c^2\pi x^2}}\,dx &= \frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^2\pi}-\frac{\left(b\sqrt{1+c^2x^2}\right)\int 1\,dx}{c\sqrt{\pi+c^2\pi x^2}}\\ &= -\frac{bx\sqrt{1+c^2x^2}}{c\sqrt{\pi+c^2\pi x^2}}+\frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^2\pi}\end{aligned}$$

Mathematica [A] time = 0.08, size = 49, normalized size = 1.17

$$\frac{a\sqrt{c^2x^2+1}+b\sqrt{c^2x^2+1}\,\sinh^{-1}(cx)-bcx}{\sqrt{\pi}\,c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] (-(b*c*x) + a*Sqrt[1 + c^2*x^2] + b*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(c^2*Sq
rt[Pi])
```

fricas [B] time = 0.60, size = 96, normalized size = 2.29

$$\frac{\sqrt{\pi + \pi c^2 x^2}\left(bc^2x^2 + b\right)\log\left(cx + \sqrt{c^2x^2 + 1}\right) + \sqrt{\pi + \pi c^2 x^2}\left(ac^2x^2 - \sqrt{c^2x^2 + 1}\,bcx + a\right)}{\pi c^4 x^2 + \pi c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(pi + pi*c^2*x^2)*(b*c^2*x^2 + b)*log(c*x + sqrt(c^2*x^2 + 1)) + sqrt(
pi + pi*c^2*x^2)*(a*c^2*x^2 - sqrt(c^2*x^2 + 1)*b*c*x + a))/(pi*c^4*x^2 + p
i*c^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arsinh}\,(cx) + a)x}{\sqrt{\pi + \pi c^2 x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x/sqrt(pi + pi*c^2*x^2), x)
```

maple [A] time = 0.06, size = 72, normalized size = 1.71

$$\frac{a\sqrt{\pi\, c^2x^2 + \pi}}{\pi\, c^2} + \frac{b\left(\mathrm{arcsinh}\,(cx)\, c^2x^2 + \mathrm{arcsinh}\,(cx) - cx\sqrt{c^2x^2 + 1}\right)}{c^2\sqrt{\pi}\,\sqrt{c^2x^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(1/2),x)
```

```
[Out] a/Pi/c^2*(Pi*c^2*x^2+Pi)^(1/2)+b/c^2/Pi^(1/2)/(c^2*x^2+1)^(1/2)*(arcsinh(c*
x)*c^2*x^2+arcsinh(c*x)-c*x*(c^2*x^2+1)^(1/2))
```

maxima [A] time = 0.51, size = 55, normalized size = 1.31

$$-\frac{bx}{\sqrt{\pi}\,c} + \frac{\sqrt{\pi + \pi c^2 x^2}\, b\,\mathrm{arsinh}\,(cx)}{\pi c^2} + \frac{\sqrt{\pi + \pi c^2 x^2}\, a}{\pi c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima")
```

```
[Out] -b*x/(sqrt(pi)*c) + sqrt(pi + pi*c^2*x^2)*b*arcsinh(c*x)/(pi*c^2) + sqrt(pi
 + pi*c^2*x^2)*a/(pi*c^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\left(a + b\,\mathrm{asinh}\left(c\,x\right)\right)}{\sqrt{\Pi\,c^2\,x^2 + \Pi}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(1/2),x)
```

```
[Out] int((x*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(1/2), x)
```

sympy [**A**] time = 2.44, size = 60, normalized size = 1.43

$$\frac{a\left(\begin{cases} \frac{x^2}{2} & \text{for } c^2 = 0 \\ \frac{\sqrt{c^2x^2+1}}{c^2} & \text{otherwise} \end{cases}\right)}{\sqrt{\pi}} + \frac{b\left(\begin{cases} -\frac{x}{c} + \frac{\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{c^2} & \text{for } c \neq 0 \\ 0 & \text{otherwise} \end{cases}\right)}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] a*Piecewise((x**2/2, Eq(c**2, 0)), (sqrt(c**2*x**2 + 1)/c**2, True))/sqrt(p
i) + b*Piecewise((-x/c + sqrt(c**2*x**2 + 1)*asinh(c*x)/c**2, Ne(c, 0)), (0
, True))/sqrt(pi)
```

3.85 $\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal. Leaf size=25

$$\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2\sqrt{\pi}\,bc}$$

```
[Out] 1/2*(a+b*arcsinh(c*x))^2/b/c/Pi^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {5675}

$$\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2\sqrt{\pi}\,bc}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] (a + b*ArcSinh[c*x])^2/(2*b*c*Sqrt[Pi])
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx=\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{\pi}}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 1.00

$$\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2\sqrt{\pi}\,bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] (a + b*ArcSinh[c*x])^2/(2*b*c*Sqrt[Pi])
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{\pi+\pi c^2x^2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/sqrt(pi + pi*c^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\mathrm{arsinh}\,(cx) + a}{\sqrt{\pi + \pi c^2 x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/sqrt(pi + pi*c^2*x^2), x)
```

maple [B] time = 0.05, size = 53, normalized size = 2.12

$$\frac{a\ln\left(\frac{\pi x\,c^2}{\sqrt{\pi\,c^2}} + \sqrt{\pi\,c^2 x^2 + \pi}\right)}{\sqrt{\pi\,c^2}} + \frac{b\,\mathrm{arcsinh}\,(cx)^2}{2c\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(1/2),x)
```

```
[Out] a*ln(Pi*x*c^2/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/2*b/c/
Pi^(1/2)*arcsinh(c*x)^2
```

maxima [A] time = 0.50, size = 28, normalized size = 1.12

$$\frac{b\,\mathrm{arsinh}\,(cx)^2}{2\,\sqrt{\pi}\,c} + \frac{a\,\mathrm{arsinh}\,(cx)}{\sqrt{\pi}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*arcsinh(c*x)^2/(sqrt(pi)*c) + a*arcsinh(c*x)/(sqrt(pi)*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b\,\mathrm{asinh}\,(c\,x)}{\sqrt{\Pi\,c^2\,x^2 + \Pi}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(Pi + Pi*c^2*x^2)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))/(Pi + Pi*c^2*x^2)^(1/2), x)
```

sympy [A] time = 2.52, size = 85, normalized size = 3.40

$$\begin{cases} a\left(\begin{cases} \frac{\sqrt{-\frac{1}{c^2}}\,\mathrm{asin}\left(x\sqrt{-c^2}\right)}{\sqrt{\pi}} & \text{for}\ \pi c^2 < 0 \\ \frac{\sqrt{\frac{1}{c^2}}\,\mathrm{asinh}\left(x\sqrt{c^2}\right)}{\sqrt{\pi}} & \text{for}\ \pi c^2 > 0 \end{cases}\right) & \text{for}\ b = 0 \\ \frac{ax}{\sqrt{\pi}} & \text{for}\ c = 0 \\ \frac{(a + b\,\mathrm{asinh}\,(cx))^2}{2\sqrt{\pi}\,bc} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] Piecewise((a*Piecewise((sqrt(-1/c**2)*asin(x*sqrt(-c**2))/sqrt(pi), pi*c**2
 < 0), (sqrt(c**(-2))*asinh(x*sqrt(c**2))/sqrt(pi), pi*c**2 > 0)), Eq(b, 0)
), (a*x/sqrt(pi), Eq(c, 0)), ((a + b*asinh(c*x))**2/(2*sqrt(pi)*b*c), True)
)
```

3.86 $\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal. Leaf size=56

$$-\frac{2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi}}-\frac{b\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)}{\sqrt{\pi}}+\frac{b\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{\pi}}$$

```
[Out] -2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))/Pi^(1/2)-b*polylog(2,-
c*x-(c^2*x^2+1)^(1/2))/Pi^(1/2)+b*polylog(2,c*x+(c^2*x^2+1)^(1/2))/Pi^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5760, 4182, 2279, 2391}

$$-\frac{b\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{\sqrt{\pi}}+\frac{b\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{\sqrt{\pi}}-\frac{2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x*Sqrt[Pi + c^2*Pi*x^2]),x]
```

```
[Out] (-2*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[Pi] - (b*PolyLog[2,
-E^ArcSinh[c*x]])/Sqrt[Pi] + (b*PolyLog[2, E^ArcSinh[c*x]])/Sqrt[Pi]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{\text{Subst}\left(\int (a+bx)\text{csch}(x)\,dx, x, \sinh^{-1}(cx)\right)}{\sqrt{\pi}}$$

$$= -\frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{\pi}} - \frac{b\,\text{Subst}\left(\int \log\left(1-e^x\right)\,dx, x, \sinh^{-1}(cx)\right)}{\sqrt{\pi}}$$

$$= -\frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{\pi}} - \frac{b\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{\sinh^{-1}(cx)}\right)}{\sqrt{\pi}} + \frac{b\,\text{S}}{}$$

$$= -\frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{\pi}} - \frac{b\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)}{\sqrt{\pi}} + \frac{b\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{\pi}}$$

Mathematica [A] time = 0.16, size = 96, normalized size = 1.71

$$\frac{-a\log\left(\pi\left(\sqrt{c^2x^2+1}+1\right)\right)+a\log(x)+b\text{Li}_2\left(-e^{-\sinh^{-1}(cx)}\right)-b\text{Li}_2\left(e^{-\sinh^{-1}(cx)}\right)+b\sinh^{-1}(cx)\log\left(1-e^{-\sinh}\right.}{\sqrt{\pi}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x*Sqrt[Pi + c^2*Pi*x^2]),x]
```

```
[Out] (b*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - b*ArcSinh[c*x]*Log[1 + E^(-Arc
Sinh[c*x])] + a*Log[x] - a*Log[Pi*(1 + Sqrt[1 + c^2*x^2])] + b*PolyLog[2, -
E^(-ArcSinh[c*x])] - b*PolyLog[2, E^(-ArcSinh[c*x])])/Sqrt[Pi]
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\pi+\pi c^2x^2}\,(b\,\text{arsinh}\,(cx)+a)}{\pi c^2x^3+\pi x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi*c^2*x^3 + pi*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{\pi+\pi c^2x^2}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(pi + pi*c^2*x^2)*x), x)
```

maple [A] time = 0.04, size = 72, normalized size = 1.29

$$-\frac{a\,\text{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi\,c^2x^2+\pi}}\right)}{\sqrt{\pi}}+\frac{b\left(4\,\text{dilog}\left(\frac{1}{cx+\sqrt{c^2x^2+1}}\right)-\text{dilog}\left(\frac{1}{\left(cx+\sqrt{c^2x^2+1}\right)^2}\right)\right)}{2\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x/(Pi*c^2*x^2+Pi)^(1/2),x)
```

```
[Out] -a/Pi^(1/2)*arctanh(Pi^(1/2)/(Pi*c^2*x^2+Pi)^(1/2))+1/2*b*(4*dilog(1/(c*x+(
c^2*x^2+1)^(1/2)))-dilog(1/(c*x+(c^2*x^2+1)^(1/2))^2))/Pi^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{\sqrt{\pi+\pi c^2x^2}\,x}\,dx-\frac{a\,\mathrm{arsinh}\left(\frac{1}{c|x|}\right)}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima")
```

```
[Out] b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(sqrt(pi + pi*c^2*x^2)*x), x) - a*
arcsinh(1/(c*abs(x)))/sqrt(pi)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{a+b\,\mathrm{asinh}\left(c\,x\right)}{x\,\sqrt{\Pi\,c^2\,x^2+\Pi}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x*(Pi + Pi*c^2*x^2)^(1/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x*(Pi + Pi*c^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{a}{x\sqrt{c^2x^2+1}}\,dx+\int\frac{b\,\mathrm{asinh}\left(cx\right)}{x\sqrt{c^2x^2+1}}\,dx}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x/(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] (Integral(a/(x*sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(x*sqrt(c**
2*x**2 + 1)), x))/sqrt(pi)
```

3.87 $\int \frac{a+b\sinh^{-1}(cx)}{x^2\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal. Leaf size=41

$$\frac{bc\log(x)}{\sqrt{\pi}}-\frac{\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{\pi x}$$

```
[Out] b*c*ln(x)/Pi^(1/2)-(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/Pi/x
```

Rubi [A] time = 0.09, antiderivative size = 63, normalized size of antiderivative = 1.54, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5723, 29}

$$\frac{bc\sqrt{c^2x^2+1}\log(x)}{\sqrt{\pi c^2x^2+\pi}}-\frac{\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{\pi x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^2*Sqrt[Pi + c^2*Pi*x^2]),x]
```

```
[Out] -((Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(Pi*x)) + (b*c*Sqrt[1 + c^2*
x^2]*Log[x])/Sqrt[Pi + c^2*Pi*x^2]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sinh^{-1}(cx)}{x^2\sqrt{\pi+c^2\pi x^2}}\,dx &= -\frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{\pi x}+\frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{x}\,dx}{\sqrt{\pi+c^2\pi x^2}}\\ &= -\frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{\pi x}+\frac{bc\sqrt{1+c^2x^2}\log(x)}{\sqrt{\pi+c^2\pi x^2}}\end{aligned}$$

Mathematica [A] time = 0.11, size = 42, normalized size = 1.02

$$\frac{bc\log(x)}{\sqrt{\pi}}-\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^2*Sqrt[Pi + c^2*Pi*x^2]),x]
```

```
[Out] -((Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(Sqrt[Pi]*x)) + (b*c*Log[x])/Sqr
t[Pi]
```

fricas [B] time = 0.63, size = 132, normalized size = 3.22

$$\frac{\sqrt{\pi}\, bcx \log\left(\frac{\pi+\pi c^2x^6+\pi c^2x^2+\pi x^4+\sqrt{\pi}\,\sqrt{\pi+\pi c^2x^2}\,\sqrt{c^2x^2+1}\,(x^4-1)}{c^2x^4+x^2}\right) - 2\sqrt{\pi+\pi c^2x^2}\, b\log\left(cx+\sqrt{c^2x^2+1}\right) - 2\sqrt{\pi+\pi c^2x^2}}{2\,\pi x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/2*(sqrt(pi)*b*c*x*log((pi + pi*c^2*x^6 + pi*c^2*x^2 + pi*x^4 + sqrt(pi)*s
qrt(pi + pi*c^2*x^2)*sqrt(c^2*x^2 + 1)*(x^4 - 1))/(c^2*x^4 + x^2)) - 2*sqrt
(pi + pi*c^2*x^2)*b*log(c*x + sqrt(c^2*x^2 + 1)) - 2*sqrt(pi + pi*c^2*x^2)*
a)/(pi*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\sqrt{\pi + \pi c^2x^2}\, x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(pi + pi*c^2*x^2)*x^2), x)
```

maple [B] time = 0.09, size = 84, normalized size = 2.05

$$-\frac{a\sqrt{\pi\, c^2x^2+\pi}}{\pi x} - \frac{bc \operatorname{arcsinh}(cx)}{\sqrt{\pi}} - \frac{b \operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}}{\sqrt{\pi}\, x} + \frac{bc \ln\left(\left(cx+\sqrt{c^2x^2+1}\right)^2 - 1\right)}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^2/(Pi*c^2*x^2+Pi)^(1/2),x)
```

```
[Out] -a/Pi/x*(Pi*c^2*x^2+Pi)^(1/2)-b*c/Pi^(1/2)*arcsinh(c*x)-b/Pi^(1/2)*arcsinh(
c*x)/x*(c^2*x^2+1)^(1/2)+b*c/Pi^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)
```

maxima [B] time = 0.46, size = 101, normalized size = 2.46

$$-\frac{\left(\sqrt{\pi}\,(-1)^{2\pi+2\pi c^2x^2}\log\left(2\pi c^2 + \frac{2\pi}{x^2}\right) - \sqrt{\pi}\log\left(x^2+\frac{1}{c^2}\right)\right)bc}{2\pi} - \frac{\sqrt{\pi+\pi c^2x^2}\, b \operatorname{arsinh}(cx)}{\pi x} - \frac{\sqrt{\pi+\pi c^2x^2}\, a}{\pi x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/2*(sqrt(pi)*(-1)^(2*pi + 2*pi*c^2*x^2)*log(2*pi*c^2 + 2*pi/x^2) - sqrt(p
i)*log(x^2 + 1/c^2))*b*c/pi - sqrt(pi + pi*c^2*x^2)*b*arcsinh(c*x)/(pi*x) -
 sqrt(pi + pi*c^2*x^2)*a/(pi*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a + b \operatorname{asinh}(c\,x)}{x^2\sqrt{\Pi\, c^2\, x^2 + \Pi}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^2*(Pi + Pi*c^2*x^2)^(1/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^2*(Pi + Pi*c^2*x^2)^(1/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{x^2\sqrt{c^2x^2+1}}\,dx + \int \frac{b\,\mathrm{asinh}\,(cx)}{x^2\sqrt{c^2x^2+1}}\,dx}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**2/(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] (Integral(a/(x**2*sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(x**2*sq
rt(c**2*x**2 + 1)), x))/sqrt(pi)
```

3.88 $\int \frac{a+b\sinh^{-1}(cx)}{x^3\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal. Leaf size=115

$$-\frac{\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{2\pi x^2}+\frac{c^2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi}}+\frac{bc^2\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)}{2\sqrt{\pi}}-\frac{bc^2\text{Li}_2\left(e^{\sinh^{-1}\right.}}{2\sqrt{\pi}}$$

```
[Out] -1/2*b*c/x/Pi^(1/2)+c^2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))/P
i^(1/2)+1/2*b*c^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))/Pi^(1/2)-1/2*b*c^2*poly
log(2,c*x+(c^2*x^2+1)^(1/2))/Pi^(1/2)-1/2*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi
)^(1/2)/Pi/x^2
```

Rubi [A] time = 0.21, antiderivative size = 137, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5747, 5760, 4182, 2279, 2391, 30}

$$\frac{bc^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{2\sqrt{\pi}}-\frac{bc^2\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{2\sqrt{\pi}}-\frac{\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{2\pi x^2}+\frac{c^2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^3*Sqrt[Pi + c^2*Pi*x^2]),x]
```

```
[Out] -(b*c*Sqrt[1 + c^2*x^2])/(2*x*Sqrt[Pi + c^2*Pi*x^2]) - (Sqrt[Pi + c^2*Pi*x^
2]*(a + b*ArcSinh[c*x]))/(2*Pi*x^2) + (c^2*(a + b*ArcSinh[c*x])*ArcTanh[E^A
rcSinh[c*x]])/Sqrt[Pi] + (b*c^2*PolyLog[2, -E^ArcSinh[c*x]])/(2*Sqrt[Pi]) -
 (b*c^2*PolyLog[2, E^ArcSinh[c*x]])/(2*Sqrt[Pi])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
```

```
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^3\sqrt{\pi+c^2\pi x^2}}\,dx = -\frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{2\pi x^2} - \frac{1}{2}c^2\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{\pi+c^2\pi x^2}}\,dx + \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{x}}{2\sqrt{\pi+c^2\pi x^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2x\sqrt{\pi+c^2\pi x^2}} - \frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{2\pi x^2} - \frac{c^2\,\mathrm{Subst}\left(\int (a+bx)\mathrm{csch}(x)\,dx\right.}{2\sqrt{\pi}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2x\sqrt{\pi+c^2\pi x^2}} - \frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{2\pi x^2} + \frac{c^2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(\right.}{\sqrt{\pi}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2x\sqrt{\pi+c^2\pi x^2}} - \frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{2\pi x^2} + \frac{c^2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(\right.}{\sqrt{\pi}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2x\sqrt{\pi+c^2\pi x^2}} - \frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{2\pi x^2} + \frac{c^2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(\right.}{\sqrt{\pi}}$$

Mathematica [A] time = 2.59, size = 185, normalized size = 1.61

$$\frac{-\frac{4a\sqrt{c^2x^2+1}}{x^2} + 4ac^2\log\left(\pi\left(\sqrt{c^2x^2+1}+1\right)\right) - 4ac^2\log(x) + bc^2\left(-4\mathrm{Li}_2\left(-e^{-\sinh^{-1}(cx)}\right) + 4\mathrm{Li}_2\left(e^{-\sinh^{-1}(cx)}\right) - 4\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^3*Sqrt[Pi + c^2*Pi*x^2]),x]
```

```
[Out] ((-4*a*Sqrt[1 + c^2*x^2])/x^2 - 4*a*c^2*Log[x] + 4*a*c^2*Log[Pi*(1 + Sqrt[1
 + c^2*x^2])] + b*c^2*(-2*Coth[ArcSinh[c*x]/2] - ArcSinh[c*x]*Csch[ArcSinh[
c*x]/2]^2 - 4*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] + 4*ArcSinh[c*x]*Log[
1 + E^(-ArcSinh[c*x])] - 4*PolyLog[2, -E^(-ArcSinh[c*x])] + 4*PolyLog[2, E^
(-ArcSinh[c*x])] - ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSinh[c*x
]/2]))/(8*Sqrt[Pi])
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{\pi+\pi c^2x^2}\,(b\,\mathrm{arsinh}\,(cx)+a)}{\pi c^2x^5+\pi x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi*c^2*x^5 + pi*x^3),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\sqrt{\pi + \pi c^2 x^2}\, x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(pi + pi*c^2*x^2)*x^3), x)
```

maple [A] time = 0.19, size = 225, normalized size = 1.96

$$-\frac{a\sqrt{\pi c^2 x^2 + \pi}}{2\pi x^2} + \frac{a c^2 \operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2 x^2 + \pi}}\right)}{2\sqrt{\pi}} - \frac{b \operatorname{arcsinh}(cx) c^2}{2\sqrt{\pi}\sqrt{c^2 x^2 + 1}} - \frac{bc}{2x\sqrt{\pi}} - \frac{b \operatorname{arcsinh}(cx)}{2\sqrt{\pi}\sqrt{c^2 x^2 + 1}\, x^2} + \frac{b c^2 \operatorname{arcsinh}(cx) \ln\Big(}{2\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^3/(Pi*c^2*x^2+Pi)^(1/2),x)
```

```
[Out] -1/2*a/Pi/x^2*(Pi*c^2*x^2+Pi)^(1/2)+1/2*a/Pi^(1/2)*c^2*arctanh(Pi^(1/2)/(Pi
*c^2*x^2+Pi)^(1/2))-1/2*b/Pi^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^2-1/2*b
*c/x/Pi^(1/2)-1/2*b/Pi^(1/2)/(c^2*x^2+1)^(1/2)/x^2*arcsinh(c*x)+1/2*b*c^2/P
i^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+1/2*b*c^2*polylog(2,-c*x-(
c^2*x^2+1)^(1/2))/Pi^(1/2)-1/2*b*c^2/Pi^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^
2+1)^(1/2))-1/2*b*c^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))/Pi^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(\frac{c^2 \operatorname{arsinh}\left(\frac{1}{c|x|}\right)}{\sqrt{\pi}} - \frac{\sqrt{\pi + \pi c^2 x^2}}{\pi x^2}\right)a + b\int \frac{\log\left(cx + \sqrt{c^2 x^2 + 1}\right)}{\sqrt{\pi + \pi c^2 x^2}\, x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/2*(c^2*arcsinh(1/(c*abs(x)))/sqrt(pi) - sqrt(pi + pi*c^2*x^2)/(pi*x^2))*a
 + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(sqrt(pi + pi*c^2*x^2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{asinh}(c\,x)}{x^3\sqrt{\Pi\,c^2\,x^2 + \Pi}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^3*(Pi + Pi*c^2*x^2)^(1/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^3*(Pi + Pi*c^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{x^3\sqrt{c^2x^2+1}}\, dx + \int \frac{b \operatorname{asinh}(cx)}{x^3\sqrt{c^2x^2+1}}\, dx}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**3/(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] (Integral(a/(x**3*sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(x**3*sq
rt(c**2*x**2 + 1)), x))/sqrt(pi)
```

3.89 $\int \frac{a+b\sinh^{-1}(cx)}{x^4\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal. Leaf size=97

$$\frac{2c^2\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{3\pi x}-\frac{\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{3\pi x^3}-\frac{2bc^3\log(x)}{3\sqrt{\pi}}-\frac{bc}{6\sqrt{\pi}\,x^2}$$

```
[Out] -1/6*b*c/x^2/Pi^(1/2)-2/3*b*c^3*ln(x)/Pi^(1/2)-1/3*(a+b*arcsinh(c*x))*(Pi*c
^2*x^2+Pi)^(1/2)/Pi/x^3+2/3*c^2*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/Pi
/x
```

Rubi [A] time = 0.18, antiderivative size = 141, normalized size of antiderivative = 1.45, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5747, 5723, 29, 30}

$$\frac{2c^2\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{3\pi x}-\frac{\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{3\pi x^3}-\frac{bc\sqrt{c^2x^2+1}}{6x^2\sqrt{\pi c^2x^2+\pi}}-\frac{2bc^3\sqrt{c^2x^2+1}\log(x)}{3\sqrt{\pi c^2x^2+\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^4*Sqrt[Pi + c^2*Pi*x^2]),x]
```

```
[Out] -(b*c*Sqrt[1 + c^2*x^2])/(6*x^2*Sqrt[Pi + c^2*Pi*x^2]) - (Sqrt[Pi + c^2*Pi*
x^2]*(a + b*ArcSinh[c*x]))/(3*Pi*x^3) + (2*c^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b
*ArcSinh[c*x]))/(3*Pi*x) - (2*b*c^3*Sqrt[1 + c^2*x^2]*Log[x])/(3*Sqrt[Pi +
c^2*Pi*x^2])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^4\sqrt{\pi+c^2\pi x^2}}\,dx = -\frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{3\pi x^3} - \frac{1}{3}\left(2c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{x^2\sqrt{\pi+c^2\pi x^2}}\,dx + \frac{\left(bc\sqrt{1+c^2x^2}\right)}{3\sqrt{\pi+c^2\pi}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6x^2\sqrt{\pi+c^2\pi x^2}} - \frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{3\pi x^3} + \frac{2c^2\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-}\right.}{3\pi x}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6x^2\sqrt{\pi+c^2\pi x^2}} - \frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{3\pi x^3} + \frac{2c^2\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-}\right.}{3\pi x}$$

Mathematica [A] time = 0.16, size = 99, normalized size = 1.02

$$\frac{2a\sqrt{c^2x^2+1}\left(2c^2x^2-1\right)+bcx\left(6c^2x^2-1\right)+2b\sqrt{c^2x^2+1}\left(2c^2x^2-1\right)\sinh^{-1}(cx)}{6\sqrt{\pi}\,x^3}-\frac{2bc^3\log(x)}{3\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^4*Sqrt[Pi + c^2*Pi*x^2]),x]
```

```
[Out] (2*a*Sqrt[1 + c^2*x^2]*(-1 + 2*c^2*x^2) + b*c*x*(-1 + 6*c^2*x^2) + 2*b*Sqrt
[1 + c^2*x^2]*(-1 + 2*c^2*x^2)*ArcSinh[c*x])/(6*Sqrt[Pi]*x^3) - (2*b*c^3*Lo
g[x])/(3*Sqrt[Pi])
```

fricas [B] time = 0.58, size = 222, normalized size = 2.29

$$\frac{2\sqrt{\pi+\pi c^2x^2}\left(2\,bc^4x^4+bc^2x^2-b\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+2\sqrt{\pi}\left(bc^5x^5+bc^3x^3\right)\log\left(\frac{\pi+\pi c^2x^6+\pi c^2x^2+\pi x^4-\sqrt{\pi}\sqrt{}}{c^2x^4+}\right.}{6\left(\pi c^2x^5+\pi x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/6*(2*sqrt(pi + pi*c^2*x^2)*(2*b*c^4*x^4 + b*c^2*x^2 - b)*log(c*x + sqrt(c
^2*x^2 + 1)) + 2*sqrt(pi)*(b*c^5*x^5 + b*c^3*x^3)*log((pi + pi*c^2*x^6 + pi
*c^2*x^2 + pi*x^4 - sqrt(pi)*sqrt(pi + pi*c^2*x^2)*sqrt(c^2*x^2 + 1)*(x^4 -
 1))/(c^2*x^4 + x^2)) + sqrt(pi + pi*c^2*x^2)*(4*a*c^4*x^4 + 2*a*c^2*x^2 +
(b*c*x^3 - b*c*x)*sqrt(c^2*x^2 + 1) - 2*a))/(pi*c^2*x^5 + pi*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\operatorname{arsinh}(cx)+a}{\sqrt{\pi+\pi c^2x^2}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(pi + pi*c^2*x^2)*x^4), x)
```

maple [B] time = 0.12, size = 372, normalized size = 3.84

$$-\frac{a\sqrt{\pi\,c^2x^2+\pi}}{3\pi\,x^3}+\frac{2a\,c^2\sqrt{\pi\,c^2x^2+\pi}}{3\pi x}+\frac{4b\,c^3\,\operatorname{arcsinh}(cx)}{3\sqrt{\pi}}-\frac{2b\,x^4c^7}{3\sqrt{\pi}\,\left(3c^2x^2-1\right)}+\frac{2b\,x^2\left(c^2x^2+1\right)c^5}{3\sqrt{\pi}\,\left(3c^2x^2-1\right)}-\frac{2b\,x^2\,\operatorname{arcsin}}{\sqrt{\pi}\,\left(3c^2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^4/(Pi*c^2*x^2+Pi)^(1/2),x)
```

```
[Out] -1/3*a/Pi/x^3*(Pi*c^2*x^2+Pi)^(1/2)+2/3*a/Pi*c^2/x*(Pi*c^2*x^2+Pi)^(1/2)+4/
3*b*c^3/Pi^(1/2)*arcsinh(c*x)-2/3*b/Pi^(1/2)/(3*c^2*x^2-1)*x^4*c^7+2/3*b/Pi
^(1/2)/(3*c^2*x^2-1)*x^2*(c^2*x^2+1)*c^5-2*b/Pi^(1/2)/(3*c^2*x^2-1)*x^2*arc
sinh(c*x)*c^5+2*b/Pi^(1/2)/(3*c^2*x^2-1)*x*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c
^4-2/3*b/Pi^(1/2)/(3*c^2*x^2-1)*(c^2*x^2+1)*c^3+2/3*b/Pi^(1/2)/(3*c^2*x^2-1
)*arcsinh(c*x)*c^3-5/3*b/Pi^(1/2)/(3*c^2*x^2-1)/x*arcsinh(c*x)*(c^2*x^2+1)^
(1/2)*c^2+1/6*b/Pi^(1/2)/(3*c^2*x^2-1)/x^2*(c^2*x^2+1)*c+1/3*b/Pi^(1/2)/(3*
c^2*x^2-1)/x^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-2/3*b*c^3/Pi^(1/2)*ln((c*x+(c
^2*x^2+1)^(1/2))^2-1)
```

maxima [A] time = 0.45, size = 121, normalized size = 1.25

$$-\frac{1}{6}\left(\frac{4\,c^2\log(x)}{\sqrt{\pi}}+\frac{1}{\sqrt{\pi}\,x^2}\right)bc+\frac{1}{3}\,b\left(\frac{2\,\sqrt{\pi+\pi c^2x^2}\,c^2}{\pi x}-\frac{\sqrt{\pi+\pi c^2x^2}}{\pi x^3}\right)\text{arsinh}\,(cx)+\frac{1}{3}\,a\left(\frac{2\,\sqrt{\pi+\pi c^2x^2}\,c^2}{\pi x}-\frac{\sqrt{\pi+\pi c^2x^2}}{\pi x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/6*(4*c^2*log(x)/sqrt(pi) + 1/(sqrt(pi)*x^2))*b*c + 1/3*b*(2*sqrt(pi + pi
*c^2*x^2)*c^2/(pi*x) - sqrt(pi + pi*c^2*x^2)/(pi*x^3))*arcsinh(c*x) + 1/3*a
*(2*sqrt(pi + pi*c^2*x^2)*c^2/(pi*x) - sqrt(pi + pi*c^2*x^2)/(pi*x^3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\text{asinh}\,(c\,x)}{x^4\,\sqrt{\Pi\,c^2\,x^2+\Pi}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^4*(Pi + Pi*c^2*x^2)^(1/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^4*(Pi + Pi*c^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{x^4\sqrt{c^2x^2+1}}\,dx+\int \frac{b\,\text{asinh}\,(cx)}{x^4\sqrt{c^2x^2+1}}\,dx}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**4/(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] (Integral(a/(x**4*sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(x**4*sq
rt(c**2*x**2 + 1)), x))/sqrt(pi)
```

3.90 $\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=137

$$\frac{\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3\pi^3c^6}-\frac{2\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{\pi^2c^6}-\frac{a+b\sinh^{-1}(cx)}{\pi c^6\sqrt{\pi c^2x^2+\pi}}+\frac{b\tan^{-1}(cx)}{\pi^{3/2}c^6}+\frac{5bx}{3\pi^{3/2}c^5}$$

```
[Out] 5/3*b*x/c^5/Pi^(3/2)-1/9*b*x^3/c^3/Pi^(3/2)+1/3*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*
arcsinh(c*x))/c^6/Pi^3+b*arctan(c*x)/c^6/Pi^(3/2)+(-a-b*arcsinh(c*x))/c^6/P
i/(Pi*c^2*x^2+Pi)^(1/2)-2*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/c^6/Pi^2
```

Rubi [A] time = 0.17, antiderivative size = 140, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {266, 43, 5732, 1153, 205}

$$\frac{\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{3/2}c^6}-\frac{2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{\pi^{3/2}c^6}-\frac{a+b\sinh^{-1}(cx)}{\pi^{3/2}c^6\sqrt{c^2x^2+1}}-\frac{bx^3}{9\pi^{3/2}c^3}+\frac{5bx}{3\pi^{3/2}c^5}+\frac{b\tan^{-1}(cx)}{\pi^{3/2}c^6}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] (5*b*x)/(3*c^5*Pi^(3/2)) - (b*x^3)/(9*c^3*Pi^(3/2)) - (a + b*ArcSinh[c*x])/
(c^6*Pi^(3/2)*Sqrt[1 + c^2*x^2]) - (2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]
))/(c^6*Pi^(3/2)) + ((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c^6*Pi^(3
/2)) + (b*ArcTan[c*x])/(c^6*Pi^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
```

```
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{x^5 \left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{c^6\pi^{3/2}\sqrt{1+c^2x^2}} - \frac{2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^6\pi^{3/2}} + \frac{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(\right.}{3c^6\pi^{3/2}}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{c^6\pi^{3/2}\sqrt{1+c^2x^2}} - \frac{2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^6\pi^{3/2}} + \frac{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(\right.}{3c^6\pi^{3/2}}$$

$$= \frac{5bx}{3c^5\pi^{3/2}} - \frac{bx^3}{9c^3\pi^{3/2}} - \frac{a+b\sinh^{-1}(cx)}{c^6\pi^{3/2}\sqrt{1+c^2x^2}} - \frac{2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^6\pi^{3/2}} + \frac{\left(1+c^2x\right.}{}$$

$$= \frac{5bx}{3c^5\pi^{3/2}} - \frac{bx^3}{9c^3\pi^{3/2}} - \frac{a+b\sinh^{-1}(cx)}{c^6\pi^{3/2}\sqrt{1+c^2x^2}} - \frac{2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^6\pi^{3/2}} + \frac{\left(1+c^2x\right.}{}$$

Mathematica [A] time = 0.22, size = 131, normalized size = 0.96

$$\frac{3ac^4x^4 - 12ac^2x^2 - 24a + 15bcx\sqrt{c^2x^2+1} + 9b\sqrt{c^2x^2+1}\tan^{-1}(cx) + 3b\left(c^4x^4-4c^2x^2-8\right)\sinh^{-1}(cx) - bc^3x^3\sqrt{}}{9\pi^{3/2}c^6\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] (-24*a - 12*a*c^2*x^2 + 3*a*c^4*x^4 + 15*b*c*x*Sqrt[1 + c^2*x^2] - b*c^3*x^
3*Sqrt[1 + c^2*x^2] + 3*b*(-8 - 4*c^2*x^2 + c^4*x^4)*ArcSinh[c*x] + 9*b*Sqr
t[1 + c^2*x^2]*ArcTan[c*x])/(9*c^6*Pi^(3/2)*Sqrt[1 + c^2*x^2])
```

fricas [A] time = 0.68, size = 196, normalized size = 1.43

$$-\frac{9\sqrt{\pi}\left(bc^2x^2+b\right)\arctan\left(-\frac{2\sqrt{\pi}\sqrt{\pi+\pi c^2x^2}\sqrt{c^2x^2+1}\,cx}{\pi-\pi c^4x^4}\right) - 6\sqrt{\pi+\pi c^2x^2}\left(bc^4x^4-4bc^2x^2-8b\right)\log\left(cx+\sqrt{c^2x^2+1}\right.}{18\left(\pi^2c^8x^2+\pi^2c^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas
")
```

```
[Out] -1/18*(9*sqrt(pi)*(b*c^2*x^2 + b)*arctan(-2*sqrt(pi)*sqrt(pi + pi*c^2*x^2)*
sqrt(c^2*x^2 + 1)*c*x/(pi - pi*c^4*x^4)) - 6*sqrt(pi + pi*c^2*x^2)*(b*c^4*x
^4 - 4*b*c^2*x^2 - 8*b)*log(c*x + sqrt(c^2*x^2 + 1)) - 2*sqrt(pi + pi*c^2*x
^2)*(3*a*c^4*x^4 - 12*a*c^2*x^2 - (b*c^3*x^3 - 15*b*c*x)*sqrt(c^2*x^2 + 1)
- 24*a))/(pi^2*c^8*x^2 + pi^2*c^6)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [C] time = 0.32, size = 224, normalized size = 1.64

$$\frac{a\,x^4}{3\pi\,c^2\sqrt{\pi\,c^2x^2+\pi}}-\frac{4a\,x^2}{3c^4\pi\sqrt{\pi\,c^2x^2+\pi}}-\frac{8a}{3c^6\pi\sqrt{\pi\,c^2x^2+\pi}}-\frac{5b\,\mathrm{arcsinh}\,(cx)\,\sqrt{c^2x^2+1}}{3\pi^{\frac{3}{2}}c^6}-\frac{ib\ln\left(cx+\sqrt{c^2x^2+1}\right.}{c^6\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(3/2),x)
```

```
[Out] 1/3*a*x^4/Pi/c^2/(Pi*c^2*x^2+Pi)^(1/2)-4/3*a/c^4*x^2/Pi/(Pi*c^2*x^2+Pi)^(1/
2)-8/3*a/c^6/Pi/(Pi*c^2*x^2+Pi)^(1/2)-5/3*b/Pi^(3/2)/c^6*arcsinh(c*x)*(c^2*
x^2+1)^(1/2)-I*b/c^6/Pi^(3/2)*ln(c*x+(c^2*x^2+1)^(1/2)-I)+I*b/c^6/Pi^(3/2)*
ln(c*x+(c^2*x^2+1)^(1/2)+I)-b/Pi^(3/2)/(c^2*x^2+1)^(1/2)/c^6*arcsinh(c*x)+1
/3*b/Pi^(3/2)/c^4*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2-1/9*b*x^3/c^3/Pi^(3/2)
+5/3*b*x/c^5/Pi^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}a\left(\frac{x^4}{\pi\sqrt{\pi+\pi c^2x^2}\,c^2}-\frac{4\,x^2}{\pi\sqrt{\pi+\pi c^2x^2}\,c^4}-\frac{8}{\pi\sqrt{\pi+\pi c^2x^2}\,c^6}\right)+\frac{1}{3}b\left(\frac{\left(\sqrt{\pi}\,c^4x^4-4\sqrt{\pi}\,c^2x^2-8\sqrt{\pi}\right)\log\left(cx+\right.}{\pi^2\sqrt{c^2x^2+1}\,c^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima
")
```

```
[Out] 1/3*a*(x^4/(pi*sqrt(pi + pi*c^2*x^2)*c^2) - 4*x^2/(pi*sqrt(pi + pi*c^2*x^2)
*c^4) - 8/(pi*sqrt(pi + pi*c^2*x^2)*c^6)) + 1/3*b*((sqrt(pi)*c^4*x^4 - 4*sq
rt(pi)*c^2*x^2 - 8*sqrt(pi))*log(c*x + sqrt(c^2*x^2 + 1))/(pi^2*sqrt(c^2*x^
2 + 1)*c^6) - integrate((sqrt(pi)*c^4*x^4 - 4*sqrt(pi)*c^2*x^2 - 8*sqrt(pi)
)/(sqrt(c^2*x^2 + 1)*x), x)/(pi^2*c^6) + 3*integrate(1/3*(sqrt(pi)*c^4*x^4
- 4*sqrt(pi)*c^2*x^2 - 8*sqrt(pi))/(pi^2*c^9*x^4 + pi^2*c^7*x^2 + (pi^2*c^8
*x^3 + pi^2*c^6*x)*sqrt(c^2*x^2 + 1)), x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{{\left(\Pi\,c^2\,x^2+\Pi\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(3/2),x)
```

```
[Out] int((x^5*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^5}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{bx^5\,\mathrm{asinh}\,(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(3/2),x)
```

```
[Out] (Integral(a*x**5/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x)
+ Integral(b*x**5*asinh(c*x)/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**
2 + 1)), x))/pi**(3/2)
```

3.91 $\int \frac{x^4(a+b\sinh^{-1}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx$

Optimal. Leaf size=131

$$-\frac{3\left(a+b\sinh^{-1}(cx)\right)^2}{4\pi^{3/2}bc^5}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\pi c^2\sqrt{\pi c^2x^2+\pi}}+\frac{3x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{2\pi^2c^4}-\frac{bx^2}{4\pi^{3/2}c^3}-\frac{b\log\left(c^2x^2+1\right)}{2\pi^{3/2}c^5}$$

```
[Out] -1/4*b*x^2/c^3/Pi^(3/2)-3/4*(a+b*arcsinh(c*x))^2/b/c^5/Pi^(3/2)-1/2*b*ln(c^
2*x^2+1)/c^5/Pi^(3/2)-x^3*(a+b*arcsinh(c*x))/c^2/Pi/(Pi*c^2*x^2+Pi)^(1/2)+3
/2*x*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/c^4/Pi^2
```

Rubi [A] time = 0.26, antiderivative size = 181, normalized size of antiderivative = 1.38, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5751, 5758, 5675, 30, 266, 43}

$$-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\pi c^2\sqrt{\pi c^2x^2+\pi}}+\frac{3x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{2\pi^2c^4}-\frac{3\left(a+b\sinh^{-1}(cx)\right)^2}{4\pi^{3/2}bc^5}-\frac{bx^2\sqrt{c^2x^2+1}}{4\pi c^3\sqrt{\pi c^2x^2+\pi}}-\frac{b\sqrt{c^2x^2+}}{2\pi c^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] -(b*x^2*Sqrt[1 + c^2*x^2])/(4*c^3*Pi*Sqrt[Pi + c^2*Pi*x^2]) - (x^3*(a + b*A
rcSinh[c*x]))/(c^2*Pi*Sqrt[Pi + c^2*Pi*x^2]) + (3*x*Sqrt[Pi + c^2*Pi*x^2]*(
a + b*ArcSinh[c*x]))/(2*c^4*Pi^2) - (3*(a + b*ArcSinh[c*x])^2)/(4*b*c^5*Pi^
(3/2)) - (b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(2*c^5*Pi*Sqrt[Pi + c^2*Pi*
x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
```

```
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx &= -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{c^2\pi\sqrt{\pi+c^2\pi x^2}}+\frac{3\int\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi+c^2\pi x^2}}\,dx}{c^2\pi}+\frac{\left(b\sqrt{1+c^2x^2}\right)\int\frac{x^3}{1+c^2x^2}\,dx}{c\pi\sqrt{\pi+c^2\pi x^2}}\\
&= -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{c^2\pi\sqrt{\pi+c^2\pi x^2}}+\frac{3x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^4\pi^2}-\frac{3\int\frac{a+b\sinh^{-1}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,d}{2c^4\pi}\\
&= -\frac{3bx^2\sqrt{1+c^2x^2}}{4c^3\pi\sqrt{\pi+c^2\pi x^2}}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{c^2\pi\sqrt{\pi+c^2\pi x^2}}+\frac{3x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^4\pi^2}\\
&= -\frac{bx^2\sqrt{1+c^2x^2}}{4c^3\pi\sqrt{\pi+c^2\pi x^2}}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{c^2\pi\sqrt{\pi+c^2\pi x^2}}+\frac{3x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^4\pi^2}
\end{aligned}$$

Mathematica [A] time = 0.36, size = 147, normalized size = 1.12

$$\frac{\sinh^{-1}(cx)\left(-12a\sqrt{c^2x^2+1}+9bcx+b\sinh\left(3\sinh^{-1}(cx)\right)\right)+4ac^3x^3+12acx-4b\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)-}{8\pi^{3/2}c^5\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] (12*a*c*x + 4*a*c^3*x^3 - 6*b*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2 - b*Sqrt[1 +
 c^2*x^2]*Cosh[2*ArcSinh[c*x]] - 4*b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2] + A
rcSinh[c*x]*(9*b*c*x - 12*a*Sqrt[1 + c^2*x^2] + b*Sinh[3*ArcSinh[c*x]]))/(8
*c^5*Pi^(3/2)*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{\pi+\pi c^2x^2}\left(bx^4\operatorname{arsinh}(cx)+ax^4\right)}{\pi^2c^4x^4+2\,\pi^2c^2x^2+\pi^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*x^4*arcsinh(c*x) + a*x^4)/(pi^2*c^4*x^4 +
 2*pi^2*c^2*x^2 + pi^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.33, size = 269, normalized size = 2.05

$$\frac{a x^3}{2\pi c^2\sqrt{\pi c^2x^2+\pi}}+\frac{3ax}{2c^4\pi\sqrt{\pi c^2x^2+\pi}}-\frac{3a\ln\left(\frac{\pi x c^2}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{2c^4\pi\sqrt{\pi c^2}}-\frac{3b\,\mathrm{arcsinh}(cx)^2}{4c^5\pi^{\frac{3}{2}}}+\frac{b\,\mathrm{arcsinh}(cx)\sqrt{c^2x^2+}}{2\pi^{\frac{3}{2}}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(3/2),x)
```

```
[Out] 1/2*a*x^3/Pi/c^2/(Pi*c^2*x^2+Pi)^(1/2)+3/2*a/c^4*x/Pi/(Pi*c^2*x^2+Pi)^(1/2)
-3/2*a/c^4/Pi*ln(Pi*x*c^2/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1
/2)-3/4*b/c^5/Pi^(3/2)*arcsinh(c*x)^2+1/2*b/Pi^(3/2)/c^4*arcsinh(c*x)*(c^2*
x^2+1)^(1/2)*x-1/4*b*x^2/c^3/Pi^(3/2)+2*b/c^5/Pi^(3/2)*arcsinh(c*x)-1/8*b/P
i^(3/2)/c^5-b/Pi^(3/2)*arcsinh(c*x)/c^3/(c^2*x^2+1)*x^2+b/Pi^(3/2)*arcsinh(
c*x)/c^4/(c^2*x^2+1)^(1/2)*x-b/Pi^(3/2)*arcsinh(c*x)/c^5/(c^2*x^2+1)-b/c^5/
Pi^(3/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}a\left(\frac{x^3}{\pi\sqrt{\pi+\pi c^2x^2}\,c^2}+\frac{3\,x}{\pi\sqrt{\pi+\pi c^2x^2}\,c^4}-\frac{3\,\mathrm{arsinh}(cx)}{\pi^{\frac{3}{2}}c^5}\right)+b\int\frac{x^4\log\left(cx+\sqrt{c^2x^2+1}\right)}{\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima
")
```

```
[Out] 1/2*a*(x^3/(pi*sqrt(pi + pi*c^2*x^2)*c^2) + 3*x/(pi*sqrt(pi + pi*c^2*x^2)*c
^4) - 3*arcsinh(c*x)/(pi^(3/2)*c^5)) + b*integrate(x^4*log(c*x + sqrt(c^2*x
^2 + 1))/(pi + pi*c^2*x^2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^4\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{{\left(\Pi\,c^2\,x^2+\Pi\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(3/2),x)
```

```
[Out] int((x^4*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{ax^4}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int\frac{bx^4\,\mathrm{asinh}(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(3/2),x)
```

```
[Out] (Integral(a*x**4/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x)
+ Integral(b*x**4*asinh(c*x)/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**
2 + 1)), x))/pi**(3/2)
```

3.92 $\int \frac{x^3(a+b\sinh^{-1}(cx))}{(\pi+c^2\pi x^2)^{3/2}}\,dx$

Optimal. Leaf size=86

$$\frac{\sqrt{\pi c^2x^2+\pi}\,(a+b\sinh^{-1}(cx))}{\pi^2c^4}+\frac{a+b\sinh^{-1}(cx)}{\pi c^4\sqrt{\pi c^2x^2+\pi}}-\frac{b\tan^{-1}(cx)}{\pi^{3/2}c^4}-\frac{bx}{\pi^{3/2}c^3}$$

```
[Out] -b*x/c^3/Pi^(3/2)-b*arctan(c*x)/c^4/Pi^(3/2)+(a+b*arcsinh(c*x))/c^4/Pi/(Pi*
c^2*x^2+Pi)^(1/2)+(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/c^4/Pi^2
```

Rubi [A] time = 0.14, antiderivative size = 88, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {266, 43, 5732, 388, 205}

$$\frac{\sqrt{c^2x^2+1}\,(a+b\sinh^{-1}(cx))}{\pi^{3/2}c^4}+\frac{a+b\sinh^{-1}(cx)}{\pi^{3/2}c^4\sqrt{c^2x^2+1}}-\frac{bx}{\pi^{3/2}c^3}-\frac{b\tan^{-1}(cx)}{\pi^{3/2}c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] -((b*x)/(c^3*Pi^(3/2))) + (a + b*ArcSinh[c*x])/(c^4*Pi^(3/2)*Sqrt[1 + c^2*x
^2]) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(c^4*Pi^(3/2)) - (b*ArcTan[
c*x])/(c^4*Pi^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
```

```
(-1)] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx=\frac{a+b\sinh^{-1}(cx)}{c^4\pi^{3/2}\sqrt{1+c^2x^2}}+\frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^4\pi^{3/2}}-\frac{(bc)\int\frac{2+c^2x^2}{c^4+c^6x^2}\,dx}{\pi^{3/2}}$$
$$=-\frac{bx}{c^3\pi^{3/2}}+\frac{a+b\sinh^{-1}(cx)}{c^4\pi^{3/2}\sqrt{1+c^2x^2}}+\frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^4\pi^{3/2}}-\frac{(bc)\int\frac{1}{c^4+c^6x^2}\,dx}{\pi^{3/2}}$$
$$=-\frac{bx}{c^3\pi^{3/2}}+\frac{a+b\sinh^{-1}(cx)}{c^4\pi^{3/2}\sqrt{1+c^2x^2}}+\frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^4\pi^{3/2}}-\frac{b\tan^{-1}(cx)}{c^4\pi^{3/2}}$$

Mathematica [A] time = 0.16, size = 87, normalized size = 1.01

$$\frac{ac^2x^2+2a-bcx\sqrt{c^2x^2+1}-b\sqrt{c^2x^2+1}\tan^{-1}(cx)+b\left(c^2x^2+2\right)\sinh^{-1}(cx)}{\pi^{3/2}c^4\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] (2*a + a*c^2*x^2 - b*c*x*Sqrt[1 + c^2*x^2] + b*(2 + c^2*x^2)*ArcSinh[c*x] -
 b*Sqrt[1 + c^2*x^2]*ArcTan[c*x])/(c^4*Pi^(3/2)*Sqrt[1 + c^2*x^2])
```

fricas [B] time = 0.66, size = 165, normalized size = 1.92

$$\frac{\sqrt{\pi}\left(bc^2x^2+b\right)\arctan\left(-\frac{2\sqrt{\pi}\sqrt{\pi+\pi c^2x^2}\sqrt{c^2x^2+1}cx}{\pi-\pi c^4x^4}\right)+2\sqrt{\pi+\pi c^2x^2}\left(bc^2x^2+2b\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+2\sqrt{\pi}}{2\left(\pi^2c^6x^2+\pi^2c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas
")
```

```
[Out] 1/2*(sqrt(pi)*(b*c^2*x^2 + b)*arctan(-2*sqrt(pi)*sqrt(pi + pi*c^2*x^2)*sqrt
(c^2*x^2 + 1)*c*x/(pi - pi*c^4*x^4)) + 2*sqrt(pi + pi*c^2*x^2)*(b*c^2*x^2 +
 2*b)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*sqrt(pi + pi*c^2*x^2)*(a*c^2*x^2 - s
qrt(c^2*x^2 + 1)*b*c*x + 2*a))/(pi^2*c^6*x^2 + pi^2*c^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [C] time = 0.34, size = 158, normalized size = 1.84

$$\frac{a\,x^2}{\pi\,c^2\sqrt{\pi\,c^2x^2+\pi}}+\frac{2a}{\pi\,c^4\sqrt{\pi\,c^2x^2+\pi}}+\frac{b\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}}{\pi^{\frac{3}{2}}c^4}-\frac{bx}{c^3\pi^{\frac{3}{2}}}+\frac{b\operatorname{arcsinh}(cx)}{\pi^{\frac{3}{2}}\sqrt{c^2x^2+1}\,c^4}+\frac{ib\ln\left(cx+\sqrt{c^2x^2+}\right.}{c^4\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(3/2),x)
```

```
[Out] a*x^2/Pi/c^2/(Pi*c^2*x^2+Pi)^(1/2)+2*a/Pi/c^4/(Pi*c^2*x^2+Pi)^(1/2)+b/Pi^(3
/2)/c^4*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-b*x/c^3/Pi^(3/2)+b/Pi^(3/2)/(c^2*x^2
+1)^(1/2)/c^4*arcsinh(c*x)+I*b/c^4/Pi^(3/2)*ln(c*x+(c^2*x^2+1)^(1/2)-I)-I*b
/c^4/Pi^(3/2)*ln(c*x+(c^2*x^2+1)^(1/2)+I)
```

maxima [A] time = 0.59, size = 119, normalized size = 1.38

$$-bc\left(\frac{x}{\pi^{\frac{3}{2}}c^4}+\frac{\arctan(cx)}{\pi^{\frac{3}{2}}c^5}\right)+b\left(\frac{x^2}{\pi\sqrt{\pi+\pi c^2x^2}\,c^2}+\frac{2}{\pi\sqrt{\pi+\pi c^2x^2}\,c^4}\right)\operatorname{arsinh}(cx)+a\left(\frac{x^2}{\pi\sqrt{\pi+\pi c^2x^2}\,c^2}+\frac{2}{\pi\sqrt{\pi+\pi c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima
")
```

```
[Out] -b*c*(x/(pi^(3/2)*c^4) + arctan(c*x)/(pi^(3/2)*c^5)) + b*(x^2/(pi*sqrt(pi +
 pi*c^2*x^2)*c^2) + 2/(pi*sqrt(pi + pi*c^2*x^2)*c^4))*arcsinh(c*x) + a*(x^2
/(pi*sqrt(pi + pi*c^2*x^2)*c^2) + 2/(pi*sqrt(pi + pi*c^2*x^2)*c^4))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{{\left(\Pi\,c^2\,x^2+\Pi\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(3/2),x)
```

```
[Out] int((x^3*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^3}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{bx^3\operatorname{asinh}(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(3/2),x)
```

```
[Out] (Integral(a*x**3/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x)
+ Integral(b*x**3*asinh(c*x)/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**
2 + 1)), x))/pi**(3/2)
```

3.93 $\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=80

$$\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2\pi^{3/2}bc^3}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{\pi c^2\sqrt{\pi c^2x^2+\pi}}+\frac{b\log\left(c^2x^2+1\right)}{2\pi^{3/2}c^3}$$

```
[Out] 1/2*(a+b*arcsinh(c*x))^2/b/c^3/Pi^(3/2)+1/2*b*ln(c^2*x^2+1)/c^3/Pi^(3/2)-x*
(a+b*arcsinh(c*x))/c^2/Pi/(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 105, normalized size of antiderivative = 1.31, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {5751, 5675, 260}

$$-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{\pi c^2\sqrt{\pi c^2x^2+\pi}}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2\pi^{3/2}bc^3}+\frac{b\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)}{2\pi c^3\sqrt{\pi c^2x^2+\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] -((x*(a + b*ArcSinh[c*x]))/(c^2*Pi*Sqrt[Pi + c^2*Pi*x^2])) + (a + b*ArcSinh
[c*x])^2/(2*b*c^3*Pi^(3/2)) + (b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(2*c^3
*Pi*Sqrt[Pi + c^2*Pi*x^2])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx &= -\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^2\pi\sqrt{\pi+c^2\pi x^2}}+\frac{\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx}{c^2\pi}+\frac{\left(b\sqrt{1+c^2x^2}\right)\int \frac{x}{1+c^2x^2}\,dx}{c\pi\sqrt{\pi+c^2\pi x^2}}\\
&= -\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^2\pi\sqrt{\pi+c^2\pi x^2}}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^3\pi^{3/2}}+\frac{b\sqrt{1+c^2x^2}\log\left(1+c^2x^2\right)}{2c^3\pi\sqrt{\pi+c^2\pi x^2}}
\end{aligned}$$

Mathematica [A] time = 0.29, size = 78, normalized size = 0.98

$$\frac{\sinh^{-1}(cx)\left(2a-\frac{2bcx}{\sqrt{c^2x^2+1}}\right)-\frac{2acx}{\sqrt{c^2x^2+1}}+b\log\left(c^2x^2+1\right)+b\sinh^{-1}(cx)^2}{2\pi^{3/2}c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] ((-2*a*c*x)/Sqrt[1 + c^2*x^2] + (2*a - (2*b*c*x)/Sqrt[1 + c^2*x^2])*ArcSinh
[c*x] + b*ArcSinh[c*x]^2 + b*Log[1 + c^2*x^2])/(2*c^3*Pi^(3/2))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\pi+\pi c^2x^2}\left(bx^2\operatorname{arsinh}(cx)+ax^2\right)}{\pi^2c^4x^4+2\,\pi^2c^2x^2+\pi^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*x^2*arcsinh(c*x) + a*x^2)/(pi^2*c^4*x^4 +
 2*pi^2*c^2*x^2 + pi^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\operatorname{arsinh}(cx)+a)x^2}{\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^2/(pi + pi*c^2*x^2)^(3/2), x)
```

maple [B] time = 0.16, size = 196, normalized size = 2.45

$$-\frac{ax}{\pi\,c^2\sqrt{\pi\,c^2x^2+\pi}}+\frac{a\ln\left(\frac{\pi x\,c^2}{\sqrt{\pi\,c^2}}+\sqrt{\pi\,c^2x^2+\pi}\right)}{\pi\,c^2\sqrt{\pi\,c^2}}+\frac{b\operatorname{arcsinh}(cx)^2}{2c^3\pi^{\frac{3}{2}}}-\frac{2b\operatorname{arcsinh}(cx)}{c^3\pi^{\frac{3}{2}}}+\frac{b\operatorname{arcsinh}(cx)\,x^2}{\pi^{\frac{3}{2}}c\left(c^2x^2+1\right)}-\frac{b\operatorname{arcsinh}}{\pi^{\frac{3}{2}}c^2\sqrt{c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(3/2),x)
```

```
[Out] -a*x/Pi/c^2/(Pi*c^2*x^2+Pi)^(1/2)+a/Pi/c^2*ln(Pi*x*c^2/(Pi*c^2)^(1/2)+(Pi*c
^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/2*b/c^3/Pi^(3/2)*arcsinh(c*x)^2-2*b/c^3/
Pi^(3/2)*arcsinh(c*x)+b/Pi^(3/2)*arcsinh(c*x)/c/(c^2*x^2+1)*x^2-b/Pi^(3/2)*
arcsinh(c*x)/c^2/(c^2*x^2+1)^(1/2)*x+b/Pi^(3/2)*arcsinh(c*x)/c^3/(c^2*x^2+1
)+b/c^3/Pi^(3/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-a\left(\frac{x}{\pi\sqrt{\pi+\pi c^2x^2}\,c^2}-\frac{\operatorname{arsinh}(cx)}{\pi^{\frac{3}{2}}c^3}\right)+b\int\frac{x^2\log\left(cx+\sqrt{c^2x^2+1}\right)}{\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima
")
```

```
[Out] -a*(x/(pi*sqrt(pi + pi*c^2*x^2)*c^2) - arcsinh(c*x)/(pi^(3/2)*c^3)) + b*int
egrate(x^2*log(c*x + sqrt(c^2*x^2 + 1))/(pi + pi*c^2*x^2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{{\left(\Pi\,c^2\,x^2+\Pi\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(3/2),x)
```

```
[Out] int((x^2*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^{2}}{c^{2}x^{2}\sqrt{c^{2}x^{2}+1}+\sqrt{c^{2}x^{2}+1}}\, dx + \int \frac{bx^{2}\operatorname{asinh}{\left(cx \right)}}{c^{2}x^{2}\sqrt{c^{2}x^{2}+1}+\sqrt{c^{2}x^{2}+1}}\, dx}{\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(3/2),x)
```

```
[Out] (Integral(a*x**2/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x)
+ Integral(b*x**2*asinh(c*x)/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**
2 + 1)), x))/pi**(3/2)
```

3.94 $\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=45

$$\frac{b\tan^{-1}(cx)}{\pi^{3/2}c^2}-\frac{a+b\sinh^{-1}(cx)}{\pi c^2\sqrt{\pi c^2x^2+\pi}}$$

```
[Out] b*arctan(c*x)/c^2/Pi^(3/2)+(-a-b*arcsinh(c*x))/c^2/Pi/(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 70, normalized size of antiderivative = 1.56, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5717, 203}

$$\frac{b\sqrt{c^2x^2+1}\,\tan^{-1}(cx)}{\pi c^2\sqrt{\pi c^2x^2+\pi}}-\frac{a+b\sinh^{-1}(cx)}{\pi c^2\sqrt{\pi c^2x^2+\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] -((a + b*ArcSinh[c*x])/(c^2*Pi*Sqrt[Pi + c^2*Pi*x^2])) + (b*Sqrt[1 + c^2*x^
2]*ArcTan[c*x])/(c^2*Pi*Sqrt[Pi + c^2*Pi*x^2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx &= -\frac{a+b\sinh^{-1}(cx)}{c^2\pi\sqrt{\pi+c^2\pi x^2}}+\frac{\left(b\sqrt{1+c^2x^2}\right)\int\frac{1}{1+c^2x^2}\,dx}{c\pi\sqrt{\pi+c^2\pi x^2}}\\ &= -\frac{a+b\sinh^{-1}(cx)}{c^2\pi\sqrt{\pi+c^2\pi x^2}}+\frac{b\sqrt{1+c^2x^2}\,\tan^{-1}(cx)}{c^2\pi\sqrt{\pi+c^2\pi x^2}}\end{aligned}$$

Mathematica [A] time = 0.11, size = 52, normalized size = 1.16

$$\frac{-a+b\sqrt{c^2x^2+1}\,\tan^{-1}(cx)-b\sinh^{-1}(cx)}{\pi^{3/2}c^2\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

[Out] (-a - b*ArcSinh[c*x] + b*Sqrt[1 + c^2*x^2]*ArcTan[c*x])/(c^2*Pi^(3/2)*Sqrt[
1 + c^2*x^2])

fricas [B] time = 0.83, size = 127, normalized size = 2.82

$$-\frac{\sqrt{\pi}\left(bc^2x^2+b\right)\arctan\left(-\frac{2\sqrt{\pi}\sqrt{\pi+\pi c^2x^2}\sqrt{c^2x^2+1}\,cx}{\pi-\pi c^4x^4}\right)+2\sqrt{\pi+\pi c^2x^2}\,b\log\left(cx+\sqrt{c^2x^2+1}\right)+2\sqrt{\pi+\pi c^2x^2}\,a}{2\left(\pi^2c^4x^2+\pi^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas")

[Out] -1/2*(sqrt(pi)*(b*c^2*x^2 + b)*arctan(-2*sqrt(pi)*sqrt(pi + pi*c^2*x^2)*sqr
t(c^2*x^2 + 1)*c*x/(pi - pi*c^4*x^4)) + 2*sqrt(pi + pi*c^2*x^2)*b*log(c*x +
sqrt(c^2*x^2 + 1)) + 2*sqrt(pi + pi*c^2*x^2)*a)/(pi^2*c^4*x^2 + pi^2*c^2)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)x}{\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")

[Out] integrate((b*arcsinh(c*x) + a)*x/(pi + pi*c^2*x^2)^(3/2), x)

maple [C] time = 0.09, size = 103, normalized size = 2.29

$$-\frac{a}{\pi c^2\sqrt{\pi c^2x^2+\pi}}-\frac{b\,\mathrm{arcsinh}\,(cx)}{\pi^{\frac{3}{2}}\sqrt{c^2x^2+1}\,c^2}+\frac{ib\ln\left(cx+\sqrt{c^2x^2+1}\,+i\right)}{c^2\pi^{\frac{3}{2}}}-\frac{ib\ln\left(cx+\sqrt{c^2x^2+1}\,-i\right)}{c^2\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(3/2),x)

[Out] -a/Pi/c^2/(Pi*c^2*x^2+Pi)^(1/2)-b/Pi^(3/2)/(c^2*x^2+1)^(1/2)/c^2*arcsinh(c*
x)+I*b/c^2/Pi^(3/2)*ln(c*x+(c^2*x^2+1)^(1/2)+I)-I*b/c^2/Pi^(3/2)*ln(c*x+(c^
2*x^2+1)^(1/2)-I)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\left(\frac{-\,\mathrm{arsinh}\left(\frac{1}{c|x|}\right)}{\pi^{\frac{3}{2}}c^2}-\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{\pi^{\frac{3}{2}}\sqrt{c^2x^2+1}\,c^2}-\int\frac{1}{\pi^{\frac{3}{2}}c^5x^4+\pi^{\frac{3}{2}}c^3x^2+\left(\pi^{\frac{3}{2}}c^4x^3+\pi^{\frac{3}{2}}c^2x\right)\sqrt{c^2x^2+1}}\,dx\right)-\frac{\quad}{\pi\sqrt{\pi+}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima")

[Out] b*(integrate(1/(sqrt(c^2*x^2 + 1)*x), x)/(pi^(3/2)*c^2) - log(c*x + sqrt(c^
2*x^2 + 1))/(pi^(3/2)*sqrt(c^2*x^2 + 1)*c^2) - integrate(1/(pi^(3/2)*c^5*x^
4 + pi^(3/2)*c^3*x^2 + (pi^(3/2)*c^4*x^3 + pi^(3/2)*c^2*x)*sqrt(c^2*x^2 + 1
)), x)) - a/(pi*sqrt(pi + pi*c^2*x^2)*c^2)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{{\left(\Pi\,c^2\,x^2+\Pi\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(3/2),x)
```

```
[Out] int((x*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{bx\,\mathrm{asinh}\,(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(3/2),x)
```

```
[Out] (Integral(a*x/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x) + I
ntegral(b*x*asinh(c*x)/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)
), x))/pi**(3/2)
```

3.95 $\int \frac{a+b\sinh^{-1}(cx)}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=51

$$\frac{x\left(a+b\sinh^{-1}(cx)\right)}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{b\log\left(c^2x^2+1\right)}{2\pi^{3/2}c}$$

```
[Out] -1/2*b*ln(c^2*x^2+1)/c/Pi^(3/2)+x*(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+Pi)^(1/
2)
```

Rubi [A] time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.49, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5687, 260}

$$\frac{x\left(a+b\sinh^{-1}(cx)\right)}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{b\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)}{2\pi c\sqrt{\pi c^2x^2+\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcSinh[c*x]))/(Pi*Sqrt[Pi + c^2*Pi*x^2]) - (b*Sqrt[1 + c^2*x^2]*
Log[1 + c^2*x^2])/(2*c*Pi*Sqrt[Pi + c^2*Pi*x^2])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sinh^{-1}(cx)}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx &= \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\pi\sqrt{\pi+c^2\pi x^2}}-\frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{x}{1+c^2x^2}\,dx}{\pi\sqrt{\pi+c^2\pi x^2}}\\ &= \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\pi\sqrt{\pi+c^2\pi x^2}}-\frac{b\sqrt{1+c^2x^2}\log\left(1+c^2x^2\right)}{2c\pi\sqrt{\pi+c^2\pi x^2}}\end{aligned}$$

Mathematica [A] time = 0.10, size = 66, normalized size = 1.29

$$\frac{2acx-b\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)+2bcx\sinh^{-1}(cx)}{2\pi^{3/2}c\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] (2*a*c*x + 2*b*c*x*ArcSinh[c*x] - b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(2*
c*Pi^(3/2)*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\pi + \pi c^2 x^2}\,(b\,\text{arsinh}\,(cx) + a)}{\pi^2 c^4 x^4 + 2\,\pi^2 c^2 x^2 + \pi^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^2*c^4*x^4 + 2*pi^2*
c^2*x^2 + pi^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arsinh}\,(cx) + a}{\left(\pi + \pi c^2 x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(pi + pi*c^2*x^2)^(3/2), x)
```

maple [B] time = 0.08, size = 132, normalized size = 2.59

$$\frac{ax}{\pi\sqrt{\pi\,c^2 x^2 + \pi}} + \frac{2b\,\text{arcsinh}\,(cx)}{c\,\pi^{\frac{3}{2}}} - \frac{b\,\text{arcsinh}\,(cx)\,c\,x^2}{\pi^{\frac{3}{2}}\left(c^2 x^2 + 1\right)} + \frac{b\,\text{arcsinh}\,(cx)\,x}{\pi^{\frac{3}{2}}\sqrt{c^2 x^2 + 1}} - \frac{b\,\text{arcsinh}\,(cx)}{\pi^{\frac{3}{2}} c\left(c^2 x^2 + 1\right)} - \frac{b\ln\left(1 + \left(cx + \sqrt{c^2 x^2 + \right.\right.}{c\,\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(3/2),x)
```

```
[Out] a/Pi*x/(Pi*c^2*x^2+Pi)^(1/2)+2*b/c/Pi^(3/2)*arcsinh(c*x)-b/Pi^(3/2)*arcsinh
(c*x)*c/(c^2*x^2+1)*x^2+b/Pi^(3/2)*arcsinh(c*x)/(c^2*x^2+1)^(1/2)*x-b/Pi^(3
/2)*arcsinh(c*x)/c/(c^2*x^2+1)-b/c/Pi^(3/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
```

maxima [A] time = 0.55, size = 58, normalized size = 1.14

$$\frac{bx\,\text{arsinh}\,(cx)}{\pi\sqrt{\pi + \pi c^2 x^2}} + \frac{ax}{\pi\sqrt{\pi + \pi c^2 x^2}} - \frac{b\log\left(x^2 + \frac{1}{c^2}\right)}{2\,\pi^{\frac{3}{2}} c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima")
```

```
[Out] b*x*arcsinh(c*x)/(pi*sqrt(pi + pi*c^2*x^2)) + a*x/(pi*sqrt(pi + pi*c^2*x^2)
) - 1/2*b*log(x^2 + 1/c^2)/(pi^(3/2)*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a + b\,\text{asinh}\,(c\,x)}{\left(\Pi\,c^2\,x^2 + \Pi\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(Pi + Pi*c^2*x^2)^(3/2),x)
```

```
[Out] int((a + b*asinh(c*x))/(Pi + Pi*c^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx + \int \frac{b\operatorname{asinh}(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(3/2),x)
```

```
[Out] (Integral(a/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x) + Int
egral(b*asinh(c*x)/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x
))/pi**(3/2)
```

3.96 $\int \frac{a+b\sinh^{-1}(cx)}{x(\pi+c^2\pi x^2)^{3/2}}\,dx$

Optimal. Leaf size=94

$$\frac{a+b\sinh^{-1}(cx)}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\pi^{3/2}}-\frac{b\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)}{\pi^{3/2}}+\frac{b\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)}{\pi^{3/2}}-\frac{b\tan^{-1}(cx)}{\pi^{3/2}}$$

```
[Out] -b*arctan(c*x)/Pi^(3/2)-2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))
/Pi^(3/2)-b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))/Pi^(3/2)+b*polylog(2,c*x+(c^2
*x^2+1)^(1/2))/Pi^(3/2)+(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 119, normalized size of antiderivative = 1.27, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5755, 5760, 4182, 2279, 2391, 203}

$$-\frac{b\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{\pi^{3/2}}+\frac{b\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{\pi^{3/2}}+\frac{a+b\sinh^{-1}(cx)}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\pi^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x*(Pi + c^2*Pi*x^2)^(3/2)),x]
```

```
[Out] (a + b*ArcSinh[c*x])/(Pi*Sqrt[Pi + c^2*Pi*x^2]) - (b*Sqrt[1 + c^2*x^2]*ArcT
an[c*x])/(Pi*Sqrt[Pi + c^2*Pi*x^2]) - (2*(a + b*ArcSinh[c*x])*ArcTanh[E^Arc
Sinh[c*x]])/Pi^(3/2) - (b*PolyLog[2, -E^ArcSinh[c*x]])/Pi^(3/2) + (b*PolyLo
g[2, E^ArcSinh[c*x]])/Pi^(3/2)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
```

```
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx = \frac{a+b\sinh^{-1}(cx)}{\pi\sqrt{\pi+c^2\pi x^2}} + \frac{\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{\pi+c^2\pi x^2}}\,dx}{\pi} - \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{1+c^2x^2}\,dx}{\pi\sqrt{\pi+c^2\pi x^2}}$$

$$= \frac{a+b\sinh^{-1}(cx)}{\pi\sqrt{\pi+c^2\pi x^2}} - \frac{b\sqrt{1+c^2x^2}\,\tan^{-1}(cx)}{\pi\sqrt{\pi+c^2\pi x^2}} + \frac{\text{Subst}\left(\int (a+bx)\text{csch}(x)\,dx, x, \sinh^{-1}(c\right.}{\pi^{3/2}}$$

$$= \frac{a+b\sinh^{-1}(cx)}{\pi\sqrt{\pi+c^2\pi x^2}} - \frac{b\sqrt{1+c^2x^2}\,\tan^{-1}(cx)}{\pi\sqrt{\pi+c^2\pi x^2}} - \frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\pi^{3/2}}$$

$$= \frac{a+b\sinh^{-1}(cx)}{\pi\sqrt{\pi+c^2\pi x^2}} - \frac{b\sqrt{1+c^2x^2}\,\tan^{-1}(cx)}{\pi\sqrt{\pi+c^2\pi x^2}} - \frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\pi^{3/2}}$$

$$= \frac{a+b\sinh^{-1}(cx)}{\pi\sqrt{\pi+c^2\pi x^2}} - \frac{b\sqrt{1+c^2x^2}\,\tan^{-1}(cx)}{\pi\sqrt{\pi+c^2\pi x^2}} - \frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\pi^{3/2}}$$

Mathematica [A] time = 0.32, size = 143, normalized size = 1.52

$$\frac{\frac{a}{\sqrt{c^2x^2+1}} - a\log\left(\pi\left(\sqrt{c^2x^2+1}+1\right)\right) + a\log(x) + \frac{b\sinh^{-1}(cx)}{\sqrt{c^2x^2+1}} + b\text{Li}_2\left(-e^{-\sinh^{-1}(cx)}\right) - b\text{Li}_2\left(e^{-\sinh^{-1}(cx)}\right) + b\sinh}{\pi^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x*(Pi + c^2*Pi*x^2)^(3/2)),x]
```

```
[Out] (a/Sqrt[1 + c^2*x^2] + (b*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - 2*b*ArcTan[Tanh
[ArcSinh[c*x]/2]] + b*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - b*ArcSinh[c
*x]*Log[1 + E^(-ArcSinh[c*x])] + a*Log[x] - a*Log[Pi*(1 + Sqrt[1 + c^2*x^2]
)] + b*PolyLog[2, -E^(-ArcSinh[c*x])] - b*PolyLog[2, E^(-ArcSinh[c*x])])/Pi
^(3/2)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\pi+\pi c^2x^2}\,(b\,\text{arsinh}\,(cx)+a)}{\pi^2c^4x^5+2\,\pi^2c^2x^3+\pi^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^2*c^4*x^5 + 2*pi^2*
c^2*x^3 + pi^2*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\left(\pi + \pi c^2 x^2\right)^{\frac{3}{2}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((pi + pi*c^2*x^2)^(3/2)*x), x)
```

maple [A] time = 0.23, size = 156, normalized size = 1.66

$$\frac{a}{\pi \sqrt{\pi c^2 x^2 + \pi}} - \frac{a \operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2 x^2 + \pi}}\right)}{\pi^{\frac{3}{2}}} + \frac{b \operatorname{arcsinh}(cx)}{\pi^{\frac{3}{2}} \sqrt{c^2 x^2 + 1}} - \frac{2b \arctan\left(cx + \sqrt{c^2 x^2 + 1}\right)}{\pi^{\frac{3}{2}}} - \frac{b \operatorname{dilog}\left(cx + \sqrt{c^2 x^2 + 1}\right)}{\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x/(Pi*c^2*x^2+Pi)^(3/2),x)
```

```
[Out] a/Pi/(Pi*c^2*x^2+Pi)^(1/2)-a/Pi^(3/2)*arctanh(Pi^(1/2)/(Pi*c^2*x^2+Pi)^(1/2
))+b/Pi^(3/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)-2*b/Pi^(3/2)*arctan(c*x+(c^2*x
^2+1)^(1/2))-b/Pi^(3/2)*dilog(c*x+(c^2*x^2+1)^(1/2))-b/Pi^(3/2)*dilog(1+c*x
+(c^2*x^2+1)^(1/2))-b/Pi^(3/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-a\left(\frac{\operatorname{arsinh}\left(\frac{1}{c|x|}\right)}{\pi^{\frac{3}{2}}} - \frac{1}{\pi \sqrt{\pi + \pi c^2 x^2}}\right) + b \int \frac{\log\left(cx + \sqrt{c^2 x^2 + 1}\right)}{\left(\pi + \pi c^2 x^2\right)^{\frac{3}{2}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima")
```

```
[Out] -a*(arcsinh(1/(c*abs(x)))/pi^(3/2) - 1/(pi*sqrt(pi + pi*c^2*x^2))) + b*inte
grate(log(c*x + sqrt(c^2*x^2 + 1))/((pi + pi*c^2*x^2)^(3/2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{asinh}(c\, x)}{x \left(\Pi\, c^2\, x^2 + \Pi\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x*(Pi + Pi*c^2*x^2)^(3/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x*(Pi + Pi*c^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^2 x^3 \sqrt{c^2 x^2 + 1} + x \sqrt{c^2 x^2 + 1}}\, dx + \int \frac{b \operatorname{asinh}(cx)}{c^2 x^3 \sqrt{c^2 x^2 + 1} + x \sqrt{c^2 x^2 + 1}}\, dx}{\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x/(pi*c**2*x**2+pi)**(3/2),x)
```

```
[Out] (Integral(a/(c**2*x**3*sqrt(c**2*x**2 + 1) + x*sqrt(c**2*x**2 + 1)), x) + I
ntegral(b*asinh(c*x)/(c**2*x**3*sqrt(c**2*x**2 + 1) + x*sqrt(c**2*x**2 + 1)
), x))/pi**(3/2)
```

3.97 $\int \frac{a+b\sinh^{-1}(cx)}{x^2(\pi+c^2\pi x^2)^{3/2}}\,dx$

Optimal. Leaf size=93

$$-\frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{a+b\sinh^{-1}(cx)}{\pi x\sqrt{\pi c^2x^2+\pi}}+\frac{bc\log\left(c^2x^2+1\right)}{2\pi^{3/2}}+\frac{bc\log(x)}{\pi^{3/2}}$$

```
[Out] b*c*ln(x)/Pi^(3/2)+1/2*b*c*ln(c^2*x^2+1)/Pi^(3/2)+(-a-b*arcsinh(c*x))/Pi/x/
(Pi*c^2*x^2+Pi)^(1/2)-2*c^2*x*(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 95, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {271, 191, 5732, 446, 72}

$$-\frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)}{\pi^{3/2}\sqrt{c^2x^2+1}}-\frac{a+b\sinh^{-1}(cx)}{\pi^{3/2}x\sqrt{c^2x^2+1}}+\frac{bc\log\left(c^2x^2+1\right)}{2\pi^{3/2}}+\frac{bc\log(x)}{\pi^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^2*(Pi + c^2*Pi*x^2)^(3/2)),x]
```

```
[Out] -((a + b*ArcSinh[c*x])/(Pi^(3/2)*x*Sqrt[1 + c^2*x^2])) - (2*c^2*x*(a + b*Ar
cSinh[c*x]))/(Pi^(3/2)*Sqrt[1 + c^2*x^2]) + (b*c*Log[x])/Pi^(3/2) + (b*c*Lo
g[1 + c^2*x^2])/(2*Pi^(3/2))
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{\pi^{3/2}x\sqrt{1+c^2x^2}}-\frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)}{\pi^{3/2}\sqrt{1+c^2x^2}}-\frac{(bc)\int\frac{-1-2c^2x^2}{x\left(1+c^2x^2\right)}\,dx}{\pi^{3/2}}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{\pi^{3/2}x\sqrt{1+c^2x^2}}-\frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)}{\pi^{3/2}\sqrt{1+c^2x^2}}-\frac{(bc)\,\text{Subst}\left(\int\frac{-1-2c^2x}{x\left(1+c^2x\right)}\,dx,x,x^2\right)}{2\pi^{3/2}}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{\pi^{3/2}x\sqrt{1+c^2x^2}}-\frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)}{\pi^{3/2}\sqrt{1+c^2x^2}}-\frac{(bc)\,\text{Subst}\left(\int\left(-\frac{1}{x}-\frac{c^2}{1+c^2x}\right)dx,x,x^2\right)}{2\pi^{3/2}}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{\pi^{3/2}x\sqrt{1+c^2x^2}}-\frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)}{\pi^{3/2}\sqrt{1+c^2x^2}}+\frac{bc\log(x)}{\pi^{3/2}}+\frac{bc\log\left(1+c^2x^2\right)}{2\pi^{3/2}}$$

Mathematica [A] time = 0.15, size = 69, normalized size = 0.74

$$\frac{b\left(\frac{1}{2}c\log\left(c^2x^2+1\right)+c\log(x)\right)}{\pi^{3/2}}-\frac{\left(2c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{\pi^{3/2}x\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^2*(Pi + c^2*Pi*x^2)^(3/2)),x]
```

```
[Out] -(((1 + 2*c^2*x^2)*(a + b*ArcSinh[c*x]))/(Pi^(3/2)*x*Sqrt[1 + c^2*x^2])) +
(b*(c*Log[x] + (c*Log[1 + c^2*x^2])/2))/Pi^(3/2)
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\pi+\pi c^2x^2}\,(b\,\text{arsinh}\,(cx)+a)}{\pi^2c^4x^6+2\,\pi^2c^2x^4+\pi^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^2*c^4*x^6 + 2*pi^2*
c^2*x^4 + pi^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{b\,\text{arsinh}\,(cx)+a}{\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((pi + pi*c^2*x^2)^(3/2)*x^2), x)
```

maple [B] time = 0.15, size = 180, normalized size = 1.94

$$-\frac{a}{\pi x\sqrt{\pi c^2x^2+\pi}}-\frac{2a\,c^2x}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{4bc\,\text{arcsinh}\,(cx)}{\pi^{\frac{3}{2}}}+\frac{2b\,\text{arcsinh}\,(cx)\,x^2c^3}{\pi^{\frac{3}{2}}\left(c^2x^2+1\right)}-\frac{2b\,\text{arcsinh}\,(cx)\,x\,c^2}{\pi^{\frac{3}{2}}\sqrt{c^2x^2+1}}+\frac{2b\,\text{arcsinh}\,(}{\pi^{\frac{3}{2}}\left(c^2x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^2/(Pi*c^2*x^2+Pi)^(3/2),x)
```

```
[Out] -a/Pi/x/(Pi*c^2*x^2+Pi)^(1/2)-2*a/Pi*c^2*x/(Pi*c^2*x^2+Pi)^(1/2)-4*b*c/Pi^(
3/2)*arcsinh(c*x)+2*b/Pi^(3/2)*arcsinh(c*x)*x^2/(c^2*x^2+1)*c^3-2*b/Pi^(3/2
)*arcsinh(c*x)*x/(c^2*x^2+1)^(1/2)*c^2+2*b/Pi^(3/2)*arcsinh(c*x)/(c^2*x^2+1
)*c-b/Pi^(3/2)*arcsinh(c*x)/x/(c^2*x^2+1)^(1/2)+b*c/Pi^(3/2)*ln((c*x+(c^2*x
^2+1)^(1/2))^4-1)
```

maxima [A] time = 0.56, size = 119, normalized size = 1.28

$$\frac{1}{2}\,bc\left(\frac{\log\left(c^2x^2+1\right)}{\pi^{\frac{3}{2}}}+\frac{2\,\log(x)}{\pi^{\frac{3}{2}}}\right)-\left(\frac{2\,c^2x}{\pi\sqrt{\pi+\pi c^2x^2}}+\frac{1}{\pi\sqrt{\pi+\pi c^2x^2}\,x}\right)b\,\mathrm{arsinh}\,(cx)-\left(\frac{2\,c^2x}{\pi\sqrt{\pi+\pi c^2x^2}}+\frac{1}{\pi\sqrt{\pi+\pi}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima
")
```

```
[Out] 1/2*b*c*(log(c^2*x^2 + 1)/pi^(3/2) + 2*log(x)/pi^(3/2)) - (2*c^2*x/(pi*sqrt
(pi + pi*c^2*x^2)) + 1/(pi*sqrt(pi + pi*c^2*x^2)*x))*b*arcsinh(c*x) - (2*c^
2*x/(pi*sqrt(pi + pi*c^2*x^2)) + 1/(pi*sqrt(pi + pi*c^2*x^2)*x))*a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{asinh}\,(c\,x)}{x^2\,{\left(\Pi\,c^2\,x^2+\Pi\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^2*(Pi + Pi*c^2*x^2)^(3/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^2*(Pi + Pi*c^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^2x^4\sqrt{c^2x^2+1}+x^2\sqrt{c^2x^2+1}}\,dx+\int \frac{b\,\mathrm{asinh}\,(cx)}{c^2x^4\sqrt{c^2x^2+1}+x^2\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**2/(pi*c**2*x**2+pi)**(3/2),x)
```

```
[Out] (Integral(a/(c**2*x**4*sqrt(c**2*x**2 + 1) + x**2*sqrt(c**2*x**2 + 1)), x)
+ Integral(b*asinh(c*x)/(c**2*x**4*sqrt(c**2*x**2 + 1) + x**2*sqrt(c**2*x**
2 + 1)), x))/pi**(3/2)
```

3.98 $\int \frac{a+b\sinh^{-1}(cx)}{x^3\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=162

$$-\frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2\pi\sqrt{\pi c^2x^2+\pi}}-\frac{a+b\sinh^{-1}(cx)}{2\pi x^2\sqrt{\pi c^2x^2+\pi}}+\frac{3c^2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\pi^{3/2}}+\frac{3bc^2\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)}{2\pi^{3/2}}$$

```
[Out] -1/2*b*c/Pi^(3/2)/x+b*c^2*arctan(c*x)/Pi^(3/2)+3*c^2*(a+b*arcsinh(c*x))*arc
tanh(c*x+(c^2*x^2+1)^(1/2))/Pi^(3/2)+3/2*b*c^2*polylog(2,-c*x-(c^2*x^2+1)^(
1/2))/Pi^(3/2)-3/2*b*c^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))/Pi^(3/2)-3/2*c^2*
(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+Pi)^(1/2)+1/2*(-a-b*arcsinh(c*x))/Pi/x^2/
(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 212, normalized size of antiderivative = 1.31, number of steps used = 11, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5747, 5755, 5760, 4182, 2279, 2391, 203, 325}

$$\frac{3bc^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{2\pi^{3/2}}-\frac{3bc^2\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{2\pi^{3/2}}-\frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2\pi\sqrt{\pi c^2x^2+\pi}}-\frac{a+b\sinh^{-1}(cx)}{2\pi x^2\sqrt{\pi c^2x^2+\pi}}+\frac{3c^2\ \text{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^3*(Pi + c^2*Pi*x^2)^(3/2)),x]
```

```
[Out] -(b*c*Sqrt[1 + c^2*x^2])/(2*Pi*x*Sqrt[Pi + c^2*Pi*x^2]) - (3*c^2*(a + b*Arc
Sinh[c*x]))/(2*Pi*Sqrt[Pi + c^2*Pi*x^2]) - (a + b*ArcSinh[c*x])/(2*Pi*x^2*S
qrt[Pi + c^2*Pi*x^2]) + (b*c^2*Sqrt[1 + c^2*x^2]*ArcTan[c*x])/(Pi*Sqrt[Pi +
 c^2*Pi*x^2]) + (3*c^2*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Pi^(3/
2) + (3*b*c^2*PolyLog[2, -E^ArcSinh[c*x]])/(2*Pi^(3/2)) - (3*b*c^2*PolyLog[
2, E^ArcSinh[c*x]])/(2*Pi^(3/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^3\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{2\pi x^2\sqrt{\pi+c^2\pi x^2}} - \frac{1}{2}\left(3c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{x\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx + \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{x^2\left(1+c^2x^2\right)}\,dx}{2\pi\sqrt{\pi+c^2\pi x^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2\pi x\sqrt{\pi+c^2\pi x^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2\pi\sqrt{\pi+c^2\pi x^2}} - \frac{a+b\sinh^{-1}(cx)}{2\pi x^2\sqrt{\pi+c^2\pi x^2}} - \frac{\left(3c^2\right)\int \frac{a+b\sinh^{-1}(c}{x\sqrt{\pi+c^2\pi x^2}}}{2\pi}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2\pi x\sqrt{\pi+c^2\pi x^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2\pi\sqrt{\pi+c^2\pi x^2}} - \frac{a+b\sinh^{-1}(cx)}{2\pi x^2\sqrt{\pi+c^2\pi x^2}} + \frac{bc^2\sqrt{1+c^2x^2}\,\tan}{\pi\sqrt{\pi+c^2\pi x^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2\pi x\sqrt{\pi+c^2\pi x^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2\pi\sqrt{\pi+c^2\pi x^2}} - \frac{a+b\sinh^{-1}(cx)}{2\pi x^2\sqrt{\pi+c^2\pi x^2}} + \frac{bc^2\sqrt{1+c^2x^2}\,\tan}{\pi\sqrt{\pi+c^2\pi x^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2\pi x\sqrt{\pi+c^2\pi x^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2\pi\sqrt{\pi+c^2\pi x^2}} - \frac{a+b\sinh^{-1}(cx)}{2\pi x^2\sqrt{\pi+c^2\pi x^2}} + \frac{bc^2\sqrt{1+c^2x^2}\,\tan}{\pi\sqrt{\pi+c^2\pi x^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2\pi x\sqrt{\pi+c^2\pi x^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2\pi\sqrt{\pi+c^2\pi x^2}} - \frac{a+b\sinh^{-1}(cx)}{2\pi x^2\sqrt{\pi+c^2\pi x^2}} + \frac{bc^2\sqrt{1+c^2x^2}\,\tan}{\pi\sqrt{\pi+c^2\pi x^2}}$$

Mathematica [A] time = 3.83, size = 269, normalized size = 1.66

$$-\frac{8ac^2}{\sqrt{c^2x^2+1}}-\frac{4a\sqrt{c^2x^2+1}}{x^2}+12ac^2\log\left(\pi\left(\sqrt{c^2x^2+1}+1\right)\right)-12ac^2\log(x)-12bc^2\text{Li}_2\left(-e^{-\sinh^{-1}(cx)}\right)+12bc^2\text{Li}_2\left(e\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^3*(Pi + c^2*Pi*x^2)^(3/2)),x]
```

```
[Out] ((-8*a*c^2)/Sqrt[1 + c^2*x^2] - (4*a*Sqrt[1 + c^2*x^2])/x^2 - (8*b*c^2*ArcS
inh[c*x])/Sqrt[1 + c^2*x^2] + 16*b*c^2*ArcTan[Tanh[ArcSinh[c*x]/2]] - 2*b*c
^2*Coth[ArcSinh[c*x]/2] - b*c^2*ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - 12*b*
c^2*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] + 12*b*c^2*ArcSinh[c*x]*Log[1 +
 E^(-ArcSinh[c*x])] - 12*a*c^2*Log[x] + 12*a*c^2*Log[Pi*(1 + Sqrt[1 + c^2*x
^2])] - 12*b*c^2*PolyLog[2, -E^(-ArcSinh[c*x])] + 12*b*c^2*PolyLog[2, E^(-A
rcSinh[c*x])] - b*c^2*ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*b*c^2*Tanh[Ar
cSinh[c*x]/2])/(8*Pi^(3/2))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\pi+\pi c^2x^2}\,(b\,\text{arsinh}\,(cx)+a)}{\pi^2c^4x^7+2\,\pi^2c^2x^5+\pi^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^2*c^4*x^7 + 2*pi^2*
c^2*x^5 + pi^2*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{b\,\text{arsinh}\,(cx)+a}{\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((pi + pi*c^2*x^2)^(3/2)*x^3), x)
```

maple [A] time = 0.30, size = 234, normalized size = 1.44

$$-\frac{a}{2\pi x^2\sqrt{\pi c^2x^2+\pi}}-\frac{3a c^2}{2\pi\sqrt{\pi c^2x^2+\pi}}+\frac{3a c^2\,\text{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)}{2\pi^{\frac{3}{2}}}-\frac{3b\,\text{arcsinh}\,(cx)\,c^2}{2\pi^{\frac{3}{2}}\sqrt{c^2x^2+1}}-\frac{bc}{2\pi^{\frac{3}{2}}x}-\frac{b\,\text{arcsinh}\,(cx}{2\pi^{\frac{3}{2}}\sqrt{c^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^3/(Pi*c^2*x^2+Pi)^(3/2),x)
```

```
[Out] -1/2*a/Pi/x^2/(Pi*c^2*x^2+Pi)^(1/2)-3/2*a*c^2/Pi/(Pi*c^2*x^2+Pi)^(1/2)+3/2*
a*c^2/Pi^(3/2)*arctanh(Pi^(1/2)/(Pi*c^2*x^2+Pi)^(1/2))-3/2*b/Pi^(3/2)/(c^2*
x^2+1)^(1/2)*arcsinh(c*x)*c^2-1/2*b*c/Pi^(3/2)/x-1/2*b/Pi^(3/2)/(c^2*x^2+1)
^(1/2)/x^2*arcsinh(c*x)+2*b*c^2/Pi^(3/2)*arctan(c*x+(c^2*x^2+1)^(1/2))+3/2*
b*c^2/Pi^(3/2)*dilog(c*x+(c^2*x^2+1)^(1/2))+3/2*b*c^2/Pi^(3/2)*dilog(1+c*x+
(c^2*x^2+1)^(1/2))+3/2*b*c^2/Pi^(3/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/
2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(\frac{3\,c^2\operatorname{arsinh}\left(\frac{1}{c|x|}\right)}{\pi^{\frac{3}{2}}} - \frac{3\,c^2}{\pi\sqrt{\pi+\pi c^2x^2}} - \frac{1}{\pi\sqrt{\pi+\pi c^2x^2}\,x^2}\right)a + b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima
")
```

```
[Out] 1/2*(3*c^2*arcsinh(1/(c*abs(x)))/pi^(3/2) - 3*c^2/(pi*sqrt(pi + pi*c^2*x^2)
) - 1/(pi*sqrt(pi + pi*c^2*x^2)*x^2))*a + b*integrate(log(c*x + sqrt(c^2*x^
2 + 1))/((pi + pi*c^2*x^2)^(3/2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{a+b\operatorname{asinh}(c\,x)}{x^3\left(\Pi\,c^2\,x^2+\Pi\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^3*(Pi + Pi*c^2*x^2)^(3/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^3*(Pi + Pi*c^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{a}{c^2x^5\sqrt{c^2x^2+1}+x^3\sqrt{c^2x^2+1}}\,dx + \int\frac{b\operatorname{asinh}(cx)}{c^2x^5\sqrt{c^2x^2+1}+x^3\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**3/(pi*c**2*x**2+pi)**(3/2),x)
```

```
[Out] (Integral(a/(c**2*x**5*sqrt(c**2*x**2 + 1) + x**3*sqrt(c**2*x**2 + 1)), x)
+ Integral(b*asinh(c*x)/(c**2*x**5*sqrt(c**2*x**2 + 1) + x**3*sqrt(c**2*x**
2 + 1)), x))/pi**(3/2)
```

3.99 $\int \frac{a+b\sinh^{-1}(cx)}{x^4\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=153

$$\frac{4c^2\left(a+b\sinh^{-1}(cx)\right)}{3\pi x\sqrt{\pi c^2x^2+\pi}}-\frac{a+b\sinh^{-1}(cx)}{3\pi x^3\sqrt{\pi c^2x^2+\pi}}+\frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3\pi\sqrt{\pi c^2x^2+\pi}}-\frac{5bc^3\log(x)}{3\pi^{3/2}}-\frac{bc^3\log\left(c^2x^2+1\right)}{2\pi^{3/2}}-\frac{bc}{6\pi^{3/2}x^2}$$

```
[Out] -1/6*b*c/Pi^(3/2)/x^2-5/3*b*c^3*ln(x)/Pi^(3/2)-1/2*b*c^3*ln(c^2*x^2+1)/Pi^(
3/2)+1/3*(-a-b*arcsinh(c*x))/Pi/x^3/(Pi*c^2*x^2+Pi)^(1/2)+4/3*c^2*(a+b*arcs
inh(c*x))/Pi/x/(Pi*c^2*x^2+Pi)^(1/2)+8/3*c^4*x*(a+b*arcsinh(c*x))/Pi/(Pi*c^
2*x^2+Pi)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 156, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {271, 191, 5732, 12, 1251, 893}

$$\frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{3/2}\sqrt{c^2x^2+1}}+\frac{4c^2\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{3/2}x\sqrt{c^2x^2+1}}-\frac{a+b\sinh^{-1}(cx)}{3\pi^{3/2}x^3\sqrt{c^2x^2+1}}-\frac{bc^3\log\left(c^2x^2+1\right)}{2\pi^{3/2}}-\frac{5bc^3\log(x)}{3\pi^{3/2}}-\frac{bc}{6\pi^{3/2}x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^4*(Pi + c^2*Pi*x^2)^(3/2)),x]
```

```
[Out] -(b*c)/(6*Pi^(3/2)*x^2) - (a + b*ArcSinh[c*x])/(3*Pi^(3/2)*x^3*Sqrt[1 + c^2
*x^2]) + (4*c^2*(a + b*ArcSinh[c*x]))/(3*Pi^(3/2)*x*Sqrt[1 + c^2*x^2]) + (8
*c^4*x*(a + b*ArcSinh[c*x]))/(3*Pi^(3/2)*Sqrt[1 + c^2*x^2]) - (5*b*c^3*Log[
x])/(3*Pi^(3/2)) - (b*c^3*Log[1 + c^2*x^2])/(2*Pi^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
```

```
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^4\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{3\pi^{3/2}x^3\sqrt{1+c^2x^2}} + \frac{4c^2\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{3/2}x\sqrt{1+c^2x^2}} + \frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{3/2}\sqrt{1+c^2x^2}} - \frac{(bc)\int \frac{-1+4c}{3x^3(}}{\pi^3}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{3\pi^{3/2}x^3\sqrt{1+c^2x^2}} + \frac{4c^2\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{3/2}x\sqrt{1+c^2x^2}} + \frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{3/2}\sqrt{1+c^2x^2}} - \frac{(bc)\int \frac{-1+4c}{x^3(}}{3\pi}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{3\pi^{3/2}x^3\sqrt{1+c^2x^2}} + \frac{4c^2\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{3/2}x\sqrt{1+c^2x^2}} + \frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{3/2}\sqrt{1+c^2x^2}} - \frac{(bc)\,\mathrm{Subst}\Big(}{}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{3\pi^{3/2}x^3\sqrt{1+c^2x^2}} + \frac{4c^2\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{3/2}x\sqrt{1+c^2x^2}} + \frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{3/2}\sqrt{1+c^2x^2}} - \frac{(bc)\,\mathrm{Subst}\Big(}{}$$

$$= -\frac{bc}{6\pi^{3/2}x^2} - \frac{a+b\sinh^{-1}(cx)}{3\pi^{3/2}x^3\sqrt{1+c^2x^2}} + \frac{4c^2\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{3/2}x\sqrt{1+c^2x^2}} + \frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{3/2}\sqrt{1+c^2x^2}} - \frac{5}{}$$

Mathematica [A] time = 0.20, size = 127, normalized size = 0.83

$$\frac{2a\left(8c^4x^4+4c^2x^2-1\right)-bcx\sqrt{c^2x^2+1}+2b\left(8c^4x^4+4c^2x^2-1\right)\sinh^{-1}(cx)}{6\pi^{3/2}x^3\sqrt{c^2x^2+1}}+\frac{-5bc^3\log(x)-\frac{3}{2}bc^3\log\left(c^2x^2+1\right)}{3\pi^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^4*(Pi + c^2*Pi*x^2)^(3/2)),x]
```

```
[Out] (-(b*c*x*Sqrt[1 + c^2*x^2]) + 2*a*(-1 + 4*c^2*x^2 + 8*c^4*x^4) + 2*b*(-1 +
4*c^2*x^2 + 8*c^4*x^4)*ArcSinh[c*x])/(6*Pi^(3/2)*x^3*Sqrt[1 + c^2*x^2]) + (
-5*b*c^3*Log[x] - (3*b*c^3*Log[1 + c^2*x^2])/2)/(3*Pi^(3/2))
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{\pi+\pi c^2x^2}\,(b\,\mathrm{arsinh}\,(cx)+a)}{\pi^2c^4x^8+2\,\pi^2c^2x^6+\pi^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^2*c^4*x^8 + 2*pi^2*
c^2*x^6 + pi^2*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\mathrm{arsinh}\,(cx) + a}{\left(\pi + \pi c^2x^2\right)^{\frac{3}{2}} x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((pi + pi*c^2*x^2)^(3/2)*x^4), x)
```

maple [B] time = 0.32, size = 601, normalized size = 3.93

$$-\frac{a}{3\pi\,x^3\sqrt{\pi\,c^2x^2+\pi}}+\frac{4a\,c^2}{3\pi x\sqrt{\pi\,c^2x^2+\pi}}+\frac{8a\,c^4x}{3\pi\sqrt{\pi\,c^2x^2+\pi}}+\frac{16b\,c^3\,\mathrm{arcsinh}\,(cx)}{3\pi^{\frac{3}{2}}}-\frac{32b\,x^8c^{11}}{3\pi^{\frac{3}{2}}\left(8c^2x^2-1\right)\left(c^2x^2+1\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^4/(Pi*c^2*x^2+Pi)^(3/2),x)
```

```
[Out] -1/3*a/Pi/x^3/(Pi*c^2*x^2+Pi)^(1/2)+4/3*a*c^2/Pi/x/(Pi*c^2*x^2+Pi)^(1/2)+8/
3*a*c^4/Pi*x/(Pi*c^2*x^2+Pi)^(1/2)+16/3*b*c^3/Pi^(3/2)*arcsinh(c*x)-32/3*b/
Pi^(3/2)/(8*c^2*x^2-1)*x^8/(c^2*x^2+1)*c^11+32/3*b/Pi^(3/2)/(8*c^2*x^2-1)*x
^6*c^9-64/3*b/Pi^(3/2)/(8*c^2*x^2-1)*x^6/(c^2*x^2+1)*c^9+32/3*b/Pi^(3/2)/(8
*c^2*x^2-1)*x^4*c^7-64/3*b/Pi^(3/2)/(8*c^2*x^2-1)*x^4/(c^2*x^2+1)*arcsinh(c
*x)*c^7+64/3*b/Pi^(3/2)/(8*c^2*x^2-1)*x^3/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^
6-32/3*b/Pi^(3/2)/(8*c^2*x^2-1)*x^4/(c^2*x^2+1)*c^7-56/3*b/Pi^(3/2)/(8*c^2*
x^2-1)*x^2/(c^2*x^2+1)*arcsinh(c*x)*c^5+8*b/Pi^(3/2)/(8*c^2*x^2-1)*x/(c^2*x
^2+1)^(1/2)*arcsinh(c*x)*c^4-4/3*b/Pi^(3/2)/(8*c^2*x^2-1)*c^3+8/3*b/Pi^(3/2
)/(8*c^2*x^2-1)/(c^2*x^2+1)*arcsinh(c*x)*c^3-4*b/Pi^(3/2)/(8*c^2*x^2-1)/x/(
c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^2+1/6*b/Pi^(3/2)/(8*c^2*x^2-1)/x^2*c+1/3*b/
Pi^(3/2)/(8*c^2*x^2-1)/x^3/(c^2*x^2+1)^(1/2)*arcsinh(c*x)-b*c^3/Pi^(3/2)*ln
(1+(c*x+(c^2*x^2+1)^(1/2))^2)-5/3*b*c^3/Pi^(3/2)*ln((c*x+(c^2*x^2+1)^(1/2))
^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\left(\frac{8\,c^4x}{\pi\sqrt{\pi+\pi c^2x^2}}+\frac{4\,c^2}{\pi\sqrt{\pi+\pi c^2x^2}\,x}-\frac{1}{\pi\sqrt{\pi+\pi c^2x^2}\,x^3}\right)a+b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima
")
```

```
[Out] 1/3*(8*c^4*x/(pi*sqrt(pi + pi*c^2*x^2)) + 4*c^2/(pi*sqrt(pi + pi*c^2*x^2)*x
) - 1/(pi*sqrt(pi + pi*c^2*x^2)*x^3))*a + b*integrate(log(c*x + sqrt(c^2*x^
2 + 1))/((pi + pi*c^2*x^2)^(3/2)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{asinh}\,(c\,x)}{x^4\left(\Pi\,c^2\,x^2+\Pi\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^4*(Pi + Pi*c^2*x^2)^(3/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^4*(Pi + Pi*c^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^2x^6\sqrt{c^2x^2+1}+x^4\sqrt{c^2x^2+1}}\,dx + \int \frac{b\operatorname{asinh}(cx)}{c^2x^6\sqrt{c^2x^2+1}+x^4\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**4/(pi*c**2*x**2+pi)**(3/2),x)
```

```
[Out] (Integral(a/(c**2*x**6*sqrt(c**2*x**2 + 1) + x**4*sqrt(c**2*x**2 + 1)), x)
+ Integral(b*asinh(c*x)/(c**2*x**6*sqrt(c**2*x**2 + 1) + x**4*sqrt(c**2*x**
2 + 1)), x))/pi**(3/2)
```

3.100 $\int \frac{x^6\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=192

$$-\frac{5\left(a+b\sinh^{-1}(cx)\right)^2}{4\pi^{5/2}bc^7}-\frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{3\pi c^2\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{5x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{2\pi^3c^6}-\frac{5x^3\left(a+b\sinh^{-1}(cx)\right)}{3\pi^2c^4\sqrt{\pi c^2x^2+\pi}}-\frac{}{4\pi}$$

```
[Out] -1/4*b*x^2/c^5/Pi^(5/2)-1/6*b/c^7/Pi^(5/2)/(c^2*x^2+1)-1/3*x^5*(a+b*arcsinh
(c*x))/c^2/Pi/(Pi*c^2*x^2+Pi)^(3/2)-5/4*(a+b*arcsinh(c*x))^2/b/c^7/Pi^(5/2)
-7/6*b*ln(c^2*x^2+1)/c^7/Pi^(5/2)-5/3*x^3*(a+b*arcsinh(c*x))/c^4/Pi^2/(Pi*c
^2*x^2+Pi)^(1/2)+5/2*x*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/c^6/Pi^3
```

Rubi [A] time = 0.43, antiderivative size = 256, normalized size of antiderivative = 1.33, number of steps used = 11, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5751, 5758, 5675, 30, 266, 43}

$$-\frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{3\pi c^2\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{5x^3\left(a+b\sinh^{-1}(cx)\right)}{3\pi^2c^4\sqrt{\pi c^2x^2+\pi}}+\frac{5x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{2\pi^3c^6}-\frac{5\left(a+b\sinh^{-1}(cx)\right)^2}{4\pi^{5/2}bc^7}-\frac{}{4\pi}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] -b/(6*c^7*Pi^2*Sqrt[1 + c^2*x^2]*Sqrt[Pi + c^2*Pi*x^2]) - (b*x^2*Sqrt[1 + c
^2*x^2])/(4*c^5*Pi^2*Sqrt[Pi + c^2*Pi*x^2]) - (x^5*(a + b*ArcSinh[c*x]))/(3
*c^2*Pi*(Pi + c^2*Pi*x^2)^(3/2)) - (5*x^3*(a + b*ArcSinh[c*x]))/(3*c^4*Pi^2
*Sqrt[Pi + c^2*Pi*x^2]) + (5*x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/
(2*c^6*Pi^3) - (5*(a + b*ArcSinh[c*x])^2)/(4*b*c^7*Pi^(5/2)) - (7*b*Sqrt[1
+ c^2*x^2]*Log[1 + c^2*x^2])/(6*c^7*Pi^2*Sqrt[Pi + c^2*Pi*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^6\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx = -\frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}}+\frac{5\int\frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx}{3c^2\pi}+\frac{\left(b\sqrt{1+c^2x^2}\right)\int\frac{x^5}{\left(1+c^2x^2\right)^2}\,dx}{3c\pi^2\sqrt{\pi+c^2\pi x^2}}$$

$$= -\frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}}-\frac{5x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^4\pi^2\sqrt{\pi+c^2\pi x^2}}+\frac{5\int\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi+c^2\pi x^2}}\,dx}{c^4\pi^2}+\frac{\left(5b\sqrt{1\right.}{3c}$$

$$= -\frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}}-\frac{5x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^4\pi^2\sqrt{\pi+c^2\pi x^2}}+\frac{5x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(c\right.}{2c^6\pi^3}$$

$$= -\frac{b}{6c^7\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}}-\frac{13bx^2\sqrt{1+c^2x^2}}{12c^5\pi^2\sqrt{\pi+c^2\pi x^2}}-\frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}}-\frac{5x}{}$$

$$= -\frac{b}{6c^7\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}}-\frac{bx^2\sqrt{1+c^2x^2}}{4c^5\pi^2\sqrt{\pi+c^2\pi x^2}}-\frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}}-\frac{5x^3}{3}$$

Mathematica [A] time = 0.47, size = 202, normalized size = 1.05

$$\frac{4\sinh^{-1}(cx)\left(bcx\left(3c^4x^4+20c^2x^2+15\right)-15a\left(c^2x^2+1\right)^{3/2}\right)+12ac^5x^5+80ac^3x^3+60acx-9bc^2x^2\sqrt{c^2x^2+1}-}{24\pi^{5/2}c^7\left(c^2x^2+1\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] (60*a*c*x + 80*a*c^3*x^3 + 12*a*c^5*x^5 - 7*b*Sqrt[1 + c^2*x^2] - 9*b*c^2*x
^2*Sqrt[1 + c^2*x^2] - 6*b*c^4*x^4*Sqrt[1 + c^2*x^2] + 4*(-15*a*(1 + c^2*x^
2)^(3/2) + b*c*x*(15 + 20*c^2*x^2 + 3*c^4*x^4))*ArcSinh[c*x] - 30*b*(1 + c^
2*x^2)^(3/2)*ArcSinh[c*x]^2 - 28*b*(1 + c^2*x^2)^(3/2)*Log[1 + c^2*x^2])/(2
4*c^7*Pi^(5/2)*(1 + c^2*x^2)^(3/2))
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\pi + \pi c^2 x^2}\left(b x^6 \operatorname{arsinh}\left(c x\right) + a x^6\right)}{\pi^3 c^6 x^6 + 3\,\pi^3 c^4 x^4 + 3\,\pi^3 c^2 x^2 + \pi^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*x^6*arcsinh(c*x) + a*x^6)/(pi^3*c^6*x^6 +
 3*pi^3*c^4*x^4 + 3*pi^3*c^2*x^2 + pi^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.51, size = 970, normalized size = 5.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(5/2),x)
```

```
[Out] 1/2*b/Pi^(5/2)/c^6*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x-49/6*b/Pi^(5/2)/(63*c^4
*x^4+111*c^2*x^2+49)/(c^2*x^2+1)^2*c*x^8-98/3*b/Pi^(5/2)/(63*c^4*x^4+111*c^
2*x^2+49)/(c^2*x^2+1)^2/c*x^6-49*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^
2*x^2+1)^2/c^3*x^4-98/3*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)^
2/c^5*x^2-343/3*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)^2/c^7*ar
csinh(c*x)+49/6*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)/c*x^6+14
*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)/c^3*x^4+6*b/Pi^(5/2)/(6
3*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)/c^5*x^2+147*b/Pi^(5/2)/(63*c^4*x^4+11
1*c^2*x^2+49)/(c^2*x^2+1)^(3/2)*arcsinh(c*x)*x^7-1/4*b*x^2/c^5/Pi^(5/2)+14/
3*b/c^7/Pi^(5/2)*arcsinh(c*x)-5/4*b/c^7/Pi^(5/2)*arcsinh(c*x)^2-7/3*b/c^7/P
i^(5/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+1/2*a*x^5/Pi/c^2/(Pi*c^2*x^2+Pi)^(3
/2)-1463/3*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)^2/c^5*arcsinh
(c*x)*x^2+385*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)^(3/2)/c^2*
arcsinh(c*x)*x^5+1009/3*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)^
(3/2)/c^4*arcsinh(c*x)*x^3+98*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x
^2+1)^(3/2)/c^6*arcsinh(c*x)*x-147*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(
c^2*x^2+1)^2*c*arcsinh(c*x)*x^8-553*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/
(c^2*x^2+1)^2/c*arcsinh(c*x)*x^6-2338/3*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+
49)/(c^2*x^2+1)^2/c^3*arcsinh(c*x)*x^4+5/6*a/c^4*x^3/Pi/(Pi*c^2*x^2+Pi)^(3/
2)+5/2*a/c^6/Pi^2*x/(Pi*c^2*x^2+Pi)^(1/2)-5/2*a/c^6/Pi^2*ln(Pi*x*c^2/(Pi*c^
2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)-49/6*b/Pi^(5/2)/(63*c^4*x^4+
111*c^2*x^2+49)/(c^2*x^2+1)^2/c^7-1/8*b/Pi^(5/2)/c^7
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}\,a\left(\frac{3\,x^5}{\pi\left(\pi + \pi c^2 x^2\right)^{\frac{3}{2}} c^2} + \frac{5\,x\left(\frac{3\,x^2}{\pi\left(\pi+\pi c^2 x^2\right)^{\frac{3}{2}} c^2} + \frac{2}{\pi\left(\pi+\pi c^2 x^2\right)^{\frac{3}{2}} c^4}\right)}{c^2} + \frac{5\,x}{\pi^2\sqrt{\pi + \pi c^2 x^2}\, c^6} - \frac{15\,\operatorname{arsinh}\left(c x\right)}{\pi^{\frac{5}{2}} c^7}\right) + b\int \frac{x^6 \log\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima
")
```

```
[Out] 1/6*a*(3*x^5/(pi*(pi + pi*c^2*x^2)^(3/2)*c^2) + 5*x*(3*x^2/(pi*(pi + pi*c^2
*x^2)^(3/2)*c^2) + 2/(pi*(pi + pi*c^2*x^2)^(3/2)*c^4))/c^2 + 5*x/(pi^2*sqrt
(pi + pi*c^2*x^2)*c^6) - 15*arcsinh(c*x)/(pi^(5/2)*c^7)) + b*integrate(x^6*
log(c*x + sqrt(c^2*x^2 + 1))/(pi + pi*c^2*x^2)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{{\left(\Pi\,c^2\,x^2+\Pi\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2),x)
```

```
[Out] int((x^6*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^{6}}{c^{4}x^{4}\sqrt{c^{2}x^{2}+1}+2c^{2}x^{2}\sqrt{c^{2}x^{2}+1}+\sqrt{c^{2}x^{2}+1}}\, dx + \int \frac{bx^{6}\operatorname{asinh}{\left(cx \right)}}{c^{4}x^{4}\sqrt{c^{2}x^{2}+1}+2c^{2}x^{2}\sqrt{c^{2}x^{2}+1}+\sqrt{c^{2}x^{2}+1}}\, dx}{\pi^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] (Integral(a*x**6/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**
2 + 1) + sqrt(c**2*x**2 + 1)), x) + Integral(b*x**6*asinh(c*x)/(c**4*x**4*s
qrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1))
, x))/pi**(5/2)
```

3.101 $\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=146

$$\frac{\sqrt{\pi c^2x^2+\pi}\,\left(a+b\sinh^{-1}(cx)\right)}{\pi^3c^6}+\frac{2\left(a+b\sinh^{-1}(cx)\right)}{\pi^2c^6\sqrt{\pi c^2x^2+\pi}}-\frac{a+b\sinh^{-1}(cx)}{3\pi c^6\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{11b\tan^{-1}(cx)}{6\pi^{5/2}c^6}-\frac{bx}{\pi^{5/2}c^5}+\frac{b}{6\pi^{5/2}c^5\left(\right.}$$

```
[Out] -b*x/c^5/Pi^(5/2)+1/6*b*x/c^5/Pi^(5/2)/(c^2*x^2+1)+1/3*(-a-b*arcsinh(c*x))/
c^6/Pi/(Pi*c^2*x^2+Pi)^(3/2)-11/6*b*arctan(c*x)/c^6/Pi^(5/2)+2*(a+b*arcsinh
(c*x))/c^6/Pi^2/(Pi*c^2*x^2+Pi)^(1/2)+(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1
/2)/c^6/Pi^3
```

Rubi [A] time = 0.18, antiderivative size = 149, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {266, 43, 5732, 12, 1157, 388, 203}

$$\frac{\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)}{\pi^{5/2}c^6}+\frac{2\left(a+b\sinh^{-1}(cx)\right)}{\pi^{5/2}c^6\sqrt{c^2x^2+1}}-\frac{a+b\sinh^{-1}(cx)}{3\pi^{5/2}c^6\left(c^2x^2+1\right)^{3/2}}+\frac{bx}{6\pi^{5/2}c^5\left(c^2x^2+1\right)}-\frac{bx}{\pi^{5/2}c^5}-\frac{11b\,\mathrm{ta}}{6\pi}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] -((b*x)/(c^5*Pi^(5/2))) + (b*x)/(6*c^5*Pi^(5/2)*(1 + c^2*x^2)) - (a + b*Arc
Sinh[c*x])/(3*c^6*Pi^(5/2)*(1 + c^2*x^2)^(3/2)) + (2*(a + b*ArcSinh[c*x]))/
(c^6*Pi^(5/2)*Sqrt[1 + c^2*x^2]) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))
/(c^6*Pi^(5/2)) - (11*b*ArcTan[c*x])/(6*c^6*Pi^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx=-\frac{a+b\sinh^{-1}(cx)}{3c^6\pi^{5/2}\left(1+c^2x^2\right)^{3/2}}+\frac{2\left(a+b\sinh^{-1}(cx)\right)}{c^6\pi^{5/2}\sqrt{1+c^2x^2}}+\frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^6\pi^{5/2}}-\ldots$$

$$=-\frac{a+b\sinh^{-1}(cx)}{3c^6\pi^{5/2}\left(1+c^2x^2\right)^{3/2}}+\frac{2\left(a+b\sinh^{-1}(cx)\right)}{c^6\pi^{5/2}\sqrt{1+c^2x^2}}+\frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^6\pi^{5/2}}-\ldots$$

$$=\frac{bx}{6c^5\pi^{5/2}\left(1+c^2x^2\right)}-\frac{a+b\sinh^{-1}(cx)}{3c^6\pi^{5/2}\left(1+c^2x^2\right)^{3/2}}+\frac{2\left(a+b\sinh^{-1}(cx)\right)}{c^6\pi^{5/2}\sqrt{1+c^2x^2}}+\frac{\sqrt{1+c^2x^2}\left(a\ldots\right.}{c^6\ldots}$$

$$=-\frac{bx}{c^5\pi^{5/2}}+\frac{bx}{6c^5\pi^{5/2}\left(1+c^2x^2\right)}-\frac{a+b\sinh^{-1}(cx)}{3c^6\pi^{5/2}\left(1+c^2x^2\right)^{3/2}}+\frac{2\left(a+b\sinh^{-1}(cx)\right)}{c^6\pi^{5/2}\sqrt{1+c^2x^2}}+\frac{\sqrt{1}\ldots}{\ldots}$$

$$=-\frac{bx}{c^5\pi^{5/2}}+\frac{bx}{6c^5\pi^{5/2}\left(1+c^2x^2\right)}-\frac{a+b\sinh^{-1}(cx)}{3c^6\pi^{5/2}\left(1+c^2x^2\right)^{3/2}}+\frac{2\left(a+b\sinh^{-1}(cx)\right)}{c^6\pi^{5/2}\sqrt{1+c^2x^2}}+\frac{\sqrt{1}\ldots}{\ldots}$$

Mathematica [A] time = 0.21, size = 132, normalized size = 0.90

$$\frac{6ac^4x^4+24ac^2x^2+16a-5bcx\sqrt{c^2x^2+1}-11b\left(c^2x^2+1\right)^{3/2}\tan^{-1}(cx)+2b\left(3c^4x^4+12c^2x^2+8\right)\sinh^{-1}(cx)-6b\ldots}{6\pi^{5/2}c^6\left(c^2x^2+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] (16*a + 24*a*c^2*x^2 + 6*a*c^4*x^4 - 5*b*c*x*Sqrt[1 + c^2*x^2] - 6*b*c^3*x^
3*Sqrt[1 + c^2*x^2] + 2*b*(8 + 12*c^2*x^2 + 3*c^4*x^4)*ArcSinh[c*x] - 11*b*
(1 + c^2*x^2)^(3/2)*ArcTan[c*x])/(6*c^6*Pi^(5/2)*(1 + c^2*x^2)^(3/2))
```

fricas [A] time = 0.89, size = 218, normalized size = 1.49

$$\frac{11\sqrt{\pi}\left(bc^4x^4+2\,bc^2x^2+b\right)\arctan\left(-\frac{2\sqrt{\pi}\sqrt{\pi+\pi c^2x^2}\sqrt{c^2x^2+1}\,cx}{\pi-\pi c^4x^4}\right)+4\sqrt{\pi+\pi c^2x^2}\left(3\,bc^4x^4+12\,bc^2x^2+8\,b\right)\log\left(\right.}{12\left(\pi^3c^{10}x^4+2\,\pi^3c^8x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas
")
```

```
[Out] 1/12*(11*sqrt(pi)*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*arctan(-2*sqrt(pi)*sqrt(pi
+ pi*c^2*x^2)*sqrt(c^2*x^2 + 1)*c*x/(pi - pi*c^4*x^4)) + 4*sqrt(pi + pi*c^2
*x^2)*(3*b*c^4*x^4 + 12*b*c^2*x^2 + 8*b)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*s
qrt(pi + pi*c^2*x^2)*(6*a*c^4*x^4 + 24*a*c^2*x^2 - (6*b*c^3*x^3 + 5*b*c*x)*
sqrt(c^2*x^2 + 1) + 16*a))/(pi^3*c^10*x^4 + 2*pi^3*c^8*x^2 + pi^3*c^6)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [C] time = 0.48, size = 231, normalized size = 1.58

$$\frac{a\,x^4}{\pi\,c^2\left(\pi\,c^2x^2+\pi\right)^{\frac{3}{2}}}+\frac{4a\,x^2}{c^4\pi\left(\pi\,c^2x^2+\pi\right)^{\frac{3}{2}}}+\frac{8a}{3c^6\pi\left(\pi\,c^2x^2+\pi\right)^{\frac{3}{2}}}+\frac{b\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}}{\pi^{\frac{5}{2}}c^6}-\frac{bx}{c^5\pi^{\frac{5}{2}}}+\frac{2b\operatorname{arcsinh}(c}{\pi^{\frac{5}{2}}\left(c^2x^2+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(5/2),x)
```

```
[Out] a*x^4/Pi/c^2/(Pi*c^2*x^2+Pi)^(3/2)+4*a/c^4*x^2/Pi/(Pi*c^2*x^2+Pi)^(3/2)+8/3
*a/c^6/Pi/(Pi*c^2*x^2+Pi)^(3/2)+b/Pi^(5/2)/c^6*arcsinh(c*x)*(c^2*x^2+1)^(1/
2)-b*x/c^5/Pi^(5/2)+2*b/Pi^(5/2)/(c^2*x^2+1)^(3/2)/c^4*arcsinh(c*x)*x^2+1/6
*b*x/c^5/Pi^(5/2)/(c^2*x^2+1)+5/3*b/Pi^(5/2)/(c^2*x^2+1)^(3/2)/c^6*arcsinh(
c*x)+11/6*I*b/c^6/Pi^(5/2)*ln(c*x+(c^2*x^2+1)^(1/2)-I)-11/6*I*b/c^6/Pi^(5/2
)*ln(c*x+(c^2*x^2+1)^(1/2)+I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}b\left(\frac{\left(3\sqrt{\pi}\,c^4x^4+12\sqrt{\pi}\,c^2x^2+8\sqrt{\pi}\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}{\left(\pi^3c^8x^2+\pi^3c^6\right)\sqrt{c^2x^2+1}}+3\int\frac{3\sqrt{\pi}\,c^4x^4+12\sqrt{\pi}}{3\left(\pi^3c^{11}x^6+2\,\pi^3c^9x^4+\pi^3c^7x^2+\left(\pi^3c^{10}\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima
")
```

```
[Out] 1/3*b*((3*sqrt(pi)*c^4*x^4 + 12*sqrt(pi)*c^2*x^2 + 8*sqrt(pi))*log(c*x + sq
rt(c^2*x^2 + 1))/((pi^3*c^8*x^2 + pi^3*c^6)*sqrt(c^2*x^2 + 1)) + 3*integrat
e(1/3*(3*sqrt(pi)*c^4*x^4 + 12*sqrt(pi)*c^2*x^2 + 8*sqrt(pi))/(pi^3*c^11*x^
6 + 2*pi^3*c^9*x^4 + pi^3*c^7*x^2 + (pi^3*c^10*x^5 + 2*pi^3*c^8*x^3 + pi^3*
```

```
c^6*x)*sqrt(c^2*x^2 + 1)), x) - 3*integrate(1/3*(3*sqrt(pi)*c^4*x^4 + 12*sq
rt(pi)*c^2*x^2 + 8*sqrt(pi))/((pi^3*c^8*x^3 + pi^3*c^6*x)*sqrt(c^2*x^2 + 1)
), x)) + 1/3*a*(3*x^4/(pi*(pi + pi*c^2*x^2)^(3/2)*c^2) + 12*x^2/(pi*(pi + p
i*c^2*x^2)^(3/2)*c^4) + 8/(pi*(pi + pi*c^2*x^2)^(3/2)*c^6))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{{\left(\Pi\,c^2\,x^2+\Pi\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2),x)
```

```
[Out] int((x^5*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^5}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{bx^5\,\mathrm{asinh}\,(cx)}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] (Integral(a*x**5/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**
2 + 1) + sqrt(c**2*x**2 + 1)), x) + Integral(b*x**5*asinh(c*x)/(c**4*x**4*s
qrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1))
, x))/pi**(5/2)
```

3.102 $\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=139

$$\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2\pi^{5/2}bc^5}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3\pi c^2\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{\pi^2c^4\sqrt{\pi c^2x^2+\pi}}+\frac{b}{6\pi^{5/2}c^5\left(c^2x^2+1\right)}+\frac{2b\log\left(c^2x^2+1\right)}{3\pi^{5/2}c^5}$$

```
[Out] 1/6*b/c^5/Pi^(5/2)/(c^2*x^2+1)-1/3*x^3*(a+b*arcsinh(c*x))/c^2/Pi/(Pi*c^2*x^
2+Pi)^(3/2)+1/2*(a+b*arcsinh(c*x))^2/b/c^5/Pi^(5/2)+2/3*b*ln(c^2*x^2+1)/c^5
/Pi^(5/2)-x*(a+b*arcsinh(c*x))/c^4/Pi^2/(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 178, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5751, 5675, 260, 266, 43}

$$-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3\pi c^2\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{\pi^2c^4\sqrt{\pi c^2x^2+\pi}}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2\pi^{5/2}bc^5}+\frac{b}{6\pi^2c^5\sqrt{c^2x^2+1}\sqrt{\pi c^2x^2+\pi}}+\frac{2b\sqrt{c^2x^2+}}{3\pi^2c^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] b/(6*c^5*Pi^2*Sqrt[1 + c^2*x^2]*Sqrt[Pi + c^2*Pi*x^2]) - (x^3*(a + b*ArcSin
h[c*x]))/(3*c^2*Pi*(Pi + c^2*Pi*x^2)^(3/2)) - (x*(a + b*ArcSinh[c*x]))/(c^4
*Pi^2*Sqrt[Pi + c^2*Pi*x^2]) + (a + b*ArcSinh[c*x])^2/(2*b*c^5*Pi^(5/2)) +
(2*b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(3*c^5*Pi^2*Sqrt[Pi + c^2*Pi*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
```

```
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rubi steps

$$\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx = -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}}+\frac{\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx}{c^2\pi}+\frac{\left(b\sqrt{1+c^2x^2}\right)\int \frac{x^3}{\left(1+c^2x^2\right)^2}\,dx}{3c\pi^2\sqrt{\pi+c^2\pi x^2}}$$

$$= -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4\pi^2\sqrt{\pi+c^2\pi x^2}}+\frac{\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx}{c^4\pi^2}+\frac{\left(b\sqrt{1+c^2x^2}\right.}{c^3\pi^2\sqrt{\pi\,-}}$$

$$= -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4\pi^2\sqrt{\pi+c^2\pi x^2}}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^5\pi^{5/2}}+\frac{b\sqrt{1+c^2x^2}}{2c^5\pi^2\sqrt{}}$$

$$= \frac{b}{6c^5\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4\pi^2\sqrt{\pi+c^2\pi x^2}}+\frac{(a}{}$$

Mathematica [A] time = 0.36, size = 166, normalized size = 1.19

$$\frac{2\sinh^{-1}(cx)\left(3a\left(c^2x^2+1\right)^2-bcx\sqrt{c^2x^2+1}\left(4c^2x^2+3\right)\right)-6acx\sqrt{c^2x^2+1}-8ac^3x^3\sqrt{c^2x^2+1}+bc^2x^2+4b\left(c^2x\right.}{6\pi^{5/2}c^5\left(c^2x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] (b + b*c^2*x^2 - 6*a*c*x*Sqrt[1 + c^2*x^2] - 8*a*c^3*x^3*Sqrt[1 + c^2*x^2]
+ 2*(3*a*(1 + c^2*x^2)^2 - b*c*x*Sqrt[1 + c^2*x^2]*(3 + 4*c^2*x^2))*ArcSinh
[c*x] + 3*b*(1 + c^2*x^2)^2*ArcSinh[c*x]^2 + 4*b*(1 + c^2*x^2)^2*Log[1 + c^
2*x^2])/(6*c^5*Pi^(5/2)*(1 + c^2*x^2)^2)
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\pi+\pi c^2x^2}\left(bx^4\operatorname{arsinh}(cx)+ax^4\right)}{\pi^3c^6x^6+3\,\pi^3c^4x^4+3\,\pi^3c^2x^2+\pi^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*x^4*arcsinh(c*x) + a*x^4)/(pi^3*c^6*x^6 +
 3*pi^3*c^4*x^4 + 3*pi^3*c^2*x^2 + pi^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arsinh}(cx)+a)x^4}{\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^4/(pi + pi*c^2*x^2)^(5/2), x)
```

maple [B] time = 0.36, size = 897, normalized size = 6.45

$$-\frac{a\,x^3}{3\pi\,c^2\left(\pi\,c^2x^2+\pi\right)^{\frac{3}{2}}}-\frac{ax}{\pi^2c^4\sqrt{\pi\,c^2x^2+\pi}}+\frac{a\ln\left(\frac{\pi x\,c^2}{\sqrt{\pi\,c^2}}+\sqrt{\pi\,c^2x^2+\pi}\right)}{\pi^2c^4\sqrt{\pi\,c^2}}+\frac{b\,\mathrm{arcsinh}\,(cx)^2}{2c^5\pi^{\frac{5}{2}}}-\frac{8b\,\mathrm{arcsinh}\,(cx)}{3c^5\pi^{\frac{5}{2}}}+\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(5/2),x)
```

```
[Out] -1/3*a*x^3/Pi/c^2/(Pi*c^2*x^2+Pi)^(3/2)-a/Pi^2/c^4*x/(Pi*c^2*x^2+Pi)^(1/2)+
a/Pi^2/c^4*ln(Pi*x*c^2/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)
+1/2*b/c^5/Pi^(5/2)*arcsinh(c*x)^2-8/3*b/c^5/Pi^(5/2)*arcsinh(c*x)+32*b/Pi^
(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^2*c^3*arcsinh(c*x)*x^8-32*b/Pi
^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^(3/2)*c^2*arcsinh(c*x)*x^7+8/
3*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^2*c^3*x^8-8/3*b/Pi^(5/2
)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)*c*x^6+116*b/Pi^(5/2)/(24*c^4*x^4+3
9*c^2*x^2+16)/(c^2*x^2+1)^2*c*arcsinh(c*x)*x^6-76*b/Pi^(5/2)/(24*c^4*x^4+39
*c^2*x^2+16)/(c^2*x^2+1)^(3/2)*arcsinh(c*x)*x^5+32/3*b/Pi^(5/2)/(24*c^4*x^4
+39*c^2*x^2+16)/(c^2*x^2+1)^2*c*x^6-4*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)
/(c^2*x^2+1)/c*x^4+472/3*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^
2/c*arcsinh(c*x)*x^4-181/3*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1
)^(3/2)/c^2*arcsinh(c*x)*x^3+16*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*
x^2+1)^2/c*x^4-3/2*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)/c^3*x^
2+284/3*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^2/c^3*arcsinh(c*x
)*x^2-16*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^(3/2)/c^4*arcsin
h(c*x)*x+32/3*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^2/c^3*x^2+6
4/3*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^2/c^5*arcsinh(c*x)+8/
3*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^2/c^5+4/3*b/c^5/Pi^(5/2
)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}\left(x\left(\frac{3\,x^2}{\pi\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}c^2}+\frac{2}{\pi\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}c^4}\right)+\frac{x}{\pi^2\sqrt{\pi+\pi c^2x^2}\,c^4}-\frac{3\,\mathrm{arsinh}\,(cx)}{\pi^{\frac{5}{2}}c^5}\right)a+b\int\frac{x^4\log\left(cx+\sqrt{c^2x\ldots}\right.}{\left(\pi+\pi c^2x^2\right)\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima
")
```

```
[Out] -1/3*(x*(3*x^2/(pi*(pi + pi*c^2*x^2)^(3/2)*c^2) + 2/(pi*(pi + pi*c^2*x^2)^(
3/2)*c^4)) + x/(pi^2*sqrt(pi + pi*c^2*x^2)*c^4) - 3*arcsinh(c*x)/(pi^(5/2)*
c^5))*a + b*integrate(x^4*log(c*x + sqrt(c^2*x^2 + 1))/(pi + pi*c^2*x^2)^(5
/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^4\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{{\left(\Pi\,c^2\,x^2+\Pi\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2),x)
```

```
[Out] int((x^4*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^4}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{bx^4\operatorname{asinh}(cx)}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] (Integral(a*x**4/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**
2 + 1) + sqrt(c**2*x**2 + 1)), x) + Integral(b*x**4*asinh(c*x)/(c**4*x**4*s
qrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1))
, x))/pi**(5/2)
```

3.103 $\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=105

$$-\frac{a+b\sinh^{-1}(cx)}{\pi^2c^4\sqrt{\pi c^2x^2+\pi}}+\frac{a+b\sinh^{-1}(cx)}{3\pi c^4\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{5b\tan^{-1}(cx)}{6\pi^{5/2}c^4}-\frac{bx}{6\pi^{5/2}c^3\left(c^2x^2+1\right)}$$

```
[Out] -1/6*b*x/c^3/Pi^(5/2)/(c^2*x^2+1)+1/3*(a+b*arcsinh(c*x))/c^4/Pi/(Pi*c^2*x^2
+Pi)^(3/2)+5/6*b*arctan(c*x)/c^4/Pi^(5/2)+(-a-b*arcsinh(c*x))/c^4/Pi^2/(Pi*
c^2*x^2+Pi)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 107, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {266, 43, 5732, 12, 385, 203}

$$-\frac{a+b\sinh^{-1}(cx)}{\pi^{5/2}c^4\sqrt{c^2x^2+1}}+\frac{a+b\sinh^{-1}(cx)}{3\pi^{5/2}c^4\left(c^2x^2+1\right)^{3/2}}-\frac{bx}{6\pi^{5/2}c^3\left(c^2x^2+1\right)}+\frac{5b\tan^{-1}(cx)}{6\pi^{5/2}c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] -(b*x)/(6*c^3*Pi^(5/2)*(1 + c^2*x^2)) + (a + b*ArcSinh[c*x])/(3*c^4*Pi^(5/2
)*(1 + c^2*x^2)^(3/2)) - (a + b*ArcSinh[c*x])/(c^4*Pi^(5/2)*Sqrt[1 + c^2*x^
2]) + (5*b*ArcTan[c*x])/(6*c^4*Pi^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx = \frac{a+b\sinh^{-1}(cx)}{3c^4\pi^{5/2}\left(1+c^2x^2\right)^{3/2}} - \frac{a+b\sinh^{-1}(cx)}{c^4\pi^{5/2}\sqrt{1+c^2x^2}} - \frac{(bc)\int \frac{-2-3c^2x^2}{3c^4\left(1+c^2x^2\right)^2}\,dx}{\pi^{5/2}}$$

$$= \frac{a+b\sinh^{-1}(cx)}{3c^4\pi^{5/2}\left(1+c^2x^2\right)^{3/2}} - \frac{a+b\sinh^{-1}(cx)}{c^4\pi^{5/2}\sqrt{1+c^2x^2}} - \frac{b\int \frac{-2-3c^2x^2}{\left(1+c^2x^2\right)^2}\,dx}{3c^3\pi^{5/2}}$$

$$= -\frac{bx}{6c^3\pi^{5/2}\left(1+c^2x^2\right)} + \frac{a+b\sinh^{-1}(cx)}{3c^4\pi^{5/2}\left(1+c^2x^2\right)^{3/2}} - \frac{a+b\sinh^{-1}(cx)}{c^4\pi^{5/2}\sqrt{1+c^2x^2}} + \frac{(5b)\int \frac{1}{1+c^2x^2}\,dx}{6c^3\pi^{5/2}}$$

$$= -\frac{bx}{6c^3\pi^{5/2}\left(1+c^2x^2\right)} + \frac{a+b\sinh^{-1}(cx)}{3c^4\pi^{5/2}\left(1+c^2x^2\right)^{3/2}} - \frac{a+b\sinh^{-1}(cx)}{c^4\pi^{5/2}\sqrt{1+c^2x^2}} + \frac{5b\tan^{-1}(cx)}{6c^4\pi^{5/2}}$$

Mathematica [A] time = 0.18, size = 93, normalized size = 0.89

$$\frac{-6ac^2x^2-4a-bcx\sqrt{c^2x^2+1}+5b\left(c^2x^2+1\right)^{3/2}\tan^{-1}(cx)-2b\left(3c^2x^2+2\right)\sinh^{-1}(cx)}{6\pi^{5/2}c^4\left(c^2x^2+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] (-4*a - 6*a*c^2*x^2 - b*c*x*Sqrt[1 + c^2*x^2] - 2*b*(2 + 3*c^2*x^2)*ArcSinh
[c*x] + 5*b*(1 + c^2*x^2)^(3/2)*ArcTan[c*x])/(6*c^4*Pi^(5/2)*(1 + c^2*x^2)^
(3/2))
```

fricas [B] time = 0.60, size = 187, normalized size = 1.78

$$-\frac{5\sqrt{\pi}\left(bc^4x^4+2bc^2x^2+b\right)\arctan\left(-\frac{2\sqrt{\pi}\sqrt{\pi+\pi c^2x^2}\sqrt{c^2x^2+1}\,cx}{\pi-\pi c^4x^4}\right)+4\sqrt{\pi+\pi c^2x^2}\left(3bc^2x^2+2b\right)\log\left(cx+\sqrt{c^2x^2+\right.}{12\left(\pi^3c^8x^4+2\pi^3c^6x^2+\pi^3c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas
")
```

```
[Out] -1/12*(5*sqrt(pi)*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*arctan(-2*sqrt(pi)*sqrt(pi
+ pi*c^2*x^2)*sqrt(c^2*x^2 + 1)*c*x/(pi - pi*c^4*x^4)) + 4*sqrt(pi + pi*c^2
*x^2)*(3*b*c^2*x^2 + 2*b)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*sqrt(pi + pi*c^2
*x^2)*(6*a*c^2*x^2 + sqrt(c^2*x^2 + 1)*b*c*x + 4*a))/(pi^3*c^8*x^4 + 2*pi^3
*c^6*x^2 + pi^3*c^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [C] time = 0.52, size = 175, normalized size = 1.67

$$-\frac{a\,x^2}{\pi\,c^2\left(\pi\,c^2x^2+\pi\right)^{\frac{3}{2}}}-\frac{2a}{3\pi\,c^4\left(\pi\,c^2x^2+\pi\right)^{\frac{3}{2}}}-\frac{b\,\mathrm{arcsinh}\,(cx)\,x^2}{\pi^{\frac{5}{2}}\left(c^2x^2+1\right)^{\frac{3}{2}}c^2}-\frac{bx}{6c^3\pi^{\frac{5}{2}}\left(c^2x^2+1\right)}-\frac{2b\,\mathrm{arcsinh}\,(cx)}{3\pi^{\frac{5}{2}}\left(c^2x^2+1\right)^{\frac{3}{2}}c^4}+\frac{5ib\ln\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(5/2),x)
```

```
[Out] -a*x^2/Pi/c^2/(Pi*c^2*x^2+Pi)^(3/2)-2/3*a/Pi/c^4/(Pi*c^2*x^2+Pi)^(3/2)-b/Pi
^(5/2)/(c^2*x^2+1)^(3/2)/c^2*arcsinh(c*x)*x^2-1/6*b*x/c^3/Pi^(5/2)/(c^2*x^2
+1)-2/3*b/Pi^(5/2)/(c^2*x^2+1)^(3/2)/c^4*arcsinh(c*x)+5/6*I*b/c^4/Pi^(5/2)*
ln(c*x+(c^2*x^2+1)^(1/2)+I)-5/6*I*b/c^4/Pi^(5/2)*ln(c*x+(c^2*x^2+1)^(1/2)-I
)
```

maxima [A] time = 0.61, size = 138, normalized size = 1.31

$$-\frac{1}{6}\,bc\left(\frac{x}{\pi^{\frac{5}{2}}c^6x^2+\pi^{\frac{5}{2}}c^4}-\frac{5\,\arctan\,(cx)}{\pi^{\frac{5}{2}}c^5}\right)-\frac{1}{3}\,b\left(\frac{3\,x^2}{\pi\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}c^2}+\frac{2}{\pi\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}c^4}\right)\mathrm{arsinh}\,(cx)-\frac{1}{3}\,a\left(\frac{}{\pi\left(\pi+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima
")
```

```
[Out] -1/6*b*c*(x/(pi^(5/2)*c^6*x^2 + pi^(5/2)*c^4) - 5*arctan(c*x)/(pi^(5/2)*c^5
)) - 1/3*b*(3*x^2/(pi*(pi + pi*c^2*x^2)^(3/2)*c^2) + 2/(pi*(pi + pi*c^2*x^2
)^(3/2)*c^4))*arcsinh(c*x) - 1/3*a*(3*x^2/(pi*(pi + pi*c^2*x^2)^(3/2)*c^2)
+ 2/(pi*(pi + pi*c^2*x^2)^(3/2)*c^4))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{{\left(\Pi\,c^2\,x^2+\Pi\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2),x)
```

```
[Out] int((x^3*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^3}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{bx^3\,\mathrm{asinh}\,(cx)}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] (Integral(a*x**3/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**
2 + 1) + sqrt(c**2*x**2 + 1)), x) + Integral(b*x**3*asinh(c*x)/(c**4*x**4*s
qrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1))
, x))/pi**(5/2)
```

3.104 $\int \frac{x^2(a+b\sinh^{-1}(cx))}{(\pi+c^2\pi x^2)^{5/2}}\,dx$

Optimal. Leaf size=80

$$\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{b}{6\pi^{5/2}c^3\left(c^2x^2+1\right)}-\frac{b\log\left(c^2x^2+1\right)}{6\pi^{5/2}c^3}$$

```
[Out] -1/6*b/c^3/Pi^(5/2)/(c^2*x^2+1)+1/3*x^3*(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+P
i)^(3/2)-1/6*b*ln(c^2*x^2+1)/c^3/Pi^(5/2)
```

Rubi [A] time = 0.13, antiderivative size = 119, normalized size of antiderivative = 1.49, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {5723, 266, 43}

$$\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{b}{6\pi^2c^3\sqrt{c^2x^2+1}\sqrt{\pi c^2x^2+\pi}}-\frac{b\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)}{6\pi^2c^3\sqrt{\pi c^2x^2+\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] -b/(6*c^3*Pi^2*Sqrt[1 + c^2*x^2]*Sqrt[Pi + c^2*Pi*x^2]) + (x^3*(a + b*ArcSi
nh[c*x]))/(3*Pi*(Pi + c^2*Pi*x^2)^(3/2)) - (b*Sqrt[1 + c^2*x^2]*Log[1 + c^2
*x^2])/(6*c^3*Pi^2*Sqrt[Pi + c^2*Pi*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx = \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} - \frac{\left(bc\sqrt{1+c^2x^2}\right)\int\frac{x^3}{\left(1+c^2x^2\right)^2}\,dx}{3\pi^2\sqrt{\pi+c^2\pi x^2}}$$

$$= \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} - \frac{\left(bc\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{x}{\left(1+c^2x\right)^2}\,dx,x,x^2\right)}{6\pi^2\sqrt{\pi+c^2\pi x^2}}$$

$$= \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} - \frac{\left(bc\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\left(-\frac{1}{c^2\left(1+c^2x\right)^2}+\frac{1}{c^2\left(1+c^2x\right)}\right)dx,x,x^2\right)}{6\pi^2\sqrt{\pi+c^2\pi x^2}}$$

$$= -\frac{b}{6c^3\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} - \frac{b\sqrt{1+c^2x^2}\log\left(1+c^2x^2\right)}{6c^3\pi^2\sqrt{\pi+c^2\pi x^2}}$$

Mathematica [A] time = 0.17, size = 88, normalized size = 1.10

$$-\frac{-2ac^3x^3-2bc^3x^3\sinh^{-1}(cx)+b\sqrt{c^2x^2+1}+b\left(c^2x^2+1\right)^{3/2}\log\left(c^2x^2+1\right)}{6\pi^{5/2}c^3\left(c^2x^2+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] -1/6*(-2*a*c^3*x^3 + b*Sqrt[1 + c^2*x^2] - 2*b*c^3*x^3*ArcSinh[c*x] + b*(1
+ c^2*x^2)^(3/2)*Log[1 + c^2*x^2])/(c^3*Pi^(5/2)*(1 + c^2*x^2)^(3/2))
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\pi+\pi c^2x^2}\left(bx^2\operatorname{arsinh}(cx)+ax^2\right)}{\pi^3c^6x^6+3\pi^3c^4x^4+3\pi^3c^2x^2+\pi^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*x^2*arcsinh(c*x) + a*x^2)/(pi^3*c^6*x^6 +
 3*pi^3*c^4*x^4 + 3*pi^3*c^2*x^2 + pi^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\operatorname{arsinh}(cx)+a)x^2}{\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^2/(pi + pi*c^2*x^2)^(5/2), x)
```

maple [B] time = 0.30, size = 707, normalized size = 8.84

$$-\frac{ax}{3c^2\pi\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}}+\frac{ax}{3c^2\pi^2\sqrt{\pi c^2x^2+\pi}}+\frac{2b\operatorname{arcsinh}(cx)}{3c^3\pi^{\frac{5}{2}}}-\frac{b\,c^5\operatorname{arcsinh}(cx)\,x^8}{\pi^{\frac{5}{2}}\left(3c^4x^4+3c^2x^2+1\right)\left(c^2x^2+1\right)^2}+\frac{b\,c^4\operatorname{ar}}{\pi^{\frac{5}{2}}\left(3c^4x^4+3c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(5/2),x)
```

```
[Out] -1/3*a/c^2/Pi*x/(Pi*c^2*x^2+Pi)^(3/2)+1/3*a/c^2/Pi^2*x/(Pi*c^2*x^2+Pi)^(1/2
)+2/3*b/c^3/Pi^(5/2)*arcsinh(c*x)-b/Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x
^2+1)^2*c^5*arcsinh(c*x)*x^8+b/Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)
^(3/2)*c^4*arcsinh(c*x)*x^7-1/6*b/Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2
+1)^2*c^5*x^8+1/6*b/Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)*c^3*x^6-3*
b/Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^2*c^3*arcsinh(c*x)*x^6+b/Pi^
(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^(3/2)*c^2*arcsinh(c*x)*x^5-2/3*b/
Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^2*c^3*x^6-10/3*b/Pi^(5/2)/(3*c
^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^2*c*arcsinh(c*x)*x^4+1/3*b/Pi^(5/2)/(3*c^4*
x^4+3*c^2*x^2+1)/(c^2*x^2+1)^(3/2)*arcsinh(c*x)*x^3-b/Pi^(5/2)/(3*c^4*x^4+3
*c^2*x^2+1)/(c^2*x^2+1)^2*c*x^4-5/3*b/Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2
*x^2+1)^2/c*arcsinh(c*x)*x^2-2/3*b/Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^
2+1)^2/c*x^2-1/3*b/Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^2/c^3*arcsi
nh(c*x)-1/6*b/Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^2/c^3-1/3*b/c^3/
Pi^(5/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
```

maxima [B] time = 0.53, size = 137, normalized size = 1.71

$$-\frac{1}{6}bc\left(\frac{1}{\pi^{\frac{5}{2}}c^6x^2+\pi^{\frac{5}{2}}c^4}+\frac{\log\left(c^2x^2+1\right)}{\pi^{\frac{5}{2}}c^4}\right)-\frac{1}{3}b\left(\frac{x}{\pi\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}c^2}-\frac{x}{\pi^2\sqrt{\pi+\pi c^2x^2}\,c^2}\right)\operatorname{arsinh}(cx)-\frac{1}{3}a\left(\frac{}{\pi(\pi+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima
")
```

```
[Out] -1/6*b*c*(1/(pi^(5/2)*c^6*x^2 + pi^(5/2)*c^4) + log(c^2*x^2 + 1)/(pi^(5/2)*
c^4)) - 1/3*b*(x/(pi*(pi + pi*c^2*x^2)^(3/2)*c^2) - x/(pi^2*sqrt(pi + pi*c^
2*x^2)*c^2))*arcsinh(c*x) - 1/3*a*(x/(pi*(pi + pi*c^2*x^2)^(3/2)*c^2) - x/(
pi^2*sqrt(pi + pi*c^2*x^2)*c^2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{{\left(\Pi\,c^2\,x^2+\Pi\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2),x)
```

```
[Out] int((x^2*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^2}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{bx^2\operatorname{asinh}(cx)}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] (Integral(a*x**2/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**
2 + 1) + sqrt(c**2*x**2 + 1)), x) + Integral(b*x**2*asinh(c*x)/(c**4*x**4*s
qrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1))
, x))/pi**(5/2)
```

3.105 $\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=75

$$-\frac{a+b\sinh^{-1}(cx)}{3\pi c^2\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{bx}{6\pi^{5/2}c\left(c^2x^2+1\right)}+\frac{b\tan^{-1}(cx)}{6\pi^{5/2}c^2}$$

```
[Out] 1/6*b*x/c/Pi^(5/2)/(c^2*x^2+1)+1/3*(-a-b*arcsinh(c*x))/c^2/Pi/(Pi*c^2*x^2+P
i)^(3/2)+1/6*b*arctan(c*x)/c^2/Pi^(5/2)
```

Rubi [A] time = 0.08, antiderivative size = 114, normalized size of antiderivative = 1.52, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5717, 199, 203}

$$-\frac{a+b\sinh^{-1}(cx)}{3\pi c^2\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{bx}{6\pi^2c\sqrt{c^2x^2+1}\sqrt{\pi c^2x^2+\pi}}+\frac{b\sqrt{c^2x^2+1}\tan^{-1}(cx)}{6\pi^2c^2\sqrt{\pi c^2x^2+\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] (b*x)/(6*c*Pi^2*Sqrt[1 + c^2*x^2]*Sqrt[Pi + c^2*Pi*x^2]) - (a + b*ArcSinh[c
*x])/(3*c^2*Pi*(Pi + c^2*Pi*x^2)^(3/2)) + (b*Sqrt[1 + c^2*x^2]*ArcTan[c*x])
/(6*c^2*Pi^2*Sqrt[Pi + c^2*Pi*x^2])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{\left(b\sqrt{1+c^2x^2}\right)\int\frac{1}{\left(1+c^2x^2\right)^2}\,dx}{3c\pi^2\sqrt{\pi+c^2\pi x^2}}$$

$$= \frac{bx}{6c\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} - \frac{a+b\sinh^{-1}(cx)}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{\left(b\sqrt{1+c^2x^2}\right)\int\frac{1}{1+c^2x^2}\,dx}{6c\pi^2\sqrt{\pi+c^2\pi x^2}}$$

$$= \frac{bx}{6c\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} - \frac{a+b\sinh^{-1}(cx)}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{b\sqrt{1+c^2x^2}\tan^{-1}(cx)}{6c^2\pi^2\sqrt{\pi+c^2\pi x^2}}$$

Mathematica [A] time = 0.14, size = 72, normalized size = 0.96

$$\frac{-2a+bcx\sqrt{c^2x^2+1}+b\left(c^2x^2+1\right)^{3/2}\tan^{-1}(cx)-2b\sinh^{-1}(cx)}{6\pi^{5/2}c^2\left(c^2x^2+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] (-2*a + b*c*x*Sqrt[1 + c^2*x^2] - 2*b*ArcSinh[c*x] + b*(1 + c^2*x^2)^(3/2)*
ArcTan[c*x])/(6*c^2*Pi^(5/2)*(1 + c^2*x^2)^(3/2))
```

fricas [B] time = 0.63, size = 165, normalized size = 2.20

$$-\frac{\sqrt{\pi}\left(bc^4x^4+2\,bc^2x^2+b\right)\arctan\left(-\frac{2\,\sqrt{\pi}\,\sqrt{\pi+\pi c^2x^2}\,\sqrt{c^2x^2+1}\,cx}{\pi-\pi c^4x^4}\right)+4\,\sqrt{\pi+\pi c^2x^2}\,b\log\left(cx+\sqrt{c^2x^2+1}\right)-2\,\sqrt{\pi}}{12\left(\pi^3c^6x^4+2\,\pi^3c^4x^2+\pi^3c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(sqrt(pi)*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*arctan(-2*sqrt(pi)*sqrt(pi +
pi*c^2*x^2)*sqrt(c^2*x^2 + 1)*c*x/(pi - pi*c^4*x^4)) + 4*sqrt(pi + pi*c^2*x
^2)*b*log(c*x + sqrt(c^2*x^2 + 1)) - 2*sqrt(pi + pi*c^2*x^2)*(sqrt(c^2*x^2
+ 1)*b*c*x - 2*a))/(pi^3*c^6*x^4 + 2*pi^3*c^4*x^2 + pi^3*c^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)x}{\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x/(pi + pi*c^2*x^2)^(5/2), x)
```

maple [C] time = 0.11, size = 124, normalized size = 1.65

$$-\frac{a}{3\pi\,c^2\left(\pi\,c^2x^2+\pi\right)^{\frac{3}{2}}}+\frac{bx}{6c\,\pi^{\frac{5}{2}}\left(c^2x^2+1\right)}-\frac{b\,\mathrm{arcsinh}\,(cx)}{3\pi^{\frac{5}{2}}\left(c^2x^2+1\right)^{\frac{3}{2}}c^2}+\frac{ib\ln\left(cx+\sqrt{c^2x^2+1}+i\right)}{6c^2\pi^{\frac{5}{2}}}-\frac{ib\ln\left(cx+\sqrt{c^2x^2+\right.}{6c^2\pi^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(5/2),x)
```

```
[Out] -1/3*a/Pi/c^2/(Pi*c^2*x^2+Pi)^(3/2)+1/6*b*x/c/Pi^(5/2)/(c^2*x^2+1)-1/3*b/Pi
^(5/2)/(c^2*x^2+1)^(3/2)/c^2*arcsinh(c*x)+1/6*I*b/c^2/Pi^(5/2)*ln(c*x+(c^2*
x^2+1)^(1/2)+I)-1/6*I*b/c^2/Pi^(5/2)*ln(c*x+(c^2*x^2+1)^(1/2)-I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\int \frac{x\log\left(cx+\sqrt{c^2x^2+1}\right)}{\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}}\,dx-\frac{a}{3\,\pi\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima")
```

```
[Out] b*integrate(x*log(c*x + sqrt(c^2*x^2 + 1))/(pi + pi*c^2*x^2)^(5/2), x) - 1/
3*a/(pi*(pi + pi*c^2*x^2)^(3/2)*c^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{{\left(\Pi\,c^2\,x^2+\Pi\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2),x)
```

```
[Out] int((x*(a + b*asinh(c*x)))/(Pi + Pi*c^2*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{bx\,\mathrm{asinh}\,(cx)}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] (Integral(a*x/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 +
 1) + sqrt(c**2*x**2 + 1)), x) + Integral(b*x*asinh(c*x)/(c**4*x**4*sqrt(c*
*2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x))/
pi**(5/2)
```

3.106 $\int \frac{a+b\sinh^{-1}(cx)}{(\pi+c^2\pi x^2)^{5/2}}\,dx$

Optimal. Leaf size=108

$$\frac{2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^2\sqrt{\pi c^2x^2+\pi}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{b}{6\pi^{5/2}c\left(c^2x^2+1\right)}-\frac{b\log\left(c^2x^2+1\right)}{3\pi^{5/2}c}$$

```
[Out] 1/6*b/c/Pi^(5/2)/(c^2*x^2+1)+1/3*x*(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+Pi)^(3
/2)-1/3*b*ln(c^2*x^2+1)/c/Pi^(5/2)+2/3*x*(a+b*arcsinh(c*x))/Pi^2/(Pi*c^2*x^
2+Pi)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 147, normalized size of antiderivative = 1.36, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5690, 5687, 260, 261}

$$\frac{2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^2\sqrt{\pi c^2x^2+\pi}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{b}{6\pi^2c\sqrt{c^2x^2+1}\sqrt{\pi c^2x^2+\pi}}-\frac{b\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)}{3\pi^2c\sqrt{\pi c^2x^2+\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] b/(6*c*Pi^2*Sqrt[1 + c^2*x^2]*Sqrt[Pi + c^2*Pi*x^2]) + (x*(a + b*ArcSinh[c*
x]))/(3*Pi*(Pi + c^2*Pi*x^2)^(3/2)) + (2*x*(a + b*ArcSinh[c*x]))/(3*Pi^2*Sq
rt[Pi + c^2*Pi*x^2]) - (b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(3*c*Pi^2*Sqr
t[Pi + c^2*Pi*x^2])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx = \frac{x\left(a+b\sinh^{-1}(cx)\right)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{2\int \frac{a+b\sinh^{-1}(cx)}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx}{3\pi} - \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{x}{\left(1+c^2x^2\right)^2}\,dx}{3\pi^2\sqrt{\pi+c^2\pi x^2}}$$

$$= \frac{b}{6c\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^2\sqrt{\pi+c^2\pi x^2}} - \frac{\left(2bc\sqrt{1}\right.}{3\pi}$$

$$= \frac{b}{6c\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^2\sqrt{\pi+c^2\pi x^2}} - \frac{b\sqrt{1+c}}{3c\pi}$$

Mathematica [A] time = 0.14, size = 100, normalized size = 0.93

$$\frac{4ac^3x^3+6acx+b\sqrt{c^2x^2+1}-2b\left(c^2x^2+1\right)^{3/2}\log\left(c^2x^2+1\right)+2bcx\left(2c^2x^2+3\right)\sinh^{-1}(cx)}{6\pi^{5/2}c\left(c^2x^2+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] (6*a*c*x + 4*a*c^3*x^3 + b*Sqrt[1 + c^2*x^2] + 2*b*c*x*(3 + 2*c^2*x^2)*ArcS
inh[c*x] - 2*b*(1 + c^2*x^2)^(3/2)*Log[1 + c^2*x^2])/(6*c*Pi^(5/2)*(1 + c^2
*x^2)^(3/2))
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{\pi+\pi c^2x^2}\,(b\,\mathrm{arsinh}\,(cx)+a)}{\pi^3c^6x^6+3\,\pi^3c^4x^4+3\,\pi^3c^2x^2+\pi^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^3*c^6*x^6 + 3*pi^3*
c^4*x^4 + 3*pi^3*c^2*x^2 + pi^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\mathrm{arsinh}\,(cx)+a}{\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(pi + pi*c^2*x^2)^(5/2), x)
```

maple [B] time = 0.12, size = 618, normalized size = 5.72

$$\frac{ax}{3\pi\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}}+\frac{2ax}{3\pi^2\sqrt{\pi c^2x^2+\pi}}+\frac{4b\,\mathrm{arcsinh}\,(cx)}{3c\,\pi^{\frac{5}{2}}}+\frac{2b\,c^7x^8}{3\pi^{\frac{5}{2}}\left(3c^2x^2+4\right)\left(c^2x^2+1\right)^2}-\frac{2b\,c^5x^6}{3\pi^{\frac{5}{2}}\left(3c^2x^2+4\right)\left(c^2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(5/2),x)
```

```
[Out] 1/3*a/Pi*x/(Pi*c^2*x^2+Pi)^(3/2)+2/3*a/Pi^2*x/(Pi*c^2*x^2+Pi)^(1/2)+4/3*b/c
/Pi^(5/2)*arcsinh(c*x)+2/3*b/Pi^(5/2)*c^7/(3*c^2*x^2+4)/(c^2*x^2+1)^2*x^8-2
/3*b/Pi^(5/2)*c^5/(3*c^2*x^2+4)/(c^2*x^2+1)*x^6-2*b/Pi^(5/2)*c^5/(3*c^2*x^2
+4)/(c^2*x^2+1)^2*arcsinh(c*x)*x^6+2*b/Pi^(5/2)*c^4/(3*c^2*x^2+4)/(c^2*x^2+
1)^(3/2)*arcsinh(c*x)*x^5+8/3*b/Pi^(5/2)*c^5/(3*c^2*x^2+4)/(c^2*x^2+1)^2*x^
6-2*b/Pi^(5/2)*c^3/(3*c^2*x^2+4)/(c^2*x^2+1)*x^4-20/3*b/Pi^(5/2)*c^3/(3*c^2
*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)*x^4+17/3*b/Pi^(5/2)*c^2/(3*c^2*x^2+4)/(c
^2*x^2+1)^(3/2)*arcsinh(c*x)*x^3+4*b/Pi^(5/2)*c^3/(3*c^2*x^2+4)/(c^2*x^2+1)
^2*x^4-3/2*b/Pi^(5/2)*c/(3*c^2*x^2+4)/(c^2*x^2+1)*x^2-22/3*b/Pi^(5/2)*c/(3*
c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)*x^2+4*b/Pi^(5/2)/(3*c^2*x^2+4)/(c^2*x
^2+1)^(3/2)*arcsinh(c*x)*x+8/3*b/Pi^(5/2)*c/(3*c^2*x^2+4)/(c^2*x^2+1)^2*x^2
-8/3*b/Pi^(5/2)/c/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)+2/3*b/Pi^(5/2)/c
/(3*c^2*x^2+4)/(c^2*x^2+1)^2-2/3*b/c/Pi^(5/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^
2)
```

maxima [A] time = 0.49, size = 126, normalized size = 1.17

$$\frac{1}{6}\,bc\left(\frac{1}{\pi^{\frac{5}{2}}c^4x^2+\pi^{\frac{5}{2}}c^2}-\frac{2\,\log\left(c^2x^2+1\right)}{\pi^{\frac{5}{2}}c^2}\right)+\frac{1}{3}\,b\left(\frac{x}{\pi\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}}+\frac{2\,x}{\pi^2\sqrt{\pi+\pi c^2x^2}}\right)\mathrm{arsinh}\left(cx\right)+\frac{1}{3}\,a\left(\frac{x}{\pi\left(\pi+\pi c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*b*c*(1/(pi^(5/2)*c^4*x^2 + pi^(5/2)*c^2) - 2*log(c^2*x^2 + 1)/(pi^(5/2)
*c^2)) + 1/3*b*(x/(pi*(pi + pi*c^2*x^2)^(3/2)) + 2*x/(pi^2*sqrt(pi + pi*c^2
*x^2)))*arcsinh(c*x) + 1/3*a*(x/(pi*(pi + pi*c^2*x^2)^(3/2)) + 2*x/(pi^2*sq
rt(pi + pi*c^2*x^2)))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{asinh}\left(c\,x\right)}{{\left(\Pi\,c^2\,x^2+\Pi\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(Pi + Pi*c^2*x^2)^(5/2),x)
```

```
[Out] int((a + b*asinh(c*x))/(Pi + Pi*c^2*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{b\,\mathrm{asinh}\left(cx\right)}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] (Integral(a/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1
) + sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(c**4*x**4*sqrt(c**2*x
**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x))/pi**
(5/2)
```

3.107 $\int \frac{a+b\sinh^{-1}(cx)}{x\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=148

$$\frac{a+b\sinh^{-1}(cx)}{\pi^2\sqrt{\pi c^2x^2+\pi}}+\frac{a+b\sinh^{-1}(cx)}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\pi^{5/2}}-\frac{bcx}{6\pi^{5/2}\left(c^2x^2+1\right)}-\frac{b\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)}{\pi^{5/2}}$$

```
[Out] -1/6*b*c*x/Pi^(5/2)/(c^2*x^2+1)+1/3*(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+Pi)^(
3/2)-7/6*b*arctan(c*x)/Pi^(5/2)-2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1
)^(1/2))/Pi^(5/2)-b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))/Pi^(5/2)+b*polylog(2,
c*x+(c^2*x^2+1)^(1/2))/Pi^(5/2)+(a+b*arcsinh(c*x))/Pi^2/(Pi*c^2*x^2+Pi)^(1/
2)
```

Rubi [A] time = 0.34, antiderivative size = 187, normalized size of antiderivative = 1.26, number of steps used = 11, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {5755, 5760, 4182, 2279, 2391, 203, 199}

$$-\frac{b\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{\pi^{5/2}}+\frac{b\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{\pi^{5/2}}+\frac{a+b\sinh^{-1}(cx)}{\pi^2\sqrt{\pi c^2x^2+\pi}}+\frac{a+b\sinh^{-1}(cx)}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\pi}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x*(Pi + c^2*Pi*x^2)^(5/2)),x]
```

```
[Out] -(b*c*x)/(6*Pi^2*Sqrt[1 + c^2*x^2]*Sqrt[Pi + c^2*Pi*x^2]) + (a + b*ArcSinh[
c*x])/(3*Pi*(Pi + c^2*Pi*x^2)^(3/2)) + (a + b*ArcSinh[c*x])/(Pi^2*Sqrt[Pi +
 c^2*Pi*x^2]) - (7*b*Sqrt[1 + c^2*x^2]*ArcTan[c*x])/(6*Pi^2*Sqrt[Pi + c^2*P
i*x^2]) - (2*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Pi^(5/2) - (b*Po
lyLog[2, -E^ArcSinh[c*x]])/Pi^(5/2) + (b*PolyLog[2, E^ArcSinh[c*x]])/Pi^(5/
2)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx = \frac{a+b\sinh^{-1}(cx)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{\int \frac{a+b\sinh^{-1}(cx)}{x\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx}{\pi} - \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{\left(1+c^2x^2\right)^2}\,dx}{3\pi^2\sqrt{\pi+c^2\pi x^2}}$$

$$= -\frac{bcx}{6\pi^2\sqrt{1+c^2x^2}\,\sqrt{\pi+c^2\pi x^2}} + \frac{a+b\sinh^{-1}(cx)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{a+b\sinh^{-1}(cx)}{\pi^2\sqrt{\pi+c^2\pi x^2}} + \frac{\int \frac{a+b\sinh^{-1}\cdots}{x\sqrt{\pi+c^2\pi\cdots}}}{\pi^2}$$

$$= -\frac{bcx}{6\pi^2\sqrt{1+c^2x^2}\,\sqrt{\pi+c^2\pi x^2}} + \frac{a+b\sinh^{-1}(cx)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{a+b\sinh^{-1}(cx)}{\pi^2\sqrt{\pi+c^2\pi x^2}} - \frac{7b\sqrt{1+c^2x\cdots}}{6\pi^2\sqrt{\pi\cdots}}$$

$$= -\frac{bcx}{6\pi^2\sqrt{1+c^2x^2}\,\sqrt{\pi+c^2\pi x^2}} + \frac{a+b\sinh^{-1}(cx)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{a+b\sinh^{-1}(cx)}{\pi^2\sqrt{\pi+c^2\pi x^2}} - \frac{7b\sqrt{1+c^2x\cdots}}{6\pi^2\sqrt{\pi\cdots}}$$

$$= -\frac{bcx}{6\pi^2\sqrt{1+c^2x^2}\,\sqrt{\pi+c^2\pi x^2}} + \frac{a+b\sinh^{-1}(cx)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{a+b\sinh^{-1}(cx)}{\pi^2\sqrt{\pi+c^2\pi x^2}} - \frac{7b\sqrt{1+c^2x\cdots}}{6\pi^2\sqrt{\pi\cdots}}$$

$$= -\frac{bcx}{6\pi^2\sqrt{1+c^2x^2}\,\sqrt{\pi+c^2\pi x^2}} + \frac{a+b\sinh^{-1}(cx)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{a+b\sinh^{-1}(cx)}{\pi^2\sqrt{\pi+c^2\pi x^2}} - \frac{7b\sqrt{1+c^2x\cdots}}{6\pi^2\sqrt{\pi\cdots}}$$

Mathematica [A] time = 0.77, size = 209, normalized size = 1.41

$$\frac{6a}{\sqrt{c^2x^2+1}} + \frac{2a}{\left(c^2x^2+1\right)^{3/2}} - 6a\log\left(\pi\left(\sqrt{c^2x^2+1}+1\right)\right) + 6a\log(x) - \frac{bcx}{c^2x^2+1} + \frac{6bc^2x^2\sinh^{-1}(cx)}{\left(c^2x^2+1\right)^{3/2}} + \frac{8b\sinh^{-1}(cx)}{\left(c^2x^2+1\right)^{3/2}} + 6b\,\mathrm{Li}_2\cdots$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*ArcSinh[c*x])/(x*(Pi + c^2*Pi*x^2)^(5/2)),x]

[Out] ((2*a)/(1 + c^2*x^2)^(3/2) - (b*c*x)/(1 + c^2*x^2) + (6*a)/Sqrt[1 + c^2*x^2] + (8*b*ArcSinh[c*x])/(1 + c^2*x^2)^(3/2) + (6*b*c^2*x^2*ArcSinh[c*x])/(1 + c^2*x^2)^(3/2) - 14*b*ArcTan[Tanh[ArcSinh[c*x]/2]] + 6*b*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 6*b*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 6*a*Log[x] - 6*a*Log[Pi*(1 + Sqrt[1 + c^2*x^2])] + 6*b*PolyLog[2, -E^(-ArcSinh[c*x])] - 6*b*PolyLog[2, E^(-ArcSinh[c*x])])/(6*Pi^(5/2))

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\pi+\pi c^2x^2}\,(b\,\text{arsinh}\,(cx)+a)}{\pi^3c^6x^7+3\,\pi^3c^4x^5+3\,\pi^3c^2x^3+\pi^3x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^3*c^6*x^7 + 3*pi^3*c^4*x^5 + 3*pi^3*c^2*x^3 + pi^3*x), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arsinh}\,(cx)+a}{\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")

[Out] integrate((b*arcsinh(c*x) + a)/((pi + pi*c^2*x^2)^(5/2)*x), x)

maple [A] time = 0.26, size = 220, normalized size = 1.49

$$\frac{a}{3\pi\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}}+\frac{a}{\pi^2\sqrt{\pi c^2x^2+\pi}}-\frac{a\,\text{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2x^2+\pi}}\right)}{\pi^{\frac{5}{2}}}+\frac{b\,\text{arcsinh}\,(cx)\,x^2c^2}{\pi^{\frac{5}{2}}\left(c^2x^2+1\right)^{\frac{3}{2}}}-\frac{bcx}{6\pi^{\frac{5}{2}}\left(c^2x^2+1\right)}+\frac{4b\,\text{arcsinh}\,(c}{3\pi^{\frac{5}{2}}\left(c^2x^2+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsinh(c*x))/x/(Pi*c^2*x^2+Pi)^(5/2),x)

[Out] 1/3*a/Pi/(Pi*c^2*x^2+Pi)^(3/2)+a/Pi^2/(Pi*c^2*x^2+Pi)^(1/2)-a/Pi^(5/2)*arctanh(Pi^(1/2)/(Pi*c^2*x^2+Pi)^(1/2))+b/Pi^(5/2)/(c^2*x^2+1)^(3/2)*arcsinh(c*x)*x^2*c^2-1/6*b*c*x/Pi^(5/2)/(c^2*x^2+1)+4/3*b/Pi^(5/2)/(c^2*x^2+1)^(3/2)*arcsinh(c*x)-7/3*b/Pi^(5/2)*arctan(c*x+(c^2*x^2+1)^(1/2))-b/Pi^(5/2)*dilog(c*x+(c^2*x^2+1)^(1/2))-b/Pi^(5/2)*dilog(1+c*x+(c^2*x^2+1)^(1/2))-b/Pi^(5/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}a\left(\frac{3\,\text{arsinh}\left(\frac{1}{c|x|}\right)}{\pi^{\frac{5}{2}}}-\frac{1}{\pi\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}}-\frac{3}{\pi^2\sqrt{\pi+\pi c^2x^2}}\right)+b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima")

```
[Out] -1/3*a*(3*arcsinh(1/(c*abs(x)))/pi^(5/2) - 1/(pi*(pi + pi*c^2*x^2)^(3/2)) -
 3/(pi^2*sqrt(pi + pi*c^2*x^2))) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))
/((pi + pi*c^2*x^2)^(5/2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{asinh}\left(c\,x\right)}{x\,{\left(\Pi\,c^2\,x^2+\Pi\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x*(Pi + Pi*c^2*x^2)^(5/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x*(Pi + Pi*c^2*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^4x^5\sqrt{c^2x^2+1}+2c^2x^3\sqrt{c^2x^2+1}+x\sqrt{c^2x^2+1}}\,dx+\int \frac{b\,\mathrm{asinh}\,(cx)}{c^4x^5\sqrt{c^2x^2+1}+2c^2x^3\sqrt{c^2x^2+1}+x\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] (Integral(a/(c**4*x**5*sqrt(c**2*x**2 + 1) + 2*c**2*x**3*sqrt(c**2*x**2 + 1
) + x*sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(c**4*x**5*sqrt(c**2
*x**2 + 1) + 2*c**2*x**3*sqrt(c**2*x**2 + 1) + x*sqrt(c**2*x**2 + 1)), x))/
pi**(5/2)
```

3.108 $\int \frac{a+b\sinh^{-1}(cx)}{x^2(\pi+c^2\pi x^2)^{5/2}}\,dx$

Optimal. Leaf size=150

$$-\frac{8c^2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^2\sqrt{\pi c^2x^2+\pi}}-\frac{4c^2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{a+b\sinh^{-1}(cx)}{\pi x\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{bc}{6\pi^{5/2}\left(c^2x^2+1\right)}+\frac{5bc\log\left(c^2x^2+1\right)}{6\pi^{5/2}}+\frac{bc}{}$$

```
[Out] -1/6*b*c/Pi^(5/2)/(c^2*x^2+1)+(-a-b*arcsinh(c*x))/Pi/x/(Pi*c^2*x^2+Pi)^(3/2
)-4/3*c^2*x*(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+Pi)^(3/2)+b*c*ln(x)/Pi^(5/2)+
5/6*b*c*ln(c^2*x^2+1)/Pi^(5/2)-8/3*c^2*x*(a+b*arcsinh(c*x))/Pi^2/(Pi*c^2*x^
2+Pi)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 153, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {271, 192, 191, 5732, 12, 1251, 893}

$$-\frac{8c^2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\sqrt{c^2x^2+1}}-\frac{4c^2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\left(c^2x^2+1\right)^{3/2}}-\frac{a+b\sinh^{-1}(cx)}{\pi^{5/2}x\left(c^2x^2+1\right)^{3/2}}-\frac{bc}{6\pi^{5/2}\left(c^2x^2+1\right)}+\frac{5bc\log\left(c^2x^2+1\right)}{6\pi^{5/2}}+\frac{bc}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^2*(Pi + c^2*Pi*x^2)^(5/2)),x]
```

```
[Out] -(b*c)/(6*Pi^(5/2)*(1 + c^2*x^2)) - (a + b*ArcSinh[c*x])/(Pi^(5/2)*x*(1 + c
^2*x^2)^(3/2)) - (4*c^2*x*(a + b*ArcSinh[c*x]))/(3*Pi^(5/2)*(1 + c^2*x^2)^(
3/2)) - (8*c^2*x*(a + b*ArcSinh[c*x]))/(3*Pi^(5/2)*Sqrt[1 + c^2*x^2]) + (b*
c*Log[x])/Pi^(5/2) + (5*b*c*Log[1 + c^2*x^2])/(6*Pi^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
```

```
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{\pi^{5/2}x\left(1+c^2x^2\right)^{3/2}} - \frac{4c^2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\left(1+c^2x^2\right)^{3/2}} - \frac{8c^2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\sqrt{1+c^2x^2}} - \frac{(bc)\int \cdots}{\cdots}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{\pi^{5/2}x\left(1+c^2x^2\right)^{3/2}} - \frac{4c^2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\left(1+c^2x^2\right)^{3/2}} - \frac{8c^2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\sqrt{1+c^2x^2}} - \frac{(bc)\int \cdots}{\cdots}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{\pi^{5/2}x\left(1+c^2x^2\right)^{3/2}} - \frac{4c^2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\left(1+c^2x^2\right)^{3/2}} - \frac{8c^2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\sqrt{1+c^2x^2}} - \frac{(bc)\,\mathrm{Su}\cdots}{\cdots}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{\pi^{5/2}x\left(1+c^2x^2\right)^{3/2}} - \frac{4c^2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\left(1+c^2x^2\right)^{3/2}} - \frac{8c^2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\sqrt{1+c^2x^2}} - \frac{(bc)\,\mathrm{Su}\cdots}{\cdots}$$

$$= -\frac{bc}{6\pi^{5/2}\left(1+c^2x^2\right)} - \frac{a+b\sinh^{-1}(cx)}{\pi^{5/2}x\left(1+c^2x^2\right)^{3/2}} - \frac{4c^2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\left(1+c^2x^2\right)^{3/2}} - \frac{8c^2x\left(a+b\sin\cdots\right.}{3\pi^{5/2}\sqrt{1+\cdots}}$$

Mathematica [A] time = 0.20, size = 123, normalized size = 0.82

$$\frac{-2a\left(8c^4x^4+12c^2x^2+3\right)-bcx\sqrt{c^2x^2+1}-2b\left(8c^4x^4+12c^2x^2+3\right)\sinh^{-1}(cx)}{6\pi^{5/2}x\left(c^2x^2+1\right)^{3/2}}+\frac{\frac{5}{2}bc\log\left(c^2x^2+1\right)+3bc\log\cdots}{3\pi^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^2*(Pi + c^2*Pi*x^2)^(5/2)),x]
```

```
[Out] (-(b*c*x*Sqrt[1 + c^2*x^2]) - 2*a*(3 + 12*c^2*x^2 + 8*c^4*x^4) - 2*b*(3 + 1
2*c^2*x^2 + 8*c^4*x^4)*ArcSinh[c*x])/(6*Pi^(5/2)*x*(1 + c^2*x^2)^(3/2)) + (
3*b*c*Log[x] + (5*b*c*Log[1 + c^2*x^2])/2)/(3*Pi^(5/2))
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\pi+\pi c^2x^2}\,(b\,\text{arsinh}\,(cx)+a)}{\pi^3c^6x^8+3\,\pi^3c^4x^6+3\,\pi^3c^2x^4+\pi^3x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^3*c^6*x^8 + 3*pi^3*
c^4*x^6 + 3*pi^3*c^2*x^4 + pi^3*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arsinh}\,(cx)+a}{\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((pi + pi*c^2*x^2)^(5/2)*x^2), x)
```

maple [B] time = 0.29, size = 778, normalized size = 5.19

$$-\frac{a}{\pi x\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}}-\frac{4a\,c^2x}{3\pi\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}}-\frac{8a\,c^2x}{3\pi^2\sqrt{\pi c^2x^2+\pi}}-\frac{16bc\,\text{arcsinh}\,(cx)}{3\pi^{\frac{5}{2}}}+\frac{32b\,x^{10}c^{11}}{3\pi^{\frac{5}{2}}\left(8c^2x^2+9\right)\left(c^2x^2+1\right)^2}-\frac{}{3\pi^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^2/(Pi*c^2*x^2+Pi)^(5/2),x)
```

```
[Out] -a/Pi/x/(Pi*c^2*x^2+Pi)^(3/2)-4/3*a*c^2/Pi*x/(Pi*c^2*x^2+Pi)^(3/2)-8/3*a*c^
2/Pi^2*x/(Pi*c^2*x^2+Pi)^(1/2)-16/3*b*c/Pi^(5/2)*arcsinh(c*x)+32/3*b/Pi^(5/
2)/(8*c^2*x^2+9)/(c^2*x^2+1)^2*x^10*c^11-32/3*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2
*x^2+1)*x^8*c^9+128/3*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^2+1)^2*x^8*c^9-32*b/P
i^(5/2)/(8*c^2*x^2+9)/(c^2*x^2+1)*x^6*c^7+64/3*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^
2*x^2+1)^2*x^6*arcsinh(c*x)*c^7-64/3*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^2+1)^(
3/2)*x^5*arcsinh(c*x)*c^6+64*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^2+1)^2*x^6*c^7
-32*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^2+1)*x^4*c^5+200/3*b/Pi^(5/2)/(8*c^2*x^
2+9)/(c^2*x^2+1)^2*x^4*arcsinh(c*x)*c^5-56*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^
2+1)^(3/2)*x^3*arcsinh(c*x)*c^4+128/3*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^2+1)^
2*x^4*c^5-12*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^2+1)*x^2*c^3+208/3*b/Pi^(5/2)/
(8*c^2*x^2+9)/(c^2*x^2+1)^2*x^2*arcsinh(c*x)*c^3-44*b/Pi^(5/2)/(8*c^2*x^2+9
)/(c^2*x^2+1)^(3/2)*x*arcsinh(c*x)*c^2+32/3*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x
^2+1)^2*x^2*c^3-3/2*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^2+1)*c+24*b/Pi^(5/2)/(8
*c^2*x^2+9)/(c^2*x^2+1)^2*arcsinh(c*x)*c-9*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^
2+1)^(3/2)/x*arcsinh(c*x)+5/3*b*c/Pi^(5/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+
b*c/Pi^(5/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}\,a\left(\frac{4\,c^2x}{\pi\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}}+\frac{8\,c^2x}{\pi^2\sqrt{\pi+\pi c^2x^2}}+\frac{3}{\pi\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}x}\right)+b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima
")
```

```
[Out] -1/3*a*(4*c^2*x/(pi*(pi + pi*c^2*x^2)^(3/2)) + 8*c^2*x/(pi^2*sqrt(pi + pi*c
^2*x^2)) + 3/(pi*(pi + pi*c^2*x^2)^(3/2)*x)) + b*integrate(log(c*x + sqrt(c
^2*x^2 + 1))/((pi + pi*c^2*x^2)^(5/2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{asinh}\left(c\,x\right)}{x^2\,{\left(\Pi\,c^2\,x^2+\Pi\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^2*(Pi + Pi*c^2*x^2)^(5/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^2*(Pi + Pi*c^2*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^{4}x^{6}\sqrt{c^{2}x^{2}+1}+2c^{2}x^{4}\sqrt{c^{2}x^{2}+1}+x^{2}\sqrt{c^{2}x^{2}+1}}\,dx+\int \frac{b\,\mathrm{asinh}\,(cx)}{c^{4}x^{6}\sqrt{c^{2}x^{2}+1}+2c^{2}x^{4}\sqrt{c^{2}x^{2}+1}+x^{2}\sqrt{c^{2}x^{2}+1}}\,dx}{\pi^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**2/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] (Integral(a/(c**4*x**6*sqrt(c**2*x**2 + 1) + 2*c**2*x**4*sqrt(c**2*x**2 + 1
) + x**2*sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(c**4*x**6*sqrt(c
**2*x**2 + 1) + 2*c**2*x**4*sqrt(c**2*x**2 + 1) + x**2*sqrt(c**2*x**2 + 1))
, x))/pi**(5/2)
```

3.109 $\int \frac{a+b\sinh^{-1}(cx)}{x^3(\pi+c^2\pi x^2)^{5/2}}\,dx$

Optimal. Leaf size=247

$$-\frac{5c^2\left(a+b\sinh^{-1}(cx)\right)}{2\pi^2\sqrt{\pi c^2x^2+\pi}}-\frac{5c^2\left(a+b\sinh^{-1}(cx)\right)}{6\pi\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{a+b\sinh^{-1}(cx)}{2\pi x^2\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{5c^2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\pi^{5/2}}$$

```
[Out] -3/4*b*c/Pi^(5/2)/x+1/4*b*c/Pi^(5/2)/x/(c^2*x^2+1)+5/12*b*c^3*x/Pi^(5/2)/(c
^2*x^2+1)-5/6*c^2*(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+Pi)^(3/2)+1/2*(-a-b*arc
sinh(c*x))/Pi/x^2/(Pi*c^2*x^2+Pi)^(3/2)+13/6*b*c^2*arctan(c*x)/Pi^(5/2)+5*c
^2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))/Pi^(5/2)+5/2*b*c^2*pol
ylog(2,-c*x-(c^2*x^2+1)^(1/2))/Pi^(5/2)-5/2*b*c^2*polylog(2,c*x+(c^2*x^2+1)
^(1/2))/Pi^(5/2)-5/2*c^2*(a+b*arcsinh(c*x))/Pi^2/(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 325, normalized size of antiderivative = 1.32, number of steps used = 15, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5747, 5755, 5760, 4182, 2279, 2391, 203, 199, 290, 325}

$$\frac{5bc^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{2\pi^{5/2}}-\frac{5bc^2\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{2\pi^{5/2}}-\frac{5c^2\left(a+b\sinh^{-1}(cx)\right)}{2\pi^2\sqrt{\pi c^2x^2+\pi}}-\frac{5c^2\left(a+b\sinh^{-1}(cx)\right)}{6\pi\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{a}{2\pi x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^3*(Pi + c^2*Pi*x^2)^(5/2)),x]
```

```
[Out] (b*c)/(4*Pi^2*x*Sqrt[1 + c^2*x^2]*Sqrt[Pi + c^2*Pi*x^2]) + (5*b*c^3*x)/(12*
Pi^2*Sqrt[1 + c^2*x^2]*Sqrt[Pi + c^2*Pi*x^2]) - (3*b*c*Sqrt[1 + c^2*x^2])/(
4*Pi^2*x*Sqrt[Pi + c^2*Pi*x^2]) - (5*c^2*(a + b*ArcSinh[c*x]))/(6*Pi*(Pi +
c^2*Pi*x^2)^(3/2)) - (a + b*ArcSinh[c*x])/(2*Pi*x^2*(Pi + c^2*Pi*x^2)^(3/2)
) - (5*c^2*(a + b*ArcSinh[c*x]))/(2*Pi^2*Sqrt[Pi + c^2*Pi*x^2]) + (13*b*c^2
*Sqrt[1 + c^2*x^2]*ArcTan[c*x])/(6*Pi^2*Sqrt[Pi + c^2*Pi*x^2]) + (5*c^2*(a
+ b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Pi^(5/2) + (5*b*c^2*PolyLog[2, -
E^ArcSinh[c*x]])/(2*Pi^(5/2)) - (5*b*c^2*PolyLog[2, E^ArcSinh[c*x]])/(2*Pi^
(5/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^3\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{2\pi x^2\left(\pi+c^2\pi x^2\right)^{3/2}} - \frac{1}{2}\left(5c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{x\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx + \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{x^2\left(1+c^2x^2\right)^2}}{2\pi^2\sqrt{\pi+c^2\pi x^2}}$$

$$= \frac{bc}{4\pi^2 x\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} - \frac{5c^2\left(a+b\sinh^{-1}(cx)\right)}{6\pi\left(\pi+c^2\pi x^2\right)^{3/2}} - \frac{a+b\sinh^{-1}(cx)}{2\pi x^2\left(\pi+c^2\pi x^2\right)^{3/2}} - \frac{\left(5c^2\right)}{}$$

$$= \frac{bc}{4\pi^2 x\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{5bc^3x}{12\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} - \frac{3bc\sqrt{1+c^2x^2}}{4\pi^2 x\sqrt{\pi+c^2\pi x^2}} - \frac{5c}{}$$

$$= \frac{bc}{4\pi^2 x\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{5bc^3x}{12\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} - \frac{3bc\sqrt{1+c^2x^2}}{4\pi^2 x\sqrt{\pi+c^2\pi x^2}} - \frac{5c}{}$$

$$= \frac{bc}{4\pi^2 x\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{5bc^3x}{12\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} - \frac{3bc\sqrt{1+c^2x^2}}{4\pi^2 x\sqrt{\pi+c^2\pi x^2}} - \frac{5c}{}$$

$$= \frac{bc}{4\pi^2 x\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{5bc^3x}{12\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} - \frac{3bc\sqrt{1+c^2x^2}}{4\pi^2 x\sqrt{\pi+c^2\pi x^2}} - \frac{5c}{}$$

$$= \frac{bc}{4\pi^2 x\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{5bc^3x}{12\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} - \frac{3bc\sqrt{1+c^2x^2}}{4\pi^2 x\sqrt{\pi+c^2\pi x^2}} - \frac{5c}{}$$

Mathematica [A] time = 6.31, size = 331, normalized size = 1.34

$$-\frac{48ac^2}{\sqrt{c^2x^2+1}} - \frac{8ac^2}{\left(c^2x^2+1\right)^{3/2}} - \frac{12a\sqrt{c^2x^2+1}}{x^2} + 60ac^2\log\left(\pi\left(\sqrt{c^2x^2+1}+1\right)\right) - 60ac^2\log(x) - 60bc^2\text{Li}_2\left(-e^{-\sinh^{-1}(cx)}\right) + 6$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^3*(Pi + c^2*Pi*x^2)^(5/2)),x]
```

```
[Out] ((-8*a*c^2)/(1 + c^2*x^2)^(3/2) + (4*b*c^3*x)/(1 + c^2*x^2) - (48*a*c^2)/Sq
rt[1 + c^2*x^2] - (12*a*Sqrt[1 + c^2*x^2])/x^2 - (56*b*c^2*ArcSinh[c*x])/(1
 + c^2*x^2)^(3/2) - (48*b*c^4*x^2*ArcSinh[c*x])/(1 + c^2*x^2)^(3/2) + 104*b
*c^2*ArcTan[Tanh[ArcSinh[c*x]/2]] - 6*b*c^2*Coth[ArcSinh[c*x]/2] - 3*b*c^2*
ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - 60*b*c^2*ArcSinh[c*x]*Log[1 - E^(-Arc
Sinh[c*x])] + 60*b*c^2*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] - 60*a*c^2*L
og[x] + 60*a*c^2*Log[Pi*(1 + Sqrt[1 + c^2*x^2])] - 60*b*c^2*PolyLog[2, -E^(
-ArcSinh[c*x])] + 60*b*c^2*PolyLog[2, E^(-ArcSinh[c*x])] + 6*b*c^2*Tanh[Arc
Sinh[c*x]/2] - (6*b*c*ArcSinh[c*x]*Tanh[ArcSinh[c*x]/2])/x)/(24*Pi^(5/2))
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\pi+\pi c^2x^2}\,(b\,\text{arsinh}\,(cx)+a)}{\pi^3c^6x^9+3\,\pi^3c^4x^7+3\,\pi^3c^2x^5+\pi^3x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas
")
```

[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^3*c^6*x^9 + 3*pi^3*c^4*x^7 + 3*pi^3*c^2*x^5 + pi^3*x^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\left(\pi + \pi c^2 x^2\right)^{\frac{5}{2}} x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")

[Out] integrate((b*arcsinh(c*x) + a)/((pi + pi*c^2*x^2)^(5/2)*x^3), x)

maple [A] time = 0.34, size = 314, normalized size = 1.27

$$-\frac{a}{2\pi x^2 \left(\pi c^2 x^2 + \pi\right)^{\frac{3}{2}}} - \frac{5a c^2}{6\pi \left(\pi c^2 x^2 + \pi\right)^{\frac{3}{2}}} - \frac{5a c^2}{2\pi^2 \sqrt{\pi c^2 x^2 + \pi}} + \frac{5a c^2 \operatorname{arctanh}\left(\frac{\sqrt{\pi}}{\sqrt{\pi c^2 x^2 + \pi}}\right)}{2\pi^{\frac{5}{2}}} - \frac{5b x^2 \operatorname{arcsinh}(cx) c^4}{2\pi^{\frac{5}{2}} \left(c^2 x^2 + 1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsinh(c*x))/x^3/(Pi*c^2*x^2+Pi)^(5/2),x)

[Out] -1/2*a/Pi/x^2/(Pi*c^2*x^2+Pi)^(3/2)-5/6*a*c^2/Pi/(Pi*c^2*x^2+Pi)^(3/2)-5/2*a*c^2/Pi^2/(Pi*c^2*x^2+Pi)^(1/2)+5/2*a*c^2/Pi^(5/2)*arctanh(Pi^(1/2)/(Pi*c^2*x^2+Pi)^(1/2))-5/2*b/Pi^(5/2)/(c^2*x^2+1)^(3/2)*x^2*arcsinh(c*x)*c^4-1/3*b*c^3*x/Pi^(5/2)/(c^2*x^2+1)-10/3*b/Pi^(5/2)/(c^2*x^2+1)^(3/2)*arcsinh(c*x)*c^2-1/2*b*c/Pi^(5/2)/x/(c^2*x^2+1)-1/2*b/Pi^(5/2)/(c^2*x^2+1)^(3/2)/x^2*arcsinh(c*x)+13/3*b*c^2/Pi^(5/2)*arctan(c*x+(c^2*x^2+1)^(1/2))+5/2*b*c^2/Pi^(5/2)*dilog(c*x+(c^2*x^2+1)^(1/2))+5/2*b*c^2/Pi^(5/2)*dilog(1+c*x+(c^2*x^2+1)^(1/2))+5/2*b*c^2/Pi^(5/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}a\left(\frac{15c^2 \operatorname{arsinh}\left(\frac{1}{c|x|}\right)}{\pi^{\frac{5}{2}}} - \frac{5c^2}{\pi\left(\pi + \pi c^2 x^2\right)^{\frac{3}{2}}} - \frac{15c^2}{\pi^2\sqrt{\pi + \pi c^2 x^2}} - \frac{3}{\pi\left(\pi + \pi c^2 x^2\right)^{\frac{3}{2}} x^2}\right) + b\int \frac{\log\left(cx + \sqrt{c^2x^2 + 1}\right)}{\left(\pi + \pi c^2 x^2\right)^{\frac{5}{2}} x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima")

[Out] 1/6*a*(15*c^2*arcsinh(1/(c*abs(x)))/pi^(5/2) - 5*c^2/(pi*(pi + pi*c^2*x^2)^(3/2)) - 15*c^2/(pi^2*sqrt(pi + pi*c^2*x^2)) - 3/(pi*(pi + pi*c^2*x^2)^(3/2)*x^2)) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/((pi + pi*c^2*x^2)^(5/2)*x^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{asinh}(c\, x)}{x^3 \left(\Pi\, c^2\, x^2 + \Pi\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*asinh(c*x))/(x^3*(Pi + Pi*c^2*x^2)^(5/2)),x)

[Out] int((a + b*asinh(c*x))/(x^3*(Pi + Pi*c^2*x^2)^(5/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^4x^7\sqrt{c^2x^2+1}+2c^2x^5\sqrt{c^2x^2+1}+x^3\sqrt{c^2x^2+1}}\,dx+\int \frac{b\operatorname{asinh}(cx)}{c^4x^7\sqrt{c^2x^2+1}+2c^2x^5\sqrt{c^2x^2+1}+x^3\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**3/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] (Integral(a/(c**4*x**7*sqrt(c**2*x**2 + 1) + 2*c**2*x**5*sqrt(c**2*x**2 + 1
) + x**3*sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(c**4*x**7*sqrt(c
**2*x**2 + 1) + 2*c**2*x**5*sqrt(c**2*x**2 + 1) + x**3*sqrt(c**2*x**2 + 1))
, x))/pi**(5/2)
```

3.110 $\int \frac{a+b\sinh^{-1}(cx)}{x^4\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=208

$$\frac{2c^2\left(a+b\sinh^{-1}(cx)\right)}{\pi x\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{a+b\sinh^{-1}(cx)}{3\pi x^3\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{16c^4x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^2\sqrt{\pi c^2x^2+\pi}}+\frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{8bc^3\log(x)}{3\pi^{5/2}}$$

```
[Out] -1/6*b*c/Pi^(5/2)/x^2+1/6*b*c^3/Pi^(5/2)/(c^2*x^2+1)+1/3*(-a-b*arcsinh(c*x)
)/Pi/x^3/(Pi*c^2*x^2+Pi)^(3/2)+2*c^2*(a+b*arcsinh(c*x))/Pi/x/(Pi*c^2*x^2+Pi
)^(3/2)+8/3*c^4*x*(a+b*arcsinh(c*x))/Pi/(Pi*c^2*x^2+Pi)^(3/2)-8/3*b*c^3*ln(
x)/Pi^(5/2)-4/3*b*c^3*ln(c^2*x^2+1)/Pi^(5/2)+16/3*c^4*x*(a+b*arcsinh(c*x))/
Pi^2/(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 212, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {271, 192, 191, 5732, 12, 1799, 1620}

$$\frac{16c^4x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\sqrt{c^2x^2+1}}+\frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\left(c^2x^2+1\right)^{3/2}}+\frac{2c^2\left(a+b\sinh^{-1}(cx)\right)}{\pi^{5/2}x\left(c^2x^2+1\right)^{3/2}}-\frac{a+b\sinh^{-1}(cx)}{3\pi^{5/2}x^3\left(c^2x^2+1\right)^{3/2}}+\frac{bc^3}{6\pi^{5/2}\left(c^2x^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^4*(Pi + c^2*Pi*x^2)^(5/2)),x]
```

```
[Out] -(b*c)/(6*Pi^(5/2)*x^2) + (b*c^3)/(6*Pi^(5/2)*(1 + c^2*x^2)) - (a + b*ArcSi
nh[c*x])/(3*Pi^(5/2)*x^3*(1 + c^2*x^2)^(3/2)) + (2*c^2*(a + b*ArcSinh[c*x])
)/(Pi^(5/2)*x*(1 + c^2*x^2)^(3/2)) + (8*c^4*x*(a + b*ArcSinh[c*x]))/(3*Pi^(
5/2)*(1 + c^2*x^2)^(3/2)) + (16*c^4*x*(a + b*ArcSinh[c*x]))/(3*Pi^(5/2)*Sqr
t[1 + c^2*x^2]) - (8*b*c^3*Log[x])/(3*Pi^(5/2)) - (4*b*c^3*Log[1 + c^2*x^2]
)/(3*Pi^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^4\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{3\pi^{5/2}x^3\left(1+c^2x^2\right)^{3/2}}+\frac{2c^2\left(a+b\sinh^{-1}(cx)\right)}{\pi^{5/2}x\left(1+c^2x^2\right)^{3/2}}+\frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\left(1+c^2x^2\right)^{3/2}}+\frac{16c^4x\left(a\right.}{3\pi^{5/}}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{3\pi^{5/2}x^3\left(1+c^2x^2\right)^{3/2}}+\frac{2c^2\left(a+b\sinh^{-1}(cx)\right)}{\pi^{5/2}x\left(1+c^2x^2\right)^{3/2}}+\frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\left(1+c^2x^2\right)^{3/2}}+\frac{16c^4x\left(a\right.}{3\pi^{5/}}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{3\pi^{5/2}x^3\left(1+c^2x^2\right)^{3/2}}+\frac{2c^2\left(a+b\sinh^{-1}(cx)\right)}{\pi^{5/2}x\left(1+c^2x^2\right)^{3/2}}+\frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\left(1+c^2x^2\right)^{3/2}}+\frac{16c^4x\left(a\right.}{3\pi^{5/}}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{3\pi^{5/2}x^3\left(1+c^2x^2\right)^{3/2}}+\frac{2c^2\left(a+b\sinh^{-1}(cx)\right)}{\pi^{5/2}x\left(1+c^2x^2\right)^{3/2}}+\frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\left(1+c^2x^2\right)^{3/2}}+\frac{16c^4x\left(a\right.}{3\pi^{5/}}$$

$$= -\frac{bc}{6\pi^{5/2}x^2}+\frac{bc^3}{6\pi^{5/2}\left(1+c^2x^2\right)}-\frac{a+b\sinh^{-1}(cx)}{3\pi^{5/2}x^3\left(1+c^2x^2\right)^{3/2}}+\frac{2c^2\left(a+b\sinh^{-1}(cx)\right)}{\pi^{5/2}x\left(1+c^2x^2\right)^{3/2}}+\frac{8c^4x\left(\right.}{3\pi}$$

Mathematica [A] time = 0.23, size = 142, normalized size = 0.68

$$\frac{2a\left(16c^6x^6+24c^4x^4+6c^2x^2-1\right)-bcx\sqrt{c^2x^2+1}+2b\left(16c^6x^6+24c^4x^4+6c^2x^2-1\right)\sinh^{-1}(cx)}{6\pi^{5/2}x^3\left(c^2x^2+1\right)^{3/2}}-\frac{8\left(bc^3\log(x)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^4*(Pi + c^2*Pi*x^2)^(5/2)),x]
```

```
[Out] (-(b*c*x*Sqrt[1 + c^2*x^2]) + 2*a*(-1 + 6*c^2*x^2 + 24*c^4*x^4 + 16*c^6*x^6
) + 2*b*(-1 + 6*c^2*x^2 + 24*c^4*x^4 + 16*c^6*x^6)*ArcSinh[c*x])/(6*Pi^(5/2
)*x^3*(1 + c^2*x^2)^(3/2)) - (8*(b*c^3*Log[x] + (b*c^3*Log[1 + c^2*x^2])/2)
)/(3*Pi^(5/2))
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\pi+\pi c^2x^2}\,(b\,\text{arsinh}\,(cx)+a)}{\pi^3c^6x^{10}+3\,\pi^3c^4x^8+3\,\pi^3c^2x^6+\pi^3x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^3*c^6*x^10 + 3*pi^3
*c^4*x^8 + 3*pi^3*c^2*x^6 + pi^3*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arsinh}\,(cx)+a}{\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((pi + pi*c^2*x^2)^(5/2)*x^4), x)
```

maple [B] time = 0.25, size = 1153, normalized size = 5.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^4/(Pi*c^2*x^2+Pi)^(5/2),x)
```

```
[Out] -1/3*a/Pi/x^3/(Pi*c^2*x^2+Pi)^(3/2)-8/3*b*c^3/Pi^(5/2)*ln((c*x+(c^2*x^2+1)^
(1/2))^4-1)+32/3*b*c^3/Pi^(5/2)*arcsinh(c*x)-64*b/Pi^(5/2)/(12*c^4*x^4+12*c
^2*x^2-1)/(c^2*x^2+1)^2*x^8*arcsinh(c*x)*c^11-192*b/Pi^(5/2)/(12*c^4*x^4+12
*c^2*x^2-1)/(c^2*x^2+1)^2*x^6*arcsinh(c*x)*c^9-560/3*b/Pi^(5/2)/(12*c^4*x^4
+12*c^2*x^2-1)/(c^2*x^2+1)^2*x^4*arcsinh(c*x)*c^7-160/3*b/Pi^(5/2)/(12*c^4*
x^4+12*c^2*x^2-1)/(c^2*x^2+1)^2*x^2*arcsinh(c*x)*c^5+160*b/Pi^(5/2)/(12*c^4
*x^4+12*c^2*x^2-1)/(c^2*x^2+1)^(3/2)*x^5*arcsinh(c*x)*c^8+344/3*b/Pi^(5/2)/
(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)^(3/2)*x^3*arcsinh(c*x)*c^6+12*b/Pi^(5
/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)^(3/2)*x*arcsinh(c*x)*c^4-6*b/Pi^(
5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)^(3/2)/x*arcsinh(c*x)*c^2+64*b/Pi
^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)^(3/2)*x^7*arcsinh(c*x)*c^10+12
8/3*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)*x^10*c^13+128*b/Pi^(5/
2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)*x^8*c^11+128*b/Pi^(5/2)/(12*c^4*x^
4+12*c^2*x^2-1)/(c^2*x^2+1)*x^6*c^9+128/3*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2
-1)/(c^2*x^2+1)*x^4*c^7-2*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)*
x^2*c^5+1/6*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)/x^2*c+1/3*b/Pi
^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)^(3/2)/x^3*arcsinh(c*x)+16/3*b/
Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)^2*arcsinh(c*x)*c^3-128/3*b/P
i^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)^2*x^12*c^15-512/3*b/Pi^(5/2)/
(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)^2*x^10*c^13-256*b/Pi^(5/2)/(12*c^4*x^
4+12*c^2*x^2-1)/(c^2*x^2+1)^2*x^8*c^11-512/3*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*
x^2-1)/(c^2*x^2+1)^2*x^6*c^9-128/3*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^
2*x^2+1)^2*x^4*c^7-2*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)*c^3+2
*a*c^2/Pi/x/(Pi*c^2*x^2+Pi)^(3/2)+8/3*a*c^4/Pi*x/(Pi*c^2*x^2+Pi)^(3/2)+16/3
*a*c^4/Pi^2*x/(Pi*c^2*x^2+Pi)^(1/2)
```

maxima [A] time = 0.52, size = 236, normalized size = 1.13

$$-\frac{1}{6}\,bc\left(\frac{8\,c^2\log\left(c^2x^2+1\right)}{\pi^{\frac{5}{2}}}+\frac{16\,c^2\log(x)}{\pi^{\frac{5}{2}}}+\frac{1}{\pi^{\frac{5}{2}}c^2x^4+\pi^{\frac{5}{2}}x^2}\right)+\frac{1}{3}\left(\frac{8\,c^4x}{\pi\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}}+\frac{16\,c^4x}{\pi^2\sqrt{\pi+\pi c^2x^2}}+\frac{6}{\pi\left(\pi+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima
")
```

```
[Out] -1/6*b*c*(8*c^2*log(c^2*x^2 + 1)/pi^(5/2) + 16*c^2*log(x)/pi^(5/2) + 1/(pi^
(5/2)*c^2*x^4 + pi^(5/2)*x^2)) + 1/3*(8*c^4*x/(pi*(pi + pi*c^2*x^2)^(3/2))
+ 16*c^4*x/(pi^2*sqrt(pi + pi*c^2*x^2)) + 6*c^2/(pi*(pi + pi*c^2*x^2)^(3/2)
*x) - 1/(pi*(pi + pi*c^2*x^2)^(3/2)*x^3))*b*arcsinh(c*x) + 1/3*(8*c^4*x/(pi
*(pi + pi*c^2*x^2)^(3/2)) + 16*c^4*x/(pi^2*sqrt(pi + pi*c^2*x^2)) + 6*c^2/(
pi*(pi + pi*c^2*x^2)^(3/2)*x) - 1/(pi*(pi + pi*c^2*x^2)^(3/2)*x^3))*a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}\left(c\,x\right)}{x^4\,{\left(\Pi\,c^2\,x^2+\Pi\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^4*(Pi + Pi*c^2*x^2)^(5/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^4*(Pi + Pi*c^2*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^4x^8\sqrt{c^2x^2+1}+2c^2x^6\sqrt{c^2x^2+1}+x^4\sqrt{c^2x^2+1}}\,dx+\int \frac{b\,\mathrm{asinh}\left(cx\right)}{c^4x^8\sqrt{c^2x^2+1}+2c^2x^6\sqrt{c^2x^2+1}+x^4\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**4/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] (Integral(a/(c**4*x**8*sqrt(c**2*x**2 + 1) + 2*c**2*x**6*sqrt(c**2*x**2 + 1
) + x**4*sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(c**4*x**8*sqrt(c
**2*x**2 + 1) + 2*c**2*x**6*sqrt(c**2*x**2 + 1) + x**4*sqrt(c**2*x**2 + 1))
, x))/pi**(5/2)
```

3.111 $\int \frac{\sinh^{-1}(ax)}{\left(c+a^2cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=200

$$\frac{2}{15ac^3\sqrt{a^2x^2+1}\,\sqrt{a^2cx^2+c}}+\frac{1}{20ac^3\left(a^2x^2+1\right)^{3/2}\sqrt{a^2cx^2+c}}-\frac{4\sqrt{a^2x^2+1}\,\log\left(a^2x^2+1\right)}{15ac^3\sqrt{a^2cx^2+c}}+\frac{8x\sinh^{-1}(ax)}{15c^3\sqrt{a^2cx^2+c}}+$$

```
[Out] 1/5*x*arcsinh(a*x)/c/(a^2*c*x^2+c)^(5/2)+4/15*x*arcsinh(a*x)/c^2/(a^2*c*x^2
+c)^(3/2)+1/20/a/c^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2)+8/15*x*arcsinh(a
*x)/c^3/(a^2*c*x^2+c)^(1/2)+2/15/a/c^3/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2
)-4/15*ln(a^2*x^2+1)*(a^2*x^2+1)^(1/2)/a/c^3/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {5690, 5687, 260, 261}

$$\frac{2}{15ac^3\sqrt{a^2x^2+1}\,\sqrt{a^2cx^2+c}}+\frac{1}{20ac^3\left(a^2x^2+1\right)^{3/2}\sqrt{a^2cx^2+c}}-\frac{4\sqrt{a^2x^2+1}\,\log\left(a^2x^2+1\right)}{15ac^3\sqrt{a^2cx^2+c}}+\frac{8x\sinh^{-1}(ax)}{15c^3\sqrt{a^2cx^2+c}}+$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]/(c + a^2*c*x^2)^(7/2),x]
```

```
[Out] 1/(20*a*c^3*(1 + a^2*x^2)^(3/2)*Sqrt[c + a^2*c*x^2]) + 2/(15*a*c^3*Sqrt[1 +
 a^2*x^2]*Sqrt[c + a^2*c*x^2]) + (x*ArcSinh[a*x])/(5*c*(c + a^2*c*x^2)^(5/2
)) + (4*x*ArcSinh[a*x])/(15*c^2*(c + a^2*c*x^2)^(3/2)) + (8*x*ArcSinh[a*x])
/(15*c^3*Sqrt[c + a^2*c*x^2]) - (4*Sqrt[1 + a^2*x^2]*Log[1 + a^2*x^2])/(15*
a*c^3*Sqrt[c + a^2*c*x^2])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)}{\left(c+a^2cx^2\right)^{7/2}}\,dx = \frac{x\sinh^{-1}(ax)}{5c\left(c+a^2cx^2\right)^{5/2}} + \frac{4\int \frac{\sinh^{-1}(ax)}{\left(c+a^2cx^2\right)^{5/2}}\,dx}{5c} - \frac{\left(a\sqrt{1+a^2x^2}\right)\int \frac{x}{\left(1+a^2x^2\right)^3}\,dx}{5c^3\sqrt{c+a^2cx^2}}$$

$$= \frac{1}{20ac^3\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)}{5c\left(c+a^2cx^2\right)^{5/2}} + \frac{4x\sinh^{-1}(ax)}{15c^2\left(c+a^2cx^2\right)^{3/2}} + \frac{8\int \frac{\sinh^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}}}{15c^2}$$

$$= \frac{1}{20ac^3\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}} + \frac{2}{15ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)}{5c\left(c+a^2cx^2\right)^{5/2}} + \frac{4x}{15c^2}$$

$$= \frac{1}{20ac^3\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}} + \frac{2}{15ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)}{5c\left(c+a^2cx^2\right)^{5/2}} + \frac{4x}{15c^2}$$

Mathematica [A] time = 0.18, size = 121, normalized size = 0.60

$$\frac{\sqrt{a^2cx^2+c}\left(4ax\sqrt{a^2x^2+1}\left(8a^4x^4+20a^2x^2+15\right)\sinh^{-1}(ax)-\left(a^2x^2+1\right)\left(-8a^2x^2+16\left(a^2x^2+1\right)^2\log\left(a^2x^2+\right.\right.\right.}{60ac^4\left(a^2x^2+1\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]/(c + a^2*c*x^2)^(7/2),x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(4*a*x*Sqrt[1 + a^2*x^2]*(15 + 20*a^2*x^2 + 8*a^4*x^4)
*ArcSinh[a*x] - (1 + a^2*x^2)*(-11 - 8*a^2*x^2 + 16*(1 + a^2*x^2)^2*Log[1 +
 a^2*x^2])))/(60*a*c^4*(1 + a^2*x^2)^(7/2))
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{a^2cx^2+c}\ \mathrm{arsinh}\,(ax)}{a^8c^4x^8+4\,a^6c^4x^6+6\,a^4c^4x^4+4\,a^2c^4x^2+c^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/(a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)/(a^8*c^4*x^8 + 4*a^6*c^4*x^6 + 6*
a^4*c^4*x^4 + 4*a^2*c^4*x^2 + c^4), x)
```

giac [A] time = 0.39, size = 124, normalized size = 0.62

$$-\frac{1}{60}\sqrt{c}\left(\frac{16\,\log\left(a^2x^2+1\right)}{ac^4}-\frac{24\,a^4x^4+56\,a^2x^2+35}{\left(a^2x^2+1\right)^2ac^4}\right)+\frac{\left(4\left(\frac{2\,a^4x^2}{c}+\frac{5\,a^2}{c}\right)x^2+\frac{15}{c}\right)x\log\left(ax+\sqrt{a^2x^2+1}\right)}{15\left(a^2cx^2+c\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/(a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] -1/60*sqrt(c)*(16*log(a^2*x^2 + 1)/(a*c^4) - (24*a^4*x^4 + 56*a^2*x^2 + 35)
/((a^2*x^2 + 1)^2*a*c^4)) + 1/15*(4*(2*a^4*x^2/c + 5*a^2/c)*x^2 + 15/c)*x*l
og(a*x + sqrt(a^2*x^2 + 1))/(a^2*c*x^2 + c)^(5/2)
```

maple [B] time = 0.22, size = 363, normalized size = 1.82

$$\frac{16\sqrt{c\left(a^2x^2+1\right)}\,\operatorname{arcsinh}(ax)}{15\sqrt{a^2x^2+1}\;a\,c^4}+\frac{\sqrt{c\left(a^2x^2+1\right)}\;\left(8x^5a^5-8\sqrt{a^2x^2+1}\;x^4a^4+20x^3a^3-16\sqrt{a^2x^2+1}\;x^2a^2+15a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)/(a^2*c*x^2+c)^(7/2),x)
```

```
[Out] 16/15*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a/c^4*arcsinh(a*x)+1/60*(c*(a
^2*x^2+1))^(1/2)*(8*x^5*a^5-8*(a^2*x^2+1)^(1/2)*x^4*a^4+20*x^3*a^3-16*(a^2*
x^2+1)^(1/2)*x^2*a^2+15*a*x-8*(a^2*x^2+1)^(1/2))*(-64*x^8*a^8-64*(a^2*x^2+1
)^(1/2)*x^7*a^7-280*x^6*a^6-248*(a^2*x^2+1)^(1/2)*x^5*a^5+160*arcsinh(a*x)*
x^4*a^4-456*x^4*a^4-340*(a^2*x^2+1)^(1/2)*x^3*a^3+380*arcsinh(a*x)*x^2*a^2-
328*a^2*x^2-165*(a^2*x^2+1)^(1/2)*x*a+256*arcsinh(a*x)-88)/(40*a^10*x^10+21
5*a^8*x^8+469*a^6*x^6+517*a^4*x^4+287*a^2*x^2+64)/a/c^4-8/15*(c*(a^2*x^2+1)
)^(1/2)/(a^2*x^2+1)^(1/2)/a/c^4*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2)
```

maxima [A] time = 0.41, size = 143, normalized size = 0.72

$$\frac{1}{60}a\left(\frac{3}{\left(a^6c^{\frac{5}{2}}x^4+2\,a^4c^{\frac{5}{2}}x^2+a^2c^{\frac{5}{2}}\right)c}+\frac{8}{\left(a^4c^{\frac{3}{2}}x^2+a^2c^{\frac{3}{2}}\right)c^2}-\frac{16\,\log\left(x^2+\frac{1}{a^2}\right)}{a^2c^{\frac{7}{2}}}\right)+\frac{1}{15}\left(\frac{8\,x}{\sqrt{a^2cx^2+c}\,c^3}+\frac{4\,x}{\left(a^2cx^2+c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/(a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/60*a*(3/((a^6*c^(5/2)*x^4 + 2*a^4*c^(5/2)*x^2 + a^2*c^(5/2))*c) + 8/((a^4
*c^(3/2)*x^2 + a^2*c^(3/2))*c^2) - 16*log(x^2 + 1/a^2)/(a^2*c^(7/2))) + 1/1
5*(8*x/(sqrt(a^2*c*x^2 + c)*c^3) + 4*x/((a^2*c*x^2 + c)^(3/2)*c^2) + 3*x/((
a^2*c*x^2 + c)^(5/2)*c))*arcsinh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{asinh}(a\,x)}{\left(c\,a^2\,x^2+c\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)/(c + a^2*c*x^2)^(7/2),x)
```

```
[Out] int(asinh(a*x)/(c + a^2*c*x^2)^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}(ax)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)/(a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Integral(asinh(a*x)/(c*(a**2*x**2 + 1))**(7/2), x)
```

3.112 $\int \frac{x^4 \sinh^{-1}(ax)}{\sqrt{1+a^2x^2}} \, dx$

Optimal. Leaf size=86

$$\frac{3\sinh^{-1}(ax)^2}{16a^5}+\frac{3x^2}{16a^3}+\frac{x^3\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{4a^2}-\frac{3x\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{8a^4}-\frac{x^4}{16a}$$

```
[Out] 3/16*x^2/a^3-1/16*x^4/a+3/16*arcsinh(a*x)^2/a^5-3/8*x*arcsinh(a*x)*(a^2*x^2
+1)^(1/2)/a^4+1/4*x^3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.15, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5758, 5675, 30}

$$\frac{3x^2}{16a^3}+\frac{x^3\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{4a^2}-\frac{3x\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{8a^4}+\frac{3\sinh^{-1}(ax)^2}{16a^5}-\frac{x^4}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (3*x^2)/(16*a^3) - x^4/(16*a) - (3*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(8*a^4
) + (x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(4*a^2) + (3*ArcSinh[a*x]^2)/(16*a
^5)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx &= \frac{x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{4a^2}-\frac{3\int\frac{x^2\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{4a^2}-\frac{\int x^3\,dx}{4a}\\
&= -\frac{x^4}{16a}-\frac{3x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{8a^4}+\frac{x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{4a^2}+\frac{3\int\frac{\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{8a^4}+\frac{3\int x\,dx}{8a^3}\\
&= \frac{3x^2}{16a^3}-\frac{x^4}{16a}-\frac{3x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{8a^4}+\frac{x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{4a^2}+\frac{3\sinh^{-1}(ax)^2}{16a^5}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 63, normalized size = 0.73

$$\frac{-a^4x^4+3a^2x^2+2ax\sqrt{a^2x^2+1}\left(2a^2x^2-3\right)\sinh^{-1}(ax)+3\sinh^{-1}(ax)^2}{16a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (3*a^2*x^2 - a^4*x^4 + 2*a*x*Sqrt[1 + a^2*x^2]*(-3 + 2*a^2*x^2)*ArcSinh[a*x
] + 3*ArcSinh[a*x]^2)/(16*a^5)
```

fricas [A] time = 0.52, size = 83, normalized size = 0.97

$$-\frac{a^4x^4-3\,a^2x^2-2\left(2\,a^3x^3-3\,ax\right)\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1}\right)-3\,\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{16\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/16*(a^4*x^4 - 3*a^2*x^2 - 2*(2*a^3*x^3 - 3*a*x)*sqrt(a^2*x^2 + 1)*log(a*
x + sqrt(a^2*x^2 + 1)) - 3*log(a*x + sqrt(a^2*x^2 + 1))^2)/a^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\,\mathrm{arsinh}\,(ax)}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4*arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

maple [A] time = 0.08, size = 74, normalized size = 0.86

$$\frac{4\,\mathrm{arcsinh}\,(ax)\,\sqrt{a^2x^2+1}\;a^3x^3-x^4a^4-6\,\mathrm{arcsinh}\,(ax)\,\sqrt{a^2x^2+1}\;ax+3a^2x^2+3\,\mathrm{arcsinh}\,(ax)^2+3}{16a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/16*(4*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a^3*x^3-x^4*a^4-6*arcsinh(a*x)*(a^2*
x^2+1)^(1/2)*a*x+3*a^2*x^2+3*arcsinh(a*x)^2+3)/a^5
```

maxima [A] time = 0.37, size = 83, normalized size = 0.97

$$-\frac{1}{16}\left(\frac{x^4}{a^2}-\frac{3\,x^2}{a^4}+\frac{3\,\mathrm{arsinh}\,(ax)^2}{a^6}\right)a+\frac{1}{8}\left(\frac{2\,\sqrt{a^2x^2+1}\,x^3}{a^2}-\frac{3\,\sqrt{a^2x^2+1}\,x}{a^4}+\frac{3\,\mathrm{arsinh}\,(ax)}{a^5}\right)\mathrm{arsinh}\,(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*(x^4/a^2 - 3*x^2/a^4 + 3*arcsinh(a*x)^2/a^6)*a + 1/8*(2*sqrt(a^2*x^2
+ 1)*x^3/a^2 - 3*sqrt(a^2*x^2 + 1)*x/a^4 + 3*arcsinh(a*x)/a^5)*arcsinh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\mathrm{asinh}\,(a\,x)}{\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*asinh(a*x))/(a^2*x^2 + 1)^(1/2),x)
```

```
[Out] int((x^4*asinh(a*x))/(a^2*x^2 + 1)^(1/2), x)
```

sympy [A] time = 2.29, size = 82, normalized size = 0.95

$$\begin{cases} -\frac{x^4}{16a} + \frac{x^3\sqrt{a^2x^2+1}\,\operatorname{asinh}(ax)}{4a^2} + \frac{3x^2}{16a^3} - \frac{3x\sqrt{a^2x^2+1}\,\operatorname{asinh}(ax)}{8a^4} + \frac{3\operatorname{asinh}^2(ax)}{16a^5} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-x**4/(16*a) + x**3*sqrt(a**2*x**2 + 1)*asinh(a*x)/(4*a**2) + 3*
x**2/(16*a**3) - 3*x*sqrt(a**2*x**2 + 1)*asinh(a*x)/(8*a**4) + 3*asinh(a*x)
**2/(16*a**5), Ne(a, 0)), (0, True))
```

3.113 $\int \frac{x^3 \sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=70

$$\frac{2x}{3a^3}+\frac{x^2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{3a^2}-\frac{2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{3a^4}-\frac{x^3}{9a}$$

```
[Out] 2/3*x/a^3-1/9*x^3/a-2/3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^4+1/3*x^2*arcsinh(
a*x)*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.11, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5758, 5717, 8, 30}

$$\frac{x^2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{3a^2}-\frac{2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{3a^4}+\frac{2x}{3a^3}-\frac{x^3}{9a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (2*x)/(3*a^3) - x^3/(9*a) - (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(3*a^4) + (x
^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(3*a^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^3\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{3a^2} - \frac{2\int \frac{x\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{3a^2} - \frac{\int x^2\,dx}{3a}$$
$$= -\frac{x^3}{9a} - \frac{2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{3a^4} + \frac{x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{3a^2} + \frac{2\int 1\,dx}{3a^3}$$
$$= \frac{2x}{3a^3} - \frac{x^3}{9a} - \frac{2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{3a^4} + \frac{x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{3a^2}$$

Mathematica [A] time = 0.04, size = 48, normalized size = 0.69

$$\frac{-a^3x^3+3\left(a^2x^2-2\right)\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)+6ax}{9a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (6*a*x - a^3*x^3 + 3*(-2 + a^2*x^2)*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(9*a^4)
```

fricas [A] time = 0.66, size = 55, normalized size = 0.79

$$-\frac{a^3x^3-3\sqrt{a^2x^2+1}\left(a^2x^2-2\right)\log\left(ax+\sqrt{a^2x^2+1}\right)-6\,ax}{9\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/9*(a^3*x^3 - 3*sqrt(a^2*x^2 + 1)*(a^2*x^2 - 2)*log(a*x + sqrt(a^2*x^2 +
1)) - 6*a*x)/a^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.05, size = 82, normalized size = 1.17

$$\frac{3\operatorname{arcsinh}(ax)\,x^4a^4-3\operatorname{arcsinh}(ax)\,x^2a^2-\sqrt{a^2x^2+1}\;x^3a^3-6\operatorname{arcsinh}(ax)+6\sqrt{a^2x^2+1}\;xa}{9a^4\sqrt{a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/9/a^4*(3*arcsinh(a*x)*x^4*a^4-3*arcsinh(a*x)*x^2*a^2-(a^2*x^2+1)^(1/2)*x^
3*a^3-6*arcsinh(a*x)+6*(a^2*x^2+1)^(1/2)*x*a)/(a^2*x^2+1)^(1/2)
```

maxima [A] time = 0.32, size = 59, normalized size = 0.84

$$-\frac{1}{9}\,a\left(\frac{x^3}{a^2}-\frac{6\,x}{a^4}\right)+\frac{1}{3}\left(\frac{\sqrt{a^2x^2+1}\,x^2}{a^2}-\frac{2\,\sqrt{a^2x^2+1}}{a^4}\right)\operatorname{arsinh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/9*a*(x^3/a^2 - 6*x/a^4) + 1/3*(sqrt(a^2*x^2 + 1)*x^2/a^2 - 2*sqrt(a^2*x^
2 + 1)/a^4)*arcsinh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \operatorname{asinh}(a\,x)}{\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*asinh(a*x))/(a^2*x^2 + 1)^(1/2),x)
```

```
[Out] int((x^3*asinh(a*x))/(a^2*x^2 + 1)^(1/2), x)
```

sympy [A] time = 1.20, size = 65, normalized size = 0.93

$$\begin{cases} -\frac{x^3}{9a} + \frac{x^2\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{3a^2} + \frac{2x}{3a^3} - \frac{2\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{3a^4} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-x**3/(9*a) + x**2*sqrt(a**2*x**2 + 1)*asinh(a*x)/(3*a**2) + 2*x
/(3*a**3) - 2*sqrt(a**2*x**2 + 1)*asinh(a*x)/(3*a**4), Ne(a, 0)), (0, True)
)
```

3.114 $\int \frac{x^2 \sinh^{-1}(ax)}{\sqrt{1+a^2x^2}} \, dx$

Optimal. Leaf size=49

$$-\frac{\sinh^{-1}(ax)^2}{4a^3}+\frac{x\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{2a^2}-\frac{x^2}{4a}$$

```
[Out] -1/4*x^2/a-1/4*arcsinh(a*x)^2/a^3+1/2*x*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.10, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5758, 5675, 30}

$$\frac{x\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{2a^2}-\frac{\sinh^{-1}(ax)^2}{4a^3}-\frac{x^2}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] -x^2/(4*a) + (x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*a^2) - ArcSinh[a*x]^2/(4
*a^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2 \sinh^{-1}(ax)}{\sqrt{1+a^2x^2}} \, dx &= \frac{x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{2a^2}-\frac{\int \frac{\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}} \, dx}{2a^2}-\frac{\int x \, dx}{2a}\\ &= -\frac{x^2}{4a}+\frac{x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{2a^2}-\frac{\sinh^{-1}(ax)^2}{4a^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 42, normalized size = 0.86

$$-\frac{a^2x^2-2ax\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)+\sinh^{-1}(ax)^2}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
[Out] -1/4*(a^2*x^2 - 2*a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] + ArcSinh[a*x]^2)/a^3
```

fricas [A] time = 0.63, size = 62, normalized size = 1.27

$$-\frac{a^2x^2 - 2\sqrt{a^2x^2+1}\,ax\log\left(ax+\sqrt{a^2x^2+1}\right) + \log\left(ax+\sqrt{a^2x^2+1}\right)^2}{4\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] -1/4*(a^2*x^2 - 2*sqrt(a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 + 1)) + log(
a*x + sqrt(a^2*x^2 + 1))^2)/a^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^2*arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

maple [A] time = 0.04, size = 40, normalized size = 0.82

$$-\frac{-2\operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}\;ax + a^2x^2 + \operatorname{arcsinh}(ax)^2 + 1}{4a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
[Out] -1/4*(-2*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a*x+a^2*x^2+arcsinh(a*x)^2+1)/a^3
```

maxima [A] time = 0.33, size = 55, normalized size = 1.12

$$-\frac{1}{4}a\left(\frac{x^2}{a^2} - \frac{\operatorname{arsinh}(ax)^2}{a^4}\right) + \frac{1}{2}\left(\frac{\sqrt{a^2x^2+1}\,x}{a^2} - \frac{\operatorname{arsinh}(ax)}{a^3}\right)\operatorname{arsinh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] -1/4*a*(x^2/a^2 - arcsinh(a*x)^2/a^4) + 1/2*(sqrt(a^2*x^2 + 1)*x/a^2 - arcs
inh(a*x)/a^3)*arcsinh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2 \operatorname{asinh}(a\,x)}{\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*asinh(a*x))/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x^2*asinh(a*x))/(a^2*x^2 + 1)^(1/2), x)
```

sympy [A] time = 0.76, size = 42, normalized size = 0.86

$$\begin{cases} -\frac{x^2}{4a} + \frac{x\sqrt{a^2x^2+1}\,\operatorname{asinh}(ax)}{2a^2} - \frac{\operatorname{asinh}^2(ax)}{4a^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-x**2/(4*a) + x*sqrt(a**2*x**2 + 1)*asinh(a*x)/(2*a**2) - asinh(
a*x)**2/(4*a**3), Ne(a, 0)), (0, True))
```

3.115 $\int \frac{x \sinh^{-1}(ax)}{\sqrt{1+a^2x^2}} \, dx$

Optimal. Leaf size=28

$$\frac{\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{a^2}-\frac{x}{a}$$

```
[Out] -x/a+arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.05, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5717, 8}

$$\frac{\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{a^2}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] -(x/a) + (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx &= \frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{a^2}-\frac{\int 1\,dx}{a}\\ &= -\frac{x}{a}+\frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 28, normalized size = 1.00

$$\frac{\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{a^2}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] -(x/a) + (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^2
```

fricas [A] time = 0.54, size = 38, normalized size = 1.36

$$-\frac{ax-\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(a*x - sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/a^2
```

giac [A] time = 0.41, size = 38, normalized size = 1.36

$$-\frac{x}{a}+\frac{\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -x/a + sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))/a^2
```

maple [A] time = 0.04, size = 47, normalized size = 1.68

$$\frac{\operatorname{arcsinh}(ax)\,x^2a^2+\operatorname{arcsinh}(ax)-\sqrt{a^2x^2+1}\;xa}{a^2\sqrt{a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/a^2/(a^2*x^2+1)^(1/2)*(arcsinh(a*x)*x^2*a^2+arcsinh(a*x)-(a^2*x^2+1)^(1/2
)*x*a)
```

maxima [A] time = 0.36, size = 26, normalized size = 0.93

$$-\frac{x}{a}+\frac{\sqrt{a^2x^2+1}\;\operatorname{arsinh}(ax)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -x/a + sqrt(a^2*x^2 + 1)*arcsinh(a*x)/a^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\operatorname{asinh}(a\,x)}{\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*asinh(a*x))/(a^2*x^2 + 1)^(1/2),x)
```

```
[Out] int((x*asinh(a*x))/(a^2*x^2 + 1)^(1/2), x)
```

sympy [A] time = 0.44, size = 24, normalized size = 0.86

$$\begin{cases} -\frac{x}{a}+\frac{\sqrt{a^2x^2+1}\;\operatorname{asinh}(ax)}{a^2} & \text{for}\ a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-x/a + sqrt(a**2*x**2 + 1)*asinh(a*x)/a**2, Ne(a, 0)), (0, True)
)
```

3.116 $\int \frac{\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=13

$$\frac{\sinh^{-1}(ax)^2}{2a}$$

[Out] 1/2*arcsinh(a*x)^2/a

Rubi [A] time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {5675}

$$\frac{\sinh^{-1}(ax)^2}{2a}$$

Antiderivative was successfully verified.

[In] Int[ArcSinh[a*x]/Sqrt[1 + a^2*x^2],x]

[Out] ArcSinh[a*x]^2/(2*a)

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{\sinh^{-1}(ax)^2}{2a}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{\sinh^{-1}(ax)^2}{2a}$$

Antiderivative was successfully verified.

[In] Integrate[ArcSinh[a*x]/Sqrt[1 + a^2*x^2],x]

[Out] ArcSinh[a*x]^2/(2*a)

fricas [B] time = 0.66, size = 23, normalized size = 1.77

$$\frac{\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")

[Out] 1/2*log(a*x + sqrt(a^2*x^2 + 1))^2/a

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$\frac{\operatorname{arcsinh}(ax)^2}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/2*arcsinh(a*x)^2/a
```

maxima [A] time = 0.35, size = 11, normalized size = 0.85

$$\frac{\operatorname{arsinh}(ax)^2}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*arcsinh(a*x)^2/a
```

mupad [B] time = 0.11, size = 11, normalized size = 0.85

$$\frac{\operatorname{asinh}(a\,x)^2}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)/(a^2*x^2 + 1)^(1/2),x)
```

```
[Out] asinh(a*x)^2/(2*a)
```

sympy [A] time = 0.34, size = 10, normalized size = 0.77

$$\begin{cases} \frac{\operatorname{asinh}^2(ax)}{2a} & \text{for}\ a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((asinh(a*x)**2/(2*a), Ne(a, 0)), (0, True))
```

3.117 $\int \frac{\sinh^{-1}(ax)}{x\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=34

$$-\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right)+\text{Li}_2\left(e^{\sinh^{-1}(ax)}\right)-2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)$$

```
[Out] -2*arcsinh(a*x)*arctanh(a*x+(a^2*x^2+1)^(1/2))-polylog(2,-a*x-(a^2*x^2+1)^(
1/2))+polylog(2,a*x+(a^2*x^2+1)^(1/2))
```

Rubi [A] time = 0.09, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5760, 4182, 2279, 2391}

$$-\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)+\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)-2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]/(x*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -2*ArcSinh[a*x]*ArcTanh[E^ArcSinh[a*x]] - PolyLog[2, -E^ArcSinh[a*x]] + Pol
yLog[2, E^ArcSinh[a*x]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)}{x\sqrt{1+a^2x^2}}\,dx = \text{Subst}\left(\int x\text{csch}(x)\,dx, x, \sinh^{-1}(ax)\right)$$
$$= -2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - \text{Subst}\left(\int \log\left(1-e^x\right)\,dx, x, \sinh^{-1}(ax)\right) + \text{Subst}\left(\int \text{lo}\right.$$
$$= -2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - \text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{\sinh^{-1}(ax)}\right) + \text{Subst}\left(\int \frac{\log(}{}\right.$$
$$= -2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - \text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + \text{Li}_2\left(e^{\sinh^{-1}(ax)}\right)$$

Mathematica [A] time = 0.10, size = 57, normalized size = 1.68

$$\text{Li}_2\left(-e^{-\sinh^{-1}(ax)}\right) - \text{Li}_2\left(e^{-\sinh^{-1}(ax)}\right) + \sinh^{-1}(ax)\left(\log\left(1-e^{-\sinh^{-1}(ax)}\right) - \log\left(e^{-\sinh^{-1}(ax)}+1\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]/(x*Sqrt[1 + a^2*x^2]),x]
[Out] ArcSinh[a*x]*(Log[1 - E^(-ArcSinh[a*x])] - Log[1 + E^(-ArcSinh[a*x])]) + Po
lyLog[2, -E^(-ArcSinh[a*x])] - PolyLog[2, E^(-ArcSinh[a*x])]
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)}{a^2x^3+x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)/(a^2*x^3 + x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arsinh}\,(ax)}{\sqrt{a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arcsinh(a*x)/(sqrt(a^2*x^2 + 1)*x), x)
```

maple [A] time = 0.06, size = 42, normalized size = 1.24

$$2\,\text{dilog}\left(\frac{1}{ax+\sqrt{a^2x^2+1}}\right) - \frac{\text{dilog}\left(\frac{1}{\left(ax+\sqrt{a^2x^2+1}\right)^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)/x/(a^2*x^2+1)^(1/2),x)
[Out] 2*dilog(1/(a*x+(a^2*x^2+1)^(1/2)))-1/2*dilog(1/(a*x+(a^2*x^2+1)^(1/2))^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arsinh}\,(ax)}{\sqrt{a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(arcsinh(a*x)/(sqrt(a^2*x^2 + 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{asinh}(a\,x)}{x\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)/(x*(a^2*x^2 + 1)^(1/2)),x)
[Out] int(asinh(a*x)/(x*(a^2*x^2 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}(ax)}{x\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)/x/(a**2*x**2+1)**(1/2),x)
[Out] Integral(asinh(a*x)/(x*sqrt(a**2*x**2 + 1)), x)
```

3.118 $\int \frac{\sinh^{-1}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=27

$$a\log(x)-\frac{\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{x}$$

```
[Out] a*ln(x)-arcsinh(a*x)*(a^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.06, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5723, 29}

$$a\log(x)-\frac{\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]/(x^2*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -((Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/x) + a*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^{-1}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx &= -\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{x}+a\int \frac{1}{x}\,dx\\ &= -\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{x}+a\log(x)\end{aligned}$$

Mathematica [A] time = 0.04, size = 29, normalized size = 1.07

$$a\log(ax)-\frac{\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]/(x^2*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -((Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/x) + a*Log[a*x]
```

fricas [A] time = 0.43, size = 39, normalized size = 1.44

$$\frac{ax\log(x)-\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (a*x*log(x) - sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/x
```

giac [B] time = 0.33, size = 71, normalized size = 2.63

$$-a\log\left(-x|a|+\sqrt{a^2x^2+1}\right)+a\log\left(|x|\right)+\frac{2|a|\log\left(ax+\sqrt{a^2x^2+1}\right)}{\left(x|a|-\sqrt{a^2x^2+1}\right)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -a*log(-x*abs(a) + sqrt(a^2*x^2 + 1)) + a*log(abs(x)) + 2*abs(a)*log(a*x +
sqrt(a^2*x^2 + 1))/((x*abs(a) - sqrt(a^2*x^2 + 1))^2 - 1)
```

maple [B] time = 0.08, size = 56, normalized size = 2.07

$$-2a\,\mathrm{arcsinh}\left(ax\right)+\frac{\left(ax-\sqrt{a^2x^2+1}\right)\mathrm{arcsinh}\left(ax\right)}{x}+a\ln\left(\left(ax+\sqrt{a^2x^2+1}\right)^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)/x^2/(a^2*x^2+1)^(1/2),x)
```

```
[Out] -2*a*arcsinh(a*x)+(a*x-(a^2*x^2+1)^(1/2))/x*arcsinh(a*x)+a*ln((a*x+(a^2*x^2
+1)^(1/2))^2-1)
```

maxima [A] time = 0.34, size = 25, normalized size = 0.93

$$a\log(x)-\frac{\sqrt{a^2x^2+1}\,\mathrm{arsinh}\left(ax\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] a*log(x) - sqrt(a^2*x^2 + 1)*arcsinh(a*x)/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{asinh}\left(a\,x\right)}{x^2\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)/(x^2*(a^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(asinh(a*x)/(x^2*(a^2*x^2 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{asinh}\left(ax\right)}{x^2\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)/x**2/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(asinh(a*x)/(x**2*sqrt(a**2*x**2 + 1)), x)
```

3.119 $\int \frac{\sinh^{-1}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=80

$$\frac{1}{2}a^2\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right)-\frac{1}{2}a^2\text{Li}_2\left(e^{\sinh^{-1}(ax)}\right)-\frac{\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{2x^2}+a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)-\frac{a}{2x}$$

```
[Out] -1/2*a/x+a^2*arcsinh(a*x)*arctanh(a*x+(a^2*x^2+1)^(1/2))+1/2*a^2*polylog(2,
-a*x-(a^2*x^2+1)^(1/2))-1/2*a^2*polylog(2,a*x+(a^2*x^2+1)^(1/2))-1/2*arcsin
h(a*x)*(a^2*x^2+1)^(1/2)/x^2
```

Rubi [A] time = 0.15, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5747, 5760, 4182, 2279, 2391, 30}

$$\frac{1}{2}a^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)-\frac{1}{2}a^2\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)-\frac{\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{2x^2}+a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(a\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]/(x^3*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -a/(2*x) - (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*x^2) + a^2*ArcSinh[a*x]*ArcT
anh[E^ArcSinh[a*x]] + (a^2*PolyLog[2, -E^ArcSinh[a*x]])/2 - (a^2*PolyLog[2,
 E^ArcSinh[a*x]])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
```

```
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx = -\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{2x^2} + \frac{1}{2}a\int \frac{1}{x^2}\,dx - \frac{1}{2}a^2\int \frac{\sinh^{-1}(ax)}{x\sqrt{1+a^2x^2}}\,dx$$
$$= -\frac{a}{2x} - \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{2x^2} - \frac{1}{2}a^2\,\mathrm{Subst}\left(\int x\operatorname{csch}(x)\,dx, x, \sinh^{-1}(ax)\right)$$
$$= -\frac{a}{2x} - \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{2x^2} + a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) + \frac{1}{2}a^2\,\mathrm{Subst}\left(\int \log(\right.$$
$$= -\frac{a}{2x} - \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{2x^2} + a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) + \frac{1}{2}a^2\,\mathrm{Subst}\left(\int \frac{\log(}{}\right.$$
$$= -\frac{a}{2x} - \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{2x^2} + a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) + \frac{1}{2}a^2\mathrm{Li}_2\left(-e^{\sinh^{-1}(ax)}\right.$$

Mathematica [A] time = 0.66, size = 126, normalized size = 1.58

$$\frac{1}{8}a^2\left(-4\mathrm{Li}_2\left(-e^{-\sinh^{-1}(ax)}\right) + 4\mathrm{Li}_2\left(e^{-\sinh^{-1}(ax)}\right) - 4\sinh^{-1}(ax)\log\left(1-e^{-\sinh^{-1}(ax)}\right) + 4\sinh^{-1}(ax)\log\left(e^{-\sinh^{-1}}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]/(x^3*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] (a^2*(-2*Coth[ArcSinh[a*x]/2] - ArcSinh[a*x]*Csch[ArcSinh[a*x]/2]^2 - 4*Arc
Sinh[a*x]*Log[1 - E^(-ArcSinh[a*x])] + 4*ArcSinh[a*x]*Log[1 + E^(-ArcSinh[a
*x])] - 4*PolyLog[2, -E^(-ArcSinh[a*x])] + 4*PolyLog[2, E^(-ArcSinh[a*x])]
- ArcSinh[a*x]*Sech[ArcSinh[a*x]/2]^2 + 2*Tanh[ArcSinh[a*x]/2]))/8
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{a^2x^2+1}\,\operatorname{arsinh}(ax)}{a^2x^5+x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)/(a^2*x^5 + x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)/(sqrt(a^2*x^2 + 1)*x^3), x)
```

maple [A] time = 0.09, size = 150, normalized size = 1.88

$$-\frac{\operatorname{arcsinh}(ax)\,x^2a^2+\sqrt{a^2x^2+1}\;xa+\operatorname{arcsinh}(ax)}{2\sqrt{a^2x^2+1}\;x^2}+\frac{a^2\operatorname{arcsinh}(ax)\ln\left(1+ax+\sqrt{a^2x^2+1}\right)}{2}+\frac{a^2\operatorname{polylog}\left(2,-a\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)/x^3/(a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/2/(a^2*x^2+1)^(1/2)*(arcsinh(a*x)*x^2*a^2+(a^2*x^2+1)^(1/2)*x*a+arcsinh(
a*x))/x^2+1/2*a^2*arcsinh(a*x)*ln(1+a*x+(a^2*x^2+1)^(1/2))+1/2*a^2*polylog(
2,-a*x-(a^2*x^2+1)^(1/2))-1/2*a^2*arcsinh(a*x)*ln(1-a*x-(a^2*x^2+1)^(1/2))-
1/2*a^2*polylog(2,a*x+(a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)/(sqrt(a^2*x^2 + 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}(a\,x)}{x^3\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)/(x^3*(a^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(asinh(a*x)/(x^3*(a^2*x^2 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}(ax)}{x^3\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)/x**3/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(asinh(a*x)/(x**3*sqrt(a**2*x**2 + 1)), x)
```

3.120 $\int x^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)dx$

Optimal. Leaf size=175

$$\frac{\left(c^2dx^2+d\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5c^4d^2}-\frac{\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c^4d}-\frac{bcx^5\sqrt{c^2dx^2+d}}{25\sqrt{c^2x^2+1}}-\frac{bx^3\sqrt{c^2dx^2+d}}{45c\sqrt{c^2x^2+1}}+\frac{2bx}{15c^3}$$

```
[Out] -1/3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/c^4/d+1/5*(c^2*d*x^2+d)^(5/2)*(
a+b*arcsinh(c*x))/c^4/d^2+2/15*b*x*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)^(1/2
)-1/45*b*x^3*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-1/25*b*c*x^5*(c^2*d*x^
2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43, 5734, 12}

$$\frac{\left(c^2dx^2+d\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5c^4d^2}-\frac{\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c^4d}-\frac{bcx^5\sqrt{c^2dx^2+d}}{25\sqrt{c^2x^2+1}}-\frac{bx^3\sqrt{c^2dx^2+d}}{45c\sqrt{c^2x^2+1}}+\frac{2bx}{15c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (2*b*x*Sqrt[d + c^2*d*x^2])/(15*c^3*Sqrt[1 + c^2*x^2]) - (b*x^3*Sqrt[d + c^
2*d*x^2])/(45*c*Sqrt[1 + c^2*x^2]) - (b*c*x^5*Sqrt[d + c^2*d*x^2])/(25*Sqrt
[1 + c^2*x^2]) - ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c^4*d) + (
(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(5*c^4*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5734

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[a + b*ArcS
inh[c*x], Int[x^m*(d + e*x^2)^p, x], x] - Dist[(b*c*d^(p - 1/2)*Sqrt[d + e*
x^2])/Sqrt[1 + c^2*x^2], Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x],
 x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p + 1/2, 0] && (
IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\int x^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)dx = -\frac{\left(bc\sqrt{d+c^2dx^2}\right)\int\frac{-2+c^2x^2+3c^4x^4}{15c^4}dx}{\sqrt{1+c^2x^2}} + \left(a+b\sinh^{-1}(cx)\right)\int x^3\sqrt{d+c^2$$

$$= -\frac{\left(b\sqrt{d+c^2dx^2}\right)\int\left(-2+c^2x^2+3c^4x^4\right)dx}{15c^3\sqrt{1+c^2x^2}} + \frac{1}{2}\left(a+b\sinh^{-1}(cx)\right)\text{Sub}$$

$$= \frac{2bx\sqrt{d+c^2dx^2}}{15c^3\sqrt{1+c^2x^2}} - \frac{bx^3\sqrt{d+c^2dx^2}}{45c\sqrt{1+c^2x^2}} - \frac{bcx^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}} + \frac{1}{2}\left(a+b\sinh\right.$$

$$= \frac{2bx\sqrt{d+c^2dx^2}}{15c^3\sqrt{1+c^2x^2}} - \frac{bx^3\sqrt{d+c^2dx^2}}{45c\sqrt{1+c^2x^2}} - \frac{bcx^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}} - \frac{\left(d+c^2dx^2\right)^{3/2}}{}$$

Mathematica [A] time = 0.11, size = 120, normalized size = 0.69

$$\frac{\sqrt{c^2dx^2+d}\left(15a\left(3c^2x^2-2\right)\left(c^2x^2+1\right)^2+15b\left(3c^2x^2-2\right)\left(c^2x^2+1\right)^2\sinh^{-1}(cx)+bcx\left(-9c^4x^4-5c^2x^2+30\right)\sqrt{}\right.}{225c^4\left(c^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(15*a*(1 + c^2*x^2)^2*(-2 + 3*c^2*x^2) + b*c*x*Sqrt[1
+ c^2*x^2]*(30 - 5*c^2*x^2 - 9*c^4*x^4) + 15*b*(1 + c^2*x^2)^2*(-2 + 3*c^2*
x^2)*ArcSinh[c*x]))/(225*c^4*(1 + c^2*x^2))
```

fricas [A] time = 0.46, size = 158, normalized size = 0.90

$$\frac{15\left(3\,bc^6x^6+4\,bc^4x^4-bc^2x^2-2\,b\right)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(45\,ac^6x^6+60\,ac^4x^4-15\,ac^2x^2-\left(9\,bc^5x\right.\right.}{225\left(c^6x^2+c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/225*(15*(3*b*c^6*x^6 + 4*b*c^4*x^4 - b*c^2*x^2 - 2*b)*sqrt(c^2*d*x^2 + d)
*log(c*x + sqrt(c^2*x^2 + 1)) + (45*a*c^6*x^6 + 60*a*c^4*x^4 - 15*a*c^2*x^2
 - (9*b*c^5*x^5 + 5*b*c^3*x^3 - 30*b*c*x)*sqrt(c^2*x^2 + 1) - 30*a)*sqrt(c^
2*d*x^2 + d))/(c^6*x^2 + c^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.26, size = 578, normalized size = 3.30

$$a\left(\frac{x^2\left(c^2d\,x^2+d\right)^{\frac{3}{2}}}{5c^2d}-\frac{2\left(c^2d\,x^2+d\right)^{\frac{3}{2}}}{15d\,c^4}\right)+b\left(\frac{\sqrt{d\left(c^2x^2+1\right)}\left(16c^6x^6+16c^5x^5\sqrt{c^2x^2+1}+28c^4x^4+20c^3x^3\sqrt{c^2x^2+}\right.}{800c^4\left(c^2x^2+1\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a*(1/5*x^2*(c^2*d*x^2+d)^(3/2)/c^2/d-2/15/d/c^4*(c^2*d*x^2+d)^(3/2))+b*(1/8
00*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^
4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+5*
arcsinh(c*x))/c^4/(c^2*x^2+1)-1/288*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*
x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arcsinh(c*
x))/c^4/(c^2*x^2+1)-1/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/
2)+1)*(-1+arcsinh(c*x))/c^4/(c^2*x^2+1)-1/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2
-c*x*(c^2*x^2+1)^(1/2)+1)*(1+arcsinh(c*x))/c^4/(c^2*x^2+1)-1/288*(d*(c^2*x^
2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2
+1)^(1/2)+1)*(1+3*arcsinh(c*x))/c^4/(c^2*x^2+1)+1/800*(d*(c^2*x^2+1))^(1/2)
*(16*c^6*x^6-16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)
^(1/2)+13*c^2*x^2-5*c*x*(c^2*x^2+1)^(1/2)+1)*(1+5*arcsinh(c*x))/c^4/(c^2*x^
2+1))
```

maxima [A] time = 0.40, size = 134, normalized size = 0.77

$$\frac{1}{15}b\left(\frac{3\left(c^2dx^2+d\right)^{\frac{3}{2}}x^2}{c^2d}-\frac{2\left(c^2dx^2+d\right)^{\frac{3}{2}}}{c^4d}\right)\operatorname{arsinh}(cx)+\frac{1}{15}a\left(\frac{3\left(c^2dx^2+d\right)^{\frac{3}{2}}x^2}{c^2d}-\frac{2\left(c^2dx^2+d\right)^{\frac{3}{2}}}{c^4d}\right)-\frac{\left(9c^4\sqrt{d}x^5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/15*b*(3*(c^2*d*x^2 + d)^(3/2)*x^2/(c^2*d) - 2*(c^2*d*x^2 + d)^(3/2)/(c^4*
d))*arcsinh(c*x) + 1/15*a*(3*(c^2*d*x^2 + d)^(3/2)*x^2/(c^2*d) - 2*(c^2*d*x
^2 + d)^(3/2)/(c^4*d)) - 1/225*(9*c^4*sqrt(d)*x^5 + 5*c^2*sqrt(d)*x^3 - 30*
sqrt(d)*x)*b/c^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\sqrt{d\,c^2\,x^2+d}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\sqrt{d\left(c^{2}x^{2}+1\right)}\left(a+b\operatorname{asinh}(cx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x)), x)
```

3.121 $\int x^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)dx$

Optimal. Leaf size=181

$$\frac{x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{8c^2}+\frac{1}{4}x^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)-\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc^3\sqrt{c^2x^2+1}}-\frac{bx^2\sqrt{c^2d}}{16c\sqrt{c^2x}}$$

```
[Out] 1/8*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2+1/4*x^3*(a+b*arcsinh(c*x))
*(c^2*d*x^2+d)^(1/2)-1/16*b*x^2*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-1/1
6*b*c*x^4*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/16*(a+b*arcsinh(c*x))^2*(
c^2*d*x^2+d)^(1/2)/b/c^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5742, 5758, 5675, 30}

$$\frac{1}{4}x^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{8c^2}-\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc^3\sqrt{c^2x^2+1}}-\frac{bcx^4\sqrt{c^2d}}{16\sqrt{c^2x}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(b*x^2*Sqrt[d + c^2*d*x^2])/(16*c*Sqrt[1 + c^2*x^2]) - (b*c*x^4*Sqrt[d + c
^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) + (x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c
*x]))/(8*c^2) + (x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/4 - (Sqrt[d
+ c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(16*b*c^3*Sqrt[1 + c^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int x^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)dx=\frac{1}{4}x^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{\sqrt{d+c^2dx^2}\int\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,d}{4\sqrt{1+c^2x^2}}$$

$$=-\frac{bcx^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}+\frac{x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{8c^2}+\frac{1}{4}x^3\sqrt{d+c^2}$$

$$=-\frac{bx^2\sqrt{d+c^2dx^2}}{16c\sqrt{1+c^2x^2}}-\frac{bcx^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}+\frac{x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(\right.}{8c^2}$$

Mathematica [A] time = 0.81, size = 129, normalized size = 0.71

$$-\frac{-16acx\left(2c^2x^2+1\right)\sqrt{c^2dx^2+d}+16a\sqrt{d}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)+\frac{b\sqrt{c^2dx^2+d}\left(8\sinh^{-1}(cx)^2-4\sinh\left(4\sinh^{-1}(cx)\right.\right.}{\sqrt{c^2x^2+1}}}{128c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/128*(-16*a*c*x*(1 + 2*c^2*x^2)*Sqrt[d + c^2*d*x^2] + 16*a*Sqrt[d]*Log[c*
d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (b*Sqrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]
^2 + Cosh[4*ArcSinh[c*x]] - 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]))/Sqrt[1 +
c^2*x^2])/c^3
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{c^2dx^2+d}\left(bx^2\operatorname{arsinh}(cx)+ax^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*x^2*arcsinh(c*x) + a*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{c^2dx^2+d}\,(b\operatorname{arsinh}(cx)+a)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)*x^2, x)
```

maple [B] time = 0.23, size = 320, normalized size = 1.77

$$\frac{ax\left(c^2dx^2+d\right)^{\frac{3}{2}}}{4c^2d}-\frac{ax\sqrt{c^2dx^2+d}}{8c^2}-\frac{ad\ln\left(\frac{xc^2d}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{8c^2\sqrt{c^2d}}-\frac{b\sqrt{d\left(c^2x^2+1\right)}}{128c^3\sqrt{c^2x^2+1}}-\frac{b\sqrt{d\left(c^2x^2+1\right)}\operatorname{arcsinh}(c}{16\sqrt{c^2x^2+1}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] 1/4*a*x*(c^2*d*x^2+d)^(3/2)/c^2/d-1/8*a/c^2*x*(c^2*d*x^2+d)^(1/2)-1/8*a/c^2
*d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)-1/128*b*(d*(
```

```
c^2*x^2+1))^(1/2)/c^3/(c^2*x^2+1)^(1/2)-1/16*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x
^2+1)^(1/2)/c^3*arcsinh(c*x)^2+1/4*b*(d*(c^2*x^2+1))^(1/2)*c^2/(c^2*x^2+1)*
arcsinh(c*x)*x^5-1/16*b*(d*(c^2*x^2+1))^(1/2)*c/(c^2*x^2+1)^(1/2)*x^4+3/8*b
*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)*arcsinh(c*x)*x^3-1/16*b*(d*(c^2*x^2+1))^
(1/2)/c/(c^2*x^2+1)^(1/2)*x^2+1/8*b*(d*(c^2*x^2+1))^(1/2)/c^2/(c^2*x^2+1)*a
rcsinh(c*x)*x
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\sqrt{d\,c^2\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2),x)
[Out] int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2} \sqrt{d \left(c^{2} x^{2} + 1\right)} \left(a + b \operatorname{asinh}{\left(c x \right)}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2),x)
[Out] Integral(x**2*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x)), x)
```

3.122 $\int x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)dx$

Optimal. Leaf size=105

$$\frac{\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c^2d}-\frac{bx\sqrt{c^2dx^2+d}}{3c\sqrt{c^2x^2+1}}-\frac{bcx^3\sqrt{c^2dx^2+d}}{9\sqrt{c^2x^2+1}}$$

```
[Out] 1/3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/c^2/d-1/3*b*x*(c^2*d*x^2+d)^(1/2
)/c/(c^2*x^2+1)^(1/2)-1/9*b*c*x^3*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {5717}

$$\frac{\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c^2d}-\frac{bcx^3\sqrt{c^2dx^2+d}}{9\sqrt{c^2x^2+1}}-\frac{bx\sqrt{c^2dx^2+d}}{3c\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(b*x*Sqrt[d + c^2*d*x^2])/(3*c*Sqrt[1 + c^2*x^2]) - (b*c*x^3*Sqrt[d + c^2*
d*x^2])/(9*Sqrt[1 + c^2*x^2]) + ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])
)/(3*c^2*d)
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)dx &= \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c^2d}-\frac{\left(b\sqrt{d+c^2dx^2}\right)\int\left(1+c^2x^2\right)dx}{3c\sqrt{1+c^2x^2}}\\ &= -\frac{bx\sqrt{d+c^2dx^2}}{3c\sqrt{1+c^2x^2}}-\frac{bcx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}+\frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(c\right.}{3c^2d}\end{aligned}$$

Mathematica [A] time = 0.09, size = 92, normalized size = 0.88

$$\frac{\sqrt{c^2dx^2+d}\left(3a\left(c^2x^2+1\right)^2-bcx\left(c^2x^2+3\right)\sqrt{c^2x^2+1}+3b\left(c^2x^2+1\right)^2\sinh^{-1}(cx)\right)}{9c^2\left(c^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(3*a*(1 + c^2*x^2)^2 - b*c*x*Sqrt[1 + c^2*x^2]*(3 + c^
2*x^2) + 3*b*(1 + c^2*x^2)^2*ArcSinh[c*x]))/(9*c^2*(1 + c^2*x^2))
```

fricas [A] time = 0.48, size = 127, normalized size = 1.21

$$\frac{3\left(bc^4x^4+2\,bc^2x^2+b\right)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(3\,ac^4x^4+6\,ac^2x^2-\left(bc^3x^3+3\,bcx\right)\sqrt{c^2x^2+1}\,+3\,a\right)}{9\left(c^4x^2+c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*(3*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2
*x^2 + 1)) + (3*a*c^4*x^4 + 6*a*c^2*x^2 - (b*c^3*x^3 + 3*b*c*x)*sqrt(c^2*x^
2 + 1) + 3*a)*sqrt(c^2*d*x^2 + d))/(c^4*x^2 + c^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.14, size = 321, normalized size = 3.06

$$\frac{a\left(c^2d\,x^2+d\right)^{\frac{3}{2}}}{3c^2d}+b\left(\frac{\sqrt{d\left(c^2x^2+1\right)}\,\left(4c^4x^4+4c^3x^3\sqrt{c^2x^2+1}\,+5c^2x^2+3cx\sqrt{c^2x^2+1}\,+1\right)\left(-1+3\,\mathrm{arcsinh}\,(cx)\right)}{72c^2\left(c^2x^2+1\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] 1/3*a/c^2/d*(c^2*d*x^2+d)^(3/2)+b*(1/72*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*
c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arcsin
h(c*x))/c^2/(c^2*x^2+1)+1/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^
(1/2)+1)*(-1+arcsinh(c*x))/c^2/(c^2*x^2+1)+1/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x
^2-c*x*(c^2*x^2+1)^(1/2)+1)*(1+arcsinh(c*x))/c^2/(c^2*x^2+1)+1/72*(d*(c^2*x
^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^
2+1)^(1/2)+1)*(1+3*arcsinh(c*x))/c^2/(c^2*x^2+1))
```

maxima [A] time = 0.43, size = 73, normalized size = 0.70

$$\frac{\left(c^2dx^2+d\right)^{\frac{3}{2}}b\,\mathrm{arsinh}\,(cx)}{3\,c^2d}-\frac{\left(c^2d^{\frac{3}{2}}x^3+3\,d^{\frac{3}{2}}x\right)b}{9\,cd}+\frac{\left(c^2dx^2+d\right)^{\frac{3}{2}}a}{3\,c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(c^2*d*x^2 + d)^(3/2)*b*arcsinh(c*x)/(c^2*d) - 1/9*(c^2*d^(3/2)*x^3 + 3
*d^(3/2)*x)*b/(c*d) + 1/3*(c^2*d*x^2 + d)^(3/2)*a/(c^2*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\left(a+b\,\mathrm{asinh}\,(c\,x)\right)\,\sqrt{d\,c^2\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2),x)
[Out] int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{d\left(c^2x^2+1\right)}\,(a+b\operatorname{asinh}(cx))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2),x)
[Out] Integral(x*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x)), x)
```

3.123 $\int \sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)dx$

Optimal. Leaf size=111

$$\frac{1}{2}x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{c^2x^2+1}}-\frac{bcx^2\sqrt{c^2dx^2+d}}{4\sqrt{c^2x^2+1}}$$

```
[Out] 1/2*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)-1/4*b*c*x^2*(c^2*d*x^2+d)^(1/2
)/(c^2*x^2+1)^(1/2)+1/4*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x
^2+1)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5682, 5675, 30}

$$\frac{1}{2}x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{c^2x^2+1}}-\frac{bcx^2\sqrt{c^2dx^2+d}}{4\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(b*c*x^2*Sqrt[d + c^2*d*x^2])/(4*Sqrt[1 + c^2*x^2]) + (x*Sqrt[d + c^2*d*x^
2]*(a + b*ArcSinh[c*x]))/2 + (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(
4*b*c*Sqrt[1 + c^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rubi steps

$$\int \sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)dx=\frac{1}{2}x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{\sqrt{d+c^2dx^2}\int\frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{2\sqrt{1+c^2x^2}}-\frac{\left(bc\sqrt{\ldots}\right.}{\ldots}$$

$$=-\frac{bcx^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}}+\frac{1}{2}x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{\sqrt{d+c^2dx^2}\left(a\ldots\right.}{4bc\sqrt{1\ldots}}$$

Mathematica [A] time = 0.45, size = 120, normalized size = 1.08

$$\frac{1}{8}\left(4ax\sqrt{c^2dx^2+d}+\frac{4a\sqrt{d}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)}{c}+\frac{b\sqrt{c^2dx^2+d}\left(2\sinh^{-1}(cx)\left(\sinh^{-1}(cx)+\sinh\left(2s\right.\right.\right.}{c\sqrt{c^2x^2+1}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (4*a*x*Sqrt[d + c^2*d*x^2] + (4*a*Sqrt[d]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*
d*x^2]])/c + (b*Sqrt[d + c^2*d*x^2]*(-Cosh[2*ArcSinh[c*x]] + 2*ArcSinh[c*x]
*(ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])))/(c*Sqrt[1 + c^2*x^2]))/8
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{c^2dx^2+d}\,(b\,\text{arsinh}\,(cx)+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.14, size = 222, normalized size = 2.00

$$\frac{ax\sqrt{c^2d\,x^2+d}}{2}+\frac{ad\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2\sqrt{c^2d}}+\frac{b\sqrt{d\left(c^2x^2+1\right)}\,\text{arcsinh}\,(cx)^2}{4\sqrt{c^2x^2+1}\,c}+\frac{b\sqrt{d\left(c^2x^2+1\right)}\,c^2\,\text{arcsinh}\,(cx)}{2c^2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] 1/2*a*x*(c^2*d*x^2+d)^(1/2)+1/2*a*d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^
(1/2))/(c^2*d)^(1/2)+1/4*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsin
h(c*x)^2+1/2*b*(d*(c^2*x^2+1))^(1/2)*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^3-1/4*b
*(d*(c^2*x^2+1))^(1/2)*c/(c^2*x^2+1)^(1/2)*x^2+1/2*b*(d*(c^2*x^2+1))^(1/2)/
(c^2*x^2+1)*arcsinh(c*x)*x-1/8*b*(d*(c^2*x^2+1))^(1/2)/c/(c^2*x^2+1)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\mathrm{asinh}\,(c\,x))\,\sqrt{d\,c^2\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2),x)
[Out] int((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\left(c^2x^2+1\right)}\,(a+b\,\mathrm{asinh}\,(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2),x)
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x)), x)
```

3.124 $\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=177

$$\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)-\frac{2\sqrt{c^2dx^2+d}\,\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}-\frac{b\sqrt{c^2dx^2+d}\,\mathrm{Li}_2\left(-e^{\sinh^{-}}\right.}{\sqrt{c^2x^2+1}}$$

```
[Out] (a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)-b*c*x*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1
)^(1/2)-2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(
1/2)/(c^2*x^2+1)^(1/2)-b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1
/2)/(c^2*x^2+1)^(1/2)+b*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2
)/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5742, 5760, 4182, 2279, 2391, 8}

$$-\frac{b\sqrt{c^2dx^2+d}\,\mathrm{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+\frac{b\sqrt{c^2dx^2+d}\,\mathrm{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] -((b*c*x*Sqrt[d + c^2*d*x^2])/Sqrt[1 + c^2*x^2]) + Sqrt[d + c^2*d*x^2]*(a +
 b*ArcSinh[c*x]) - (2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^Ar
cSinh[c*x]])/Sqrt[1 + c^2*x^2] - (b*Sqrt[d + c^2*d*x^2]*PolyLog[2, -E^ArcSi
nh[c*x]])/Sqrt[1 + c^2*x^2] + (b*Sqrt[d + c^2*d*x^2]*PolyLog[2, E^ArcSinh[c
*x]])/Sqrt[1 + c^2*x^2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
```

```
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x}\,dx = \sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{\sqrt{d+c^2dx^2}\int\frac{a+b\sinh^{-1}(cx)}{x\sqrt{1+c^2x^2}}\,dx}{\sqrt{1+c^2x^2}}-\frac{\left(bc\sqrt{d\right.}}{\sqrt{}}$$

$$= -\frac{bcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}+\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\right.}{}$$

$$= -\frac{bcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}+\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{2\sqrt{d+c^2dx^2}\left(a+b\right.}{}$$

$$= -\frac{bcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}+\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{2\sqrt{d+c^2dx^2}\left(a+b\right.}{}$$

$$= -\frac{bcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}+\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{2\sqrt{d+c^2dx^2}\left(a+b\right.}{}$$

Mathematica [A] time = 0.45, size = 168, normalized size = 0.95

$$a\sqrt{c^2dx^2+d}-a\sqrt{d}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+d\right)+a\sqrt{d}\log(x)+\frac{b\sqrt{c^2dx^2+d}\left(\sqrt{c^2x^2+1}\sinh^{-1}(cx)+\text{Li}_2\left(-e^{-\sinh^{-1}}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] a*Sqrt[d + c^2*d*x^2] + a*Sqrt[d]*Log[x] - a*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d
 + c^2*d*x^2]] + (b*Sqrt[d + c^2*d*x^2]*(-(c*x) + Sqrt[1 + c^2*x^2]*ArcSinh
[c*x] + ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Log[1 + E^(-
ArcSinh[c*x])] + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*
x])]))/Sqrt[1 + c^2*x^2]
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\,(b\,\text{arsinh}\,(cx)+a)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/x, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.18, size = 331, normalized size = 1.87

$$-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2d\,x^2+d}}{x}\right)a+a\sqrt{c^2d\,x^2+d}+\frac{b\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsinh}\left(cx\right)x^2c^2}{c^2x^2+1}-\frac{b\sqrt{d\left(c^2x^2+1\right)}\,cx}{\sqrt{c^2x^2+1}}+-\frac{b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x,x)
```

```
[Out] -d^(1/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)*a+a*(c^2*d*x^2+d)^(1/2)+
b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)*arcsinh(c*x)*x^2*c^2-b*(d*(c^2*x^2+1))^
(1/2)/(c^2*x^2+1)^(1/2)*c*x+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)*arcsinh(c*x
)+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+
1)^(1/2))+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+
1)^(1/2))-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(
c^2*x^2+1)^(1/2))-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-
(c^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\left(\sqrt{d}\,\operatorname{arsinh}\left(\frac{1}{c|x|}\right)-\sqrt{c^2dx^2+d}\right)a+b\int\frac{\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -(sqrt(d)*arcsinh(1/(c*abs(x))) - sqrt(c^2*d*x^2 + d))*a + b*integrate(sqrt
(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(a+b\,\operatorname{asinh}\left(c\,x\right)\right)\,\sqrt{d\,c^2\,x^2+d}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/x,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{d\left(c^2x^2+1\right)}\,\left(a+b\operatorname{asinh}\left(cx\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))/x, x)
```

3.125 $\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=105

$$\frac{c\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{2b\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{x}+\frac{bc\log(x)\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}}$$

```
[Out] -(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x+1/2*c*(a+b*arcsinh(c*x))^2*(c^2*d
*x^2+d)^(1/2)/b/(c^2*x^2+1)^(1/2)+b*c*ln(x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)
^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {5737, 29, 5675}

$$\frac{c\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{2b\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{x}+\frac{bc\log(x)\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] -((Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x) + (c*Sqrt[d + c^2*d*x^2]*(a
 + b*ArcSinh[c*x])^2)/(2*b*Sqrt[1 + c^2*x^2]) + (b*c*Sqrt[d + c^2*d*x^2]*Lo
g[x])/Sqrt[1 + c^2*x^2]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5737

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 1)*S
qrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x] -
 Dist[(c^2*Sqrt[d + e*x^2])/(f^2*(m + 1)*Sqrt[1 + c^2*x^2]), Int[((f*x)^(m
+ 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x^2}\,dx=-\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x}+\frac{\left(bc\sqrt{d+c^2dx^2}\right)\int\frac{1}{x}\,dx}{\sqrt{1+c^2x^2}}+\frac{\left(c^2\sqrt{d+c^2\right.}{}$$

$$=-\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x}+\frac{c\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2b\sqrt{1+c^2x^2}}+\frac{bc}{}$$

Mathematica [A] time = 0.32, size = 129, normalized size = 1.23

$$-\frac{a\sqrt{d\left(c^2x^2+1\right)}}{x}+ac\sqrt{d}\log\left(\sqrt{d}\sqrt{d\left(c^2x^2+1\right)}+cdx\right)+\frac{bc\sqrt{d\left(c^2x^2+1\right)}\left(-\frac{2\sqrt{c^2x^2+1}\sinh^{-1}(cx)}{cx}+2\log(cx)+s\right.}{2\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] -((a*Sqrt[d*(1 + c^2*x^2)])/x) + (b*c*Sqrt[d*(1 + c^2*x^2)]*((-2*Sqrt[1 + c
^2*x^2]*ArcSinh[c*x])/(c*x) + ArcSinh[c*x]^2 + 2*Log[c*x]))/(2*Sqrt[1 + c^2
*x^2]) + a*c*Sqrt[d]*Log[c*d*x + Sqrt[d]*Sqrt[d*(1 + c^2*x^2)]]
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\,(b\,\text{arsinh}\,(cx)+a)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/x^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.18, size = 263, normalized size = 2.50

$$-\frac{a\left(c^2d\,x^2+d\right)^{\frac{3}{2}}}{dx}+a\,c^2x\sqrt{c^2d\,x^2+d}+\frac{a\,c^2d\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{\sqrt{c^2d}}+\frac{b\sqrt{d\left(c^2x^2+1\right)}\,\text{arcsinh}\,(cx)^2\,c}{2\sqrt{c^2x^2+1}}-\frac{b\sqrt{d\,(c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^2,x)
```

```
[Out] -a/d/x*(c^2*d*x^2+d)^(3/2)+a*c^2*x*(c^2*d*x^2+d)^(1/2)+a*c^2*d*ln(x*c^2*d/(
c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/2*b*(d*(c^2*x^2+1))^(1/2)
/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/
2)*arcsinh(c*x)*c-b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)*x/(c^2*x^2+1)*c^2-b*
(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/x/(c^2*x^2+1)+b*(d*(c^2*x^2+1))^(1/2)/(c
^2*x^2+1)^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(c\sqrt{d}\,\text{arsinh}\,(cx)-\frac{\sqrt{c^2dx^2+d}}{x}\right)a+b\int\frac{\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] (c*sqrt(d)*arcsinh(c*x) - sqrt(c^2*d*x^2 + d)/x)*a + b*integrate(sqrt(c^2*d
*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{asinh}\,(c\,x))\,\sqrt{d\,c^2\,x^2 + d}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\left(c^2x^2+1\right)}\,(a + b\,\mathrm{asinh}\,(cx))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))/x**2, x)
```

3.126 $\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=201

$$-\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{2x^2}-\frac{c^2\sqrt{c^2dx^2+d}\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}-\frac{bc^2\sqrt{c^2dx^2+d}\,\mathrm{Li}_2\left(-e\right.}{2\sqrt{c^2x^2+1}}$$

```
[Out] -1/2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^2-1/2*b*c*(c^2*d*x^2+d)^(1/2)
/x/(c^2*x^2+1)^(1/2)-c^2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))*
(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/2*b*c^2*polylog(2,-c*x-(c^2*x^2+1)^
(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+1/2*b*c^2*polylog(2,c*x+(c^2*x
^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5737, 30, 5760, 4182, 2279, 2391}

$$-\frac{bc^2\sqrt{c^2dx^2+d}\,\mathrm{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2x^2+1}}+\frac{bc^2\sqrt{c^2dx^2+d}\,\mathrm{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(\right.}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] -(b*c*Sqrt[d + c^2*d*x^2])/(2*x*Sqrt[1 + c^2*x^2]) - (Sqrt[d + c^2*d*x^2]*(
a + b*ArcSinh[c*x]))/(2*x^2) - (c^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]
)*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (b*c^2*Sqrt[d + c^2*d*x^2]*P
olyLog[2, -E^ArcSinh[c*x]])/(2*Sqrt[1 + c^2*x^2]) + (b*c^2*Sqrt[d + c^2*d*x
^2]*PolyLog[2, E^ArcSinh[c*x]])/(2*Sqrt[1 + c^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5737

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
```

```
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 1)*S
qrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x] -
 Dist[(c^2*Sqrt[d + e*x^2])/(f^2*(m + 1)*Sqrt[1 + c^2*x^2]), Int[((f*x)^(m
+ 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x^3}\,dx = -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2x^2}+\frac{\left(bc\sqrt{d+c^2dx^2}\right)\int\frac{1}{x^2}\,dx}{2\sqrt{1+c^2x^2}}+\frac{\left(c^2\sqrt{d+c^2\right.}{}$$

$$= -\frac{bc\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}}-\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2x^2}+\frac{\left(c^2\sqrt{d+c^2dx^2}\right)\mathrm{Sub}}{}$$

$$= -\frac{bc\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}}-\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2x^2}-\frac{c^2\sqrt{d+c^2dx^2}\left(a+b\right.}{}$$

$$= -\frac{bc\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}}-\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2x^2}-\frac{c^2\sqrt{d+c^2dx^2}\left(a+b\right.}{}$$

$$= -\frac{bc\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}}-\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2x^2}-\frac{c^2\sqrt{d+c^2dx^2}\left(a+b\right.}{}$$

Mathematica [A] time = 2.93, size = 223, normalized size = 1.11

$$\frac{1}{8}\left(-\frac{4a\sqrt{c^2dx^2+d}}{x^2}-4ac^2\sqrt{d}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+d\right)+4ac^2\sqrt{d}\log(x)+\frac{bc^2\sqrt{c^2dx^2+d}\left(4\mathrm{Li}_2\left(-e^{-\sinh^{-1}(cx)}\right)\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] ((-4*a*Sqrt[d + c^2*d*x^2])/x^2 + 4*a*c^2*Sqrt[d]*Log[x] - 4*a*c^2*Sqrt[d]*
Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (b*c^2*Sqrt[d + c^2*d*x^2]*(-2*Coth[
ArcSinh[c*x]/2] - ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*x]*Log[
1 - E^(-ArcSinh[c*x])] - 4*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 4*Poly
Log[2, -E^(-ArcSinh[c*x])] - 4*PolyLog[2, E^(-ArcSinh[c*x])] - ArcSinh[c*x]
*Sech[ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSinh[c*x]/2]))/Sqrt[1 + c^2*x^2])/8
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{c^2dx^2+d}\,(b\,\mathrm{arsinh}\,(cx)+a)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/x^3, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.24, size = 377, normalized size = 1.88

$$-\frac{a\left(c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}{2 d\,x^{2}}-\frac{a\sqrt{d}\,\ln\left(\frac{2 d+2\sqrt{d}\,\sqrt{c^{2} d\,x^{2}+d}}{x}\right) c^{2}}{2}+\frac{a\sqrt{c^{2} d\,x^{2}+d}\,c^{2}}{2}-\frac{b\sqrt{d\left(c^{2} x^{2}+1\right)}\,\operatorname{arcsinh}\left(c x\right) c^{2}}{2\left(c^{2} x^{2}+1\right)}-\frac{b\sqrt{d\left(c^{2}x\right.}}{2x\sqrt{c^{2}x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^3,x)
```

```
[Out] -1/2*a/d/x^2*(c^2*d*x^2+d)^(3/2)-1/2*a*d^(1/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2
+d)^(1/2))/x)*c^2+1/2*a*(c^2*d*x^2+d)^(1/2)*c^2-1/2*b*(d*(c^2*x^2+1))^(1/2)
/(c^2*x^2+1)*arcsinh(c*x)*c^2-1/2*b*(d*(c^2*x^2+1))^(1/2)/x/(c^2*x^2+1)^(1/
2)*c-1/2*b*(d*(c^2*x^2+1))^(1/2)/x^2/(c^2*x^2+1)*arcsinh(c*x)+1/2*b*(d*(c^2
*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c
^2+1/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^
(1/2))*c^2-1/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+
c*x+(c^2*x^2+1)^(1/2))*c^2-1/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*po
lylog(2,-c*x-(c^2*x^2+1)^(1/2))*c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\left(c^{2}\sqrt{d}\,\operatorname{arsinh}\left(\frac{1}{c|x|}\right)-\sqrt{c^{2}dx^{2}+d}\,c^{2}+\frac{\left(c^{2}dx^{2}+d\right)^{\frac{3}{2}}}{dx^{2}}\right)a+b\int\frac{\sqrt{c^{2}dx^{2}+d}\,\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c^2*sqrt(d)*arcsinh(1/(c*abs(x))) - sqrt(c^2*d*x^2 + d)*c^2 + (c^2*d*
x^2 + d)^(3/2)/(d*x^2))*a + b*integrate(sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(
c^2*x^2 + 1))/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\sqrt{d\,c^{2}\,x^{2}+d}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/x^3,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{d\left(c^{2}x^{2}+1\right)}\,\left(a+b\,\operatorname{asinh}\left(cx\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))/x**3, x)
```

3.127 $\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=106

$$-\frac{\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^3}-\frac{bc\sqrt{c^2dx^2+d}}{6x^2\sqrt{c^2x^2+1}}+\frac{bc^3\log(x)\sqrt{c^2dx^2+d}}{3\sqrt{c^2x^2+1}}$$

```
[Out] -1/3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/d/x^3-1/6*b*c*(c^2*d*x^2+d)^(1/
2)/x^2/(c^2*x^2+1)^(1/2)+1/3*b*c^3*ln(x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1
/2)
```

Rubi [A] time = 0.09, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {5723, 14}

$$-\frac{\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^3}-\frac{bc\sqrt{c^2dx^2+d}}{6x^2\sqrt{c^2x^2+1}}+\frac{bc^3\log(x)\sqrt{c^2dx^2+d}}{3\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] -(b*c*Sqrt[d + c^2*d*x^2])/(6*x^2*Sqrt[1 + c^2*x^2]) - ((d + c^2*d*x^2)^(3/
2)*(a + b*ArcSinh[c*x]))/(3*d*x^3) + (b*c^3*Sqrt[d + c^2*d*x^2]*Log[x])/(3*
Sqrt[1 + c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x^4}\,dx &= -\frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^3}+\frac{\left(bc\sqrt{d+c^2dx^2}\right)\int \frac{1+c^2x^2}{x^3}\,dx}{3\sqrt{1+c^2x^2}}\\
&= -\frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^3}+\frac{\left(bc\sqrt{d+c^2dx^2}\right)\int \left(\frac{1}{x^3}+\frac{c^2}{x}\right)dx}{3\sqrt{1+c^2x^2}}\\
&= -\frac{bc\sqrt{d+c^2dx^2}}{6x^2\sqrt{1+c^2x^2}}-\frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^3}+\frac{bc^3\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x}}
\end{aligned}$$

Mathematica [A] time = 0.24, size = 131, normalized size = 1.24

$$\frac{bc^3\log(x)\sqrt{d\left(c^2x^2+1\right)}}{3\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}\left(2a\left(c^2x^2+1\right)^2+bcx\left(3c^2x^2+1\right)\sqrt{c^2x^2+1}+2b\left(c^2x^2+1\right)^2\sinh^{-1}(cx)\right)}{6x^3\left(c^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] -1/6*(Sqrt[d + c^2*d*x^2]*(2*a*(1 + c^2*x^2)^2 + b*c*x*Sqrt[1 + c^2*x^2]*(1
 + 3*c^2*x^2) + 2*b*(1 + c^2*x^2)^2*ArcSinh[c*x]))/(x^3*(1 + c^2*x^2)) + (b
*c^3*Sqrt[d*(1 + c^2*x^2)]*Log[x])/(3*Sqrt[1 + c^2*x^2])
```

fricas [B] time = 0.73, size = 217, normalized size = 2.05

$$-\frac{2\left(bc^4x^4+2\,bc^2x^2+b\right)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)-\left(bc^5x^5+bc^3x^3\right)\sqrt{d}\,\log\left(\frac{c^2dx^6+c^2dx^2+dx^4+\sqrt{c^2dx^2+d}\,\sqrt{c^2x}}{c^2x^4+x^2}\right.}{6\left(c^2x^5+x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(2*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^
2*x^2 + 1)) - (b*c^5*x^5 + b*c^3*x^3)*sqrt(d)*log((c^2*d*x^6 + c^2*d*x^2 +
d*x^4 + sqrt(c^2*d*x^2 + d)*sqrt(c^2*x^2 + 1)*(x^4 - 1)*sqrt(d) + d)/(c^2*x
^4 + x^2)) + (2*a*c^4*x^4 + 4*a*c^2*x^2 - (b*c*x^3 - b*c*x)*sqrt(c^2*x^2 +
1) + 2*a)*sqrt(c^2*d*x^2 + d))/(c^2*x^5 + x^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.26, size = 946, normalized size = 8.92

$$-\frac{a\left(c^2d\,x^2+d\right)^{\frac{3}{2}}}{3d\,x^3}-\frac{2b\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsinh}(cx)\,c^3}{3\sqrt{c^2x^2+1}}-\frac{b\sqrt{d\left(c^2x^2+1\right)}\,x^5\operatorname{arcsinh}(cx)\,c^8}{\left(3c^4x^4+3c^2x^2+1\right)\left(c^2x^2+1\right)}+\frac{b\sqrt{d\left(c^2x^2+1\right)}\,x^4\operatorname{arcsi}}{\left(3c^4x^4+3c^2x^2+1\right)\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^4,x)
```

```
[Out] -1/3*a/d/x^3*(c^2*d*x^2+d)^(3/2)-2/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1
/2)*arcsinh(c*x)*c^3-b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^5/(c
^2*x^2+1)*arcsinh(c*x)*c^8+b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*
x^4/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^7-1/6*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x
^4+3*c^2*x^2+1)*x^5/(c^2*x^2+1)*c^8+1/6*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+
3*c^2*x^2+1)*x^3*c^6-3*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^3/
(c^2*x^2+1)*arcsinh(c*x)*c^6+b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1
)*x^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^5-1/3*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4
*x^4+3*c^2*x^2+1)*x^3/(c^2*x^2+1)*c^6-1/2*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^
```

```
4+3*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*c^5+1/6*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4
*x^4+3*c^2*x^2+1)*x*c^4-10/3*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1
)*x/(c^2*x^2+1)*arcsinh(c*x)*c^4+1/3*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c
^2*x^2+1)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^3-1/6*b*(d*(c^2*x^2+1))^(1/2)/(3
*c^4*x^4+3*c^2*x^2+1)*x/(c^2*x^2+1)*c^4-1/2*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*
x^4+3*c^2*x^2+1)/(c^2*x^2+1)^(1/2)*c^3-5/3*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x
^4+3*c^2*x^2+1)/x/(c^2*x^2+1)*arcsinh(c*x)*c^2-1/6*b*(d*(c^2*x^2+1))^(1/2)/
(3*c^4*x^4+3*c^2*x^2+1)/x^2/(c^2*x^2+1)^(1/2)*c-1/3*b*(d*(c^2*x^2+1))^(1/2)
/(3*c^4*x^4+3*c^2*x^2+1)/x^3/(c^2*x^2+1)*arcsinh(c*x)+1/3*b*(d*(c^2*x^2+1))
^(1/2)/(c^2*x^2+1)^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c^3
```

maxima [A] time = 0.53, size = 133, normalized size = 1.25

$$-\frac{\left((-1)^{2\,c^2dx^2+2\,d}\,c^2d^{\frac{3}{2}}\log\left(2\,c^2d+\frac{2\,d}{x^2}\right)-c^2d^{\frac{3}{2}}\log\left(x^2+\frac{1}{c^2}\right)+\frac{\sqrt{c^4dx^4+2\,c^2dx^2+d}\,d}{x^2}\right)bc}{6\,d}-\frac{\left(c^2dx^2+d\right)^{\frac{3}{2}}b\,\mathrm{arsinh}\left(cx\right)}{3\,dx^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*((-1)^(2*c^2*d*x^2 + 2*d)*c^2*d^(3/2)*log(2*c^2*d + 2*d/x^2) - c^2*d^(
3/2)*log(x^2 + 1/c^2) + sqrt(c^4*d*x^4 + 2*c^2*d*x^2 + d)*d/x^2)*b*c/d - 1/
3*(c^2*d*x^2 + d)^(3/2)*b*arcsinh(c*x)/(d*x^3) - 1/3*(c^2*d*x^2 + d)^(3/2)*
a/(d*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))\,\sqrt{d\,c^2\,x^2+d}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/x^4,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\left(c^2x^2+1\right)}\,\left(a+b\,\mathrm{asinh}\,(cx)\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))/x**4, x)
```

3.128 $\int x^3 \left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=217

$$\frac{\left(c^2dx^2+d\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7c^4d^2}-\frac{\left(c^2dx^2+d\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5c^4d}-\frac{8bcdx^5\sqrt{c^2dx^2+d}}{175\sqrt{c^2x^2+1}}-\frac{bdx^3\sqrt{c^2dx^2+d}}{105c\sqrt{c^2x^2+1}}+\frac{2bd\ldots}{35\ldots}$$

```
[Out] -1/5*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/c^4/d+1/7*(c^2*d*x^2+d)^(7/2)*(
a+b*arcsinh(c*x))/c^4/d^2+2/35*b*d*x*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)^(1
/2)-1/105*b*d*x^3*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-8/175*b*c*d*x^5*(
c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/49*b*c^3*d*x^7*(c^2*d*x^2+d)^(1/2)/(
c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {266, 43, 5734, 12, 373}

$$\frac{\left(c^2dx^2+d\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7c^4d^2}-\frac{\left(c^2dx^2+d\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5c^4d}-\frac{bc^3dx^7\sqrt{c^2dx^2+d}}{49\sqrt{c^2x^2+1}}-\frac{8bcdx^5\sqrt{c^2dx^2+d}}{175\sqrt{c^2x^2+1}}-\frac{bc\ldots}{1\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (2*b*d*x*Sqrt[d + c^2*d*x^2])/(35*c^3*Sqrt[1 + c^2*x^2]) - (b*d*x^3*Sqrt[d
 + c^2*d*x^2])/(105*c*Sqrt[1 + c^2*x^2]) - (8*b*c*d*x^5*Sqrt[d + c^2*d*x^2])
/(175*Sqrt[1 + c^2*x^2]) - (b*c^3*d*x^7*Sqrt[d + c^2*d*x^2])/(49*Sqrt[1 + c
^2*x^2]) - ((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(5*c^4*d) + ((d + c
^2*d*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7*c^4*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 5734

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[a + b*ArcS
inh[c*x], Int[x^m*(d + e*x^2)^p, x], x] - Dist[(b*c*d^(p - 1/2)*Sqrt[d + e*
```

```
x^2])/Sqrt[1 + c^2*x^2], Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x],
 x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p + 1/2, 0] && (
IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\int x^3\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)dx=-\frac{\left(bcd\sqrt{d+c^2dx^2}\right)\int\frac{\left(1+c^2x^2\right)^2\left(-2+5c^2x^2\right)}{35c^4}dx}{\sqrt{1+c^2x^2}}+\left(a+b\sinh^{-1}(cx)\right)\int$$

$$=-\frac{\left(bd\sqrt{d+c^2dx^2}\right)\int\left(1+c^2x^2\right)^2\left(-2+5c^2x^2\right)dx}{35c^3\sqrt{1+c^2x^2}}+\frac{1}{2}\left(a+b\sinh\right.$$

$$=-\frac{\left(bd\sqrt{d+c^2dx^2}\right)\int\left(-2+c^2x^2+8c^4x^4+5c^6x^6\right)dx}{35c^3\sqrt{1+c^2x^2}}+\frac{1}{2}\left(a+b\,\mathrm{s}\right.$$

$$=\frac{2bdx\sqrt{d+c^2dx^2}}{35c^3\sqrt{1+c^2x^2}}-\frac{bdx^3\sqrt{d+c^2dx^2}}{105c\sqrt{1+c^2x^2}}-\frac{8bcdx^5\sqrt{d+c^2dx^2}}{175\sqrt{1+c^2x^2}}-\frac{bc^3}{4}$$

Mathematica [A] time = 0.16, size = 130, normalized size = 0.60

$$\frac{d\sqrt{c^2dx^2+d}\left(105a\left(5c^2x^2-2\right)\left(c^2x^2+1\right)^3+105b\left(5c^2x^2-2\right)\left(c^2x^2+1\right)^3\sinh^{-1}(cx)-bcx\left(75c^6x^6+168c^4x^4\right.\right.}{3675c^4\left(c^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d*Sqrt[d + c^2*d*x^2]*(105*a*(1 + c^2*x^2)^3*(-2 + 5*c^2*x^2) - b*c*x*Sqrt
[1 + c^2*x^2]*(-210 + 35*c^2*x^2 + 168*c^4*x^4 + 75*c^6*x^6) + 105*b*(1 + c
^2*x^2)^3*(-2 + 5*c^2*x^2)*ArcSinh[c*x]))/(3675*c^4*(1 + c^2*x^2))
```

fricas [A] time = 0.67, size = 199, normalized size = 0.92

$$\frac{105\left(5\,bc^8dx^8+13\,bc^6dx^6+9\,bc^4dx^4-bc^2dx^2-2\,bd\right)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(525\,ac^8dx^8+1365\right.}{367}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/3675*(105*(5*b*c^8*d*x^8 + 13*b*c^6*d*x^6 + 9*b*c^4*d*x^4 - b*c^2*d*x^2 -
 2*b*d)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + (525*a*c^8*d*x^8
 + 1365*a*c^6*d*x^6 + 945*a*c^4*d*x^4 - 105*a*c^2*d*x^2 - 210*a*d - (75*b*c
^7*d*x^7 + 168*b*c^5*d*x^5 + 35*b*c^3*d*x^3 - 210*b*c*d*x)*sqrt(c^2*x^2 + 1
))*sqrt(c^2*d*x^2 + d))/(c^6*x^2 + c^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.27, size = 872, normalized size = 4.02

$$a\left(\frac{x^2\left(c^2d\,x^2+d\right)^{\frac{5}{2}}}{7c^2d}-\frac{2\left(c^2d\,x^2+d\right)^{\frac{5}{2}}}{35d\,c^4}\right)+b\left(\frac{\sqrt{d\left(c^2x^2+1\right)}\,\left(64c^8x^8+64c^7x^7\sqrt{c^2x^2+1}+144c^6x^6+112c^5x^5\sqrt{c^2x^2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] a*(1/7*x^2*(c^2*d*x^2+d)^(5/2)/c^2/d-2/35/d/c^4*(c^2*d*x^2+d)^(5/2))+b*(1/6
272*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8+64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*
x^6+112*c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4+56*c^3*x^3*(c^2*x^2+1)^(1/2)+
25*c^2*x^2+7*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+7*arcsinh(c*x))*d/c^4/(c^2*x^2+1)
+1/3200*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c
^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(
-1+5*arcsinh(c*x))*d/c^4/(c^2*x^2+1)-1/384*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4
+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arc
sinh(c*x))*d/c^4/(c^2*x^2+1)-3/128*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*
x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))*d/c^4/(c^2*x^2+1)-3/128*(d*(c^2*x^2+1))^(
1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(1+arcsinh(c*x))*d/c^4/(c^2*x^2+1)-1
/384*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2
-3*c*x*(c^2*x^2+1)^(1/2)+1)*(1+3*arcsinh(c*x))*d/c^4/(c^2*x^2+1)+1/3200*(d*
(c^2*x^2+1))^(1/2)*(16*c^6*x^6-16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c
^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2-5*c*x*(c^2*x^2+1)^(1/2)+1)*(1+5*arcsinh
(c*x))*d/c^4/(c^2*x^2+1)+1/6272*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8-64*c^7*x^
7*(c^2*x^2+1)^(1/2)+144*c^6*x^6-112*c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4-5
6*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2-7*c*x*(c^2*x^2+1)^(1/2)+1)*(1+7*arcs
inh(c*x))*d/c^4/(c^2*x^2+1))
```

maxima [A] time = 0.39, size = 145, normalized size = 0.67

$$\frac{1}{35}\left(\frac{5\left(c^2dx^2+d\right)^{\frac{5}{2}}x^2}{c^2d}-\frac{2\left(c^2dx^2+d\right)^{\frac{5}{2}}}{c^4d}\right)b\,\mathrm{arsinh}\,(cx)+\frac{1}{35}\left(\frac{5\left(c^2dx^2+d\right)^{\frac{5}{2}}x^2}{c^2d}-\frac{2\left(c^2dx^2+d\right)^{\frac{5}{2}}}{c^4d}\right)a-\frac{\left(75\,c^6d^{\frac{3}{2}}x^7+1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/35*(5*(c^2*d*x^2 + d)^(5/2)*x^2/(c^2*d) - 2*(c^2*d*x^2 + d)^(5/2)/(c^4*d)
)*b*arcsinh(c*x) + 1/35*(5*(c^2*d*x^2 + d)^(5/2)*x^2/(c^2*d) - 2*(c^2*d*x^2
 + d)^(5/2)/(c^4*d))*a - 1/3675*(75*c^6*d^(3/2)*x^7 + 168*c^4*d^(3/2)*x^5 +
 35*c^2*d^(3/2)*x^3 - 210*d^(3/2)*x)*b/c^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(d\,c^2\,x^2+d\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}\left(a+b\,\mathrm{asinh}\,(cx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x**3*(d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x)), x)
```

3.129 $\int x^2 \left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=254

$$\frac{dx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{16c^2}+\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{8}dx^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)-\frac{d}{-}$$

```
[Out] 1/6*x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))+1/16*d*x*(a+b*arcsinh(c*x))*
(c^2*d*x^2+d)^(1/2)/c^2+1/8*d*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)-1/
32*b*d*x^2*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-7/96*b*c*d*x^4*(c^2*d*x^
2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/36*b*c^3*d*x^6*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+
1)^(1/2)-1/32*d*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c^3/(c^2*x^2+1)^
(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5744, 5742, 5758, 5675, 30, 14}

$$\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{8}dx^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{dx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{16c^2}-\frac{d}{-}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(b*d*x^2*Sqrt[d + c^2*d*x^2])/(32*c*Sqrt[1 + c^2*x^2]) - (7*b*c*d*x^4*Sqrt
[d + c^2*d*x^2])/(96*Sqrt[1 + c^2*x^2]) - (b*c^3*d*x^6*Sqrt[d + c^2*d*x^2])
/(36*Sqrt[1 + c^2*x^2]) + (d*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(1
6*c^2) + (d*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/8 + (x^3*(d + c^2
*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/6 - (d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSi
nh[c*x])^2)/(32*b*c^3*Sqrt[1 + c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int x^2 \left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right) dx = \frac{1}{6}x^3 \left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right) + \frac{1}{2}d \int x^2\sqrt{d+c^2dx^2} \left(a+\right.$$

$$= \frac{1}{8}dx^3\sqrt{d+c^2dx^2} \left(a+b\sinh^{-1}(cx)\right) + \frac{1}{6}x^3 \left(d+c^2dx^2\right)^{3/2} \left(a+b\,\mathrm{s}\right.$$

$$= -\frac{7bcdx^4\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}} - \frac{bc^3dx^6\sqrt{d+c^2dx^2}}{36\sqrt{1+c^2x^2}} + \frac{dx\sqrt{d+c^2dx^2} \left(a+\right.}{16c^2}$$

$$= -\frac{bdx^2\sqrt{d+c^2dx^2}}{32c\sqrt{1+c^2x^2}} - \frac{7bcdx^4\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}} - \frac{bc^3dx^6\sqrt{d+c^2dx^2}}{36\sqrt{1+c^2x^2}} + $$

Mathematica [A] time = 0.73, size = 251, normalized size = 0.99

$$\frac{-144ad^{3/2}\sqrt{c^2x^2+1}\,\log\left(\sqrt{d}\,\sqrt{c^2dx^2+d}\,+cdx\right)+48acdx\sqrt{c^2x^2+1}\,\left(8c^4x^4+14c^2x^2+3\right)\sqrt{c^2dx^2+d}\,-18bc}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (48*a*c*d*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(3 + 14*c^2*x^2 + 8*c^4*x
^4) - 144*a*d^(3/2)*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^
2]] - 18*b*d*Sqrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]^2 + Cosh[4*ArcSinh[c*x]] -
 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]) + b*d*Sqrt[d + c^2*d*x^2]*(72*ArcSinh
[c*x]^2 + 18*Cosh[2*ArcSinh[c*x]] + 9*Cosh[4*ArcSinh[c*x]] - 2*Cosh[6*ArcSi
nh[c*x]] + 12*ArcSinh[c*x]*(-3*Sinh[2*ArcSinh[c*x]] - 3*Sinh[4*ArcSinh[c*x]
] + Sinh[6*ArcSinh[c*x]])))/(2304*c^3*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(ac^2dx^4+adx^2+\left(bc^2dx^4+bdx^2\right)\mathrm{arsinh}\left(cx\right)\right)\sqrt{c^2dx^2+d}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*c^2*d*x^4 + a*d*x^2 + (b*c^2*d*x^4 + b*d*x^2)*arcsinh(c*x))*sqr
t(c^2*d*x^2 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c^2dx^2+d\right)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)*x^2, x)
```

maple [A] time = 0.30, size = 421, normalized size = 1.66

$$\frac{ax\left(c^2d\,x^2+d\right)^{\frac{5}{2}}}{6c^2d}-\frac{ax\left(c^2d\,x^2+d\right)^{\frac{3}{2}}}{24c^2}-\frac{adx\sqrt{c^2d\,x^2+d}}{16c^2}-\frac{a\,d^2\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{16c^2\sqrt{c^2d}}+\frac{b\sqrt{d\left(c^2x^2+1\right)}\,d\,c^4\operatorname{arcsinh}}{6c^2x^2+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/6*a*x*(c^2*d*x^2+d)^(5/2)/c^2/d-1/24*a/c^2*x*(c^2*d*x^2+d)^(3/2)-1/16*a/c
^2*d*x*(c^2*d*x^2+d)^(1/2)-1/16*a/c^2*d^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x
^2+d)^(1/2))/(c^2*d)^(1/2)+1/6*b*(d*(c^2*x^2+1))^(1/2)*d*c^4/(c^2*x^2+1)*ar
csinh(c*x)*x^7-1/36*b*(d*(c^2*x^2+1))^(1/2)*d*c^3/(c^2*x^2+1)^(1/2)*x^6+11/
24*b*(d*(c^2*x^2+1))^(1/2)*d*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^5-7/96*b*(d*(c^
2*x^2+1))^(1/2)*d*c/(c^2*x^2+1)^(1/2)*x^4+17/48*b*(d*(c^2*x^2+1))^(1/2)*d/(
c^2*x^2+1)*arcsinh(c*x)*x^3-1/32*b*(d*(c^2*x^2+1))^(1/2)*d/c/(c^2*x^2+1)^(1
/2)*x^2+1/16*b*(d*(c^2*x^2+1))^(1/2)*d/c^2/(c^2*x^2+1)*arcsinh(c*x)*x-1/32*
b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3*arcsinh(c*x)^2*d+7/2304*b*(d*
(c^2*x^2+1))^(1/2)*d/c^3/(c^2*x^2+1)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(d\,c^2\,x^2+d\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}(a+b\operatorname{asinh}(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x**2*(d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x)), x)
```

3.130 $\int x \left(d+c^2 d x^2\right)^{3/2} \left(a+b \sinh^{-1}(c x)\right) \, dx$

Optimal. Leaf size=146

$$\frac{\left(c^2 d x^2+d\right)^{5/2} \left(a+b \sinh^{-1}(c x)\right)}{5 c^2 d}-\frac{b d x \sqrt{c^2 d x^2+d}}{5 c \sqrt{c^2 x^2+1}}-\frac{2 b c d x^3 \sqrt{c^2 d x^2+d}}{15 \sqrt{c^2 x^2+1}}-\frac{b c^3 d x^5 \sqrt{c^2 d x^2+d}}{25 \sqrt{c^2 x^2+1}}$$

```
[Out] 1/5*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/c^2/d-1/5*b*d*x*(c^2*d*x^2+d)^(1
/2)/c/(c^2*x^2+1)^(1/2)-2/15*b*c*d*x^3*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2
)-1/25*b*c^3*d*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5717, 194}

$$\frac{\left(c^2 d x^2+d\right)^{5/2} \left(a+b \sinh^{-1}(c x)\right)}{5 c^2 d}-\frac{b c^3 d x^5 \sqrt{c^2 d x^2+d}}{25 \sqrt{c^2 x^2+1}}-\frac{2 b c d x^3 \sqrt{c^2 d x^2+d}}{15 \sqrt{c^2 x^2+1}}-\frac{b d x \sqrt{c^2 d x^2+d}}{5 c \sqrt{c^2 x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(b*d*x*Sqrt[d + c^2*d*x^2])/(5*c*Sqrt[1 + c^2*x^2]) - (2*b*c*d*x^3*Sqrt[d
 + c^2*d*x^2])/(15*Sqrt[1 + c^2*x^2]) - (b*c^3*d*x^5*Sqrt[d + c^2*d*x^2])/(2
5*Sqrt[1 + c^2*x^2]) + ((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(5*c^2*
d)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\int x \left(d+c^2 d x^2\right)^{3/2} \left(a+b \sinh^{-1}(c x)\right) \, dx = \frac{\left(d+c^2 d x^2\right)^{5/2} \left(a+b \sinh^{-1}(c x)\right)}{5 c^2 d}-\frac{\left(b d \sqrt{d+c^2 d x^2}\right) \int \left(1+c^2 x^2\right)^2 \, dx}{5 c \sqrt{1+c^2 x^2}}$$

$$= \frac{\left(d+c^2 d x^2\right)^{5/2} \left(a+b \sinh^{-1}(c x)\right)}{5 c^2 d}-\frac{\left(b d \sqrt{d+c^2 d x^2}\right) \int \left(1+2 c^2 x^2+c\right.}{5 c \sqrt{1+c^2 x^2}}$$

$$= -\frac{b d x \sqrt{d+c^2 d x^2}}{5 c \sqrt{1+c^2 x^2}}-\frac{2 b c d x^3 \sqrt{d+c^2 d x^2}}{15 \sqrt{1+c^2 x^2}}-\frac{b c^3 d x^5 \sqrt{d+c^2 d x^2}}{25 \sqrt{1+c^2 x^2}}+\frac{(d+}{}$$

Mathematica [A] time = 0.13, size = 102, normalized size = 0.70

$$\frac{d \sqrt{c^2 d x^2+d} \left(15 a \left(c^2 x^2+1\right)^3+15 b \left(c^2 x^2+1\right)^3 \sinh^{-1}(c x)-b c x \left(3 c^4 x^4+10 c^2 x^2+15\right) \sqrt{c^2 x^2+1}\right)}{75 c^2 \left(c^2 x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d*Sqrt[d + c^2*d*x^2]*(15*a*(1 + c^2*x^2)^3 - b*c*x*Sqrt[1 + c^2*x^2]*(15
+ 10*c^2*x^2 + 3*c^4*x^4) + 15*b*(1 + c^2*x^2)^3*ArcSinh[c*x]))/(75*c^2*(1
+ c^2*x^2))
```

fricas [A] time = 0.70, size = 167, normalized size = 1.14

$$\frac{15\left(bc^6dx^6+3\,bc^4dx^4+3\,bc^2dx^2+bd\right)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(15\,ac^6dx^6+45\,ac^4dx^4+45\,ac^2dx\right.}{75\left(c^4x^2+c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/75*(15*(b*c^6*d*x^6 + 3*b*c^4*d*x^4 + 3*b*c^2*d*x^2 + b*d)*sqrt(c^2*d*x^2
 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + (15*a*c^6*d*x^6 + 45*a*c^4*d*x^4 + 45*
a*c^2*d*x^2 + 15*a*d - (3*b*c^5*d*x^5 + 10*b*c^3*d*x^3 + 15*b*c*d*x)*sqrt(c
^2*x^2 + 1))*sqrt(c^2*d*x^2 + d))/(c^4*x^2 + c^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.17, size = 559, normalized size = 3.83

$$\frac{a\left(c^2d\,x^2+d\right)^{\frac{5}{2}}}{5c^2d}+b\left(\frac{\sqrt{d\left(c^2x^2+1\right)}\,\left(16c^6x^6+16c^5x^5\sqrt{c^2x^2+1}\,+28c^4x^4+20c^3x^3\sqrt{c^2x^2+1}\,+13c^2x^2+5cx\sqrt{}\right.}{800c^2\left(c^2x^2+1\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/5*a/c^2/d*(c^2*d*x^2+d)^(5/2)+b*(1/800*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6+
16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2
*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+5*arcsinh(c*x))*d/c^2/(c^2*x^2+1)+1/96*
(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*
x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arcsinh(c*x))*d/c^2/(c^2*x^2+1)+1/16*(d*(c^2*x
^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))*d/c^2/(c^2
*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(1+arc
sinh(c*x))*d/c^2/(c^2*x^2+1)+1/96*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^
3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(1+3*arcsinh(c*x))
*d/c^2/(c^2*x^2+1)+1/800*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6-16*c^5*x^5*(c^2*
x^2+1)^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2-5*c*x*(c^2*
x^2+1)^(1/2)+1)*(1+5*arcsinh(c*x))*d/c^2/(c^2*x^2+1))
```

maxima [A] time = 0.44, size = 85, normalized size = 0.58

$$\frac{\left(c^2dx^2+d\right)^{\frac{5}{2}}b\,\mathrm{arsinh}\left(cx\right)}{5\,c^2d}+\frac{\left(c^2dx^2+d\right)^{\frac{5}{2}}a}{5\,c^2d}-\frac{\left(3\,c^4d^{\frac{5}{2}}x^5+10\,c^2d^{\frac{5}{2}}x^3+15\,d^{\frac{5}{2}}x\right)b}{75\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*(c^2*d*x^2 + d)^(5/2)*b*arcsinh(c*x)/(c^2*d) + 1/5*(c^2*d*x^2 + d)^(5/2
)*a/(c^2*d) - 1/75*(3*c^4*d^(5/2)*x^5 + 10*c^2*d^(5/2)*x^3 + 15*d^(5/2)*x)*
b/(c*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(d\,c^2\,x^2+d\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(d\left(c^{2}x^{2}+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{asinh}\left(cx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x*(d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x)), x)
```

3.131 $\int \left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=180

$$\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{8}dx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{3d\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc\sqrt{c^2x^2+1}}-$$

```
[Out] 1/4*x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))+3/8*d*x*(a+b*arcsinh(c*x))*(c^
2*d*x^2+d)^(1/2)-5/16*b*c*d*x^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/16*
b*c^3*d*x^4*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+3/16*d*(a+b*arcsinh(c*x))
^2*(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5684, 5682, 5675, 30, 14}

$$\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{8}dx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{3d\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc\sqrt{c^2x^2+1}}-$$

Antiderivative was successfully verified.

```
[In] Int[(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-5*b*c*d*x^2*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) - (b*c^3*d*x^4*Sq
rt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) + (3*d*x*Sqrt[d + c^2*d*x^2]*(a +
 b*ArcSinh[c*x]))/8 + (x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/4 + (3
*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(16*b*c*Sqrt[1 + c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
```

```
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rubi steps

$$\int \left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)dx=\frac{1}{4}x\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}(3d)\int\sqrt{d+c^2dx^2}\left(a+b\sinh\right.$$

$$=\frac{3}{8}dx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}x\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(c\right.$$

$$=-\frac{5bcdx^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}-\frac{bc^3dx^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}+\frac{3}{8}dx\sqrt{d+c^2dx^2}\left(a+b\sin\right.$$

Mathematica [A] time = 0.81, size = 200, normalized size = 1.11

$$\frac{3ad^{3/2}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)}{8c}+\frac{1}{8}adx\left(2c^2x^2+5\right)\sqrt{c^2dx^2+d}+\frac{bd\sqrt{c^2dx^2+d}\left(2\sinh^{-1}(cx)\left(\sinh^{-1}(cx)+\text{si}\right.\right.}{8c\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (a*d*x*(5 + 2*c^2*x^2)*Sqrt[d + c^2*d*x^2])/8 + (3*a*d^(3/2)*Log[c*d*x + Sq
rt[d]*Sqrt[d + c^2*d*x^2]])/(8*c) + (b*d*Sqrt[d + c^2*d*x^2]*(-Cosh[2*ArcSi
nh[c*x]] + 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])))/(8*c*Sqrt
[1 + c^2*x^2]) - (b*d*Sqrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]^2 + Cosh[4*ArcSin
h[c*x]] - 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]))/(128*c*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ac^2dx^2+ad+\left(bc^2dx^2+bd\right)\text{arsinh}\,(cx)\right)\sqrt{c^2dx^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*sqrt(c^2*d*
x^2 + d), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.17, size = 318, normalized size = 1.77

$$\frac{x\left(c^2d\,x^2+d\right)^{\frac{3}{2}}a}{4}+\frac{3adx\sqrt{c^2d\,x^2+d}}{8}+\frac{3a\,d^2\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{8\sqrt{c^2d}}+\frac{b\sqrt{d\left(c^2x^2+1\right)}\,d\,c^4\,\text{arcsinh}\,(cx)\,x^5}{4c^2x^2+4}-\frac{b\sqrt{d}}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/4*x*(c^2*d*x^2+d)^(3/2)*a+3/8*a*d*x*(c^2*d*x^2+d)^(1/2)+3/8*a*d^2*ln(x*c^
2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/4*b*(d*(c^2*x^2+1))^
(1/2)*d*c^4/(c^2*x^2+1)*arcsinh(c*x)*x^5-1/16*b*(d*(c^2*x^2+1))^(1/2)*d*c^3
/(c^2*x^2+1)^(1/2)*x^4+7/8*b*(d*(c^2*x^2+1))^(1/2)*d*c^2/(c^2*x^2+1)*arcsin
h(c*x)*x^3-5/16*b*(d*(c^2*x^2+1))^(1/2)*d*c/(c^2*x^2+1)^(1/2)*x^2+5/8*b*(d*
(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*arcsinh(c*x)*x+3/16*b*(d*(c^2*x^2+1))^(1/2
)/(c^2*x^2+1)^(1/2)/c*arcsinh(c*x)^2*d-17/128*b*(d*(c^2*x^2+1))^(1/2)*d/c/(
c^2*x^2+1)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \operatorname{asinh}(c\,x))\,\left(d\,c^2\,x^2 + d\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\left(c^2x^2 + 1\right)\right)^{\frac{3}{2}} (a + b \operatorname{asinh}(cx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x)), x)
```

3.132 $\int \frac{(d+c^2dx^2)^{3/2}(a+b\sinh^{-1}(cx))}{x}\,dx$

Optimal. Leaf size=249

$$\frac{1}{3}\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+d\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)-\frac{2d\sqrt{c^2dx^2+d}\,\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}$$

```
[Out] 1/3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))+d*(a+b*arcsinh(c*x))*(c^2*d*x^2+
d)^(1/2)-4/3*b*c*d*x*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/9*b*c^3*d*x^3*
(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2*d*(a+b*arcsinh(c*x))*arctanh(c*x+(c
^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-b*d*polylog(2,-c*x-(
c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+b*d*polylog(2,c*x+(
c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5744, 5742, 5760, 4182, 2279, 2391, 8}

$$-\frac{bd\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+\frac{bd\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+\frac{1}{3}\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (-4*b*c*d*x*Sqrt[d + c^2*d*x^2])/(3*Sqrt[1 + c^2*x^2]) - (b*c^3*d*x^3*Sqrt[
d + c^2*d*x^2])/(9*Sqrt[1 + c^2*x^2]) + d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSin
h[c*x]) + ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/3 - (2*d*Sqrt[d + c^
2*d*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] -
(b*d*Sqrt[d + c^2*d*x^2]*PolyLog[2, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (
b*d*Sqrt[d + c^2*d*x^2]*PolyLog[2, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x}\,dx = \frac{1}{3}\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+d\int \frac{\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}\right.}{x}$$

$$= -\frac{bcdx\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}}-\frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}+d\sqrt{d+c^2dx^2}\,\left(a+b\sinh\right.$$

$$= -\frac{4bcdx\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}}-\frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}+d\sqrt{d+c^2dx^2}\,\left(a+b\sin\right.$$

$$= -\frac{4bcdx\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}}-\frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}+d\sqrt{d+c^2dx^2}\,\left(a+b\sin\right.$$

$$= -\frac{4bcdx\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}}-\frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}+d\sqrt{d+c^2dx^2}\,\left(a+b\sin\right.$$

$$= -\frac{4bcdx\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}}-\frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}+d\sqrt{d+c^2dx^2}\,\left(a+b\sin\right.$$

Mathematica [A] time = 0.77, size = 248, normalized size = 1.00

$$-ad^{3/2}\log\left(\sqrt{d}\,\sqrt{c^2dx^2+d}+d\right)+\frac{1}{3}ad\left(c^2x^2+4\right)\sqrt{c^2dx^2+d}+ad^{3/2}\log(x)+\frac{bd\sqrt{c^2dx^2+d}\,\left(\sqrt{c^2x^2+1}\,\sinh^{-1}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (a*d*(4 + c^2*x^2)*Sqrt[d + c^2*d*x^2])/3 + (b*d*Sqrt[d + c^2*d*x^2]*(-(c*x
*(3 + c^2*x^2)) + 3*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]))/(9*Sqrt[1 + c^2*x^2]
) + a*d^(3/2)*Log[x] - a*d^(3/2)*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (b*
d*Sqrt[d + c^2*d*x^2]*(-(c*x) + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + ArcSinh[c*
x]*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + P
olyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])]))/Sqrt[1 + c^
2*x^2]
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ac^2dx^2 + ad + \left(bc^2dx^2 + bd\right)\operatorname{arsinh}\left(cx\right)\right)\sqrt{c^2dx^2 + d}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*sqrt(c^2*d*
x^2 + d)/x, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.21, size = 428, normalized size = 1.72

$$\frac{\left(c^2d\,x^2+d\right)^{\frac{3}{2}}a}{3}-a\,d^{\frac{3}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2d\,x^2+d}}{x}\right)+a\sqrt{c^2d\,x^2+d}\,d+\frac{4b\sqrt{d\left(c^2x^2+1\right)}\,d\operatorname{arcsinh}\left(cx\right)}{3\left(c^2x^2+1\right)}-\frac{b\sqrt{d\left(c^2x^2\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x,x)
```

```
[Out] 1/3*(c^2*d*x^2+d)^(3/2)*a-a*d^(3/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/
x)+a*(c^2*d*x^2+d)^(1/2)*d+4/3*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*arcsin
h(c*x)-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)
^(1/2))*d+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+
1)^(1/2))*d+5/3*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*arcsinh(c*x)*x^2*c^2-
4/3*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)^(1/2)*c*x+1/3*b*(d*(c^2*x^2+1))^(
1/2)*d/(c^2*x^2+1)*arcsinh(c*x)*x^4*c^4-1/9*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*
x^2+1)^(1/2)*c^3*x^3+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)
*ln(1-c*x-(c^2*x^2+1)^(1/2))*d-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*ar
csinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}\left(3\,d^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{1}{c|x|}\right)-\left(c^2dx^2+d\right)^{\frac{3}{2}}-3\,\sqrt{c^2dx^2+d}\,d\right)a+b\int\frac{\left(c^2dx^2+d\right)^{\frac{3}{2}}\log\left(cx+\sqrt{c^2x^2+1}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x,x, algorithm="maxima")
```

```
[Out] -1/3*(3*d^(3/2)*arcsinh(1/(c*abs(x))) - (c^2*d*x^2 + d)^(3/2) - 3*sqrt(c^2*
d*x^2 + d)*d)*a + b*integrate((c^2*d*x^2 + d)^(3/2)*log(c*x + sqrt(c^2*x^2
+ 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{asinh}(c\,x))\,\left(d\,c^2\,x^2 + d\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/x,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\left(c^2x^2 + 1\right)\right)^{\frac{3}{2}}\,(a + b \operatorname{asinh}(cx))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))/x,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))/x, x)
```

3.133 $\int \frac{(d+c^2dx^2)^{3/2}(a+b\sinh^{-1}(cx))}{x^2}\,dx$

Optimal. Leaf size=177

$$\frac{3}{2}c^2dx\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)+\frac{3cd\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2}{4b\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x}+\frac{bc}{}$$

```
[Out] -(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x+3/2*c^2*d*x*(a+b*arcsinh(c*x))*(c
^2*d*x^2+d)^(1/2)-1/4*b*c^3*d*x^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+3/4
*c*d*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/(c^2*x^2+1)^(1/2)+b*c*d*ln(
x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5739, 5682, 5675, 30, 14}

$$\frac{3}{2}c^2dx\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)+\frac{3cd\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2}{4b\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{bc}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] -(b*c^3*d*x^2*Sqrt[d + c^2*d*x^2])/(4*Sqrt[1 + c^2*x^2]) + (3*c^2*d*x*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/2 - ((d + c^2*d*x^2)^(3/2)*(a + b*ArcS
inh[c*x]))/x + (3*c*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*Sqrt
[1 + c^2*x^2]) + (b*c*d*Sqrt[d + c^2*d*x^2]*Log[x])/Sqrt[1 + c^2*x^2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
```

```
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x^2}\,dx = -\frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x} + \left(3c^2d\right)\int \sqrt{d+c^2dx^2}\,\left(a+b\,\mathrm{si}\right.$$

$$= \frac{3}{2}c^2dx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(\right.}{x}$$

$$= -\frac{bc^3dx^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}} + \frac{3}{2}c^2dx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{\left(d+\right.}{}$$

Mathematica [A] time = 0.79, size = 200, normalized size = 1.13

$$\frac{1}{8}\left(12acd^{3/2}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right) + \frac{4ad\left(c^2x^2-2\right)\sqrt{c^2dx^2+d}}{x} + \frac{4bd\sqrt{c^2dx^2+d}\left(-2\sqrt{c^2x^2+1}\sinh^{-1}\right.}{x\sqrt{c^2x}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] ((4*a*d*(-2 + c^2*x^2)*Sqrt[d + c^2*d*x^2])/x + (4*b*d*Sqrt[d + c^2*d*x^2]*
(-2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + c*x*ArcSinh[c*x]^2 + 2*c*x*Log[c*x]))/
(x*Sqrt[1 + c^2*x^2]) + 12*a*c*d^(3/2)*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x
^2]] + (b*c*d*Sqrt[d + c^2*d*x^2]*(-Cosh[2*ArcSinh[c*x]] + 2*ArcSinh[c*x]*(
ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])))/Sqrt[1 + c^2*x^2])/8
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(ac^2dx^2+ad+\left(bc^2dx^2+bd\right)\mathrm{arsinh}\left(cx\right)\right)\sqrt{c^2dx^2+d}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*sqrt(c^2*d*
x^2 + d)/x^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.21, size = 392, normalized size = 2.21

$$-\frac{a\left(c^{2}d\,x^{2}+d\right)^{\frac{5}{2}}}{dx}+a\,c^{2}x\left(c^{2}d\,x^{2}+d\right)^{\frac{3}{2}}+\frac{3a\,c^{2}dx\sqrt{c^{2}d\,x^{2}+d}}{2}+\frac{3a\,c^{2}d^{2}\ln\left(\frac{x\,c^{2}d}{\sqrt{c^{2}d}}+\sqrt{c^{2}d\,x^{2}+d}\right)}{2\sqrt{c^{2}d}}+\frac{3b\sqrt{d\left(c^{2}x^{2}+1\right)}}{4\sqrt{c^{2}x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^2,x)
```

```
[Out] -a/d/x*(c^2*d*x^2+d)^(5/2)+a*c^2*x*(c^2*d*x^2+d)^(3/2)+3/2*a*c^2*d*x*(c^2*d
*x^2+d)^(1/2)+3/2*a*c^2*d^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(
c^2*d)^(1/2)+3/4*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*d
*c+1/2*b*(d*(c^2*x^2+1))^(1/2)*d*c^4/(c^2*x^2+1)*arcsinh(c*x)*x^3-1/4*b*(d*
(c^2*x^2+1))^(1/2)*d*c^3/(c^2*x^2+1)^(1/2)*x^2-1/2*b*(d*(c^2*x^2+1))^(1/2)*
d*c^2/(c^2*x^2+1)*arcsinh(c*x)*x-b*(d*(c^2*x^2+1))^(1/2)*d*c/(c^2*x^2+1)^(1
/2)*arcsinh(c*x)-1/8*b*(d*(c^2*x^2+1))^(1/2)*d*c/(c^2*x^2+1)^(1/2)-b*(d*(c^
2*x^2+1))^(1/2)*arcsinh(c*x)*d/x/(c^2*x^2+1)+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x
^2+1)^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*d*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))\,\left(d\,c^{2}\,x^{2}+d\right)^{3/2}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\left(c^{2}x^{2}+1\right)\right)^{\frac{3}{2}}\,(a+b\,\mathrm{asinh}\,(cx))}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))/x**2,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))/x**2, x)
```

3.134 $\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=270

$$\frac{3}{2}c^2d\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)-\frac{\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{2x^2}-\frac{3c^2d\sqrt{c^2dx^2+d}\,\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a\right.}{\sqrt{c^2x^2+1}}$$

```
[Out] -1/2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^2+3/2*c^2*d*(a+b*arcsinh(c*x)
)*(c^2*d*x^2+d)^(1/2)-1/2*b*c*d*(c^2*d*x^2+d)^(1/2)/x/(c^2*x^2+1)^(1/2)-b*c
^3*d*x*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-3*c^2*d*(a+b*arcsinh(c*x))*arc
tanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-3/2*b*c^2
*d*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+
3/2*b*c^2*d*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1
)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5739, 5742, 5760, 4182, 2279, 2391, 8, 14}

$$-\frac{3bc^2d\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2x^2+1}}+\frac{3bc^2d\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2x^2+1}}+\frac{3}{2}c^2d\sqrt{c^2dx^2+d}\,\left(a+\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] -(b*c*d*Sqrt[d + c^2*d*x^2])/(2*x*Sqrt[1 + c^2*x^2]) - (b*c^3*d*x*Sqrt[d +
c^2*d*x^2])/Sqrt[1 + c^2*x^2] + (3*c^2*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh
[c*x]))/2 - ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(2*x^2) - (3*c^2*d
*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 +
 c^2*x^2] - (3*b*c^2*d*Sqrt[d + c^2*d*x^2]*PolyLog[2, -E^ArcSinh[c*x]])/(2*
Sqrt[1 + c^2*x^2]) + (3*b*c^2*d*Sqrt[d + c^2*d*x^2]*PolyLog[2, E^ArcSinh[c*
x]])/(2*Sqrt[1 + c^2*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x^3}\,dx = -\frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{2x^2}+\frac{1}{2}\left(3c^2d\right)\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sin\right.}{x}$$

$$=\frac{3}{2}c^2d\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{2x^2}$$

$$=-\frac{bcd\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}}-\frac{bc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}+\frac{3}{2}c^2d\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}\right.$$

$$=-\frac{bcd\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}}-\frac{bc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}+\frac{3}{2}c^2d\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}\right.$$

$$=-\frac{bcd\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}}-\frac{bc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}+\frac{3}{2}c^2d\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}\right.$$

$$=-\frac{bcd\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}}-\frac{bc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}+\frac{3}{2}c^2d\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}\right.$$

Mathematica [A] time = 4.46, size = 352, normalized size = 1.30

$$-\frac{3}{2}ac^2d^{3/2}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+d\right)+\frac{3}{2}ac^2d^{3/2}\log(x)+a\left(c^2d-\frac{d}{2x^2}\right)\sqrt{c^2dx^2+d}+\frac{bc^2d\sqrt{c^2dx^2+d}\left(\sqrt{c^2x^2+1}\right.}{}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^3,x]

[Out] a*(c^2*d - d/(2*x^2))*Sqrt[d + c^2*d*x^2] + (3*a*c^2*d^(3/2)*Log[x])/2 - (3*a*c^2*d^(3/2)*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/2 + (b*c^2*d*Sqrt[d + c^2*d*x^2]*(-(c*x) + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2] + (b*c^2*d*Sqrt[d + c^2*d*x^2]*(-2*Coth[ArcSinh[c*x]/2] - ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 4*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 4*PolyLog[2, -E^(-ArcSinh[c*x])] - 4*PolyLog[2, E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSinh[c*x]/2]))/(8*Sqrt[1 + c^2*x^2])

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ac^2dx^2+ad+\left(bc^2dx^2+bd\right)\operatorname{arsinh}(cx)\right)\sqrt{c^2dx^2+d}}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="fricas")

[Out] integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^3, x)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

maple [A] time = 0.27, size = 472, normalized size = 1.75

$$-\frac{a\left(c^2d\,x^2+d\right)^{\frac{5}{2}}}{2d\,x^2}+\frac{a\,c^2\left(c^2d\,x^2+d\right)^{\frac{3}{2}}}{2}-\frac{3a\,c^2d^{\frac{3}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2d\,x^2+d}}{x}\right)}{2}+\frac{3a\,c^2\sqrt{c^2d\,x^2+d}\,d}{2}+\frac{b\sqrt{d\left(c^2x^2+1\right)}\,c^4d}{c^2x^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^3,x)

[Out] -1/2*a/d/x^2*(c^2*d*x^2+d)^(5/2)+1/2*a*c^2*(c^2*d*x^2+d)^(3/2)-3/2*a*c^2*d^(3/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)+3/2*a*c^2*(c^2*d*x^2+d)^(1/2)*d+b*(d*(c^2*x^2+1))^(1/2)*c^4*d/(c^2*x^2+1)*arcsinh(c*x)*x^2-b*(d*(c^2*x^2+1))^(1/2)*c^3*d/(c^2*x^2+1)^(1/2)*x+1/2*b*(d*(c^2*x^2+1))^(1/2)*c^2*d/(c^2*x^2+1)*arcsinh(c*x)-1/2*b*(d*(c^2*x^2+1))^(1/2)*d/x/(c^2*x^2+1)^(1/2)*c-1/2*b*(d*(c^2*x^2+1))^(1/2)*d/x^2/(c^2*x^2+1)*arcsinh(c*x)+3/2*b*(d*(c^2*x^

```
2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^2*
d+3/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(
1/2))*c^2*d-3/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1
+c*x+(c^2*x^2+1)^(1/2))*c^2*d-3/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)
*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c^2*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\left(3\,c^2 d^{\frac{3}{2}}\,\mathrm{arsinh}\left(\frac{1}{c|x|}\right)-\left(c^2dx^2+d\right)^{\frac{3}{2}}c^2-3\,\sqrt{c^2dx^2+d}\,c^2d+\frac{\left(c^2dx^2+d\right)^{\frac{5}{2}}}{dx^2}\right)a+b\int\frac{\left(c^2dx^2+d\right)^{\frac{3}{2}}\log\left(cx+\sqrt{c\right.}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(3*c^2*d^(3/2)*arcsinh(1/(c*abs(x))) - (c^2*d*x^2 + d)^(3/2)*c^2 - 3*s
qrt(c^2*d*x^2 + d)*c^2*d + (c^2*d*x^2 + d)^(5/2)/(d*x^2))*a + b*integrate((
c^2*d*x^2 + d)^(3/2)*log(c*x + sqrt(c^2*x^2 + 1))/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))\,\left(d\,c^2\,x^2+d\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/x^3,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}(a+b\,\mathrm{asinh}\,(cx))}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))/x**3,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))/x**3, x)
```

3.135 $\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=184

$$-\frac{c^2d\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3x^3}+\frac{c^3d\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2b\sqrt{c^2x^2+1}}-$$

```
[Out] -1/3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^3-c^2*d*(a+b*arcsinh(c*x))*(c
^2*d*x^2+d)^(1/2)/x-1/6*b*c*d*(c^2*d*x^2+d)^(1/2)/x^2/(c^2*x^2+1)^(1/2)+1/2
*c^3*d*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/(c^2*x^2+1)^(1/2)+4/3*b*c
^3*d*ln(x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5739, 5737, 29, 5675, 14}

$$\frac{c^3d\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2b\sqrt{c^2x^2+1}}-\frac{c^2d\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] -(b*c*d*Sqrt[d + c^2*d*x^2])/(6*x^2*Sqrt[1 + c^2*x^2]) - (c^2*d*Sqrt[d + c^
2*d*x^2]*(a + b*ArcSinh[c*x]))/x - ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*
x]))/(3*x^3) + (c^3*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*Sqrt
[1 + c^2*x^2]) + (4*b*c^3*d*Sqrt[d + c^2*d*x^2]*Log[x])/(3*Sqrt[1 + c^2*x^2
])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5737

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 1)*S
qrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x] -
 Dist[(c^2*Sqrt[d + e*x^2])/(f^2*(m + 1)*Sqrt[1 + c^2*x^2]), Int[((f*x)^(m
+ 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
```

```
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x^4}\,dx = -\frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3x^3}+\left(c^2d\right)\int \frac{\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}\right.}{x^2}$$

$$= -\frac{c^2d\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right.}{3x^3}$$

$$= -\frac{bcd\sqrt{d+c^2dx^2}}{6x^2\sqrt{1+c^2x^2}}-\frac{c^2d\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{\left(d+c^2dx^2\right)^{3/2}}{}$$

Mathematica [A] time = 0.77, size = 214, normalized size = 1.16

$$\frac{1}{6}d\left(-\frac{2a\left(4c^2x^2+1\right)\sqrt{c^2dx^2+d}}{x^3}+6ac^3\sqrt{d}\,\log\left(\sqrt{d}\,\sqrt{c^2dx^2+d}+cdx\right)+\frac{3bc^2\sqrt{c^2dx^2+d}\,\left(-2\sqrt{c^2x^2+1}\,\sinh^{-1}\right.}{x\sqrt{c^2x}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] (d*((-2*a*(1 + 4*c^2*x^2)*Sqrt[d + c^2*d*x^2])/x^3 + (3*b*c^2*Sqrt[d + c^2*
d*x^2]*(-2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + c*x*ArcSinh[c*x]^2 + 2*c*x*Log[
c*x]))/(x*Sqrt[1 + c^2*x^2]) - (b*Sqrt[d + c^2*d*x^2]*(c*x + 2*(1 + c^2*x^2
)^(3/2)*ArcSinh[c*x] - 2*c^3*x^3*Log[c*x]))/(x^3*Sqrt[1 + c^2*x^2]) + 6*a*c
^3*Sqrt[d]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]]))/6
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ac^2dx^2+ad+\left(bc^2dx^2+bd\right)\text{arsinh}\left(cx\right)\right)\sqrt{c^2dx^2+d}}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*sqrt(c^2*d*
x^2 + d)/x^4, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.26, size = 1107, normalized size = 6.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^4,x)
```

```
[Out] -1/3*a/d/x^3*(c^2*d*x^2+d)^(5/2)-2/3*a*c^2/d/x*(c^2*d*x^2+d)^(5/2)+2/3*a*c^
4*x*(c^2*d*x^2+d)^(3/2)+a*c^4*d*x*(c^2*d*x^2+d)^(1/2)+a*c^4*d^2*ln(x*c^2*d/
(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/2*b*(d*(c^2*x^2+1))^(1/2
)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*d*c^3-8/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x
^2+1)^(1/2)*arcsinh(c*x)*d*c^3-32*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c
^2*x^2+1)*x^5/(c^2*x^2+1)*arcsinh(c*x)*c^8+32*b*(d*(c^2*x^2+1))^(1/2)*d/(24
*c^4*x^4+9*c^2*x^2+1)*x^4/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^7-8/3*b*(d*(c^2*
x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^5/(c^2*x^2+1)*c^8+8/3*b*(d*(c^2*
x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^3*c^6-52*b*(d*(c^2*x^2+1))^(1/2)
*d/(24*c^4*x^4+9*c^2*x^2+1)*x^3/(c^2*x^2+1)*arcsinh(c*x)*c^6+12*b*(d*(c^2*x
^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*
c^5-10/3*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^3/(c^2*x^2+1)
*c^6-4*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^2/(c^2*x^2+1)^(
1/2)*c^5+2/3*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x*c^4-73/3*
b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x/(c^2*x^2+1)*arcsinh(c*
x)*c^4+4/3*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)/(c^2*x^2+1)^(
1/2)*arcsinh(c*x)*c^3-2/3*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1
)*x/(c^2*x^2+1)*c^4-3/2*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)/
(c^2*x^2+1)^(1/2)*c^3-14/3*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+
1)/x/(c^2*x^2+1)*arcsinh(c*x)*c^2-1/6*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4
+9*c^2*x^2+1)/x^2/(c^2*x^2+1)^(1/2)*c-1/3*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4
*x^4+9*c^2*x^2+1)/x^3/(c^2*x^2+1)*arcsinh(c*x)+4/3*b*(d*(c^2*x^2+1))^(1/2)/
(c^2*x^2+1)^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*d*c^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{asinh}\,(c\,x))\,\left(d\,c^2\,x^2 + d\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/x^4,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}(a+b\,\mathrm{asinh}\,(cx))}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))/x**4,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))/x**4, x)
```

3.136 $\int x^3 \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=266

$$\frac{\left(c^2dx^2+d\right)^{9/2}\left(a+b\sinh^{-1}(cx)\right)}{9c^4d^2}-\frac{\left(c^2dx^2+d\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7c^4d}-\frac{bcd^2x^5\sqrt{c^2dx^2+d}}{21\sqrt{c^2x^2+1}}-\frac{bd^2x^3\sqrt{c^2dx^2+d}}{189c\sqrt{c^2x^2+1}}-\frac{bc^5}{}$$

```
[Out] -1/7*(c^2*d*x^2+d)^(7/2)*(a+b*arcsinh(c*x))/c^4/d+1/9*(c^2*d*x^2+d)^(9/2)*(
a+b*arcsinh(c*x))/c^4/d^2+2/63*b*d^2*x*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)^
(1/2)-1/189*b*d^2*x^3*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-1/21*b*c*d^2*
x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-19/441*b*c^3*d^2*x^7*(c^2*d*x^2+d
)^(1/2)/(c^2*x^2+1)^(1/2)-1/81*b*c^5*d^2*x^9*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1
)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {266, 43, 5734, 12, 373}

$$\frac{\left(c^2dx^2+d\right)^{9/2}\left(a+b\sinh^{-1}(cx)\right)}{9c^4d^2}-\frac{\left(c^2dx^2+d\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7c^4d}-\frac{bc^5d^2x^9\sqrt{c^2dx^2+d}}{81\sqrt{c^2x^2+1}}-\frac{19bc^3d^2x^7\sqrt{c^2dx^2+d}}{441\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (2*b*d^2*x*Sqrt[d + c^2*d*x^2])/(63*c^3*Sqrt[1 + c^2*x^2]) - (b*d^2*x^3*Sqr
t[d + c^2*d*x^2])/(189*c*Sqrt[1 + c^2*x^2]) - (b*c*d^2*x^5*Sqrt[d + c^2*d*x
^2])/(21*Sqrt[1 + c^2*x^2]) - (19*b*c^3*d^2*x^7*Sqrt[d + c^2*d*x^2])/(441*S
qrt[1 + c^2*x^2]) - (b*c^5*d^2*x^9*Sqrt[d + c^2*d*x^2])/(81*Sqrt[1 + c^2*x^
2]) - ((d + c^2*d*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7*c^4*d) + ((d + c^2*d*
x^2)^(9/2)*(a + b*ArcSinh[c*x]))/(9*c^4*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 5734

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[a + b*ArcS
inh[c*x], Int[x^m*(d + e*x^2)^p, x], x] - Dist[(b*c*d^(p - 1/2)*Sqrt[d + e*
x^2])/Sqrt[1 + c^2*x^2], Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x],
 x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p + 1/2, 0] && (
IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\int x^3\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)dx=-\frac{\left(bcd^2\sqrt{d+c^2dx^2}\right)\int\frac{\left(1+c^2x^2\right)^3\left(-2+7c^2x^2\right)}{63c^4}dx}{\sqrt{1+c^2x^2}}+\left(a+b\sinh^{-1}(cx)\right)$$

$$=-\frac{\left(bd^2\sqrt{d+c^2dx^2}\right)\int\left(1+c^2x^2\right)^3\left(-2+7c^2x^2\right)dx}{63c^3\sqrt{1+c^2x^2}}+\frac{1}{2}\left(a+b\sin\right.$$

$$=-\frac{\left(bd^2\sqrt{d+c^2dx^2}\right)\int\left(-2+c^2x^2+15c^4x^4+19c^6x^6+7c^8x^8\right)dx}{63c^3\sqrt{1+c^2x^2}}$$

$$=\frac{2bd^2x\sqrt{d+c^2dx^2}}{63c^3\sqrt{1+c^2x^2}}-\frac{bd^2x^3\sqrt{d+c^2dx^2}}{189c\sqrt{1+c^2x^2}}-\frac{bcd^2x^5\sqrt{d+c^2dx^2}}{21\sqrt{1+c^2x^2}}-\frac{19}{}$$

Mathematica [A] time = 0.19, size = 140, normalized size = 0.53

$$\frac{d^2\sqrt{c^2dx^2+d}\left(63a\left(7c^2x^2-2\right)\left(c^2x^2+1\right)^4+63b\left(7c^2x^2-2\right)\left(c^2x^2+1\right)^4\sinh^{-1}(cx)-bcx\left(49c^8x^8+171c^6x^6\right.\right.}{3969c^4\left(c^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*Sqrt[d + c^2*d*x^2]*(63*a*(1 + c^2*x^2)^4*(-2 + 7*c^2*x^2) - b*c*x*Sqr
t[1 + c^2*x^2]*(-126 + 21*c^2*x^2 + 189*c^4*x^4 + 171*c^6*x^6 + 49*c^8*x^8)
 + 63*b*(1 + c^2*x^2)^4*(-2 + 7*c^2*x^2)*ArcSinh[c*x]))/(3969*c^4*(1 + c^2*
x^2))
```

fricas [A] time = 0.78, size = 263, normalized size = 0.99

$$\frac{63\left(7\,bc^{10}d^2x^{10}+26\,bc^8d^2x^8+34\,bc^6d^2x^6+16\,bc^4d^2x^4-bc^2d^2x^2-2\,bd^2\right)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/3969*(63*(7*b*c^10*d^2*x^10 + 26*b*c^8*d^2*x^8 + 34*b*c^6*d^2*x^6 + 16*b*
c^4*d^2*x^4 - b*c^2*d^2*x^2 - 2*b*d^2)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c
^2*x^2 + 1)) + (441*a*c^10*d^2*x^10 + 1638*a*c^8*d^2*x^8 + 2142*a*c^6*d^2*x
^6 + 1008*a*c^4*d^2*x^4 - 63*a*c^2*d^2*x^2 - 126*a*d^2 - (49*b*c^9*d^2*x^9
+ 171*b*c^7*d^2*x^7 + 189*b*c^5*d^2*x^5 + 21*b*c^3*d^2*x^3 - 126*b*c*d^2*x)
*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d))/(c^6*x^2 + c^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.28, size = 996, normalized size = 3.74

$$a\left(\frac{x^2\left(c^2d\,x^2+d\right)^{\frac{7}{2}}}{9c^2d}-\frac{2\left(c^2d\,x^2+d\right)^{\frac{7}{2}}}{63d\,c^4}\right)+b\left(\frac{\sqrt{d\left(c^2x^2+1\right)}\,\left(256c^{10}x^{10}+256c^9x^9\sqrt{c^2x^2+1}+704c^8x^8+576c^7x^7\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] a*(1/9*x^2*(c^2*d*x^2+d)^(7/2)/c^2/d-2/63/d/c^4*(c^2*d*x^2+d)^(7/2))+b*(1/4
1472*(d*(c^2*x^2+1))^(1/2)*(256*c^10*x^10+256*c^9*x^9*(c^2*x^2+1)^(1/2)+704
*c^8*x^8+576*c^7*x^7*(c^2*x^2+1)^(1/2)+688*c^6*x^6+432*c^5*x^5*(c^2*x^2+1)^
(1/2)+280*c^4*x^4+120*c^3*x^3*(c^2*x^2+1)^(1/2)+41*c^2*x^2+9*c*x*(c^2*x^2+1
)^(1/2)+1)*(-1+9*arcsinh(c*x))*d^2/c^4/(c^2*x^2+1)+3/25088*(d*(c^2*x^2+1))^
(1/2)*(64*c^8*x^8+64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6+112*c^5*x^5*(c^2
*x^2+1)^(1/2)+104*c^4*x^4+56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2+7*c*x*(c^
2*x^2+1)^(1/2)+1)*(-1+7*arcsinh(c*x))*d^2/c^4/(c^2*x^2+1)-1/576*(d*(c^2*x^2
+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+
1)^(1/2)+1)*(-1+3*arcsinh(c*x))*d^2/c^4/(c^2*x^2+1)-3/256*(d*(c^2*x^2+1))^(
1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))*d^2/c^4/(c^2*x^2+1
)-3/256*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(1+arcsinh(
c*x))*d^2/c^4/(c^2*x^2+1)-1/576*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*
(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(1+3*arcsinh(c*x))*d
^2/c^4/(c^2*x^2+1)+3/25088*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8-64*c^7*x^7*(c^
2*x^2+1)^(1/2)+144*c^6*x^6-112*c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4-56*c^3
*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2-7*c*x*(c^2*x^2+1)^(1/2)+1)*(1+7*arcsinh(c
*x))*d^2/c^4/(c^2*x^2+1)+1/41472*(d*(c^2*x^2+1))^(1/2)*(256*c^10*x^10-256*c
^9*x^9*(c^2*x^2+1)^(1/2)+704*c^8*x^8-576*c^7*x^7*(c^2*x^2+1)^(1/2)+688*c^6*
x^6-432*c^5*x^5*(c^2*x^2+1)^(1/2)+280*c^4*x^4-120*c^3*x^3*(c^2*x^2+1)^(1/2)
+41*c^2*x^2-9*c*x*(c^2*x^2+1)^(1/2)+1)*(1+9*arcsinh(c*x))*d^2/c^4/(c^2*x^2+
1))
```

maxima [A] time = 0.44, size = 156, normalized size = 0.59

$$\frac{1}{63}\left(\frac{7\left(c^2dx^2+d\right)^{\frac{7}{2}}x^2}{c^2d}-\frac{2\left(c^2dx^2+d\right)^{\frac{7}{2}}}{c^4d}\right)b\,\mathrm{arsinh}\,(cx)+\frac{1}{63}\left(\frac{7\left(c^2dx^2+d\right)^{\frac{7}{2}}x^2}{c^2d}-\frac{2\left(c^2dx^2+d\right)^{\frac{7}{2}}}{c^4d}\right)a-\frac{\left(49\,c^8d^{\frac{5}{2}}x^9+1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/63*(7*(c^2*d*x^2 + d)^(7/2)*x^2/(c^2*d) - 2*(c^2*d*x^2 + d)^(7/2)/(c^4*d)
)*b*arcsinh(c*x) + 1/63*(7*(c^2*d*x^2 + d)^(7/2)*x^2/(c^2*d) - 2*(c^2*d*x^2
 + d)^(7/2)/(c^4*d))*a - 1/3969*(49*c^8*d^(5/2)*x^9 + 171*c^6*d^(5/2)*x^7 +
 189*c^4*d^(5/2)*x^5 + 21*c^2*d^(5/2)*x^3 - 126*d^(5/2)*x)*b/c^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(d\,c^2\,x^2+d\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int(x^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

3.137 $\int x^2 \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=337

$$\frac{5d^2x\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{128c^2}+\frac{5}{64}d^2x^3\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{8}x^3\left(c^2dx^2+d\right)^{5/2}\left(a+b\sinh^{-1}(cx\right.$$

```
[Out] 5/48*d*x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))+1/8*x^3*(c^2*d*x^2+d)^(5/
2)*(a+b*arcsinh(c*x))+5/128*d^2*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^
2+5/64*d^2*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)-5/256*b*d^2*x^2*(c^2*
d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-59/768*b*c*d^2*x^4*(c^2*d*x^2+d)^(1/2)/(
c^2*x^2+1)^(1/2)-17/288*b*c^3*d^2*x^6*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
-1/64*b*c^5*d^2*x^8*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-5/256*d^2*(a+b*ar
csinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5744, 5742, 5758, 5675, 30, 14, 266, 43}

$$\frac{5}{64}d^2x^3\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)+\frac{5d^2x\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{128c^2}-\frac{5d^2\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2}{256bc^3\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-5*b*d^2*x^2*Sqrt[d + c^2*d*x^2])/(256*c*Sqrt[1 + c^2*x^2]) - (59*b*c*d^2*
x^4*Sqrt[d + c^2*d*x^2])/(768*Sqrt[1 + c^2*x^2]) - (17*b*c^3*d^2*x^6*Sqrt[d
 + c^2*d*x^2])/(288*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*x^8*Sqrt[d + c^2*d*x^2]
)/(64*Sqrt[1 + c^2*x^2]) + (5*d^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]
))/(128*c^2) + (5*d^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/64 + (5
*d*x^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/48 + (x^3*(d + c^2*d*x^2
)^(5/2)*(a + b*ArcSinh[c*x]))/8 - (5*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh
[c*x])^2)/(256*b*c^3*Sqrt[1 + c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int x^2 \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right) dx = \frac{1}{8}x^3 \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right) + \frac{1}{8}(5d) \int x^2 \left(d+c^2dx^2\right)^{3/2}$$

$$= \frac{5}{48}dx^3 \left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right) + \frac{1}{8}x^3 \left(d+c^2dx^2\right)^{5/2} (a+$$

$$= \frac{5}{64}d^2x^3\sqrt{d+c^2dx^2} \left(a+b\sinh^{-1}(cx)\right) + \frac{5}{48}dx^3 \left(d+c^2dx^2\right)^{3/2} (a$$

$$= -\frac{59bcd^2x^4\sqrt{d+c^2dx^2}}{768\sqrt{1+c^2x^2}} - \frac{17bc^3d^2x^6\sqrt{d+c^2dx^2}}{288\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^8\sqrt{d+c}}{64\sqrt{1+c^2x}}$$

$$= -\frac{5bd^2x^2\sqrt{d+c^2dx^2}}{256c\sqrt{1+c^2x^2}} - \frac{59bcd^2x^4\sqrt{d+c^2dx^2}}{768\sqrt{1+c^2x^2}} - \frac{17bc^3d^2x^6\sqrt{d+c^2}}{288\sqrt{1+c^2x^2}}$$

Mathematica [A] time = 0.94, size = 388, normalized size = 1.15

$$\frac{d^2\left(2880acx\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}-2880a\sqrt{d}\sqrt{c^2x^2+1}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)+9216ac^7x^7\sqrt{c^2x^2+1}\sqrt{c^2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*(2880*a*c*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 22656*a*c^3*x^3*Sq
rt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 26112*a*c^5*x^5*Sqrt[1 + c^2*x^2]*Sqr
t[d + c^2*d*x^2] + 9216*a*c^7*x^7*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] - 1
440*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^2 + 576*b*Sqrt[d + c^2*d*x^2]*Cosh[2
*ArcSinh[c*x]] - 144*b*Sqrt[d + c^2*d*x^2]*Cosh[4*ArcSinh[c*x]] - 64*b*Sqrt
[d + c^2*d*x^2]*Cosh[6*ArcSinh[c*x]] - 9*b*Sqrt[d + c^2*d*x^2]*Cosh[8*ArcSi
nh[c*x]] - 2880*a*Sqrt[d]*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^
2*d*x^2]] + 24*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*(-48*Sinh[2*ArcSinh[c*x]]
 + 24*Sinh[4*ArcSinh[c*x]] + 16*Sinh[6*ArcSinh[c*x]] + 3*Sinh[8*ArcSinh[c*x
]])))/(73728*c^3*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(ac^4d^2x^6+2\,ac^2d^2x^4+ad^2x^2+\left(bc^4d^2x^6+2\,bc^2d^2x^4+bd^2x^2\right)\mathrm{arsinh}\left(cx\right)\right)\sqrt{c^2dx^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^6 + 2*a*c^2*d^2*x^4 + a*d^2*x^2 + (b*c^4*d^2*x^6 + 2*
b*c^2*d^2*x^4 + b*d^2*x^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c^2dx^2+d\right)^{\frac{5}{2}}\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)*x^2, x)
```

maple [A] time = 0.34, size = 537, normalized size = 1.59

$$\frac{ax\left(c^2d\,x^2+d\right)^{\frac{7}{2}}}{8c^2d}-\frac{ax\left(c^2d\,x^2+d\right)^{\frac{5}{2}}}{48c^2}-\frac{5adx\left(c^2d\,x^2+d\right)^{\frac{3}{2}}}{192c^2}-\frac{5a\,d^2x\sqrt{c^2d\,x^2+d}}{128c^2}-\frac{5a\,d^3\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{128c^2\sqrt{c^2d}}-5b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/8*a*x*(c^2*d*x^2+d)^(7/2)/c^2/d-1/48*a/c^2*x*(c^2*d*x^2+d)^(5/2)-5/192*a/
c^2*d*x*(c^2*d*x^2+d)^(3/2)-5/128*a/c^2*d^2*x*(c^2*d*x^2+d)^(1/2)-5/128*a/c
^2*d^3*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)-5/256*b*
(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3*arcsinh(c*x)^2*d^2+359/73728*b*
(d*(c^2*x^2+1))^(1/2)*d^2/c^3/(c^2*x^2+1)^(1/2)+1/8*b*(d*(c^2*x^2+1))^(1/2)
*d^2*c^6/(c^2*x^2+1)*arcsinh(c*x)*x^9-1/64*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^5/
(c^2*x^2+1)^(1/2)*x^8+23/48*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^4/(c^2*x^2+1)*arc
sinh(c*x)*x^7-17/288*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^3/(c^2*x^2+1)^(1/2)*x^6+
127/192*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^5-59/768
```

```
*b*(d*(c^2*x^2+1))^(1/2)*d^2*c/(c^2*x^2+1)^(1/2)*x^4+133/384*b*(d*(c^2*x^2+
1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^3-5/256*b*(d*(c^2*x^2+1))^(1/2)*d^
2/c/(c^2*x^2+1)^(1/2)*x^2+5/128*b*(d*(c^2*x^2+1))^(1/2)*d^2/c^2/(c^2*x^2+1)
*arcsinh(c*x)*x
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(d\,c^2\,x^2+d\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int(x^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

3.138 $\int x \left(d+c^2 d x^2\right)^{5/2} \left(a+b \sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=193

$$\frac{\left(c^2 d x^2+d\right)^{7/2} \left(a+b \sinh^{-1}(cx)\right)}{7 c^2 d}-\frac{b d^2 x \sqrt{c^2 d x^2+d}}{7 c \sqrt{c^2 x^2+1}}-\frac{b c d^2 x^3 \sqrt{c^2 d x^2+d}}{7 \sqrt{c^2 x^2+1}}-\frac{b c^5 d^2 x^7 \sqrt{c^2 d x^2+d}}{49 \sqrt{c^2 x^2+1}}-\frac{3 b c^3 d^2 x^5 \sqrt{c^2 d x^2+d}}{35 \sqrt{c^2 x^2+1}}$$

```
[Out] 1/7*(c^2*d*x^2+d)^(7/2)*(a+b*arcsinh(c*x))/c^2/d-1/7*b*d^2*x*(c^2*d*x^2+d)^
(1/2)/c/(c^2*x^2+1)^(1/2)-1/7*b*c*d^2*x^3*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(
1/2)-3/35*b*c^3*d^2*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/49*b*c^5*d^
2*x^7*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5717, 194}

$$\frac{\left(c^2 d x^2+d\right)^{7/2} \left(a+b \sinh^{-1}(cx)\right)}{7 c^2 d}-\frac{b c^5 d^2 x^7 \sqrt{c^2 d x^2+d}}{49 \sqrt{c^2 x^2+1}}-\frac{3 b c^3 d^2 x^5 \sqrt{c^2 d x^2+d}}{35 \sqrt{c^2 x^2+1}}-\frac{b c d^2 x^3 \sqrt{c^2 d x^2+d}}{7 \sqrt{c^2 x^2+1}}-\frac{b d^2 x \sqrt{c^2 d x^2+d}}{7 c \sqrt{c^2 x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(b*d^2*x*Sqrt[d + c^2*d*x^2])/(7*c*Sqrt[1 + c^2*x^2]) - (b*c*d^2*x^3*Sqrt[
d + c^2*d*x^2])/(7*Sqrt[1 + c^2*x^2]) - (3*b*c^3*d^2*x^5*Sqrt[d + c^2*d*x^2
])/(35*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*x^7*Sqrt[d + c^2*d*x^2])/(49*Sqrt[1
+ c^2*x^2]) + ((d + c^2*d*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7*c^2*d)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x \left(d+c^2 d x^2\right)^{5/2} \left(a+b \sinh^{-1}(cx)\right) dx &= \frac{\left(d+c^2 d x^2\right)^{7/2} \left(a+b \sinh^{-1}(cx)\right)}{7 c^2 d}-\frac{\left(b d^2 \sqrt{d+c^2 d x^2}\right) \int \left(1+c^2 x^2\right)^3 d}{7 c \sqrt{1+c^2 x^2}}\\ &= \frac{\left(d+c^2 d x^2\right)^{7/2} \left(a+b \sinh^{-1}(cx)\right)}{7 c^2 d}-\frac{\left(b d^2 \sqrt{d+c^2 d x^2}\right) \int \left(1+3 c^2 x^2+\right.}{7 c \sqrt{1+c^2 x^2}}\\ &= -\frac{b d^2 x \sqrt{d+c^2 d x^2}}{7 c \sqrt{1+c^2 x^2}}-\frac{b c d^2 x^3 \sqrt{d+c^2 d x^2}}{7 \sqrt{1+c^2 x^2}}-\frac{3 b c^3 d^2 x^5 \sqrt{d+c^2 d x^2}}{35 \sqrt{1+c^2 x^2}}-\frac{bc}{}\end{aligned}$$

Mathematica [A] time = 0.15, size = 112, normalized size = 0.58

$$\frac{d^2\sqrt{c^2dx^2+d}\left(35a\left(c^2x^2+1\right)^4+35b\left(c^2x^2+1\right)^4\sinh^{-1}(cx)-bcx\left(5c^6x^6+21c^4x^4+35c^2x^2+35\right)\sqrt{c^2x^2+1}\right)}{245c^2\left(c^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*Sqrt[d + c^2*d*x^2]*(35*a*(1 + c^2*x^2)^4 - b*c*x*Sqrt[1 + c^2*x^2]*(3
5 + 35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6) + 35*b*(1 + c^2*x^2)^4*ArcSinh[c*x
]))/(245*c^2*(1 + c^2*x^2))
```

fricas [A] time = 0.66, size = 225, normalized size = 1.17

$$\frac{35\left(bc^8d^2x^8+4\,bc^6d^2x^6+6\,bc^4d^2x^4+4\,bc^2d^2x^2+bd^2\right)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(35\,ac^8d^2x^8+140\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/245*(35*(b*c^8*d^2*x^8 + 4*b*c^6*d^2*x^6 + 6*b*c^4*d^2*x^4 + 4*b*c^2*d^2*
x^2 + b*d^2)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + (35*a*c^8*d
^2*x^8 + 140*a*c^6*d^2*x^6 + 210*a*c^4*d^2*x^4 + 140*a*c^2*d^2*x^2 + 35*a*d
^2 - (5*b*c^7*d^2*x^7 + 21*b*c^5*d^2*x^5 + 35*b*c^3*d^2*x^3 + 35*b*c*d^2*x)
*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d))/(c^4*x^2 + c^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.21, size = 863, normalized size = 4.47

$$\frac{a\left(c^2d\,x^2+d\right)^{\frac{7}{2}}}{7c^2d}+b\left(\frac{\sqrt{d\left(c^2x^2+1\right)}\,\left(64c^8x^8+64c^7x^7\sqrt{c^2x^2+1}+144c^6x^6+112c^5x^5\sqrt{c^2x^2+1}+104c^4x^4+56\right.}{6272c^2\left(c^2x^2+1\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/7*a/c^2/d*(c^2*d*x^2+d)^(7/2)+b*(1/6272*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8
+64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6+112*c^5*x^5*(c^2*x^2+1)^(1/2)+104
*c^4*x^4+56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2+7*c*x*(c^2*x^2+1)^(1/2)+1)
*(-1+7*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+1/640*(d*(c^2*x^2+1))^(1/2)*(16*c^
6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+
13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+5*arcsinh(c*x))*d^2/c^2/(c^2*x^2+
1)+1/128*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2
*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+5/1
28*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x)
)*d^2/c^2/(c^2*x^2+1)+5/128*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^
```

```
(1/2)+1)*(1+arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+1/128*(d*(c^2*x^2+1))^(1/2)*(
4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*
(1+3*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+1/640*(d*(c^2*x^2+1))^(1/2)*(16*c^6*
x^6-16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+13
*c^2*x^2-5*c*x*(c^2*x^2+1)^(1/2)+1)*(1+5*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+
1/6272*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8-64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c
^6*x^6-112*c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4-56*c^3*x^3*(c^2*x^2+1)^(1/
2)+25*c^2*x^2-7*c*x*(c^2*x^2+1)^(1/2)+1)*(1+7*arcsinh(c*x))*d^2/c^2/(c^2*x^
2+1))
```

maxima [A] time = 0.37, size = 96, normalized size = 0.50

$$\frac{\left(c^2dx^2+d\right)^{\frac{7}{2}}b\operatorname{arsinh}\left(cx\right)}{7\,c^2d}+\frac{\left(c^2dx^2+d\right)^{\frac{7}{2}}a}{7\,c^2d}-\frac{\left(5\,c^6d^{\frac{7}{2}}x^7+21\,c^4d^{\frac{7}{2}}x^5+35\,c^2d^{\frac{7}{2}}x^3+35\,d^{\frac{7}{2}}x\right)b}{245\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*(c^2*d*x^2 + d)^(7/2)*b*arcsinh(c*x)/(c^2*d) + 1/7*(c^2*d*x^2 + d)^(7/2
)*a/(c^2*d) - 1/245*(5*c^6*d^(7/2)*x^7 + 21*c^4*d^(7/2)*x^5 + 35*c^2*d^(7/2
)*x^3 + 35*d^(7/2)*x)*b/(c*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(d\,c^2\,x^2+d\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int(x*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

3.139 $\int \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=254

$$\frac{5}{16}d^2x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{5d^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{32bc\sqrt{c^2x^2+1}}+\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}\left(a+b\sinh^{-1}(cx\right.$$

```
[Out] 5/24*d*x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))+1/6*x*(c^2*d*x^2+d)^(5/2)*(
a+b*arcsinh(c*x))-1/36*b*d^2*(c^2*x^2+1)^(5/2)*(c^2*d*x^2+d)^(1/2)/c+5/16*d
^2*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)-25/96*b*c*d^2*x^2*(c^2*d*x^2+d)
^(1/2)/(c^2*x^2+1)^(1/2)-5/96*b*c^3*d^2*x^4*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)
^(1/2)+5/32*d^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1
/2)
```

Rubi [A] time = 0.16, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5684, 5682, 5675, 30, 14, 261}

$$\frac{5}{16}d^2x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{5d^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{32bc\sqrt{c^2x^2+1}}+\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}\left(a+b\sinh^{-1}(cx\right.$$

Antiderivative was successfully verified.

```
[In] Int[(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-25*b*c*d^2*x^2*Sqrt[d + c^2*d*x^2])/(96*Sqrt[1 + c^2*x^2]) - (5*b*c^3*d^2
*x^4*Sqrt[d + c^2*d*x^2])/(96*Sqrt[1 + c^2*x^2]) - (b*d^2*(1 + c^2*x^2)^(5/
2)*Sqrt[d + c^2*d*x^2])/(36*c) + (5*d^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSin
h[c*x]))/16 + (5*d*x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/24 + (x*(d
 + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/6 + (5*d^2*Sqrt[d + c^2*d*x^2]*(a
 + b*ArcSinh[c*x])^2)/(32*b*c*Sqrt[1 + c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
```

```
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rubi steps

$$\int \left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)dx = \frac{1}{6}x\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{6}(5d)\int\left(d+c^2dx^2\right)^{3/2}\left(a+b\,\mathrm{si}\right.$$

$$= -\frac{bd^2\left(1+c^2x^2\right)^{5/2}\sqrt{d+c^2dx^2}}{36c}+\frac{5}{24}dx\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right.$$

$$= -\frac{bd^2\left(1+c^2x^2\right)^{5/2}\sqrt{d+c^2dx^2}}{36c}+\frac{5}{16}d^2x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)$$

$$= -\frac{25bcd^2x^2\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}}-\frac{5bc^3d^2x^4\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}}-\frac{bd^2\left(1+c^2x^2\right)^{5/2}\sqrt{d\,\cdot}}{36c}$$

Mathematica [A] time = 0.72, size = 317, normalized size = 1.25

$$\frac{d^2\left(1584acx\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}+720a\sqrt{d}\sqrt{c^2x^2+1}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)+384ac^5x^5\sqrt{c^2x^2+1}\sqrt{c^2dx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*(1584*a*c*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 1248*a*c^3*x^3*Sqr
t[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 384*a*c^5*x^5*Sqrt[1 + c^2*x^2]*Sqrt[d
 + c^2*d*x^2] + 360*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^2 - 270*b*Sqrt[d + c
^2*d*x^2]*Cosh[2*ArcSinh[c*x]] - 27*b*Sqrt[d + c^2*d*x^2]*Cosh[4*ArcSinh[c*
x]] - 2*b*Sqrt[d + c^2*d*x^2]*Cosh[6*ArcSinh[c*x]] + 720*a*Sqrt[d]*Sqrt[1 +
 c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + 12*b*Sqrt[d + c^2*d*x^
2]*ArcSinh[c*x]*(45*Sinh[2*ArcSinh[c*x]] + 9*Sinh[4*ArcSinh[c*x]] + Sinh[6*
ArcSinh[c*x]])))/(2304*c*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(ac^4d^2x^4+2\,ac^2d^2x^2+ad^2+\left(bc^4d^2x^4+2\,bc^2d^2x^2+bd^2\right)\mathrm{arsinh}\left(cx\right)\right)\sqrt{c^2dx^2+d}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c^
2*d^2*x^2 + b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.21, size = 427, normalized size = 1.68

$$\frac{x\left(c^2d\,x^2+d\right)^{\frac{5}{2}}a}{6}+\frac{5adx\left(c^2d\,x^2+d\right)^{\frac{3}{2}}}{24}+\frac{5a\,d^2x\sqrt{c^2d\,x^2+d}}{16}+\frac{5a\,d^3\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{16\sqrt{c^2d}}-\frac{299b\sqrt{d\left(c^2x^2+1\right)}}{2304c\sqrt{c^2x^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/6*x*(c^2*d*x^2+d)^(5/2)*a+5/24*a*d*x*(c^2*d*x^2+d)^(3/2)+5/16*a*d^2*x*(c^
2*d*x^2+d)^(1/2)+5/16*a*d^3*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(
c^2*d)^(1/2)-299/2304*b*(d*(c^2*x^2+1))^(1/2)*d^2/c/(c^2*x^2+1)^(1/2)+1/6*b
*(d*(c^2*x^2+1))^(1/2)*d^2*c^6/(c^2*x^2+1)*arcsinh(c*x)*x^7-1/36*b*(d*(c^2*
x^2+1))^(1/2)*d^2*c^5/(c^2*x^2+1)^(1/2)*x^6+17/24*b*(d*(c^2*x^2+1))^(1/2)*d
^2*c^4/(c^2*x^2+1)*arcsinh(c*x)*x^5-13/96*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^3/(
c^2*x^2+1)^(1/2)*x^4+59/48*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^2/(c^2*x^2+1)*arcs
inh(c*x)*x^3-11/32*b*(d*(c^2*x^2+1))^(1/2)*d^2*c/(c^2*x^2+1)^(1/2)*x^2+11/1
6*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)*x+5/32*b*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsinh(c*x)^2*d^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{asinh}\,(c\,x))\,\left(d\,c^2\,x^2+d\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\left(c^2x^2+1\right)\right)^{\frac{5}{2}}(a+b\,\mathrm{asinh}\,(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x)), x)
```

3.140 $\int \frac{(d+c^2dx^2)^{5/2}(a+b\sinh^{-1}(cx))}{x}\,dx$

Optimal. Leaf size=329

$$d^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)-\frac{2d^2\sqrt{c^2dx^2+d}\,\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}+\frac{1}{5}\left(c^2dx^2+d\right)^{5/2}\left(a+b\right.$$

```
[Out] 1/3*d*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))+1/5*(c^2*d*x^2+d)^(5/2)*(a+b*a
rcsinh(c*x))+d^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)-23/15*b*c*d^2*x*(c^
2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-11/45*b*c^3*d^2*x^3*(c^2*d*x^2+d)^(1/2)/
(c^2*x^2+1)^(1/2)-1/25*b*c^5*d^2*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-
2*d^2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)
/(c^2*x^2+1)^(1/2)-b*d^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1
/2)/(c^2*x^2+1)^(1/2)+b*d^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^
(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5744, 5742, 5760, 4182, 2279, 2391, 8, 194}

$$-\frac{bd^2\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+\frac{bd^2\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+d^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (-23*b*c*d^2*x*Sqrt[d + c^2*d*x^2])/(15*Sqrt[1 + c^2*x^2]) - (11*b*c^3*d^2*
x^3*Sqrt[d + c^2*d*x^2])/(45*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*x^5*Sqrt[d + c
^2*d*x^2])/(25*Sqrt[1 + c^2*x^2]) + d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[
c*x]) + (d*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/3 + ((d + c^2*d*x^2)
^(5/2)*(a + b*ArcSinh[c*x]))/5 - (2*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[
c*x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (b*d^2*Sqrt[d + c^2*d*x^
2]*PolyLog[2, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (b*d^2*Sqrt[d + c^2*d*x
^2]*PolyLog[2, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x}\,dx = \frac{1}{5}\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)+d\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(\right.}{x}$$

$$= \frac{1}{3}d\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(c\right.$$

$$= -\frac{8bcd^2x\sqrt{d+c^2dx^2}}{15\sqrt{1+c^2x^2}}-\frac{11bc^3d^2x^3\sqrt{d+c^2dx^2}}{45\sqrt{1+c^2x^2}}-\frac{bc^5d^2x^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}}+$$

$$= -\frac{23bcd^2x\sqrt{d+c^2dx^2}}{15\sqrt{1+c^2x^2}}-\frac{11bc^3d^2x^3\sqrt{d+c^2dx^2}}{45\sqrt{1+c^2x^2}}-\frac{bc^5d^2x^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}}-$$

$$= -\frac{23bcd^2x\sqrt{d+c^2dx^2}}{15\sqrt{1+c^2x^2}}-\frac{11bc^3d^2x^3\sqrt{d+c^2dx^2}}{45\sqrt{1+c^2x^2}}-\frac{bc^5d^2x^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}}-$$

$$= -\frac{23bcd^2x\sqrt{d+c^2dx^2}}{15\sqrt{1+c^2x^2}}-\frac{11bc^3d^2x^3\sqrt{d+c^2dx^2}}{45\sqrt{1+c^2x^2}}-\frac{bc^5d^2x^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}}-$$

$$= -\frac{23bcd^2x\sqrt{d+c^2dx^2}}{15\sqrt{1+c^2x^2}}-\frac{11bc^3d^2x^3\sqrt{d+c^2dx^2}}{45\sqrt{1+c^2x^2}}-\frac{bc^5d^2x^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}}-$$

Mathematica [A] time = 1.27, size = 361, normalized size = 1.10

$$-ad^{5/2}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+d\right)+\frac{1}{15}ad^2\left(3c^4x^4+11c^2x^2+23\right)\sqrt{c^2dx^2+d}+ad^{5/2}\log(x)+\frac{bd^2\sqrt{c^2dx^2+d}\left(\sqrt{c^2x^2}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (-8*b*c*d^2*x*(3 + c^2*x^2)*Sqrt[d + c^2*d*x^2])/(45*Sqrt[1 + c^2*x^2]) - (
b*c^3*d^2*x^3*(5 + 3*c^2*x^2)*Sqrt[d + c^2*d*x^2])/(75*Sqrt[1 + c^2*x^2]) +
 (a*d^2*Sqrt[d + c^2*d*x^2]*(23 + 11*c^2*x^2 + 3*c^4*x^4))/15 + (2*b*d^2*(1
 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/3 + (b*d*(-2 + 3*c^2*x^2)*(d
+ c^2*d*x^2)^(3/2)*ArcSinh[c*x])/15 + a*d^(5/2)*Log[x] - a*d^(5/2)*Log[d +
Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (b*d^2*Sqrt[d + c^2*d*x^2]*(-(c*x) + Sqrt[1
+ c^2*x^2]*ArcSinh[c*x] + ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh
[c*x]*Log[1 + E^(-ArcSinh[c*x])] + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog
[2, E^(-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2]
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(ac^4d^2x^4+2\,ac^2d^2x^2+ad^2+\left(bc^4d^2x^4+2\,bc^2d^2x^2+bd^2\right)\mathrm{arsinh}\left(cx\right)\right)\sqrt{c^2dx^2+d}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c^
2*d^2*x^2 + b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.25, size = 540, normalized size = 1.64

$$\frac{\left(c^2 d\,x^2+d\right)^{\frac{5}{2}} a}{5}+\frac{ad\left(c^2 d\,x^2+d\right)^{\frac{3}{2}}}{3}-a\,d^{\frac{5}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2 d\,x^2+d}}{x}\right)+a\sqrt{c^2 d\,x^2+d}\;d^2+\frac{b\sqrt{d\left(c^2x^2+1\right)}\,d^2\,\mathrm{arc}}{5c^2x^2+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x,x)
```

```
[Out] 1/5*(c^2*d*x^2+d)^(5/2)*a+1/3*a*d*(c^2*d*x^2+d)^(3/2)-a*d^(5/2)*ln((2*d+2*d
^(1/2)*(c^2*d*x^2+d)^(1/2))/x)+a*(c^2*d*x^2+d)^(1/2)*d^2+1/5*b*(d*(c^2*x^2+
1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^6*c^6-1/25*b*(d*(c^2*x^2+1))^(1/2)
*d^2/(c^2*x^2+1)^(1/2)*c^5*x^5+14/15*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1
)*arcsinh(c*x)*x^4*c^4-11/45*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)*
c^3*x^3+34/15*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^2*c^2-
23/15*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)*c*x+23/15*b*(d*(c^2*x^2
+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)
^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*d^2+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x
^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*d^2+b*(d*(c^2*x^2+1))^(1/2)/(c
^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*d^2-b*(d*(c^2*x^2+
1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*d^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{15}\left(15\,d^{\frac{5}{2}}\,\mathrm{arsinh}\left(\frac{1}{c|x|}\right)-3\left(c^2dx^2+d\right)^{\frac{5}{2}}-5\left(c^2dx^2+d\right)^{\frac{3}{2}}d-15\sqrt{c^2dx^2+d}\,d^2\right)a+b\int\frac{\left(c^2dx^2+d\right)^{\frac{5}{2}}\log\left(cx\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x,x, algorithm="maxima")
```

```
[Out] -1/15*(15*d^(5/2)*arcsinh(1/(c*abs(x))) - 3*(c^2*d*x^2 + d)^(5/2) - 5*(c^2*
d*x^2 + d)^(3/2)*d - 15*sqrt(c^2*d*x^2 + d)*d^2)*a + b*integrate((c^2*d*x^2
 + d)^(5/2)*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(d\,c^2\,x^2+d\right)}^{5/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/x,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(d\left(c^2x^2+1\right)\right)^{\frac{5}{2}}\left(a+b\,\mathrm{asinh}\left(cx\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))/x,x)
[Out] Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))/x, x)
```

3.141 $\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=257

$$\frac{15}{8}c^2d^2x\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)+\frac{15cd^2\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2}{16b\sqrt{c^2x^2+1}}+\frac{5}{4}c^2dx\left(c^2dx^2+d\right)^{3/2}\left(a+b\sin\right.$$

```
[Out] 5/4*c^2*d*x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))-(c^2*d*x^2+d)^(5/2)*(a+b
*arcsinh(c*x))/x+15/8*c^2*d^2*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)-9/16
*b*c^3*d^2*x^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/16*b*c^5*d^2*x^4*(c^
2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+15/16*c*d^2*(a+b*arcsinh(c*x))^2*(c^2*d*
x^2+d)^(1/2)/b/(c^2*x^2+1)^(1/2)+b*c*d^2*ln(x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2
+1)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5739, 5684, 5682, 5675, 30, 14, 266, 43}

$$\frac{15}{8}c^2d^2x\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)+\frac{15cd^2\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2}{16b\sqrt{c^2x^2+1}}+\frac{5}{4}c^2dx\left(c^2dx^2+d\right)^{3/2}\left(a+b\sin\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] (-9*b*c^3*d^2*x^2*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*
x^4*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) + (15*c^2*d^2*x*Sqrt[d + c^
2*d*x^2]*(a + b*ArcSinh[c*x]))/8 + (5*c^2*d*x*(d + c^2*d*x^2)^(3/2)*(a + b*
ArcSinh[c*x]))/4 - ((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x + (15*c*d
^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(16*b*Sqrt[1 + c^2*x^2]) + (
b*c*d^2*Sqrt[d + c^2*d*x^2]*Log[x])/Sqrt[1 + c^2*x^2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x^2}\,dx = -\frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x} + \left(5c^2d\right)\int \left(d+c^2dx^2\right)^{3/2}\left(a+b\sin\right.$$

$$= \frac{5}{4}c^2dx\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right) - \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(\right.}{x}$$

$$= \frac{15}{8}c^2d^2x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right) + \frac{5}{4}c^2dx\left(d+c^2dx^2\right)^{3/2}\left(a+b\,\mathrm{si}\right.$$

$$= -\frac{9bc^3d^2x^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} + \frac{15}{8}c^2d^2x\sqrt{d+c^2dx^2}\left(a\right.$$

Mathematica [A] time = 1.47, size = 270, normalized size = 1.05

$$\frac{1}{128}d^2\left(240ac\sqrt{d}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right) + \frac{16a\left(2c^4x^4+9c^2x^2-8\right)\sqrt{c^2dx^2+d}}{x} + \frac{64b\sqrt{c^2dx^2+d}\left(-2\sqrt{c^2x^2}\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] (d^2*((16*a*Sqrt[d + c^2*d*x^2]*(-8 + 9*c^2*x^2 + 2*c^4*x^4))/x + (64*b*Sqr
t[d + c^2*d*x^2]*(-2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + c*x*ArcSinh[c*x]^2 +
2*c*x*Log[c*x]))/(x*Sqrt[1 + c^2*x^2]) + 240*a*c*Sqrt[d]*Log[c*d*x + Sqrt[d
]*Sqrt[d + c^2*d*x^2]] + (32*b*c*Sqrt[d + c^2*d*x^2]*(-Cosh[2*ArcSinh[c*x]]
 + 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])))/Sqrt[1 + c^2*x^2]
 - (b*c*Sqrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]^2 + Cosh[4*ArcSinh[c*x]] - 4*Ar
cSinh[c*x]*Sinh[4*ArcSinh[c*x]]))/Sqrt[1 + c^2*x^2]))/128
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ac^4d^2x^4 + 2\,ac^2d^2x^2 + ad^2 + \left(bc^4d^2x^4 + 2\,bc^2d^2x^2 + bd^2\right)\text{arsinh}\,(cx)\right)\sqrt{c^2dx^2 + d}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c^
2*d^2*x^2 + b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.24, size = 506, normalized size = 1.97

$$-\frac{a\left(c^2d\,x^2+d\right)^{\frac{7}{2}}}{dx}+a\,c^2x\left(c^2d\,x^2+d\right)^{\frac{5}{2}}+\frac{5a\,c^2dx\left(c^2d\,x^2+d\right)^{\frac{3}{2}}}{4}+\frac{15a\,c^2d^2x\sqrt{c^2d\,x^2+d}}{8}+\frac{15a\,c^2d^3\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2}\right.}{8\sqrt{c^2d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^2,x)
```

```
[Out] -a/d/x*(c^2*d*x^2+d)^(7/2)+a*c^2*x*(c^2*d*x^2+d)^(5/2)+5/4*a*c^2*d*x*(c^2*d
*x^2+d)^(3/2)+15/8*a*c^2*d^2*x*(c^2*d*x^2+d)^(1/2)+15/8*a*c^2*d^3*ln(x*c^2*
d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)-b*(d*(c^2*x^2+1))^(1/2)/
(c^2*x^2+1)^(1/2)*arcsinh(c*x)*d^2*c+15/16*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2
+1)^(1/2)*arcsinh(c*x)^2*d^2*c+1/4*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^6/(c^2*x^2
+1)*arcsinh(c*x)*x^5-1/16*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^5/(c^2*x^2+1)^(1/2)
*x^4+11/8*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^4/(c^2*x^2+1)*arcsinh(c*x)*x^3-9/16
*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^3/(c^2*x^2+1)^(1/2)*x^2+1/8*b*(d*(c^2*x^2+1)
)^(1/2)*d^2*c^2/(c^2*x^2+1)*arcsinh(c*x)*x-b*(d*(c^2*x^2+1))^(1/2)*arcsinh(
c*x)*d^2/x/(c^2*x^2+1)+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*ln((c*x+(c
^2*x^2+1)^(1/2))^2-1)*d^2*c-33/128*b*(d*(c^2*x^2+1))^(1/2)*d^2*c/(c^2*x^2+1
)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{asinh}\,(c\,x))\,\left(d\,c^2\,x^2 + d\right)^{5/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\left(c^2x^2+1\right)\right)^{\frac{5}{2}}\,(a + b\,\mathrm{asinh}\,(cx))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))/x**2,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))/x**2, x)
```

3.142 $\int \frac{(d+c^2dx^2)^{5/2}(a+b\sinh^{-1}(cx))}{x^3}\,dx$

Optimal. Leaf size=355

$$\frac{5}{2}c^2d^2\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)-\frac{5c^2d^2\sqrt{c^2dx^2+d}\,\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}+\frac{5}{6}c^2d\left(c^2dx^2+\right.$$

```
[Out] 5/6*c^2*d*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))-1/2*(c^2*d*x^2+d)^(5/2)*(a
+b*arcsinh(c*x))/x^2+5/2*c^2*d^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)-1/2
*b*c*d^2*(c^2*d*x^2+d)^(1/2)/x/(c^2*x^2+1)^(1/2)-7/3*b*c^3*d^2*x*(c^2*d*x^2
+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/9*b*c^5*d^2*x^3*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+
1)^(1/2)-5*c^2*d^2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*d
*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-5/2*b*c^2*d^2*polylog(2,-c*x-(c^2*x^2+1)^(1
/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+5/2*b*c^2*d^2*polylog(2,c*x+(c^2
*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5739, 5744, 5742, 5760, 4182, 2279, 2391, 8, 270}

$$-\frac{5bc^2d^2\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2x^2+1}}+\frac{5bc^2d^2\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2x^2+1}}+\frac{5}{2}c^2d^2\sqrt{c^2dx^2+d}\,\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] -(b*c*d^2*Sqrt[d + c^2*d*x^2])/(2*x*Sqrt[1 + c^2*x^2]) - (7*b*c^3*d^2*x*Sqr
t[d + c^2*d*x^2])/(3*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*x^3*Sqrt[d + c^2*d*x^2
])/(9*Sqrt[1 + c^2*x^2]) + (5*c^2*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x]))/2 + (5*c^2*d*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/6 - ((d + c^2
*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(2*x^2) - (5*c^2*d^2*Sqrt[d + c^2*d*x^2
]*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (5*b*c^
2*d^2*Sqrt[d + c^2*d*x^2]*PolyLog[2, -E^ArcSinh[c*x]])/(2*Sqrt[1 + c^2*x^2]
) + (5*b*c^2*d^2*Sqrt[d + c^2*d*x^2]*PolyLog[2, E^ArcSinh[c*x]])/(2*Sqrt[1
+ c^2*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x^3}\,dx = -\frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{2x^2}+\frac{1}{2}\left(5c^2d\right)\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+\right.}{x}$$

$$= \frac{5}{6}c^2d\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)-\frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-}\right.}{2x^2}$$

$$= -\frac{bcd^2\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}}+\frac{bc^3d^2x\sqrt{d+c^2dx^2}}{6\sqrt{1+c^2x^2}}-\frac{bc^5d^2x^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}+\frac{5}{2}$$

$$= -\frac{bcd^2\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}}-\frac{7bc^3d^2x\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}}-\frac{bc^5d^2x^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}+\frac{5}{2}$$

$$= -\frac{bcd^2\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}}-\frac{7bc^3d^2x\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}}-\frac{bc^5d^2x^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}+\frac{5}{2}$$

$$= -\frac{bcd^2\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}}-\frac{7bc^3d^2x\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}}-\frac{bc^5d^2x^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}+\frac{5}{2}$$

$$= -\frac{bcd^2\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}}-\frac{7bc^3d^2x\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}}-\frac{bc^5d^2x^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}+\frac{5}{2}$$

Mathematica [A] time = 6.74, size = 467, normalized size = 1.32

$$-\frac{5}{2}ac^2d^{5/2}\log\left(\sqrt{d}\sqrt{d\left(c^2x^2+1\right)}+d\right)+\frac{5}{2}ac^2d^{5/2}\log(x)+\sqrt{d\left(c^2x^2+1\right)}\left(\frac{1}{3}ac^4d^2x^2+\frac{7}{3}ac^2d^2-\frac{ad^2}{2x^2}\right)+\frac{2bc^2d^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] Sqrt[d*(1 + c^2*x^2)]*((7*a*c^2*d^2)/3 - (a*d^2)/(2*x^2) + (a*c^4*d^2*x^2)/
3) + b*c^2*d^2*(-1/9*(c*x*Sqrt[d*(1 + c^2*x^2)]*(3 + c^2*x^2))/Sqrt[1 + c^2
*x^2] + ((1 + c^2*x^2)*Sqrt[d*(1 + c^2*x^2)]*ArcSinh[c*x])/3) + (5*a*c^2*d^
(5/2)*Log[x])/2 - (5*a*c^2*d^(5/2)*Log[d + Sqrt[d]*Sqrt[d*(1 + c^2*x^2)]])/
2 + (2*b*c^2*d^2*Sqrt[d*(1 + c^2*x^2)]*(-(c*x) + Sqrt[1 + c^2*x^2]*ArcSinh[
c*x] + ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Log[1 + E^(-A
rcSinh[c*x])] + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x
])]))/Sqrt[1 + c^2*x^2] + (b*c^2*d^2*Sqrt[d*(1 + c^2*x^2)]*(-2*Coth[ArcSinh
[c*x]/2] - ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*x]*Log[1 - E^(
-ArcSinh[c*x])] - 4*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 4*PolyLog[2,
-E^(-ArcSinh[c*x])] - 4*PolyLog[2, E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Sech[A
rcSinh[c*x]/2]^2 + 2*Tanh[ArcSinh[c*x]/2]))/(8*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(ac^4d^2x^4+2\,ac^2d^2x^2+ad^2+\left(bc^4d^2x^4+2\,bc^2d^2x^2+bd^2\right)\mathrm{arsinh}\left(cx\right)\right)\sqrt{c^2dx^2+d}}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c^
2*d^2*x^2 + b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^3, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.30, size = 588, normalized size = 1.66

$$-\frac{a\left(c^{2} d\,x^{2}+d\right)^{\frac{7}{2}}}{2 d\,x^{2}}+\frac{a\,c^{2}\left(c^{2} d\,x^{2}+d\right)^{\frac{5}{2}}}{2}+\frac{5 a\,c^{2} d\left(c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}{6}-\frac{5 a\,c^{2} d^{\frac{5}{2}} \ln\left(\frac{2 d+2 \sqrt{d}\,\sqrt{c^{2} d\,x^{2}+d}}{x}\right)}{2}+\frac{5 a\,c^{2} \sqrt{c^{2} d\,x^{2}+d}\,d^{2}}{2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^3,x)
```

```
[Out] -1/2*a/d/x^2*(c^2*d*x^2+d)^(7/2)+1/2*a*c^2*(c^2*d*x^2+d)^(5/2)+5/6*a*c^2*d*
(c^2*d*x^2+d)^(3/2)-5/2*a*c^2*d^(5/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2)
)/x)+5/2*a*c^2*(c^2*d*x^2+d)^(1/2)*d^2+5/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2
+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^2*d^2-5/2*b*(d*(c^2*x^
2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^2*
d^2+5/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)
^(1/2))*c^2*d^2-5/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*
x-(c^2*x^2+1)^(1/2))*c^2*d^2+11/6*b*(d*(c^2*x^2+1))^(1/2)*c^2*d^2/(c^2*x^2+
1)*arcsinh(c*x)-1/2*b*(d*(c^2*x^2+1))^(1/2)*d^2/x^2/(c^2*x^2+1)*arcsinh(c*x
)+1/3*b*(d*(c^2*x^2+1))^(1/2)*c^6*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^4-1/9*b*(d
*(c^2*x^2+1))^(1/2)*c^5*d^2/(c^2*x^2+1)^(1/2)*x^3+8/3*b*(d*(c^2*x^2+1))^(1/
2)*c^4*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^2-7/3*b*(d*(c^2*x^2+1))^(1/2)*c^3*d^2
/(c^2*x^2+1)^(1/2)*x-1/2*b*(d*(c^2*x^2+1))^(1/2)*d^2/x/(c^2*x^2+1)^(1/2)*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{6}\left(15\,c^{2} d^{\frac{5}{2}} \operatorname{arsinh}\left(\frac{1}{c|x|}\right)-3\left(c^{2} d x^{2}+d\right)^{\frac{5}{2}} c^{2}-5\left(c^{2} d x^{2}+d\right)^{\frac{3}{2}} c^{2} d-15\sqrt{c^{2} d x^{2}+d}\,c^{2} d^{2}+\frac{3\left(c^{2} d x^{2}+d\right)^{\frac{7}{2}}}{d x^{2}}\right) a+b\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] -1/6*(15*c^2*d^(5/2)*arcsinh(1/(c*abs(x))) - 3*(c^2*d*x^2 + d)^(5/2)*c^2 -
5*(c^2*d*x^2 + d)^(3/2)*c^2*d - 15*sqrt(c^2*d*x^2 + d)*c^2*d^2 + 3*(c^2*d*x
^2 + d)^(7/2)/(d*x^2))*a + b*integrate((c^2*d*x^2 + d)^(5/2)*log(c*x + sqrt
(c^2*x^2 + 1))/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\operatorname{asinh}(c\,x))\left(d\,c^{2}\,x^{2}+d\right)^{5/2}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/x^3,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\left(c^2x^2+1\right)\right)^{\frac{5}{2}}(a+b\operatorname{asinh}(cx))}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))/x**3,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))/x**3, x)
```

3.143 $\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=266

$$-\frac{5c^2d\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3x}-\frac{\left(c^2dx^2+d\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{3x^3}+\frac{5}{2}c^4d^2x\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right.$$

```
[Out] -5/3*c^2*d*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x-1/3*(c^2*d*x^2+d)^(5/2)
*(a+b*arcsinh(c*x))/x^3+5/2*c^4*d^2*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2
)-1/6*b*c*d^2*(c^2*d*x^2+d)^(1/2)/x^2/(c^2*x^2+1)^(1/2)-1/4*b*c^5*d^2*x^2*(
c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+5/4*c^3*d^2*(a+b*arcsinh(c*x))^2*(c^2*
d*x^2+d)^(1/2)/b/(c^2*x^2+1)^(1/2)+7/3*b*c^3*d^2*ln(x)*(c^2*d*x^2+d)^(1/2)/
(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {5739, 5682, 5675, 30, 14, 266, 43}

$$\frac{5}{2}c^4d^2x\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)+\frac{5c^3d^2\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2}{4b\sqrt{c^2x^2+1}}-\frac{5c^2d\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(\right.}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] -(b*c*d^2*Sqrt[d + c^2*d*x^2])/(6*x^2*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*x^2*S
qrt[d + c^2*d*x^2])/(4*Sqrt[1 + c^2*x^2]) + (5*c^4*d^2*x*Sqrt[d + c^2*d*x^2
]*(a + b*ArcSinh[c*x]))/2 - (5*c^2*d*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c
*x]))/(3*x) - ((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(3*x^3) + (5*c^3
*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*Sqrt[1 + c^2*x^2]) +
(7*b*c^3*d^2*Sqrt[d + c^2*d*x^2]*Log[x])/(3*Sqrt[1 + c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x^4}\,dx = -\frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{3x^3}+\frac{1}{3}\left(5c^2d\right)\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+\right.}{x^2}$$

$$= -\frac{5c^2d\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3x}-\frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh\right.}{3x^3}$$

$$= \frac{5}{2}c^4d^2x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{5c^2d\left(d+c^2dx^2\right)^{3/2}\left(a+b\,\mathrm{s}\right.}{3x}$$

$$= -\frac{bcd^2\sqrt{d+c^2dx^2}}{6x^2\sqrt{1+c^2x^2}}-\frac{bc^5d^2x^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}}+\frac{5}{2}c^4d^2x\sqrt{d+c^2dx^2}\left(a+\right.$$

Mathematica [A] time = 0.97, size = 287, normalized size = 1.08

$$\frac{d^2\left(4a\sqrt{c^2x^2+1}\left(3c^4x^4-14c^2x^2-2\right)\sqrt{c^2dx^2+d}+60ac^3\sqrt{d}\,x^3\sqrt{c^2x^2+1}\,\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)+24b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] (d^2*(4*a*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(-2 - 14*c^2*x^2 + 3*c^4*x^
4) + 24*b*c^2*x^2*Sqrt[d + c^2*d*x^2]*(-2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] +
c*x*ArcSinh[c*x]^2 + 2*c*x*Log[c*x]) + 4*b*Sqrt[d + c^2*d*x^2]*(-(c*x) - 2*
(1 + c^2*x^2)^(3/2)*ArcSinh[c*x] + 2*c^3*x^3*Log[c*x]) + 60*a*c^3*Sqrt[d]*x
^3*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] - 3*b*c^3*x^3
*Sqrt[d + c^2*d*x^2]*(Cosh[2*ArcSinh[c*x]] - 2*ArcSinh[c*x]*(ArcSinh[c*x] +
 Sinh[2*ArcSinh[c*x]]))))/(24*x^3*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ac^4d^2x^4 + 2\,ac^2d^2x^2 + ad^2 + \left(bc^4d^2x^4 + 2\,bc^2d^2x^2 + bd^2\right)\text{arsinh}\left(cx\right)\right)\sqrt{c^2dx^2 + d}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c^
2*d^2*x^2 + b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^4, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.30, size = 1316, normalized size = 4.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^4,x)
```

```
[Out] -1/3*a/d/x^3*(c^2*d*x^2+d)^(7/2)+4/3*a*c^4*x*(c^2*d*x^2+d)^(5/2)-4/3*a*c^2/
d/x*(c^2*d*x^2+d)^(7/2)+5/3*a*c^4*d*x*(c^2*d*x^2+d)^(3/2)+5/2*a*c^4*d^2*x*(
c^2*d*x^2+d)^(1/2)+5/2*a*c^4*d^3*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/
2))/(c^2*d)^(1/2)-1/8*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^3/(c^2*x^2+1)^(1/2)+147
*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^4/(c^2*x^2+1)^(1/2
)*arcsinh(c*x)*c^7+35*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)
*x^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^5-147*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63
*c^4*x^4+15*c^2*x^2+1)*x^5/(c^2*x^2+1)*arcsinh(c*x)*c^8-203*b*(d*(c^2*x^2+1
))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^3/(c^2*x^2+1)*arcsinh(c*x)*c^6-190
/3*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x/(c^2*x^2+1)*arcs
inh(c*x)*c^4-23/3*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)/x/(
c^2*x^2+1)*arcsinh(c*x)*c^2-1/3*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*
c^2*x^2+1)/x^3/(c^2*x^2+1)*arcsinh(c*x)+1/2*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^6
/(c^2*x^2+1)*arcsinh(c*x)*x^3+1/2*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^4/(c^2*x^2+
1)*arcsinh(c*x)*x-49/6*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1
)*x^5/(c^2*x^2+1)*c^8-28/3*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x
^2+1)*x^3/(c^2*x^2+1)*c^6-7/6*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^
2*x^2+1)*x/(c^2*x^2+1)*c^4-21/2*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*
c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*c^5-1/6*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^
4*x^4+15*c^2*x^2+1)/x^2/(c^2*x^2+1)^(1/2)*c+7/3*b*(d*(c^2*x^2+1))^(1/2)*d^2
/(63*c^4*x^4+15*c^2*x^2+1)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^3-5/2*b*(d*(c^2
*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)/(c^2*x^2+1)^(1/2)*c^3-1/4*b*(d
*(c^2*x^2+1))^(1/2)*d^2*c^5/(c^2*x^2+1)^(1/2)*x^2+49/6*b*(d*(c^2*x^2+1))^(1
/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^3*c^6+7/6*b*(d*(c^2*x^2+1))^(1/2)*d^2/(
63*c^4*x^4+15*c^2*x^2+1)*x*c^4+5/4*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2
)*arcsinh(c*x)^2*d^2*c^3+7/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*ln((
c*x+(c^2*x^2+1)^(1/2))^2-1)*d^2*c^3-14/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1
)^(1/2)*arcsinh(c*x)*d^2*c^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{asinh}\,(c\,x))\,\left(d\,c^2\,x^2 + d\right)^{5/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/x^4,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\left(c^2x^2 + 1\right)\right)^{\frac{5}{2}}(a + b\,\mathrm{asinh}\,(cx))}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))/x**4,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))/x**4, x)
```

3.144 $\int \sqrt{1+x^2} \sinh^{-1}(x) \, dx$

Optimal. Leaf size=32

$$-\frac{x^2}{4}+\frac{1}{2}\sqrt{x^2+1}\,x\sinh^{-1}(x)+\frac{1}{4}\sinh^{-1}(x)^2$$

```
[Out] -1/4*x^2+1/4*arcsinh(x)^2+1/2*x*arcsinh(x)*(x^2+1)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5682, 5675, 30}

$$-\frac{x^2}{4}+\frac{1}{2}\sqrt{x^2+1}\,x\sinh^{-1}(x)+\frac{1}{4}\sinh^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2]*ArcSinh[x],x]
```

```
[Out] -x^2/4 + (x*Sqrt[1 + x^2]*ArcSinh[x])/2 + ArcSinh[x]^2/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1+x^2}\sinh^{-1}(x)\,dx &= \frac{1}{2}x\sqrt{1+x^2}\sinh^{-1}(x)-\frac{\int x\,dx}{2}+\frac{1}{2}\int\frac{\sinh^{-1}(x)}{\sqrt{1+x^2}}\,dx\\ &= -\frac{x^2}{4}+\frac{1}{2}x\sqrt{1+x^2}\sinh^{-1}(x)+\frac{1}{4}\sinh^{-1}(x)^2\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.88

$$\frac{1}{4}\left(-x^2+2\sqrt{x^2+1}\,x\sinh^{-1}(x)+\sinh^{-1}(x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2]*ArcSinh[x],x]
```

```
[Out] (-x^2 + 2*x*Sqrt[1 + x^2]*ArcSinh[x] + ArcSinh[x]^2)/4
```

fricas [A] time = 0.60, size = 40, normalized size = 1.25

$$\frac{1}{2}\sqrt{x^2+1}\,x\log\left(x+\sqrt{x^2+1}\right)-\frac{1}{4}\,x^2+\frac{1}{4}\,\log\left(x+\sqrt{x^2+1}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x)*(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(x^2 + 1)*x*log(x + sqrt(x^2 + 1)) - 1/4*x^2 + 1/4*log(x + sqrt(x^2
 + 1))^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^2+1}\,\operatorname{arsinh}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x)*(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 1)*arcsinh(x), x)
```

maple [A] time = 0.00, size = 26, normalized size = 0.81

$$\frac{x\,\operatorname{arcsinh}(x)\sqrt{x^2+1}}{2}+\frac{\operatorname{arcsinh}(x)^2}{4}-\frac{x^2}{4}-\frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(x)*(x^2+1)^(1/2),x)
```

```
[Out] 1/2*x*arcsinh(x)*(x^2+1)^(1/2)+1/4*arcsinh(x)^2-1/4*x^2-1/4
```

maxima [A] time = 0.47, size = 28, normalized size = 0.88

$$-\frac{1}{4}\,x^2+\frac{1}{2}\left(\sqrt{x^2+1}\,x+\operatorname{arsinh}(x)\right)\operatorname{arsinh}(x)-\frac{1}{4}\,\operatorname{arsinh}(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x)*(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*x^2 + 1/2*(sqrt(x^2 + 1)*x + arcsinh(x))*arcsinh(x) - 1/4*arcsinh(x)^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \operatorname{asinh}(x)\,\sqrt{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(x)*(x^2 + 1)^(1/2),x)
```

```
[Out] int(asinh(x)*(x^2 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^2+1}\,\operatorname{asinh}{\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x)*(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + 1)*asinh(x), x)
```

3.145 $\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=215

$$\frac{x^4\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{5c^2d}+\frac{8\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{15c^6d}-\frac{4x^2\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{15c^4d}-\frac{bx^5\sqrt{c^2}}{25c\sqrt{c^2}}$$

```
[Out] -8/15*b*x*(c^2*x^2+1)^(1/2)/c^5/(c^2*d*x^2+d)^(1/2)+4/45*b*x^3*(c^2*x^2+1)^
(1/2)/c^3/(c^2*d*x^2+d)^(1/2)-1/25*b*x^5*(c^2*x^2+1)^(1/2)/c/(c^2*d*x^2+d)^
(1/2)+8/15*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^6/d-4/15*x^2*(a+b*arcsi
nh(c*x))*(c^2*d*x^2+d)^(1/2)/c^4/d+1/5*x^4*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)
^(1/2)/c^2/d
```

Rubi [A] time = 0.26, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5758, 5717, 8, 30}

$$\frac{x^4\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{5c^2d}-\frac{4x^2\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{15c^4d}+\frac{8\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{15c^6d}-\frac{bx^5\sqrt{c^2}}{25c\sqrt{c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (-8*b*x*Sqrt[1 + c^2*x^2])/(15*c^5*Sqrt[d + c^2*d*x^2]) + (4*b*x^3*Sqrt[1 +
 c^2*x^2])/(45*c^3*Sqrt[d + c^2*d*x^2]) - (b*x^5*Sqrt[1 + c^2*x^2])/(25*c*S
qrt[d + c^2*d*x^2]) + (8*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(15*c^6*
d) - (4*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(15*c^4*d) + (x^4*Sqr
t[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(5*c^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx=\frac{x^4\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{5c^2d}-\frac{4\int\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx}{5c^2}-\frac{\left(b\sqrt{1+c^2x^2}\right)\int x^4}{5c\sqrt{d+c^2dx^2}}$$

$$=-\frac{bx^5\sqrt{1+c^2x^2}}{25c\sqrt{d+c^2dx^2}}-\frac{4x^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{15c^4d}+\frac{x^4\sqrt{d+c^2dx^2}\left(a+b\,\mathrm{s}\right.}{5c^2d}$$

$$=\frac{4bx^3\sqrt{1+c^2x^2}}{45c^3\sqrt{d+c^2dx^2}}-\frac{bx^5\sqrt{1+c^2x^2}}{25c\sqrt{d+c^2dx^2}}+\frac{8\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{15c^6d}-\frac{4x^2\sqrt{d}}{}$$

$$=-\frac{8bx\sqrt{1+c^2x^2}}{15c^5\sqrt{d+c^2dx^2}}+\frac{4bx^3\sqrt{1+c^2x^2}}{45c^3\sqrt{d+c^2dx^2}}-\frac{bx^5\sqrt{1+c^2x^2}}{25c\sqrt{d+c^2dx^2}}+\frac{8\sqrt{d+c^2dx^2}\left(a+b\right.}{15c^6d}$$

Mathematica [A] time = 0.17, size = 119, normalized size = 0.55

$$\frac{15a\left(3c^6x^6-c^4x^4+4c^2x^2+8\right)+bcx\sqrt{c^2x^2+1}\left(-9c^4x^4+20c^2x^2-120\right)+15b\left(3c^6x^6-c^4x^4+4c^2x^2+8\right)\sin}{225c^6\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (b*c*x*Sqrt[1 + c^2*x^2]*(-120 + 20*c^2*x^2 - 9*c^4*x^4) + 15*a*(8 + 4*c^2*
x^2 - c^4*x^4 + 3*c^6*x^6) + 15*b*(8 + 4*c^2*x^2 - c^4*x^4 + 3*c^6*x^6)*Arc
Sinh[c*x])/(225*c^6*Sqrt[d + c^2*d*x^2])
```

fricas [A] time = 0.54, size = 161, normalized size = 0.75

$$\frac{15\left(3\,bc^6x^6-bc^4x^4+4\,bc^2x^2+8\,b\right)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(45\,ac^6x^6-15\,ac^4x^4+60\,ac^2x^2-\left(9\,b\right.\right.}{225\left(c^8dx^2+c^6d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/225*(15*(3*b*c^6*x^6 - b*c^4*x^4 + 4*b*c^2*x^2 + 8*b)*sqrt(c^2*d*x^2 + d)
*log(c*x + sqrt(c^2*x^2 + 1)) + (45*a*c^6*x^6 - 15*a*c^4*x^4 + 60*a*c^2*x^2
 - (9*b*c^5*x^5 - 20*b*c^3*x^3 + 120*b*c*x)*sqrt(c^2*x^2 + 1) + 120*a)*sqrt
(c^2*d*x^2 + d))/(c^8*d*x^2 + c^6*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.29, size = 625, normalized size = 2.91

$$a\left(\frac{x^4\sqrt{c^2d\,x^2+d}}{5c^2d}-\frac{4\left(\frac{x^2\sqrt{c^2d\,x^2+d}}{3c^2d}-\frac{2\sqrt{c^2d\,x^2+d}}{3d\,c^4}\right)}{5c^2}\right)+b\left(\frac{\sqrt{d\left(c^2x^2+1\right)}\left(16c^6x^6+16c^5x^5\sqrt{c^2x^2+1}+28c^4x^4+2\right.}{80}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a*(1/5*x^4/c^2/d*(c^2*d*x^2+d)^(1/2)-4/5/c^2*(1/3*x^2/c^2/d*(c^2*d*x^2+d)^(
1/2)-2/3/d/c^4*(c^2*d*x^2+d)^(1/2)))+b*(1/800*(d*(c^2*x^2+1))^(1/2)*(16*c^6
*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+1
3*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+5*arcsinh(c*x))/c^6/d/(c^2*x^2+1)-
5/288*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^
2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arcsinh(c*x))/c^6/d/(c^2*x^2+1)+5/16*(d*
(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))/c^6/
d/(c^2*x^2+1)+5/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*
(1+arcsinh(c*x))/c^6/d/(c^2*x^2+1)-5/288*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4
*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(1+3*arcsin
h(c*x))/c^6/d/(c^2*x^2+1)+1/800*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6-16*c^5*x^
5*(c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2-5*c*
x*(c^2*x^2+1)^(1/2)+1)*(1+5*arcsinh(c*x))/c^6/d/(c^2*x^2+1))
```

maxima [A] time = 0.38, size = 174, normalized size = 0.81

$$\frac{1}{15}\left(\frac{3\sqrt{c^2dx^2+d}\,x^4}{c^2d}-\frac{4\sqrt{c^2dx^2+d}\,x^2}{c^4d}+\frac{8\sqrt{c^2dx^2+d}}{c^6d}\right)b\operatorname{arsinh}(cx)+\frac{1}{15}\left(\frac{3\sqrt{c^2dx^2+d}\,x^4}{c^2d}-\frac{4\sqrt{c^2dx^2+d}\,x^2}{c^4d}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*sqrt(c^2*d*x^2 + d)*x^4/(c^2*d) - 4*sqrt(c^2*d*x^2 + d)*x^2/(c^4*d)
 + 8*sqrt(c^2*d*x^2 + d)/(c^6*d))*b*arcsinh(c*x) + 1/15*(3*sqrt(c^2*d*x^2 +
 d)*x^4/(c^2*d) - 4*sqrt(c^2*d*x^2 + d)*x^2/(c^4*d) + 8*sqrt(c^2*d*x^2 + d)
/(c^6*d))*a - 1/225*(9*c^4*x^5 - 20*c^2*x^3 + 120*x)*b/(c^5*sqrt(d))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int((x^5*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\left(a+b\operatorname{asinh}(cx)\right)}{\sqrt{d\left(c^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**5*(a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

3.146 $\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=192

$$\frac{x^3\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{4c^2d}+\frac{3\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)^2}{16bc^5\sqrt{c^2dx^2+d}}-\frac{3x\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{8c^4d}-\frac{bx^4\sqrt{\ldots}}{16c\sqrt{\ldots}}$$

```
[Out] 3/16*b*x^2*(c^2*x^2+1)^(1/2)/c^3/(c^2*d*x^2+d)^(1/2)-1/16*b*x^4*(c^2*x^2+1)
^(1/2)/c/(c^2*d*x^2+d)^(1/2)+3/16*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/
c^5/(c^2*d*x^2+d)^(1/2)-3/8*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^4/d+
1/4*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2/d
```

Rubi [A] time = 0.25, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5758, 5677, 5675, 30}

$$\frac{x^3\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{4c^2d}-\frac{3x\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{8c^4d}+\frac{3\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)^2}{16bc^5\sqrt{c^2dx^2+d}}-\frac{bx^4\sqrt{\ldots}}{16c\sqrt{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (3*b*x^2*Sqrt[1 + c^2*x^2])/(16*c^3*Sqrt[d + c^2*d*x^2]) - (b*x^4*Sqrt[1 +
c^2*x^2])/(16*c*Sqrt[d + c^2*d*x^2]) - (3*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcS
inh[c*x]))/(8*c^4*d) + (x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(4*c^
2*d) + (3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(16*b*c^5*Sqrt[d + c^2*
d*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] &&  !GtQ[d, 0]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx = \frac{x^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{4c^2d} - \frac{3\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx}{4c^2} - \frac{\left(b\sqrt{1+c^2x^2}\right)\int x^3\,dx}{4c\sqrt{d+c^2dx^2}}$$

$$= -\frac{bx^4\sqrt{1+c^2x^2}}{16c\sqrt{d+c^2dx^2}} - \frac{3x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{8c^4d} + \frac{x^3\sqrt{d+c^2dx^2}\left(a+b\sinh\right.}{4c^2d}$$

$$= \frac{3bx^2\sqrt{1+c^2x^2}}{16c^3\sqrt{d+c^2dx^2}} - \frac{bx^4\sqrt{1+c^2x^2}}{16c\sqrt{d+c^2dx^2}} - \frac{3x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{8c^4d} + \frac{x^3\sqrt{d+}}{}$$

$$= \frac{3bx^2\sqrt{1+c^2x^2}}{16c^3\sqrt{d+c^2dx^2}} - \frac{bx^4\sqrt{1+c^2x^2}}{16c\sqrt{d+c^2dx^2}} - \frac{3x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{8c^4d} + \frac{x^3\sqrt{d+}}{}$$

Mathematica [A] time = 0.62, size = 151, normalized size = 0.79

$$\frac{\frac{16acx\left(2c^2x^2-3\right)\sqrt{c^2dx^2+d}}{d} + \frac{48a\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)}{\sqrt{d}} + \frac{b\sqrt{c^2x^2+1}\left(4\sinh^{-1}(cx)\left(6\sinh^{-1}(cx)-8\sinh\left(2\sinh^{-1}(cx)\right)+\sinh\left(4\sinh^{-1}(cx)\right)\right)+16\right.}{\sqrt{c^2dx^2+d}}}{128c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] ((16*a*c*x*(-3 + 2*c^2*x^2)*Sqrt[d + c^2*d*x^2])/d + (48*a*Log[c*d*x + Sqrt
[d]*Sqrt[d + c^2*d*x^2]])/Sqrt[d] + (b*Sqrt[1 + c^2*x^2]*(16*Cosh[2*ArcSinh
[c*x]] - Cosh[4*ArcSinh[c*x]] + 4*ArcSinh[c*x]*(6*ArcSinh[c*x] - 8*Sinh[2*A
rcSinh[c*x]] + Sinh[4*ArcSinh[c*x]])))/Sqrt[d + c^2*d*x^2])/(128*c^5)
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^4\,\text{arsinh}\,(cx)+ax^4}{\sqrt{c^2dx^2+d}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arcsinh(c*x) + a*x^4)/sqrt(c^2*d*x^2 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arsinh}\,(cx)+a)x^4}{\sqrt{c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^4/sqrt(c^2*d*x^2 + d), x)
```

maple [B] time = 0.34, size = 347, normalized size = 1.81

$$\frac{a\,x^3\sqrt{c^2d\,x^2+d}}{4c^2d} - \frac{3ax\sqrt{c^2d\,x^2+d}}{8c^4d} + \frac{3a\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{8c^4\sqrt{c^2d}} + \frac{b\sqrt{d\left(c^2x^2+1\right)}\,\text{arcsinh}\,(cx)\,x^5}{4d\left(c^2x^2+1\right)} - \frac{b\sqrt{d\left(c^2x^2+\right.}}{16cd\sqrt{c^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] 1/4*a*x^3/c^2/d*(c^2*d*x^2+d)^(1/2)-3/8*a/c^4*x/d*(c^2*d*x^2+d)^(1/2)+3/8*a
/c^4*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/4*b*(d*(
c^2*x^2+1))^(1/2)/d/(c^2*x^2+1)*arcsinh(c*x)*x^5-1/16*b*(d*(c^2*x^2+1))^(1/
2)/c/d/(c^2*x^2+1)^(1/2)*x^4-1/8*b*(d*(c^2*x^2+1))^(1/2)/c^2/d/(c^2*x^2+1)*
arcsinh(c*x)*x^3+3/16*b*(d*(c^2*x^2+1))^(1/2)/c^3/d/(c^2*x^2+1)^(1/2)*x^2-3
/8*b*(d*(c^2*x^2+1))^(1/2)/c^4/d/(c^2*x^2+1)*arcsinh(c*x)*x+3/16*b*(d*(c^2*
x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d*arcsinh(c*x)^2+15/128*b*(d*(c^2*x^2+1
))^(1/2)/c^5/d/(c^2*x^2+1)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \left(a + b \operatorname{asinh}{\left(cx \right)}\right)}{\sqrt{d \left(c^2x^2 + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**4*(a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

3.147 $\int \frac{x^3(a+b\sinh^{-1}(cx))}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=142

$$\frac{x^2\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{3c^2d}-\frac{2\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{3c^4d}-\frac{bx^3\sqrt{c^2x^2+1}}{9c\sqrt{c^2dx^2+d}}+\frac{2bx\sqrt{c^2x^2+1}}{3c^3\sqrt{c^2dx^2+d}}$$

```
[Out] 2/3*b*x*(c^2*x^2+1)^(1/2)/c^3/(c^2*d*x^2+d)^(1/2)-1/9*b*x^3*(c^2*x^2+1)^(1/
2)/c/(c^2*d*x^2+d)^(1/2)-2/3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^4/d+1
/3*x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2/d
```

Rubi [A] time = 0.16, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5758, 5717, 8, 30}

$$\frac{x^2\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{3c^2d}-\frac{2\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{3c^4d}-\frac{bx^3\sqrt{c^2x^2+1}}{9c\sqrt{c^2dx^2+d}}+\frac{2bx\sqrt{c^2x^2+1}}{3c^3\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (2*b*x*Sqrt[1 + c^2*x^2])/(3*c^3*Sqrt[d + c^2*d*x^2]) - (b*x^3*Sqrt[1 + c^2
*x^2])/(9*c*Sqrt[d + c^2*d*x^2]) - (2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x]))/(3*c^4*d) + (x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*c^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}dx = \frac{x^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3c^2d} - \frac{2\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}dx}{3c^2} - \frac{\left(b\sqrt{1+c^2x^2}\right)\int x^2 \ldots}{3c\sqrt{d+c^2dx^2}}$$

$$= -\frac{bx^3\sqrt{1+c^2x^2}}{9c\sqrt{d+c^2dx^2}} - \frac{2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3c^4d} + \frac{x^2\sqrt{d+c^2dx^2}\left(a+b\sinh \ldots}{3c^2d}$$

$$= \frac{2bx\sqrt{1+c^2x^2}}{3c^3\sqrt{d+c^2dx^2}} - \frac{bx^3\sqrt{1+c^2x^2}}{9c\sqrt{d+c^2dx^2}} - \frac{2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3c^4d} + \frac{x^2\sqrt{d+c \ldots}}{}$$

Mathematica [A] time = 0.13, size = 93, normalized size = 0.65

$$\frac{3a\left(c^4x^4-c^2x^2-2\right)+bcx\sqrt{c^2x^2+1}\left(6-c^2x^2\right)+3b\left(c^4x^4-c^2x^2-2\right)\sinh^{-1}(cx)}{9c^4\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (b*c*x*(6 - c^2*x^2)*Sqrt[1 + c^2*x^2] + 3*a*(-2 - c^2*x^2 + c^4*x^4) + 3*b
*(-2 - c^2*x^2 + c^4*x^4)*ArcSinh[c*x])/(9*c^4*Sqrt[d + c^2*d*x^2])
```

fricas [A] time = 0.55, size = 132, normalized size = 0.93

$$\frac{3\left(bc^4x^4-bc^2x^2-2\,b\right)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(3\,ac^4x^4-3\,ac^2x^2-\left(bc^3x^3-6\,bcx\right)\sqrt{c^2x^2+1}-6 \ldots}{9\left(c^6dx^2+c^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*(3*(b*c^4*x^4 - b*c^2*x^2 - 2*b)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2
*x^2 + 1)) + (3*a*c^4*x^4 - 3*a*c^2*x^2 - (b*c^3*x^3 - 6*b*c*x)*sqrt(c^2*x^
2 + 1) - 6*a)*sqrt(c^2*d*x^2 + d))/(c^6*d*x^2 + c^4*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.23, size = 358, normalized size = 2.52

$$a\left(\frac{x^2\sqrt{c^2d\,x^2+d}}{3c^2d}-\frac{2\sqrt{c^2d\,x^2+d}}{3d\,c^4}\right)+b\left(\frac{\sqrt{d\left(c^2x^2+1\right)}\left(4c^4x^4+4c^3x^3\sqrt{c^2x^2+1}+5c^2x^2+3cx\sqrt{c^2x^2+1}+1 \ldots}{72c^4d\left(c^2x^2+1\right)} \ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a*(1/3*x^2/c^2/d*(c^2*d*x^2+d)^(1/2)-2/3/d/c^4*(c^2*d*x^2+d)^(1/2))+b*(1/72
*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c
*x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arcsinh(c*x))/c^4/d/(c^2*x^2+1)-3/8*(d*(c^2*x
^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))/c^4/d/(c^2
*x^2+1)-3/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(1+arcs
inh(c*x))/c^4/d/(c^2*x^2+1)+1/72*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3
*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(1+3*arcsinh(c*x))/
c^4/d/(c^2*x^2+1))
```

maxima [A] time = 0.36, size = 117, normalized size = 0.82

$$\frac{1}{3}b\left(\frac{\sqrt{c^2dx^2+d}\,x^2}{c^2d}-\frac{2\sqrt{c^2dx^2+d}}{c^4d}\right)\operatorname{arsinh}(cx)+\frac{1}{3}a\left(\frac{\sqrt{c^2dx^2+d}\,x^2}{c^2d}-\frac{2\sqrt{c^2dx^2+d}}{c^4d}\right)-\frac{\left(c^2x^3-6x\right)b}{9c^3\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*b*(sqrt(c^2*d*x^2 + d)*x^2/(c^2*d) - 2*sqrt(c^2*d*x^2 + d)/(c^4*d))*arc
sinh(c*x) + 1/3*a*(sqrt(c^2*d*x^2 + d)*x^2/(c^2*d) - 2*sqrt(c^2*d*x^2 + d)/
(c^4*d)) - 1/9*(c^2*x^3 - 6*x)*b/(c^3*sqrt(d))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(a+b\operatorname{asinh}(cx)\right)}{\sqrt{d\left(c^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

3.148 $\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=119

$$\frac{x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{2c^2d}-\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc^3\sqrt{c^2dx^2+d}}-\frac{bx^2\sqrt{c^2x^2+1}}{4c\sqrt{c^2dx^2+d}}$$

```
[Out] -1/4*b*x^2*(c^2*x^2+1)^(1/2)/c/(c^2*d*x^2+d)^(1/2)-1/4*(a+b*arcsinh(c*x))^2
*(c^2*x^2+1)^(1/2)/b/c^3/(c^2*d*x^2+d)^(1/2)+1/2*x*(a+b*arcsinh(c*x))*(c^2*
d*x^2+d)^(1/2)/c^2/d
```

Rubi [A] time = 0.15, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5758, 5677, 5675, 30}

$$\frac{x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{2c^2d}-\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc^3\sqrt{c^2dx^2+d}}-\frac{bx^2\sqrt{c^2x^2+1}}{4c\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] -(b*x^2*Sqrt[1 + c^2*x^2])/(4*c*Sqrt[d + c^2*d*x^2]) + (x*Sqrt[d + c^2*d*x^
2]*(a + b*ArcSinh[c*x]))/(2*c^2*d) - (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]
)^2)/(4*b*c^3*Sqrt[d + c^2*d*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] &&  !GtQ[d, 0]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx = \frac{x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^2d} - \frac{\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+c^2dx^2}}\,dx}{2c^2} - \frac{\left(b\sqrt{1+c^2x^2}\right)\int x\,dx}{2c\sqrt{d+c^2dx^2}}$$
$$= -\frac{bx^2\sqrt{1+c^2x^2}}{4c\sqrt{d+c^2dx^2}} + \frac{x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^2d} - \frac{\sqrt{1+c^2x^2}\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{2c^2\sqrt{d+c^2dx^2}}$$
$$= -\frac{bx^2\sqrt{1+c^2x^2}}{4c\sqrt{d+c^2dx^2}} + \frac{x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^2d} - \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{4bc^3\sqrt{d+c^2dx^2}}$$

Mathematica [A] time = 0.73, size = 121, normalized size = 1.02

$$-\frac{-\frac{4acx\sqrt{c^2dx^2+d}}{d} + \frac{4a\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)}{\sqrt{d}} + \frac{b\sqrt{c^2x^2+1}\left(2\sinh^{-1}(cx)\left(\sinh^{-1}(cx)-\sinh\left(2\sinh^{-1}(cx)\right)\right)+\cosh\left(2\sinh^{-1}(cx)\right)\right)}{\sqrt{c^2dx^2+d}}}{8c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] -1/8*((-4*a*c*x*Sqrt[d + c^2*d*x^2])/d + (4*a*Log[c*d*x + Sqrt[d]*Sqrt[d +
c^2*d*x^2]])/Sqrt[d] + (b*Sqrt[1 + c^2*x^2]*(Cosh[2*ArcSinh[c*x]] + 2*ArcSi
nh[c*x]*(ArcSinh[c*x] - Sinh[2*ArcSinh[c*x]]))))/Sqrt[d + c^2*d*x^2])/c^3
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^2\,\text{arsinh}\,(cx) + ax^2}{\sqrt{c^2dx^2+d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arcsinh(c*x) + a*x^2)/sqrt(c^2*d*x^2 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arsinh}\,(cx) + a)x^2}{\sqrt{c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^2/sqrt(c^2*d*x^2 + d), x)
```

maple [B] time = 0.25, size = 247, normalized size = 2.08

$$\frac{ax\sqrt{c^2d\,x^2+d}}{2c^2d} - \frac{a\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}} + \sqrt{c^2d\,x^2+d}\right)}{2c^2\sqrt{c^2d}} - \frac{b\sqrt{d\left(c^2x^2+1\right)}\,\text{arcsinh}\,(cx)^2}{4\sqrt{c^2x^2+1}\,c^3d} + \frac{b\sqrt{d\left(c^2x^2+1\right)}\,\text{arcsinh}\,(cx)\,x^3}{2d\left(c^2x^2+1\right)} - \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] 1/2*a*x/c^2/d*(c^2*d*x^2+d)^(1/2)-1/2*a/c^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d
*x^2+d)^(1/2))/(c^2*d)^(1/2)-1/4*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/
c^3/d*arcsinh(c*x)^2+1/2*b*(d*(c^2*x^2+1))^(1/2)/d/(c^2*x^2+1)*arcsinh(c*x)
```

```
*x^3-1/4*b*(d*(c^2*x^2+1))^(1/2)/c/d/(c^2*x^2+1)^(1/2)*x^2+1/2*b*(d*(c^2*x^
2+1))^(1/2)/c^2/d/(c^2*x^2+1)*arcsinh(c*x)*x-1/8*b*(d*(c^2*x^2+1))^(1/2)/c^
3/d/(c^2*x^2+1)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(a + b \operatorname{asinh}{\left(c x \right)}\right)}{\sqrt{d \left(c^{2} x^{2} + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

3.149 $\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=64

$$\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{c^2d}-\frac{bx\sqrt{c^2x^2+1}}{c\sqrt{c^2dx^2+d}}$$

```
[Out] -b*x*(c^2*x^2+1)^(1/2)/c/(c^2*d*x^2+d)^(1/2)+(a+b*arcsinh(c*x))*(c^2*d*x^2+
d)^(1/2)/c^2/d
```

Rubi [A] time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5717, 8}

$$\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{c^2d}-\frac{bx\sqrt{c^2x^2+1}}{c\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] -((b*x*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + c^2*d*x^2])) + (Sqrt[d + c^2*d*x^2]*(
a + b*ArcSinh[c*x]))/(c^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx &= \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{c^2d}-\frac{\left(b\sqrt{1+c^2x^2}\right)\int 1\,dx}{c\sqrt{d+c^2dx^2}}\\ &= -\frac{bx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}}+\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{c^2d}\end{aligned}$$

Mathematica [A] time = 0.10, size = 74, normalized size = 1.16

$$\frac{\sqrt{c^2dx^2+d}\left(a\sqrt{c^2x^2+1}+b\sqrt{c^2x^2+1}\,\sinh^{-1}(cx)-bcx\right)}{c^2d\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

[Out] (Sqrt[d + c^2*d*x^2]*(-(b*c*x) + a*Sqrt[1 + c^2*x^2] + b*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]))/(c^2*d*Sqrt[1 + c^2*x^2])

fricas [A] time = 0.42, size = 96, normalized size = 1.50

$$\frac{\left(bc^2x^2+b\right)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(ac^2x^2-\sqrt{c^2x^2+1}\,bcx+a\right)\sqrt{c^2dx^2+d}}{c^4dx^2+c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")

[Out] ((b*c^2*x^2 + b)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + (a*c^2*x^2 - sqrt(c^2*x^2 + 1)*b*c*x + a)*sqrt(c^2*d*x^2 + d))/(c^4*d*x^2 + c^2*d)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\operatorname{arsinh}(cx)+a)x}{\sqrt{c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate((b*arcsinh(c*x) + a)*x/sqrt(c^2*d*x^2 + d), x)

maple [B] time = 0.12, size = 148, normalized size = 2.31

$$\frac{a\sqrt{c^2d\,x^2+d}}{c^2d}+b\left(\frac{\sqrt{d\left(c^2x^2+1\right)}\,\left(c^2x^2+cx\sqrt{c^2x^2+1}\,+1\right)(-1+\operatorname{arcsinh}(cx))}{2c^2d\left(c^2x^2+1\right)}+\frac{\sqrt{d\left(c^2x^2+1\right)}\,\left(c^2x^2-cx\sqrt{}\right.}{2c^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x)

[Out] a/c^2/d*(c^2*d*x^2+d)^(1/2)+b*(1/2*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))/c^2/d/(c^2*x^2+1)+1/2*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(1+arcsinh(c*x))/c^2/d/(c^2*x^2+1))

maxima [A] time = 0.41, size = 55, normalized size = 0.86

$$-\frac{bx}{c\sqrt{d}}+\frac{\sqrt{c^2dx^2+d}\,b\operatorname{arsinh}(cx)}{c^2d}+\frac{\sqrt{c^2dx^2+d}\,a}{c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")

[Out] -b*x/(c*sqrt(d)) + sqrt(c^2*d*x^2 + d)*b*arcsinh(c*x)/(c^2*d) + sqrt(c^2*d*x^2 + d)*a/(c^2*d)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2),x)

[Out] int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(a+b\operatorname{asinh}\left(cx\right)\right)}{\sqrt{d\left(c^{2}x^{2}+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x*(a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

3.150 $\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=47

$$\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{c^2dx^2+d}}$$

```
[Out] 1/2*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5677, 5675}

$$\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*Sqrt[d + c^2*d*x^2])
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] && !GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+c^2dx^2}}\,dx &= \frac{\sqrt{1+c^2x^2}\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}\\
&= \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{d+c^2dx^2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 48, normalized size = 1.02

$$\frac{\sqrt{c^2x^2+1}\sinh^{-1}(cx)\left(2a+b\sinh^{-1}(cx)\right)}{2c\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*(2*a + b*ArcSinh[c*x]))/(2*c*Sqrt[d + c^2*d
*x^2])
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b \operatorname{arsinh}(cx) + a}{\sqrt{c^2dx^2 + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral((b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\sqrt{c^2dx^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

maple [A] time = 0.05, size = 77, normalized size = 1.64

$$\frac{a \ln\left(\frac{x c^2 d}{\sqrt{c^2 d}} + \sqrt{c^2 d\, x^2 + d}\right)}{\sqrt{c^2 d}} + \frac{b \sqrt{d\left(c^2x^2 + 1\right)}\, \operatorname{arcsinh}(cx)^2}{2\sqrt{c^2x^2 + 1}\, cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x)
[Out] a*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/2*b*(d*(c^2
*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d*arcsinh(c*x)^2
```

maxima [A] time = 0.38, size = 28, normalized size = 0.60

$$\frac{b \operatorname{arsinh}(cx)^2}{2\, c\sqrt{d}} + \frac{a \operatorname{arsinh}(cx)}{c\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
[Out] 1/2*b*arcsinh(c*x)^2/(c*sqrt(d)) + a*arcsinh(c*x)/(c*sqrt(d))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a + b \operatorname{asinh}(c\, x)}{\sqrt{d\, c^2\, x^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(1/2),x)
[Out] int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asinh}(cx)}{\sqrt{d\left(c^2x^2 + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
[Out] Integral((a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

3.151 $\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=122

$$-\frac{2\sqrt{c^2x^2+1}\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\,\mathrm{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2dx^2+d}}+\frac{b\sqrt{c^2x^2+1}\,\mathrm{Li}_2\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2dx^2+d}}$$

```
[Out] -2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2
*d*x^2+d)^(1/2)-b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*
d*x^2+d)^(1/2)+b*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*
x^2+d)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5764, 5760, 4182, 2279, 2391}

$$-\frac{b\sqrt{c^2x^2+1}\,\mathrm{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2dx^2+d}}+\frac{b\sqrt{c^2x^2+1}\,\mathrm{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2dx^2+d}}-\frac{2\sqrt{c^2x^2+1}\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] (-2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[d
+ c^2*d*x^2] - (b*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^ArcSinh[c*x]])/Sqrt[d + c
^2*d*x^2] + (b*Sqrt[1 + c^2*x^2]*PolyLog[2, E^ArcSinh[c*x]])/Sqrt[d + c^2*d
*x^2]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 5764

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[
```

```
((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&  !GtQ[d, 0] && (Integer
Q[m] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{d+c^2dx^2}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}$$
$$= \frac{\sqrt{1+c^2x^2}\ \mathrm{Subst}\left(\int (a+bx)\mathrm{csch}(x)\,dx, x, \sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}$$
$$= -\frac{2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{d+c^2dx^2}} - \frac{\left(b\sqrt{1+c^2x^2}\right)\mathrm{Subst}\left(\int \log(1-e\right.}{\sqrt{d+c^2dx^2}}$$
$$= -\frac{2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{d+c^2dx^2}} - \frac{\left(b\sqrt{1+c^2x^2}\right)\mathrm{Subst}\left(\int \frac{\log(1-x)}{x}\,d\right.}{\sqrt{d+c^2dx^2}}$$
$$= -\frac{2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{d+c^2dx^2}} - \frac{b\sqrt{1+c^2x^2}\,\mathrm{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)}{\sqrt{d+c^2dx^2}} + \frac{b}{}$$

Mathematica [A] time = 0.33, size = 129, normalized size = 1.06

$$-\frac{a\log\left(\sqrt{d}\sqrt{d\left(c^2x^2+1\right)}+d\right)}{\sqrt{d}}+\frac{a\log(x)}{\sqrt{d}}+\frac{b\sqrt{c^2x^2+1}\left(\mathrm{Li}_2\left(-e^{-\sinh^{-1}(cx)}\right)-\mathrm{Li}_2\left(e^{-\sinh^{-1}(cx)}\right)+\sinh^{-1}(cx)\left(\log\right.\right.}{\sqrt{d\left(c^2x^2+1\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] (a*Log[x])/Sqrt[d] - (a*Log[d + Sqrt[d]*Sqrt[d*(1 + c^2*x^2)]])/Sqrt[d] + (
b*Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(Log[1 - E^(-ArcSinh[c*x])] - Log[1 + E^(
-ArcSinh[c*x])]) + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[
c*x])]))/Sqrt[d*(1 + c^2*x^2)]
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{c^2dx^2+d}\,(b\,\mathrm{arsinh}\,(cx)+a)}{c^2dx^3+dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^2*d*x^3 + d*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\mathrm{arsinh}\,(cx)+a}{\sqrt{c^2dx^2+d}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*x), x)
```

maple [A] time = 0.16, size = 234, normalized size = 1.92

$$-\frac{a \ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2 d\,x^2+d}}{x}\right)}{\sqrt{d}}+\frac{b\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsinh}(cx)\ln\left(1-cx-\sqrt{c^2x^2+1}\right)}{\sqrt{c^2x^2+1}\,d}+\frac{b\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{polylog}\left(2,\right.}{\sqrt{c^2x^2+1}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -a/d^(1/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)+b*(d*(c^2*x^2+1))^(1/2
)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+b*(d*(c^2*x^
2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,c*x+(c^2*x^2+1)^(1/2))-b*(d*(c^2*
x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-
b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,-c*x-(c^2*x^2+1)^(1/2
))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{\sqrt{c^2dx^2+d}\,x}\,dx-\frac{a\,\operatorname{arsinh}\left(\frac{1}{c|x|}\right)}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(sqrt(c^2*d*x^2 + d)*x), x) - a*ar
csinh(1/(c*abs(x)))/sqrt(d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{a+b\,\mathrm{asinh}\left(c\,x\right)}{x\,\sqrt{d\,c^2\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\,\operatorname{asinh}(cx)}{x\sqrt{d\left(c^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(x*sqrt(d*(c**2*x**2 + 1))), x)
```

3.152 $\int \frac{a+b\sinh^{-1}(cx)}{x^2\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=63

$$\frac{bc\sqrt{c^2x^2+1}\,\log(x)}{\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{dx}$$

```
[Out] b*c*ln(x)*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-(a+b*arcsinh(c*x))*(c^2*d*x
^2+d)^(1/2)/d/x
```

Rubi [A] time = 0.09, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5723, 29}

$$\frac{bc\sqrt{c^2x^2+1}\,\log(x)}{\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^2*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] -((Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(d*x)) + (b*c*Sqrt[1 + c^2*x^2
]*Log[x])/Sqrt[d + c^2*d*x^2]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sinh^{-1}(cx)}{x^2\sqrt{d+c^2dx^2}}\,dx &= -\frac{\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)}{dx}+\frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{x}\,dx}{\sqrt{d+c^2dx^2}}\\ &= -\frac{\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)}{dx}+\frac{bc\sqrt{1+c^2x^2}\,\log(x)}{\sqrt{d+c^2dx^2}}\end{aligned}$$

Mathematica [A] time = 0.16, size = 67, normalized size = 1.06

$$\frac{bc\log(x)\sqrt{d\left(c^2x^2+1\right)}}{d\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^2*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] -((Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(d*x)) + (b*c*Sqrt[d*(1 + c^2*
x^2)]*Log[x])/(d*Sqrt[1 + c^2*x^2])
```

fricas [B] time = 0.58, size = 132, normalized size = 2.10

$$\frac{bc\sqrt{d}\,x\log\left(\frac{c^2dx^6+c^2dx^2+dx^4+\sqrt{c^2dx^2+d}\,\sqrt{c^2x^2+1}\,(x^4-1)\sqrt{d}+d}{c^2x^4+x^2}\right)-2\sqrt{c^2dx^2+d}\,b\log\left(cx+\sqrt{c^2x^2+1}\right)-2\sqrt{c^2dx^2+d}}{2\,dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*c*sqrt(d)*x*log((c^2*d*x^6 + c^2*d*x^2 + d*x^4 + sqrt(c^2*d*x^2 + d)
*sqrt(c^2*x^2 + 1)*(x^4 - 1)*sqrt(d) + d)/(c^2*x^4 + x^2)) - 2*sqrt(c^2*d*x
^2 + d)*b*log(c*x + sqrt(c^2*x^2 + 1)) - 2*sqrt(c^2*d*x^2 + d)*a)/(d*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\operatorname{arsinh}(cx)+a}{\sqrt{c^2dx^2+d}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*x^2), x)
```

maple [B] time = 0.17, size = 183, normalized size = 2.90

$$-\frac{a\sqrt{c^2d\,x^2+d}}{dx}-\frac{b\sqrt{d\left(c^2x^2+1\right)}\ \operatorname{arcsinh}(cx)\,c}{\sqrt{c^2x^2+1}\ d}-\frac{b\sqrt{d\left(c^2x^2+1\right)}\ \operatorname{arcsinh}(cx)\,x\,c^2}{\left(c^2x^2+1\right)d}-\frac{b\sqrt{d\left(c^2x^2+1\right)}\ \operatorname{arcsinh}(}{\left(c^2x^2+1\right)dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -a/d/x*(c^2*d*x^2+d)^(1/2)-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcs
inh(c*x)*c-b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/(c^2*x^2+1)/d*x*c^2-b*(d*(c
^2*x^2+1))^(1/2)*arcsinh(c*x)/(c^2*x^2+1)/d/x+b*(d*(c^2*x^2+1))^(1/2)/(c^2*
x^2+1)^(1/2)/d*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c
```

maxima [A] time = 0.34, size = 101, normalized size = 1.60

$$-\frac{\left((-1)^{2\,c^2dx^2+2\,d}\sqrt{d}\,\log\left(2\,c^2d+\frac{2\,d}{x^2}\right)-\sqrt{d}\,\log\left(x^2+\frac{1}{c^2}\right)\right)bc}{2\,d}-\frac{\sqrt{c^2dx^2+d}\,b\operatorname{arsinh}(cx)}{dx}-\frac{\sqrt{c^2dx^2+d}\,a}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*((-1)^(2*c^2*d*x^2 + 2*d)*sqrt(d)*log(2*c^2*d + 2*d/x^2) - sqrt(d)*log
(x^2 + 1/c^2))*b*c/d - sqrt(c^2*d*x^2 + d)*b*arcsinh(c*x)/(d*x) - sqrt(c^2*
d*x^2 + d)*a/(d*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\operatorname{asinh}(c\,x)}{x^2\,\sqrt{d\,c^2\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)^(1/2)),x)
[Out] int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asinh}(cx)}{x^2 \sqrt{d(c^2x^2 + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**2/(c**2*d*x**2+d)**(1/2),x)
[Out] Integral((a + b*asinh(c*x))/(x**2*sqrt(d*(c**2*x**2 + 1))), x)
```

3.153 $\int \frac{a+b\sinh^{-1}(cx)}{x^3\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=203

$$-\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{2dx^2}+\frac{c^2\sqrt{c^2x^2+1}\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2dx^2+d}}+\frac{bc^2\sqrt{c^2x^2+1}\,\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2dx^2+d}}$$

```
[Out] -1/2*b*c*(c^2*x^2+1)^(1/2)/x/(c^2*d*x^2+d)^(1/2)+c^2*(a+b*arcsinh(c*x))*arc
tanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+1/2*b*c^2
*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-1/
2*b*c^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1
/2)-1/2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/d/x^2
```

Rubi [A] time = 0.29, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5747, 5764, 5760, 4182, 2279, 2391, 30}

$$\frac{bc^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2dx^2+d}}-\frac{bc^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{2dx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^3*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] -(b*c*Sqrt[1 + c^2*x^2])/(2*x*Sqrt[d + c^2*d*x^2]) - (Sqrt[d + c^2*d*x^2]*(
a + b*ArcSinh[c*x]))/(2*d*x^2) + (c^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]
)*ArcTanh[E^ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2] + (b*c^2*Sqrt[1 + c^2*x^2]*P
olyLog[2, -E^ArcSinh[c*x]])/(2*Sqrt[d + c^2*d*x^2]) - (b*c^2*Sqrt[1 + c^2*x
^2]*PolyLog[2, E^ArcSinh[c*x]])/(2*Sqrt[d + c^2*d*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
```

```
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 5764

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[
((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&  !GtQ[d, 0] && (Integer
Q[m] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^3\sqrt{d+c^2dx^2}}\,dx = -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2dx^2} - \frac{1}{2}c^2\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{d+c^2dx^2}}\,dx + \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{x^2}\,dx}{2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2x\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2dx^2} - \frac{\left(c^2\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{1+c^2x^2}}\,dx}{2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2x\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2dx^2} - \frac{\left(c^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int (a+bx)\operatorname{cs}\right.}{2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2x\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2dx^2} + \frac{c^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tan}{\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2x\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2dx^2} + \frac{c^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tan}{\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2x\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2dx^2} + \frac{c^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tan}{\sqrt{d+c^2dx^2}}$$

Mathematica [A] time = 3.02, size = 229, normalized size = 1.13

$$-\frac{4a\sqrt{c^2dx^2+d}}{x^2} + 4ac^2\sqrt{d}\,\log\left(\sqrt{d}\,\sqrt{c^2dx^2+d}\,+d\right) - 4ac^2\sqrt{d}\,\log(x) + \frac{bc^2d^2\left(c^2x^2+1\right)^{3/2}\left(-4\text{Li}_2\left(-e^{-\sinh^{-1}(cx)}\right)+4\text{Li}_2\left(e^{-\sinh^{-1}(cx)}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^3*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] ((-4*a*Sqrt[d + c^2*d*x^2])/x^2 - 4*a*c^2*Sqrt[d]*Log[x] + 4*a*c^2*Sqrt[d]*
Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (b*c^2*d^2*(1 + c^2*x^2)^(3/2)*(-2*C
```

```
oth[ArcSinh[c*x]/2] - ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - 4*ArcSinh[c*x]*
Log[1 - E^(-ArcSinh[c*x])] + 4*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] - 4*
PolyLog[2, -E^(-ArcSinh[c*x])] + 4*PolyLog[2, E^(-ArcSinh[c*x])] - ArcSinh[
c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSinh[c*x]/2]))/(d + c^2*d*x^2)^(3/2
))/(8*d)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{c^2dx^2+d}\,(b\operatorname{arsinh}(cx)+a)}{c^2dx^5+dx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^2*d*x^5 + d*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\operatorname{arsinh}(cx)+a}{\sqrt{c^2dx^2+d}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*x^3), x)
```

maple [A] time = 0.33, size = 380, normalized size = 1.87

$$-\frac{a\sqrt{c^2dx^2+d}}{2dx^2}+\frac{ac^2\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2dx^2+d}}{x}\right)}{2\sqrt{d}}-\frac{b\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsinh}(cx)\,c^2}{2d\left(c^2x^2+1\right)}-\frac{b\sqrt{d\left(c^2x^2+1\right)}\,c}{2xd\sqrt{c^2x^2+1}}-\frac{b\sqrt{d\left(c^2x^2+1\right.}}{2x^2d\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -1/2*a/d/x^2*(c^2*d*x^2+d)^(1/2)+1/2*a*c^2/d^(1/2)*ln((2*d+2*d^(1/2)*(c^2*d
*x^2+d)^(1/2))/x)-1/2*b*(d*(c^2*x^2+1))^(1/2)/d/(c^2*x^2+1)*arcsinh(c*x)*c^
2-1/2*b*(d*(c^2*x^2+1))^(1/2)/x/d/(c^2*x^2+1)^(1/2)*c-1/2*b*(d*(c^2*x^2+1))
^(1/2)/x^2/d/(c^2*x^2+1)*arcsinh(c*x)-1/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+
1)^(1/2)/d*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^2-1/2*b*(d*(c^2*x^2+1
))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c^2+1/2*b*(d*
(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1
/2))*c^2+1/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,-c*x-(c^
2*x^2+1)^(1/2))*c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(\frac{c^2\operatorname{arsinh}\left(\frac{1}{c|x|}\right)}{\sqrt{d}}-\frac{\sqrt{c^2dx^2+d}}{dx^2}\right)a+b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{\sqrt{c^2dx^2+d}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(c^2*arcsinh(1/(c*abs(x)))/sqrt(d) - sqrt(c^2*d*x^2 + d)/(d*x^2))*a + b
*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(sqrt(c^2*d*x^2 + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}\left(c\,x\right)}{x^3\,\sqrt{d\,c^2\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)^(1/2)),x)
[Out] int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{asinh}\,(cx)}{x^3\sqrt{d\left(c^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**3/(c**2*d*x**2+d)**(1/2),x)
[Out] Integral((a + b*asinh(c*x))/(x**3*sqrt(d*(c**2*x**2 + 1))), x)
```

3.154 $\int \frac{a+b\sinh^{-1}(cx)}{x^4\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=141

$$\frac{2c^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{3dx}-\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{3dx^3}-\frac{bc\sqrt{c^2x^2+1}}{6x^2\sqrt{c^2dx^2+d}}-\frac{2bc^3\sqrt{c^2x^2+1}\log(x)}{3\sqrt{c^2dx^2+d}}$$

```
[Out] -1/6*b*c*(c^2*x^2+1)^(1/2)/x^2/(c^2*d*x^2+d)^(1/2)-2/3*b*c^3*ln(x)*(c^2*x^2
+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-1/3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/d/
x^3+2/3*c^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/d/x
```

Rubi [A] time = 0.18, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5747, 5723, 29, 30}

$$\frac{2c^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{3dx}-\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{3dx^3}-\frac{bc\sqrt{c^2x^2+1}}{6x^2\sqrt{c^2dx^2+d}}-\frac{2bc^3\sqrt{c^2x^2+1}\log(x)}{3\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^4*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] -(b*c*Sqrt[1 + c^2*x^2])/(6*x^2*Sqrt[d + c^2*d*x^2]) - (Sqrt[d + c^2*d*x^2]
*(a + b*ArcSinh[c*x]))/(3*d*x^3) + (2*c^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSin
h[c*x]))/(3*d*x) - (2*b*c^3*Sqrt[1 + c^2*x^2]*Log[x])/(3*Sqrt[d + c^2*d*x^2
])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^4\sqrt{d+c^2dx^2}}\,dx = -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^3} - \frac{1}{3}\left(2c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{x^2\sqrt{d+c^2dx^2}}\,dx + \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{x}}{3\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6x^2\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^3} + \frac{2c^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6x^2\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^3} + \frac{2c^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx}$$

Mathematica [A] time = 0.20, size = 135, normalized size = 0.96

$$\frac{2a\left(2c^4x^4+c^2x^2-1\right)+bcx\sqrt{c^2x^2+1}\left(6c^2x^2-1\right)+2b\left(2c^4x^4+c^2x^2-1\right)\sinh^{-1}(cx)}{6x^3\sqrt{c^2dx^2+d}} - \frac{2bc^3\log(x)\sqrt{d\left(c^2x^2+1\right)}}{3d\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^4*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] (b*c*x*Sqrt[1 + c^2*x^2]*(-1 + 6*c^2*x^2) + 2*a*(-1 + c^2*x^2 + 2*c^4*x^4)
+ 2*b*(-1 + c^2*x^2 + 2*c^4*x^4)*ArcSinh[c*x])/(6*x^3*Sqrt[d + c^2*d*x^2])
- (2*b*c^3*Sqrt[d*(1 + c^2*x^2)]*Log[x])/(3*d*Sqrt[1 + c^2*x^2])
```

fricas [A] time = 0.71, size = 222, normalized size = 1.57

$$\frac{2\left(2\,bc^4x^4+bc^2x^2-b\right)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)+2\left(bc^5x^5+bc^3x^3\right)\sqrt{d}\,\log\left(\frac{c^2dx^6+c^2dx^2+dx^4-\sqrt{c^2dx^2+d}\,\sqrt{c^2x}}{c^2x^4+x^2}\right.}{6\left(c^2dx^5+dx^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*(2*b*c^4*x^4 + b*c^2*x^2 - b)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2
*x^2 + 1)) + 2*(b*c^5*x^5 + b*c^3*x^3)*sqrt(d)*log((c^2*d*x^6 + c^2*d*x^2 +
 d*x^4 - sqrt(c^2*d*x^2 + d)*sqrt(c^2*x^2 + 1)*(x^4 - 1)*sqrt(d) + d)/(c^2*
x^4 + x^2)) + (4*a*c^4*x^4 + 2*a*c^2*x^2 + (b*c*x^3 - b*c*x)*sqrt(c^2*x^2 +
 1) - 2*a)*sqrt(c^2*d*x^2 + d))/(c^2*d*x^5 + d*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\mathrm{arsinh}\,(cx)+a}{\sqrt{c^2dx^2+d}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*x^4), x)
```

maple [B] time = 0.28, size = 791, normalized size = 5.61

$$-\frac{a\sqrt{c^2d\,x^2+d}}{3d\,x^3}+\frac{2a\,c^2\sqrt{c^2d\,x^2+d}}{3dx}+\frac{4b\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsinh}(cx)\,c^3}{3\sqrt{c^2x^2+1}\,d}+\frac{2b\sqrt{d\left(c^2x^2+1\right)}\,x^5c^8}{3\left(3c^4x^4+2c^2x^2-1\right)d}-\frac{2b\sqrt{d\left(c^2x^2+1\right)}}{3\left(3c^4x^4+2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -1/3*a/d/x^3*(c^2*d*x^2+d)^(1/2)+2/3*a*c^2/d/x*(c^2*d*x^2+d)^(1/2)+4/3*b*(d
*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*c^3+2/3*b*(d*(c^2*x^2+
1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x^5*c^8-2/3*b*(d*(c^2*x^2+1))^(1/2)/(3*
c^4*x^4+2*c^2*x^2-1)/d*x^3*(c^2*x^2+1)*c^6+2*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4
*x^4+2*c^2*x^2-1)/d*x^3*arcsinh(c*x)*c^6-2*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x
^4+2*c^2*x^2-1)/d*x^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^5+1/3*b*(d*(c^2*x^2+
1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x^3*c^6+1/3*b*(d*(c^2*x^2+1))^(1/2)/(3*
c^4*x^4+2*c^2*x^2-1)/d*x*(c^2*x^2+1)*c^4+1/3*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4
*x^4+2*c^2*x^2-1)/d*x*arcsinh(c*x)*c^4+2/3*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x
^4+2*c^2*x^2-1)/d*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^3-1/3*b*(d*(c^2*x^2+1))^
(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x*c^4-1/2*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^
4+2*c^2*x^2-1)/d*c^3*(c^2*x^2+1)^(1/2)-4/3*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x
^4+2*c^2*x^2-1)/d/x*arcsinh(c*x)*c^2+1/6*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4
+2*c^2*x^2-1)/d/x^2*c*(c^2*x^2+1)^(1/2)+1/3*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*
x^4+2*c^2*x^2-1)/d/x^3*arcsinh(c*x)-2/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)
^(1/2)/d*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c^3
```

maxima [A] time = 0.35, size = 121, normalized size = 0.86

$$-\frac{1}{6}\left(\frac{4\,c^2\log(x)}{\sqrt{d}}+\frac{1}{\sqrt{d}\,x^2}\right)bc+\frac{1}{3}\,b\left(\frac{2\,\sqrt{c^2dx^2+d}\,c^2}{dx}-\frac{\sqrt{c^2dx^2+d}}{dx^3}\right)\operatorname{arsinh}(cx)+\frac{1}{3}\,a\left(\frac{2\,\sqrt{c^2dx^2+d}\,c^2}{dx}-\frac{\sqrt{c^2dx^2+d}}{dx^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*(4*c^2*log(x)/sqrt(d) + 1/(sqrt(d)*x^2))*b*c + 1/3*b*(2*sqrt(c^2*d*x^2
 + d)*c^2/(d*x) - sqrt(c^2*d*x^2 + d)/(d*x^3))*arcsinh(c*x) + 1/3*a*(2*sqrt
(c^2*d*x^2 + d)*c^2/(d*x) - sqrt(c^2*d*x^2 + d)/(d*x^3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{asinh}\left(c\,x\right)}{x^4\,\sqrt{d\,c^2\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\operatorname{asinh}(cx)}{x^4\sqrt{d\left(c^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**4/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(x**4*sqrt(d*(c**2*x**2 + 1))), x)
```

3.155 $\int \frac{x^5(a+b\sinh^{-1}(cx))}{(d+c^2dx^2)^{3/2}}\,dx$

Optimal. Leaf size=212

$$\frac{(c^2dx^2+d)^{3/2}(a+b\sinh^{-1}(cx))}{3c^6d^3}-\frac{2\sqrt{c^2dx^2+d}\,(a+b\sinh^{-1}(cx))}{c^6d^2}-\frac{a+b\sinh^{-1}(cx)}{c^6d\sqrt{c^2dx^2+d}}+\frac{b\sqrt{c^2dx^2+d}\tan^{-1}(cx)}{c^6d^2\sqrt{c^2x^2+1}}+\frac{5}{3}$$

```
[Out] 1/3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/c^6/d^3+(-a-b*arcsinh(c*x))/c^6/
d/(c^2*d*x^2+d)^(1/2)-2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^6/d^2+5/3*
b*x*(c^2*d*x^2+d)^(1/2)/c^5/d^2/(c^2*x^2+1)^(1/2)-1/9*b*x^3*(c^2*d*x^2+d)^(
1/2)/c^3/d^2/(c^2*x^2+1)^(1/2)+b*arctan(c*x)*(c^2*d*x^2+d)^(1/2)/c^6/d^2/(c
^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 220, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5751, 5758, 5717, 8, 30, 302, 203}

$$\frac{4x^2\sqrt{c^2dx^2+d}\,(a+b\sinh^{-1}(cx))}{3c^4d^2}-\frac{8\sqrt{c^2dx^2+d}\,(a+b\sinh^{-1}(cx))}{3c^6d^2}-\frac{x^4(a+b\sinh^{-1}(cx))}{c^2d\sqrt{c^2dx^2+d}}-\frac{bx^3\sqrt{c^2x^2+1}}{9c^3d\sqrt{c^2dx^2+d}}+\frac{}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (5*b*x*Sqrt[1 + c^2*x^2])/(3*c^5*d*Sqrt[d + c^2*d*x^2]) - (b*x^3*Sqrt[1 + c
^2*x^2])/(9*c^3*d*Sqrt[d + c^2*d*x^2]) - (x^4*(a + b*ArcSinh[c*x]))/(c^2*d*
Sqrt[d + c^2*d*x^2]) - (8*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*c^6*
d^2) + (4*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*c^4*d^2) + (b*Sq
rt[1 + c^2*x^2]*ArcTan[c*x])/(c^6*d*Sqrt[d + c^2*d*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
```

```
0] && NeQ[p, -1]
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{3/2}}\,dx = -\frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}} + \frac{4\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx}{c^2d} + \frac{\left(b\sqrt{1+c^2x^2}\right)\int \frac{x^4}{1+c^2x^2}\,dx}{cd\sqrt{d+c^2dx^2}}$$

$$= -\frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}} + \frac{4x^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3c^4d^2} - \frac{8\int \frac{x\left(a+b\sinh^{-1}(cx\right.}{\sqrt{d+c^2dx^2}}}{3c^4d}$$

$$= -\frac{bx\sqrt{1+c^2x^2}}{c^5d\sqrt{d+c^2dx^2}} - \frac{bx^3\sqrt{1+c^2x^2}}{9c^3d\sqrt{d+c^2dx^2}} - \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}} - \frac{8\sqrt{d+c^2dx^2}\left(a\right.}{3c^6}$$

$$= \frac{5bx\sqrt{1+c^2x^2}}{3c^5d\sqrt{d+c^2dx^2}} - \frac{bx^3\sqrt{1+c^2x^2}}{9c^3d\sqrt{d+c^2dx^2}} - \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}} - \frac{8\sqrt{d+c^2dx^2}\left(a\right.}{3c^6}$$

Mathematica [A] time = 0.27, size = 148, normalized size = 0.70

$$\frac{\sqrt{c^2dx^2+d}\left(3a\left(c^4x^4-4c^2x^2-8\right)+bcx\sqrt{c^2x^2+1}\left(15-c^2x^2\right)+3b\left(c^4x^4-4c^2x^2-8\right)\sinh^{-1}(cx)\right)}{9c^6d^2\left(c^2x^2+1\right)}+\frac{b\sqrt{d\left(c^2\right.}}{c^6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(b*c*x*(15 - c^2*x^2)*Sqrt[1 + c^2*x^2] + 3*a*(-8 - 4*
c^2*x^2 + c^4*x^4) + 3*b*(-8 - 4*c^2*x^2 + c^4*x^4)*ArcSinh[c*x]))/(9*c^6*d
^2*(1 + c^2*x^2)) + (b*Sqrt[d*(1 + c^2*x^2)]*ArcTan[c*x])/(c^6*d^2*Sqrt[1 +
 c^2*x^2])
```

fricas [A] time = 0.61, size = 197, normalized size = 0.93

$$-\frac{9\left(bc^2x^2+b\right)\sqrt{d}\,\arctan\left(\frac{2\sqrt{c^2dx^2+d}\sqrt{c^2x^2+1}\,c\sqrt{d}\,x}{c^4dx^4-d}\right)-6\left(bc^4x^4-4\,bc^2x^2-8\,b\right)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)}{18\left(c^8d^2x^2+c^6d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/18*(9*(b*c^2*x^2 + b)*sqrt(d)*arctan(2*sqrt(c^2*d*x^2 + d)*sqrt(c^2*x^2
+ 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)) - 6*(b*c^4*x^4 - 4*b*c^2*x^2 - 8*b)*sqrt(
c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) - 2*(3*a*c^4*x^4 - 12*a*c^2*x^2
 - (b*c^3*x^3 - 15*b*c*x)*sqrt(c^2*x^2 + 1) - 24*a)*sqrt(c^2*d*x^2 + d))/(c
^8*d^2*x^2 + c^6*d^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [C] time = 0.29, size = 362, normalized size = 1.71

$$\frac{a\,x^4}{3c^2d\sqrt{c^2d\,x^2+d}}-\frac{4a\,x^2}{3c^4d\sqrt{c^2d\,x^2+d}}-\frac{8a}{3c^6d\sqrt{c^2d\,x^2+d}}-\frac{8b\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsinh}(cx)}{3c^6d^2\left(c^2x^2+1\right)}+\frac{ib\sqrt{d\left(c^2x^2+1\right)}\,\ln\left(cx\right.}{\sqrt{c^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] 1/3*a*x^4/c^2/d/(c^2*d*x^2+d)^(1/2)-4/3*a/c^4*x^2/d/(c^2*d*x^2+d)^(1/2)-8/3
*a/c^6/d/(c^2*d*x^2+d)^(1/2)-8/3*b*(d*(c^2*x^2+1))^(1/2)/c^6/d^2/(c^2*x^2+1
)*arcsinh(c*x)+I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^6/d^2*ln(c*x+(
c^2*x^2+1)^(1/2)+I)-I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^6/d^2*ln(
c*x+(c^2*x^2+1)^(1/2)-I)+1/3*b*(d*(c^2*x^2+1))^(1/2)/c^2/d^2/(c^2*x^2+1)*ar
csinh(c*x)*x^4-1/9*b*(d*(c^2*x^2+1))^(1/2)/c^3/d^2/(c^2*x^2+1)^(1/2)*x^3-4/
3*b*(d*(c^2*x^2+1))^(1/2)/c^4/d^2/(c^2*x^2+1)*arcsinh(c*x)*x^2+5/3*b*(d*(c^
2*x^2+1))^(1/2)/c^5/d^2/(c^2*x^2+1)^(1/2)*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}a\left(\frac{x^4}{\sqrt{c^2dx^2+d}\,c^2d}-\frac{4x^2}{\sqrt{c^2dx^2+d}\,c^4d}-\frac{8}{\sqrt{c^2dx^2+d}\,c^6d}\right)+\frac{1}{3}b\left(\frac{\left(c^4\sqrt{d}\,x^4-4c^2\sqrt{d}\,x^2-8\sqrt{d}\right)\log\left(cx+\sqrt{c^2x^2+\right.}{\sqrt{c^2x^2+1}\,c^6d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*(x^4/(sqrt(c^2*d*x^2 + d)*c^2*d) - 4*x^2/(sqrt(c^2*d*x^2 + d)*c^4*d)
- 8/(sqrt(c^2*d*x^2 + d)*c^6*d)) + 1/3*b*((c^4*sqrt(d)*x^4 - 4*c^2*sqrt(d)*
x^2 - 8*sqrt(d))*log(c*x + sqrt(c^2*x^2 + 1))/(sqrt(c^2*x^2 + 1)*c^6*d^2) -
 integrate((c^4*sqrt(d)*x^4 - 4*c^2*sqrt(d)*x^2 - 8*sqrt(d))/(sqrt(c^2*x^2
+ 1)*x), x)/(c^6*d^2) + 3*integrate(1/3*(c^4*sqrt(d)*x^4 - 4*c^2*sqrt(d)*x^
2 - 8*sqrt(d))/(c^9*d^2*x^4 + c^7*d^2*x^2 + (c^8*d^2*x^3 + c^6*d^2*x)*sqrt(
c^2*x^2 + 1)), x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{{\left(d\,c^2\,x^2+d\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int((x^5*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 (a + b \operatorname{asinh}(cx))}{\left(d \left(c^2 x^2 + 1\right)\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**5*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(3/2), x)
```

3.156 $\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=206

$$-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{c^2dx^2+d}}-\frac{3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc^5d\sqrt{c^2dx^2+d}}+\frac{3x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{2c^4d^2}-\frac{b\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)}{2c^5d\sqrt{c^2dx^2+d}}$$

```
[Out] -x^3*(a+b*arcsinh(c*x))/c^2/d/(c^2*d*x^2+d)^(1/2)-1/4*b*x^2*(c^2*x^2+1)^(1/
2)/c^3/d/(c^2*d*x^2+d)^(1/2)-3/4*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c
^5/d/(c^2*d*x^2+d)^(1/2)-1/2*b*ln(c^2*x^2+1)*(c^2*x^2+1)^(1/2)/c^5/d/(c^2*d
*x^2+d)^(1/2)+3/2*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^4/d^2
```

Rubi [A] time = 0.28, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {5751, 5758, 5677, 5675, 30, 266, 43}

$$\frac{3x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{2c^4d^2}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{c^2dx^2+d}}-\frac{3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc^5d\sqrt{c^2dx^2+d}}-\frac{bx^2\sqrt{c^2x^2+1}}{4c^3d\sqrt{c^2dx^2+d}}-\frac{b\sqrt{\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] -(b*x^2*Sqrt[1 + c^2*x^2])/(4*c^3*d*Sqrt[d + c^2*d*x^2]) - (x^3*(a + b*ArcS
inh[c*x]))/(c^2*d*Sqrt[d + c^2*d*x^2]) + (3*x*Sqrt[d + c^2*d*x^2]*(a + b*Ar
cSinh[c*x]))/(2*c^4*d^2) - (3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(4*
b*c^5*d*Sqrt[d + c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(2*c^
5*d*Sqrt[d + c^2*d*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
```

```
d] && !GtQ[d, 0]
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{3/2}}\,dx = -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}}+\frac{3\int\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx}{c^2d}+\frac{\left(b\sqrt{1+c^2x^2}\right)\int\frac{x^3}{1+c^2x^2}\,dx}{cd\sqrt{d+c^2dx^2}}$$

$$= -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}}+\frac{3x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^4d^2}-\frac{3\int\frac{a+b\sinh^{-1}(cx)}{\sqrt{d+c^2dx^2}}\,d}{2c^4d}$$

$$= -\frac{3bx^2\sqrt{1+c^2x^2}}{4c^3d\sqrt{d+c^2dx^2}}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}}+\frac{3x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^4d^2}$$

$$= -\frac{bx^2\sqrt{1+c^2x^2}}{4c^3d\sqrt{d+c^2dx^2}}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}}+\frac{3x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^4d^2}$$

Mathematica [A] time = 0.47, size = 161, normalized size = 0.78

$$\frac{4ac\sqrt{d}\,x\left(c^2x^2+3\right)-12a\sqrt{c^2dx^2+d}\,\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)+b\sqrt{d}\left(8cx\sinh^{-1}(cx)-\sqrt{c^2x^2+1}\left(4\log(\right.\right.}{8c^5d^{3/2}\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (4*a*c*Sqrt[d]*x*(3 + c^2*x^2) - 12*a*Sqrt[d + c^2*d*x^2]*Log[c*d*x + Sqrt[
d]*Sqrt[d + c^2*d*x^2]] + b*Sqrt[d]*(8*c*x*ArcSinh[c*x] - Sqrt[1 + c^2*x^2]
*(6*ArcSinh[c*x]^2 + Cosh[2*ArcSinh[c*x]] + 4*Log[1 + c^2*x^2] - 2*ArcSinh[
c*x]*Sinh[2*ArcSinh[c*x]])))/(8*c^5*d^(3/2)*Sqrt[d + c^2*d*x^2])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4\operatorname{arsinh}(cx)+ax^4\right)\sqrt{c^2dx^2+d}}{c^4d^2x^4+2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arcsinh(c*x) + a*x^4)*sqrt(c^2*d*x^2 + d)/(c^4*d^2*x^4 + 2*
c^2*d^2*x^2 + d^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.33, size = 366, normalized size = 1.78

$$\frac{a\,x^{3}}{2c^{2}d\sqrt{c^{2}d\,x^{2}+d}}+\frac{3ax}{2c^{4}d\sqrt{c^{2}d\,x^{2}+d}}-\frac{3a\ln\left(\frac{x\,c^{2}d}{\sqrt{c^{2}d}}+\sqrt{c^{2}d\,x^{2}+d}\right)}{2c^{4}d\sqrt{c^{2}d}}-\frac{3b\sqrt{d\left(c^{2}x^{2}+1\right)}\,\operatorname{arcsinh}(cx)^{2}}{4\sqrt{c^{2}x^{2}+1}\,c^{5}d^{2}}+\frac{b\sqrt{d\left(c^{2}x^{2}+\right.}}{2c^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] 1/2*a*x^3/c^2/d/(c^2*d*x^2+d)^(1/2)+3/2*a/c^4*x/d/(c^2*d*x^2+d)^(1/2)-3/2*a
/c^4/d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)-3/4*b*(d
*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d^2*arcsinh(c*x)^2+1/2*b*(d*(c^2*
x^2+1))^(1/2)/c^2/d^2/(c^2*x^2+1)*arcsinh(c*x)*x^3-1/4*b*(d*(c^2*x^2+1))^(1
/2)/c^3/d^2/(c^2*x^2+1)^(1/2)*x^2+3/2*b*(d*(c^2*x^2+1))^(1/2)/c^4/d^2/(c^2*
x^2+1)*arcsinh(c*x)*x+b*(d*(c^2*x^2+1))^(1/2)/c^5/d^2/(c^2*x^2+1)^(1/2)*arc
sinh(c*x)-1/8*b*(d*(c^2*x^2+1))^(1/2)/c^5/d^2/(c^2*x^2+1)^(1/2)-b*(d*(c^2*x
^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}a\left(\frac{x^{3}}{\sqrt{c^{2}dx^{2}+d}\,c^{2}d}+\frac{3\,x}{\sqrt{c^{2}dx^{2}+d}\,c^{4}d}-\frac{3\,\operatorname{arsinh}(cx)}{c^{5}d^{\frac{3}{2}}}\right)+b\int\frac{x^{4}\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)}{\left(c^{2}dx^{2}+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*a*(x^3/(sqrt(c^2*d*x^2 + d)*c^2*d) + 3*x/(sqrt(c^2*d*x^2 + d)*c^4*d) -
3*arcsinh(c*x)/(c^5*d^(3/2))) + b*integrate(x^4*log(c*x + sqrt(c^2*x^2 + 1)
)/(c^2*d*x^2 + d)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x^{4}\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{{\left(d\,c^{2}\,x^{2}+d\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 (a + b \operatorname{asinh}(cx))}{\left(d\left(c^2x^2 + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**4*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(3/2), x)
```

3.157 $\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=136

$$\frac{\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{c^4d^2}+\frac{a+b\sinh^{-1}(cx)}{c^4d\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2dx^2+d}\,\tan^{-1}(cx)}{c^4d^2\sqrt{c^2x^2+1}}-\frac{bx\sqrt{c^2dx^2+d}}{c^3d^2\sqrt{c^2x^2+1}}$$

```
[Out] (a+b*arcsinh(c*x))/c^4/d/(c^2*d*x^2+d)^(1/2)+(a+b*arcsinh(c*x))*(c^2*d*x^2+
d)^(1/2)/c^4/d^2-b*x*(c^2*d*x^2+d)^(1/2)/c^3/d^2/(c^2*x^2+1)^(1/2)-b*arctan
(c*x)*(c^2*d*x^2+d)^(1/2)/c^4/d^2/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 141, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5751, 5717, 8, 321, 203}

$$\frac{2\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{c^4d^2}-\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{c^2dx^2+d}}-\frac{bx\sqrt{c^2x^2+1}}{c^3d\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\,\tan^{-1}(cx)}{c^4d\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] -((b*x*Sqrt[1 + c^2*x^2])/(c^3*d*Sqrt[d + c^2*d*x^2])) - (x^2*(a + b*ArcSin
h[c*x]))/(c^2*d*Sqrt[d + c^2*d*x^2]) + (2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSin
h[c*x]))/(c^4*d^2) - (b*Sqrt[1 + c^2*x^2]*ArcTan[c*x])/(c^4*d*Sqrt[d + c^2*
d*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
```

```
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rubi steps

$$\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{3/2}}\,dx=-\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}}+\frac{2\int\frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx}{c^2d}+\frac{\left(b\sqrt{1+c^2x^2}\right)\int\frac{x^2}{1+c^2x^2}\,dx}{cd\sqrt{d+c^2dx^2}}$$

$$=\frac{bx\sqrt{1+c^2x^2}}{c^3d\sqrt{d+c^2dx^2}}-\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}}+\frac{2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{c^4d^2}-\ldots$$

$$=-\frac{bx\sqrt{1+c^2x^2}}{c^3d\sqrt{d+c^2dx^2}}-\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}}+\frac{2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{c^4d^2}-\ldots$$

Mathematica [A] time = 0.24, size = 143, normalized size = 1.05

$$\frac{\sqrt{c^2dx^2+d}\left(a\sqrt{c^2x^2+1}\left(c^2x^2+2\right)-b\left(c^3x^3+cx\right)+b\sqrt{c^2x^2+1}\left(c^2x^2+2\right)\sinh^{-1}(cx)\right)}{c^4d^2\left(c^2x^2+1\right)^{3/2}}-\frac{b\sqrt{d\left(c^2x^2+1\right)}\tan\ldots}{c^4d^2\sqrt{c^2x^2+\ldots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(a*Sqrt[1 + c^2*x^2]*(2 + c^2*x^2) - b*(c*x + c^3*x^3)
 + b*Sqrt[1 + c^2*x^2]*(2 + c^2*x^2)*ArcSinh[c*x]))/(c^4*d^2*(1 + c^2*x^2)^
(3/2)) - (b*Sqrt[d*(1 + c^2*x^2)]*ArcTan[c*x])/(c^4*d^2*Sqrt[1 + c^2*x^2])
```

fricas [A] time = 0.76, size = 166, normalized size = 1.22

$$\frac{\left(bc^2x^2+b\right)\sqrt{d}\arctan\left(\frac{2\sqrt{c^2dx^2+d}\sqrt{c^2x^2+1}c\sqrt{d}x}{c^4dx^4-d}\right)+2\left(bc^2x^2+2b\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)+2\left(ac^2x^2-\ldots\right.}{2\left(c^6d^2x^2+c^4d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((b*c^2*x^2 + b)*sqrt(d)*arctan(2*sqrt(c^2*d*x^2 + d)*sqrt(c^2*x^2 + 1)
*c*sqrt(d)*x/(c^4*d*x^4 - d)) + 2*(b*c^2*x^2 + 2*b)*sqrt(c^2*d*x^2 + d)*log
(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*c^2*x^2 - sqrt(c^2*x^2 + 1)*b*c*x + 2*a)*s
qrt(c^2*d*x^2 + d))/(c^6*d^2*x^2 + c^4*d^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [C] time = 0.22, size = 260, normalized size = 1.91

$$\frac{a\,x^{2}}{c^{2}d\sqrt{c^{2}d\,x^{2}+d}}+\frac{2a}{d\,c^{4}\sqrt{c^{2}d\,x^{2}+d}}+\frac{b\sqrt{d\left(c^{2}x^{2}+1\right)}\,\operatorname{arcsinh}\left(cx\right)x^{2}}{c^{2}d^{2}\left(c^{2}x^{2}+1\right)}-\frac{b\sqrt{d\left(c^{2}x^{2}+1\right)}\,x}{c^{3}d^{2}\sqrt{c^{2}x^{2}+1}}+\frac{2b\sqrt{d\left(c^{2}x^{2}+1\right)}\,\operatorname{arcsin}}{c^{4}d^{2}\left(c^{2}x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] a*x^2/c^2/d/(c^2*d*x^2+d)^(1/2)+2*a/d/c^4/(c^2*d*x^2+d)^(1/2)+b*(d*(c^2*x^2
+1))^(1/2)/c^2/d^2/(c^2*x^2+1)*arcsinh(c*x)*x^2-b*(d*(c^2*x^2+1))^(1/2)/c^3
/d^2/(c^2*x^2+1)^(1/2)*x+2*b*(d*(c^2*x^2+1))^(1/2)/c^4/d^2/(c^2*x^2+1)*arcs
inh(c*x)+I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d^2*ln(c*x+(c^2*x^
2+1)^(1/2)-I)-I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d^2*ln(c*x+(c
^2*x^2+1)^(1/2)+I)
```

maxima [A] time = 0.49, size = 119, normalized size = 0.88

$$-bc\left(\frac{x}{c^{4}d^{\frac{3}{2}}}+\frac{\arctan\left(cx\right)}{c^{5}d^{\frac{3}{2}}}\right)+b\left(\frac{x^{2}}{\sqrt{c^{2}dx^{2}+d}\,c^{2}d}+\frac{2}{\sqrt{c^{2}dx^{2}+d}\,c^{4}d}\right)\operatorname{arsinh}\left(cx\right)+a\left(\frac{x^{2}}{\sqrt{c^{2}dx^{2}+d}\,c^{2}d}+\frac{2}{\sqrt{c^{2}dx^{2}+d}\,c^{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] -b*c*(x/(c^4*d^(3/2)) + arctan(c*x)/(c^5*d^(3/2))) + b*(x^2/(sqrt(c^2*d*x^2
 + d)*c^2*d) + 2/(sqrt(c^2*d*x^2 + d)*c^4*d))*arcsinh(c*x) + a*(x^2/(sqrt(c
^2*d*x^2 + d)*c^2*d) + 2/(sqrt(c^2*d*x^2 + d)*c^4*d))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3}\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{{\left(d\,c^{2}\,x^{2}+d\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}\left(a+b\operatorname{asinh}\left(cx\right)\right)}{\left(d\left(c^{2}x^{2}+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**3*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(3/2), x)
```

3.158 $\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=130

$$-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^3d\sqrt{c^2dx^2+d}}+\frac{b\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)}{2c^3d\sqrt{c^2dx^2+d}}$$

```
[Out] -x*(a+b*arcsinh(c*x))/c^2/d/(c^2*d*x^2+d)^(1/2)+1/2*(a+b*arcsinh(c*x))^2*(c
^2*x^2+1)^(1/2)/b/c^3/d/(c^2*d*x^2+d)^(1/2)+1/2*b*ln(c^2*x^2+1)*(c^2*x^2+1)
^(1/2)/c^3/d/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5751, 5677, 5675, 260}

$$\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^3d\sqrt{c^2dx^2+d}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{c^2dx^2+d}}+\frac{b\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)}{2c^3d\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] -((x*(a + b*ArcSinh[c*x]))/(c^2*d*Sqrt[d + c^2*d*x^2])) + (Sqrt[1 + c^2*x^2
]*(a + b*ArcSinh[c*x])^2)/(2*b*c^3*d*Sqrt[d + c^2*d*x^2]) + (b*Sqrt[1 + c^2
*x^2]*Log[1 + c^2*x^2])/(2*c^3*d*Sqrt[d + c^2*d*x^2])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] &&  !GtQ[d, 0]
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rubi steps

$$\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{3/2}}\,dx = -\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}} + \frac{\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+c^2dx^2}}\,dx}{c^2d} + \frac{\left(b\sqrt{1+c^2x^2}\right)\int \frac{x}{1+c^2x^2}\,dx}{cd\sqrt{d+c^2dx^2}}$$

$$= -\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}} + \frac{b\sqrt{1+c^2x^2}\log\left(1+c^2x^2\right)}{2c^3d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{c^2d\sqrt{d+c^2dx^2}}$$

$$= -\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^3d\sqrt{d+c^2dx^2}} + \frac{b\sqrt{1+c^2x^2}\log\left(1+c^2x\right.}{2c^3d\sqrt{d+c^2dx^2}}$$

Mathematica [A] time = 0.20, size = 146, normalized size = 1.12

$$-\frac{ax\sqrt{d\left(c^2x^2+1\right)}}{c^2d^2\left(c^2x^2+1\right)}+\frac{a\log\left(\sqrt{d}\sqrt{d\left(c^2x^2+1\right)}+cdx\right)}{c^3d^{3/2}}+\frac{b\left(\sqrt{c^2x^2+1}\left(2\log\left(\sqrt{c^2x^2+1}\right)+\sinh^{-1}(cx)^2\right)-2cx\sin\right.}{2c^3d\sqrt{d\left(c^2x^2+1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] -((a*x*Sqrt[d*(1 + c^2*x^2)])/(c^2*d^2*(1 + c^2*x^2))) + (b*(-2*c*x*ArcSinh
[c*x] + Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]^2 + 2*Log[Sqrt[1 + c^2*x^2]])))/(2*
c^3*d*Sqrt[d*(1 + c^2*x^2)]) + (a*Log[c*d*x + Sqrt[d]*Sqrt[d*(1 + c^2*x^2)]
])/(c^3*d^(3/2))
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(bx^2\operatorname{arsinh}(cx)+ax^2\right)}{c^4d^2x^4+2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*x^2*arcsinh(c*x) + a*x^2)/(c^4*d^2*x^4 + 2*
c^2*d^2*x^2 + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arsinh}(cx)+a)x^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^2/(c^2*d*x^2 + d)^(3/2), x)
```

maple [A] time = 0.25, size = 232, normalized size = 1.78

$$-\frac{ax}{c^2d\sqrt{c^2d\,x^2+d}}+\frac{a\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{c^2d\sqrt{c^2d}}+\frac{b\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsinh}(cx)^2}{2\sqrt{c^2x^2+1}\,c^3d^2}-\frac{b\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsinh}(cx)}{\sqrt{c^2x^2+1}\,c^3d^2}-\frac{b\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] -a*x/c^2/d/(c^2*d*x^2+d)^(1/2)+a/c^2/d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+
d)^(1/2))/(c^2*d)^(1/2)+1/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3/d
^2*arcsinh(c*x)^2-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3/d^2*arcsinh
(c*x)-b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/c^2/d^2/(c^2*x^2+1)*x+b*(d*(c^2*
x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3/d^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-a\left(\frac{x}{\sqrt{c^2dx^2+d}\,c^2d}-\frac{\operatorname{arsinh}(cx)}{c^3d^{\frac{3}{2}}}\right)+b\int\frac{x^2\log\left(cx+\sqrt{c^2x^2+1}\right)}{\left(c^2dx^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] -a*(x/(sqrt(c^2*d*x^2 + d)*c^2*d) - arcsinh(c*x)/(c^3*d^(3/2))) + b*integra
te(x^2*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^2\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{{\left(d\,c^2\,x^2+d\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2\left(a+b\operatorname{asinh}(cx)\right)}{\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(3/2), x)
```

3.159 $\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=70

$$\frac{b\sqrt{c^2x^2+1}\,\tan^{-1}(cx)}{c^2d\sqrt{c^2dx^2+d}}-\frac{a+b\sinh^{-1}(cx)}{c^2d\sqrt{c^2dx^2+d}}$$

```
[Out] (-a-b*arcsinh(c*x))/c^2/d/(c^2*d*x^2+d)^(1/2)+b*arctan(c*x)*(c^2*x^2+1)^(1/
2)/c^2/d/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5717, 203}

$$\frac{b\sqrt{c^2x^2+1}\,\tan^{-1}(cx)}{c^2d\sqrt{c^2dx^2+d}}-\frac{a+b\sinh^{-1}(cx)}{c^2d\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] -((a + b*ArcSinh[c*x])/(c^2*d*Sqrt[d + c^2*d*x^2])) + (b*Sqrt[1 + c^2*x^2]*
ArcTan[c*x])/(c^2*d*Sqrt[d + c^2*d*x^2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{3/2}}\,dx &= -\frac{a+b\sinh^{-1}(cx)}{c^2d\sqrt{d+c^2dx^2}}+\frac{\left(b\sqrt{1+c^2x^2}\right)\int \frac{1}{1+c^2x^2}\,dx}{cd\sqrt{d+c^2dx^2}}\\ &= -\frac{a+b\sinh^{-1}(cx)}{c^2d\sqrt{d+c^2dx^2}}+\frac{b\sqrt{1+c^2x^2}\,\tan^{-1}(cx)}{c^2d\sqrt{d+c^2dx^2}}\end{aligned}$$

Mathematica [A] time = 0.16, size = 82, normalized size = 1.17

$$\frac{b\sqrt{d\left(c^2x^2+1\right)}\,\tan^{-1}(cx)}{c^2d^2\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{c^2d^2\left(c^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] -((Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(c^2*d^2*(1 + c^2*x^2))) + (b*
Sqrt[d*(1 + c^2*x^2)]*ArcTan[c*x])/(c^2*d^2*Sqrt[1 + c^2*x^2])
```

fricas [A] time = 0.63, size = 128, normalized size = 1.83

$$-\frac{\left(bc^2x^2+b\right)\sqrt{d}\arctan\left(\frac{2\sqrt{c^2dx^2+d}\sqrt{c^2x^2+1}c\sqrt{d}x}{c^4dx^4-d}\right)+2\sqrt{c^2dx^2+d}\,b\log\left(cx+\sqrt{c^2x^2+1}\right)+2\sqrt{c^2dx^2+d}\,a}{2\left(c^4d^2x^2+c^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*((b*c^2*x^2 + b)*sqrt(d)*arctan(2*sqrt(c^2*d*x^2 + d)*sqrt(c^2*x^2 + 1
)*c*sqrt(d)*x/(c^4*d*x^4 - d)) + 2*sqrt(c^2*d*x^2 + d)*b*log(c*x + sqrt(c^2
*x^2 + 1)) + 2*sqrt(c^2*d*x^2 + d)*a)/(c^4*d^2*x^2 + c^2*d^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\,\mathrm{arsinh}\,(cx)+a)x}{\left(c^2dx^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x/(c^2*d*x^2 + d)^(3/2), x)
```

maple [C] time = 0.14, size = 164, normalized size = 2.34

$$-\frac{a}{c^2d\sqrt{c^2d\,x^2+d}}-\frac{b\sqrt{d\left(c^2x^2+1\right)}\,\mathrm{arcsinh}\,(cx)}{c^2d^2\left(c^2x^2+1\right)}+\frac{ib\sqrt{d\left(c^2x^2+1\right)}\,\ln\left(cx+\sqrt{c^2x^2+1}+i\right)}{\sqrt{c^2x^2+1}\,c^2d^2}-\frac{ib\sqrt{d\left(c^2x^2+1\right)}}{\sqrt{c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] -a/c^2/d/(c^2*d*x^2+d)^(1/2)-b*(d*(c^2*x^2+1))^(1/2)/c^2/d^2/(c^2*x^2+1)*ar
csinh(c*x)+I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^2*ln(c*x+(c^2*
x^2+1)^(1/2)+I)-I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^2*ln(c*x+
(c^2*x^2+1)^(1/2)-I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\left(\frac{-\mathrm{arsinh}\left(\frac{1}{c|x|}\right)}{c^2d^{\frac{3}{2}}}-\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{\sqrt{c^2x^2+1}\,c^2d^{\frac{3}{2}}}-\int\frac{1}{c^5d^{\frac{3}{2}}x^4+c^3d^{\frac{3}{2}}x^2+\left(c^4d^{\frac{3}{2}}x^3+c^2d^{\frac{3}{2}}x\right)\sqrt{c^2x^2+1}}\,dx\right)-\frac{a}{\sqrt{c^2dx^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] b*(integrate(1/(sqrt(c^2*x^2 + 1)*x), x)/(c^2*d^(3/2)) - log(c*x + sqrt(c^2
*x^2 + 1))/(sqrt(c^2*x^2 + 1)*c^2*d^(3/2)) - integrate(1/(c^5*d^(3/2)*x^4 +
 c^3*d^(3/2)*x^2 + (c^4*d^(3/2)*x^3 + c^2*d^(3/2)*x)*sqrt(c^2*x^2 + 1)), x)
) - a/(sqrt(c^2*d*x^2 + d)*c^2*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\ (a+b\,\mathrm{asinh}\,(c\,x))}{\left(d\,c^2\,x^2+d\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2),x)
[Out] int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\,(a+b\,\mathrm{asinh}\,(cx))}{\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(3/2),x)
[Out] Integral(x*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(3/2), x)
```

3.160 $\int \frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=76

$$\frac{x\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\,\log\left(c^2x^2+1\right)}{2cd\sqrt{c^2dx^2+d}}$$

```
[Out] x*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(1/2)-1/2*b*ln(c^2*x^2+1)*(c^2*x^2+1)^
(1/2)/c/d/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5687, 260}

$$\frac{x\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\,\log\left(c^2x^2+1\right)}{2cd\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcSinh[c*x]))/(d*Sqrt[d + c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*Log
[1 + c^2*x^2])/(2*c*d*Sqrt[d + c^2*d*x^2])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^{3/2}}\,dx &= \frac{x\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+c^2dx^2}}-\frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{x}{1+c^2x^2}\,dx}{d\sqrt{d+c^2dx^2}}\\ &= \frac{x\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+c^2dx^2}}-\frac{b\sqrt{1+c^2x^2}\,\log\left(1+c^2x^2\right)}{2cd\sqrt{d+c^2dx^2}}\end{aligned}$$

Mathematica [A] time = 0.17, size = 100, normalized size = 1.32

$$\frac{\sqrt{c^2dx^2+d}\left(2acx\sqrt{c^2x^2+1}-\left(bc^2x^2+b\right)\log\left(c^2x^2+1\right)+2bcx\sqrt{c^2x^2+1}\,\sinh^{-1}(cx)\right)}{2cd^2\left(c^2x^2+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(2*a*c*x*Sqrt[1 + c^2*x^2] + 2*b*c*x*Sqrt[1 + c^2*x^2]
*ArcSinh[c*x] - (b + b*c^2*x^2)*Log[1 + c^2*x^2]))/(2*c*d^2*(1 + c^2*x^2)^(
3/2))
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{c^2dx^2+d}\,(b\operatorname{arsinh}(cx)+a)}{c^4d^2x^4+2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^4*d^2*x^4 + 2*c^2*d^2*
x^2 + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{b\operatorname{arsinh}(cx)+a}{\left(c^2dx^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(c^2*d*x^2 + d)^(3/2), x)
```

maple [B] time = 0.11, size = 143, normalized size = 1.88

$$\frac{ax}{d\sqrt{c^2d\,x^2+d}}+\frac{b\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsinh}(cx)}{\sqrt{c^2x^2+1}\,c\,d^2}+\frac{b\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsinh}(cx)\,x}{d^2\left(c^2x^2+1\right)}-\frac{b\sqrt{d\left(c^2x^2+1\right)}\,\ln\left(1+\left(cx+\sqrt{c^2\right.\right.}}{\sqrt{c^2x^2+1}\,c\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] a*x/d/(c^2*d*x^2+d)^(1/2)+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d^2*a
rcsinh(c*x)+b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/d^2/(c^2*x^2+1)*x-b*(d*(c^
2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
```

maxima [A] time = 0.35, size = 58, normalized size = 0.76

$$\frac{bx\operatorname{arsinh}(cx)}{\sqrt{c^2dx^2+d}\,d}+\frac{ax}{\sqrt{c^2dx^2+d}\,d}-\frac{b\log\left(x^2+\frac{1}{c^2}\right)}{2\,cd^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] b*x*arcsinh(c*x)/(sqrt(c^2*d*x^2 + d)*d) + a*x/(sqrt(c^2*d*x^2 + d)*d) - 1/
2*b*log(x^2 + 1/c^2)/(c*d^(3/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{a+b\operatorname{asinh}(c\,x)}{\left(d\,c^2\,x^2+d\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{asinh}(cx)}{\left(d\left(c^2x^2 + 1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(3/2), x)
```

3.161 $\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=194

$$\frac{a+b\sinh^{-1}(cx)}{d\sqrt{c^2dx^2+d}}-\frac{2\sqrt{c^2x^2+1}\,\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\,\mathrm{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)}{d\sqrt{c^2dx^2+d}}+\frac{b\sqrt{c^2x^2+1}\,\mathrm{L}}{d\sqrt{c^2d}}$$

```
[Out] (a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(1/2)-b*arctan(c*x)*(c^2*x^2+1)^(1/2)/d/
(c^2*d*x^2+d)^(1/2)-2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^
2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)-b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c
^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)+b*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c
^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5755, 5764, 5760, 4182, 2279, 2391, 203}

$$-\frac{b\sqrt{c^2x^2+1}\,\mathrm{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{d\sqrt{c^2dx^2+d}}+\frac{b\sqrt{c^2x^2+1}\,\mathrm{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{d\sqrt{c^2dx^2+d}}+\frac{a+b\sinh^{-1}(cx)}{d\sqrt{c^2dx^2+d}}-\frac{2\sqrt{c^2x^2+1}\,\tanh^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] (a + b*ArcSinh[c*x])/(d*Sqrt[d + c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*ArcTan[
c*x])/(d*Sqrt[d + c^2*d*x^2]) - (2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*A
rcTanh[E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*Poly
Log[2, -E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) + (b*Sqrt[1 + c^2*x^2]*Pol
yLog[2, E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 5764

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[
((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&  !GtQ[d, 0] && (Integer
Q[m] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)^{3/2}}\,dx = \frac{a+b\sinh^{-1}(cx)}{d\sqrt{d+c^2dx^2}} + \frac{\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{d+c^2dx^2}}\,dx}{d} - \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{1+c^2x^2}\,dx}{d\sqrt{d+c^2dx^2}}$$

$$= \frac{a+b\sinh^{-1}(cx)}{d\sqrt{d+c^2dx^2}} - \frac{b\sqrt{1+c^2x^2}\tan^{-1}(cx)}{d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{1+c^2x^2}}\,dx}{d\sqrt{d+c^2dx^2}}$$

$$= \frac{a+b\sinh^{-1}(cx)}{d\sqrt{d+c^2dx^2}} - \frac{b\sqrt{1+c^2x^2}\tan^{-1}(cx)}{d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}\,\mathrm{Subst}\left(\int (a+bx)\operatorname{csch}(x)\,dx,\right.}{d\sqrt{d+c^2dx^2}}$$

$$= \frac{a+b\sinh^{-1}(cx)}{d\sqrt{d+c^2dx^2}} - \frac{b\sqrt{1+c^2x^2}\tan^{-1}(cx)}{d\sqrt{d+c^2dx^2}} - \frac{2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(\right.}{d\sqrt{d+c^2dx^2}}$$

$$= \frac{a+b\sinh^{-1}(cx)}{d\sqrt{d+c^2dx^2}} - \frac{b\sqrt{1+c^2x^2}\tan^{-1}(cx)}{d\sqrt{d+c^2dx^2}} - \frac{2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(\right.}{d\sqrt{d+c^2dx^2}}$$

$$= \frac{a+b\sinh^{-1}(cx)}{d\sqrt{d+c^2dx^2}} - \frac{b\sqrt{1+c^2x^2}\tan^{-1}(cx)}{d\sqrt{d+c^2dx^2}} - \frac{2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(\right.}{d\sqrt{d+c^2dx^2}}$$

Mathematica [A] time = 0.85, size = 231, normalized size = 1.19

$$\frac{\frac{a\sqrt{c^2dx^2+d}}{c^2x^2+1} - a\sqrt{d}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+d\right) + a\sqrt{d}\log(x) + \frac{bd\left(\sqrt{c^2x^2+1}\,\mathrm{Li}_2\left(-e^{-\sinh^{-1}(cx)}\right) - \sqrt{c^2x^2+1}\,\mathrm{Li}_2\left(e^{-\sinh^{-1}(cx)}\right) + \sqrt{c^2\right.}{}}{d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] ((a*Sqrt[d + c^2*d*x^2])/(1 + c^2*x^2) + a*Sqrt[d]*Log[x] - a*Sqrt[d]*Log[d
 + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (b*d*(ArcSinh[c*x] - 2*Sqrt[1 + c^2*x^2]*
ArcTan[Tanh[ArcSinh[c*x]/2]] + Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 - E^(-A
rcSinh[c*x])] - Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] +
 Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-ArcSinh[c*x])] - Sqrt[1 + c^2*x^2]*PolyL
og[2, E^(-ArcSinh[c*x])]))/Sqrt[d + c^2*d*x^2])/d^2
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\,(b\,\text{arsinh}\,(cx)+a)}{c^4d^2x^5+2\,c^2d^2x^3+d^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^4*d^2*x^5 + 2*c^2*d^2*
x^3 + d^2*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{b\,\text{arsinh}\,(cx)+a}{\left(c^2dx^2+d\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^(3/2)*x), x)
```

maple [A] time = 0.18, size = 274, normalized size = 1.41

$$\frac{a}{d\sqrt{c^2d\,x^2+d}}-\frac{a\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2d\,x^2+d}}{x}\right)}{d^{\frac{3}{2}}}+\frac{b\sqrt{d\left(c^2x^2+1\right)}\;\text{arcsinh}\,(cx)}{d^2\left(c^2x^2+1\right)}-\frac{2b\sqrt{d\left(c^2x^2+1\right)}\;\text{arctan}\left(cx+\sqrt{c^2x^2+1}\right.}{\sqrt{c^2x^2+1}\;d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] a/d/(c^2*d*x^2+d)^(1/2)-a/d^(3/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)
+b*(d*(c^2*x^2+1))^(1/2)/d^2/(c^2*x^2+1)*arcsinh(c*x)-2*b*(d*(c^2*x^2+1))^(
1/2)/(c^2*x^2+1)^(1/2)/d^2*arctan(c*x+(c^2*x^2+1)^(1/2))-b*(d*(c^2*x^2+1))^
(1/2)/(c^2*x^2+1)^(1/2)/d^2*dilog(c*x+(c^2*x^2+1)^(1/2))-b*(d*(c^2*x^2+1))^
(1/2)/(c^2*x^2+1)^(1/2)/d^2*dilog(1+c*x+(c^2*x^2+1)^(1/2))-b*(d*(c^2*x^2+1)
)^(1/2)/(c^2*x^2+1)^(1/2)/d^2*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-a\left(\frac{\text{arsinh}\left(\frac{1}{c|x|}\right)}{d^{\frac{3}{2}}}-\frac{1}{\sqrt{c^2dx^2+d}\,d}\right)+b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{\left(c^2dx^2+d\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] -a*(arcsinh(1/(c*abs(x)))/d^(3/2) - 1/(sqrt(c^2*d*x^2 + d)*d)) + b*integrat
e(log(c*x + sqrt(c^2*x^2 + 1))/((c^2*d*x^2 + d)^(3/2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\operatorname{asinh}(c\,x)}{x\,\left(d\,c^2\,x^2+d\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)^(3/2)),x)
[Out] int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\operatorname{asinh}(cx)}{x\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x/(c**2*d*x**2+d)**(3/2),x)
[Out] Integral((a + b*asinh(c*x))/(x*(d*(c**2*x**2 + 1))**(3/2)), x)
```

3.162 $\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=143

$$-\frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{c^2dx^2+d}}-\frac{a+b\sinh^{-1}(cx)}{dx\sqrt{c^2dx^2+d}}+\frac{bc\log(x)\sqrt{c^2dx^2+d}}{d^2\sqrt{c^2x^2+1}}+\frac{bc\sqrt{c^2dx^2+d}\,\log\left(c^2x^2+1\right)}{2d^2\sqrt{c^2x^2+1}}$$

```
[Out] (-a-b*arcsinh(c*x))/d/x/(c^2*d*x^2+d)^(1/2)-2*c^2*x*(a+b*arcsinh(c*x))/d/(c
^2*d*x^2+d)^(1/2)+b*c*ln(x)*(c^2*d*x^2+d)^(1/2)/d^2/(c^2*x^2+1)^(1/2)+1/2*b
*c*ln(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)/d^2/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5747, 5687, 260, 266, 36, 29, 31}

$$-\frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{c^2dx^2+d}}-\frac{a+b\sinh^{-1}(cx)}{dx\sqrt{c^2dx^2+d}}+\frac{bc\sqrt{c^2x^2+1}\,\log(x)}{d\sqrt{c^2dx^2+d}}+\frac{bc\sqrt{c^2x^2+1}\,\log\left(c^2x^2+1\right)}{2d\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] -((a + b*ArcSinh[c*x])/(d*x*Sqrt[d + c^2*d*x^2])) - (2*c^2*x*(a + b*ArcSinh
[c*x]))/(d*Sqrt[d + c^2*d*x^2]) + (b*c*Sqrt[1 + c^2*x^2]*Log[x])/(d*Sqrt[d
+ c^2*d*x^2]) + (b*c*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(2*d*Sqrt[d + c^2*
d*x^2])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
```

```
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(d+c^2dx^2\right)^{3/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{dx\sqrt{d+c^2dx^2}} - \left(2c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^{3/2}}\,dx + \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{x\left(1+c^2x^2\right)}\,dx}{d\sqrt{d+c^2dx^2}}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{dx\sqrt{d+c^2dx^2}} - \frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+c^2dx^2}} + \frac{\left(bc\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{1}{x\left(1+c^2x\right)}\,dx,\right.}{2d\sqrt{d+c^2dx^2}}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{dx\sqrt{d+c^2dx^2}} - \frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+c^2dx^2}} + \frac{bc\sqrt{1+c^2x^2}\,\log\left(1+c^2x^2\right)}{d\sqrt{d+c^2dx^2}} + \frac{\left(bc\sqrt{1\right.}{}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{dx\sqrt{d+c^2dx^2}} - \frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+c^2dx^2}} + \frac{bc\sqrt{1+c^2x^2}\,\log(x)}{d\sqrt{d+c^2dx^2}} + \frac{bc\sqrt{1+c^2x^2}\,\mathrm{l}}{2d\sqrt{d+}}$$

Mathematica [A] time = 0.28, size = 163, normalized size = 1.14

$$-\frac{\sqrt{c^2dx^2+d}\left(4ac^2x^2\sqrt{c^2x^2+1}+2a\sqrt{c^2x^2+1}+bcx\left(c^2x^2+1\right)\log\left(\frac{1}{c^2x^2}+1\right)-2bcx\log\left(c^2x^2+1\right)+2b\sqrt{c^2}\right.}{2d^2x\left(c^2x^2+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] -1/2*(Sqrt[d + c^2*d*x^2]*(2*a*Sqrt[1 + c^2*x^2] + 4*a*c^2*x^2*Sqrt[1 + c^2
*x^2] + 2*b*Sqrt[1 + c^2*x^2]*(1 + 2*c^2*x^2)*ArcSinh[c*x] + b*c*x*(1 + c^2
*x^2)*Log[1 + 1/(c^2*x^2)] - 2*b*c*x*Log[1 + c^2*x^2] - 2*b*c^3*x^3*Log[1 +
 c^2*x^2]))/(d^2*x*(1 + c^2*x^2)^(3/2))
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\,(b\,\text{arsinh}\,(cx)+a)}{c^4d^2x^6+2\,c^2d^2x^4+d^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^4*d^2*x^6 + 2*c^2*d^2*
x^4 + d^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\left(c^2dx^2 + d\right)^{\frac{3}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^(3/2)*x^2), x)
```

maple [A] time = 0.17, size = 205, normalized size = 1.43

$$-\frac{a}{dx\sqrt{c^2d\,x^2+d}}-\frac{2a\,c^2x}{d\sqrt{c^2d\,x^2+d}}-\frac{2b\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsinh}(cx)\,c}{\sqrt{c^2x^2+1}\,d^2}-\frac{2b\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsinh}(cx)\,x\,c^2}{\left(c^2x^2+1\right)d^2}-\frac{b\sqrt{d\left(c^2x\right.}}{\left(c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] -a/d/x/(c^2*d*x^2+d)^(1/2)-2*a*c^2/d*x/(c^2*d*x^2+d)^(1/2)-2*b*(d*(c^2*x^2+
1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*arcsinh(c*x)*c-2*b*(d*(c^2*x^2+1))^(1/2)*ar
csinh(c*x)/(c^2*x^2+1)/d^2*x*c^2-b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/(c^2*
x^2+1)/d^2/x+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*ln((c*x+(c^2*x^2
+1)^(1/2))^4-1)*c
```

maxima [A] time = 0.41, size = 119, normalized size = 0.83

$$\frac{1}{2}\,bc\left(\frac{\log\left(c^2x^2+1\right)}{d^{\frac{3}{2}}}+\frac{2\,\log(x)}{d^{\frac{3}{2}}}\right)-\left(\frac{2\,c^2x}{\sqrt{c^2dx^2+d}\,d}+\frac{1}{\sqrt{c^2dx^2+d}\,dx}\right)b\operatorname{arsinh}(cx)-\left(\frac{2\,c^2x}{\sqrt{c^2dx^2+d}\,d}+\frac{1}{\sqrt{c^2dx^2+d}\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*c*(log(c^2*x^2 + 1)/d^(3/2) + 2*log(x)/d^(3/2)) - (2*c^2*x/(sqrt(c^2*
d*x^2 + d)*d) + 1/(sqrt(c^2*d*x^2 + d)*d*x))*b*arcsinh(c*x) - (2*c^2*x/(sqr
t(c^2*d*x^2 + d)*d) + 1/(sqrt(c^2*d*x^2 + d)*d*x))*a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\operatorname{asinh}(c\,x)}{x^2\left(d\,c^2\,x^2 + d\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)^(3/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{asinh}(cx)}{x^2\left(d\left(c^2x^2 + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**2/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(x**2*(d*(c**2*x**2 + 1))**(3/2)), x)
```

3.163 $\int \frac{a+b\sinh^{-1}(cx)}{x^3(d+c^2dx^2)^{3/2}}\,dx$

Optimal. Leaf size=287

$$-\frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2d\sqrt{c^2dx^2+d}}-\frac{a+b\sinh^{-1}(cx)}{2dx^2\sqrt{c^2dx^2+d}}+\frac{3c^2\sqrt{c^2x^2+1}\,\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{c^2dx^2+d}}+\frac{3bc^2\sqrt{c^2x^2+1}}{2d}$$

```
[Out] -3/2*c^2*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(1/2)+1/2*(-a-b*arcsinh(c*x))/d
/x^2/(c^2*d*x^2+d)^(1/2)-1/2*b*c*(c^2*x^2+1)^(1/2)/d/x/(c^2*d*x^2+d)^(1/2)+
b*c^2*arctan(c*x)*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)+3*c^2*(a+b*arcsin
h(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1
/2)+3/2*b*c^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*
x^2+d)^(1/2)-3/2*b*c^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d
/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5747, 5755, 5764, 5760, 4182, 2279, 2391, 203, 325}

$$\frac{3bc^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{2d\sqrt{c^2dx^2+d}}-\frac{3bc^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{2d\sqrt{c^2dx^2+d}}-\frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2d\sqrt{c^2dx^2+d}}-\frac{a+}{2dx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] -(b*c*Sqrt[1 + c^2*x^2])/(2*d*x*Sqrt[d + c^2*d*x^2]) - (3*c^2*(a + b*ArcSin
h[c*x]))/(2*d*Sqrt[d + c^2*d*x^2]) - (a + b*ArcSinh[c*x])/(2*d*x^2*Sqrt[d +
 c^2*d*x^2]) + (b*c^2*Sqrt[1 + c^2*x^2]*ArcTan[c*x])/(d*Sqrt[d + c^2*d*x^2]
) + (3*c^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/
(d*Sqrt[d + c^2*d*x^2]) + (3*b*c^2*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^ArcSinh[
c*x]])/(2*d*Sqrt[d + c^2*d*x^2]) - (3*b*c^2*Sqrt[1 + c^2*x^2]*PolyLog[2, E^
ArcSinh[c*x]])/(2*d*Sqrt[d + c^2*d*x^2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 5764

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[
((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&  !GtQ[d, 0] && (Integer
Q[m] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^3\left(d+c^2dx^2\right)^{3/2}}dx = -\frac{a+b\sinh^{-1}(cx)}{2dx^2\sqrt{d+c^2dx^2}} - \frac{1}{2}\left(3c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)^{3/2}}dx + \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{x^2(1+c^2x^2)}dx}{2d\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2dx\sqrt{d+c^2dx^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2d\sqrt{d+c^2dx^2}} - \frac{a+b\sinh^{-1}(cx)}{2dx^2\sqrt{d+c^2dx^2}} - \frac{\left(3c^2\right)\int \frac{a+b\sinh^{-1}(cx}{x\sqrt{d+c^2dx^2}}}{2d}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2dx\sqrt{d+c^2dx^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2d\sqrt{d+c^2dx^2}} - \frac{a+b\sinh^{-1}(cx)}{2dx^2\sqrt{d+c^2dx^2}} + \frac{bc^2\sqrt{1+c^2x^2}\tan^{-}}{d\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2dx\sqrt{d+c^2dx^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2d\sqrt{d+c^2dx^2}} - \frac{a+b\sinh^{-1}(cx)}{2dx^2\sqrt{d+c^2dx^2}} + \frac{bc^2\sqrt{1+c^2x^2}\tan^{-}}{d\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2dx\sqrt{d+c^2dx^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2d\sqrt{d+c^2dx^2}} - \frac{a+b\sinh^{-1}(cx)}{2dx^2\sqrt{d+c^2dx^2}} + \frac{bc^2\sqrt{1+c^2x^2}\tan^{-}}{d\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2dx\sqrt{d+c^2dx^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2d\sqrt{d+c^2dx^2}} - \frac{a+b\sinh^{-1}(cx)}{2dx^2\sqrt{d+c^2dx^2}} + \frac{bc^2\sqrt{1+c^2x^2}\tan^{-}}{d\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2dx\sqrt{d+c^2dx^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2d\sqrt{d+c^2dx^2}} - \frac{a+b\sinh^{-1}(cx)}{2dx^2\sqrt{d+c^2dx^2}} + \frac{bc^2\sqrt{1+c^2x^2}\tan^{-}}{d\sqrt{d+c^2dx^2}}$$

Mathematica [A] time = 6.37, size = 381, normalized size = 1.33

$$\frac{3ac^2\log\left(\sqrt{d}\sqrt{d\left(c^2x^2+1\right)}+d\right)}{2d^{3/2}} - \frac{3ac^2\log(x)}{2d^{3/2}} + \sqrt{d\left(c^2x^2+1\right)}\left(-\frac{ac^2}{d^2\left(c^2x^2+1\right)} - \frac{a}{2d^2x^2}\right) + \frac{bc^2\left(-12\sqrt{c^2x^2+1}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] Sqrt[d*(1 + c^2*x^2)]*(-1/2*a/(d^2*x^2) - (a*c^2)/(d^2*(1 + c^2*x^2))) - (3
*a*c^2*Log[x])/(2*d^(3/2)) + (3*a*c^2*Log[d + Sqrt[d]*Sqrt[d*(1 + c^2*x^2)]
])/(2*d^(3/2)) + (b*c^2*(-8*ArcSinh[c*x] + 16*Sqrt[1 + c^2*x^2]*ArcTan[Tanh
[ArcSinh[c*x]/2]] - 2*Sqrt[1 + c^2*x^2]*Coth[ArcSinh[c*x]/2] - Sqrt[1 + c^2
*x^2]*ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - 12*Sqrt[1 + c^2*x^2]*ArcSinh[c*
x]*Log[1 - E^(-ArcSinh[c*x])] + 12*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + E
^(-ArcSinh[c*x])] - 12*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-ArcSinh[c*x])] + 1
2*Sqrt[1 + c^2*x^2]*PolyLog[2, E^(-ArcSinh[c*x])] - Sqrt[1 + c^2*x^2]*ArcSi
nh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Sqrt[1 + c^2*x^2]*Tanh[ArcSinh[c*x]/2]))
/(8*d*Sqrt[d*(1 + c^2*x^2)])
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\,(b\,\text{arsinh}\,(cx)+a)}{c^4d^2x^7+2\,c^2d^2x^5+d^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^4*d^2*x^7 + 2*c^2*d^2*
x^5 + d^2*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\left(c^2dx^2 + d\right)^{\frac{3}{2}} x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^(3/2)*x^3), x)
```

maple [A] time = 0.30, size = 389, normalized size = 1.36

$$-\frac{a}{2d\,x^2\sqrt{c^2d\,x^2+d}} - \frac{3a\,c^2}{2d\sqrt{c^2d\,x^2+d}} + \frac{3a\,c^2 \ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2d\,x^2+d}}{x}\right)}{2d^{\frac{3}{2}}} - \frac{3b\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsinh}(cx)\,c^2}{2d^2\left(c^2x^2+1\right)} - \frac{b\sqrt{d\left(c^2x^2+\right.}}{2d^2\sqrt{c^2x^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] -1/2*a/d/x^2/(c^2*d*x^2+d)^(1/2)-3/2*a*c^2/d/(c^2*d*x^2+d)^(1/2)+3/2*a*c^2/
d^(3/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)-3/2*b*(d*(c^2*x^2+1))^(1/
2)/d^2/(c^2*x^2+1)*arcsinh(c*x)*c^2-1/2*b*(d*(c^2*x^2+1))^(1/2)/d^2/(c^2*x^
2+1)^(1/2)/x*c-1/2*b*(d*(c^2*x^2+1))^(1/2)/d^2/(c^2*x^2+1)/x^2*arcsinh(c*x)
+2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*arctan(c*x+(c^2*x^2+1)^(1/
2))*c^2+3/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*dilog(c*x+(c^2*x^
2+1)^(1/2))*c^2+3/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*dilog(1+c
*x+(c^2*x^2+1)^(1/2))*c^2+3/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2
*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(\frac{3\,c^2 \operatorname{arsinh}\left(\frac{1}{c|x|}\right)}{d^{\frac{3}{2}}} - \frac{3\,c^2}{\sqrt{c^2dx^2+d}\,d} - \frac{1}{\sqrt{c^2dx^2+d}\,dx^2}\right)a + b\int \frac{\log\left(cx + \sqrt{c^2x^2+1}\right)}{\left(c^2dx^2+d\right)^{\frac{3}{2}} x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(3*c^2*arcsinh(1/(c*abs(x)))/d^(3/2) - 3*c^2/(sqrt(c^2*d*x^2 + d)*d) -
1/(sqrt(c^2*d*x^2 + d)*d*x^2))*a + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))
/((c^2*d*x^2 + d)^(3/2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{asinh}(c\,x)}{x^3\left(d\,c^2\,x^2 + d\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)^(3/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asinh}(cx)}{x^3\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**3/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(x**3*(d*(c**2*x**2 + 1))**(3/2)), x)
```

3.164 $\int \frac{a+b\sinh^{-1}(cx)}{x^4(d+c^2dx^2)^{3/2}}\,dx$

Optimal. Leaf size=228

$$\frac{4c^2\left(a+b\sinh^{-1}(cx)\right)}{3dx\sqrt{c^2dx^2+d}}-\frac{a+b\sinh^{-1}(cx)}{3dx^3\sqrt{c^2dx^2+d}}+\frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3d\sqrt{c^2dx^2+d}}-\frac{bc\sqrt{c^2dx^2+d}}{6d^2x^2\sqrt{c^2x^2+1}}-\frac{5bc^3\log(x)\sqrt{c^2dx^2+d}}{3d^2\sqrt{c^2x^2+1}}-\frac{b}{}$$

```
[Out] 1/3*(-a-b*arcsinh(c*x))/d/x^3/(c^2*d*x^2+d)^(1/2)+4/3*c^2*(a+b*arcsinh(c*x)
)/d/x/(c^2*d*x^2+d)^(1/2)+8/3*c^4*x*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(1/2
)-1/6*b*c*(c^2*d*x^2+d)^(1/2)/d^2/x^2/(c^2*x^2+1)^(1/2)-5/3*b*c^3*ln(x)*(c^
2*d*x^2+d)^(1/2)/d^2/(c^2*x^2+1)^(1/2)-1/2*b*c^3*ln(c^2*x^2+1)*(c^2*d*x^2+d
)^(1/2)/d^2/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5747, 5687, 260, 266, 36, 29, 31, 44}

$$\frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3d\sqrt{c^2dx^2+d}}+\frac{4c^2\left(a+b\sinh^{-1}(cx)\right)}{3dx\sqrt{c^2dx^2+d}}-\frac{a+b\sinh^{-1}(cx)}{3dx^3\sqrt{c^2dx^2+d}}-\frac{bc\sqrt{c^2x^2+1}}{6dx^2\sqrt{c^2dx^2+d}}-\frac{5bc^3\sqrt{c^2x^2+1}\log(x)}{3d\sqrt{c^2dx^2+d}}-\frac{bc}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] -(b*c*Sqrt[1 + c^2*x^2])/(6*d*x^2*Sqrt[d + c^2*d*x^2]) - (a + b*ArcSinh[c*x
])/(3*d*x^3*Sqrt[d + c^2*d*x^2]) + (4*c^2*(a + b*ArcSinh[c*x]))/(3*d*x*Sqrt
[d + c^2*d*x^2]) + (8*c^4*x*(a + b*ArcSinh[c*x]))/(3*d*Sqrt[d + c^2*d*x^2])
 - (5*b*c^3*Sqrt[1 + c^2*x^2]*Log[x])/(3*d*Sqrt[d + c^2*d*x^2]) - (b*c^3*Sq
rt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(2*d*Sqrt[d + c^2*d*x^2])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^4\left(d+c^2dx^2\right)^{3/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{3dx^3\sqrt{d+c^2dx^2}}-\frac{1}{3}\left(4c^2\right)\int\frac{a+b\sinh^{-1}(cx)}{x^2\left(d+c^2dx^2\right)^{3/2}}\,dx+\frac{\left(bc\sqrt{1+c^2x^2}\right)\int\frac{1}{x^3\left(1+c^2x^2\right)}\,dx}{3d\sqrt{d+c^2dx^2}}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{3dx^3\sqrt{d+c^2dx^2}}+\frac{4c^2\left(a+b\sinh^{-1}(cx)\right)}{3dx\sqrt{d+c^2dx^2}}+\frac{1}{3}\left(8c^4\right)\int\frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^{3/2}}\,dx+\frac{\left(bc\right.}{}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{3dx^3\sqrt{d+c^2dx^2}}+\frac{4c^2\left(a+b\sinh^{-1}(cx)\right)}{3dx\sqrt{d+c^2dx^2}}+\frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3d\sqrt{d+c^2dx^2}}+\frac{\left(bc\sqrt{1+c^2\right.}}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6dx^2\sqrt{d+c^2dx^2}}-\frac{a+b\sinh^{-1}(cx)}{3dx^3\sqrt{d+c^2dx^2}}+\frac{4c^2\left(a+b\sinh^{-1}(cx)\right)}{3dx\sqrt{d+c^2dx^2}}+\frac{8c^4x\left(a+b\sinh^{-1}\right.}{3d\sqrt{d+c^2dx}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6dx^2\sqrt{d+c^2dx^2}}-\frac{a+b\sinh^{-1}(cx)}{3dx^3\sqrt{d+c^2dx^2}}+\frac{4c^2\left(a+b\sinh^{-1}(cx)\right)}{3dx\sqrt{d+c^2dx^2}}+\frac{8c^4x\left(a+b\sinh^{-1}\right.}{3d\sqrt{d+c^2dx}}$$

Mathematica [A] time = 0.32, size = 216, normalized size = 0.95

$$\frac{\sqrt{c^2dx^2+d}\left(8ac^2x^2\sqrt{c^2x^2+1}-2a\sqrt{c^2x^2+1}+16ac^4x^4\sqrt{c^2x^2+1}-bc^3x^3-8bc^5x^5\log\left(c^2x^2+1\right)+2b\sqrt{c^2x\right.}{6d^2x^3\left(c^2x^2+\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(-(b*c*x) - b*c^3*x^3 - 2*a*Sqrt[1 + c^2*x^2] + 8*a*c^
2*x^2*Sqrt[1 + c^2*x^2] + 16*a*c^4*x^4*Sqrt[1 + c^2*x^2] + 2*b*Sqrt[1 + c^2
*x^2]*(-1 + 4*c^2*x^2 + 8*c^4*x^4)*ArcSinh[c*x] + 5*b*c^3*x^3*(1 + c^2*x^2)
```

```
*Log[1 + 1/(c^2*x^2)] - 8*b*c^3*x^3*Log[1 + c^2*x^2] - 8*b*c^5*x^5*Log[1 +
c^2*x^2]))/(6*d^2*x^3*(1 + c^2*x^2)^(3/2))
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\,(b\,\text{arsinh}\,(cx)+a)}{c^4d^2x^8+2\,c^2d^2x^6+d^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^4*d^2*x^8 + 2*c^2*d^2*
x^6 + d^2*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arsinh}\,(cx)+a}{\left(c^2dx^2+d\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^(3/2)*x^4), x)
```

maple [B] time = 0.31, size = 965, normalized size = 4.23

$$-\frac{a}{3d\,x^3\sqrt{c^2d\,x^2+d}}+\frac{4a\,c^2}{3dx\sqrt{c^2d\,x^2+d}}+\frac{8a\,c^4x}{3d\sqrt{c^2d\,x^2+d}}+\frac{16b\sqrt{d\left(c^2x^2+1\right)}\;\text{arcsinh}\,(cx)\,c^3}{3\sqrt{c^2x^2+1}\;d^2}+\frac{32b\sqrt{d\left(c^2x^2+1\right)}\;x^7}{3\left(8c^4x^4+7c^2x^2-1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] -1/3*a/d/x^3/(c^2*d*x^2+d)^(1/2)+4/3*a*c^2/d/x/(c^2*d*x^2+d)^(1/2)+8/3*a*c^
4/d*x/(c^2*d*x^2+d)^(1/2)+16/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^
2*arcsinh(c*x)*c^3+32/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2
*x^7*c^10-32/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^5*(c^2
*x^2+1)*c^8+16*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^5*c^8-
16/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^3*(c^2*x^2+1)*c^
6+64/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^3*arcsinh(c*x)
*c^6-64/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^2*arcsinh(c
*x)*(c^2*x^2+1)^(1/2)*c^5+4*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)
/d^2*x^3*c^6+4/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x*(c^2
*x^2+1)*c^4+8*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x*arcsinh
(c*x)*c^4+8/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*arcsinh(c
*x)*(c^2*x^2+1)^(1/2)*c^3-4/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-
1)/d^2*x*c^4-4/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*c^3*(c
^2*x^2+1)^(1/2)-4*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2/x*arc
sinh(c*x)*c^2+1/6*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2/x^2*c
*(c^2*x^2+1)^(1/2)+1/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2/
x^3*arcsinh(c*x)-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*ln(1+(c*x+(c
^2*x^2+1)^(1/2))^2)*c^3-5/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*l
n((c*x+(c^2*x^2+1)^(1/2))^2-1)*c^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\left(\frac{8\,c^4x}{\sqrt{c^2dx^2+d}\,d}+\frac{4\,c^2}{\sqrt{c^2dx^2+d}\,dx}-\frac{1}{\sqrt{c^2dx^2+d}\,dx^3}\right)a+b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{\left(c^2dx^2+d\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*(8*c^4*x/(sqrt(c^2*d*x^2 + d)*d) + 4*c^2/(sqrt(c^2*d*x^2 + d)*d*x) - 1/
(sqrt(c^2*d*x^2 + d)*d*x^3))*a + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(
(c^2*d*x^2 + d)^(3/2)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\,\mathrm{asinh}\,(c\,x)}{x^4\,\left(d\,c^2\,x^2 + d\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)^(3/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{asinh}\,(cx)}{x^4\,\left(d\left(c^2x^2 + 1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**4/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(x**4*(d*(c**2*x**2 + 1))**(3/2)), x)
```

3.165 $\int \frac{x^6\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=281

$$-\frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(c^2dx^2+d\right)^{3/2}}-\frac{5\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc^7d^2\sqrt{c^2dx^2+d}}+\frac{5x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{2c^6d^3}-\frac{5x^3\left(a+b\sinh^{-1}(c\right.}{3c^4d^2\sqrt{c^2dx^2+d}}$$

```
[Out] -1/3*x^5*(a+b*arcsinh(c*x))/c^2/d/(c^2*d*x^2+d)^(3/2)-5/3*x^3*(a+b*arcsinh(
c*x))/c^4/d^2/(c^2*d*x^2+d)^(1/2)-1/6*b/c^7/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^
2+d)^(1/2)-1/4*b*x^2*(c^2*x^2+1)^(1/2)/c^5/d^2/(c^2*d*x^2+d)^(1/2)-5/4*(a+b
*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c^7/d^2/(c^2*d*x^2+d)^(1/2)-7/6*b*ln(c
^2*x^2+1)*(c^2*x^2+1)^(1/2)/c^7/d^2/(c^2*d*x^2+d)^(1/2)+5/2*x*(a+b*arcsinh(
c*x))*(c^2*d*x^2+d)^(1/2)/c^6/d^3
```

Rubi [A] time = 0.43, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5751, 5758, 5677, 5675, 30, 266, 43}

$$-\frac{5x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^4d^2\sqrt{c^2dx^2+d}}+\frac{5x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{2c^6d^3}-\frac{5\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc^7d^2\sqrt{c^2dx^2+d}}-\frac{x^5\left(a+b\sinh^{-1}(c\right.}{3c^2d\left(c^2dx^2+d\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] -b/(6*c^7*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (b*x^2*Sqrt[1 + c^2*
x^2])/(4*c^5*d^2*Sqrt[d + c^2*d*x^2]) - (x^5*(a + b*ArcSinh[c*x]))/(3*c^2*d
*(d + c^2*d*x^2)^(3/2)) - (5*x^3*(a + b*ArcSinh[c*x]))/(3*c^4*d^2*Sqrt[d +
c^2*d*x^2]) + (5*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(2*c^6*d^3) -
(5*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*c^7*d^2*Sqrt[d + c^2*d*x^
2]) - (7*b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(6*c^7*d^2*Sqrt[d + c^2*d*x^
2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] &&  !GtQ[d, 0]
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^6\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{5/2}}\,dx = -\frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}}+\frac{5\int\frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{3/2}}\,dx}{3c^2d}+\frac{\left(b\sqrt{1+c^2x^2}\right)\int\frac{x^5}{\left(1+c^2x^2\right)^2}\,dx}{3cd^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}}-\frac{5x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^4d^2\sqrt{d+c^2dx^2}}+\frac{5\int\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx}{c^4d^2}+\frac{(5b}{}$$

$$= -\frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}}-\frac{5x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^4d^2\sqrt{d+c^2dx^2}}+\frac{5x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}\right.}{2c^6d^3}$$

$$= -\frac{b}{6c^7d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{13bx^2\sqrt{1+c^2x^2}}{12c^5d^2\sqrt{d+c^2dx^2}}-\frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}}-\frac{5}{}$$

$$= -\frac{b}{6c^7d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{bx^2\sqrt{1+c^2x^2}}{4c^5d^2\sqrt{d+c^2dx^2}}-\frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}}-\frac{5x}{}$$

Mathematica [A] time = 1.00, size = 222, normalized size = 0.79

$$\frac{-60a\sqrt{d}\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)+4acdx\left(3c^4x^4+20c^2x^2+15\right)+bd\left(-30\left(c^2x^2+\right.\right.}{24c^7d^3\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (4*a*c*d*x*(15 + 20*c^2*x^2 + 3*c^4*x^4) + b*d*(4*c*x*(15 + 20*c^2*x^2 + 3*
c^4*x^4)*ArcSinh[c*x] - 30*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]^2 - Sqrt[1 + c^
2*x^2]*(7 + 9*c^2*x^2 + 6*c^4*x^4 + 28*(1 + c^2*x^2)*Log[1 + c^2*x^2])) - 6
0*a*Sqrt[d]*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d +
c^2*d*x^2]])/(24*c^7*d^3*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2])
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^6 \operatorname{arsinh}(cx) + ax^6\right)\sqrt{c^2dx^2 + d}}{c^6d^3x^6 + 3\,c^4d^3x^4 + 3\,c^2d^3x^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^6*arcsinh(c*x) + a*x^6)*sqrt(c^2*d*x^2 + d)/(c^6*d^3*x^6 + 3*
c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.48, size = 1607, normalized size = 5.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] 1/2*a*x^5/c^2/d/(c^2*d*x^2+d)^(3/2)+147*b*(d*(c^2*x^2+1))^(1/2)/(63*c^8*x^8
+237*c^6*x^6+334*c^4*x^4+209*c^2*x^2+49)/d^3*arcsinh(c*x)*x^7+5/6*a/c^4*x^3
/d/(c^2*d*x^2+d)^(3/2)+5/2*a/c^6/d^2*x/(c^2*d*x^2+d)^(1/2)-5/2*a/c^6/d^2*ln
(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+49/6*b*(d*(c^2*x^
2+1))^(1/2)/(63*c^8*x^8+237*c^6*x^6+334*c^4*x^4+209*c^2*x^2+49)/d^3*x^7-1/8
*b*(d*(c^2*x^2+1))^(1/2)/c^7/d^3/(c^2*x^2+1)^(1/2)-1/4*b*(d*(c^2*x^2+1))^(1
/2)/c^5/d^3/(c^2*x^2+1)^(1/2)*x^2+14/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^
(1/2)/c^7/d^3*arcsinh(c*x)-7/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^
7/d^3*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+70/3*b*(d*(c^2*x^2+1))^(1/2)/(63*c^8*
x^8+237*c^6*x^6+334*c^4*x^4+209*c^2*x^2+49)/c^2/d^3*x^5+133/6*b*(d*(c^2*x^2
+1))^(1/2)/(63*c^8*x^8+237*c^6*x^6+334*c^4*x^4+209*c^2*x^2+49)/c^4/d^3*x^3+
7*b*(d*(c^2*x^2+1))^(1/2)/(63*c^8*x^8+237*c^6*x^6+334*c^4*x^4+209*c^2*x^2+4
9)/c^6/d^3*x-49/6*b*(d*(c^2*x^2+1))^(1/2)/(63*c^8*x^8+237*c^6*x^6+334*c^4*x
^4+209*c^2*x^2+49)/c^7/d^3*(c^2*x^2+1)^(1/2)-5/4*b*(d*(c^2*x^2+1))^(1/2)/(c
^2*x^2+1)^(1/2)/c^7/d^3*arcsinh(c*x)^2-147*b*(d*(c^2*x^2+1))^(1/2)/(63*c^8*
x^8+237*c^6*x^6+334*c^4*x^4+209*c^2*x^2+49)/c/d^3*arcsinh(c*x)*(c^2*x^2+1)^
(1/2)*x^6-406*b*(d*(c^2*x^2+1))^(1/2)/(63*c^8*x^8+237*c^6*x^6+334*c^4*x^4+2
09*c^2*x^2+49)/c^3/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^4-1120/3*b*(d*(c^2*
x^2+1))^(1/2)/(63*c^8*x^8+237*c^6*x^6+334*c^4*x^4+209*c^2*x^2+49)/c^5/d^3*a
rcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2+1/2*b*(d*(c^2*x^2+1))^(1/2)/c^4/d^3/(c^2*
```

```
x^2+1)*arcsinh(c*x)*x^3+1/2*b*(d*(c^2*x^2+1))^(1/2)/c^6/d^3/(c^2*x^2+1)*arc
sinh(c*x)*x-49/6*b*(d*(c^2*x^2+1))^(1/2)/(63*c^8*x^8+237*c^6*x^6+334*c^4*x^
4+209*c^2*x^2+49)/c^2/d^3*(c^2*x^2+1)*x^5+385*b*(d*(c^2*x^2+1))^(1/2)/(63*c
^8*x^8+237*c^6*x^6+334*c^4*x^4+209*c^2*x^2+49)/c^2/d^3*arcsinh(c*x)*x^5-21/
2*b*(d*(c^2*x^2+1))^(1/2)/(63*c^8*x^8+237*c^6*x^6+334*c^4*x^4+209*c^2*x^2+4
9)/c^3/d^3*(c^2*x^2+1)^(1/2)*x^4-91/6*b*(d*(c^2*x^2+1))^(1/2)/(63*c^8*x^8+2
37*c^6*x^6+334*c^4*x^4+209*c^2*x^2+49)/c^4/d^3*(c^2*x^2+1)*x^3+1009/3*b*(d*
(c^2*x^2+1))^(1/2)/(63*c^8*x^8+237*c^6*x^6+334*c^4*x^4+209*c^2*x^2+49)/c^4/
d^3*arcsinh(c*x)*x^3-37/2*b*(d*(c^2*x^2+1))^(1/2)/(63*c^8*x^8+237*c^6*x^6+3
34*c^4*x^4+209*c^2*x^2+49)/c^5/d^3*x^2*(c^2*x^2+1)^(1/2)-7*b*(d*(c^2*x^2+1)
)^(1/2)/(63*c^8*x^8+237*c^6*x^6+334*c^4*x^4+209*c^2*x^2+49)/c^6/d^3*(c^2*x^
2+1)*x+98*b*(d*(c^2*x^2+1))^(1/2)/(63*c^8*x^8+237*c^6*x^6+334*c^4*x^4+209*c
^2*x^2+49)/c^6/d^3*arcsinh(c*x)*x-343/3*b*(d*(c^2*x^2+1))^(1/2)/(63*c^8*x^8
+237*c^6*x^6+334*c^4*x^4+209*c^2*x^2+49)/c^7/d^3*arcsinh(c*x)*(c^2*x^2+1)^(
1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}a\left(\frac{3x^5}{\left(c^2dx^2+d\right)^{\frac{3}{2}}c^2d}+\frac{5x\left(\frac{3x^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}c^2d}+\frac{2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}c^4d}\right)}{c^2}+\frac{5x}{\sqrt{c^2dx^2+d}\,c^6d^2}-\frac{15\,\operatorname{arsinh}(cx)}{c^7d^{\frac{5}{2}}}\right)+b\int\frac{x^6\log\left(c\cdots\right.}{\left(c^2\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*a*(3*x^5/((c^2*d*x^2 + d)^(3/2)*c^2*d) + 5*x*(3*x^2/((c^2*d*x^2 + d)^(3
/2)*c^2*d) + 2/((c^2*d*x^2 + d)^(3/2)*c^4*d))/c^2 + 5*x/(sqrt(c^2*d*x^2 + d
)*c^6*d^2) - 15*arcsinh(c*x)/(c^7*d^(5/2))) + b*integrate(x^6*log(c*x + sqr
t(c^2*x^2 + 1))/(c^2*d*x^2 + d)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{{\left(d\,c^2\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int((x^6*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{6} \left(a + b \operatorname{asinh}{\left(c x \right)}\right)}{\left(d \left(c^{2} x^{2} + 1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**6*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(5/2), x)
```

3.166 $\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=210

$$\frac{\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{c^6d^3}+\frac{2\left(a+b\sinh^{-1}(cx)\right)}{c^6d^2\sqrt{c^2dx^2+d}}-\frac{a+b\sinh^{-1}(cx)}{3c^6d\left(c^2dx^2+d\right)^{3/2}}-\frac{11b\sqrt{c^2dx^2+d}\,\tan^{-1}(cx)}{6c^6d^3\sqrt{c^2x^2+1}}-\frac{bx\sqrt{c^2dx^2}}{c^5d^3\sqrt{c^2x^2}}$$

```
[Out] 1/3*(-a-b*arcsinh(c*x))/c^6/d/(c^2*d*x^2+d)^(3/2)+2*(a+b*arcsinh(c*x))/c^6/
d^2/(c^2*d*x^2+d)^(1/2)+1/6*b*x*(c^2*d*x^2+d)^(1/2)/c^5/d^3/(c^2*x^2+1)^(3/
2)+(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^6/d^3-b*x*(c^2*d*x^2+d)^(1/2)/c
^5/d^3/(c^2*x^2+1)^(1/2)-11/6*b*arctan(c*x)*(c^2*d*x^2+d)^(1/2)/c^6/d^3/(c^
2*x^2+1)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 225, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5751, 5717, 8, 321, 203, 288}

$$-\frac{4x^2\left(a+b\sinh^{-1}(cx)\right)}{3c^4d^2\sqrt{c^2dx^2+d}}+\frac{8\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{3c^6d^3}-\frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(c^2dx^2+d\right)^{3/2}}-\frac{bx^3}{6c^3d^2\sqrt{c^2x^2+1}\,\sqrt{c^2dx^2+d}}-$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] -(b*x^3)/(6*c^3*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (5*b*x*Sqrt[1
+ c^2*x^2])/(6*c^5*d^2*Sqrt[d + c^2*d*x^2]) - (x^4*(a + b*ArcSinh[c*x]))/(3
*c^2*d*(d + c^2*d*x^2)^(3/2)) - (4*x^2*(a + b*ArcSinh[c*x]))/(3*c^4*d^2*Sqr
t[d + c^2*d*x^2]) + (8*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*c^6*d^3
) - (11*b*Sqrt[1 + c^2*x^2]*ArcTan[c*x])/(6*c^6*d^2*Sqrt[d + c^2*d*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rubi steps

$$\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{5/2}}\,dx = -\frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}}+\frac{4\int\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{3/2}}\,dx}{3c^2d}+\frac{\left(b\sqrt{1+c^2x^2}\right)\int\frac{x^4}{\left(1+c^2x^2\right)^2}\,dx}{3cd^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bx^3}{6c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}}-\frac{4x^2\left(a+b\sinh^{-1}(cx)\right)}{3c^4d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bx^3}{6c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}+\frac{11bx\sqrt{1+c^2x^2}}{6c^5d^2\sqrt{d+c^2dx^2}}-\frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}}-4x$$

$$= -\frac{bx^3}{6c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{5bx\sqrt{1+c^2x^2}}{6c^5d^2\sqrt{d+c^2dx^2}}-\frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}}-4x$$

Mathematica [A] time = 0.27, size = 154, normalized size = 0.73

$$\frac{\sqrt{c^2dx^2+d}\left(2a\left(3c^4x^4+12c^2x^2+8\right)-bcx\sqrt{c^2x^2+1}\left(6c^2x^2+5\right)+2b\left(3c^4x^4+12c^2x^2+8\right)\sinh^{-1}(cx)\right)}{6c^6d^3\left(c^2x^2+1\right)^2}-11b$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(-(b*c*x*Sqrt[1 + c^2*x^2]*(5 + 6*c^2*x^2)) + 2*a*(8 +
 12*c^2*x^2 + 3*c^4*x^4) + 2*b*(8 + 12*c^2*x^2 + 3*c^4*x^4)*ArcSinh[c*x]))/
(6*c^6*d^3*(1 + c^2*x^2)^2) - (11*b*Sqrt[d*(1 + c^2*x^2)]*ArcTan[c*x])/(6*c
^6*d^3*Sqrt[1 + c^2*x^2])
```

fricas [A] time = 0.71, size = 219, normalized size = 1.04

$$\frac{11\left(bc^4x^4+2\,bc^2x^2+b\right)\sqrt{d}\,\arctan\left(\frac{2\,\sqrt{c^2dx^2+d}\,\sqrt{c^2x^2+1}\,c\sqrt{d}\,x}{c^4dx^4-d}\right)+4\left(3\,bc^4x^4+12\,bc^2x^2+8\,b\right)\sqrt{c^2dx^2+d}\,\log\left(cx\right.}{12\left(c^{10}d^3x^4+2\,c^8d^3x^2+c^6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*(11*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*sqrt(d)*arctan(2*sqrt(c^2*d*x^2 + d)
*sqrt(c^2*x^2 + 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)) + 4*(3*b*c^4*x^4 + 12*b*c^2
*x^2 + 8*b)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(6*a*c^4*x
^4 + 24*a*c^2*x^2 - (6*b*c^3*x^3 + 5*b*c*x)*sqrt(c^2*x^2 + 1) + 16*a)*sqrt(
c^2*d*x^2 + d))/(c^10*d^3*x^4 + 2*c^8*d^3*x^2 + c^6*d^3)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [**C**] time = 0.28, size = 394, normalized size = 1.88

$$\frac{a\,x^{4}}{c^{2} d \left(c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}+\frac{4 a\,x^{2}}{c^{4} d \left(c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}+\frac{8 a}{3 c^{6} d \left(c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}+\frac{b \sqrt{d \left(c^{2} x^{2}+1\right)}\,\operatorname{arcsinh}\left(c x\right) x^{2}}{c^{4} d^{3} \left(c^{2} x^{2}+1\right)}-\frac{b \sqrt{d \left(c^{2} x^{2}+1\right)}\,x}{c^{5} d^{3} \sqrt{c^{2} x^{2}+1}}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] a*x^4/c^2/d/(c^2*d*x^2+d)^(3/2)+4*a/c^4*x^2/d/(c^2*d*x^2+d)^(3/2)+8/3*a/c^6
/d/(c^2*d*x^2+d)^(3/2)+b*(d*(c^2*x^2+1))^(1/2)/c^4/d^3/(c^2*x^2+1)*arcsinh(
c*x)*x^2-b*(d*(c^2*x^2+1))^(1/2)/c^5/d^3/(c^2*x^2+1)^(1/2)*x+b*(d*(c^2*x^2+
1))^(1/2)/c^6/d^3/(c^2*x^2+1)*arcsinh(c*x)+2*b*(d*(c^2*x^2+1))^(1/2)/d^3/(c
^2*x^2+1)^2/c^4*arcsinh(c*x)*x^2+1/6*b*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1
)^(3/2)/c^5*x+5/3*b*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2/c^6*arcsinh(c*x
)+11/6*I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^6/d^3*ln(c*x+(c^2*x^2+
1)^(1/2)-I)-11/6*I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^6/d^3*ln(c*x
+(c^2*x^2+1)^(1/2)+I)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\,b\left(\frac{\left(3\,c^{4}\sqrt{d}\,x^{4}+12\,c^{2}\sqrt{d}\,x^{2}+8\,\sqrt{d}\right)\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)}{\left(c^{8}d^{3}x^{2}+c^{6}d^{3}\right)\sqrt{c^{2}x^{2}+1}}+3\int\frac{3\,c^{4}\sqrt{d}\,x^{4}+12\,c^{2}\sqrt{d}\,x^{2}+\cdots}{3\left(c^{11}d^{3}x^{6}+2\,c^{9}d^{3}x^{4}+c^{7}d^{3}x^{2}+\left(c^{10}d^{3}x^{5}+\cdots\right.\right.}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*b*((3*c^4*sqrt(d)*x^4 + 12*c^2*sqrt(d)*x^2 + 8*sqrt(d))*log(c*x + sqrt(
c^2*x^2 + 1))/((c^8*d^3*x^2 + c^6*d^3)*sqrt(c^2*x^2 + 1)) + 3*integrate(1/3
*(3*c^4*sqrt(d)*x^4 + 12*c^2*sqrt(d)*x^2 + 8*sqrt(d))/(c^11*d^3*x^6 + 2*c^9
*d^3*x^4 + c^7*d^3*x^2 + (c^10*d^3*x^5 + 2*c^8*d^3*x^3 + c^6*d^3*x)*sqrt(c^
2*x^2 + 1)), x) - 3*integrate(1/3*(3*c^4*sqrt(d)*x^4 + 12*c^2*sqrt(d)*x^2 +
 8*sqrt(d))/((c^8*d^3*x^3 + c^6*d^3*x)*sqrt(c^2*x^2 + 1)), x)) + 1/3*a*(3*x
^4/((c^2*d*x^2 + d)^(3/2)*c^2*d) + 12*x^2/((c^2*d*x^2 + d)^(3/2)*c^4*d) + 8
/((c^2*d*x^2 + d)^(3/2)*c^6*d))
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{{\left(d\,c^2\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2),x)
[Out] int((x^5*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 (a + b \operatorname{asinh}(cx))}{\left(d \left(c^2x^2 + 1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(5/2),x)
[Out] Integral(x**5*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(5/2), x)
```

3.167 $\int \frac{x^4(a+b\sinh^{-1}(cx))}{(d+c^2dx^2)^{5/2}}\,dx$

Optimal. Leaf size=203

$$-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(c^2dx^2+d\right)^{3/2}}+\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^5d^2\sqrt{c^2dx^2+d}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4d^2\sqrt{c^2dx^2+d}}+\frac{b}{6c^5d^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}+\frac{2b\sqrt{}}{}$$

```
[Out] -1/3*x^3*(a+b*arcsinh(c*x))/c^2/d/(c^2*d*x^2+d)^(3/2)-x*(a+b*arcsinh(c*x))/
c^4/d^2/(c^2*d*x^2+d)^(1/2)+1/6*b/c^5/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(
1/2)+1/2*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c^5/d^2/(c^2*d*x^2+d)^(1/
2)+2/3*b*ln(c^2*x^2+1)*(c^2*x^2+1)^(1/2)/c^5/d^2/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5751, 5677, 5675, 260, 266, 43}

$$-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4d^2\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^5d^2\sqrt{c^2dx^2+d}}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(c^2dx^2+d\right)^{3/2}}+\frac{b}{6c^5d^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}+\frac{2b\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] b/(6*c^5*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (x^3*(a + b*ArcSinh[c
*x]))/(3*c^2*d*(d + c^2*d*x^2)^(3/2)) - (x*(a + b*ArcSinh[c*x]))/(c^4*d^2*S
qrt[d + c^2*d*x^2]) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c^5*d
^2*Sqrt[d + c^2*d*x^2]) + (2*b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(3*c^5*d
^2*Sqrt[d + c^2*d*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
```

```
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] &&  !GtQ[d, 0]
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rubi steps

$$\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{5/2}}\,dx=-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}}+\frac{\int\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{3/2}}\,dx}{c^2d}+\frac{\left(b\sqrt{1+c^2x^2}\right)\int\frac{x^3}{\left(1+c^2x^2\right)^2}\,dx}{3cd^2\sqrt{d+c^2dx^2}}$$

$$=-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4d^2\sqrt{d+c^2dx^2}}+\frac{\int\frac{a+b\sinh^{-1}(cx)}{\sqrt{d+c^2dx^2}}\,dx}{c^4d^2}+\frac{\left(b\sqrt{1+c^2}\right.}{c^3d^2\sqrt{}}$$

$$=-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4d^2\sqrt{d+c^2dx^2}}+\frac{b\sqrt{1+c^2x^2}\,\log\left(1+c^2x^2\right)}{2c^5d^2\sqrt{d+c^2dx^2}}+$$

$$=\frac{b}{6c^5d^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4d^2\sqrt{d+c^2dx^2}}+$$

Mathematica [A] time = 0.53, size = 191, normalized size = 0.94

$$\frac{-2ac\sqrt{d}\,x\left(4c^2x^2+3\right)+6a\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}\,\log\left(\sqrt{d}\,\sqrt{c^2dx^2+d}+cdx\right)+b\sqrt{d}\left(\sqrt{c^2x^2+1}-8cx\left(c^2x^2\right.\right.}{6c^5d^{5/2}\left(c^2x^2+1\right)\sqrt{c^2dx^2+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (-2*a*c*Sqrt[d]*x*(3 + 4*c^2*x^2) + b*Sqrt[d]*(Sqrt[1 + c^2*x^2] + 2*c*x*Ar
cSinh[c*x] - 8*c*x*(1 + c^2*x^2)*ArcSinh[c*x] + (1 + c^2*x^2)^(3/2)*(3*ArcS
inh[c*x]^2 + 4*Log[1 + c^2*x^2])) + 6*a*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*L
og[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/(6*c^5*d^(5/2)*(1 + c^2*x^2)*Sqrt[
d + c^2*d*x^2])
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4\,\text{arsinh}\,(cx)+ax^4\right)\sqrt{c^2dx^2+d}}{c^6d^3x^6+3\,c^4d^3x^4+3\,c^2d^3x^2+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arcsinh(c*x) + a*x^4)*sqrt(c^2*d*x^2 + d)/(c^6*d^3*x^6 + 3*
c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(cx) + a)x^4}{(c^2dx^2 + d)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^4/(c^2*d*x^2 + d)^(5/2), x)
```

maple [B] time = 0.37, size = 1430, normalized size = 7.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] -1/3*a*x^3/c^2/d/(c^2*d*x^2+d)^(3/2)-a/c^4/d^2*x/(c^2*d*x^2+d)^(1/2)+a/c^4/
d^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/2*b*(d*(c
^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d^3*arcsinh(c*x)^2-8/3*b*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d^3*arcsinh(c*x)-32*b*(d*(c^2*x^2+1))^(1/2
)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)*c^2/d^3*arcsinh(c*x)*x^
7+32*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+
16)*c/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^6-8/3*b*(d*(c^2*x^2+1))^(1/2)/(2
4*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)*c^2/d^3*x^7+8/3*b*(d*(c^2*x
^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/d^3*(c^2*x^2
+1)*x^5-76*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^
2*x^2+16)/d^3*arcsinh(c*x)*x^5+84*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^
6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^4-2
2/3*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+1
6)/d^3*x^5+4*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*
c^2*x^2+16)/c/d^3*(c^2*x^2+1)^(1/2)*x^4+14/3*b*(d*(c^2*x^2+1))^(1/2)/(24*c^
8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^2/d^3*(c^2*x^2+1)*x^3-181/3*b
*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^
2/d^3*arcsinh(c*x)*x^3+220/3*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6
+118*c^4*x^4+71*c^2*x^2+16)/c^3/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2-20/3
*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/
c^2/d^3*x^3+13/2*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4
+71*c^2*x^2+16)/c^3/d^3*x^2*(c^2*x^2+1)^(1/2)+2*b*(d*(c^2*x^2+1))^(1/2)/(24
*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^4/d^3*(c^2*x^2+1)*x-16*b*(
d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^4/
d^3*arcsinh(c*x)*x+64/3*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*
c^4*x^4+71*c^2*x^2+16)/c^5/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-2*b*(d*(c^2*x
^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^4/d^3*x+8/
3*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)
/c^5/d^3*(c^2*x^2+1)^(1/2)+4/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^
5/d^3*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}\left(x\left(\frac{3\,x^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}c^2d}+\frac{2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}c^4d}\right)+\frac{x}{\sqrt{c^2dx^2+d}\,c^4d^2}-\frac{3\,\operatorname{arsinh}(cx)}{c^5d^{\frac{5}{2}}}\right)a+b\int\frac{x^4\log\left(cx+\sqrt{c^2x^2+1}\right)}{\left(c^2dx^2+d\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(x*(3*x^2/((c^2*d*x^2 + d)^(3/2)*c^2*d) + 2/((c^2*d*x^2 + d)^(3/2)*c^4
*d)) + x/(sqrt(c^2*d*x^2 + d)*c^4*d^2) - 3*arcsinh(c*x)/(c^5*d^(5/2)))*a +
b*integrate(x^4*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{{\left(d\,c^2\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int((x^4*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4} \left(a + b \operatorname{asinh}{\left(c x \right)}\right)}{\left(d \left(c^{2} x^{2} + 1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**4*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(5/2), x)
```

3.168 $\int \frac{x^3(a+b\sinh^{-1}(cx))}{(d+c^2dx^2)^{5/2}}\,dx$

Optimal. Leaf size=144

$$-\frac{a+b\sinh^{-1}(cx)}{c^4d^2\sqrt{c^2dx^2+d}}+\frac{a+b\sinh^{-1}(cx)}{3c^4d\left(c^2dx^2+d\right)^{3/2}}+\frac{5b\sqrt{c^2dx^2+d}\,\tan^{-1}(cx)}{6c^4d^3\sqrt{c^2x^2+1}}-\frac{bx\sqrt{c^2dx^2+d}}{6c^3d^3\left(c^2x^2+1\right)^{3/2}}$$

```
[Out] 1/3*(a+b*arcsinh(c*x))/c^4/d/(c^2*d*x^2+d)^(3/2)+(-a-b*arcsinh(c*x))/c^4/d^
2/(c^2*d*x^2+d)^(1/2)-1/6*b*x*(c^2*d*x^2+d)^(1/2)/c^3/d^3/(c^2*x^2+1)^(3/2)
+5/6*b*arctan(c*x)*(c^2*d*x^2+d)^(1/2)/c^4/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 149, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5751, 5717, 203, 288}

$$-\frac{2\left(a+b\sinh^{-1}(cx)\right)}{3c^4d^2\sqrt{c^2dx^2+d}}-\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(c^2dx^2+d\right)^{3/2}}-\frac{bx}{6c^3d^2\sqrt{c^2x^2+1}\,\sqrt{c^2dx^2+d}}+\frac{5b\sqrt{c^2x^2+1}\,\tan^{-1}(cx)}{6c^4d^2\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] -(b*x)/(6*c^3*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (x^2*(a + b*ArcS
inh[c*x]))/(3*c^2*d*(d + c^2*d*x^2)^(3/2)) - (2*(a + b*ArcSinh[c*x]))/(3*c^
4*d^2*Sqrt[d + c^2*d*x^2]) + (5*b*Sqrt[1 + c^2*x^2]*ArcTan[c*x])/(6*c^4*d^2
*Sqrt[d + c^2*d*x^2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
```

```
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rubi steps

$$\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{5/2}}\,dx=-\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}}+\frac{2\int\frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{3/2}}\,dx}{3c^2d}+\frac{\left(b\sqrt{1+c^2x^2}\right)\int\frac{x^2}{\left(1+c^2x^2\right)^2}\,dx}{3cd^2\sqrt{d+c^2dx^2}}$$

$$=-\frac{bx}{6c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}}-\frac{2\left(a+b\sinh^{-1}(cx)\right)}{3c^4d^2\sqrt{d+c^2dx^2}}+$$

$$=-\frac{bx}{6c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}}-\frac{2\left(a+b\sinh^{-1}(cx)\right)}{3c^4d^2\sqrt{d+c^2dx^2}}+$$

Mathematica [A] time = 0.25, size = 151, normalized size = 1.05

$$\frac{5b\sqrt{d\left(c^2x^2+1\right)}\tan^{-1}(cx)}{6c^4d^3\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}\left(2a\sqrt{c^2x^2+1}\left(3c^2x^2+2\right)+b\left(c^3x^3+cx\right)+2b\sqrt{c^2x^2+1}\left(3c^2x^2+2\right)\right.}{6c^4d^3\left(c^2x^2+1\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] -1/6*(Sqrt[d + c^2*d*x^2]*(2*a*Sqrt[1 + c^2*x^2]*(2 + 3*c^2*x^2) + b*(c*x +
 c^3*x^3) + 2*b*Sqrt[1 + c^2*x^2]*(2 + 3*c^2*x^2)*ArcSinh[c*x]))/(c^4*d^3*(
1 + c^2*x^2)^(5/2)) + (5*b*Sqrt[d*(1 + c^2*x^2)]*ArcTan[c*x])/(6*c^4*d^3*Sq
rt[1 + c^2*x^2])
```

fricas [A] time = 0.64, size = 188, normalized size = 1.31

$$-\frac{5\left(bc^4x^4+2\,bc^2x^2+b\right)\sqrt{d}\,\arctan\left(\frac{2\,\sqrt{c^2dx^2+d}\,\sqrt{c^2x^2+1}\,c\sqrt{d}\,x}{c^4dx^4-d}\right)+4\left(3\,bc^2x^2+2\,b\right)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+\right.}{12\left(c^8d^3x^4+2\,c^6d^3x^2+c^4d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(5*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*sqrt(d)*arctan(2*sqrt(c^2*d*x^2 + d)
*sqrt(c^2*x^2 + 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)) + 4*(3*b*c^2*x^2 + 2*b)*sqr
t(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(6*a*c^2*x^2 + sqrt(c^2*x
^2 + 1)*b*c*x + 4*a)*sqrt(c^2*d*x^2 + d))/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^
4*d^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [C] time = 0.23, size = 262, normalized size = 1.82

$$-\frac{a\,x^2}{c^2 d\left(c^2 d\,x^2+d\right)^{\frac{3}{2}}}-\frac{2a}{3d\,c^4\left(c^2 d\,x^2+d\right)^{\frac{3}{2}}}-\frac{b\sqrt{d\left(c^2x^2+1\right)}\;\operatorname{arcsinh}(cx)\,x^2}{d^3\left(c^2x^2+1\right)^2 c^2}-\frac{b\sqrt{d\left(c^2x^2+1\right)}\;x}{6d^3\left(c^2x^2+1\right)^{\frac{3}{2}} c^3}-\frac{2b\sqrt{d\left(c^2x^2+1\right)}\;\text{a}}{3d^3\left(c^2x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] -a*x^2/c^2/d/(c^2*d*x^2+d)^(3/2)-2/3*a/d/c^4/(c^2*d*x^2+d)^(3/2)-b*(d*(c^2*
x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2/c^2*arcsinh(c*x)*x^2-1/6*b*(d*(c^2*x^2+1))^
(1/2)/d^3/(c^2*x^2+1)^(3/2)/c^3*x-2/3*b*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+
1)^2/c^4*arcsinh(c*x)+5/6*I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d
^3*ln(c*x+(c^2*x^2+1)^(1/2)+I)-5/6*I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1
/2)/c^4/d^3*ln(c*x+(c^2*x^2+1)^(1/2)-I)
```

maxima [A] time = 0.59, size = 138, normalized size = 0.96

$$-\frac{1}{6}bc\left(\frac{x}{c^6 d^{\frac{5}{2}}x^2+c^4 d^{\frac{5}{2}}}-\frac{5\arctan(cx)}{c^5 d^{\frac{5}{2}}}\right)-\frac{1}{3}b\left(\frac{3x^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}c^2 d}+\frac{2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}c^4 d}\right)\operatorname{arsinh}(cx)-\frac{1}{3}a\left(\frac{3x^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/6*b*c*(x/(c^6*d^(5/2)*x^2 + c^4*d^(5/2)) - 5*arctan(c*x)/(c^5*d^(5/2)))
- 1/3*b*(3*x^2/((c^2*d*x^2 + d)^(3/2)*c^2*d) + 2/((c^2*d*x^2 + d)^(3/2)*c^4
*d))*arcsinh(c*x) - 1/3*a*(3*x^2/((c^2*d*x^2 + d)^(3/2)*c^2*d) + 2/((c^2*d*
x^2 + d)^(3/2)*c^4*d))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{{\left(d\,c^2\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int((x^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(a+b\operatorname{asinh}(cx)\right)}{\left(d\left(c^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**3*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(5/2), x)
```

3.169 $\int \frac{x^2(a+b\sinh^{-1}(cx))}{(d+c^2dx^2)^{5/2}}\,dx$

Optimal. Leaf size=119

$$\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{b}{6c^3d^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)}{6c^3d^2\sqrt{c^2dx^2+d}}$$

```
[Out] 1/3*x^3*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(3/2)-1/6*b/c^3/d^2/(c^2*x^2+1)^
(1/2)/(c^2*d*x^2+d)^(1/2)-1/6*b*ln(c^2*x^2+1)*(c^2*x^2+1)^(1/2)/c^3/d^2/(c^
2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {5723, 266, 43}

$$\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{b}{6c^3d^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)}{6c^3d^2\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] -b/(6*c^3*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) + (x^3*(a + b*ArcSinh[
c*x]))/(3*d*(d + c^2*d*x^2)^(3/2)) - (b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])
/(6*c^3*d^2*Sqrt[d + c^2*d*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{5/2}}\,dx = \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}} - \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{x^3}{\left(1+c^2x^2\right)^2}\,dx}{3d^2\sqrt{d+c^2dx^2}}$$

$$= \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}} - \frac{\left(bc\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{x}{\left(1+c^2x\right)^2}\,dx, x, x^2\right)}{6d^2\sqrt{d+c^2dx^2}}$$

$$= \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}} - \frac{\left(bc\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \left(-\frac{1}{c^2\left(1+c^2x\right)^2}+\frac{1}{c^2\left(1+c^2x\right)}\right)dx, x, x^2\right.}{6d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{b}{6c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}} - \frac{b\sqrt{1+c^2x^2}\log\left(1+c^2x^2\right.}{6c^3d^2\sqrt{d+c^2dx^2}}$$

Mathematica [A] time = 0.20, size = 118, normalized size = 0.99

$$-\frac{\sqrt{c^2dx^2+d}\left(-2ac^3x^3\sqrt{c^2x^2+1}+bc^2x^2+b\left(c^2x^2+1\right)^2\log\left(c^2x^2+1\right)-2bc^3x^3\sqrt{c^2x^2+1}\sinh^{-1}(cx)+b\right)}{6c^3d^3\left(c^2x^2+1\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] -1/6*(Sqrt[d + c^2*d*x^2]*(b + b*c^2*x^2 - 2*a*c^3*x^3*Sqrt[1 + c^2*x^2] -
2*b*c^3*x^3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + b*(1 + c^2*x^2)^2*Log[1 + c^2*
x^2]))/(c^3*d^3*(1 + c^2*x^2)^(5/2))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(bx^2\,\text{arsinh}\,(cx)+ax^2\right)}{c^6d^3x^6+3\,c^4d^3x^4+3\,c^2d^3x^2+d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*x^2*arcsinh(c*x) + a*x^2)/(c^6*d^3*x^6 + 3*
c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arsinh}\,(cx)+a)x^2}{\left(c^2dx^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^2/(c^2*d*x^2 + d)^(5/2), x)
```

maple [B] time = 0.28, size = 1153, normalized size = 9.69

$$-\frac{ax}{3c^2d\left(c^2d\,x^2+d\right)^{\frac{3}{2}}}+\frac{ax}{3c^2d^2\sqrt{c^2d\,x^2+d}}+\frac{2b\sqrt{d\left(c^2x^2+1\right)}\,\text{arcsinh}\,(cx)}{3\sqrt{c^2x^2+1}\,c^3d^3}+\frac{b\sqrt{d\left(c^2x^2+1\right)}\,c^4\,\text{arcsinh}\,(cx)\,x^7}{\left(3c^8x^8+9c^6x^6+10c^4x^4+5c^2x^2+1\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] -1/3*a/c^2*x/d/(c^2*d*x^2+d)^(3/2)+1/3*a/c^2/d^2*x/(c^2*d*x^2+d)^(1/2)+2/3*
b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3/d^3*arcsinh(c*x)+b*(d*(c^2*x^
2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^4/d^3*arcsinh(c*
x)*x^7-b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)
*c^3/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^6+1/6*b*(d*(c^2*x^2+1))^(1/2)/(3*
c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^4/d^3*x^7-1/6*b*(d*(c^2*x^2+1))
^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^2/d^3*(c^2*x^2+1)*x^5
+b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^2/d
^3*arcsinh(c*x)*x^5-2*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x
^4+5*c^2*x^2+1)*c/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^4+1/3*b*(d*(c^2*x^2+
1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^2/d^3*x^5-1/2*b*(d
*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c/d^3*(c^2
*x^2+1)^(1/2)*x^4-1/6*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x
^4+5*c^2*x^2+1)/d^3*(c^2*x^2+1)*x^3+1/3*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+
9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/d^3*arcsinh(c*x)*x^3-4/3*b*(d*(c^2*x^2+1)
)^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/c/d^3*arcsinh(c*x)*(c^
2*x^2+1)^(1/2)*x^2+1/6*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*
x^4+5*c^2*x^2+1)/d^3*x^3-1/2*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+1
0*c^4*x^4+5*c^2*x^2+1)/c/d^3*x^2*(c^2*x^2+1)^(1/2)-1/3*b*(d*(c^2*x^2+1))^(1
/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/c^3/d^3*arcsinh(c*x)*(c^2*
x^2+1)^(1/2)-1/6*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*
c^2*x^2+1)/c^3/d^3*(c^2*x^2+1)^(1/2)-1/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1
)^(1/2)/c^3/d^3*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
```

maxima [A] time = 0.39, size = 137, normalized size = 1.15

$$-\frac{1}{6}\,bc\left(\frac{1}{c^6d^{\frac{5}{2}}x^2+c^4d^{\frac{5}{2}}}+\frac{\log\left(c^2x^2+1\right)}{c^4d^{\frac{5}{2}}}\right)+\frac{1}{3}\,b\left(\frac{x}{\sqrt{c^2dx^2+d}\,c^2d^2}-\frac{x}{\left(c^2dx^2+d\right)^{\frac{3}{2}}c^2d}\right)\operatorname{arsinh}(cx)+\frac{1}{3}\,a\left(\frac{x}{\sqrt{c^2dx^2 -}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/6*b*c*(1/(c^6*d^(5/2)*x^2 + c^4*d^(5/2)) + log(c^2*x^2 + 1)/(c^4*d^(5/2)
)) + 1/3*b*(x/(sqrt(c^2*d*x^2 + d)*c^2*d^2) - x/((c^2*d*x^2 + d)^(3/2)*c^2*
d))*arcsinh(c*x) + 1/3*a*(x/(sqrt(c^2*d*x^2 + d)*c^2*d^2) - x/((c^2*d*x^2 +
 d)^(3/2)*c^2*d))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{{\left(d\,c^2\,x^2+d\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int((x^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(a+b\operatorname{asinh}(cx)\right)}{\left(d\left(c^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**2*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(5/2), x)
```

3.170 $\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=114

$$-\frac{a+b\sinh^{-1}(cx)}{3c^2d\left(c^2dx^2+d\right)^{3/2}}+\frac{bx}{6cd^2\sqrt{c^2x^2+1}\,\sqrt{c^2dx^2+d}}+\frac{b\sqrt{c^2x^2+1}\,\tan^{-1}(cx)}{6c^2d^2\sqrt{c^2dx^2+d}}$$

```
[Out] 1/3*(-a-b*arcsinh(c*x))/c^2/d/(c^2*d*x^2+d)^(3/2)+1/6*b*x/c/d^2/(c^2*x^2+1)
^(1/2)/(c^2*d*x^2+d)^(1/2)+1/6*b*arctan(c*x)*(c^2*x^2+1)^(1/2)/c^2/d^2/(c^2
*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5717, 199, 203}

$$-\frac{a+b\sinh^{-1}(cx)}{3c^2d\left(c^2dx^2+d\right)^{3/2}}+\frac{bx}{6cd^2\sqrt{c^2x^2+1}\,\sqrt{c^2dx^2+d}}+\frac{b\sqrt{c^2x^2+1}\,\tan^{-1}(cx)}{6c^2d^2\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (b*x)/(6*c*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (a + b*ArcSinh[c*x]
)/(3*c^2*d*(d + c^2*d*x^2)^(3/2)) + (b*Sqrt[1 + c^2*x^2]*ArcTan[c*x])/(6*c^
2*d^2*Sqrt[d + c^2*d*x^2])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{5/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{3c^2d\left(d+c^2dx^2\right)^{3/2}} + \frac{\left(b\sqrt{1+c^2x^2}\right)\int \frac{1}{\left(1+c^2x^2\right)^2}\,dx}{3cd^2\sqrt{d+c^2dx^2}}$$

$$= \frac{bx}{6cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{a+b\sinh^{-1}(cx)}{3c^2d\left(d+c^2dx^2\right)^{3/2}} + \frac{\left(b\sqrt{1+c^2x^2}\right)\int \frac{1}{1+c^2x^2}\,dx}{6cd^2\sqrt{d+c^2dx^2}}$$

$$= \frac{bx}{6cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{a+b\sinh^{-1}(cx)}{3c^2d\left(d+c^2dx^2\right)^{3/2}} + \frac{b\sqrt{1+c^2x^2}\tan^{-1}(cx)}{6c^2d^2\sqrt{d+c^2dx^2}}$$

Mathematica [A] time = 0.19, size = 130, normalized size = 1.14

$$\frac{\sqrt{c^2dx^2+d}\left(-2a\sqrt{c^2x^2+1}+bc^3x^3-2b\sqrt{c^2x^2+1}\sinh^{-1}(cx)+bcx\right)}{6c^2d^3\left(c^2x^2+1\right)^{5/2}}+\frac{b\sqrt{d\left(c^2x^2+1\right)}\tan^{-1}(cx)}{6c^2d^3\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(b*c*x + b*c^3*x^3 - 2*a*Sqrt[1 + c^2*x^2] - 2*b*Sqrt[
1 + c^2*x^2]*ArcSinh[c*x]))/(6*c^2*d^3*(1 + c^2*x^2)^(5/2)) + (b*Sqrt[d*(1
+ c^2*x^2)]*ArcTan[c*x])/(6*c^2*d^3*Sqrt[1 + c^2*x^2])
```

fricas [A] time = 0.69, size = 166, normalized size = 1.46

$$-\frac{\left(bc^4x^4+2\,bc^2x^2+b\right)\sqrt{d}\,\arctan\left(\frac{2\,\sqrt{c^2dx^2+d}\,\sqrt{c^2x^2+1}\,c\sqrt{d}\,x}{c^4dx^4-d}\right)+4\,\sqrt{c^2dx^2+d}\,b\log\left(cx+\sqrt{c^2x^2+1}\right)-2\,\sqrt{c^2dx^2+d}}{12\left(c^6d^3x^4+2\,c^4d^3x^2+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*((b*c^4*x^4 + 2*b*c^2*x^2 + b)*sqrt(d)*arctan(2*sqrt(c^2*d*x^2 + d)*s
qrt(c^2*x^2 + 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)) + 4*sqrt(c^2*d*x^2 + d)*b*log
(c*x + sqrt(c^2*x^2 + 1)) - 2*sqrt(c^2*d*x^2 + d)*(sqrt(c^2*x^2 + 1)*b*c*x
- 2*a))/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)x}{\left(c^2dx^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x/(c^2*d*x^2 + d)^(5/2), x)
```

maple [C] time = 0.14, size = 198, normalized size = 1.74

$$-\frac{a}{3c^2d\left(c^2d\,x^2+d\right)^{\frac{3}{2}}}+\frac{b\sqrt{d\left(c^2x^2+1\right)}\,x}{6d^3\left(c^2x^2+1\right)^{\frac{3}{2}}c}-\frac{b\sqrt{d\left(c^2x^2+1\right)}\,\mathrm{arcsinh}\,(cx)}{3d^3\left(c^2x^2+1\right)^2c^2}+\frac{ib\sqrt{d\left(c^2x^2+1\right)}\,\ln\left(cx+\sqrt{c^2x^2+1}+i\right)}{6\sqrt{c^2x^2+1}\,c^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] -1/3*a/c^2/d/(c^2*d*x^2+d)^(3/2)+1/6*b*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1
)^(3/2)/c*x-1/3*b*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2/c^2*arcsinh(c*x)+
1/6*I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^3*ln(c*x+(c^2*x^2+1)^
(1/2)+I)-1/6*I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^3*ln(c*x+(c^
2*x^2+1)^(1/2)-I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\int \frac{x \log\left(cx + \sqrt{c^2x^2+1}\right)}{\left(c^2dx^2+d\right)^{\frac{5}{2}}}\,dx - \frac{a}{3\left(c^2dx^2+d\right)^{\frac{3}{2}}c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] b*integrate(x*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d)^(5/2), x) - 1/3*
a/((c^2*d*x^2 + d)^(3/2)*c^2*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{{\left(d\,c^2\,x^2+d\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int((x*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(a + b\operatorname{asinh}\left(cx\right)\right)}{\left(d\left(c^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(5/2), x)
```

3.171 $\int \frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=147

$$\frac{2x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(c^2dx^2+d\right)^{3/2}}+\frac{b}{6cd^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)}{3cd^2\sqrt{c^2dx^2+d}}$$

```
[Out] 1/3*x*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(3/2)+2/3*x*(a+b*arcsinh(c*x))/d^2
/(c^2*d*x^2+d)^(1/2)+1/6*b/c/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-1/3*
b*ln(c^2*x^2+1)*(c^2*x^2+1)^(1/2)/c/d^2/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5690, 5687, 260, 261}

$$\frac{2x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(c^2dx^2+d\right)^{3/2}}+\frac{b}{6cd^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)}{3cd^2\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] b/(6*c*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) + (x*(a + b*ArcSinh[c*x])
)/(3*d*(d + c^2*d*x^2)^(3/2)) + (2*x*(a + b*ArcSinh[c*x]))/(3*d^2*Sqrt[d +
c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(3*c*d^2*Sqrt[d + c^2*
d*x^2])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{(d+c^2dx^2)^{5/2}}\,dx = \frac{x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{2\int \frac{a+b\sinh^{-1}(cx)}{(d+c^2dx^2)^{3/2}}\,dx}{3d} - \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{x}{(1+c^2x^2)^2}\,dx}{3d^2\sqrt{d+c^2dx^2}}$$

$$= \frac{b}{6cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{2x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{d+c^2dx^2}} - \frac{\left(2bc\sqrt{\ldots}\right.}{\ldots}$$

$$= \frac{b}{6cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{2x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{d+c^2dx^2}} - \frac{b\sqrt{1+\ldots}}{3\ldots}$$

Mathematica [A] time = 0.20, size = 143, normalized size = 0.97

$$\frac{\sqrt{c^2dx^2+d}\left(6acx\sqrt{c^2x^2+1}+4ac^3x^3\sqrt{c^2x^2+1}+bc^2x^2-2b\left(c^2x^2+1\right)^2\log\left(c^2x^2+1\right)+2bcx\sqrt{c^2x^2+1}\left(2c\ldots\right.\right.}{6cd^3\left(c^2x^2+1\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(b + b*c^2*x^2 + 6*a*c*x*Sqrt[1 + c^2*x^2] + 4*a*c^3*x
^3*Sqrt[1 + c^2*x^2] + 2*b*c*x*Sqrt[1 + c^2*x^2]*(3 + 2*c^2*x^2)*ArcSinh[c*
x] - 2*b*(1 + c^2*x^2)^2*Log[1 + c^2*x^2]))/(6*c*d^3*(1 + c^2*x^2)^(5/2))
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\,(b\,\text{arsinh}\,(cx)+a)}{c^6d^3x^6+3\,c^4d^3x^4+3\,c^2d^3x^2+d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^6*d^3*x^6 + 3*c^4*d^3*
x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arsinh}\,(cx)+a}{\left(c^2dx^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(c^2*d*x^2 + d)^(5/2), x)
```

maple [B] time = 0.15, size = 1005, normalized size = 6.84

$$\frac{ax}{3d\left(c^2d\,x^2+d\right)^{\frac{3}{2}}}+\frac{2ax}{3d^2\sqrt{c^2d\,x^2+d}}+\frac{4b\sqrt{d\left(c^2x^2+1\right)}\,\text{arcsinh}\,(cx)}{3\sqrt{c^2x^2+1}\,c\,d^3}-\frac{2b\sqrt{d\left(c^2x^2+1\right)}\,c^6x^7}{3\left(3c^6x^6+10c^4x^4+11c^2x^2+4\right)d^3}+\frac{2b\ldots}{3\left(3c\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] 1/3*a*x/d/(c^2*d*x^2+d)^(3/2)+2/3*a/d^2*x/(c^2*d*x^2+d)^(1/2)+4/3*b*(d*(c^2
*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d^3*arcsinh(c*x)-2/3*b*(d*(c^2*x^2+1))^(
1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^6/d^3*x^7+2/3*b*(d*(c^2*x^2+1))^
(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^4/d^3*(c^2*x^2+1)*x^5+2*b*(d*(c
^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^4/d^3*arcsinh(c*x)*x
^5-2*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^3/d^3*ar
csinh(c*x)*(c^2*x^2+1)^(1/2)*x^4-7/3*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*
c^4*x^4+11*c^2*x^2+4)*c^4/d^3*x^5+5/3*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10
*c^4*x^4+11*c^2*x^2+4)*c^2/d^3*(c^2*x^2+1)*x^3+17/3*b*(d*(c^2*x^2+1))^(1/2)
/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^2/d^3*arcsinh(c*x)*x^3-14/3*b*(d*(c^
2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c/d^3*arcsinh(c*x)*(c^2
*x^2+1)^(1/2)*x^2-8/3*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*
x^2+4)*c^2/d^3*x^3+1/2*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2
*x^2+4)*c/d^3*x^2*(c^2*x^2+1)^(1/2)+b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c
^4*x^4+11*c^2*x^2+4)/d^3*(c^2*x^2+1)*x+4*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6
+10*c^4*x^4+11*c^2*x^2+4)/d^3*arcsinh(c*x)*x-8/3*b*(d*(c^2*x^2+1))^(1/2)/(3
*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/c/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-b*(d
*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/d^3*x+2/3*b*(d*(c^2
*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/c/d^3*(c^2*x^2+1)^(1/2)-
2/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d^3*ln(1+(c*x+(c^2*x^2+1)^(
1/2))^2)
```

maxima [A] time = 0.36, size = 126, normalized size = 0.86

$$\frac{1}{6}\,bc\left(\frac{1}{c^4d^{\frac{5}{2}}x^2+c^2d^{\frac{5}{2}}}-\frac{2\,\log\left(c^2x^2+1\right)}{c^2d^{\frac{5}{2}}}\right)+\frac{1}{3}\,b\left(\frac{2\,x}{\sqrt{c^2dx^2+d}\,d^2}+\frac{x}{\left(c^2dx^2+d\right)^{\frac{3}{2}}d}\right)\operatorname{arsinh}\left(cx\right)+\frac{1}{3}\,a\left(\frac{2\,x}{\sqrt{c^2dx^2+d}\,d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*b*c*(1/(c^4*d^(5/2)*x^2 + c^2*d^(5/2)) - 2*log(c^2*x^2 + 1)/(c^2*d^(5/2
))) + 1/3*b*(2*x/(sqrt(c^2*d*x^2 + d)*d^2) + x/((c^2*d*x^2 + d)^(3/2)*d))*a
rcsinh(c*x) + 1/3*a*(2*x/(sqrt(c^2*d*x^2 + d)*d^2) + x/((c^2*d*x^2 + d)^(3/
2)*d))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{asinh}\left(c\,x\right)}{{\left(d\,c^2\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\operatorname{asinh}\left(cx\right)}{\left(d\left(c^2x^2+1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(5/2), x)
```

3.172 $\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=262

$$\frac{a+b\sinh^{-1}(cx)}{d^2\sqrt{c^2dx^2+d}}-\frac{2\sqrt{c^2x^2+1}\,\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{c^2dx^2+d}}+\frac{a+b\sinh^{-1}(cx)}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{b\sqrt{c^2x^2+1}\,\mathrm{Li}_2\left(-e^{\sin\cdots}\right.}{d^2\sqrt{c^2dx^2+d}}$$

```
[Out] 1/3*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(3/2)+(a+b*arcsinh(c*x))/d^2/(c^2*d*
x^2+d)^(1/2)-1/6*b*c*x/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-7/6*b*arct
an(c*x)*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)-2*(a+b*arcsinh(c*x))*arct
anh(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)-b*poly
log(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)+b*p
olylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5755, 5764, 5760, 4182, 2279, 2391, 203, 199}

$$-\frac{b\sqrt{c^2x^2+1}\,\mathrm{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{d^2\sqrt{c^2dx^2+d}}+\frac{b\sqrt{c^2x^2+1}\,\mathrm{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{d^2\sqrt{c^2dx^2+d}}+\frac{a+b\sinh^{-1}(cx)}{d^2\sqrt{c^2dx^2+d}}-\frac{2\sqrt{c^2x^2+1}\,\tan\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] -(b*c*x)/(6*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) + (a + b*ArcSinh[c*x
])/(3*d*(d + c^2*d*x^2)^(3/2)) + (a + b*ArcSinh[c*x])/(d^2*Sqrt[d + c^2*d*x
^2]) - (7*b*Sqrt[1 + c^2*x^2]*ArcTan[c*x])/(6*d^2*Sqrt[d + c^2*d*x^2]) - (2
*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/(d^2*Sqrt[
d + c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^ArcSinh[c*x]])/(d^2*Sq
rt[d + c^2*d*x^2]) + (b*Sqrt[1 + c^2*x^2]*PolyLog[2, E^ArcSinh[c*x]])/(d^2*
Sqrt[d + c^2*d*x^2])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 5764

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[
((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&  !GtQ[d, 0] && (Integer
Q[m] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)^{5/2}}\,dx = \frac{a+b\sinh^{-1}(cx)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)^{3/2}}\,dx}{d} - \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{\left(1+c^2x^2\right)^2}\,dx}{3d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bcx}{6d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{a+b\sinh^{-1}(cx)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{a+b\sinh^{-1}(cx)}{d^2\sqrt{d+c^2dx^2}} + \frac{\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{d+c^2dx^2}}}{d^2}$$

$$= -\frac{bcx}{6d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{a+b\sinh^{-1}(cx)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{a+b\sinh^{-1}(cx)}{d^2\sqrt{d+c^2dx^2}} - \frac{7b\sqrt{1+c^2x^2}}{6d^2\sqrt{d+c}}$$

$$= -\frac{bcx}{6d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{a+b\sinh^{-1}(cx)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{a+b\sinh^{-1}(cx)}{d^2\sqrt{d+c^2dx^2}} - \frac{7b\sqrt{1+c^2x^2}}{6d^2\sqrt{d+c}}$$

$$= -\frac{bcx}{6d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{a+b\sinh^{-1}(cx)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{a+b\sinh^{-1}(cx)}{d^2\sqrt{d+c^2dx^2}} - \frac{7b\sqrt{1+c^2x^2}}{6d^2\sqrt{d+c}}$$

$$= -\frac{bcx}{6d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{a+b\sinh^{-1}(cx)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{a+b\sinh^{-1}(cx)}{d^2\sqrt{d+c^2dx^2}} - \frac{7b\sqrt{1+c^2x^2}}{6d^2\sqrt{d+c}}$$

$$= -\frac{bcx}{6d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{a+b\sinh^{-1}(cx)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{a+b\sinh^{-1}(cx)}{d^2\sqrt{d+c^2dx^2}} - \frac{7b\sqrt{1+c^2x^2}}{6d^2\sqrt{d+c}}$$

Mathematica [A] time = 1.20, size = 247, normalized size = 0.94

$$\frac{\frac{2a\left(3c^2x^2+4\right)\sqrt{c^2dx^2+d}}{\left(c^2x^2+1\right)^2} - 6a\sqrt{d}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+d\right) + 6a\sqrt{d}\log(x) + \frac{bd^2\left(c^2x^2+1\right)^{3/2}\left(-\frac{cx}{c^2x^2+1}+\frac{6\sinh^{-1}(cx)}{\sqrt{c^2x^2+1}}+\frac{2\sinh^{-1}(cx)}{\left(c^2x^2+1\right)^{3/2}}\right.}{}}{6d^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] ((2*a*(4 + 3*c^2*x^2)*Sqrt[d + c^2*d*x^2])/(1 + c^2*x^2)^2 + 6*a*Sqrt[d]*Lo
g[x] - 6*a*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (b*d^2*(1 + c^2*x
^2)^(3/2)*(-((c*x)/(1 + c^2*x^2)) + (2*ArcSinh[c*x])/(1 + c^2*x^2)^(3/2) +
(6*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - 14*ArcTan[Tanh[ArcSinh[c*x]/2]] + 6*Ar
cSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 6*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[
c*x])] + 6*PolyLog[2, -E^(-ArcSinh[c*x])] - 6*PolyLog[2, E^(-ArcSinh[c*x])]
))/(d + c^2*d*x^2)^(3/2))/(6*d^3)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\,(b\,\text{arsinh}\,(cx)+a)}{c^6d^3x^7+3\,c^4d^3x^5+3\,c^2d^3x^3+d^3x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^6*d^3*x^7 + 3*c^4*d^3*
x^5 + 3*c^2*d^3*x^3 + d^3*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\mathrm{arsinh}\,(cx)+a}{\left(c^2dx^2+d\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^(5/2)*x), x)
```

maple [A] time = 0.20, size = 364, normalized size = 1.39

$$\frac{a}{3d\left(c^2d\,x^2+d\right)^{\frac{3}{2}}}+\frac{a}{d^2\sqrt{c^2d\,x^2+d}}-\frac{a\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2d\,x^2+d}}{x}\right)}{d^{\frac{5}{2}}}+\frac{b\sqrt{d\left(c^2x^2+1\right)}\,\mathrm{arcsinh}\,(cx)\,x^2c^2}{d^3\left(c^2x^2+1\right)^2}-\frac{b\sqrt{d\left(c^2x^2+1\right)}\,cx}{6d^3\left(c^2x^2+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] 1/3*a/d/(c^2*d*x^2+d)^(3/2)+a/d^2/(c^2*d*x^2+d)^(1/2)-a/d^(5/2)*ln((2*d+2*d
^(1/2)*(c^2*d*x^2+d)^(1/2))/x)+b*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2*ar
csinh(c*x)*x^2*c^2-1/6*b*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^(3/2)*c*x+4/
3*b*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2*arcsinh(c*x)-7/3*b*(d*(c^2*x^2+
1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*arctan(c*x+(c^2*x^2+1)^(1/2))-b*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*dilog(c*x+(c^2*x^2+1)^(1/2))-b*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*dilog(1+c*x+(c^2*x^2+1)^(1/2))-b*(d*(c^2*x
^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}a\left(\frac{3\,\mathrm{arsinh}\left(\frac{1}{c|x|}\right)}{d^{\frac{5}{2}}}-\frac{3}{\sqrt{c^2dx^2+d}\,d^2}-\frac{1}{\left(c^2dx^2+d\right)^{\frac{3}{2}}d}\right)+b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{\left(c^2dx^2+d\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*a*(3*arcsinh(1/(c*abs(x)))/d^(5/2) - 3/(sqrt(c^2*d*x^2 + d)*d^2) - 1/(
(c^2*d*x^2 + d)^(3/2)*d)) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/((c^2*
d*x^2 + d)^(5/2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}\,(c\,x)}{x\left(d\,c^2\,x^2+d\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)^(5/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x*(d + c^2*d*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{asinh}\,(cx)}{x\left(d\left(c^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(x*(d*(c**2*x**2 + 1))**(5/2)), x)
```

3.173 $\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=214

$$-\frac{8c^2x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{c^2dx^2+d}}-\frac{4c^2x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{a+b\sinh^{-1}(cx)}{dx\left(c^2dx^2+d\right)^{3/2}}-\frac{bc\sqrt{c^2dx^2+d}}{6d^3\left(c^2x^2+1\right)^{3/2}}+\frac{bc\log(x)\sqrt{c^2dx^2+d}}{d^3\sqrt{c^2x^2+1}}+$$

```
[Out] (-a-b*arcsinh(c*x))/d/x/(c^2*d*x^2+d)^(3/2)-4/3*c^2*x*(a+b*arcsinh(c*x))/d/
(c^2*d*x^2+d)^(3/2)-8/3*c^2*x*(a+b*arcsinh(c*x))/d^2/(c^2*d*x^2+d)^(1/2)-1/
6*b*c*(c^2*d*x^2+d)^(1/2)/d^3/(c^2*x^2+1)^(3/2)+b*c*ln(x)*(c^2*d*x^2+d)^(1/
2)/d^3/(c^2*x^2+1)^(1/2)+5/6*b*c*ln(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)/d^3/(c^2
*x^2+1)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5747, 5690, 5687, 260, 261, 266, 44}

$$-\frac{8c^2x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{c^2dx^2+d}}-\frac{4c^2x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{a+b\sinh^{-1}(cx)}{dx\left(c^2dx^2+d\right)^{3/2}}-\frac{bc}{6d^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}+\frac{bc\sqrt{c^2x^2+1}\,\mathrm{l}}{d^2\sqrt{c^2dx^2+}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] -(b*c)/(6*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (a + b*ArcSinh[c*x])
/(d*x*(d + c^2*d*x^2)^(3/2)) - (4*c^2*x*(a + b*ArcSinh[c*x]))/(3*d*(d + c^2
*d*x^2)^(3/2)) - (8*c^2*x*(a + b*ArcSinh[c*x]))/(3*d^2*Sqrt[d + c^2*d*x^2])
 + (b*c*Sqrt[1 + c^2*x^2]*Log[x])/(d^2*Sqrt[d + c^2*d*x^2]) + (5*b*c*Sqrt[1
 + c^2*x^2]*Log[1 + c^2*x^2])/(6*d^2*Sqrt[d + c^2*d*x^2])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
```

```
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(d+c^2dx^2\right)^{5/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{dx\left(d+c^2dx^2\right)^{3/2}} - \left(4c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^{5/2}}\,dx + \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{x\left(1+c^2x^2\right)^2}\,dx}{d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{dx\left(d+c^2dx^2\right)^{3/2}} - \frac{4c^2x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}} - \frac{\left(8c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^{3/2}}\,dx}{3d} + \frac{\left(bc\sqrt{1+c\right.}}{}$$

$$= -\frac{2bc}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{a+b\sinh^{-1}(cx)}{dx\left(d+c^2dx^2\right)^{3/2}} - \frac{4c^2x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}} - \frac{8c^2x\left(a\right.}{3d}$$

$$= -\frac{bc}{6d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{a+b\sinh^{-1}(cx)}{dx\left(d+c^2dx^2\right)^{3/2}} - \frac{4c^2x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}} - \frac{8c^2x\left(a\right.}{3d}$$

Mathematica [A] time = 0.31, size = 227, normalized size = 1.06

$$-\frac{\sqrt{c^2dx^2+d}\left(24ac^2x^2\sqrt{c^2x^2+1}+6a\sqrt{c^2x^2+1}+16ac^4x^4\sqrt{c^2x^2+1}+bc^3x^3+3bcx\left(c^2x^2+1\right)^2\log\left(\frac{1}{c^2x^2}+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] -1/6*(Sqrt[d + c^2*d*x^2]*(b*c*x + b*c^3*x^3 + 6*a*Sqrt[1 + c^2*x^2] + 24*a
*c^2*x^2*Sqrt[1 + c^2*x^2] + 16*a*c^4*x^4*Sqrt[1 + c^2*x^2] + 2*b*Sqrt[1 +
c^2*x^2]*(3 + 12*c^2*x^2 + 8*c^4*x^4)*ArcSinh[c*x] + 3*b*c*x*(1 + c^2*x^2)^
```

```
2*Log[1 + 1/(c^2*x^2)] - 8*b*c*x*Log[1 + c^2*x^2] - 16*b*c^3*x^3*Log[1 + c^
2*x^2] - 8*b*c^5*x^5*Log[1 + c^2*x^2]))/(d^3*x*(1 + c^2*x^2)^(5/2))
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\,(b\,\text{arsinh}\,(cx)+a)}{c^6d^3x^8+3\,c^4d^3x^6+3\,c^2d^3x^4+d^3x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^6*d^3*x^8 + 3*c^4*d^3*
x^6 + 3*c^2*d^3*x^4 + d^3*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arsinh}\,(cx)+a}{\left(c^2dx^2+d\right)^{\frac{5}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^(5/2)*x^2), x)
```

maple [B] time = 0.21, size = 1257, normalized size = 5.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] -a/d/x/(c^2*d*x^2+d)^(3/2)-4/3*a*c^2*x/d/(c^2*d*x^2+d)^(3/2)-8/3*a*c^2/d^2*
x/(c^2*d*x^2+d)^(1/2)-16/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*ar
csinh(c*x)*c-32/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+
9)/d^3*x^9*c^10+32/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x
^2+9)/d^3*x^7*(c^2*x^2+1)*c^8-112/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c
^4*x^4+26*c^2*x^2+9)/d^3*x^7*c^8+80/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25
*c^4*x^4+26*c^2*x^2+9)/d^3*x^5*(c^2*x^2+1)*c^6-64/3*b*(d*(c^2*x^2+1))^(1/2)
/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^5*arcsinh(c*x)*c^6+64/3*b*(d*(c^
2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^4*arcsinh(c*x)*(c
^2*x^2+1)^(1/2)*c^5-140/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*
c^2*x^2+9)/d^3*x^5*c^6+20*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*
c^2*x^2+9)/d^3*x^3*(c^2*x^2+1)*c^4-56*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25
*c^4*x^4+26*c^2*x^2+9)/d^3*x^3*arcsinh(c*x)*c^4+136/3*b*(d*(c^2*x^2+1))^(1/
2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^2*arcsinh(c*x)*(c^2*x^2+1)^(1/
2)*c^3-24*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x
^3*c^4-4/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*
x^2*c^3*(c^2*x^2+1)^(1/2)+4*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+2
6*c^2*x^2+9)/d^3*x*(c^2*x^2+1)*c^2-44*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25
*c^4*x^4+26*c^2*x^2+9)/d^3*x*arcsinh(c*x)*c^2+24*b*(d*(c^2*x^2+1))^(1/2)/(8
*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c-4*b*
(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x*c^2-3/2*b*(
d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*c*(c^2*x^2+1)^
(1/2)-9*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3/x*a
rcsinh(c*x)+5/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*ln(1+(c*x+(c^
2*x^2+1)^(1/2))^2)*c+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*ln((c*x+
(c^2*x^2+1)^(1/2))^2-1)*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}\,a\left(\frac{8\,c^2x}{\sqrt{c^2dx^2+d}\,d^2}+\frac{4\,c^2x}{\left(c^2dx^2+d\right)^{\frac{3}{2}}d}+\frac{3}{\left(c^2dx^2+d\right)^{\frac{3}{2}}dx}\right)+b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{\left(c^2dx^2+d\right)^{\frac{5}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*a*(8*c^2*x/(sqrt(c^2*d*x^2 + d)*d^2) + 4*c^2*x/((c^2*d*x^2 + d)^(3/2)*
d) + 3/((c^2*d*x^2 + d)^(3/2)*d*x)) + b*integrate(log(c*x + sqrt(c^2*x^2 +
1))/((c^2*d*x^2 + d)^(5/2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{a+b\,\mathrm{asinh}\left(c\,x\right)}{x^2\,{\left(d\,c^2\,x^2+d\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)^(5/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^2*(d + c^2*d*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\,\mathrm{asinh}\,(cx)}{x^2\left(d\left(c^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**2/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(x**2*(d*(c**2*x**2 + 1))**(5/2)), x)
```

3.174 $\int \frac{a+b\sinh^{-1}(cx)}{x^3(d+c^2dx^2)^{5/2}}\,dx$

Optimal. Leaf size=400

$$-\frac{5c^2\left(a+b\sinh^{-1}(cx)\right)}{2d^2\sqrt{c^2dx^2+d}}+\frac{5c^2\sqrt{c^2x^2+1}\,\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{c^2dx^2+d}}-\frac{5c^2\left(a+b\sinh^{-1}(cx)\right)}{6d\left(c^2dx^2+d\right)^{3/2}}-\frac{a+b\sin}{2dx^2\left(c^2d\right.}$$

```
[Out] -5/6*c^2*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(3/2)+1/2*(-a-b*arcsinh(c*x))/d
/x^2/(c^2*d*x^2+d)^(3/2)-5/2*c^2*(a+b*arcsinh(c*x))/d^2/(c^2*d*x^2+d)^(1/2)
+1/4*b*c/d^2/x/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+5/12*b*c^3*x/d^2/(c^2*
x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-3/4*b*c*(c^2*x^2+1)^(1/2)/d^2/x/(c^2*d*x^2
+d)^(1/2)+13/6*b*c^2*arctan(c*x)*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)+
5*c^2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d
^2/(c^2*d*x^2+d)^(1/2)+5/2*b*c^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2
+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)-5/2*b*c^2*polylog(2,c*x+(c^2*x^2+1)^(1/2)
)*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 400, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {5747, 5755, 5764, 5760, 4182, 2279, 2391, 203, 199, 290, 325}

$$\frac{5bc^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{2d^2\sqrt{c^2dx^2+d}}-\frac{5bc^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{2d^2\sqrt{c^2dx^2+d}}-\frac{5c^2\left(a+b\sinh^{-1}(cx)\right)}{2d^2\sqrt{c^2dx^2+d}}+\frac{5c^2\sqrt{c^2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] (b*c)/(4*d^2*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) + (5*b*c^3*x)/(12*d^2
*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (3*b*c*Sqrt[1 + c^2*x^2])/(4*d^2*
x*Sqrt[d + c^2*d*x^2]) - (5*c^2*(a + b*ArcSinh[c*x]))/(6*d*(d + c^2*d*x^2)^
(3/2)) - (a + b*ArcSinh[c*x])/(2*d*x^2*(d + c^2*d*x^2)^(3/2)) - (5*c^2*(a +
 b*ArcSinh[c*x]))/(2*d^2*Sqrt[d + c^2*d*x^2]) + (13*b*c^2*Sqrt[1 + c^2*x^2]
*ArcTan[c*x])/(6*d^2*Sqrt[d + c^2*d*x^2]) + (5*c^2*Sqrt[1 + c^2*x^2]*(a + b
*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2]) + (5*b*c^
2*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^ArcSinh[c*x]])/(2*d^2*Sqrt[d + c^2*d*x^2]
) - (5*b*c^2*Sqrt[1 + c^2*x^2]*PolyLog[2, E^ArcSinh[c*x]])/(2*d^2*Sqrt[d +
c^2*d*x^2])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
```

```
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 5764

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[
((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&  !GtQ[d, 0] && (Integer
Q[m] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^3\left(d+c^2dx^2\right)^{5/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{2dx^2\left(d+c^2dx^2\right)^{3/2}} - \frac{1}{2}\left(5c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)^{5/2}}\,dx + \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{x^2\left(1+c^2x^2\right)^2}\,dx}{2d^2\sqrt{d+c^2dx^2}}$$

$$= \frac{bc}{4d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{5c^2\left(a+b\sinh^{-1}(cx)\right)}{6d\left(d+c^2dx^2\right)^{3/2}} - \frac{a+b\sinh^{-1}(cx)}{2dx^2\left(d+c^2dx^2\right)^{3/2}} - \frac{\left(5c^2\right)\int \frac{a}{x}}{}$$

$$= \frac{bc}{4d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{5bc^3x}{12d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{3bc\sqrt{1+c^2x^2}}{4d^2x\sqrt{d+c^2dx^2}} - \frac{5c^2\left(a\right.}{6d\left(\right.}$$

$$= \frac{bc}{4d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{5bc^3x}{12d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{3bc\sqrt{1+c^2x^2}}{4d^2x\sqrt{d+c^2dx^2}} - \frac{5c^2\left(a\right.}{6d\left(\right.}$$

$$= \frac{bc}{4d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{5bc^3x}{12d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{3bc\sqrt{1+c^2x^2}}{4d^2x\sqrt{d+c^2dx^2}} - \frac{5c^2\left(a\right.}{6d\left(\right.}$$

$$= \frac{bc}{4d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{5bc^3x}{12d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{3bc\sqrt{1+c^2x^2}}{4d^2x\sqrt{d+c^2dx^2}} - \frac{5c^2\left(a\right.}{6d\left(\right.}$$

$$= \frac{bc}{4d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{5bc^3x}{12d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{3bc\sqrt{1+c^2x^2}}{4d^2x\sqrt{d+c^2dx^2}} - \frac{5c^2\left(a\right.}{6d\left(\right.}$$

$$= \frac{bc}{4d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{5bc^3x}{12d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{3bc\sqrt{1+c^2x^2}}{4d^2x\sqrt{d+c^2dx^2}} - \frac{5c^2\left(a\right.}{6d\left(\right.}$$

Mathematica [A] time = 6.56, size = 437, normalized size = 1.09

$$\frac{5ac^2\log\left(\sqrt{d}\sqrt{d\left(c^2x^2+1\right)}+d\right)}{2d^{5/2}} - \frac{5ac^2\log(x)}{2d^{5/2}} + \sqrt{d\left(c^2x^2+1\right)}\left(-\frac{2ac^2}{d^3\left(c^2x^2+1\right)} - \frac{ac^2}{3d^3\left(c^2x^2+1\right)^2} - \frac{a}{2d^3x^2}\right) + \frac{bc^2}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] Sqrt[d*(1 + c^2*x^2)]*(-1/2*a/(d^3*x^2) - (a*c^2)/(3*d^3*(1 + c^2*x^2)^2) -
 (2*a*c^2)/(d^3*(1 + c^2*x^2))) - (5*a*c^2*Log[x])/(2*d^(5/2)) + (5*a*c^2*L
```

```
og[d + Sqrt[d]*Sqrt[d*(1 + c^2*x^2)]])/(2*d^(5/2)) + (b*c^2*((4*c*x)/Sqrt[1
 + c^2*x^2] - 48*ArcSinh[c*x] - (8*ArcSinh[c*x])/(1 + c^2*x^2) + 104*Sqrt[1
 + c^2*x^2]*ArcTan[Tanh[ArcSinh[c*x]/2]] - 6*Sqrt[1 + c^2*x^2]*Coth[ArcSinh
[c*x]/2] - 3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - 60*Sqr
t[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] + 60*Sqrt[1 + c^2*x^
2]*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] - 60*Sqrt[1 + c^2*x^2]*PolyLog[2
, -E^(-ArcSinh[c*x])] + 60*Sqrt[1 + c^2*x^2]*PolyLog[2, E^(-ArcSinh[c*x])]
- 3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 6*Sqrt[1 + c^2*
x^2]*Tanh[ArcSinh[c*x]/2]))/(24*d^2*Sqrt[d*(1 + c^2*x^2)])
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\,(b\operatorname{arsinh}(cx)+a)}{c^6d^3x^9+3\,c^4d^3x^7+3\,c^2d^3x^5+d^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^6*d^3*x^9 + 3*c^4*d^3*
x^7 + 3*c^2*d^3*x^5 + d^3*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{b\operatorname{arsinh}(cx)+a}{\left(c^2dx^2+d\right)^{\frac{5}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^(5/2)*x^3), x)
```

maple [A] time = 0.31, size = 546, normalized size = 1.36

$$-\frac{a}{2d\,x^2\left(c^2d\,x^2+d\right)^{\frac{3}{2}}}-\frac{5a\,c^2}{6d\left(c^2d\,x^2+d\right)^{\frac{3}{2}}}-\frac{5a\,c^2}{2d^2\sqrt{c^2d\,x^2+d}}+\frac{5a\,c^2\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2d\,x^2+d}}{x}\right)}{2d^{\frac{5}{2}}}-\frac{5b\sqrt{d\left(c^2x^2+1\right)}\,x^2\operatorname{arc}}{2\left(c^4x^4+2c^2x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] -1/2*a/d/x^2/(c^2*d*x^2+d)^(3/2)-5/6*a*c^2/d/(c^2*d*x^2+d)^(3/2)-5/2*a*c^2/
d^2/(c^2*d*x^2+d)^(1/2)+5/2*a*c^2/d^(5/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(
1/2))/x)-5/2*b*(d*(c^2*x^2+1))^(1/2)/(c^4*x^4+2*c^2*x^2+1)/d^3*x^2*arcsinh(
c*x)*c^4-1/3*b*(d*(c^2*x^2+1))^(1/2)/(c^4*x^4+2*c^2*x^2+1)/d^3*x*c^3*(c^2*x
^2+1)^(1/2)-10/3*b*(d*(c^2*x^2+1))^(1/2)/(c^4*x^4+2*c^2*x^2+1)/d^3*arcsinh(
c*x)*c^2-1/2*b*(d*(c^2*x^2+1))^(1/2)/(c^4*x^4+2*c^2*x^2+1)/d^3/x*c*(c^2*x^2
+1)^(1/2)-1/2*b*(d*(c^2*x^2+1))^(1/2)/(c^4*x^4+2*c^2*x^2+1)/d^3/x^2*arcsinh
(c*x)+13/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*arctan(c*x+(c^2*x^
2+1)^(1/2))*c^2+5/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*dilog(c*x
+(c^2*x^2+1)^(1/2))*c^2+5/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*d
ilog(1+c*x+(c^2*x^2+1)^(1/2))*c^2+5/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(
1/2)/d^3*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}\,a\left(\frac{15\,c^2\operatorname{arsinh}\left(\frac{1}{c|x|}\right)}{d^{\frac{5}{2}}}-\frac{15\,c^2}{\sqrt{c^2dx^2+d}\,d^2}-\frac{5\,c^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}d}-\frac{3}{\left(c^2dx^2+d\right)^{\frac{3}{2}}dx^2}\right)+b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{\left(c^2dx^2+d\right)^{\frac{5}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*a*(15*c^2*arcsinh(1/(c*abs(x)))/d^(5/2) - 15*c^2/(sqrt(c^2*d*x^2 + d)*d
^2) - 5*c^2/((c^2*d*x^2 + d)^(3/2)*d) - 3/((c^2*d*x^2 + d)^(3/2)*d*x^2)) +
b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/((c^2*d*x^2 + d)^(5/2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}\left(c\,x\right)}{x^3\,{\left(d\,c^2\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)^(5/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^3*(d + c^2*d*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asinh}{\left(c x \right)}}{x^{3} \left(d \left(c^{2} x^{2} + 1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**3/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(x**3*(d*(c**2*x**2 + 1))**(5/2)), x)
```

3.175 $\int \frac{a+b\sinh^{-1}(cx)}{x^4\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=297

$$\frac{2c^2\left(a+b\sinh^{-1}(cx)\right)}{dx\left(c^2dx^2+d\right)^{3/2}}-\frac{a+b\sinh^{-1}(cx)}{3dx^3\left(c^2dx^2+d\right)^{3/2}}+\frac{16c^4x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{bc\sqrt{c^2dx^2+}}{6d^3x^2\sqrt{c^2x^2}}$$

```
[Out] 1/3*(-a-b*arcsinh(c*x))/d/x^3/(c^2*d*x^2+d)^(3/2)+2*c^2*(a+b*arcsinh(c*x))/
d/x/(c^2*d*x^2+d)^(3/2)+8/3*c^4*x*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(3/2)+
16/3*c^4*x*(a+b*arcsinh(c*x))/d^2/(c^2*d*x^2+d)^(1/2)+1/6*b*c^3*(c^2*d*x^2+
d)^(1/2)/d^3/(c^2*x^2+1)^(3/2)-1/6*b*c*(c^2*d*x^2+d)^(1/2)/d^3/x^2/(c^2*x^2
+1)^(1/2)-8/3*b*c^3*ln(x)*(c^2*d*x^2+d)^(1/2)/d^3/(c^2*x^2+1)^(1/2)-4/3*b*c
^3*ln(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5747, 5690, 5687, 260, 261, 266, 44}

$$\frac{16c^4x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(c^2dx^2+d\right)^{3/2}}+\frac{2c^2\left(a+b\sinh^{-1}(cx)\right)}{dx\left(c^2dx^2+d\right)^{3/2}}-\frac{a+b\sinh^{-1}(cx)}{3dx^3\left(c^2dx^2+d\right)^{3/2}}+\frac{b}{6d^2\sqrt{c^2x^2+}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] (b*c^3)/(6*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (b*c*Sqrt[1 + c^2*x
^2])/(6*d^2*x^2*Sqrt[d + c^2*d*x^2]) - (a + b*ArcSinh[c*x])/(3*d*x^3*(d + c
^2*d*x^2)^(3/2)) + (2*c^2*(a + b*ArcSinh[c*x]))/(d*x*(d + c^2*d*x^2)^(3/2))
 + (8*c^4*x*(a + b*ArcSinh[c*x]))/(3*d*(d + c^2*d*x^2)^(3/2)) + (16*c^4*x*(
a + b*ArcSinh[c*x]))/(3*d^2*Sqrt[d + c^2*d*x^2]) - (8*b*c^3*Sqrt[1 + c^2*x^
2]*Log[x])/(3*d^2*Sqrt[d + c^2*d*x^2]) - (4*b*c^3*Sqrt[1 + c^2*x^2]*Log[1 +
 c^2*x^2])/(3*d^2*Sqrt[d + c^2*d*x^2])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^4\left(d+c^2dx^2\right)^{5/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{3dx^3\left(d+c^2dx^2\right)^{3/2}} - \left(2c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(d+c^2dx^2\right)^{5/2}}\,dx + \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{x^3\left(1+c^2x^2\right)^2}\,dx}{3d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{3dx^3\left(d+c^2dx^2\right)^{3/2}} + \frac{2c^2\left(a+b\sinh^{-1}(cx)\right)}{dx\left(d+c^2dx^2\right)^{3/2}} + \left(8c^4\right)\int \frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^{5/2}}\,dx + \frac{\left(bc\sqrt{1\right.}}{}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{3dx^3\left(d+c^2dx^2\right)^{3/2}} + \frac{2c^2\left(a+b\sinh^{-1}(cx)\right)}{dx\left(d+c^2dx^2\right)^{3/2}} + \frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{\left(16c^4\right)\int \frac{a+}{(}}{3}$$

$$= \frac{7bc^3}{6d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}}{6d^2x^2\sqrt{d+c^2dx^2}} - \frac{a+b\sinh^{-1}(cx)}{3dx^3\left(d+c^2dx^2\right)^{3/2}} + \frac{2c^2\left(a+b\sinh\right.}{dx\left(d+c^2dx\right.}$$

$$= \frac{bc^3}{6d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}}{6d^2x^2\sqrt{d+c^2dx^2}} - \frac{a+b\sinh^{-1}(cx)}{3dx^3\left(d+c^2dx^2\right)^{3/2}} + \frac{2c^2\left(a+b\sinh\right.}{dx\left(d+c^2dx\right.}$$

Mathematica [A] time = 0.36, size = 267, normalized size = 0.90

$$\sqrt{c^2dx^2+d}\left(12ac^2x^2\sqrt{c^2x^2+1} - 2a\sqrt{c^2x^2+1} + 32ac^6x^6\sqrt{c^2x^2+1} + 48ac^4x^4\sqrt{c^2x^2+1} - bc^3x^3 - 16bc^7x^7\log\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(-(b*c*x) - b*c^3*x^3 - 2*a*Sqrt[1 + c^2*x^2] + 12*a*c
^2*x^2*Sqrt[1 + c^2*x^2] + 48*a*c^4*x^4*Sqrt[1 + c^2*x^2] + 32*a*c^6*x^6*Sq
rt[1 + c^2*x^2] + 2*b*Sqrt[1 + c^2*x^2]*(-1 + 6*c^2*x^2 + 24*c^4*x^4 + 16*c
^6*x^6)*ArcSinh[c*x] + 8*b*c^3*x^3*(1 + c^2*x^2)^2*Log[1 + 1/(c^2*x^2)] - 1
6*b*c^3*x^3*Log[1 + c^2*x^2] - 32*b*c^5*x^5*Log[1 + c^2*x^2] - 16*b*c^7*x^7
*Log[1 + c^2*x^2]))/(6*d^3*x^3*(1 + c^2*x^2)^(5/2))
```

fricas [F] time = 1.10, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\,(b\,\text{arsinh}\,(cx)+a)}{c^6d^3x^{10}+3\,c^4d^3x^8+3\,c^2d^3x^6+d^3x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^6*d^3*x^10 + 3*c^4*d^3
*x^8 + 3*c^2*d^3*x^6 + d^3*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{b\,\text{arsinh}\,(cx)+a}{\left(c^2dx^2+d\right)^{\frac{5}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^(5/2)*x^4), x)
```

maple [B] time = 0.29, size = 1790, normalized size = 6.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] 32/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*arcsinh(c*x)*c^3+2*a*c^2
/d/x/(c^2*d*x^2+d)^(3/2)-1/3*a/d/x^3/(c^2*d*x^2+d)^(3/2)-64*b*(d*(c^2*x^2+1
))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^6*arcsinh(c*
x)*(c^2*x^2+1)^(1/2)*c^9-128*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6
+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^4*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^7-176/3*
b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3
*x^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^5-128/3*b*(d*(c^2*x^2+1))^(1/2)/(12*c
^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^9*(c^2*x^2+1)*c^12-320/3*b
*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*
x^7*(c^2*x^2+1)*c^10+64*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c
^4*x^4+10*c^2*x^2-1)/d^3*x^7*arcsinh(c*x)*c^10-80*b*(d*(c^2*x^2+1))^(1/2)/(
12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^5*(c^2*x^2+1)*c^8+160*
b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3
*x^5*arcsinh(c*x)*c^8-40/3*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+3
5*c^4*x^4+10*c^2*x^2-1)/d^3*x^3*(c^2*x^2+1)*c^6+344/3*b*(d*(c^2*x^2+1))^(1/
2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^3*arcsinh(c*x)*c^6
-2*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/
d^3*x^2*c^5*(c^2*x^2+1)^(1/2)+16/3*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c
^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^3+8/3*
```

```
b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3
*x*(c^2*x^2+1)*c^4+12*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4
*x^4+10*c^2*x^2-1)/d^3*x*arcsinh(c*x)*c^4-6*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8
*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3/x*arcsinh(c*x)*c^2+1/6*b*(d*(c
^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3/x^2*c*
(c^2*x^2+1)^(1/2)-2*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x
^4+10*c^2*x^2-1)/d^3*c^3*(c^2*x^2+1)^(1/2)+128/3*b*(d*(c^2*x^2+1))^(1/2)/(1
2*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^11*c^14+448/3*b*(d*(c^2
*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^9*c^12
+560/3*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2
-1)/d^3*x^7*c^10+280/3*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^
4*x^4+10*c^2*x^2-1)/d^3*x^5*c^8+32/3*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36
*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^3*c^6-8/3*b*(d*(c^2*x^2+1))^(1/2)/(
12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x*c^4+1/3*b*(d*(c^2*x^2+
1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3/x^3*arcsinh(c
*x)-8/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*ln((c*x+(c^2*x^2+1)^(
1/2))^4-1)*c^3+16/3*a*c^4/d^2*x/(c^2*d*x^2+d)^(1/2)+8/3*a*c^4*x/d/(c^2*d*x^
2+d)^(3/2)
```

maxima [A] time = 0.44, size = 236, normalized size = 0.79

$$-\frac{1}{6}\,bc\left(\frac{8\,c^2\log\left(c^2x^2+1\right)}{d^{\frac{5}{2}}}+\frac{16\,c^2\log(x)}{d^{\frac{5}{2}}}+\frac{1}{c^2d^{\frac{5}{2}}x^4+d^{\frac{5}{2}}x^2}\right)+\frac{1}{3}\left(\frac{16\,c^4x}{\sqrt{c^2dx^2+d}\,d^2}+\frac{8\,c^4x}{\left(c^2dx^2+d\right)^{\frac{3}{2}}d}+\frac{6\,c^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/6*b*c*(8*c^2*log(c^2*x^2 + 1)/d^(5/2) + 16*c^2*log(x)/d^(5/2) + 1/(c^2*d
^(5/2)*x^4 + d^(5/2)*x^2)) + 1/3*(16*c^4*x/(sqrt(c^2*d*x^2 + d)*d^2) + 8*c^
4*x/((c^2*d*x^2 + d)^(3/2)*d) + 6*c^2/((c^2*d*x^2 + d)^(3/2)*d*x) - 1/((c^2
*d*x^2 + d)^(3/2)*d*x^3))*b*arcsinh(c*x) + 1/3*(16*c^4*x/(sqrt(c^2*d*x^2 +
d)*d^2) + 8*c^4*x/((c^2*d*x^2 + d)^(3/2)*d) + 6*c^2/((c^2*d*x^2 + d)^(3/2)*
d*x) - 1/((c^2*d*x^2 + d)^(3/2)*d*x^3))*a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}\left(c\,x\right)}{x^4\,{\left(d\,c^2\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)^(5/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/(x^4*(d + c^2*d*x^2)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**4/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

3.176 $\int \frac{\sinh^{-1}(ax)}{\left(c+a^2cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=200

$$\frac{2}{15ac^3\sqrt{a^2x^2+1}\,\sqrt{a^2cx^2+c}}+\frac{1}{20ac^3\left(a^2x^2+1\right)^{3/2}\sqrt{a^2cx^2+c}}-\frac{4\sqrt{a^2x^2+1}\,\log\left(a^2x^2+1\right)}{15ac^3\sqrt{a^2cx^2+c}}+\frac{8x\sinh^{-1}(ax)}{15c^3\sqrt{a^2cx^2+c}}+$$

```
[Out] 1/5*x*arcsinh(a*x)/c/(a^2*c*x^2+c)^(5/2)+4/15*x*arcsinh(a*x)/c^2/(a^2*c*x^2
+c)^(3/2)+1/20/a/c^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2)+8/15*x*arcsinh(a
*x)/c^3/(a^2*c*x^2+c)^(1/2)+2/15/a/c^3/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2
)-4/15*ln(a^2*x^2+1)*(a^2*x^2+1)^(1/2)/a/c^3/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {5690, 5687, 260, 261}

$$\frac{2}{15ac^3\sqrt{a^2x^2+1}\,\sqrt{a^2cx^2+c}}+\frac{1}{20ac^3\left(a^2x^2+1\right)^{3/2}\sqrt{a^2cx^2+c}}-\frac{4\sqrt{a^2x^2+1}\,\log\left(a^2x^2+1\right)}{15ac^3\sqrt{a^2cx^2+c}}+\frac{8x\sinh^{-1}(ax)}{15c^3\sqrt{a^2cx^2+c}}+$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]/(c + a^2*c*x^2)^(7/2),x]
```

```
[Out] 1/(20*a*c^3*(1 + a^2*x^2)^(3/2)*Sqrt[c + a^2*c*x^2]) + 2/(15*a*c^3*Sqrt[1 +
 a^2*x^2]*Sqrt[c + a^2*c*x^2]) + (x*ArcSinh[a*x])/(5*c*(c + a^2*c*x^2)^(5/2
)) + (4*x*ArcSinh[a*x])/(15*c^2*(c + a^2*c*x^2)^(3/2)) + (8*x*ArcSinh[a*x])
/(15*c^3*Sqrt[c + a^2*c*x^2]) - (4*Sqrt[1 + a^2*x^2]*Log[1 + a^2*x^2])/(15*
a*c^3*Sqrt[c + a^2*c*x^2])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)}{\left(c+a^2cx^2\right)^{7/2}}\,dx = \frac{x\sinh^{-1}(ax)}{5c\left(c+a^2cx^2\right)^{5/2}} + \frac{4\int \frac{\sinh^{-1}(ax)}{\left(c+a^2cx^2\right)^{5/2}}\,dx}{5c} - \frac{\left(a\sqrt{1+a^2x^2}\right)\int \frac{x}{\left(1+a^2x^2\right)^3}\,dx}{5c^3\sqrt{c+a^2cx^2}}$$

$$= \frac{1}{20ac^3\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)}{5c\left(c+a^2cx^2\right)^{5/2}} + \frac{4x\sinh^{-1}(ax)}{15c^2\left(c+a^2cx^2\right)^{3/2}} + \frac{8\int \frac{\sinh^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}}}{15c^2}$$

$$= \frac{1}{20ac^3\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}} + \frac{2}{15ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)}{5c\left(c+a^2cx^2\right)^{5/2}} + \frac{4x}{15c^2}$$

$$= \frac{1}{20ac^3\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}} + \frac{2}{15ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)}{5c\left(c+a^2cx^2\right)^{5/2}} + \frac{4x}{15c^2}$$

Mathematica [A] time = 0.15, size = 121, normalized size = 0.60

$$\frac{\sqrt{a^2cx^2+c}\left(4ax\sqrt{a^2x^2+1}\left(8a^4x^4+20a^2x^2+15\right)\sinh^{-1}(ax)-\left(a^2x^2+1\right)\left(-8a^2x^2+16\left(a^2x^2+1\right)^2\log\left(a^2x^2+\right.\right.\right.}{60ac^4\left(a^2x^2+1\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]/(c + a^2*c*x^2)^(7/2),x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(4*a*x*Sqrt[1 + a^2*x^2]*(15 + 20*a^2*x^2 + 8*a^4*x^4)
*ArcSinh[a*x] - (1 + a^2*x^2)*(-11 - 8*a^2*x^2 + 16*(1 + a^2*x^2)^2*Log[1 +
 a^2*x^2])))/(60*a*c^4*(1 + a^2*x^2)^(7/2))
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\ \text{arsinh}\,(ax)}{a^8c^4x^8+4\,a^6c^4x^6+6\,a^4c^4x^4+4\,a^2c^4x^2+c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/(a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)/(a^8*c^4*x^8 + 4*a^6*c^4*x^6 + 6*
a^4*c^4*x^4 + 4*a^2*c^4*x^2 + c^4), x)
```

giac [A] time = 0.68, size = 124, normalized size = 0.62

$$-\frac{1}{60}\sqrt{c}\left(\frac{16\,\log\left(a^2x^2+1\right)}{ac^4}-\frac{24\,a^4x^4+56\,a^2x^2+35}{\left(a^2x^2+1\right)^2ac^4}\right)+\frac{\left(4\left(\frac{2\,a^4x^2}{c}+\frac{5\,a^2}{c}\right)x^2+\frac{15}{c}\right)x\log\left(ax+\sqrt{a^2x^2+1}\right)}{15\left(a^2cx^2+c\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/(a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] -1/60*sqrt(c)*(16*log(a^2*x^2 + 1)/(a*c^4) - (24*a^4*x^4 + 56*a^2*x^2 + 35)
/((a^2*x^2 + 1)^2*a*c^4)) + 1/15*(4*(2*a^4*x^2/c + 5*a^2/c)*x^2 + 15/c)*x*l
og(a*x + sqrt(a^2*x^2 + 1))/(a^2*c*x^2 + c)^(5/2)
```

maple [B] time = 0.00, size = 363, normalized size = 1.82

$$\frac{16\sqrt{c\left(a^2x^2+1\right)}\ \mathrm{arcsinh}\left(ax\right)}{15\sqrt{a^2x^2+1}\ a\,c^4}+\frac{\sqrt{c\left(a^2x^2+1\right)}\ \left(8x^5a^5-8\sqrt{a^2x^2+1}\ x^4a^4+20x^3a^3-16\sqrt{a^2x^2+1}\ x^2a^2+15a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)/(a^2*c*x^2+c)^(7/2),x)
```

```
[Out] 16/15*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a/c^4*arcsinh(a*x)+1/60*(c*(a
^2*x^2+1))^(1/2)*(8*x^5*a^5-8*(a^2*x^2+1)^(1/2)*x^4*a^4+20*x^3*a^3-16*(a^2*
x^2+1)^(1/2)*x^2*a^2+15*a*x-8*(a^2*x^2+1)^(1/2))*(-64*x^8*a^8-64*(a^2*x^2+1
)^(1/2)*x^7*a^7-280*x^6*a^6-248*(a^2*x^2+1)^(1/2)*x^5*a^5+160*arcsinh(a*x)*
x^4*a^4-456*x^4*a^4-340*(a^2*x^2+1)^(1/2)*x^3*a^3+380*arcsinh(a*x)*x^2*a^2-
328*a^2*x^2-165*(a^2*x^2+1)^(1/2)*x*a+256*arcsinh(a*x)-88)/(40*a^10*x^10+21
5*a^8*x^8+469*a^6*x^6+517*a^4*x^4+287*a^2*x^2+64)/a/c^4-8/15*(c*(a^2*x^2+1)
)^(1/2)/(a^2*x^2+1)^(1/2)/a/c^4*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2)
```

maxima [A] time = 0.42, size = 143, normalized size = 0.72

$$\frac{1}{60}a\left(\frac{3}{\left(a^6c^{\frac{5}{2}}x^4+2\,a^4c^{\frac{5}{2}}x^2+a^2c^{\frac{5}{2}}\right)c}+\frac{8}{\left(a^4c^{\frac{3}{2}}x^2+a^2c^{\frac{3}{2}}\right)c^2}-\frac{16\,\log\left(x^2+\frac{1}{a^2}\right)}{a^2c^{\frac{7}{2}}}\right)+\frac{1}{15}\left(\frac{8\,x}{\sqrt{a^2cx^2+c}\,c^3}+\frac{4\,x}{\left(a^2cx^2+c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/(a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/60*a*(3/((a^6*c^(5/2)*x^4 + 2*a^4*c^(5/2)*x^2 + a^2*c^(5/2))*c) + 8/((a^4
*c^(3/2)*x^2 + a^2*c^(3/2))*c^2) - 16*log(x^2 + 1/a^2)/(a^2*c^(7/2))) + 1/1
5*(8*x/(sqrt(a^2*c*x^2 + c)*c^3) + 4*x/((a^2*c*x^2 + c)^(3/2)*c^2) + 3*x/((
a^2*c*x^2 + c)^(5/2)*c))*arcsinh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{asinh}\left(a\,x\right)}{{\left(c\,a^2\,x^2+c\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)/(c + a^2*c*x^2)^(7/2),x)
```

```
[Out] int(asinh(a*x)/(c + a^2*c*x^2)^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{asinh}\left(ax\right)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)/(a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Integral(asinh(a*x)/(c*(a**2*x**2 + 1))**(7/2), x)
```

3.177 $\int \frac{x^4 \sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=86

$$\frac{3\sinh^{-1}(ax)^2}{16a^5}+\frac{3x^2}{16a^3}+\frac{x^3\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{4a^2}-\frac{3x\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{8a^4}-\frac{x^4}{16a}$$

```
[Out] 3/16*x^2/a^3-1/16*x^4/a+3/16*arcsinh(a*x)^2/a^5-3/8*x*arcsinh(a*x)*(a^2*x^2
+1)^(1/2)/a^4+1/4*x^3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.15, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5758, 5675, 30}

$$\frac{3x^2}{16a^3}+\frac{x^3\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{4a^2}-\frac{3x\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{8a^4}+\frac{3\sinh^{-1}(ax)^2}{16a^5}-\frac{x^4}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (3*x^2)/(16*a^3) - x^4/(16*a) - (3*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(8*a^4
) + (x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(4*a^2) + (3*ArcSinh[a*x]^2)/(16*a
^5)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx &= \frac{x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{4a^2}-\frac{3\int \frac{x^2\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{4a^2}-\frac{\int x^3\,dx}{4a}\\
&= -\frac{x^4}{16a}-\frac{3x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{8a^4}+\frac{x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{4a^2}+\frac{3\int \frac{\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{8a^4}+\frac{3\int x\,dx}{8a^3}\\
&= \frac{3x^2}{16a^3}-\frac{x^4}{16a}-\frac{3x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{8a^4}+\frac{x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{4a^2}+\frac{3\sinh^{-1}(ax)^2}{16a^5}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 63, normalized size = 0.73

$$\frac{-a^4x^4+3a^2x^2+2ax\sqrt{a^2x^2+1}\left(2a^2x^2-3\right)\sinh^{-1}(ax)+3\sinh^{-1}(ax)^2}{16a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (3*a^2*x^2 - a^4*x^4 + 2*a*x*Sqrt[1 + a^2*x^2]*(-3 + 2*a^2*x^2)*ArcSinh[a*x
] + 3*ArcSinh[a*x]^2)/(16*a^5)
```

fricas [A] time = 0.59, size = 83, normalized size = 0.97

$$-\frac{a^4x^4-3\,a^2x^2-2\left(2\,a^3x^3-3\,ax\right)\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1}\right)-3\,\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{16\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/16*(a^4*x^4 - 3*a^2*x^2 - 2*(2*a^3*x^3 - 3*a*x)*sqrt(a^2*x^2 + 1)*log(a*
x + sqrt(a^2*x^2 + 1)) - 3*log(a*x + sqrt(a^2*x^2 + 1))^2)/a^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4*arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

maple [A] time = 0.00, size = 74, normalized size = 0.86

$$\frac{4\operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}\,a^3x^3-x^4a^4-6\operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}\,ax+3a^2x^2+3\operatorname{arcsinh}(ax)^2+3}{16a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/16*(4*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a^3*x^3-x^4*a^4-6*arcsinh(a*x)*(a^2*
x^2+1)^(1/2)*a*x+3*a^2*x^2+3*arcsinh(a*x)^2+3)/a^5
```

maxima [A] time = 0.42, size = 83, normalized size = 0.97

$$-\frac{1}{16}\left(\frac{x^4}{a^2}-\frac{3\,x^2}{a^4}+\frac{3\,\operatorname{arsinh}(ax)^2}{a^6}\right)a+\frac{1}{8}\left(\frac{2\,\sqrt{a^2x^2+1}\,x^3}{a^2}-\frac{3\,\sqrt{a^2x^2+1}\,x}{a^4}+\frac{3\,\operatorname{arsinh}(ax)}{a^5}\right)\operatorname{arsinh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*(x^4/a^2 - 3*x^2/a^4 + 3*arcsinh(a*x)^2/a^6)*a + 1/8*(2*sqrt(a^2*x^2
+ 1)*x^3/a^2 - 3*sqrt(a^2*x^2 + 1)*x/a^4 + 3*arcsinh(a*x)/a^5)*arcsinh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\operatorname{asinh}(a\,x)}{\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*asinh(a*x))/(a^2*x^2 + 1)^(1/2),x)
```

```
[Out] int((x^4*asinh(a*x))/(a^2*x^2 + 1)^(1/2), x)
```

sympy [A]   time = 2.27, size = 82, normalized size = 0.95

$$\begin{cases} -\frac{x^4}{16a} + \frac{x^3\sqrt{a^2x^2+1}\,\operatorname{asinh}(ax)}{4a^2} + \frac{3x^2}{16a^3} - \frac{3x\sqrt{a^2x^2+1}\,\operatorname{asinh}(ax)}{8a^4} + \frac{3\operatorname{asinh}^2(ax)}{16a^5} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-x**4/(16*a) + x**3*sqrt(a**2*x**2 + 1)*asinh(a*x)/(4*a**2) + 3*
x**2/(16*a**3) - 3*x*sqrt(a**2*x**2 + 1)*asinh(a*x)/(8*a**4) + 3*asinh(a*x)
**2/(16*a**5), Ne(a, 0)), (0, True))
```

3.178 $\int \frac{x^3 \sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=70

$$\frac{2x}{3a^3}+\frac{x^2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{3a^2}-\frac{2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{3a^4}-\frac{x^3}{9a}$$

```
[Out] 2/3*x/a^3-1/9*x^3/a-2/3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^4+1/3*x^2*arcsinh(
a*x)*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.10, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5758, 5717, 8, 30}

$$\frac{x^2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{3a^2}-\frac{2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{3a^4}+\frac{2x}{3a^3}-\frac{x^3}{9a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (2*x)/(3*a^3) - x^3/(9*a) - (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(3*a^4) + (x
^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(3*a^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^3 \sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{3a^2} - \frac{2\int \frac{x\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{3a^2} - \frac{\int x^2\,dx}{3a}$$
$$= -\frac{x^3}{9a} - \frac{2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{3a^4} + \frac{x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{3a^2} + \frac{2\int 1\,dx}{3a^3}$$
$$= \frac{2x}{3a^3} - \frac{x^3}{9a} - \frac{2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{3a^4} + \frac{x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{3a^2}$$

Mathematica [A] time = 0.04, size = 48, normalized size = 0.69

$$\frac{-a^3x^3 + 3\left(a^2x^2-2\right)\sqrt{a^2x^2+1}\,\sinh^{-1}(ax) + 6ax}{9a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (6*a*x - a^3*x^3 + 3*(-2 + a^2*x^2)*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(9*a^4)
```

fricas [A] time = 0.71, size = 55, normalized size = 0.79

$$-\frac{a^3x^3 - 3\sqrt{a^2x^2+1}\left(a^2x^2-2\right)\log\left(ax+\sqrt{a^2x^2+1}\right) - 6\,ax}{9\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/9*(a^3*x^3 - 3*sqrt(a^2*x^2 + 1)*(a^2*x^2 - 2)*log(a*x + sqrt(a^2*x^2 +
1)) - 6*a*x)/a^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.00, size = 82, normalized size = 1.17

$$\frac{3\,\mathrm{arcsinh}\,(ax)\,x^4a^4 - 3\,\mathrm{arcsinh}\,(ax)\,x^2a^2 - \sqrt{a^2x^2+1}\;x^3a^3 - 6\,\mathrm{arcsinh}\,(ax) + 6\sqrt{a^2x^2+1}\;xa}{9a^4\sqrt{a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/9/a^4*(3*arcsinh(a*x)*x^4*a^4-3*arcsinh(a*x)*x^2*a^2-(a^2*x^2+1)^(1/2)*x^
3*a^3-6*arcsinh(a*x)+6*(a^2*x^2+1)^(1/2)*x*a)/(a^2*x^2+1)^(1/2)
```

maxima [A] time = 0.40, size = 59, normalized size = 0.84

$$-\frac{1}{9}\,a\left(\frac{x^3}{a^2} - \frac{6\,x}{a^4}\right) + \frac{1}{3}\left(\frac{\sqrt{a^2x^2+1}\,x^2}{a^2} - \frac{2\,\sqrt{a^2x^2+1}}{a^4}\right)\mathrm{arsinh}\,(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/9*a*(x^3/a^2 - 6*x/a^4) + 1/3*(sqrt(a^2*x^2 + 1)*x^2/a^2 - 2*sqrt(a^2*x^
2 + 1)/a^4)*arcsinh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \operatorname{asinh}(a\,x)}{\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*asinh(a*x))/(a^2*x^2 + 1)^(1/2),x)
```

```
[Out] int((x^3*asinh(a*x))/(a^2*x^2 + 1)^(1/2), x)
```

sympy [A] time = 1.20, size = 65, normalized size = 0.93

$$\begin{cases} -\frac{x^3}{9a} + \frac{x^2\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{3a^2} + \frac{2x}{3a^3} - \frac{2\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{3a^4} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-x**3/(9*a) + x**2*sqrt(a**2*x**2 + 1)*asinh(a*x)/(3*a**2) + 2*x
/(3*a**3) - 2*sqrt(a**2*x**2 + 1)*asinh(a*x)/(3*a**4), Ne(a, 0)), (0, True)
)
```

3.179 $\int \frac{x^2 \sinh^{-1}(ax)}{\sqrt{1+a^2x^2}} \, dx$

Optimal. Leaf size=49

$$-\frac{\sinh^{-1}(ax)^2}{4a^3}+\frac{x\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{2a^2}-\frac{x^2}{4a}$$

```
[Out] -1/4*x^2/a-1/4*arcsinh(a*x)^2/a^3+1/2*x*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.08, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5758, 5675, 30}

$$\frac{x\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{2a^2}-\frac{\sinh^{-1}(ax)^2}{4a^3}-\frac{x^2}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] -x^2/(4*a) + (x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*a^2) - ArcSinh[a*x]^2/(4
*a^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx &= \frac{x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{2a^2}-\frac{\int \frac{\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{2a^2}-\frac{\int x\,dx}{2a}\\ &= -\frac{x^2}{4a}+\frac{x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{2a^2}-\frac{\sinh^{-1}(ax)^2}{4a^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 42, normalized size = 0.86

$$-\frac{a^2x^2-2ax\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)+\sinh^{-1}(ax)^2}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
[Out] -1/4*(a^2*x^2 - 2*a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] + ArcSinh[a*x]^2)/a^3
```

fricas [A] time = 0.52, size = 62, normalized size = 1.27

$$-\frac{a^2x^2 - 2\sqrt{a^2x^2+1}\,ax\log\left(ax+\sqrt{a^2x^2+1}\right) + \log\left(ax+\sqrt{a^2x^2+1}\right)^2}{4\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] -1/4*(a^2*x^2 - 2*sqrt(a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 + 1)) + log(
a*x + sqrt(a^2*x^2 + 1))^2)/a^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^2*arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

maple [A] time = 0.00, size = 40, normalized size = 0.82

$$-\frac{-2\operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}\;ax + a^2x^2 + \operatorname{arcsinh}(ax)^2 + 1}{4a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
[Out] -1/4*(-2*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a*x+a^2*x^2+arcsinh(a*x)^2+1)/a^3
```

maxima [A] time = 0.54, size = 55, normalized size = 1.12

$$-\frac{1}{4}a\left(\frac{x^2}{a^2} - \frac{\operatorname{arsinh}(ax)^2}{a^4}\right) + \frac{1}{2}\left(\frac{\sqrt{a^2x^2+1}\,x}{a^2} - \frac{\operatorname{arsinh}(ax)}{a^3}\right)\operatorname{arsinh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] -1/4*a*(x^2/a^2 - arcsinh(a*x)^2/a^4) + 1/2*(sqrt(a^2*x^2 + 1)*x/a^2 - arcs
inh(a*x)/a^3)*arcsinh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2 \operatorname{asinh}(a\,x)}{\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*asinh(a*x))/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x^2*asinh(a*x))/(a^2*x^2 + 1)^(1/2), x)
```

sympy [A] time = 0.75, size = 42, normalized size = 0.86

$$\begin{cases} -\frac{x^2}{4a} + \frac{x\sqrt{a^2x^2+1}\,\operatorname{asinh}(ax)}{2a^2} - \frac{\operatorname{asinh}^2(ax)}{4a^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-x**2/(4*a) + x*sqrt(a**2*x**2 + 1)*asinh(a*x)/(2*a**2) - asinh(
a*x)**2/(4*a**3), Ne(a, 0)), (0, True))
```

3.180 $\int \frac{x\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=28

$$\frac{\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{a^2}-\frac{x}{a}$$

[Out] -x/a+arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^2

Rubi [A] time = 0.04, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5717, 8}

$$\frac{\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{a^2}-\frac{x}{a}$$

Antiderivative was successfully verified.

[In] Int[(x*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]

[Out] -(x/a) + (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^2

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx &= \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{a^2}-\frac{\int 1\,dx}{a}\\ &= -\frac{x}{a}+\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 28, normalized size = 1.00

$$\frac{\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{a^2}-\frac{x}{a}$$

Antiderivative was successfully verified.

[In] Integrate[(x*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]

[Out] -(x/a) + (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^2

fricas [A] time = 0.50, size = 38, normalized size = 1.36

$$-\frac{ax-\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(a*x - sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/a^2
```

giac [A] time = 0.51, size = 38, normalized size = 1.36

$$-\frac{x}{a} + \frac{\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -x/a + sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))/a^2
```

maple [A] time = 0.00, size = 47, normalized size = 1.68

$$\frac{\operatorname{arcsinh}(ax)\,x^2a^2 + \operatorname{arcsinh}(ax) - \sqrt{a^2x^2+1}\,xa}{a^2\sqrt{a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/a^2/(a^2*x^2+1)^(1/2)*(arcsinh(a*x)*x^2*a^2+arcsinh(a*x)-(a^2*x^2+1)^(1/2
)*x*a)
```

maxima [A] time = 0.41, size = 26, normalized size = 0.93

$$-\frac{x}{a} + \frac{\sqrt{a^2x^2+1}\,\operatorname{arsinh}(ax)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -x/a + sqrt(a^2*x^2 + 1)*arcsinh(a*x)/a^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\,\operatorname{asinh}(a\,x)}{\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*asinh(a*x))/(a^2*x^2 + 1)^(1/2),x)
```

```
[Out] int((x*asinh(a*x))/(a^2*x^2 + 1)^(1/2), x)
```

sympy [A] time = 0.43, size = 24, normalized size = 0.86

$$\begin{cases} -\frac{x}{a} + \frac{\sqrt{a^2x^2+1}\,\operatorname{asinh}(ax)}{a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-x/a + sqrt(a**2*x**2 + 1)*asinh(a*x)/a**2, Ne(a, 0)), (0, True)
)
```

3.181 $\int \frac{\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}} dx$

Optimal. Leaf size=13

$$\frac{\sinh^{-1}(ax)^2}{2a}$$

[Out] 1/2*arcsinh(a*x)^2/a

Rubi [A] time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {5675}

$$\frac{\sinh^{-1}(ax)^2}{2a}$$

Antiderivative was successfully verified.

[In] Int[ArcSinh[a*x]/Sqrt[1 + a^2*x^2],x]

[Out] ArcSinh[a*x]^2/(2*a)

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}} dx = \frac{\sinh^{-1}(ax)^2}{2a}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{\sinh^{-1}(ax)^2}{2a}$$

Antiderivative was successfully verified.

[In] Integrate[ArcSinh[a*x]/Sqrt[1 + a^2*x^2],x]

[Out] ArcSinh[a*x]^2/(2*a)

fricas [B] time = 0.65, size = 23, normalized size = 1.77

$$\frac{\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")

[Out] 1/2*log(a*x + sqrt(a^2*x^2 + 1))^2/a

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$\frac{\operatorname{arcsinh}(ax)^2}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/2*arcsinh(a*x)^2/a
```

maxima [A] time = 0.40, size = 11, normalized size = 0.85

$$\frac{\operatorname{arsinh}(ax)^2}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*arcsinh(a*x)^2/a
```

mupad [B] time = 0.10, size = 11, normalized size = 0.85

$$\frac{\operatorname{asinh}(a\,x)^2}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)/(a^2*x^2 + 1)^(1/2),x)
```

```
[Out] asinh(a*x)^2/(2*a)
```

sympy [A] time = 0.34, size = 10, normalized size = 0.77

$$\begin{cases} \frac{\operatorname{asinh}^{2}(ax)}{2a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((asinh(a*x)**2/(2*a), Ne(a, 0)), (0, True))
```

3.182 $\int \frac{\sinh^{-1}(ax)}{x\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=34

$$-\mathrm{Li}_2\left(-e^{\sinh^{-1}(ax)}\right)+\mathrm{Li}_2\left(e^{\sinh^{-1}(ax)}\right)-2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)$$

```
[Out] -2*arcsinh(a*x)*arctanh(a*x+(a^2*x^2+1)^(1/2))-polylog(2,-a*x-(a^2*x^2+1)^(
1/2))+polylog(2,a*x+(a^2*x^2+1)^(1/2))
```

Rubi [A] time = 0.09, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5760, 4182, 2279, 2391}

$$-\mathrm{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)+\mathrm{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)-2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]/(x*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -2*ArcSinh[a*x]*ArcTanh[E^ArcSinh[a*x]] - PolyLog[2, -E^ArcSinh[a*x]] + Pol
yLog[2, E^ArcSinh[a*x]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)}{x\sqrt{1+a^2x^2}}\,dx = \text{Subst}\left(\int x\text{csch}(x)\,dx, x, \sinh^{-1}(ax)\right)$$
$$= -2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - \text{Subst}\left(\int \log\left(1-e^x\right)\,dx, x, \sinh^{-1}(ax)\right) + \text{Subst}\left(\int \text{lo}\right.$$
$$= -2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - \text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{\sinh^{-1}(ax)}\right) + \text{Subst}\left(\int \frac{\log(}{}\right.$$
$$= -2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - \text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + \text{Li}_2\left(e^{\sinh^{-1}(ax)}\right)$$

Mathematica [A] time = 0.04, size = 57, normalized size = 1.68

$$\text{Li}_2\left(-e^{-\sinh^{-1}(ax)}\right) - \text{Li}_2\left(e^{-\sinh^{-1}(ax)}\right) + \sinh^{-1}(ax)\left(\log\left(1-e^{-\sinh^{-1}(ax)}\right) - \log\left(e^{-\sinh^{-1}(ax)}+1\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]/(x*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] ArcSinh[a*x]*(Log[1 - E^(-ArcSinh[a*x])] - Log[1 + E^(-ArcSinh[a*x])]) + Po
lyLog[2, -E^(-ArcSinh[a*x])] - PolyLog[2, E^(-ArcSinh[a*x])]
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a^2x^2+1}\,\text{arsinh}(ax)}{a^2x^3+x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)/(a^2*x^3 + x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arsinh}(ax)}{\sqrt{a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)/(sqrt(a^2*x^2 + 1)*x), x)
```

maple [A] time = 0.00, size = 42, normalized size = 1.24

$$2\,\text{dilog}\left(\frac{1}{ax+\sqrt{a^2x^2+1}}\right) - \frac{\text{dilog}\left(\frac{1}{\left(ax+\sqrt{a^2x^2+1}\right)^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)/x/(a^2*x^2+1)^(1/2),x)
```

```
[Out] 2*dilog(1/(a*x+(a^2*x^2+1)^(1/2)))-1/2*dilog(1/(a*x+(a^2*x^2+1)^(1/2))^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arsinh}(ax)}{\sqrt{a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(arcsinh(a*x)/(sqrt(a^2*x^2 + 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{asinh}(a\,x)}{x\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)/(x*(a^2*x^2 + 1)^(1/2)),x)
[Out] int(asinh(a*x)/(x*(a^2*x^2 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}(ax)}{x\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)/x/(a**2*x**2+1)**(1/2),x)
[Out] Integral(asinh(a*x)/(x*sqrt(a**2*x**2 + 1)), x)
```

3.183 $\int \frac{\sinh^{-1}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=27

$$a\log(x)-\frac{\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{x}$$

```
[Out] a*ln(x)-arcsinh(a*x)*(a^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.06, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5723, 29}

$$a\log(x)-\frac{\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]/(x^2*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -((Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/x) + a*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^{-1}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx &= -\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{x}+a\int \frac{1}{x}\,dx\\ &= -\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{x}+a\log(x)\end{aligned}$$

Mathematica [A] time = 0.04, size = 29, normalized size = 1.07

$$a\log(ax)-\frac{\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]/(x^2*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -((Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/x) + a*Log[a*x]
```

fricas [A] time = 0.58, size = 39, normalized size = 1.44

$$\frac{ax\log(x)-\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] (a*x*log(x) - sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/x
```

giac [B] time = 0.44, size = 71, normalized size = 2.63

$$-a\log\left(-x|a|+\sqrt{a^2x^2+1}\right)+a\log\left(|x|\right)+\frac{2\,|a|\log\left(ax+\sqrt{a^2x^2+1}\right)}{\left(x|a|-\sqrt{a^2x^2+1}\right)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] -a*log(-x*abs(a) + sqrt(a^2*x^2 + 1)) + a*log(abs(x)) + 2*abs(a)*log(a*x +
sqrt(a^2*x^2 + 1))/((x*abs(a) - sqrt(a^2*x^2 + 1))^2 - 1)
```

maple [B] time = 0.00, size = 56, normalized size = 2.07

$$-2a\,\mathrm{arcsinh}\left(ax\right)+\frac{\left(ax-\sqrt{a^2x^2+1}\right)\mathrm{arcsinh}\left(ax\right)}{x}+a\ln\left(\left(ax+\sqrt{a^2x^2+1}\right)^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)/x^2/(a^2*x^2+1)^(1/2),x)
[Out] -2*a*arcsinh(a*x)+(a*x-(a^2*x^2+1)^(1/2))/x*arcsinh(a*x)+a*ln((a*x+(a^2*x^2
+1)^(1/2))^2-1)
```

maxima [A] time = 0.47, size = 25, normalized size = 0.93

$$a\log(x)-\frac{\sqrt{a^2x^2+1}\,\mathrm{arsinh}\left(ax\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] a*log(x) - sqrt(a^2*x^2 + 1)*arcsinh(a*x)/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{asinh}\left(a\,x\right)}{x^2\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)/(x^2*(a^2*x^2 + 1)^(1/2)),x)
[Out] int(asinh(a*x)/(x^2*(a^2*x^2 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{asinh}\left(ax\right)}{x^2\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)/x**2/(a**2*x**2+1)**(1/2),x)
[Out] Integral(asinh(a*x)/(x**2*sqrt(a**2*x**2 + 1)), x)
```

3.184 $\int \frac{\sinh^{-1}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=80

$$\frac{1}{2}a^2\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right)-\frac{1}{2}a^2\text{Li}_2\left(e^{\sinh^{-1}(ax)}\right)-\frac{\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{2x^2}+a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)-\frac{a}{2x}$$

```
[Out] -1/2*a/x+a^2*arcsinh(a*x)*arctanh(a*x+(a^2*x^2+1)^(1/2))+1/2*a^2*polylog(2,
-a*x-(a^2*x^2+1)^(1/2))-1/2*a^2*polylog(2,a*x+(a^2*x^2+1)^(1/2))-1/2*arcsin
h(a*x)*(a^2*x^2+1)^(1/2)/x^2
```

Rubi [A] time = 0.14, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5747, 5760, 4182, 2279, 2391, 30}

$$\frac{1}{2}a^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)-\frac{1}{2}a^2\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)-\frac{\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{2x^2}+a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(a\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]/(x^3*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -a/(2*x) - (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*x^2) + a^2*ArcSinh[a*x]*ArcT
anh[E^ArcSinh[a*x]] + (a^2*PolyLog[2, -E^ArcSinh[a*x]])/2 - (a^2*PolyLog[2,
 E^ArcSinh[a*x]])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
```

```
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx = -\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{2x^2}+\frac{1}{2}a\int\frac{1}{x^2}\,dx-\frac{1}{2}a^2\int\frac{\sinh^{-1}(ax)}{x\sqrt{1+a^2x^2}}\,dx$$
$$= -\frac{a}{2x}-\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{2x^2}-\frac{1}{2}a^2\,\text{Subst}\left(\int x\text{csch}(x)\,dx,x,\sinh^{-1}(ax)\right)$$
$$= -\frac{a}{2x}-\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{2x^2}+a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)+\frac{1}{2}a^2\,\text{Subst}\left(\int \log(\right.$$
$$= -\frac{a}{2x}-\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{2x^2}+a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)+\frac{1}{2}a^2\,\text{Subst}\left(\int \frac{\log(}{}\right.$$
$$= -\frac{a}{2x}-\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{2x^2}+a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)+\frac{1}{2}a^2\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right.$$

Mathematica [A] time = 0.03, size = 126, normalized size = 1.58

$$\frac{1}{8}a^2\left(-4\text{Li}_2\left(-e^{-\sinh^{-1}(ax)}\right)+4\text{Li}_2\left(e^{-\sinh^{-1}(ax)}\right)-4\sinh^{-1}(ax)\log\left(1-e^{-\sinh^{-1}(ax)}\right)+4\sinh^{-1}(ax)\log\left(e^{-\sinh^{-1}}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]/(x^3*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] (a^2*(-2*Coth[ArcSinh[a*x]/2] - ArcSinh[a*x]*Csch[ArcSinh[a*x]/2]^2 - 4*Arc
Sinh[a*x]*Log[1 - E^(-ArcSinh[a*x])] + 4*ArcSinh[a*x]*Log[1 + E^(-ArcSinh[a
*x])] - 4*PolyLog[2, -E^(-ArcSinh[a*x])] + 4*PolyLog[2, E^(-ArcSinh[a*x])]
- ArcSinh[a*x]*Sech[ArcSinh[a*x]/2]^2 + 2*Tanh[ArcSinh[a*x]/2]))/8
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a^2x^2+1}\,\text{arsinh}(ax)}{a^2x^5+x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)/(a^2*x^5 + x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arsinh}(ax)}{\sqrt{a^2x^2+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

[Out] integrate(arcsinh(a*x)/(sqrt(a^2*x^2 + 1)*x^3), x)

maple [A] time = 0.00, size = 150, normalized size = 1.88

$$-\frac{\operatorname{arcsinh}(ax)\,x^2a^2+\sqrt{a^2x^2+1}\;xa+\operatorname{arcsinh}(ax)}{2\sqrt{a^2x^2+1}\;x^2}+\frac{a^2\operatorname{arcsinh}(ax)\ln\left(1+ax+\sqrt{a^2x^2+1}\right)}{2}+\frac{a^2\operatorname{polylog}\left(2,-a\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsinh(a*x)/x^3/(a^2*x^2+1)^(1/2),x)

[Out] -1/2/(a^2*x^2+1)^(1/2)*(arcsinh(a*x)*x^2*a^2+(a^2*x^2+1)^(1/2)*x*a+arcsinh(a*x))/x^2+1/2*a^2*arcsinh(a*x)*ln(1+a*x+(a^2*x^2+1)^(1/2))+1/2*a^2*polylog(2,-a*x-(a^2*x^2+1)^(1/2))-1/2*a^2*arcsinh(a*x)*ln(1-a*x-(a^2*x^2+1)^(1/2))-1/2*a^2*polylog(2,a*x+(a^2*x^2+1)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsinh(a*x)/x^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")

[Out] integrate(arcsinh(a*x)/(sqrt(a^2*x^2 + 1)*x^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}(a\,x)}{x^3\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(asinh(a*x)/(x^3*(a^2*x^2 + 1)^(1/2)),x)

[Out] int(asinh(a*x)/(x^3*(a^2*x^2 + 1)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}(ax)}{x^3\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asinh(a*x)/x**3/(a**2*x**2+1)**(1/2),x)

[Out] Integral(asinh(a*x)/(x**3*sqrt(a**2*x**2 + 1)), x)

3.185 $\int x^m \left(d+c^2dx^2\right)^3 \left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=313

$$\frac{c^6d^3x^{m+7}\left(a+b\sinh^{-1}(cx)\right)}{m+7}+\frac{3c^4d^3x^{m+5}\left(a+b\sinh^{-1}(cx)\right)}{m+5}+\frac{3c^2d^3x^{m+3}\left(a+b\sinh^{-1}(cx)\right)}{m+3}+\frac{d^3x^{m+1}\left(a+b\sin\right.}{m+1}$$

```
[Out] d^3*x^(1+m)*(a+b*arcsinh(c*x))/(1+m)+3*c^2*d^3*x^(3+m)*(a+b*arcsinh(c*x))/(
3+m)+3*c^4*d^3*x^(5+m)*(a+b*arcsinh(c*x))/(5+m)+c^6*d^3*x^(7+m)*(a+b*arcsin
h(c*x))/(7+m)-3*b*c*d^3*(35*m^3+455*m^2+1813*m+2161)*x^(2+m)*hypergeom([1/2
, 1+1/2*m],[2+1/2*m],-c^2*x^2)/(m^2+3*m+2)/(m^3+15*m^2+71*m+105)^2-b*c*d^3*
(m^4+27*m^3+284*m^2+1329*m+2271)*x^(2+m)*(c^2*x^2+1)^(1/2)/(7+m)^2/(m^2+8*m
+15)^2-b*c^3*d^3*(9+m)*(13+2*m)*x^(4+m)*(c^2*x^2+1)^(1/2)/(5+m)^2/(7+m)^2-b
*c^5*d^3*x^(6+m)*(c^2*x^2+1)^(1/2)/(7+m)^2
```

Rubi [A] time = 2.17, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {270, 5730, 12, 1809, 1267, 459, 364}

$$\frac{3c^2d^3x^{m+3}\left(a+b\sinh^{-1}(cx)\right)}{m+3}+\frac{3c^4d^3x^{m+5}\left(a+b\sinh^{-1}(cx)\right)}{m+5}+\frac{c^6d^3x^{m+7}\left(a+b\sinh^{-1}(cx)\right)}{m+7}+\frac{d^3x^{m+1}\left(a+b\sin\right.}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -((b*c*d^3*(2271 + 1329*m + 284*m^2 + 27*m^3 + m^4)*x^(2 + m)*Sqrt[1 + c^2*
x^2])/((3 + m)^2*(5 + m)^2*(7 + m)^2)) - (b*c^3*d^3*(9 + m)*(13 + 2*m)*x^(4
 + m)*Sqrt[1 + c^2*x^2])/((5 + m)^2*(7 + m)^2) - (b*c^5*d^3*x^(6 + m)*Sqrt[
1 + c^2*x^2])/(7 + m)^2 + (d^3*x^(1 + m)*(a + b*ArcSinh[c*x]))/(1 + m) + (3
*c^2*d^3*x^(3 + m)*(a + b*ArcSinh[c*x]))/(3 + m) + (3*c^4*d^3*x^(5 + m)*(a
+ b*ArcSinh[c*x]))/(5 + m) + (c^6*d^3*x^(7 + m)*(a + b*ArcSinh[c*x]))/(7 +
m) - (3*b*c*d^3*(2161 + 1813*m + 455*m^2 + 35*m^3)*x^(2 + m)*Hypergeometric
2F1[1/2, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/((1 + m)*(2 + m)*(3 + m)^2*(5 +
 m)^2*(7 + m)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
```

```
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1267

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\int x^m \left(d+c^2dx^2\right)^3 \left(a+b\sinh^{-1}(cx)\right) dx = \frac{d^3x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{1+m} + \frac{3c^2d^3x^{3+m}\left(a+b\sinh^{-1}(cx)\right)}{3+m} + \frac{3c^4d^3x^5}{}$$

$$= \frac{d^3x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{1+m} + \frac{3c^2d^3x^{3+m}\left(a+b\sinh^{-1}(cx)\right)}{3+m} + \frac{3c^4d^3x^5}{}$$

$$= -\frac{bc^5d^3x^{6+m}\sqrt{1+c^2x^2}}{(7+m)^2} + \frac{d^3x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{1+m} + \frac{3c^2d^3x^{3+m}\left(a\right.}{3+}$$

$$= -\frac{bc^3d^3(9+m)(13+2m)x^{4+m}\sqrt{1+c^2x^2}}{(5+m)^2(7+m)^2} - \frac{bc^5d^3x^{6+m}\sqrt{1+c^2x^2}}{(7+m)^2} + \frac{d^3}{}$$

$$= -\frac{bcd^3\left(2271+1329m+284m^2+27m^3+m^4\right)x^{2+m}\sqrt{1+c^2x^2}}{(3+m)^2(5+m)^2(7+m)^2} - \frac{bc^3}{}$$

$$= -\frac{bcd^3\left(2271+1329m+284m^2+27m^3+m^4\right)x^{2+m}\sqrt{1+c^2x^2}}{(3+m)^2(5+m)^2(7+m)^2} - \frac{bc^3}{}$$

Mathematica [A] time = 0.49, size = 257, normalized size = 0.82

$$\frac{x^{m+1}\left(\frac{6d\left(\frac{4d^2\left((m+2)(c^2mx^2+c^2x^2+m+3)\left(a+b\sinh^{-1}(cx)\right)-bc(m+1)x\,{}_2F_1\left(-\frac{1}{2},\frac{m}{2}+1;\frac{m}{2}+2;-c^2x^2\right)-2bcx\,{}_2F_1\left(\frac{1}{2},\frac{m}{2}+1;\frac{m}{2}+2;-c^2x^2\right)\right)}{(m+1)(m+2)(m+3)}+\left(c^2dx^2+d\right)^2\left(a+b\sinh^{-1}(cx)\right)\cdots}{m+5}\cdots\right.}{m+7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (x^(1 + m)*((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]) - (b*c*d^3*x*Hypergeomet
ric2F1[-5/2, 1 + m/2, 2 + m/2, -(c^2*x^2)])/(2 + m) + (6*d*((d + c^2*d*x^2)
^2*(a + b*ArcSinh[c*x]) - (b*c*d^2*x*Hypergeometric2F1[-3/2, 1 + m/2, 2 + m
/2, -(c^2*x^2)])/(2 + m) + (4*d^2*((2 + m)*(3 + m + c^2*x^2 + c^2*m*x^2)*(a
 + b*ArcSinh[c*x]) - b*c*(1 + m)*x*Hypergeometric2F1[-1/2, 1 + m/2, 2 + m/2
, -(c^2*x^2)] - 2*b*c*x*Hypergeometric2F1[1/2, 1 + m/2, 2 + m/2, -(c^2*x^2)
]))/((1 + m)*(2 + m)*(3 + m))))/(5 + m)))/(7 + m)
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ac^6d^3x^6 + 3\,ac^4d^3x^4 + 3\,ac^2d^3x^2 + ad^3 + \left(bc^6d^3x^6 + 3\,bc^4d^3x^4 + 3\,bc^2d^3x^2 + bd^3\right)\text{arsinh}\left(cx\right)\right)x^m, x\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*c^6*d^3*x^6 + 3*a*c^4*d^3*x^4 + 3*a*c^2*d^3*x^2 + a*d^3 + (b*c^
6*d^3*x^6 + 3*b*c^4*d^3*x^4 + 3*b*c^2*d^3*x^2 + b*d^3)*arcsinh(c*x))*x^m, x
)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int x^m\left(c^2d\,x^2+d\right)^3\left(a+b\,\text{arcsinh}\left(cx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x^m*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ac^6d^3x^{m+7}}{m+7}+\frac{3\,ac^4d^3x^{m+5}}{m+5}+\frac{3\,ac^2d^3x^{m+3}}{m+3}+\frac{ad^3x^{m+1}}{m+1}+\frac{\left(\left(m^3+9\,m^2+23\,m+15\right)bc^6d^3x^7+3\left(m^3+11\,m^2+31\,m\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] a*c^6*d^3*x^(m + 7)/(m + 7) + 3*a*c^4*d^3*x^(m + 5)/(m + 5) + 3*a*c^2*d^3*x
^(m + 3)/(m + 3) + a*d^3*x^(m + 1)/(m + 1) + ((m^3 + 9*m^2 + 23*m + 15)*b*c
^6*d^3*x^7 + 3*(m^3 + 11*m^2 + 31*m + 21)*b*c^4*d^3*x^5 + 3*(m^3 + 13*m^2 +
 47*m + 35)*b*c^2*d^3*x^3 + (m^3 + 15*m^2 + 71*m + 105)*b*d^3*x)*x^m*log(c*
x + sqrt(c^2*x^2 + 1))/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105) - integrate(((
m^3 + 9*m^2 + 23*m + 15)*b*c^7*d^3*x^7 + 3*(m^3 + 11*m^2 + 31*m + 21)*b*c^5
*d^3*x^5 + 3*(m^3 + 13*m^2 + 47*m + 35)*b*c^3*d^3*x^3 + (m^3 + 15*m^2 + 71*
m + 105)*b*c*d^3*x)*x^m/((m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*c^3*x^3 + (m
^4 + 16*m^3 + 86*m^2 + 176*m + 105)*c*x + ((m^4 + 16*m^3 + 86*m^2 + 176*m +
 105)*c^2*x^2 + m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*sqrt(c^2*x^2 + 1)), x)
 - integrate(((m^3 + 9*m^2 + 23*m + 15)*b*c^8*d^3*x^8 + 3*(m^3 + 11*m^2 + 3
1*m + 21)*b*c^6*d^3*x^6 + 3*(m^3 + 13*m^2 + 47*m + 35)*b*c^4*d^3*x^4 + (m^3
 + 15*m^2 + 71*m + 105)*b*c^2*d^3*x^2)*x^m/((m^4 + 16*m^3 + 86*m^2 + 176*m
+ 105)*c^2*x^2 + m^4 + 16*m^3 + 86*m^2 + 176*m + 105), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^m\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(d\,c^2\,x^2+d\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2)^3,x)
```

```
[Out] int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^3\left(\int ax^m\,dx + \int bx^m \operatorname{asinh}(cx)\,dx + \int 3ac^2x^2x^m\,dx + \int 3ac^4x^4x^m\,dx + \int ac^6x^6x^m\,dx + \int 3bc^2x^2x^m \operatorname{asinh}(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*d*x**2+d)**3*(a+b*asinh(c*x)),x)
```

```
[Out] d**3*(Integral(a*x**m, x) + Integral(b*x**m*asinh(c*x), x) + Integral(3*a*c
**2*x**2*x**m, x) + Integral(3*a*c**4*x**4*x**m, x) + Integral(a*c**6*x**6*
x**m, x) + Integral(3*b*c**2*x**2*x**m*asinh(c*x), x) + Integral(3*b*c**4*x
**4*x**m*asinh(c*x), x) + Integral(b*c**6*x**6*x**m*asinh(c*x), x))
```

3.186 $\int x^m (d+c^2dx^2)^2 (a+b\sinh^{-1}(cx))\,dx$

Optimal. Leaf size=217

$$\frac{c^4d^2x^{m+5}\left(a+b\sinh^{-1}(cx)\right)}{m+5}+\frac{2c^2d^2x^{m+3}\left(a+b\sinh^{-1}(cx)\right)}{m+3}+\frac{d^2x^{m+1}\left(a+b\sinh^{-1}(cx)\right)}{m+1}-\frac{bcd^2\left(15m^2+100m\right.}{(m+1)(}$$

```
[Out] d^2*x^(1+m)*(a+b*arcsinh(c*x))/(1+m)+2*c^2*d^2*x^(3+m)*(a+b*arcsinh(c*x))/(
3+m)+c^4*d^2*x^(5+m)*(a+b*arcsinh(c*x))/(5+m)-b*c*d^2*(15*m^2+100*m+149)*x^
(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],-c^2*x^2)/(m^2+3*m+2)/(m^2+8*m+15)
^2-b*c*d^2*(m^2+13*m+38)*x^(2+m)*(c^2*x^2+1)^(1/2)/(3+m)^2/(5+m)^2-b*c^3*d^
2*x^(4+m)*(c^2*x^2+1)^(1/2)/(5+m)^2
```

Rubi [A] time = 0.30, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {270, 5730, 12, 1267, 459, 364}

$$\frac{2c^2d^2x^{m+3}\left(a+b\sinh^{-1}(cx)\right)}{m+3}+\frac{c^4d^2x^{m+5}\left(a+b\sinh^{-1}(cx)\right)}{m+5}+\frac{d^2x^{m+1}\left(a+b\sinh^{-1}(cx)\right)}{m+1}-\frac{bcd^2\left(15m^2+100m\right.}{(m+1)(}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -((b*c*d^2*(38 + 13*m + m^2)*x^(2 + m)*Sqrt[1 + c^2*x^2])/((3 + m)^2*(5 + m
)^2)) - (b*c^3*d^2*x^(4 + m)*Sqrt[1 + c^2*x^2])/(5 + m)^2 + (d^2*x^(1 + m)*
(a + b*ArcSinh[c*x]))/(1 + m) + (2*c^2*d^2*x^(3 + m)*(a + b*ArcSinh[c*x]))/
(3 + m) + (c^4*d^2*x^(5 + m)*(a + b*ArcSinh[c*x]))/(5 + m) - (b*c*d^2*(149
+ 100*m + 15*m^2)*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(
c^2*x^2)])/((1 + m)*(2 + m)*(3 + m)^2*(5 + m)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1267

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rubi steps

$$\int x^m \left(d+c^2dx^2\right)^2 \left(a+b\sinh^{-1}(cx)\right)\,dx = \frac{d^2x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{1+m} + \frac{2c^2d^2x^{3+m}\left(a+b\sinh^{-1}(cx)\right)}{3+m} + \frac{c^4d^2x^{5+}}{}$$

$$= \frac{d^2x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{1+m} + \frac{2c^2d^2x^{3+m}\left(a+b\sinh^{-1}(cx)\right)}{3+m} + \frac{c^4d^2x^{5+}}{}$$

$$= -\frac{bc^3d^2x^{4+m}\sqrt{1+c^2x^2}}{(5+m)^2} + \frac{d^2x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{1+m} + \frac{2c^2d^2x^{3+m}\left(a\right.}{3+}$$

$$= -\frac{bcd^2\left(38+13m+m^2\right)x^{2+m}\sqrt{1+c^2x^2}}{(3+m)^2(5+m)^2} - \frac{bc^3d^2x^{4+m}\sqrt{1+c^2x^2}}{(5+m)^2} + \frac{d^2}{}$$

$$= -\frac{bcd^2\left(38+13m+m^2\right)x^{2+m}\sqrt{1+c^2x^2}}{(3+m)^2(5+m)^2} - \frac{bc^3d^2x^{4+m}\sqrt{1+c^2x^2}}{(5+m)^2} + \frac{d^2}{}$$

Mathematica [A] time = 0.06, size = 188, normalized size = 0.87

$$\frac{x^{m+1}\left(\frac{4d^2\left((m+2)\left(c^2mx^2+c^2x^2+m+3\right)\left(a+b\sinh^{-1}(cx)\right)-bc(m+1)x\,{}_2F_1\left(-\frac{1}{2},\frac{m}{2}+1;\frac{m}{2}+2;-c^2x^2\right)-2bcx\,{}_2F_1\left(\frac{1}{2},\frac{m}{2}+1;\frac{m}{2}+2;-c^2x^2\right)\right)}{(m+1)(m+2)(m+3)} + \left(c^2dx^2+d\right)^2 \left(\right.}{m+5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (x^(1 + m)*((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]) - (b*c*d^2*x*Hypergeomet
ric2F1[-3/2, 1 + m/2, 2 + m/2, -(c^2*x^2)])/(2 + m) + (4*d^2*((2 + m)*(3 +
m + c^2*x^2 + c^2*m*x^2)*(a + b*ArcSinh[c*x]) - b*c*(1 + m)*x*Hypergeometri
c2F1[-1/2, 1 + m/2, 2 + m/2, -(c^2*x^2)] - 2*b*c*x*Hypergeometric2F1[1/2, 1
 + m/2, 2 + m/2, -(c^2*x^2)]))/((1 + m)*(2 + m)*(3 + m))))/(5 + m)
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(ac^4d^2x^4 + 2\,ac^2d^2x^2 + ad^2 + \left(bc^4d^2x^4 + 2\,bc^2d^2x^2 + bd^2\right)\mathrm{arsinh}\left(cx\right)\right)x^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c^
2*d^2*x^2 + b*d^2)*arcsinh(c*x))*x^m, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int x^{m} \left(c^{2} d\,x^{2}+d\right)^{2} \left(a +b \,\mathrm{arcsinh}\left(c x \right)\right)d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x^m*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ac^4d^2x^{m+5}}{m+5}+\frac{2\,ac^2d^2x^{m+3}}{m+3}+\frac{ad^2x^{m+1}}{m+1}+\frac{\left(\left(m^2+4\,m+3\right)bc^4d^2x^5+2\left(m^2+6\,m+5\right)bc^2d^2x^3+\left(m^2+8\,m+15\right)bc}{m^3+9\,m^2+23\,m+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] a*c^4*d^2*x^(m + 5)/(m + 5) + 2*a*c^2*d^2*x^(m + 3)/(m + 3) + a*d^2*x^(m +
1)/(m + 1) + ((m^2 + 4*m + 3)*b*c^4*d^2*x^5 + 2*(m^2 + 6*m + 5)*b*c^2*d^2*x
^3 + (m^2 + 8*m + 15)*b*d^2*x)*x^m*log(c*x + sqrt(c^2*x^2 + 1))/(m^3 + 9*m^
2 + 23*m + 15) - integrate(((m^2 + 4*m + 3)*b*c^5*d^2*x^5 + 2*(m^2 + 6*m +
5)*b*c^3*d^2*x^3 + (m^2 + 8*m + 15)*b*c*d^2*x)*x^m/((m^3 + 9*m^2 + 23*m + 1
5)*c^3*x^3 + (m^3 + 9*m^2 + 23*m + 15)*c*x + ((m^3 + 9*m^2 + 23*m + 15)*c^2
*x^2 + m^3 + 9*m^2 + 23*m + 15)*sqrt(c^2*x^2 + 1)), x) - integrate(((m^2 +
4*m + 3)*b*c^6*d^2*x^6 + 2*(m^2 + 6*m + 5)*b*c^4*d^2*x^4 + (m^2 + 8*m + 15)
*b*c^2*d^2*x^2)*x^m/((m^3 + 9*m^2 + 23*m + 15)*c^2*x^2 + m^3 + 9*m^2 + 23*m
 + 15), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^m\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(d\,c^2\,x^2+d\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2)^2,x)
```

```
[Out] int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^{2}\left(\int ax^{m}\,dx+\int bx^{m}\operatorname{asinh}{\left(cx \right)}\,dx+\int 2ac^{2}x^{2}x^{m}\,dx+\int ac^{4}x^{4}x^{m}\,dx+\int 2bc^{2}x^{2}x^{m}\operatorname{asinh}{\left(cx \right)}\,dx+\int bc^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*d*x**2+d)**2*(a+b*asinh(c*x)),x)
```

```
[Out] d**2*(Integral(a*x**m, x) + Integral(b*x**m*asinh(c*x), x) + Integral(2*a*c
**2*x**2*x**m, x) + Integral(a*c**4*x**4*x**m, x) + Integral(2*b*c**2*x**2*
x**m*asinh(c*x), x) + Integral(b*c**4*x**4*x**m*asinh(c*x), x))
```

3.187 $\int x^m \left(d+c^2 d x^2\right) \left(a+b \sinh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=128

$$\frac{c^2 d x^{m+3} \left(a+b \sinh^{-1}(cx)\right)}{m+3}+\frac{d x^{m+1} \left(a+b \sinh^{-1}(cx)\right)}{m+1}-\frac{bcd(3m+7)x^{m+2} {}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-c^2x^2\right)}{(m+1)(m+2)(m+3)^2}-\frac{bcd\sqrt{c^2x^2}}{(m+}$$

```
[Out] d*x^(1+m)*(a+b*arcsinh(c*x))/(1+m)+c^2*d*x^(3+m)*(a+b*arcsinh(c*x))/(3+m)-b
*c*d*(7+3*m)*x^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],-c^2*x^2)/(3+m)^2/(
m^2+3*m+2)-b*c*d*x^(2+m)*(c^2*x^2+1)^(1/2)/(3+m)^2
```

Rubi [A] time = 0.13, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {14, 5730, 12, 459, 364}

$$\frac{c^2 d x^{m+3} \left(a+b \sinh^{-1}(cx)\right)}{m+3}+\frac{d x^{m+1} \left(a+b \sinh^{-1}(cx)\right)}{m+1}-\frac{bcd(3m+7)x^{m+2} {}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-c^2x^2\right)}{(m+1)(m+2)(m+3)^2}-\frac{bcd\sqrt{c^2x^2}}{(m+}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -((b*c*d*x^(2 + m)*Sqrt[1 + c^2*x^2])/(3 + m)^2) + (d*x^(1 + m)*(a + b*ArcS
inh[c*x]))/(1 + m) + (c^2*d*x^(3 + m)*(a + b*ArcSinh[c*x]))/(3 + m) - (b*c*
d*(7 + 3*m)*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(c^2*x^
2)])/((1 + m)*(2 + m)*(3 + m)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
```

GtQ[p, 0]

Rubi steps

$$\int x^m\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)dx=\frac{dx^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{1+m}+\frac{c^2dx^{3+m}\left(a+b\sinh^{-1}(cx)\right)}{3+m}-(bc)\int\frac{dx^{1+}}{}$$

$$=\frac{dx^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{1+m}+\frac{c^2dx^{3+m}\left(a+b\sinh^{-1}(cx)\right)}{3+m}-(bcd)\int\frac{x^{1+}}{}$$

$$=-\frac{bcdx^{2+m}\sqrt{1+c^2x^2}}{(3+m)^2}+\frac{dx^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{1+m}+\frac{c^2dx^{3+m}\left(a+b\sin\right.}{3+m}$$

$$=-\frac{bcdx^{2+m}\sqrt{1+c^2x^2}}{(3+m)^2}+\frac{dx^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{1+m}+\frac{c^2dx^{3+m}\left(a+b\sin\right.}{3+m}$$

Mathematica [A] time = 0.04, size = 118, normalized size = 0.92

$$\frac{dx^{m+1}\left((m+2)\left(c^2mx^2+c^2x^2+m+3\right)\left(a+b\sinh^{-1}(cx)\right)-bc(m+1)x\,{}_2F_1\left(-\frac{1}{2},\frac{m}{2}+1;\frac{m}{2}+2;-c^2x^2\right)-2bcx\,{}_2F_1\right.}{(m+1)(m+2)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d*x^(1 + m)*((2 + m)*(3 + m + c^2*x^2 + c^2*m*x^2)*(a + b*ArcSinh[c*x]) -
b*c*(1 + m)*x*Hypergeometric2F1[-1/2, 1 + m/2, 2 + m/2, -(c^2*x^2)] - 2*b*c
*x*Hypergeometric2F1[1/2, 1 + m/2, 2 + m/2, -(c^2*x^2)]))/((1 + m)*(2 + m)*
(3 + m))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ac^2dx^2+ad+\left(bc^2dx^2+bd\right)\text{arsinh}\left(cx\right)\right)x^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*x^m, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int x^m\left(c^2d\,x^2+d\right)\left(a+b\,\text{arcsinh}\left(cx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x^m*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ac^2dx^{m+3}}{m+3}+\frac{adx^{m+1}}{m+1}+\frac{\left(bc^2d(m+1)x^3+bd(m+3)x\right)x^m\log\left(cx+\sqrt{c^2x^2+1}\right)}{m^2+4\,m+3}-\int\frac{}{\left(m^2+4\,m+3\right)c^3x^3+\left(m^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] a*c^2*d*x^(m + 3)/(m + 3) + a*d*x^(m + 1)/(m + 1) + (b*c^2*d*(m + 1)*x^3 +
b*d*(m + 3)*x)*x^m*log(c*x + sqrt(c^2*x^2 + 1))/(m^2 + 4*m + 3) - integrate
((b*c^3*d*(m + 1)*x^3 + b*c*d*(m + 3)*x)*x^m/((m^2 + 4*m + 3)*c^3*x^3 + (m^
2 + 4*m + 3)*c*x + ((m^2 + 4*m + 3)*c^2*x^2 + m^2 + 4*m + 3)*sqrt(c^2*x^2 +
 1)), x) - integrate((b*c^4*d*(m + 1)*x^4 + b*c^2*d*(m + 3)*x^2)*x^m/((m^2
+ 4*m + 3)*c^2*x^2 + m^2 + 4*m + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^m\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\left(d\,c^2\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2),x)
```

```
[Out] int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int ax^m\,dx+\int bx^m\operatorname{asinh}\left(cx\right)dx+\int ac^2x^2x^m\,dx+\int bc^2x^2x^m\operatorname{asinh}\left(cx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*d*x**2+d)*(a+b*asinh(c*x)),x)
```

```
[Out] d*(Integral(a*x**m, x) + Integral(b*x**m*asinh(c*x), x) + Integral(a*c**2*x
**2*x**m, x) + Integral(b*c**2*x**2*x**m*asinh(c*x), x))
```

3.188 $\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{c^2dx^2+d},x\right)$$

```
[Out] Unintegrable(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x)
```

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
[Out] Defer[Int][(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2), x]
```

Rubi steps

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx = \int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx$$

Mathematica [A] time = 3.81, size = 0, normalized size = 0.00

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
[Out] Integrate[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2), x]
```

fricas [A] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b\operatorname{arsinh}(cx)+a)x^m}{c^2dx^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="fricas")
[Out] integral((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arsinh}(cx)+a)x^m}{c^2dx^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d), x)
```

maple [A] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{x^{m} \left(a + b \,\mathrm{arcsinh}\left(c x\right)\right)}{c^{2} d \,x^{2} + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x)
```

```
[Out] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \,\mathrm{arsinh}\left(cx\right) + a)x^{m}}{c^{2}dx^{2} + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^{m}\ \left(a + b \,\mathrm{asinh}\left(c\,x\right)\right)}{d\,c^{2}\,x^{2} + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2),x)
```

```
[Out] int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^{m}}{c^{2}x^{2}+1}\, dx + \int \frac{bx^{m}\,\mathrm{asinh}\left(cx\right)}{c^{2}x^{2}+1}\, dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*asinh(c*x))/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a*x**m/(c**2*x**2 + 1), x) + Integral(b*x**m*asinh(c*x)/(c**2*x**
2 + 1), x))/d
```

3.189 $\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=116

$$\frac{(1-m)\mathrm{Int}\left(\frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{c^2dx^2+d},x\right)}{2d}+\frac{x^{m+1}\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(c^2x^2+1\right)}-\frac{bcx^{m+2}\,{}_2F_1\left(\frac{3}{2},\frac{m+2}{2};\frac{m+4}{2};-c^2x^2\right)}{2d^2(m+2)}$$

```
[Out] 1/2*x^(1+m)*(a+b*arcsinh(c*x))/d^2/(c^2*x^2+1)-1/2*b*c*x^(2+m)*hypergeom([3
/2, 1+1/2*m],[2+1/2*m],-c^2*x^2)/d^2/(2+m)+1/2*(1-m)*Unintegrable(x^m*(a+b*
arcsinh(c*x))/(c^2*d*x^2+d),x)/d
```

Rubi [A] time = 0.16, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

```
[Out] (x^(1 + m)*(a + b*ArcSinh[c*x]))/(2*d^2*(1 + c^2*x^2)) - (b*c*x^(2 + m)*Hyp
ergeometric2F1[3/2, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/(2*d^2*(2 + m)) + ((
1 - m)*Defer[Int][(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2), x])/(2*d)
```

Rubi steps

$$\begin{aligned}\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^2}\,dx &= \frac{x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(1+c^2x^2\right)}-\frac{(bc)\int \frac{x^{1+m}}{\left(1+c^2x^2\right)^{3/2}}\,dx}{2d^2}+\frac{(1-m)\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx}{2d}\\ &= \frac{x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(1+c^2x^2\right)}-\frac{bcx^{2+m}\,{}_2F_1\left(\frac{3}{2},\frac{2+m}{2};\frac{4+m}{2};-c^2x^2\right)}{2d^2(2+m)}+\frac{(1-m)\int \frac{x^m\left(a+b\sinh\right.}{d+c^2dx}}{2d}\end{aligned}$$

Mathematica [A] time = 5.63, size = 0, normalized size = 0.00

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

```
[Out] Integrate[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2, x]
```

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(b\,\mathrm{arsinh}\,(cx)+a)x^m}{c^4d^2x^4+2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)*x^m/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(cx) + a)x^m}{\left(c^2dx^2 + d\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d)^2, x)
```

maple [A] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{x^m\,(a + b \operatorname{arcsinh}(cx))}{\left(c^2d\,x^2 + d\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x)
[Out] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(cx) + a)x^m}{\left(c^2dx^2 + d\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="maxima")
[Out] integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,\left(a + b \operatorname{asinh}(c\,x)\right)}{\left(d\,c^2\,x^2 + d\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^2,x)
[Out] int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^m}{c^4x^4+2c^2x^2+1}\, dx + \int \frac{bx^m \operatorname{asinh}(cx)}{c^4x^4+2c^2x^2+1}\, dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*asinh(c*x))/(c**2*d*x**2+d)**2,x)
[Out] (Integral(a*x**m/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b*x**m*asinh(
c*x)/(c**4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

3.190 $\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=207

$$\frac{(1-m)(3-m)\text{Int}\left(\frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{c^2dx^2+d},x\right)}{8d^2}+\frac{(3-m)x^{m+1}\left(a+b\sinh^{-1}(cx)\right)}{8d^3\left(c^2x^2+1\right)}+\frac{x^{m+1}\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(c^2x^2+1\right)^2}-\frac{bc(3-m)x^{m+}}{}$$

```
[Out] 1/4*x^(1+m)*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^2+1/8*(3-m)*x^(1+m)*(a+b*arc
sinh(c*x))/d^3/(c^2*x^2+1)-1/8*b*c*(3-m)*x^(2+m)*hypergeom([3/2, 1+1/2*m],[
2+1/2*m],-c^2*x^2)/d^3/(2+m)-1/4*b*c*x^(2+m)*hypergeom([5/2, 1+1/2*m],[2+1/
2*m],-c^2*x^2)/d^3/(2+m)+1/8*(1-m)*(3-m)*Unintegrable(x^m*(a+b*arcsinh(c*x)
)/(c^2*d*x^2+d),x)/d^2
```

Rubi [A] time = 0.25, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

```
[Out] (x^(1 + m)*(a + b*ArcSinh[c*x]))/(4*d^3*(1 + c^2*x^2)^2) + ((3 - m)*x^(1 +
m)*(a + b*ArcSinh[c*x]))/(8*d^3*(1 + c^2*x^2)) - (b*c*(3 - m)*x^(2 + m)*Hyp
ergeometric2F1[3/2, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/(8*d^3*(2 + m)) - (b
*c*x^(2 + m)*Hypergeometric2F1[5/2, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/(4*d
^3*(2 + m)) + ((1 - m)*(3 - m)*Defer[Int][(x^m*(a + b*ArcSinh[c*x]))/(d + c
^2*d*x^2), x])/(8*d^2)
```

Rubi steps

$$\begin{aligned}
\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^3}\,dx &= \frac{x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2}-\frac{(bc)\int \frac{x^{1+m}}{\left(1+c^2x^2\right)^{5/2}}\,dx}{4d^3}+\frac{(3-m)\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^2}\,dx}{4d}\\
&= \frac{x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2}+\frac{(3-m)x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{8d^3\left(1+c^2x^2\right)}-\frac{bcx^{2+m}\,{}_2F_1\left(\frac{5}{2},\frac{2+m}{2};\frac{4}{}\right)}{4d^3(2+m}\\
&= \frac{x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2}+\frac{(3-m)x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{8d^3\left(1+c^2x^2\right)}-\frac{bc(3-m)x^{2+m}\,{}_2F_1\left(\frac{3}{2}\right)}{8d^3(2}
\end{aligned}$$

Mathematica [A] time = 5.96, size = 0, normalized size = 0.00

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

```
[Out] Integrate[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3, x]
```

fricas [A] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b \operatorname{arsinh}(cx) + a)x^m}{c^6d^3x^6 + 3\,c^4d^3x^4 + 3\,c^2d^3x^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)*x^m/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*
x^2 + d^3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(cx) + a)x^m}{\left(c^2dx^2 + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d)^3, x)
```

maple [A] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{x^m\,(a + b \operatorname{arcsinh}(cx))}{\left(c^2d\,x^2 + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x)
```

```
[Out] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(cx) + a)x^m}{\left(c^2dx^2 + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d)^3, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^m\,\left(a + b \operatorname{asinh}(c\,x)\right)}{\left(d\,c^2\,x^2 + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^3,x)
```

```
[Out] int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*asinh(c*x))/(c**2*d*x**2+d)**3,x)
```

```
[Out] Timed out
```

3.191 $\int x^m \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=618

$$-\frac{15bcd^2x^{m+2}\sqrt{c^2dx^2+d}\ {}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-c^2x^2\right)}{(m+1)(m+2)^2(m+4)(m+6)\sqrt{c^2x^2+1}}+\frac{15d^2x^{m+1}\sqrt{c^2dx^2+d}\ {}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-c^2\right.}{(m+6)\left(m^3+7m^2+14m+8\right)}$$

```
[Out] 5*d*x^(1+m)*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/(4+m)/(6+m)+x^(1+m)*(c^2
*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/(6+m)+15*d^2*x^(1+m)*(a+b*arcsinh(c*x))*
(c^2*d*x^2+d)^(1/2)/(6+m)/(m^2+6*m+8)-15*b*c*d^2*x^(2+m)*(c^2*d*x^2+d)^(1/2
)/(2+m)^2/(4+m)/(6+m)/(c^2*x^2+1)^(1/2)-5*b*c*d^2*x^(2+m)*(c^2*d*x^2+d)^(1/
2)/(6+m)/(m^2+6*m+8)/(c^2*x^2+1)^(1/2)-b*c*d^2*x^(2+m)*(c^2*d*x^2+d)^(1/2)/
(m^2+8*m+12)/(c^2*x^2+1)^(1/2)-5*b*c^3*d^2*x^(4+m)*(c^2*d*x^2+d)^(1/2)/(4+m
)^2/(6+m)/(c^2*x^2+1)^(1/2)-2*b*c^3*d^2*x^(4+m)*(c^2*d*x^2+d)^(1/2)/(4+m)/(
6+m)/(c^2*x^2+1)^(1/2)-b*c^5*d^2*x^(6+m)*(c^2*d*x^2+d)^(1/2)/(6+m)^2/(c^2*x
^2+1)^(1/2)+15*d^2*x^(1+m)*(a+b*arcsinh(c*x))*hypergeom([1/2, 1/2+1/2*m],[3
/2+1/2*m],-c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(6+m)/(m^3+7*m^2+14*m+8)/(c^2*x^2+1
)^(1/2)-15*b*c*d^2*x^(2+m)*HypergeometricPFQ([1, 1+1/2*m, 1+1/2*m],[3/2+1/2
*m, 2+1/2*m],-c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(2+m)^2/(6+m)/(m^2+5*m+4)/(c^2*x
^2+1)^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 618, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5744, 5742, 5762, 30, 14, 270}

$$-\frac{15bcd^2x^{m+2}\sqrt{c^2dx^2+d}\ {}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-c^2x^2\right)}{(m+1)(m+2)^2(m+4)(m+6)\sqrt{c^2x^2+1}}+\frac{15d^2x^{m+1}\sqrt{c^2dx^2+d}\ {}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-c^2\right.}{(m+6)\left(m^3+7m^2+14m+8\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-15*b*c*d^2*x^(2 + m)*Sqrt[d + c^2*d*x^2])/((2 + m)^2*(4 + m)*(6 + m)*Sqrt
[1 + c^2*x^2]) - (5*b*c*d^2*x^(2 + m)*Sqrt[d + c^2*d*x^2])/((6 + m)*(8 + 6*
m + m^2)*Sqrt[1 + c^2*x^2]) - (b*c*d^2*x^(2 + m)*Sqrt[d + c^2*d*x^2])/((12
+ 8*m + m^2)*Sqrt[1 + c^2*x^2]) - (5*b*c^3*d^2*x^(4 + m)*Sqrt[d + c^2*d*x^2
])/((4 + m)^2*(6 + m)*Sqrt[1 + c^2*x^2]) - (2*b*c^3*d^2*x^(4 + m)*Sqrt[d +
c^2*d*x^2])/((4 + m)*(6 + m)*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*x^(6 + m)*Sqrt
[d + c^2*d*x^2])/((6 + m)^2*Sqrt[1 + c^2*x^2]) + (15*d^2*x^(1 + m)*Sqrt[d +
 c^2*d*x^2]*(a + b*ArcSinh[c*x]))/((6 + m)*(8 + 6*m + m^2)) + (5*d*x^(1 + m
)*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/((4 + m)*(6 + m)) + (x^(1 + m
)*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(6 + m) + (15*d^2*x^(1 + m)*S
qrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (
3 + m)/2, -(c^2*x^2)])/((6 + m)*(8 + 14*m + 7*m^2 + m^3)*Sqrt[1 + c^2*x^2])
 - (15*b*c*d^2*x^(2 + m)*Sqrt[d + c^2*d*x^2]*HypergeometricPFQ[{1, 1 + m/2,
 1 + m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)])/((1 + m)*(2 + m)^2*(4 + m)*(6
 + m)*Sqrt[1 + c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5762

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*ArcSinh[c*x])*Hypergeom
etric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(Sqrt[d]*f*(m + 1)), x] -
Simp[(b*c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2
, 2 + m/2}, -(c^2*x^2)])/(Sqrt[d]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int x^m \left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)dx = \frac{x^{1+m}\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{6+m} + \frac{(5d)\int x^m\left(d+c^2dx^2\right)^{3/}}{6+}$$

$$= \frac{5dx^{1+m}\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{(4+m)(6+m)} + \frac{x^{1+m}\left(d+c^2dx^2\right)^{5/2}\left(\right.}{6+}$$

$$= -\frac{bcd^2x^{2+m}\sqrt{d+c^2dx^2}}{\left(12+8m+m^2\right)\sqrt{1+c^2x^2}} - \frac{2bc^3d^2x^{4+m}\sqrt{d+c^2dx^2}}{(4+m)(6+m)\sqrt{1+c^2x^2}} - \frac{bc^5}{(6}$$

$$= -\frac{15bcd^2x^{2+m}\sqrt{d+c^2dx^2}}{(2+m)^2(4+m)(6+m)\sqrt{1+c^2x^2}} - \frac{5bcd^2x^{2+m}\sqrt{d+c^2d}}{(2+m)(4+m)(6+m)\sqrt{1}}$$

Mathematica [A] time = 1.28, size = 332, normalized size = 0.54

$$d^2x^{m+1}\sqrt{c^2dx^2+d}\left(-\frac{5\left(3(m+4)\left(bcx\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-c^2x^2\right)-(m+2)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)-(m+1)(m+2)\sqrt{}\right.\right.}{(m+1)(m+}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*x^(1 + m)*Sqrt[d + c^2*d*x^2]*(-((b*c*x*((4 + m)*(6 + m) + 2*c^2*(2 +
m)*(6 + m)*x^2 + c^4*(2 + m)*(4 + m)*x^4))/((2 + m)*(4 + m)*(6 + m)*Sqrt[1
+ c^2*x^2])) + (1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]) - (5*(b*c*(1 + m)*(2 +
m)*x*(4 + m + c^2*(2 + m)*x^2) - (1 + m)*(2 + m)^2*(4 + m)*(1 + c^2*x^2)^(3
/2)*(a + b*ArcSinh[c*x]) + 3*(4 + m)*(b*c*(1 + m)*x - (1 + m)*(2 + m)*Sqrt[
1 + c^2*x^2]*(a + b*ArcSinh[c*x]) - (2 + m)*(a + b*ArcSinh[c*x])*Hypergeome
tric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)] + b*c*x*HypergeometricPFQ[{1
, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)])))/((1 + m)*(2 + m)^
2*(4 + m)^2*Sqrt[1 + c^2*x^2])))/(6 + m)
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ac^4d^2x^4 + 2\,ac^2d^2x^2 + ad^2 + \left(bc^4d^2x^4 + 2\,bc^2d^2x^2 + bd^2\right)\text{arsinh}\,(cx)\right)\sqrt{c^2dx^2 + d}\,x^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c^
2*d^2*x^2 + b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)*x^m, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 1.64, size = 0, normalized size = 0.00

$$\int x^m \left(c^2d\,x^2 + d\right)^{\frac{5}{2}} (a + b\,\text{arcsinh}\,(cx))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c^2dx^2 + d\right)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx) + a)x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)*x^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^m\,\left(a + b\,\text{asinh}\,(c\,x)\right)\,\left(d\,c^2\,x^2 + d\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

3.192 $\int x^m \left(d+c^2 d x^2\right)^{3/2} \left(a+b \sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=390

$$-\frac{3bcdx^{m+2}\sqrt{c^2dx^2+d}\ {}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-c^2x^2\right)}{(m+1)(m+2)^2(m+4)\sqrt{c^2x^2+1}}+\frac{3dx^{m+1}\sqrt{c^2dx^2+d}\ {}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-c^2x^2\right)\left(\right.}{\left(m^3+7m^2+14m+8\right)\sqrt{c^2x^2+\right.}$$

```
[Out] x^(1+m)*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/(4+m)+3*d*x^(1+m)*(a+b*arcsi
nh(c*x))*(c^2*d*x^2+d)^(1/2)/(m^2+6*m+8)-3*b*c*d*x^(2+m)*(c^2*d*x^2+d)^(1/2
)/(2+m)^2/(4+m)/(c^2*x^2+1)^(1/2)-b*c*d*x^(2+m)*(c^2*d*x^2+d)^(1/2)/(m^2+6*
m+8)/(c^2*x^2+1)^(1/2)-b*c^3*d*x^(4+m)*(c^2*d*x^2+d)^(1/2)/(4+m)^2/(c^2*x^2
+1)^(1/2)+3*d*x^(1+m)*(a+b*arcsinh(c*x))*hypergeom([1/2, 1/2+1/2*m],[3/2+1/
2*m],-c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(m^3+7*m^2+14*m+8)/(c^2*x^2+1)^(1/2)-3*b
*c*d*x^(2+m)*HypergeometricPFQ([1, 1+1/2*m, 1+1/2*m],[3/2+1/2*m, 2+1/2*m],-
c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(2+m)^2/(m^2+5*m+4)/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 390, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5744, 5742, 5762, 30, 14}

$$-\frac{3bcdx^{m+2}\sqrt{c^2dx^2+d}\ {}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-c^2x^2\right)}{(m+1)(m+2)^2(m+4)\sqrt{c^2x^2+1}}+\frac{3dx^{m+1}\sqrt{c^2dx^2+d}\ {}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-c^2x^2\right)\left(\right.}{\left(m^3+7m^2+14m+8\right)\sqrt{c^2x^2+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-3*b*c*d*x^(2 + m)*Sqrt[d + c^2*d*x^2])/((2 + m)^2*(4 + m)*Sqrt[1 + c^2*x^
2]) - (b*c*d*x^(2 + m)*Sqrt[d + c^2*d*x^2])/((8 + 6*m + m^2)*Sqrt[1 + c^2*x
^2]) - (b*c^3*d*x^(4 + m)*Sqrt[d + c^2*d*x^2])/((4 + m)^2*Sqrt[1 + c^2*x^2]
) + (3*d*x^(1 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(8 + 6*m + m^2
) + (x^(1 + m)*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(4 + m) + (3*d*x
^(1 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1
 + m)/2, (3 + m)/2, -(c^2*x^2)])/((8 + 14*m + 7*m^2 + m^3)*Sqrt[1 + c^2*x^2
]) - (3*b*c*d*x^(2 + m)*Sqrt[d + c^2*d*x^2]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)])/((1 + m)*(2 + m)^2*(4 + m)*Sqr
t[1 + c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
```

```
& EqQ[e, c^2*d] && GtQ[n, 0] && !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] && !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5762

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*ArcSinh[c*x])*Hypergeom
etric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(Sqrt[d]*f*(m + 1)), x] -
Simp[(b*c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2
, 2 + m/2}, -(c^2*x^2)])/(Sqrt[d]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && !IntegerQ[m]
```

Rubi steps

$$\int x^m \left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right)\,dx = \frac{x^{1+m}\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{4+m} + \frac{(3d)\int x^m\sqrt{d+c^2dx^2}\,\left(\ldots\right.}{4+\ldots}$$

$$= \frac{3dx^{1+m}\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)}{8+6m+m^2} + \frac{x^{1+m}\left(d+c^2dx^2\right)^{3/2}\left(a\ldots\right.}{4+m}$$

$$= -\frac{3bcdx^{2+m}\sqrt{d+c^2dx^2}}{(2+m)^2(4+m)\sqrt{1+c^2x^2}} - \frac{bcdx^{2+m}\sqrt{d+c^2dx^2}}{\left(8+6m+m^2\right)\sqrt{1+c^2x^2}} - \frac{bc^3\ldots}{(4\ldots}$$

Mathematica [A] time = 0.52, size = 233, normalized size = 0.60

$$\frac{dx^{m+1}\sqrt{c^2dx^2+d}\left(-\frac{3\left(bcx\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-c^2x^2\right)-(m+2)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)-(m+1)(m+2)\sqrt{c^2x^2+1}\left(a\ldots\right.\right.}{(m+1)(m+2)^2\sqrt{c^2x^2+1}}\ldots\right.}{m+4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d*x^(1 + m)*Sqrt[d + c^2*d*x^2]*(-((b*c*x*(4 + m + c^2*(2 + m)*x^2))/((2 +
 m)*(4 + m)*Sqrt[1 + c^2*x^2])) + (1 + c^2*x^2)*(a + b*ArcSinh[c*x]) - (3*(
b*c*(1 + m)*x - (1 + m)*(2 + m)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]) - (2
 + m)*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c
^2*x^2)] + b*c*x*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m
/2}, -(c^2*x^2)]))/((1 + m)*(2 + m)^2*Sqrt[1 + c^2*x^2])))/(4 + m)
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ac^2dx^2+ad+\left(bc^2dx^2+bd\right)\operatorname{arsinh}(cx)\right)\sqrt{c^2dx^2+d}\,x^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*sqrt(c^2*d*
x^2 + d)*x^m, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 1.42, size = 0, normalized size = 0.00

$$\int x^{m} \left(c^{2} d \,x^{2}+d \right)^{\frac{3}{2}} \left(a +b \,\mathrm{arcsinh}\left(c x \right)\right)d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int {\left(c^{2} d x^{2} + d\right)}^{\frac{3}{2}} {\left(b \operatorname{arsinh}\left(c x\right) + a\right)} x^{m}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)*x^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^m\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(d\,c^2\,x^2+d\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

3.193 $\int x^m \sqrt{d+c^2 d x^2} \left(a+b \sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=240

$$-\frac{bcx^{m+2}\sqrt{c^2dx^2+d}\, {}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-c^2x^2\right)}{(m+1)(m+2)^2\sqrt{c^2x^2+1}}+\frac{x^{m+1}\sqrt{c^2dx^2+d}\, {}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-c^2x^2\right)\left(a\right.}{\left(m^2+3m+2\right)\sqrt{c^2x^2+1}}$$

```
[Out] x^(1+m)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(2+m)-b*c*x^(2+m)*(c^2*d*x^2
+d)^(1/2)/(2+m)^2/(c^2*x^2+1)^(1/2)+x^(1+m)*(a+b*arcsinh(c*x))*hypergeom([1
/2, 1/2+1/2*m],[3/2+1/2*m],-c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(m^2+3*m+2)/(c^2*x
^2+1)^(1/2)-b*c*x^(2+m)*HypergeometricPFQ([1, 1+1/2*m, 1+1/2*m],[3/2+1/2*m,
 2+1/2*m],-c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(1+m)/(2+m)^2/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {5742, 5762, 30}

$$-\frac{bcx^{m+2}\sqrt{c^2dx^2+d}\, {}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-c^2x^2\right)}{(m+1)(m+2)^2\sqrt{c^2x^2+1}}+\frac{x^{m+1}\sqrt{c^2dx^2+d}\, {}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-c^2x^2\right)\left(a\right.}{\left(m^2+3m+2\right)\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -((b*c*x^(2 + m)*Sqrt[d + c^2*d*x^2])/((2 + m)^2*Sqrt[1 + c^2*x^2])) + (x^(
1 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(2 + m) + (x^(1 + m)*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
m)/2, -(c^2*x^2)])/((2 + 3*m + m^2)*Sqrt[1 + c^2*x^2]) - (b*c*x^(2 + m)*Sqr
t[d + c^2*d*x^2]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m
/2}, -(c^2*x^2)])/((1 + m)*(2 + m)^2*Sqrt[1 + c^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5762

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*ArcSinh[c*x])*Hypergeom
etric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(Sqrt[d]*f*(m + 1)), x] -
Simp[(b*c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2
, 2 + m/2}, -(c^2*x^2)])/(Sqrt[d]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int x^m\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)dx=\frac{x^{1+m}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2+m}+\frac{\sqrt{d+c^2dx^2}\int\frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}d}{(2+m)\sqrt{1+c^2x^2}}$$

$$=-\frac{bcx^{2+m}\sqrt{d+c^2dx^2}}{(2+m)^2\sqrt{1+c^2x^2}}+\frac{x^{1+m}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2+m}+\frac{x^{1+m}\sqrt{c}}{}$$

Mathematica [A] time = 0.07, size = 179, normalized size = 0.75

$$\frac{x^{m+1}\sqrt{c^2dx^2+d}\left(-bcx\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-c^2x^2\right)+(m+2)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-c^2x^2\right)\left(a+b\sinh\right.\right.}{(m+1)(m+2)^2\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (x^(1 + m)*Sqrt[d + c^2*d*x^2]*((1 + m)*(-(b*c*x) + a*(2 + m)*Sqrt[1 + c^2*
x^2] + b*(2 + m)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]) + (2 + m)*(a + b*ArcSinh[c
*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)] - b*c*x*Hyper
geometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)]))/((1
 + m)*(2 + m)^2*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{c^2dx^2+d}\,(b\,\text{arsinh}\,(cx)+a)x^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)*x^m, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 1.34, size = 0, normalized size = 0.00

$$\int x^m\sqrt{c^2d\,x^2+d}\,\left(a+b\,\text{arcsinh}\,(cx)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{c^2dx^2+d}\,(b\,\text{arsinh}\,(cx)+a)x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)*x^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^m\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\sqrt{d\,c^2\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int(x^m*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m}\sqrt{d\left(c^{2}x^{2}+1\right)}\,\left(a+b\,\mathrm{asinh}\,(cx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*d*x**2+d)**(1/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x**m*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x)), x)
```

3.194 $\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=161

$$\frac{\sqrt{c^2x^2+1}\,x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{(m+1)\sqrt{c^2dx^2+d}}-\frac{bc\sqrt{c^2x^2+1}\,x^{m+2}\,{}_3F_2\Bigl(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2}{\left(m^2+3m+2\right)\sqrt{c^2dx^2+d}}$$

```
[Out] x^(1+m)*(a+b*arcsinh(c*x))*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],-c^2*x^2)
*(c^2*x^2+1)^(1/2)/(1+m)/(c^2*d*x^2+d)^(1/2)-b*c*x^(2+m)*HypergeometricPFQ(
[1, 1+1/2*m, 1+1/2*m],[3/2+1/2*m, 2+1/2*m],-c^2*x^2)*(c^2*x^2+1)^(1/2)/(m^2
+3*m+2)/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5764, 5762}

$$\frac{\sqrt{c^2x^2+1}\,x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{(m+1)\sqrt{c^2dx^2+d}}-\frac{bc\sqrt{c^2x^2+1}\,x^{m+2}\,{}_3F_2\Bigl(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2}{\left(m^2+3m+2\right)\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1
 + m)/2, (3 + m)/2, -(c^2*x^2)])/((1 + m)*Sqrt[d + c^2*d*x^2]) - (b*c*x^(2
+ m)*Sqrt[1 + c^2*x^2]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2,
 2 + m/2}, -(c^2*x^2)])/((2 + 3*m + m^2)*Sqrt[d + c^2*d*x^2])
```

Rule 5762

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*ArcSinh[c*x])*Hypergeom
etric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(Sqrt[d]*f*(m + 1)), x] -
Simp[(b*c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2
, 2 + m/2}, -(c^2*x^2)])/(Sqrt[d]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] &&  !IntegerQ[m]
```

Rule 5764

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[
((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&  !GtQ[d, 0] && (Integer
Q[m] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx &= \frac{\sqrt{1+c^2x^2}\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}\\ &= \frac{x^{1+m}\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-c^2x^2\right)}{(1+m)\sqrt{d+c^2dx^2}}-\frac{bcx^{2+m}\sqrt{1+c^2x^2}\,{}_3}{\bigl(2}\end{aligned}$$

Mathematica [A] time = 0.05, size = 129, normalized size = 0.80

$$\frac{\sqrt{c^2x^2+1}\,x^{m+1}\left((m+2)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)-bcx\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;\cdot\right.\right.}{(m+1)(m+2)\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + c^2*x^2]*((2 + m)*(a + b*ArcSinh[c*x])*Hypergeometric2F
1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)] - b*c*x*HypergeometricPFQ[{1, 1 +
m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)]))/((1 + m)*(2 + m)*Sqrt[d
+ c^2*d*x^2])
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b\,\text{arsinh}\,(cx)+a)x^m}{\sqrt{c^2dx^2+d}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)*x^m/sqrt(c^2*d*x^2 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\,\text{arsinh}\,(cx)+a)x^m}{\sqrt{c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^m/sqrt(c^2*d*x^2 + d), x)
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int\frac{x^m\,(a+b\,\text{arcsinh}\,(cx))}{\sqrt{c^2d\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\,\text{arsinh}\,(cx)+a)x^m}{\sqrt{c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^m/sqrt(c^2*d*x^2 + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^m\,\left(a+b\,\text{asinh}\left(c\,x\right)\right)}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2),x)
[Out] int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m (a + b \operatorname{asinh}(cx))}{\sqrt{d (c^2x^2 + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
[Out] Integral(x**m*(a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

3.195 $\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=268

$$\frac{bcm\sqrt{c^2x^2+1}\,x^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-c^2x^2\right)}{d\left(m^2+3m+2\right)\sqrt{c^2dx^2+d}}-\frac{m\sqrt{c^2x^2+1}\,x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-c^2x^2\right)\left(a-\right.}{d(m+1)\sqrt{c^2dx^2+d}}$$

```
[Out] x^(1+m)*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(1/2)-m*x^(1+m)*(a+b*arcsinh(c*x
))*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],-c^2*x^2)*(c^2*x^2+1)^(1/2)/d/(1+
m)/(c^2*d*x^2+d)^(1/2)-b*c*x^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],-c^2*x^
2)*(c^2*x^2+1)^(1/2)/d/(2+m)/(c^2*d*x^2+d)^(1/2)+b*c*m*x^(2+m)*Hypergeometr
icPFQ([1, 1+1/2*m, 1+1/2*m],[3/2+1/2*m, 2+1/2*m],-c^2*x^2)*(c^2*x^2+1)^(1/2
)/d/(m^2+3*m+2)/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5755, 5764, 5762, 364}

$$\frac{bcm\sqrt{c^2x^2+1}\,x^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-c^2x^2\right)}{d\left(m^2+3m+2\right)\sqrt{c^2dx^2+d}}-\frac{m\sqrt{c^2x^2+1}\,x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-c^2x^2\right)\left(a-\right.}{d(m+1)\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (x^(1 + m)*(a + b*ArcSinh[c*x]))/(d*Sqrt[d + c^2*d*x^2]) - (m*x^(1 + m)*Sqr
t[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
m)/2, -(c^2*x^2)])/(d*(1 + m)*Sqrt[d + c^2*d*x^2]) - (b*c*x^(2 + m)*Sqrt[1
+ c^2*x^2]*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/(d*(2 +
m)*Sqrt[d + c^2*d*x^2]) + (b*c*m*x^(2 + m)*Sqrt[1 + c^2*x^2]*Hypergeometric
PFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)])/(d*(2 + 3*m +
 m^2)*Sqrt[d + c^2*d*x^2])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5762

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*ArcSinh[c*x])*Hypergeom
```

```
etric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(Sqrt[d]*f*(m + 1)), x] -
Simp[(b*c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2
, 2 + m/2}, -(c^2*x^2)])/(Sqrt[d]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] &&  !IntegerQ[m]
```

Rule 5764

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[
((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&  !GtQ[d, 0] && (Integer
Q[m] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{x^m \left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{3/2}}\,dx = \frac{x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+c^2dx^2}} - \frac{m\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx}{d} - \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{x^{1+m}}{1+c^2x^2}\,dx}{d\sqrt{d+c^2dx^2}}$$

$$= \frac{x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+c^2dx^2}} - \frac{bcx^{2+m}\sqrt{1+c^2x^2}\;{}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};-c^2x^2\right)}{d(2+m)\sqrt{d+c^2dx^2}} - \frac{\left(m\sqrt{1+c\right.}{}$$

$$= \frac{x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+c^2dx^2}} - \frac{mx^{1+m}\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\;{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-\right.}{d(1+m)\sqrt{d+c^2dx^2}}$$

Mathematica [A] time = 0.23, size = 206, normalized size = 0.77

$$\frac{x^{m+1}\left(bcmx\sqrt{c^2x^2+1}\;{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-c^2x^2\right)-m(m+2)\sqrt{c^2x^2+1}\;{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-c^2x^2\right)\right.}{d(m+1)(m+2)\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (x^(1 + m)*(-(m*(2 + m)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Hypergeometr
ic2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)]) + (1 + m)*((2 + m)*(a + b*Arc
Sinh[c*x]) - b*c*x*Sqrt[1 + c^2*x^2]*Hypergeometric2F1[1, 1 + m/2, 2 + m/2,
 -(c^2*x^2)]) + b*c*m*x*Sqrt[1 + c^2*x^2]*HypergeometricPFQ[{1, 1 + m/2, 1
+ m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)]))/(d*(1 + m)*(2 + m)*Sqrt[d + c^2
*d*x^2])
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\,(b\,\mathrm{arsinh}\,(cx)+a)x^m}{c^4d^2x^4+2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)*x^m/(c^4*d^2*x^4 + 2*c^2*
d^2*x^2 + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)x^m}{\left(c^2dx^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d)^(3/2), x)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{x^{m} \left(a+b \,\mathrm{arcsinh}\left(c x\right)\right)}{\left(c^{2} d \,x^{2}+d \right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x)
[Out] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \,\mathrm{arsinh}\left(cx\right) + a)x^{m}}{\left(c^{2}dx^{2} + d\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^m\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{{\left(d\,c^2\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2),x)
[Out] int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m} \left(a + b \operatorname{asinh}{\left(c x \right)}\right)}{\left(d \left(c^{2} x^{2} + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(3/2),x)
[Out] Integral(x**m*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(3/2), x)
```

3.196 $\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=402

$$\frac{bc(2-m)m\sqrt{c^2x^2+1}\,x^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-c^2x^2\right)}{3d^2\left(m^2+3m+2\right)\sqrt{c^2dx^2+d}}-\frac{(2-m)m\sqrt{c^2x^2+1}\,x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2}\right.}{3d^2(m+1)\sqrt{c^2dx^2}}$$

```
[Out] 1/3*x^(1+m)*(a+b*arcsinh(c*x))/d/(c^2*d*x^2+d)^(3/2)+1/3*(2-m)*x^(1+m)*(a+b
*arcsinh(c*x))/d^2/(c^2*d*x^2+d)^(1/2)-1/3*(2-m)*m*x^(1+m)*(a+b*arcsinh(c*x
))*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],-c^2*x^2)*(c^2*x^2+1)^(1/2)/d^2/(
1+m)/(c^2*d*x^2+d)^(1/2)-1/3*b*c*(2-m)*x^(2+m)*hypergeom([1, 1+1/2*m],[2+1/
2*m],-c^2*x^2)*(c^2*x^2+1)^(1/2)/d^2/(2+m)/(c^2*d*x^2+d)^(1/2)-1/3*b*c*x^(2
+m)*hypergeom([2, 1+1/2*m],[2+1/2*m],-c^2*x^2)*(c^2*x^2+1)^(1/2)/d^2/(2+m)/
(c^2*d*x^2+d)^(1/2)+1/3*b*c*(2-m)*m*x^(2+m)*HypergeometricPFQ([1, 1+1/2*m,
1+1/2*m],[3/2+1/2*m, 2+1/2*m],-c^2*x^2)*(c^2*x^2+1)^(1/2)/d^2/(m^2+3*m+2)/(
c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5755, 5764, 5762, 364}

$$\frac{bc(2-m)m\sqrt{c^2x^2+1}\,x^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-c^2x^2\right)}{3d^2\left(m^2+3m+2\right)\sqrt{c^2dx^2+d}}-\frac{(2-m)m\sqrt{c^2x^2+1}\,x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2}\right.}{3d^2(m+1)\sqrt{c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (x^(1 + m)*(a + b*ArcSinh[c*x]))/(3*d*(d + c^2*d*x^2)^(3/2)) + ((2 - m)*x^(
1 + m)*(a + b*ArcSinh[c*x]))/(3*d^2*Sqrt[d + c^2*d*x^2]) - ((2 - m)*m*x^(1
+ m)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, -(c^2*x^2)])/(3*d^2*(1 + m)*Sqrt[d + c^2*d*x^2]) - (b*c*(2 -
m)*x^(2 + m)*Sqrt[1 + c^2*x^2]*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -
(c^2*x^2)])/(3*d^2*(2 + m)*Sqrt[d + c^2*d*x^2]) - (b*c*x^(2 + m)*Sqrt[1 + c
^2*x^2]*Hypergeometric2F1[2, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/(3*d^2*(2 +
 m)*Sqrt[d + c^2*d*x^2]) + (b*c*(2 - m)*m*x^(2 + m)*Sqrt[1 + c^2*x^2]*Hyper
geometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)])/(3*d
^2*(2 + 3*m + m^2)*Sqrt[d + c^2*d*x^2])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
```

)

Rule 5762

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*ArcSinh[c*x])*Hypergeom
etric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(Sqrt[d]*f*(m + 1)), x] -
Simp[(b*c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2
, 2 + m/2}, -(c^2*x^2)])/(Sqrt[d]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] &&  !IntegerQ[m]
```

Rule 5764

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[
((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&  !GtQ[d, 0] && (Integer
Q[m] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{5/2}}\,dx = \frac{x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{(2-m)\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{3/2}}\,dx}{3d} - \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{x}{(1+}}{3d^2\sqrt{d+c^2dx}}$$

$$= \frac{x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{(2-m)x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{d+c^2dx^2}} - \frac{bcx^{2+m}\sqrt{1+c^2x^2}}{3d^2(2+}$$

$$= \frac{x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{(2-m)x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc(2-m)x^{2+m}\sqrt{1}}{3d^2(}$$

$$= \frac{x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{(2-m)x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{d+c^2dx^2}} - \frac{(2-m)mx^{1+m}\sqrt{1}}{}$$

Mathematica [A] time = 0.42, size = 286, normalized size = 0.71

$$\frac{x^{m+1}\left((2-m)\left(c^2x^2+1\right)\left(-m\sqrt{c^2x^2+1}\left((m+2)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)-bcx\,{}_3F_2\left(1,\frac{m}{2}+\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (x^(1 + m)*((1 + m)*(2 + m)*(a + b*ArcSinh[c*x]) - b*c*(1 + m)*x*(1 + c^2*x
^2)^(3/2)*Hypergeometric2F1[2, 1 + m/2, 2 + m/2, -(c^2*x^2)] + (2 - m)*(1 +
 c^2*x^2)*((1 + m)*(2 + m)*(a + b*ArcSinh[c*x]) - b*c*(1 + m)*x*Sqrt[1 + c^
2*x^2]*Hypergeometric2F1[1, 1 + m/2, 2 + m/2, -(c^2*x^2)] - m*Sqrt[1 + c^2*
x^2]*((2 + m)*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m
)/2, -(c^2*x^2)] - b*c*x*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/
2, 2 + m/2}, -(c^2*x^2)]))))/(3*d^2*(1 + m)*(2 + m)*(1 + c^2*x^2)*Sqrt[d +
c^2*d*x^2])
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\,(b\,\text{arsinh}\,(cx)+a)x^m}{c^6d^3x^6+3\,c^4d^3x^4+3\,c^2d^3x^2+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)*x^m/(c^6*d^3*x^6 + 3*c^4*
d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arsinh}\,(cx)+a)x^m}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d)^(5/2), x)
```

maple [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{x^m\,(a+b\,\text{arcsinh}\,(cx))}{(c^2d\,x^2+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arsinh}\,(cx)+a)x^m}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^m\,\left(a+b\,\text{asinh}\,(c\,x)\right)}{{\left(d\,c^2\,x^2+d\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int((x^m*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

3.197 $\int \frac{x^m \sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=102

$$\frac{x^{m+1}\sinh^{-1}(ax)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-a^2x^2\right)}{m+1}-\frac{ax^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-a^2x^2\right)}{m^2+3m+2}$$

```
[Out] x^(1+m)*arcsinh(a*x)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],-a^2*x^2)/(1+m)
-a*x^(2+m)*HypergeometricPFQ([1, 1+1/2*m, 1+1/2*m],[3/2+1/2*m, 2+1/2*m],-a^
2*x^2)/(m^2+3*m+2)
```

Rubi [A] time = 0.07, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {5762}

$$\frac{x^{m+1}\sinh^{-1}(ax)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-a^2x^2\right)}{m+1}-\frac{ax^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-a^2x^2\right)}{m^2+3m+2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (x^(1 + m)*ArcSinh[a*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(a^2*
x^2)])/(1 + m) - (a*x^(2 + m)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2
 + m/2, 2 + m/2}, -(a^2*x^2)])/(2 + 3*m + m^2)
```

Rule 5762

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*ArcSinh[c*x])*Hypergeom
etric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(Sqrt[d]*f*(m + 1)), x] -
Simp[(b*c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2
, 2 + m/2}, -(c^2*x^2)])/(Sqrt[d]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{x^m \sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx=\frac{x^{1+m}\sinh^{-1}(ax)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-a^2x^2\right)}{1+m}-\frac{ax^{2+m}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};\right.}{2+3m+m^2}$$

Mathematica [A] time = 0.03, size = 97, normalized size = 0.95

$$\frac{x^{m+1}\left((m+2)\sinh^{-1}(ax)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-a^2x^2\right)-ax\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-a^2x^2\right)\right)}{(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (x^(1 + m)*((2 + m)*ArcSinh[a*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/
2, -(a^2*x^2)] - a*x*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2
 + m/2}, -(a^2*x^2)]))/((1 + m)*(2 + m))
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m \operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(x^m*arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^m*arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{x^m \operatorname{arcsinh}(ax)}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
[Out] int(x^m*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^m*arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m \operatorname{asinh}(a\,x)}{\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*asinh(a*x))/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x^m*asinh(a*x))/(a^2*x^2 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \operatorname{asinh}(ax)}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
[Out] Integral(x**m*asinh(a*x)/sqrt(a**2*x**2 + 1), x)
```

3.198 $\int x^4 \left(d+c^2dx^2\right) \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=283

$$\frac{1}{7}dx^5\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2-\frac{4bdx^4\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{175c}-\frac{2bd\left(c^2x^2+1\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{49c^5}$$

```
[Out] 304/3675*b^2*d*x/c^4-152/11025*b^2*d*x^3/c^2+38/6125*b^2*d*x^5+2/343*b^2*c^
2*d*x^7-2/21*b*d*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/c^5+4/35*b*d*(c^2*x^2
+1)^(5/2)*(a+b*arcsinh(c*x))/c^5-2/49*b*d*(c^2*x^2+1)^(7/2)*(a+b*arcsinh(c*
x))/c^5+2/35*d*x^5*(a+b*arcsinh(c*x))^2+1/7*d*x^5*(c^2*x^2+1)*(a+b*arcsinh(
c*x))^2-32/525*b*d*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^5+16/525*b*d*x^2*
(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-4/175*b*d*x^4*(a+b*arcsinh(c*x))*(
c^2*x^2+1)^(1/2)/c
```

Rubi [A] time = 0.48, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5744, 5661, 5758, 5717, 8, 30, 266, 43, 5732, 12}

$$\frac{1}{7}dx^5\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2-\frac{4bdx^4\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{175c}+\frac{16bdx^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{525c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (304*b^2*d*x)/(3675*c^4) - (152*b^2*d*x^3)/(11025*c^2) + (38*b^2*d*x^5)/612
5 + (2*b^2*c^2*d*x^7)/343 - (32*b*d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))
/(525*c^5) + (16*b*d*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(525*c^3)
- (4*b*d*x^4*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(175*c) - (2*b*d*(1 +
c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(21*c^5) + (4*b*d*(1 + c^2*x^2)^(5/2)*
(a + b*ArcSinh[c*x]))/(35*c^5) - (2*b*d*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[
c*x]))/(49*c^5) + (2*d*x^5*(a + b*ArcSinh[c*x])^2)/35 + (d*x^5*(1 + c^2*x^2
)*(a + b*ArcSinh[c*x])^2)/7
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int x^4 \left(d + c^2dx^2\right)\left(a + b\sinh^{-1}(cx)\right)^2 dx = \frac{1}{7}dx^5\left(1 + c^2x^2\right)\left(a + b\sinh^{-1}(cx)\right)^2 + \frac{1}{7}(2d)\int x^4 \left(a + b\sinh^{-1}(cx\right.$$

$$= -\frac{2bd\left(1 + c^2x^2\right)^{3/2}\left(a + b\sinh^{-1}(cx)\right)}{21c^5} + \frac{4bd\left(1 + c^2x^2\right)^{5/2}\left(a + b\sin\right.}{35c^5}$$

$$= -\frac{4bdx^4\sqrt{1 + c^2x^2}\left(a + b\sinh^{-1}(cx)\right)}{175c} - \frac{2bd\left(1 + c^2x^2\right)^{3/2}\left(a + b\sin\right.}{21c^5}$$

$$= \frac{16b^2dx}{735c^4} - \frac{8b^2dx^3}{2205c^2} + \frac{38b^2dx^5}{6125} + \frac{2}{343}b^2c^2dx^7 + \frac{16bdx^2\sqrt{1 + c^2x^2}\left(a\right.}{525c}$$

$$= \frac{16b^2dx}{735c^4} - \frac{152b^2dx^3}{11025c^2} + \frac{38b^2dx^5}{6125} + \frac{2}{343}b^2c^2dx^7 - \frac{32bd\sqrt{1 + c^2x^2}\left(a\right.}{525}$$

$$= \frac{304b^2dx}{3675c^4} - \frac{152b^2dx^3}{11025c^2} + \frac{38b^2dx^5}{6125} + \frac{2}{343}b^2c^2dx^7 - \frac{32bd\sqrt{1 + c^2x^2}\left(\right.}{52}$$

Mathematica [A] time = 0.29, size = 201, normalized size = 0.71

$$\frac{d\left(11025a^2c^5x^5\left(5c^2x^2 + 7\right) - 210ab\sqrt{c^2x^2 + 1}\left(75c^6x^6 + 57c^4x^4 - 76c^2x^2 + 152\right) - 210b\sinh^{-1}(cx)\left(b\sqrt{c^2x^2 +}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d*(11025*a^2*c^5*x^5*(7 + 5*c^2*x^2) - 210*a*b*Sqrt[1 + c^2*x^2]*(152 - 76
*c^2*x^2 + 57*c^4*x^4 + 75*c^6*x^6) + b^2*(31920*c*x - 5320*c^3*x^3 + 2394*
c^5*x^5 + 2250*c^7*x^7) - 210*b*(-105*a*c^5*x^5*(7 + 5*c^2*x^2) + b*Sqrt[1
+ c^2*x^2]*(152 - 76*c^2*x^2 + 57*c^4*x^4 + 75*c^6*x^6))*ArcSinh[c*x] + 110
25*b^2*c^5*x^5*(7 + 5*c^2*x^2)*ArcSinh[c*x]^2))/(385875*c^5)
```

fricas [A] time = 0.45, size = 260, normalized size = 0.92

$$\frac{1125\left(49\,a^2 + 2\,b^2\right)c^7dx^7 + 63\left(1225\,a^2 + 38\,b^2\right)c^5dx^5 - 5320\,b^2c^3dx^3 + 31920\,b^2cdx + 11025\left(5\,b^2c^7dx^7 + 7\,b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/385875*(1125*(49*a^2 + 2*b^2)*c^7*d*x^7 + 63*(1225*a^2 + 38*b^2)*c^5*d*x^
5 - 5320*b^2*c^3*d*x^3 + 31920*b^2*c*d*x + 11025*(5*b^2*c^7*d*x^7 + 7*b^2*c
^5*d*x^5)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 210*(525*a*b*c^7*d*x^7 + 735*a*b
*c^5*d*x^5 - (75*b^2*c^6*d*x^6 + 57*b^2*c^4*d*x^4 - 76*b^2*c^2*d*x^2 + 152*
b^2*d)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 210*(75*a*b*c^6*d*
x^6 + 57*a*b*c^4*d*x^4 - 76*a*b*c^2*d*x^2 + 152*a*b*d)*sqrt(c^2*x^2 + 1))/c
^5
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.06, size = 330, normalized size = 1.17

$$d\,a^2\left(\frac{1}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+d\,b^2\left(\frac{\mathrm{arcsinh}(cx)^2c^3x^3\left(c^2x^2+1\right)^2}{7}-\frac{3\,\mathrm{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^2}{35}+\frac{2\,\mathrm{arcsinh}(cx)^2cx}{35}+\frac{\mathrm{arcsinh}(cx)^2cx\left(c^2x^2+1\right)}{35}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c^5*(d*a^2*(1/7*c^7*x^7+1/5*c^5*x^5)+d*b^2*(1/7*arcsinh(c*x)^2*c^3*x^3*(c
^2*x^2+1)^2-3/35*arcsinh(c*x)^2*c*x*(c^2*x^2+1)^2+2/35*arcsinh(c*x)^2*c*x+1
/35*arcsinh(c*x)^2*c*x*(c^2*x^2+1)-2/49*arcsinh(c*x)*c^2*x^2*(c^2*x^2+1)^(5
/2)+62/1225*arcsinh(c*x)*(c^2*x^2+1)^(5/2)+2/343*c*x*(c^2*x^2+1)^3+37384/38
5875*c*x-484/42875*c*x*(c^2*x^2+1)^2-3358/385875*c*x*(c^2*x^2+1)-4/35*arcsi
nh(c*x)*(c^2*x^2+1)^(1/2)-2/105*arcsinh(c*x)*(c^2*x^2+1)^(3/2))+2*d*a*b*(1/
7*arcsinh(c*x)*c^7*x^7+1/5*arcsinh(c*x)*c^5*x^5-1/49*c^6*x^6*(c^2*x^2+1)^(1
/2)-19/1225*c^4*x^4*(c^2*x^2+1)^(1/2)+76/3675*c^2*x^2*(c^2*x^2+1)^(1/2)-152
/3675*(c^2*x^2+1)^(1/2)))
```

maxima [A] time = 0.54, size = 441, normalized size = 1.56

$$\frac{1}{7}\,b^2c^2dx^7\,\mathrm{arsinh}\,(cx)^2+\frac{1}{7}\,a^2c^2dx^7+\frac{1}{5}\,b^2dx^5\,\mathrm{arsinh}\,(cx)^2+\frac{1}{5}\,a^2dx^5+\frac{2}{245}\left(35\,x^7\,\mathrm{arsinh}\,(cx)-\left(\frac{5\,\sqrt{c^2x^2+1}\,x^6}{c^2}-\frac{6}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*c^2*d*x^7*arcsinh(c*x)^2 + 1/7*a^2*c^2*d*x^7 + 1/5*b^2*d*x^5*arcsin
h(c*x)^2 + 1/5*a^2*d*x^5 + 2/245*(35*x^7*arcsinh(c*x) - (5*sqrt(c^2*x^2 + 1
)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*
sqrt(c^2*x^2 + 1)/c^8)*c)*a*b*c^2*d - 2/25725*(105*(5*sqrt(c^2*x^2 + 1)*x^6
/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*sqrt(
c^2*x^2 + 1)/c^8)*c*arcsinh(c*x) - (75*c^6*x^7 - 126*c^4*x^5 + 280*c^2*x^3
- 1680*x)/c^6)*b^2*c^2*d + 2/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)
*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*d
- 2/1125*(15*(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8
*sqrt(c^2*x^2 + 1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c
^4)*b^2*d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,\left(d\,c^2\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*asinh(c*x))^2*(d + c^2*d*x^2),x)
```

```
[Out] int(x^4*(a + b*asinh(c*x))^2*(d + c^2*d*x^2), x)
```

sympy [A] time = 11.40, size = 388, normalized size = 1.37

$$\begin{cases}\frac{a^2c^2dx^7}{7}+\frac{a^2dx^5}{5}+\frac{2abc^2dx^7\,\mathrm{asinh}\,(cx)}{7}-\frac{2abcdx^6\sqrt{c^2x^2+1}}{49}+\frac{2abdx^5\,\mathrm{asinh}\,(cx)}{5}-\frac{38abdx^4\sqrt{c^2x^2+1}}{1225c}+\frac{152abdx^2\sqrt{c^2x^2+1}}{3675c^3}-\frac{304abd\sqrt{c^2x^2}}{3675c^5} \\ \frac{a^2dx^5}{5}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c**2*d*x**2+d)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**2*d*x**7/7 + a**2*d*x**5/5 + 2*a*b*c**2*d*x**7*asinh(c*x
)/7 - 2*a*b*c*d*x**6*sqrt(c**2*x**2 + 1)/49 + 2*a*b*d*x**5*asinh(c*x)/5 - 3
8*a*b*d*x**4*sqrt(c**2*x**2 + 1)/(1225*c) + 152*a*b*d*x**2*sqrt(c**2*x**2 +
 1)/(3675*c**3) - 304*a*b*d*sqrt(c**2*x**2 + 1)/(3675*c**5) + b**2*c**2*d*x
**7*asinh(c*x)**2/7 + 2*b**2*c**2*d*x**7/343 - 2*b**2*c*d*x**6*sqrt(c**2*x*
*2 + 1)*asinh(c*x)/49 + b**2*d*x**5*asinh(c*x)**2/5 + 38*b**2*d*x**5/6125 -
 38*b**2*d*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/(1225*c) - 152*b**2*d*x**3/(
11025*c**2) + 152*b**2*d*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(3675*c**3) +
304*b**2*d*x/(3675*c**4) - 304*b**2*d*sqrt(c**2*x**2 + 1)*asinh(c*x)/(3675*
c**5), Ne(c, 0)), (a**2*d*x**5/5, True))
```

3.199 $\int x^3 \left(d+c^2 d x^2\right) \left(a+b \sinh^{-1}(c x)\right)^2 \, dx$

Optimal. Leaf size=198

$$-\frac{d\left(a+b\sinh^{-1}(cx)\right)^2}{24c^4}-\frac{1}{18}bcdx^5\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{6}dx^4\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2-\frac{bdx^3\sqrt{c^2x^2}}{}$$

```
[Out] -1/24*b^2*d*x^2/c^2+1/72*b^2*d*x^4+1/108*b^2*c^2*d*x^6-1/24*d*(a+b*arcsinh(
c*x))^2/c^4+1/12*d*x^4*(a+b*arcsinh(c*x))^2+1/6*d*x^4*(c^2*x^2+1)*(a+b*arcs
inh(c*x))^2+1/12*b*d*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-1/18*b*d*x^
3*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c-1/18*b*c*d*x^5*(a+b*arcsinh(c*x))*
(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.57, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5744, 5661, 5758, 5675, 30, 5742}

$$-\frac{1}{18}bcdx^5\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{6}dx^4\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2-\frac{bdx^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{18c}-$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -(b^2*d*x^2)/(24*c^2) + (b^2*d*x^4)/72 + (b^2*c^2*d*x^6)/108 + (b*d*x*Sqrt[
1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(12*c^3) - (b*d*x^3*Sqrt[1 + c^2*x^2]*(a
 + b*ArcSinh[c*x]))/(18*c) - (b*c*d*x^5*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*
x]))/18 - (d*(a + b*ArcSinh[c*x])^2)/(24*c^4) + (d*x^4*(a + b*ArcSinh[c*x])
^2)/12 + (d*x^4*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/6
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int x^3\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{1}{6}dx^4\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{3}d\int x^3\left(a+b\sinh^{-1}(cx)\right)^2$$

$$=-\frac{1}{18}bcdx^5\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{12}dx^4\left(a+b\sinh^{-1}(cx)\right)$$

$$=\frac{1}{108}b^2c^2dx^6-\frac{bdx^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{18c}-\frac{1}{18}bcdx^5\sqrt{1+c}$$

$$=\frac{1}{72}b^2dx^4+\frac{1}{108}b^2c^2dx^6+\frac{bdx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{12c^3}-\frac{bdx^3}{}$$

$$=-\frac{b^2dx^2}{24c^2}+\frac{1}{72}b^2dx^4+\frac{1}{108}b^2c^2dx^6+\frac{bdx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(c\right.}{12c^3}$$

Mathematica [A] time = 0.27, size = 186, normalized size = 0.94

$$\frac{d\left(cx\left(18a^2c^3x^3\left(2c^2x^2+3\right)-6ab\sqrt{c^2x^2+1}\left(2c^4x^4+2c^2x^2-3\right)+b^2cx\left(2c^4x^4+3c^2x^2-9\right)\right)+6b\sinh^{-1}(cx)\right.}{216c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d*(c*x*(18*a^2*c^3*x^3*(3 + 2*c^2*x^2) - 6*a*b*Sqrt[1 + c^2*x^2]*(-3 + 2*c
^2*x^2 + 2*c^4*x^4) + b^2*c*x*(-9 + 3*c^2*x^2 + 2*c^4*x^4)) + 6*b*(b*c*x*Sq
rt[1 + c^2*x^2]*(3 - 2*c^2*x^2 - 2*c^4*x^4) + 3*a*(-1 + 6*c^4*x^4 + 4*c^6*x
^6))*ArcSinh[c*x] + 9*b^2*(-1 + 6*c^4*x^4 + 4*c^6*x^6)*ArcSinh[c*x]^2))/(21
6*c^4)
```

fricas [A] time = 0.53, size = 240, normalized size = 1.21

$$\frac{2\left(18\,a^2+b^2\right)c^6dx^6+3\left(18\,a^2+b^2\right)c^4dx^4-9\,b^2c^2dx^2+9\left(4\,b^2c^6dx^6+6\,b^2c^4dx^4-b^2d\right)\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/216*(2*(18*a^2 + b^2)*c^6*d*x^6 + 3*(18*a^2 + b^2)*c^4*d*x^4 - 9*b^2*c^2*
d*x^2 + 9*(4*b^2*c^6*d*x^6 + 6*b^2*c^4*d*x^4 - b^2*d)*log(c*x + sqrt(c^2*x^
2 + 1))^2 + 6*(12*a*b*c^6*d*x^6 + 18*a*b*c^4*d*x^4 - 3*a*b*d - (2*b^2*c^5*d
*x^5 + 2*b^2*c^3*d*x^3 - 3*b^2*c*d*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2
*x^2 + 1)) - 6*(2*a*b*c^5*d*x^5 + 2*a*b*c^3*d*x^3 - 3*a*b*c*d*x)*sqrt(c^2*x
^2 + 1))/c^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.04, size = 267, normalized size = 1.35

$$d\,a^{2}\left(\frac{1}{6}c^{6}x^{6}+\frac{1}{4}c^{4}x^{4}\right)+d\,b^{2}\left(\frac{\operatorname{arcsinh}(cx)^{2}c^{2}x^{2}\left(c^{2}x^{2}+1\right)^{2}}{6}-\frac{\operatorname{arcsinh}(cx)^{2}\left(c^{2}x^{2}+1\right)^{2}}{12}-\frac{\operatorname{arcsinh}(cx)cx\left(c^{2}x^{2}+1\right)^{\frac{5}{2}}}{18}+\frac{\operatorname{arcsinh}(cx)cx\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}}{18}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c^4*(d*a^2*(1/6*c^6*x^6+1/4*c^4*x^4)+d*b^2*(1/6*arcsinh(c*x)^2*c^2*x^2*(c
^2*x^2+1)^2-1/12*arcsinh(c*x)^2*(c^2*x^2+1)^2-1/18*arcsinh(c*x)*c*x*(c^2*x^
2+1)^(5/2)+1/18*arcsinh(c*x)*c*x*(c^2*x^2+1)^(3/2)+1/12*arcsinh(c*x)*(c^2*x
^2+1)^(1/2)*c*x+1/24*arcsinh(c*x)^2+1/108*(c^2*x^2+1)^3-1/72*(c^2*x^2+1)^2-
1/24*c^2*x^2-1/24)+2*d*a*b*(1/6*arcsinh(c*x)*c^6*x^6+1/4*arcsinh(c*x)*c^4*x
^4-1/36*c^5*x^5*(c^2*x^2+1)^(1/2)-1/36*c^3*x^3*(c^2*x^2+1)^(1/2)+1/24*c*x*(
c^2*x^2+1)^(1/2)-1/24*arcsinh(c*x)))
```

maxima [B] time = 0.47, size = 442, normalized size = 2.23

$$\frac{1}{6}\,b^{2}c^{2}dx^{6}\,\operatorname{arsinh}\left(cx\right)^{2}+\frac{1}{6}\,a^{2}c^{2}dx^{6}+\frac{1}{4}\,b^{2}dx^{4}\,\operatorname{arsinh}\left(cx\right)^{2}+\frac{1}{4}\,a^{2}dx^{4}+\frac{1}{144}\left(48\,x^{6}\,\operatorname{arsinh}\left(cx\right)-\left(\frac{8\,\sqrt{c^{2}x^{2}+1}\,x^{5}}{c^{2}}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*c^2*d*x^6*arcsinh(c*x)^2 + 1/6*a^2*c^2*d*x^6 + 1/4*b^2*d*x^4*arcsin
h(c*x)^2 + 1/4*a^2*d*x^4 + 1/144*(48*x^6*arcsinh(c*x) - (8*sqrt(c^2*x^2 + 1
)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 + 1)*x/c^6 - 15*
arcsinh(c*x)/c^7)*c)*a*b*c^2*d + 1/864*((8*x^6/c^2 - 15*x^4/c^4 + 45*x^2/c^
6 - 45*log(c*x + sqrt(c^2*x^2 + 1))^2/c^8)*c^2 - 6*(8*sqrt(c^2*x^2 + 1)*x^5
/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 + 1)*x/c^6 - 15*arcsi
nh(c*x)/c^7)*c*arcsinh(c*x))*b^2*c^2*d + 1/16*(8*x^4*arcsinh(c*x) - (2*sqrt
(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*
a*b*d + 1/32*((x^4/c^2 - 3*x^2/c^4 + 3*log(c*x + sqrt(c^2*x^2 + 1))^2/c^6)*
c^2 - 2*(2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsin
h(c*x)/c^5)*c*arcsinh(c*x))*b^2*d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 (a + b \operatorname{asinh}(c x))^2 \left(d c^2 x^2 + d\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2),x)
```

```
[Out] int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2), x)
```

sympy [A] time = 7.86, size = 332, normalized size = 1.68

$$\begin{cases} \frac{a^2c^2dx^6}{6} + \frac{a^2dx^4}{4} + \frac{abc^2dx^6 \operatorname{asinh}(cx)}{3} - \frac{abcdx^5\sqrt{c^2x^2+1}}{18} + \frac{abdx^4 \operatorname{asinh}(cx)}{2} - \frac{abdx^3\sqrt{c^2x^2+1}}{18c} + \frac{abdx\sqrt{c^2x^2+1}}{12c^3} - \frac{abd \operatorname{asinh}(cx)}{12c^4} + \frac{b^2 \\ \frac{a^2dx^4}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*d*x**2+d)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**2*d*x**6/6 + a**2*d*x**4/4 + a*b*c**2*d*x**6*asinh(c*x)/
3 - a*b*c*d*x**5*sqrt(c**2*x**2 + 1)/18 + a*b*d*x**4*asinh(c*x)/2 - a*b*d*x
**3*sqrt(c**2*x**2 + 1)/(18*c) + a*b*d*x*sqrt(c**2*x**2 + 1)/(12*c**3) - a*
b*d*asinh(c*x)/(12*c**4) + b**2*c**2*d*x**6*asinh(c*x)**2/6 + b**2*c**2*d*x
**6/108 - b**2*c*d*x**5*sqrt(c**2*x**2 + 1)*asinh(c*x)/18 + b**2*d*x**4*asi
nh(c*x)**2/4 + b**2*d*x**4/72 - b**2*d*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/
(18*c) - b**2*d*x**2/(24*c**2) + b**2*d*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/(1
2*c**3) - b**2*d*asinh(c*x)**2/(24*c**4), Ne(c, 0)), (a**2*d*x**4/4, True))
```

3.200 $\int x^2 \left(d+c^2 d x^2\right) \left(a+b \sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=206

$$-\frac{4bdx^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{45c}+\frac{1}{5}dx^3\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2-\frac{2bd\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{25c^3}+$$

```
[Out] -52/225*b^2*d*x/c^2+26/675*b^2*d*x^3+2/125*b^2*c^2*d*x^5+2/15*b*d*(c^2*x^2+
1)^(3/2)*(a+b*arcsinh(c*x))/c^3-2/25*b*d*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x
))/c^3+2/15*d*x^3*(a+b*arcsinh(c*x))^2+1/5*d*x^3*(c^2*x^2+1)*(a+b*arcsinh(c
*x))^2+8/45*b*d*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-4/45*b*d*x^2*(a+b*
arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] time = 0.34, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5744, 5661, 5758, 5717, 8, 30, 266, 43, 5732, 12}

$$\frac{1}{5}dx^3\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2-\frac{4bdx^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{45c}-\frac{2bd\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{25c^3}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-52*b^2*d*x)/(225*c^2) + (26*b^2*d*x^3)/675 + (2*b^2*c^2*d*x^5)/125 + (8*b
*d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(45*c^3) - (4*b*d*x^2*Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x]))/(45*c) + (2*b*d*(1 + c^2*x^2)^(3/2)*(a + b*Ar
cSinh[c*x]))/(15*c^3) - (2*b*d*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(2
5*c^3) + (2*d*x^3*(a + b*ArcSinh[c*x])^2)/15 + (d*x^3*(1 + c^2*x^2)*(a + b*
ArcSinh[c*x])^2)/5
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int x^2\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{1}{5}dx^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{5}(2d)\int x^2\left(a+b\sinh^{-1}(cx)\right)^2$$
$$=\frac{2bd\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{15c^3}-\frac{2bd\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(\right.}{25c^3}$$
$$=-\frac{4bdx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{45c}+\frac{2bd\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-}\right.}{15c^3}$$
$$=-\frac{4b^2dx}{75c^2}+\frac{26}{675}b^2dx^3+\frac{2}{125}b^2c^2dx^5+\frac{8bd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx\right.}{45c^3}$$
$$=-\frac{52b^2dx}{225c^2}+\frac{26}{675}b^2dx^3+\frac{2}{125}b^2c^2dx^5+\frac{8bd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(c\right.}{45c^3}$$

Mathematica [A] time = 0.25, size = 177, normalized size = 0.86

$$\frac{d\left(225a^2c^3x^3\left(3c^2x^2+5\right)-30ab\sqrt{c^2x^2+1}\left(9c^4x^4+13c^2x^2-26\right)-30b\sinh^{-1}(cx)\left(b\sqrt{c^2x^2+1}\left(9c^4x^4+13c^2x^2\right.\right.\right.}{3375c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d*(225*a^2*c^3*x^3*(5 + 3*c^2*x^2) - 30*a*b*Sqrt[1 + c^2*x^2]*(-26 + 13*c^
2*x^2 + 9*c^4*x^4) + 2*b^2*c*x*(-390 + 65*c^2*x^2 + 27*c^4*x^4) - 30*b*(-15
*a*c^3*x^3*(5 + 3*c^2*x^2) + b*Sqrt[1 + c^2*x^2]*(-26 + 13*c^2*x^2 + 9*c^4*
x^4))*ArcSinh[c*x] + 225*b^2*c^3*x^3*(5 + 3*c^2*x^2)*ArcSinh[c*x]^2))/(3375
*c^3)
```

fricas [A] time = 0.67, size = 225, normalized size = 1.09

$$\frac{27\left(25a^2+2b^2\right)c^5dx^5+5\left(225a^2+26b^2\right)c^3dx^3-780b^2cdx+225\left(3b^2c^5dx^5+5b^2c^3dx^3\right)\log\left(cx+\sqrt{c^2x^2+1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/3375*(27*(25*a^2 + 2*b^2)*c^5*d*x^5 + 5*(225*a^2 + 26*b^2)*c^3*d*x^3 - 78
0*b^2*c*d*x + 225*(3*b^2*c^5*d*x^5 + 5*b^2*c^3*d*x^3)*log(c*x + sqrt(c^2*x^
2 + 1))^2 + 30*(45*a*b*c^5*d*x^5 + 75*a*b*c^3*d*x^3 - (9*b^2*c^4*d*x^4 + 13
*b^2*c^2*d*x^2 - 26*b^2*d)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))
- 30*(9*a*b*c^4*d*x^4 + 13*a*b*c^2*d*x^2 - 26*a*b*d)*sqrt(c^2*x^2 + 1))/c^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.04, size = 248, normalized size = 1.20

$$d\,a^2\left(\frac{1}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+d\,b^2\left(\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^2}{5}-\frac{2\operatorname{arcsinh}(cx)^2cx}{15}-\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)}{15}-\frac{2\operatorname{arcsinh}(cx)\left(c^2x^2+1\right)^{\frac{5}{2}}}{25}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c^3*(d*a^2*(1/5*c^5*x^5+1/3*c^3*x^3)+d*b^2*(1/5*arcsinh(c*x)^2*c*x*(c^2*x
^2+1)^2-2/15*arcsinh(c*x)^2*c*x-1/15*arcsinh(c*x)^2*c*x*(c^2*x^2+1)-2/25*ar
csinh(c*x)*(c^2*x^2+1)^(5/2)-856/3375*c*x+2/125*c*x*(c^2*x^2+1)^2+22/3375*c
*x*(c^2*x^2+1)+4/15*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2/45*arcsinh(c*x)*(c^2*x
^2+1)^(3/2))+2*d*a*b*(1/5*arcsinh(c*x)*c^5*x^5+1/3*arcsinh(c*x)*c^3*x^3-1/2
5*c^4*x^4*(c^2*x^2+1)^(1/2)-13/225*c^2*x^2*(c^2*x^2+1)^(1/2)+26/225*(c^2*x^
2+1)^(1/2)))
```

maxima [A] time = 0.44, size = 346, normalized size = 1.68

$$\frac{1}{5}\,b^2c^2dx^5\,\operatorname{arsinh}(cx)^2+\frac{1}{5}\,a^2c^2dx^5+\frac{1}{3}\,b^2dx^3\,\operatorname{arsinh}(cx)^2+\frac{2}{75}\left(15\,x^5\,\operatorname{arsinh}(cx)-\left(\frac{3\,\sqrt{c^2x^2+1}\,x^4}{c^2}-\frac{4\,\sqrt{c^2x^2+}}{c^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*c^2*d*x^5*arcsinh(c*x)^2 + 1/5*a^2*c^2*d*x^5 + 1/3*b^2*d*x^3*arcsin
h(c*x)^2 + 2/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqr
t(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*c^2*d - 2/1125*(15
*(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^
2 + 1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c^4)*b^2*c^2*
d + 1/3*a^2*d*x^3 + 2/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2
- 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*d - 2/27*(3*c*(sqrt(c^2*x^2 + 1)*x^2/c^2 -
2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3 - 6*x)/c^2)*b^2*d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,(a+b\,\mathrm{asinh}\,(c\,x))^2\,\left(d\,c^2\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2),x)
```

```
[Out] int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2), x)
```

sympy [A] time = 5.21, size = 313, normalized size = 1.52

$$\begin{cases}\frac{a^2c^2dx^5}{5}+\frac{a^2dx^3}{3}+\frac{2abc^2dx^5\,\mathrm{asinh}\,(cx)}{5}-\frac{2abcdx^4\sqrt{c^2x^2+1}}{25}+\frac{2abdx^3\,\mathrm{asinh}\,(cx)}{3}-\frac{26abdx^2\sqrt{c^2x^2+1}}{225c}+\frac{52abd\sqrt{c^2x^2+1}}{225c^3}+\frac{b^2c^2dx^5\,\mathrm{asin}}{5}\\ \frac{a^2dx^3}{3}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**2*d*x**5/5 + a**2*d*x**3/3 + 2*a*b*c**2*d*x**5*asinh(c*x
)/5 - 2*a*b*c*d*x**4*sqrt(c**2*x**2 + 1)/25 + 2*a*b*d*x**3*asinh(c*x)/3 - 2
6*a*b*d*x**2*sqrt(c**2*x**2 + 1)/(225*c) + 52*a*b*d*sqrt(c**2*x**2 + 1)/(22
```

```
5*c**3) + b**2*c**2*d*x**5*asinh(c*x)**2/5 + 2*b**2*c**2*d*x**5/125 - 2*b**
2*c*d*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/25 + b**2*d*x**3*asinh(c*x)**2/3
+ 26*b**2*d*x**3/675 - 26*b**2*d*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(225*c
) - 52*b**2*d*x/(225*c**2) + 52*b**2*d*sqrt(c**2*x**2 + 1)*asinh(c*x)/(225*
c**3), Ne(c, 0)), (a**2*d*x**3/3, True))
```

3.201 $\int x \left(d+c^2dx^2\right) \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=135

$$-\frac{bdx\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{8c}-\frac{3bdx\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{16c}+\frac{d\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2}-$$

```
[Out] 5/32*b^2*d*x^2+1/32*b^2*c^2*d*x^4-1/8*b*d*x*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(
c*x))/c-3/32*d*(a+b*arcsinh(c*x))^2/c^2+1/4*d*(c^2*x^2+1)^2*(a+b*arcsinh(c*
x))^2/c^2-3/16*b*d*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] time = 0.13, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5717, 5684, 5682, 5675, 30, 14}

$$-\frac{bdx\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{8c}-\frac{3bdx\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{16c}+\frac{d\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2}-$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (5*b^2*d*x^2)/32 + (b^2*c^2*d*x^4)/32 - (3*b*d*x*Sqrt[1 + c^2*x^2]*(a + b*A
rcSinh[c*x]))/(16*c) - (b*d*x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(8*
c) - (3*d*(a + b*ArcSinh[c*x])^2)/(32*c^2) + (d*(1 + c^2*x^2)^2*(a + b*ArcS
inh[c*x])^2)/(4*c^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
```

```
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\int x\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{d\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2}-\frac{(bd)\int\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(c\right.}{2c}$$

$$=-\frac{bdx\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{8c}+\frac{d\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right.}{4c^2}$$

$$=-\frac{3bdx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{16c}-\frac{bdx\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(\right.}{8c}$$

$$=\frac{5}{32}b^2dx^2+\frac{1}{32}b^2c^2dx^4-\frac{3bdx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{16c}-\frac{bdx\left(1+\right.}{}$$

Mathematica [A] time = 0.26, size = 155, normalized size = 1.15

$$\frac{d\left(cx\left(8a^2cx\left(c^2x^2+2\right)-2ab\sqrt{c^2x^2+1}\left(2c^2x^2+5\right)+b^2cx\left(c^2x^2+5\right)\right)+2b\sinh^{-1}(cx)\left(a\left(8c^4x^4+16c^2x^2+5\right)-\right.\right.}{32c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d*(c*x*(8*a^2*c*x*(2 + c^2*x^2) + b^2*c*x*(5 + c^2*x^2) - 2*a*b*Sqrt[1 + c
^2*x^2]*(5 + 2*c^2*x^2)) + 2*b*(-(b*c*x*Sqrt[1 + c^2*x^2]*(5 + 2*c^2*x^2))
+ a*(5 + 16*c^2*x^2 + 8*c^4*x^4))*ArcSinh[c*x] + b^2*(5 + 16*c^2*x^2 + 8*c^
4*x^4)*ArcSinh[c*x]^2))/(32*c^2)
```

fricas [A] time = 0.75, size = 204, normalized size = 1.51

$$\frac{\left(8a^2+b^2\right)c^4dx^4+\left(16a^2+5b^2\right)c^2dx^2+\left(8b^2c^4dx^4+16b^2c^2dx^2+5b^2d\right)\log\left(cx+\sqrt{c^2x^2+1}\right)^2+2\left(8abc^4dx^4\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/32*((8*a^2 + b^2)*c^4*d*x^4 + (16*a^2 + 5*b^2)*c^2*d*x^2 + (8*b^2*c^4*d*x
^4 + 16*b^2*c^2*d*x^2 + 5*b^2*d)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*(8*a*b*
c^4*d*x^4 + 16*a*b*c^2*d*x^2 + 5*a*b*d - (2*b^2*c^3*d*x^3 + 5*b^2*c*d*x)*sq
rt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 2*(2*a*b*c^3*d*x^3 + 5*a*b*
c*d*x)*sqrt(c^2*x^2 + 1))/c^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.04, size = 191, normalized size = 1.41

$$\frac{d\,a^2\left(\frac{1}{4}c^4x^4+\frac{1}{2}c^2x^2\right)+d\,b^2\left(\frac{\operatorname{arcsinh}(cx)^2\left(c^2x^2+1\right)^2}{4}-\frac{\operatorname{arcsinh}(cx)cx\left(c^2x^2+1\right)^{\frac{3}{2}}}{8}-\frac{3\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,cx}{16}-\frac{3\operatorname{arcsinh}(cx)^2}{32}+\frac{(c^2}{}\right.}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c^2*(d*a^2*(1/4*c^4*x^4+1/2*c^2*x^2)+d*b^2*(1/4*arcsinh(c*x)^2*(c^2*x^2+1
)^2-1/8*arcsinh(c*x)*c*x*(c^2*x^2+1)^(3/2)-3/16*arcsinh(c*x)*(c^2*x^2+1)^(1
/2)*c*x-3/32*arcsinh(c*x)^2+1/32*(c^2*x^2+1)^2+3/32*c^2*x^2+3/32)+2*d*a*b*(
1/4*arcsinh(c*x)*c^4*x^4+1/2*arcsinh(c*x)*c^2*x^2-1/16*c^3*x^3*(c^2*x^2+1)^
(1/2)-5/32*c*x*(c^2*x^2+1)^(1/2)+5/32*arcsinh(c*x)))
```

maxima [B] time = 0.37, size = 347, normalized size = 2.57

$$\frac{1}{4}\,b^2c^2dx^4\operatorname{arsinh}(cx)^2+\frac{1}{4}\,a^2c^2dx^4+\frac{1}{2}\,b^2dx^2\operatorname{arsinh}(cx)^2+\frac{1}{16}\left(8\,x^4\operatorname{arsinh}(cx)-\left(\frac{2\sqrt{c^2x^2+1}\,x^3}{c^2}-\frac{3\sqrt{c^2x^2+}}{c^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*c^2*d*x^4*arcsinh(c*x)^2 + 1/4*a^2*c^2*d*x^4 + 1/2*b^2*d*x^2*arcsin
h(c*x)^2 + 1/16*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt
(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*a*b*c^2*d + 1/32*((x^4/c^2 - 3
*x^2/c^4 + 3*log(c*x + sqrt(c^2*x^2 + 1))^2/c^6)*c^2 - 2*(2*sqrt(c^2*x^2 +
1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c*arcsinh(c*x)
)*b^2*c^2*d + 1/2*a^2*d*x^2 + 1/2*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1
)*x/c^2 - arcsinh(c*x)/c^3))*a*b*d + 1/4*(c^2*(x^2/c^2 - log(c*x + sqrt(c^2
*x^2 + 1))^2/c^4) - 2*c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsinh(c*x)/c^3)*arcsin
h(c*x))*b^2*d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,(a+b\operatorname{asinh}(c\,x))^2\,\left(d\,c^2\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2),x)
```

```
[Out] int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2), x)
```

sympy [A] time = 3.33, size = 269, normalized size = 1.99

$$\begin{cases}\frac{a^2c^2dx^4}{4}+\frac{a^2dx^2}{2}+\frac{abc^2dx^4\operatorname{asinh}(cx)}{2}-\frac{abcdx^3\sqrt{c^2x^2+1}}{8}+abdx^2\operatorname{asinh}(cx)-\frac{5abdx\sqrt{c^2x^2+1}}{16c}+\frac{5abd\operatorname{asinh}(cx)}{16c^2}+\frac{b^2c^2dx^4\operatorname{asinh}}{4} \\ \frac{a^2dx^2}{2}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*d*x**2+d)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**2*d*x**4/4 + a**2*d*x**2/2 + a*b*c**2*d*x**4*asinh(c*x)/
2 - a*b*c*d*x**3*sqrt(c**2*x**2 + 1)/8 + a*b*d*x**2*asinh(c*x) - 5*a*b*d*x*
sqrt(c**2*x**2 + 1)/(16*c) + 5*a*b*d*asinh(c*x)/(16*c**2) + b**2*c**2*d*x**
4*asinh(c*x)**2/4 + b**2*c**2*d*x**4/32 - b**2*c*d*x**3*sqrt(c**2*x**2 + 1)
*asinh(c*x)/8 + b**2*d*x**2*asinh(c*x)**2/2 + 5*b**2*d*x**2/32 - 5*b**2*d*x
*sqrt(c**2*x**2 + 1)*asinh(c*x)/(16*c) + 5*b**2*d*asinh(c*x)**2/(32*c**2),
Ne(c, 0)), (a**2*d*x**2/2, True))
```

3.202 $\int \left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=125

$$\frac{1}{3}dx\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2-\frac{2bd\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{9c}-\frac{4bd\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{3c}+\frac{2}{3}$$

```
[Out] 14/9*b^2*d*x+2/27*b^2*c^2*d*x^3-2/9*b*d*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x)
)/c+2/3*d*x*(a+b*arcsinh(c*x))^2+1/3*d*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2-4
/3*b*d*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] time = 0.14, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5684, 5653, 5717, 8}

$$\frac{1}{3}dx\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2-\frac{2bd\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{9c}-\frac{4bd\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{3c}+\frac{2}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (14*b^2*d*x)/9 + (2*b^2*c^2*d*x^3)/27 - (4*b*d*Sqrt[1 + c^2*x^2]*(a + b*Arc
Sinh[c*x]))/(3*c) - (2*b*d*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(9*c)
+ (2*d*x*(a + b*ArcSinh[c*x])^2)/3 + (d*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]
)^2)/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)^2\,dx = \frac{1}{3}dx\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{3}(2d)\int \left(a+b\sinh^{-1}(cx)\right)^2\,dx - $$
$$= -\frac{2bd\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{9c}+\frac{2}{3}dx\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{3}dx\Big($$
$$= \frac{2}{9}b^2dx+\frac{2}{27}b^2c^2dx^3-\frac{4bd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3c}-\frac{2bd\left(1+c^2x^2\right.}{}$$
$$= \frac{14}{9}b^2dx+\frac{2}{27}b^2c^2dx^3-\frac{4bd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3c}-\frac{2bd\left(1+c^2x\right.}{}$$

Mathematica [A] time = 0.21, size = 135, normalized size = 1.08

$$\frac{d\left(9a^2cx\left(c^2x^2+3\right)-6ab\sqrt{c^2x^2+1}\left(c^2x^2+7\right)-6b\sinh^{-1}(cx)\left(b\sqrt{c^2x^2+1}\left(c^2x^2+7\right)-3acx\left(c^2x^2+3\right)\right)+2b^2c\right.}{27c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d*(9*a^2*c*x*(3 + c^2*x^2) - 6*a*b*Sqrt[1 + c^2*x^2]*(7 + c^2*x^2) + 2*b^2
*c*x*(21 + c^2*x^2) - 6*b*(-3*a*c*x*(3 + c^2*x^2) + b*Sqrt[1 + c^2*x^2]*(7
+ c^2*x^2))*ArcSinh[c*x] + 9*b^2*c*x*(3 + c^2*x^2)*ArcSinh[c*x]^2))/(27*c)
```

fricas [A] time = 0.63, size = 178, normalized size = 1.42

$$\frac{\left(9\,a^2+2\,b^2\right)c^3dx^3+3\left(9\,a^2+14\,b^2\right)cdx+9\left(b^2c^3dx^3+3\,b^2cdx\right)\log\left(cx+\sqrt{c^2x^2+1}\right)^2+6\left(3\,abc^3dx^3+9\,abcdx\right.}{27\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/27*((9*a^2 + 2*b^2)*c^3*d*x^3 + 3*(9*a^2 + 14*b^2)*c*d*x + 9*(b^2*c^3*d*x
^3 + 3*b^2*c*d*x)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 6*(3*a*b*c^3*d*x^3 + 9*a
*b*c*d*x - (b^2*c^2*d*x^2 + 7*b^2*d)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*
x^2 + 1)) - 6*(a*b*c^2*d*x^2 + 7*a*b*d)*sqrt(c^2*x^2 + 1))/c
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.05, size = 166, normalized size = 1.33

$$\frac{d\,a^2\left(\frac{1}{3}c^3x^3+cx\right)+d\,b^2\left(\frac{2\operatorname{arcsinh}(cx)^2cx}{3}+\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)}{3}-\frac{4\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}}{3}+\frac{40cx}{27}-\frac{2\operatorname{arcsinh}(cx)\left(c^2x^2+1\right)^{\frac{3}{2}}}{9}+\frac{2c}{}\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c*(d*a^2*(1/3*c^3*x^3+c*x)+d*b^2*(2/3*arcsinh(c*x)^2*c*x+1/3*arcsinh(c*x)
^2*c*x*(c^2*x^2+1)-4/3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+40/27*c*x-2/9*arcsinh
(c*x)*(c^2*x^2+1)^(3/2)+2/27*c*x*(c^2*x^2+1))+2*d*a*b*(1/3*arcsinh(c*x)*c^3
*x^3+arcsinh(c*x)*c*x-1/9*c^2*x^2*(c^2*x^2+1)^(1/2)-7/9*(c^2*x^2+1)^(1/2)))
```

maxima [B] time = 0.72, size = 230, normalized size = 1.84

$$\frac{1}{3}\,b^2c^2dx^3\,\mathrm{arsinh}\,(cx)^2+\frac{1}{3}\,a^2c^2dx^3+\frac{2}{9}\left(3\,x^3\,\mathrm{arsinh}\,(cx)-c\left(\frac{\sqrt{c^2x^2+1}\,x^2}{c^2}-\frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)abc^2d-\frac{2}{27}\left(3\,c\left(\frac{\sqrt{c^2}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*c^2*d*x^3*arcsinh(c*x)^2 + 1/3*a^2*c^2*d*x^3 + 2/9*(3*x^3*arcsinh(c
*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*c^2*d -
2/27*(3*c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x
) - (c^2*x^3 - 6*x)/c^2)*b^2*c^2*d + b^2*d*x*arcsinh(c*x)^2 + 2*b^2*d*(x -
sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*d*x + 2*(c*x*arcsinh(c*x) - sqrt(c^
2*x^2 + 1))*a*b*d/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\mathrm{asinh}\,(c\,x))^2\,\left(d\,c^2\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + c^2*d*x^2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + c^2*d*x^2), x)
```

sympy [A] time = 1.45, size = 224, normalized size = 1.79

$$\begin{cases} \frac{a^2c^2dx^3}{3}+a^2dx+\frac{2abc^2dx^3\,\mathrm{asinh}\,(cx)}{3}-\frac{2abcdx^2\sqrt{c^2x^2+1}}{9}+2abdx\,\mathrm{asinh}\,(cx)-\frac{14abd\sqrt{c^2x^2+1}}{9c}+\frac{b^2c^2dx^3\,\mathrm{asinh}^2\,(cx)}{3}+\frac{2b^2c^2d}{27} \\ a^2dx \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**2*d*x**3/3 + a**2*d*x + 2*a*b*c**2*d*x**3*asinh(c*x)/3 -
 2*a*b*c*d*x**2*sqrt(c**2*x**2 + 1)/9 + 2*a*b*d*x*asinh(c*x) - 14*a*b*d*sqr
t(c**2*x**2 + 1)/(9*c) + b**2*c**2*d*x**3*asinh(c*x)**2/3 + 2*b**2*c**2*d*x
**3/27 - 2*b**2*c*d*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/9 + b**2*d*x*asinh(
c*x)**2 + 14*b**2*d*x/9 - 14*b**2*d*sqrt(c**2*x**2 + 1)*asinh(c*x)/(9*c), N
e(c, 0)), (a**2*d*x, True))
```

3.203 $\int \frac{(d+c^2dx^2)(a+b\sinh^{-1}(cx))^2}{x}\,dx$

Optimal. Leaf size=166

$$\frac{1}{2}d\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2-\frac{1}{2}bcdx\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)-bd\text{Li}_2\left(e^{-2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)+$$

```
[Out] 1/4*b^2*c^2*d*x^2-1/4*d*(a+b*arcsinh(c*x))^2+1/2*d*(c^2*x^2+1)*(a+b*arcsinh
(c*x))^2+1/3*d*(a+b*arcsinh(c*x))^3/b+d*(a+b*arcsinh(c*x))^2*ln(1-1/(c*x+(c
^2*x^2+1)^(1/2))^2)-b*d*(a+b*arcsinh(c*x))*polylog(2,1/(c*x+(c^2*x^2+1)^(1/
2))^2)-1/2*b^2*d*polylog(3,1/(c*x+(c^2*x^2+1)^(1/2))^2)-1/2*b*c*d*x*(a+b*ar
csinh(c*x))*(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 165, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5744, 5659, 3716, 2190, 2531, 2282, 6589, 5682, 5675, 30}

$$bd\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{2}b^2d\text{PolyLog}\left(3,e^{2\sinh^{-1}(cx)}\right)+\frac{1}{2}d\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2-$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] (b^2*c^2*d*x^2)/4 - (b*c*d*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/2 - (d
*(a + b*ArcSinh[c*x])^2)/4 + (d*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/2 - (
d*(a + b*ArcSinh[c*x])^3)/(3*b) + d*(a + b*ArcSinh[c*x])^2*Log[1 - E^(2*Arc
Sinh[c*x])] + b*d*(a + b*ArcSinh[c*x])*PolyLog[2, E^(2*ArcSinh[c*x])] - (b^
2*d*PolyLog[3, E^(2*ArcSinh[c*x])])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(d+c^2dx^2)(a+b\sinh^{-1}(cx))^2}{x}\,dx = \frac{1}{2}d(1+c^2x^2)(a+b\sinh^{-1}(cx))^2 + d\int \frac{(a+b\sinh^{-1}(cx))^2}{x}\,dx - (bcd)$$
$$= -\frac{1}{2}bcdx\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx)) + \frac{1}{2}d(1+c^2x^2)(a+b\sinh^{-1}(cx))$$
$$= \frac{1}{4}b^2c^2dx^2 - \frac{1}{2}bcdx\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx)) - \frac{1}{4}d(a+b\sinh^{-1}(cx))$$
$$= \frac{1}{4}b^2c^2dx^2 - \frac{1}{2}bcdx\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx)) - \frac{1}{4}d(a+b\sinh^{-1}(cx))$$
$$= \frac{1}{4}b^2c^2dx^2 - \frac{1}{2}bcdx\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx)) - \frac{1}{4}d(a+b\sinh^{-1}(cx))$$
$$= \frac{1}{4}b^2c^2dx^2 - \frac{1}{2}bcdx\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx)) - \frac{1}{4}d(a+b\sinh^{-1}(cx))$$
$$= \frac{1}{4}b^2c^2dx^2 - \frac{1}{2}bcdx\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx)) - \frac{1}{4}d(a+b\sinh^{-1}(cx))$$

Mathematica [A] time = 0.39, size = 209, normalized size = 1.26

$$\frac{1}{8}d\left(4a^2c^2x^2 + 8a^2\log(x) - 4ab\left(cx\sqrt{c^2x^2+1} - \sinh^{-1}(cx)\right) + 8abc^2x^2\sinh^{-1}(cx) - 8ab\text{Li}_2\left(e^{-2\sinh^{-1}(cx)}\right) + 8ab\,\mathrm{s}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] (d*(4*a^2*c^2*x^2 - 4*a*b*(c*x*Sqrt[1 + c^2*x^2] - ArcSinh[c*x]) + 8*a*b*c^
2*x^2*ArcSinh[c*x] + b^2*(1 + 2*ArcSinh[c*x]^2)*Cosh[2*ArcSinh[c*x]] + 8*a*
b*ArcSinh[c*x]*(ArcSinh[c*x] + 2*Log[1 - E^(-2*ArcSinh[c*x])]) + 8*a^2*Log[
x] - 8*a*b*PolyLog[2, E^(-2*ArcSinh[c*x])] + 8*b^2*(-1/3*ArcSinh[c*x]^3 + A
rcSinh[c*x]^2*Log[1 - E^(2*ArcSinh[c*x])] + ArcSinh[c*x]*PolyLog[2, E^(2*Ar
cSinh[c*x])] - PolyLog[3, E^(2*ArcSinh[c*x])]/2) - 2*b^2*ArcSinh[c*x]*Sinh[
2*ArcSinh[c*x]]))/8
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^2c^2dx^2 + a^2d + (b^2c^2dx^2 + b^2d)\,\text{arsinh}(cx)^2 + 2\,(abc^2dx^2 + abd)\,\text{arsinh}(cx)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))/x, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.17, size = 425, normalized size = 2.56

$$\frac{d\,a^2c^2x^2}{2}+d\,a^2\ln(cx)-\frac{d\,b^2\,\mathrm{arcsinh}\,(cx)^3}{3}+\frac{d\,b^2\,\mathrm{arcsinh}\,(cx)^2\,c^2x^2}{2}-\frac{d\,b^2\,\mathrm{arcsinh}\,(cx)\,\sqrt{c^2x^2+1}\;cx}{2}+\frac{d\,b^2\,\mathrm{arcsin}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x,x)
```

```
[Out] 1/2*d*a^2*c^2*x^2+d*a^2*ln(c*x)-1/3*d*b^2*arcsinh(c*x)^3+1/2*d*b^2*arcsinh(
c*x)^2*c^2*x^2-1/2*d*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c*x+1/4*d*b^2*arcsi
nh(c*x)^2+1/4*b^2*c^2*d*x^2+1/8*d*b^2+d*b^2*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^
2+1)^(1/2))+2*d*b^2*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))-2*d*b^2*
polylog(3,-c*x-(c^2*x^2+1)^(1/2))+d*b^2*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)
^(1/2))+2*d*b^2*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))-2*d*b^2*polyl
og(3,c*x+(c^2*x^2+1)^(1/2))-d*a*b*arcsinh(c*x)^2+d*a*b*arcsinh(c*x)*c^2*x^2
-1/2*d*a*b*c*x*(c^2*x^2+1)^(1/2)+1/2*d*a*b*arcsinh(c*x)+2*d*a*b*arcsinh(c*x
)*ln(1+c*x+(c^2*x^2+1)^(1/2))+2*d*a*b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+2*d
*a*b*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*d*a*b*polylog(2,c*x+(c^2*x^
2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,a^2c^2dx^2+a^2d\log(x)+\int b^2c^2dx\log\left(cx+\sqrt{c^2x^2+1}\right)^2+2\,abc^2dx\log\left(cx+\sqrt{c^2x^2+1}\right)+\frac{b^2d\log\left(cx+\sqrt{c^2x^2}\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*c^2*d*x^2 + a^2*d*log(x) + integrate(b^2*c^2*d*x*log(c*x + sqrt(c^2
*x^2 + 1))^2 + 2*a*b*c^2*d*x*log(c*x + sqrt(c^2*x^2 + 1)) + b^2*d*log(c*x +
 sqrt(c^2*x^2 + 1))^2/x + 2*a*b*d*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))^2\,\left(d\,c^2\,x^2+d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2))/x,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int\frac{a^2}{x}\,dx+\int a^2c^2x\,dx+\int\frac{b^2\,\mathrm{asinh}^2\,(cx)}{x}\,dx+\int\frac{2ab\,\mathrm{asinh}\,(cx)}{x}\,dx+\int b^2c^2x\,\mathrm{asinh}^2\,(cx)\,dx+\int 2abc^2x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))**2/x,x)
```

```
[Out] d*(Integral(a**2/x, x) + Integral(a**2*c**2*x, x) + Integral(b**2*asinh(c*x
)**2/x, x) + Integral(2*a*b*asinh(c*x)/x, x) + Integral(b**2*c**2*x*asinh(c
*x)**2, x) + Integral(2*a*b*c**2*x*asinh(c*x), x))
```

3.204 $\int \frac{(d+c^2dx^2)(a+b\sinh^{-1}(cx))^2}{x^2}\,dx$

Optimal. Leaf size=131

$$-2bcd\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)-\frac{d\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{x}+2c^2dx\left(a+b\sinh^{-1}(cx)\right)^2-4bcd\tanh^{-1}\left(e^{s}\right.$$

[Out] 2*b^2*c^2*d*x+2*c^2*d*x*(a+b*arcsinh(c*x))^2-d*(c^2*x^2+1)*(a+b*arcsinh(c*x
))^2/x-4*b*c*d*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))-2*b^2*c*d*
polylog(2,-c*x-(c^2*x^2+1)^(1/2))+2*b^2*c*d*polylog(2,c*x+(c^2*x^2+1)^(1/2)
)-2*b*c*d*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)

Rubi [A] time = 0.32, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5739, 5653, 5717, 8, 5742, 5760, 4182, 2279, 2391}

$$-2b^2cd\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)+2b^2cd\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)-2bcd\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)-\frac{d\left(c^2x^2+1\right)\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] 2*b^2*c^2*d*x - 2*b*c*d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]) + 2*c^2*d*x*
(a + b*ArcSinh[c*x])^2 - (d*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/x - 4*b*c
*d*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] - 2*b^2*c*d*PolyLog[2, -E^A
rcSinh[c*x]] + 2*b^2*c*d*PolyLog[2, E^ArcSinh[c*x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{x^2}\,dx = -\frac{d\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{x} + (2bcd)\int \frac{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}\right.}{x}$$

$$= 2bcd\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right) + 2c^2dx\left(a+b\sinh^{-1}(cx)\right)^2 - \frac{d\left(1\right.}{}$$

$$= -2b^2c^2dx - 2bcd\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right) + 2c^2dx\left(a+b\sinh^{-1}(\right.$$

$$= 2b^2c^2dx - 2bcd\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right) + 2c^2dx\left(a+b\sinh^{-1}(cx\right.$$

$$= 2b^2c^2dx - 2bcd\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right) + 2c^2dx\left(a+b\sinh^{-1}(cx\right.$$

$$= 2b^2c^2dx - 2bcd\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right) + 2c^2dx\left(a+b\sinh^{-1}(cx\right.$$

Mathematica [A] time = 0.43, size = 192, normalized size = 1.47

$$\frac{d\left(a^2c^2x^2 - a^2 + 2abcx\left(cx\sinh^{-1}(cx) - \sqrt{c^2x^2+1}\right) - 2ab\left(cx\tanh^{-1}\left(\sqrt{c^2x^2+1}\right) + \sinh^{-1}(cx)\right) + b^2cx\left(-2\sqrt{c^2\cdots\right.\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] (d*(-a^2 + a^2*c^2*x^2 + 2*a*b*c*x*(-Sqrt[1 + c^2*x^2] + c*x*ArcSinh[c*x])
+ b^2*c*x*(2*c*x - 2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + c*x*ArcSinh[c*x]^2) -
 2*a*b*(ArcSinh[c*x] + c*x*ArcTanh[Sqrt[1 + c^2*x^2]]) - b^2*(ArcSinh[c*x]*
(ArcSinh[c*x] + 2*c*x*(-Log[1 - E^(-ArcSinh[c*x])] + Log[1 + E^(-ArcSinh[c*
x])])) - 2*c*x*PolyLog[2, -E^(-ArcSinh[c*x])] + 2*c*x*PolyLog[2, E^(-ArcSin
h[c*x])])))/x
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^2c^2dx^2 + a^2d + \left(b^2c^2dx^2 + b^2d\right)\text{arsinh}\left(cx\right)^2 + 2\left(abc^2dx^2 + abd\right)\text{arsinh}\left(cx\right)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))/x^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.18, size = 252, normalized size = 1.92

$$d\,a^2c^2x - \frac{d\,a^2}{x} + d\,b^2\,\text{arcsinh}\left(cx\right)^2c^2x - 2cd\,b^2\,\text{arcsinh}\left(cx\right)\sqrt{c^2x^2+1} + 2b^2c^2dx - \frac{d\,b^2\,\text{arcsinh}\left(cx\right)^2}{x} - 2cd\,b^2\,\text{arcsinh}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^2,x)
```

```
[Out] d*a^2*c^2*x-d*a^2/x+d*b^2*arcsinh(c*x)^2*c^2*x-2*c*d*b^2*arcsinh(c*x)*(c^2*
x^2+1)^(1/2)+2*b^2*c^2*d*x-d*b^2*arcsinh(c*x)^2/x-2*c*d*b^2*arcsinh(c*x)*ln
(1+c*x+(c^2*x^2+1)^(1/2))-2*b^2*c*d*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+2*c*d
*b^2*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*b^2*c*d*polylog(2,c*x+(c^2*
x^2+1)^(1/2))+2*d*a*b*arcsinh(c*x)*c^2*x-2*d*a*b*arcsinh(c*x)/x-2*c*d*a*b*(
c^2*x^2+1)^(1/2)-2*c*d*a*b*arctanh(1/(c^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^2c^2dx\,\text{arsinh}\left(cx\right)^2 + 2\,b^2c^2d\left(x - \frac{\sqrt{c^2x^2+1}\,\text{arsinh}\left(cx\right)}{c}\right) + a^2c^2dx + 2\left(cx\,\text{arsinh}\left(cx\right) - \sqrt{c^2x^2+1}\right)abcd - 2\left(c\,\text{arsin}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="maxima")
```

```
[Out] b^2*c^2*d*x*arcsinh(c*x)^2 + 2*b^2*c^2*d*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x
)/c) + a^2*c^2*d*x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*c*d - 2*(
c*arcsinh(1/(c*abs(x))) + arcsinh(c*x)/x)*a*b*d - b^2*d*(log(c*x + sqrt(c^2
*x^2 + 1))^2/x - integrate(2*(c^3*x^2 + sqrt(c^2*x^2 + 1)*c^2*x + c)*log(c*
x + sqrt(c^2*x^2 + 1))/(c^3*x^4 + c*x^2 + (c^2*x^3 + x)*sqrt(c^2*x^2 + 1)),
 x)) - a^2*d/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))^2\,\left(d\,c^2\,x^2+d\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2))/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int a^2c^2\,dx+\int \frac{a^2}{x^2}\,dx+\int b^2c^2\,\mathrm{asinh}^2\,(cx)\,dx+\int \frac{b^2\,\mathrm{asinh}^2\,(cx)}{x^2}\,dx+\int 2abc^2\,\mathrm{asinh}\,(cx)\,dx+\int \frac{2ab\,\mathrm{asi}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))**2/x**2,x)
```

```
[Out] d*(Integral(a**2*c**2, x) + Integral(a**2/x**2, x) + Integral(b**2*c**2*asi
nh(c*x)**2, x) + Integral(b**2*asinh(c*x)**2/x**2, x) + Integral(2*a*b*c**2
*asinh(c*x), x) + Integral(2*a*b*asinh(c*x)/x**2, x))
```

3.205 $\int \frac{(d+c^2dx^2)(a+b\sinh^{-1}(cx))^2}{x^3}\,dx$

Optimal. Leaf size=180

$$-bc^2d\mathrm{Li}_2\left(e^{-2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{d\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{2x^2}-\frac{bcd\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{x}+\cdots$$

```
[Out] 1/2*c^2*d*(a+b*arcsinh(c*x))^2-1/2*d*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/x^2+1
/3*c^2*d*(a+b*arcsinh(c*x))^3/b+c^2*d*(a+b*arcsinh(c*x))^2*ln(1-1/(c*x+(c^2
*x^2+1)^(1/2))^2)+b^2*c^2*d*ln(x)-b*c^2*d*(a+b*arcsinh(c*x))*polylog(2,1/(c
*x+(c^2*x^2+1)^(1/2))^2)-1/2*b^2*c^2*d*polylog(3,1/(c*x+(c^2*x^2+1)^(1/2))^
2)-b*c*d*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.31, antiderivative size = 179, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5739, 5659, 3716, 2190, 2531, 2282, 6589, 5737, 29, 5675}

$$bc^2d\mathrm{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{2}b^2c^2d\mathrm{PolyLog}\left(3,e^{2\sinh^{-1}(cx)}\right)-\frac{d\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{2x^2}\cdots$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] -((b*c*d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/x) + (c^2*d*(a + b*ArcSinh
[c*x])^2)/2 - (d*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(2*x^2) - (c^2*d*(a
+ b*ArcSinh[c*x])^3)/(3*b) + c^2*d*(a + b*ArcSinh[c*x])^2*Log[1 - E^(2*ArcS
inh[c*x])] + b^2*c^2*d*Log[x] + b*c^2*d*(a + b*ArcSinh[c*x])*PolyLog[2, E^(
2*ArcSinh[c*x])] - (b^2*c^2*d*PolyLog[3, E^(2*ArcSinh[c*x])])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5737

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 1)*S
qrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x] -
 Dist[(c^2*Sqrt[d + e*x^2])/(f^2*(m + 1)*Sqrt[1 + c^2*x^2]), Int[((f*x)^(m
+ 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(d+c^2dx^2)(a+b\sinh^{-1}(cx))^2}{x^3}dx = -\frac{d(1+c^2x^2)(a+b\sinh^{-1}(cx))^2}{2x^2} + (bcd)\int \frac{\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx$$

$$= -\frac{bcd\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))}{x} - \frac{d(1+c^2x^2)(a+b\sinh^{-1}(cx))^2}{2x^2} +$$

$$= -\frac{bcd\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))}{x} + \frac{1}{2}c^2d(a+b\sinh^{-1}(cx))^2 - \frac{d(1+c}{}$$

$$= -\frac{bcd\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))}{x} + \frac{1}{2}c^2d(a+b\sinh^{-1}(cx))^2 - \frac{d(1+c}{}$$

$$= -\frac{bcd\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))}{x} + \frac{1}{2}c^2d(a+b\sinh^{-1}(cx))^2 - \frac{d(1+c}{}$$

$$= -\frac{bcd\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))}{x} + \frac{1}{2}c^2d(a+b\sinh^{-1}(cx))^2 - \frac{d(1+c}{}$$

$$= -\frac{bcd\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))}{x} + \frac{1}{2}c^2d(a+b\sinh^{-1}(cx))^2 - \frac{d(1+c}{}$$

Mathematica [A] time = 0.38, size = 212, normalized size = 1.18

$$\frac{1}{2}d\left(2a^2c^2\log(x) - \frac{a^2}{x^2} + 2abc^2\left(\sinh^{-1}(cx)\left(\sinh^{-1}(cx) + 2\log\left(1-e^{-2\sinh^{-1}(cx)}\right)\right) - \text{Li}_2\left(e^{-2\sinh^{-1}(cx)}\right)\right) - \frac{2ab(cx}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] (d*(-(a^2/x^2) - (2*a*b*(c*x*Sqrt[1 + c^2*x^2] + ArcSinh[c*x]))/x^2 + 2*a^2
*c^2*Log[x] - (b^2*(2*c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + ArcSinh[c*x]^2 -
 2*c^2*x^2*Log[c*x]))/x^2 + 2*a*b*c^2*(ArcSinh[c*x]*(ArcSinh[c*x] + 2*Log[1
 - E^(-2*ArcSinh[c*x])]) - PolyLog[2, E^(-2*ArcSinh[c*x])]) - (b^2*c^2*(2*A
rcSinh[c*x]^2*(ArcSinh[c*x] - 3*Log[1 - E^(2*ArcSinh[c*x])]) - 6*ArcSinh[c*
x]*PolyLog[2, E^(2*ArcSinh[c*x])] + 3*PolyLog[3, E^(2*ArcSinh[c*x])]))/3))/
2
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^2c^2dx^2 + a^2d + (b^2c^2dx^2 + b^2d)\,\text{arsinh}\,(cx)^2 + 2\,(abc^2dx^2 + abd)\,\text{arsinh}\,(cx)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))/x^3, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.37, size = 515, normalized size = 2.86

$$c^2 d\,a^2 \ln(cx) - \frac{d\,a^2}{2x^2} - \frac{c^2 d\,b^2 \operatorname{arcsinh}(cx)^3}{3} - \frac{cd\,b^2 \operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}}{x} + c^2 d\,b^2 \operatorname{arcsinh}(cx) - \frac{d\,b^2 \operatorname{arcsinh}(cx)^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^3,x)
```

```
[Out] c^2*d*a^2*ln(c*x)-1/2*d*a^2/x^2-1/3*c^2*d*b^2*arcsinh(c*x)^3-c*d*b^2*arcsin
h(c*x)/x*(c^2*x^2+1)^(1/2)+c^2*d*b^2*arcsinh(c*x)-1/2*d*b^2*arcsinh(c*x)^2/
x^2-2*c^2*d*b^2*ln(c*x+(c^2*x^2+1)^(1/2))+c^2*d*b^2*ln(c*x+(c^2*x^2+1)^(1/2
)-1)+c^2*d*b^2*ln(1+c*x+(c^2*x^2+1)^(1/2))+c^2*d*b^2*arcsinh(c*x)^2*ln(1+c*
x+(c^2*x^2+1)^(1/2))+2*c^2*d*b^2*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1
/2))-2*c^2*d*b^2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))+c^2*d*b^2*arcsinh(c*x)^2
*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*c^2*d*b^2*arcsinh(c*x)*polylog(2,c*x+(c^2*x^
2+1)^(1/2))-2*c^2*d*b^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))-c^2*d*a*b*arcsinh(
c*x)^2-c*d*a*b/x*(c^2*x^2+1)^(1/2)+c^2*d*a*b-d*a*b*arcsinh(c*x)/x^2+2*c^2*d
*a*b*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+2*c^2*d*a*b*polylog(2,-c*x-(c
^2*x^2+1)^(1/2))+2*c^2*d*a*b*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*c^2
*d*a*b*polylog(2,c*x+(c^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2c^2d\log(x) - abd\left(\frac{\sqrt{c^2x^2+1}\,c}{x} + \frac{\operatorname{arsinh}(cx)}{x^2}\right) - \frac{a^2d}{2\,x^2} + \int \frac{b^2c^2d\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{x} + \frac{2\,abc^2d\log\left(cx+\sqrt{c^2x^2+1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="maxima")
```

```
[Out] a^2*c^2*d*log(x) - a*b*d*(sqrt(c^2*x^2 + 1)*c/x + arcsinh(c*x)/x^2) - 1/2*a
^2*d/x^2 + integrate(b^2*c^2*d*log(c*x + sqrt(c^2*x^2 + 1))^2/x + 2*a*b*c^2
*d*log(c*x + sqrt(c^2*x^2 + 1))/x + b^2*d*log(c*x + sqrt(c^2*x^2 + 1))^2/x^
3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\operatorname{asinh}(c\,x))^2\,\left(d\,c^2\,x^2+d\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2))/x^3,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int \frac{a^2}{x^3}\,dx + \int \frac{a^2c^2}{x}\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{x^3}\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{x^3}\,dx + \int \frac{b^2c^2\operatorname{asinh}^2(cx)}{x}\,dx + \int \frac{2abc^2\operatorname{as}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))**2/x**3,x)
```

```
[Out] d*(Integral(a**2/x**3, x) + Integral(a**2*c**2/x, x) + Integral(b**2*asinh(
c*x)**2/x**3, x) + Integral(2*a*b*asinh(c*x)/x**3, x) + Integral(b**2*c**2*
asinh(c*x)**2/x, x) + Integral(2*a*b*c**2*asinh(c*x)/x, x))
```

3.206 $\int \frac{\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{x^4}\,dx$

Optimal. Leaf size=158

$$-\frac{10}{3}bc^3d\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{bcd\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{3x^2}-\frac{d\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right.}{3x^3}$$

```
[Out] -1/3*b^2*c^2*d/x-2/3*c^2*d*(a+b*arcsinh(c*x))^2/x-1/3*d*(c^2*x^2+1)*(a+b*ar
csinh(c*x))^2/x^3-10/3*b*c^3*d*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(
1/2))-5/3*b^2*c^3*d*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+5/3*b^2*c^3*d*polylog
(2,c*x+(c^2*x^2+1)^(1/2))-1/3*b*c*d*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/x^
2
```

Rubi [A] time = 0.39, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5739, 5661, 5760, 4182, 2279, 2391, 5737, 30}

$$-\frac{5}{3}b^2c^3d\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)+\frac{5}{3}b^2c^3d\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)-\frac{bcd\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{3x^2}-\frac{d\left(c^2x^2+1\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] -(b^2*c^2*d)/(3*x) - (b*c*d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(3*x^2)
 - (2*c^2*d*(a + b*ArcSinh[c*x])^2)/(3*x) - (d*(1 + c^2*x^2)*(a + b*ArcSinh
[c*x])^2)/(3*x^3) - (10*b*c^3*d*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]
])/3 - (5*b^2*c^3*d*PolyLog[2, -E^ArcSinh[c*x]])/3 + (5*b^2*c^3*d*PolyLog[2
, E^ArcSinh[c*x]])/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
```

```
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5737

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 1)*S
qrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x] -
 Dist[(c^2*Sqrt[d + e*x^2])/(f^2*(m + 1)*Sqrt[1 + c^2*x^2]), Int[((f*x)^(m
+ 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{x^4}\,dx = -\frac{d\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3x^3}+\frac{1}{3}(2bcd)\int \frac{\sqrt{1+c^2x^2}\,\left(a+b\sinh\right.}{x^3}$$

$$= -\frac{bcd\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{3x^2}-\frac{2c^2d\left(a+b\sinh^{-1}(cx)\right)^2}{3x}-\frac{d\left(1\right.}{}$$

$$= -\frac{b^2c^2d}{3x}-\frac{bcd\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{3x^2}-\frac{2c^2d\left(a+b\sinh^{-1}(cx)\right)}{3x}$$

$$= -\frac{b^2c^2d}{3x}-\frac{bcd\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{3x^2}-\frac{2c^2d\left(a+b\sinh^{-1}(cx)\right)}{3x}$$

$$= -\frac{b^2c^2d}{3x}-\frac{bcd\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{3x^2}-\frac{2c^2d\left(a+b\sinh^{-1}(cx)\right)}{3x}$$

$$= -\frac{b^2c^2d}{3x}-\frac{bcd\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{3x^2}-\frac{2c^2d\left(a+b\sinh^{-1}(cx)\right)}{3x}$$

Mathematica [A] time = 0.80, size = 245, normalized size = 1.55

$$-\frac{d\left(3a^2c^2x^2+a^2+abcx\sqrt{c^2x^2+1}+6abc^2x^2\sinh^{-1}(cx)+5abc^3x^3\tanh^{-1}\left(\sqrt{c^2x^2+1}\right)+2ab\sinh^{-1}(cx)-5b\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] -1/3*(d*(a^2 + 3*a^2*c^2*x^2 + b^2*c^2*x^2 + a*b*c*x*Sqrt[1 + c^2*x^2] + 2*
a*b*ArcSinh[c*x] + 6*a*b*c^2*x^2*ArcSinh[c*x] + b^2*c*x*Sqrt[1 + c^2*x^2]*A
rcSinh[c*x] + b^2*ArcSinh[c*x]^2 + 3*b^2*c^2*x^2*ArcSinh[c*x]^2 + 5*a*b*c^3
*x^3*ArcTanh[Sqrt[1 + c^2*x^2]] - 5*b^2*c^3*x^3*ArcSinh[c*x]*Log[1 - E^(-Ar
cSinh[c*x])] + 5*b^2*c^3*x^3*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] - 5*b^
2*c^3*x^3*PolyLog[2, -E^(-ArcSinh[c*x])] + 5*b^2*c^3*x^3*PolyLog[2, E^(-Arc
Sinh[c*x])]))/x^3
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^2c^2dx^2 + a^2d + \left(b^2c^2dx^2 + b^2d\right)\operatorname{arsinh}(cx)^2 + 2\left(abc^2dx^2 + abd\right)\operatorname{arsinh}(cx)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))/x^4, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.36, size = 278, normalized size = 1.76

$$-\frac{c^2d\,a^2}{x}-\frac{d\,a^2}{3x^3}-\frac{c^2d\,b^2\operatorname{arcsinh}(cx)^2}{x}-\frac{cd\,b^2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}}{3x^2}-\frac{d\,b^2\operatorname{arcsinh}(cx)^2}{3x^3}-\frac{b^2c^2d}{3x}-\frac{5c^3d\,b^2\operatorname{arcsinh}(c\dots}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^4,x)
```

```
[Out] -c^2*d*a^2/x-1/3*d*a^2/x^3-c^2*d*b^2*arcsinh(c*x)^2/x-1/3*c*d*b^2/x^2*arcsi
nh(c*x)*(c^2*x^2+1)^(1/2)-1/3*d*b^2/x^3*arcsinh(c*x)^2-1/3*b^2*c^2*d/x-5/3*
c^3*d*b^2*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-5/3*b^2*c^3*d*polylog(2,
-c*x-(c^2*x^2+1)^(1/2))+5/3*c^3*d*b^2*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/
2))+5/3*b^2*c^3*d*polylog(2,c*x+(c^2*x^2+1)^(1/2))-2*c^2*d*a*b*arcsinh(c*x)
/x-2/3*d*a*b*arcsinh(c*x)/x^3-5/3*c^3*d*a*b*arctanh(1/(c^2*x^2+1)^(1/2))-1/
3*c*d*a*b/x^2*(c^2*x^2+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-2\left(c\operatorname{arsinh}\left(\frac{1}{c|x|}\right)+\frac{\operatorname{arsinh}(cx)}{x}\right)abc^2d+\frac{1}{3}\left(\left(c^2\operatorname{arsinh}\left(\frac{1}{c|x|}\right)-\frac{\sqrt{c^2x^2+1}}{x^2}\right)c-\frac{2\operatorname{arsinh}(cx)}{x^3}\right)abd-\frac{a^2c^2d}{x}-\frac{a^2d}{3x^3}\dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="maxima")
```

```
[Out] -2*(c*arcsinh(1/(c*abs(x))) + arcsinh(c*x)/x)*a*b*c^2*d + 1/3*((c^2*arcsinh
(1/(c*abs(x))) - sqrt(c^2*x^2 + 1)/x^2)*c - 2*arcsinh(c*x)/x^3)*a*b*d - a^2
*c^2*d/x - 1/3*a^2*d/x^3 - 1/3*(3*b^2*c^2*d*x^2 + b^2*d)*log(c*x + sqrt(c^2
*x^2 + 1))^2/x^3 + integrate(2/3*(3*b^2*c^5*d*x^4 + 4*b^2*c^3*d*x^2 + b^2*c
*d + (3*b^2*c^4*d*x^3 + b^2*c^2*d*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*
x^2 + 1))/(c^3*x^6 + c*x^4 + (c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{asinh}\,(c\,x))^2\,\left(d\,c^2\,x^2 + d\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2))/x^4,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int \frac{a^2}{x^4}\,dx + \int \frac{a^2c^2}{x^2}\,dx + \int \frac{b^2\,\mathrm{asinh}^2\,(cx)}{x^4}\,dx + \int \frac{2ab\,\mathrm{asinh}\,(cx)}{x^4}\,dx + \int \frac{b^2c^2\,\mathrm{asinh}^2\,(cx)}{x^2}\,dx + \int \frac{2abc^2\,\mathrm{as}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))**2/x**4,x)
```

```
[Out] d*(Integral(a**2/x**4, x) + Integral(a**2*c**2/x**2, x) + Integral(b**2*asi
nh(c*x)**2/x**4, x) + Integral(2*a*b*asinh(c*x)/x**4, x) + Integral(b**2*c*
*2*asinh(c*x)**2/x**2, x) + Integral(2*a*b*c**2*asinh(c*x)/x**2, x))
```

3.207 $\int x^4 \left(d+c^2 d x^2\right)^2 \left(a+b \sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=386

$$\frac{1}{9}d^2x^5\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)^2+\frac{4}{63}d^2x^5\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2-\frac{16bd^2x^4\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(c\right.}{1575c}$$

```
[Out] 4208/99225*b^2*d^2*x/c^4-2104/297675*b^2*d^2*x^3/c^2+526/165375*b^2*d^2*x^5
+212/27783*b^2*c^2*d^2*x^7+2/729*b^2*c^4*d^2*x^9-8/189*b*d^2*(c^2*x^2+1)^(3
/2)*(a+b*arcsinh(c*x))/c^5+2/315*b*d^2*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))
/c^5+20/441*b*d^2*(c^2*x^2+1)^(7/2)*(a+b*arcsinh(c*x))/c^5-2/81*b*d^2*(c^2*
x^2+1)^(9/2)*(a+b*arcsinh(c*x))/c^5+8/315*d^2*x^5*(a+b*arcsinh(c*x))^2+4/63
*d^2*x^5*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2+1/9*d^2*x^5*(c^2*x^2+1)^2*(a+b*ar
csinh(c*x))^2-128/4725*b*d^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^5+64/47
25*b*d^2*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-16/1575*b*d^2*x^4*(a+
b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] time = 0.74, antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {5744, 5661, 5758, 5717, 8, 30, 266, 43, 5732, 12, 1153}

$$\frac{1}{9}d^2x^5\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)^2+\frac{4}{63}d^2x^5\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2-\frac{16bd^2x^4\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(c\right.}{1575c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (4208*b^2*d^2*x)/(99225*c^4) - (2104*b^2*d^2*x^3)/(297675*c^2) + (526*b^2*d
^2*x^5)/165375 + (212*b^2*c^2*d^2*x^7)/27783 + (2*b^2*c^4*d^2*x^9)/729 - (1
28*b*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(4725*c^5) + (64*b*d^2*x^2
*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(4725*c^3) - (16*b*d^2*x^4*Sqrt[1
+ c^2*x^2]*(a + b*ArcSinh[c*x]))/(1575*c) - (8*b*d^2*(1 + c^2*x^2)^(3/2)*(a
 + b*ArcSinh[c*x]))/(189*c^5) + (2*b*d^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh
[c*x]))/(315*c^5) + (20*b*d^2*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(44
1*c^5) - (2*b*d^2*(1 + c^2*x^2)^(9/2)*(a + b*ArcSinh[c*x]))/(81*c^5) + (8*d
^2*x^5*(a + b*ArcSinh[c*x])^2)/315 + (4*d^2*x^5*(1 + c^2*x^2)*(a + b*ArcSin
h[c*x])^2)/63 + (d^2*x^5*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/9
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int x^4 \left(d+c^2dx^2\right)^2 \left(a+b\sinh^{-1}(cx)\right)^2 \, dx = \frac{1}{9}d^2x^5\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2 + \frac{1}{9}(4d)\int x^4\left(d+c^2dx^2\right)\left(a+\right.$$

$$= -\frac{2bd^2\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{45c^5} + \frac{4bd^2\left(1+c^2x^2\right)^{7/2}\left(a+b\,\mathrm{sir}\right.}{63c^5}$$

$$= -\frac{8bd^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{189c^5} + \frac{2bd^2\left(1+c^2x^2\right)^{5/2}\left(a+b\,\mathrm{sir}\right.}{315c^5}$$

$$= -\frac{16bd^2x^4\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{1575c} - \frac{8bd^2\left(1+c^2x^2\right)^{3/2}\left(a+b\,\mathrm{si}\right.}{189c^5}$$

$$= \frac{304b^2d^2x}{19845c^4} - \frac{152b^2d^2x^3}{59535c^2} + \frac{526b^2d^2x^5}{165375} + \frac{212b^2c^2d^2x^7}{27783} + \frac{2}{729}b^2c^4d^2x^9$$

$$= \frac{304b^2d^2x}{19845c^4} - \frac{2104b^2d^2x^3}{297675c^2} + \frac{526b^2d^2x^5}{165375} + \frac{212b^2c^2d^2x^7}{27783} + \frac{2}{729}b^2c^4d^2x$$

$$= \frac{4208b^2d^2x}{99225c^4} - \frac{2104b^2d^2x^3}{297675c^2} + \frac{526b^2d^2x^5}{165375} + \frac{212b^2c^2d^2x^7}{27783} + \frac{2}{729}b^2c^4d^2$$

Mathematica [A] time = 0.41, size = 251, normalized size = 0.65

$$\frac{d^2\left(99225a^2c^5x^5\left(35c^4x^4+90c^2x^2+63\right)-630ab\sqrt{c^2x^2+1}\left(1225c^8x^8+2650c^6x^6+789c^4x^4-1052c^2x^2+2104\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^2*(99225*a^2*c^5*x^5*(63 + 90*c^2*x^2 + 35*c^4*x^4) - 630*a*b*Sqrt[1 + c
^2*x^2]*(2104 - 1052*c^2*x^2 + 789*c^4*x^4 + 2650*c^6*x^6 + 1225*c^8*x^8) +
 2*b^2*c*x*(662760 - 110460*c^2*x^2 + 49707*c^4*x^4 + 119250*c^6*x^6 + 4287
5*c^8*x^8) - 630*b*(-315*a*c^5*x^5*(63 + 90*c^2*x^2 + 35*c^4*x^4) + b*Sqrt[
1 + c^2*x^2]*(2104 - 1052*c^2*x^2 + 789*c^4*x^4 + 2650*c^6*x^6 + 1225*c^8*x
^8))*ArcSinh[c*x] + 99225*b^2*c^5*x^5*(63 + 90*c^2*x^2 + 35*c^4*x^4)*ArcSin
h[c*x]^2))/(31255875*c^5)
```

fricas [A] time = 0.68, size = 368, normalized size = 0.95

$$\frac{42875\left(81\,a^2+2\,b^2\right)c^9d^2x^9+2250\left(3969\,a^2+106\,b^2\right)c^7d^2x^7+189\left(33075\,a^2+526\,b^2\right)c^5d^2x^5-220920\,b^2c^3d^2x^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/31255875*(42875*(81*a^2 + 2*b^2)*c^9*d^2*x^9 + 2250*(3969*a^2 + 106*b^2)*
c^7*d^2*x^7 + 189*(33075*a^2 + 526*b^2)*c^5*d^2*x^5 - 220920*b^2*c^3*d^2*x^
3 + 1325520*b^2*c*d^2*x + 99225*(35*b^2*c^9*d^2*x^9 + 90*b^2*c^7*d^2*x^7 +
63*b^2*c^5*d^2*x^5)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 630*(11025*a*b*c^9*d^2
*x^9 + 28350*a*b*c^7*d^2*x^7 + 19845*a*b*c^5*d^2*x^5 - (1225*b^2*c^8*d^2*x^
8 + 2650*b^2*c^6*d^2*x^6 + 789*b^2*c^4*d^2*x^4 - 1052*b^2*c^2*d^2*x^2 + 210
4*b^2*d^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 630*(1225*a*b*
c^8*d^2*x^8 + 2650*a*b*c^6*d^2*x^6 + 789*a*b*c^4*d^2*x^4 - 1052*a*b*c^2*d^2
*x^2 + 2104*a*b*d^2)*sqrt(c^2*x^2 + 1))/c^5
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.06, size = 428, normalized size = 1.11

$$d^2a^2\left(\frac{1}{9}c^9x^9+\frac{2}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+d^2b^2\left(\frac{\operatorname{arcsinh}(cx)^2c^3x^3\left(c^2x^2+1\right)^3}{9}-\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^3}{21}+\frac{8\operatorname{arcsinh}(cx)^2cx}{315}+\frac{\operatorname{arcsinh}(cx)^2}{10\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c^5*(d^2*a^2*(1/9*c^9*x^9+2/7*c^7*x^7+1/5*c^5*x^5)+d^2*b^2*(1/9*arcsinh(c
*x)^2*c^3*x^3*(c^2*x^2+1)^3-1/21*arcsinh(c*x)^2*c*x*(c^2*x^2+1)^3+8/315*arc
sinh(c*x)^2*c*x+1/105*arcsinh(c*x)^2*c*x*(c^2*x^2+1)^2+4/315*arcsinh(c*x)^2
*c*x*(c^2*x^2+1)-2/81*arcsinh(c*x)*c^2*x^2*(c^2*x^2+1)^(7/2)+82/3969*arcsin
h(c*x)*(c^2*x^2+1)^(7/2)+2/729*c*x*(c^2*x^2+1)^4+1493104/31255875*c*x-836/2
50047*c*x*(c^2*x^2+1)^3-33862/10418625*c*x*(c^2*x^2+1)^2-47248/31255875*c*x
*(c^2*x^2+1)-16/315*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-2/525*arcsinh(c*x)*(c^2*
x^2+1)^(5/2)-8/945*arcsinh(c*x)*(c^2*x^2+1)^(3/2))+2*d^2*a*b*(1/9*arcsinh(c
*x)*c^9*x^9+2/7*arcsinh(c*x)*c^7*x^7+1/5*arcsinh(c*x)*c^5*x^5-1/81*c^8*x^8*
(c^2*x^2+1)^(1/2)-106/3969*c^6*x^6*(c^2*x^2+1)^(1/2)-263/33075*c^4*x^4*(c^2
*x^2+1)^(1/2)+1052/99225*c^2*x^2*(c^2*x^2+1)^(1/2)-2104/99225*(c^2*x^2+1)^(
1/2)))
```

maxima [B] time = 0.45, size = 760, normalized size = 1.97

$$\frac{1}{9}b^2c^4d^2x^9\operatorname{arsinh}(cx)^2+\frac{1}{9}a^2c^4d^2x^9+\frac{2}{7}b^2c^2d^2x^7\operatorname{arsinh}(cx)^2+\frac{2}{7}a^2c^2d^2x^7+\frac{1}{5}b^2d^2x^5\operatorname{arsinh}(cx)^2+\frac{2}{2835}\left(315\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^2*c^4*d^2*x^9*arcsinh(c*x)^2 + 1/9*a^2*c^4*d^2*x^9 + 2/7*b^2*c^2*d^2*
x^7*arcsinh(c*x)^2 + 2/7*a^2*c^2*d^2*x^7 + 1/5*b^2*d^2*x^5*arcsinh(c*x)^2 +
 2/2835*(315*x^9*arcsinh(c*x) - (35*sqrt(c^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c^2
*x^2 + 1)*x^6/c^4 + 48*sqrt(c^2*x^2 + 1)*x^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x^2
/c^8 + 128*sqrt(c^2*x^2 + 1)/c^10)*c)*a*b*c^4*d^2 - 2/893025*(315*(35*sqrt(
c^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(c^2*x^2 + 1)*
x^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(c^2*x^2 + 1)/c^10)*c*arcs
inh(c*x) - (1225*c^8*x^9 - 1800*c^6*x^7 + 3024*c^4*x^5 - 6720*c^2*x^3 + 403
20*x)/c^8)*b^2*c^4*d^2 + 1/5*a^2*d^2*x^5 + 4/245*(35*x^7*arcsinh(c*x) - (5*
sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 +
1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*a*b*c^2*d^2 - 4/25725*(105*(5*sqr
t(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*
x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c*arcsinh(c*x) - (75*c^6*x^7 - 126*c^4*
x^5 + 280*c^2*x^3 - 1680*x)/c^6)*b^2*c^2*d^2 + 2/75*(15*x^5*arcsinh(c*x) -
(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2
 + 1)/c^6)*c)*a*b*d^2 - 2/1125*(15*(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^
```

```
2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 -
 20*c^2*x^3 + 120*x)/c^4)*b^2*d^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 (a + b \operatorname{asinh}(c x))^2 \left(d c^2 x^2 + d\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2,x)
```

```
[Out] int(x^4*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2, x)
```

sympy [A] time = 30.82, size = 563, normalized size = 1.46

$$\begin{cases} \frac{a^2c^4d^2x^9}{9} + \frac{2a^2c^2d^2x^7}{7} + \frac{a^2d^2x^5}{5} + \frac{2abc^4d^2x^9 \operatorname{asinh}(cx)}{9} - \frac{2abc^3d^2x^8\sqrt{c^2x^2+1}}{81} + \frac{4abc^2d^2x^7 \operatorname{asinh}(cx)}{7} - \frac{212abcd^2x^6\sqrt{c^2x^2+1}}{3969} + \frac{2abd^2x^5 \operatorname{a}}{5} \\ \frac{a^2d^2x^5}{5} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**4*d**2*x**9/9 + 2*a**2*c**2*d**2*x**7/7 + a**2*d**2*x**5
/5 + 2*a*b*c**4*d**2*x**9*asinh(c*x)/9 - 2*a*b*c**3*d**2*x**8*sqrt(c**2*x**
2 + 1)/81 + 4*a*b*c**2*d**2*x**7*asinh(c*x)/7 - 212*a*b*c*d**2*x**6*sqrt(c*
*2*x**2 + 1)/3969 + 2*a*b*d**2*x**5*asinh(c*x)/5 - 526*a*b*d**2*x**4*sqrt(c
**2*x**2 + 1)/(33075*c) + 2104*a*b*d**2*x**2*sqrt(c**2*x**2 + 1)/(99225*c**
3) - 4208*a*b*d**2*sqrt(c**2*x**2 + 1)/(99225*c**5) + b**2*c**4*d**2*x**9*a
sinh(c*x)**2/9 + 2*b**2*c**4*d**2*x**9/729 - 2*b**2*c**3*d**2*x**8*sqrt(c**
2*x**2 + 1)*asinh(c*x)/81 + 2*b**2*c**2*d**2*x**7*asinh(c*x)**2/7 + 212*b**
2*c**2*d**2*x**7/27783 - 212*b**2*c*d**2*x**6*sqrt(c**2*x**2 + 1)*asinh(c*x
)/3969 + b**2*d**2*x**5*asinh(c*x)**2/5 + 526*b**2*d**2*x**5/165375 - 526*b
**2*d**2*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/(33075*c) - 2104*b**2*d**2*x**
3/(297675*c**2) + 2104*b**2*d**2*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(99225
*c**3) + 4208*b**2*d**2*x/(99225*c**4) - 4208*b**2*d**2*sqrt(c**2*x**2 + 1)
*asinh(c*x)/(99225*c**5), Ne(c, 0)), (a**2*d**2*x**5/5, True))
```

3.208 $\int x^3 \left(d+c^2 d x^2\right)^2 \left(a+b \sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=296

$$-\frac{73 d^2 \left(a+b \sinh^{-1}(cx)\right)^2}{3072 c^4}-\frac{1}{32} b c d^2 x^5 \left(c^2 x^2+1\right)^{3/2} \left(a+b \sinh^{-1}(cx)\right)-\frac{25}{576} b c d^2 x^5 \sqrt{c^2 x^2+1} \left(a+b \sinh^{-1}(cx)\right.$$

```
[Out] -73/3072*b^2*d^2*x^2/c^2+73/9216*b^2*d^2*x^4+43/3456*b^2*c^2*d^2*x^6+1/256*
b^2*c^4*d^2*x^8-1/32*b*c*d^2*x^5*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))-73/30
72*d^2*(a+b*arcsinh(c*x))^2/c^4+1/24*d^2*x^4*(a+b*arcsinh(c*x))^2+1/12*d^2*
x^4*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2+1/8*d^2*x^4*(c^2*x^2+1)^2*(a+b*arcsinh
(c*x))^2+73/1536*b*d^2*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-73/2304*b
*d^2*x^3*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c-25/576*b*c*d^2*x^5*(a+b*arc
sinh(c*x))*(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 1.04, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5744, 5661, 5758, 5675, 30, 5742, 14}

$$-\frac{1}{32} b c d^2 x^5 \left(c^2 x^2+1\right)^{3/2} \left(a+b \sinh^{-1}(cx)\right)-\frac{25}{576} b c d^2 x^5 \sqrt{c^2 x^2+1} \left(a+b \sinh^{-1}(cx)\right)+\frac{1}{8} d^2 x^4 \left(c^2 x^2+1\right)^2 \left(a+b\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-73*b^2*d^2*x^2)/(3072*c^2) + (73*b^2*d^2*x^4)/9216 + (43*b^2*c^2*d^2*x^6)
/3456 + (b^2*c^4*d^2*x^8)/256 + (73*b*d^2*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSin
h[c*x]))/(1536*c^3) - (73*b*d^2*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))
/(2304*c) - (25*b*c*d^2*x^5*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/576 - (
b*c*d^2*x^5*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/32 - (73*d^2*(a + b*A
rcSinh[c*x])^2)/(3072*c^4) + (d^2*x^4*(a + b*ArcSinh[c*x])^2)/24 + (d^2*x^4
*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/12 + (d^2*x^4*(1 + c^2*x^2)^2*(a + b
*ArcSinh[c*x])^2)/8
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int x^3\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{1}{8}d^2x^4\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{2}d\int x^3\left(d+c^2dx^2\right)\left(a+b\,\mathrm{s}\right.$$

$$=-\frac{1}{32}bcd^2x^5\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{12}d^2x^4\left(1+c^2x^2\right)\left(a+\right.$$

$$=-\frac{25}{576}bcd^2x^5\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{32}bcd^2x^5\left(1+c^2x^2\right)^{3/2}\left(\right.$$

$$=\frac{43b^2c^2d^2x^6}{3456}+\frac{1}{256}b^2c^4d^2x^8-\frac{73bd^2x^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2304c}$$

$$=\frac{73b^2d^2x^4}{9216}+\frac{43b^2c^2d^2x^6}{3456}+\frac{1}{256}b^2c^4d^2x^8+\frac{73bd^2x\sqrt{1+c^2x^2}\left(a+b\,\mathrm{s}\right.}{1536c^3}$$

$$=-\frac{73b^2d^2x^2}{3072c^2}+\frac{73b^2d^2x^4}{9216}+\frac{43b^2c^2d^2x^6}{3456}+\frac{1}{256}b^2c^4d^2x^8+\frac{73bd^2x\sqrt{1+}}{}$$

Mathematica [**A**] time = 0.36, size = 237, normalized size = 0.80

$$\frac{d^2\left(cx\left(1152a^2c^3x^3\left(3c^4x^4+8c^2x^2+6\right)-6ab\sqrt{c^2x^2+1}\left(144c^6x^6+344c^4x^4+146c^2x^2-219\right)+b^2cx\left(108c^6x^6+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^2*(c*x*(1152*a^2*c^3*x^3*(6 + 8*c^2*x^2 + 3*c^4*x^4) + b^2*c*x*(-657 + 2
19*c^2*x^2 + 344*c^4*x^4 + 108*c^6*x^6) - 6*a*b*Sqrt[1 + c^2*x^2]*(-219 + 1
46*c^2*x^2 + 344*c^4*x^4 + 144*c^6*x^6)) + 6*b*(-(b*c*x*Sqrt[1 + c^2*x^2]*(
-219 + 146*c^2*x^2 + 344*c^4*x^4 + 144*c^6*x^6)) + 3*a*(-73 + 768*c^4*x^4 +
 1024*c^6*x^6 + 384*c^8*x^8))*ArcSinh[c*x] + 9*b^2*(-73 + 768*c^4*x^4 + 102
4*c^6*x^6 + 384*c^8*x^8)*ArcSinh[c*x]^2))/(27648*c^4)
```

fricas [A] time = 0.59, size = 348, normalized size = 1.18

$$108\left(32\,a^{2}+b^{2}\right)c^{8}d^{2}x^{8}+8\left(1152\,a^{2}+43\,b^{2}\right)c^{6}d^{2}x^{6}+3\left(2304\,a^{2}+73\,b^{2}\right)c^{4}d^{2}x^{4}-657\,b^{2}c^{2}d^{2}x^{2}+9\left(384\,b^{2}c^{8}d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/27648*(108*(32*a^2 + b^2)*c^8*d^2*x^8 + 8*(1152*a^2 + 43*b^2)*c^6*d^2*x^6
 + 3*(2304*a^2 + 73*b^2)*c^4*d^2*x^4 - 657*b^2*c^2*d^2*x^2 + 9*(384*b^2*c^8
*d^2*x^8 + 1024*b^2*c^6*d^2*x^6 + 768*b^2*c^4*d^2*x^4 - 73*b^2*d^2)*log(c*x
 + sqrt(c^2*x^2 + 1))^2 + 6*(1152*a*b*c^8*d^2*x^8 + 3072*a*b*c^6*d^2*x^6 +
2304*a*b*c^4*d^2*x^4 - 219*a*b*d^2 - (144*b^2*c^7*d^2*x^7 + 344*b^2*c^5*d^2
*x^5 + 146*b^2*c^3*d^2*x^3 - 219*b^2*c*d^2*x)*sqrt(c^2*x^2 + 1))*log(c*x +
sqrt(c^2*x^2 + 1)) - 6*(144*a*b*c^7*d^2*x^7 + 344*a*b*c^5*d^2*x^5 + 146*a*b
*c^3*d^2*x^3 - 219*a*b*c*d^2*x)*sqrt(c^2*x^2 + 1))/c^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.05, size = 344, normalized size = 1.16

$$d^{2}a^{2}\left(\frac{1}{8}c^{8}x^{8}+\frac{1}{3}c^{6}x^{6}+\frac{1}{4}c^{4}x^{4}\right)+d^{2}b^{2}\left(\frac{\operatorname{arcsinh}(cx)^{2}c^{2}x^{2}\left(c^{2}x^{2}+1\right)^{3}}{8}-\frac{\operatorname{arcsinh}(cx)^{2}\left(c^{2}x^{2}+1\right)^{3}}{24}-\frac{\operatorname{arcsinh}(cx)cx\left(c^{2}x^{2}+1\right)^{\frac{7}{2}}}{32}+\frac{11\operatorname{arcs}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c^4*(d^2*a^2*(1/8*c^8*x^8+1/3*c^6*x^6+1/4*c^4*x^4)+d^2*b^2*(1/8*arcsinh(c
*x)^2*c^2*x^2*(c^2*x^2+1)^3-1/24*arcsinh(c*x)^2*(c^2*x^2+1)^3-1/32*arcsinh(
c*x)*c*x*(c^2*x^2+1)^(7/2)+11/576*arcsinh(c*x)*c*x*(c^2*x^2+1)^(5/2)+55/230
4*arcsinh(c*x)*c*x*(c^2*x^2+1)^(3/2)+55/1536*arcsinh(c*x)*(c^2*x^2+1)^(1/2)
*c*x+55/3072*arcsinh(c*x)^2+1/256*(c^2*x^2+1)^4-11/3456*(c^2*x^2+1)^3-55/92
16*(c^2*x^2+1)^2-55/3072*c^2*x^2-55/3072)+2*d^2*a*b*(1/8*arcsinh(c*x)*c^8*x
^8+1/3*arcsinh(c*x)*c^6*x^6+1/4*arcsinh(c*x)*c^4*x^4-1/64*c^7*x^7*(c^2*x^2+
1)^(1/2)-43/1152*c^5*x^5*(c^2*x^2+1)^(1/2)-73/4608*c^3*x^3*(c^2*x^2+1)^(1/2
)+73/3072*c*x*(c^2*x^2+1)^(1/2)-73/3072*arcsinh(c*x)))
```

maxima [B] time = 0.61, size = 762, normalized size = 2.57

$$\frac{1}{8}\,b^{2}c^{4}d^{2}x^{8}\,\operatorname{arsinh}\left(cx\right)^{2}+\frac{1}{8}\,a^{2}c^{4}d^{2}x^{8}+\frac{1}{3}\,b^{2}c^{2}d^{2}x^{6}\,\operatorname{arsinh}\left(cx\right)^{2}+\frac{1}{3}\,a^{2}c^{2}d^{2}x^{6}+\frac{1}{4}\,b^{2}d^{2}x^{4}\,\operatorname{arsinh}\left(cx\right)^{2}+\frac{1}{1536}\left(384\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*c^4*d^2*x^8*arcsinh(c*x)^2 + 1/8*a^2*c^4*d^2*x^8 + 1/3*b^2*c^2*d^2*
x^6*arcsinh(c*x)^2 + 1/3*a^2*c^2*d^2*x^6 + 1/4*b^2*d^2*x^4*arcsinh(c*x)^2 +
 1/1536*(384*x^8*arcsinh(c*x) - (48*sqrt(c^2*x^2 + 1)*x^7/c^2 - 56*sqrt(c^2
*x^2 + 1)*x^5/c^4 + 70*sqrt(c^2*x^2 + 1)*x^3/c^6 - 105*sqrt(c^2*x^2 + 1)*x/
c^8 + 105*arcsinh(c*x)/c^9)*c)*a*b*c^4*d^2 + 1/9216*((36*x^8/c^2 - 56*x^6/c
^4 + 105*x^4/c^6 - 315*x^2/c^8 + 315*log(c*x + sqrt(c^2*x^2 + 1))^2/c^10)*c
^2 - 6*(48*sqrt(c^2*x^2 + 1)*x^7/c^2 - 56*sqrt(c^2*x^2 + 1)*x^5/c^4 + 70*sq
rt(c^2*x^2 + 1)*x^3/c^6 - 105*sqrt(c^2*x^2 + 1)*x/c^8 + 105*arcsinh(c*x)/c^
9)*c*arcsinh(c*x))*b^2*c^4*d^2 + 1/4*a^2*d^2*x^4 + 1/72*(48*x^6*arcsinh(c*x
) - (8*sqrt(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c
^2*x^2 + 1)*x/c^6 - 15*arcsinh(c*x)/c^7)*c)*a*b*c^2*d^2 + 1/432*((8*x^6/c^2
 - 15*x^4/c^4 + 45*x^2/c^6 - 45*log(c*x + sqrt(c^2*x^2 + 1))^2/c^8)*c^2 - 6
*(8*sqrt(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*
x^2 + 1)*x/c^6 - 15*arcsinh(c*x)/c^7)*c*arcsinh(c*x))*b^2*c^2*d^2 + 1/16*(8
*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^
4 + 3*arcsinh(c*x)/c^5)*c)*a*b*d^2 + 1/32*((x^4/c^2 - 3*x^2/c^4 + 3*log(c*x
 + sqrt(c^2*x^2 + 1))^2/c^6)*c^2 - 2*(2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(
c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c*arcsinh(c*x))*b^2*d^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,(a+b\,\mathrm{asinh}\,(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2,x)
```

```
[Out] int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2, x)
```

sympy [A] time = 22.95, size = 515, normalized size = 1.74

$$\begin{cases} \frac{a^2c^4d^2x^8}{8} + \frac{a^2c^2d^2x^6}{3} + \frac{a^2d^2x^4}{4} + \frac{abc^4d^2x^8\,\mathrm{asinh}\,(cx)}{4} - \frac{abc^3d^2x^7\sqrt{c^2x^2+1}}{32} + \frac{2abc^2d^2x^6\,\mathrm{asinh}\,(cx)}{3} - \frac{43abcd^2x^5\sqrt{c^2x^2+1}}{576} + \frac{abd^2x^4\,\mathrm{asinh}\,(\\ \frac{a^2d^2x^4}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**4*d**2*x**8/8 + a**2*c**2*d**2*x**6/3 + a**2*d**2*x**4/4
 + a*b*c**4*d**2*x**8*asinh(c*x)/4 - a*b*c**3*d**2*x**7*sqrt(c**2*x**2 + 1)
/32 + 2*a*b*c**2*d**2*x**6*asinh(c*x)/3 - 43*a*b*c*d**2*x**5*sqrt(c**2*x**2
 + 1)/576 + a*b*d**2*x**4*asinh(c*x)/2 - 73*a*b*d**2*x**3*sqrt(c**2*x**2 +
1)/(2304*c) + 73*a*b*d**2*x*sqrt(c**2*x**2 + 1)/(1536*c**3) - 73*a*b*d**2*a
sinh(c*x)/(1536*c**4) + b**2*c**4*d**2*x**8*asinh(c*x)**2/8 + b**2*c**4*d**
2*x**8/256 - b**2*c**3*d**2*x**7*sqrt(c**2*x**2 + 1)*asinh(c*x)/32 + b**2*c
**2*d**2*x**6*asinh(c*x)**2/3 + 43*b**2*c**2*d**2*x**6/3456 - 43*b**2*c*d**
2*x**5*sqrt(c**2*x**2 + 1)*asinh(c*x)/576 + b**2*d**2*x**4*asinh(c*x)**2/4
+ 73*b**2*d**2*x**4/9216 - 73*b**2*d**2*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)
/(2304*c) - 73*b**2*d**2*x**2/(3072*c**2) + 73*b**2*d**2*x*sqrt(c**2*x**2 +
 1)*asinh(c*x)/(1536*c**3) - 73*b**2*d**2*asinh(c*x)**2/(3072*c**4), Ne(c,
0)), (a**2*d**2*x**4/4, True))
```

3.209 $\int x^2 \left(d+c^2 d x^2\right)^2 \left(a+b \sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=303

$$-\frac{16 b d^2 x^2 \sqrt{c^2 x^2+1} \left(a+b \sinh^{-1}(cx)\right)}{315 c}+\frac{1}{7} d^2 x^3 \left(c^2 x^2+1\right)^2 \left(a+b \sinh^{-1}(cx)\right)^2+\frac{4}{35} d^2 x^3 \left(c^2 x^2+1\right) \left(a+b \sinh\right.$$

```
[Out] -1636/11025*b^2*d^2*x/c^2+818/33075*b^2*d^2*x^3+136/6125*b^2*c^2*d^2*x^5+2/
343*b^2*c^4*d^2*x^7+8/105*b*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/c^3+2/
175*b*d^2*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))/c^3-2/49*b*d^2*(c^2*x^2+1)^(
7/2)*(a+b*arcsinh(c*x))/c^3+8/105*d^2*x^3*(a+b*arcsinh(c*x))^2+4/35*d^2*x^3
*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2+1/7*d^2*x^3*(c^2*x^2+1)^2*(a+b*arcsinh(c*
x))^2+32/315*b*d^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-16/315*b*d^2*x^
2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] time = 0.59, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {5744, 5661, 5758, 5717, 8, 30, 266, 43, 5732, 12, 373}

$$\frac{1}{7} d^2 x^3 \left(c^2 x^2+1\right)^2 \left(a+b \sinh^{-1}(cx)\right)^2+\frac{4}{35} d^2 x^3 \left(c^2 x^2+1\right) \left(a+b \sinh^{-1}(cx)\right)^2-\frac{16 b d^2 x^2 \sqrt{c^2 x^2+1} \left(a+b \sinh^{-1}\right.}{315 c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-1636*b^2*d^2*x)/(11025*c^2) + (818*b^2*d^2*x^3)/33075 + (136*b^2*c^2*d^2*
x^5)/6125 + (2*b^2*c^4*d^2*x^7)/343 + (32*b*d^2*Sqrt[1 + c^2*x^2]*(a + b*Ar
cSinh[c*x]))/(315*c^3) - (16*b*d^2*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x
]))/(315*c) + (8*b*d^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(105*c^3)
+ (2*b*d^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(175*c^3) - (2*b*d^2*(
1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(49*c^3) + (8*d^2*x^3*(a + b*ArcSi
nh[c*x])^2)/105 + (4*d^2*x^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/35 + (d^
2*x^3*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/7
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int x^2\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{1}{7}d^2x^3\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{7}(4d)\int x^2\left(d+c^2dx^2\right)\left(\right.$$

$$=\frac{2bd^2\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{35c^3}-\frac{2bd^2\left(1+c^2x^2\right)^{7/2}\left(a+b\,\mathrm{s}\right.}{49c^3}$$

$$=\frac{8bd^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{105c^3}+\frac{2bd^2\left(1+c^2x^2\right)^{5/2}\left(a+b\,\mathrm{s}\right.}{175c^3}$$

$$=-\frac{16bd^2x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{315c}+\frac{8bd^2\left(1+c^2x^2\right)^{3/2}\left(a+\right.}{105c^3}$$

$$=-\frac{172b^2d^2x}{3675c^2}+\frac{818b^2d^2x^3}{33075}+\frac{136b^2c^2d^2x^5}{6125}+\frac{2}{343}b^2c^4d^2x^7+\frac{32bd^2}{}$$

$$=-\frac{1636b^2d^2x}{11025c^2}+\frac{818b^2d^2x^3}{33075}+\frac{136b^2c^2d^2x^5}{6125}+\frac{2}{343}b^2c^4d^2x^7+\frac{32bd^2}{}$$

Mathematica [A] time = 0.40, size = 227, normalized size = 0.75

$$\frac{d^2\left(11025a^2c^3x^3\left(15c^4x^4+42c^2x^2+35\right)-210ab\sqrt{c^2x^2+1}\left(225c^6x^6+612c^4x^4+409c^2x^2-818\right)-210b\sinh\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^2*(11025*a^2*c^3*x^3*(35 + 42*c^2*x^2 + 15*c^4*x^4) - 210*a*b*Sqrt[1 + c
^2*x^2]*(-818 + 409*c^2*x^2 + 612*c^4*x^4 + 225*c^6*x^6) + 2*b^2*c*x*(-8589
0 + 14315*c^2*x^2 + 12852*c^4*x^4 + 3375*c^6*x^6) - 210*b*(-105*a*c^3*x^3*(
35 + 42*c^2*x^2 + 15*c^4*x^4) + b*Sqrt[1 + c^2*x^2]*(-818 + 409*c^2*x^2 + 6
12*c^4*x^4 + 225*c^6*x^6))*ArcSinh[c*x] + 11025*b^2*c^3*x^3*(35 + 42*c^2*x^
2 + 15*c^4*x^4)*ArcSinh[c*x]^2))/(1157625*c^3)
```

fricas [A] time = 0.67, size = 327, normalized size = 1.08

$$\frac{3375\left(49\,a^2+2\,b^2\right)c^7d^2x^7+378\left(1225\,a^2+68\,b^2\right)c^5d^2x^5+35\left(11025\,a^2+818\,b^2\right)c^3d^2x^3-171780\,b^2cd^2x+11}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/1157625*(3375*(49*a^2 + 2*b^2)*c^7*d^2*x^7 + 378*(1225*a^2 + 68*b^2)*c^5*
d^2*x^5 + 35*(11025*a^2 + 818*b^2)*c^3*d^2*x^3 - 171780*b^2*c*d^2*x + 11025
*(15*b^2*c^7*d^2*x^7 + 42*b^2*c^5*d^2*x^5 + 35*b^2*c^3*d^2*x^3)*log(c*x + s
qrt(c^2*x^2 + 1))^2 + 210*(1575*a*b*c^7*d^2*x^7 + 4410*a*b*c^5*d^2*x^5 + 36
75*a*b*c^3*d^2*x^3 - (225*b^2*c^6*d^2*x^6 + 612*b^2*c^4*d^2*x^4 + 409*b^2*c
^2*d^2*x^2 - 818*b^2*d^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) -
 210*(225*a*b*c^6*d^2*x^6 + 612*a*b*c^4*d^2*x^4 + 409*a*b*c^2*d^2*x^2 - 818
*a*b*d^2)*sqrt(c^2*x^2 + 1))/c^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.05, size = 346, normalized size = 1.14

$$d^2a^2\left(\frac{1}{7}c^7x^7+\frac{2}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+d^2b^2\left(\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^3}{7}-\frac{8\operatorname{arcsinh}(cx)^2cx}{105}-\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^2}{35}-\frac{4\operatorname{arcsinh}(cx)^2cx\left(c^2\right.}{105}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c^3*(d^2*a^2*(1/7*c^7*x^7+2/5*c^5*x^5+1/3*c^3*x^3)+d^2*b^2*(1/7*arcsinh(c
*x)^2*c*x*(c^2*x^2+1)^3-8/105*arcsinh(c*x)^2*c*x-1/35*arcsinh(c*x)^2*c*x*(c
^2*x^2+1)^2-4/105*arcsinh(c*x)^2*c*x*(c^2*x^2+1)-2/49*arcsinh(c*x)*(c^2*x^2
+1)^(7/2)-181456/1157625*c*x+2/343*c*x*(c^2*x^2+1)^3+202/42875*c*x*(c^2*x^2
+1)^2-2528/1157625*c*x*(c^2*x^2+1)+16/105*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2/
175*arcsinh(c*x)*(c^2*x^2+1)^(5/2)+8/315*arcsinh(c*x)*(c^2*x^2+1)^(3/2))+2*
d^2*a*b*(1/7*arcsinh(c*x)*c^7*x^7+2/5*arcsinh(c*x)*c^5*x^5+1/3*arcsinh(c*x)
*c^3*x^3-1/49*c^6*x^6*(c^2*x^2+1)^(1/2)-68/1225*c^4*x^4*(c^2*x^2+1)^(1/2)-4
09/11025*c^2*x^2*(c^2*x^2+1)^(1/2)+818/11025*(c^2*x^2+1)^(1/2)))
```

maxima [B] time = 0.38, size = 619, normalized size = 2.04

$$\frac{1}{7}b^2c^4d^2x^7\operatorname{arsinh}(cx)^2+\frac{1}{7}a^2c^4d^2x^7+\frac{2}{5}b^2c^2d^2x^5\operatorname{arsinh}(cx)^2+\frac{2}{5}a^2c^2d^2x^5+\frac{2}{245}\left(35x^7\operatorname{arsinh}(cx)-\left(\frac{5\sqrt{c^2x^2+}}{c^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*c^4*d^2*x^7*arcsinh(c*x)^2 + 1/7*a^2*c^4*d^2*x^7 + 2/5*b^2*c^2*d^2*
x^5*arcsinh(c*x)^2 + 2/5*a^2*c^2*d^2*x^5 + 2/245*(35*x^7*arcsinh(c*x) - (5*
sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 +
1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*a*b*c^4*d^2 - 2/25725*(105*(5*sqr
t(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*
x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c*arcsinh(c*x) - (75*c^6*x^7 - 126*c^4*
x^5 + 280*c^2*x^3 - 1680*x)/c^6)*b^2*c^4*d^2 + 1/3*b^2*d^2*x^3*arcsinh(c*x)
^2 + 4/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*
x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*c^2*d^2 - 4/1125*(15*(3*
sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 +
1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c^4)*b^2*c^2*d^2
+ 1/3*a^2*d^2*x^3 + 2/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2
- 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*d^2 - 2/27*(3*c*(sqrt(c^2*x^2 + 1)*x^2/c^2
- 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3 - 6*x)/c^2)*b^2*d^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(d\,c^2\,x^2+d\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2,x)
```

```
[Out] int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2, x)
```

sympy [A] time = 12.70, size = 483, normalized size = 1.59

$$\begin{cases} \frac{a^2c^4d^2x^7}{7} + \frac{2a^2c^2d^2x^5}{5} + \frac{a^2d^2x^3}{3} + \frac{2abc^4d^2x^7 \operatorname{asinh}(cx)}{7} - \frac{2abc^3d^2x^6\sqrt{c^2x^2+1}}{49} + \frac{4abc^2d^2x^5 \operatorname{asinh}(cx)}{5} - \frac{136abcd^2x^4\sqrt{c^2x^2+1}}{1225} + \frac{2abd^2 \\ \frac{a^2d^2x^3}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**4*d**2*x**7/7 + 2*a**2*c**2*d**2*x**5/5 + a**2*d**2*x**3
/3 + 2*a*b*c**4*d**2*x**7*asinh(c*x)/7 - 2*a*b*c**3*d**2*x**6*sqrt(c**2*x**
2 + 1)/49 + 4*a*b*c**2*d**2*x**5*asinh(c*x)/5 - 136*a*b*c*d**2*x**4*sqrt(c*
*2*x**2 + 1)/1225 + 2*a*b*d**2*x**3*asinh(c*x)/3 - 818*a*b*d**2*x**2*sqrt(c
**2*x**2 + 1)/(11025*c) + 1636*a*b*d**2*sqrt(c**2*x**2 + 1)/(11025*c**3) +
b**2*c**4*d**2*x**7*asinh(c*x)**2/7 + 2*b**2*c**4*d**2*x**7/343 - 2*b**2*c*
*3*d**2*x**6*sqrt(c**2*x**2 + 1)*asinh(c*x)/49 + 2*b**2*c**2*d**2*x**5*asin
h(c*x)**2/5 + 136*b**2*c**2*d**2*x**5/6125 - 136*b**2*c*d**2*x**4*sqrt(c**2
*x**2 + 1)*asinh(c*x)/1225 + b**2*d**2*x**3*asinh(c*x)**2/3 + 818*b**2*d**2
*x**3/33075 - 818*b**2*d**2*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(11025*c) -
 1636*b**2*d**2*x/(11025*c**2) + 1636*b**2*d**2*sqrt(c**2*x**2 + 1)*asinh(c
*x)/(11025*c**3), Ne(c, 0)), (a**2*d**2*x**3/3, True))
```

3.210 $\int x\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=204

$$-\frac{bd^2x\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{18c}-\frac{5bd^2x\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{72c}-\frac{5bd^2x\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx\right.}{48c}$$

```
[Out] 25/288*b^2*d^2*x^2+5/288*b^2*c^2*d^2*x^4+1/108*b^2*d^2*(c^2*x^2+1)^3/c^2-5/
72*b*d^2*x*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/c-1/18*b*d^2*x*(c^2*x^2+1)^
(5/2)*(a+b*arcsinh(c*x))/c-5/96*d^2*(a+b*arcsinh(c*x))^2/c^2+1/6*d^2*(c^2*x
^2+1)^3*(a+b*arcsinh(c*x))^2/c^2-5/48*b*d^2*x*(a+b*arcsinh(c*x))*(c^2*x^2+1
)^(1/2)/c
```

Rubi [A] time = 0.21, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {5717, 5684, 5682, 5675, 30, 14, 261}

$$-\frac{bd^2x\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{18c}-\frac{5bd^2x\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{72c}-\frac{5bd^2x\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx\right.}{48c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (25*b^2*d^2*x^2)/288 + (5*b^2*c^2*d^2*x^4)/288 + (b^2*d^2*(1 + c^2*x^2)^3)/
(108*c^2) - (5*b*d^2*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(48*c) - (5*
b*d^2*x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(72*c) - (b*d^2*x*(1 + c^
2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(18*c) - (5*d^2*(a + b*ArcSinh[c*x])^2)/
(96*c^2) + (d^2*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x])^2)/(6*c^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
```

```
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\int x\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{d^2\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2}{6c^2}-\frac{\left(bd^2\right)\int\left(1+c^2x^2\right)^{5/2}\left(a+b\sin\right.}{3c}$$

$$=-\frac{bd^2x\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{18c}+\frac{d^2\left(1+c^2x^2\right)^3\left(a+b\sinh\right.}{6c^2}$$

$$=\frac{b^2d^2\left(1+c^2x^2\right)^3}{108c^2}-\frac{5bd^2x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{72c}-\frac{bd^2x\left(1\right.}{}$$

$$=\frac{b^2d^2\left(1+c^2x^2\right)^3}{108c^2}-\frac{5bd^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{48c}-\frac{5bd^2x\left(1\right.}{}$$

$$=\frac{25}{288}b^2d^2x^2+\frac{5}{288}b^2c^2d^2x^4+\frac{b^2d^2\left(1+c^2x^2\right)^3}{108c^2}-\frac{5bd^2x\sqrt{1+c^2x^2}\left(\right.}{48}$$

Mathematica [A] time = 0.50, size = 208, normalized size = 1.02

$$d^2\left(cx\left(144a^2cx\left(c^4x^4+3c^2x^2+3\right)-6ab\sqrt{c^2x^2+1}\left(8c^4x^4+26c^2x^2+33\right)+b^2cx\left(8c^4x^4+39c^2x^2+99\right)\right)+6b\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^2*(c*x*(144*a^2*c*x*(3 + 3*c^2*x^2 + c^4*x^4) - 6*a*b*Sqrt[1 + c^2*x^2]*
(33 + 26*c^2*x^2 + 8*c^4*x^4) + b^2*c*x*(99 + 39*c^2*x^2 + 8*c^4*x^4)) + 6*
b*(-(b*c*x*Sqrt[1 + c^2*x^2]*(33 + 26*c^2*x^2 + 8*c^4*x^4)) + 3*a*(11 + 48*
c^2*x^2 + 48*c^4*x^4 + 16*c^6*x^6))*ArcSinh[c*x] + 9*b^2*(11 + 48*c^2*x^2 +
 48*c^4*x^4 + 16*c^6*x^6)*ArcSinh[c*x]^2))/(864*c^2)
```

fricas [A] time = 0.56, size = 307, normalized size = 1.50

$$8\left(18\,a^2+b^2\right)c^6d^2x^6+3\left(144\,a^2+13\,b^2\right)c^4d^2x^4+9\left(48\,a^2+11\,b^2\right)c^2d^2x^2+9\left(16\,b^2c^6d^2x^6+48\,b^2c^4d^2x^4+48\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/864*(8*(18*a^2 + b^2)*c^6*d^2*x^6 + 3*(144*a^2 + 13*b^2)*c^4*d^2*x^4 + 9*
(48*a^2 + 11*b^2)*c^2*d^2*x^2 + 9*(16*b^2*c^6*d^2*x^6 + 48*b^2*c^4*d^2*x^4
+ 48*b^2*c^2*d^2*x^2 + 11*b^2*d^2)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 6*(48*a
*b*c^6*d^2*x^6 + 144*a*b*c^4*d^2*x^4 + 144*a*b*c^2*d^2*x^2 + 33*a*b*d^2 - (
8*b^2*c^5*d^2*x^5 + 26*b^2*c^3*d^2*x^3 + 33*b^2*c*d^2*x)*sqrt(c^2*x^2 + 1))
*log(c*x + sqrt(c^2*x^2 + 1)) - 6*(8*a*b*c^5*d^2*x^5 + 26*a*b*c^3*d^2*x^3 +
 33*a*b*c*d^2*x)*sqrt(c^2*x^2 + 1))/c^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.04, size = 268, normalized size = 1.31

$$d^2a^2\left(\frac{1}{6}c^6x^6+\frac{1}{2}c^4x^4+\frac{1}{2}c^2x^2\right)+d^2b^2\left(\frac{\operatorname{arcsinh}(cx)^2\left(c^2x^2+1\right)^3}{6}-\frac{\operatorname{arcsinh}(cx)cx\left(c^2x^2+1\right)^{\frac{5}{2}}}{18}-\frac{5\operatorname{arcsinh}(cx)cx\left(c^2x^2+1\right)^{\frac{3}{2}}}{72}-\frac{5\operatorname{arcsinh}(cx}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c^2*(d^2*a^2*(1/6*c^6*x^6+1/2*c^4*x^4+1/2*c^2*x^2)+d^2*b^2*(1/6*arcsinh(c
*x)^2*(c^2*x^2+1)^3-1/18*arcsinh(c*x)*c*x*(c^2*x^2+1)^(5/2)-5/72*arcsinh(c*
x)*c*x*(c^2*x^2+1)^(3/2)-5/48*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c*x-5/96*arcsi
nh(c*x)^2+1/108*(c^2*x^2+1)^3+5/288*(c^2*x^2+1)^2+5/96*c^2*x^2+5/96)+2*d^2*
a*b*(1/6*arcsinh(c*x)*c^6*x^6+1/2*arcsinh(c*x)*c^4*x^4+1/2*arcsinh(c*x)*c^2
*x^2-1/36*c^5*x^5*(c^2*x^2+1)^(1/2)-13/144*c^3*x^3*(c^2*x^2+1)^(1/2)-11/96*
c*x*(c^2*x^2+1)^(1/2)+11/96*arcsinh(c*x)))
```

maxima [B] time = 0.55, size = 621, normalized size = 3.04

$$\frac{1}{6}b^2c^4d^2x^6\operatorname{arsinh}(cx)^2+\frac{1}{6}a^2c^4d^2x^6+\frac{1}{2}b^2c^2d^2x^4\operatorname{arsinh}(cx)^2+\frac{1}{2}a^2c^2d^2x^4+\frac{1}{144}\left(48x^6\operatorname{arsinh}(cx)-\left(\frac{8\sqrt{c^2x^2+1}}{c^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*c^4*d^2*x^6*arcsinh(c*x)^2 + 1/6*a^2*c^4*d^2*x^6 + 1/2*b^2*c^2*d^2*
x^4*arcsinh(c*x)^2 + 1/2*a^2*c^2*d^2*x^4 + 1/144*(48*x^6*arcsinh(c*x) - (8*
sqrt(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2
+ 1)*x/c^6 - 15*arcsinh(c*x)/c^7)*c)*a*b*c^4*d^2 + 1/864*((8*x^6/c^2 - 15*x
^4/c^4 + 45*x^2/c^6 - 45*log(c*x + sqrt(c^2*x^2 + 1))^2/c^8)*c^2 - 6*(8*sqr
t(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 + 1
)*x/c^6 - 15*arcsinh(c*x)/c^7)*c*arcsinh(c*x))*b^2*c^4*d^2 + 1/2*b^2*d^2*x^
2*arcsinh(c*x)^2 + 1/8*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^2 + 1)*x^3/c^2 -
 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*a*b*c^2*d^2 + 1/16*((x^
```

```
4/c^2 - 3*x^2/c^4 + 3*log(c*x + sqrt(c^2*x^2 + 1))^2/c^6)*c^2 - 2*(2*sqrt(c
^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c*arc
sinh(c*x))*b^2*c^2*d^2 + 1/2*a^2*d^2*x^2 + 1/2*(2*x^2*arcsinh(c*x) - c*(sqr
t(c^2*x^2 + 1)*x/c^2 - arcsinh(c*x)/c^3))*a*b*d^2 + 1/4*(c^2*(x^2/c^2 - log
(c*x + sqrt(c^2*x^2 + 1))^2/c^4) - 2*c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsinh(c
*x)/c^3)*arcsinh(c*x))*b^2*d^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,(a+b\,\mathrm{asinh}\,(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2,x)
```

```
[Out] int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2, x)
```

sympy [A] time = 8.82, size = 430, normalized size = 2.11

$$\begin{cases} \frac{a^2c^4d^2x^6}{6} + \frac{a^2c^2d^2x^4}{2} + \frac{a^2d^2x^2}{2} + \frac{abc^4d^2x^6\,\mathrm{asinh}\,(cx)}{3} - \frac{abc^3d^2x^5\sqrt{c^2x^2+1}}{18} + abc^2d^2x^4\,\mathrm{asinh}\,(cx) - \frac{13abcd^2x^3\sqrt{c^2x^2+1}}{72} + abd^2 \\ \frac{a^2d^2x^2}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**4*d**2*x**6/6 + a**2*c**2*d**2*x**4/2 + a**2*d**2*x**2/2
 + a*b*c**4*d**2*x**6*asinh(c*x)/3 - a*b*c**3*d**2*x**5*sqrt(c**2*x**2 + 1)
/18 + a*b*c**2*d**2*x**4*asinh(c*x) - 13*a*b*c*d**2*x**3*sqrt(c**2*x**2 + 1
)/72 + a*b*d**2*x**2*asinh(c*x) - 11*a*b*d**2*x*sqrt(c**2*x**2 + 1)/(48*c)
+ 11*a*b*d**2*asinh(c*x)/(48*c**2) + b**2*c**4*d**2*x**6*asinh(c*x)**2/6 +
b**2*c**4*d**2*x**6/108 - b**2*c**3*d**2*x**5*sqrt(c**2*x**2 + 1)*asinh(c*x
)/18 + b**2*c**2*d**2*x**4*asinh(c*x)**2/2 + 13*b**2*c**2*d**2*x**4/288 - 1
3*b**2*c*d**2*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/72 + b**2*d**2*x**2*asinh
(c*x)**2/2 + 11*b**2*d**2*x**2/96 - 11*b**2*d**2*x*sqrt(c**2*x**2 + 1)*asin
h(c*x)/(48*c) + 11*b**2*d**2*asinh(c*x)**2/(96*c**2), Ne(c, 0)), (a**2*d**2
*x**2/2, True))
```

3.211 $\int \left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=214

$$\frac{1}{5}d^2x\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)^2+\frac{4}{15}d^2x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2-\frac{2bd^2\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{25c}$$

```
[Out] 298/225*b^2*d^2*x+76/675*b^2*c^2*d^2*x^3+2/125*b^2*c^4*d^2*x^5-8/45*b*d^2*(
c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/c-2/25*b*d^2*(c^2*x^2+1)^(5/2)*(a+b*arc
sinh(c*x))/c+8/15*d^2*x*(a+b*arcsinh(c*x))^2+4/15*d^2*x*(c^2*x^2+1)*(a+b*ar
csinh(c*x))^2+1/5*d^2*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2-16/15*b*d^2*(a+b
*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] time = 0.26, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5684, 5653, 5717, 8, 194}

$$\frac{1}{5}d^2x\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)^2+\frac{4}{15}d^2x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2-\frac{2bd^2\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{25c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (298*b^2*d^2*x)/225 + (76*b^2*c^2*d^2*x^3)/675 + (2*b^2*c^4*d^2*x^5)/125 -
(16*b*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(15*c) - (8*b*d^2*(1 + c^
2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(45*c) - (2*b*d^2*(1 + c^2*x^2)^(5/2)*(a
 + b*ArcSinh[c*x]))/(25*c) + (8*d^2*x*(a + b*ArcSinh[c*x])^2)/15 + (4*d^2*x
*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/15 + (d^2*x*(1 + c^2*x^2)^2*(a + b*A
rcSinh[c*x])^2)/5
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
```

```
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(d+c^2dx^2\right)^2 \left(a+b\sinh^{-1}(cx)\right)^2\,dx = \frac{1}{5}d^2x\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{5}(4d)\int \left(d+c^2dx^2\right)\left(a+b\,\mathrm{s}\right.$$

$$= -\frac{2bd^2\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{25c}+\frac{4}{15}d^2x\left(1+c^2x^2\right)\left(a+b\sin\right.$$

$$= -\frac{8bd^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{45c}-\frac{2bd^2\left(1+c^2x^2\right)^{5/2}\left(a+b\sin\right.}{25c}$$

$$= \frac{58}{225}b^2d^2x+\frac{76}{675}b^2c^2d^2x^3+\frac{2}{125}b^2c^4d^2x^5-\frac{16bd^2\sqrt{1+c^2x^2}\left(a+b\,\mathrm{s}\right.}{15c}$$

$$= \frac{298}{225}b^2d^2x+\frac{76}{675}b^2c^2d^2x^3+\frac{2}{125}b^2c^4d^2x^5-\frac{16bd^2\sqrt{1+c^2x^2}\left(a+b\,\mathrm{s}\right.}{15c}$$

Mathematica [A] time = 0.41, size = 191, normalized size = 0.89

$$d^2\left(225a^2cx\left(3c^4x^4+10c^2x^2+15\right)-30ab\sqrt{c^2x^2+1}\left(9c^4x^4+38c^2x^2+149\right)-30b\sinh^{-1}(cx)\left(b\sqrt{c^2x^2+1}\left(9\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^2*(225*a^2*c*x*(15 + 10*c^2*x^2 + 3*c^4*x^4) - 30*a*b*Sqrt[1 + c^2*x^2]*
(149 + 38*c^2*x^2 + 9*c^4*x^4) + 2*b^2*c*x*(2235 + 190*c^2*x^2 + 27*c^4*x^4
) - 30*b*(-15*a*c*x*(15 + 10*c^2*x^2 + 3*c^4*x^4) + b*Sqrt[1 + c^2*x^2]*(14
9 + 38*c^2*x^2 + 9*c^4*x^4))*ArcSinh[c*x] + 225*b^2*c*x*(15 + 10*c^2*x^2 +
3*c^4*x^4)*ArcSinh[c*x]^2))/(3375*c)
```

fricas [A] time = 0.62, size = 278, normalized size = 1.30

$$27\left(25\,a^2+2\,b^2\right)c^5d^2x^5+10\left(225\,a^2+38\,b^2\right)c^3d^2x^3+15\left(225\,a^2+298\,b^2\right)cd^2x+225\left(3\,b^2c^5d^2x^5+10\,b^2c^3d^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/3375*(27*(25*a^2 + 2*b^2)*c^5*d^2*x^5 + 10*(225*a^2 + 38*b^2)*c^3*d^2*x^3
 + 15*(225*a^2 + 298*b^2)*c*d^2*x + 225*(3*b^2*c^5*d^2*x^5 + 10*b^2*c^3*d^2
*x^3 + 15*b^2*c*d^2*x)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 30*(45*a*b*c^5*d^2*
x^5 + 150*a*b*c^3*d^2*x^3 + 225*a*b*c*d^2*x - (9*b^2*c^4*d^2*x^4 + 38*b^2*c
^2*d^2*x^2 + 149*b^2*d^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) -
 30*(9*a*b*c^4*d^2*x^4 + 38*a*b*c^2*d^2*x^2 + 149*a*b*d^2)*sqrt(c^2*x^2 + 1
))/c
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.05, size = 264, normalized size = 1.23

$$d^2a^2\left(\frac{1}{5}c^5x^5+\frac{2}{3}c^3x^3+cx\right)+d^2b^2\left(\frac{8\operatorname{arcsinh}(cx)^2cx}{15}+\frac{\operatorname{arcsinh}(cx)^2cx(c^2x^2+1)^2}{5}+\frac{4\operatorname{arcsinh}(cx)^2cx(c^2x^2+1)}{15}-\frac{16\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}}{15}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c*(d^2*a^2*(1/5*c^5*x^5+2/3*c^3*x^3+c*x)+d^2*b^2*(8/15*arcsinh(c*x)^2*c*x
+1/5*arcsinh(c*x)^2*c*x*(c^2*x^2+1)^2+4/15*arcsinh(c*x)^2*c*x*(c^2*x^2+1)-1
6/15*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+4144/3375*c*x-2/25*arcsinh(c*x)*(c^2*x^
2+1)^(5/2)+2/125*c*x*(c^2*x^2+1)^2+272/3375*c*x*(c^2*x^2+1)-8/45*arcsinh(c*
x)*(c^2*x^2+1)^(3/2))+2*d^2*a*b*(1/5*arcsinh(c*x)*c^5*x^5+2/3*arcsinh(c*x)*
c^3*x^3+arcsinh(c*x)*c*x-1/25*c^4*x^4*(c^2*x^2+1)^(1/2)-38/225*c^2*x^2*(c^2
*x^2+1)^(1/2)-149/225*(c^2*x^2+1)^(1/2)))
```

maxima [B] time = 0.54, size = 457, normalized size = 2.14

$$\frac{1}{5}\,b^2c^4d^2x^5\,\operatorname{arsinh}(cx)^2+\frac{1}{5}\,a^2c^4d^2x^5+\frac{2}{3}\,b^2c^2d^2x^3\,\operatorname{arsinh}(cx)^2+\frac{2}{75}\left(15\,x^5\,\operatorname{arsinh}(cx)-\left(\frac{3\,\sqrt{c^2x^2+1}\,x^4}{c^2}-\frac{4\,\sqrt{c^2x^2}}{c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*c^4*d^2*x^5*arcsinh(c*x)^2 + 1/5*a^2*c^4*d^2*x^5 + 2/3*b^2*c^2*d^2*
x^3*arcsinh(c*x)^2 + 2/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c
^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*c^4*d^2
- 2/1125*(15*(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8
*sqrt(c^2*x^2 + 1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c
^4)*b^2*c^4*d^2 + 2/3*a^2*c^2*d^2*x^3 + 4/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c
^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*c^2*d^2 - 4/27*(3*c*(sq
rt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3
- 6*x)/c^2)*b^2*c^2*d^2 + b^2*d^2*x*arcsinh(c*x)^2 + 2*b^2*d^2*(x - sqrt(c^
2*x^2 + 1)*arcsinh(c*x)/c) + a^2*d^2*x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2
 + 1))*a*b*d^2/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{asinh}\,(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2,x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2, x)
```

sympy [A] time = 4.77, size = 389, normalized size = 1.82

$$\begin{cases}\frac{a^2c^4d^2x^5}{5}+\frac{2a^2c^2d^2x^3}{3}+a^2d^2x+\frac{2abc^4d^2x^5\,\operatorname{asinh}(cx)}{5}-\frac{2abc^3d^2x^4\sqrt{c^2x^2+1}}{25}+\frac{4abc^2d^2x^3\,\operatorname{asinh}(cx)}{3}-\frac{76abcd^2x^2\sqrt{c^2x^2+1}}{225}+2abd^2x\,\mathrm{a} \\ a^2d^2x\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**4*d**2*x**5/5 + 2*a**2*c**2*d**2*x**3/3 + a**2*d**2*x +
2*a*b*c**4*d**2*x**5*asinh(c*x)/5 - 2*a*b*c**3*d**2*x**4*sqrt(c**2*x**2 + 1
)/25 + 4*a*b*c**2*d**2*x**3*asinh(c*x)/3 - 76*a*b*c*d**2*x**2*sqrt(c**2*x**
2 + 1)/225 + 2*a*b*d**2*x*asinh(c*x) - 298*a*b*d**2*sqrt(c**2*x**2 + 1)/(22
5*c) + b**2*c**4*d**2*x**5*asinh(c*x)**2/5 + 2*b**2*c**4*d**2*x**5/125 - 2*
b**2*c**3*d**2*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/25 + 2*b**2*c**2*d**2*x*
*3*asinh(c*x)**2/3 + 76*b**2*c**2*d**2*x**3/675 - 76*b**2*c*d**2*x**2*sqrt(
c**2*x**2 + 1)*asinh(c*x)/225 + b**2*d**2*x*asinh(c*x)**2 + 298*b**2*d**2*x
/225 - 298*b**2*d**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(225*c), Ne(c, 0)), (a*
*2*d**2*x, True))
```

3.212 $\int \frac{(d+c^2dx^2)^2(a+b\sinh^{-1}(cx))^2}{x}\,dx$

Optimal. Leaf size=257

$$-\frac{1}{8}bcd^2x\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)-\frac{11}{16}bcd^2x\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}d^2\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(c\right.$$

```
[Out] 13/32*b^2*c^2*d^2*x^2+1/32*b^2*c^4*d^2*x^4-1/8*b*c*d^2*x*(c^2*x^2+1)^(3/2)*
(a+b*arcsinh(c*x))-11/32*d^2*(a+b*arcsinh(c*x))^2+1/2*d^2*(c^2*x^2+1)*(a+b*
arcsinh(c*x))^2+1/4*d^2*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2+1/3*d^2*(a+b*arc
sinh(c*x))^3/b+d^2*(a+b*arcsinh(c*x))^2*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^2)-b
*d^2*(a+b*arcsinh(c*x))*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)-1/2*b^2*d^2*
polylog(3,1/(c*x+(c^2*x^2+1)^(1/2))^2)-11/16*b*c*d^2*x*(a+b*arcsinh(c*x))*(
c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {5744, 5659, 3716, 2190, 2531, 2282, 6589, 5682, 5675, 30, 5684, 14}

$$bd^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{2}b^2d^2\text{PolyLog}\left(3,e^{2\sinh^{-1}(cx)}\right)-\frac{1}{8}bcd^2x\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] (13*b^2*c^2*d^2*x^2)/32 + (b^2*c^4*d^2*x^4)/32 - (11*b*c*d^2*x*Sqrt[1 + c^2
*x^2]*(a + b*ArcSinh[c*x]))/16 - (b*c*d^2*x*(1 + c^2*x^2)^(3/2)*(a + b*ArcS
inh[c*x]))/8 - (11*d^2*(a + b*ArcSinh[c*x])^2)/32 + (d^2*(1 + c^2*x^2)*(a +
 b*ArcSinh[c*x])^2)/2 + (d^2*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/4 - (d
^2*(a + b*ArcSinh[c*x])^3)/(3*b) + d^2*(a + b*ArcSinh[c*x])^2*Log[1 - E^(2*
ArcSinh[c*x])] + b*d^2*(a + b*ArcSinh[c*x])*PolyLog[2, E^(2*ArcSinh[c*x])]
- (b^2*d^2*PolyLog[3, E^(2*ArcSinh[c*x])])/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
```

```
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{x}\,dx = \frac{1}{4}d^2\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2+d\int \frac{\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx\right.}{x}$$

$$= -\frac{1}{8}bcd^2x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{2}d^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}\right.$$

$$= -\frac{11}{16}bcd^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{8}bcd^2x\left(1+c^2x^2\right)^{3/2}\left(a+b\,\mathrm{si}\right.$$

$$= \frac{13}{32}b^2c^2d^2x^2+\frac{1}{32}b^2c^4d^2x^4-\frac{11}{16}bcd^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{8}$$

$$= \frac{13}{32}b^2c^2d^2x^2+\frac{1}{32}b^2c^4d^2x^4-\frac{11}{16}bcd^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{8}$$

$$= \frac{13}{32}b^2c^2d^2x^2+\frac{1}{32}b^2c^4d^2x^4-\frac{11}{16}bcd^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{8}$$

$$= \frac{13}{32}b^2c^2d^2x^2+\frac{1}{32}b^2c^4d^2x^4-\frac{11}{16}bcd^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{8}$$

$$= \frac{13}{32}b^2c^2d^2x^2+\frac{1}{32}b^2c^4d^2x^4-\frac{11}{16}bcd^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{8}$$

Mathematica [A] time = 0.40, size = 323, normalized size = 1.26

$$\frac{1}{768}d^2\left(192a^2c^4x^4+768a^2c^2x^2+768a^2\log(cx)+384abc^4x^4\sinh^{-1}(cx)-624abcx\sqrt{c^2x^2+1}+1536abc^2x^2\,\mathrm{sinh}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] (d^2*(768*a^2*c^2*x^2 + 192*a^2*c^4*x^4 - 624*a*b*c*x*Sqrt[1 + c^2*x^2] - 9
6*a*b*c^3*x^3*Sqrt[1 + c^2*x^2] + 624*a*b*ArcSinh[c*x] + 1536*a*b*c^2*x^2*A
rcSinh[c*x] + 384*a*b*c^4*x^4*ArcSinh[c*x] + 768*a*b*ArcSinh[c*x]^2 - 256*b
^2*ArcSinh[c*x]^3 + 144*b^2*Cosh[2*ArcSinh[c*x]] + 288*b^2*ArcSinh[c*x]^2*C
osh[2*ArcSinh[c*x]] + 3*b^2*Cosh[4*ArcSinh[c*x]] + 24*b^2*ArcSinh[c*x]^2*Co
sh[4*ArcSinh[c*x]] + 1536*a*b*ArcSinh[c*x]*Log[1 - E^(-2*ArcSinh[c*x])] + 7
68*b^2*ArcSinh[c*x]^2*Log[1 - E^(2*ArcSinh[c*x])] + 768*a^2*Log[c*x] - 768*
a*b*PolyLog[2, E^(-2*ArcSinh[c*x])] + 768*b^2*ArcSinh[c*x]*PolyLog[2, E^(2*
ArcSinh[c*x])] - 384*b^2*PolyLog[3, E^(2*ArcSinh[c*x])] - 288*b^2*ArcSinh[c
*x]*Sinh[2*ArcSinh[c*x]] - 12*b^2*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]))/768
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{a^2c^4d^2x^4+2\,a^2c^2d^2x^2+a^2d^2+\left(b^2c^4d^2x^4+2\,b^2c^2d^2x^2+b^2d^2\right)\mathrm{arsinh}\,(cx)^2+2\left(abc^4d^2x^4+2\,abc^2d^2x^2\right.}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
+ 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))/x, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.23, size = 586, normalized size = 2.28

$$-\frac{13d^2b^2 \operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,cx}{16}+d^2a^2c^2x^2-d^2ab\operatorname{arcsinh}(cx)^2+\frac{13d^2ab\operatorname{arcsinh}(cx)}{16}+2d^2ab\operatorname{polylog}\left(2,cx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x,x)
```

```
[Out] d^2*a^2*c^2*x^2-d^2*a*b*arcsinh(c*x)^2+13/16*d^2*a*b*arcsinh(c*x)+2*d^2*a*b
*polylog(2,c*x+(c^2*x^2+1)^(1/2))+2*d^2*a*b*polylog(2,-c*x-(c^2*x^2+1)^(1/2
))+d^2*b^2*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*d^2*b^2*arcsinh(c*x
)*polylog(2,c*x+(c^2*x^2+1)^(1/2))+d^2*b^2*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2
+1)^(1/2))+2*d^2*b^2*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+1/4*d^2
*a^2*c^4*x^4+13/32*b^2*c^2*d^2*x^2+1/32*b^2*c^4*d^2*x^4+49/256*d^2*b^2+d^2*
b^2*arcsinh(c*x)^2*c^2*x^2+2*d^2*a*b*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2
))+2*d^2*a*b*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+1/4*d^2*b^2*arcsinh(c
*x)^2*c^4*x^4+13/32*d^2*b^2*arcsinh(c*x)^2-2*d^2*b^2*polylog(3,c*x+(c^2*x^2
+1)^(1/2))-1/3*d^2*b^2*arcsinh(c*x)^3-2*d^2*b^2*polylog(3,-c*x-(c^2*x^2+1)^
(1/2))+d^2*a^2*ln(c*x)-13/16*d^2*a*b*c*x*(c^2*x^2+1)^(1/2)+2*d^2*a*b*arcsin
h(c*x)*c^2*x^2+1/2*d^2*a*b*arcsinh(c*x)*c^4*x^4-1/8*d^2*b^2*arcsinh(c*x)*(c
^2*x^2+1)^(1/2)*c^3*x^3-13/16*d^2*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c*x-1/
8*d^2*a*b*c^3*x^3*(c^2*x^2+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}a^2c^4d^2x^4+a^2c^2d^2x^2+a^2d^2\log(x)+\int b^2c^4d^2x^3\log\left(cx+\sqrt{c^2x^2+1}\right)^2+2\,abc^4d^2x^3\log\left(cx+\sqrt{c^2x^2+1}\right)+2\,b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*c^4*d^2*x^4 + a^2*c^2*d^2*x^2 + a^2*d^2*log(x) + integrate(b^2*c^4*
d^2*x^3*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*a*b*c^4*d^2*x^3*log(c*x + sqrt(c
^2*x^2 + 1)) + 2*b^2*c^2*d^2*x*log(c*x + sqrt(c^2*x^2 + 1))^2 + 4*a*b*c^2*d
^2*x*log(c*x + sqrt(c^2*x^2 + 1)) + b^2*d^2*log(c*x + sqrt(c^2*x^2 + 1))^2/
x + 2*a*b*d^2*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2\,\left(d\,c^2\,x^2+d\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2)/x,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int \frac{a^2}{x}\,dx + \int 2a^2c^2x\,dx + \int a^2c^4x^3\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{x}\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{x}\,dx + \int 2b^2c^2x\operatorname{asinh}^2(cx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2/x,x)
```

```
[Out] d**2*(Integral(a**2/x, x) + Integral(2*a**2*c**2*x, x) + Integral(a**2*c**4
*x**3, x) + Integral(b**2*asinh(c*x)**2/x, x) + Integral(2*a*b*asinh(c*x)/x
, x) + Integral(2*b**2*c**2*x*asinh(c*x)**2, x) + Integral(b**2*c**4*x**3*a
sinh(c*x)**2, x) + Integral(4*a*b*c**2*x*asinh(c*x), x) + Integral(2*a*b*c*
*4*x**3*asinh(c*x), x))
```

3.213 $\int \frac{(d+c^2dx^2)^2(a+b\sinh^{-1}(cx))^2}{x^2}\,dx$

Optimal. Leaf size=229

$$\frac{4}{3}c^2d^2x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2-\frac{2}{9}bcd^2\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)-\frac{10}{3}bcd^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}\right.$$

```
[Out] 32/9*b^2*c^2*d^2*x+2/27*b^2*c^4*d^2*x^3-2/9*b*c*d^2*(c^2*x^2+1)^(3/2)*(a+b*
arcsinh(c*x))+8/3*c^2*d^2*x*(a+b*arcsinh(c*x))^2+4/3*c^2*d^2*x*(c^2*x^2+1)*
(a+b*arcsinh(c*x))^2-d^2*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/x-4*b*c*d^2*(a+
b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))-2*b^2*c*d^2*polylog(2,-c*x-(
c^2*x^2+1)^(1/2))+2*b^2*c*d^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))-10/3*b*c*d^2
*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {5739, 5684, 5653, 5717, 8, 5744, 5742, 5760, 4182, 2279, 2391}

$$-2b^2cd^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)+2b^2cd^2\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)+\frac{4}{3}c^2d^2x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2-\frac{2}{9}bcd$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] (32*b^2*c^2*d^2*x)/9 + (2*b^2*c^4*d^2*x^3)/27 - (10*b*c*d^2*Sqrt[1 + c^2*x^
2]*(a + b*ArcSinh[c*x]))/3 - (2*b*c*d^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[
c*x]))/9 + (8*c^2*d^2*x*(a + b*ArcSinh[c*x])^2)/3 + (4*c^2*d^2*x*(1 + c^2*x
^2)*(a + b*ArcSinh[c*x])^2)/3 - (d^2*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2
)/x - 4*b*c*d^2*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] - 2*b^2*c*d^2*
PolyLog[2, -E^ArcSinh[c*x]] + 2*b^2*c*d^2*PolyLog[2, E^ArcSinh[c*x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
```

```
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{x^2}\,dx = -\frac{d^2\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{x} + \left(4c^2d\right)\int\left(d+c^2dx^2\right)\left(a+b\,\mathrm{si}\right.$$

$$= \frac{2}{3}bcd^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right) + \frac{4}{3}c^2d^2x\left(1+c^2x^2\right)\left(a+b\,\mathrm{sir}\right.$$

$$= -\frac{2}{3}b^2c^2d^2x - \frac{2}{9}b^2c^4d^2x^3 + 2bcd^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{2}{9}bc$$

$$= -\frac{16}{9}b^2c^2d^2x + \frac{2}{27}b^2c^4d^2x^3 - \frac{10}{3}bcd^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) -$$

$$= \frac{32}{9}b^2c^2d^2x + \frac{2}{27}b^2c^4d^2x^3 - \frac{10}{3}bcd^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{2}{9}$$

$$= \frac{32}{9}b^2c^2d^2x + \frac{2}{27}b^2c^4d^2x^3 - \frac{10}{3}bcd^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{2}{9}$$

$$= \frac{32}{9}b^2c^2d^2x + \frac{2}{27}b^2c^4d^2x^3 - \frac{10}{3}bcd^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{2}{9}$$

Mathematica [A] time = 1.17, size = 306, normalized size = 1.34

$$\frac{1}{54}d^2\left(18a^2c^4x^3 + 108a^2c^2x - \frac{54a^2}{x} + 36abc^4x^3\sinh^{-1}(cx) - 12abc\left(c^2x^2-2\right)\sqrt{c^2x^2+1} + 216abc\left(cx\sinh^{-1}(\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] (d^2*((-54*a^2)/x + 108*a^2*c^2*x + 18*a^2*c^4*x^3 - 12*a*b*c*(-2 + c^2*x^2
)*Sqrt[1 + c^2*x^2] + 36*a*b*c^4*x^3*ArcSinh[c*x] - 189*b^2*c*Sqrt[1 + c^2*
x^2]*ArcSinh[c*x] + 216*a*b*c*(-Sqrt[1 + c^2*x^2] + c*x*ArcSinh[c*x]) + 108
*b^2*c^2*x*(2 + ArcSinh[c*x]^2) + 2*b^2*c^2*x*(-12 + 2*c^2*x^2 + 9*c^2*x^2*
ArcSinh[c*x]^2) - (108*a*b*(ArcSinh[c*x] + c*x*ArcTanh[Sqrt[1 + c^2*x^2]]))
/x - 3*b^2*c*ArcSinh[c*x]*Cosh[3*ArcSinh[c*x]] - (54*b^2*ArcSinh[c*x]*(ArcS
inh[c*x] + 2*c*x*(-Log[1 - E^(-ArcSinh[c*x])] + Log[1 + E^(-ArcSinh[c*x])])
))/x + 108*b^2*c*PolyLog[2, -E^(-ArcSinh[c*x])] - 108*b^2*c*PolyLog[2, E^(-
ArcSinh[c*x])]))/54
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{a^2c^4d^2x^4 + 2\,a^2c^2d^2x^2 + a^2d^2 + \left(b^2c^4d^2x^4 + 2\,b^2c^2d^2x^2 + b^2d^2\right)\mathrm{arsinh}\,(cx)^2 + 2\left(abc^4d^2x^4 + 2\,abc^2d\right.}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
+ 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))/x^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.22, size = 400, normalized size = 1.75

$$\frac{d^2a^2c^4x^3}{3}+2d^2a^2c^2x-\frac{d^2a^2}{x}+\frac{32b^2c^2d^2x}{9}+\frac{2b^2c^4d^2x^3}{27}+\frac{d^2b^2\operatorname{arcsinh}(cx)^2c^4x^3}{3}+2d^2b^2\operatorname{arcsinh}(cx)^2c^2x-\frac{32c\,d^2b^2\,\mathrm{a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^2,x)
```

```
[Out] 1/3*d^2*a^2*c^4*x^3+2*d^2*a^2*c^2*x-d^2*a^2/x+32/9*b^2*c^2*d^2*x+2/27*b^2*c
^4*d^2*x^3+1/3*d^2*b^2*arcsinh(c*x)^2*c^4*x^3+2*d^2*b^2*arcsinh(c*x)^2*c^2*
x-32/9*c*d^2*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-d^2*b^2*arcsinh(c*x)^2/x-2*
c*d^2*b^2*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+2*c*d^2*b^2*arcsinh(c*x)
*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*b^2*c*d^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))-2
/9*d^2*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^3*x^2-2*b^2*c*d^2*polylog(2,-c*
x-(c^2*x^2+1)^(1/2))+2/3*d^2*a*b*arcsinh(c*x)*c^4*x^3+4*d^2*a*b*arcsinh(c*x
)*c^2*x-2*d^2*a*b*arcsinh(c*x)/x-2/9*d^2*a*b*c^3*x^2*(c^2*x^2+1)^(1/2)-32/9
*c*d^2*a*b*(c^2*x^2+1)^(1/2)-2*c*d^2*a*b*arctanh(1/(c^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}a^2c^4d^2x^3+\frac{2}{9}\left(3x^3\operatorname{arsinh}(cx)-c\left(\frac{\sqrt{c^2x^2+1}\,x^2}{c^2}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)abc^4d^2+2b^2c^2d^2x\operatorname{arsinh}(cx)^2+4b^2c^2d^2\left(x-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*c^4*d^2*x^3 + 2/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^
2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*c^4*d^2 + 2*b^2*c^2*d^2*x*arcsinh(c*x)^2
+ 4*b^2*c^2*d^2*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + 2*a^2*c^2*d^2*x +
4*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*c*d^2 - 2*(c*arcsinh(1/(c*abs(
x))) + arcsinh(c*x)/x)*a*b*d^2 - a^2*d^2/x + 1/3*(b^2*c^4*d^2*x^4 - 3*b^2*d
^2)*log(c*x + sqrt(c^2*x^2 + 1))^2/x - integrate(2/3*(b^2*c^7*d^2*x^6 + b^2
*c^5*d^2*x^4 - 3*b^2*c^3*d^2*x^2 - 3*b^2*c*d^2 + (b^2*c^6*d^2*x^5 - 3*b^2*c
^2*d^2*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))/(c^3*x^4 + c*x^2
+ (c^2*x^3 + x)*sqrt(c^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\operatorname{asinh}(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2)/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int 2a^2c^2\,dx + \int \frac{a^2}{x^2}\,dx + \int a^2c^4x^2\,dx + \int 2b^2c^2\,\mathrm{asinh}^2\,(cx)\,dx + \int \frac{b^2\,\mathrm{asinh}^2\,(cx)}{x^2}\,dx + \int 4abc^2\,\mathrm{asinh}\,(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2/x**2,x)
```

```
[Out] d**2*(Integral(2*a**2*c**2, x) + Integral(a**2/x**2, x) + Integral(a**2*c**
4*x**2, x) + Integral(2*b**2*c**2*asinh(c*x)**2, x) + Integral(b**2*asinh(c
*x)**2/x**2, x) + Integral(4*a*b*c**2*asinh(c*x), x) + Integral(2*a*b*asinh
(c*x)/x**2, x) + Integral(b**2*c**4*x**2*asinh(c*x)**2, x) + Integral(2*a*b
*c**4*x**2*asinh(c*x), x))
```

3.214 $\int \frac{(d+c^2dx^2)^2 (a+b\sinh^{-1}(cx))^2}{x^3} dx$

Optimal. Leaf size=272

$$-2bc^2d^2\text{Li}_2\left(e^{-2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)+c^2d^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2-\frac{bcd^2\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}\right.}{x}$$

```
[Out] 1/4*b^2*c^4*d^2*x^2-b*c*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/x+1/4*c^2*
d^2*(a+b*arcsinh(c*x))^2+c^2*d^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2-1/2*d^2*(
c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/x^2+2/3*c^2*d^2*(a+b*arcsinh(c*x))^3/b+2*
c^2*d^2*(a+b*arcsinh(c*x))^2*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^2)+b^2*c^2*d^2*
ln(x)-2*b*c^2*d^2*(a+b*arcsinh(c*x))*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)
-b^2*c^2*d^2*polylog(3,1/(c*x+(c^2*x^2+1)^(1/2))^2)+1/2*b*c^3*d^2*x*(a+b*ar
csinh(c*x))*(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.50, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {5739, 5744, 5659, 3716, 2190, 2531, 2282, 6589, 5682, 5675, 30, 14}

$$2bc^2d^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)-b^2c^2d^2\text{PolyLog}\left(3,e^{2\sinh^{-1}(cx)}\right)+\frac{1}{2}bc^3d^2x\sqrt{c^2x^2+1}\left(a+b\sin\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] (b^2*c^4*d^2*x^2)/4 + (b*c^3*d^2*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/
2 - (b*c*d^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x + (c^2*d^2*(a + b*
ArcSinh[c*x])^2)/4 + c^2*d^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2 - (d^2*(1
 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(2*x^2) - (2*c^2*d^2*(a + b*ArcSinh[c
*x])^3)/(3*b) + 2*c^2*d^2*(a + b*ArcSinh[c*x])^2*Log[1 - E^(2*ArcSinh[c*x])
] + b^2*c^2*d^2*Log[x] + 2*b*c^2*d^2*(a + b*ArcSinh[c*x])*PolyLog[2, E^(2*A
rcSinh[c*x])] - b^2*c^2*d^2*PolyLog[3, E^(2*ArcSinh[c*x])]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
```

```
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{x^3}\,dx = -\frac{d^2\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{2x^2}+\left(2c^2d\right)\int\frac{\left(d+c^2dx^2\right)\left(a+b\sinh\right.}{x}$$

$$= -\frac{bcd^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x}+c^2d^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(c\right.$$

$$= \frac{1}{2}bc^3d^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{bcd^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}\right.}{x}$$

$$= \frac{1}{4}b^2c^4d^2x^2+\frac{1}{2}bc^3d^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{bcd^2\left(1+c^2x^2\right)^{3/2}}{}$$

$$= \frac{1}{4}b^2c^4d^2x^2+\frac{1}{2}bc^3d^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{bcd^2\left(1+c^2x^2\right)^{3/2}}{}$$

$$= \frac{1}{4}b^2c^4d^2x^2+\frac{1}{2}bc^3d^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{bcd^2\left(1+c^2x^2\right)^{3/2}}{}$$

$$= \frac{1}{4}b^2c^4d^2x^2+\frac{1}{2}bc^3d^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{bcd^2\left(1+c^2x^2\right)^{3/2}}{}$$

$$= \frac{1}{4}b^2c^4d^2x^2+\frac{1}{2}bc^3d^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{bcd^2\left(1+c^2x^2\right)^{3/2}}{}$$

Mathematica [A] time = 0.88, size = 305, normalized size = 1.12

$$\frac{1}{2}d^2\left(a^2c^4x^2+4a^2c^2\log(x)-\frac{a^2}{x^2}+4abc^2\left(\sinh^{-1}(cx)\left(\sinh^{-1}(cx)+2\log\left(1-e^{-2\sinh^{-1}(cx)}\right)\right)-\text{Li}_2\left(e^{-2\sinh^{-1}(cx)}\right)\right)\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] (d^2*(-(a^2/x^2) + a^2*c^4*x^2 - (2*a*b*(c*x*Sqrt[1 + c^2*x^2] + ArcSinh[c*
x]))/x^2 + a*b*c^2*(-(c*x*Sqrt[1 + c^2*x^2]) + (1 + 2*c^2*x^2)*ArcSinh[c*x]
) + 4*a^2*c^2*Log[x] - (b^2*(2*c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + ArcSinh
[c*x]^2 - 2*c^2*x^2*Log[c*x]))/x^2 + 4*a*b*c^2*(ArcSinh[c*x]*(ArcSinh[c*x]
+ 2*Log[1 - E^(-2*ArcSinh[c*x])]) - PolyLog[2, E^(-2*ArcSinh[c*x])]) - (2*b
^2*c^2*(2*ArcSinh[c*x]^2*(ArcSinh[c*x] - 3*Log[1 - E^(2*ArcSinh[c*x])]) - 6
*ArcSinh[c*x]*PolyLog[2, E^(2*ArcSinh[c*x])] + 3*PolyLog[3, E^(2*ArcSinh[c*
x])]))/3 + (b^2*c^2*((1 + 2*ArcSinh[c*x]^2)*Cosh[2*ArcSinh[c*x]] - 2*ArcSin
h[c*x]*Sinh[2*ArcSinh[c*x]]))/4))/2
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^2c^4d^2x^4+2\,a^2c^2d^2x^2+a^2d^2+\left(b^2c^4d^2x^4+2\,b^2c^2d^2x^2+b^2d^2\right)\text{arsinh}\,(cx)^2+2\left(abc^4d^2x^4+2\,abc^2d^2x^2\right.}{x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
+ 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))/x^3, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.48, size = 719, normalized size = 2.64

$$\frac{b^2c^4d^2x^2}{4}-\frac{d^2a^2}{2x^2}+2c^2d^2b^2\operatorname{arcsinh}(cx)^2\ln\left(1-cx-\sqrt{c^2x^2+1}\right)+4c^2d^2b^2\operatorname{arcsinh}(cx)\operatorname{polylog}\left(2,cx+\sqrt{c^2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^3,x)
```

```
[Out] 1/4*b^2*c^4*d^2*x^2-1/2*d^2*a^2/x^2+2*c^2*d^2*b^2*arcsinh(c*x)^2*ln(1-c*x-(
c^2*x^2+1)^(1/2))+4*c^2*d^2*b^2*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2
))+2*c^2*d^2*b^2*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))+4*c^2*d^2*b^2*a
rcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))-2*c^2*d^2*a*b*arcsinh(c*x)^2+
1/2*c^2*d^2*a*b*arcsinh(c*x)+4*c^2*d^2*a*b*polylog(2,c*x+(c^2*x^2+1)^(1/2))
+4*c^2*d^2*a*b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))-d^2*a*b*arcsinh(c*x)/x^2+1
/2*c^4*d^2*b^2*arcsinh(c*x)^2*x^2+c^4*d^2*a*b*arcsinh(c*x)*x^2-c*d^2*b^2*ar
csinh(c*x)/x*(c^2*x^2+1)^(1/2)-1/2*c^3*d^2*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/
2)*x-c*d^2*a*b/x*(c^2*x^2+1)^(1/2)+4*c^2*d^2*a*b*arcsinh(c*x)*ln(1+c*x+(c^2
*x^2+1)^(1/2))+4*c^2*d^2*a*b*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-1/2*c
^3*d^2*a*b*x*(c^2*x^2+1)^(1/2)+1/8*d^2*b^2*c^2+d^2*a*b*c^2+1/2*c^4*d^2*a^2*
x^2-1/2*d^2*b^2*arcsinh(c*x)^2/x^2-2*c^2*d^2*b^2*ln(c*x+(c^2*x^2+1)^(1/2))+
c^2*d^2*b^2*ln(c*x+(c^2*x^2+1)^(1/2)-1)-4*c^2*d^2*b^2*polylog(3,-c*x-(c^2*x
^2+1)^(1/2))+c^2*d^2*b^2*arcsinh(c*x)+c^2*d^2*b^2*ln(1+c*x+(c^2*x^2+1)^(1/2
))+1/4*c^2*d^2*b^2*arcsinh(c*x)^2-4*c^2*d^2*b^2*polylog(3,c*x+(c^2*x^2+1)^(
1/2))-2/3*c^2*d^2*b^2*arcsinh(c*x)^3+2*c^2*d^2*a^2*ln(c*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}a^2c^4d^2x^2+2a^2c^2d^2\log(x)-abd^2\left(\frac{\sqrt{c^2x^2+1}\,c}{x}+\frac{\operatorname{arsinh}(cx)}{x^2}\right)-\frac{a^2d^2}{2x^2}+\int b^2c^4d^2x\log\left(cx+\sqrt{c^2x^2+1}\right)^2+2\,ab$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*c^4*d^2*x^2 + 2*a^2*c^2*d^2*log(x) - a*b*d^2*(sqrt(c^2*x^2 + 1)*c/x
 + arcsinh(c*x)/x^2) - 1/2*a^2*d^2/x^2 + integrate(b^2*c^4*d^2*x*log(c*x +
sqrt(c^2*x^2 + 1))^2 + 2*a*b*c^4*d^2*x*log(c*x + sqrt(c^2*x^2 + 1)) + 2*b^2
*c^2*d^2*log(c*x + sqrt(c^2*x^2 + 1))^2/x + 4*a*b*c^2*d^2*log(c*x + sqrt(c^
2*x^2 + 1))/x + b^2*d^2*log(c*x + sqrt(c^2*x^2 + 1))^2/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2)/x^3,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int \frac{a^2}{x^3}\,dx + \int \frac{2a^2c^2}{x}\,dx + \int a^2c^4x\,dx + \int \frac{b^2\,\mathrm{asinh}^2\,(cx)}{x^3}\,dx + \int \frac{2ab\,\mathrm{asinh}\,(cx)}{x^3}\,dx + \int \frac{2b^2c^2\,\mathrm{asinh}^2\,(cx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2/x**3,x)
```

```
[Out] d**2*(Integral(a**2/x**3, x) + Integral(2*a**2*c**2/x, x) + Integral(a**2*c
**4*x, x) + Integral(b**2*asinh(c*x)**2/x**3, x) + Integral(2*a*b*asinh(c*x
)/x**3, x) + Integral(2*b**2*c**2*asinh(c*x)**2/x, x) + Integral(b**2*c**4*
x*asinh(c*x)**2, x) + Integral(4*a*b*c**2*asinh(c*x)/x, x) + Integral(2*a*b
*c**4*x*asinh(c*x), x))
```

3.215 $\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{x^4}\,dx$

Optimal. Leaf size=248

$$\frac{8}{3}c^4d^2x\left(a+b\sinh^{-1}(cx)\right)^2-\frac{22}{3}bc^3d^2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{4c^2d^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right.}{3x}$$

```
[Out] -1/3*b^2*c^2*d^2/x+2*b^2*c^4*d^2*x-1/3*b*c*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsi
nh(c*x))/x^2+8/3*c^4*d^2*x*(a+b*arcsinh(c*x))^2-4/3*c^2*d^2*(c^2*x^2+1)*(a+
b*arcsinh(c*x))^2/x-1/3*d^2*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/x^3-22/3*b*c
^3*d^2*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))-11/3*b^2*c^3*d^2*p
olylog(2,-c*x-(c^2*x^2+1)^(1/2))+11/3*b^2*c^3*d^2*polylog(2,c*x+(c^2*x^2+1)
^(1/2))-5/3*b*c^3*d^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.71, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5739, 5653, 5717, 8, 5742, 5760, 4182, 2279, 2391, 14}

$$-\frac{11}{3}b^2c^3d^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)+\frac{11}{3}b^2c^3d^2\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)-\frac{5}{3}bc^3d^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)-\frac{4}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] -(b^2*c^2*d^2)/(3*x) + 2*b^2*c^4*d^2*x - (5*b*c^3*d^2*Sqrt[1 + c^2*x^2]*(a
+ b*ArcSinh[c*x]))/3 - (b*c*d^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(
3*x^2) + (8*c^4*d^2*x*(a + b*ArcSinh[c*x])^2)/3 - (4*c^2*d^2*(1 + c^2*x^2)*
(a + b*ArcSinh[c*x])^2)/(3*x) - (d^2*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2
)/(3*x^3) - (22*b*c^3*d^2*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/3 -
 (11*b^2*c^3*d^2*PolyLog[2, -E^ArcSinh[c*x]])/3 + (11*b^2*c^3*d^2*PolyLog[2
, E^ArcSinh[c*x]])/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
```

```
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(d+c^2dx^2)^2 (a+b\sinh^{-1}(cx))^2}{x^4}\,dx = -\frac{d^2(1+c^2x^2)^2 (a+b\sinh^{-1}(cx))^2}{3x^3} + \frac{1}{3}(4c^2d)\int \frac{(d+c^2dx^2)(a+b}{x^2}$$

$$= -\frac{bcd^2(1+c^2x^2)^{3/2}(a+b\sinh^{-1}(cx))}{3x^2} - \frac{4c^2d^2(1+c^2x^2)(a+b\sinh}{3x}$$

$$= \frac{11}{3}bc^3d^2\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx)) - \frac{bcd^2(1+c^2x^2)^{3/2}(a+b\sinh}{3x^2}$$

$$= -\frac{b^2c^2d^2}{3x} - \frac{10}{3}b^2c^4d^2x - \frac{5}{3}bc^3d^2\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx)) - \frac{bcd^2}{}$$

$$= -\frac{b^2c^2d^2}{3x} + 2b^2c^4d^2x - \frac{5}{3}bc^3d^2\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx)) - \frac{bcd^2(}{}$$

$$= -\frac{b^2c^2d^2}{3x} + 2b^2c^4d^2x - \frac{5}{3}bc^3d^2\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx)) - \frac{bcd^2(}{}$$

$$= -\frac{b^2c^2d^2}{3x} + 2b^2c^4d^2x - \frac{5}{3}bc^3d^2\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx)) - \frac{bcd^2(}{}$$

Mathematica [A] time = 0.91, size = 357, normalized size = 1.44

$$\frac{d^2\left(3a^2c^4x^4 - 6a^2c^2x^2 - a^2 + 6abc^4x^4\sinh^{-1}(cx) - abcx\sqrt{c^2x^2+1} - 12abc^2x^2\sinh^{-1}(cx) - 6abc^3x^3\sqrt{c^2x^2+1}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] (d^2*(-a^2 - 6*a^2*c^2*x^2 - b^2*c^2*x^2 + 3*a^2*c^4*x^4 + 6*b^2*c^4*x^4 -
a*b*c*x*Sqrt[1 + c^2*x^2] - 6*a*b*c^3*x^3*Sqrt[1 + c^2*x^2] - 2*a*b*ArcSinh
[c*x] - 12*a*b*c^2*x^2*ArcSinh[c*x] + 6*a*b*c^4*x^4*ArcSinh[c*x] - b^2*c*x*
Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - 6*b^2*c^3*x^3*Sqrt[1 + c^2*x^2]*ArcSinh[c*
x] - b^2*ArcSinh[c*x]^2 - 6*b^2*c^2*x^2*ArcSinh[c*x]^2 + 3*b^2*c^4*x^4*ArcS
inh[c*x]^2 - 11*a*b*c^3*x^3*ArcTanh[Sqrt[1 + c^2*x^2]] + 11*b^2*c^3*x^3*Arc
Sinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 11*b^2*c^3*x^3*ArcSinh[c*x]*Log[1 +
E^(-ArcSinh[c*x])] + 11*b^2*c^3*x^3*PolyLog[2, -E^(-ArcSinh[c*x])] - 11*b^2
*c^3*x^3*PolyLog[2, E^(-ArcSinh[c*x])]))/(3*x^3)
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^2c^4d^2x^4 + 2\,a^2c^2d^2x^2 + a^2d^2 + \left(b^2c^4d^2x^4 + 2\,b^2c^2d^2x^2 + b^2d^2\right)\operatorname{arsinh}(cx)^2 + 2\left(abc^4d^2x^4 + 2\,abc^2d\right.}{x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
+ 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))/x^4, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.42, size = 408, normalized size = 1.65

$$c^4d^2a^2x-\frac{2c^2d^2a^2}{x}-\frac{d^2a^2}{3x^3}+c^4d^2b^2\operatorname{arcsinh}(cx)^2x-2c^3d^2b^2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}+2b^2c^4d^2x-\frac{2c^2d^2b^2\operatorname{arcsinh}(cx)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^4,x)
```

```
[Out] c^4*d^2*a^2*x-2*c^2*d^2*a^2/x-1/3*d^2*a^2/x^3+c^4*d^2*b^2*arcsinh(c*x)^2*x-
2*c^3*d^2*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*b^2*c^4*d^2*x-2*c^2*d^2*b^2*
arcsinh(c*x)^2/x-1/3*c*d^2*b^2/x^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-1/3*d^2*b
^2/x^3*arcsinh(c*x)^2-1/3*b^2*c^2*d^2/x-11/3*c^3*d^2*b^2*arcsinh(c*x)*ln(1+
c*x+(c^2*x^2+1)^(1/2))-11/3*b^2*c^3*d^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+1
1/3*c^3*d^2*b^2*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+11/3*b^2*c^3*d^2*p
olylog(2,c*x+(c^2*x^2+1)^(1/2))+2*c^4*d^2*a*b*arcsinh(c*x)*x-4*c^2*d^2*a*b*
arcsinh(c*x)/x-2/3*d^2*a*b*arcsinh(c*x)/x^3-2*c^3*d^2*a*b*(c^2*x^2+1)^(1/2)
-11/3*c^3*d^2*a*b*arctanh(1/(c^2*x^2+1)^(1/2))-1/3*c*d^2*a*b/x^2*(c^2*x^2+1
)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^2c^4d^2x\operatorname{arsinh}(cx)^2+2\,b^2c^4d^2\left(x-\frac{\sqrt{c^2x^2+1}\,\operatorname{arsinh}(cx)}{c}\right)+a^2c^4d^2x+2\left(cx\operatorname{arsinh}(cx)-\sqrt{c^2x^2+1}\right)abc^3d^2-4\,(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="maxima")
```

```
[Out] b^2*c^4*d^2*x*arcsinh(c*x)^2 + 2*b^2*c^4*d^2*(x - sqrt(c^2*x^2 + 1)*arcsinh
(c*x)/c) + a^2*c^4*d^2*x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*c^3
*d^2 - 4*(c*arcsinh(1/(c*abs(x))) + arcsinh(c*x)/x)*a*b*c^2*d^2 + 1/3*((c^2
*arcsinh(1/(c*abs(x))) - sqrt(c^2*x^2 + 1)/x^2)*c - 2*arcsinh(c*x)/x^3)*a*b
*d^2 - 2*a^2*c^2*d^2/x - 1/3*a^2*d^2/x^3 - 1/3*(6*b^2*c^2*d^2*x^2 + b^2*d^2
)*log(c*x + sqrt(c^2*x^2 + 1))^2/x^3 + integrate(2/3*(6*b^2*c^5*d^2*x^4 + 7
*b^2*c^3*d^2*x^2 + b^2*c*d^2 + (6*b^2*c^4*d^2*x^3 + b^2*c^2*d^2*x)*sqrt(c^2
*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))/(c^3*x^6 + c*x^4 + (c^2*x^5 + x^3)*
sqrt(c^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\operatorname{asinh}(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2)/x^4,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int a^2c^4\,dx+\int\frac{a^2}{x^4}\,dx+\int\frac{2a^2c^2}{x^2}\,dx+\int b^2c^4\operatorname{asinh}^2(cx)\,dx+\int\frac{b^2\operatorname{asinh}^2(cx)}{x^4}\,dx+\int 2abc^4\operatorname{asinh}(cx)\,dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2/x**4,x)
```

```
[Out] d**2*(Integral(a**2*c**4, x) + Integral(a**2/x**4, x) + Integral(2*a**2*c**
2/x**2, x) + Integral(b**2*c**4*asinh(c*x)**2, x) + Integral(b**2*asinh(c*x
)**2/x**4, x) + Integral(2*a*b*c**4*asinh(c*x), x) + Integral(2*a*b*asinh(c
*x)/x**4, x) + Integral(2*b**2*c**2*asinh(c*x)**2/x**2, x) + Integral(4*a*b
*c**2*asinh(c*x)/x**2, x))
```

3.216 $\int x^4 \left(d+c^2dx^2\right)^3 \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=465

$$\frac{1}{11}d^3x^5\left(c^2x^2+1\right)^3\left(a+b\sinh^{-1}(cx)\right)^2+\frac{2}{33}d^3x^5\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)^2+\frac{8}{231}d^3x^5\left(c^2x^2+1\right)\left(a+b\sinh^{-1}\right.$$

```
[Out] 100976/4002075*b^2*d^3*x/c^4-50488/12006225*b^2*d^3*x^3/c^2+12622/6670125*b
^2*d^3*x^5+9410/1120581*b^2*c^2*d^3*x^7+182/29403*b^2*c^4*d^3*x^9+2/1331*b^
2*c^6*d^3*x^11-16/693*b*d^3*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/c^5+4/1155
*b*d^3*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))/c^5-2/1617*b*d^3*(c^2*x^2+1)^(7
/2)*(a+b*arcsinh(c*x))/c^5+8/297*b*d^3*(c^2*x^2+1)^(9/2)*(a+b*arcsinh(c*x))
/c^5-2/121*b*d^3*(c^2*x^2+1)^(11/2)*(a+b*arcsinh(c*x))/c^5+16/1155*d^3*x^5*
(a+b*arcsinh(c*x))^2+8/231*d^3*x^5*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2+2/33*d^
3*x^5*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2+1/11*d^3*x^5*(c^2*x^2+1)^3*(a+b*ar
csinh(c*x))^2-256/17325*b*d^3*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^5+128/
17325*b*d^3*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-32/5775*b*d^3*x^4*
(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] time = 1.06, antiderivative size = 465, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {5744, 5661, 5758, 5717, 8, 30, 266, 43, 5732, 12, 1153}

$$\frac{1}{11}d^3x^5\left(c^2x^2+1\right)^3\left(a+b\sinh^{-1}(cx)\right)^2+\frac{2}{33}d^3x^5\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)^2+\frac{8}{231}d^3x^5\left(c^2x^2+1\right)\left(a+b\sinh^{-1}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (100976*b^2*d^3*x)/(4002075*c^4) - (50488*b^2*d^3*x^3)/(12006225*c^2) + (12
622*b^2*d^3*x^5)/6670125 + (9410*b^2*c^2*d^3*x^7)/1120581 + (182*b^2*c^4*d^
3*x^9)/29403 + (2*b^2*c^6*d^3*x^11)/1331 - (256*b*d^3*Sqrt[1 + c^2*x^2]*(a
+ b*ArcSinh[c*x]))/(17325*c^5) + (128*b*d^3*x^2*Sqrt[1 + c^2*x^2]*(a + b*Ar
cSinh[c*x]))/(17325*c^3) - (32*b*d^3*x^4*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c
*x]))/(5775*c) - (16*b*d^3*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(693*c
^5) + (4*b*d^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(1155*c^5) - (2*b*
d^3*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(1617*c^5) + (8*b*d^3*(1 + c^
2*x^2)^(9/2)*(a + b*ArcSinh[c*x]))/(297*c^5) - (2*b*d^3*(1 + c^2*x^2)^(11/2
)*(a + b*ArcSinh[c*x]))/(121*c^5) + (16*d^3*x^5*(a + b*ArcSinh[c*x])^2)/115
5 + (8*d^3*x^5*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/231 + (2*d^3*x^5*(1 +
c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/33 + (d^3*x^5*(1 + c^2*x^2)^3*(a + b*Arc
Sinh[c*x])^2)/11
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
```

```
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int x^4 \left(d + c^2dx^2\right)^3 \left(a + b\sinh^{-1}(cx)\right)^2 dx = \frac{1}{11}d^3x^5\left(1 + c^2x^2\right)^3\left(a + b\sinh^{-1}(cx)\right)^2 + \frac{1}{11}(6d)\int x^4\left(d + c^2dx^2\right)^2 (a$$

$$= -\frac{2bd^3\left(1 + c^2x^2\right)^{7/2}\left(a + b\sinh^{-1}(cx)\right)}{77c^5} + \frac{4bd^3\left(1 + c^2x^2\right)^{9/2}\left(a + b\sin\right.}{99c^5}$$

$$= -\frac{4bd^3\left(1 + c^2x^2\right)^{5/2}\left(a + b\sinh^{-1}(cx)\right)}{165c^5} + \frac{2bd^3\left(1 + c^2x^2\right)^{7/2}\left(a + b\sin\right.}{231c^5}$$

$$= -\frac{16bd^3\left(1 + c^2x^2\right)^{3/2}\left(a + b\sinh^{-1}(cx)\right)}{693c^5} + \frac{4bd^3\left(1 + c^2x^2\right)^{5/2}\left(a + b\,\mathrm{si}\right.}{1155c^5}$$

$$= \frac{16b^2d^3x}{7623c^4} - \frac{8b^2d^3x^3}{22869c^2} + \frac{2b^2d^3x^5}{12705} + \frac{226b^2c^2d^3x^7}{53361} + \frac{46b^2c^4d^3x^9}{9801} + \frac{2b^2c}{1}$$

$$= \frac{8368b^2d^3x}{800415c^4} - \frac{4184b^2d^3x^3}{2401245c^2} + \frac{12622b^2d^3x^5}{6670125} + \frac{9410b^2c^2d^3x^7}{1120581} + \frac{182b^2c}{294}$$

$$= \frac{8368b^2d^3x}{800415c^4} - \frac{50488b^2d^3x^3}{12006225c^2} + \frac{12622b^2d^3x^5}{6670125} + \frac{9410b^2c^2d^3x^7}{1120581} + \frac{182b^2}{29}$$

$$= \frac{100976b^2d^3x}{4002075c^4} - \frac{50488b^2d^3x^3}{12006225c^2} + \frac{12622b^2d^3x^5}{6670125} + \frac{9410b^2c^2d^3x^7}{1120581} + \frac{182}{}$$

Mathematica [A] time = 0.48, size = 299, normalized size = 0.64

$$\frac{d^3\left(12006225a^2c^5x^5\left(105c^6x^6 + 385c^4x^4 + 495c^2x^2 + 231\right) - 6930ab\sqrt{c^2x^2+1}\left(33075c^{10}x^{10} + 111475c^8x^8 + 117\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^3*(12006225*a^2*c^5*x^5*(231 + 495*c^2*x^2 + 385*c^4*x^4 + 105*c^6*x^6)
- 6930*a*b*Sqrt[1 + c^2*x^2]*(50488 - 25244*c^2*x^2 + 18933*c^4*x^4 + 11762
5*c^6*x^6 + 111475*c^8*x^8 + 33075*c^10*x^10) + 2*b^2*c*x*(174940920 - 2915
6820*c^2*x^2 + 13120569*c^4*x^4 + 58224375*c^6*x^6 + 42917875*c^8*x^8 + 104
18625*c^10*x^10) - 6930*b*(-3465*a*c^5*x^5*(231 + 495*c^2*x^2 + 385*c^4*x^4
 + 105*c^6*x^6) + b*Sqrt[1 + c^2*x^2]*(50488 - 25244*c^2*x^2 + 18933*c^4*x^
4 + 117625*c^6*x^6 + 111475*c^8*x^8 + 33075*c^10*x^10))*ArcSinh[c*x] + 1200
6225*b^2*c^5*x^5*(231 + 495*c^2*x^2 + 385*c^4*x^4 + 105*c^6*x^6)*ArcSinh[c*
x]^2))/(13867189875*c^5)
```

fricas [A] time = 0.60, size = 444, normalized size = 0.95

$$\frac{10418625\left(121\,a^2 + 2\,b^2\right)c^{11}d^3x^{11} + 471625\left(9801\,a^2 + 182\,b^2\right)c^9d^3x^9 + 12375\left(480249\,a^2 + 9410\,b^2\right)c^7d^3x^7 + 20}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/13867189875*(10418625*(121*a^2 + 2*b^2)*c^11*d^3*x^11 + 471625*(9801*a^2
+ 182*b^2)*c^9*d^3*x^9 + 12375*(480249*a^2 + 9410*b^2)*c^7*d^3*x^7 + 2079*(
1334025*a^2 + 12622*b^2)*c^5*d^3*x^5 - 58313640*b^2*c^3*d^3*x^3 + 349881840
*b^2*c*d^3*x + 12006225*(105*b^2*c^11*d^3*x^11 + 385*b^2*c^9*d^3*x^9 + 495*
b^2*c^7*d^3*x^7 + 231*b^2*c^5*d^3*x^5)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 693
0*(363825*a*b*c^11*d^3*x^11 + 1334025*a*b*c^9*d^3*x^9 + 1715175*a*b*c^7*d^3
*x^7 + 800415*a*b*c^5*d^3*x^5 - (33075*b^2*c^10*d^3*x^10 + 111475*b^2*c^8*d
^3*x^8 + 117625*b^2*c^6*d^3*x^6 + 18933*b^2*c^4*d^3*x^4 - 25244*b^2*c^2*d^3
*x^2 + 50488*b^2*d^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 693
0*(33075*a*b*c^10*d^3*x^10 + 111475*a*b*c^8*d^3*x^8 + 117625*a*b*c^6*d^3*x^
6 + 18933*a*b*c^4*d^3*x^4 - 25244*a*b*c^2*d^3*x^2 + 50488*a*b*d^3)*sqrt(c^2
*x^2 + 1))/c^5
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.10, size = 520, normalized size = 1.12

$$d^3a^2\left(\frac{1}{11}c^{11}x^{11}+\frac{1}{3}c^9x^9+\frac{3}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+d^3b^2\left(\frac{\operatorname{arcsinh}(cx)^2c^3x^3\left(c^2x^2+1\right)^4}{11}-\frac{\operatorname{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^4}{33}+\frac{16\operatorname{arcsinh}(cx)^2cx}{1155}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c^5*(d^3*a^2*(1/11*c^11*x^11+1/3*c^9*x^9+3/7*c^7*x^7+1/5*c^5*x^5)+d^3*b^2
*(1/11*arcsinh(c*x)^2*c^3*x^3*(c^2*x^2+1)^4-1/33*arcsinh(c*x)^2*c*x*(c^2*x^
2+1)^4+16/1155*arcsinh(c*x)^2*c*x+1/231*arcsinh(c*x)^2*c*x*(c^2*x^2+1)^3+2/
385*arcsinh(c*x)^2*c*x*(c^2*x^2+1)^2+8/1155*arcsinh(c*x)^2*c*x*(c^2*x^2+1)-
5487704/4622396625*c*x*(c^2*x^2+1)^2-16/3465*arcsinh(c*x)*(c^2*x^2+1)^(3/2)
-606416/13867189875*c*x*(c^2*x^2+1)-428/323433*c*x*(c^2*x^2+1)^4-2/1617*arc
sinh(c*x)*(c^2*x^2+1)^(7/2)-4/1925*arcsinh(c*x)*(c^2*x^2+1)^(5/2)-148174/11
0937519*c*x*(c^2*x^2+1)^3+34/3267*arcsinh(c*x)*(c^2*x^2+1)^(9/2)+2/1331*c*x
*(c^2*x^2+1)^5-32/1155*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-2/121*arcsinh(c*x)*c^
2*x^2*(c^2*x^2+1)^(9/2)+382986368/13867189875*c*x)+2*d^3*a*b*(1/11*arcsinh(
c*x)*c^11*x^11+1/3*arcsinh(c*x)*c^9*x^9+3/7*arcsinh(c*x)*c^7*x^7+1/5*arcsin
h(c*x)*c^5*x^5-1/121*c^10*x^10*(c^2*x^2+1)^(1/2)-91/3267*c^8*x^8*(c^2*x^2+1
)^(1/2)-4705/160083*c^6*x^6*(c^2*x^2+1)^(1/2)-6311/1334025*c^4*x^4*(c^2*x^2
+1)^(1/2)+25244/4002075*c^2*x^2*(c^2*x^2+1)^(1/2)-50488/4002075*(c^2*x^2+1)
^(1/2)))
```

maxima [B] time = 0.47, size = 1109, normalized size = 2.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/11*b^2*c^6*d^3*x^11*arcsinh(c*x)^2 + 1/11*a^2*c^6*d^3*x^11 + 1/3*b^2*c^4*
d^3*x^9*arcsinh(c*x)^2 + 1/3*a^2*c^4*d^3*x^9 + 3/7*b^2*c^2*d^3*x^7*arcsinh(
```

```
c*x)^2 + 3/7*a^2*c^2*d^3*x^7 + 2/7623*(693*x^11*arcsinh(c*x) - (63*sqrt(c^2
*x^2 + 1)*x^10/c^2 - 70*sqrt(c^2*x^2 + 1)*x^8/c^4 + 80*sqrt(c^2*x^2 + 1)*x^
6/c^6 - 96*sqrt(c^2*x^2 + 1)*x^4/c^8 + 128*sqrt(c^2*x^2 + 1)*x^2/c^10 - 256
*sqrt(c^2*x^2 + 1)/c^12)*c)*a*b*c^6*d^3 - 2/26413695*(3465*(63*sqrt(c^2*x^2
 + 1)*x^10/c^2 - 70*sqrt(c^2*x^2 + 1)*x^8/c^4 + 80*sqrt(c^2*x^2 + 1)*x^6/c^
6 - 96*sqrt(c^2*x^2 + 1)*x^4/c^8 + 128*sqrt(c^2*x^2 + 1)*x^2/c^10 - 256*sqr
t(c^2*x^2 + 1)/c^12)*c*arcsinh(c*x) - (19845*c^10*x^11 - 26950*c^8*x^9 + 39
600*c^6*x^7 - 66528*c^4*x^5 + 147840*c^2*x^3 - 887040*x)/c^10)*b^2*c^6*d^3
+ 1/5*b^2*d^3*x^5*arcsinh(c*x)^2 + 2/945*(315*x^9*arcsinh(c*x) - (35*sqrt(c
^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(c^2*x^2 + 1)*x
^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(c^2*x^2 + 1)/c^10)*c)*a*b*
c^4*d^3 - 2/297675*(315*(35*sqrt(c^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c^2*x^2 + 1
)*x^6/c^4 + 48*sqrt(c^2*x^2 + 1)*x^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x^2/c^8 + 1
28*sqrt(c^2*x^2 + 1)/c^10)*c*arcsinh(c*x) - (1225*c^8*x^9 - 1800*c^6*x^7 +
3024*c^4*x^5 - 6720*c^2*x^3 + 40320*x)/c^8)*b^2*c^4*d^3 + 1/5*a^2*d^3*x^5 +
 6/245*(35*x^7*arcsinh(c*x) - (5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2
 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*
a*b*c^2*d^3 - 2/8575*(105*(5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1
)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c*arcsi
nh(c*x) - (75*c^6*x^7 - 126*c^4*x^5 + 280*c^2*x^3 - 1680*x)/c^6)*b^2*c^2*d^
3 + 2/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x
^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*d^3 - 2/1125*(15*(3*sqrt(
c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^
6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c^4)*b^2*d^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,(a+b\,\mathrm{asinh}\,(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3,x)
```

```
[Out] int(x^4*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3, x)
```

sympy [A] time = 71.19, size = 702, normalized size = 1.51

$$\begin{cases} \frac{a^2c^6d^3x^{11}}{11} + \frac{a^2c^4d^3x^9}{3} + \frac{3a^2c^2d^3x^7}{7} + \frac{a^2d^3x^5}{5} + \frac{2abc^6d^3x^{11}\,\mathrm{asinh}\,(cx)}{11} - \frac{2abc^5d^3x^{10}\sqrt{c^2x^2+1}}{121} + \frac{2abc^4d^3x^9\,\mathrm{asinh}\,(cx)}{3} - \frac{182abc^3d^3x^8\sqrt{c^2x^2} \\ \frac{a^2d^3x^5}{5} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**6*d**3*x**11/11 + a**2*c**4*d**3*x**9/3 + 3*a**2*c**2*d*
*3*x**7/7 + a**2*d**3*x**5/5 + 2*a*b*c**6*d**3*x**11*asinh(c*x)/11 - 2*a*b*
c**5*d**3*x**10*sqrt(c**2*x**2 + 1)/121 + 2*a*b*c**4*d**3*x**9*asinh(c*x)/3
 - 182*a*b*c**3*d**3*x**8*sqrt(c**2*x**2 + 1)/3267 + 6*a*b*c**2*d**3*x**7*a
sinh(c*x)/7 - 9410*a*b*c*d**3*x**6*sqrt(c**2*x**2 + 1)/160083 + 2*a*b*d**3*
x**5*asinh(c*x)/5 - 12622*a*b*d**3*x**4*sqrt(c**2*x**2 + 1)/(1334025*c) + 5
0488*a*b*d**3*x**2*sqrt(c**2*x**2 + 1)/(4002075*c**3) - 100976*a*b*d**3*sqr
t(c**2*x**2 + 1)/(4002075*c**5) + b**2*c**6*d**3*x**11*asinh(c*x)**2/11 + 2
*b**2*c**6*d**3*x**11/1331 - 2*b**2*c**5*d**3*x**10*sqrt(c**2*x**2 + 1)*asi
nh(c*x)/121 + b**2*c**4*d**3*x**9*asinh(c*x)**2/3 + 182*b**2*c**4*d**3*x**9
/29403 - 182*b**2*c**3*d**3*x**8*sqrt(c**2*x**2 + 1)*asinh(c*x)/3267 + 3*b*
*2*c**2*d**3*x**7*asinh(c*x)**2/7 + 9410*b**2*c**2*d**3*x**7/1120581 - 9410
*b**2*c*d**3*x**6*sqrt(c**2*x**2 + 1)*asinh(c*x)/160083 + b**2*d**3*x**5*as
inh(c*x)**2/5 + 12622*b**2*d**3*x**5/6670125 - 12622*b**2*d**3*x**4*sqrt(c*
```

```
*2*x**2 + 1)*asinh(c*x)/(1334025*c) - 50488*b**2*d**3*x**3/(12006225*c**2)
+ 50488*b**2*d**3*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(4002075*c**3) + 1009
76*b**2*d**3*x/(4002075*c**4) - 100976*b**2*d**3*sqrt(c**2*x**2 + 1)*asinh(
c*x)/(4002075*c**5), Ne(c, 0)), (a**2*d**3*x**5/5, True))
```

3.217 $\int x^3 \left(d+c^2dx^2\right)^3 \left(a+b\sinh^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=376

$$-\frac{79d^3\left(a+b\sinh^{-1}(cx)\right)^2}{5120c^4}-\frac{1}{50}bcd^3x^5\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{32}bcd^3x^5\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)-$$

```
[Out] -79/5120*b^2*d^3*x^2/c^2+79/15360*b^2*d^3*x^4+401/28800*b^2*c^2*d^3*x^6+57/
6400*b^2*c^4*d^3*x^8+1/500*b^2*c^6*d^3*x^10-1/32*b*c*d^3*x^5*(c^2*x^2+1)^(3
/2)*(a+b*arcsinh(c*x))-1/50*b*c*d^3*x^5*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x)
)-79/5120*d^3*(a+b*arcsinh(c*x))^2/c^4+1/40*d^3*x^4*(a+b*arcsinh(c*x))^2+1/
20*d^3*x^4*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2+3/40*d^3*x^4*(c^2*x^2+1)^2*(a+b
*arcsinh(c*x))^2+1/10*d^3*x^4*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))^2+79/2560*b*
d^3*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-79/3840*b*d^3*x^3*(a+b*arcsi
nh(c*x))*(c^2*x^2+1)^(1/2)/c-31/960*b*c*d^3*x^5*(a+b*arcsinh(c*x))*(c^2*x^2
+1)^(1/2)
```

Rubi [A] time = 1.66, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 40, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {5744, 5661, 5758, 5675, 30, 5742, 14, 266, 43}

$$-\frac{1}{50}bcd^3x^5\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{32}bcd^3x^5\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)-\frac{31}{960}bcd^3x^5\sqrt{c^2x^2+1}\left(a+\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-79*b^2*d^3*x^2)/(5120*c^2) + (79*b^2*d^3*x^4)/15360 + (401*b^2*c^2*d^3*x^
6)/28800 + (57*b^2*c^4*d^3*x^8)/6400 + (b^2*c^6*d^3*x^10)/500 + (79*b*d^3*x
*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2560*c^3) - (79*b*d^3*x^3*Sqrt[1
+ c^2*x^2]*(a + b*ArcSinh[c*x]))/(3840*c) - (31*b*c*d^3*x^5*Sqrt[1 + c^2*x^
2]*(a + b*ArcSinh[c*x]))/960 - (b*c*d^3*x^5*(1 + c^2*x^2)^(3/2)*(a + b*ArcS
inh[c*x]))/32 - (b*c*d^3*x^5*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/50 -
 (79*d^3*(a + b*ArcSinh[c*x])^2)/(5120*c^4) + (d^3*x^4*(a + b*ArcSinh[c*x])
^2)/40 + (d^3*x^4*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/20 + (3*d^3*x^4*(1
+ c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/40 + (d^3*x^4*(1 + c^2*x^2)^3*(a + b*A
rcSinh[c*x])^2)/10
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int x^3\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2\,dx = \frac{1}{10}d^3x^4\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{5}(3d)\int x^3\left(d+c^2dx^2\right)^2\left(a\right.$$

$$= -\frac{1}{50}bcd^3x^5\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{40}d^3x^4\left(1+c^2x^2\right)^2\left(a\right.$$

$$= -\frac{1}{32}bcd^3x^5\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{50}bcd^3x^5\left(1+c^2x^2\right)^{5/2}$$

$$= -\frac{31}{960}bcd^3x^5\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{32}bcd^3x^5\left(1+c^2x^2\right)^{3/2}\left(\right.$$

$$= \frac{401b^2c^2d^3x^6}{28800}+\frac{57b^2c^4d^3x^8}{6400}+\frac{1}{500}b^2c^6d^3x^{10}-\frac{79bd^3x^3\sqrt{1+c^2x^2}\left(a\right.}{3840c}$$

$$= \frac{79b^2d^3x^4}{15360}+\frac{401b^2c^2d^3x^6}{28800}+\frac{57b^2c^4d^3x^8}{6400}+\frac{1}{500}b^2c^6d^3x^{10}+\frac{79bd^3x\sqrt{}}{}$$

$$= -\frac{79b^2d^3x^2}{5120c^2}+\frac{79b^2d^3x^4}{15360}+\frac{401b^2c^2d^3x^6}{28800}+\frac{57b^2c^4d^3x^8}{6400}+\frac{1}{500}b^2c^6d^3x$$

Mathematica [A] time = 0.46, size = 285, normalized size = 0.76

$$d^3\left(cx\left(28800a^2c^3x^3\left(4c^6x^6+15c^4x^4+20c^2x^2+10\right)-30ab\sqrt{c^2x^2+1}\left(768c^8x^8+2736c^6x^6+3208c^4x^4+790c^2\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^3*(c*x*(28800*a^2*c^3*x^3*(10 + 20*c^2*x^2 + 15*c^4*x^4 + 4*c^6*x^6) - 3
0*a*b*Sqrt[1 + c^2*x^2]*(-1185 + 790*c^2*x^2 + 3208*c^4*x^4 + 2736*c^6*x^6
+ 768*c^8*x^8) + b^2*c*x*(-17775 + 5925*c^2*x^2 + 16040*c^4*x^4 + 10260*c^6
*x^6 + 2304*c^8*x^8)) + 30*b*(-(b*c*x*Sqrt[1 + c^2*x^2]*(-1185 + 790*c^2*x^
2 + 3208*c^4*x^4 + 2736*c^6*x^6 + 768*c^8*x^8)) + 15*a*(-79 + 1280*c^4*x^4
+ 2560*c^6*x^6 + 1920*c^8*x^8 + 512*c^10*x^10))*ArcSinh[c*x] + 225*b^2*(-79
 + 1280*c^4*x^4 + 2560*c^6*x^6 + 1920*c^8*x^8 + 512*c^10*x^10)*ArcSinh[c*x]
^2))/(1152000*c^4)
```

fricas [A] time = 0.65, size = 424, normalized size = 1.13

$$2304\left(50\,a^2+b^2\right)c^{10}d^3x^{10}+540\left(800\,a^2+19\,b^2\right)c^8d^3x^8+40\left(14400\,a^2+401\,b^2\right)c^6d^3x^6+75\left(3840\,a^2+79\,b^2\right)c^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/1152000*(2304*(50*a^2 + b^2)*c^10*d^3*x^10 + 540*(800*a^2 + 19*b^2)*c^8*d
^3*x^8 + 40*(14400*a^2 + 401*b^2)*c^6*d^3*x^6 + 75*(3840*a^2 + 79*b^2)*c^4*
d^3*x^4 - 17775*b^2*c^2*d^3*x^2 + 225*(512*b^2*c^10*d^3*x^10 + 1920*b^2*c^8
*d^3*x^8 + 2560*b^2*c^6*d^3*x^6 + 1280*b^2*c^4*d^3*x^4 - 79*b^2*d^3)*log(c*
x + sqrt(c^2*x^2 + 1))^2 + 30*(7680*a*b*c^10*d^3*x^10 + 28800*a*b*c^8*d^3*x
^8 + 38400*a*b*c^6*d^3*x^6 + 19200*a*b*c^4*d^3*x^4 - 1185*a*b*d^3 - (768*b^
2*c^9*d^3*x^9 + 2736*b^2*c^7*d^3*x^7 + 3208*b^2*c^5*d^3*x^5 + 790*b^2*c^3*d
^3*x^3 - 1185*b^2*c*d^3*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))
- 30*(768*a*b*c^9*d^3*x^9 + 2736*a*b*c^7*d^3*x^7 + 3208*a*b*c^5*d^3*x^5 + 7
90*a*b*c^3*d^3*x^3 - 1185*a*b*c*d^3*x)*sqrt(c^2*x^2 + 1))/c^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.09, size = 415, normalized size = 1.10

$$d^3a^2\left(\frac{1}{10}c^{10}x^{10}+\frac{3}{8}c^8x^8+\frac{1}{2}c^6x^6+\frac{1}{4}c^4x^4\right)+d^3b^2\left(\frac{\operatorname{arcsinh}(cx)^2c^2x^2(c^2x^2+1)^4}{10}-\frac{\operatorname{arcsinh}(cx)^2(c^2x^2+1)^4}{40}-\frac{\operatorname{arcsinh}(cx)cx(c^2x^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c^4*(d^3*a^2*(1/10*c^10*x^10+3/8*c^8*x^8+1/2*c^6*x^6+1/4*c^4*x^4)+d^3*b^2
*(1/10*arcsinh(c*x)^2*c^2*x^2*(c^2*x^2+1)^4-1/40*arcsinh(c*x)^2*(c^2*x^2+1)
^4-1/50*arcsinh(c*x)*c*x*(c^2*x^2+1)^(9/2)+7/800*arcsinh(c*x)*c*x*(c^2*x^2+
1)^(7/2)+49/4800*arcsinh(c*x)*c*x*(c^2*x^2+1)^(5/2)+49/3840*arcsinh(c*x)*c*
x*(c^2*x^2+1)^(3/2)+49/2560*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c*x+49/5120*arcs
inh(c*x)^2+1/500*(c^2*x^2+1)^5-7/6400*(c^2*x^2+1)^4-49/28800*(c^2*x^2+1)^3-
49/15360*(c^2*x^2+1)^2-49/5120*c^2*x^2-49/5120)+2*d^3*a*b*(1/10*arcsinh(c*x
)*c^10*x^10+3/8*arcsinh(c*x)*c^8*x^8+1/2*arcsinh(c*x)*c^6*x^6+1/4*arcsinh(c
*x)*c^4*x^4-1/100*c^9*x^9*(c^2*x^2+1)^(1/2)-57/1600*c^7*x^7*(c^2*x^2+1)^(1/
2)-401/9600*c^5*x^5*(c^2*x^2+1)^(1/2)-79/7680*c^3*x^3*(c^2*x^2+1)^(1/2)+79/
5120*c*x*(c^2*x^2+1)^(1/2)-79/5120*arcsinh(c*x)))
```

maxima [B] time = 0.60, size = 1112, normalized size = 2.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/10*b^2*c^6*d^3*x^10*arcsinh(c*x)^2 + 1/10*a^2*c^6*d^3*x^10 + 3/8*b^2*c^4*
d^3*x^8*arcsinh(c*x)^2 + 3/8*a^2*c^4*d^3*x^8 + 1/2*b^2*c^2*d^3*x^6*arcsinh(
c*x)^2 + 1/2*a^2*c^2*d^3*x^6 + 1/6400*(1280*x^10*arcsinh(c*x) - (128*sqrt(c
^2*x^2 + 1)*x^9/c^2 - 144*sqrt(c^2*x^2 + 1)*x^7/c^4 + 168*sqrt(c^2*x^2 + 1)
*x^5/c^6 - 210*sqrt(c^2*x^2 + 1)*x^3/c^8 + 315*sqrt(c^2*x^2 + 1)*x/c^10 - 3
15*arcsinh(c*x)/c^11)*c)*a*b*c^6*d^3 + 1/64000*((128*x^10/c^2 - 180*x^8/c^4
 + 280*x^6/c^6 - 525*x^4/c^8 + 1575*x^2/c^10 - 1575*log(c*x + sqrt(c^2*x^2
+ 1))^2/c^12)*c^2 - 10*(128*sqrt(c^2*x^2 + 1)*x^9/c^2 - 144*sqrt(c^2*x^2 +
1)*x^7/c^4 + 168*sqrt(c^2*x^2 + 1)*x^5/c^6 - 210*sqrt(c^2*x^2 + 1)*x^3/c^8
+ 315*sqrt(c^2*x^2 + 1)*x/c^10 - 315*arcsinh(c*x)/c^11)*c*arcsinh(c*x))*b^2
*c^6*d^3 + 1/4*b^2*d^3*x^4*arcsinh(c*x)^2 + 1/512*(384*x^8*arcsinh(c*x) - (
48*sqrt(c^2*x^2 + 1)*x^7/c^2 - 56*sqrt(c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(c^2*x
^2 + 1)*x^3/c^6 - 105*sqrt(c^2*x^2 + 1)*x/c^8 + 105*arcsinh(c*x)/c^9)*c)*a*
b*c^4*d^3 + 1/3072*((36*x^8/c^2 - 56*x^6/c^4 + 105*x^4/c^6 - 315*x^2/c^8 +
315*log(c*x + sqrt(c^2*x^2 + 1))^2/c^10)*c^2 - 6*(48*sqrt(c^2*x^2 + 1)*x^7/
c^2 - 56*sqrt(c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(c^2*x^2 + 1)*x^3/c^6 - 105*sqr
t(c^2*x^2 + 1)*x/c^8 + 105*arcsinh(c*x)/c^9)*c*arcsinh(c*x))*b^2*c^4*d^3 +
1/4*a^2*d^3*x^4 + 1/48*(48*x^6*arcsinh(c*x) - (8*sqrt(c^2*x^2 + 1)*x^5/c^2
- 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 + 1)*x/c^6 - 15*arcsinh(c*
```

```
x)/c^7)*c)*a*b*c^2*d^3 + 1/288*((8*x^6/c^2 - 15*x^4/c^4 + 45*x^2/c^6 - 45*l
og(c*x + sqrt(c^2*x^2 + 1))^2/c^8)*c^2 - 6*(8*sqrt(c^2*x^2 + 1)*x^5/c^2 - 1
0*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 + 1)*x/c^6 - 15*arcsinh(c*x)/
c^7)*c*arcsinh(c*x))*b^2*c^2*d^3 + 1/16*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x
^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*a*b*d^
3 + 1/32*((x^4/c^2 - 3*x^2/c^4 + 3*log(c*x + sqrt(c^2*x^2 + 1))^2/c^6)*c^2
- 2*(2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*
x)/c^5)*c*arcsinh(c*x))*b^2*d^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 (a + b \operatorname{asinh}(c x))^2 \left(d c^2 x^2 + d\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3,x)
```

```
[Out] int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3, x)
```

sympy [A] time = 55.92, size = 654, normalized size = 1.74

$$\begin{cases} \frac{a^2c^6d^3x^{10}}{10} + \frac{3a^2c^4d^3x^8}{8} + \frac{a^2c^2d^3x^6}{2} + \frac{a^2d^3x^4}{4} + \frac{abc^6d^3x^{10}\operatorname{asinh}(cx)}{5} - \frac{abc^5d^3x^9\sqrt{c^2x^2+1}}{50} + \frac{3abc^4d^3x^8\operatorname{asinh}(cx)}{4} - \frac{57abc^3d^3x^7\sqrt{c^2x^2+1}}{800} \\ \frac{a^2d^3x^4}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**6*d**3*x**10/10 + 3*a**2*c**4*d**3*x**8/8 + a**2*c**2*d*
*3*x**6/2 + a**2*d**3*x**4/4 + a*b*c**6*d**3*x**10*asinh(c*x)/5 - a*b*c**5*
d**3*x**9*sqrt(c**2*x**2 + 1)/50 + 3*a*b*c**4*d**3*x**8*asinh(c*x)/4 - 57*a
*b*c**3*d**3*x**7*sqrt(c**2*x**2 + 1)/800 + a*b*c**2*d**3*x**6*asinh(c*x) -
 401*a*b*c*d**3*x**5*sqrt(c**2*x**2 + 1)/4800 + a*b*d**3*x**4*asinh(c*x)/2
- 79*a*b*d**3*x**3*sqrt(c**2*x**2 + 1)/(3840*c) + 79*a*b*d**3*x*sqrt(c**2*x
**2 + 1)/(2560*c**3) - 79*a*b*d**3*asinh(c*x)/(2560*c**4) + b**2*c**6*d**3*
x**10*asinh(c*x)**2/10 + b**2*c**6*d**3*x**10/500 - b**2*c**5*d**3*x**9*sqr
t(c**2*x**2 + 1)*asinh(c*x)/50 + 3*b**2*c**4*d**3*x**8*asinh(c*x)**2/8 + 57
*b**2*c**4*d**3*x**8/6400 - 57*b**2*c**3*d**3*x**7*sqrt(c**2*x**2 + 1)*asin
h(c*x)/800 + b**2*c**2*d**3*x**6*asinh(c*x)**2/2 + 401*b**2*c**2*d**3*x**6/
28800 - 401*b**2*c*d**3*x**5*sqrt(c**2*x**2 + 1)*asinh(c*x)/4800 + b**2*d**
3*x**4*asinh(c*x)**2/4 + 79*b**2*d**3*x**4/15360 - 79*b**2*d**3*x**3*sqrt(c
**2*x**2 + 1)*asinh(c*x)/(3840*c) - 79*b**2*d**3*x**2/(5120*c**2) + 79*b**2
*d**3*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/(2560*c**3) - 79*b**2*d**3*asinh(c*x
)**2/(5120*c**4), Ne(c, 0)), (a**2*d**3*x**4/4, True))
```

3.218 $\int x^2 \left(d+c^2 d x^2\right)^3 \left(a+b \sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=382

$$-\frac{32 b d^3 x^2 \sqrt{c^2x^2+1} \left(a+b\sinh^{-1}(cx)\right)}{945c}+\frac{1}{9} d^3 x^3 \left(c^2x^2+1\right)^3 \left(a+b\sinh^{-1}(cx)\right)^2+\frac{2}{21} d^3 x^3 \left(c^2x^2+1\right)^2 \left(a+b\sinh\right.$$

```
[Out] -10516/99225*b^2*d^3*x/c^2+5258/297675*b^2*d^3*x^3+4198/165375*b^2*c^2*d^3*
x^5+374/27783*b^2*c^4*d^3*x^7+2/729*b^2*c^6*d^3*x^9+16/315*b*d^3*(c^2*x^2+1
)^(3/2)*(a+b*arcsinh(c*x))/c^3+4/525*b*d^3*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c
*x))/c^3+2/441*b*d^3*(c^2*x^2+1)^(7/2)*(a+b*arcsinh(c*x))/c^3-2/81*b*d^3*(c
^2*x^2+1)^(9/2)*(a+b*arcsinh(c*x))/c^3+16/315*d^3*x^3*(a+b*arcsinh(c*x))^2+
8/105*d^3*x^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2+2/21*d^3*x^3*(c^2*x^2+1)^2*(
a+b*arcsinh(c*x))^2+1/9*d^3*x^3*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))^2+64/945*b
*d^3*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-32/945*b*d^3*x^2*(a+b*arcsinh
(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] time = 0.86, antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {5744, 5661, 5758, 5717, 8, 30, 266, 43, 5732, 12, 373}

$$\frac{1}{9} d^3 x^3 \left(c^2x^2+1\right)^3 \left(a+b\sinh^{-1}(cx)\right)^2+\frac{2}{21} d^3 x^3 \left(c^2x^2+1\right)^2 \left(a+b\sinh^{-1}(cx)\right)^2+\frac{8}{105} d^3 x^3 \left(c^2x^2+1\right) \left(a+b\sinh\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-10516*b^2*d^3*x)/(99225*c^2) + (5258*b^2*d^3*x^3)/297675 + (4198*b^2*c^2*
d^3*x^5)/165375 + (374*b^2*c^4*d^3*x^7)/27783 + (2*b^2*c^6*d^3*x^9)/729 + (
64*b*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(945*c^3) - (32*b*d^3*x^2*
Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(945*c) + (16*b*d^3*(1 + c^2*x^2)^(
3/2)*(a + b*ArcSinh[c*x]))/(315*c^3) + (4*b*d^3*(1 + c^2*x^2)^(5/2)*(a + b*
ArcSinh[c*x]))/(525*c^3) + (2*b*d^3*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]
))/(441*c^3) - (2*b*d^3*(1 + c^2*x^2)^(9/2)*(a + b*ArcSinh[c*x]))/(81*c^3)
+ (16*d^3*x^3*(a + b*ArcSinh[c*x])^2)/315 + (8*d^3*x^3*(1 + c^2*x^2)*(a + b
*ArcSinh[c*x])^2)/105 + (2*d^3*x^3*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/
21 + (d^3*x^3*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x])^2)/9
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int x^2\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{1}{9}d^3x^3\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{3}(2d)\int x^2\left(d+c^2dx^2\right)^2$$
$$=\frac{2bd^3\left(1+c^2x^2\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{63c^3}-\frac{2bd^3\left(1+c^2x^2\right)^{9/2}\left(a+b\,\mathrm{s}\right.}{81c^3}$$
$$=\frac{4bd^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{105c^3}+\frac{2bd^3\left(1+c^2x^2\right)^{7/2}\left(a+b\,\mathrm{s}\right.}{441c^3}$$
$$=\frac{16bd^3\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{315c^3}+\frac{4bd^3\left(1+c^2x^2\right)^{5/2}\left(a+b\right.}{525c^3}$$
$$=-\frac{4b^2d^3x}{567c^2}+\frac{2b^2d^3x^3}{1701}+\frac{2}{189}b^2c^2d^3x^5+\frac{38b^2c^4d^3x^7}{3969}+\frac{2}{729}b^2c^6d^3x^9$$
$$=-\frac{3796b^2d^3x}{99225c^2}+\frac{5258b^2d^3x^3}{297675}+\frac{4198b^2c^2d^3x^5}{165375}+\frac{374b^2c^4d^3x^7}{27783}+\frac{2}{72}$$
$$=-\frac{10516b^2d^3x}{99225c^2}+\frac{5258b^2d^3x^3}{297675}+\frac{4198b^2c^2d^3x^5}{165375}+\frac{374b^2c^4d^3x^7}{27783}+\frac{}{7}$$

Mathematica [A] time = 0.51, size = 275, normalized size = 0.72

$$\frac{d^3\left(99225a^2c^3x^3\left(35c^6x^6+135c^4x^4+189c^2x^2+105\right)-630ab\sqrt{c^2x^2+1}\left(1225c^8x^8+4675c^6x^6+6297c^4x^4+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^3*(99225*a^2*c^3*x^3*(105 + 189*c^2*x^2 + 135*c^4*x^4 + 35*c^6*x^6) - 63
0*a*b*Sqrt[1 + c^2*x^2]*(-5258 + 2629*c^2*x^2 + 6297*c^4*x^4 + 4675*c^6*x^6
 + 1225*c^8*x^8) + b^2*(-3312540*c*x + 552090*c^3*x^3 + 793422*c^5*x^5 + 42
0750*c^7*x^7 + 85750*c^9*x^9) - 630*b*(-315*a*c^3*x^3*(105 + 189*c^2*x^2 +
135*c^4*x^4 + 35*c^6*x^6) + b*Sqrt[1 + c^2*x^2]*(-5258 + 2629*c^2*x^2 + 629
7*c^4*x^4 + 4675*c^6*x^6 + 1225*c^8*x^8))*ArcSinh[c*x] + 99225*b^2*c^3*x^3*
(105 + 189*c^2*x^2 + 135*c^4*x^4 + 35*c^6*x^6)*ArcSinh[c*x]^2))/(31255875*c
^3)
```

fricas [A] time = 0.58, size = 403, normalized size = 1.05

$$\frac{42875\left(81\,a^2+2\,b^2\right)c^9d^3x^9+1125\left(11907\,a^2+374\,b^2\right)c^7d^3x^7+189\left(99225\,a^2+4198\,b^2\right)c^5d^3x^5+105\left(99225\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/31255875*(42875*(81*a^2 + 2*b^2)*c^9*d^3*x^9 + 1125*(11907*a^2 + 374*b^2)
*c^7*d^3*x^7 + 189*(99225*a^2 + 4198*b^2)*c^5*d^3*x^5 + 105*(99225*a^2 + 52
58*b^2)*c^3*d^3*x^3 - 3312540*b^2*c*d^3*x + 99225*(35*b^2*c^9*d^3*x^9 + 135
*b^2*c^7*d^3*x^7 + 189*b^2*c^5*d^3*x^5 + 105*b^2*c^3*d^3*x^3)*log(c*x + sqr
t(c^2*x^2 + 1))^2 + 630*(11025*a*b*c^9*d^3*x^9 + 42525*a*b*c^7*d^3*x^7 + 59
535*a*b*c^5*d^3*x^5 + 33075*a*b*c^3*d^3*x^3 - (1225*b^2*c^8*d^3*x^8 + 4675*
b^2*c^6*d^3*x^6 + 6297*b^2*c^4*d^3*x^4 + 2629*b^2*c^2*d^3*x^2 - 5258*b^2*d^
3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 630*(1225*a*b*c^8*d^3*
x^8 + 4675*a*b*c^6*d^3*x^6 + 6297*a*b*c^4*d^3*x^4 + 2629*a*b*c^2*d^3*x^2 -
5258*a*b*d^3)*sqrt(c^2*x^2 + 1))/c^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.05, size = 438, normalized size = 1.15

$$d^3a^2\left(\frac{1}{9}c^9x^9+\frac{3}{7}c^7x^7+\frac{3}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+d^3b^2\left(\frac{\operatorname{arcsinh}(cx)^2cx(c^2x^2+1)^4}{9}-\frac{16\operatorname{arcsinh}(cx)^2cx}{315}-\frac{\operatorname{arcsinh}(cx)^2cx(c^2x^2+1)^3}{63}-\frac{2\operatorname{arcsi}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c^3*(d^3*a^2*(1/9*c^9*x^9+3/7*c^7*x^7+3/5*c^5*x^5+1/3*c^3*x^3)+d^3*b^2*(1
/9*arcsinh(c*x)^2*c*x*(c^2*x^2+1)^4-16/315*arcsinh(c*x)^2*c*x-1/63*arcsinh(
c*x)^2*c*x*(c^2*x^2+1)^3-2/105*arcsinh(c*x)^2*c*x*(c^2*x^2+1)^2-8/315*arcsi
nh(c*x)^2*c*x*(c^2*x^2+1)-2/81*arcsinh(c*x)*(c^2*x^2+1)^(9/2)-3406208/31255
875*c*x+2/729*c*x*(c^2*x^2+1)^4+622/250047*c*x*(c^2*x^2+1)^3+15224/10418625
*c*x*(c^2*x^2+1)^2-115504/31255875*c*x*(c^2*x^2+1)+32/315*arcsinh(c*x)*(c^2
*x^2+1)^(1/2)+2/441*arcsinh(c*x)*(c^2*x^2+1)^(7/2)+4/525*arcsinh(c*x)*(c^2*
x^2+1)^(5/2)+16/945*arcsinh(c*x)*(c^2*x^2+1)^(3/2))+2*d^3*a*b*(1/9*arcsinh(
c*x)*c^9*x^9+3/7*arcsinh(c*x)*c^7*x^7+3/5*arcsinh(c*x)*c^5*x^5+1/3*arcsinh(
c*x)*c^3*x^3-1/81*c^8*x^8*(c^2*x^2+1)^(1/2)-187/3969*c^6*x^6*(c^2*x^2+1)^(1
/2)-2099/33075*c^4*x^4*(c^2*x^2+1)^(1/2)-2629/99225*c^2*x^2*(c^2*x^2+1)^(1/
2)+5258/99225*(c^2*x^2+1)^(1/2)))
```

maxima [B] time = 0.45, size = 922, normalized size = 2.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^2*c^6*d^3*x^9*arcsinh(c*x)^2 + 1/9*a^2*c^6*d^3*x^9 + 3/7*b^2*c^4*d^3*
x^7*arcsinh(c*x)^2 + 3/7*a^2*c^4*d^3*x^7 + 3/5*b^2*c^2*d^3*x^5*arcsinh(c*x)
^2 + 2/2835*(315*x^9*arcsinh(c*x) - (35*sqrt(c^2*x^2 + 1)*x^8/c^2 - 40*sqrt
(c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(c^2*x^2 + 1)*x^4/c^6 - 64*sqrt(c^2*x^2 + 1)
*x^2/c^8 + 128*sqrt(c^2*x^2 + 1)/c^10)*c)*a*b*c^6*d^3 - 2/893025*(315*(35*s
qrt(c^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(c^2*x^2 +
 1)*x^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(c^2*x^2 + 1)/c^10)*c*
arcsinh(c*x) - (1225*c^8*x^9 - 1800*c^6*x^7 + 3024*c^4*x^5 - 6720*c^2*x^3 +
 40320*x)/c^8)*b^2*c^6*d^3 + 3/5*a^2*c^2*d^3*x^5 + 6/245*(35*x^7*arcsinh(c*
x) - (5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^
2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*a*b*c^4*d^3 - 2/8575*(105
*(5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^
2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c*arcsinh(c*x) - (75*c^6*x^7 - 1
26*c^4*x^5 + 280*c^2*x^3 - 1680*x)/c^6)*b^2*c^4*d^3 + 1/3*b^2*d^3*x^3*arcsi
nh(c*x)^2 + 2/25*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sq
rt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*c^2*d^3 - 2/375*(
15*(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*
x^2 + 1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c^4)*b^2*c^
```

```
2*d^3 + 1/3*a^2*d^3*x^3 + 2/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^
2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*d^3 - 2/27*(3*c*(sqrt(c^2*x^2 + 1)*x^
2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3 - 6*x)/c^2)*b^2*d^
3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,(a+b\,\mathrm{asinh}\,(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3,x)
```

```
[Out] int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3, x)
```

sympy [A] time = 32.87, size = 626, normalized size = 1.64

$$\begin{cases} \frac{a^2c^6d^3x^9}{9} + \frac{3a^2c^4d^3x^7}{7} + \frac{3a^2c^2d^3x^5}{5} + \frac{a^2d^3x^3}{3} + \frac{2abc^6d^3x^9\,\mathrm{asinh}\,(cx)}{9} - \frac{2abc^5d^3x^8\sqrt{c^2x^2+1}}{81} + \frac{6abc^4d^3x^7\,\mathrm{asinh}\,(cx)}{7} - \frac{374abc^3d^3x^6\sqrt{c^2 \\ \frac{a^2d^3x^3}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**6*d**3*x**9/9 + 3*a**2*c**4*d**3*x**7/7 + 3*a**2*c**2*d*
*3*x**5/5 + a**2*d**3*x**3/3 + 2*a*b*c**6*d**3*x**9*asinh(c*x)/9 - 2*a*b*c*
*5*d**3*x**8*sqrt(c**2*x**2 + 1)/81 + 6*a*b*c**4*d**3*x**7*asinh(c*x)/7 - 3
74*a*b*c**3*d**3*x**6*sqrt(c**2*x**2 + 1)/3969 + 6*a*b*c**2*d**3*x**5*asinh
(c*x)/5 - 4198*a*b*c*d**3*x**4*sqrt(c**2*x**2 + 1)/33075 + 2*a*b*d**3*x**3*
asinh(c*x)/3 - 5258*a*b*d**3*x**2*sqrt(c**2*x**2 + 1)/(99225*c) + 10516*a*b
*d**3*sqrt(c**2*x**2 + 1)/(99225*c**3) + b**2*c**6*d**3*x**9*asinh(c*x)**2/
9 + 2*b**2*c**6*d**3*x**9/729 - 2*b**2*c**5*d**3*x**8*sqrt(c**2*x**2 + 1)*a
sinh(c*x)/81 + 3*b**2*c**4*d**3*x**7*asinh(c*x)**2/7 + 374*b**2*c**4*d**3*x
**7/27783 - 374*b**2*c**3*d**3*x**6*sqrt(c**2*x**2 + 1)*asinh(c*x)/3969 + 3
*b**2*c**2*d**3*x**5*asinh(c*x)**2/5 + 4198*b**2*c**2*d**3*x**5/165375 - 41
98*b**2*c*d**3*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/33075 + b**2*d**3*x**3*a
sinh(c*x)**2/3 + 5258*b**2*d**3*x**3/297675 - 5258*b**2*d**3*x**2*sqrt(c**2
*x**2 + 1)*asinh(c*x)/(99225*c) - 10516*b**2*d**3*x/(99225*c**2) + 10516*b*
*2*d**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/(99225*c**3), Ne(c, 0)), (a**2*d**3*
x**3/3, True))
```

3.219 $\int x\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=261

$$-\frac{bd^3x\left(c^2x^2+1\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{32c}-\frac{7bd^3x\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{192c}-\frac{35bd^3x\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}\right.}{768c}$$

```
[Out] 175/3072*b^2*d^3*x^2+35/3072*b^2*c^2*d^3*x^4+7/1152*b^2*d^3*(c^2*x^2+1)^3/c
^2+1/256*b^2*d^3*(c^2*x^2+1)^4/c^2-35/768*b*d^3*x*(c^2*x^2+1)^(3/2)*(a+b*ar
csinh(c*x))/c-7/192*b*d^3*x*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))/c-1/32*b*d
^3*x*(c^2*x^2+1)^(7/2)*(a+b*arcsinh(c*x))/c-35/1024*d^3*(a+b*arcsinh(c*x))^
2/c^2+1/8*d^3*(c^2*x^2+1)^4*(a+b*arcsinh(c*x))^2/c^2-35/512*b*d^3*x*(a+b*ar
csinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] time = 0.26, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {5717, 5684, 5682, 5675, 30, 14, 261}

$$-\frac{bd^3x\left(c^2x^2+1\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{32c}-\frac{7bd^3x\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{192c}-\frac{35bd^3x\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}\right.}{768c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (175*b^2*d^3*x^2)/3072 + (35*b^2*c^2*d^3*x^4)/3072 + (7*b^2*d^3*(1 + c^2*x^
2)^3)/(1152*c^2) + (b^2*d^3*(1 + c^2*x^2)^4)/(256*c^2) - (35*b*d^3*x*Sqrt[1
 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(512*c) - (35*b*d^3*x*(1 + c^2*x^2)^(3/2)
*(a + b*ArcSinh[c*x]))/(768*c) - (7*b*d^3*x*(1 + c^2*x^2)^(5/2)*(a + b*ArcS
inh[c*x]))/(192*c) - (b*d^3*x*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(32
*c) - (35*d^3*(a + b*ArcSinh[c*x])^2)/(1024*c^2) + (d^3*(1 + c^2*x^2)^4*(a
+ b*ArcSinh[c*x])^2)/(8*c^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
```

```
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\int x\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{d^3\left(1+c^2x^2\right)^4\left(a+b\sinh^{-1}(cx)\right)^2}{8c^2}-\frac{\left(bd^3\right)\int\left(1+c^2x^2\right)^{7/2}\left(a+b\sin\ldots\right.}{4c}$$

$$=-\frac{bd^3x\left(1+c^2x^2\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{32c}+\frac{d^3\left(1+c^2x^2\right)^4\left(a+b\sinh\ldots\right.}{8c^2}$$

$$=\frac{b^2d^3\left(1+c^2x^2\right)^4}{256c^2}-\frac{7bd^3x\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{192c}-\frac{bd^3x\left(1\ldots\right.}{\ldots}$$

$$=\frac{7b^2d^3\left(1+c^2x^2\right)^3}{1152c^2}+\frac{b^2d^3\left(1+c^2x^2\right)^4}{256c^2}-\frac{35bd^3x\left(1+c^2x^2\right)^{3/2}\left(a+b\ldots\right.}{768c}$$

$$=\frac{7b^2d^3\left(1+c^2x^2\right)^3}{1152c^2}+\frac{b^2d^3\left(1+c^2x^2\right)^4}{256c^2}-\frac{35bd^3x\sqrt{1+c^2x^2}\left(a+b\sin\ldots\right.}{512c}$$

$$=\frac{175b^2d^3x^2}{3072}+\frac{35b^2c^2d^3x^4}{3072}+\frac{7b^2d^3\left(1+c^2x^2\right)^3}{1152c^2}+\frac{b^2d^3\left(1+c^2x^2\right)^4}{256c^2}-\ldots$$

Mathematica [A] time = 0.66, size = 256, normalized size = 0.98

$$\frac{d^3\left(cx\left(1152a^2cx\left(c^6x^6+4c^4x^4+6c^2x^2+4\right)-6ab\sqrt{c^2x^2+1}\left(48c^6x^6+200c^4x^4+326c^2x^2+279\right)+b^2cx\left(36c\ldots\right.\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^3*(c*x*(1152*a^2*c*x*(4 + 6*c^2*x^2 + 4*c^4*x^4 + c^6*x^6) + b^2*c*x*(83
7 + 489*c^2*x^2 + 200*c^4*x^4 + 36*c^6*x^6) - 6*a*b*Sqrt[1 + c^2*x^2]*(279
+ 326*c^2*x^2 + 200*c^4*x^4 + 48*c^6*x^6)) + 6*b*(-(b*c*x*Sqrt[1 + c^2*x^2]
*(279 + 326*c^2*x^2 + 200*c^4*x^4 + 48*c^6*x^6)) + 3*a*(93 + 512*c^2*x^2 +
```

```
768*c^4*x^4 + 512*c^6*x^6 + 128*c^8*x^8))*ArcSinh[c*x] + 9*b^2*(93 + 512*c^
2*x^2 + 768*c^4*x^4 + 512*c^6*x^6 + 128*c^8*x^8)*ArcSinh[c*x]^2))/(9216*c^2
)
```

fricas [A] time = 0.54, size = 383, normalized size = 1.47

$$\frac{36\left(32a^2+b^2\right)c^8d^3x^8+8\left(576a^2+25b^2\right)c^6d^3x^6+3\left(2304a^2+163b^2\right)c^4d^3x^4+9\left(512a^2+93b^2\right)c^2d^3x^2+9\left(12\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/9216*(36*(32*a^2 + b^2)*c^8*d^3*x^8 + 8*(576*a^2 + 25*b^2)*c^6*d^3*x^6 +
3*(2304*a^2 + 163*b^2)*c^4*d^3*x^4 + 9*(512*a^2 + 93*b^2)*c^2*d^3*x^2 + 9*(
128*b^2*c^8*d^3*x^8 + 512*b^2*c^6*d^3*x^6 + 768*b^2*c^4*d^3*x^4 + 512*b^2*c
^2*d^3*x^2 + 93*b^2*d^3)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 6*(384*a*b*c^8*d^
3*x^8 + 1536*a*b*c^6*d^3*x^6 + 2304*a*b*c^4*d^3*x^4 + 1536*a*b*c^2*d^3*x^2
+ 279*a*b*d^3 - (48*b^2*c^7*d^3*x^7 + 200*b^2*c^5*d^3*x^5 + 326*b^2*c^3*d^3
*x^3 + 279*b^2*c*d^3*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 6
*(48*a*b*c^7*d^3*x^7 + 200*a*b*c^5*d^3*x^5 + 326*a*b*c^3*d^3*x^3 + 279*a*b*
c*d^3*x)*sqrt(c^2*x^2 + 1))/c^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.05, size = 339, normalized size = 1.30

$$\frac{d^3a^2\left(\frac{1}{8}c^8x^8+\frac{1}{2}c^6x^6+\frac{3}{4}c^4x^4+\frac{1}{2}c^2x^2\right)+d^3b^2\left(\frac{\operatorname{arcsinh}(cx)^2\left(c^2x^2+1\right)^4}{8}-\frac{\operatorname{arcsinh}(cx)cx\left(c^2x^2+1\right)^{\frac{7}{2}}}{32}-\frac{7\operatorname{arcsinh}(cx)cx\left(c^2x^2+1\right)^{\frac{5}{2}}}{192}-\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c^2*(d^3*a^2*(1/8*c^8*x^8+1/2*c^6*x^6+3/4*c^4*x^4+1/2*c^2*x^2)+d^3*b^2*(1
/8*arcsinh(c*x)^2*(c^2*x^2+1)^4-1/32*arcsinh(c*x)*c*x*(c^2*x^2+1)^(7/2)-7/1
92*arcsinh(c*x)*c*x*(c^2*x^2+1)^(5/2)-35/768*arcsinh(c*x)*c*x*(c^2*x^2+1)^(
3/2)-35/512*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c*x-35/1024*arcsinh(c*x)^2+1/256
*(c^2*x^2+1)^4+7/1152*(c^2*x^2+1)^3+35/3072*(c^2*x^2+1)^2+35/1024*c^2*x^2+3
5/1024)+2*d^3*a*b*(1/8*arcsinh(c*x)*c^8*x^8+1/2*arcsinh(c*x)*c^6*x^6+3/4*ar
csinh(c*x)*c^4*x^4+1/2*arcsinh(c*x)*c^2*x^2-1/64*c^7*x^7*(c^2*x^2+1)^(1/2)-
25/384*c^5*x^5*(c^2*x^2+1)^(1/2)-163/1536*c^3*x^3*(c^2*x^2+1)^(1/2)-93/1024
*c*x*(c^2*x^2+1)^(1/2)+93/1024*arcsinh(c*x)))
```

maxima [B] time = 0.45, size = 925, normalized size = 3.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*c^6*d^3*x^8*arcsinh(c*x)^2 + 1/8*a^2*c^6*d^3*x^8 + 1/2*b^2*c^4*d^3*
x^6*arcsinh(c*x)^2 + 1/2*a^2*c^4*d^3*x^6 + 3/4*b^2*c^2*d^3*x^4*arcsinh(c*x)
^2 + 1/1536*(384*x^8*arcsinh(c*x) - (48*sqrt(c^2*x^2 + 1)*x^7/c^2 - 56*sqrt
(c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(c^2*x^2 + 1)*x^3/c^6 - 105*sqrt(c^2*x^2 + 1
)*x/c^8 + 105*arcsinh(c*x)/c^9)*c)*a*b*c^6*d^3 + 1/9216*((36*x^8/c^2 - 56*x
^6/c^4 + 105*x^4/c^6 - 315*x^2/c^8 + 315*log(c*x + sqrt(c^2*x^2 + 1))^2/c^1
0)*c^2 - 6*(48*sqrt(c^2*x^2 + 1)*x^7/c^2 - 56*sqrt(c^2*x^2 + 1)*x^5/c^4 + 7
0*sqrt(c^2*x^2 + 1)*x^3/c^6 - 105*sqrt(c^2*x^2 + 1)*x/c^8 + 105*arcsinh(c*x
)/c^9)*c*arcsinh(c*x))*b^2*c^6*d^3 + 3/4*a^2*c^2*d^3*x^4 + 1/48*(48*x^6*arc
sinh(c*x) - (8*sqrt(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 1
5*sqrt(c^2*x^2 + 1)*x/c^6 - 15*arcsinh(c*x)/c^7)*c)*a*b*c^4*d^3 + 1/288*((8
*x^6/c^2 - 15*x^4/c^4 + 45*x^2/c^6 - 45*log(c*x + sqrt(c^2*x^2 + 1))^2/c^8)
*c^2 - 6*(8*sqrt(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*s
qrt(c^2*x^2 + 1)*x/c^6 - 15*arcsinh(c*x)/c^7)*c*arcsinh(c*x))*b^2*c^4*d^3 +
 1/2*b^2*d^3*x^2*arcsinh(c*x)^2 + 3/16*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^
2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*a*b*c^2
*d^3 + 3/32*((x^4/c^2 - 3*x^2/c^4 + 3*log(c*x + sqrt(c^2*x^2 + 1))^2/c^6)*c
^2 - 2*(2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh
(c*x)/c^5)*c*arcsinh(c*x))*b^2*c^2*d^3 + 1/2*a^2*d^3*x^2 + 1/2*(2*x^2*arcsi
nh(c*x) - c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsinh(c*x)/c^3))*a*b*d^3 + 1/4*(c^
2*(x^2/c^2 - log(c*x + sqrt(c^2*x^2 + 1))^2/c^4) - 2*c*(sqrt(c^2*x^2 + 1)*x
/c^2 - arcsinh(c*x)/c^3)*arcsinh(c*x))*b^2*d^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,(a+b\,\mathrm{asinh}\,(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3,x)
```

```
[Out] int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3, x)
```

sympy [A] time = 24.28, size = 573, normalized size = 2.20

$$\begin{cases} \frac{a^2c^6d^3x^8}{8} + \frac{a^2c^4d^3x^6}{2} + \frac{3a^2c^2d^3x^4}{4} + \frac{a^2d^3x^2}{2} + \frac{abc^6d^3x^8\,\mathrm{asinh}\,(cx)}{4} - \frac{abc^5d^3x^7\sqrt{c^2x^2+1}}{32} + abc^4d^3x^6\,\mathrm{asinh}\,(cx) - \frac{25abc^3d^3x^5\sqrt{c^2}}{192} \\ \frac{a^2d^3x^2}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**6*d**3*x**8/8 + a**2*c**4*d**3*x**6/2 + 3*a**2*c**2*d**3
*x**4/4 + a**2*d**3*x**2/2 + a*b*c**6*d**3*x**8*asinh(c*x)/4 - a*b*c**5*d**
3*x**7*sqrt(c**2*x**2 + 1)/32 + a*b*c**4*d**3*x**6*asinh(c*x) - 25*a*b*c**3
*d**3*x**5*sqrt(c**2*x**2 + 1)/192 + 3*a*b*c**2*d**3*x**4*asinh(c*x)/2 - 16
3*a*b*c*d**3*x**3*sqrt(c**2*x**2 + 1)/768 + a*b*d**3*x**2*asinh(c*x) - 93*a
*b*d**3*x*sqrt(c**2*x**2 + 1)/(512*c) + 93*a*b*d**3*asinh(c*x)/(512*c**2) +
 b**2*c**6*d**3*x**8*asinh(c*x)**2/8 + b**2*c**6*d**3*x**8/256 - b**2*c**5*
d**3*x**7*sqrt(c**2*x**2 + 1)*asinh(c*x)/32 + b**2*c**4*d**3*x**6*asinh(c*x
)**2/2 + 25*b**2*c**4*d**3*x**6/1152 - 25*b**2*c**3*d**3*x**5*sqrt(c**2*x**
2 + 1)*asinh(c*x)/192 + 3*b**2*c**2*d**3*x**4*asinh(c*x)**2/4 + 163*b**2*c*
*2*d**3*x**4/3072 - 163*b**2*c*d**3*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/768
 + b**2*d**3*x**2*asinh(c*x)**2/2 + 93*b**2*d**3*x**2/1024 - 93*b**2*d**3*x
*sqrt(c**2*x**2 + 1)*asinh(c*x)/(512*c) + 93*b**2*d**3*asinh(c*x)**2/(1024*
c**2), Ne(c, 0)), (a**2*d**3*x**2/2, True))
```

3.220 $\int \left(d+c^2 d x^2\right)^3 \left(a+b \sinh^{-1}(c x)\right)^2 \, dx$

Optimal. Leaf size=291

$$\frac{1}{7}d^3x\left(c^2x^2+1\right)^3\left(a+b\sinh^{-1}(cx)\right)^2+\frac{6}{35}d^3x\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)^2+\frac{8}{35}d^3x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right.$$

```
[Out] 4322/3675*b^2*d^3*x+1514/11025*b^2*c^2*d^3*x^3+234/6125*b^2*c^4*d^3*x^5+2/3
43*b^2*c^6*d^3*x^7-16/105*b*d^3*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/c-12/1
75*b*d^3*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))/c-2/49*b*d^3*(c^2*x^2+1)^(7/2
)*(a+b*arcsinh(c*x))/c+16/35*d^3*x*(a+b*arcsinh(c*x))^2+8/35*d^3*x*(c^2*x^2
+1)*(a+b*arcsinh(c*x))^2+6/35*d^3*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2+1/7*
d^3*x*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))^2-32/35*b*d^3*(a+b*arcsinh(c*x))*(c^
2*x^2+1)^(1/2)/c
```

Rubi [A] time = 0.40, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5684, 5653, 5717, 8, 194}

$$\frac{1}{7}d^3x\left(c^2x^2+1\right)^3\left(a+b\sinh^{-1}(cx)\right)^2+\frac{6}{35}d^3x\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)^2+\frac{8}{35}d^3x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right.$$

Antiderivative was successfully verified.

```
[In] Int[(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (4322*b^2*d^3*x)/3675 + (1514*b^2*c^2*d^3*x^3)/11025 + (234*b^2*c^4*d^3*x^5
)/6125 + (2*b^2*c^6*d^3*x^7)/343 - (32*b*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSi
nh[c*x]))/(35*c) - (16*b*d^3*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(105
*c) - (12*b*d^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(175*c) - (2*b*d^
3*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(49*c) + (16*d^3*x*(a + b*ArcSi
nh[c*x])^2)/35 + (8*d^3*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/35 + (6*d^3
*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/35 + (d^3*x*(1 + c^2*x^2)^3*(a +
 b*ArcSinh[c*x])^2)/7
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(d+c^2dx^2\right)^3 \left(a+b\sinh^{-1}(cx)\right)^2 dx = \frac{1}{7}d^3x\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{7}(6d)\int \left(d+c^2dx^2\right)^2\left(a+b\right.$$

$$= -\frac{2bd^3\left(1+c^2x^2\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{49c}+\frac{6}{35}d^3x\left(1+c^2x^2\right)^2\left(a+b\sin\right.$$

$$= -\frac{12bd^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{175c}-\frac{2bd^3\left(1+c^2x^2\right)^{7/2}\left(a+b\right.}{49c}$$

$$= \frac{2}{49}b^2d^3x+\frac{2}{49}b^2c^2d^3x^3+\frac{6}{245}b^2c^4d^3x^5+\frac{2}{343}b^2c^6d^3x^7-\frac{16bd^3\left(1+\right.}{}$$

$$= \frac{962b^2d^3x}{3675}+\frac{1514b^2c^2d^3x^3}{11025}+\frac{234b^2c^4d^3x^5}{6125}+\frac{2}{343}b^2c^6d^3x^7-\frac{32bd^3\sqrt{}}{}$$

$$= \frac{4322b^2d^3x}{3675}+\frac{1514b^2c^2d^3x^3}{11025}+\frac{234b^2c^4d^3x^5}{6125}+\frac{2}{343}b^2c^6d^3x^7-\frac{32bd^3}{}$$

Mathematica [A] time = 0.56, size = 239, normalized size = 0.82

$$\frac{d^3\left(11025a^2cx\left(5c^6x^6+21c^4x^4+35c^2x^2+35\right)-210ab\sqrt{c^2x^2+1}\left(75c^6x^6+351c^4x^4+757c^2x^2+2161\right)-210\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^3*(11025*a^2*c*x*(35 + 35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6) - 210*a*b*Sq
rt[1 + c^2*x^2]*(2161 + 757*c^2*x^2 + 351*c^4*x^4 + 75*c^6*x^6) + 2*b^2*c*x
*(226905 + 26495*c^2*x^2 + 7371*c^4*x^4 + 1125*c^6*x^6) - 210*b*(-105*a*c*x
*(35 + 35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6) + b*Sqrt[1 + c^2*x^2]*(2161 + 7
57*c^2*x^2 + 351*c^4*x^4 + 75*c^6*x^6))*ArcSinh[c*x] + 11025*b^2*c*x*(35 +
35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6)*ArcSinh[c*x]^2))/(385875*c)
```

fricas [A] time = 0.73, size = 354, normalized size = 1.22

$$\frac{1125\left(49a^2+2b^2\right)c^7d^3x^7+189\left(1225a^2+78b^2\right)c^5d^3x^5+35\left(11025a^2+1514b^2\right)c^3d^3x^3+105\left(3675a^2+43\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/385875*(1125*(49*a^2 + 2*b^2)*c^7*d^3*x^7 + 189*(1225*a^2 + 78*b^2)*c^5*d
^3*x^5 + 35*(11025*a^2 + 1514*b^2)*c^3*d^3*x^3 + 105*(3675*a^2 + 4322*b^2)*
c*d^3*x + 11025*(5*b^2*c^7*d^3*x^7 + 21*b^2*c^5*d^3*x^5 + 35*b^2*c^3*d^3*x^
3 + 35*b^2*c*d^3*x)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 210*(525*a*b*c^7*d^3*x
```

```
^7 + 2205*a*b*c^5*d^3*x^5 + 3675*a*b*c^3*d^3*x^3 + 3675*a*b*c*d^3*x - (75*b
^2*c^6*d^3*x^6 + 351*b^2*c^4*d^3*x^4 + 757*b^2*c^2*d^3*x^2 + 2161*b^2*d^3)*
sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 210*(75*a*b*c^6*d^3*x^6 +
 351*a*b*c^4*d^3*x^4 + 757*a*b*c^2*d^3*x^2 + 2161*a*b*d^3)*sqrt(c^2*x^2 + 1
))/c
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.06, size = 354, normalized size = 1.22

$$d^3a^2\left(\frac{1}{7}c^7x^7+\frac{3}{5}c^5x^5+c^3x^3+cx\right)+d^3b^2\left(\frac{16\,\mathrm{arcsinh}(cx)^2cx}{35}+\frac{\mathrm{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^3}{7}+\frac{6\,\mathrm{arcsinh}(cx)^2cx\left(c^2x^2+1\right)^2}{35}+\frac{8\,\mathrm{arcsinh}(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c*(d^3*a^2*(1/7*c^7*x^7+3/5*c^5*x^5+c^3*x^3+c*x)+d^3*b^2*(16/35*arcsinh(c
*x)^2*c*x+1/7*arcsinh(c*x)^2*c*x*(c^2*x^2+1)^3+6/35*arcsinh(c*x)^2*c*x*(c^2
*x^2+1)^2+8/35*arcsinh(c*x)^2*c*x*(c^2*x^2+1)-32/35*arcsinh(c*x)*(c^2*x^2+1
)^(1/2)+413312/385875*c*x-2/49*arcsinh(c*x)*(c^2*x^2+1)^(7/2)+2/343*c*x*(c^
2*x^2+1)^3+888/42875*c*x*(c^2*x^2+1)^2+30256/385875*c*x*(c^2*x^2+1)-12/175*
arcsinh(c*x)*(c^2*x^2+1)^(5/2)-16/105*arcsinh(c*x)*(c^2*x^2+1)^(3/2))+2*d^3
*a*b*(1/7*arcsinh(c*x)*c^7*x^7+3/5*arcsinh(c*x)*c^5*x^5+arcsinh(c*x)*c^3*x^
3+arcsinh(c*x)*c*x-1/49*c^6*x^6*(c^2*x^2+1)^(1/2)-117/1225*c^4*x^4*(c^2*x^2
+1)^(1/2)-757/3675*c^2*x^2*(c^2*x^2+1)^(1/2)-2161/3675*(c^2*x^2+1)^(1/2)))
```

maxima [B] time = 0.47, size = 712, normalized size = 2.45

$$\frac{1}{7}\,b^2c^6d^3x^7\,\mathrm{arsinh}\,(cx)^2+\frac{1}{7}\,a^2c^6d^3x^7+\frac{3}{5}\,b^2c^4d^3x^5\,\mathrm{arsinh}\,(cx)^2+\frac{3}{5}\,a^2c^4d^3x^5+\frac{2}{245}\left(35\,x^7\,\mathrm{arsinh}\,(cx)-\left(\frac{5\,\sqrt{c^2x^2+}}{c^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*c^6*d^3*x^7*arcsinh(c*x)^2 + 1/7*a^2*c^6*d^3*x^7 + 3/5*b^2*c^4*d^3*
x^5*arcsinh(c*x)^2 + 3/5*a^2*c^4*d^3*x^5 + 2/245*(35*x^7*arcsinh(c*x) - (5*
sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 +
1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*a*b*c^6*d^3 - 2/25725*(105*(5*sqr
t(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*
x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c*arcsinh(c*x) - (75*c^6*x^7 - 126*c^4*
x^5 + 280*c^2*x^3 - 1680*x)/c^6)*b^2*c^6*d^3 + b^2*c^2*d^3*x^3*arcsinh(c*x)
^2 + 2/25*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*
x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*c^4*d^3 - 2/375*(15*(3*s
qrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1
)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c^4)*b^2*c^4*d^3 +
 a^2*c^2*d^3*x^3 + 2/3*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 -
 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*c^2*d^3 - 2/9*(3*c*(sqrt(c^2*x^2 + 1)*x^2/c^
```

```
2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3 - 6*x)/c^2)*b^2*c^2*d^
3 + b^2*d^3*x*arcsinh(c*x)^2 + 2*b^2*d^3*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x
)/c) + a^2*d^3*x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*d^3/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,\mathrm{asinh}\,(c\,x))^2\,\left(d\,c^2\,x^2 + d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3,x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3, x)
```

sympy [A] time = 13.36, size = 524, normalized size = 1.80

$$\begin{cases} \frac{a^2c^6d^3x^7}{7} + \frac{3a^2c^4d^3x^5}{5} + a^2c^2d^3x^3 + a^2d^3x + \frac{2abc^6d^3x^7\,\mathrm{asinh}\,(cx)}{7} - \frac{2abc^5d^3x^6\sqrt{c^2x^2+1}}{49} + \frac{6abc^4d^3x^5\,\mathrm{asinh}\,(cx)}{5} - \frac{234abc^3d^3x^4\sqrt{c^2 \\ a^2d^3x \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**6*d**3*x**7/7 + 3*a**2*c**4*d**3*x**5/5 + a**2*c**2*d**3
*x**3 + a**2*d**3*x + 2*a*b*c**6*d**3*x**7*asinh(c*x)/7 - 2*a*b*c**5*d**3*x
**6*sqrt(c**2*x**2 + 1)/49 + 6*a*b*c**4*d**3*x**5*asinh(c*x)/5 - 234*a*b*c*
*3*d**3*x**4*sqrt(c**2*x**2 + 1)/1225 + 2*a*b*c**2*d**3*x**3*asinh(c*x) - 1
514*a*b*c*d**3*x**2*sqrt(c**2*x**2 + 1)/3675 + 2*a*b*d**3*x*asinh(c*x) - 43
22*a*b*d**3*sqrt(c**2*x**2 + 1)/(3675*c) + b**2*c**6*d**3*x**7*asinh(c*x)**
2/7 + 2*b**2*c**6*d**3*x**7/343 - 2*b**2*c**5*d**3*x**6*sqrt(c**2*x**2 + 1)
*asinh(c*x)/49 + 3*b**2*c**4*d**3*x**5*asinh(c*x)**2/5 + 234*b**2*c**4*d**3
*x**5/6125 - 234*b**2*c**3*d**3*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/1225 +
b**2*c**2*d**3*x**3*asinh(c*x)**2 + 1514*b**2*c**2*d**3*x**3/11025 - 1514*b
**2*c*d**3*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/3675 + b**2*d**3*x*asinh(c*x
)**2 + 4322*b**2*d**3*x/3675 - 4322*b**2*d**3*sqrt(c**2*x**2 + 1)*asinh(c*x
)/(3675*c), Ne(c, 0)), (a**2*d**3*x, True))
```

3.221 $\int \frac{(d+c^2dx^2)^3(a+b\sinh^{-1}(cx))^2}{x}\,dx$

Optimal. Leaf size=337

$$-\frac{1}{18}bcd^3x\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)-\frac{7}{36}bcd^3x\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)-\frac{19}{24}bcd^3x\sqrt{c^2x^2+1}\left(a+b\,\mathrm{si}\right.$$

```
[Out] 71/144*b^2*c^2*d^3*x^2+7/144*b^2*c^4*d^3*x^4+1/108*b^2*d^3*(c^2*x^2+1)^3-7/
36*b*c*d^3*x*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))-1/18*b*c*d^3*x*(c^2*x^2+1
)^(5/2)*(a+b*arcsinh(c*x))-19/48*d^3*(a+b*arcsinh(c*x))^2+1/2*d^3*(c^2*x^2+
1)*(a+b*arcsinh(c*x))^2+1/4*d^3*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2+1/6*d^3*
(c^2*x^2+1)^3*(a+b*arcsinh(c*x))^2+1/3*d^3*(a+b*arcsinh(c*x))^3/b+d^3*(a+b*
arcsinh(c*x))^2*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^2)-b*d^3*(a+b*arcsinh(c*x))*
polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)-1/2*b^2*d^3*polylog(3,1/(c*x+(c^2*x^
2+1)^(1/2))^2)-19/24*b*c*d^3*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.68, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 13, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5744, 5659, 3716, 2190, 2531, 2282, 6589, 5682, 5675, 30, 5684, 14, 261}

$$bd^3\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{2}b^2d^3\text{PolyLog}\left(3,e^{2\sinh^{-1}(cx)}\right)-\frac{1}{18}bcd^3x\left(c^2x^2+1\right)^{5/2}\left(a+b\,\mathrm{sinh}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] (71*b^2*c^2*d^3*x^2)/144 + (7*b^2*c^4*d^3*x^4)/144 + (b^2*d^3*(1 + c^2*x^2)
^3)/108 - (19*b*c*d^3*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/24 - (7*b*c
*d^3*x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/36 - (b*c*d^3*x*(1 + c^2*x
^2)^(5/2)*(a + b*ArcSinh[c*x]))/18 - (19*d^3*(a + b*ArcSinh[c*x])^2)/48 + (
d^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/2 + (d^3*(1 + c^2*x^2)^2*(a + b*A
rcSinh[c*x])^2)/4 + (d^3*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x])^2)/6 - (d^3*(
a + b*ArcSinh[c*x])^3)/(3*b) + d^3*(a + b*ArcSinh[c*x])^2*Log[1 - E^(2*ArcS
inh[c*x])] + b*d^3*(a + b*ArcSinh[c*x])*PolyLog[2, E^(2*ArcSinh[c*x])] - (b
^2*d^3*PolyLog[3, E^(2*ArcSinh[c*x])])/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2}{x}\,dx = \frac{1}{6}d^3\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2 + d\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(c\right.}{x}$$

$$= -\frac{1}{18}bcd^3x\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right) + \frac{1}{4}d^3\left(1+c^2x^2\right)^2\left(a+b\sinh\right.$$

$$= \frac{1}{108}b^2d^3\left(1+c^2x^2\right)^3 - \frac{7}{36}bcd^3x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right) - \frac{1}{18}bc$$

$$= \frac{1}{108}b^2d^3\left(1+c^2x^2\right)^3 - \frac{19}{24}bcd^3x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{7}{36}bcd^3$$

$$= \frac{71}{144}b^2c^2d^3x^2 + \frac{7}{144}b^2c^4d^3x^4 + \frac{1}{108}b^2d^3\left(1+c^2x^2\right)^3 - \frac{19}{24}bcd^3x\sqrt{1+c^2x}$$

$$= \frac{71}{144}b^2c^2d^3x^2 + \frac{7}{144}b^2c^4d^3x^4 + \frac{1}{108}b^2d^3\left(1+c^2x^2\right)^3 - \frac{19}{24}bcd^3x\sqrt{1+c^2x}$$

$$= \frac{71}{144}b^2c^2d^3x^2 + \frac{7}{144}b^2c^4d^3x^4 + \frac{1}{108}b^2d^3\left(1+c^2x^2\right)^3 - \frac{19}{24}bcd^3x\sqrt{1+c^2x}$$

$$= \frac{71}{144}b^2c^2d^3x^2 + \frac{7}{144}b^2c^4d^3x^4 + \frac{1}{108}b^2d^3\left(1+c^2x^2\right)^3 - \frac{19}{24}bcd^3x\sqrt{1+c^2x}$$

$$= \frac{71}{144}b^2c^2d^3x^2 + \frac{7}{144}b^2c^4d^3x^4 + \frac{1}{108}b^2d^3\left(1+c^2x^2\right)^3 - \frac{19}{24}bcd^3x\sqrt{1+c^2x}$$

Mathematica [A] time = 0.69, size = 416, normalized size = 1.23

$$\frac{d^3\left(576a^2c^6x^6 + 2592a^2c^4x^4 + 5184a^2c^2x^2 + 3456a^2\log(cx) + 1152abc^6x^6\sinh^{-1}(cx) + 5184abc^4x^4\sinh^{-1}(cx) - \right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] (d^3*(5184*a^2*c^2*x^2 + 2592*a^2*c^4*x^4 + 576*a^2*c^6*x^6 - 3600*a*b*c*x*
Sqrt[1 + c^2*x^2] - 1056*a*b*c^3*x^3*Sqrt[1 + c^2*x^2] - 192*a*b*c^5*x^5*Sq
rt[1 + c^2*x^2] + 3600*a*b*ArcSinh[c*x] + 10368*a*b*c^2*x^2*ArcSinh[c*x] +
5184*a*b*c^4*x^4*ArcSinh[c*x] + 1152*a*b*c^6*x^6*ArcSinh[c*x] + 3456*a*b*Ar
```

```
cSinh[c*x]^2 - 1152*b^2*ArcSinh[c*x]^3 + 783*b^2*Cosh[2*ArcSinh[c*x]] + 156
6*b^2*ArcSinh[c*x]^2*Cosh[2*ArcSinh[c*x]] + 27*b^2*Cosh[4*ArcSinh[c*x]] + 2
16*b^2*ArcSinh[c*x]^2*Cosh[4*ArcSinh[c*x]] + b^2*Cosh[6*ArcSinh[c*x]] + 18*
b^2*ArcSinh[c*x]^2*Cosh[6*ArcSinh[c*x]] + 6912*a*b*ArcSinh[c*x]*Log[1 - E^(
-2*ArcSinh[c*x])] + 3456*b^2*ArcSinh[c*x]^2*Log[1 - E^(2*ArcSinh[c*x])] + 3
456*a^2*Log[c*x] - 3456*a*b*PolyLog[2, E^(-2*ArcSinh[c*x])] + 3456*b^2*ArcS
inh[c*x]*PolyLog[2, E^(2*ArcSinh[c*x])] - 1728*b^2*PolyLog[3, E^(2*ArcSinh[
c*x])] - 1566*b^2*ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]] - 108*b^2*ArcSinh[c*x]*
Sinh[4*ArcSinh[c*x]] - 6*b^2*ArcSinh[c*x]*Sinh[6*ArcSinh[c*x]]))/3456
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{a^2c^6d^3x^6 + 3\,a^2c^4d^3x^4 + 3\,a^2c^2d^3x^2 + a^2d^3 + \left(b^2c^6d^3x^6 + 3\,b^2c^4d^3x^4 + 3\,b^2c^2d^3x^2 + b^2d^3\right)\mathrm{arsinh}\,(cx}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^6*d^3*x^6 + 3*a^2*c^4*d^3*x^4 + 3*a^2*c^2*d^3*x^2 + a^2*d^3
 + (b^2*c^6*d^3*x^6 + 3*b^2*c^4*d^3*x^4 + 3*b^2*c^2*d^3*x^2 + b^2*d^3)*arcs
inh(c*x)^2 + 2*(a*b*c^6*d^3*x^6 + 3*a*b*c^4*d^3*x^4 + 3*a*b*c^2*d^3*x^2 + a
*b*d^3)*arcsinh(c*x))/x, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.30, size = 706, normalized size = 2.09

$$\frac{d^3a^2c^6x^6}{6}+\frac{3d^3a^2c^4x^4}{4}+\frac{3d^3a^2c^2x^2}{2}+\frac{d^3b^2c^6x^6}{108}-d^3ab\,\mathrm{arcsinh}\,(cx)^2+\frac{25d^3ab\,\mathrm{arcsinh}\,(cx)}{24}+2d^3ab\,\mathrm{polylog}\left(2,-cx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x,x)
```

```
[Out] -d^3*a*b*arcsinh(c*x)^2+25/24*d^3*a*b*arcsinh(c*x)+2*d^3*a*b*polylog(2,-c*x
-(c^2*x^2+1)^(1/2))+2*d^3*a*b*polylog(2,c*x+(c^2*x^2+1)^(1/2))+1/6*d^3*a^2*
c^6*x^6+3/4*d^3*a^2*c^4*x^4+3/2*d^3*a^2*c^2*x^2+1/108*d^3*b^2*c^6*x^6+2*d^3
*b^2*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+d^3*b^2*arcsinh(c*x)^2*
ln(1+c*x+(c^2*x^2+1)^(1/2))+d^3*b^2*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/
2))+2*d^3*b^2*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))-25/24*d^3*b^2*a
rcsinh(c*x)*(c^2*x^2+1)^(1/2)*c*x-1/18*d^3*a*b*c^5*x^5*(c^2*x^2+1)^(1/2)-11
/36*d^3*a*b*c^3*x^3*(c^2*x^2+1)^(1/2)-25/24*d^3*a*b*c*x*(c^2*x^2+1)^(1/2)-1
/18*d^3*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^5*x^5+1/3*d^3*a*b*arcsinh(c*x)
*c^6*x^6+3/2*d^3*a*b*arcsinh(c*x)*c^4*x^4+3*d^3*a*b*arcsinh(c*x)*c^2*x^2-11
/36*d^3*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^3*x^3+25/48*b^2*c^2*d^3*x^2+11
/144*b^2*c^4*d^3*x^4+811/3456*d^3*b^2+1/6*d^3*b^2*arcsinh(c*x)^2*c^6*x^6+2*
d^3*a*b*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+d^3*a^2*ln(c*x)+25/48*d^3*
b^2*arcsinh(c*x)^2-1/3*d^3*b^2*arcsinh(c*x)^3-2*d^3*b^2*polylog(3,c*x+(c^2*
x^2+1)^(1/2))-2*d^3*b^2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))+2*d^3*a*b*arcsinh
(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+3/4*d^3*b^2*arcsinh(c*x)^2*c^4*x^4+3/2*d^
3*b^2*arcsinh(c*x)^2*c^2*x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}a^2c^6d^3x^6+\frac{3}{4}a^2c^4d^3x^4+\frac{3}{2}a^2c^2d^3x^2+a^2d^3\log(x)+\int b^2c^6d^3x^5\log\left(cx+\sqrt{c^2x^2+1}\right)^2+2\,abc^6d^3x^5\log\left(cx+\sqrt{c^2x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] 1/6*a^2*c^6*d^3*x^6 + 3/4*a^2*c^4*d^3*x^4 + 3/2*a^2*c^2*d^3*x^2 + a^2*d^3*l
og(x) + integrate(b^2*c^6*d^3*x^5*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*a*b*c^
6*d^3*x^5*log(c*x + sqrt(c^2*x^2 + 1)) + 3*b^2*c^4*d^3*x^3*log(c*x + sqrt(c
^2*x^2 + 1))^2 + 6*a*b*c^4*d^3*x^3*log(c*x + sqrt(c^2*x^2 + 1)) + 3*b^2*c^2
*d^3*x*log(c*x + sqrt(c^2*x^2 + 1))^2 + 6*a*b*c^2*d^3*x*log(c*x + sqrt(c^2*
x^2 + 1)) + b^2*d^3*log(c*x + sqrt(c^2*x^2 + 1))^2/x + 2*a*b*d^3*log(c*x +
sqrt(c^2*x^2 + 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3)/x,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^3\left(\int\frac{a^2}{x}\,dx+\int 3a^2c^2x\,dx+\int 3a^2c^4x^3\,dx+\int a^2c^6x^5\,dx+\int\frac{b^2\,\mathrm{asinh}^2\,(cx)}{x}\,dx+\int\frac{2ab\,\mathrm{asinh}\,(cx)}{x}\,dx+\int 3b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2/x,x)
```

```
[Out] d**3*(Integral(a**2/x, x) + Integral(3*a**2*c**2*x, x) + Integral(3*a**2*c*
*4*x**3, x) + Integral(a**2*c**6*x**5, x) + Integral(b**2*asinh(c*x)**2/x,
x) + Integral(2*a*b*asinh(c*x)/x, x) + Integral(3*b**2*c**2*x*asinh(c*x)**2
, x) + Integral(3*b**2*c**4*x**3*asinh(c*x)**2, x) + Integral(b**2*c**6*x**
5*asinh(c*x)**2, x) + Integral(6*a*b*c**2*x*asinh(c*x), x) + Integral(6*a*b
*c**4*x**3*asinh(c*x), x) + Integral(2*a*b*c**6*x**5*asinh(c*x), x))
```

3.222 $\int \frac{(d+c^2dx^2)^3(a+b\sinh^{-1}(cx))^2}{x^2}\,dx$

Optimal. Leaf size=307

$$\frac{6}{5}c^2d^3x\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)^2+\frac{8}{5}c^2d^3x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2-\frac{2}{25}bcd^3\left(c^2x^2+1\right)^{5/2}\left(a+b\sin\right.$$

```
[Out] 122/25*b^2*c^2*d^3*x+14/75*b^2*c^4*d^3*x^3+2/125*b^2*c^6*d^3*x^5-2/5*b*c*d^
3*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))-2/25*b*c*d^3*(c^2*x^2+1)^(5/2)*(a+b*
arcsinh(c*x))+16/5*c^2*d^3*x*(a+b*arcsinh(c*x))^2+8/5*c^2*d^3*x*(c^2*x^2+1)
*(a+b*arcsinh(c*x))^2+6/5*c^2*d^3*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2-d^3*
(c^2*x^2+1)^3*(a+b*arcsinh(c*x))^2/x-4*b*c*d^3*(a+b*arcsinh(c*x))*arctanh(c
*x+(c^2*x^2+1)^(1/2))-2*b^2*c*d^3*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+2*b^2*c
*d^3*polylog(2,c*x+(c^2*x^2+1)^(1/2))-22/5*b*c*d^3*(a+b*arcsinh(c*x))*(c^2*
x^2+1)^(1/2)
```

Rubi [A] time = 0.74, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 12, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {5739, 5684, 5653, 5717, 8, 194, 5744, 5742, 5760, 4182, 2279, 2391}

$$-2b^2cd^3\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)+2b^2cd^3\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)+\frac{6}{5}c^2d^3x\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)^2+\frac{8}{5}c^2\ldots$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] (122*b^2*c^2*d^3*x)/25 + (14*b^2*c^4*d^3*x^3)/75 + (2*b^2*c^6*d^3*x^5)/125
- (22*b*c*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/5 - (2*b*c*d^3*(1 + c
^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/5 - (2*b*c*d^3*(1 + c^2*x^2)^(5/2)*(a +
 b*ArcSinh[c*x]))/25 + (16*c^2*d^3*x*(a + b*ArcSinh[c*x])^2)/5 + (8*c^2*d^3
*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/5 + (6*c^2*d^3*x*(1 + c^2*x^2)^2*(
a + b*ArcSinh[c*x])^2)/5 - (d^3*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x])^2)/x -
 4*b*c*d^3*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] - 2*b^2*c*d^3*PolyL
og[2, -E^ArcSinh[c*x]] + 2*b^2*c*d^3*PolyLog[2, E^ArcSinh[c*x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
```

```
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2}{x^2}\,dx = -\frac{d^3\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2}{x} + \left(6c^2d\right)\int\left(d+c^2dx^2\right)^2\left(a+b\text{s}\right.$$

$$= \frac{2}{5}bcd^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right) + \frac{6}{5}c^2d^3x\left(1+c^2x^2\right)^2\left(a+b\,\text{si}\right.$$

$$= \frac{2}{3}bcd^3\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right) - \frac{2}{25}bcd^3\left(1+c^2x^2\right)^{5/2}\left(a+b\,\text{s}\right.$$

$$= -\frac{16}{15}b^2c^2d^3x - \frac{22}{45}b^2c^4d^3x^3 - \frac{2}{25}b^2c^6d^3x^5 + 2bcd^3\sqrt{1+c^2x^2}\left(a+b\,\text{si}\right.$$

$$= -\frac{38}{25}b^2c^2d^3x + \frac{14}{75}b^2c^4d^3x^3 + \frac{2}{125}b^2c^6d^3x^5 - \frac{22}{5}bcd^3\sqrt{1+c^2x^2}\left(a+\right.$$

$$= \frac{122}{25}b^2c^2d^3x + \frac{14}{75}b^2c^4d^3x^3 + \frac{2}{125}b^2c^6d^3x^5 - \frac{22}{5}bcd^3\sqrt{1+c^2x^2}\left(a+l\right.$$

$$= \frac{122}{25}b^2c^2d^3x + \frac{14}{75}b^2c^4d^3x^3 + \frac{2}{125}b^2c^6d^3x^5 - \frac{22}{5}bcd^3\sqrt{1+c^2x^2}\left(a+l\right.$$

$$= \frac{122}{25}b^2c^2d^3x + \frac{14}{75}b^2c^4d^3x^3 + \frac{2}{125}b^2c^6d^3x^5 - \frac{22}{5}bcd^3\sqrt{1+c^2x^2}\left(a+l\right.$$

Mathematica [A] time = 1.43, size = 466, normalized size = 1.52

$$\frac{1}{720}d^3\left(144a^2c^6x^5 + 720a^2c^4x^3 + 2160a^2c^2x - \frac{720a^2}{x} + 288abc^6x^5\sinh^{-1}(cx) + 1440abc^4x^3\sinh^{-1}(cx) - \frac{175\text{6}}{5}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] (d^3*((-720*a^2)/x + 2160*a^2*c^2*x + 3460*b^2*c^2*x + 720*a^2*c^4*x^3 + 14
4*a^2*c^6*x^5 - (17568*a*b*c*Sqrt[1 + c^2*x^2])/5 - (2016*a*b*c^3*x^2*Sqrt[
1 + c^2*x^2])/5 - (288*a*b*c^5*x^4*Sqrt[1 + c^2*x^2])/5 - (1440*a*b*ArcSinh
[c*x])/x + 4320*a*b*c^2*x*ArcSinh[c*x] + 1440*a*b*c^4*x^3*ArcSinh[c*x] + 28
8*a*b*c^6*x^5*ArcSinh[c*x] - 3420*b^2*c*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - (7
20*b^2*ArcSinh[c*x]^2)/x + 1890*b^2*c^2*x*ArcSinh[c*x]^2 - 1440*a*b*c*ArcTa
nh[Sqrt[1 + c^2*x^2]] + 80*b^2*c^2*x*Cosh[2*ArcSinh[c*x]] + 360*b^2*c^2*x*A
rcSinh[c*x]^2*Cosh[2*ArcSinh[c*x]] - 90*b^2*c*ArcSinh[c*x]*Cosh[3*ArcSinh[c
*x]] - (18*b^2*c*ArcSinh[c*x]*Cosh[5*ArcSinh[c*x]])/5 + 1440*b^2*c*ArcSinh[
c*x]*Log[1 - E^(-ArcSinh[c*x])] - 1440*b^2*c*ArcSinh[c*x]*Log[1 + E^(-ArcSi
nh[c*x])] + 1440*b^2*c*PolyLog[2, -E^(-ArcSinh[c*x])] - 1440*b^2*c*PolyLog[
2, E^(-ArcSinh[c*x])] - 10*b^2*c*Sinh[3*ArcSinh[c*x]] - 45*b^2*c*ArcSinh[c*
```

```
x]^2*Sinh[3*ArcSinh[c*x]] + (18*b^2*c*Sinh[5*ArcSinh[c*x]])/25 + 9*b^2*c*Ar
cSinh[c*x]^2*Sinh[5*ArcSinh[c*x]]))/720
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^2c^6d^3x^6 + 3\,a^2c^4d^3x^4 + 3\,a^2c^2d^3x^2 + a^2d^3 + \left(b^2c^6d^3x^6 + 3\,b^2c^4d^3x^4 + 3\,b^2c^2d^3x^2 + b^2d^3\right)\text{arsinh}\,(cx)^2}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^6*d^3*x^6 + 3*a^2*c^4*d^3*x^4 + 3*a^2*c^2*d^3*x^2 + a^2*d^3
 + (b^2*c^6*d^3*x^6 + 3*b^2*c^4*d^3*x^4 + 3*b^2*c^2*d^3*x^2 + b^2*d^3)*arcs
inh(c*x)^2 + 2*(a*b*c^6*d^3*x^6 + 3*a*b*c^4*d^3*x^4 + 3*a*b*c^2*d^3*x^2 + a
*b*d^3)*arcsinh(c*x))/x^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.30, size = 516, normalized size = 1.68

$$\frac{2d^3ab\,\text{arcsinh}\,(cx)\,c^6x^5}{5} + 2d^3ab\,\text{arcsinh}\,(cx)\,c^4x^3 + 6d^3ab\,\text{arcsinh}\,(cx)\,c^2x - \frac{2d^3b^2\,\text{arcsinh}\,(cx)\,\sqrt{c^2x^2+1}\,c^5x^4}{25} - \frac{14}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^2,x)
```

```
[Out] -2*b^2*c*d^3*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+2*b^2*c*d^3*polylog(2,c*x+(c
^2*x^2+1)^(1/2))+2/5*d^3*a*b*arcsinh(c*x)*c^6*x^5+2*d^3*a*b*arcsinh(c*x)*c^
4*x^3+6*d^3*a*b*arcsinh(c*x)*c^2*x-2/25*d^3*a*b*c^5*x^4*(c^2*x^2+1)^(1/2)-1
4/25*d^3*a*b*c^3*x^2*(c^2*x^2+1)^(1/2)-2/25*d^3*b^2*arcsinh(c*x)*(c^2*x^2+1
)^(1/2)*c^5*x^4-14/25*d^3*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^3*x^2+1/5*d^
3*a^2*c^6*x^5+d^3*a^2*c^4*x^3+3*d^3*a^2*c^2*x-d^3*b^2*arcsinh(c*x)^2/x+122/
25*b^2*c^2*d^3*x+14/75*b^2*c^4*d^3*x^3+2/125*b^2*c^6*d^3*x^5-d^3*a^2/x-2*d^
3*a*b*arcsinh(c*x)/x-122/25*c*d^3*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*d^
3*b^2*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-2*c*d^3*b^2*arcsinh(c*x)*ln(
1+c*x+(c^2*x^2+1)^(1/2))-122/25*c*d^3*a*b*(c^2*x^2+1)^(1/2)-2*c*d^3*a*b*arc
tanh(1/(c^2*x^2+1)^(1/2))+1/5*d^3*b^2*arcsinh(c*x)^2*c^6*x^5+d^3*b^2*arcsin
h(c*x)^2*c^4*x^3+3*d^3*b^2*arcsinh(c*x)^2*c^2*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{5}\,a^2c^6d^3x^5 + \frac{2}{75}\left(15\,x^5\,\text{arsinh}\,(cx) - \left(\frac{3\,\sqrt{c^2x^2+1}\,x^4}{c^2} - \frac{4\,\sqrt{c^2x^2+1}\,x^2}{c^4} + \frac{8\,\sqrt{c^2x^2+1}}{c^6}\right)c\right)abc^6d^3 + a^2c^4d^3x^3 + \frac{2}{3}\left(3\,x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^2*c^6*d^3*x^5 + 2/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/
c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*c^6*d^3
 + a^2*c^4*d^3*x^3 + 2/3*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2
 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*c^4*d^3 + 3*b^2*c^2*d^3*x*arcsinh(c*x)^2 +
 6*b^2*c^2*d^3*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + 3*a^2*c^2*d^3*x + 6
*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*c*d^3 - 2*(c*arcsinh(1/(c*abs(x
))) + arcsinh(c*x)/x)*a*b*d^3 - a^2*d^3/x + 1/5*(b^2*c^6*d^3*x^6 + 5*b^2*c^
4*d^3*x^4 - 5*b^2*d^3)*log(c*x + sqrt(c^2*x^2 + 1))^2/x - integrate(2/5*(b^
2*c^9*d^3*x^8 + 6*b^2*c^7*d^3*x^6 + 5*b^2*c^5*d^3*x^4 - 5*b^2*c^3*d^3*x^2 -
 5*b^2*c*d^3 + (b^2*c^8*d^3*x^7 + 5*b^2*c^6*d^3*x^5 - 5*b^2*c^2*d^3*x)*sqrt
(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))/(c^3*x^4 + c*x^2 + (c^2*x^3 + x
)*sqrt(c^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3)/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^3\left(\int 3a^2c^2\,dx+\int \frac{a^2}{x^2}\,dx+\int 3a^2c^4x^2\,dx+\int a^2c^6x^4\,dx+\int 3b^2c^2\,\mathrm{asinh}^2\,(cx)\,dx+\int \frac{b^2\,\mathrm{asinh}^2\,(cx)}{x^2}\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2/x**2,x)
```

```
[Out] d**3*(Integral(3*a**2*c**2, x) + Integral(a**2/x**2, x) + Integral(3*a**2*c
**4*x**2, x) + Integral(a**2*c**6*x**4, x) + Integral(3*b**2*c**2*asinh(c*x
)**2, x) + Integral(b**2*asinh(c*x)**2/x**2, x) + Integral(6*a*b*c**2*asinh
(c*x), x) + Integral(2*a*b*asinh(c*x)/x**2, x) + Integral(3*b**2*c**4*x**2*
asinh(c*x)**2, x) + Integral(b**2*c**6*x**4*asinh(c*x)**2, x) + Integral(6*
a*b*c**4*x**2*asinh(c*x), x) + Integral(2*a*b*c**6*x**4*asinh(c*x), x))
```

3.223 $\int \frac{(d+c^2dx^2)^3(a+b\sinh^{-1}(cx))^2}{x^3}\,dx$

Optimal. Leaf size=354

$$-3bc^2d^3\mathrm{Li}_2\left(e^{-2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{4}c^2d^3\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)^2+\frac{3}{2}c^2d^3\left(c^2x^2+1\right)\left(a+b\sinh\right.$$

```
[Out] 21/32*b^2*c^4*d^3*x^2+1/32*b^2*c^6*d^3*x^4+7/8*b*c^3*d^3*x*(c^2*x^2+1)^(3/2
)*(a+b*arcsinh(c*x))-b*c*d^3*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))/x-3/32*c^
2*d^3*(a+b*arcsinh(c*x))^2+3/2*c^2*d^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2+3/4
*c^2*d^3*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2-1/2*d^3*(c^2*x^2+1)^3*(a+b*arcs
inh(c*x))^2/x^2+c^2*d^3*(a+b*arcsinh(c*x))^3/b+3*c^2*d^3*(a+b*arcsinh(c*x))
^2*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^2)+b^2*c^2*d^3*ln(x)-3*b*c^2*d^3*(a+b*arc
sinh(c*x))*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)-3/2*b^2*c^2*d^3*polylog(3
,1/(c*x+(c^2*x^2+1)^(1/2))^2)-3/16*b*c^3*d^3*x*(a+b*arcsinh(c*x))*(c^2*x^2+
1)^(1/2)
```

Rubi [A] time = 0.75, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 15, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.577, Rules used = {5739, 5744, 5659, 3716, 2190, 2531, 2282, 6589, 5682, 5675, 30, 5684, 14, 266, 43}

$$3bc^2d^3\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{3}{2}b^2c^2d^3\text{PolyLog}\left(3,e^{2\sinh^{-1}(cx)}\right)+\frac{7}{8}bc^3d^3x\left(c^2x^2+1\right)^{3/2}\left(a+b\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] (21*b^2*c^4*d^3*x^2)/32 + (b^2*c^6*d^3*x^4)/32 - (3*b*c^3*d^3*x*Sqrt[1 + c^
2*x^2]*(a + b*ArcSinh[c*x]))/16 + (7*b*c^3*d^3*x*(1 + c^2*x^2)^(3/2)*(a + b
*ArcSinh[c*x]))/8 - (b*c*d^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x -
(3*c^2*d^3*(a + b*ArcSinh[c*x])^2)/32 + (3*c^2*d^3*(1 + c^2*x^2)*(a + b*Arc
Sinh[c*x])^2)/2 + (3*c^2*d^3*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/4 - (d
^3*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x])^2)/(2*x^2) - (c^2*d^3*(a + b*ArcSin
h[c*x])^3)/b + 3*c^2*d^3*(a + b*ArcSinh[c*x])^2*Log[1 - E^(2*ArcSinh[c*x])]
 + b^2*c^2*d^3*Log[x] + 3*b*c^2*d^3*(a + b*ArcSinh[c*x])*PolyLog[2, E^(2*Ar
cSinh[c*x])] - (3*b^2*c^2*d^3*PolyLog[3, E^(2*ArcSinh[c*x])])/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2}{x^3}dx = -\frac{d^3\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2}{2x^2} + \left(3c^2d\right)\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\right.}{x}$$

$$= -\frac{bcd^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x} + \frac{3}{4}c^2d^3\left(1+c^2x^2\right)^2\left(a+b\sin\right.$$

$$= \frac{7}{8}bc^3d^3x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right) - \frac{bcd^3\left(1+c^2x^2\right)^{5/2}\left(a+b\right.}{x}$$

$$= -\frac{3}{16}bc^3d^3x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) + \frac{7}{8}bc^3d^3x\left(1+c^2x^2\right)^{3/2}\left(a\right.$$

$$= \frac{21}{32}b^2c^4d^3x^2 + \frac{1}{32}b^2c^6d^3x^4 - \frac{3}{16}bc^3d^3x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)$$

$$= \frac{21}{32}b^2c^4d^3x^2 + \frac{1}{32}b^2c^6d^3x^4 - \frac{3}{16}bc^3d^3x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)$$

$$= \frac{21}{32}b^2c^4d^3x^2 + \frac{1}{32}b^2c^6d^3x^4 - \frac{3}{16}bc^3d^3x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)$$

$$= \frac{21}{32}b^2c^4d^3x^2 + \frac{1}{32}b^2c^6d^3x^4 - \frac{3}{16}bc^3d^3x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)$$

$$= \frac{21}{32}b^2c^4d^3x^2 + \frac{1}{32}b^2c^6d^3x^4 - \frac{3}{16}bc^3d^3x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)$$

Mathematica [A] time = 1.20, size = 459, normalized size = 1.30

$$\frac{1}{256}d^3\left(64a^2c^6x^4 + 384a^2c^4x^2 + 768a^2c^2\log(x) - \frac{128a^2}{x^2} + 128abc^6x^4\sinh^{-1}(cx) + 768abc^4x^2\sinh^{-1}(cx) - 76\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] (d^3*((-128*a^2)/x^2 + 384*a^2*c^4*x^2 + 64*a^2*c^6*x^4 - (256*a*b*c*Sqrt[1
 + c^2*x^2])/x - 336*a*b*c^3*x*Sqrt[1 + c^2*x^2] - 32*a*b*c^5*x^3*Sqrt[1 +
c^2*x^2] + 336*a*b*c^2*ArcSinh[c*x] - (256*a*b*ArcSinh[c*x])/x^2 + 768*a*b*
c^4*x^2*ArcSinh[c*x] + 128*a*b*c^6*x^4*ArcSinh[c*x] - (256*b^2*c*Sqrt[1 + c
^2*x^2]*ArcSinh[c*x])/x + 768*a*b*c^2*ArcSinh[c*x]^2 - (128*b^2*ArcSinh[c*x
]^2)/x^2 - 256*b^2*c^2*ArcSinh[c*x]^3 + 80*b^2*c^2*Cosh[2*ArcSinh[c*x]] + 1
60*b^2*c^2*ArcSinh[c*x]^2*Cosh[2*ArcSinh[c*x]] + b^2*c^2*Cosh[4*ArcSinh[c*x
]] + 8*b^2*c^2*ArcSinh[c*x]^2*Cosh[4*ArcSinh[c*x]] + 1536*a*b*c^2*ArcSinh[c
*x]*Log[1 - E^(-2*ArcSinh[c*x])] + 768*b^2*c^2*ArcSinh[c*x]^2*Log[1 - E^(2*
ArcSinh[c*x])] + 768*a^2*c^2*Log[x] + 256*b^2*c^2*Log[c*x] - 768*a*b*c^2*Po
lyLog[2, E^(-2*ArcSinh[c*x])] + 768*b^2*c^2*ArcSinh[c*x]*PolyLog[2, E^(2*Ar
cSinh[c*x])] - 384*b^2*c^2*PolyLog[3, E^(2*ArcSinh[c*x])] - 160*b^2*c^2*Arc
Sinh[c*x]*Sinh[2*ArcSinh[c*x]] - 4*b^2*c^2*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]
]))/256
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{a^2c^6d^3x^6 + 3\,a^2c^4d^3x^4 + 3\,a^2c^2d^3x^2 + a^2d^3 + \left(b^2c^6d^3x^6 + 3\,b^2c^4d^3x^4 + 3\,b^2c^2d^3x^2 + b^2d^3\right)\mathrm{arsinh}\,(c\right.}{x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^6*d^3*x^6 + 3*a^2*c^4*d^3*x^4 + 3*a^2*c^2*d^3*x^2 + a^2*d^3
 + (b^2*c^6*d^3*x^6 + 3*b^2*c^4*d^3*x^4 + 3*b^2*c^2*d^3*x^2 + b^2*d^3)*arcs
inh(c*x)^2 + 2*(a*b*c^6*d^3*x^6 + 3*a*b*c^4*d^3*x^4 + 3*a*b*c^2*d^3*x^2 + a
*b*d^3)*arcsinh(c*x))/x^3, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.80, size = 838, normalized size = 2.37

$$\frac{21b^2c^4d^3x^2}{32}+\frac{b^2c^6d^3x^4}{32}+6c^2d^3b^2\,\mathrm{arcsinh}\,(cx)\,\mathrm{polylog}\left(2,-cx-\sqrt{c^2x^2+1}\right)+\frac{c^6d^3b^2\,\mathrm{arcsinh}\,(cx)^2\,x^4}{4}+\frac{3c^4d^3b^2\,\mathrm{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^3,x)
```

```
[Out] 21/32*b^2*c^4*d^3*x^2+1/32*b^2*c^6*d^3*x^4+6*c^2*d^3*b^2*arcsinh(c*x)*polyl
og(2,-c*x-(c^2*x^2+1)^(1/2))+1/4*c^6*d^3*b^2*arcsinh(c*x)^2*x^4+3/2*c^4*d^3
*b^2*arcsinh(c*x)^2*x^2+3*c^2*d^3*b^2*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(
1/2))+3*c^2*d^3*b^2*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))+6*c^2*d^3*b^
2*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))-3*c^2*d^3*a*b*arcsinh(c*x)^
2+21/16*c^2*d^3*a*b*arcsinh(c*x)+6*c^2*d^3*a*b*polylog(2,-c*x-(c^2*x^2+1)^(
1/2))-d^3*a*b*arcsinh(c*x)/x^2+6*c^2*d^3*a*b*polylog(2,c*x+(c^2*x^2+1)^(1/2
))+81/256*d^3*b^2*c^2-1/2*d^3*a^2/x^2+3*c^4*d^3*a*b*arcsinh(c*x)*x^2-1/8*c^
5*d^3*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^3-21/16*c^3*d^3*b^2*arcsinh(c*x)
*(c^2*x^2+1)^(1/2)*x-1/8*c^5*d^3*a*b*x^3*(c^2*x^2+1)^(1/2)-21/16*c^3*d^3*a*
b*x*(c^2*x^2+1)^(1/2)-c*d^3*a*b/x*(c^2*x^2+1)^(1/2)+6*c^2*d^3*a*b*arcsinh(c
*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+6*c^2*d^3*a*b*arcsinh(c*x)*ln(1+c*x+(c^2*x^
2+1)^(1/2))-c*d^3*b^2*arcsinh(c*x)/x*(c^2*x^2+1)^(1/2)+1/2*c^6*d^3*a*b*arcs
inh(c*x)*x^4+1/4*c^6*d^3*a^2*x^4+3/2*c^4*d^3*a^2*x^2-1/2*d^3*b^2*arcsinh(c*
x)^2/x^2+3*c^2*d^3*a^2*ln(c*x)+21/32*c^2*d^3*b^2*arcsinh(c*x)^2-c^2*d^3*b^2
*arcsinh(c*x)^3-6*c^2*d^3*b^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))-6*c^2*d^3*b^
2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))+c^2*d^3*b^2*arcsinh(c*x)+c^2*d^3*b^2*ln
(c*x+(c^2*x^2+1)^(1/2)-1)+c^2*d^3*b^2*ln(1+c*x+(c^2*x^2+1)^(1/2))-2*c^2*d^3
*b^2*ln(c*x+(c^2*x^2+1)^(1/2))+d^3*a*b*c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\,a^2c^6d^3x^4+\frac{3}{2}\,a^2c^4d^3x^2+3\,a^2c^2d^3\log(x)-abd^3\left(\frac{\sqrt{c^2x^2+1}\,c}{x}+\frac{\mathrm{arcsinh}\,(cx)}{x^2}\right)-\frac{a^2d^3}{2\,x^2}+\int b^2c^6d^3x^3\log\left(cx+\sqrt{c^2x^2+\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*c^6*d^3*x^4 + 3/2*a^2*c^4*d^3*x^2 + 3*a^2*c^2*d^3*log(x) - a*b*d^3*
(sqrt(c^2*x^2 + 1)*c/x + arcsinh(c*x)/x^2) - 1/2*a^2*d^3/x^2 + integrate(b^
2*c^6*d^3*x^3*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*a*b*c^6*d^3*x^3*log(c*x +
sqrt(c^2*x^2 + 1)) + 3*b^2*c^4*d^3*x*log(c*x + sqrt(c^2*x^2 + 1))^2 + 6*a*b
```

```
*c^4*d^3*x*log(c*x + sqrt(c^2*x^2 + 1)) + 3*b^2*c^2*d^3*log(c*x + sqrt(c^2*
x^2 + 1))^2/x + 6*a*b*c^2*d^3*log(c*x + sqrt(c^2*x^2 + 1))/x + b^2*d^3*log(
c*x + sqrt(c^2*x^2 + 1))^2/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3)/x^3,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^3\left(\int \frac{a^2}{x^3}\,dx + \int \frac{3a^2c^2}{x}\,dx + \int 3a^2c^4x\,dx + \int a^2c^6x^3\,dx + \int \frac{b^2\,\mathrm{asinh}^2\,(cx)}{x^3}\,dx + \int \frac{2ab\,\mathrm{asinh}\,(cx)}{x^3}\,dx + \int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2/x**3,x)
```

```
[Out] d**3*(Integral(a**2/x**3, x) + Integral(3*a**2*c**2/x, x) + Integral(3*a**2
*c**4*x, x) + Integral(a**2*c**6*x**3, x) + Integral(b**2*asinh(c*x)**2/x**
3, x) + Integral(2*a*b*asinh(c*x)/x**3, x) + Integral(3*b**2*c**2*asinh(c*x
)**2/x, x) + Integral(3*b**2*c**4*x*asinh(c*x)**2, x) + Integral(b**2*c**6*
x**3*asinh(c*x)**2, x) + Integral(6*a*b*c**2*asinh(c*x)/x, x) + Integral(6*
a*b*c**4*x*asinh(c*x), x) + Integral(2*a*b*c**6*x**3*asinh(c*x), x))
```

3.224 $\int \frac{\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2}{x^4}\,dx$

Optimal. Leaf size=326

$$\frac{16}{3}c^4d^3x\left(a+b\sinh^{-1}(cx)\right)^2-\frac{34}{3}bc^3d^3\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{2c^2d^3\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right.}{x}$$

```
[Out] -1/3*b^2*c^2*d^3/x+50/9*b^2*c^4*d^3*x+2/27*b^2*c^6*d^3*x^3+1/9*b*c^3*d^3*(c
^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))-1/3*b*c*d^3*(c^2*x^2+1)^(5/2)*(a+b*arcsi
nh(c*x))/x^2+16/3*c^4*d^3*x*(a+b*arcsinh(c*x))^2+8/3*c^4*d^3*x*(c^2*x^2+1)*
(a+b*arcsinh(c*x))^2-2*c^2*d^3*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/x-1/3*d^3
*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))^2/x^3-34/3*b*c^3*d^3*(a+b*arcsinh(c*x))*a
rctanh(c*x+(c^2*x^2+1)^(1/2))-17/3*b^2*c^3*d^3*polylog(2,-c*x-(c^2*x^2+1)^(
1/2))+17/3*b^2*c^3*d^3*polylog(2,c*x+(c^2*x^2+1)^(1/2))-5*b*c^3*d^3*(a+b*ar
csinh(c*x))*(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 1.02, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 12, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {5739, 5684, 5653, 5717, 8, 5744, 5742, 5760, 4182, 2279, 2391, 270}

$$-\frac{17}{3}b^2c^3d^3\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)+\frac{17}{3}b^2c^3d^3\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)+\frac{8}{3}c^4d^3x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{9}b$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] -(b^2*c^2*d^3)/(3*x) + (50*b^2*c^4*d^3*x)/9 + (2*b^2*c^6*d^3*x^3)/27 - 5*b*
c^3*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]) + (b*c^3*d^3*(1 + c^2*x^2)^(
3/2)*(a + b*ArcSinh[c*x]))/9 - (b*c*d^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[
c*x]))/(3*x^2) + (16*c^4*d^3*x*(a + b*ArcSinh[c*x])^2)/3 + (8*c^4*d^3*x*(1
+ c^2*x^2)*(a + b*ArcSinh[c*x])^2)/3 - (2*c^2*d^3*(1 + c^2*x^2)^2*(a + b*Ar
cSinh[c*x])^2)/x - (d^3*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x])^2)/(3*x^3) - (
34*b*c^3*d^3*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/3 - (17*b^2*c^3*
d^3*PolyLog[2, -E^ArcSinh[c*x]])/3 + (17*b^2*c^3*d^3*PolyLog[2, E^ArcSinh[c
*x]])/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
```

```
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2}{x^4}\,dx = -\frac{d^3\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2}{3x^3} + \left(2c^2d\right)\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\sin\right.}{x^2}$$

$$= -\frac{bcd^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2} - \frac{2c^2d^3\left(1+c^2x^2\right)^2\left(a+b\sinh^{-}\right.}{x}$$

$$= \frac{17}{9}bc^3d^3\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right) - \frac{bcd^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh\right.}{3x^2}$$

$$= -\frac{b^2c^2d^3}{3x} - \frac{11}{9}b^2c^4d^3x - \frac{14}{27}b^2c^6d^3x^3 + \frac{17}{3}bc^3d^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}\right.$$

$$= -\frac{b^2c^2d^3}{3x} - \frac{46}{9}b^2c^4d^3x + \frac{2}{27}b^2c^6d^3x^3 - 5bc^3d^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(c\right.$$

$$= -\frac{b^2c^2d^3}{3x} + \frac{50}{9}b^2c^4d^3x + \frac{2}{27}b^2c^6d^3x^3 - 5bc^3d^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(c\right.$$

$$= -\frac{b^2c^2d^3}{3x} + \frac{50}{9}b^2c^4d^3x + \frac{2}{27}b^2c^6d^3x^3 - 5bc^3d^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(c\right.$$

$$= -\frac{b^2c^2d^3}{3x} + \frac{50}{9}b^2c^4d^3x + \frac{2}{27}b^2c^6d^3x^3 - 5bc^3d^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(c\right.$$

Mathematica [A] time = 1.10, size = 461, normalized size = 1.41

$$d^3\left(9a^2c^6x^6 + 81a^2c^4x^4 - 81a^2c^2x^2 - 9a^2 + 18abc^6x^6\sinh^{-1}(cx) + 162abc^4x^4\sinh^{-1}(cx) - 9abcx\sqrt{c^2x^2+1} - 162\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] (d^3*(-9*a^2 - 81*a^2*c^2*x^2 - 9*b^2*c^2*x^2 + 81*a^2*c^4*x^4 + 150*b^2*c^
4*x^4 + 9*a^2*c^6*x^6 + 2*b^2*c^6*x^6 - 9*a*b*c*x*Sqrt[1 + c^2*x^2] - 150*a
*b*c^3*x^3*Sqrt[1 + c^2*x^2] - 6*a*b*c^5*x^5*Sqrt[1 + c^2*x^2] - 18*a*b*Arc
Sinh[c*x] - 162*a*b*c^2*x^2*ArcSinh[c*x] + 162*a*b*c^4*x^4*ArcSinh[c*x] + 1
8*a*b*c^6*x^6*ArcSinh[c*x] - 9*b^2*c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - 150
*b^2*c^3*x^3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - 6*b^2*c^5*x^5*Sqrt[1 + c^2*x^
2]*ArcSinh[c*x] - 9*b^2*ArcSinh[c*x]^2 - 81*b^2*c^2*x^2*ArcSinh[c*x]^2 + 81
*b^2*c^4*x^4*ArcSinh[c*x]^2 + 9*b^2*c^6*x^6*ArcSinh[c*x]^2 - 153*a*b*c^3*x^
3*ArcTanh[Sqrt[1 + c^2*x^2]] + 153*b^2*c^3*x^3*ArcSinh[c*x]*Log[1 - E^(-Arc
Sinh[c*x])] - 153*b^2*c^3*x^3*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 153
```

```
*b^2*c^3*x^3*PolyLog[2, -E^(-ArcSinh[c*x])] - 153*b^2*c^3*x^3*PolyLog[2, E^
(-ArcSinh[c*x])]))/(27*x^3)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^2c^6d^3x^6 + 3\,a^2c^4d^3x^4 + 3\,a^2c^2d^3x^2 + a^2d^3 + \left(b^2c^6d^3x^6 + 3\,b^2c^4d^3x^4 + 3\,b^2c^2d^3x^2 + b^2d^3\right)\operatorname{arsinh}\left(cx\right.}{x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^6*d^3*x^6 + 3*a^2*c^4*d^3*x^4 + 3*a^2*c^2*d^3*x^2 + a^2*d^3
 + (b^2*c^6*d^3*x^6 + 3*b^2*c^4*d^3*x^4 + 3*b^2*c^2*d^3*x^2 + b^2*d^3)*arcs
inh(c*x)^2 + 2*(a*b*c^6*d^3*x^6 + 3*a*b*c^4*d^3*x^4 + 3*a*b*c^2*d^3*x^2 + a
*b*d^3)*arcsinh(c*x))/x^4, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.67, size = 528, normalized size = 1.62

$$-\frac{b^2c^2d^3}{3x}+\frac{50b^2c^4d^3x}{9}+\frac{2b^2c^6d^3x^3}{27}+\frac{2c^6d^3ab\operatorname{arcsinh}\left(cx\right)x^3}{3}+6c^4d^3ab\operatorname{arcsinh}\left(cx\right)x-\frac{6c^2d^3ab\operatorname{arcsinh}\left(cx\right)}{x}-\frac{c\,d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^4,x)
```

```
[Out] -17/3*b^2*c^3*d^3*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+17/3*b^2*c^3*d^3*polylo
g(2,c*x+(c^2*x^2+1)^(1/2))-1/3*b^2*c^2*d^3/x+50/9*b^2*c^4*d^3*x+2/27*b^2*c^
6*d^3*x^3-1/3*d^3*a^2/x^3+2/3*c^6*d^3*a*b*arcsinh(c*x)*x^3+6*c^4*d^3*a*b*ar
csinh(c*x)*x-6*c^2*d^3*a*b*arcsinh(c*x)/x-2/9*c^5*d^3*a*b*x^2*(c^2*x^2+1)^(
1/2)-1/3*c*d^3*a*b/x^2*(c^2*x^2+1)^(1/2)-1/3*c*d^3*b^2/x^2*arcsinh(c*x)*(c^
2*x^2+1)^(1/2)-2/9*c^5*d^3*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2-1/3*d^3*b
^2/x^3*arcsinh(c*x)^2+1/3*c^6*d^3*a^2*x^3+3*c^4*d^3*a^2*x-3*c^2*d^3*a^2/x-2
/3*d^3*a*b*arcsinh(c*x)/x^3+3*c^4*d^3*b^2*arcsinh(c*x)^2*x-3*c^2*d^3*b^2*ar
csinh(c*x)^2/x-50/9*c^3*d^3*a*b*(c^2*x^2+1)^(1/2)-17/3*c^3*d^3*a*b*arctanh(
1/(c^2*x^2+1)^(1/2))-50/9*c^3*d^3*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+17/3*c
^3*d^3*b^2*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-17/3*c^3*d^3*b^2*arcsin
h(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+1/3*c^6*d^3*b^2*arcsinh(c*x)^2*x^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\,a^2c^6d^3x^3+\frac{2}{9}\left(3\,x^3\operatorname{arsinh}\left(cx\right)-c\left(\frac{\sqrt{c^2x^2+1}\,x^2}{c^2}-\frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)abc^6d^3+3\,b^2c^4d^3x\operatorname{arsinh}\left(cx\right)^2+6\,b^2c^4d^3\left(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*c^6*d^3*x^3 + 2/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^
2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*c^6*d^3 + 3*b^2*c^4*d^3*x*arcsinh(c*x)^2
+ 6*b^2*c^4*d^3*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + 3*a^2*c^4*d^3*x +
6*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*c^3*d^3 - 6*(c*arcsinh(1/(c*ab
s(x))) + arcsinh(c*x)/x)*a*b*c^2*d^3 + 1/3*((c^2*arcsinh(1/(c*abs(x))) - sq
rt(c^2*x^2 + 1)/x^2)*c - 2*arcsinh(c*x)/x^3)*a*b*d^3 - 3*a^2*c^2*d^3/x - 1/
3*a^2*d^3/x^3 + 1/3*(b^2*c^6*d^3*x^6 - 9*b^2*c^2*d^3*x^2 - b^2*d^3)*log(c*x
 + sqrt(c^2*x^2 + 1))^2/x^3 - integrate(2/3*(b^2*c^9*d^3*x^8 + b^2*c^7*d^3*
x^6 - 9*b^2*c^5*d^3*x^4 - 10*b^2*c^3*d^3*x^2 - b^2*c*d^3 + (b^2*c^8*d^3*x^7
 - 9*b^2*c^4*d^3*x^3 - b^2*c^2*d^3*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2
*x^2 + 1))/(c^3*x^6 + c*x^4 + (c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3)/x^4,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^3)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^3\left(\int 3a^2c^4\,dx+\int \frac{a^2}{x^4}\,dx+\int \frac{3a^2c^2}{x^2}\,dx+\int a^2c^6x^2\,dx+\int 3b^2c^4\,\mathrm{asinh}^2\,(cx)\,dx+\int \frac{b^2\,\mathrm{asinh}^2\,(cx)}{x^4}\,dx+\int 6a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2/x**4,x)
```

```
[Out] d**3*(Integral(3*a**2*c**4, x) + Integral(a**2/x**4, x) + Integral(3*a**2*c
**2/x**2, x) + Integral(a**2*c**6*x**2, x) + Integral(3*b**2*c**4*asinh(c*x
)**2, x) + Integral(b**2*asinh(c*x)**2/x**4, x) + Integral(6*a*b*c**4*asinh
(c*x), x) + Integral(2*a*b*asinh(c*x)/x**4, x) + Integral(3*b**2*c**2*asinh
(c*x)**2/x**2, x) + Integral(b**2*c**6*x**2*asinh(c*x)**2, x) + Integral(6*
a*b*c**2*asinh(c*x)/x**2, x) + Integral(2*a*b*c**6*x**2*asinh(c*x), x))
```

3.225 $\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx$

Optimal. Leaf size=277

$$-\frac{2ib\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^5d}+\frac{2ib\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^5d}+\frac{2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right.}{c^5d}$$

```
[Out] -22/9*b^2*x/c^4/d+2/27*b^2*x^3/c^2/d-x*(a+b*arcsinh(c*x))^2/c^4/d+1/3*x^3*(
a+b*arcsinh(c*x))^2/c^2/d+2*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(1/
2))/c^5/d-2*I*b*(a+b*arcsinh(c*x))*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c^
5/d+2*I*b*(a+b*arcsinh(c*x))*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c^5/d+2*I
*b^2*polylog(3,-I*(c*x+(c^2*x^2+1)^(1/2)))/c^5/d-2*I*b^2*polylog(3,I*(c*x+(
c^2*x^2+1)^(1/2)))/c^5/d+22/9*b*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^5/d-
2/9*b*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3/d
```

Rubi [A] time = 0.55, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5767, 5693, 4180, 2531, 2282, 6589, 5717, 8, 5758, 30}

$$-\frac{2ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^5d}+\frac{2ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^5d}+\frac{2ib^2\text{PolyLo}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2),x]
```

```
[Out] (-22*b^2*x)/(9*c^4*d) + (2*b^2*x^3)/(27*c^2*d) + (22*b*Sqrt[1 + c^2*x^2]*(a
 + b*ArcSinh[c*x]))/(9*c^5*d) - (2*b*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c
*x]))/(9*c^3*d) - (x*(a + b*ArcSinh[c*x])^2)/(c^4*d) + (x^3*(a + b*ArcSinh[
c*x])^2)/(3*c^2*d) + (2*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]])/(c^5
*d) - ((2*I)*b*(a + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^5*d
) + ((2*I)*b*(a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])/(c^5*d) + (
(2*I)*b^2*PolyLog[3, (-I)*E^ArcSinh[c*x]])/(c^5*d) - ((2*I)*b^2*PolyLog[3,
I*E^ArcSinh[c*x]])/(c^5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5767

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(e*(m + 2*p + 1)), x] + (-Dist[(f^2*(m - 1))/(c^2*(m +
 2*p + 1)), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(c*(m + 2*p + 1)*(1 + c
^2*x^2)^FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcS
inh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2
*d] && GtQ[n, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^4 \left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx = \frac{x^3 \left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d} - \frac{\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}dx}{c^2} - \frac{(2b)\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}dx}{3cd}$$

$$= -\frac{2bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{9c^3d} - \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^4d} + \frac{x^3\left(a+b\sinh^{-1}\right.}{3c^2d}$$

$$= \frac{2b^2x^3}{27c^2d} + \frac{22b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{9c^5d} - \frac{2bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{9c^3d}$$

$$= -\frac{22b^2x}{9c^4d} + \frac{2b^2x^3}{27c^2d} + \frac{22b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{9c^5d} - \frac{2bx^2\sqrt{1+c^2x^2}\left(a+b\,\mathrm{s}\right.}{9c^3d}$$

$$= -\frac{22b^2x}{9c^4d} + \frac{2b^2x^3}{27c^2d} + \frac{22b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{9c^5d} - \frac{2bx^2\sqrt{1+c^2x^2}\left(a+b\,\mathrm{s}\right.}{9c^3d}$$

$$= -\frac{22b^2x}{9c^4d} + \frac{2b^2x^3}{27c^2d} + \frac{22b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{9c^5d} - \frac{2bx^2\sqrt{1+c^2x^2}\left(a+b\,\mathrm{s}\right.}{9c^3d}$$

$$= -\frac{22b^2x}{9c^4d} + \frac{2b^2x^3}{27c^2d} + \frac{22b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{9c^5d} - \frac{2bx^2\sqrt{1+c^2x^2}\left(a+b\,\mathrm{s}\right.}{9c^3d}$$

Mathematica [A] time = 1.32, size = 365, normalized size = 1.32

$$\frac{a^2c^3x^3 - 3a^2cx + 3a^2\tan^{-1}(cx) - \frac{2}{3}ab\left(-3c^3x^3\sinh^{-1}(cx) + c^2x^2\sqrt{c^2x^2+1} - 11\sqrt{c^2x^2+1} + 9i\mathrm{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2),x]
```

```
[Out] (-3*a^2*c*x + a^2*c^3*x^3 + 3*a^2*ArcTan[c*x] - (2*a*b*(-11*Sqrt[1 + c^2*x^
2] + c^2*x^2*Sqrt[1 + c^2*x^2] + 9*c*x*ArcSinh[c*x] - 3*c^3*x^3*ArcSinh[c*x
] - (9*I)*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + (9*I)*ArcSinh[c*x]*Log[1
 + I*E^ArcSinh[c*x]] + (9*I)*PolyLog[2, (-I)*E^ArcSinh[c*x]] - (9*I)*PolyLo
g[2, I*E^ArcSinh[c*x]]))/3 + 3*b^2*((5*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/2 -
(5*c*x*(2 + ArcSinh[c*x]^2))/4 - (ArcSinh[c*x]*Cosh[3*ArcSinh[c*x]])/18 + I
*(-(ArcSinh[c*x]^2*(Log[1 - I/E^ArcSinh[c*x]] - Log[1 + I/E^ArcSinh[c*x]]))
 - 2*ArcSinh[c*x]*(PolyLog[2, (-I)/E^ArcSinh[c*x]] - PolyLog[2, I/E^ArcSinh
[c*x]]) - 2*PolyLog[3, (-I)/E^ArcSinh[c*x]] + 2*PolyLog[3, I/E^ArcSinh[c*x]
]) + ((2 + 9*ArcSinh[c*x]^2)*Sinh[3*ArcSinh[c*x]])/108))/(3*c^5*d)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b^2x^4\,\mathrm{arsinh}\,(cx)^2 + 2\,abx^4\,\mathrm{arsinh}\,(cx) + a^2x^4}{c^2dx^2+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4*arcsinh(c*x)^2 + 2*a*b*x^4*arcsinh(c*x) + a^2*x^4)/(c^2*d
*x^2 + d), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{x^4 (a + b \operatorname{arcsinh}(cx))^2}{c^2 d x^2 + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x)
```

```
[Out] int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3} a^2 \left(\frac{c^2 x^3 - 3x}{c^4 d} + \frac{3 \arctan(cx)}{c^5 d} \right) + \int \frac{b^2 x^4 \log\left(cx + \sqrt{c^2x^2 + 1}\right)^2}{c^2 d x^2 + d} + \frac{2abx^4 \log\left(cx + \sqrt{c^2x^2 + 1}\right)}{c^2 d x^2 + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/3*a^2*((c^2*x^3 - 3*x)/(c^4*d) + 3*arctan(c*x)/(c^5*d)) + integrate(b^2*x
^4*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d*x^2 + d) + 2*a*b*x^4*log(c*x + sqr
t(c^2*x^2 + 1))/(c^2*d*x^2 + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 (a + b \operatorname{asinh}(c x))^2}{d c^2 x^2 + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2),x)
```

```
[Out] int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2 x^4}{c^2x^2+1}\, dx + \int \frac{b^2 x^4 \operatorname{asinh}^2(cx)}{c^2x^2+1}\, dx + \int \frac{2abx^4 \operatorname{asinh}(cx)}{c^2x^2+1}\, dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))**2/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a**2*x**4/(c**2*x**2 + 1), x) + Integral(b**2*x**4*asinh(c*x)**2/
(c**2*x**2 + 1), x) + Integral(2*a*b*x**4*asinh(c*x)/(c**2*x**2 + 1), x))/d
```

3.226 $\int \frac{x^3(a+b\sinh^{-1}(cx))^2}{d+c^2dx^2}\,dx$

Optimal. Leaf size=199

$$-\frac{b\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^4d}+\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3bc^4d}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4c^4d}-\frac{\log\left(e^{2\sinh^{-1}(cx)}+1\right)\left(a+\right.}{c^4d}$$

```
[Out] 1/4*b^2*x^2/c^2/d+1/4*(a+b*arcsinh(c*x))^2/c^4/d+1/2*x^2*(a+b*arcsinh(c*x))
^2/c^2/d+1/3*(a+b*arcsinh(c*x))^3/b/c^4/d-(a+b*arcsinh(c*x))^2*ln(1+(c*x+(c
^2*x^2+1)^(1/2))^2)/c^4/d-b*(a+b*arcsinh(c*x))*polylog(2,-(c*x+(c^2*x^2+1)^
(1/2))^2)/c^4/d+1/2*b^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/c^4/d-1/2*b*x
*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3/d
```

Rubi [A] time = 0.41, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5767, 5714, 3718, 2190, 2531, 2282, 6589, 5758, 5675, 30}

$$-\frac{b\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^4d}+\frac{b^2\text{PolyLog}\left(3,-e^{2\sinh^{-1}(cx)}\right)}{2c^4d}+\frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d}-\frac{bx\sqrt{c^2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2),x]
```

```
[Out] (b^2*x^2)/(4*c^2*d) - (b*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2*c^3*d
) + (a + b*ArcSinh[c*x])^2/(4*c^4*d) + (x^2*(a + b*ArcSinh[c*x])^2)/(2*c^2*
d) + (a + b*ArcSinh[c*x])^3/(3*b*c^4*d) - ((a + b*ArcSinh[c*x])^2*Log[1 + E
^(2*ArcSinh[c*x])])/(c^4*d) - (b*(a + b*ArcSinh[c*x])*PolyLog[2, -E^(2*ArcS
inh[c*x])])/(c^4*d) + (b^2*PolyLog[3, -E^(2*ArcSinh[c*x])])/(2*c^4*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5767

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(e*(m + 2*p + 1)), x] + (-Dist[(f^2*(m - 1))/(c^2*(m +
 2*p + 1)), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(c*(m + 2*p + 1)*(1 + c
^2*x^2)^FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcS
inh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2
*d] && GtQ[n, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^3 \left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx = \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d} - \frac{\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}dx}{c^2} - \frac{b\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}dx}{cd}$$

$$= -\frac{bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^3d} + \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d} - \text{Subst}\left(\int(a+bx\right.$$

$$= \frac{b^2x^2}{4c^2d} - \frac{bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^3d} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4c^4d} + \frac{x^2\left(a+b\sinh\right.}{2c^2d}$$

$$= \frac{b^2x^2}{4c^2d} - \frac{bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^3d} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4c^4d} + \frac{x^2\left(a+b\sinh\right.}{2c^2d}$$

$$= \frac{b^2x^2}{4c^2d} - \frac{bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^3d} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4c^4d} + \frac{x^2\left(a+b\sinh\right.}{2c^2d}$$

$$= \frac{b^2x^2}{4c^2d} - \frac{bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^3d} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4c^4d} + \frac{x^2\left(a+b\sinh\right.}{2c^2d}$$

$$= \frac{b^2x^2}{4c^2d} - \frac{bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^3d} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4c^4d} + \frac{x^2\left(a+b\sinh\right.}{2c^2d}$$

Mathematica [C] time = 0.46, size = 279, normalized size = 1.40

$$\frac{12a^2c^2x^2 - 12a^2\log\left(c^2x^2+1\right) - 12abcx\sqrt{c^2x^2+1} + 24abc^2x^2\sinh^{-1}(cx) - 48ab\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right) - 48ab\text{Li}_2\left(\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2),x]
```

```
[Out] (12*a^2*c^2*x^2 - 12*a*b*c*x*Sqrt[1 + c^2*x^2] + 12*a*b*ArcSinh[c*x] + 24*a
*b*c^2*x^2*ArcSinh[c*x] + 24*a*b*ArcSinh[c*x]^2 - 8*b^2*ArcSinh[c*x]^3 + 3*
b^2*Cosh[2*ArcSinh[c*x]] + 6*b^2*ArcSinh[c*x]^2*Cosh[2*ArcSinh[c*x]] - 24*b
^2*ArcSinh[c*x]^2*Log[1 + E^(-2*ArcSinh[c*x])] - 48*a*b*ArcSinh[c*x]*Log[1
- I*E^ArcSinh[c*x]] - 48*a*b*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] - 12*a^
2*Log[1 + c^2*x^2] + 24*b^2*ArcSinh[c*x]*PolyLog[2, -E^(-2*ArcSinh[c*x])] -
 48*a*b*PolyLog[2, (-I)*E^ArcSinh[c*x]] - 48*a*b*PolyLog[2, I*E^ArcSinh[c*x
]] + 12*b^2*PolyLog[3, -E^(-2*ArcSinh[c*x])] - 6*b^2*ArcSinh[c*x]*Sinh[2*Ar
cSinh[c*x]])/(24*c^4*d)
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2x^3\,\text{arsinh}\,(cx)^2 + 2\,abx^3\,\text{arsinh}\,(cx) + a^2x^3}{c^2dx^2+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^3*arcsinh(c*x)^2 + 2*a*b*x^3*arcsinh(c*x) + a^2*x^3)/(c^2*d
*x^2 + d), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.20, size = 380, normalized size = 1.91

$$\frac{a^2x^2}{2c^2d}-\frac{a^2\ln\left(c^2x^2+1\right)}{2c^4d}+\frac{b^2\operatorname{arcsinh}(cx)^3}{3c^4d}+\frac{b^2\operatorname{arcsinh}(cx)^2x^2}{2c^2d}-\frac{b^2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,x}{2c^3d}+\frac{b^2\operatorname{arcsinh}(cx)^2}{4c^4d}+\frac{b^2}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x)
```

```
[Out] 1/2/c^2*a^2/d*x^2-1/2/c^4*a^2/d*ln(c^2*x^2+1)+1/3/c^4*b^2/d*arcsinh(c*x)^3+
1/2/c^2*b^2/d*arcsinh(c*x)^2*x^2-1/2/c^3*b^2/d*arcsinh(c*x)*(c^2*x^2+1)^(1/
2)*x+1/4/c^4*b^2/d*arcsinh(c*x)^2+1/4*b^2*x^2/c^2/d+1/8/c^4*b^2/d-1/c^4*b^2
/d*arcsinh(c*x)^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-1/c^4*b^2/d*arcsinh(c*x)*
polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)+1/2*b^2*polylog(3,-(c*x+(c^2*x^2+1)^(
1/2))^2)/c^4/d+1/c^4*a*b/d*arcsinh(c*x)^2+1/c^2*a*b/d*arcsinh(c*x)*x^2-1/2/
c^3*a*b/d*x*(c^2*x^2+1)^(1/2)+1/2/c^4*a*b/d*arcsinh(c*x)-2/c^4*a*b/d*arcsin
h(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-1/c^4*a*b/d*polylog(2,-(c*x+(c^2*x^2
+1)^(1/2))^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}a^2\left(\frac{x^2}{c^2d}-\frac{\log\left(c^2x^2+1\right)}{c^4d}\right)+\frac{\left(b^2c^2x^2-b^2\log\left(c^2x^2+1\right)\right)\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{2\,c^4d}+\int-\frac{\left(b^2c^2x^2-\left(2\,abc^4-b^2c^4\right)x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(x^2/(c^2*d) - log(c^2*x^2 + 1)/(c^4*d)) + 1/2*(b^2*c^2*x^2 - b^2*l
og(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^4*d) + integrate(-(b^2*c
^2*x^2 - (2*a*b*c^4 - b^2*c^4)*x^4 - (b^2*c^2*x^2 + b^2)*log(c^2*x^2 + 1) -
 (b^2*c*x*log(c^2*x^2 + 1) + (2*a*b*c^3 - b^2*c^3)*x^3)*sqrt(c^2*x^2 + 1))*
log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d*x^3 + c^4*d*x + (c^5*d*x^2 + c^3*d)*sqr
t(c^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2}{d\,c^2\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2),x)
```

```
[Out] int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2x^3}{c^2x^2+1}\,dx+\int \frac{b^2x^3\operatorname{asinh}^2(cx)}{c^2x^2+1}\,dx+\int \frac{2abx^3\operatorname{asinh}(cx)}{c^2x^2+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))**2/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a**2*x**3/(c**2*x**2 + 1), x) + Integral(b**2*x**3*asinh(c*x)**2/
(c**2*x**2 + 1), x) + Integral(2*a*b*x**3*asinh(c*x)/(c**2*x**2 + 1), x))/d
```

3.227 $\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx$

Optimal. Leaf size=198

$$\frac{2ib\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^3d}-\frac{2ib\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^3d}-\frac{2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sin\right.}{c^3d}$$

```
[Out] 2*b^2*x/c^2/d+x*(a+b*arcsinh(c*x))^2/c^2/d-2*(a+b*arcsinh(c*x))^2*arctan(c*
x+(c^2*x^2+1)^(1/2))/c^3/d+2*I*b*(a+b*arcsinh(c*x))*polylog(2,-I*(c*x+(c^2*
x^2+1)^(1/2)))/c^3/d-2*I*b*(a+b*arcsinh(c*x))*polylog(2,I*(c*x+(c^2*x^2+1)^
(1/2)))/c^3/d-2*I*b^2*polylog(3,-I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d+2*I*b^2*p
olylog(3,I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d-2*b*(a+b*arcsinh(c*x))*(c^2*x^2+1
)^(1/2)/c^3/d
```

Rubi [A] time = 0.29, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5767, 5693, 4180, 2531, 2282, 6589, 5717, 8}

$$\frac{2ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^3d}-\frac{2ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^3d}-\frac{2ib^2\text{PolyLog}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2),x]
```

```
[Out] (2*b^2*x)/(c^2*d) - (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(c^3*d) +
(x*(a + b*ArcSinh[c*x])^2)/(c^2*d) - (2*(a + b*ArcSinh[c*x])^2*ArcTan[E^Arc
Sinh[c*x]])/(c^3*d) + ((2*I)*b*(a + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSi
nh[c*x]])/(c^3*d) - ((2*I)*b*(a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*
x]])/(c^3*d) - ((2*I)*b^2*PolyLog[3, (-I)*E^ArcSinh[c*x]])/(c^3*d) + ((2*I)
*b^2*PolyLog[3, I*E^ArcSinh[c*x]])/(c^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
```

```
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5767

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(e*(m + 2*p + 1)), x] + (-Dist[(f^2*(m - 1))/(c^2*(m +
 2*p + 1)), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(c*(m + 2*p + 1)*(1 + c
^2*x^2)^FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcS
inh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2
*d] && GtQ[n, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx = \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d} - \frac{\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx}{c^2} - \frac{(2b)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{cd}$$

$$= -\frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^3d} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d} - \frac{\text{Subst}\left(\int (a+bx)^2\text{sec}\cdots\right.}{c\cdots}$$

$$= \frac{2b^2x}{c^2d} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^3d} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d} - \frac{2\left(a+b\sinh^{-1}\cdots\right.}{\cdots}$$

$$= \frac{2b^2x}{c^2d} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^3d} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d} - \frac{2\left(a+b\sinh^{-1}\cdots\right.}{\cdots}$$

$$= \frac{2b^2x}{c^2d} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^3d} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d} - \frac{2\left(a+b\sinh^{-1}\cdots\right.}{\cdots}$$

$$= \frac{2b^2x}{c^2d} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^3d} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d} - \frac{2\left(a+b\sinh^{-1}\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.59, size = 293, normalized size = 1.48

$$-\frac{a^2\tan^{-1}(cx)}{c^3d}+\frac{a^2x}{c^2d}+\frac{2ab\left(-\sqrt{c^2x^2+1}+i\left(\text{Li}_2\left(-ie^{-\sinh^{-1}(cx)}\right)-\text{Li}_2\left(ie^{-\sinh^{-1}(cx)}\right)\right)+cx\sinh^{-1}(cx)+i\sinh^{-1}}{c^3d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2),x]
```

```
[Out] (a^2*x)/(c^2*d) - (a^2*ArcTan[c*x])/(c^3*d) + (2*a*b*(-Sqrt[1 + c^2*x^2] +
c*x*ArcSinh[c*x] + I*ArcSinh[c*x]*(Log[1 - I/E^ArcSinh[c*x]] - Log[1 + I/E^
ArcSinh[c*x]]) + I*(PolyLog[2, (-I)/E^ArcSinh[c*x]] - PolyLog[2, I/E^ArcSin
h[c*x]])))/(c^3*d) + (b^2*(-2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + c*x*(2 + Arc
Sinh[c*x]^2) - I*(-(ArcSinh[c*x]^2*(Log[1 - I/E^ArcSinh[c*x]] - Log[1 + I/E
^ArcSinh[c*x]])) - 2*ArcSinh[c*x]*(PolyLog[2, (-I)/E^ArcSinh[c*x]] - PolyLo
g[2, I/E^ArcSinh[c*x]]) - 2*(PolyLog[3, (-I)/E^ArcSinh[c*x]] - PolyLog[3, I
/E^ArcSinh[c*x]]))))/(c^3*d)
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2x^2\operatorname{arsinh}(cx)^2+2\,abx^2\operatorname{arsinh}(cx)+a^2x^2}{c^2dx^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*x) + a^2*x^2)/(c^2*d
*x^2 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\operatorname{arsinh}(cx)+a)^2x^2}{c^2dx^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^2/(c^2*d*x^2 + d), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int\frac{x^2\,(a+b\operatorname{arcsinh}(cx))^2}{c^2d\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x)
```

```
[Out] int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\frac{x}{c^2d}-\frac{\arctan(cx)}{c^3d}\right)+\int\frac{b^2x^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^2dx^2+d}+\frac{2\,abx^2\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^2dx^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] a^2*(x/(c^2*d) - arctan(c*x)/(c^3*d)) + integrate(b^2*x^2*log(c*x + sqrt(c^
2*x^2 + 1))^2/(c^2*d*x^2 + d) + 2*a*b*x^2*log(c*x + sqrt(c^2*x^2 + 1))/(c^2
*d*x^2 + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,(a+b\,\mathrm{asinh}\,(c\,x))^2}{d\,c^2\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2),x)
```

```
[Out] int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2x^2}{c^2x^2+1}\,dx + \int \frac{b^2x^2\,\mathrm{asinh}^2\,(cx)}{c^2x^2+1}\,dx + \int \frac{2abx^2\,\mathrm{asinh}\,(cx)}{c^2x^2+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))**2/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a**2*x**2/(c**2*x**2 + 1), x) + Integral(b**2*x**2*asinh(c*x)**2/
(c**2*x**2 + 1), x) + Integral(2*a*b*x**2*asinh(c*x)/(c**2*x**2 + 1), x))/d
```

3.228 $\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx$

Optimal. Leaf size=105

$$\frac{b\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^2d}-\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3bc^2d}+\frac{\log\left(e^{2\sinh^{-1}(cx)}+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d}-\frac{b^2\text{Li}_3\left(\right.}{}$$

```
[Out] -1/3*(a+b*arcsinh(c*x))^3/b/c^2/d+(a+b*arcsinh(c*x))^2*ln(1+(c*x+(c^2*x^2+1
)^(1/2))^2)/c^2/d+b*(a+b*arcsinh(c*x))*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2
)/c^2/d-1/2*b^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/c^2/d
```

Rubi [A] time = 0.18, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5714, 3718, 2190, 2531, 2282, 6589}

$$\frac{b\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^2d}-\frac{b^2\text{PolyLog}\left(3,-e^{2\sinh^{-1}(cx)}\right)}{2c^2d}-\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3bc^2d}+\frac{\log\left(e^{2\sinh\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2),x]
```

```
[Out] -(a + b*ArcSinh[c*x])^3/(3*b*c^2*d) + ((a + b*ArcSinh[c*x])^2*Log[1 + E^(2*
ArcSinh[c*x])])/(c^2*d) + (b*(a + b*ArcSinh[c*x])*PolyLog[2, -E^(2*ArcSinh[
c*x])])/(c^2*d) - (b^2*PolyLog[3, -E^(2*ArcSinh[c*x])])/(2*c^2*d)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx = \frac{\text{Subst}\left(\int (a+bx)^2\tanh(x)\,dx, x, \sinh^{-1}(cx)\right)}{c^2d}$$
$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3bc^2d} + \frac{2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)^2}{1+e^{2x}}\,dx, x, \sinh^{-1}(cx)\right)}{c^2d}$$
$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3bc^2d} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+e^{2\sinh^{-1}(cx)}\right)}{c^2d} - \frac{(2b)\,\text{Subst}\left(\int (a\right.}{}$$
$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3bc^2d} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+e^{2\sinh^{-1}(cx)}\right)}{c^2d} + \frac{b\left(a+b\sinh^{-1}\right.}{}$$
$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3bc^2d} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+e^{2\sinh^{-1}(cx)}\right)}{c^2d} + \frac{b\left(a+b\sinh^{-1}\right.}{}$$
$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3bc^2d} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+e^{2\sinh^{-1}(cx)}\right)}{c^2d} + \frac{b\left(a+b\sinh^{-1}\right.}{}$$

Mathematica [B] time = 0.23, size = 281, normalized size = 2.68

$$\frac{3a^2\log\left(c^2x^2+1\right)+12b\text{Li}_2\left(\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right)\left(a+b\sinh^{-1}(cx)\right)+12b\text{Li}_2\left(\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right)\left(a+b\sinh^{-1}(cx)\right)+12ab\sin}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2),x]
```

```
[Out] (-6*a*b*ArcSinh[c*x]^2 - 2*b^2*ArcSinh[c*x]^3 + 12*a*b*ArcSinh[c*x]*Log[1 +
 (c*E^ArcSinh[c*x])/Sqrt[-c^2]] + 6*b^2*ArcSinh[c*x]^2*Log[1 + (c*E^ArcSinh
[c*x])/Sqrt[-c^2]] + 12*a*b*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x]
)/c] + 6*b^2*ArcSinh[c*x]^2*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + 3*a^2*
Log[1 + c^2*x^2] + 12*b*(a + b*ArcSinh[c*x])*PolyLog[2, (c*E^ArcSinh[c*x])/
Sqrt[-c^2]] + 12*b*(a + b*ArcSinh[c*x])*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*
x])/c] - 12*b^2*PolyLog[3, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - 12*b^2*PolyLog[
3, (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/(6*c^2*d)
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2x\,\text{arsinh}\,(cx)^2+2\,abx\,\text{arsinh}\,(cx)+a^2x}{c^2dx^2+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x*arcsinh(c*x)^2 + 2*a*b*x*arcsinh(c*x) + a^2*x)/(c^2*d*x^2 +
 d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2 x}{c^2 dx^2 + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x/(c^2*d*x^2 + d), x)
```

maple [A] time = 0.07, size = 223, normalized size = 2.12

$$\frac{a^2 \ln\left(c^2x^2+1\right)}{2c^2d} - \frac{b^2 \operatorname{arcsinh}(cx)^3}{3c^2d} + \frac{b^2 \operatorname{arcsinh}(cx)^2 \ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)}{c^2d} + \frac{b^2 \operatorname{arcsinh}(cx) \operatorname{polylog}\left(2, -\right.}{c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x)
```

```
[Out] 1/2/c^2*a^2/d*ln(c^2*x^2+1)-1/3/c^2*b^2/d*arcsinh(c*x)^3+1/c^2*b^2/d*arcsin
h(c*x)^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+1/c^2*b^2/d*arcsinh(c*x)*polylog(2
,-(c*x+(c^2*x^2+1)^(1/2))^2)-1/2*b^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/
c^2/d-1/c^2*a*b/d*arcsinh(c*x)^2+2/c^2*a*b/d*arcsinh(c*x)*ln(1+(c*x+(c^2*x^
2+1)^(1/2))^2)+1/c^2*a*b/d*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{b^2 \log\left(c^2x^2+1\right) \log\left(cx+\sqrt{c^2x^2+1}\right)^2}{2\,c^2d} + \frac{a^2 \log\left(c^2dx^2+d\right)}{2\,c^2d} - \int -\frac{\left(2\,abc^2x^2 - \left(b^2c^2x^2+b^2\right) \log\left(c^2x^2+1\right) - \left(\right.\right.}{c^4dx^3+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/2*b^2*log(c^2*x^2 + 1)*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d) + 1/2*a^2*l
og(c^2*d*x^2 + d)/(c^2*d) - integrate(-(2*a*b*c^2*x^2 - (b^2*c^2*x^2 + b^2)
*log(c^2*x^2 + 1) - (b^2*c*x*log(c^2*x^2 + 1) - 2*a*b*c*x)*sqrt(c^2*x^2 + 1
))*log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d*x^3 + c^2*d*x + (c^3*d*x^2 + c*d)*sq
rt(c^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2}{d\,c^2\,x^2+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2),x)
```

```
[Out] int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2x}{c^2x^2+1}\, dx + \int \frac{b^2x \operatorname{asinh}^2(cx)}{c^2x^2+1}\, dx + \int \frac{2abx \operatorname{asinh}(cx)}{c^2x^2+1}\, dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))**2/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a**2*x/(c**2*x**2 + 1), x) + Integral(b**2*x*asinh(c*x)**2/(c**2*
x**2 + 1), x) + Integral(2*a*b*x*asinh(c*x)/(c**2*x**2 + 1), x))/d
```

3.229 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx$

Optimal. Leaf size=138

$$-\frac{2ib\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{cd}+\frac{2ib\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{cd}+\frac{2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\,\text{si}\right.}{cd}$$

```
[Out] 2*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(1/2))/c/d-2*I*b*(a+b*arcsinh
(c*x))*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/d+2*I*b*(a+b*arcsinh(c*x))*p
olylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/d+2*I*b^2*polylog(3,-I*(c*x+(c^2*x^2+
1)^(1/2)))/c/d-2*I*b^2*polylog(3,I*(c*x+(c^2*x^2+1)^(1/2)))/c/d
```

Rubi [A] time = 0.12, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5693, 4180, 2531, 2282, 6589}

$$-\frac{2ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{cd}+\frac{2ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{cd}+\frac{2ib^2\text{PolyLo}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2),x]
```

```
[Out] (2*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]])/(c*d) - ((2*I)*b*(a + b*A
rcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*d) + ((2*I)*b*(a + b*ArcSi
nh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])/(c*d) + ((2*I)*b^2*PolyLog[3, (-I)*E
^ArcSinh[c*x]])/(c*d) - ((2*I)*b^2*PolyLog[3, I*E^ArcSinh[c*x]])/(c*d)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx = \frac{\text{Subst}\left(\int (a+bx)^2\text{sech}(x)\,dx,x,\sinh^{-1}(cx)\right)}{cd}$$
$$= \frac{2\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{cd} - \frac{(2ib)\,\text{Subst}\left(\int (a+bx)\log\left(1-ie^x\right)\,dx,x,s\right.}{cd}$$
$$= \frac{2\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{cd} - \frac{2ib\left(a+b\sinh^{-1}(cx)\right)\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{cd} +$$
$$= \frac{2\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{cd} - \frac{2ib\left(a+b\sinh^{-1}(cx)\right)\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{cd} +$$
$$= \frac{2\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{cd} - \frac{2ib\left(a+b\sinh^{-1}(cx)\right)\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{cd} +$$

Mathematica [A] time = 0.24, size = 274, normalized size = 1.99

$$-\frac{c\left(a^2\sqrt{-c^2}\tan^{-1}(cx)+2bc\text{Li}_2\left(\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right)\left(a+b\sinh^{-1}(cx)\right)-2bc\text{Li}_2\left(\frac{\sqrt{-c^2}\,e^{\sinh^{-1}(cx)}}{c}\right)\left(a+b\sinh^{-1}(cx)\right)-2abc\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2),x]
```

```
[Out] -((c*(a^2*Sqrt[-c^2]*ArcTan[c*x] - 2*a*b*c*ArcSinh[c*x]*Log[1 + (c*E^ArcSin
h[c*x])/Sqrt[-c^2]] - b^2*c*ArcSinh[c*x]^2*Log[1 + (c*E^ArcSinh[c*x])/Sqrt[
-c^2]] + 2*a*b*c*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + b^2*
c*ArcSinh[c*x]^2*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + 2*b*c*(a + b*ArcS
inh[c*x])*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - 2*b*c*(a + b*ArcSinh[
c*x])*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - 2*b^2*c*PolyLog[3, (c*E^A
rcSinh[c*x])/Sqrt[-c^2]] + 2*b^2*c*PolyLog[3, (Sqrt[-c^2]*E^ArcSinh[c*x])/c
]))/((-c^2)^(3/2)*d))
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\,\text{arsinh}\,(cx)^2+2\,ab\,\text{arsinh}\,(cx)+a^2}{c^2dx^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^2*d*x^2 + d), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\text{arsinh}\,(cx)+a\right)^2}{c^2dx^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(c^2*d*x^2 + d), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2}{c^2d\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2\arctan(cx)}{cd}+\int \frac{b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^2dx^2+d}+\frac{2\,ab\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^2dx^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] a^2*arctan(c*x)/(c*d) + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d
*x^2 + d) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))^2}{d\,c^2\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(d + c^2*d*x^2),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(d + c^2*d*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^2x^2+1}\,dx+\int \frac{b^2\,\mathrm{asinh}^2\,(cx)}{c^2x^2+1}\,dx+\int \frac{2ab\,\mathrm{asinh}\,(cx)}{c^2x^2+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a**2/(c**2*x**2 + 1), x) + Integral(b**2*asinh(c*x)**2/(c**2*x**2
 + 1), x) + Integral(2*a*b*asinh(c*x)/(c**2*x**2 + 1), x))/d
```

3.230 $\int \frac{(a+b\sinh^{-1}(cx))^2}{x(d+c^2dx^2)}\,dx$

Optimal. Leaf size=116

$$-\frac{b\mathrm{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}+\frac{b\mathrm{Li}_2\left(e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}-\frac{2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh\right.}{d}$$

```
[Out] -2*(a+b*arcsinh(c*x))^2*arctanh((c*x+(c^2*x^2+1)^(1/2))^2)/d-b*(a+b*arcsinh
(c*x))*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d+b*(a+b*arcsinh(c*x))*polylog
(2,(c*x+(c^2*x^2+1)^(1/2))^2)/d+1/2*b^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^
2)/d-1/2*b^2*polylog(3,(c*x+(c^2*x^2+1)^(1/2))^2)/d
```

Rubi [A] time = 0.21, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5720, 5461, 4182, 2531, 2282, 6589}

$$-\frac{b\mathrm{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}+\frac{b\mathrm{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}+\frac{b^2\mathrm{PolyLog}\left(3,-e\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)),x]
```

```
[Out] (-2*(a + b*ArcSinh[c*x])^2*ArcTanh[E^(2*ArcSinh[c*x])])/d - (b*(a + b*ArcSi
nh[c*x])*PolyLog[2, -E^(2*ArcSinh[c*x])])/d + (b*(a + b*ArcSinh[c*x])*PolyL
og[2, E^(2*ArcSinh[c*x])])/d + (b^2*PolyLog[3, -E^(2*ArcSinh[c*x])])/(2*d)
- (b^2*PolyLog[3, E^(2*ArcSinh[c*x])])/(2*d)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5720

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)}\,dx = \frac{\text{Subst}\left(\int (a+bx)^2\text{csch}(x)\text{sech}(x)\,dx, x, \sinh^{-1}(cx)\right)}{d}$$

$$= \frac{2\,\text{Subst}\left(\int (a+bx)^2\text{csch}(2x)\,dx, x, \sinh^{-1}(cx)\right)}{d}$$

$$= -\frac{2\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d} - \frac{(2b)\,\text{Subst}\left(\int (a+bx)\log\left(1-e^{2x}\right)\right.}{d}$$

$$= -\frac{2\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d} - \frac{b\left(a+b\sinh^{-1}(cx)\right)\text{Li}_2\left(-e^{2\sinh^{-1}(c\right.}}{d}$$

$$= -\frac{2\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d} - \frac{b\left(a+b\sinh^{-1}(cx)\right)\text{Li}_2\left(-e^{2\sinh^{-1}(c\right.}}{d}$$

$$= -\frac{2\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d} - \frac{b\left(a+b\sinh^{-1}(cx)\right)\text{Li}_2\left(-e^{2\sinh^{-1}(c\right.}}{d}$$

Mathematica [B] time = 0.36, size = 400, normalized size = 3.45

$$-\frac{2a^3+3a^2b\log\left(c^2x^2+1\right)+6a^2b\sinh^{-1}(cx)-6a^2b\log\left(1-e^{2\sinh^{-1}(cx)}\right)+12b^2\text{Li}_2\left(\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right)\left(a+b\sinh^{-1}(\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)),x]
```

```
[Out] -1/6*(2*a^3 + 6*a^2*b*ArcSinh[c*x] + 12*a*b^2*ArcSinh[c*x]*Log[1 + (c*E^Arc
Sinh[c*x])/Sqrt[-c^2]] + 6*b^3*ArcSinh[c*x]^2*Log[1 + (c*E^ArcSinh[c*x])/Sq
rt[-c^2]] + 12*a*b^2*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] +
6*b^3*ArcSinh[c*x]^2*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - 6*a^2*b*Log[1
 - E^(2*ArcSinh[c*x])] - 12*a*b^2*ArcSinh[c*x]*Log[1 - E^(2*ArcSinh[c*x])]
- 6*b^3*ArcSinh[c*x]^2*Log[1 - E^(2*ArcSinh[c*x])] + 3*a^2*b*Log[1 + c^2*x^
2] + 12*b^2*(a + b*ArcSinh[c*x])*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]]
+ 12*b^2*(a + b*ArcSinh[c*x])*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - 6
*a*b^2*PolyLog[2, E^(2*ArcSinh[c*x])] - 6*b^3*ArcSinh[c*x]*PolyLog[2, E^(2*
ArcSinh[c*x])] - 12*b^3*PolyLog[3, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - 12*b^3*
PolyLog[3, (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + 3*b^3*PolyLog[3, E^(2*ArcSinh[c
*x])])/(b*d)
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\,\text{arsinh}\,(cx)^2+2\,ab\,\text{arsinh}\,(cx)+a^2}{c^2dx^3+dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^2*d*x^3 + d*x),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2}{(c^2dx^2 + d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)*x), x)
```

maple [B] time = 0.11, size = 354, normalized size = 3.05

$$\frac{a^2\ln(cx)}{d} - \frac{a^2\ln\left(c^2x^2+1\right)}{2d} + \frac{b^2\operatorname{arcsinh}(cx)^2\ln\left(1+cx+\sqrt{c^2x^2+1}\right)}{d} + \frac{2b^2\operatorname{arcsinh}(cx)\operatorname{polylog}\left(2,-cx-\sqrt{c^2x^2}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d),x)
```

```
[Out] a^2/d*ln(c*x)-1/2*a^2/d*ln(c^2*x^2+1)+b^2/d*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^
2+1)^(1/2))+2*b^2/d*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))-2*b^2/d*
polylog(3,-c*x-(c^2*x^2+1)^(1/2))+b^2/d*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)
^(1/2))+2*b^2/d*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))-2*b^2/d*polyl
og(3,c*x+(c^2*x^2+1)^(1/2))-b^2/d*arcsinh(c*x)^2*ln(1+(c*x+(c^2*x^2+1)^(1/2
))^2)-b^2/d*arcsinh(c*x)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)+1/2*b^2*poly
log(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/d+2*a*b/d*dilog(1/(c*x+(c^2*x^2+1)^(1/2))
^2)-1/2*a*b/d*dilog(1/(c*x+(c^2*x^2+1)^(1/2))^4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}a^2\left(\frac{\log\left(c^2x^2+1\right)}{d} - \frac{2\log(x)}{d}\right) + \int \frac{b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^2dx^3+dx} + \frac{2ab\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^2dx^3+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/2*a^2*(log(c^2*x^2 + 1)/d - 2*log(x)/d) + integrate(b^2*log(c*x + sqrt(c
^2*x^2 + 1))^2/(c^2*d*x^3 + d*x) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*
d*x^3 + d*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\operatorname{asinh}(c\,x))^2}{x\,\left(d\,c^2\,x^2 + d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^2x^3+x}\,dx + \int \frac{b^2 \operatorname{asinh}^2(cx)}{c^2x^3+x}\,dx + \int \frac{2ab \operatorname{asinh}(cx)}{c^2x^3+x}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a**2/(c**2*x**3 + x), x) + Integral(b**2*asinh(c*x)**2/(c**2*x**3
 + x), x) + Integral(2*a*b*asinh(c*x)/(c**2*x**3 + x), x))/d
```

3.231 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)}\,dx$

Optimal. Leaf size=204

$$\frac{2ibc\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}-\frac{2ibc\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx}-\frac{2c\tan^{-1}(}{}$$

```
[Out] -(a+b*arcsinh(c*x))^2/d/x-2*c*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(
1/2))/d-4*b*c*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))/d-2*b^2*c*p
olylog(2,-c*x-(c^2*x^2+1)^(1/2))/d+2*I*b*c*(a+b*arcsinh(c*x))*polylog(2,-I*
(c*x+(c^2*x^2+1)^(1/2)))/d-2*I*b*c*(a+b*arcsinh(c*x))*polylog(2,I*(c*x+(c^2
*x^2+1)^(1/2)))/d+2*b^2*c*polylog(2,c*x+(c^2*x^2+1)^(1/2))/d-2*I*b^2*c*poly
log(3,-I*(c*x+(c^2*x^2+1)^(1/2)))/d+2*I*b^2*c*polylog(3,I*(c*x+(c^2*x^2+1)^
(1/2)))/d
```

Rubi [A] time = 0.34, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5747, 5693, 4180, 2531, 2282, 6589, 5760, 4182, 2279, 2391}

$$\frac{2ibc\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}-\frac{2ibc\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}-\frac{2b^2c\text{PolyLog}(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)),x]
```

```
[Out] -((a + b*ArcSinh[c*x])^2/(d*x)) - (2*c*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcS
inh[c*x]])/d - (4*b*c*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/d - (2*
b^2*c*PolyLog[2, -E^ArcSinh[c*x]])/d + ((2*I)*b*c*(a + b*ArcSinh[c*x])*Poly
Log[2, (-I)*E^ArcSinh[c*x]])/d - ((2*I)*b*c*(a + b*ArcSinh[c*x])*PolyLog[2,
 I*E^ArcSinh[c*x]])/d + (2*b^2*c*PolyLog[2, E^ArcSinh[c*x]])/d - ((2*I)*b^2
*c*PolyLog[3, (-I)*E^ArcSinh[c*x]])/d + ((2*I)*b^2*c*PolyLog[3, I*E^ArcSinh
[c*x]])/d
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx} - c^2\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx + \frac{(2bc)\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{1+c^2x^2}}\,dx}{d}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx} - \frac{c\,\text{Subst}\left(\int (a+bx)^2\text{sech}(x)\,dx, x, \sinh^{-1}(cx)\right)}{d} + \frac{(2bc)\,\text{Subst}}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx} - \frac{2c\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{d} - \frac{4bc\left(a+b\sinh^{-1}\right.}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx} - \frac{2c\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{d} - \frac{4bc\left(a+b\sinh^{-1}\right.}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx} - \frac{2c\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{d} - \frac{4bc\left(a+b\sinh^{-1}\right.}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx} - \frac{2c\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{d} - \frac{4bc\left(a+b\sinh^{-1}\right.}{}$$

Mathematica [A] time = 1.06, size = 363, normalized size = 1.78

$$-\frac{a^2c\tan^{-1}(cx) + \frac{a^2}{x} + 2abc\tanh^{-1}\left(\sqrt{c^2x^2+1}\right) + \frac{1}{2}iabc\left(\sinh^{-1}(cx)\left(\sinh^{-1}(cx) - 4\log\left(1+ie^{\sinh^{-1}(cx)}\right)\right) - 4\text{Li}_2\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)),x]
```

```
[Out] -((a^2/x + (2*a*b*ArcSinh[c*x])/x + a^2*c*ArcTan[c*x] + 2*a*b*c*ArcTanh[Sqr
t[1 + c^2*x^2]] + (I/2)*a*b*c*(ArcSinh[c*x]*(ArcSinh[c*x] - 4*Log[1 + I*E^A
rcSinh[c*x]]) - 4*PolyLog[2, (-I)*E^ArcSinh[c*x]]) - (I/2)*a*b*c*(ArcSinh[c
*x]*(ArcSinh[c*x] - 4*Log[1 - I*E^ArcSinh[c*x]]) - 4*PolyLog[2, I*E^ArcSinh
[c*x]]) - b^2*c*(-(ArcSinh[c*x]^2/(c*x)) + 2*ArcSinh[c*x]*Log[1 - E^(-ArcSi
nh[c*x])] + I*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] - I*ArcSinh[c*x]^2*L
og[1 + I/E^ArcSinh[c*x]] - 2*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 2*Po
lyLog[2, -E^(-ArcSinh[c*x])] + (2*I)*ArcSinh[c*x]*PolyLog[2, (-I)/E^ArcSinh
[c*x]] - (2*I)*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[c*x]] - 2*PolyLog[2, E^(
-ArcSinh[c*x])] + (2*I)*PolyLog[3, (-I)/E^ArcSinh[c*x]] - (2*I)*PolyLog[3,
I/E^ArcSinh[c*x]]))/d)
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\,\text{arsinh}(cx)^2 + 2\,ab\,\text{arsinh}(cx) + a^2}{c^2dx^4 + dx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^2*d*x^4 + d*x^2
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arsinh}(cx) + a)^2}{\left(c^2dx^2 + d\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)*x^2), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2}{x^2\left(c^2d\,x^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\frac{c\arctan\,(cx)}{d}+\frac{1}{dx}\right)+\int \frac{b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^2dx^4+dx^2}+\frac{2\,ab\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^2dx^4+dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] -a^2*(c*arctan(c*x)/d + 1/(d*x)) + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1
))^2/(c^2*d*x^4 + d*x^2) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^4 +
d*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))^2}{x^2\,\left(d\,c^2\,x^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^2x^4+x^2}\,dx+\int \frac{b^2\,\mathrm{asinh}^2\,(cx)}{c^2x^4+x^2}\,dx+\int \frac{2ab\,\mathrm{asinh}\,(cx)}{c^2x^4+x^2}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**2/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a**2/(c**2*x**4 + x**2), x) + Integral(b**2*asinh(c*x)**2/(c**2*x
**4 + x**2), x) + Integral(2*a*b*asinh(c*x)/(c**2*x**4 + x**2), x))/d
```

3.232 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^3\left(d+c^2dx^2\right)}\,dx$

Optimal. Leaf size=194

$$\frac{bc^2\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}-\frac{bc^2\text{Li}_2\left(e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}-\frac{bc\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{dx}$$

```
[Out] -1/2*(a+b*arcsinh(c*x))^2/d/x^2+2*c^2*(a+b*arcsinh(c*x))^2*arctanh((c*x+(c^
2*x^2+1)^(1/2))^2)/d+b^2*c^2*ln(x)/d+b*c^2*(a+b*arcsinh(c*x))*polylog(2,-(c
*x+(c^2*x^2+1)^(1/2))^2)/d-b*c^2*(a+b*arcsinh(c*x))*polylog(2,(c*x+(c^2*x^2
+1)^(1/2))^2)/d-1/2*b^2*c^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/d+1/2*b^2
*c^2*polylog(3,(c*x+(c^2*x^2+1)^(1/2))^2)/d-b*c*(a+b*arcsinh(c*x))*(c^2*x^2
+1)^(1/2)/d/x
```

Rubi [A] time = 0.39, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5747, 5720, 5461, 4182, 2531, 2282, 6589, 5723, 29}

$$\frac{bc^2\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}-\frac{bc^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}-\frac{b^2c^2\text{PolyLog}(\ldots}{2\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)),x]
```

```
[Out] -((b*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(d*x)) - (a + b*ArcSinh[c*x]
)^2/(2*d*x^2) + (2*c^2*(a + b*ArcSinh[c*x])^2*ArcTanh[E^(2*ArcSinh[c*x])])/
d + (b^2*c^2*Log[x])/d + (b*c^2*(a + b*ArcSinh[c*x])*PolyLog[2, -E^(2*ArcSi
nh[c*x])])/d - (b*c^2*(a + b*ArcSinh[c*x])*PolyLog[2, E^(2*ArcSinh[c*x])])/
d - (b^2*c^2*PolyLog[3, -E^(2*ArcSinh[c*x])])/(2*d) + (b^2*c^2*PolyLog[3, E
^(2*ArcSinh[c*x])])/(2*d)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
```

```
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5720

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^3\left(d+c^2dx^2\right)}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} - c^2\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)}\,dx + \frac{(bc)\int \frac{a+b\sinh^{-1}(cx)}{x^2\sqrt{1+c^2x^2}}\,dx}{d}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{dx} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} - \frac{c^2\,\text{Subst}\left(\int (a+bx)^2\text{csch}\right.}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{dx} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} + \frac{b^2c^2\log(x)}{d} - \frac{\left(2c^2\right)\text{Subst}}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{dx} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} + \frac{2c^2\left(a+b\sinh^{-1}(cx)\right)^2\text{ta}}{d}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{dx} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} + \frac{2c^2\left(a+b\sinh^{-1}(cx)\right)^2\text{ta}}{d}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{dx} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} + \frac{2c^2\left(a+b\sinh^{-1}(cx)\right)^2\text{ta}}{d}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{dx} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} + \frac{2c^2\left(a+b\sinh^{-1}(cx)\right)^2\text{ta}}{d}$$

Mathematica [C] time = 1.05, size = 419, normalized size = 2.16

$$-\frac{-a^2c^2\log\left(c^2x^2+1\right)+2a^2c^2\log(x)+\frac{a^2}{x^2}-4abc^2\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)-4abc^2\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)+2abc^2\text{Li}_2\left(e^{2\sinh^{-1}(c}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)),x]
```

```
[Out] -1/2*((I/12)*b^2*c^2*Pi^3 + a^2/x^2 + (2*a*b*c*Sqrt[1 + c^2*x^2])/x + (2*a*
b*ArcSinh[c*x])/x^2 + (2*b^2*c*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/x + (b^2*Arc
Sinh[c*x]^2)/x^2 - (4*b^2*c^2*ArcSinh[c*x]^3)/3 - 2*b^2*c^2*ArcSinh[c*x]^2*
Log[1 + E^(-2*ArcSinh[c*x])] - 4*a*b*c^2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c
*x]] - 4*a*b*c^2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] + 4*a*b*c^2*ArcSinh
[c*x]*Log[1 - E^(2*ArcSinh[c*x])] + 2*b^2*c^2*ArcSinh[c*x]^2*Log[1 - E^(2*A
rcSinh[c*x])] + 2*a^2*c^2*Log[x] - 2*b^2*c^2*Log[c*x] - a^2*c^2*Log[1 + c^2
*x^2] + 2*b^2*c^2*ArcSinh[c*x]*PolyLog[2, -E^(-2*ArcSinh[c*x])] - 4*a*b*c^2
*PolyLog[2, (-I)*E^ArcSinh[c*x]] - 4*a*b*c^2*PolyLog[2, I*E^ArcSinh[c*x]] +
 2*a*b*c^2*PolyLog[2, E^(2*ArcSinh[c*x])] + 2*b^2*c^2*ArcSinh[c*x]*PolyLog[
2, E^(2*ArcSinh[c*x])] + b^2*c^2*PolyLog[3, -E^(-2*ArcSinh[c*x])] - b^2*c^2
*PolyLog[3, E^(2*ArcSinh[c*x])])/d
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\,\text{arsinh}\,(cx)^2+2\,ab\,\text{arsinh}\,(cx)+a^2}{c^2dx^5+dx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^2*d*x^5 + d*x^3
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\text{arsinh}\,(cx)+a\right)^2}{\left(c^2dx^2+d\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)*x^3), x)
```

maple [B] time = 0.21, size = 719, normalized size = 3.71

$$\frac{2c^2b^2\,\mathrm{polylog}\left(3,cx+\sqrt{c^2x^2+1}\right)}{d}+\frac{c^2b^2\,\mathrm{arcsinh}(cx)}{d}+\frac{c^2a^2\ln\left(c^2x^2+1\right)}{2d}-\frac{c^2a^2\ln(cx)}{d}-\frac{2c^2b^2\ln\left(cx+\sqrt{c^2x^2}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d),x)
```

```
[Out] -1/2*b^2*c^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/d-1/2*b^2/d*arcsinh(c*x)
^2/x^2+1/2*c^2*a^2/d*ln(c^2*x^2+1)-c^2*a^2/d*ln(c*x)+2*c^2*b^2/d*polylog(3,
-c*x-(c^2*x^2+1)^(1/2))+2*c^2*b^2/d*polylog(3,c*x+(c^2*x^2+1)^(1/2))+c^2*b^
2/d*arcsinh(c*x)-2*c^2*b^2/d*ln(c*x+(c^2*x^2+1)^(1/2))+c^2*b^2/d*ln(c*x+(c^
2*x^2+1)^(1/2)-1)+c^2*b^2/d*ln(1+c*x+(c^2*x^2+1)^(1/2))+c^2*a*b/d+2*c^2*a*b
/d*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-c*b^2/d*arcsinh(c*x)/x*(c^2
*x^2+1)^(1/2)-c*a*b/d/x*(c^2*x^2+1)^(1/2)-2*c^2*a*b/d*arcsinh(c*x)*ln(1+c*x
+(c^2*x^2+1)^(1/2))-2*c^2*a*b/d*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-2*
c^2*b^2/d*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))-2*c^2*a*b/d*polylog
(2,-c*x-(c^2*x^2+1)^(1/2))-2*c^2*a*b/d*polylog(2,c*x+(c^2*x^2+1)^(1/2))-a*b
/d*arcsinh(c*x)/x^2+c^2*b^2/d*arcsinh(c*x)^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2
)+c^2*b^2/d*arcsinh(c*x)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)+c^2*a*b/d*po
lylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)-c^2*b^2/d*arcsinh(c*x)^2*ln(1+c*x+(c^2*
x^2+1)^(1/2))-2*c^2*b^2/d*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))-1/
2*a^2/d/x^2-c^2*b^2/d*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(\frac{c^2\log\left(c^2x^2+1\right)}{d}-\frac{2\,c^2\log(x)}{d}-\frac{1}{dx^2}\right)a^2+\int\frac{b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^2dx^5+dx^3}+\frac{2\,ab\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^2dx^5+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/2*(c^2*log(c^2*x^2 + 1)/d - 2*c^2*log(x)/d - 1/(d*x^2))*a^2 + integrate(b
^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d*x^5 + d*x^3) + 2*a*b*log(c*x + sqr
t(c^2*x^2 + 1))/(c^2*d*x^5 + d*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{x^3\,\left(d\,c^2\,x^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{a^2}{c^2x^5+x^3}\,dx+\int\frac{b^2\,\mathrm{asinh}^2(cx)}{c^2x^5+x^3}\,dx+\int\frac{2ab\,\mathrm{asinh}(cx)}{c^2x^5+x^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**3/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a**2/(c**2*x**5 + x**3), x) + Integral(b**2*asinh(c*x)**2/(c**2*x
**5 + x**3), x) + Integral(2*a*b*asinh(c*x)/(c**2*x**5 + x**3), x))/d
```

3.233 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^4\left(d+c^2dx^2\right)}\,dx$

Optimal. Leaf size=297

$$-\frac{2ibc^3\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}+\frac{2ibc^3\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}+\frac{2c^3\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a\right.}{d}$$

```
[Out] -1/3*b^2*c^2/d/x-1/3*(a+b*arcsinh(c*x))^2/d/x^3+c^2*(a+b*arcsinh(c*x))^2/d/
x+2*c^3*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(1/2))/d+14/3*b*c^3*(a+
b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))/d+7/3*b^2*c^3*polylog(2,-c*x
-(c^2*x^2+1)^(1/2))/d-2*I*b*c^3*(a+b*arcsinh(c*x))*polylog(2,-I*(c*x+(c^2*x
^2+1)^(1/2)))/d+2*I*b*c^3*(a+b*arcsinh(c*x))*polylog(2,I*(c*x+(c^2*x^2+1)^(
1/2)))/d-7/3*b^2*c^3*polylog(2,c*x+(c^2*x^2+1)^(1/2))/d+2*I*b^2*c^3*polylog
(3,-I*(c*x+(c^2*x^2+1)^(1/2)))/d-2*I*b^2*c^3*polylog(3,I*(c*x+(c^2*x^2+1)^(
1/2)))/d-1/3*b*c*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/d/x^2
```

Rubi [A] time = 0.64, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {5747, 5693, 4180, 2531, 2282, 6589, 5760, 4182, 2279, 2391, 30}

$$-\frac{2ibc^3\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}+\frac{2ibc^3\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}+\frac{7b^2c^3\text{Po}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)),x]
```

```
[Out] -(b^2*c^2)/(3*d*x) - (b*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(3*d*x^2)
 - (a + b*ArcSinh[c*x])^2/(3*d*x^3) + (c^2*(a + b*ArcSinh[c*x])^2)/(d*x) +
(2*c^3*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]])/d + (14*b*c^3*(a + b*
ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/(3*d) + (7*b^2*c^3*PolyLog[2, -E^Arc
Sinh[c*x]])/(3*d) - ((2*I)*b*c^3*(a + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^Arc
Sinh[c*x]])/d + ((2*I)*b*c^3*(a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*
x]])/d - (7*b^2*c^3*PolyLog[2, E^ArcSinh[c*x]])/(3*d) + ((2*I)*b^2*c^3*Poly
Log[3, (-I)*E^ArcSinh[c*x]])/d - ((2*I)*b^2*c^3*PolyLog[3, I*E^ArcSinh[c*x]
])/d
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^4\left(d+c^2dx^2\right)}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3} - c^2\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)}\,dx + \frac{(2bc)\int \frac{a+b\sinh^{-1}(cx)}{x^3\sqrt{1+c^2x^2}}\,dx}{3d}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3} + \frac{c^2\left(a+b\sinh^{-1}(cx)\right)^2}{dx}$$

$$= -\frac{b^2c^2}{3dx} - \frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3} + \frac{c^2\left(a+b\sinh^{-1}\right.}{dx}$$

$$= -\frac{b^2c^2}{3dx} - \frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3} + \frac{c^2\left(a+b\sinh^{-1}\right.}{dx}$$

$$= -\frac{b^2c^2}{3dx} - \frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3} + \frac{c^2\left(a+b\sinh^{-1}\right.}{dx}$$

$$= -\frac{b^2c^2}{3dx} - \frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3} + \frac{c^2\left(a+b\sinh^{-1}\right.}{dx}$$

$$= -\frac{b^2c^2}{3dx} - \frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3} + \frac{c^2\left(a+b\sinh^{-1}\right.}{dx}$$

Mathematica [B] time = 7.96, size = 602, normalized size = 2.03

$$\frac{a^2c^3\tan^{-1}(cx)}{d} + \frac{a^2c^2}{dx} - \frac{a^2}{3dx^3} + \frac{2ab\left(-\frac{1}{2}ic^4\left(\frac{2\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{c} - \frac{\sinh^{-1}(cx)^2}{2c} + \frac{2\sinh^{-1}(cx)\log\left(1+ie^{\sinh^{-1}(cx)}\right)}{c}\right) + \frac{1}{2}ic^4\left(\frac{2\text{Li}_2(\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)),x]
```

```
[Out] -1/3*a^2/(d*x^3) + (a^2*c^2)/(d*x) + (a^2*c^3*ArcTan[c*x])/d + (2*a*b*(-1/6
*(c*Sqrt[1 + c^2*x^2])/x^2 - ArcSinh[c*x]/(3*x^3) + (c^3*ArcTanh[Sqrt[1 + c
^2*x^2]])/6 - c^2*(-(ArcSinh[c*x]/x) - c*ArcTanh[Sqrt[1 + c^2*x^2]]) - (I/2
)*c^4*(-1/2*ArcSinh[c*x]^2/c + (2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]])/c
 + (2*PolyLog[2, (-I)*E^ArcSinh[c*x]])/c) + (I/2)*c^4*(-1/2*ArcSinh[c*x]^2/
c + (2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]])/c + (2*PolyLog[2, I*E^ArcSin
h[c*x]])/c)))/d + (b^2*c^3*(-4*Coth[ArcSinh[c*x]/2] + 14*ArcSinh[c*x]^2*Cot
h[ArcSinh[c*x]/2] - 2*ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - (c*x*ArcSinh[c*
x]^2*Csch[ArcSinh[c*x]/2]^4)/2 - 56*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])]
 - (24*I)*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] + (24*I)*ArcSinh[c*x]^2*
Log[1 + I/E^ArcSinh[c*x]] + 56*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] - 56
*PolyLog[2, -E^(-ArcSinh[c*x])] - (48*I)*ArcSinh[c*x]*PolyLog[2, (-I)/E^Arc
Sinh[c*x]] + (48*I)*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[c*x]] + 56*PolyLog[
2, E^(-ArcSinh[c*x])] - (48*I)*PolyLog[3, (-I)/E^ArcSinh[c*x]] + (48*I)*Pol
yLog[3, I/E^ArcSinh[c*x]] - 2*ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 - (8*ArcS
inh[c*x]^2*Sinh[ArcSinh[c*x]/2]^4)/(c^3*x^3) + 4*Tanh[ArcSinh[c*x]/2] - 14*
ArcSinh[c*x]^2*Tanh[ArcSinh[c*x]/2]))/(24*d)
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\operatorname{arsinh}(cx)^2 + 2\,ab\operatorname{arsinh}(cx) + a^2}{c^2dx^6 + dx^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^2*d*x^6 + d*x^4
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2}{(c^2dx^2 + d)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)*x^4), x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arcsinh}(cx))^2}{x^4 (c^2 d\, x^2 + d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\left(\frac{3\, c^3 \arctan(cx)}{d} + \frac{3\, c^2x^2 - 1}{dx^3}\right)a^2 + \int \frac{b^2 \log\left(cx + \sqrt{c^2x^2 + 1}\right)^2}{c^2dx^6 + dx^4} + \frac{2\, ab \log\left(cx + \sqrt{c^2x^2 + 1}\right)}{c^2dx^6 + dx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/3*(3*c^3*arctan(c*x)/d + (3*c^2*x^2 - 1)/(d*x^3))*a^2 + integrate(b^2*log
(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d*x^6 + d*x^4) + 2*a*b*log(c*x + sqrt(c^2*
x^2 + 1))/(c^2*d*x^6 + d*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{asinh}(c\, x))^2}{x^4 \left(d\, c^2\, x^2 + d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^2x^6+x^4}\,dx + \int \frac{b^2 \operatorname{asinh}^2(cx)}{c^2x^6+x^4}\,dx + \int \frac{2ab \operatorname{asinh}(cx)}{c^2x^6+x^4}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**4/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a**2/(c**2*x**6 + x**4), x) + Integral(b**2*asinh(c*x)**2/(c**2*x
**6 + x**4), x) + Integral(2*a*b*asinh(c*x)/(c**2*x**6 + x**4), x))/d
```

3.234 $\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=279

$$\frac{3ib\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2}-\frac{3ib\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2}-\frac{3\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh\right.}{c^5d^2}$$

```
[Out] 2*b^2*x/c^4/d^2+3/2*x*(a+b*arcsinh(c*x))^2/c^4/d^2-1/2*x^3*(a+b*arcsinh(c*x
))^2/c^2/d^2/(c^2*x^2+1)-3*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(1/2
))/c^5/d^2-b^2*arctan(c*x)/c^5/d^2+3*I*b*(a+b*arcsinh(c*x))*polylog(2,-I*(c
*x+(c^2*x^2+1)^(1/2)))/c^5/d^2-3*I*b*(a+b*arcsinh(c*x))*polylog(2,I*(c*x+(c
^2*x^2+1)^(1/2)))/c^5/d^2-3*I*b^2*polylog(3,-I*(c*x+(c^2*x^2+1)^(1/2)))/c^5
/d^2+3*I*b^2*polylog(3,I*(c*x+(c^2*x^2+1)^(1/2)))/c^5/d^2+b*(a+b*arcsinh(c*
x))/c^5/d^2/(c^2*x^2+1)^(1/2)-2*b*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^5/
d^2
```

Rubi [A] time = 0.53, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 14, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {5751, 5767, 5693, 4180, 2531, 2282, 6589, 5717, 8, 266, 43, 5732, 388, 205}

$$\frac{3ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2}-\frac{3ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2}-\frac{3ib^2\text{PolyLog}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^2,x]
```

```
[Out] (2*b^2*x)/(c^4*d^2) + (b*(a + b*ArcSinh[c*x]))/(c^5*d^2*Sqrt[1 + c^2*x^2])
- (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(c^5*d^2) + (3*x*(a + b*ArcS
inh[c*x])^2)/(2*c^4*d^2) - (x^3*(a + b*ArcSinh[c*x])^2)/(2*c^2*d^2*(1 + c^2
*x^2)) - (3*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]])/(c^5*d^2) - (b^2
*ArcTan[c*x])/(c^5*d^2) + ((3*I)*b*(a + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^A
rcSinh[c*x]])/(c^5*d^2) - ((3*I)*b*(a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcS
inh[c*x]])/(c^5*d^2) - ((3*I)*b^2*PolyLog[3, (-I)*E^ArcSinh[c*x]])/(c^5*d^2
) + ((3*I)*b^2*PolyLog[3, I*E^ArcSinh[c*x]])/(c^5*d^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5767

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(e*(m + 2*p + 1)), x] + (-Dist[(f^2*(m - 1))/(c^2*(m +
 2*p + 1)), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(c*(m + 2*p + 1)*(1 + c
^2*x^2)^FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcS
inh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2
*d] && GtQ[n, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^2}\,dx = -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)} + \frac{b\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{cd^2} + \frac{3\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx}{2c^2d}$$

$$= \frac{b\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2\sqrt{1+c^2x^2}} + \frac{b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2} + \frac{3x\left(a+b\sinh^{-1}(cx)\right)^2}{2c^4d^2}$$

$$= -\frac{b^2x}{c^4d^2} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2\sqrt{1+c^2x^2}} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2} + \frac{3x\left(a+b\sinh^{-1}(cx)\right)}{2c^4}$$

$$= \frac{2b^2x}{c^4d^2} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2\sqrt{1+c^2x^2}} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2} + \frac{3x\left(a+b\sin\right.}{2c^4d}$$

$$= \frac{2b^2x}{c^4d^2} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2\sqrt{1+c^2x^2}} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2} + \frac{3x\left(a+b\sin\right.}{2c^4d}$$

$$= \frac{2b^2x}{c^4d^2} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2\sqrt{1+c^2x^2}} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2} + \frac{3x\left(a+b\sin\right.}{2c^4d}$$

$$= \frac{2b^2x}{c^4d^2} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2\sqrt{1+c^2x^2}} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2} + \frac{3x\left(a+b\sin\right.}{2c^4d}$$

Mathematica [A] time = 2.23, size = 482, normalized size = 1.73

$$-\frac{3a^2\tan^{-1}(cx)}{c^5}+\frac{2a^2x}{c^4}+\frac{a^2x}{c^6x^2+c^4}-\frac{2ab\left(-2c^3x^3\sinh^{-1}(cx)-3i\left(c^2x^2+1\right)\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)+3i\left(c^2x^2+1\right)\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^2,x]
```

```
[Out] ((2*a^2*x)/c^4 + (a^2*x)/(c^4 + c^6*x^2) - (3*a^2*ArcTan[c*x])/c^5 - (2*a*b
*(Sqrt[1 + c^2*x^2] + 2*c^2*x^2*Sqrt[1 + c^2*x^2] - 3*c*x*ArcSinh[c*x] - 2*
c^3*x^3*ArcSinh[c*x] + (3*I)*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + (3*I)
*c^2*x^2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - (3*I)*ArcSinh[c*x]*Log[1
+ I*E^ArcSinh[c*x]] - (3*I)*c^2*x^2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]]
- (3*I)*(1 + c^2*x^2)*PolyLog[2, (-I)*E^ArcSinh[c*x]] + (3*I)*(1 + c^2*x^2)
*PolyLog[2, I*E^ArcSinh[c*x]]))/(c^5 + c^7*x^2) + (2*b^2*(ArcSinh[c*x]/Sqrt
[1 + c^2*x^2] - 2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + (c*x*ArcSinh[c*x]^2)/(2
+ 2*c^2*x^2) + c*x*(2 + ArcSinh[c*x]^2) + (I/2)*((4*I)*ArcTan[Tanh[ArcSinh[
c*x]/2]] + 3*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] - 3*ArcSinh[c*x]^2*Lo
g[1 + I/E^ArcSinh[c*x]] + 6*ArcSinh[c*x]*PolyLog[2, (-I)/E^ArcSinh[c*x]] -
6*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[c*x]] + 6*PolyLog[3, (-I)/E^ArcSinh[c
*x]] - 6*PolyLog[3, I/E^ArcSinh[c*x]])))/c^5)/(2*d^2)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2x^4\,\text{arsinh}\,(cx)^2+2\,abx^4\,\text{arsinh}\,(cx)+a^2x^4}{c^4d^2x^4+2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4*arcsinh(c*x)^2 + 2*a*b*x^4*arcsinh(c*x) + a^2*x^4)/(c^4*d
^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(a+b\,\text{arcsinh}\,(cx)\right)^2}{\left(c^2d\,x^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x)
```

```
[Out] int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,a^2\left(\frac{x}{c^6d^2x^2+c^4d^2}+\frac{2\,x}{c^4d^2}-\frac{3\,\arctan\,(cx)}{c^5d^2}\right)+\int\frac{b^2x^4\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^4d^2x^4+2\,c^2d^2x^2+d^2}+\frac{2\,abx^4\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^4d^2x^4+2\,c^2d^2x^2+d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(x/(c^6*d^2*x^2 + c^4*d^2) + 2*x/(c^4*d^2) - 3*arctan(c*x)/(c^5*d^2
)) + integrate(b^2*x^4*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^4*d^2*x^4 + 2*c^2*
d^2*x^2 + d^2) + 2*a*b*x^4*log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^4 + 2*c^
2*d^2*x^2 + d^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d\,c^2\,x^2+d\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^2,x)
```

```
[Out] int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^{2} x^{4}}{c^{4} x^{4} + 2 c^{2} x^{2} + 1}\, dx + \int \frac{b^{2} x^{4} \operatorname{asinh}^{2}{\left(c x \right)}}{c^{4} x^{4} + 2 c^{2} x^{2} + 1}\, dx + \int \frac{2 a b x^{4} \operatorname{asinh}{\left(c x \right)}}{c^{4} x^{4} + 2 c^{2} x^{2} + 1}\, dx}{d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a**2*x**4/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b**2*x**4*
asinh(c*x)**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(2*a*b*x**4*asinh
(c*x)/(c**4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

3.235 $\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=213

$$\frac{b\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^4d^2}-\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3bc^4d^2}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2c^4d^2}+\frac{\log\left(e^{2\sinh^{-1}(cx)}+1\right)\left(a+b\sin\right.}{c^4d^2}$$

```
[Out] 1/2*(a+b*arcsinh(c*x))^2/c^4/d^2-1/2*x^2*(a+b*arcsinh(c*x))^2/c^2/d^2/(c^2*
x^2+1)-1/3*(a+b*arcsinh(c*x))^3/b/c^4/d^2+(a+b*arcsinh(c*x))^2*ln(1+(c*x+(c
^2*x^2+1)^(1/2))^2)/c^4/d^2+1/2*b^2*ln(c^2*x^2+1)/c^4/d^2+b*(a+b*arcsinh(c*
x))*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c^4/d^2-1/2*b^2*polylog(3,-(c*x+(
c^2*x^2+1)^(1/2))^2)/c^4/d^2-b*x*(a+b*arcsinh(c*x))/c^3/d^2/(c^2*x^2+1)^(1/
2)
```

Rubi [A] time = 0.40, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5751, 5714, 3718, 2190, 2531, 2282, 6589, 5675, 260}

$$\frac{b\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^4d^2}-\frac{b^2\text{PolyLog}\left(3,-e^{2\sinh^{-1}(cx)}\right)}{2c^4d^2}-\frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(c^2x^2+1\right)}-\frac{bx\left(a+b\,\text{si}\right.}{c^3d^2\sqrt{c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^2,x]
```

```
[Out] -((b*x*(a + b*ArcSinh[c*x]))/(c^3*d^2*Sqrt[1 + c^2*x^2])) + (a + b*ArcSinh[
c*x])^2/(2*c^4*d^2) - (x^2*(a + b*ArcSinh[c*x])^2)/(2*c^2*d^2*(1 + c^2*x^2)
) - (a + b*ArcSinh[c*x])^3/(3*b*c^4*d^2) + ((a + b*ArcSinh[c*x])^2*Log[1 +
E^(2*ArcSinh[c*x])])/(c^4*d^2) + (b^2*Log[1 + c^2*x^2])/(2*c^4*d^2) + (b*(a
 + b*ArcSinh[c*x])*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c^4*d^2) - (b^2*PolyLo
g[3, -E^(2*ArcSinh[c*x])])/(2*c^4*d^2)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^3 \left(a + b\sinh^{-1}(cx)\right)^2}{\left(d + c^2dx^2\right)^2}\,dx = -\frac{x^2\left(a + b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1 + c^2x^2\right)} + \frac{b\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}dx}{cd^2} + \frac{\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}dx}{c^2d}$$

$$= -\frac{bx\left(a + b\sinh^{-1}(cx)\right)}{c^3d^2\sqrt{1 + c^2x^2}} - \frac{x^2\left(a + b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1 + c^2x^2\right)} + \frac{\text{Subst}\left(\int (a + bx)^2\tanh(x)\,dx,\right.}{c^4d^2}$$

$$= -\frac{bx\left(a + b\sinh^{-1}(cx)\right)}{c^3d^2\sqrt{1 + c^2x^2}} + \frac{\left(a + b\sinh^{-1}(cx)\right)^2}{2c^4d^2} - \frac{x^2\left(a + b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1 + c^2x^2\right)} - \frac{(a + b\,\text{si}}{3b}$$

$$= -\frac{bx\left(a + b\sinh^{-1}(cx)\right)}{c^3d^2\sqrt{1 + c^2x^2}} + \frac{\left(a + b\sinh^{-1}(cx)\right)^2}{2c^4d^2} - \frac{x^2\left(a + b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1 + c^2x^2\right)} - \frac{(a + b\,\text{si}}{3b}$$

$$= -\frac{bx\left(a + b\sinh^{-1}(cx)\right)}{c^3d^2\sqrt{1 + c^2x^2}} + \frac{\left(a + b\sinh^{-1}(cx)\right)^2}{2c^4d^2} - \frac{x^2\left(a + b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1 + c^2x^2\right)} - \frac{(a + b\,\text{si}}{3b}$$

$$= -\frac{bx\left(a + b\sinh^{-1}(cx)\right)}{c^3d^2\sqrt{1 + c^2x^2}} + \frac{\left(a + b\sinh^{-1}(cx)\right)^2}{2c^4d^2} - \frac{x^2\left(a + b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1 + c^2x^2\right)} - \frac{(a + b\,\text{si}}{3b}$$

$$= -\frac{bx\left(a + b\sinh^{-1}(cx)\right)}{c^3d^2\sqrt{1 + c^2x^2}} + \frac{\left(a + b\sinh^{-1}(cx)\right)^2}{2c^4d^2} - \frac{x^2\left(a + b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1 + c^2x^2\right)} - \frac{(a + b\,\text{si}}{3b}$$

Mathematica [C] time = 1.10, size = 320, normalized size = 1.50

$$\frac{\frac{a^2}{c^2x^2+1} + a^2\log\left(c^2x^2 + 1\right) - \frac{ab\left(\sqrt{c^2x^2+1} - i\sinh^{-1}(cx)\right)}{cx+i} - \frac{ab\left(\sqrt{c^2x^2+1} + i\sinh^{-1}(cx)\right)}{cx-i} + 4ab\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right) + 4ab\text{Li}_2\left(ie^{\sinh}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^2,x]
```

```
[Out] (a^2/(1 + c^2*x^2) - (a*b*(Sqrt[1 + c^2*x^2] - I*ArcSinh[c*x]))/(I + c*x) -
 (a*b*(Sqrt[1 + c^2*x^2] + I*ArcSinh[c*x]))/(-I + c*x) - a*b*ArcSinh[c*x]*(
ArcSinh[c*x] - 4*Log[1 - I*E^ArcSinh[c*x]]) - a*b*ArcSinh[c*x]*(ArcSinh[c*x
] - 4*Log[1 + I*E^ArcSinh[c*x]]) + a^2*Log[1 + c^2*x^2] + 4*a*b*PolyLog[2,
(-I)*E^ArcSinh[c*x]] + 4*a*b*PolyLog[2, I*E^ArcSinh[c*x]] + 2*b^2*(-((c*x*A
rcSinh[c*x])/Sqrt[1 + c^2*x^2]) + ArcSinh[c*x]^2/(2 + 2*c^2*x^2) + ArcSinh[
c*x]^3/3 + ArcSinh[c*x]^2*Log[1 + E^(-2*ArcSinh[c*x])] + Log[1 + c^2*x^2]/2
 - ArcSinh[c*x]*PolyLog[2, -E^(-2*ArcSinh[c*x])] - PolyLog[3, -E^(-2*ArcSin
h[c*x])]/2))/(2*c^4*d^2)
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2x^3\,\text{arsinh}\,(cx)^2 + 2\,abx^3\,\text{arsinh}\,(cx) + a^2x^3}{c^4d^2x^4 + 2\,c^2d^2x^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^3*arcsinh(c*x)^2 + 2*a*b*x^3*arcsinh(c*x) + a^2*x^3)/(c^4*d
^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.34, size = 499, normalized size = 2.34

$$\frac{a^2}{2c^4d^2\left(c^2x^2+1\right)}+\frac{a^2\ln\left(c^2x^2+1\right)}{2c^4d^2}-\frac{b^2\operatorname{arcsinh}(cx)^3}{3c^4d^2}-\frac{b^2\operatorname{arcsinh}(cx)\,x}{c^3d^2\sqrt{c^2x^2+1}}+\frac{b^2\operatorname{arcsinh}(cx)\,x^2}{c^2d^2\left(c^2x^2+1\right)}+\frac{b^2\operatorname{arcsinh}(cx)^2}{2c^4d^2\left(c^2x^2+1\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x)
```

```
[Out] 1/2/c^4*a^2/d^2/(c^2*x^2+1)+1/2/c^4*a^2/d^2*ln(c^2*x^2+1)-1/3/c^4*b^2/d^2*a
rcsinh(c*x)^3-1/c^3*b^2/d^2*arcsinh(c*x)/(c^2*x^2+1)^(1/2)*x+1/c^2*b^2/d^2*
arcsinh(c*x)/(c^2*x^2+1)*x^2+1/2/c^4*b^2/d^2*arcsinh(c*x)^2/(c^2*x^2+1)+1/c
^4*b^2/d^2*arcsinh(c*x)/(c^2*x^2+1)-2/c^4*b^2/d^2*ln(c*x+(c^2*x^2+1)^(1/2))
+1/c^4*b^2/d^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+1/c^4*b^2/d^2*arcsinh(c*x)^2
*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+1/c^4*b^2/d^2*arcsinh(c*x)*polylog(2,-(c*x
+(c^2*x^2+1)^(1/2))^2)-1/2*b^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/c^4/d^
2-1/c^4*a*b/d^2*arcsinh(c*x)^2-1/c^3*a*b/d^2/(c^2*x^2+1)^(1/2)*x+1/c^2*a*b/
d^2*x^2/(c^2*x^2+1)+1/c^4*a*b/d^2*arcsinh(c*x)/(c^2*x^2+1)+1/c^4*a*b/d^2/(c
^2*x^2+1)+2/c^4*a*b/d^2*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+1/c^4*
a*b/d^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}a^2\left(\frac{1}{c^6d^2x^2+c^4d^2}+\frac{\log\left(c^2x^2+1\right)}{c^4d^2}\right)+\frac{\left(b^2+\left(b^2c^2x^2+b^2\right)\log\left(c^2x^2+1\right)\right)\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{2\left(c^6d^2x^2+c^4d^2\right)}-\int-\frac{(2\,abc$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(1/(c^6*d^2*x^2 + c^4*d^2) + log(c^2*x^2 + 1)/(c^4*d^2)) + 1/2*(b^2
 + (b^2*c^2*x^2 + b^2)*log(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^
6*d^2*x^2 + c^4*d^2) - integrate(-(2*a*b*c^4*x^4 - b^2*c^2*x^2 - b^2 - (b^2
*c^4*x^4 + 2*b^2*c^2*x^2 + b^2)*log(c^2*x^2 + 1) + (2*a*b*c^3*x^3 - b^2*c*x
 - (b^2*c^3*x^3 + b^2*c*x)*log(c^2*x^2 + 1))*sqrt(c^2*x^2 + 1))*log(c*x + s
qrt(c^2*x^2 + 1))/(c^8*d^2*x^5 + 2*c^6*d^2*x^3 + c^4*d^2*x + (c^7*d^2*x^4 +
 2*c^5*d^2*x^2 + c^3*d^2)*sqrt(c^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d\,c^2\,x^2+d\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^2,x)
```

```
[Out] int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2x^3}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{b^2x^3\operatorname{asinh}^2(cx)}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{2abx^3\operatorname{asinh}(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a**2*x**3/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b**2*x**3*
asinh(c*x)**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(2*a*b*x**3*asinh
(c*x)/(c**4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

3.236 $\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=213

$$-\frac{ib\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^3d^2}+\frac{ib\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^3d^2}+\frac{\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}\right.}{c^3d^2}$$

```
[Out] -1/2*x*(a+b*arcsinh(c*x))^2/c^2/d^2/(c^2*x^2+1)+(a+b*arcsinh(c*x))^2*arctan
(c*x+(c^2*x^2+1)^(1/2))/c^3/d^2+b^2*arctan(c*x)/c^3/d^2-I*b*(a+b*arcsinh(c*
x))*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d^2+I*b*(a+b*arcsinh(c*x))*po
lylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d^2+I*b^2*polylog(3,-I*(c*x+(c^2*x^2
+1)^(1/2)))/c^3/d^2-I*b^2*polylog(3,I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d^2-b*(a
+b*arcsinh(c*x))/c^3/d^2/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5751, 5693, 4180, 2531, 2282, 6589, 5717, 203}

$$-\frac{ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^3d^2}+\frac{ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^3d^2}+\frac{ib^2\text{PolyLog}\left(\right.}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^2,x]
```

```
[Out] -((b*(a + b*ArcSinh[c*x]))/(c^3*d^2*Sqrt[1 + c^2*x^2])) - (x*(a + b*ArcSinh
[c*x])^2)/(2*c^2*d^2*(1 + c^2*x^2)) + ((a + b*ArcSinh[c*x])^2*ArcTan[E^ArcS
inh[c*x]])/(c^3*d^2) + (b^2*ArcTan[c*x])/(c^3*d^2) - (I*b*(a + b*ArcSinh[c*
x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^3*d^2) + (I*b*(a + b*ArcSinh[c*x])*
PolyLog[2, I*E^ArcSinh[c*x]])/(c^3*d^2) + (I*b^2*PolyLog[3, (-I)*E^ArcSinh[
c*x]])/(c^3*d^2) - (I*b^2*PolyLog[3, I*E^ArcSinh[c*x]])/(c^3*d^2)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^2}\,dx = -\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)} + \frac{b\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{cd^2} + \frac{\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx}{2c^2d}$$

$$= -\frac{b\left(a+b\sinh^{-1}(cx)\right)}{c^3d^2\sqrt{1+c^2x^2}} - \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)} + \frac{\text{Subst}\left(\int (a+bx)^2\text{sech}(x)\,dx,\right.}{2c^3d^2}$$

$$= -\frac{b\left(a+b\sinh^{-1}(cx)\right)}{c^3d^2\sqrt{1+c^2x^2}} - \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sin}\right.}{c^3d^2}$$

$$= -\frac{b\left(a+b\sinh^{-1}(cx)\right)}{c^3d^2\sqrt{1+c^2x^2}} - \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sin}\right.}{c^3d^2}$$

$$= -\frac{b\left(a+b\sinh^{-1}(cx)\right)}{c^3d^2\sqrt{1+c^2x^2}} - \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sin}\right.}{c^3d^2}$$

$$= -\frac{b\left(a+b\sinh^{-1}(cx)\right)}{c^3d^2\sqrt{1+c^2x^2}} - \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sin}\right.}{c^3d^2}$$

Mathematica [A] time = 1.86, size = 385, normalized size = 1.81

$$-\frac{\frac{a^2cx}{c^2x^2+1} + a^2\left(-\tan^{-1}(cx)\right) + \frac{ab\left(\sinh^{-1}(cx)-i\sqrt{c^2x^2+1}\right)}{cx-i} + \frac{ab\left(\sinh^{-1}(cx)+i\sqrt{c^2x^2+1}\right)}{cx+i} - \frac{1}{2}iab\left(\sinh^{-1}(cx)\left(\sinh^{-1}(cx)-4\,\mathrm{l}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^2,x]
```

```
[Out] -1/2*((a^2*c*x)/(1 + c^2*x^2) + (2*b^2*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] + (b
^2*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2) + (a*b*((-I)*Sqrt[1 + c^2*x^2] + ArcSi
nh[c*x]))/(-I + c*x) + (a*b*(I*Sqrt[1 + c^2*x^2] + ArcSinh[c*x]))/(I + c*x)
 - a^2*ArcTan[c*x] - (I/2)*a*b*(ArcSinh[c*x]*(ArcSinh[c*x] - 4*Log[1 + I*E^
ArcSinh[c*x]]) - 4*PolyLog[2, (-I)*E^ArcSinh[c*x]]) + (I/2)*a*b*(ArcSinh[c*
x]*(ArcSinh[c*x] - 4*Log[1 - I*E^ArcSinh[c*x]]) - 4*PolyLog[2, I*E^ArcSinh[
c*x]]) + I*b^2*((4*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + ArcSinh[c*x]^2*Log[1 -
 I/E^ArcSinh[c*x]] - ArcSinh[c*x]^2*Log[1 + I/E^ArcSinh[c*x]] + 2*ArcSinh[c
*x]*PolyLog[2, (-I)/E^ArcSinh[c*x]] - 2*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh
[c*x]] + 2*PolyLog[3, (-I)/E^ArcSinh[c*x]] - 2*PolyLog[3, I/E^ArcSinh[c*x]]
))/(c^3*d^2)
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2x^2\,\text{arsinh}\,(cx)^2 + 2\,abx^2\,\text{arsinh}\,(cx) + a^2x^2}{c^4d^2x^4 + 2\,c^2d^2x^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*x) + a^2*x^2)/(c^4*d
^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\text{arsinh}\,(cx)+a\right)^2x^2}{\left(c^2dx^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^2/(c^2*d*x^2 + d)^2, x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + b \operatorname{arcsinh}(cx))^2}{\left(c^2 d x^2 + d\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x)
```

```
[Out] int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2} a^2 \left(\frac{x}{c^4 d^2 x^2 + c^2 d^2} - \frac{\arctan(cx)}{c^3 d^2}\right) + \int \frac{b^2 x^2 \log\left(cx + \sqrt{c^2 x^2 + 1}\right)^2}{c^4 d^2 x^4 + 2 c^2 d^2 x^2 + d^2} + \frac{2 abx^2 \log\left(cx + \sqrt{c^2 x^2 + 1}\right)}{c^4 d^2 x^4 + 2 c^2 d^2 x^2 + d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a^2*(x/(c^4*d^2*x^2 + c^2*d^2) - arctan(c*x)/(c^3*d^2)) + integrate(b^
2*x^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2) +
2*a*b*x^2*log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 (a + b \operatorname{asinh}(c x))^2}{\left(d c^2 x^2 + d\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^2,x)
```

```
[Out] int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2 x^2}{c^4 x^4 + 2c^2 x^2 + 1} dx + \int \frac{b^2 x^2 \operatorname{asinh}^2(cx)}{c^4 x^4 + 2c^2 x^2 + 1} dx + \int \frac{2abx^2 \operatorname{asinh}(cx)}{c^4 x^4 + 2c^2 x^2 + 1} dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a**2*x**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b**2*x**2*
asinh(c*x)**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(2*a*b*x**2*asinh
(c*x)/(c**4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

3.237 $\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=85

$$\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{cd^2\sqrt{c^2x^2+1}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(c^2x^2+1\right)}-\frac{b^2\log\left(c^2x^2+1\right)}{2c^2d^2}$$

```
[Out] -1/2*(a+b*arcsinh(c*x))^2/c^2/d^2/(c^2*x^2+1)-1/2*b^2*ln(c^2*x^2+1)/c^2/d^2
+b*x*(a+b*arcsinh(c*x))/c/d^2/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5717, 5687, 260}

$$\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{cd^2\sqrt{c^2x^2+1}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(c^2x^2+1\right)}-\frac{b^2\log\left(c^2x^2+1\right)}{2c^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^2,x]
```

```
[Out] (b*x*(a + b*ArcSinh[c*x]))/(c*d^2*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])
^2/(2*c^2*d^2*(1 + c^2*x^2)) - (b^2*Log[1 + c^2*x^2])/(2*c^2*d^2)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^2}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)}+\frac{b\int \frac{a+b\sinh^{-1}(cx)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{cd^2}$$
$$=\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{cd^2\sqrt{1+c^2x^2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)}-\frac{b^2\int \frac{x}{1+c^2x^2}\,dx}{d^2}$$
$$=\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{cd^2\sqrt{1+c^2x^2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)}-\frac{b^2\log\left(1+c^2x^2\right)}{2c^2d^2}$$

Mathematica [A] time = 0.22, size = 145, normalized size = 1.71

$$-\frac{a^2}{2c^2d^2\left(c^2x^2+1\right)}+\frac{abx}{cd^2\sqrt{c^2x^2+1}}+\frac{b\sinh^{-1}(cx)\left(bcx\sqrt{c^2x^2+1}-a\right)}{c^2d^2\left(c^2x^2+1\right)}-\frac{b^2\log\left(c^2x^2+1\right)}{2c^2d^2}-\frac{b^2\sinh^{-1}(cx)^2}{2c^2d^2\left(c^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^2,x]
```

```
[Out] -1/2*a^2/(c^2*d^2*(1 + c^2*x^2)) + (a*b*x)/(c*d^2*Sqrt[1 + c^2*x^2]) + (b*(
-a + b*c*x*Sqrt[1 + c^2*x^2])*ArcSinh[c*x])/(c^2*d^2*(1 + c^2*x^2)) - (b^2*
ArcSinh[c*x]^2)/(2*c^2*d^2*(1 + c^2*x^2)) - (b^2*Log[1 + c^2*x^2])/(2*c^2*d
^2)
```

fricas [B] time = 0.61, size = 185, normalized size = 2.18

$$\frac{2\,abc^2x^2+2\,\sqrt{c^2x^2+1}\,abcx-b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2-a^2+2\,ab-\left(b^2c^2x^2+b^2\right)\log\left(c^2x^2+1\right)+2\left(abc^2x^2+\right.}{2\left(c^4d^2x^2+c^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*c^2*x^2 + 2*sqrt(c^2*x^2 + 1)*a*b*c*x - b^2*log(c*x + sqrt(c^2*x
^2 + 1))^2 - a^2 + 2*a*b - (b^2*c^2*x^2 + b^2)*log(c^2*x^2 + 1) + 2*(a*b*c^
2*x^2 + sqrt(c^2*x^2 + 1)*b^2*c*x)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^
2*x^2 + a*b)*log(-c*x + sqrt(c^2*x^2 + 1)))/(c^4*d^2*x^2 + c^2*d^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^2x}{\left(c^2dx^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x/(c^2*d*x^2 + d)^2, x)
```

maple [B] time = 0.10, size = 222, normalized size = 2.61

$$-\frac{a^2}{2c^2d^2\left(c^2x^2+1\right)}+\frac{2b^2\,\mathrm{arcsinh}\,(cx)}{c^2d^2}+\frac{b^2\,\mathrm{arcsinh}\,(cx)\,x}{c\,d^2\sqrt{c^2x^2+1}}-\frac{b^2\,\mathrm{arcsinh}\,(cx)\,x^2}{d^2\left(c^2x^2+1\right)}-\frac{b^2\,\mathrm{arcsinh}\,(cx)^2}{2c^2d^2\left(c^2x^2+1\right)}-\frac{b^2\,\mathrm{arcsinh}\,(cx)}{c^2d^2\left(c^2x^2+1\right)}-\frac{b}{}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x)
```

```
[Out] -1/2/c^2*a^2/d^2/(c^2*x^2+1)+2/c^2*b^2/d^2*arcsinh(c*x)+1/c*b^2/d^2*arcsinh
(c*x)/(c^2*x^2+1)^(1/2)*x-b^2/d^2*arcsinh(c*x)/(c^2*x^2+1)*x^2-1/2/c^2*b^2/
d^2*arcsinh(c*x)^2/(c^2*x^2+1)-1/c^2*b^2/d^2*arcsinh(c*x)/(c^2*x^2+1)-1/c^2
*b^2/d^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-1/c^2*a*b/d^2/(c^2*x^2+1)*arcsinh(
c*x)+1/c*a*b/d^2*x/(c^2*x^2+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{2\left(c^4d^2x^2+c^2d^2\right)}-\frac{a^2}{2\left(c^4d^2x^2+c^2d^2\right)}+\int\frac{\left(\left(2\,abc^2+b^2c^2\right)x^2+\sqrt{c^2x^2+1}\left(2\,abc+b^2c\right)x+b^2\right)\log\left(c\right.}{c^6d^2x^5+2\,c^4d^2x^3+c^2d^2x+\left(c^5d^2x^4+2\,c^3d^2x^2+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^4*d^2*x^2 + c^2*d^2) - 1/2*a^2/(
c^4*d^2*x^2 + c^2*d^2) + integrate(((2*a*b*c^2 + b^2*c^2)*x^2 + sqrt(c^2*x^
2 + 1)*(2*a*b*c + b^2*c)*x + b^2)*log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d^2*x^5
 + 2*c^4*d^2*x^3 + c^2*d^2*x + (c^5*d^2*x^4 + 2*c^3*d^2*x^2 + c*d^2)*sqrt(c
^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d\,c^2\,x^2+d\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^2,x)
```

```
[Out] int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{a^2x}{c^4x^4+2c^2x^2+1}\,dx+\int\frac{b^2x\operatorname{asinh}^2(cx)}{c^4x^4+2c^2x^2+1}\,dx+\int\frac{2abx\operatorname{asinh}(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a**2*x/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b**2*x*asinh(
c*x)**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(2*a*b*x*asinh(c*x)/(c*
*4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

3.238 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=210

$$\frac{b\left(a+b\sinh^{-1}(cx)\right)}{cd^2\sqrt{c^2x^2+1}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(c^2x^2+1\right)}-\frac{ib\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{cd^2}+\frac{ib\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)\left(a+b\sin\right.}{cd^2}$$

[Out] 1/2*x*(a+b*arcsinh(c*x))^2/d^2/(c^2*x^2+1)+(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(1/2))/c/d^2-b^2*arctan(c*x)/c/d^2-I*b*(a+b*arcsinh(c*x))*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^2+I*b*(a+b*arcsinh(c*x))*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^2+I*b^2*polylog(3,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^2-I*b^2*polylog(3,I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^2+b*(a+b*arcsinh(c*x))/c/d^2/(c^2*x^2+1)^(1/2)

Rubi [A] time = 0.25, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5690, 5693, 4180, 2531, 2282, 6589, 5717, 203}

$$-\frac{ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{cd^2}+\frac{ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{cd^2}+\frac{ib^2\text{PolyLog}\left(3,-\right.}{cd^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^2,x]

[Out] (b*(a + b*ArcSinh[c*x]))/(c*d^2*Sqrt[1 + c^2*x^2]) + (x*(a + b*ArcSinh[c*x])^2)/(2*d^2*(1 + c^2*x^2)) + ((a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]])/(c*d^2) - (b^2*ArcTan[c*x])/(c*d^2) - (I*b*(a + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*d^2) + (I*b*(a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])/(c*d^2) + (I*b^2*PolyLog[3, (-I)*E^ArcSinh[c*x]])/(c*d^2) - (I*b^2*PolyLog[3, I*E^ArcSinh[c*x]])/(c*d^2)

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^2}\,dx = \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} - \frac{(bc)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{d^2} + \frac{\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx}{2d}$$

$$= \frac{b\left(a+b\sinh^{-1}(cx)\right)}{cd^2\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} - \frac{b^2\int \frac{1}{1+c^2x^2}\,dx}{d^2} + \frac{\text{Subst}\left(\int (a+bx)^2\text{se}\right.}{2}$$

$$= \frac{b\left(a+b\sinh^{-1}(cx)\right)}{cd^2\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{cd^2}$$

$$= \frac{b\left(a+b\sinh^{-1}(cx)\right)}{cd^2\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{cd^2}$$

$$= \frac{b\left(a+b\sinh^{-1}(cx)\right)}{cd^2\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{cd^2}$$

$$= \frac{b\left(a+b\sinh^{-1}(cx)\right)}{cd^2\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{cd^2}$$

Mathematica [A] time = 1.42, size = 403, normalized size = 1.92

$$\frac{\frac{a^2x}{c^2x^2+1} + \frac{a^2\tan^{-1}(cx)}{c} + \frac{2ab\left(-i\left(c^2x^2+1\right)\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)+i\left(c^2x^2+1\right)\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)+\sqrt{c^2x^2+1}+ic^2x^2\sinh^{-1}(cx)\log\left(1-ie^{\sinh^{-1}(cx)}\right)-ic^2x^2\sinh\right.}{c^3x^2+c}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^2,x]
```

```
[Out] ((a^2*x)/(1 + c^2*x^2) + (a^2*ArcTan[c*x])/c + (2*a*b*(Sqrt[1 + c^2*x^2] +
c*x*ArcSinh[c*x] + I*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + I*c^2*x^2*Arc
Sinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - I*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*
x]] - I*c^2*x^2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] - I*(1 + c^2*x^2)*Po
lyLog[2, (-I)*E^ArcSinh[c*x]] + I*(1 + c^2*x^2)*PolyLog[2, I*E^ArcSinh[c*x]
]))/(c + c^3*x^2) + (2*b^2*(ArcSinh[c*x]/Sqrt[1 + c^2*x^2] + (c*x*ArcSinh[c
*x]^2)/(2 + 2*c^2*x^2) - (I/2)*((-4*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + ArcSi
nh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] - ArcSinh[c*x]^2*Log[1 + I/E^ArcSinh[c*
x]] + 2*ArcSinh[c*x]*PolyLog[2, (-I)/E^ArcSinh[c*x]] - 2*ArcSinh[c*x]*PolyL
og[2, I/E^ArcSinh[c*x]] + 2*PolyLog[3, (-I)/E^ArcSinh[c*x]] - 2*PolyLog[3,
I/E^ArcSinh[c*x]])))/c)/(2*d^2)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\,\text{arsinh}(cx)^2 + 2\,ab\,\text{arsinh}(cx) + a^2}{c^4d^2x^4 + 2\,c^2d^2x^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^4*d^2*x^4 + 2*c
^2*d^2*x^2 + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\text{arsinh}(cx) + a\right)^2}{\left(c^2dx^2 + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(c^2*d*x^2 + d)^2, x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arcsinh}(cx))^2}{\left(c^2 d x^2 + d\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} a^2 \left(\frac{x}{c^2 d^2 x^2 + d^2} + \frac{\arctan(cx)}{cd^2} \right) + \int \frac{b^2 \log\left(cx + \sqrt{c^2x^2 + 1}\right)^2}{c^4 d^2 x^4 + 2 c^2 d^2 x^2 + d^2} + \frac{2 ab \log\left(cx + \sqrt{c^2x^2 + 1}\right)}{c^4 d^2 x^4 + 2 c^2 d^2 x^2 + d^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(x/(c^2*d^2*x^2 + d^2) + arctan(c*x)/(c*d^2)) + integrate(b^2*log(c
*x + sqrt(c^2*x^2 + 1))^2/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2) + 2*a*b*log(c
*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{asinh}(c\,x))^2}{\left(d\,c^2\,x^2 + d\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^2,x)
```

```
[Out] int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^4x^4+2c^2x^2+1}\, dx + \int \frac{b^2 \operatorname{asinh}^2(cx)}{c^4x^4+2c^2x^2+1}\, dx + \int \frac{2ab \operatorname{asinh}(cx)}{c^4x^4+2c^2x^2+1}\, dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b**2*asinh(c*x)
**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(2*a*b*asinh(c*x)/(c**4*x**
4 + 2*c**2*x**2 + 1), x))/d**2
```

3.239 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)^2} dx$

Optimal. Leaf size=193

$$-\frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{c^2x^2+1}}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(c^2x^2+1\right)}-\frac{b\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}+\frac{b\text{Li}_2\left(e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}$$

```
[Out] 1/2*(a+b*arcsinh(c*x))^2/d^2/(c^2*x^2+1)-2*(a+b*arcsinh(c*x))^2*arctanh((c*
x+(c^2*x^2+1)^(1/2))^2)/d^2+1/2*b^2*ln(c^2*x^2+1)/d^2-b*(a+b*arcsinh(c*x))*
polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^2+b*(a+b*arcsinh(c*x))*polylog(2,(c
*x+(c^2*x^2+1)^(1/2))^2)/d^2+1/2*b^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/
d^2-1/2*b^2*polylog(3,(c*x+(c^2*x^2+1)^(1/2))^2)/d^2-b*c*x*(a+b*arcsinh(c*x
))/d^2/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5755, 5720, 5461, 4182, 2531, 2282, 6589, 5687, 260}

$$-\frac{b\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}+\frac{b\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}+\frac{b^2\text{PolyLog}\left(3,-e\right.}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)^2),x]
```

```
[Out] -((b*c*x*(a + b*ArcSinh[c*x]))/(d^2*Sqrt[1 + c^2*x^2])) + (a + b*ArcSinh[c*
x])^2/(2*d^2*(1 + c^2*x^2)) - (2*(a + b*ArcSinh[c*x])^2*ArcTanh[E^(2*ArcSin
h[c*x])])/d^2 + (b^2*Log[1 + c^2*x^2])/(2*d^2) - (b*(a + b*ArcSinh[c*x])*Po
lyLog[2, -E^(2*ArcSinh[c*x])])/d^2 + (b*(a + b*ArcSinh[c*x])*PolyLog[2, E^(
2*ArcSinh[c*x])])/d^2 + (b^2*PolyLog[3, -E^(2*ArcSinh[c*x])])/(2*d^2) - (b^
2*PolyLog[3, E^(2*ArcSinh[c*x])])/(2*d^2)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
```

```
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5720

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)^2}\,dx = \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} - \frac{(bc)\int \frac{a+b\sinh^{-1}(cx)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{d^2} + \frac{\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)}\,dx}{d}$$

$$= -\frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} + \frac{\text{Subst}\left(\int (a+bx)^2\text{csch}(x)\text{sech}(x)\,d\right.}{d^2}$$

$$= -\frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} + \frac{b^2\log\left(1+c^2x^2\right)}{2d^2} + \frac{2\,\text{Subst}\left(\int (a+\right.}{}$$

$$= -\frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} - \frac{2\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(e^{2\sinh}\right.}{d^2}$$

$$= -\frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} - \frac{2\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(e^{2\sinh}\right.}{d^2}$$

$$= -\frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} - \frac{2\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(e^{2\sinh}\right.}{d^2}$$

$$= -\frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} - \frac{2\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(e^{2\sinh}\right.}{d^2}$$

Mathematica [C] time = 1.86, size = 428, normalized size = 2.22

$$\frac{\frac{a^2}{c^2x^2+1} - a^2\log\left(c^2x^2+1\right) + 2a^2\log(cx) - \frac{ab\left(\sqrt{c^2x^2+1}-i\sinh^{-1}(cx)\right)}{cx+i} - \frac{ab\left(\sqrt{c^2x^2+1}+i\sinh^{-1}(cx)\right)}{cx-i} + 2ab\text{Li}_2\left(e^{2\sinh^{-1}(cx)}\right) + a}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)^2),x]
```

```
[Out] (a^2/(1 + c^2*x^2) - (a*b*(Sqrt[1 + c^2*x^2] - I*ArcSinh[c*x]))/(I + c*x) -
 (a*b*(Sqrt[1 + c^2*x^2] + I*ArcSinh[c*x]))/(-I + c*x) - 2*a*b*ArcSinh[c*x]
^2 + 4*a*b*ArcSinh[c*x]*Log[1 - E^(2*ArcSinh[c*x])] + 2*a^2*Log[c*x] - a^2*
Log[1 + c^2*x^2] + a*b*(ArcSinh[c*x]*(ArcSinh[c*x] - 4*Log[1 + I*E^ArcSinh[
c*x]]) - 4*PolyLog[2, (-I)*E^ArcSinh[c*x]]) + a*b*(ArcSinh[c*x]*(ArcSinh[c*
x] - 4*Log[1 - I*E^ArcSinh[c*x]]) - 4*PolyLog[2, I*E^ArcSinh[c*x]]) + 2*a*b
*PolyLog[2, E^(2*ArcSinh[c*x])] + 2*b^2*((I/24)*Pi^3 - (c*x*ArcSinh[c*x])/S
qrt[1 + c^2*x^2] + ArcSinh[c*x]^2/(2 + 2*c^2*x^2) - (2*ArcSinh[c*x]^3)/3 -
ArcSinh[c*x]^2*Log[1 + E^(-2*ArcSinh[c*x])] + ArcSinh[c*x]^2*Log[1 - E^(2*A
rcSinh[c*x])] + Log[1 + c^2*x^2]/2 + ArcSinh[c*x]*PolyLog[2, -E^(-2*ArcSinh
[c*x])] + ArcSinh[c*x]*PolyLog[2, E^(2*ArcSinh[c*x])] + PolyLog[3, -E^(-2*A
rcSinh[c*x])]/2 - PolyLog[3, E^(2*ArcSinh[c*x])]/2))/(2*d^2)
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\,\text{arsinh}\,(cx)^2 + 2\,ab\,\text{arsinh}\,(cx) + a^2}{c^4d^2x^5 + 2\,c^2d^2x^3 + d^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^4*d^2*x^5 + 2*c
^2*d^2*x^3 + d^2*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2}{\left(c^2dx^2 + d\right)^2 x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^2*x), x)
```

maple [B] time = 0.19, size = 724, normalized size = 3.75

$$\frac{ab\,c^2x^2}{d^2\left(c^2x^2+1\right)}-\frac{abcx}{d^2\sqrt{c^2x^2+1}}-\frac{b^2\operatorname{arcsinh}(cx)\,cx}{d^2\sqrt{c^2x^2+1}}+\frac{b^2\operatorname{arcsinh}(cx)\,c^2x^2}{d^2\left(c^2x^2+1\right)}+\frac{ab}{d^2\left(c^2x^2+1\right)}+\frac{2ab\operatorname{polylog}\left(2,-cx-\right.}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^2,x)
```

```
[Out] -b^2/d^2*arcsinh(c*x)/(c^2*x^2+1)^(1/2)*c*x+a*b/d^2*c^2*x^2/(c^2*x^2+1)+b^2
/d^2*arcsinh(c*x)/(c^2*x^2+1)*c^2*x^2-a*b/d^2/(c^2*x^2+1)^(1/2)*c*x+2*a*b/d
^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+2*a*b/d^2*polylog(2,c*x+(c^2*x^2+1)^(1
/2))+b^2/d^2*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))+2*b^2/d^2*arcsinh(c
*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+b^2/d^2*arcsinh(c*x)^2*ln(1-c*x-(c^2*
x^2+1)^(1/2))+2*b^2/d^2*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))+1/2*b
^2/d^2*arcsinh(c*x)^2/(c^2*x^2+1)+b^2/d^2*arcsinh(c*x)/(c^2*x^2+1)-b^2/d^2*
arcsinh(c*x)^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-b^2/d^2*arcsinh(c*x)*polylog
(2,-(c*x+(c^2*x^2+1)^(1/2))^2)+a*b/d^2/(c^2*x^2+1)-a*b/d^2*polylog(2,-(c*x+
(c^2*x^2+1)^(1/2))^2)+1/2*b^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^2+a*b
/d^2*arcsinh(c*x)/(c^2*x^2+1)-2*a*b/d^2*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^
(1/2))^2)+2*a*b/d^2*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+2*a*b/d^2*arcs
inh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+1/2*a^2/d^2/(c^2*x^2+1)-1/2*a^2/d^2*ln
(c^2*x^2+1)+a^2/d^2*ln(c*x)-2*b^2/d^2*ln(c*x+(c^2*x^2+1)^(1/2))+b^2/d^2*ln(
1+(c*x+(c^2*x^2+1)^(1/2))^2)-2*b^2/d^2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))-2*
b^2/d^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,a^2\left(\frac{1}{c^2d^2x^2+d^2}-\frac{\log\left(c^2x^2+1\right)}{d^2}+\frac{2\,\log(x)}{d^2}\right)+\int \frac{b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^4d^2x^5+2\,c^2d^2x^3+d^2x}+\frac{2\,ab\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^4d^2x^5+2\,c^2d^2x^3+d^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(1/(c^2*d^2*x^2 + d^2) - log(c^2*x^2 + 1)/d^2 + 2*log(x)/d^2) + int
egrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^
2*x) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^
2*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\operatorname{asinh}(c\,x))^2}{x\left(d\,c^2\,x^2+d\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)^2),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)^2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^4x^5+2c^2x^3+x}\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{c^4x^5+2c^2x^3+x}\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{c^4x^5+2c^2x^3+x}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a**2/(c**4*x**5 + 2*c**2*x**3 + x), x) + Integral(b**2*asinh(c*x)
**2/(c**4*x**5 + 2*c**2*x**3 + x), x) + Integral(2*a*b*asinh(c*x)/(c**4*x**
5 + 2*c**2*x**3 + x), x))/d**2
```

3.240 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=287

$$-\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{c^2x^2+1}}-\frac{3c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(c^2x^2+1\right)}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^2x\left(c^2x^2+1\right)}+\frac{3ibc\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx\right.}{d^2}$$

```
[Out] -(a+b*arcsinh(c*x))^2/d^2/x/(c^2*x^2+1)-3/2*c^2*x*(a+b*arcsinh(c*x))^2/d^2/
(c^2*x^2+1)-3*c*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(1/2))/d^2+b^2*
c*arctan(c*x)/d^2-4*b*c*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))/d
^2-2*b^2*c*polylog(2,-c*x-(c^2*x^2+1)^(1/2))/d^2+3*I*b*c*(a+b*arcsinh(c*x))
*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/d^2-3*I*b*c*(a+b*arcsinh(c*x))*polyl
og(2,I*(c*x+(c^2*x^2+1)^(1/2)))/d^2+2*b^2*c*polylog(2,c*x+(c^2*x^2+1)^(1/2)
)/d^2-3*I*b^2*c*polylog(3,-I*(c*x+(c^2*x^2+1)^(1/2)))/d^2+3*I*b^2*c*polylog
(3,I*(c*x+(c^2*x^2+1)^(1/2)))/d^2-b*c*(a+b*arcsinh(c*x))/d^2/(c^2*x^2+1)^(1
/2)
```

Rubi [A] time = 0.54, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 14, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {5747, 5690, 5693, 4180, 2531, 2282, 6589, 5717, 203, 5755, 5760, 4182, 2279, 2391}

$$\frac{3ibc\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}-\frac{3ibc\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}-\frac{2b^2c\text{PolyL}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)^2),x]
```

```
[Out] -((b*c*(a + b*ArcSinh[c*x]))/(d^2*Sqrt[1 + c^2*x^2])) - (a + b*ArcSinh[c*x]
)^2/(d^2*x*(1 + c^2*x^2)) - (3*c^2*x*(a + b*ArcSinh[c*x])^2)/(2*d^2*(1 + c^
2*x^2)) - (3*c*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]])/d^2 + (b^2*c*
ArcTan[c*x])/d^2 - (4*b*c*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/d^2
 - (2*b^2*c*PolyLog[2, -E^ArcSinh[c*x]])/d^2 + ((3*I)*b*c*(a + b*ArcSinh[c*
x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/d^2 - ((3*I)*b*c*(a + b*ArcSinh[c*x])*
PolyLog[2, I*E^ArcSinh[c*x]])/d^2 + (2*b^2*c*PolyLog[2, E^ArcSinh[c*x]])/d^
2 - ((3*I)*b^2*c*PolyLog[3, (-I)*E^ArcSinh[c*x]])/d^2 + ((3*I)*b^2*c*PolyLo
g[3, I*E^ArcSinh[c*x]])/d^2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)^2}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^2x\left(1+c^2x^2\right)} - \left(3c^2\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^2}\,dx + \frac{(2bc)\int \frac{a+b\sinh^{-1}(cx)}{x\left(1+c^2x^2\right)^{3/2}}\,dx}{d^2}$$

$$= \frac{2bc\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^2x\left(1+c^2x^2\right)} - \frac{3c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} + \frac{(2bc)\int \frac{a}{}}{}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^2x\left(1+c^2x^2\right)} - \frac{3c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} - \frac{2b^2c\tan}{d^2}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^2x\left(1+c^2x^2\right)} - \frac{3c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} - \frac{3c\left(a+b\right.}{}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^2x\left(1+c^2x^2\right)} - \frac{3c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} - \frac{3c\left(a+b\right.}{}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^2x\left(1+c^2x^2\right)} - \frac{3c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} - \frac{3c\left(a+b\right.}{}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^2x\left(1+c^2x^2\right)} - \frac{3c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} - \frac{3c\left(a+b\right.}{}$$

Mathematica [A] time = 7.21, size = 549, normalized size = 1.91

$$-\frac{a^2c^2x}{2d^2\left(c^2x^2+1\right)} - \frac{3a^2c\tan^{-1}(cx)}{2d^2} - \frac{a^2}{d^2x} + \frac{2abc\left(\frac{\sqrt{c^2x^2+1}+i\sinh^{-1}(cx)}{4(-1-icx)} - \frac{\sinh^{-1}(cx)+i\sqrt{c^2x^2+1}}{4(cx+i)} - \tanh^{-1}\left(\sqrt{c^2x^2+1}\right) + \frac{3}{4}i\left(\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)^2),x]
```

```
[Out] -(a^2/(d^2*x)) - (a^2*c^2*x)/(2*d^2*(1 + c^2*x^2)) - (3*a^2*c*ArcTan[c*x])/
(2*d^2) + (2*a*b*c*((Sqrt[1 + c^2*x^2] + I*ArcSinh[c*x])/(4*(-1 - I*c*x)) -
 ArcSinh[c*x]/(c*x) - (I*Sqrt[1 + c^2*x^2] + ArcSinh[c*x])/(4*(I + c*x)) -
ArcTanh[Sqrt[1 + c^2*x^2]] + ((3*I)/4)*(-1/2*ArcSinh[c*x]^2 + 2*ArcSinh[c*x
]*Log[1 + I*E^ArcSinh[c*x]] + 2*PolyLog[2, (-I)*E^ArcSinh[c*x]]) - ((3*I)/4
)*(-1/2*ArcSinh[c*x]^2 + 2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + 2*PolyL
og[2, I*E^ArcSinh[c*x]])))/d^2 + (b^2*c*((-2*ArcSinh[c*x])/Sqrt[1 + c^2*x^2
] - (c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2) + 4*ArcTan[Tanh[ArcSinh[c*x]/2]] - A
rcSinh[c*x]^2*Coth[ArcSinh[c*x]/2] + 4*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x
])] + (3*I)*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] - (3*I)*ArcSinh[c*x]^2
*Log[1 + I/E^ArcSinh[c*x]] - 4*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 4*
PolyLog[2, -E^(-ArcSinh[c*x])] + (6*I)*ArcSinh[c*x]*PolyLog[2, (-I)/E^ArcSi
nh[c*x]] - (6*I)*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[c*x]] - 4*PolyLog[2, E
^(-ArcSinh[c*x])] + (6*I)*PolyLog[3, (-I)/E^ArcSinh[c*x]] - (6*I)*PolyLog[3
, I/E^ArcSinh[c*x]] + ArcSinh[c*x]^2*Tanh[ArcSinh[c*x]/2]))/(2*d^2)
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\operatorname{arsinh}(cx)^2 + 2\,ab\operatorname{arsinh}(cx) + a^2}{c^4d^2x^6 + 2\,c^2d^2x^4 + d^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^4*d^2*x^6 + 2*c
^2*d^2*x^4 + d^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^2}{\left(c^2dx^2+d\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^2*x^2), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2}{x^2\left(c^2d\,x^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^2,x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\,a^2\left(\frac{3\,c^2x^2+2}{c^2d^2x^3+d^2x}+\frac{3\,c\arctan(cx)}{d^2}\right)+\int \frac{b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^4d^2x^6+2\,c^2d^2x^4+d^2x^2}+\frac{2\,ab\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^4d^2x^6+2\,c^2d^2x^4+d^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a^2*((3*c^2*x^2 + 2)/(c^2*d^2*x^3 + d^2*x) + 3*c*arctan(c*x)/d^2) + in
tegrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d
^2*x^2) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 +
 d^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))^2}{x^2\left(d\,c^2\,x^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)^2),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^4x^6+2c^2x^4+x^2}\,dx+\int \frac{b^2\,\mathrm{asinh}^2\,(cx)}{c^4x^6+2c^2x^4+x^2}\,dx+\int \frac{2ab\,\mathrm{asinh}\,(cx)}{c^4x^6+2c^2x^4+x^2}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**2/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a**2/(c**4*x**6 + 2*c**2*x**4 + x**2), x) + Integral(b**2*asinh(c
*x)**2/(c**4*x**6 + 2*c**2*x**4 + x**2), x) + Integral(2*a*b*asinh(c*x)/(c*
*4*x**6 + 2*c**2*x**4 + x**2), x))/d**2
```

3.241 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^3\left(d+c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=253

$$\frac{2bc^2\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}-\frac{2bc^2\text{Li}_2\left(e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}-\frac{c^2\left(a+b\sinh^{-1}(cx)\right)^2}{d^2\left(c^2x^2+1\right)}-\frac{bc\left(a\right.}{d^{\cdots}}$$

```
[Out] -c^2*(a+b*arcsinh(c*x))^2/d^2/(c^2*x^2+1)-1/2*(a+b*arcsinh(c*x))^2/d^2/x^2/
(c^2*x^2+1)+4*c^2*(a+b*arcsinh(c*x))^2*arctanh((c*x+(c^2*x^2+1)^(1/2))^2)/d
^2+b^2*c^2*ln(x)/d^2-1/2*b^2*c^2*ln(c^2*x^2+1)/d^2+2*b*c^2*(a+b*arcsinh(c*x
))*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^2-2*b*c^2*(a+b*arcsinh(c*x))*pol
ylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)/d^2-b^2*c^2*polylog(3,-(c*x+(c^2*x^2+1)^(
1/2))^2)/d^2+b^2*c^2*polylog(3,(c*x+(c^2*x^2+1)^(1/2))^2)/d^2-b*c*(a+b*arcs
inh(c*x))/d^2/x/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.58, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 15, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.577, Rules used = {5747, 5755, 5720, 5461, 4182, 2531, 2282, 6589, 5687, 260, 271, 191, 5732, 446, 72}

$$\frac{2bc^2\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}-\frac{2bc^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}-\frac{b^2c^2\text{PolyLog}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)^2),x]
```

```
[Out] -((b*c*(a + b*ArcSinh[c*x]))/(d^2*x*Sqrt[1 + c^2*x^2])) - (c^2*(a + b*ArcSi
nh[c*x])^2)/(d^2*(1 + c^2*x^2)) - (a + b*ArcSinh[c*x])^2/(2*d^2*x^2*(1 + c^
2*x^2)) + (4*c^2*(a + b*ArcSinh[c*x])^2*ArcTanh[E^(2*ArcSinh[c*x])])/d^2 +
(b^2*c^2*Log[x])/d^2 - (b^2*c^2*Log[1 + c^2*x^2])/(2*d^2) + (2*b*c^2*(a + b
*ArcSinh[c*x])*PolyLog[2, -E^(2*ArcSinh[c*x])])/d^2 - (2*b*c^2*(a + b*ArcSi
nh[c*x])*PolyLog[2, E^(2*ArcSinh[c*x])])/d^2 - (b^2*c^2*PolyLog[3, -E^(2*Ar
cSinh[c*x])])/d^2 + (b^2*c^2*PolyLog[3, E^(2*ArcSinh[c*x])])/d^2
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
```

```
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5720

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
```

```
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^3\left(d+c^2dx^2\right)^2}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2x^2\left(1+c^2x^2\right)} - \left(2c^2\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)^2}\,dx + \frac{(bc)\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(1+c^2x^2\right)^{3/2}}\,dx}{d^2}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}} - \frac{2bc^3x\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}} - \frac{c^2\left(a+b\sinh^{-1}(cx)\right)^2}{d^2\left(1+c^2x^2\right)} - \frac{(a}{2a}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}} - \frac{c^2\left(a+b\sinh^{-1}(cx)\right)^2}{d^2\left(1+c^2x^2\right)} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2x^2\left(1+c^2x^2\right)} - \left(2c^2\right)\text{Su}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}} - \frac{c^2\left(a+b\sinh^{-1}(cx)\right)^2}{d^2\left(1+c^2x^2\right)} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2x^2\left(1+c^2x^2\right)} - b^2c^2\log$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}} - \frac{c^2\left(a+b\sinh^{-1}(cx)\right)^2}{d^2\left(1+c^2x^2\right)} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2x^2\left(1+c^2x^2\right)} + 4c^2\left(a+\right.$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}} - \frac{c^2\left(a+b\sinh^{-1}(cx)\right)^2}{d^2\left(1+c^2x^2\right)} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2x^2\left(1+c^2x^2\right)} + 4c^2\left(a+\right.$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}} - \frac{c^2\left(a+b\sinh^{-1}(cx)\right)^2}{d^2\left(1+c^2x^2\right)} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2x^2\left(1+c^2x^2\right)} + 4c^2\left(a+\right.$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}} - \frac{c^2\left(a+b\sinh^{-1}(cx)\right)^2}{d^2\left(1+c^2x^2\right)} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2x^2\left(1+c^2x^2\right)} + 4c^2\left(a+\right.$$

Mathematica [B] time = 0.91, size = 594, normalized size = 2.35

$$2a^2c^2\log\left(c^2x^2+1\right) + \frac{a^2}{c^2x^4+x^2} + 4a^2c^2\sinh^{-1}(cx) - 4a^2c^2\log\left(1-e^{2\sinh^{-1}(cx)}\right) - \frac{2a^2}{x^2} - 4abc^2\text{Li}_2\left(e^{2\sinh^{-1}(cx)}\right) +$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)^2),x]
```

```
[Out] ((-2*a^2)/x^2 - (2*a*b*c)/(x*Sqrt[1 + c^2*x^2]) + a^2/(x^2 + c^2*x^4) + 4*a
^2*c^2*ArcSinh[c*x] - (4*a*b*ArcSinh[c*x])/x^2 - (2*b^2*c*ArcSinh[c*x])/(x*
Sqrt[1 + c^2*x^2]) + (2*a*b*ArcSinh[c*x])/(x^2 + c^2*x^4) - (2*b^2*ArcSinh[
c*x]^2)/x^2 + (b^2*ArcSinh[c*x]^2)/(x^2 + c^2*x^4) + 8*a*b*c^2*ArcSinh[c*x]
*Log[1 + (c*E^ArcSinh[c*x])/Sqrt[-c^2]] + 4*b^2*c^2*ArcSinh[c*x]^2*Log[1 +
(c*E^ArcSinh[c*x])/Sqrt[-c^2]] + 8*a*b*c^2*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]
*E^ArcSinh[c*x])/c] + 4*b^2*c^2*ArcSinh[c*x]^2*Log[1 + (Sqrt[-c^2]*E^ArcSin
h[c*x])/c] - 4*a^2*c^2*Log[1 - E^(2*ArcSinh[c*x])] - 8*a*b*c^2*ArcSinh[c*x]
*Log[1 - E^(2*ArcSinh[c*x])] - 4*b^2*c^2*ArcSinh[c*x]^2*Log[1 - E^(2*ArcSin
h[c*x])] + 2*b^2*c^2*Log[x] + 2*a^2*c^2*Log[1 + c^2*x^2] - b^2*c^2*Log[1 +
c^2*x^2] + 8*b*c^2*(a + b*ArcSinh[c*x])*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[
-c^2]] + 8*b*c^2*(a + b*ArcSinh[c*x])*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x]
)/c] - 4*a*b*c^2*PolyLog[2, E^(2*ArcSinh[c*x])] - 4*b^2*c^2*ArcSinh[c*x]*Po
lyLog[2, E^(2*ArcSinh[c*x])] - 8*b^2*c^2*PolyLog[3, (c*E^ArcSinh[c*x])/Sqrt
[-c^2]] - 8*b^2*c^2*PolyLog[3, (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + 2*b^2*c^2*P
olyLog[3, E^(2*ArcSinh[c*x])])/(2*d^2)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\,\text{arsinh}\,(cx)^2+2\,ab\,\text{arsinh}\,(cx)+a^2}{c^4d^2x^7+2\,c^2d^2x^5+d^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^4*d^2*x^7 + 2*c
^2*d^2*x^5 + d^2*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{\left(c^2dx^2+d\right)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^2*x^3), x)
```

maple [B] time = 0.23, size = 799, normalized size = 3.16

$$-\frac{c^2b^2\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)}{d^2}+\frac{c^2b^2\ln\left(cx+\sqrt{c^2x^2+1}\,-1\right)}{d^2}+\frac{c^2b^2\ln\left(1+cx+\sqrt{c^2x^2+1}\right)}{d^2}-\frac{c^2a^2}{2d^2\left(c^2x^2+1\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^2,x)
```

```
[Out] -b^2*c^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^2-c^2*b^2/d^2*ln(1+(c*x+(c
^2*x^2+1)^(1/2))^2)+c^2*b^2/d^2*ln(c*x+(c^2*x^2+1)^(1/2)-1)+c^2*b^2/d^2*ln(
1+c*x+(c^2*x^2+1)^(1/2))+4*c^2*b^2/d^2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))+4*
c^2*b^2/d^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))-1/2*c^2*a^2/d^2/(c^2*x^2+1)+c^
2*a^2/d^2*ln(c^2*x^2+1)-2*c^2*a^2/d^2*ln(c*x)-4*c^2*a*b/d^2*arcsinh(c*x)*ln
(1+c*x+(c^2*x^2+1)^(1/2))-4*c^2*a*b/d^2*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(
1/2))-a*b/d^2*arcsinh(c*x)/x^2/(c^2*x^2+1)-c*a*b/d^2/x/(c^2*x^2+1)^(1/2)-c*
b^2/d^2*arcsinh(c*x)/x/(c^2*x^2+1)^(1/2)-2*c^2*a*b/d^2*arcsinh(c*x)/(c^2*x^
2+1)+4*c^2*a*b/d^2*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-1/2*a^2/d^2
/x^2-2*c^2*b^2/d^2*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))-4*c^2*b^2/d^2
*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))-2*c^2*b^2/d^2*arcsinh(c*x)^
2*ln(1-c*x-(c^2*x^2+1)^(1/2))-4*c^2*b^2/d^2*arcsinh(c*x)*polylog(2,c*x+(c^2
*x^2+1)^(1/2))-c^2*b^2/d^2*arcsinh(c*x)^2/(c^2*x^2+1)+2*c^2*b^2/d^2*arcsinh
(c*x)^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+2*c^2*b^2/d^2*arcsinh(c*x)*polylog(
2,-(c*x+(c^2*x^2+1)^(1/2))^2)-4*c^2*a*b/d^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2
))-4*c^2*a*b/d^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))+2*c^2*a*b/d^2*polylog(2,-
(c*x+(c^2*x^2+1)^(1/2))^2)-1/2*b^2/d^2*arcsinh(c*x)^2/x^2/(c^2*x^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,a^2\left(\frac{2\,c^2\log\left(c^2x^2+1\right)}{d^2}-\frac{4\,c^2\log(x)}{d^2}-\frac{2\,c^2x^2+1}{c^2d^2x^4+d^2x^2}\right)+\int\frac{b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^4d^2x^7+2\,c^2d^2x^5+d^2x^3}+\frac{2\,ab\log\left(cx+\sqrt{c^2x^2+1}\right.}{c^4d^2x^7+2\,c^2d^2x^5+d^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(2*c^2*log(c^2*x^2 + 1)/d^2 - 4*c^2*log(x)/d^2 - (2*c^2*x^2 + 1)/(c
^2*d^2*x^4 + d^2*x^2)) + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^4*
d^2*x^7 + 2*c^2*d^2*x^5 + d^2*x^3) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^
4*d^2*x^7 + 2*c^2*d^2*x^5 + d^2*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{asinh}\,(c\,x))^2}{x^3\,\left(d\,c^2\,x^2 + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)^2),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^4x^7+2c^2x^5+x^3}\,dx + \int \frac{b^2\,\mathrm{asinh}^2\,(cx)}{c^4x^7+2c^2x^5+x^3}\,dx + \int \frac{2ab\,\mathrm{asinh}\,(cx)}{c^4x^7+2c^2x^5+x^3}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**3/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a**2/(c**4*x**7 + 2*c**2*x**5 + x**3), x) + Integral(b**2*asinh(c
*x)**2/(c**4*x**7 + 2*c**2*x**5 + x**3), x) + Integral(2*a*b*asinh(c*x)/(c*
*4*x**7 + 2*c**2*x**5 + x**3), x))/d**2
```

3.242 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^4\left(d+c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=401

$$-\frac{5ibc^3\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}+\frac{5ibc^3\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}+\frac{5c^3\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+\right.}{d^2}$$

```
[Out] -1/3*b^2*c^2/d^2/x-1/3*(a+b*arcsinh(c*x))^2/d^2/x^3/(c^2*x^2+1)+5/3*c^2*(a+
b*arcsinh(c*x))^2/d^2/x/(c^2*x^2+1)+5/2*c^4*x*(a+b*arcsinh(c*x))^2/d^2/(c^2
*x^2+1)+5*c^3*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(1/2))/d^2-b^2*c^
3*arctan(c*x)/d^2+26/3*b*c^3*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/
2))/d^2+13/3*b^2*c^3*polylog(2,-c*x-(c^2*x^2+1)^(1/2))/d^2-5*I*b*c^3*(a+b*a
rcsinh(c*x))*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/d^2+5*I*b*c^3*(a+b*arcsi
nh(c*x))*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/d^2-13/3*b^2*c^3*polylog(2,c*
x+(c^2*x^2+1)^(1/2))/d^2+5*I*b^2*c^3*polylog(3,-I*(c*x+(c^2*x^2+1)^(1/2)))/
d^2-5*I*b^2*c^3*polylog(3,I*(c*x+(c^2*x^2+1)^(1/2)))/d^2+2/3*b*c^3*(a+b*arc
sinh(c*x))/d^2/(c^2*x^2+1)^(1/2)-1/3*b*c*(a+b*arcsinh(c*x))/d^2/x^2/(c^2*x^
2+1)^(1/2)
```

Rubi [A] time = 0.96, antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 15, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.577, Rules used = {5747, 5690, 5693, 4180, 2531, 2282, 6589, 5717, 203, 5755, 5760, 4182, 2279, 2391, 325}

$$-\frac{5ibc^3\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}+\frac{5ibc^3\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}+13b^2c^3\text{Pol}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)^2),x]
```

```
[Out] -(b^2*c^2)/(3*d^2*x) + (2*b*c^3*(a + b*ArcSinh[c*x]))/(3*d^2*Sqrt[1 + c^2*x
^2]) - (b*c*(a + b*ArcSinh[c*x]))/(3*d^2*x^2*Sqrt[1 + c^2*x^2]) - (a + b*Ar
cSinh[c*x])^2/(3*d^2*x^3*(1 + c^2*x^2)) + (5*c^2*(a + b*ArcSinh[c*x])^2)/(3
*d^2*x*(1 + c^2*x^2)) + (5*c^4*x*(a + b*ArcSinh[c*x])^2)/(2*d^2*(1 + c^2*x^
2)) + (5*c^3*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]])/d^2 - (b^2*c^3*
ArcTan[c*x])/d^2 + (26*b*c^3*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/
(3*d^2) + (13*b^2*c^3*PolyLog[2, -E^ArcSinh[c*x]])/(3*d^2) - ((5*I)*b*c^3*(
a + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/d^2 + ((5*I)*b*c^3*(a
+ b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])/d^2 - (13*b^2*c^3*PolyLog[2
, E^ArcSinh[c*x]])/(3*d^2) + ((5*I)*b^2*c^3*PolyLog[3, (-I)*E^ArcSinh[c*x]]
)/d^2 - ((5*I)*b^2*c^3*PolyLog[3, I*E^ArcSinh[c*x]])/d^2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^4\left(d+c^2dx^2\right)^2}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2x^3\left(1+c^2x^2\right)} - \frac{1}{3}\left(5c^2\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)^2}\,dx + \frac{(2bc)\int \frac{a+b\sinh^{-1}(cx)}{x^3\left(1+c^2x^2\right)^{3/2}}\,d}{3d^2}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2x^3\left(1+c^2x^2\right)} + \frac{5c^2\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2x\left(1+c^2x^2\right)} + \left(5c^4\right)\int$$

$$= -\frac{b^2c^2}{3d^2x} - \frac{13bc^3\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2x^3\left(1+c^2x^2\right)}$$

$$= -\frac{b^2c^2}{3d^2x} + \frac{2bc^3\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2x^3\left(1+c^2x^2\right)} +$$

$$= -\frac{b^2c^2}{3d^2x} + \frac{2bc^3\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2x^3\left(1+c^2x^2\right)} +$$

$$= -\frac{b^2c^2}{3d^2x} + \frac{2bc^3\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2x^3\left(1+c^2x^2\right)} +$$

$$= -\frac{b^2c^2}{3d^2x} + \frac{2bc^3\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2x^3\left(1+c^2x^2\right)} +$$

$$= -\frac{b^2c^2}{3d^2x} + \frac{2bc^3\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2x^3\left(1+c^2x^2\right)} +$$

Mathematica [A] time = 9.04, size = 764, normalized size = 1.91

$$\frac{5a^2c^3\tan^{-1}(cx)}{2d^2} + \frac{2a^2c^2}{d^2x} + \frac{a^2c^4x}{2d^2\left(c^2x^2+1\right)} - \frac{a^2}{3d^2x^3} + \frac{2ab\left(-\frac{5}{4}ic^4\left(\frac{2\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{c} - \frac{\sinh^{-1}(cx)^2}{2c} + \frac{2\sinh^{-1}(cx)\log\left(1+ie^{\sinh^{-1}(cx)}\right)}{c}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)^2),x]
```

```
[Out] -1/3*a^2/(d^2*x^3) + (2*a^2*c^2)/(d^2*x) + (a^2*c^4*x)/(2*d^2*(1 + c^2*x^2)
) + (5*a^2*c^3*ArcTan[c*x])/(2*d^2) + (2*a*b*(-1/6*(c*Sqrt[1 + c^2*x^2])/x^
2 - (c^3*(Sqrt[1 + c^2*x^2] + I*ArcSinh[c*x]))/(4*(-1 - I*c*x)) - ArcSinh[c
*x]/(3*x^3) + (c^4*(I*Sqrt[1 + c^2*x^2] + ArcSinh[c*x]))/(4*(I*c + c^2*x))
+ (c^3*ArcTanh[Sqrt[1 + c^2*x^2]])/6 - 2*c^2*(-(ArcSinh[c*x]/x) - c*ArcTanh
[Sqrt[1 + c^2*x^2]]) - ((5*I)/4)*c^4*(-1/2*ArcSinh[c*x]^2/c + (2*ArcSinh[c*
x]*Log[1 + I*E^ArcSinh[c*x]])/c + (2*PolyLog[2, (-I)*E^ArcSinh[c*x]])/c) +
((5*I)/4)*c^4*(-1/2*ArcSinh[c*x]^2/c + (2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[
c*x]])/c + (2*PolyLog[2, I*E^ArcSinh[c*x]])/c)))/d^2 + (b^2*c^3*((24*ArcSin
h[c*x])/Sqrt[1 + c^2*x^2] + (12*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2) - 48*ArcT
an[Tanh[ArcSinh[c*x]/2]] - 4*Coth[ArcSinh[c*x]/2] + 26*ArcSinh[c*x]^2*Coth[
ArcSinh[c*x]/2] - 2*ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - (c*x*ArcSinh[c*x]
^2*Csch[ArcSinh[c*x]/2]^4)/2 - 104*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])]
- (60*I)*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] + (60*I)*ArcSinh[c*x]^2*L
og[1 + I/E^ArcSinh[c*x]] + 104*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] - 10
4*PolyLog[2, -E^(-ArcSinh[c*x])] - (120*I)*ArcSinh[c*x]*PolyLog[2, (-I)/E^A
```

```
rcSinh[c*x]] + (120*I)*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[c*x]] + 104*Poly
Log[2, E^(-ArcSinh[c*x])] - (120*I)*PolyLog[3, (-I)/E^ArcSinh[c*x]] + (120*
I)*PolyLog[3, I/E^ArcSinh[c*x]] - 2*ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 - (
8*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2]^4)/(c^3*x^3) + 4*Tanh[ArcSinh[c*x]/2]
 - 26*ArcSinh[c*x]^2*Tanh[ArcSinh[c*x]/2]))/(24*d^2)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2 \operatorname{arsinh}(cx)^2 + 2\,ab \operatorname{arsinh}(cx) + a^2}{c^4d^2x^8 + 2\,c^2d^2x^6 + d^2x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^4*d^2*x^8 + 2*c
^2*d^2*x^6 + d^2*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2}{(c^2dx^2 + d)^2 x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^2*x^4), x)
```

maple [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arcsinh}(cx))^2}{x^4 (c^2d\,x^2 + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^2,x)
[Out] int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}\left(\frac{15\,c^3 \arctan(cx)}{d^2} + \frac{15\,c^4x^4 + 10\,c^2x^2 - 2}{c^2d^2x^5 + d^2x^3}\right)a^2 + \int \frac{b^2 \log\left(cx + \sqrt{c^2x^2 + 1}\right)^2}{c^4d^2x^8 + 2\,c^2d^2x^6 + d^2x^4} + \frac{2\,ab \log\left(cx + \sqrt{c^2x^2 + 1}\right)}{c^4d^2x^8 + 2\,c^2d^2x^6 + d^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^2,x, algorithm="maxima")
[Out] 1/6*(15*c^3*arctan(c*x)/d^2 + (15*c^4*x^4 + 10*c^2*x^2 - 2)/(c^2*d^2*x^5 +
d^2*x^3))*a^2 + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^4*d^2*x^8 +
 2*c^2*d^2*x^6 + d^2*x^4) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^8
 + 2*c^2*d^2*x^6 + d^2*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{asinh}(c\,x))^2}{x^4\,(d\,c^2\,x^2 + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)^2),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^4x^8+2c^2x^6+x^4}\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{c^4x^8+2c^2x^6+x^4}\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{c^4x^8+2c^2x^6+x^4}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**4/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a**2/(c**4*x**8 + 2*c**2*x**6 + x**4), x) + Integral(b**2*asinh(c
*x)**2/(c**4*x**8 + 2*c**2*x**6 + x**4), x) + Integral(2*a*b*asinh(c*x)/(c*
*4*x**8 + 2*c**2*x**6 + x**4), x))/d**2
```

3.243 $\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=320

$$-\frac{3ib\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4c^5d^3}+\frac{3ib\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4c^5d^3}+\frac{3\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh\right.}{4c^5d^3}$$

```
[Out] -1/12*b^2*x/c^4/d^3/(c^2*x^2+1)+1/6*b*(a+b*arcsinh(c*x))/c^5/d^3/(c^2*x^2+1
)^(3/2)-1/4*x^3*(a+b*arcsinh(c*x))^2/c^2/d^3/(c^2*x^2+1)^2-3/8*x*(a+b*arcsi
nh(c*x))^2/c^4/d^3/(c^2*x^2+1)+3/4*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2
+1)^(1/2))/c^5/d^3+7/6*b^2*arctan(c*x)/c^5/d^3-3/4*I*b*(a+b*arcsinh(c*x))*p
olylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c^5/d^3+3/4*I*b*(a+b*arcsinh(c*x))*pol
ylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c^5/d^3+3/4*I*b^2*polylog(3,-I*(c*x+(c^2*
x^2+1)^(1/2)))/c^5/d^3-3/4*I*b^2*polylog(3,I*(c*x+(c^2*x^2+1)^(1/2)))/c^5/d
^3-5/4*b*(a+b*arcsinh(c*x))/c^5/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5751, 5693, 4180, 2531, 2282, 6589, 5717, 203, 266, 43, 5732, 12, 385}

$$-\frac{3ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4c^5d^3}+\frac{3ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4c^5d^3}+\frac{3ib^2\text{PolyLog}\left(\right.}{4c^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^3,x]
```

```
[Out] -(b^2*x)/(12*c^4*d^3*(1 + c^2*x^2)) + (b*(a + b*ArcSinh[c*x]))/(6*c^5*d^3*(
1 + c^2*x^2)^(3/2)) - (5*b*(a + b*ArcSinh[c*x]))/(4*c^5*d^3*Sqrt[1 + c^2*x^
2]) - (x^3*(a + b*ArcSinh[c*x])^2)/(4*c^2*d^3*(1 + c^2*x^2)^2) - (3*x*(a +
b*ArcSinh[c*x])^2)/(8*c^4*d^3*(1 + c^2*x^2)) + (3*(a + b*ArcSinh[c*x])^2*Ar
cTan[E^ArcSinh[c*x]])/(4*c^5*d^3) + (7*b^2*ArcTan[c*x])/(6*c^5*d^3) - (((3*
I)/4)*b*(a + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^5*d^3) + (
((3*I)/4)*b*(a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])/(c^5*d^3) +
(((3*I)/4)*b^2*PolyLog[3, (-I)*E^ArcSinh[c*x]])/(c^5*d^3) - (((3*I)/4)*b^2*
PolyLog[3, I*E^ArcSinh[c*x]])/(c^5*d^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
```

```
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^3}\,dx=-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d^3\left(1+c^2x^2\right)^2}+\frac{b\int\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{5/2}}\,dx}{2cd^3}+\frac{3\int\frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^2}\,dx}{4c^2d}$$

$$=\frac{b\left(a+b\sinh^{-1}(cx)\right)}{6c^5d^3\left(1+c^2x^2\right)^{3/2}}-\frac{b\left(a+b\sinh^{-1}(cx)\right)}{2c^5d^3\sqrt{1+c^2x^2}}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d^3\left(1+c^2x^2\right)^2}-\frac{3x\left(a+b\,\mathrm{s}\right.}{8c^4d^3\left(\right.}$$

$$=\frac{b\left(a+b\sinh^{-1}(cx)\right)}{6c^5d^3\left(1+c^2x^2\right)^{3/2}}-\frac{5b\left(a+b\sinh^{-1}(cx)\right)}{4c^5d^3\sqrt{1+c^2x^2}}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d^3\left(1+c^2x^2\right)^2}-\frac{3x\left(a+b\right.}{8c^4d^3}$$

$$=-\frac{b^2x}{12c^4d^3\left(1+c^2x^2\right)}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{6c^5d^3\left(1+c^2x^2\right)^{3/2}}-\frac{5b\left(a+b\sinh^{-1}(cx)\right)}{4c^5d^3\sqrt{1+c^2x^2}}-\frac{x^3\left(a+b\sin\right.}{4c^2d^3\left(1+\right.}$$

$$=-\frac{b^2x}{12c^4d^3\left(1+c^2x^2\right)}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{6c^5d^3\left(1+c^2x^2\right)^{3/2}}-\frac{5b\left(a+b\sinh^{-1}(cx)\right)}{4c^5d^3\sqrt{1+c^2x^2}}-\frac{x^3\left(a+b\sin\right.}{4c^2d^3\left(1+\right.}$$

$$=-\frac{b^2x}{12c^4d^3\left(1+c^2x^2\right)}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{6c^5d^3\left(1+c^2x^2\right)^{3/2}}-\frac{5b\left(a+b\sinh^{-1}(cx)\right)}{4c^5d^3\sqrt{1+c^2x^2}}-\frac{x^3\left(a+b\sin\right.}{4c^2d^3\left(1+\right.}$$

$$=-\frac{b^2x}{12c^4d^3\left(1+c^2x^2\right)}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{6c^5d^3\left(1+c^2x^2\right)^{3/2}}-\frac{5b\left(a+b\sinh^{-1}(cx)\right)}{4c^5d^3\sqrt{1+c^2x^2}}-\frac{x^3\left(a+b\sin\right.}{4c^2d^3\left(1+\right.}$$

Mathematica [A] time = 3.40, size = 552, normalized size = 1.72

$$-\frac{15a^2cx}{c^2x^2+1}+\frac{6a^2cx}{\left(c^2x^2+1\right)^2}+9a^2\tan^{-1}(cx)+\frac{15ab\left(\sqrt{c^2x^2+1}-i\sinh^{-1}(cx)\right)}{-1+icx}+\frac{15ab\left(\sqrt{c^2x^2+1}+i\sinh^{-1}(cx)\right)}{-1-icx}-\frac{iab\left(3\sinh^{-1}(cx)+\sqrt{c^2x^2+1}\,(cx-\right.}{(cx-i)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^3,x]
```

```
[Out] ((6*a^2*c*x)/(1 + c^2*x^2)^2 - (15*a^2*c*x)/(1 + c^2*x^2) + (15*a*b*(Sqrt[1
 + c^2*x^2] - I*ArcSinh[c*x]))/(-1 + I*c*x) + (15*a*b*(Sqrt[1 + c^2*x^2] +
I*ArcSinh[c*x]))/(-1 - I*c*x) - (I*a*b*((-2*I + c*x)*Sqrt[1 + c^2*x^2] + 3*
ArcSinh[c*x]))/(-I + c*x)^2 + (I*a*b*((2*I + c*x)*Sqrt[1 + c^2*x^2] + 3*Arc
Sinh[c*x]))/(I + c*x)^2 + 9*a^2*ArcTan[c*x] + ((9*I)/2)*a*b*(ArcSinh[c*x]*(
ArcSinh[c*x] - 4*Log[1 + I*E^ArcSinh[c*x]]) - 4*PolyLog[2, (-I)*E^ArcSinh[c
*x]]) - ((9*I)/2)*a*b*(ArcSinh[c*x]*(ArcSinh[c*x] - 4*Log[1 - I*E^ArcSinh[c
*x]]) - 4*PolyLog[2, I*E^ArcSinh[c*x]]) + b^2*((-2*c*x)/(1 + c^2*x^2) + (4*
ArcSinh[c*x])/(1 + c^2*x^2)^(3/2) - (30*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] + (
6*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2)^2 - (15*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^
2) + 56*ArcTan[Tanh[ArcSinh[c*x]/2]] - (9*I)*ArcSinh[c*x]^2*Log[1 - I/E^Arc
Sinh[c*x]] + (9*I)*ArcSinh[c*x]^2*Log[1 + I/E^ArcSinh[c*x]] - (18*I)*ArcSin
h[c*x]*PolyLog[2, (-I)/E^ArcSinh[c*x]] + (18*I)*ArcSinh[c*x]*PolyLog[2, I/E
^ArcSinh[c*x]] - (18*I)*PolyLog[3, (-I)/E^ArcSinh[c*x]] + (18*I)*PolyLog[3,
 I/E^ArcSinh[c*x]]))/(24*c^5*d^3)
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2x^4\operatorname{arsinh}(cx)^2+2\,abx^4\operatorname{arsinh}(cx)+a^2x^4}{c^6d^3x^6+3\,c^4d^3x^4+3\,c^2d^3x^2+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4*arcsinh(c*x)^2 + 2*a*b*x^4*arcsinh(c*x) + a^2*x^4)/(c^6*d
^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\operatorname{arsinh}(cx)+a)^2x^4}{(c^2dx^2+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^4/(c^2*d*x^2 + d)^3, x)
```

maple [F] time = 0.54, size = 0, normalized size = 0.00

$$\int\frac{x^4(a+b\operatorname{arcsinh}(cx))^2}{(c^2d\,x^2+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x)
```

```
[Out] int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{8}\,a^2\left(\frac{5\,c^2x^3+3\,x}{c^8d^3x^4+2\,c^6d^3x^2+c^4d^3}-\frac{3\,\arctan(cx)}{c^5d^3}\right)+\int\frac{b^2x^4\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^6d^3x^6+3\,c^4d^3x^4+3\,c^2d^3x^2+d^3}+\frac{2\,abx^4\log\left(cx+\right.}{c^6d^3x^6+3\,c^4d^3x^4+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*a^2*((5*c^2*x^3 + 3*x)/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3) - 3*arc
tan(c*x)/(c^5*d^3)) + integrate(b^2*x^4*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^6
*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3) + 2*a*b*x^4*log(c*x + sqrt(
c^2*x^2 + 1))/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d\,c^2\,x^2+d\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^3,x)
```

```
[Out] int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2x^4}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{b^2x^4\,\mathrm{asinh}^2\,(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{2abx^4\,\mathrm{asinh}\,(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a**2*x**4/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integ
ral(b**2*x**4*asinh(c*x)**2/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x)
 + Integral(2*a*b*x**4*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 +
1), x))/d**3
```

3.244 $\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=167

$$-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4c^4d^3}+\frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(c^2x^2+1\right)^2}+\frac{bx^3\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(c^2x^2+1\right)^{3/2}}+\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{2c^3d^3\sqrt{c^2x^2+1}}-\frac{b^2}{12c^4d^3\left(c^2x^2+\right.}$$

```
[Out] -1/12*b^2/c^4/d^3/(c^2*x^2+1)+1/6*b*x^3*(a+b*arcsinh(c*x))/c/d^3/(c^2*x^2+1
)^(3/2)-1/4*(a+b*arcsinh(c*x))^2/c^4/d^3+1/4*x^4*(a+b*arcsinh(c*x))^2/d^3/(
c^2*x^2+1)^2-1/3*b^2*ln(c^2*x^2+1)/c^4/d^3+1/2*b*x*(a+b*arcsinh(c*x))/c^3/d
^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5723, 5751, 5675, 260, 266, 43}

$$\frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(c^2x^2+1\right)^2}+\frac{bx^3\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(c^2x^2+1\right)^{3/2}}+\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{2c^3d^3\sqrt{c^2x^2+1}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4c^4d^3}-\frac{b^2}{12c^4d^3\left(c^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^3,x]
```

```
[Out] -b^2/(12*c^4*d^3*(1 + c^2*x^2)) + (b*x^3*(a + b*ArcSinh[c*x]))/(6*c*d^3*(1
+ c^2*x^2)^(3/2)) + (b*x*(a + b*ArcSinh[c*x]))/(2*c^3*d^3*Sqrt[1 + c^2*x^2]
) - (a + b*ArcSinh[c*x])^2/(4*c^4*d^3) + (x^4*(a + b*ArcSinh[c*x])^2)/(4*d^
3*(1 + c^2*x^2)^2) - (b^2*Log[1 + c^2*x^2])/(3*c^4*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
```

```
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rubi steps

$$\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^3}\,dx=\frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(1+c^2x^2\right)^2}-\frac{(bc)\int\frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{5/2}}\,dx}{2d^3}$$

$$=\frac{bx^3\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(1+c^2x^2\right)^{3/2}}+\frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(1+c^2x^2\right)^2}-\frac{b^2\int\frac{x^3}{\left(1+c^2x^2\right)^2}\,dx}{6d^3}-\frac{b\int\frac{x^2\left(a+b\sin\right.}{\left(1+c^2x\right.}}{2cd}$$

$$=\frac{bx^3\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(1+c^2x^2\right)^{3/2}}+\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{2c^3d^3\sqrt{1+c^2x^2}}+\frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(1+c^2x^2\right)^2}-\frac{b^2\,\mathrm{Sub}}{}$$

$$=\frac{bx^3\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(1+c^2x^2\right)^{3/2}}+\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{2c^3d^3\sqrt{1+c^2x^2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4c^4d^3}+\frac{x^4\left(a+b\right.}{4d^3\left(1\right.}$$

$$=-\frac{b^2}{12c^4d^3\left(1+c^2x^2\right)}+\frac{bx^3\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(1+c^2x^2\right)^{3/2}}+\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{2c^3d^3\sqrt{1+c^2x^2}}-\frac{\left(a+b\sin\right.}{4c^4}$$

Mathematica [A] time = 0.40, size = 186, normalized size = 1.11

$$-\frac{6a^2c^2x^2+3a^2-6abcx\sqrt{c^2x^2+1}+2b\sinh^{-1}(cx)\left(a\left(6c^2x^2+3\right)-bcx\sqrt{c^2x^2+1}\left(4c^2x^2+3\right)\right)-8abc^3x^3\sqrt{c^2x^2}}{12c^4d^3\left(c^2x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^3,x]
```

```
[Out] -1/12*(3*a^2 + b^2 + 6*a^2*c^2*x^2 + b^2*c^2*x^2 - 6*a*b*c*x*Sqrt[1 + c^2*x
^2] - 8*a*b*c^3*x^3*Sqrt[1 + c^2*x^2] + 2*b*(-(b*c*x*Sqrt[1 + c^2*x^2]*(3 +
 4*c^2*x^2)) + a*(3 + 6*c^2*x^2))*ArcSinh[c*x] + 3*b^2*(1 + 2*c^2*x^2)*ArcS
inh[c*x]^2 + 4*(b + b*c^2*x^2)^2*Log[1 + c^2*x^2])/(c^4*d^3*(1 + c^2*x^2)^2
)
```

fricas [A] time = 0.43, size = 283, normalized size = 1.69

$$\frac{8\,abc^4x^4-\left(6\,a^2-16\,ab+b^2\right)c^2x^2-3\left(2\,b^2c^2x^2+b^2\right)\log\left(cx+\sqrt{c^2x^2+1}\right)^2-3\,a^2+8\,ab-b^2-4\left(b^2c^4x^4+2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(8*a*b*c^4*x^4 - (6*a^2 - 16*a*b + b^2)*c^2*x^2 - 3*(2*b^2*c^2*x^2 + b
^2)*log(c*x + sqrt(c^2*x^2 + 1))^2 - 3*a^2 + 8*a*b - b^2 - 4*(b^2*c^4*x^4 +
 2*b^2*c^2*x^2 + b^2)*log(c^2*x^2 + 1) + 2*(3*a*b*c^4*x^4 + (4*b^2*c^3*x^3
+ 3*b^2*c*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 6*(a*b*c^4*x
^4 + 2*a*b*c^2*x^2 + a*b)*log(-c*x + sqrt(c^2*x^2 + 1)) + 2*(4*a*b*c^3*x^3
+ 3*a*b*c*x)*sqrt(c^2*x^2 + 1))/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.38, size = 523, normalized size = 3.13

$$-\frac{a^2}{2c^4d^3\left(c^2x^2+1\right)}+\frac{a^2}{4c^4d^3\left(c^2x^2+1\right)^2}+\frac{4b^2\operatorname{arcsinh}(cx)}{3c^4d^3}+\frac{2b^2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\;x^3}{3c\,d^3\left(c^4x^4+2c^2x^2+1\right)}-\frac{2b^2\operatorname{arcsinh}(cx)\,x^4}{3d^3\left(c^4x^4+2c^2x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x)
```

```
[Out] -1/2/c^4*a^2/d^3/(c^2*x^2+1)+1/4/c^4*a^2/d^3/(c^2*x^2+1)^2+4/3/c^4*b^2/d^3*
arcsinh(c*x)+2/3/c*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)*(c^2*x^2+1)^(
1/2)*x^3-2/3*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)*x^4-1/2/c^2*b^2/d^3
/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)^2*x^2+1/2/c^3*b^2/d^3/(c^4*x^4+2*c^2*x^
2+1)*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x-4/3/c^2*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)
*arcsinh(c*x)*x^2-1/4/c^4*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)^2-1/12
/c^2*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*x^2-2/3/c^4*b^2/d^3/(c^4*x^4+2*c^2*x^2+1
)*arcsinh(c*x)-1/12/c^4*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)-2/3/c^4*b^2/d^3*ln(1+
(c*x+(c^2*x^2+1)^(1/2))^2)-1/c^4*a*b/d^3/(c^2*x^2+1)*arcsinh(c*x)+1/2/c^4*a
*b/d^3/(c^2*x^2+1)^2*arcsinh(c*x)-1/6/c^3*a*b/d^3/(c^2*x^2+1)^(3/2)*x+2/3/c
^3*a*b/d^3*x/(c^2*x^2+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(2\,c^2x^2+1\right)a^2}{4\left(c^8d^3x^4+2\,c^6d^3x^2+c^4d^3\right)}-\frac{\left(2\,b^2c^2x^2+b^2\right)\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{4\left(c^8d^3x^4+2\,c^6d^3x^2+c^4d^3\right)}+\int\frac{\left(3\,b^2c^2x^2+2\left(2\,abc^4+b^2c^4\right)x^4+b\right.}{2\left(c^{10}d^3x^7+3\,c^8d^3x^5+3\,c^6d^3x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*c^2*x^2 + 1)*a^2/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3) - 1/4*(2*b
^2*c^2*x^2 + b^2)*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^8*d^3*x^4 + 2*c^6*d^3*x
^2 + c^4*d^3) + integrate(1/2*(3*b^2*c^2*x^2 + 2*(2*a*b*c^4 + b^2*c^4)*x^4
+ b^2 + (b^2*c*x + 2*(2*a*b*c^3 + b^2*c^3)*x^3)*sqrt(c^2*x^2 + 1))*log(c*x
+ sqrt(c^2*x^2 + 1))/(c^10*d^3*x^7 + 3*c^8*d^3*x^5 + 3*c^6*d^3*x^3 + c^4*d^
3*x + (c^9*d^3*x^6 + 3*c^7*d^3*x^4 + 3*c^5*d^3*x^2 + c^3*d^3)*sqrt(c^2*x^2
+ 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d\,c^2\,x^2+d\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^3,x)
```

```
[Out] int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2x^3}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{b^2x^3\operatorname{asinh}^2(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{2abx^3\operatorname{asinh}(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a**2*x**3/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integ
ral(b**2*x**3*asinh(c*x)**2/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x)
 + Integral(2*a*b*x**3*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 +
1), x))/d**3
```

3.245 $\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=318

$$-\frac{ib\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4c^3d^3}+\frac{ib\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4c^3d^3}+\frac{\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}\right.}{4c^3d^3}$$

```
[Out] 1/12*b^2*x/c^2/d^3/(c^2*x^2+1)-1/6*b*(a+b*arcsinh(c*x))/c^3/d^3/(c^2*x^2+1)
^(3/2)-1/4*x*(a+b*arcsinh(c*x))^2/c^2/d^3/(c^2*x^2+1)^2+1/8*x*(a+b*arcsinh(
c*x))^2/c^2/d^3/(c^2*x^2+1)+1/4*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)
^(1/2))/c^3/d^3-1/6*b^2*arctan(c*x)/c^3/d^3-1/4*I*b*(a+b*arcsinh(c*x))*poly
log(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d^3+1/4*I*b*(a+b*arcsinh(c*x))*polylo
g(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d^3+1/4*I*b^2*polylog(3,-I*(c*x+(c^2*x^2
+1)^(1/2)))/c^3/d^3-1/4*I*b^2*polylog(3,I*(c*x+(c^2*x^2+1)^(1/2)))/c^3/d^3+
1/4*b*(a+b*arcsinh(c*x))/c^3/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5751, 5690, 5693, 4180, 2531, 2282, 6589, 5717, 203, 199}

$$-\frac{ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4c^3d^3}+\frac{ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4c^3d^3}+\frac{ib^2\text{PolyLog}\left(\right.}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^3,x]
```

```
[Out] (b^2*x)/(12*c^2*d^3*(1 + c^2*x^2)) - (b*(a + b*ArcSinh[c*x]))/(6*c^3*d^3*(1
 + c^2*x^2)^(3/2)) + (b*(a + b*ArcSinh[c*x]))/(4*c^3*d^3*Sqrt[1 + c^2*x^2])
 - (x*(a + b*ArcSinh[c*x])^2)/(4*c^2*d^3*(1 + c^2*x^2)^2) + (x*(a + b*ArcSi
nh[c*x])^2)/(8*c^2*d^3*(1 + c^2*x^2)) + ((a + b*ArcSinh[c*x])^2*ArcTan[E^Ar
cSinh[c*x]])/(4*c^3*d^3) - (b^2*ArcTan[c*x])/(6*c^3*d^3) - ((I/4)*b*(a + b*
ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^3*d^3) + ((I/4)*b*(a + b*
ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])/(c^3*d^3) + ((I/4)*b^2*PolyLog[
3, (-I)*E^ArcSinh[c*x]])/(c^3*d^3) - ((I/4)*b^2*PolyLog[3, I*E^ArcSinh[c*x]
])/(c^3*d^3)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^3}\,dx=-\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d^3\left(1+c^2x^2\right)^2}+\frac{b\int\frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{5/2}}\,dx}{2cd^3}+\frac{\int\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^2}\,dx}{4c^2d}$$

$$=-\frac{b\left(a+b\sinh^{-1}(cx)\right)}{6c^3d^3\left(1+c^2x^2\right)^{3/2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d^3\left(1+c^2x^2\right)^2}+\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{8c^2d^3\left(1+c^2x^2\right)}+\frac{b^2\int\cdots}{\cdots}$$

$$=\frac{b^2x}{12c^2d^3\left(1+c^2x^2\right)}-\frac{b\left(a+b\sinh^{-1}(cx)\right)}{6c^3d^3\left(1+c^2x^2\right)^{3/2}}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{4c^3d^3\sqrt{1+c^2x^2}}-\frac{x\left(a+b\sinh\cdots\right.}{4c^2d^3\left(1+\cdots\right.}$$

$$=\frac{b^2x}{12c^2d^3\left(1+c^2x^2\right)}-\frac{b\left(a+b\sinh^{-1}(cx)\right)}{6c^3d^3\left(1+c^2x^2\right)^{3/2}}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{4c^3d^3\sqrt{1+c^2x^2}}-\frac{x\left(a+b\sinh\cdots\right.}{4c^2d^3\left(1+\cdots\right.}$$

$$=\frac{b^2x}{12c^2d^3\left(1+c^2x^2\right)}-\frac{b\left(a+b\sinh^{-1}(cx)\right)}{6c^3d^3\left(1+c^2x^2\right)^{3/2}}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{4c^3d^3\sqrt{1+c^2x^2}}-\frac{x\left(a+b\sinh\cdots\right.}{4c^2d^3\left(1+\cdots\right.}$$

$$=\frac{b^2x}{12c^2d^3\left(1+c^2x^2\right)}-\frac{b\left(a+b\sinh^{-1}(cx)\right)}{6c^3d^3\left(1+c^2x^2\right)^{3/2}}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{4c^3d^3\sqrt{1+c^2x^2}}-\frac{x\left(a+b\sinh\cdots\right.}{4c^2d^3\left(1+\cdots\right.}$$

$$=\frac{b^2x}{12c^2d^3\left(1+c^2x^2\right)}-\frac{b\left(a+b\sinh^{-1}(cx)\right)}{6c^3d^3\left(1+c^2x^2\right)^{3/2}}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{4c^3d^3\sqrt{1+c^2x^2}}-\frac{x\left(a+b\sinh\cdots\right.}{4c^2d^3\left(1+\cdots\right.}$$

Mathematica [A] time = 2.51, size = 550, normalized size = 1.73

$$\frac{\frac{3a^2cx}{c^2x^2+1}-\frac{6a^2cx}{\left(c^2x^2+1\right)^2}+3a^2\tan^{-1}(cx)+\frac{ab\left(\sqrt{c^2x^2+1}\,(2+icx)+3i\sinh^{-1}(cx)\right)}{(cx-i)^2}+\frac{3ab\left(\sinh^{-1}(cx)-i\sqrt{c^2x^2+1}\right)}{cx-i}+\frac{3ab\left(\sinh^{-1}(cx)+i\sqrt{c^2x^2+\cdots}\right.}{cx+i}\cdots}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^3,x]
```

```
[Out] ((-6*a^2*c*x)/(1 + c^2*x^2)^2 + (3*a^2*c*x)/(1 + c^2*x^2) + (a*b*((2 + I*c*
x)*Sqrt[1 + c^2*x^2] + (3*I)*ArcSinh[c*x]))/(-I + c*x)^2 + (3*a*b*((-I)*Sqr
t[1 + c^2*x^2] + ArcSinh[c*x]))/(-I + c*x) + (3*a*b*(I*Sqrt[1 + c^2*x^2] +
ArcSinh[c*x]))/(I + c*x) - (I*a*b*((2*I + c*x)*Sqrt[1 + c^2*x^2] + 3*ArcSin
h[c*x]))/(I + c*x)^2 + 3*a^2*ArcTan[c*x] + ((3*I)/2)*a*b*(ArcSinh[c*x]*(Arc
Sinh[c*x] - 4*Log[1 + I*E^ArcSinh[c*x]]) - 4*PolyLog[2, (-I)*E^ArcSinh[c*x]
]) - ((3*I)/2)*a*b*(ArcSinh[c*x]*(ArcSinh[c*x] - 4*Log[1 - I*E^ArcSinh[c*x]
]) - 4*PolyLog[2, I*E^ArcSinh[c*x]]) + b^2*((2*c*x)/(1 + c^2*x^2) - (4*ArcS
inh[c*x])/(1 + c^2*x^2)^(3/2) + (6*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - (6*c*x
*ArcSinh[c*x]^2)/(1 + c^2*x^2)^2 + (3*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2) - 8
*ArcTan[Tanh[ArcSinh[c*x]/2]] - (3*I)*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*
x]] + (3*I)*ArcSinh[c*x]^2*Log[1 + I/E^ArcSinh[c*x]] - (6*I)*ArcSinh[c*x]*P
olyLog[2, (-I)/E^ArcSinh[c*x]] + (6*I)*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[
```

```
c*x]] - (6*I)*PolyLog[3, (-I)/E^ArcSinh[c*x]] + (6*I)*PolyLog[3, I/E^ArcSin
h[c*x]]))/(24*c^3*d^3)
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2x^2\,\text{arsinh}\,(cx)^2+2\,abx^2\,\text{arsinh}\,(cx)+a^2x^2}{c^6d^3x^6+3\,c^4d^3x^4+3\,c^2d^3x^2+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*x) + a^2*x^2)/(c^6*d
^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\,\text{arsinh}\,(cx)+a)^2x^2}{\left(c^2dx^2+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^2/(c^2*d*x^2 + d)^3, x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int\frac{x^2\,(a+b\,\text{arcsinh}\,(cx))^2}{\left(c^2d\,x^2+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x)
```

```
[Out] int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8}\,a^2\left(\frac{c^2x^3-x}{c^6d^3x^4+2\,c^4d^3x^2+c^2d^3}+\frac{\arctan\,(cx)}{c^3d^3}\right)+\int\frac{b^2x^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^6d^3x^6+3\,c^4d^3x^4+3\,c^2d^3x^2+d^3}+\frac{2\,abx^2\log\left(cx+\sqrt{c^2x^2}\right.}{c^6d^3x^6+3\,c^4d^3x^4+3\,c^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*a^2*((c^2*x^3 - x)/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3) + arctan(c*x
)/(c^3*d^3)) + integrate(b^2*x^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^6*d^3*x^
6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3) + 2*a*b*x^2*log(c*x + sqrt(c^2*x^2
 + 1))/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x^2\,(a+b\,\text{asinh}\,(c\,x))^2}{\left(d\,c^2\,x^2+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^3,x)
```

```
[Out] int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2x^2}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{b^2x^2\operatorname{asinh}^2(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{2abx^2\operatorname{asinh}(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a**2*x**2/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integ
ral(b**2*x**2*asinh(c*x)**2/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x)
 + Integral(2*a*b*x**2*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 +
1), x))/d**3
```

3.246 $\int \frac{x(a+b\sinh^{-1}(cx))^2}{(d+c^2dx^2)^3}\,dx$

Optimal. Leaf size=145

$$\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3cd^3\sqrt{c^2x^2+1}}+\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(c^2x^2+1\right)^{3/2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d^3\left(c^2x^2+1\right)^2}+\frac{b^2}{12c^2d^3\left(c^2x^2+1\right)}-\frac{b^2\log\left(c^2x^2+1\right)}{6c^2d^3}$$

```
[Out] 1/12*b^2/c^2/d^3/(c^2*x^2+1)+1/6*b*x*(a+b*arcsinh(c*x))/c/d^3/(c^2*x^2+1)^(
3/2)-1/4*(a+b*arcsinh(c*x))^2/c^2/d^3/(c^2*x^2+1)^2-1/6*b^2*ln(c^2*x^2+1)/c
^2/d^3+1/3*b*x*(a+b*arcsinh(c*x))/c/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5717, 5690, 5687, 260, 261}

$$\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3cd^3\sqrt{c^2x^2+1}}+\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(c^2x^2+1\right)^{3/2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d^3\left(c^2x^2+1\right)^2}+\frac{b^2}{12c^2d^3\left(c^2x^2+1\right)}-\frac{b^2\log\left(c^2x^2+1\right)}{6c^2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^3,x]
```

```
[Out] b^2/(12*c^2*d^3*(1 + c^2*x^2)) + (b*x*(a + b*ArcSinh[c*x]))/(6*c*d^3*(1 + c
^2*x^2)^(3/2)) + (b*x*(a + b*ArcSinh[c*x]))/(3*c*d^3*Sqrt[1 + c^2*x^2]) - (
a + b*ArcSinh[c*x])^2/(4*c^2*d^3*(1 + c^2*x^2)^2) - (b^2*Log[1 + c^2*x^2])/
(6*c^2*d^3)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^3}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d^3\left(1+c^2x^2\right)^2}+\frac{b\int\frac{a+b\sinh^{-1}(cx)}{\left(1+c^2x^2\right)^{5/2}}\,dx}{2cd^3}$$

$$=\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(1+c^2x^2\right)^{3/2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d^3\left(1+c^2x^2\right)^2}-\frac{b^2\int\frac{x}{\left(1+c^2x^2\right)^2}\,dx}{6d^3}+\frac{b\int\frac{a+b\sinh^{-1}(c}{\left(1+c^2x^2\right)^{3/2}}}{3cd^3}$$

$$=\frac{b^2}{12c^2d^3\left(1+c^2x^2\right)}+\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(1+c^2x^2\right)^{3/2}}+\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3cd^3\sqrt{1+c^2x^2}}-\frac{\left(a+b\sinh\right.}{4c^2d^3\left(1-\right.}$$

$$=\frac{b^2}{12c^2d^3\left(1+c^2x^2\right)}+\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(1+c^2x^2\right)^{3/2}}+\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3cd^3\sqrt{1+c^2x^2}}-\frac{\left(a+b\sinh\right.}{4c^2d^3\left(1-\right.}$$

Mathematica [A] time = 0.22, size = 152, normalized size = 1.05

$$\frac{-3a^2+6abcx\sqrt{c^2x^2+1}+2b\sinh^{-1}(cx)\left(bcx\sqrt{c^2x^2+1}\left(2c^2x^2+3\right)-3a\right)+4abc^3x^3\sqrt{c^2x^2+1}+b^2c^2x^2-3b}{12d^3\left(c^3x^2+c\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^3,x]
```

```
[Out] (-3*a^2 + b^2 + b^2*c^2*x^2 + 6*a*b*c*x*Sqrt[1 + c^2*x^2] + 4*a*b*c^3*x^3*S
qrt[1 + c^2*x^2] + 2*b*(-3*a + b*c*x*Sqrt[1 + c^2*x^2]*(3 + 2*c^2*x^2))*Arc
Sinh[c*x] - 3*b^2*ArcSinh[c*x]^2 - 2*(b + b*c^2*x^2)^2*Log[1 + c^2*x^2])/(1
2*d^3*(c + c^3*x^2)^2)
```

fricas [B] time = 0.58, size = 273, normalized size = 1.88

$$4\,abc^4x^4+\left(8\,ab+b^2\right)c^2x^2-3\,b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2-3\,a^2+4\,ab+b^2-2\left(b^2c^4x^4+2\,b^2c^2x^2+b^2\right)\log\left(c^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*a*b*c^4*x^4 + (8*a*b + b^2)*c^2*x^2 - 3*b^2*log(c*x + sqrt(c^2*x^2
+ 1))^2 - 3*a^2 + 4*a*b + b^2 - 2*(b^2*c^4*x^4 + 2*b^2*c^2*x^2 + b^2)*log(c
^2*x^2 + 1) + 2*(3*a*b*c^4*x^4 + 6*a*b*c^2*x^2 + (2*b^2*c^3*x^3 + 3*b^2*c*x
)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 6*(a*b*c^4*x^4 + 2*a*b*
c^2*x^2 + a*b)*log(-c*x + sqrt(c^2*x^2 + 1)) + 2*(2*a*b*c^3*x^3 + 3*a*b*c*x
)*sqrt(c^2*x^2 + 1))/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2 x}{(c^2 d x^2 + d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x/(c^2*d*x^2 + d)^3, x)
```

maple [B] time = 0.15, size = 432, normalized size = 2.98

$$-\frac{a^2}{4c^2d^3\left(c^2x^2+1\right)^2}+\frac{2b^2 \operatorname{arcsinh}(cx)}{3c^2d^3}+\frac{c\,b^2 \operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\;x^3}{3d^3\left(c^4x^4+2c^2x^2+1\right)}-\frac{c^2b^2 \operatorname{arcsinh}(cx)\,x^4}{3d^3\left(c^4x^4+2c^2x^2+1\right)}+\frac{b^2 \operatorname{arcsinh}(cx)\sqrt{}}{2c\,d^3\left(c^4x^4+2c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x)
```

```
[Out] -1/4/c^2*a^2/d^3/(c^2*x^2+1)^2+2/3/c^2*b^2/d^3*arcsinh(c*x)+1/3*c*b^2/d^3/(
c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^3-1/3*c^2*b^2/d^3/(c^
4*x^4+2*c^2*x^2+1)*arcsinh(c*x)*x^4+1/2/c*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arc
sinh(c*x)*(c^2*x^2+1)^(1/2)*x-2/3*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x
)*x^2-1/4/c^2*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)^2+1/12*b^2/d^3/(c^
4*x^4+2*c^2*x^2+1)*x^2-1/3/c^2*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)+1
/12/c^2*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)-1/3/c^2*b^2/d^3*ln(1+(c*x+(c^2*x^2+1)
^(1/2))^2)-1/2/c^2*a*b/d^3/(c^2*x^2+1)^2*arcsinh(c*x)+1/6/c*a*b/d^3/(c^2*x^
2+1)^(3/2)*x+1/3/c*a*b/d^3*x/(c^2*x^2+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^2 \log\left(cx+\sqrt{c^2x^2+1}\right)^2}{4\left(c^6d^3x^4+2\,c^4d^3x^2+c^2d^3\right)}-\frac{a^2}{4\left(c^6d^3x^4+2\,c^4d^3x^2+c^2d^3\right)}+\int \frac{\left(\left(4\,abc^2+b^2c^2\right)x^2+\sqrt{c^2x^2+1}\left(4\,abc+\right.\right.}{2\left(c^8d^3x^7+3\,c^6d^3x^5+3\,c^4d^3x^3+c^2d^3x+\left(c^7d^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/4*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*
d^3) - 1/4*a^2/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3) + integrate(1/2*((4*
a*b*c^2 + b^2*c^2)*x^2 + sqrt(c^2*x^2 + 1)*(4*a*b*c + b^2*c)*x + b^2)*log(c
*x + sqrt(c^2*x^2 + 1))/(c^8*d^3*x^7 + 3*c^6*d^3*x^5 + 3*c^4*d^3*x^3 + c^2*
d^3*x + (c^7*d^3*x^6 + 3*c^5*d^3*x^4 + 3*c^3*d^3*x^2 + c*d^3)*sqrt(c^2*x^2
+ 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,(a+b\,\mathrm{asinh}(c\,x))^2}{(d\,c^2\,x^2+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^3,x)
```

```
[Out] int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2x}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{b^2x\operatorname{asinh}^2(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{2abx\operatorname{asinh}(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a**2*x/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integral
(b**2*x*asinh(c*x)**2/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Int
egral(2*a*b*x*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x))/d
**3
```

3.247 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=309

$$\frac{3b\left(a+b\sinh^{-1}(cx)\right)}{4cd^3\sqrt{c^2x^2+1}}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(c^2x^2+1\right)^{3/2}}+\frac{3x\left(a+b\sinh^{-1}(cx)\right)^2}{8d^3\left(c^2x^2+1\right)}+\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(c^2x^2+1\right)^2}-\frac{3ib\mathrm{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{4cc}$$

```
[Out] -1/12*b^2*x/d^3/(c^2*x^2+1)+1/6*b*(a+b*arcsinh(c*x))/c/d^3/(c^2*x^2+1)^(3/2
)+1/4*x*(a+b*arcsinh(c*x))^2/d^3/(c^2*x^2+1)^2+3/8*x*(a+b*arcsinh(c*x))^2/d
^3/(c^2*x^2+1)+3/4*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(1/2))/c/d^3
-5/6*b^2*arctan(c*x)/c/d^3-3/4*I*b*(a+b*arcsinh(c*x))*polylog(2,-I*(c*x+(c^
2*x^2+1)^(1/2)))/c/d^3+3/4*I*b*(a+b*arcsinh(c*x))*polylog(2,I*(c*x+(c^2*x^2
+1)^(1/2)))/c/d^3+3/4*I*b^2*polylog(3,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^3-3/4
*I*b^2*polylog(3,I*(c*x+(c^2*x^2+1)^(1/2)))/c/d^3+3/4*b*(a+b*arcsinh(c*x))/
c/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5690, 5693, 4180, 2531, 2282, 6589, 5717, 203, 199}

$$-\frac{3ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4cd^3}+\frac{3ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4cd^3}+\frac{3ib^2\text{PolyLog}\left(\right.}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^3,x]
```

```
[Out] -(b^2*x)/(12*d^3*(1 + c^2*x^2)) + (b*(a + b*ArcSinh[c*x]))/(6*c*d^3*(1 + c^
2*x^2)^(3/2)) + (3*b*(a + b*ArcSinh[c*x]))/(4*c*d^3*Sqrt[1 + c^2*x^2]) + (x
*(a + b*ArcSinh[c*x])^2)/(4*d^3*(1 + c^2*x^2)^2) + (3*x*(a + b*ArcSinh[c*x]
)^2)/(8*d^3*(1 + c^2*x^2)) + (3*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x
]])/(4*c*d^3) - (5*b^2*ArcTan[c*x])/(6*c*d^3) - (((3*I)/4)*b*(a + b*ArcSinh
[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*d^3) + (((3*I)/4)*b*(a + b*ArcSi
nh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])/(c*d^3) + (((3*I)/4)*b^2*PolyLog[3,
(-I)*E^ArcSinh[c*x]])/(c*d^3) - (((3*I)/4)*b^2*PolyLog[3, I*E^ArcSinh[c*x]]
)/(c*d^3)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^3}\,dx = \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(1+c^2x^2\right)^2} - \frac{(bc)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{5/2}}\,dx}{2d^3} + \frac{3\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^2}\,dx}{4d}$$

$$= \frac{b\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(1+c^2x^2\right)^2} + \frac{3x\left(a+b\sinh^{-1}(cx)\right)^2}{8d^3\left(1+c^2x^2\right)} - \frac{b^2\int \frac{1}{(1+c^2x^2}}{6d^3}$$

$$= -\frac{b^2x}{12d^3\left(1+c^2x^2\right)} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{3b\left(a+b\sinh^{-1}(cx)\right)}{4cd^3\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh^{-1}(c\right.}{4d^3\left(1+c^2x^2\right.}$$

$$= -\frac{b^2x}{12d^3\left(1+c^2x^2\right)} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{3b\left(a+b\sinh^{-1}(cx)\right)}{4cd^3\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh^{-1}(c\right.}{4d^3\left(1+c^2x^2\right.}$$

$$= -\frac{b^2x}{12d^3\left(1+c^2x^2\right)} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{3b\left(a+b\sinh^{-1}(cx)\right)}{4cd^3\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh^{-1}(c\right.}{4d^3\left(1+c^2x^2\right.}$$

$$= -\frac{b^2x}{12d^3\left(1+c^2x^2\right)} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{3b\left(a+b\sinh^{-1}(cx)\right)}{4cd^3\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh^{-1}(c\right.}{4d^3\left(1+c^2x^2\right.}$$

$$= -\frac{b^2x}{12d^3\left(1+c^2x^2\right)} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{3b\left(a+b\sinh^{-1}(cx)\right)}{4cd^3\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh^{-1}(c\right.}{4d^3\left(1+c^2x^2\right.}$$

Mathematica [A] time = 2.34, size = 546, normalized size = 1.77

$$\frac{\frac{9a^2x}{c^2x^2+1} + \frac{6a^2x}{\left(c^2x^2+1\right)^2} + \frac{9a^2\tan^{-1}(cx)}{c} + \frac{ab\left(\frac{9\left(\sinh^{-1}(cx)-i\sqrt{c^2x^2+1}\right)}{cx-i} + \frac{9\left(\sinh^{-1}(cx)+i\sqrt{c^2x^2+1}\right)}{cx+i} - \frac{i\left(3\sinh^{-1}(cx)+\sqrt{c^2x^2+1}\,(cx-2i)\right)}{(cx-i)^2} + \frac{i\left(3\sinh^{-1}(cx)+\sqrt{c^2x^2+1}\,(c\right.}{(cx+i)^2}\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^3,x]
```

```
[Out] ((6*a^2*x)/(1 + c^2*x^2)^2 + (9*a^2*x)/(1 + c^2*x^2) + (9*a^2*ArcTan[c*x])/
c + (a*b*((9*((-I)*Sqrt[1 + c^2*x^2] + ArcSinh[c*x]))/(-I + c*x) + (9*(I*Sq
rt[1 + c^2*x^2] + ArcSinh[c*x]))/(I + c*x) - (I*((-2*I + c*x)*Sqrt[1 + c^2*
x^2] + 3*ArcSinh[c*x]))/(-I + c*x)^2 + (I*((2*I + c*x)*Sqrt[1 + c^2*x^2] +
3*ArcSinh[c*x]))/(I + c*x)^2 + ((9*I)/2)*(ArcSinh[c*x]*(ArcSinh[c*x] - 4*Lo
g[1 + I*E^ArcSinh[c*x]]) - 4*PolyLog[2, (-I)*E^ArcSinh[c*x]]) - ((9*I)/2)*(
ArcSinh[c*x]*(ArcSinh[c*x] - 4*Log[1 - I*E^ArcSinh[c*x]]) - 4*PolyLog[2, I*
E^ArcSinh[c*x]])))/c + (b^2*((-2*c*x)/(1 + c^2*x^2) + (4*ArcSinh[c*x])/(1 +
 c^2*x^2)^(3/2) + (18*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] + (6*c*x*ArcSinh[c*x]
^2)/(1 + c^2*x^2)^2 + (9*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2) - 40*ArcTan[Tanh
[ArcSinh[c*x]/2]] - (9*I)*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] + (9*I)*
ArcSinh[c*x]^2*Log[1 + I/E^ArcSinh[c*x]] - (18*I)*ArcSinh[c*x]*PolyLog[2, (
-I)/E^ArcSinh[c*x]] + (18*I)*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[c*x]] - (1
8*I)*PolyLog[3, (-I)/E^ArcSinh[c*x]] + (18*I)*PolyLog[3, I/E^ArcSinh[c*x]])
)/c)/(24*d^3)
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\,\mathrm{arsinh}\,(cx)^2 + 2\,ab\,\mathrm{arsinh}\,(cx) + a^2}{c^6d^3x^6 + 3\,c^4d^3x^4 + 3\,c^2d^3x^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^6*d^3*x^6 + 3*c
^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2}{(c^2dx^2 + d)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(c^2*d*x^2 + d)^3, x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arcsinh}(cx))^2}{(c^2d\,x^2 + d)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8}\,a^2\left(\frac{3\,c^2x^3 + 5\,x}{c^4d^3x^4 + 2\,c^2d^3x^2 + d^3} + \frac{3\,\arctan(cx)}{cd^3}\right) + \int \frac{b^2\log\left(cx + \sqrt{c^2x^2 + 1}\right)^2}{c^6d^3x^6 + 3\,c^4d^3x^4 + 3\,c^2d^3x^2 + d^3} + \frac{2\,ab\log\left(cx + \sqrt{c^2x\right.}}{c^6d^3x^6 + 3\,c^4d^3x^4 + 3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*a^2*((3*c^2*x^3 + 5*x)/(c^4*d^3*x^4 + 2*c^2*d^3*x^2 + d^3) + 3*arctan(c
*x)/(c*d^3)) + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^6*d^3*x^6 +
3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(
c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{asinh}(c\,x))^2}{(d\,c^2\,x^2 + d)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^3,x)
```

```
[Out] int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^6x^6+3c^4x^4+3c^2x^2+1}\, dx + \int \frac{b^2 \operatorname{asinh}^2(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\, dx + \int \frac{2ab \operatorname{asinh}(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\, dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a**2/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integral(b
**2*asinh(c*x)**2/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integra
l(2*a*b*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x))/d**3
```

3.248 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=275

$$-\frac{4bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^3\sqrt{c^2x^2+1}}-\frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(c^2x^2+1\right)^{3/2}}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^3\left(c^2x^2+1\right)}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(c^2x^2+1\right)^2}-\frac{b\text{Li}_2\left(-e^{2\sinh^{-1}(c}\right.}{}$$

```
[Out] -1/12*b^2/d^3/(c^2*x^2+1)-1/6*b*c*x*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^(3/2
)+1/4*(a+b*arcsinh(c*x))^2/d^3/(c^2*x^2+1)^2+1/2*(a+b*arcsinh(c*x))^2/d^3/(
c^2*x^2+1)-2*(a+b*arcsinh(c*x))^2*arctanh((c*x+(c^2*x^2+1)^(1/2))^2)/d^3+2/
3*b^2*ln(c^2*x^2+1)/d^3-b*(a+b*arcsinh(c*x))*polylog(2,-(c*x+(c^2*x^2+1)^(1
/2))^2)/d^3+b*(a+b*arcsinh(c*x))*polylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)/d^3+1
/2*b^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^3-1/2*b^2*polylog(3,(c*x+(c^
2*x^2+1)^(1/2))^2)/d^3-4/3*b*c*x*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {5755, 5720, 5461, 4182, 2531, 2282, 6589, 5687, 260, 5690, 261}

$$-\frac{b\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^3}+\frac{b\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^3}+\frac{b^2\text{PolyLog}\left(3,\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)^3),x]
```

```
[Out] -b^2/(12*d^3*(1 + c^2*x^2)) - (b*c*x*(a + b*ArcSinh[c*x]))/(6*d^3*(1 + c^2*
x^2)^(3/2)) - (4*b*c*x*(a + b*ArcSinh[c*x]))/(3*d^3*Sqrt[1 + c^2*x^2]) + (a
 + b*ArcSinh[c*x])^2/(4*d^3*(1 + c^2*x^2)^2) + (a + b*ArcSinh[c*x])^2/(2*d^
3*(1 + c^2*x^2)) - (2*(a + b*ArcSinh[c*x])^2*ArcTanh[E^(2*ArcSinh[c*x])])/d
^3 + (2*b^2*Log[1 + c^2*x^2])/(3*d^3) - (b*(a + b*ArcSinh[c*x])*PolyLog[2,
-E^(2*ArcSinh[c*x])])/d^3 + (b*(a + b*ArcSinh[c*x])*PolyLog[2, E^(2*ArcSinh
[c*x])])/d^3 + (b^2*PolyLog[3, -E^(2*ArcSinh[c*x])])/(2*d^3) - (b^2*PolyLog
[3, E^(2*ArcSinh[c*x])])/(2*d^3)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5720

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
```

```
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)^3}\,dx = \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(1+c^2x^2\right)^2} - \frac{(bc)\int \frac{a+b\sinh^{-1}(cx)}{\left(1+c^2x^2\right)^{5/2}}\,dx}{2d^3} + \frac{\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)^2}\,dx}{d}$$

$$= -\frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(1+c^2x^2\right)^2} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^3\left(1+c^2x^2\right)} - \frac{(bc)\int \frac{a+b}{(1}}{3}$$

$$= -\frac{b^2}{12d^3\left(1+c^2x^2\right)} - \frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{4bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^3\sqrt{1+c^2x^2}} + \frac{\left(a+b\sin\right.}{4d^3\left(1\right.}$$

$$= -\frac{b^2}{12d^3\left(1+c^2x^2\right)} - \frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{4bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^3\sqrt{1+c^2x^2}} + \frac{\left(a+b\sin\right.}{4d^3\left(1\right.}$$

$$= -\frac{b^2}{12d^3\left(1+c^2x^2\right)} - \frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{4bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^3\sqrt{1+c^2x^2}} + \frac{\left(a+b\sin\right.}{4d^3\left(1\right.}$$

$$= -\frac{b^2}{12d^3\left(1+c^2x^2\right)} - \frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{4bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^3\sqrt{1+c^2x^2}} + \frac{\left(a+b\sin\right.}{4d^3\left(1\right.}$$

$$= -\frac{b^2}{12d^3\left(1+c^2x^2\right)} - \frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{4bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^3\sqrt{1+c^2x^2}} + \frac{\left(a+b\sin\right.}{4d^3\left(1\right.}$$

$$= -\frac{b^2}{12d^3\left(1+c^2x^2\right)} - \frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{4bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^3\sqrt{1+c^2x^2}} + \frac{\left(a+b\sin\right.}{4d^3\left(1\right.}$$

Mathematica [C] time = 3.78, size = 560, normalized size = 2.04

$$\frac{12a^2}{c^2x^2+1} + \frac{6a^2}{\left(c^2x^2+1\right)^2} - 12a^2\log\left(c^2x^2+1\right) + 24a^2\log(cx) + ab\left(-\frac{15\left(\sqrt{c^2x^2+1}-i\sinh^{-1}(cx)\right)}{cx+i} - \frac{15\left(\sqrt{c^2x^2+1}+i\sinh^{-1}(cx)\right)}{cx-i} - \right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)^3),x]
```

```
[Out] ((6*a^2)/(1 + c^2*x^2)^2 + (12*a^2)/(1 + c^2*x^2) + 24*a^2*Log[c*x] - 12*a^
2*Log[1 + c^2*x^2] + a*b*((-15*(Sqrt[1 + c^2*x^2] - I*ArcSinh[c*x]))/(I + c
*x) - (15*(Sqrt[1 + c^2*x^2] + I*ArcSinh[c*x]))/(-I + c*x) - 24*ArcSinh[c*x
]^2 - ((-2*I + c*x)*Sqrt[1 + c^2*x^2] + 3*ArcSinh[c*x])/(-I + c*x)^2 - ((2*
I + c*x)*Sqrt[1 + c^2*x^2] + 3*ArcSinh[c*x])/(I + c*x)^2 + 48*ArcSinh[c*x]*
Log[1 - E^(2*ArcSinh[c*x])] + 12*(ArcSinh[c*x]*(ArcSinh[c*x] - 4*Log[1 + I*
E^ArcSinh[c*x]]) - 4*PolyLog[2, (-I)*E^ArcSinh[c*x]]) + 12*(ArcSinh[c*x]*(A
rcSinh[c*x] - 4*Log[1 - I*E^ArcSinh[c*x]]) - 4*PolyLog[2, I*E^ArcSinh[c*x]]
) + 24*PolyLog[2, E^(2*ArcSinh[c*x])]) + b^2*(I*Pi^3 - 2/(1 + c^2*x^2) - (4
*c*x*ArcSinh[c*x])/(1 + c^2*x^2)^(3/2) - (32*c*x*ArcSinh[c*x])/Sqrt[1 + c^2
*x^2] + (6*ArcSinh[c*x]^2)/(1 + c^2*x^2)^2 + (12*ArcSinh[c*x]^2)/(1 + c^2*x
```

```
^2) - 16*ArcSinh[c*x]^3 - 24*ArcSinh[c*x]^2*Log[1 + E^(-2*ArcSinh[c*x])] +
24*ArcSinh[c*x]^2*Log[1 - E^(2*ArcSinh[c*x])] + 16*Log[1 + c^2*x^2] + 24*Ar
cSinh[c*x]*PolyLog[2, -E^(-2*ArcSinh[c*x])] + 24*ArcSinh[c*x]*PolyLog[2, E^
(2*ArcSinh[c*x])] + 12*PolyLog[3, -E^(-2*ArcSinh[c*x])] - 12*PolyLog[3, E^(
2*ArcSinh[c*x])]))/(24*d^3)
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\,\text{arsinh}\,(cx)^2 + 2\,ab\,\text{arsinh}\,(cx) + a^2}{c^6d^3x^7 + 3\,c^4d^3x^5 + 3\,c^2d^3x^3 + d^3x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^6*d^3*x^7 + 3*c
^4*d^3*x^5 + 3*c^2*d^3*x^3 + d^3*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arsinh}\,(cx) + a)^2}{\left(c^2dx^2 + d\right)^3 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^3*x), x)
```

maple [B] time = 0.34, size = 1129, normalized size = 4.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^3,x)
```

```
[Out] b^2/d^3*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))+2*b^2/d^3*arcsinh(c*x)*p
olylog(2,-c*x-(c^2*x^2+1)^(1/2))-b^2/d^3*arcsinh(c*x)^2*ln(1+(c*x+(c^2*x^2+
1)^(1/2))^2)-b^2/d^3*arcsinh(c*x)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)+b^2
/d^3*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*b^2/d^3*arcsinh(c*x)*poly
log(2,c*x+(c^2*x^2+1)^(1/2))-a*b/d^3*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)+
2*a*b/d^3*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+2*a*b/d^3*polylog(2,c*x+(c^2*x^
2+1)^(1/2))+4/3*a*b/d^3/(c^4*x^4+2*c^2*x^2+1)+3/4*b^2/d^3/(c^4*x^4+2*c^2*x^
2+1)*arcsinh(c*x)^2+4/3*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)+4/3*a*b/
d^3/(c^4*x^4+2*c^2*x^2+1)*c^4*x^4+8/3*a*b/d^3/(c^4*x^4+2*c^2*x^2+1)*c^2*x^2
+4/3*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)*c^4*x^4+1/2*b^2/d^3/(c^4*x^
4+2*c^2*x^2+1)*arcsinh(c*x)^2*c^2*x^2+8/3*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arc
sinh(c*x)*c^2*x^2+1/2*b^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^3+1/4*a^2
/d^3/(c^2*x^2+1)^2+a^2/d^3*ln(c*x)-1/2*a^2/d^3*ln(c^2*x^2+1)+1/2*a^2/d^3/(c
^2*x^2+1)-8/3*b^2/d^3*ln(c*x+(c^2*x^2+1)^(1/2))-2*b^2/d^3*polylog(3,-c*x-(c
^2*x^2+1)^(1/2))-2*b^2/d^3*polylog(3,c*x+(c^2*x^2+1)^(1/2))-1/12*b^2/d^3/(c
^4*x^4+2*c^2*x^2+1)+4/3*b^2/d^3*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+a*b/d^3/(c^
4*x^4+2*c^2*x^2+1)*arcsinh(c*x)*c^2*x^2-3/2*a*b/d^3/(c^4*x^4+2*c^2*x^2+1)*c
*x*(c^2*x^2+1)^(1/2)-4/3*a*b/d^3/(c^4*x^4+2*c^2*x^2+1)*c^3*x^3*(c^2*x^2+1)^
(1/2)-4/3*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^3*
x^3-3/2*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c*x-2*
a*b/d^3*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+2*a*b/d^3*arcsinh(c*x)
*ln(1+c*x+(c^2*x^2+1)^(1/2))+2*a*b/d^3*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1
/2))-1/12*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*c^2*x^2+3/2*a*b/d^3/(c^4*x^4+2*c^2*
x^2+1)*arcsinh(c*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\,a^2\left(\frac{2\,c^2x^2+3}{c^4d^3x^4+2\,c^2d^3x^2+d^3}-\frac{2\,\log\left(c^2x^2+1\right)}{d^3}+\frac{4\,\log(x)}{d^3}\right)+\int\frac{b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^6d^3x^7+3\,c^4d^3x^5+3\,c^2d^3x^3+d^3x}+\frac{2\,a}{c^6d^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*((2*c^2*x^2 + 3)/(c^4*d^3*x^4 + 2*c^2*d^3*x^2 + d^3) - 2*log(c^2*x^
2 + 1)/d^3 + 4*log(x)/d^3) + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(
c^6*d^3*x^7 + 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 + d^3*x) + 2*a*b*log(c*x + sqrt
(c^2*x^2 + 1))/(c^6*d^3*x^7 + 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 + d^3*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))^2}{x\,\left(d\,c^2\,x^2+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)^3),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^6x^7+3c^4x^5+3c^2x^3+x}\,dx+\int \frac{b^2\,\mathrm{asinh}^2\,(cx)}{c^6x^7+3c^4x^5+3c^2x^3+x}\,dx+\int \frac{2ab\,\mathrm{asinh}\,(cx)}{c^6x^7+3c^4x^5+3c^2x^3+x}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a**2/(c**6*x**7 + 3*c**4*x**5 + 3*c**2*x**3 + x), x) + Integral(b
**2*asinh(c*x)**2/(c**6*x**7 + 3*c**4*x**5 + 3*c**2*x**3 + x), x) + Integra
l(2*a*b*asinh(c*x)/(c**6*x**7 + 3*c**4*x**5 + 3*c**2*x**3 + x), x))/d**3
```

3.249 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=389

$$-\frac{7bc\left(a+b\sinh^{-1}(cx)\right)}{4d^3\sqrt{c^2x^2+1}}-\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(c^2x^2+1\right)^{3/2}}-\frac{15c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{8d^3\left(c^2x^2+1\right)}-\frac{5c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(c^2x^2+1\right)^2}-\frac{\left(a+b\sinh\right.}{d^3x\left(c^2x^2\right.}$$

```
[Out] 1/12*b^2*c^2*x/d^3/(c^2*x^2+1)-1/6*b*c*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^(
3/2)-(a+b*arcsinh(c*x))^2/d^3/x/(c^2*x^2+1)^2-5/4*c^2*x*(a+b*arcsinh(c*x))^
2/d^3/(c^2*x^2+1)^2-15/8*c^2*x*(a+b*arcsinh(c*x))^2/d^3/(c^2*x^2+1)-15/4*c*
(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(1/2))/d^3+11/6*b^2*c*arctan(c*
x)/d^3-4*b*c*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)^(1/2))/d^3-2*b^2*c*
polylog(2,-c*x-(c^2*x^2+1)^(1/2))/d^3+15/4*I*b*c*(a+b*arcsinh(c*x))*polylog
(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/d^3-15/4*I*b*c*(a+b*arcsinh(c*x))*polylog(2,
I*(c*x+(c^2*x^2+1)^(1/2)))/d^3+2*b^2*c*polylog(2,c*x+(c^2*x^2+1)^(1/2))/d^3
-15/4*I*b^2*c*polylog(3,-I*(c*x+(c^2*x^2+1)^(1/2)))/d^3+15/4*I*b^2*c*polylo
g(3,I*(c*x+(c^2*x^2+1)^(1/2)))/d^3-7/4*b*c*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+
1)^(1/2)
```

Rubi [A] time = 0.77, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 15, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.577, Rules used = {5747, 5690, 5693, 4180, 2531, 2282, 6589, 5717, 203, 199, 5755, 5760, 4182, 2279, 2391}

$$\frac{15ibc\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4d^3}-\frac{15ibc\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4d^3}-\frac{2b^2c\text{PolyLo}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)^3),x]
```

```
[Out] (b^2*c^2*x)/(12*d^3*(1 + c^2*x^2)) - (b*c*(a + b*ArcSinh[c*x]))/(6*d^3*(1 +
 c^2*x^2)^(3/2)) - (7*b*c*(a + b*ArcSinh[c*x]))/(4*d^3*Sqrt[1 + c^2*x^2]) -
 (a + b*ArcSinh[c*x])^2/(d^3*x*(1 + c^2*x^2)^2) - (5*c^2*x*(a + b*ArcSinh[c
*x])^2)/(4*d^3*(1 + c^2*x^2)^2) - (15*c^2*x*(a + b*ArcSinh[c*x])^2)/(8*d^3*
(1 + c^2*x^2)) - (15*c*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]])/(4*d^
3) + (11*b^2*c*ArcTan[c*x])/(6*d^3) - (4*b*c*(a + b*ArcSinh[c*x])*ArcTanh[E
^ArcSinh[c*x]])/d^3 - (2*b^2*c*PolyLog[2, -E^ArcSinh[c*x]])/d^3 + (((15*I)/
4)*b*c*(a + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/d^3 - (((15*I)
/4)*b*c*(a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])/d^3 + (2*b^2*c*P
olyLog[2, E^ArcSinh[c*x]])/d^3 - (((15*I)/4)*b^2*c*PolyLog[3, (-I)*E^ArcSin
h[c*x]])/d^3 + (((15*I)/4)*b^2*c*PolyLog[3, I*E^ArcSinh[c*x]])/d^3
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
```

```
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)^3}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^3x\left(1+c^2x^2\right)^2} - \left(5c^2\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^3}\,dx + \frac{(2bc)\int \frac{a+b\sinh^{-1}(cx)}{x\left(1+c^2x^2\right)^{5/2}}\,dx}{d^3}$$

$$= \frac{2bc\left(a+b\sinh^{-1}(cx)\right)}{3d^3\left(1+c^2x^2\right)^{3/2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^3x\left(1+c^2x^2\right)^2} - \frac{5c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(1+c^2x^2\right)^2} + \frac{(2bc)}{}$$

$$= -\frac{b^2c^2x}{3d^3\left(1+c^2x^2\right)} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} + \frac{2bc\left(a+b\sinh^{-1}(cx)\right)}{d^3\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}\right.}{d^3x\left(1+c^2\right.}$$

$$= \frac{b^2c^2x}{12d^3\left(1+c^2x^2\right)} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{7bc\left(a+b\sinh^{-1}(cx)\right)}{4d^3\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}\right.}{d^3x\left(1+c^2\right.}$$

$$= \frac{b^2c^2x}{12d^3\left(1+c^2x^2\right)} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{7bc\left(a+b\sinh^{-1}(cx)\right)}{4d^3\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}\right.}{d^3x\left(1+c^2\right.}$$

$$= \frac{b^2c^2x}{12d^3\left(1+c^2x^2\right)} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{7bc\left(a+b\sinh^{-1}(cx)\right)}{4d^3\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}\right.}{d^3x\left(1+c^2\right.}$$

$$= \frac{b^2c^2x}{12d^3\left(1+c^2x^2\right)} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{7bc\left(a+b\sinh^{-1}(cx)\right)}{4d^3\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}\right.}{d^3x\left(1+c^2\right.}$$

$$= \frac{b^2c^2x}{12d^3\left(1+c^2x^2\right)} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{7bc\left(a+b\sinh^{-1}(cx)\right)}{4d^3\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}\right.}{d^3x\left(1+c^2\right.}$$

Mathematica [A] time = 7.70, size = 716, normalized size = 1.84

$$-\frac{7a^2c^2x}{8d^3\left(c^2x^2+1\right)} - \frac{a^2c^2x}{4d^3\left(c^2x^2+1\right)^2} - \frac{15a^2c\tan^{-1}(cx)}{8d^3} - \frac{a^2}{d^3x} + \frac{2abc\left(\frac{7\left(\sqrt{c^2x^2+1}+i\sinh^{-1}(cx)\right)}{16(-1-icx)} - \frac{7\left(\sinh^{-1}(cx)+i\sqrt{c^2x^2+1}\right)}{16(cx+i)} + \right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)^3),x]
```

```
[Out] -(a^2/(d^3*x)) - (a^2*c^2*x)/(4*d^3*(1 + c^2*x^2)^2) - (7*a^2*c^2*x)/(8*d^3
*(1 + c^2*x^2)) - (15*a^2*c*ArcTan[c*x])/(8*d^3) + (2*a*b*c*((7*(Sqrt[1 + c
^2*x^2] + I*ArcSinh[c*x]))/(16*(-1 - I*c*x)) - ArcSinh[c*x]/(c*x) - (7*(I*S
qrt[1 + c^2*x^2] + ArcSinh[c*x]))/(16*(I + c*x)) + ((I/48)*((-2*I + c*x)*Sq
rt[1 + c^2*x^2] + 3*ArcSinh[c*x]))/(-I + c*x)^2 - ((I/48)*((2*I + c*x)*Sqrt
[1 + c^2*x^2] + 3*ArcSinh[c*x]))/(I + c*x)^2 - ArcTanh[Sqrt[1 + c^2*x^2]] +
 ((15*I)/16)*(-1/2*ArcSinh[c*x]^2 + 2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]
] + 2*PolyLog[2, (-I)*E^ArcSinh[c*x]]) - ((15*I)/16)*(-1/2*ArcSinh[c*x]^2 +
 2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + 2*PolyLog[2, I*E^ArcSinh[c*x]])
))/d^3 + (b^2*c*((2*c*x)/(1 + c^2*x^2) - (4*ArcSinh[c*x])/(1 + c^2*x^2)^(3/
2) - (42*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - (6*c*x*ArcSinh[c*x]^2)/(1 + c^2*
x^2)^2 - (21*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2) + 88*ArcTan[Tanh[ArcSinh[c*x
]/2]] - 12*ArcSinh[c*x]^2*Coth[ArcSinh[c*x]/2] + 48*ArcSinh[c*x]*Log[1 - E^
(-ArcSinh[c*x])] + (45*I)*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] - (45*I)
```

```
*ArcSinh[c*x]^2*Log[1 + I/E^ArcSinh[c*x]] - 48*ArcSinh[c*x]*Log[1 + E^(-Arc
Sinh[c*x])] + 48*PolyLog[2, -E^(-ArcSinh[c*x])] + (90*I)*ArcSinh[c*x]*PolyL
og[2, (-I)/E^ArcSinh[c*x]] - (90*I)*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[c*x
]] - 48*PolyLog[2, E^(-ArcSinh[c*x])] + (90*I)*PolyLog[3, (-I)/E^ArcSinh[c*
x]] - (90*I)*PolyLog[3, I/E^ArcSinh[c*x]] + 12*ArcSinh[c*x]^2*Tanh[ArcSinh[
c*x]/2]))/(24*d^3)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2 \operatorname{arsinh}(cx)^2 + 2\,ab \operatorname{arsinh}(cx) + a^2}{c^6 d^3 x^8 + 3\,c^4 d^3 x^6 + 3\,c^2 d^3 x^4 + d^3 x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^6*d^3*x^8 + 3*c
^4*d^3*x^6 + 3*c^2*d^3*x^4 + d^3*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2}{\left(c^2 dx^2 + d\right)^3 x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^3*x^2), x)
```

maple [F] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arcsinh}(cx))^2}{x^2 \left(c^2 d\, x^2 + d\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^3,x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{8}\,a^2\left(\frac{15\,c^4x^4 + 25\,c^2x^2 + 8}{c^4d^3x^5 + 2\,c^2d^3x^3 + d^3x} + \frac{15\,c \arctan(cx)}{d^3}\right) + \int \frac{b^2 \log\left(cx + \sqrt{c^2x^2+1}\right)^2}{c^6d^3x^8 + 3\,c^4d^3x^6 + 3\,c^2d^3x^4 + d^3x^2} + \frac{2\,ab \log\left(cx +\right.}{c^6d^3x^8 + 3\,c^4d^3x^6 +}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*a^2*((15*c^4*x^4 + 25*c^2*x^2 + 8)/(c^4*d^3*x^5 + 2*c^2*d^3*x^3 + d^3*
x) + 15*c*arctan(c*x)/d^3) + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(
c^6*d^3*x^8 + 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4 + d^3*x^2) + 2*a*b*log(c*x + sq
rt(c^2*x^2 + 1))/(c^6*d^3*x^8 + 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4 + d^3*x^2), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{asinh}(c\,x))^2}{x^2 \left(d\,c^2\,x^2 + d\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)^3),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^6x^8+3c^4x^6+3c^2x^4+x^2}\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{c^6x^8+3c^4x^6+3c^2x^4+x^2}\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{c^6x^8+3c^4x^6+3c^2x^4+x^2}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**2/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a**2/(c**6*x**8 + 3*c**4*x**6 + 3*c**2*x**4 + x**2), x) + Integra
l(b**2*asinh(c*x)**2/(c**6*x**8 + 3*c**4*x**6 + 3*c**2*x**4 + x**2), x) + I
ntegral(2*a*b*asinh(c*x)/(c**6*x**8 + 3*c**4*x**6 + 3*c**2*x**4 + x**2), x)
)/d**3
```

3.250 $\int \frac{(a+b\sinh^{-1}(cx))^2}{x^3(d+c^2dx^2)^3}\,dx$

Optimal. Leaf size=381

$$\frac{3bc^2\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^3}-\frac{3bc^2\text{Li}_2\left(e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^3}-\frac{3c^2\left(a+b\sinh^{-1}(cx)\right)^2}{2d^3\left(c^2x^2+1\right)}-\frac{3c^2}{}$$

```
[Out] 1/12*b^2*c^2/d^3/(c^2*x^2+1)-b*c*(a+b*arcsinh(c*x))/d^3/x/(c^2*x^2+1)^(3/2)
-5/6*b*c^3*x*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^(3/2)-3/4*c^2*(a+b*arcsinh(
c*x))^2/d^3/(c^2*x^2+1)^2-1/2*(a+b*arcsinh(c*x))^2/d^3/x^2/(c^2*x^2+1)^2-3/
2*c^2*(a+b*arcsinh(c*x))^2/d^3/(c^2*x^2+1)+6*c^2*(a+b*arcsinh(c*x))^2*arcta
nh((c*x+(c^2*x^2+1)^(1/2))^2)/d^3+b^2*c^2*ln(x)/d^3-7/6*b^2*c^2*ln(c^2*x^2+
1)/d^3+3*b*c^2*(a+b*arcsinh(c*x))*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^3
-3*b*c^2*(a+b*arcsinh(c*x))*polylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)/d^3-3/2*b^
2*c^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^3+3/2*b^2*c^2*polylog(3,(c*x+
(c^2*x^2+1)^(1/2))^2)/d^3+4/3*b*c^3*x*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^(1
/2)
```

Rubi [A] time = 0.80, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 19, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.731, Rules used = {5747, 5755, 5720, 5461, 4182, 2531, 2282, 6589, 5687, 260, 5690, 261, 271, 192, 191, 5732, 12, 1251, 893}

$$\frac{3bc^2\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^3}-\frac{3bc^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^3}-\frac{3b^2c^2\text{PolyLo}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)^3),x]
```

```
[Out] (b^2*c^2)/(12*d^3*(1 + c^2*x^2)) - (b*c*(a + b*ArcSinh[c*x]))/(d^3*x*(1 + c
^2*x^2)^(3/2)) - (5*b*c^3*x*(a + b*ArcSinh[c*x]))/(6*d^3*(1 + c^2*x^2)^(3/2
)) + (4*b*c^3*x*(a + b*ArcSinh[c*x]))/(3*d^3*Sqrt[1 + c^2*x^2]) - (3*c^2*(a
 + b*ArcSinh[c*x])^2)/(4*d^3*(1 + c^2*x^2)^2) - (a + b*ArcSinh[c*x])^2/(2*d
^3*x^2*(1 + c^2*x^2)^2) - (3*c^2*(a + b*ArcSinh[c*x])^2)/(2*d^3*(1 + c^2*x^
2)) + (6*c^2*(a + b*ArcSinh[c*x])^2*ArcTanh[E^(2*ArcSinh[c*x])])/d^3 + (b^2
*c^2*Log[x])/d^3 - (7*b^2*c^2*Log[1 + c^2*x^2])/(6*d^3) + (3*b*c^2*(a + b*A
rcSinh[c*x])*PolyLog[2, -E^(2*ArcSinh[c*x])])/d^3 - (3*b*c^2*(a + b*ArcSinh
[c*x])*PolyLog[2, E^(2*ArcSinh[c*x])])/d^3 - (3*b^2*c^2*PolyLog[3, -E^(2*Ar
cSinh[c*x])])/(2*d^3) + (3*b^2*c^2*PolyLog[3, E^(2*ArcSinh[c*x])])/(2*d^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
```

```
 && NeQ[p, -1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
```

```
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5720

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
```

```
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^3\left(d+c^2dx^2\right)^3}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^3x^2\left(1+c^2x^2\right)^2} - \left(3c^2\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)^3}\,dx + \frac{(bc)\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(1+c^2x^2\right)^{5/2}}\,dx}{d^3}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{4bc^3x\left(a+b\sinh^{-1}(cx)\right)}{3d^3\left(1+c^2x^2\right)^{3/2}} - \frac{8bc^3x\left(a+b\sinh^{-1}(cx)\right)}{3d^3\sqrt{1+c^2x^2}} - \cdots$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{8bc^3x\left(a+b\sinh^{-1}(cx)\right)}{3d^3\sqrt{1+c^2x^2}} - \cdots$$

$$= \frac{b^2c^2}{4d^3\left(1+c^2x^2\right)} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} + \frac{4bc^3x\left(a+b\cdots\right.}{3d^3\sqrt{1\cdots}}$$

$$= \frac{b^2c^2}{4d^3\left(1+c^2x^2\right)} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} + \frac{4bc^3x\left(a+b\cdots\right.}{3d^3\sqrt{1\cdots}}$$

$$= \frac{b^2c^2}{12d^3\left(1+c^2x^2\right)} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} + \frac{4bc^3x\left(a+\cdots\right.}{3d^3\sqrt{\cdots}}$$

$$= \frac{b^2c^2}{12d^3\left(1+c^2x^2\right)} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} + \frac{4bc^3x\left(a+\cdots\right.}{3d^3\sqrt{\cdots}}$$

$$= \frac{b^2c^2}{12d^3\left(1+c^2x^2\right)} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} + \frac{4bc^3x\left(a+\cdots\right.}{3d^3\sqrt{\cdots}}$$

$$= \frac{b^2c^2}{12d^3\left(1+c^2x^2\right)} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} + \frac{4bc^3x\left(a+\cdots\right.}{3d^3\sqrt{\cdots}}$$

Mathematica [C] time = 9.46, size = 759, normalized size = 1.99

$$-\frac{a^2c^2}{d^3\left(c^2x^2+1\right)} - \frac{a^2c^2}{4d^3\left(c^2x^2+1\right)^2} + \frac{3a^2c^2\log\left(c^2x^2+1\right)}{2d^3} - \frac{3a^2c^2\log(x)}{d^3} - \frac{a^2}{2d^3x^2} + \frac{2ab\left(\frac{3}{2}c^3\left(\frac{2\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{c} - \frac{\sinh^{-}\cdots}{2\cdots}\right.\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)^3),x]
```

```
[Out] -1/2*a^2/(d^3*x^2) - (a^2*c^2)/(4*d^3*(1 + c^2*x^2)^2) - (a^2*c^2)/(d^3*(1
+ c^2*x^2)) - (3*a^2*c^2*Log[x])/d^3 + (3*a^2*c^2*Log[1 + c^2*x^2])/(2*d^3)
 + (2*a*b*(-1/48*(c^2*((2*I - c*x)*Sqrt[1 + c^2*x^2] - 3*ArcSinh[c*x]))/(-I
 + c*x)^2 - (((9*I)/16)*c^2*(Sqrt[1 + c^2*x^2] + I*ArcSinh[c*x]))/(-1 - I*c
*x) - (((9*I)/16)*c^3*(I*Sqrt[1 + c^2*x^2] + ArcSinh[c*x]))/(I*c + c^2*x) -
 (c*x*Sqrt[1 + c^2*x^2] + ArcSinh[c*x])/(2*x^2) + (c^2*((2*I + c*x)*Sqrt[1
+ c^2*x^2] + 3*ArcSinh[c*x]))/(48*(I + c*x)^2) + (3*c^3*(-1/2*ArcSinh[c*x]^
2/c + (2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]])/c + (2*PolyLog[2, (-I)*E^A
rcSinh[c*x]])/c))/2 + (3*c^3*(-1/2*ArcSinh[c*x]^2/c + (2*ArcSinh[c*x]*Log[1
 - I*E^ArcSinh[c*x]])/c + (2*PolyLog[2, I*E^ArcSinh[c*x]])/c))/2 - 3*c^2*(-
1/2*ArcSinh[c*x]^2 + ArcSinh[c*x]*Log[1 - E^(2*ArcSinh[c*x])] + PolyLog[2,
E^(2*ArcSinh[c*x])]/2)))/d^3 + (b^2*c^2*(-3*ArcSinh[c*x]*PolyLog[2, -E^(-2*
ArcSinh[c*x])] - 3*ArcSinh[c*x]*PolyLog[2, E^(2*ArcSinh[c*x])] + ((-3*I)*Pi
^3 + 2/(1 + c^2*x^2) + (4*c*x*ArcSinh[c*x])/(1 + c^2*x^2)^(3/2) + (56*c*x*A
rcSinh[c*x])/Sqrt[1 + c^2*x^2] - (24*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(c*x)
- (12*ArcSinh[c*x]^2)/(c^2*x^2) - (6*ArcSinh[c*x]^2)/(1 + c^2*x^2)^2 - (24*
ArcSinh[c*x]^2)/(1 + c^2*x^2) + 48*ArcSinh[c*x]^3 + 72*ArcSinh[c*x]^2*Log[1
 + E^(-2*ArcSinh[c*x])] - 72*ArcSinh[c*x]^2*Log[1 - E^(2*ArcSinh[c*x])] + 2
4*Log[c*x] - 56*Log[Sqrt[1 + c^2*x^2]] - 36*PolyLog[3, -E^(-2*ArcSinh[c*x])
] + 36*PolyLog[3, E^(2*ArcSinh[c*x])]])/24))/d^3
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2 \operatorname{arsinh}(cx)^2 + 2\,ab \operatorname{arsinh}(cx) + a^2}{c^6 d^3 x^9 + 3\,c^4 d^3 x^7 + 3\,c^2 d^3 x^5 + d^3 x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^6*d^3*x^9 + 3*c
^4*d^3*x^7 + 3*c^2*d^3*x^5 + d^3*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2}{(c^2 dx^2 + d)^3 x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^3*x^3), x)
```

maple [B] time = 0.43, size = 1436, normalized size = 3.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^3,x)
```

```
[Out] -3/2*b^2*c^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^3-4/3*c^6*a*b/d^3/(c^4
*x^4+2*c^2*x^2+1)*x^4-8/3*c^4*a*b/d^3/(c^4*x^4+2*c^2*x^2+1)*x^2-4/3*c^6*b^2
/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)*x^4-3/2*c^4*b^2/d^3/(c^4*x^4+2*c^2*
x^2+1)*arcsinh(c*x)^2*x^2-8/3*c^4*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x
)*x^2-6*c^2*a*b/d^3*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-6*c^2*a*b/d^3*
arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-9/2*c^2*a*b/d^3/(c^4*x^4+2*c^2*x^2
```

```
+1)*arcsinh(c*x)+6*c^2*a*b/d^3*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
-a*b/d^3/(c^4*x^4+2*c^2*x^2+1)/x^2*arcsinh(c*x)+3*c^2*b^2/d^3*arcsinh(c*x)*
polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)-9/4*c^2*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)
*arcsinh(c*x)^2-3*c^2*b^2/d^3*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))-6*
c^2*b^2/d^3*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))+3*c^2*b^2/d^3*arc
sinh(c*x)^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-4/3*c^2*b^2/d^3/(c^4*x^4+2*c^2*
x^2+1)*arcsinh(c*x)-4/3*c^2*a*b/d^3/(c^4*x^4+2*c^2*x^2+1)+1/12*c^4*b^2/d^3/
(c^4*x^4+2*c^2*x^2+1)*x^2-1/2*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)/x^2*arcsinh(c*x
)^2+3*c^2*a*b/d^3*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)-6*c^2*a*b/d^3*polyl
og(2,-c*x-(c^2*x^2+1)^(1/2))-6*c^2*a*b/d^3*polylog(2,c*x+(c^2*x^2+1)^(1/2))
-3*c^2*b^2/d^3*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))-6*c^2*b^2/d^3*arc
sinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))-1/2*a^2/d^3/x^2-1/4*c^2*a^2/d^3
/(c^2*x^2+1)^2-c^2*a^2/d^3/(c^2*x^2+1)+1/12*c^2*b^2/d^3/(c^4*x^4+2*c^2*x^2+
1)-3*c^2*a^2/d^3*ln(c*x)+3/2*c^2*a^2/d^3*ln(c^2*x^2+1)+8/3*c^2*b^2/d^3*ln(c
*x+(c^2*x^2+1)^(1/2))+6*c^2*b^2/d^3*polylog(3,-c*x-(c^2*x^2+1)^(1/2))+6*c^2
*b^2/d^3*polylog(3,c*x+(c^2*x^2+1)^(1/2))+c^2*b^2/d^3*ln(c*x+(c^2*x^2+1)^(1
/2)-1)+c^2*b^2/d^3*ln(1+c*x+(c^2*x^2+1)^(1/2))-7/3*c^2*b^2/d^3*ln(1+(c*x+(c
^2*x^2+1)^(1/2))^2)-3*c^4*a*b/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)*x^2+1/
2*c^3*a*b/d^3/(c^4*x^4+2*c^2*x^2+1)*x*(c^2*x^2+1)^(1/2)+4/3*c^5*a*b/d^3/(c^
4*x^4+2*c^2*x^2+1)*x^3*(c^2*x^2+1)^(1/2)-c*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)/x*
arcsinh(c*x)*(c^2*x^2+1)^(1/2)-c*a*b/d^3/(c^4*x^4+2*c^2*x^2+1)/x*(c^2*x^2+1
)^(1/2)+4/3*c^5*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)*(c^2*x^2+1)^(1/2
)*x^3+1/2*c^3*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*
x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{4}a^2\left(\frac{6c^4x^4+9c^2x^2+2}{c^4d^3x^6+2c^2d^3x^4+d^3x^2}-\frac{6c^2\log\left(c^2x^2+1\right)}{d^3}+\frac{12c^2\log(x)}{d^3}\right)+\int\frac{b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^6d^3x^9+3c^4d^3x^7+3c^2d^3x^5+d^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/4*a^2*((6*c^4*x^4 + 9*c^2*x^2 + 2)/(c^4*d^3*x^6 + 2*c^2*d^3*x^4 + d^3*x^
2) - 6*c^2*log(c^2*x^2 + 1)/d^3 + 12*c^2*log(x)/d^3) + integrate(b^2*log(c*
x + sqrt(c^2*x^2 + 1))^2/(c^6*d^3*x^9 + 3*c^4*d^3*x^7 + 3*c^2*d^3*x^5 + d^3
*x^3) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d^3*x^9 + 3*c^4*d^3*x^7 + 3
*c^2*d^3*x^5 + d^3*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{(a+b\,\mathrm{asinh}\,(c\,x))^2}{x^3\left(d\,c^2\,x^2+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)^3),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{a^2}{c^6x^9+3c^4x^7+3c^2x^5+x^3}\,dx+\int\frac{b^2\,\mathrm{asinh}^2\,(cx)}{c^6x^9+3c^4x^7+3c^2x^5+x^3}\,dx+\int\frac{2ab\,\mathrm{asinh}\,(cx)}{c^6x^9+3c^4x^7+3c^2x^5+x^3}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**3/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a**2/(c**6*x**9 + 3*c**4*x**7 + 3*c**2*x**5 + x**3), x) + Integra
l(b**2*asinh(c*x)**2/(c**6*x**9 + 3*c**4*x**7 + 3*c**2*x**5 + x**3), x) + I
ntegral(2*a*b*asinh(c*x)/(c**6*x**9 + 3*c**4*x**7 + 3*c**2*x**5 + x**3), x)
)/d**3
```

3.251 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^4\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=529

$$-\frac{35ibc^3\mathrm{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4d^3}+\frac{35ibc^3\mathrm{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4d^3}+\frac{35c^3\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right.}{4\ldots}$$

```
[Out] -1/2*b^2*c^2/d^3/x+1/6*b^2*c^2/d^3/x/(c^2*x^2+1)+1/12*b^2*c^4*x/d^3/(c^2*x^
2+1)-1/6*b*c^3*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^(3/2)-1/3*b*c*(a+b*arcsin
h(c*x))/d^3/x^2/(c^2*x^2+1)^(3/2)-1/3*(a+b*arcsinh(c*x))^2/d^3/x^3/(c^2*x^2
+1)^2+7/3*c^2*(a+b*arcsinh(c*x))^2/d^3/x/(c^2*x^2+1)^2+35/12*c^4*x*(a+b*arc
sinh(c*x))^2/d^3/(c^2*x^2+1)^2+35/8*c^4*x*(a+b*arcsinh(c*x))^2/d^3/(c^2*x^2
+1)+35/4*c^3*(a+b*arcsinh(c*x))^2*arctan(c*x+(c^2*x^2+1)^(1/2))/d^3-17/6*b^
2*c^3*arctan(c*x)/d^3+38/3*b*c^3*(a+b*arcsinh(c*x))*arctanh(c*x+(c^2*x^2+1)
^(1/2))/d^3+19/3*b^2*c^3*polylog(2,-c*x-(c^2*x^2+1)^(1/2))/d^3-35/4*I*b^2*c
^3*polylog(3,I*(c*x+(c^2*x^2+1)^(1/2)))/d^3+35/4*I*b^2*c^3*polylog(3,-I*(c*
x+(c^2*x^2+1)^(1/2)))/d^3-19/3*b^2*c^3*polylog(2,c*x+(c^2*x^2+1)^(1/2))/d^3
+35/4*I*b*c^3*(a+b*arcsinh(c*x))*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/d^3-3
5/4*I*b*c^3*(a+b*arcsinh(c*x))*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/d^3+29
/12*b*c^3*(a+b*arcsinh(c*x))/d^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 1.31, antiderivative size = 529, normalized size of antiderivative = 1.00, number of steps used = 43, number of rules used = 17, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.654, Rules used = {5747, 5690, 5693, 4180, 2531, 2282, 6589, 5717, 203, 199, 5755, 5760, 4182, 2279, 2391, 290, 325}

$$-\frac{35ibc^3\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4d^3}+\frac{35ibc^3\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4d^3}+\frac{19b^2\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)^3),x]
```

```
[Out] -(b^2*c^2)/(2*d^3*x) + (b^2*c^2)/(6*d^3*x*(1 + c^2*x^2)) + (b^2*c^4*x)/(12*
d^3*(1 + c^2*x^2)) - (b*c^3*(a + b*ArcSinh[c*x]))/(6*d^3*(1 + c^2*x^2)^(3/2
)) - (b*c*(a + b*ArcSinh[c*x]))/(3*d^3*x^2*(1 + c^2*x^2)^(3/2)) + (29*b*c^3
*(a + b*ArcSinh[c*x]))/(12*d^3*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])^2/
(3*d^3*x^3*(1 + c^2*x^2)^2) + (7*c^2*(a + b*ArcSinh[c*x])^2)/(3*d^3*x*(1 +
c^2*x^2)^2) + (35*c^4*x*(a + b*ArcSinh[c*x])^2)/(12*d^3*(1 + c^2*x^2)^2) +
(35*c^4*x*(a + b*ArcSinh[c*x])^2)/(8*d^3*(1 + c^2*x^2)) + (35*c^3*(a + b*Ar
cSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]])/(4*d^3) - (17*b^2*c^3*ArcTan[c*x])/(6
*d^3) + (38*b*c^3*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/(3*d^3) + (
19*b^2*c^3*PolyLog[2, -E^ArcSinh[c*x]])/(3*d^3) - (((35*I)/4)*b*c^3*(a + b*
ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/d^3 + (((35*I)/4)*b*c^3*(a +
 b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])/d^3 - (19*b^2*c^3*PolyLog[2,
 E^ArcSinh[c*x]])/(3*d^3) + (((35*I)/4)*b^2*c^3*PolyLog[3, (-I)*E^ArcSinh[c
*x]])/d^3 - (((35*I)/4)*b^2*c^3*PolyLog[3, I*E^ArcSinh[c*x]])/d^3
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
```

```
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^4\left(d+c^2dx^2\right)^3}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d^3x^3\left(1+c^2x^2\right)^2} - \frac{1}{3}\left(7c^2\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)^3}\,dx + \frac{(2bc)\int \frac{a+b\sinh^{-1}(cx)}{x^3\left(1+c^2x^2\right)^{5/2}}\,dx}{3d^3}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^3x^2\left(1+c^2x^2\right)^{3/2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d^3x^3\left(1+c^2x^2\right)^2} + \frac{7c^2\left(a+b\sinh^{-1}(cx)\right)^2}{3d^3x\left(1+c^2x^2\right)^2} + \frac{1}{3}\left(35c^4\right)$$

$$= \frac{b^2c^2}{6d^3x\left(1+c^2x^2\right)} - \frac{19bc^3\left(a+b\sinh^{-1}(cx)\right)}{9d^3\left(1+c^2x^2\right)^{3/2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^3x^2\left(1+c^2x^2\right)^{3/2}} - \frac{\left(a+b\sinh^{-1}(\right.}{3d^3x^3\left(1+c^2\right.}$$

$$= -\frac{b^2c^2}{2d^3x} + \frac{b^2c^2}{6d^3x\left(1+c^2x^2\right)} + \frac{19b^2c^4x}{18d^3\left(1+c^2x^2\right)} - \frac{bc^3\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{bc\left(a+b\sin\right.}{3d^3x^2\left(1+\right.}$$

$$= -\frac{b^2c^2}{2d^3x} + \frac{b^2c^2}{6d^3x\left(1+c^2x^2\right)} + \frac{b^2c^4x}{12d^3\left(1+c^2x^2\right)} - \frac{bc^3\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{bc\left(a+b\sin\right.}{3d^3x^2\left(1+\right.}$$

$$= -\frac{b^2c^2}{2d^3x} + \frac{b^2c^2}{6d^3x\left(1+c^2x^2\right)} + \frac{b^2c^4x}{12d^3\left(1+c^2x^2\right)} - \frac{bc^3\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{bc\left(a+b\sin\right.}{3d^3x^2\left(1+\right.}$$

$$= -\frac{b^2c^2}{2d^3x} + \frac{b^2c^2}{6d^3x\left(1+c^2x^2\right)} + \frac{b^2c^4x}{12d^3\left(1+c^2x^2\right)} - \frac{bc^3\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{bc\left(a+b\sin\right.}{3d^3x^2\left(1+\right.}$$

$$= -\frac{b^2c^2}{2d^3x} + \frac{b^2c^2}{6d^3x\left(1+c^2x^2\right)} + \frac{b^2c^4x}{12d^3\left(1+c^2x^2\right)} - \frac{bc^3\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{bc\left(a+b\sin\right.}{3d^3x^2\left(1+\right.}$$

$$= -\frac{b^2c^2}{2d^3x} + \frac{b^2c^2}{6d^3x\left(1+c^2x^2\right)} + \frac{b^2c^4x}{12d^3\left(1+c^2x^2\right)} - \frac{bc^3\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{bc\left(a+b\sin\right.}{3d^3x^2\left(1+\right.}$$

Mathematica [**A**] time = 10.29, size = 937, normalized size = 1.77

$$\frac{11a^2xc^4}{8d^3\left(c^2x^2+1\right)} + \frac{a^2xc^4}{4d^3\left(c^2x^2+1\right)^2} + \frac{35a^2\tan^{-1}(cx)c^3}{8d^3} + \frac{b^2\left(-\frac{1}{2}cx\sinh^{-1}(cx)^2\operatorname{csch}^4\left(\frac{1}{2}\sinh^{-1}(cx)\right) - 2\sinh^{-1}(cx)\operatorname{csc}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)^3),x]
```

```
[Out] -1/3*a^2/(d^3*x^3) + (3*a^2*c^2)/(d^3*x) + (a^2*c^4*x)/(4*d^3*(1 + c^2*x^2)
^2) + (11*a^2*c^4*x)/(8*d^3*(1 + c^2*x^2)) + (35*a^2*c^3*ArcTan[c*x])/(8*d^
3) + (2*a*b*(-1/6*(c*Sqrt[1 + c^2*x^2])/x^2 + ((I/48)*c^3*((2*I - c*x)*Sqrt
[1 + c^2*x^2] - 3*ArcSinh[c*x]))/(-I + c*x)^2 - (11*c^3*(Sqrt[1 + c^2*x^2]
+ I*ArcSinh[c*x]))/(16*(-1 - I*c*x)) - ArcSinh[c*x]/(3*x^3) + (11*c^4*(I*Sq
rt[1 + c^2*x^2] + ArcSinh[c*x]))/(16*(I*c + c^2*x)) + ((I/48)*c^3*((2*I + c
*x)*Sqrt[1 + c^2*x^2] + 3*ArcSinh[c*x]))/(I + c*x)^2 + (c^3*ArcTanh[Sqrt[1
+ c^2*x^2]])/6 - 3*c^2*(-(ArcSinh[c*x]/x) - c*ArcTanh[Sqrt[1 + c^2*x^2]]) -
 ((35*I)/16)*c^4*(-1/2*ArcSinh[c*x]^2/c + (2*ArcSinh[c*x]*Log[1 + I*E^ArcSi
nh[c*x]])/c + (2*PolyLog[2, (-I)*E^ArcSinh[c*x]])/c) + ((35*I)/16)*c^4*(-1/
2*ArcSinh[c*x]^2/c + (2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]])/c + (2*Poly
Log[2, I*E^ArcSinh[c*x]])/c)))/d^3 + (b^2*c^3*((-2*c*x)/(1 + c^2*x^2) + (4*
ArcSinh[c*x])/(1 + c^2*x^2)^(3/2) + (66*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] + (
6*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2)^2 + (33*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^
2) - 136*ArcTan[Tanh[ArcSinh[c*x]/2]] - 4*Coth[ArcSinh[c*x]/2] + 38*ArcSinh
[c*x]^2*Coth[ArcSinh[c*x]/2] - 2*ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - (c*x
*ArcSinh[c*x]^2*Csch[ArcSinh[c*x]/2]^4)/2 - 152*ArcSinh[c*x]*Log[1 - E^(-Ar
cSinh[c*x])] - (105*I)*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] + (105*I)*A
rcSinh[c*x]^2*Log[1 + I/E^ArcSinh[c*x]] + 152*ArcSinh[c*x]*Log[1 + E^(-ArcS
inh[c*x])] - 152*PolyLog[2, -E^(-ArcSinh[c*x])] - (210*I)*ArcSinh[c*x]*Poly
Log[2, (-I)/E^ArcSinh[c*x]] + (210*I)*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[c
*x]] + 152*PolyLog[2, E^(-ArcSinh[c*x])] - (210*I)*PolyLog[3, (-I)/E^ArcSin
h[c*x]] + (210*I)*PolyLog[3, I/E^ArcSinh[c*x]] - 2*ArcSinh[c*x]*Sech[ArcSin
h[c*x]/2]^2 - (8*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2]^4)/(c^3*x^3) + 4*Tanh[
ArcSinh[c*x]/2] - 38*ArcSinh[c*x]^2*Tanh[ArcSinh[c*x]/2]))/(24*d^3)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2 \operatorname{arsinh}(cx)^2 + 2\,ab \operatorname{arsinh}(cx) + a^2}{c^6d^3x^{10} + 3\,c^4d^3x^8 + 3\,c^2d^3x^6 + d^3x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^6*d^3*x^10 + 3*
c^4*d^3*x^8 + 3*c^2*d^3*x^6 + d^3*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2}{\left(c^2dx^2 + d\right)^3 x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^3*x^4), x)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arcsinh}(cx))^2}{x^4 \left(c^2 d\, x^2 + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^3,x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{24}a^2\left(\frac{105\,c^3\arctan(cx)}{d^3}+\frac{105\,c^6x^6+175\,c^4x^4+56\,c^2x^2-8}{c^4d^3x^7+2\,c^2d^3x^5+d^3x^3}\right)+\int\frac{b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^6d^3x^{10}+3\,c^4d^3x^8+3\,c^2d^3x^6+d^3x^4}+\frac{2}{c^6d^3x^{1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/24*a^2*(105*c^3*arctan(c*x)/d^3 + (105*c^6*x^6 + 175*c^4*x^4 + 56*c^2*x^2
 - 8)/(c^4*d^3*x^7 + 2*c^2*d^3*x^5 + d^3*x^3)) + integrate(b^2*log(c*x + sq
rt(c^2*x^2 + 1))^2/(c^6*d^3*x^10 + 3*c^4*d^3*x^8 + 3*c^2*d^3*x^6 + d^3*x^4)
 + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d^3*x^10 + 3*c^4*d^3*x^8 + 3*c^2
*d^3*x^6 + d^3*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))^2}{x^4\,\left(d\,c^2\,x^2+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)^3),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^6x^{10}+3c^4x^8+3c^2x^6+x^4}\,dx+\int \frac{b^2\,\mathrm{asinh}^2\,(cx)}{c^6x^{10}+3c^4x^8+3c^2x^6+x^4}\,dx+\int \frac{2ab\,\mathrm{asinh}\,(cx)}{c^6x^{10}+3c^4x^8+3c^2x^6+x^4}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**4/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a**2/(c**6*x**10 + 3*c**4*x**8 + 3*c**2*x**6 + x**4), x) + Integr
al(b**2*asinh(c*x)**2/(c**6*x**10 + 3*c**4*x**8 + 3*c**2*x**6 + x**4), x) +
 Integral(2*a*b*asinh(c*x)/(c**6*x**10 + 3*c**4*x**8 + 3*c**2*x**6 + x**4),
 x))/d**3
```

3.252 $\int \left(\pi + c^2\pi x^2\right)^{5/2} \left(a + b\sinh^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=300

$$-\frac{\pi^{5/2}b\left(c^2x^2+1\right)^3\left(a+b\sinh^{-1}(cx)\right)}{18c}-\frac{5\pi^{5/2}b\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{48c}+\frac{1}{6}x\left(\pi c^2x^2+\pi\right)^{5/2}\left(a+b\sinh^{-1}\right.$$

```
[Out] 65/1728*b^2*Pi^(5/2)*x*(c^2*x^2+1)^(3/2)+1/108*b^2*Pi^(5/2)*x*(c^2*x^2+1)^(
5/2)-115/1152*b^2*Pi^(5/2)*arcsinh(c*x)/c-5/16*b*c*Pi^(5/2)*x^2*(a+b*arcsin
h(c*x))-5/48*b*Pi^(5/2)*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/c-1/18*b*Pi^(5/2)*
(c^2*x^2+1)^3*(a+b*arcsinh(c*x))/c+5/24*Pi*x*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arc
sinh(c*x))^2+1/6*x*(Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))^2+5/48*Pi^(5/2)
*(a+b*arcsinh(c*x))^3/b/c+245/1152*b^2*Pi^(5/2)*x*(c^2*x^2+1)^(1/2)+5/16*Pi
^2*x*(a+b*arcsinh(c*x))^2*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 420, normalized size of antiderivative = 1.40, number of steps used = 16, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {5684, 5682, 5675, 5661, 321, 215, 5717, 195}

$$\frac{5\pi^2\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)^3}{48bc\sqrt{c^2x^2+1}}+\frac{1}{6}x\left(\pi c^2x^2+\pi\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{5}{24}\pi x\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh\right.$$

Antiderivative was successfully verified.

```
[In] Int[(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (245*b^2*Pi^2*x*Sqrt[Pi + c^2*Pi*x^2])/1152 + (65*b^2*Pi^2*x*(1 + c^2*x^2)*
Sqrt[Pi + c^2*Pi*x^2])/1728 + (b^2*Pi^2*x*(1 + c^2*x^2)^2*Sqrt[Pi + c^2*Pi*
x^2])/108 - (115*b^2*Pi^2*Sqrt[Pi + c^2*Pi*x^2]*ArcSinh[c*x])/(1152*c*Sqrt[
1 + c^2*x^2]) - (5*b*c*Pi^2*x^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))
/(16*Sqrt[1 + c^2*x^2]) - (5*b*Pi^2*(1 + c^2*x^2)^(3/2)*Sqrt[Pi + c^2*Pi*x^
2]*(a + b*ArcSinh[c*x]))/(48*c) - (b*Pi^2*(1 + c^2*x^2)^(5/2)*Sqrt[Pi + c^2
*Pi*x^2]*(a + b*ArcSinh[c*x]))/(18*c) + (5*Pi^2*x*Sqrt[Pi + c^2*Pi*x^2]*(a
+ b*ArcSinh[c*x])^2)/16 + (5*Pi*x*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*
x])^2)/24 + (x*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/6 + (5*Pi^2*
Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^3)/(48*b*c*Sqrt[1 + c^2*x^2])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(\pi + c^2\pi x^2\right)^{5/2} \left(a + b\sinh^{-1}(cx)\right)^2 dx = \frac{1}{6}x\left(\pi + c^2\pi x^2\right)^{5/2} \left(a + b\sinh^{-1}(cx)\right)^2 + \frac{1}{6}(5\pi)\int \left(\pi + c^2\pi x^2\right)^{3/2} ($$

$$= -\frac{b\pi^2\left(1 + c^2x^2\right)^{5/2}\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right)}{18c} + \frac{5}{24}\pi x\left(\pi + c\right.$$

$$= \frac{1}{108}b^2\pi^2x\left(1 + c^2x^2\right)^2\sqrt{\pi + c^2\pi x^2} - \frac{5b\pi^2\left(1 + c^2x^2\right)^{3/2}\sqrt{\pi + c^2\pi}}{48c}$$

$$= \frac{65b^2\pi^2x\left(1 + c^2x^2\right)\sqrt{\pi + c^2\pi x^2}}{1728} + \frac{1}{108}b^2\pi^2x\left(1 + c^2x^2\right)^2\sqrt{\pi + c^2}$$

$$= \frac{245b^2\pi^2x\sqrt{\pi + c^2\pi x^2}}{1152} + \frac{65b^2\pi^2x\left(1 + c^2x^2\right)\sqrt{\pi + c^2\pi x^2}}{1728} + \frac{1}{108}b$$

$$= \frac{245b^2\pi^2x\sqrt{\pi + c^2\pi x^2}}{1152} + \frac{65b^2\pi^2x\left(1 + c^2x^2\right)\sqrt{\pi + c^2\pi x^2}}{1728} + \frac{1}{108}b$$

Mathematica [A] time = 0.93, size = 284, normalized size = 0.95

$$\frac{\pi^{5/2}\left(12\sinh^{-1}(cx)\left(360a^2 + 540ab\sinh\left(2\sinh^{-1}(cx)\right) + 108ab\sinh\left(4\sinh^{-1}(cx)\right) + 12ab\sinh\left(6\sinh^{-1}(cx\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (Pi^(5/2)*(9504*a^2*c*x*Sqrt[1 + c^2*x^2] + 7488*a^2*c^3*x^3*Sqrt[1 + c^2*x
^2] + 2304*a^2*c^5*x^5*Sqrt[1 + c^2*x^2] + 1440*b^2*ArcSinh[c*x]^3 - 3240*a
*b*Cosh[2*ArcSinh[c*x]] - 324*a*b*Cosh[4*ArcSinh[c*x]] - 24*a*b*Cosh[6*ArcS
inh[c*x]] + 1620*b^2*Sinh[2*ArcSinh[c*x]] + 81*b^2*Sinh[4*ArcSinh[c*x]] + 4
*b^2*Sinh[6*ArcSinh[c*x]] + 72*b*ArcSinh[c*x]^2*(60*a + 45*b*Sinh[2*ArcSinh
[c*x]] + 9*b*Sinh[4*ArcSinh[c*x]] + b*Sinh[6*ArcSinh[c*x]]) + 12*ArcSinh[c*
x]*(360*a^2 - 270*b^2*Cosh[2*ArcSinh[c*x]] - 27*b^2*Cosh[4*ArcSinh[c*x]] -
2*b^2*Cosh[6*ArcSinh[c*x]] + 540*a*b*Sinh[2*ArcSinh[c*x]] + 108*a*b*Sinh[4*
ArcSinh[c*x]] + 12*a*b*Sinh[6*ArcSinh[c*x]])))/(13824*c)
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{\pi + \pi c^2x^2}\left(\pi^2a^2c^4x^4 + 2\,\pi^2a^2c^2x^2 + \pi^2a^2 + \left(\pi^2b^2c^4x^4 + 2\,\pi^2b^2c^2x^2 + \pi^2b^2\right)\operatorname{arsinh}(cx)^2 + 2\left(\pi^2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi^2*a^2*c^4*x^4 + 2*pi^2*a^2*c^2*x^2 + pi^
2*a^2 + (pi^2*b^2*c^4*x^4 + 2*pi^2*b^2*c^2*x^2 + pi^2*b^2)*arcsinh(c*x)^2 +
 2*(pi^2*a*b*c^4*x^4 + 2*pi^2*a*b*c^2*x^2 + pi^2*a*b)*arcsinh(c*x)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.13, size = 486, normalized size = 1.62

$$\frac{a^2x\left(\pi c^2x^2+\pi\right)^{\frac{5}{2}}}{6}+\frac{5a^2\pi x\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}}{24}+\frac{5a^2\pi^2x\sqrt{\pi c^2x^2+\pi}}{16}+\frac{5a^2\pi^3\ln\left(\frac{\pi x c^2}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)}{16\sqrt{\pi c^2}}+\frac{b^2\pi^{\frac{5}{2}}c^4\sqrt{c^2x}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/6*a^2*x*(Pi*c^2*x^2+Pi)^(5/2)+5/24*a^2*Pi*x*(Pi*c^2*x^2+Pi)^(3/2)+5/16*a^
2*Pi^2*x*(Pi*c^2*x^2+Pi)^(1/2)+5/16*a^2*Pi^3*ln(Pi*x*c^2/(Pi*c^2)^(1/2)+(Pi
*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/6*b^2*Pi^(5/2)*c^4*(c^2*x^2+1)^(1/2)*a
rcsinh(c*x)^2*x^5-1/18*b^2*Pi^(5/2)*c^5*arcsinh(c*x)*x^6+1/108*b^2*Pi^(5/2)
*c^4*x^5*(c^2*x^2+1)^(1/2)+13/24*b^2*Pi^(5/2)*c^2*(c^2*x^2+1)^(1/2)*arcsinh
(c*x)^2*x^3-13/48*b^2*Pi^(5/2)*c^3*arcsinh(c*x)*x^4+97/1728*b^2*Pi^(5/2)*c^
2*x^3*(c^2*x^2+1)^(1/2)+11/16*b^2*Pi^(5/2)*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)
*x-11/16*b^2*Pi^(5/2)*c*arcsinh(c*x)*x^2+299/1152*b^2*Pi^(5/2)*x*(c^2*x^2+1
)^(1/2)+5/48*b^2*Pi^(5/2)/c*arcsinh(c*x)^3-299/1152*b^2*Pi^(5/2)*arcsinh(c*
x)/c+1/3*a*b*Pi^(5/2)*c^4*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^5-1/18*a*b*Pi^(5
/2)*c^5*x^6+13/12*a*b*Pi^(5/2)*c^2*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^3-13/48
*a*b*Pi^(5/2)*c^3*x^4+11/8*a*b*Pi^(5/2)*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x-11
/16*a*b*Pi^(5/2)*c*x^2+5/16*a*b*Pi^(5/2)/c*arcsinh(c*x)^2-17/36*a*b*Pi^(5/2
)/c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{asinh}\,(c\,x))^2\,\left(\Pi\,c^2\,x^2+\Pi\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(Pi + Pi*c^2*x^2)^(5/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(Pi + Pi*c^2*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

3.253 $\int \left(\pi + c^2\pi x^2\right)^{3/2} \left(a + b\sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=210

$$\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{3}{8}\pi x\sqrt{\pi c^2x^2+\pi}\,\left(a+b\sinh^{-1}(cx)\right)^2-\frac{\pi^{3/2}b\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(\right.}{8c}$$

```
[Out] 1/32*b^2*Pi^(3/2)*x*(c^2*x^2+1)^(3/2)-9/64*b^2*Pi^(3/2)*arcsinh(c*x)/c-3/8*
b*c*Pi^(3/2)*x^2*(a+b*arcsinh(c*x))-1/8*b*Pi^(3/2)*(c^2*x^2+1)^2*(a+b*arcsi
nh(c*x))/c+1/4*x*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))^2+1/8*Pi^(3/2)*(a
+b*arcsinh(c*x))^3/b/c+15/64*b^2*Pi^(3/2)*x*(c^2*x^2+1)^(1/2)+3/8*Pi*x*(a+b
*arcsinh(c*x))^2*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 294, normalized size of antiderivative = 1.40, number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {5684, 5682, 5675, 5661, 321, 215, 5717, 195}

$$\frac{\pi\sqrt{\pi c^2x^2+\pi}\,\left(a+b\sinh^{-1}(cx)\right)^3}{8bc\sqrt{c^2x^2+1}}+\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{3}{8}\pi x\sqrt{\pi c^2x^2+\pi}\,\left(a+b\sinh^{-1}(cx\right.$$

Antiderivative was successfully verified.

```
[In] Int[(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (15*b^2*Pi*x*Sqrt[Pi + c^2*Pi*x^2])/64 + (b^2*Pi*x*(1 + c^2*x^2)*Sqrt[Pi +
c^2*Pi*x^2])/32 - (9*b^2*Pi*Sqrt[Pi + c^2*Pi*x^2]*ArcSinh[c*x])/(64*c*Sqrt[
1 + c^2*x^2]) - (3*b*c*Pi*x^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(
8*Sqrt[1 + c^2*x^2]) - (b*Pi*(1 + c^2*x^2)^(3/2)*Sqrt[Pi + c^2*Pi*x^2]*(a +
 b*ArcSinh[c*x]))/(8*c) + (3*Pi*x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]
)^2)/8 + (x*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/4 + (Pi*Sqrt[Pi
 + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^3)/(8*b*c*Sqrt[1 + c^2*x^2])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{1}{4}x\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{4}(3\pi)\int \sqrt{\pi+c^2\pi x^2}\,\left(a+b\right.$$

$$=-\frac{b\pi\left(1+c^2x^2\right)^{3/2}\sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{8c}+\frac{3}{8}\pi x\sqrt{\pi+c^2\pi x^2}$$

$$=\frac{1}{32}b^2\pi x\left(1+c^2x^2\right)\sqrt{\pi+c^2\pi x^2}-\frac{3bc\pi x^2\sqrt{\pi+c^2\pi x^2}\,\left(a+b\sinh^{-1}(\right.}{8\sqrt{1+c^2x^2}}$$

$$=\frac{15}{64}b^2\pi x\sqrt{\pi+c^2\pi x^2}+\frac{1}{32}b^2\pi x\left(1+c^2x^2\right)\sqrt{\pi+c^2\pi x^2}-\frac{3bc\pi x^2\sqrt{\pi}}{}$$

$$=\frac{15}{64}b^2\pi x\sqrt{\pi+c^2\pi x^2}+\frac{1}{32}b^2\pi x\left(1+c^2x^2\right)\sqrt{\pi+c^2\pi x^2}-\frac{9b^2\pi\sqrt{\pi+}}{64c}$$

Mathematica [A] time = 0.57, size = 202, normalized size = 0.96

$$\frac{\pi^{3/2}\left(160a^2cx\sqrt{c^2x^2+1}+64a^2c^3x^3\sqrt{c^2x^2+1}+4\sinh^{-1}(cx)\left(4a\left(6a+8b\sinh\left(2\sinh^{-1}(cx)\right)+b\sinh\left(4\sinh^{-1}\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (Pi^(3/2)*(160*a^2*c*x*Sqrt[1 + c^2*x^2] + 64*a^2*c^3*x^3*Sqrt[1 + c^2*x^2]
 + 32*b^2*ArcSinh[c*x]^3 - 64*a*b*Cosh[2*ArcSinh[c*x]] - 4*a*b*Cosh[4*ArcSi
nh[c*x]] + 32*b^2*Sinh[2*ArcSinh[c*x]] + b^2*Sinh[4*ArcSinh[c*x]] + 8*b*Arc
Sinh[c*x]^2*(12*a + 8*b*Sinh[2*ArcSinh[c*x]] + b*Sinh[4*ArcSinh[c*x]]) + 4*
ArcSinh[c*x]*(-16*b^2*Cosh[2*ArcSinh[c*x]] - b^2*Cosh[4*ArcSinh[c*x]] + 4*a
*(6*a + 8*b*Sinh[2*ArcSinh[c*x]] + b*Sinh[4*ArcSinh[c*x]]))))/(256*c)
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{\pi + \pi c^2 x^2}\left(\pi a^2 c^2 x^2 + \pi a^2 + \left(\pi b^2 c^2 x^2 + \pi b^2\right)\text{arsinh}(cx)^2 + 2\left(\pi abc^2 x^2 + \pi ab\right)\text{arsinh}(cx)\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi*a^2*c^2*x^2 + pi*a^2 + (pi*b^2*c^2*x^2 +
 pi*b^2)*arcsinh(c*x)^2 + 2*(pi*a*b*c^2*x^2 + pi*a*b)*arcsinh(c*x)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.12, size = 350, normalized size = 1.67

$$\frac{a^2 x\left(\pi c^2 x^2 + \pi\right)^{\frac{3}{2}}}{4} + \frac{3a^2 \pi x\sqrt{\pi c^2 x^2 + \pi}}{8} + \frac{3a^2 \pi^2 \ln\left(\frac{\pi x c^2}{\sqrt{\pi c^2}} + \sqrt{\pi c^2 x^2 + \pi}\right)}{8\sqrt{\pi c^2}} + \frac{b^2 \pi^{\frac{3}{2}} c^2 \,\text{arcsinh}(cx)^2 \sqrt{c^2 x^2 + 1}\; x}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/4*a^2*x*(Pi*c^2*x^2+Pi)^(3/2)+3/8*a^2*Pi*x*(Pi*c^2*x^2+Pi)^(1/2)+3/8*a^2*
Pi^2*ln(Pi*x*c^2/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/4*b
^2*Pi^(3/2)*c^2*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*x^3-1/8*b^2*Pi^(3/2)*c^3*a
rcsinh(c*x)*x^4+1/32*b^2*Pi^(3/2)*c^2*x^3*(c^2*x^2+1)^(1/2)+5/8*b^2*Pi^(3/2
)*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*x-5/8*b^2*Pi^(3/2)*c*arcsinh(c*x)*x^2+17
/64*b^2*Pi^(3/2)*x*(c^2*x^2+1)^(1/2)+1/8*b^2*Pi^(3/2)/c*arcsinh(c*x)^3-17/6
4*b^2*Pi^(3/2)*arcsinh(c*x)/c+1/2*a*b*Pi^(3/2)*c^2*arcsinh(c*x)*(c^2*x^2+1)
^(1/2)*x^3-1/8*a*b*Pi^(3/2)*c^3*x^4+5/4*a*b*Pi^(3/2)*arcsinh(c*x)*(c^2*x^2+
1)^(1/2)*x-5/8*a*b*Pi^(3/2)*c*x^2+3/8*a*b*Pi^(3/2)/c*arcsinh(c*x)^2-1/2*a*b
*Pi^(3/2)/c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,\text{asinh}\,(c\,x))^2\,\left(\Pi\,c^2\,x^2 + \Pi\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(Pi + Pi*c^2*x^2)^(3/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(Pi + Pi*c^2*x^2)^(3/2), x)
```

sympy [A] time = 95.26, size = 405, normalized size = 1.93

$$\begin{cases} \frac{\pi^{\frac{3}{2}}a^2c^2x^3\sqrt{c^2x^2+1}}{4} + \frac{5\pi^{\frac{3}{2}}a^2x\sqrt{c^2x^2+1}}{8} + \frac{3\pi^{\frac{3}{2}}a^2\,\text{asinh}\,(cx)}{8c} - \frac{\pi^{\frac{3}{2}}abc^3x^4}{8} + \frac{\pi^{\frac{3}{2}}abc^2x^3\sqrt{c^2x^2+1}\,\text{asinh}\,(cx)}{2} - \frac{5\pi^{\frac{3}{2}}abcx^2}{8} + \frac{5\pi^{\frac{3}{2}}abx\sqrt{c^2x^2+1}}{4} \\ \pi^{\frac{3}{2}}a^2x \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((pi**(3/2)*a**2*c**2*x**3*sqrt(c**2*x**2 + 1)/4 + 5*pi**(3/2)*a**
2*x*sqrt(c**2*x**2 + 1)/8 + 3*pi**(3/2)*a**2*asinh(c*x)/(8*c) - pi**(3/2)*a
*b*c**3*x**4/8 + pi**(3/2)*a*b*c**2*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/2 -
 5*pi**(3/2)*a*b*c*x**2/8 + 5*pi**(3/2)*a*b*x*sqrt(c**2*x**2 + 1)*asinh(c*x
)/4 + 3*pi**(3/2)*a*b*asinh(c*x)**2/(8*c) - pi**(3/2)*b**2*c**3*x**4*asinh(
c*x)/8 + pi**(3/2)*b**2*c**2*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)**2/4 + pi*
*(3/2)*b**2*c**2*x**3*sqrt(c**2*x**2 + 1)/32 - 5*pi**(3/2)*b**2*c*x**2*asin
h(c*x)/8 + 5*pi**(3/2)*b**2*x*sqrt(c**2*x**2 + 1)*asinh(c*x)**2/8 + 17*pi**
(3/2)*b**2*x*sqrt(c**2*x**2 + 1)/64 + pi**(3/2)*b**2*asinh(c*x)**3/(8*c) -
17*pi**(3/2)*b**2*asinh(c*x)/(64*c), Ne(c, 0)), (pi**(3/2)*a**2*x, True))
```

3.254 $\int \sqrt{\pi + c^2\pi x^2} \left(a + b\sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=122

$$\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{1}{2}\sqrt{\pi}\,bcx^2\left(a+b\sinh^{-1}(cx)\right)+\frac{\sqrt{\pi}\left(a+b\sinh^{-1}(cx)\right)^3}{6bc}+\frac{1}{4}\sqrt{\pi}\,b^2x\sqrt{c^2x^2+}$$

```
[Out] -1/4*b^2*arcsinh(c*x)*Pi^(1/2)/c-1/2*b*c*x^2*(a+b*arcsinh(c*x))*Pi^(1/2)+1/
6*(a+b*arcsinh(c*x))^3*Pi^(1/2)/b/c+1/4*b^2*x*Pi^(1/2)*(c^2*x^2+1)^(1/2)+1/
2*x*(a+b*arcsinh(c*x))^2*(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 184, normalized size of antiderivative = 1.51, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5682, 5675, 5661, 321, 215}

$$\frac{\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)^3}{6bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{bcx^2\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{c^2x^2+1}}+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (b^2*x*Sqrt[Pi + c^2*Pi*x^2])/4 - (b^2*Sqrt[Pi + c^2*Pi*x^2]*ArcSinh[c*x])/
(4*c*Sqrt[1 + c^2*x^2]) - (b*c*x^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x
]))/(2*Sqrt[1 + c^2*x^2]) + (x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^2
)/2 + (Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^3)/(6*b*c*Sqrt[1 + c^2*x^
2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
```

```
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rubi steps

$$\int \sqrt{\pi + c^2\pi x^2} \left(a + b\sinh^{-1}(cx)\right)^2 dx = \frac{1}{2}x\sqrt{\pi + c^2\pi x^2} \left(a + b\sinh^{-1}(cx)\right)^2 + \frac{\sqrt{\pi + c^2\pi x^2} \int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}} dx}{2\sqrt{1 + c^2x^2}}$$

$$= -\frac{bcx^2\sqrt{\pi + c^2\pi x^2} \left(a + b\sinh^{-1}(cx)\right)}{2\sqrt{1 + c^2x^2}} + \frac{1}{2}x\sqrt{\pi + c^2\pi x^2} \left(a + b\sinh^{-1}(\right.$$

$$= \frac{1}{4}b^2x\sqrt{\pi + c^2\pi x^2} - \frac{bcx^2\sqrt{\pi + c^2\pi x^2} \left(a + b\sinh^{-1}(cx)\right)}{2\sqrt{1 + c^2x^2}} + \frac{1}{2}x\sqrt{\pi + c^2}$$

$$= \frac{1}{4}b^2x\sqrt{\pi + c^2\pi x^2} - \frac{b^2\sqrt{\pi + c^2\pi x^2}\sinh^{-1}(cx)}{4c\sqrt{1 + c^2x^2}} - \frac{bcx^2\sqrt{\pi + c^2\pi x^2} (a +}{2\sqrt{1 + c^2}}$$

Mathematica [A] time = 0.35, size = 124, normalized size = 1.02

$$\frac{\sqrt{\pi}\left(3\left(4a^2cx\sqrt{c^2x^2+1} - 2ab\cosh\left(2\sinh^{-1}(cx)\right) + b^2\sinh\left(2\sinh^{-1}(cx)\right)\right) + 6\sinh^{-1}(cx)\left(2a\left(a + b\sinh\left(2\sin\right.\right.\right.}{24c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (Sqrt[Pi]*(4*b^2*ArcSinh[c*x]^3 + 6*b*ArcSinh[c*x]^2*(2*a + b*Sinh[2*ArcSin
h[c*x]]) + 3*(4*a^2*c*x*Sqrt[1 + c^2*x^2] - 2*a*b*Cosh[2*ArcSinh[c*x]] + b^
2*Sinh[2*ArcSinh[c*x]]) + 6*ArcSinh[c*x]*(-(b^2*Cosh[2*ArcSinh[c*x]]) + 2*a
*(a + b*Sinh[2*ArcSinh[c*x]]))))/(24*c)
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{\pi + \pi c^2x^2}\left(b^2\operatorname{arsinh}(cx)^2 + 2ab\operatorname{arsinh}(cx) + a^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a
^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.10, size = 213, normalized size = 1.75

$$\frac{a^{2}x\sqrt{\pi\,c^{2}x^{2}+\pi}}{2}+\frac{a^{2}\pi\ln\left(\frac{\pi x\,c^{2}}{\sqrt{\pi\,c^{2}}}+\sqrt{\pi\,c^{2}x^{2}+\pi}\right)}{2\sqrt{\pi\,c^{2}}}+\frac{b^{2}\sqrt{\pi}\ \operatorname{arcsinh}\left(cx\right)^{2}\sqrt{c^{2}x^{2}+1}\ x}{2}-\frac{b^{2}\sqrt{\pi}\ c\operatorname{arcsinh}\left(cx\right)x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi*c^2*x^2+Pi)^(1/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/2*a^2*x*(Pi*c^2*x^2+Pi)^(1/2)+1/2*a^2*Pi*ln(Pi*x*c^2/(Pi*c^2)^(1/2)+(Pi*c
^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/2*b^2*Pi^(1/2)*arcsinh(c*x)^2*(c^2*x^2+1
)^(1/2)*x-1/2*b^2*Pi^(1/2)*c*arcsinh(c*x)*x^2+1/6*b^2*Pi^(1/2)/c*arcsinh(c*
x)^3+1/4*b^2*x*Pi^(1/2)*(c^2*x^2+1)^(1/2)-1/4*b^2*arcsinh(c*x)*Pi^(1/2)/c+a
*b*Pi^(1/2)*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x-1/2*a*b*Pi^(1/2)*c*x^2+1/2*a*b
*Pi^(1/2)/c*arcsinh(c*x)^2-1/2*a*b*Pi^(1/2)/c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2\,\sqrt{\Pi\,c^2\,x^2+\Pi}\,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(Pi + Pi*c^2*x^2)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(Pi + Pi*c^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\sqrt{\pi}\left(\int a^{2}\sqrt{c^{2}x^{2}+1}\,dx+\int b^{2}\sqrt{c^{2}x^{2}+1}\operatorname{asinh}^{2}{\left(cx\right)}\,dx+\int 2ab\sqrt{c^{2}x^{2}+1}\operatorname{asinh}{\left(cx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c**2*x**2+pi)**(1/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] sqrt(pi)*(Integral(a**2*sqrt(c**2*x**2 + 1), x) + Integral(b**2*sqrt(c**2*x
**2 + 1)*asinh(c*x)**2, x) + Integral(2*a*b*sqrt(c**2*x**2 + 1)*asinh(c*x),
 x))
```

3.255 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal. Leaf size=25

$$\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3\sqrt{\pi}\,bc}$$

```
[Out] 1/3*(a+b*arcsinh(c*x))^3/b/c/Pi^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {5675}

$$\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3\sqrt{\pi}\,bc}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] (a + b*ArcSinh[c*x])^3/(3*b*c*Sqrt[Pi])
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{\pi+c^2\pi x^2}}\,dx=\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{\pi}}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 1.00

$$\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3\sqrt{\pi}\,bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] (a + b*ArcSinh[c*x])^3/(3*b*c*Sqrt[Pi])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\operatorname{arsinh}(cx)^2+2\,ab\operatorname{arsinh}(cx)+a^2}{\sqrt{\pi+\pi c^2x^2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/sqrt(pi + pi*c^2*x
^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arsinh}\,(cx) + a)^2}{\sqrt{\pi + \pi c^2 x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/sqrt(pi + pi*c^2*x^2), x)
```

maple [B] time = 0.08, size = 72, normalized size = 2.88

$$\frac{a^2 \ln\left(\frac{\pi x c^2}{\sqrt{\pi c^2}} + \sqrt{\pi c^2 x^2 + \pi}\right)}{\sqrt{\pi c^2}} + \frac{b^2 \,\text{arcsinh}\,(cx)^3}{3c\sqrt{\pi}} + \frac{ab\,\text{arcsinh}\,(cx)^2}{c\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(Pi*c^2*x^2+Pi)^(1/2),x)
```

```
[Out] a^2*ln(Pi*x*c^2/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/3*b^
2/c/Pi^(1/2)*arcsinh(c*x)^3+a*b*arcsinh(c*x)^2/c/Pi^(1/2)
```

maxima [B] time = 0.36, size = 47, normalized size = 1.88

$$\frac{b^2 \,\text{arsinh}\,(cx)^3}{3\sqrt{\pi}\,c} + \frac{ab\,\text{arsinh}\,(cx)^2}{\sqrt{\pi}\,c} + \frac{a^2 \,\text{arsinh}\,(cx)}{\sqrt{\pi}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*b^2*arcsinh(c*x)^3/(sqrt(pi)*c) + a*b*arcsinh(c*x)^2/(sqrt(pi)*c) + a^2
*arcsinh(c*x)/(sqrt(pi)*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(a + b\,\text{asinh}\,(c\,x))^2}{\sqrt{\Pi\,c^2\,x^2 + \Pi}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(Pi + Pi*c^2*x^2)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(Pi + Pi*c^2*x^2)^(1/2), x)
```

sympy [A] time = 2.81, size = 88, normalized size = 3.52

$$\begin{cases} a^2 \left(\begin{cases} \frac{\sqrt{-\frac{1}{c^2}}\,\text{asin}\left(x\sqrt{-c^2}\right)}{\sqrt{\pi}} & \text{for}\ \pi c^2 < 0 \\ \frac{\sqrt{\frac{1}{c^2}}\,\text{asinh}\left(x\sqrt{c^2}\right)}{\sqrt{\pi}} & \text{for}\ \pi c^2 > 0 \end{cases}\right) & \text{for}\ b = 0 \\ \frac{a^2 x}{\sqrt{\pi}} & \text{for}\ c = 0 \\ \frac{(a + b\,\text{asinh}\,(cx))^3}{3\sqrt{\pi}\,bc} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] Piecewise((a**2*Piecewise((sqrt(-1/c**2)*asin(x*sqrt(-c**2))/sqrt(pi), pi*c
**2 < 0), (sqrt(c**(-2))*asinh(x*sqrt(c**2))/sqrt(pi), pi*c**2 > 0)), Eq(b,
 0)), (a**2*x/sqrt(pi), Eq(c, 0)), ((a + b*asinh(c*x))**3/(3*sqrt(pi)*b*c),
 True))
```

3.256 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=104

$$\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\pi\sqrt{\pi c^2x^2+\pi}}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\pi^{3/2}c}-\frac{2b\log\left(e^{2\sinh^{-1}(cx)}+1\right)\left(a+b\sinh^{-1}(cx)\right)}{\pi^{3/2}c}-\frac{b^2\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{\pi^{3/2}c}$$

```
[Out] (a+b*arcsinh(c*x))^2/c/Pi^(3/2)-2*b*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1
)^(1/2))^2)/c/Pi^(3/2)-b^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/Pi^(3/2)
+x*(a+b*arcsinh(c*x))^2/Pi/(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 179, normalized size of antiderivative = 1.72, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {5687, 5714, 3718, 2190, 2279, 2391}

$$-\frac{b^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{\pi c\sqrt{\pi c^2x^2+\pi}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\pi\sqrt{\pi c^2x^2+\pi}}+\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{\pi c\sqrt{\pi c^2x^2+\pi}}-\frac{2b\sqrt{c^2x^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcSinh[c*x])^2)/(Pi*Sqrt[Pi + c^2*Pi*x^2]) + (Sqrt[1 + c^2*x^2]*
(a + b*ArcSinh[c*x])^2)/(c*Pi*Sqrt[Pi + c^2*Pi*x^2]) - (2*b*Sqrt[1 + c^2*x^
2]*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(c*Pi*Sqrt[Pi + c^2*Pi
*x^2]) - (b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*Pi*Sqrt
[Pi + c^2*Pi*x^2])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx = \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\pi\sqrt{\pi+c^2\pi x^2}} - \frac{\left(2bc\sqrt{1+c^2x^2}\right)\int\frac{x\left(a+b\sinh^{-1}(cx)\right)}{1+c^2x^2}\,dx}{\pi\sqrt{\pi+c^2\pi x^2}}$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\pi\sqrt{\pi+c^2\pi x^2}} - \frac{\left(2b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int(a+bx)\tanh(x)\,dx, x, \sinh^{-1}(cx)\right)}{c\pi\sqrt{\pi+c^2\pi x^2}}$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\pi\sqrt{\pi+c^2\pi x^2}} + \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{c\pi\sqrt{\pi+c^2\pi x^2}} - \frac{\left(4b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{e^{\ldots}}{\ldots}\right.}{c\pi\sqrt{\pi+\ldots}}$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\pi\sqrt{\pi+c^2\pi x^2}} + \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{c\pi\sqrt{\pi+c^2\pi x^2}} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}\ldots\right.}{c\pi\sqrt{\pi+\ldots}}$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\pi\sqrt{\pi+c^2\pi x^2}} + \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{c\pi\sqrt{\pi+c^2\pi x^2}} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}\ldots\right.}{c\pi\sqrt{\pi+\ldots}}$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\pi\sqrt{\pi+c^2\pi x^2}} + \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{c\pi\sqrt{\pi+c^2\pi x^2}} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}\ldots\right.}{c\pi\sqrt{\pi+\ldots}}$$

Mathematica [A] time = 0.39, size = 153, normalized size = 1.47

$$\frac{a\left(acx-b\sqrt{c^2x^2+1}\,\log\left(c^2x^2+1\right)\right)+2b\sinh^{-1}(cx)\left(acx-b\sqrt{c^2x^2+1}\,\log\left(e^{-2\sinh^{-1}(cx)}+1\right)\right)+b^2\sqrt{c^2x^2+1}\,\text{Li}_2\ldots}{\pi^{3/2}c\sqrt{c^2x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] (-(b^2*(-(c*x) + Sqrt[1 + c^2*x^2])*ArcSinh[c*x]^2) + 2*b*ArcSinh[c*x]*(a*c
*x - b*Sqrt[1 + c^2*x^2]*Log[1 + E^(-2*ArcSinh[c*x])]) + a*(a*c*x - b*Sqrt[
1 + c^2*x^2]*Log[1 + c^2*x^2]) + b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-2*Ar
cSinh[c*x])])/(c*Pi^(3/2)*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\pi+\pi c^2x^2}\left(b^2\,\text{arsinh}\,(cx)^2+2\,ab\,\text{arsinh}\,(cx)+a^2\right)}{\pi^2c^4x^4+2\,\pi^2c^2x^2+\pi^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a
^2)/(pi^2*c^4*x^4 + 2*pi^2*c^2*x^2 + pi^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2}{(\pi + \pi c^2 x^2)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(pi + pi*c^2*x^2)^(3/2), x)
```

maple [B] time = 0.19, size = 306, normalized size = 2.94

$$\frac{a^2 x}{\pi\sqrt{\pi c^2 x^2 + \pi}} - \frac{b^2 \operatorname{arcsinh}(cx)^2 c x^2}{\pi^{\frac{3}{2}}(c^2x^2+1)} + \frac{b^2 \operatorname{arcsinh}(cx)^2 x}{\pi^{\frac{3}{2}}\sqrt{c^2x^2+1}} - \frac{b^2 \operatorname{arcsinh}(cx)^2}{\pi^{\frac{3}{2}} c (c^2x^2+1)} + \frac{2b^2 \operatorname{arcsinh}(cx)^2}{c\,\pi^{\frac{3}{2}}} - \frac{2b^2 \operatorname{arcsinh}(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(Pi*c^2*x^2+Pi)^(3/2),x)
```

```
[Out] a^2/Pi*x/(Pi*c^2*x^2+Pi)^(1/2)-b^2/Pi^(3/2)*arcsinh(c*x)^2*c/(c^2*x^2+1)*x^
2+b^2/Pi^(3/2)*arcsinh(c*x)^2/(c^2*x^2+1)^(1/2)*x-b^2/Pi^(3/2)*arcsinh(c*x)
^2/c/(c^2*x^2+1)+2*b^2/c/Pi^(3/2)*arcsinh(c*x)^2-2*b^2/c/Pi^(3/2)*arcsinh(c
*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-b^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^
2)/c/Pi^(3/2)+4*a*b/c/Pi^(3/2)*arcsinh(c*x)-2*a*b/Pi^(3/2)*arcsinh(c*x)*c/(
c^2*x^2+1)*x^2+2*a*b/Pi^(3/2)*arcsinh(c*x)/(c^2*x^2+1)^(1/2)*x-2*a*b/Pi^(3/
2)*arcsinh(c*x)/c/(c^2*x^2+1)-2*a*b/c/Pi^(3/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))
^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^2 \int \frac{\log\left(cx + \sqrt{c^2x^2+1}\right)^2}{(\pi + \pi c^2 x^2)^{\frac{3}{2}}}\, dx + \frac{2\,abx \operatorname{arsinh}(cx)}{\pi\sqrt{\pi + \pi c^2 x^2}} + \frac{a^2 x}{\pi\sqrt{\pi + \pi c^2 x^2}} - \frac{ab \log\left(x^2 + \frac{1}{c^2}\right)}{\pi^{\frac{3}{2}} c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima")
```

```
[Out] b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/(pi + pi*c^2*x^2)^(3/2), x) +
2*a*b*x*arcsinh(c*x)/(pi*sqrt(pi + pi*c^2*x^2)) + a^2*x/(pi*sqrt(pi + pi*c^
2*x^2)) - a*b*log(x^2 + 1/c^2)/(pi^(3/2)*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{asinh}(c\,x))^2}{(\Pi\,c^2\,x^2 + \Pi)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(Pi + Pi*c^2*x^2)^(3/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(Pi + Pi*c^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{b^2\operatorname{asinh}^2(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{2ab\operatorname{asinh}(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(pi*c**2*x**2+pi)**(3/2),x)
```

```
[Out] (Integral(a**2/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x) +
Integral(b**2*asinh(c*x)**2/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2
 + 1)), x) + Integral(2*a*b*asinh(c*x)/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqr
t(c**2*x**2 + 1)), x))/pi**(3/2)
```

3.257 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=204

$$\frac{b\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}c\left(c^2x^2+1\right)}+\frac{2x\left(a+b\sinh^{-1}(cx)\right)^2}{3\pi^2\sqrt{\pi c^2x^2+\pi}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{2\left(a+b\sinh^{-1}(cx)\right)^2}{3\pi^{5/2}c}-\frac{4b\log\left(e^{2\sinh^{-1}(cx}\right.}{}$$

```
[Out] 1/3*b*(a+b*arcsinh(c*x))/c/Pi^(5/2)/(c^2*x^2+1)+2/3*(a+b*arcsinh(c*x))^2/c/
Pi^(5/2)+1/3*x*(a+b*arcsinh(c*x))^2/Pi/(Pi*c^2*x^2+Pi)^(3/2)-4/3*b*(a+b*arc
sinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/Pi^(5/2)-2/3*b^2*polylog(2,-(c
*x+(c^2*x^2+1)^(1/2))^2)/c/Pi^(5/2)-1/3*b^2*x/Pi^(5/2)/(c^2*x^2+1)^(1/2)+2/
3*x*(a+b*arcsinh(c*x))^2/Pi^2/(Pi*c^2*x^2+Pi)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 292, normalized size of antiderivative = 1.43, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {5690, 5687, 5714, 3718, 2190, 2279, 2391, 5717, 191}

$$-\frac{2b^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{3\pi^2c\sqrt{\pi c^2x^2+\pi}}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{3\pi^2c\sqrt{c^2x^2+1}\sqrt{\pi c^2x^2+\pi}}+\frac{2x\left(a+b\sinh^{-1}(cx)\right)^2}{3\pi^2\sqrt{\pi c^2x^2+\pi}}+\frac{2\sqrt{c^2x^2+1}}{3\pi^2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] -(b^2*x)/(3*Pi^2*Sqrt[Pi + c^2*Pi*x^2]) + (b*(a + b*ArcSinh[c*x]))/(3*c*Pi^
2*Sqrt[1 + c^2*x^2]*Sqrt[Pi + c^2*Pi*x^2]) + (x*(a + b*ArcSinh[c*x])^2)/(3*
Pi*(Pi + c^2*Pi*x^2)^(3/2)) + (2*x*(a + b*ArcSinh[c*x])^2)/(3*Pi^2*Sqrt[Pi
+ c^2*Pi*x^2]) + (2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c*Pi^2*Sqr
t[Pi + c^2*Pi*x^2]) - (4*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + E
^(2*ArcSinh[c*x])])/(3*c*Pi^2*Sqrt[Pi + c^2*Pi*x^2]) - (2*b^2*Sqrt[1 + c^2*
x^2]*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*c*Pi^2*Sqrt[Pi + c^2*Pi*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx = \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{2\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(\pi+c^2\pi x^2\right)^{3/2}}dx}{3\pi} - \frac{\left(2bc\sqrt{1+c^2x^2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^2}}{3\pi^2\sqrt{\pi+c^2\pi x^2}}$$

$$= \frac{b\left(a+b\sinh^{-1}(cx)\right)}{3c\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{2x\left(a+b\sinh^{-1}(cx)\right)^2}{3\pi^2\sqrt{\pi+c^2\pi x^2}} -$$

$$= -\frac{b^2x}{3\pi^2\sqrt{\pi+c^2\pi x^2}} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{3c\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{2x(}{3}$$

$$= -\frac{b^2x}{3\pi^2\sqrt{\pi+c^2\pi x^2}} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{3c\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{2x(}{3}$$

$$= -\frac{b^2x}{3\pi^2\sqrt{\pi+c^2\pi x^2}} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{3c\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{2x(}{3}$$

$$= -\frac{b^2x}{3\pi^2\sqrt{\pi+c^2\pi x^2}} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{3c\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{2x(}{3}$$

$$= -\frac{b^2x}{3\pi^2\sqrt{\pi+c^2\pi x^2}} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{3c\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{2x(}{3}$$

Mathematica [A] time = 0.63, size = 293, normalized size = 1.44

$$\frac{2a^2c^3x^3 + 3a^2cx + ab\sqrt{c^2x^2+1} - 2abc^2x^2\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right) - 2ab\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right) - b\sinh^{-1}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] (3*a^2*c*x - b^2*c*x + 2*a^2*c^3*x^3 - b^2*c^3*x^3 + a*b*Sqrt[1 + c^2*x^2]
- b^2*(-3*c*x - 2*c^3*x^3 + 2*Sqrt[1 + c^2*x^2] + 2*c^2*x^2*Sqrt[1 + c^2*x^
2])*ArcSinh[c*x]^2 - b*ArcSinh[c*x]*(-6*a*c*x - 4*a*c^3*x^3 - b*Sqrt[1 + c^
2*x^2] + 4*b*(1 + c^2*x^2)^(3/2)*Log[1 + E^(-2*ArcSinh[c*x])]) - 2*a*b*Sqrt
[1 + c^2*x^2]*Log[1 + c^2*x^2] - 2*a*b*c^2*x^2*Sqrt[1 + c^2*x^2]*Log[1 + c^
2*x^2] + 2*b^2*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(-2*ArcSinh[c*x])])/(3*c*P
i^(5/2)*(1 + c^2*x^2)^(3/2))
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\pi+\pi c^2x^2}\left(b^2\operatorname{arsinh}(cx)^2 + 2\,ab\operatorname{arsinh}(cx) + a^2\right)}{\pi^3c^6x^6 + 3\,\pi^3c^4x^4 + 3\,\pi^3c^2x^2 + \pi^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a
^2)/(pi^3*c^6*x^6 + 3*pi^3*c^4*x^4 + 3*pi^3*c^2*x^2 + pi^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2}{(\pi + \pi c^2 x^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/(pi + pi*c^2*x^2)^(5/2), x)
```

maple [B] time = 0.36, size = 1730, normalized size = 8.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(Pi*c^2*x^2+Pi)^(5/2),x)
[Out] -2/3*b^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/Pi^(5/2)+4/3*b^2/c/Pi^(5/2
)*arcsinh(c*x)^2+1/3*a^2/Pi*x/(Pi*c^2*x^2+Pi)^(3/2)+2/3*a^2/Pi^2*x/(Pi*c^2*
x^2+Pi)^(1/2)-22/3*b^2/Pi^(5/2)*c/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)^
2*x^2+2/3*b^2/Pi^(5/2)*c^7/(3*c^2*x^2+4)/(c^2*x^2+1)^2*x^8+10/3*b^2/Pi^(5/2
)*c^5/(3*c^2*x^2+4)/(c^2*x^2+1)^2*x^6+6*b^2/Pi^(5/2)*c^3/(3*c^2*x^2+4)/(c^2
*x^2+1)^2*x^4+14/3*b^2/Pi^(5/2)*c/(3*c^2*x^2+4)/(c^2*x^2+1)^2*x^2-8/3*b^2/P
i^(5/2)/c/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)^2+4/3*b^2/Pi^(5/2)/c/(3*
c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)-2/3*b^2/Pi^(5/2)*c^5/(3*c^2*x^2+4)/(c
^2*x^2+1)*x^6-b^2/Pi^(5/2)*c^4/(3*c^2*x^2+4)/(c^2*x^2+1)^(3/2)*x^5+4*b^2/Pi
^(5/2)/(3*c^2*x^2+4)/(c^2*x^2+1)^(3/2)*arcsinh(c*x)^2*x-b^2/Pi^(5/2)*c/(3*c
^2*x^2+4)/(c^2*x^2+1)*x^2-5/3*b^2/Pi^(5/2)*c^3/(3*c^2*x^2+4)/(c^2*x^2+1)*x^
4-7/3*b^2/Pi^(5/2)*c^2/(3*c^2*x^2+4)/(c^2*x^2+1)^(3/2)*x^3+4/3*a*b/Pi^(5/2)
/c/(3*c^2*x^2+4)/(c^2*x^2+1)^2+4*a*b/Pi^(5/2)*c^4/(3*c^2*x^2+4)/(c^2*x^2+1)
^(3/2)*arcsinh(c*x)*x^5+34/3*a*b/Pi^(5/2)*c^2/(3*c^2*x^2+4)/(c^2*x^2+1)^(3/
2)*arcsinh(c*x)*x^3-40/3*a*b/Pi^(5/2)*c^3/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsi
nh(c*x)*x^4-44/3*a*b/Pi^(5/2)*c/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)*x^
2-4*a*b/Pi^(5/2)*c^5/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)*x^6+16/3*a*b/
Pi^(5/2)*c/(3*c^2*x^2+4)/(c^2*x^2+1)^2*x^2-16/3*a*b/Pi^(5/2)/c/(3*c^2*x^2+4
)/(c^2*x^2+1)^2*arcsinh(c*x)-4/3*b^2/Pi^(5/2)/(3*c^2*x^2+4)/(c^2*x^2+1)^(3/
2)*x-4*a*b/Pi^(5/2)*c^3/(3*c^2*x^2+4)/(c^2*x^2+1)*x^4+16/3*b^2/Pi^(5/2)*c^5
/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)*x^6-20/3*b^2/Pi^(5/2)*c^3/(3*c^2*
x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)^2*x^4+4/3*b^2/Pi^(5/2)*c^7/(3*c^2*x^2+4)/
(c^2*x^2+1)^2*arcsinh(c*x)*x^8-2*b^2/Pi^(5/2)*c^5/(3*c^2*x^2+4)/(c^2*x^2+1)
^2*arcsinh(c*x)^2*x^6+16/3*b^2/Pi^(5/2)*c/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsi
nh(c*x)*x^2+8*b^2/Pi^(5/2)*c^3/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)*x^4
+2*b^2/Pi^(5/2)*c^4/(3*c^2*x^2+4)/(c^2*x^2+1)^(3/2)*arcsinh(c*x)^2*x^5-4*b^
2/Pi^(5/2)*c^3/(3*c^2*x^2+4)/(c^2*x^2+1)*arcsinh(c*x)*x^4+17/3*b^2/Pi^(5/2)
*c^2/(3*c^2*x^2+4)/(c^2*x^2+1)^(3/2)*arcsinh(c*x)^2*x^3-3*b^2/Pi^(5/2)*c/(3
*c^2*x^2+4)/(c^2*x^2+1)*arcsinh(c*x)*x^2-4/3*b^2/Pi^(5/2)*c^5/(3*c^2*x^2+4)
/(c^2*x^2+1)*arcsinh(c*x)*x^6+4/3*b^2/Pi^(5/2)/c/(3*c^2*x^2+4)/(c^2*x^2+1)^
2-4/3*b^2/c/Pi^(5/2)*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+8/3*a*b/c
/Pi^(5/2)*arcsinh(c*x)-4/3*a*b/c/Pi^(5/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+8
*a*b/Pi^(5/2)/(3*c^2*x^2+4)/(c^2*x^2+1)^(3/2)*arcsinh(c*x)*x-3*a*b/Pi^(5/2)
*c/(3*c^2*x^2+4)/(c^2*x^2+1)*x^2+8*a*b/Pi^(5/2)*c^3/(3*c^2*x^2+4)/(c^2*x^2+
1)^2*x^4+4/3*a*b/Pi^(5/2)*c^7/(3*c^2*x^2+4)/(c^2*x^2+1)^2*x^8+16/3*a*b/Pi^(
5/2)*c^5/(3*c^2*x^2+4)/(c^2*x^2+1)^2*x^6-4/3*a*b/Pi^(5/2)*c^5/(3*c^2*x^2+4)
/(c^2*x^2+1)*x^6
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\,abc\left(\frac{1}{\pi^{\frac{5}{2}}c^4x^2+\pi^{\frac{5}{2}}c^2}-\frac{2\,\log\left(c^2x^2+1\right)}{\pi^{\frac{5}{2}}c^2}\right)+\frac{2}{3}\,ab\left(\frac{x}{\pi\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}}+\frac{2\,x}{\pi^2\sqrt{\pi+\pi c^2x^2}}\right)\operatorname{arsinh}\left(cx\right)+\frac{1}{3}\,a^2\left(\frac{}{\pi\left(\pi+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*b*c*(1/(pi^(5/2)*c^4*x^2 + pi^(5/2)*c^2) - 2*log(c^2*x^2 + 1)/(pi^(5/
2)*c^2)) + 2/3*a*b*(x/(pi*(pi + pi*c^2*x^2)^(3/2)) + 2*x/(pi^2*sqrt(pi + pi
*c^2*x^2)))*arcsinh(c*x) + 1/3*a^2*(x/(pi*(pi + pi*c^2*x^2)^(3/2)) + 2*x/(p
i^2*sqrt(pi + pi*c^2*x^2))) + b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/
(pi + pi*c^2*x^2)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2}{\left(\Pi\,c^2\,x^2+\Pi\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(Pi + Pi*c^2*x^2)^(5/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(Pi + Pi*c^2*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{b^2\operatorname{asinh}^2\left(cx\right)}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{2ab\operatorname{asinh}\left(cx\right)}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2}}}{\pi^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] (Integral(a**2/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2
+ 1) + sqrt(c**2*x**2 + 1)), x) + Integral(b**2*asinh(c*x)**2/(c**4*x**4*sq
rt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)),
 x) + Integral(2*a*b*asinh(c*x)/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**
2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x))/pi**(5/2)
```

3.258 $\int x^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=358

$$\frac{x^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{15c^2}-\frac{2bcx^5\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{25\sqrt{c^2x^2+1}}+\frac{1}{5}x^4\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{2}{}$$

```
[Out] -52/225*b^2*(c^2*d*x^2+d)^(1/2)/c^4-26/675*b^2*(c^2*x^2+1)*(c^2*d*x^2+d)^(1
/2)/c^4+2/125*b^2*(c^2*x^2+1)^2*(c^2*d*x^2+d)^(1/2)/c^4-2/15*(a+b*arcsinh(c
*x))^2*(c^2*d*x^2+d)^(1/2)/c^4+1/15*x^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^
(1/2)/c^2+1/5*x^4*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)+4/15*a*b*x*(c^2*
d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)^(1/2)+4/15*b^2*x*arcsinh(c*x)*(c^2*d*x^2+d)^
(1/2)/c^3/(c^2*x^2+1)^(1/2)-2/45*b*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/
2)/c/(c^2*x^2+1)^(1/2)-2/25*b*c*x^5*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/
(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5742, 5758, 5717, 5653, 261, 5661, 266, 43}

$$\frac{4abx\sqrt{c^2dx^2+d}}{15c^3\sqrt{c^2x^2+1}}-\frac{2bcx^5\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{25\sqrt{c^2x^2+1}}+\frac{1}{5}x^4\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{2bx^3\sqrt{c^2dx^2+d}\,(}{45c\sqrt{c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-52*b^2*Sqrt[d + c^2*d*x^2])/(225*c^4) + (4*a*b*x*Sqrt[d + c^2*d*x^2])/(15
*c^3*Sqrt[1 + c^2*x^2]) - (26*b^2*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2])/(675*c
^4) + (2*b^2*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2])/(125*c^4) + (4*b^2*x*Sqrt
[d + c^2*d*x^2]*ArcSinh[c*x])/(15*c^3*Sqrt[1 + c^2*x^2]) - (2*b*x^3*Sqrt[d
+ c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(45*c*Sqrt[1 + c^2*x^2]) - (2*b*c*x^5*Sq
rt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(25*Sqrt[1 + c^2*x^2]) - (2*Sqrt[d
+ c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(15*c^4) + (x^2*Sqrt[d + c^2*d*x^2]*(a
 + b*ArcSinh[c*x])^2)/(15*c^2) + (x^4*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x])^2)/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int x^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2dx = \frac{1}{5}x^4\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2 + \frac{\sqrt{d+c^2dx^2}\int\frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}}{5\sqrt{1+c^2x^2}}$$

$$= -\frac{2bcx^5\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{25\sqrt{1+c^2x^2}} + \frac{x^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(\right.}{15c^2}$$

$$= -\frac{2bx^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{45c\sqrt{1+c^2x^2}} - \frac{2bcx^5\sqrt{d+c^2dx^2}\left(a+b\sinh\right.}{25\sqrt{1+c^2x^2}}$$

$$= \frac{4abx\sqrt{d+c^2dx^2}}{15c^3\sqrt{1+c^2x^2}} - \frac{2bx^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{45c\sqrt{1+c^2x^2}} - 2bcx^5\sqrt{d+}$$

$$= \frac{2b^2\sqrt{d+c^2dx^2}}{25c^4} + \frac{4abx\sqrt{d+c^2dx^2}}{15c^3\sqrt{1+c^2x^2}} - \frac{4b^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}{75c^4} + 2$$

$$= -\frac{52b^2\sqrt{d+c^2dx^2}}{225c^4} + \frac{4abx\sqrt{d+c^2dx^2}}{15c^3\sqrt{1+c^2x^2}} - \frac{26b^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}{675c^4}$$

Mathematica [A] time = 0.27, size = 222, normalized size = 0.62

$$\sqrt{c^2dx^2+d}\left(-30abcx\sqrt{c^2x^2+1}\left(9c^4x^4+5c^2x^2-30\right)-30b\sinh^{-1}(cx)\left(bcx\sqrt{c^2x^2+1}\left(9c^4x^4+5c^2x^2-30\right)-15\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(225*(-2 + 3*c^2*x^2)*(a + a*c^2*x^2)^2 - 30*a*b*c*x*S
qrt[1 + c^2*x^2]*(-30 + 5*c^2*x^2 + 9*c^4*x^4) + 2*b^2*(-428 - 439*c^2*x^2
+ 16*c^4*x^4 + 27*c^6*x^6) - 30*b*(-15*a*(1 + c^2*x^2)^2*(-2 + 3*c^2*x^2) +
 b*c*x*Sqrt[1 + c^2*x^2]*(-30 + 5*c^2*x^2 + 9*c^4*x^4))*ArcSinh[c*x] + 225*
(-2 + 3*c^2*x^2)*(b + b*c^2*x^2)^2*ArcSinh[c*x]^2))/(3375*c^4*(1 + c^2*x^2)
)
```

fricas [A] time = 0.73, size = 316, normalized size = 0.88

$$225\left(3b^2c^6x^6+4b^2c^4x^4-b^2c^2x^2-2b^2\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)^2+30\left(45abc^6x^6+60abc^4x^4-15abc\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/3375*(225*(3*b^2*c^6*x^6 + 4*b^2*c^4*x^4 - b^2*c^2*x^2 - 2*b^2)*sqrt(c^2*
d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 30*(45*a*b*c^6*x^6 + 60*a*b*c^4
*x^4 - 15*a*b*c^2*x^2 - 30*a*b - (9*b^2*c^5*x^5 + 5*b^2*c^3*x^3 - 30*b^2*c*
x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + (2
7*(25*a^2 + 2*b^2)*c^6*x^6 + 4*(225*a^2 + 8*b^2)*c^4*x^4 - (225*a^2 + 878*b
^2)*c^2*x^2 - 450*a^2 - 856*b^2 - 30*(9*a*b*c^5*x^5 + 5*a*b*c^3*x^3 - 30*a*
b*c*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d))/(c^6*x^2 + c^4)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [**B**] time = 0.41, size = 1162, normalized size = 3.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a^2*(1/5*x^2*(c^2*d*x^2+d)^(3/2)/c^2/d-2/15/d/c^4*(c^2*d*x^2+d)^(3/2))+b^2*
(1/4000*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c
^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(
25*arcsinh(c*x)^2-10*arcsinh(c*x)+2)/c^4/(c^2*x^2+1)-1/864*(d*(c^2*x^2+1))^
(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1
/2)+1)*(9*arcsinh(c*x)^2-6*arcsinh(c*x)+2)/c^4/(c^2*x^2+1)-1/16*(d*(c^2*x^2
+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2-2*arcsinh(c*x)
+2)/c^4/(c^2*x^2+1)-1/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/
2)+1)*(arcsinh(c*x)^2+2*arcsinh(c*x)+2)/c^4/(c^2*x^2+1)-1/864*(d*(c^2*x^2+1
))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)
^(1/2)+1)*(9*arcsinh(c*x)^2+6*arcsinh(c*x)+2)/c^4/(c^2*x^2+1)+1/4000*(d*(c^
2*x^2+1))^(1/2)*(16*c^6*x^6-16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c^3*
x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2-5*c*x*(c^2*x^2+1)^(1/2)+1)*(25*arcsinh(c*x
)^2+10*arcsinh(c*x)+2)/c^4/(c^2*x^2+1))+2*a*b*(1/800*(d*(c^2*x^2+1))^(1/2)*
(16*c^6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^
(1/2)+13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+5*arcsinh(c*x))/c^4/(c^2*x^
2+1)-1/288*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c
^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arcsinh(c*x))/c^4/(c^2*x^2+1)-1/16*
(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))/c
^4/(c^2*x^2+1)-1/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)
*(1+arcsinh(c*x))/c^4/(c^2*x^2+1)-1/288*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*
c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(1+3*arcsinh
(c*x))/c^4/(c^2*x^2+1)+1/800*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6-16*c^5*x^5*(
c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2-5*c*x*(
c^2*x^2+1)^(1/2)+1)*(1+5*arcsinh(c*x))/c^4/(c^2*x^2+1))
```

maxima [**A**] time = 0.46, size = 302, normalized size = 0.84

$$\frac{1}{15}\,b^2\left(\frac{3\left(c^2dx^2+d\right)^{\frac{3}{2}}x^2}{c^2d}-\frac{2\left(c^2dx^2+d\right)^{\frac{3}{2}}}{c^4d}\right)\operatorname{arsinh}\left(cx\right)^2+\frac{2}{15}\,ab\left(\frac{3\left(c^2dx^2+d\right)^{\frac{3}{2}}x^2}{c^2d}-\frac{2\left(c^2dx^2+d\right)^{\frac{3}{2}}}{c^4d}\right)\operatorname{arsinh}\left(cx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/15*b^2*(3*(c^2*d*x^2 + d)^(3/2)*x^2/(c^2*d) - 2*(c^2*d*x^2 + d)^(3/2)/(c^
4*d))*arcsinh(c*x)^2 + 2/15*a*b*(3*(c^2*d*x^2 + d)^(3/2)*x^2/(c^2*d) - 2*(c
^2*d*x^2 + d)^(3/2)/(c^4*d))*arcsinh(c*x) + 1/15*a^2*(3*(c^2*d*x^2 + d)^(3/
2)*x^2/(c^2*d) - 2*(c^2*d*x^2 + d)^(3/2)/(c^4*d)) + 2/3375*b^2*((27*sqrt(c^
```

```
2*x^2 + 1)*c^2*sqrt(d)*x^4 - 11*sqrt(c^2*x^2 + 1)*sqrt(d)*x^2 - 428*sqrt(c^
2*x^2 + 1)*sqrt(d)/c^2)/c^2 - 15*(9*c^4*sqrt(d)*x^5 + 5*c^2*sqrt(d)*x^3 - 3
0*sqrt(d)*x)*arcsinh(c*x)/c^3) - 2/225*(9*c^4*sqrt(d)*x^5 + 5*c^2*sqrt(d)*x
^3 - 30*sqrt(d)*x)*a*b/c^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 (a + b \operatorname{asinh}(c x))^2 \sqrt{d c^2 x^2 + d} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sqrt{d \left(c^2 x^2 + 1\right)} (a + b \operatorname{asinh}(cx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))**2*(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2, x)
```

3.259 $\int x^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=291

$$-\frac{bx^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{8c\sqrt{c^2x^2+1}}+\frac{x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{8c^2}-\frac{bcx^4\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{8\sqrt{c^2x^2+1}}+$$

```
[Out] 1/64*b^2*x*(c^2*d*x^2+d)^(1/2)/c^2+1/32*b^2*x^3*(c^2*d*x^2+d)^(1/2)+1/8*x*(
a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^2+1/4*x^3*(a+b*arcsinh(c*x))^2*(c
^2*d*x^2+d)^(1/2)-1/64*b^2*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)
^(1/2)-1/8*b*x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)
-1/8*b*c*x^4*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/24*
(a+b*arcsinh(c*x))^3*(c^2*d*x^2+d)^(1/2)/b/c^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5742, 5758, 5675, 5661, 321, 215}

$$-\frac{bcx^4\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{8\sqrt{c^2x^2+1}}+\frac{1}{4}x^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{bx^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{8c\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (b^2*x*Sqrt[d + c^2*d*x^2])/(64*c^2) + (b^2*x^3*Sqrt[d + c^2*d*x^2])/32 - (
b^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/(64*c^3*Sqrt[1 + c^2*x^2]) - (b*x^2*S
qrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(8*c*Sqrt[1 + c^2*x^2]) - (b*c*x^4
*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(8*Sqrt[1 + c^2*x^2]) + (x*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(8*c^2) + (x^3*Sqrt[d + c^2*d*x^2]*(
a + b*ArcSinh[c*x])^2)/4 - (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^3)/(24
*b*c^3*Sqrt[1 + c^2*x^2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int x^2\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{1}{4}x^3\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)^2+\frac{\sqrt{d+c^2dx^2}\,\int\frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}}{4\sqrt{1+c^2x^2}}$$

$$=-\frac{bcx^4\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)}{8\sqrt{1+c^2x^2}}+\frac{x\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx\right.}{8c^2}$$

$$=\frac{1}{32}b^2x^3\sqrt{d+c^2dx^2}-\frac{bx^2\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)}{8c\sqrt{1+c^2x^2}}-\frac{bcx^4\sqrt{d+}}{}$$

$$=\frac{b^2x\sqrt{d+c^2dx^2}}{64c^2}+\frac{1}{32}b^2x^3\sqrt{d+c^2dx^2}-\frac{bx^2\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}\right.}{8c\sqrt{1+c^2x^2}}$$

$$=\frac{b^2x\sqrt{d+c^2dx^2}}{64c^2}+\frac{1}{32}b^2x^3\sqrt{d+c^2dx^2}-\frac{b^2\sqrt{d+c^2dx^2}\,\sinh^{-1}(cx)}{64c^3\sqrt{1+c^2x^2}}-$$

Mathematica [A] time = 1.32, size = 207, normalized size = 0.71

$$-\frac{-96a^2cx\left(2c^2x^2+1\right)\sqrt{c^2dx^2+d}+96a^2\sqrt{d}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)+\frac{12ab\sqrt{c^2dx^2+d}\left(8\sinh^{-1}(cx)^2-4\sinh\left(4\sinh^{-1}(c\right.\right.}{\sqrt{c^2x^2+1}}}{768}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -1/768*(-96*a^2*c*x*(1 + 2*c^2*x^2)*Sqrt[d + c^2*d*x^2] + 96*a^2*Sqrt[d]*Lo
g[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (12*a*b*Sqrt[d + c^2*d*x^2]*(8*Arc
Sinh[c*x]^2 + Cosh[4*ArcSinh[c*x]] - 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]))/
Sqrt[1 + c^2*x^2] + (b^2*Sqrt[d + c^2*d*x^2]*(32*ArcSinh[c*x]^3 + 12*ArcSin
h[c*x]*Cosh[4*ArcSinh[c*x]] - 3*(1 + 8*ArcSinh[c*x]^2)*Sinh[4*ArcSinh[c*x]]
))/Sqrt[1 + c^2*x^2])/c^3
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2x^2\operatorname{arsinh}(cx)^2+2\,abx^2\operatorname{arsinh}(cx)+a^2x^2\right)\sqrt{c^2dx^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*x) + a^2*x^2)*sqrt(c
^2*d*x^2 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c^2dx^2+d}\,(b\operatorname{arsinh}(cx)+a)^2x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^2*x^2, x)
```

maple [B] time = 0.35, size = 701, normalized size = 2.41

$$\frac{a^2x\left(c^2d\,x^2+d\right)^{\frac{3}{2}}}{4c^2d}-\frac{a^2x\sqrt{c^2d\,x^2+d}}{8c^2}-\frac{a^2d\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{8c^2\sqrt{c^2d}}-\frac{b^2\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsinh}(cx)^3}{24\sqrt{c^2x^2+1}\,c^3}+\frac{b^2\sqrt{d\left(c^2\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] 1/4*a^2*x*(c^2*d*x^2+d)^(3/2)/c^2/d-1/8*a^2/c^2*x*(c^2*d*x^2+d)^(1/2)-1/8*a
^2/c^2*d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)-1/24*b
^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3*arcsinh(c*x)^3+1/4*b^2*(d*(c
^2*x^2+1))^(1/2)*c^2/(c^2*x^2+1)*arcsinh(c*x)^2*x^5+3/8*b^2*(d*(c^2*x^2+1))
^(1/2)/(c^2*x^2+1)*arcsinh(c*x)^2*x^3+1/8*b^2*(d*(c^2*x^2+1))^(1/2)/c^2/(c^
2*x^2+1)*arcsinh(c*x)^2*x-1/64*b^2*(d*(c^2*x^2+1))^(1/2)/c^3/(c^2*x^2+1)^(1
/2)*arcsinh(c*x)-1/8*b^2*(d*(c^2*x^2+1))^(1/2)/c/(c^2*x^2+1)^(1/2)*arcsinh(
c*x)*x^2+1/32*b^2*(d*(c^2*x^2+1))^(1/2)*c^2/(c^2*x^2+1)*x^5+3/64*b^2*(d*(c^
2*x^2+1))^(1/2)/(c^2*x^2+1)*x^3+1/64*b^2*(d*(c^2*x^2+1))^(1/2)/c^2/(c^2*x^2
+1)*x-1/8*b^2*(d*(c^2*x^2+1))^(1/2)*c/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^4-1/
8*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3*arcsinh(c*x)^2+1/2*a*b*(d
*(c^2*x^2+1))^(1/2)*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^5-1/8*a*b*(d*(c^2*x^2+1)
)^(1/2)*c/(c^2*x^2+1)^(1/2)*x^4+3/4*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)*a
rcsinh(c*x)*x^3-1/8*a*b*(d*(c^2*x^2+1))^(1/2)/c/(c^2*x^2+1)^(1/2)*x^2+1/4*a
*b*(d*(c^2*x^2+1))^(1/2)/c^2/(c^2*x^2+1)*arcsinh(c*x)*x-1/64*a*b*(d*(c^2*x^
2+1))^(1/2)/c^3/(c^2*x^2+1)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,\sqrt{d\,c^2\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2),x)
[Out] int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{d\left(c^2x^2+1\right)}\,\left(a+b\operatorname{asinh}\left(cx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))**2*(c**2*d*x**2+d)**(1/2),x)
[Out] Integral(x**2*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2, x)
```

3.260 $\int x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=180

$$-\frac{2bx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{3c\sqrt{c^2x^2+1}}+\frac{\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d}-\frac{2bcx^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{9\sqrt{c^2x^2+1}}$$

```
[Out] 1/3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/c^2/d+4/9*b^2*(c^2*d*x^2+d)^(1
/2)/c^2+2/27*b^2*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)/c^2-2/3*b*x*(a+b*arcsinh(c
*x))*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-2/9*b*c*x^3*(a+b*arcsinh(c*x))
*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5717, 5679, 444, 43}

$$-\frac{2bcx^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{9\sqrt{c^2x^2+1}}-\frac{2bx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{3c\sqrt{c^2x^2+1}}+\frac{\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (4*b^2*Sqrt[d + c^2*d*x^2])/(9*c^2) + (2*b^2*(1 + c^2*x^2)*Sqrt[d + c^2*d*x
^2])/(27*c^2) - (2*b*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*c*Sqrt[
1 + c^2*x^2]) - (2*b*c*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(9*Sqr
t[1 + c^2*x^2]) + ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/(3*c^2*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 5679

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2dx=\frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d}-\frac{\left(2b\sqrt{d+c^2dx^2}\right)\int\left(1+c^2x^2\right)\left(a+\right.}{3c\sqrt{1+c^2x^2}}$$

$$=-\frac{2bx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3c\sqrt{1+c^2x^2}}-\frac{2bcx^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}\right.}{9\sqrt{1+c^2x^2}}$$

$$=-\frac{2bx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3c\sqrt{1+c^2x^2}}-\frac{2bcx^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}\right.}{9\sqrt{1+c^2x^2}}$$

$$=-\frac{2bx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3c\sqrt{1+c^2x^2}}-\frac{2bcx^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}\right.}{9\sqrt{1+c^2x^2}}$$

$$=\frac{4b^2\sqrt{d+c^2dx^2}}{9c^2}+\frac{2b^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}{27c^2}-\frac{2bx\sqrt{d+c^2dx^2}\left(a+b\right.}{3c\sqrt{1+c^2x^2}}$$

Mathematica [A] time = 0.27, size = 166, normalized size = 0.92

$$\frac{\sqrt{c^2dx^2+d}\left(-6abcx\sqrt{c^2x^2+1}\left(c^2x^2+3\right)+6b\sinh^{-1}(cx)\left(3a\left(c^2x^2+1\right)^2-bcx\sqrt{c^2x^2+1}\left(c^2x^2+3\right)\right)+9\left(ac^2x^2\right.\right.}{27c^2\left(c^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(-6*a*b*c*x*Sqrt[1 + c^2*x^2]*(3 + c^2*x^2) + 9*(a + a
*c^2*x^2)^2 + 2*b^2*(7 + 8*c^2*x^2 + c^4*x^4) + 6*b*(3*a*(1 + c^2*x^2)^2 -
b*c*x*Sqrt[1 + c^2*x^2]*(3 + c^2*x^2))*ArcSinh[c*x] + 9*(b + b*c^2*x^2)^2*A
rcSinh[c*x]^2))/(27*c^2*(1 + c^2*x^2))
```

fricas [A] time = 0.67, size = 249, normalized size = 1.38

$$\frac{9\left(b^2c^4x^4+2\,b^2c^2x^2+b^2\right)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2+6\left(3\,abc^4x^4+6\,abc^2x^2+3\,ab-\left(b^2c^3x^3+3\,b^2cx\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/27*(9*(b^2*c^4*x^4 + 2*b^2*c^2*x^2 + b^2)*sqrt(c^2*d*x^2 + d)*log(c*x + s
qrt(c^2*x^2 + 1))^2 + 6*(3*a*b*c^4*x^4 + 6*a*b*c^2*x^2 + 3*a*b - (b^2*c^3*x
^3 + 3*b^2*c*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x
^2 + 1)) + ((9*a^2 + 2*b^2)*c^4*x^4 + 2*(9*a^2 + 8*b^2)*c^2*x^2 + 9*a^2 + 1
4*b^2 - 6*(a*b*c^3*x^3 + 3*a*b*c*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d))
/(c^4*x^2 + c^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.23, size = 657, normalized size = 3.65

$$\frac{a^2\left(c^2d\,x^2+d\right)^{\frac{3}{2}}}{3c^2d}+b^2\left(\frac{\sqrt{d\left(c^2x^2+1\right)}\,\left(4c^4x^4+4c^3x^3\sqrt{c^2x^2+1}\,+5c^2x^2+3cx\sqrt{c^2x^2+1}\,+1\right)\left(9\,\mathrm{arcsinh}\left(cx\right)^2\right.}{216c^2\left(c^2x^2+1\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] 1/3*a^2/c^2/d*(c^2*d*x^2+d)^(3/2)+b^2*(1/216*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x
^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arcs
inh(c*x)^2-6*arcsinh(c*x)+2)/c^2/(c^2*x^2+1)+1/8*(d*(c^2*x^2+1))^(1/2)*(c^2
*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2-2*arcsinh(c*x)+2)/c^2/(c^2*x^
2+1)+1/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c
*x)^2+2*arcsinh(c*x)+2)/c^2/(c^2*x^2+1)+1/216*(d*(c^2*x^2+1))^(1/2)*(4*c^4*
x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arc
sinh(c*x)^2+6*arcsinh(c*x)+2)/c^2/(c^2*x^2+1))+2*a*b*(1/72*(d*(c^2*x^2+1))^
(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1
/2)+1)*(-1+3*arcsinh(c*x))/c^2/(c^2*x^2+1)+1/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x
^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))/c^2/(c^2*x^2+1)+1/8*(d*(c^2*x
^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(1+arcsinh(c*x))/c^2/(c^2*x^
2+1)+1/72*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^
2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(1+3*arcsinh(c*x))/c^2/(c^2*x^2+1))
```

maxima [A] time = 0.43, size = 183, normalized size = 1.02

$$\frac{2}{27}\,b^2\left(\frac{\sqrt{c^2x^2+1}\,d^{\frac{3}{2}}x^2+\frac{7\,\sqrt{c^2x^2+1}\,d^{\frac{3}{2}}}{c^2}}{d}-\frac{3\left(c^2d^{\frac{3}{2}}x^3+3\,d^{\frac{3}{2}}x\right)\mathrm{arsinh}\left(cx\right)}{cd}\right)+\frac{\left(c^2dx^2+d\right)^{\frac{3}{2}}b^2\,\mathrm{arsinh}\left(cx\right)^2}{3\,c^2d}+\frac{2\left(c^2dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/27*b^2*((sqrt(c^2*x^2 + 1)*d^(3/2)*x^2 + 7*sqrt(c^2*x^2 + 1)*d^(3/2)/c^2)
/d - 3*(c^2*d^(3/2)*x^3 + 3*d^(3/2)*x)*arcsinh(c*x)/(c*d)) + 1/3*(c^2*d*x^2
 + d)^(3/2)*b^2*arcsinh(c*x)^2/(c^2*d) + 2/3*(c^2*d*x^2 + d)^(3/2)*a*b*arcs
inh(c*x)/(c^2*d) - 2/9*(c^2*d^(3/2)*x^3 + 3*d^(3/2)*x)*a*b/(c*d) + 1/3*(c^2
*d*x^2 + d)^(3/2)*a^2/(c^2*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,\sqrt{d\,c^2\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{d\left(c^2x^2+1\right)}\,\left(a+b\,\mathrm{asinh}\left(cx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))**2*(c**2*d*x**2+d)**(1/2),x)
[Out] Integral(x*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2, x)
```

3.261 $\int \sqrt{d+c^2dx^2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=184

$$\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^3}{6bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{bcx^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{c^2x^2+1}}+\frac{1}{4}b$$

```
[Out] 1/4*b^2*x*(c^2*d*x^2+d)^(1/2)+1/2*x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2
)-1/4*b^2*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-1/2*b*c*x^2*
(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+1/6*(a+b*arcsinh(c
*x))^3*(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5682, 5675, 5661, 321, 215}

$$\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^3}{6bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{bcx^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{c^2x^2+1}}+\frac{1}{4}b$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (b^2*x*Sqrt[d + c^2*d*x^2])/4 - (b^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/(4*c
*Sqrt[1 + c^2*x^2]) - (b*c*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(2
*Sqrt[1 + c^2*x^2]) + (x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/2 + (S
qrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^3)/(6*b*c*Sqrt[1 + c^2*x^2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
```

```
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rubi steps

$$\int \sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2 dx = \frac{1}{2}x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{\sqrt{d+c^2dx^2}\int\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}dx}{2\sqrt{1+c^2x^2}}-$$

$$=-\frac{bcx^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{1+c^2x^2}}+\frac{1}{2}x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx\right.$$

$$=\frac{1}{4}b^2x\sqrt{d+c^2dx^2}-\frac{bcx^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{1+c^2x^2}}+\frac{1}{2}x\sqrt{d+c^2dx^2}$$

$$=\frac{1}{4}b^2x\sqrt{d+c^2dx^2}-\frac{b^2\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{4c\sqrt{1+c^2x^2}}-\frac{bcx^2\sqrt{d+c^2dx^2}\left(a+b\,s\right.}{2\sqrt{1+c^2x^2}}$$

Mathematica [A] time = 1.07, size = 200, normalized size = 1.09

$$\frac{1}{24}\left(12a^2x\sqrt{c^2dx^2+d}+\frac{12a^2\sqrt{d}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)}{c}+\frac{6ab\sqrt{c^2dx^2+d}\left(2\sinh^{-1}(cx)\left(\sinh^{-1}(cx)+\sinh\right.\right.}{c\sqrt{c^2x^2+}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (12*a^2*x*Sqrt[d + c^2*d*x^2] + (12*a^2*Sqrt[d]*Log[c*d*x + Sqrt[d]*Sqrt[d
+ c^2*d*x^2]])/c + (b^2*Sqrt[d + c^2*d*x^2]*(4*ArcSinh[c*x]^3 - 6*ArcSinh[c
*x]*Cosh[2*ArcSinh[c*x]] + (3 + 6*ArcSinh[c*x]^2)*Sinh[2*ArcSinh[c*x]]))/(c
*Sqrt[1 + c^2*x^2]) + (6*a*b*Sqrt[d + c^2*d*x^2]*(-Cosh[2*ArcSinh[c*x]] + 2
*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])))/(c*Sqrt[1 + c^2*x^2])
)/24
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{c^2dx^2+d}\left(b^2\operatorname{arsinh}(cx)^2+2\,ab\operatorname{arsinh}(cx)+a^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.21, size = 482, normalized size = 2.62

$$\frac{x a^2\sqrt{c^2 d x^2+d}}{2}+\frac{a^2 d \ln\left(\frac{x c^2 d}{\sqrt{c^2 d}}+\sqrt{c^2 d x^2+d}\right)}{2\sqrt{c^2 d}}+\frac{b^2\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsinh}(cx)^3}{6\sqrt{c^2x^2+1}\,c}+\frac{b^2\sqrt{d\left(c^2x^2+1\right)}\,c^2\operatorname{arcsinh}}{2c^2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] 1/2*x*a^2*(c^2*d*x^2+d)^(1/2)+1/2*a^2*d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2
+d)^(1/2))/(c^2*d)^(1/2)+1/6*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*
arcsinh(c*x)^3+1/2*b^2*(d*(c^2*x^2+1))^(1/2)*c^2/(c^2*x^2+1)*arcsinh(c*x)^2
*x^3+1/4*b^2*(d*(c^2*x^2+1))^(1/2)*c^2/(c^2*x^2+1)*x^3+1/4*b^2*(d*(c^2*x^2+
1))^(1/2)/(c^2*x^2+1)*x-1/2*b^2*(d*(c^2*x^2+1))^(1/2)*c/(c^2*x^2+1)^(1/2)*a
rcsinh(c*x)*x^2+1/2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)*arcsinh(c*x)^2*x-
1/4*b^2*(d*(c^2*x^2+1))^(1/2)/c/(c^2*x^2+1)^(1/2)*arcsinh(c*x)+1/2*a*b*(d*(
c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsinh(c*x)^2+a*b*(d*(c^2*x^2+1))^(1
/2)*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^3-1/2*a*b*(d*(c^2*x^2+1))^(1/2)*c/(c^2*x
^2+1)^(1/2)*x^2+a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)*arcsinh(c*x)*x-1/4*a*
b*(d*(c^2*x^2+1))^(1/2)/c/(c^2*x^2+1)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\operatorname{asinh}(c\,x))^2\,\sqrt{d\,c^2\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\left(c^2x^2+1\right)}\,(a+b\operatorname{asinh}(cx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2, x)
```

3.262 $\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x}\,dx$

Optimal. Leaf size=338

$$-\frac{2b\sqrt{c^2dx^2+d}\,\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}+\frac{2b\sqrt{c^2dx^2+d}\,\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}-\frac{2abcx\sqrt{c^2}}{\sqrt{c^2x^2}}$$

```
[Out] 2*b^2*(c^2*d*x^2+d)^(1/2)+(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)-2*a*b*c*
x*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2*b^2*c*x*arcsinh(c*x)*(c^2*d*x^2+d
)^(1/2)/(c^2*x^2+1)^(1/2)-2*(a+b*arcsinh(c*x))^2*arctanh(c*x+(c^2*x^2+1)^(1
/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2*b*(a+b*arcsinh(c*x))*polylog(2
,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+2*b*(a+b*arc
sinh(c*x))*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)
^(1/2)+2*b^2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2
+1)^(1/2)-2*b^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x
^2+1)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5742, 5760, 4182, 2531, 2282, 6589, 5653, 261}

$$-\frac{2b\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}+\frac{2b\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(\right.}{\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] 2*b^2*Sqrt[d + c^2*d*x^2] - (2*a*b*c*x*Sqrt[d + c^2*d*x^2])/Sqrt[1 + c^2*x^
2] - (2*b^2*c*x*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] + Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2 - (2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSi
nh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (2*b*Sqrt[d + c^2*d
*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] +
 (2*b*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, E^ArcSinh[c*x]])/
Sqrt[1 + c^2*x^2] + (2*b^2*Sqrt[d + c^2*d*x^2]*PolyLog[3, -E^ArcSinh[c*x]])
/Sqrt[1 + c^2*x^2] - (2*b^2*Sqrt[d + c^2*d*x^2]*PolyLog[3, E^ArcSinh[c*x]])
/Sqrt[1 + c^2*x^2]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x}dx = \sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2 + \frac{\sqrt{d+c^2dx^2}\int\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\sqrt{1+c^2x^2}}dx}{\sqrt{1+c^2x^2}} - \frac{(2b$$

$$= -\frac{2abcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2 + \frac{\sqrt{d+c^2dx^2}\ \mathrm{Su}$$

$$= -\frac{2abcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{2b^2cx\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}} + \sqrt{d+c^2dx^2}\ (a+$$

$$= 2b^2\sqrt{d+c^2dx^2} - \frac{2abcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{2b^2cx\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}} + $$

$$= 2b^2\sqrt{d+c^2dx^2} - \frac{2abcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{2b^2cx\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}} + $$

$$= 2b^2\sqrt{d+c^2dx^2} - \frac{2abcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{2b^2cx\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}} + $$

Mathematica [A] time = 1.17, size = 352, normalized size = 1.04

$$a^2\sqrt{c^2dx^2+d} - a^2\sqrt{d}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+d\right) + a^2\sqrt{d}\log(cx) + \frac{2ab\sqrt{c^2dx^2+d}\left(\sqrt{c^2x^2+1}\sinh^{-1}(cx)+\mathrm{Li}_2\left(-e\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] a^2*Sqrt[d + c^2*d*x^2] + a^2*Sqrt[d]*Log[c*x] - a^2*Sqrt[d]*Log[d + Sqrt[d
]*Sqrt[d + c^2*d*x^2]] + (2*a*b*Sqrt[d + c^2*d*x^2]*(-(c*x) + Sqrt[1 + c^2*
x^2]*ArcSinh[c*x] + ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]*
Log[1 + E^(-ArcSinh[c*x])] + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^
(-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2] + (b^2*Sqrt[d + c^2*d*x^2]*(2*Sqrt[1 +
 c^2*x^2] - 2*c*x*ArcSinh[c*x] + Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2 + ArcSinh
[c*x]^2*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]^2*Log[1 + E^(-ArcSinh[c*x
])] + 2*ArcSinh[c*x]*PolyLog[2, -E^(-ArcSinh[c*x])] - 2*ArcSinh[c*x]*PolyLo
g[2, E^(-ArcSinh[c*x])] + 2*PolyLog[3, -E^(-ArcSinh[c*x])] - 2*PolyLog[3, E
^(-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2]
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(b^2\operatorname{arsinh}(cx)^2+2\,ab\operatorname{arsinh}(cx)+a^2\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/x, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

maple [B] time = 0.34, size = 823, normalized size = 2.43

$$-\sqrt{d}\ \ln\left(\frac{2d+2\sqrt{d}\ \sqrt{c^2d\,x^2+d}}{x}\right)a^2+a^2\sqrt{c^2d\,x^2+d}+\frac{b^2\sqrt{d\left(c^2x^2+1\right)}\ \operatorname{arcsinh}(cx)^2x^2c^2}{c^2x^2+1}-\frac{2b^2\sqrt{d\left(c^2x^2+1\right)}}{\sqrt{c^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x,x)

[Out] -d^(1/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)*a^2+a^2*(c^2*d*x^2+d)^(1/2)+b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)*arcsinh(c*x)^2*x^2*c^2-2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x*c+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)*c^2*x^2+b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)*arcsinh(c*x)^2+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)-b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))-2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(3,-c*x-(c^2*x^2+1)^(1/2))+b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))-2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(3,c*x+(c^2*x^2+1)^(1/2))+2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)*arcsinh(c*x)*x^2*c^2-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*c*x+2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)*arcsinh(c*x)+2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\left(\sqrt{d}\ \operatorname{arsinh}\left(\frac{1}{c|x|}\right)-\sqrt{c^2dx^2+d}\right)a^2+\int\frac{\sqrt{c^2dx^2+d}\,b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{x}+\frac{2\sqrt{c^2dx^2+d}\,ab\log\left(cx+\sqrt{\right.}\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="maxima")

[Out] -(sqrt(d)*arcsinh(1/(c*abs(x))) - sqrt(c^2*d*x^2 + d))*a^2 + integrate(sqrt(c^2*d*x^2 + d)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/x + 2*sqrt(c^2*d*x^2 + d)*a*b*log(c*x + sqrt(c^2*x^2 + 1))/x, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{(a+b\operatorname{asinh}(c\,x))^2\sqrt{d\,c^2\,x^2+d}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2))/x,x)

[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2))/x, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\left(c^2x^2+1\right)}\,(a+b\operatorname{asinh}(cx))^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*(c**2*d*x**2+d)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2/x, x)
```

3.263 $\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^2}\,dx$

Optimal. Leaf size=209

$$\frac{c\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^3}{3b\sqrt{c^2x^2+1}}+\frac{c\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{x}+\frac{2bc\sqrt{c^2}}{}$$

```
[Out] -(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x+c*(a+b*arcsinh(c*x))^2*(c^2*d*x
^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+1/3*c*(a+b*arcsinh(c*x))^3*(c^2*d*x^2+d)^(1/2
)/b/(c^2*x^2+1)^(1/2)+2*b*c*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(c^2*x^2+1)^(1/2
))^2)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-b^2*c*polylog(2,1/(c*x+(c^2*x^2
+1)^(1/2))^2)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5737, 5659, 3716, 2190, 2279, 2391, 5675}

$$\frac{b^2c\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+\frac{c\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^3}{3b\sqrt{c^2x^2+1}}-\frac{c\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{c^2x^2+1}}-$$

Warning: Unable to verify antiderivative.

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] -((Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x) - (c*Sqrt[d + c^2*d*x^2]*
(a + b*ArcSinh[c*x])^2)/Sqrt[1 + c^2*x^2] + (c*Sqrt[d + c^2*d*x^2]*(a + b*A
rcSinh[c*x])^3)/(3*b*Sqrt[1 + c^2*x^2]) + (2*b*c*Sqrt[d + c^2*d*x^2]*(a + b
*ArcSinh[c*x])*Log[1 - E^(2*ArcSinh[c*x])])/Sqrt[1 + c^2*x^2] + (b^2*c*Sqrt
[d + c^2*d*x^2]*PolyLog[2, E^(2*ArcSinh[c*x])])/Sqrt[1 + c^2*x^2]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5737

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 1)*S
qrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x] -
 Dist[(c^2*Sqrt[d + e*x^2])/(f^2*(m + 1)*Sqrt[1 + c^2*x^2]), Int[((f*x)^(m
+ 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^2}dx = -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x}+\frac{\left(2bc\sqrt{d+c^2dx^2}\right)\int\frac{a+b\sinh^{-1}(cx)}{x}dx}{\sqrt{1+c^2x^2}}$$

$$= -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x}+\frac{c\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^3}{3b\sqrt{1+c^2x^2}}+$$

$$= -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x}-\frac{c\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}+$$

$$= -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x}-\frac{c\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}+$$

$$= -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x}-\frac{c\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}+$$

$$= -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x}-\frac{c\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}+$$

Mathematica [A] time = 1.17, size = 232, normalized size = 1.11

$$-\frac{a^2\sqrt{c^2dx^2+d}}{x}+a^2c\sqrt{d}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)+\frac{ab\sqrt{c^2dx^2+d}\left(-2\sqrt{c^2x^2+1}\sinh^{-1}(cx)+2cx\log(cx)+c\right.}{x\sqrt{c^2x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] -((a^2*Sqrt[d + c^2*d*x^2])/x) + (a*b*Sqrt[d + c^2*d*x^2]*(-2*Sqrt[1 + c^2*
x^2]*ArcSinh[c*x] + c*x*ArcSinh[c*x]^2 + 2*c*x*Log[c*x]))/(x*Sqrt[1 + c^2*x
^2]) + a^2*c*Sqrt[d]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (b^2*c*Sqrt
```

```
[d + c^2*d*x^2]*(ArcSinh[c*x]*((3 - (3*Sqrt[1 + c^2*x^2])/(c*x))*ArcSinh[c*
x] + ArcSinh[c*x]^2 + 6*Log[1 - E^(-2*ArcSinh[c*x])]) - 3*PolyLog[2, E^(-2*
ArcSinh[c*x])]))/(3*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(b^2\operatorname{arsinh}(cx)^2+2\,ab\operatorname{arsinh}(cx)+a^2\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/x^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.32, size = 625, normalized size = 2.99

$$-\frac{a^2\left(c^2d\,x^2+d\right)^{\frac{3}{2}}}{dx}+a^2c^2x\sqrt{c^2d\,x^2+d}+\frac{a^2c^2d\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{\sqrt{c^2d}}+\frac{b^2\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsinh}(cx)^3c}{3\sqrt{c^2x^2+1}}-\frac{b^2\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^2,x)
```

```
[Out] -a^2/d/x*(c^2*d*x^2+d)^(3/2)+a^2*c^2*x*(c^2*d*x^2+d)^(1/2)+a^2*c^2*d*ln(x*c
^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/3*b^2*(d*(c^2*x^2+1
))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^3*c-b^2*(d*(c^2*x^2+1))^(1/2)*arcsi
nh(c*x)^2*x/(c^2*x^2+1)*c^2-b^2*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2/(c^2*x
^2+1)^(1/2)*c-b^2*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2/x/(c^2*x^2+1)+2*b^2*
(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(
1/2))*c+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x
^2+1)^(1/2))*c+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*l
n(1-c*x-(c^2*x^2+1)^(1/2))*c+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*
polylog(2,c*x+(c^2*x^2+1)^(1/2))*c+a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1
/2)*arcsinh(c*x)^2*c-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(
c*x)*c-2*a*b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)*x/(c^2*x^2+1)*c^2-2*a*b*(d*
(c^2*x^2+1))^(1/2)*arcsinh(c*x)/x/(c^2*x^2+1)+2*a*b*(d*(c^2*x^2+1))^(1/2)/(
c^2*x^2+1)^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(c\sqrt{d}\operatorname{arsinh}(cx)-\frac{\sqrt{c^2dx^2+d}}{x}\right)a^2+\int\frac{\sqrt{c^2dx^2+d}\,b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{x^2}+\frac{2\sqrt{c^2dx^2+d}\,ab\log\left(cx+\sqrt{c^2\right.}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="maxima
")
```

```
[Out] (c*sqrt(d)*arcsinh(c*x) - sqrt(c^2*d*x^2 + d)/x)*a^2 + integrate(sqrt(c^2*d
*x^2 + d)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/x^2 + 2*sqrt(c^2*d*x^2 + d)*a*
b*log(c*x + sqrt(c^2*x^2 + 1))/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))^2\,\sqrt{d\,c^2\,x^2+d}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2))/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\left(c^2x^2+1\right)}\,(a+b\,\mathrm{asinh}\,(cx))^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*(c**2*d*x**2+d)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2/x**2, x)
```

3.264 $\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^3}\,dx$

Optimal. Leaf size=358

$$-\frac{bc^2\sqrt{c^2dx^2+d}\,\mathrm{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}+\frac{bc^2\sqrt{c^2dx^2+d}\,\mathrm{Li}_2\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}-\frac{bc\sqrt{c}}{}$$

```
[Out] -1/2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^2-b*c*(a+b*arcsinh(c*x))*(c
^2*d*x^2+d)^(1/2)/x/(c^2*x^2+1)^(1/2)-c^2*(a+b*arcsinh(c*x))^2*arctanh(c*x+
(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-b^2*c^2*arctanh((c
^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-b*c^2*(a+b*arcsinh(c
*x))*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2
)+b*c^2*(a+b*arcsinh(c*x))*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(
1/2)/(c^2*x^2+1)^(1/2)+b^2*c^2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2
+d)^(1/2)/(c^2*x^2+1)^(1/2)-b^2*c^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))*(c^2*d
*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5737, 5661, 266, 63, 208, 5760, 4182, 2531, 2282, 6589}

$$-\frac{bc^2\sqrt{c^2dx^2+d}\,\mathrm{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}+\frac{bc^2\sqrt{c^2dx^2+d}\,\mathrm{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)\left(a+b\sin\right.}{\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] -((b*c*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(x*Sqrt[1 + c^2*x^2])) - (
Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*x^2) - (c^2*Sqrt[d + c^2*d*x
^2]*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (b^
2*c^2*Sqrt[d + c^2*d*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]])/Sqrt[1 + c^2*x^2] - (
b*c^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcSinh[c*x]])
/Sqrt[1 + c^2*x^2] + (b*c^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*PolyLo
g[2, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (b^2*c^2*Sqrt[d + c^2*d*x^2]*Poly
Log[3, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (b^2*c^2*Sqrt[d + c^2*d*x^2]*P
olyLog[3, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5737

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 1)*S
qrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x] -
 Dist[(c^2*Sqrt[d + e*x^2])/(f^2*(m + 1)*Sqrt[1 + c^2*x^2]), Int[((f*x)^(m
+ 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^3}\,dx = -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2x^2} + \frac{\left(bc\sqrt{d+c^2dx^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{x^2}\,d}{\sqrt{1+c^2x^2}}$$

$$= -\frac{bc\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{1+c^2x^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2x^2}$$

$$= -\frac{bc\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{1+c^2x^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2x^2}$$

$$= -\frac{bc\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{1+c^2x^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2x^2}$$

$$= -\frac{bc\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{1+c^2x^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2x^2}$$

$$= -\frac{bc\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{1+c^2x^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2x^2}$$

Mathematica [A] time = 4.88, size = 446, normalized size = 1.25

$$\frac{1}{8}\left(-\frac{4a^2\sqrt{c^2dx^2+d}}{x^2} - 4a^2c^2\sqrt{d}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+d\right) + 4a^2c^2\sqrt{d}\log(x) + \frac{2abc^2\sqrt{c^2dx^2+d}\left(4\mathrm{Li}_2\left(-e^{-\mathrm{si}}\right.\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] ((-4*a^2*Sqrt[d + c^2*d*x^2])/x^2 + 4*a^2*c^2*Sqrt[d]*Log[x] - 4*a^2*c^2*Sq
rt[d]*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (2*a*b*c^2*Sqrt[d + c^2*d*x^2]
*(-2*Coth[ArcSinh[c*x]/2] - ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 + 4*ArcSinh
[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 4*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])
] + 4*PolyLog[2, -E^(-ArcSinh[c*x])] - 4*PolyLog[2, E^(-ArcSinh[c*x])] - Ar
cSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSinh[c*x]/2]))/Sqrt[1 + c^2*x
^2] + (b^2*c^2*Sqrt[d + c^2*d*x^2]*(-4*ArcSinh[c*x]*Coth[ArcSinh[c*x]/2] -
ArcSinh[c*x]^2*Csch[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*x]^2*Log[1 - E^(-ArcSin
h[c*x])] - 4*ArcSinh[c*x]^2*Log[1 + E^(-ArcSinh[c*x])] + 8*Log[Tanh[ArcSinh
[c*x]/2]] + 8*ArcSinh[c*x]*PolyLog[2, -E^(-ArcSinh[c*x])] - 8*ArcSinh[c*x]*
PolyLog[2, E^(-ArcSinh[c*x])] + 8*PolyLog[3, -E^(-ArcSinh[c*x])] - 8*PolyLo
g[3, E^(-ArcSinh[c*x])] - ArcSinh[c*x]^2*Sech[ArcSinh[c*x]/2]^2 + 4*ArcSinh
[c*x]*Tanh[ArcSinh[c*x]/2]))/Sqrt[1 + c^2*x^2])/8
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(b^2\operatorname{arsinh}(cx)^2 + 2\,ab\operatorname{arsinh}(cx) + a^2\right)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/x^3, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.45, size = 870, normalized size = 2.43

$$-\frac{a^2\left(c^2d\,x^2+d\right)^{\frac{3}{2}}}{2d\,x^2}-\frac{a^2\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2d\,x^2+d}}{x}\right)c^2}{2}+\frac{a^2\sqrt{c^2d\,x^2+d}\,c^2}{2}-\frac{b^2\operatorname{arcsinh}(cx)^2\sqrt{d\left(c^2x^2+1\right)}\,c^2}{2\left(c^2x^2+1\right)}-\frac{b^2\operatorname{arcs}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^3,x)
```

```
[Out] -1/2*a^2/d/x^2*(c^2*d*x^2+d)^(3/2)-1/2*a^2*d^(1/2)*ln((2*d+2*d^(1/2)*(c^2*d
*x^2+d)^(1/2))/x)*c^2+1/2*a^2*(c^2*d*x^2+d)^(1/2)*c^2-1/2*b^2*arcsinh(c*x)^
2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)*c^2-b^2*arcsinh(c*x)*(d*(c^2*x^2+1))^(1
/2)/x/(c^2*x^2+1)^(1/2)*c-1/2*b^2*arcsinh(c*x)^2*(d*(c^2*x^2+1))^(1/2)/x^2/
(c^2*x^2+1)-1/2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*
ln(1+c*x+(c^2*x^2+1)^(1/2))*c^2-b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)
*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c^2+b^2*(d*(c^2*x^2+1))^(1/
2)/(c^2*x^2+1)^(1/2)*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*c^2+1/2*b^2*(d*(c^2*
x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))*
c^2+b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*polylog(2,c*x+
(c^2*x^2+1)^(1/2))*c^2-b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(
3,c*x+(c^2*x^2+1)^(1/2))*c^2-2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*
arctanh(c*x+(c^2*x^2+1)^(1/2))*c^2-a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)*ar
csinh(c*x)*c^2-a*b*(d*(c^2*x^2+1))^(1/2)/x/(c^2*x^2+1)^(1/2)*c-a*b*arcsinh(
c*x)*(d*(c^2*x^2+1))^(1/2)/x^2/(c^2*x^2+1)+a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x
^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^2+a*b*(d*(c^2*x^2+1)
)^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c^2-a*b*(d*(c^2*
x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^
2-a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1
/2))*c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\left(c^2\sqrt{d}\,\operatorname{arsinh}\left(\frac{1}{c|x|}\right)-\sqrt{c^2dx^2+d}\,c^2+\frac{\left(c^2dx^2+d\right)^{\frac{3}{2}}}{dx^2}\right)a^2+\int\frac{\sqrt{c^2dx^2+d}\,b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{x^3}+\frac{2\sqrt{c^2dx^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="maxima
")
```

```
[Out] -1/2*(c^2*sqrt(d)*arcsinh(1/(c*abs(x))) - sqrt(c^2*d*x^2 + d)*c^2 + (c^2*d*
x^2 + d)^(3/2)/(d*x^2))*a^2 + integrate(sqrt(c^2*d*x^2 + d)*b^2*log(c*x + s
qrt(c^2*x^2 + 1))^2/x^3 + 2*sqrt(c^2*d*x^2 + d)*a*b*log(c*x + sqrt(c^2*x^2
+ 1))/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{(a+b\operatorname{asinh}(c\,x))^2\,\sqrt{d\,c^2\,x^2+d}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2))/x^3,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\left(c^2x^2+1\right)}\,(a+b\operatorname{asinh}(cx))^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*(c**2*d*x**2+d)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2/x**3, x)
```

3.265 $\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^4}\,dx$

Optimal. Leaf size=294

$$-\frac{bc\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{3x^2}-\frac{\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3}+\frac{c^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx\right.}{3\sqrt{c^2x^2+1}}$$

```
[Out] -1/3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/d/x^3-1/3*b^2*c^2*(c^2*d*x^2+
d)^(1/2)/x+1/3*b^2*c^3*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+1
/3*c^3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+2/3*b*c^3
*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*d*x^2+d)^(1/2)/(
c^2*x^2+1)^(1/2)-1/3*b^2*c^3*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*d*
x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/3*b*c*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)
*(c^2*d*x^2+d)^(1/2)/x^2
```

Rubi [A] time = 0.28, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {5723, 5728, 277, 215, 5659, 3716, 2190, 2279, 2391}

$$\frac{b^2c^3\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{3\sqrt{c^2x^2+1}}-\frac{c^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{3\sqrt{c^2x^2+1}}-\frac{bc\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}\left(a+b\sin\right.}{3x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] -(b^2*c^2*Sqrt[d + c^2*d*x^2])/(3*x) + (b^2*c^3*Sqrt[d + c^2*d*x^2]*ArcSinh
[c*x])/(3*Sqrt[1 + c^2*x^2]) - (b*c*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(
a + b*ArcSinh[c*x]))/(3*x^2) - (c^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]
)^2)/(3*Sqrt[1 + c^2*x^2]) - ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)
/(3*d*x^3) + (2*b*c^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*Log[1 - E^(2
*ArcSinh[c*x])])/(3*Sqrt[1 + c^2*x^2]) + (b^2*c^3*Sqrt[d + c^2*d*x^2]*PolyL
og[2, E^(2*ArcSinh[c*x])])/(3*Sqrt[1 + c^2*x^2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rule 5728

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*ArcSinh[c
*x]))/(f*(m + 1)), x] + (-Dist[(b*c*d^p)/(f*(m + 1)), Int[(f*x)^(m + 1)*(1
+ c^2*x^2)^(p - 1/2), x], x] - Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2
)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && ILtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^4}\,dx = -\frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3} + \frac{\left(2bc\sqrt{d+c^2dx^2}\right)\int \frac{\left(1+c^2x^2\right)\left(a+b\sin\right.}{x^3}}{3\sqrt{1+c^2x^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2} - \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sin\right.}{3dx^3}$$

$$= -\frac{b^2c^2\sqrt{d+c^2dx^2}}{3x} - \frac{bc\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2} - \frac{(d+}{}$$

$$= -\frac{b^2c^2\sqrt{d+c^2dx^2}}{3x} + \frac{b^2c^3\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{3\sqrt{1+c^2x^2}} - \frac{bc\sqrt{1+c^2x^2}\sqrt{d+c^2}}{3}$$

$$= -\frac{b^2c^2\sqrt{d+c^2dx^2}}{3x} + \frac{b^2c^3\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{3\sqrt{1+c^2x^2}} - \frac{bc\sqrt{1+c^2x^2}\sqrt{d+c^2}}{3}$$

$$= -\frac{b^2c^2\sqrt{d+c^2dx^2}}{3x} + \frac{b^2c^3\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{3\sqrt{1+c^2x^2}} - \frac{bc\sqrt{1+c^2x^2}\sqrt{d+c^2}}{3}$$

$$= -\frac{b^2c^2\sqrt{d+c^2dx^2}}{3x} + \frac{b^2c^3\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{3\sqrt{1+c^2x^2}} - \frac{bc\sqrt{1+c^2x^2}\sqrt{d+c^2}}{3}$$

Mathematica [A] time = 0.80, size = 240, normalized size = 0.82

$$-\frac{\sqrt{c^2dx^2+d}\left(a^2c^2x^2\sqrt{c^2x^2+1}+a^2\sqrt{c^2x^2+1}-2abc^3x^3\log(cx)-b\sinh^{-1}(cx)\left(-2a\left(c^2x^2+1\right)^{3/2}+2bc^3x^3\log\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] -1/3*(Sqrt[d + c^2*d*x^2]*(a*b*c*x + a^2*Sqrt[1 + c^2*x^2] + a^2*c^2*x^2*Sq
rt[1 + c^2*x^2] + b^2*c^2*x^2*Sqrt[1 + c^2*x^2] + b^2*(-(c^3*x^3) + Sqrt[1
+ c^2*x^2] + c^2*x^2*Sqrt[1 + c^2*x^2])*ArcSinh[c*x]^2 - b*ArcSinh[c*x]*(-(
b*c*x) - 2*a*(1 + c^2*x^2)^(3/2) + 2*b*c^3*x^3*Log[1 - E^(-2*ArcSinh[c*x])]
) - 2*a*b*c^3*x^3*Log[c*x] + b^2*c^3*x^3*PolyLog[2, E^(-2*ArcSinh[c*x])]))/
(x^3*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(b^2\operatorname{arsinh}(cx)^2+2\,ab\operatorname{arsinh}(cx)+a^2\right)}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/x^4, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.49, size = 2557, normalized size = 8.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^4,x)
```

```
[Out] -2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^3-1/3*a^2
/d/x^3*(c^2*d*x^2+d)^(3/2)-3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2
+1)*x^3/(c^2*x^2+1)*arcsinh(c*x)^2*c^6-2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4
*x^4+3*c^2*x^2+1)*x^3/(c^2*x^2+1)*arcsinh(c*x)*c^6-10/3*b^2*(d*(c^2*x^2+1))
^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x/(c^2*x^2+1)*arcsinh(c*x)^2*c^4-1/3*b^2*(d*
(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x/(c^2*x^2+1)*arcsinh(c*x)*c^4-5
/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/x/(c^2*x^2+1)*arcsinh(
c*x)^2*c^2-a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^2/(c^2*x^2+1
)^(1/2)*c^5+2/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+
1)^(1/2)*arcsinh(c*x)*c^3-1/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^
2+1)/x^2/(c^2*x^2+1)^(1/2)*c+b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2
+1)*x^4/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^7+b^2*(d*(c^2*x^2+1))^(1/2)/(3*c
^4*x^4+3*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^5-b^2*(d*(c^2*x^
2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^5-
1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/x^2/(c^2*x^2+1)^(1/2)
*arcsinh(c*x)*c-b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^5/(c^2*
x^2+1)*arcsinh(c*x)^2*c^8-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^
2+1)*x^5/(c^2*x^2+1)*arcsinh(c*x)*c^8-1/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*
x^4+3*c^2*x^2+1)*x^5/(c^2*x^2+1)*c^8-2/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x
^4+3*c^2*x^2+1)*x^3/(c^2*x^2+1)*c^6-1/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^
4+3*c^2*x^2+1)*x/(c^2*x^2+1)*c^4-2/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3
*c^2*x^2+1)/x^3/(c^2*x^2+1)*arcsinh(c*x)-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c
^4*x^4+3*c^2*x^2+1)/x^3/(c^2*x^2+1)*arcsinh(c*x)^2+2/3*b^2*(d*(c^2*x^2+1))^
(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^3+2/3*b^
2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)
^(1/2))*c^3+2/3*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*ln((c*x+(c^2*x^
2+1)^(1/2))^2-1)*c^3-a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2
*x^2+1)^(1/2)*c^3-2*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^5/(
c^2*x^2+1)*arcsinh(c*x)*c^8-6*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^
2+1)*x^3/(c^2*x^2+1)*arcsinh(c*x)*c^6-20/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4
*x^4+3*c^2*x^2+1)*x/(c^2*x^2+1)*arcsinh(c*x)*c^4-10/3*a*b*(d*(c^2*x^2+1))^(
1/2)/(3*c^4*x^4+3*c^2*x^2+1)/x/(c^2*x^2+1)*arcsinh(c*x)*c^2+2*a*b*(d*(c^2*x
^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^4/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^7
+2*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*
arcsinh(c*x)*c^5-4/3*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*
x)*c^3+b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^4/(c^2*x^2+1)^(1
/2)*c^7+b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^2/(c^2*x^2+1)^(
1/2)*c^5+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^
(1/2)*arcsinh(c*x)^2*c^3-b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/
(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^3+1/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4
+3*c^2*x^2+1)*x^3*c^6+1/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)
*x*c^4+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^3*arcsinh(c*
x)*c^6+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x*arcsinh(c*x)
*c^4-2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^5/(c^2*x^2+1)*
c^8-5/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^3/(c^2*x^2+1)*c
^6-4/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x/(c^2*x^2+1)*c^4-
1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/x/(c^2*x^2+1)*c^2+2/3
```

```
*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/
2))*c^3+2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*
x^2+1)^(1/2))*c^3-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^3
*c^6+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^(1/2
)*c^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left((-1)^{2\,c^2dx^2+2\,d}\,c^2d^{\frac{3}{2}}\log\left(2\,c^2d+\frac{2\,d}{x^2}\right)-c^2d^{\frac{3}{2}}\log\left(x^2+\frac{1}{c^2}\right)+\frac{\sqrt{c^4dx^4+2\,c^2dx^2+d}\,d}{x^2}\right)abc}{3\,d}-\frac{1}{3}\,b^2\left(\frac{\left(c^2\sqrt{d}\,x^2+\sqrt{d}\right)\sqrt{c^2x^2}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="maxima
")
```

```
[Out] -1/3*((-1)^(2*c^2*d*x^2 + 2*d)*c^2*d^(3/2)*log(2*c^2*d + 2*d/x^2) - c^2*d^(
3/2)*log(x^2 + 1/c^2) + sqrt(c^4*d*x^4 + 2*c^2*d*x^2 + d)*d/x^2)*a*b*c/d -
1/3*b^2*((c^2*sqrt(d)*x^2 + sqrt(d))*sqrt(c^2*x^2 + 1)*log(c*x + sqrt(c^2*x
^2 + 1))^2/x^3 - 3*integrate(2/3*((c^2*x^2 + 1)*c^2*sqrt(d)*x + (c^3*sqrt(d
)*x^2 + c*sqrt(d))*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))/(c*x^4 +
 sqrt(c^2*x^2 + 1)*x^3), x)) - 2/3*(c^2*d*x^2 + d)^(3/2)*a*b*arcsinh(c*x)/(
d*x^3) - 1/3*(c^2*d*x^2 + d)^(3/2)*a^2/(d*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))^2\,\sqrt{d\,c^2\,x^2+d}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2))/x^4,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\left(c^2x^2+1\right)}\,\left(a+b\,\mathrm{asinh}\,(cx)\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*(c**2*d*x**2+d)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2/x**4, x)
```

3.266 $\int x^3 \left(d+c^2 d x^2\right)^{3/2} \left(a+b \sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=482

$$\frac{dx^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{35c^2}-\frac{16bcdx^5\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{175\sqrt{c^2x^2+1}}+\frac{1}{7}x^4\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}\right.$$

```
[Out] 1/7*x^4*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2-304/3675*b^2*d*(c^2*d*x^2+
d)^(1/2)/c^4-152/11025*b^2*d*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)/c^4-38/6125*b^
2*d*(c^2*x^2+1)^2*(c^2*d*x^2+d)^(1/2)/c^4+2/343*b^2*d*(c^2*x^2+1)^3*(c^2*d*
x^2+d)^(1/2)/c^4-2/35*d*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^4+1/35*d
*x^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^2+3/35*d*x^4*(a+b*arcsinh(c
*x))^2*(c^2*d*x^2+d)^(1/2)+4/35*a*b*d*x*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)
^(1/2)+4/35*b^2*d*x*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)^(1/2)-
2/105*b*d*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-16
/175*b*c*d*x^5*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2/4
9*b*c^3*d*x^7*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.81, antiderivative size = 482, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 14, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5744, 5742, 5758, 5717, 5653, 261, 5661, 266, 43, 14, 5730, 12, 446, 77}

$$\frac{4abdx\sqrt{c^2dx^2+d}}{35c^3\sqrt{c^2x^2+1}}-\frac{2bc^3dx^7\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{49\sqrt{c^2x^2+1}}-\frac{16bcdx^5\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{175\sqrt{c^2x^2+1}}+\frac{1}{7}x^4\left(c^2d\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-304*b^2*d*Sqrt[d + c^2*d*x^2])/(3675*c^4) + (4*a*b*d*x*Sqrt[d + c^2*d*x^2
])/(35*c^3*Sqrt[1 + c^2*x^2]) - (152*b^2*d*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2
])/(11025*c^4) - (38*b^2*d*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2])/(6125*c^4)
+ (2*b^2*d*(1 + c^2*x^2)^3*Sqrt[d + c^2*d*x^2])/(343*c^4) + (4*b^2*d*x*Sqrt
[d + c^2*d*x^2]*ArcSinh[c*x])/(35*c^3*Sqrt[1 + c^2*x^2]) - (2*b*d*x^3*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(105*c*Sqrt[1 + c^2*x^2]) - (16*b*c*d*
x^5*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(175*Sqrt[1 + c^2*x^2]) - (2*
b*c^3*d*x^7*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(49*Sqrt[1 + c^2*x^2]
) - (2*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(35*c^4) + (d*x^2*Sqrt
[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(35*c^2) + (3*d*x^4*Sqrt[d + c^2*d*
x^2]*(a + b*ArcSinh[c*x])^2)/35 + (x^4*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh
[c*x])^2)/7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
```

```
GtQ[p, 0]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int x^3 \left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx = \frac{1}{7}x^4 \left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right)^2 + \frac{1}{7}(3d) \int x^3\sqrt{d+c^2dx^2} \left(a\right.$$

$$= -\frac{2bcdx^5\sqrt{d+c^2dx^2} \left(a+b\sinh^{-1}(cx)\right)}{35\sqrt{1+c^2x^2}} - \frac{2bc^3dx^7\sqrt{d+c^2dx^2} \left(a+\right.}{49\sqrt{1+c^2x^2}}$$

$$= -\frac{16bcdx^5\sqrt{d+c^2dx^2} \left(a+b\sinh^{-1}(cx)\right)}{175\sqrt{1+c^2x^2}} - \frac{2bc^3dx^7\sqrt{d+c^2dx^2} \left(a\right.}{49\sqrt{1+c^2x^2}}$$

$$= -\frac{2bdx^3\sqrt{d+c^2dx^2} \left(a+b\sinh^{-1}(cx)\right)}{105c\sqrt{1+c^2x^2}} - \frac{16bcdx^5\sqrt{d+c^2dx^2} \left(a+b\right.}{175\sqrt{1+c^2x^2}}$$

$$= \frac{4abdx\sqrt{d+c^2dx^2}}{35c^3\sqrt{1+c^2x^2}} - \frac{2bdx^3\sqrt{d+c^2dx^2} \left(a+b\sinh^{-1}(cx)\right)}{105c\sqrt{1+c^2x^2}} - \frac{16bcdx}{}$$

$$= \frac{62b^2d\sqrt{d+c^2dx^2}}{1225c^4} + \frac{4abdx\sqrt{d+c^2dx^2}}{35c^3\sqrt{1+c^2x^2}} - \frac{74b^2d\left(1+c^2x^2\right)\sqrt{d+c^2}}{3675c^4}$$

$$= -\frac{304b^2d\sqrt{d+c^2dx^2}}{3675c^4} + \frac{4abdx\sqrt{d+c^2dx^2}}{35c^3\sqrt{1+c^2x^2}} - \frac{152b^2d\left(1+c^2x^2\right)\sqrt{d}}{11025c^4}$$

Mathematica [A] time = 0.39, size = 251, normalized size = 0.52

$$\frac{d\sqrt{c^2dx^2+d}\left(11025a^2\left(5c^2x^2-2\right)\left(c^2x^2+1\right)^3-210abcx\left(75c^6x^6+168c^4x^4+35c^2x^2-210\right)\sqrt{c^2x^2+1}-210b\,\mathrm{s}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d*Sqrt[d + c^2*d*x^2]*(11025*a^2*(1 + c^2*x^2)^3*(-2 + 5*c^2*x^2) - 210*a*
b*c*x*Sqrt[1 + c^2*x^2]*(-210 + 35*c^2*x^2 + 168*c^4*x^4 + 75*c^6*x^6) + 2*
b^2*(-18692 - 20371*c^2*x^2 + 499*c^4*x^4 + 3303*c^6*x^6 + 1125*c^8*x^8) -
210*b*(-105*a*(1 + c^2*x^2)^3*(-2 + 5*c^2*x^2) + b*c*x*Sqrt[1 + c^2*x^2]*(-
210 + 35*c^2*x^2 + 168*c^4*x^4 + 75*c^6*x^6))*ArcSinh[c*x] + 11025*b^2*(1 +
 c^2*x^2)^3*(-2 + 5*c^2*x^2)*ArcSinh[c*x]^2))/(385875*c^4*(1 + c^2*x^2))
```

fricas [A] time = 0.67, size = 402, normalized size = 0.83

$$\frac{11025\left(5\,b^2c^8dx^8+13\,b^2c^6dx^6+9\,b^2c^4dx^4-b^2c^2dx^2-2\,b^2d\right)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2+210\left(525\,abc^8\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas
")
```

```
[Out] 1/385875*(11025*(5*b^2*c^8*d*x^8 + 13*b^2*c^6*d*x^6 + 9*b^2*c^4*d*x^4 - b^2
*c^2*d*x^2 - 2*b^2*d)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))^2 +
210*(525*a*b*c^8*d*x^8 + 1365*a*b*c^6*d*x^6 + 945*a*b*c^4*d*x^4 - 105*a*b*c
^2*d*x^2 - 210*a*b*d - (75*b^2*c^7*d*x^7 + 168*b^2*c^5*d*x^5 + 35*b^2*c^3*d
```

```
*x^3 - 210*b^2*c*d*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt
(c^2*x^2 + 1)) + (1125*(49*a^2 + 2*b^2)*c^8*d*x^8 + 9*(15925*a^2 + 734*b^2)
*c^6*d*x^6 + (99225*a^2 + 998*b^2)*c^4*d*x^4 - (11025*a^2 + 40742*b^2)*c^2*
d*x^2 - 2*(11025*a^2 + 18692*b^2)*d - 210*(75*a*b*c^7*d*x^7 + 168*a*b*c^5*d
*x^5 + 35*a*b*c^3*d*x^3 - 210*a*b*c*d*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2
+ d))/(c^6*x^2 + c^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.54, size = 1766, normalized size = 3.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] a^2*(1/7*x^2*(c^2*d*x^2+d)^(5/2)/c^2/d-2/35/d/c^4*(c^2*d*x^2+d)^(5/2))+b^2*
(1/43904*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8+64*c^7*x^7*(c^2*x^2+1)^(1/2)+144
*c^6*x^6+112*c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4+56*c^3*x^3*(c^2*x^2+1)^(
1/2)+25*c^2*x^2+7*c*x*(c^2*x^2+1)^(1/2)+1)*(49*arcsinh(c*x)^2-14*arcsinh(c*
x)+2)*d/c^4/(c^2*x^2+1)+1/16000*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6+16*c^5*x^
5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2+5*c*
x*(c^2*x^2+1)^(1/2)+1)*(25*arcsinh(c*x)^2-10*arcsinh(c*x)+2)*d/c^4/(c^2*x^2
+1)-1/1152*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c
^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arcsinh(c*x)^2-6*arcsinh(c*x)+2)*d/c^4
/(c^2*x^2+1)-3/128*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*
(arcsinh(c*x)^2-2*arcsinh(c*x)+2)*d/c^4/(c^2*x^2+1)-3/128*(d*(c^2*x^2+1))^(
1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2+2*arcsinh(c*x)+2)*d/
c^4/(c^2*x^2+1)-1/1152*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+
1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arcsinh(c*x)^2+6*arcsinh(c
*x)+2)*d/c^4/(c^2*x^2+1)+1/16000*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6-16*c^5*x
^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2-5*c
*x*(c^2*x^2+1)^(1/2)+1)*(25*arcsinh(c*x)^2+10*arcsinh(c*x)+2)*d/c^4/(c^2*x^
2+1)+1/43904*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8-64*c^7*x^7*(c^2*x^2+1)^(1/2)
+144*c^6*x^6-112*c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4-56*c^3*x^3*(c^2*x^2+
1)^(1/2)+25*c^2*x^2-7*c*x*(c^2*x^2+1)^(1/2)+1)*(49*arcsinh(c*x)^2+14*arcsin
h(c*x)+2)*d/c^4/(c^2*x^2+1))+2*a*b*(1/6272*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^
8+64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6+112*c^5*x^5*(c^2*x^2+1)^(1/2)+10
4*c^4*x^4+56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2+7*c*x*(c^2*x^2+1)^(1/2)+1
)*(-1+7*arcsinh(c*x))*d/c^4/(c^2*x^2+1)+1/3200*(d*(c^2*x^2+1))^(1/2)*(16*c^
6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+
13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+5*arcsinh(c*x))*d/c^4/(c^2*x^2+1)
-1/384*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x
^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arcsinh(c*x))*d/c^4/(c^2*x^2+1)-3/128*(
d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))*d/
c^4/(c^2*x^2+1)-3/128*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+
1)*(1+arcsinh(c*x))*d/c^4/(c^2*x^2+1)-1/384*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^
4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(1+3*arc
sinh(c*x))*d/c^4/(c^2*x^2+1)+1/3200*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6-16*c^
5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2-
```

```
5*c*x*(c^2*x^2+1)^(1/2)+1)*(1+5*arcsinh(c*x))*d/c^4/(c^2*x^2+1)+1/6272*(d*(
c^2*x^2+1))^(1/2)*(64*c^8*x^8-64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6-112*
c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4-56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x
^2-7*c*x*(c^2*x^2+1)^(1/2)+1)*(1+7*arcsinh(c*x))*d/c^4/(c^2*x^2+1))
```

maxima [A] time = 0.44, size = 346, normalized size = 0.72

$$\frac{1}{35}\left(\frac{5\left(c^2dx^2+d\right)^{\frac{5}{2}}x^2}{c^2d}-\frac{2\left(c^2dx^2+d\right)^{\frac{5}{2}}}{c^4d}\right)b^2\operatorname{arsinh}(cx)^2+\frac{2}{35}\left(\frac{5\left(c^2dx^2+d\right)^{\frac{5}{2}}x^2}{c^2d}-\frac{2\left(c^2dx^2+d\right)^{\frac{5}{2}}}{c^4d}\right)ab\operatorname{arsinh}(cx)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima
")
```

```
[Out] 1/35*(5*(c^2*d*x^2 + d)^(5/2)*x^2/(c^2*d) - 2*(c^2*d*x^2 + d)^(5/2)/(c^4*d)
)*b^2*arcsinh(c*x)^2 + 2/35*(5*(c^2*d*x^2 + d)^(5/2)*x^2/(c^2*d) - 2*(c^2*d
*x^2 + d)^(5/2)/(c^4*d))*a*b*arcsinh(c*x) + 1/35*(5*(c^2*d*x^2 + d)^(5/2)*x
^2/(c^2*d) - 2*(c^2*d*x^2 + d)^(5/2)/(c^4*d))*a^2 + 2/385875*b^2*((1125*sqr
t(c^2*x^2 + 1)*c^4*d^(3/2)*x^6 + 2178*sqrt(c^2*x^2 + 1)*c^2*d^(3/2)*x^4 - 1
679*sqrt(c^2*x^2 + 1)*d^(3/2)*x^2 - 18692*sqrt(c^2*x^2 + 1)*d^(3/2)/c^2)/c^
2 - 105*(75*c^6*d^(3/2)*x^7 + 168*c^4*d^(3/2)*x^5 + 35*c^2*d^(3/2)*x^3 - 21
0*d^(3/2)*x)*arcsinh(c*x)/c^3) - 2/3675*(75*c^6*d^(3/2)*x^7 + 168*c^4*d^(3/
2)*x^5 + 35*c^2*d^(3/2)*x^3 - 210*d^(3/2)*x)*a*b/c^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,(a+b\,\mathrm{asinh}\,(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}(a+b\operatorname{asinh}(cx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x**3*(d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**2, x)
```

3.267 $\int x^2 \left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=405

$$-\frac{bdx^2\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{16c\sqrt{c^2x^2+1}}+\frac{dx\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2}{16c^2}-\frac{7bcdx^4\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(c\right.}{48\sqrt{c^2x^2+1}}$$

```
[Out] 1/6*x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2-7/1152*b^2*d*x*(c^2*d*x^2+
d)^(1/2)/c^2+43/1728*b^2*d*x^3*(c^2*d*x^2+d)^(1/2)+1/108*b^2*c^2*d*x^5*(c^2
*d*x^2+d)^(1/2)+1/16*d*x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^2+1/8*d
*x^3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)+7/1152*b^2*d*arcsinh(c*x)*(c^
2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)^(1/2)-1/16*b*d*x^2*(a+b*arcsinh(c*x))*(c^2
*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-7/48*b*c*d*x^4*(a+b*arcsinh(c*x))*(c^2*
d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/18*b*c^3*d*x^6*(a+b*arcsinh(c*x))*(c^2*d
*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/48*d*(a+b*arcsinh(c*x))^3*(c^2*d*x^2+d)^(
1/2)/b/c^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.66, antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {5744, 5742, 5758, 5675, 5661, 321, 215, 14, 5730, 12, 459}

$$-\frac{bc^3dx^6\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{18\sqrt{c^2x^2+1}}-\frac{7bcdx^4\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{48\sqrt{c^2x^2+1}}+\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-7*b^2*d*x*Sqrt[d + c^2*d*x^2])/(1152*c^2) + (43*b^2*d*x^3*Sqrt[d + c^2*d*
x^2])/1728 + (b^2*c^2*d*x^5*Sqrt[d + c^2*d*x^2])/108 + (7*b^2*d*Sqrt[d + c^
2*d*x^2]*ArcSinh[c*x])/(1152*c^3*Sqrt[1 + c^2*x^2]) - (b*d*x^2*Sqrt[d + c^2
*d*x^2]*(a + b*ArcSinh[c*x]))/(16*c*Sqrt[1 + c^2*x^2]) - (7*b*c*d*x^4*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(48*Sqrt[1 + c^2*x^2]) - (b*c^3*d*x^6*
Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(18*Sqrt[1 + c^2*x^2]) + (d*x*Sqr
t[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(16*c^2) + (d*x^3*Sqrt[d + c^2*d*x
^2]*(a + b*ArcSinh[c*x])^2)/8 + (x^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c
*x])^2)/6 - (d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^3)/(48*b*c^3*Sqrt[1
 + c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
```

```
2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int x^2\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{1}{6}x^3\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{2}d\int x^2\sqrt{d+c^2dx^2}\,\left(a\right.$$

$$=-\frac{bcdx^4\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{12\sqrt{1+c^2x^2}}-\frac{bc^3dx^6\sqrt{d+c^2dx^2}\left(a+\right.}{18\sqrt{1+c^2x}}$$

$$=-\frac{7bcdx^4\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{48\sqrt{1+c^2x^2}}-\frac{bc^3dx^6\sqrt{d+c^2dx^2}\left(a\right.}{18\sqrt{1+c^2}}$$

$$=\frac{1}{64}b^2dx^3\sqrt{d+c^2dx^2}+\frac{1}{108}b^2c^2dx^5\sqrt{d+c^2dx^2}-\frac{bdx^2\sqrt{d+c^2d}}{16c}$$

$$=\frac{b^2dx\sqrt{d+c^2dx^2}}{128c^2}+\frac{43b^2dx^3\sqrt{d+c^2dx^2}}{1728}+\frac{1}{108}b^2c^2dx^5\sqrt{d+c^2d}$$

$$=-\frac{7b^2dx\sqrt{d+c^2dx^2}}{1152c^2}+\frac{43b^2dx^3\sqrt{d+c^2dx^2}}{1728}+\frac{1}{108}b^2c^2dx^5\sqrt{d+c}$$

$$=-\frac{7b^2dx\sqrt{d+c^2dx^2}}{1152c^2}+\frac{43b^2dx^3\sqrt{d+c^2dx^2}}{1728}+\frac{1}{108}b^2c^2dx^5\sqrt{d+c}$$

Mathematica [A] time = 1.19, size = 508, normalized size = 1.25

$$\frac{-864a^2d^{3/2}\sqrt{c^2x^2+1}\,\log\left(\sqrt{d}\,\sqrt{c^2dx^2+d}+cdx\right)+864a^2cdx\sqrt{c^2x^2+1}\,\sqrt{c^2dx^2+d}+2304a^2c^5dx^5\sqrt{c^2x^2+1}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (864*a^2*c*d*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 4032*a^2*c^3*d*x^3*S
qrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 2304*a^2*c^5*d*x^5*Sqrt[1 + c^2*x^2]
*Sqrt[d + c^2*d*x^2] - 288*b^2*d*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^3 + 216*a
*b*d*Sqrt[d + c^2*d*x^2]*Cosh[2*ArcSinh[c*x]] - 108*a*b*d*Sqrt[d + c^2*d*x^
2]*Cosh[4*ArcSinh[c*x]] - 24*a*b*d*Sqrt[d + c^2*d*x^2]*Cosh[6*ArcSinh[c*x]]
 - 864*a^2*d^(3/2)*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2
]] - 108*b^2*d*Sqrt[d + c^2*d*x^2]*Sinh[2*ArcSinh[c*x]] + 27*b^2*d*Sqrt[d +
 c^2*d*x^2]*Sinh[4*ArcSinh[c*x]] + 4*b^2*d*Sqrt[d + c^2*d*x^2]*Sinh[6*ArcSi
nh[c*x]] + 12*b*d*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*(18*b*Cosh[2*ArcSinh[c*x
]] - 9*b*Cosh[4*ArcSinh[c*x]] - 2*b*Cosh[6*ArcSinh[c*x]] - 36*a*Sinh[2*ArcS
inh[c*x]] + 36*a*Sinh[4*ArcSinh[c*x]] + 12*a*Sinh[6*ArcSinh[c*x]]) + 72*b*d
*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^2*(-12*a - 3*b*Sinh[2*ArcSinh[c*x]] + 3*b
*Sinh[4*ArcSinh[c*x]] + b*Sinh[6*ArcSinh[c*x]]))/(13824*c^3*Sqrt[1 + c^2*x^
2])
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2c^2dx^4+a^2dx^2+\left(b^2c^2dx^4+b^2dx^2\right)\operatorname{arsinh}(cx)^2+2\left(abc^2dx^4+abdx^2\right)\operatorname{arsinh}(cx)\right)\sqrt{c^2dx^2+d}\,,\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas
")
```

```
[Out] integral((a^2*c^2*d*x^4 + a^2*d*x^2 + (b^2*c^2*d*x^4 + b^2*d*x^2)*arcsinh(c
*x)^2 + 2*(a*b*c^2*d*x^4 + a*b*d*x^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c^2dx^2+d\right)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)^2x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^2*x^2, x)
```

maple [B] time = 0.49, size = 934, normalized size = 2.31

$$\frac{ab\sqrt{d\left(c^2x^2+1\right)}\,d\,c^4\operatorname{arcsinh}(cx)\,x^7}{3c^2x^2+3}+\frac{7ab\sqrt{d\left(c^2x^2+1\right)}\,d}{1152c^3\sqrt{c^2x^2+1}}+\frac{59b^2\sqrt{d\left(c^2x^2+1\right)}\,d\,c^2x^5}{1728\left(c^2x^2+1\right)}-\frac{7b^2\sqrt{d\left(c^2x^2+1\right)}\,dx}{1152c^2\left(c^2x^2+1\right)}+\frac{7b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 7/1152*b^2*(d*(c^2*x^2+1))^(1/2)*d/c^3/(c^2*x^2+1)^(1/2)*arcsinh(c*x)+7/115
2*a*b*(d*(c^2*x^2+1))^(1/2)*d/c^3/(c^2*x^2+1)^(1/2)+1/108*b^2*(d*(c^2*x^2+1
))^(1/2)*d*c^4/(c^2*x^2+1)*x^7+59/1728*b^2*(d*(c^2*x^2+1))^(1/2)*d*c^2/(c^2
*x^2+1)*x^5-7/1152*b^2*(d*(c^2*x^2+1))^(1/2)*d/c^2/(c^2*x^2+1)*x-1/48*b^2*(
d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3*arcsinh(c*x)^3*d+17/48*b^2*(d*(c
^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*arcsinh(c*x)^2*x^3+1/8*a*b*(d*(c^2*x^2+1))^(
1/2)*d/c^2/(c^2*x^2+1)*arcsinh(c*x)*x+1/3*a*b*(d*(c^2*x^2+1))^(1/2)*d*c^4/(
c^2*x^2+1)*arcsinh(c*x)*x^7+11/12*a*b*(d*(c^2*x^2+1))^(1/2)*d*c^2/(c^2*x^2+
1)*arcsinh(c*x)*x^5+1/6*b^2*(d*(c^2*x^2+1))^(1/2)*d*c^4/(c^2*x^2+1)*arcsinh
(c*x)^2*x^7+11/24*b^2*(d*(c^2*x^2+1))^(1/2)*d*c^2/(c^2*x^2+1)*arcsinh(c*x)^
2*x^5+1/16*b^2*(d*(c^2*x^2+1))^(1/2)*d/c^2/(c^2*x^2+1)*arcsinh(c*x)^2*x-1/1
6*a*b*(d*(c^2*x^2+1))^(1/2)*d/c/(c^2*x^2+1)^(1/2)*x^2-1/16*a*b*(d*(c^2*x^2+
1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3*arcsinh(c*x)^2*d+17/24*a*b*(d*(c^2*x^2+1))^
(1/2)*d/(c^2*x^2+1)*arcsinh(c*x)*x^3-1/18*b^2*(d*(c^2*x^2+1))^(1/2)*d*c^3/(
c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^6-7/48*b^2*(d*(c^2*x^2+1))^(1/2)*d*c/(c^2*x
^2+1)^(1/2)*arcsinh(c*x)*x^4-1/16*b^2*(d*(c^2*x^2+1))^(1/2)*d/c/(c^2*x^2+1)
^(1/2)*arcsinh(c*x)*x^2-1/18*a*b*(d*(c^2*x^2+1))^(1/2)*d*c^3/(c^2*x^2+1)^(1
/2)*x^6-7/48*a*b*(d*(c^2*x^2+1))^(1/2)*d*c/(c^2*x^2+1)^(1/2)*x^4+1/6*a^2*x*
(c^2*d*x^2+d)^(5/2)/c^2/d-1/24*a^2/c^2*x*(c^2*d*x^2+d)^(3/2)-1/16*a^2/c^2*d
^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+65/3456*b^2*
(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*x^3-1/16*a^2/c^2*d*x*(c^2*d*x^2+d)^(1/2
)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(a + b \operatorname{asinh}(c x)\right)^2 \left(d c^2 x^2 + d\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2),x)
[Out] int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(d \left(c^2 x^2 + 1\right)\right)^{\frac{3}{2}} \left(a + b \operatorname{asinh}(cx)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2,x)
[Out] Integral(x**2*(d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**2, x)
```

3.268 $\int x \left(d+c^2 d x^2\right)^{3/2} \left(a+b \sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=267

$$-\frac{2bdx\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{5c\sqrt{c^2x^2+1}}+\frac{\left(c^2dx^2+d\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{5c^2d}-\frac{4bcdx^3\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{15\sqrt{c^2x^2+1}}$$

```
[Out] 1/5*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/c^2/d+16/75*b^2*d*(c^2*d*x^2+d
)^(1/2)/c^2+8/225*b^2*d*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)/c^2+2/125*b^2*d*(c^
2*x^2+1)^2*(c^2*d*x^2+d)^(1/2)/c^2-2/5*b*d*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+
d)^(1/2)/c/(c^2*x^2+1)^(1/2)-4/15*b*c*d*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d
)^(1/2)/(c^2*x^2+1)^(1/2)-2/25*b*c^3*d*x^5*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)
^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5717, 194, 5679, 12, 1247, 698}

$$-\frac{2bc^3dx^5\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{25\sqrt{c^2x^2+1}}-\frac{4bcdx^3\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{15\sqrt{c^2x^2+1}}-\frac{2bdx\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(\right.}{5c\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (16*b^2*d*Sqrt[d + c^2*d*x^2])/(75*c^2) + (8*b^2*d*(1 + c^2*x^2)*Sqrt[d + c
^2*d*x^2])/(225*c^2) + (2*b^2*d*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2])/(125*c
^2) - (2*b*d*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(5*c*Sqrt[1 + c^2*
x^2]) - (4*b*c*d*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(15*Sqrt[1 +
 c^2*x^2]) - (2*b*c^3*d*x^5*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(25*S
qrt[1 + c^2*x^2]) + ((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(5*c^2*d
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 5679

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\int x\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{5c^2d}-\frac{\left(2bd\sqrt{d+c^2dx^2}\right)\int\left(1+c^2x\right.}{5c\sqrt{1+}}$$

$$=-\frac{2bdx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{5c\sqrt{1+c^2x^2}}-\frac{4bcdx^3\sqrt{d+c^2dx^2}\left(a+b\right.}{15\sqrt{1+c^2x^2}}$$

$$=-\frac{2bdx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{5c\sqrt{1+c^2x^2}}-\frac{4bcdx^3\sqrt{d+c^2dx^2}\left(a+b\right.}{15\sqrt{1+c^2x^2}}$$

$$=-\frac{2bdx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{5c\sqrt{1+c^2x^2}}-\frac{4bcdx^3\sqrt{d+c^2dx^2}\left(a+b\right.}{15\sqrt{1+c^2x^2}}$$

$$=-\frac{2bdx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{5c\sqrt{1+c^2x^2}}-\frac{4bcdx^3\sqrt{d+c^2dx^2}\left(a+b\right.}{15\sqrt{1+c^2x^2}}$$

$$=\frac{16b^2d\sqrt{d+c^2dx^2}}{75c^2}+\frac{8b^2d\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}{225c^2}+\frac{2b^2d\left(1+c^2x\right.}{12}$$

Mathematica [A] time = 0.36, size = 198, normalized size = 0.74

$$\frac{d\sqrt{c^2dx^2+d}\left(225a^2\left(c^2x^2+1\right)^3-30abcx\left(3c^4x^4+10c^2x^2+15\right)\sqrt{c^2x^2+1}+30b\sinh^{-1}(cx)\left(15a\left(c^2x^2+1\right)^3\right.\right.}{1125c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d*Sqrt[d + c^2*d*x^2]*(225*a^2*(1 + c^2*x^2)^3 - 30*a*b*c*x*Sqrt[1 + c^2*x
^2]*(15 + 10*c^2*x^2 + 3*c^4*x^4) + 2*b^2*(149 + 187*c^2*x^2 + 47*c^4*x^4 +
 9*c^6*x^6) + 30*b*(15*a*(1 + c^2*x^2)^3 - b*c*x*Sqrt[1 + c^2*x^2]*(15 + 10
*c^2*x^2 + 3*c^4*x^4))*ArcSinh[c*x] + 225*b^2*(1 + c^2*x^2)^3*ArcSinh[c*x]^
2))/(1125*c^2*(1 + c^2*x^2))
```

fricas [A] time = 0.52, size = 332, normalized size = 1.24

$$\frac{225\left(b^2c^6dx^6+3\,b^2c^4dx^4+3\,b^2c^2dx^2+b^2d\right)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2+30\left(15\,abc^6dx^6+45\,abc^4dx^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/1125*(225*(b^2*c^6*d*x^6 + 3*b^2*c^4*d*x^4 + 3*b^2*c^2*d*x^2 + b^2*d)*sqr
t(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 30*(15*a*b*c^6*d*x^6 + 45
*a*b*c^4*d*x^4 + 45*a*b*c^2*d*x^2 + 15*a*b*d - (3*b^2*c^5*d*x^5 + 10*b^2*c^
3*d*x^3 + 15*b^2*c*d*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d)*log(c*x + sq
rt(c^2*x^2 + 1)) + (9*(25*a^2 + 2*b^2)*c^6*d*x^6 + (675*a^2 + 94*b^2)*c^4*d
*x^4 + (675*a^2 + 374*b^2)*c^2*d*x^2 + (225*a^2 + 298*b^2)*d - 30*(3*a*b*c^
5*d*x^5 + 10*a*b*c^3*d*x^3 + 15*a*b*c*d*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^
2 + d))/(c^4*x^2 + c^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.29, size = 1149, normalized size = 4.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/5*a^2/c^2/d*(c^2*d*x^2+d)^(5/2)+b^2*(1/4000*(d*(c^2*x^2+1))^(1/2)*(16*c^6
*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+1
3*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(25*arcsinh(c*x)^2-10*arcsinh(c*x)+2)*
d/c^2/(c^2*x^2+1)+1/288*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2
+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arcsinh(c*x)^2-6*arcsinh(
c*x)+2)*d/c^2/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+
1)^(1/2)+1)*(arcsinh(c*x)^2-2*arcsinh(c*x)+2)*d/c^2/(c^2*x^2+1)+1/16*(d*(c^
2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2+2*arcsinh
(c*x)+2)*d/c^2/(c^2*x^2+1)+1/288*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3
*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arcsinh(c*x)^2+6
*arcsinh(c*x)+2)*d/c^2/(c^2*x^2+1)+1/4000*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6
-16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^
2*x^2-5*c*x*(c^2*x^2+1)^(1/2)+1)*(25*arcsinh(c*x)^2+10*arcsinh(c*x)+2)*d/c^
2/(c^2*x^2+1))+2*a*b*(1/800*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6+16*c^5*x^5*(c
^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2+5*c*x*(c
^2*x^2+1)^(1/2)+1)*(-1+5*arcsinh(c*x))*d/c^2/(c^2*x^2+1)+1/96*(d*(c^2*x^2+1
))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)
^(1/2)+1)*(-1+3*arcsinh(c*x))*d/c^2/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*
(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))*d/c^2/(c^2*x^2+1)+1/16*
(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(1+arcsinh(c*x))*d/
c^2/(c^2*x^2+1)+1/96*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)
^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(1+3*arcsinh(c*x))*d/c^2/(c^2*x
^2+1)+1/800*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6-16*c^5*x^5*(c^2*x^2+1)^(1/2)+
28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2-5*c*x*(c^2*x^2+1)^(1/2)+
1)*(1+5*arcsinh(c*x))*d/c^2/(c^2*x^2+1))
```

maxima [A] time = 0.39, size = 230, normalized size = 0.86

$$\frac{\left(c^2dx^2+d\right)^{\frac{5}{2}}b^2\operatorname{arsinh}(cx)^2}{5\,c^2d}+\frac{2}{1125}\,b^2\left(\frac{9\,\sqrt{c^2x^2+1}\,c^2d^{\frac{5}{2}}x^4+38\,\sqrt{c^2x^2+1}\,d^{\frac{5}{2}}x^2+\frac{149\,\sqrt{c^2x^2+1}\,d^{\frac{5}{2}}}{c^2}}{d}-\frac{15\left(3\,c^4d^{\frac{5}{2}}x^5\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/5*(c^2*d*x^2 + d)^(5/2)*b^2*arcsinh(c*x)^2/(c^2*d) + 2/1125*b^2*((9*sqrt(
c^2*x^2 + 1)*c^2*d^(5/2)*x^4 + 38*sqrt(c^2*x^2 + 1)*d^(5/2)*x^2 + 149*sqrt(
c^2*x^2 + 1)*d^(5/2)/c^2)/d - 15*(3*c^4*d^(5/2)*x^5 + 10*c^2*d^(5/2)*x^3 +
15*d^(5/2)*x)*arcsinh(c*x)/(c*d)) + 2/5*(c^2*d*x^2 + d)^(5/2)*a*b*arcsinh(c
*x)/(c^2*d) + 1/5*(c^2*d*x^2 + d)^(5/2)*a^2/(c^2*d) - 2/75*(3*c^4*d^(5/2)*x
^5 + 10*c^2*d^(5/2)*x^3 + 15*d^(5/2)*x)*a*b/(c*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,(a+b\operatorname{asinh}(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}(a+b\operatorname{asinh}(cx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x*(d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**2, x)
```

3.269 $\int \left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=294

$$\frac{d\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^3}{8bc\sqrt{c^2x^2+1}}+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{3}{8}dx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{bc}{}$$

```
[Out] 1/4*x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2+15/64*b^2*d*x*(c^2*d*x^2+d)^
(1/2)+1/32*b^2*d*x*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)-1/8*b*d*(c^2*x^2+1)^(3/2
)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c+3/8*d*x*(a+b*arcsinh(c*x))^2*(c^
2*d*x^2+d)^(1/2)-9/64*b^2*d*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^
(1/2)-3/8*b*c*d*x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2
)+1/8*d*(a+b*arcsinh(c*x))^3*(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {5684, 5682, 5675, 5661, 321, 215, 5717, 195}

$$\frac{d\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^3}{8bc\sqrt{c^2x^2+1}}+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{3}{8}dx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{bc}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (15*b^2*d*x*Sqrt[d + c^2*d*x^2])/64 + (b^2*d*x*(1 + c^2*x^2)*Sqrt[d + c^2*d
*x^2])/32 - (9*b^2*d*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/(64*c*Sqrt[1 + c^2*x
^2]) - (3*b*c*d*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(8*Sqrt[1 + c
^2*x^2]) - (b*d*(1 + c^2*x^2)^(3/2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]
))/(8*c) + (3*d*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/8 + (x*(d + c
^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/4 + (d*Sqrt[d + c^2*d*x^2]*(a + b*A
rcSinh[c*x])^3)/(8*b*c*Sqrt[1 + c^2*x^2])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
```

```
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{1}{4}x\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{4}(3d)\int \sqrt{d+c^2dx^2}\,\left(a+b\right.$$

$$=-\frac{bd\left(1+c^2x^2\right)^{3/2}\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)}{8c}+\frac{3}{8}dx\sqrt{d+c^2dx^2}$$

$$=\frac{1}{32}b^2dx\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}-\frac{3bcdx^2\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(\right.}{8\sqrt{1+c^2x^2}}$$

$$=\frac{15}{64}b^2dx\sqrt{d+c^2dx^2}+\frac{1}{32}b^2dx\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}-\frac{3bcdx^2\sqrt{d}}{}$$

$$=\frac{15}{64}b^2dx\sqrt{d+c^2dx^2}+\frac{1}{32}b^2dx\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}-\frac{9b^2d\sqrt{d+c}}{64c}$$

Mathematica [A] time = 1.91, size = 329, normalized size = 1.12

$$\frac{288a^2d^{3/2}\sqrt{c^2x^2+1}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)+96a^2cdx\sqrt{c^2x^2+1}\left(2c^2x^2+5\right)\sqrt{c^2dx^2+d}-192abd\sqrt{c^2dx^2+\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (96*a^2*c*d*x*Sqrt[1 + c^2*x^2]*(5 + 2*c^2*x^2)*Sqrt[d + c^2*d*x^2] + 288*a
^2*d^(3/2)*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + 32*
b^2*d*Sqrt[d + c^2*d*x^2]*(4*ArcSinh[c*x]^3 - 6*ArcSinh[c*x]*Cosh[2*ArcSinh
[c*x]] + (3 + 6*ArcSinh[c*x]^2)*Sinh[2*ArcSinh[c*x]]) - 192*a*b*d*Sqrt[d +
c^2*d*x^2]*(Cosh[2*ArcSinh[c*x]] - 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*Ar
cSinh[c*x]])) - 12*a*b*d*Sqrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]^2 + Cosh[4*Arc
Sinh[c*x]] - 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]) - b^2*d*Sqrt[d + c^2*d*x^
2]*(32*ArcSinh[c*x]^3 + 12*ArcSinh[c*x]*Cosh[4*ArcSinh[c*x]] - 3*(1 + 8*Arc
Sinh[c*x]^2)*Sinh[4*ArcSinh[c*x]]))/(768*c*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(a^2c^2dx^2+a^2d+\left(b^2c^2dx^2+b^2d\right)\operatorname{arsinh}(cx)^2+2\left(abc^2dx^2+abd\right)\operatorname{arsinh}(cx)\right)\sqrt{c^2dx^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.28, size = 709, normalized size = 2.41

$$\frac{x\left(c^2d\,x^2+d\right)^{\frac{3}{2}}a^2}{4}+\frac{3a^2dx\sqrt{c^2d\,x^2+d}}{8}+\frac{3a^2d^2\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{8\sqrt{c^2d}}+\frac{5b^2\sqrt{d\left(c^2x^2+1\right)}\,d\operatorname{arcsinh}(cx)^2x}{8\left(c^2x^2+1\right)}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/4*x*(c^2*d*x^2+d)^(3/2)*a^2+3/8*a^2*d*x*(c^2*d*x^2+d)^(1/2)+3/8*a^2*d^2*l
n(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+5/8*b^2*(d*(c^2*
x^2+1))^(1/2)*d/(c^2*x^2+1)*arcsinh(c*x)^2*x-17/64*b^2*(d*(c^2*x^2+1))^(1/2
)*d/c/(c^2*x^2+1)^(1/2)*arcsinh(c*x)+1/4*b^2*(d*(c^2*x^2+1))^(1/2)*d*c^4/(c
^2*x^2+1)*arcsinh(c*x)^2*x^5+7/8*b^2*(d*(c^2*x^2+1))^(1/2)*d*c^2/(c^2*x^2+1
)*arcsinh(c*x)^2*x^3+1/32*b^2*(d*(c^2*x^2+1))^(1/2)*d*c^4/(c^2*x^2+1)*x^5+1
9/64*b^2*(d*(c^2*x^2+1))^(1/2)*d*c^2/(c^2*x^2+1)*x^3+17/64*b^2*(d*(c^2*x^2+
1))^(1/2)*d/(c^2*x^2+1)*x+1/8*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c
*arcsinh(c*x)^3*d-1/8*b^2*(d*(c^2*x^2+1))^(1/2)*d*c^3/(c^2*x^2+1)^(1/2)*arc
```

```
sinh(c*x)*x^4-5/8*b^2*(d*(c^2*x^2+1))^(1/2)*d*c/(c^2*x^2+1)^(1/2)*arcsinh(c
*x)*x^2+7/4*a*b*(d*(c^2*x^2+1))^(1/2)*d*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^3-5/
8*a*b*(d*(c^2*x^2+1))^(1/2)*d*c/(c^2*x^2+1)^(1/2)*x^2+5/4*a*b*(d*(c^2*x^2+1
))^(1/2)*d/(c^2*x^2+1)*arcsinh(c*x)*x+1/2*a*b*(d*(c^2*x^2+1))^(1/2)*d*c^4/(
c^2*x^2+1)*arcsinh(c*x)*x^5-1/8*a*b*(d*(c^2*x^2+1))^(1/2)*d*c^3/(c^2*x^2+1)
^(1/2)*x^4+3/8*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsinh(c*x)^2
*d-17/64*a*b*(d*(c^2*x^2+1))^(1/2)*d/c/(c^2*x^2+1)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,\mathrm{asinh}\,(c\,x))^2\,\left(d\,c^2\,x^2 + d\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\left(c^2x^2 + 1\right)\right)^{\frac{3}{2}} (a + b\,\mathrm{asinh}\,(cx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**2, x)
```

3.270 $\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x}\,dx$

Optimal. Leaf size=498

$$-\frac{2bd\sqrt{c^2dx^2+d}\,\mathrm{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}+\frac{2bd\sqrt{c^2dx^2+d}\,\mathrm{Li}_2\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}-\frac{2abcdx}{\sqrt{c}}$$

```
[Out] 1/3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2+22/9*b^2*d*(c^2*d*x^2+d)^(1/2)
+2/27*b^2*d*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)+d*(a+b*arcsinh(c*x))^2*(c^2*d*x
^2+d)^(1/2)-2*a*b*c*d*x*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2*b^2*c*d*x*a
rcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2/3*b*c*d*x*(a+b*arcsinh(
c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2/9*b*c^3*d*x^3*(a+b*arcsinh(c*
x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2*d*(a+b*arcsinh(c*x))^2*arctanh(
c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2*b*d*(a+b*arc
sinh(c*x))*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1
)^(1/2)+2*b*d*(a+b*arcsinh(c*x))*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^
2+d)^(1/2)/(c^2*x^2+1)^(1/2)+2*b^2*d*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*(c^2
*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2*b^2*d*polylog(3,c*x+(c^2*x^2+1)^(1/2))*
(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.59, antiderivative size = 498, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5744, 5742, 5760, 4182, 2531, 2282, 6589, 5653, 261, 5679, 444, 43}

$$-\frac{2bd\sqrt{c^2dx^2+d}\,\mathrm{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}+\frac{2bd\sqrt{c^2dx^2+d}\,\mathrm{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh\right.}{\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] (22*b^2*d*Sqrt[d + c^2*d*x^2])/9 - (2*a*b*c*d*x*Sqrt[d + c^2*d*x^2])/Sqrt[1
 + c^2*x^2] + (2*b^2*d*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2])/27 - (2*b^2*c*d*x
*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - (2*b*c*d*x*Sqrt[d +
c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*Sqrt[1 + c^2*x^2]) - (2*b*c^3*d*x^3*Sqr
t[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(9*Sqrt[1 + c^2*x^2]) + d*Sqrt[d + c
^2*d*x^2]*(a + b*ArcSinh[c*x])^2 + ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*
x])^2)/3 - (2*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSin
h[c*x]])/Sqrt[1 + c^2*x^2] - (2*b*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]
)*PolyLog[2, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (2*b*d*Sqrt[d + c^2*d*x^
2]*(a + b*ArcSinh[c*x])*PolyLog[2, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (2*
b^2*d*Sqrt[d + c^2*d*x^2]*PolyLog[3, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] -
(2*b^2*d*Sqrt[d + c^2*d*x^2]*PolyLog[3, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5679

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
```

```
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x}\,dx = \frac{1}{3}\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2+d\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(\right.}{x}$$

$$= -\frac{2bcdx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3\sqrt{1+c^2x^2}}-\frac{2bc^3dx^3\sqrt{d+c^2dx^2}\left(a+b\,\mathrm{si}\right.}{9\sqrt{1+c^2x^2}}$$

$$= -\frac{2abcdx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}-\frac{2bcdx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3\sqrt{1+c^2x^2}}-\frac{2bc^3dx}{}$$

$$= -\frac{2abcdx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}-\frac{2b^2cdx\sqrt{d+c^2dx^2}\,\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}-\frac{2bcdx\sqrt{d+c}}{3}$$

$$= \frac{22}{9}b^2d\sqrt{d+c^2dx^2}-\frac{2abcdx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}+\frac{2}{27}b^2d\left(1+c^2x^2\right)\sqrt{d+c^2d}$$

$$= \frac{22}{9}b^2d\sqrt{d+c^2dx^2}-\frac{2abcdx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}+\frac{2}{27}b^2d\left(1+c^2x^2\right)\sqrt{d+c^2d}$$

$$= \frac{22}{9}b^2d\sqrt{d+c^2dx^2}-\frac{2abcdx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}+\frac{2}{27}b^2d\left(1+c^2x^2\right)\sqrt{d+c^2d}$$

Mathematica [A] time = 2.29, size = 520, normalized size = 1.04

$$-a^2d^{3/2}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+d\right)+\frac{1}{3}a^2d\left(c^2x^2+4\right)\sqrt{c^2dx^2+d}+a^2d^{3/2}\log(cx)+\frac{2abd\sqrt{c^2dx^2+d}\left(\sqrt{c^2x^2+1}\,\sin\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] (a^2*d*(4 + c^2*x^2)*Sqrt[d + c^2*d*x^2])/3 - (2*a*b*d*Sqrt[d + c^2*d*x^2]*
(3*c*x + c^3*x^3 - 3*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]))/(9*Sqrt[1 + c^2*x^2
```

```
]) + a^2*d^(3/2)*Log[c*x] - a^2*d^(3/2)*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]
] + (2*a*b*d*Sqrt[d + c^2*d*x^2]*(-(c*x) + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] +
 ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Log[1 + E^(-ArcSinh
[c*x])] + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])]))/
Sqrt[1 + c^2*x^2] + (b^2*d*Sqrt[d + c^2*d*x^2]*(2*Sqrt[1 + c^2*x^2] - 2*c*x
*ArcSinh[c*x] + Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2 + ArcSinh[c*x]^2*(Log[1 -
E^(-ArcSinh[c*x])] - Log[1 + E^(-ArcSinh[c*x])]) + 2*ArcSinh[c*x]*(PolyLog[
2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])]) + 2*(PolyLog[3, -E^
(-ArcSinh[c*x])] - PolyLog[3, E^(-ArcSinh[c*x])])))/Sqrt[1 + c^2*x^2] + (b^
2*d*Sqrt[d + c^2*d*x^2]*(27*Sqrt[1 + c^2*x^2]*(2 + ArcSinh[c*x]^2) + (2 + 9
*ArcSinh[c*x]^2)*Cosh[3*ArcSinh[c*x]] - 6*ArcSinh[c*x]*(9*c*x + Sinh[3*ArcS
inh[c*x]])))/(108*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2c^2dx^2 + a^2d + \left(b^2c^2dx^2 + b^2d\right)\operatorname{arsinh}\left(cx\right)^2 + 2\left(abc^2dx^2 + abd\right)\operatorname{arsinh}\left(cx\right)\right)\sqrt{c^2dx^2 + d}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.37, size = 1053, normalized size = 2.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x,x)
```

```
[Out] 2/3*a*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*arcsinh(c*x)*x^4*c^4-a^2*d^(3/2
)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)+a^2*(c^2*d*x^2+d)^(1/2)*d+10/3*
a*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*arcsinh(c*x)*x^2*c^2+68/27*b^2*(d*(
c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)+4/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)
*arcsinh(c*x)^2-b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*
ln(1+c*x+(c^2*x^2+1)^(1/2))*d+2/27*b^2*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*
c^4*x^4+70/27*b^2*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*c^2*x^2-2*b^2*(d*(c^2
*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1
/2))*d+b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*ln(1-c*x-
(c^2*x^2+1)^(1/2))*d+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(
c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*d+8/3*a*b*(d*(c^2*x^2+1))^(1/2)*d/(c^
2*x^2+1)*arcsinh(c*x)+2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog
(2,c*x+(c^2*x^2+1)^(1/2))*d-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*p
olylog(2,-c*x-(c^2*x^2+1)^(1/2))*d+1/3*(c^2*d*x^2+d)^(3/2)*a^2+1/3*b^2*(d*(
c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*arcsinh(c*x)^2*x^4*c^4-2/9*b^2*(d*(c^2*x^2+
1))^(1/2)*d/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^3*c^3-8/3*b^2*(d*(c^2*x^2+1))^
```

```
(1/2)*d/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x*c-2/9*a*b*(d*(c^2*x^2+1))^(1/2)*d/
(c^2*x^2+1)^(1/2)*c^3*x^3-8/3*a*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)^(1/2)
*c*x+5/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*arcsinh(c*x)^2*x^2*c^2+2*a
*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1
)^(1/2))*d-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+
c*x+(c^2*x^2+1)^(1/2))*d+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*poly
log(3,-c*x-(c^2*x^2+1)^(1/2))*d-2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/
2)*polylog(3,c*x+(c^2*x^2+1)^(1/2))*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}\left(3\,d^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{1}{c|x|}\right)-\left(c^2dx^2+d\right)^{\frac{3}{2}}-3\,\sqrt{c^2dx^2+d}\,d\right)a^2+\int\frac{\left(c^2dx^2+d\right)^{\frac{3}{2}}b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{x}+\frac{2\left(c^2dx^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] -1/3*(3*d^(3/2)*arcsinh(1/(c*abs(x))) - (c^2*d*x^2 + d)^(3/2) - 3*sqrt(c^2*
d*x^2 + d)*d)*a^2 + integrate((c^2*d*x^2 + d)^(3/2)*b^2*log(c*x + sqrt(c^2*
x^2 + 1))^2/x + 2*(c^2*d*x^2 + d)^(3/2)*a*b*log(c*x + sqrt(c^2*x^2 + 1))/x,
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(d\,c^2\,x^2+d\right)}^{3/2}}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2))/x,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\left(c^{2}x^{2}+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{asinh}\left(cx\right)\right)^{2}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2/x,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**2/x, x)
```

3.271 $\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^2}\,dx$

Optimal. Leaf size=398

$$\frac{3}{2}c^2dx\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2+\frac{cd\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^3}{2b\sqrt{c^2x^2+1}}+\frac{cd\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{c^2x^2+1}}$$

```
[Out] -(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x+1/4*b^2*c^2*d*x*(c^2*d*x^2+d)^(
1/2)+3/2*c^2*d*x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)-5/4*b^2*c*d*arcsi
nh(c*x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-3/2*b*c^3*d*x^2*(a+b*arcsinh(
c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+c*d*(a+b*arcsinh(c*x))^2*(c^2*d
*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+1/2*c*d*(a+b*arcsinh(c*x))^3*(c^2*d*x^2+d)^
(1/2)/b/(c^2*x^2+1)^(1/2)+2*b*c*d*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(c^2*x^2+1
)^(1/2))^2)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-b^2*c*d*polylog(2,1/(c*x+
(c^2*x^2+1)^(1/2))^2)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+b*c*d*(a+b*arcs
inh(c*x))*(c^2*x^2+1)^(1/2)*(c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {5739, 5682, 5675, 5661, 321, 215, 5726, 5659, 3716, 2190, 2279, 2391, 195}

$$\frac{b^2cd\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}-\frac{3bc^3dx^2\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{c^2x^2+1}}+\frac{3}{2}c^2dx\sqrt{c^2dx^2+d}\,\left(a+b\,\text{si}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] (b^2*c^2*d*x*Sqrt[d + c^2*d*x^2])/4 - (5*b^2*c*d*Sqrt[d + c^2*d*x^2]*ArcSin
h[c*x])/(4*Sqrt[1 + c^2*x^2]) - (3*b*c^3*d*x^2*Sqrt[d + c^2*d*x^2]*(a + b*A
rcSinh[c*x]))/(2*Sqrt[1 + c^2*x^2]) + b*c*d*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*
d*x^2]*(a + b*ArcSinh[c*x]) + (3*c^2*d*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh
[c*x])^2)/2 - (c*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/Sqrt[1 + c^2
*x^2] - ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x + (c*d*Sqrt[d + c^
2*d*x^2]*(a + b*ArcSinh[c*x])^3)/(2*b*Sqrt[1 + c^2*x^2]) + (2*b*c*d*Sqrt[d
+ c^2*d*x^2]*(a + b*ArcSinh[c*x])*Log[1 - E^(2*ArcSinh[c*x])])/Sqrt[1 + c^2
*x^2] + (b^2*c*d*Sqrt[d + c^2*d*x^2]*PolyLog[2, E^(2*ArcSinh[c*x])])/Sqrt[1
 + c^2*x^2]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5726

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[((d + e*x^2)^p*(a + b*ArcSinh[c*x]))/(2*p), x] + (Dist[d
, Int[((d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]))/x, x], x] - Dist[(b*c*d^p)
/(2*p), Int[(1 + c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^2}\,dx = -\frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x}+\left(3c^2d\right)\int \sqrt{d+c^2dx^2}\,\left(a+b\right.$$

$$= bcd\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{2}c^2dx\sqrt{d+c^2dx^2}\,\left(\right.$$

$$= -\frac{1}{2}b^2c^2dx\sqrt{d+c^2dx^2}-\frac{3bc^3dx^2\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{1+c^2x^2}}+b$$

$$= \frac{1}{4}b^2c^2dx\sqrt{d+c^2dx^2}-\frac{b^2cd\sqrt{d+c^2dx^2}\,\sinh^{-1}(cx)}{2\sqrt{1+c^2x^2}}-\frac{3bc^3dx^2\sqrt{d+}}{}$$

$$= \frac{1}{4}b^2c^2dx\sqrt{d+c^2dx^2}-\frac{5b^2cd\sqrt{d+c^2dx^2}\,\sinh^{-1}(cx)}{4\sqrt{1+c^2x^2}}-\frac{3bc^3dx^2\sqrt{d}}{}$$

$$= \frac{1}{4}b^2c^2dx\sqrt{d+c^2dx^2}-\frac{5b^2cd\sqrt{d+c^2dx^2}\,\sinh^{-1}(cx)}{4\sqrt{1+c^2x^2}}-\frac{3bc^3dx^2\sqrt{d}}{}$$

$$= \frac{1}{4}b^2c^2dx\sqrt{d+c^2dx^2}-\frac{5b^2cd\sqrt{d+c^2dx^2}\,\sinh^{-1}(cx)}{4\sqrt{1+c^2x^2}}-\frac{3bc^3dx^2\sqrt{d}}{}$$

Mathematica [A] time = 2.97, size = 369, normalized size = 0.93

$$\frac{36a^2cd^{3/2}x\sqrt{c^2x^2+1}\,\log\left(\sqrt{d}\,\sqrt{c^2dx^2+d}+cdx\right)+12a^2d\left(c^2x^2-2\right)\sqrt{c^2x^2+1}\,\sqrt{c^2dx^2+d}+24abd\sqrt{c^2dx^2+}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] (12*a^2*d*(-2 + c^2*x^2)*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 24*a*b*d*S
qrt[d + c^2*d*x^2]*(-2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + c*x*ArcSinh[c*x]^2
+ 2*c*x*Log[c*x]) + 36*a^2*c*d^(3/2)*x*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d
]*Sqrt[d + c^2*d*x^2]] - 8*b^2*d*Sqrt[d + c^2*d*x^2]*(ArcSinh[c*x]*(3*Sqrt[
```

```
1 + c^2*x^2]*ArcSinh[c*x] - c*x*ArcSinh[c*x]*(3 + ArcSinh[c*x]) - 6*c*x*Log
[1 - E^(-2*ArcSinh[c*x])]) + 3*c*x*PolyLog[2, E^(-2*ArcSinh[c*x])]) + b^2*c
*d*x*Sqrt[d + c^2*d*x^2]*(4*ArcSinh[c*x]^3 - 6*ArcSinh[c*x]*Cosh[2*ArcSinh[
c*x]] + (3 + 6*ArcSinh[c*x]^2)*Sinh[2*ArcSinh[c*x]]) - 6*a*b*c*d*x*Sqrt[d +
 c^2*d*x^2]*(Cosh[2*ArcSinh[c*x]] - 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*A
rcSinh[c*x]])))/(24*x*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2c^2dx^2 + a^2d + \left(b^2c^2dx^2 + b^2d\right)\operatorname{arsinh}\left(cx\right)^2 + 2\left(abc^2dx^2 + abd\right)\operatorname{arsinh}\left(cx\right)\right)\sqrt{c^2dx^2 + d}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="fricas
")
```

```
[Out] integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.36, size = 954, normalized size = 2.40

$$-\frac{a^2\left(c^2d\,x^2+d\right)^{\frac{5}{2}}}{dx}+a^2c^2x\left(c^2d\,x^2+d\right)^{\frac{3}{2}}-\frac{ab\sqrt{d\left(c^2x^2+1\right)}\,dc}{4\sqrt{c^2x^2+1}}-\frac{ab\sqrt{d\left(c^2x^2+1\right)}\,d\,c^3x^2}{2\sqrt{c^2x^2+1}}+\frac{2ab\sqrt{d\left(c^2x^2+1\right)}\,\ln\left(\left(cx\right.\right.}{\sqrt{c^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^2,x)
```

```
[Out] -a^2/d/x*(c^2*d*x^2+d)^(5/2)+a^2*c^2*x*(c^2*d*x^2+d)^(3/2)-a*b*(d*(c^2*x^2+
1))^(1/2)*d*c^2/(c^2*x^2+1)*arcsinh(c*x)*x+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2
*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*d*c-b^2*(d*(c^2*x^2+1))^(1/2
)*arcsinh(c*x)^2*d/x/(c^2*x^2+1)+1/2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^
(1/2)*arcsinh(c*x)^3*d*c+1/4*b^2*(d*(c^2*x^2+1))^(1/2)*d*c^4/(c^2*x^2+1)*x^
3+1/4*b^2*(d*(c^2*x^2+1))^(1/2)*d*c^2/(c^2*x^2+1)*x+2*b^2*(d*(c^2*x^2+1))^(
1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*d*c-b^2*(d*(c^2*x^
2+1))^(1/2)*d*c/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2-1/4*b^2*(d*(c^2*x^2+1))^(1
/2)*d*c/(c^2*x^2+1)^(1/2)*arcsinh(c*x)-1/4*a*b*(d*(c^2*x^2+1))^(1/2)*d*c/(c
^2*x^2+1)^(1/2)-1/2*a*b*(d*(c^2*x^2+1))^(1/2)*d*c^3/(c^2*x^2+1)^(1/2)*x^2-2
*a*b*(d*(c^2*x^2+1))^(1/2)*d*c/(c^2*x^2+1)^(1/2)*arcsinh(c*x)-1/2*b^2*(d*(c
^2*x^2+1))^(1/2)*d*c^3/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^2+1/2*b^2*(d*(c^2*x
^2+1))^(1/2)*d*c^4/(c^2*x^2+1)*arcsinh(c*x)^2*x^3-1/2*b^2*(d*(c^2*x^2+1))^(
1/2)*d*c^2/(c^2*x^2+1)*arcsinh(c*x)^2*x+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^
2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*d*c+2*b^2*(d*(c^2*x^2+1
))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*d*c+3/2
*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*d*c-2*a*b*(d*(c
^2*x^2+1))^(1/2)*arcsinh(c*x)*d/x/(c^2*x^2+1)+2*a*b*(d*(c^2*x^2+1))^(1/2)/(
c^2*x^2+1)^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*d*c+3/2*a^2*c^2*d*x*(c^2*d
```

```
*x^2+d)^(1/2)+a*b*(d*(c^2*x^2+1))^(1/2)*d*c^4/(c^2*x^2+1)*arcsinh(c*x)*x^3+
3/2*a^2*c^2*d^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2))/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}(a+b\,\mathrm{asinh}\,(cx))^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2/x**2,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**2/x**2, x)
```

3.272 $\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^3}\,dx$

Optimal. Leaf size=541

$$-\frac{3bc^2d\sqrt{c^2dx^2+d}\,\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}+\frac{3bc^2d\sqrt{c^2dx^2+d}\,\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}+\frac{3}{2}c^2$$

```
[Out] -1/2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^2+2*b^2*c^2*d*(c^2*d*x^2+d)
^(1/2)+3/2*c^2*d*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)-3*a*b*c^3*d*x*(c^
2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-3*b^2*c^3*d*x*arcsinh(c*x)*(c^2*d*x^2+d)
^(1/2)/(c^2*x^2+1)^(1/2)-b*c*d*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x/(c^
2*x^2+1)^(1/2)+b*c^3*d*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)
^(1/2)-3*c^2*d*(a+b*arcsinh(c*x))^2*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x
^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-b^2*c^2*d*arctanh((c^2*x^2+1)^(1/2))*(c^2*d*x
^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-3*b*c^2*d*(a+b*arcsinh(c*x))*polylog(2,-c*x-(
c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+3*b*c^2*d*(a+b*arcs
inh(c*x))*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^
(1/2)+3*b^2*c^2*d*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^
2*x^2+1)^(1/2)-3*b^2*c^2*d*polylog(3,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(
1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.65, antiderivative size = 541, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.536, Rules used = {5739, 5742, 5760, 4182, 2531, 2282, 6589, 5653, 261, 14, 5730, 446, 80, 63, 208}

$$-\frac{3bc^2d\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}+\frac{3bc^2d\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)\left(a+b\text{s}\right.}{\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] 2*b^2*c^2*d*Sqrt[d + c^2*d*x^2] - (3*a*b*c^3*d*x*Sqrt[d + c^2*d*x^2])/Sqrt[
1 + c^2*x^2] - (3*b^2*c^3*d*x*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/Sqrt[1 + c^
2*x^2] - (b*c*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(x*Sqrt[1 + c^2*x
^2]) + (b*c^3*d*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/Sqrt[1 + c^2*x^
2] + (3*c^2*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/2 - ((d + c^2*d*x
^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/(2*x^2) - (3*c^2*d*Sqrt[d + c^2*d*x^2]*(a
 + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (b^2*c^2*
d*Sqrt[d + c^2*d*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]])/Sqrt[1 + c^2*x^2] - (3*b*
c^2*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcSinh[c*x]])
/Sqrt[1 + c^2*x^2] + (3*b*c^2*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*Po
lyLog[2, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (3*b^2*c^2*d*Sqrt[d + c^2*d*x
^2]*PolyLog[3, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (3*b^2*c^2*d*Sqrt[d +
c^2*d*x^2]*PolyLog[3, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^3}\,dx = -\frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{2x^2} + \frac{1}{2}\left(3c^2d\right)\int \frac{\sqrt{d+c^2dx^2}\left(a+\right.}{x}$$

$$= -\frac{bcd\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{1+c^2x^2}} + \frac{bc^3dx\sqrt{d+c^2dx^2}\left(a+b\sin\right.}{\sqrt{1+c^2x^2}}$$

$$= -\frac{3abc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{bcd\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{1+c^2x^2}} + \frac{bc^3d}{}$$

$$= -b^2c^2d\sqrt{d+c^2dx^2} - \frac{3abc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{3b^2c^3dx\sqrt{d+c^2dx^2}\,\mathrm{si}}{\sqrt{1+c^2x^2}}$$

$$= 2b^2c^2d\sqrt{d+c^2dx^2} - \frac{3abc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{3b^2c^3dx\sqrt{d+c^2dx^2}\,\mathrm{si}}{\sqrt{1+c^2x^2}}$$

$$= 2b^2c^2d\sqrt{d+c^2dx^2} - \frac{3abc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{3b^2c^3dx\sqrt{d+c^2dx^2}\,\mathrm{si}}{\sqrt{1+c^2x^2}}$$

$$= 2b^2c^2d\sqrt{d+c^2dx^2} - \frac{3abc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{3b^2c^3dx\sqrt{d+c^2dx^2}\,\mathrm{si}}{\sqrt{1+c^2x^2}}$$

Mathematica [A] time = 7.88, size = 771, normalized size = 1.43

$$-\frac{3}{2}a^2c^2d^{3/2}\log\left(\sqrt{d}\sqrt{d\left(c^2x^2+1\right)}+d\right)+\frac{3}{2}a^2c^2d^{3/2}\log(x)+\sqrt{d\left(c^2x^2+1\right)}\left(a^2c^2d-\frac{a^2d}{2x^2}\right)+\frac{2abc^2d\sqrt{d\left(c^2x^2+1\right.}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] (a^2*c^2*d - (a^2*d)/(2*x^2))*Sqrt[d*(1 + c^2*x^2)] + (3*a^2*c^2*d^(3/2)*Lo
g[x])/2 - (3*a^2*c^2*d^(3/2)*Log[d + Sqrt[d]*Sqrt[d*(1 + c^2*x^2)]])/2 + (2
*a*b*c^2*d*Sqrt[d*(1 + c^2*x^2)]*(-(c*x) + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] +
 ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Log[1 + E^(-ArcSinh
[c*x])] + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])]))/
Sqrt[1 + c^2*x^2] + b^2*c^2*d*Sqrt[d*(1 + c^2*x^2)]*(2 - (2*c*x*ArcSinh[c*x
])/Sqrt[1 + c^2*x^2] + ArcSinh[c*x]^2 + (ArcSinh[c*x]^2*(Log[1 - E^(-ArcSin
h[c*x])] - Log[1 + E^(-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2] + (2*ArcSinh[c*x]
*(PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])]))/Sqrt[1 +
 c^2*x^2] + (2*(PolyLog[3, -E^(-ArcSinh[c*x])] - PolyLog[3, E^(-ArcSinh[c*x
])]))/Sqrt[1 + c^2*x^2]) + (a*b*c^2*d*Sqrt[d*(1 + c^2*x^2)]*(-2*Coth[ArcSin
h[c*x]/2] - ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*x]*Log[1 - E^
(-ArcSinh[c*x])] - 4*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 4*PolyLog[2,
 -E^(-ArcSinh[c*x])] - 4*PolyLog[2, E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Sech[
ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSinh[c*x]/2]))/(4*Sqrt[1 + c^2*x^2]) + (b^2*c
^2*d*Sqrt[d*(1 + c^2*x^2)]*(-4*ArcSinh[c*x]*Coth[ArcSinh[c*x]/2] - ArcSinh[
c*x]^2*Csch[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*x]^2*Log[1 - E^(-ArcSinh[c*x])]
 - 4*ArcSinh[c*x]^2*Log[1 + E^(-ArcSinh[c*x])] + 8*Log[Tanh[ArcSinh[c*x]/2]
] + 8*ArcSinh[c*x]*PolyLog[2, -E^(-ArcSinh[c*x])] - 8*ArcSinh[c*x]*PolyLog[
2, E^(-ArcSinh[c*x])] + 8*PolyLog[3, -E^(-ArcSinh[c*x])] - 8*PolyLog[3, E^(
-ArcSinh[c*x])] - ArcSinh[c*x]^2*Sech[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*x]*Ta
nh[ArcSinh[c*x]/2]))/(8*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2c^2dx^2 + a^2d + \left(b^2c^2dx^2 + b^2d\right)\operatorname{arsinh}(cx)^2 + 2\left(abc^2dx^2 + abd\right)\operatorname{arsinh}(cx)\right)\sqrt{c^2dx^2 + d}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="fricas
")
```

```
[Out] integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^3, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.48, size = 1131, normalized size = 2.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^3,x)
```

```
[Out] 1/2*a^2*c^2*(c^2*d*x^2+d)^(3/2)+3/2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(
1/2)*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^2*d-3/2*b^2*(d*(c^2*x^2+1
))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^2*d
-3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*polylog(2,-c*x-
(c^2*x^2+1)^(1/2))*c^2*d+a*b*(d*(c^2*x^2+1))^(1/2)*c^2*d/(c^2*x^2+1)*arcsin
h(c*x)-a*b*arcsinh(c*x)*(d*(c^2*x^2+1))^(1/2)*d/x^2/(c^2*x^2+1)+3*a*b*(d*(c
^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c^2*d-3
*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/
2))*c^2*d+2*b^2*(d*(c^2*x^2+1))^(1/2)*c^2*d/(c^2*x^2+1)-1/2*a^2/d/x^2*(c^2*
d*x^2+d)^(5/2)-3/2*a^2*c^2*d^(3/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x
)+3/2*a^2*c^2*(c^2*d*x^2+d)^(1/2)*d+2*a*b*(d*(c^2*x^2+1))^(1/2)*c^4*d/(c^2*
x^2+1)*arcsinh(c*x)*x^2+3*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsi
nh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^2*d-3*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*
x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^2*d-2*a*b*(d*(c^2*x
^2+1))^(1/2)*c^3*d/(c^2*x^2+1)^(1/2)*x-a*b*(d*(c^2*x^2+1))^(1/2)*d/x/(c^2*x
^2+1)^(1/2)*c-2*b^2*(d*(c^2*x^2+1))^(1/2)*c^3*d/(c^2*x^2+1)^(1/2)*arcsinh(c
*x)*x-b^2*arcsinh(c*x)*(d*(c^2*x^2+1))^(1/2)*d/x/(c^2*x^2+1)^(1/2)*c+3*b^2*
(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2
+1)^(1/2))*c^2*d+b^2*(d*(c^2*x^2+1))^(1/2)*c^4*d/(c^2*x^2+1)*arcsinh(c*x)^2
*x^2+2*b^2*(d*(c^2*x^2+1))^(1/2)*c^4*d/(c^2*x^2+1)*x^2+1/2*b^2*(d*(c^2*x^2+
1))^(1/2)*c^2*d/(c^2*x^2+1)*arcsinh(c*x)^2+3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2
*x^2+1)^(1/2)*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*c^2*d-2*b^2*(d*(c^2*x^2+1))
^(1/2)/(c^2*x^2+1)^(1/2)*arctanh(c*x+(c^2*x^2+1)^(1/2))*c^2*d-3*b^2*(d*(c^2
*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(3,c*x+(c^2*x^2+1)^(1/2))*c^2*d-1/2
*b^2*arcsinh(c*x)^2*(d*(c^2*x^2+1))^(1/2)*d/x^2/(c^2*x^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\left(3\,c^2d^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{1}{c|x|}\right) - \left(c^2dx^2 + d\right)^{\frac{3}{2}}c^2 - 3\sqrt{c^2dx^2 + d}\,c^2d + \frac{\left(c^2dx^2 + d\right)^{\frac{5}{2}}}{dx^2}\right)a^2 + \int \frac{\left(c^2dx^2 + d\right)^{\frac{3}{2}}b^2\log\left(cx + \right.}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="maxima
")
```

```
[Out] -1/2*(3*c^2*d^(3/2)*arcsinh(1/(c*abs(x))) - (c^2*d*x^2 + d)^(3/2)*c^2 - 3*s
qrt(c^2*d*x^2 + d)*c^2*d + (c^2*d*x^2 + d)^(5/2)/(d*x^2))*a^2 + integrate((
c^2*d*x^2 + d)^(3/2)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/x^3 + 2*(c^2*d*x^2
+ d)^(3/2)*a*b*log(c*x + sqrt(c^2*x^2 + 1))/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2))/x^3,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}\,(a+b\,\mathrm{asinh}\,(cx))^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2/x**3,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**2/x**3, x)
```

3.273 $\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^4}\,dx$

Optimal. Leaf size=378

$$-\frac{c^2d\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{x}-\frac{bcd\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{3x^2}-\frac{\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}\right.}{3x^3}$$

```
[Out] -1/3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^3-1/3*b^2*c^2*d*(c^2*d*x^2+
d)^(1/2)/x-c^2*d*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x+1/3*b^2*c^3*d*a
rcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+4/3*c^3*d*(a+b*arcsinh(c*
x))^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+1/3*c^3*d*(a+b*arcsinh(c*x))^3*
(c^2*d*x^2+d)^(1/2)/b/(c^2*x^2+1)^(1/2)+8/3*b*c^3*d*(a+b*arcsinh(c*x))*ln(1
-1/(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-4/3*b^2
*c^3*d*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+
1)^(1/2)-1/3*b*c*d*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)*(c^2*d*x^2+d)^(1/2)
/x^2
```

Rubi [A] time = 0.58, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {5739, 5737, 5659, 3716, 2190, 2279, 2391, 5675, 5728, 277, 215}

$$\frac{4b^2c^3d\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{3\sqrt{c^2x^2+1}}+\frac{c^3d\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^3}{3b\sqrt{c^2x^2+1}}-\frac{4c^3d\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(\right.}{3\sqrt{c^2x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] -(b^2*c^2*d*Sqrt[d + c^2*d*x^2])/(3*x) + (b^2*c^3*d*Sqrt[d + c^2*d*x^2]*Arc
Sinh[c*x])/(3*Sqrt[1 + c^2*x^2]) - (b*c*d*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*
x^2]*(a + b*ArcSinh[c*x]))/(3*x^2) - (c^2*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcS
inh[c*x])^2)/x - (4*c^3*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(3*Sq
rt[1 + c^2*x^2]) - ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/(3*x^3) +
 (c^3*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^3)/(3*b*Sqrt[1 + c^2*x^2])
 + (8*b*c^3*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*Log[1 - E^(2*ArcSinh
[c*x])])/(3*Sqrt[1 + c^2*x^2]) + (4*b^2*c^3*d*Sqrt[d + c^2*d*x^2]*PolyLog[2
, E^(2*ArcSinh[c*x])])/(3*Sqrt[1 + c^2*x^2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5728

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*ArcSinh[c
*x]))/(f*(m + 1)), x] + (-Dist[(b*c*d^p)/(f*(m + 1)), Int[(f*x)^(m + 1)*(1
+ c^2*x^2)^(p - 1/2), x], x] - Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2
)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && ILtQ[(m + 1)/2, 0]
```

Rule 5737

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 1)*S
qrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x] -
 Dist[(c^2*Sqrt[d + e*x^2])/(f^2*(m + 1)*Sqrt[1 + c^2*x^2]), Int[((f*x)^(m
+ 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
```

```
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^4}\,dx = -\frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3x^3} + \left(c^2d\right)\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh\right.}{x^2}$$

$$= -\frac{bcd\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2} - \frac{c^2d\sqrt{d+c^2dx^2}\left(a+\right.}{x}$$

$$= -\frac{b^2c^2d\sqrt{d+c^2dx^2}}{3x} - \frac{bcd\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2} -$$

$$= -\frac{b^2c^2d\sqrt{d+c^2dx^2}}{3x} + \frac{b^2c^3d\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{3\sqrt{1+c^2x^2}} - \frac{bcd\sqrt{1+c^2x^2}\sqrt{}}{}$$

$$= -\frac{b^2c^2d\sqrt{d+c^2dx^2}}{3x} + \frac{b^2c^3d\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{3\sqrt{1+c^2x^2}} - \frac{bcd\sqrt{1+c^2x^2}\sqrt{}}{}$$

$$= -\frac{b^2c^2d\sqrt{d+c^2dx^2}}{3x} + \frac{b^2c^3d\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{3\sqrt{1+c^2x^2}} - \frac{bcd\sqrt{1+c^2x^2}\sqrt{}}{}$$

$$= -\frac{b^2c^2d\sqrt{d+c^2dx^2}}{3x} + \frac{b^2c^3d\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{3\sqrt{1+c^2x^2}} - \frac{bcd\sqrt{1+c^2x^2}\sqrt{}}{}$$

Mathematica [A] time = 1.42, size = 458, normalized size = 1.21

$$\frac{-4a^2c^2dx^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d} - a^2d\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d} + 3a^2c^3d^{3/2}x^3\sqrt{c^2x^2+1}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] (-(a*b*c*d*x*Sqrt[d + c^2*d*x^2]) - a^2*d*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*
x^2] - 4*a^2*c^2*d*x^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] - b^2*c^2*d*x^
2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + b*d*Sqrt[d + c^2*d*x^2]*(3*a*c^3*
x^3 - b*(-4*c^3*x^3 + Sqrt[1 + c^2*x^2] + 4*c^2*x^2*Sqrt[1 + c^2*x^2]))*Arc
Sinh[c*x]^2 + b^2*c^3*d*x^3*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^3 + b*d*Sqrt[d
 + c^2*d*x^2]*ArcSinh[c*x]*(-(b*c*x) - 2*a*Sqrt[1 + c^2*x^2]*(1 + 4*c^2*x^2
) + 8*b*c^3*x^3*Log[1 - E^(-2*ArcSinh[c*x])]) + 8*a*b*c^3*d*x^3*Sqrt[d + c^
2*d*x^2]*Log[c*x] + 3*a^2*c^3*d^(3/2)*x^3*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqr
t[d]*Sqrt[d + c^2*d*x^2]] - 4*b^2*c^3*d*x^3*Sqrt[d + c^2*d*x^2]*PolyLog[2,
E^(-2*ArcSinh[c*x])])/(3*x^3*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2c^2dx^2+a^2d+\left(b^2c^2dx^2+b^2d\right)\operatorname{arsinh}(cx)^2+2\left(abc^2dx^2+abd\right)\operatorname{arsinh}(cx)\right)\sqrt{c^2dx^2+d}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="fricas
")
```

```
[Out] integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^4, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.46, size = 2796, normalized size = 7.40

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^4,x)
```

```
[Out] -2/3*a^2*c^2/d/x*(c^2*d*x^2+d)^(5/2)-4/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^
4*x^4+9*c^2*x^2+1)*x^3*c^6+1/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^
2*x^2+1)/(c^2*x^2+1)^(1/2)*c^3+8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1
/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*d*c^3+8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c
^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*d*c^3-8/3*b^2*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*d*c^3+1/3*b^2*(d*(c^2*x^2+1))^(
1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^3*d*c^3-1/3*a^2/d/x^3*(c^2*d*x^2+d)^(5/
2)+2/3*a^2*c^4*x*(c^2*d*x^2+d)^(3/2)+64*a*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4
*x^4+9*c^2*x^2+1)*x^4/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^7+24*a*b*(d*(c^2*x^2
+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^
5-64*a*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^5/(c^2*x^2+1)*a
rcsinh(c*x)*c^8-104*a*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^
3/(c^2*x^2+1)*arcsinh(c*x)*c^6-146/3*a*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^
4+9*c^2*x^2+1)*x/(c^2*x^2+1)*arcsinh(c*x)*c^4-28/3*a*b*(d*(c^2*x^2+1))^(1/2
)*d/(24*c^4*x^4+9*c^2*x^2+1)/x/(c^2*x^2+1)*arcsinh(c*x)*c^2-20/3*b^2*(d*(c^
2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^3/(c^2*x^2+1)*arcsinh(c*x)*c^6
-73/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x/(c^2*x^2+1)*ar
csinh(c*x)^2*c^4-4/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x
/(c^2*x^2+1)*arcsinh(c*x)*c^4-14/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+
9*c^2*x^2+1)/x/(c^2*x^2+1)*arcsinh(c*x)^2*c^2-32*b^2*(d*(c^2*x^2+1))^(1/2)*
d/(24*c^4*x^4+9*c^2*x^2+1)*x^5/(c^2*x^2+1)*arcsinh(c*x)^2*c^8-16/3*b^2*(d*(
c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^5/(c^2*x^2+1)*arcsinh(c*x)*c
^8-52*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^3/(c^2*x^2+1)*
arcsinh(c*x)^2*c^6-8*a*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x
^2/(c^2*x^2+1)^(1/2)*c^5+8/3*a*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*
x^2+1)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^3-1/3*a*b*(d*(c^2*x^2+1))^(1/2)*d/(
24*c^4*x^4+9*c^2*x^2+1)/x^2/(c^2*x^2+1)^(1/2)*c+12*b^2*(d*(c^2*x^2+1))^(1/2
)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^5-8*b^2
*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*arc
sinh(c*x)*c^5-1/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)/x^2/
(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c+32*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4
+9*c^2*x^2+1)*x^4/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^7-16/3*a*b*(d*(c^2*x^2
+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^5/(c^2*x^2+1)*c^8-20/3*a*b*(d*(c^2*
x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^3/(c^2*x^2+1)*c^6-4/3*a*b*(d*(c^
2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x/(c^2*x^2+1)*c^4-2/3*a*b*(d*(c^
2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)/x^3/(c^2*x^2+1)*arcsinh(c*x)+a*b
```

```
*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*d*c^3-16/3*a*b*(d*(
c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*d*c^3+8/3*a*b*(d*(c^2*x^2+
1))^(1/2)/(c^2*x^2+1)^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*d*c^3+3*b^2*(d*
(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*c^5+4/3
*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)/(c^2*x^2+1)^(1/2)*arc
sinh(c*x)^2*c^3-3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)/(c^2
*x^2+1)^(1/2)*arcsinh(c*x)*c^3+8*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*
c^2*x^2+1)*x^4/(c^2*x^2+1)^(1/2)*c^7+16/3*a*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c
^4*x^4+9*c^2*x^2+1)*x^3*c^6+4/3*a*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c
^2*x^2+1)*x*c^4+16/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x
^3*arcsinh(c*x)*c^6+4/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1
)*x*arcsinh(c*x)*c^4-3*a*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)
/(c^2*x^2+1)^(1/2)*c^3-1/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^
2+1)/x/(c^2*x^2+1)*c^2-1/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^
2+1)/x^3/(c^2*x^2+1)*arcsinh(c*x)^2+8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+
1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*d*c^3+8/3*b^2*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*d*c^3
-20/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^5/(c^2*x^2+1)*
c^8-29/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^3/(c^2*x^2+
1)*c^6-10/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x/(c^2*x^2
+1)*c^4+a^2*c^4*d^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(
1/2)+a^2*c^4*d*x*(c^2*d*x^2+d)^(1/2)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2))/x^4,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2))/x^4, x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}\,(a+b\,\mathrm{asinh}\,(cx))^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2/x**4,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**2/x**4, x)
```

3.274 $\int x^3 \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=625

$$\frac{d^2x^2\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2}{63c^2}-\frac{2bcd^2x^5\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{21\sqrt{c^2x^2+1}}+\frac{1}{21}d^2x^4\sqrt{c^2dx^2+d}\,\left(a+b\sinh\right.$$

```
[Out] 5/63*d*x^4*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2+1/9*x^4*(c^2*d*x^2+d)^(
5/2)*(a+b*arcsinh(c*x))^2-160/3969*b^2*d^2*(c^2*d*x^2+d)^(1/2)/c^4-80/11907
*b^2*d^2*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)/c^4-4/1323*b^2*d^2*(c^2*x^2+1)^2*(
c^2*d*x^2+d)^(1/2)/c^4-50/27783*b^2*d^2*(c^2*x^2+1)^3*(c^2*d*x^2+d)^(1/2)/c
^4+2/729*b^2*d^2*(c^2*x^2+1)^4*(c^2*d*x^2+d)^(1/2)/c^4-2/63*d^2*(a+b*arcsin
h(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^4+1/63*d^2*x^2*(a+b*arcsinh(c*x))^2*(c^2*d*
x^2+d)^(1/2)/c^2+1/21*d^2*x^4*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)+4/63
*a*b*d^2*x*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)^(1/2)+4/63*b^2*d^2*x*arcsinh
(c*x)*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)^(1/2)-2/189*b*d^2*x^3*(a+b*arcsin
h(c*x))*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-2/21*b*c*d^2*x^5*(a+b*arcsi
nh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-38/441*b*c^3*d^2*x^7*(a+b*ar
csinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2/81*b*c^5*d^2*x^9*(a+b*a
rcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 1.24, antiderivative size = 625, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 18, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5744, 5742, 5758, 5717, 5653, 261, 5661, 266, 43, 14, 5730, 12, 446, 77, 270, 1251, 897, 1153}

$$\frac{4abd^2x\sqrt{c^2dx^2+d}}{63c^3\sqrt{c^2x^2+1}}-\frac{2bc^5d^2x^9\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{81\sqrt{c^2x^2+1}}-\frac{38bc^3d^2x^7\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{441\sqrt{c^2x^2+1}}-2bcd$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-160*b^2*d^2*Sqrt[d + c^2*d*x^2])/(3969*c^4) + (4*a*b*d^2*x*Sqrt[d + c^2*d
*x^2])/(63*c^3*Sqrt[1 + c^2*x^2]) - (80*b^2*d^2*(1 + c^2*x^2)*Sqrt[d + c^2*
d*x^2])/(11907*c^4) - (4*b^2*d^2*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2])/(1323
*c^4) - (50*b^2*d^2*(1 + c^2*x^2)^3*Sqrt[d + c^2*d*x^2])/(27783*c^4) + (2*b
^2*d^2*(1 + c^2*x^2)^4*Sqrt[d + c^2*d*x^2])/(729*c^4) + (4*b^2*d^2*x*Sqrt[d
 + c^2*d*x^2]*ArcSinh[c*x])/(63*c^3*Sqrt[1 + c^2*x^2]) - (2*b*d^2*x^3*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(189*c*Sqrt[1 + c^2*x^2]) - (2*b*c*d^2
*x^5*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(21*Sqrt[1 + c^2*x^2]) - (38
*b*c^3*d^2*x^7*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(441*Sqrt[1 + c^2*
x^2]) - (2*b*c^5*d^2*x^9*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(81*Sqrt
[1 + c^2*x^2]) - (2*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(63*c^4
) + (d^2*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(63*c^2) + (d^2*x^
4*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/21 + (5*d*x^4*(d + c^2*d*x^2)
^(3/2)*(a + b*ArcSinh[c*x])^2)/63 + (x^4*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSi
nh[c*x])^2)/9
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
```

```
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
```

```
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int x^3\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{1}{9}x^4\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{9}(5d)\int x^3\left(d+c^2dx^2\right)^{3/2}$$

$$=-\frac{2bcd^2x^5\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{45\sqrt{1+c^2x^2}}-\frac{4bc^3d^2x^7\sqrt{d+c^2dx^2}\left(a\right.}{63\sqrt{1+c^2}}$$

$$=-\frac{8bcd^2x^5\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{105\sqrt{1+c^2x^2}}-\frac{38bc^3d^2x^7\sqrt{d+c^2dx^2}\left(\right.}{441\sqrt{1+c}}$$

$$=-\frac{2bcd^2x^5\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{21\sqrt{1+c^2x^2}}-\frac{38bc^3d^2x^7\sqrt{d+c^2dx^2}\left(\right.}{441\sqrt{1+c}}$$

$$=-\frac{2bd^2x^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{189c\sqrt{1+c^2x^2}}-\frac{2bcd^2x^5\sqrt{d+c^2dx^2}\left(a+\right.}{21\sqrt{1+c^2x^2}}$$

$$=\frac{4abd^2x\sqrt{d+c^2dx^2}}{63c^3\sqrt{1+c^2x^2}}-\frac{2bd^2x^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{189c\sqrt{1+c^2x^2}}-\frac{2bcd}{}$$

$$=\frac{134b^2d^2\sqrt{d+c^2dx^2}}{3969c^4}+\frac{4abd^2x\sqrt{d+c^2dx^2}}{63c^3\sqrt{1+c^2x^2}}-\frac{122b^2d^2\left(1+c^2x^2\right)\sqrt{d}}{11907c^4}$$

$$=-\frac{160b^2d^2\sqrt{d+c^2dx^2}}{3969c^4}+\frac{4abd^2x\sqrt{d+c^2dx^2}}{63c^3\sqrt{1+c^2x^2}}-\frac{80b^2d^2\left(1+c^2x^2\right)\sqrt{d}}{11907c^4}$$

Mathematica [A] time = 0.46, size = 277, normalized size = 0.44

$$\frac{d^2\sqrt{c^2dx^2+d}\left(3969a^2\left(7c^2x^2-2\right)\left(c^2x^2+1\right)^4-126abcx\left(49c^8x^8+171c^6x^6+189c^4x^4+21c^2x^2-126\right)\sqrt{c^2x^2+1}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^2*Sqrt[d + c^2*d*x^2]*(3969*a^2*(1 + c^2*x^2)^4*(-2 + 7*c^2*x^2) - 126*a
*b*c*x*Sqrt[1 + c^2*x^2]*(-126 + 21*c^2*x^2 + 189*c^4*x^4 + 171*c^6*x^6 + 4
9*c^8*x^8) + 2*b^2*(-6140 - 7039*c^2*x^2 + 106*c^4*x^4 + 2152*c^6*x^6 + 149
0*c^8*x^8 + 343*c^10*x^10) - 126*b*(-63*a*(1 + c^2*x^2)^4*(-2 + 7*c^2*x^2)
```

```
+ b*c*x*Sqrt[1 + c^2*x^2]*(-126 + 21*c^2*x^2 + 189*c^4*x^4 + 171*c^6*x^6 +
49*c^8*x^8))*ArcSinh[c*x] + 3969*b^2*(1 + c^2*x^2)^4*(-2 + 7*c^2*x^2)*ArcSi
nh[c*x]^2))/(250047*c^4*(1 + c^2*x^2))
```

fricas [A] time = 0.61, size = 525, normalized size = 0.84

$$\frac{3969\left(7\,b^2c^{10}d^2x^{10}+26\,b^2c^8d^2x^8+34\,b^2c^6d^2x^6+16\,b^2c^4d^2x^4-b^2c^2d^2x^2-2\,b^2d^2\right)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas
")
```

```
[Out] 1/250047*(3969*(7*b^2*c^10*d^2*x^10 + 26*b^2*c^8*d^2*x^8 + 34*b^2*c^6*d^2*x
^6 + 16*b^2*c^4*d^2*x^4 - b^2*c^2*d^2*x^2 - 2*b^2*d^2)*sqrt(c^2*d*x^2 + d)*
log(c*x + sqrt(c^2*x^2 + 1))^2 + 126*(441*a*b*c^10*d^2*x^10 + 1638*a*b*c^8*
d^2*x^8 + 2142*a*b*c^6*d^2*x^6 + 1008*a*b*c^4*d^2*x^4 - 63*a*b*c^2*d^2*x^2
- 126*a*b*d^2 - (49*b^2*c^9*d^2*x^9 + 171*b^2*c^7*d^2*x^7 + 189*b^2*c^5*d^2
*x^5 + 21*b^2*c^3*d^2*x^3 - 126*b^2*c*d^2*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*
x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + (343*(81*a^2 + 2*b^2)*c^10*d^2*x^10
 + 2*(51597*a^2 + 1490*b^2)*c^8*d^2*x^8 + 2*(67473*a^2 + 2152*b^2)*c^6*d^2*
x^6 + 4*(15876*a^2 + 53*b^2)*c^4*d^2*x^4 - (3969*a^2 + 14078*b^2)*c^2*d^2*x
^2 - 2*(3969*a^2 + 6140*b^2)*d^2 - 126*(49*a*b*c^9*d^2*x^9 + 171*a*b*c^7*d^
2*x^7 + 189*a*b*c^5*d^2*x^5 + 21*a*b*c^3*d^2*x^3 - 126*a*b*c*d^2*x)*sqrt(c^
2*x^2 + 1))*sqrt(c^2*d*x^2 + d))/(c^6*x^2 + c^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.49, size = 2014, normalized size = 3.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] a^2*(1/9*x^2*(c^2*d*x^2+d)^(7/2)/c^2/d-2/63/d/c^4*(c^2*d*x^2+d)^(7/2))+b^2*
(1/373248*(d*(c^2*x^2+1))^(1/2)*(256*c^10*x^10+256*c^9*x^9*(c^2*x^2+1)^(1/2
)+704*c^8*x^8+576*c^7*x^7*(c^2*x^2+1)^(1/2)+688*c^6*x^6+432*c^5*x^5*(c^2*x^
2+1)^(1/2)+280*c^4*x^4+120*c^3*x^3*(c^2*x^2+1)^(1/2)+41*c^2*x^2+9*c*x*(c^2*
x^2+1)^(1/2)+1)*(81*arcsinh(c*x)^2-18*arcsinh(c*x)+2)*d^2/c^4/(c^2*x^2+1)+3
/175616*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8+64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*
c^6*x^6+112*c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4+56*c^3*x^3*(c^2*x^2+1)^(1
/2)+25*c^2*x^2+7*c*x*(c^2*x^2+1)^(1/2)+1)*(49*arcsinh(c*x)^2-14*arcsinh(c*x
)+2)*d^2/c^4/(c^2*x^2+1)-1/1728*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*
(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arcsinh(c*x)^2-6*
arcsinh(c*x)+2)*d^2/c^4/(c^2*x^2+1)-3/256*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*
x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2-2*arcsinh(c*x)+2)*d^2/c^4/(c^2*x^2+1
)-3/256*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*
x)^2+2*arcsinh(c*x)+2)*d^2/c^4/(c^2*x^2+1)-1/1728*(d*(c^2*x^2+1))^(1/2)*(4*
```

```
c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(9
*arcsinh(c*x)^2+6*arcsinh(c*x)+2)*d^2/c^4/(c^2*x^2+1)+3/175616*(d*(c^2*x^2+
1))^(1/2)*(64*c^8*x^8-64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6-112*c^5*x^5*
(c^2*x^2+1)^(1/2)+104*c^4*x^4-56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2-7*c*x
*(c^2*x^2+1)^(1/2)+1)*(49*arcsinh(c*x)^2+14*arcsinh(c*x)+2)*d^2/c^4/(c^2*x^
2+1)+1/373248*(d*(c^2*x^2+1))^(1/2)*(256*c^10*x^10-256*c^9*x^9*(c^2*x^2+1)^
(1/2)+704*c^8*x^8-576*c^7*x^7*(c^2*x^2+1)^(1/2)+688*c^6*x^6-432*c^5*x^5*(c^
2*x^2+1)^(1/2)+280*c^4*x^4-120*c^3*x^3*(c^2*x^2+1)^(1/2)+41*c^2*x^2-9*c*x*(
c^2*x^2+1)^(1/2)+1)*(81*arcsinh(c*x)^2+18*arcsinh(c*x)+2)*d^2/c^4/(c^2*x^2+
1))+2*a*b*(1/41472*(d*(c^2*x^2+1))^(1/2)*(256*c^10*x^10+256*c^9*x^9*(c^2*x^
2+1)^(1/2)+704*c^8*x^8+576*c^7*x^7*(c^2*x^2+1)^(1/2)+688*c^6*x^6+432*c^5*x^
5*(c^2*x^2+1)^(1/2)+280*c^4*x^4+120*c^3*x^3*(c^2*x^2+1)^(1/2)+41*c^2*x^2+9*
c*x*(c^2*x^2+1)^(1/2)+1)*(-1+9*arcsinh(c*x))*d^2/c^4/(c^2*x^2+1)+3/25088*(d
*(c^2*x^2+1))^(1/2)*(64*c^8*x^8+64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6+11
2*c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4+56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2
*x^2+7*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+7*arcsinh(c*x))*d^2/c^4/(c^2*x^2+1)-1/5
76*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3
*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arcsinh(c*x))*d^2/c^4/(c^2*x^2+1)-3/256*(d*
(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))*d^2/
c^4/(c^2*x^2+1)-3/256*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+
1)*(1+arcsinh(c*x))*d^2/c^4/(c^2*x^2+1)-1/576*(d*(c^2*x^2+1))^(1/2)*(4*c^4*
x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(1+3*a
rcsinh(c*x))*d^2/c^4/(c^2*x^2+1)+3/25088*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8-
64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6-112*c^5*x^5*(c^2*x^2+1)^(1/2)+104*
c^4*x^4-56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2-7*c*x*(c^2*x^2+1)^(1/2)+1)*
(1+7*arcsinh(c*x))*d^2/c^4/(c^2*x^2+1)+1/41472*(d*(c^2*x^2+1))^(1/2)*(256*c
^10*x^10-256*c^9*x^9*(c^2*x^2+1)^(1/2)+704*c^8*x^8-576*c^7*x^7*(c^2*x^2+1)^
(1/2)+688*c^6*x^6-432*c^5*x^5*(c^2*x^2+1)^(1/2)+280*c^4*x^4-120*c^3*x^3*(c^
2*x^2+1)^(1/2)+41*c^2*x^2-9*c*x*(c^2*x^2+1)^(1/2)+1)*(1+9*arcsinh(c*x))*d^2
/c^4/(c^2*x^2+1))
```

maxima [A] time = 0.42, size = 390, normalized size = 0.62

$$\frac{1}{63}\left(\frac{7\left(c^2dx^2+d\right)^{\frac{7}{2}}x^2}{c^2d}-\frac{2\left(c^2dx^2+d\right)^{\frac{7}{2}}}{c^4d}\right)b^2\operatorname{arsinh}(cx)^2+\frac{2}{63}\left(\frac{7\left(c^2dx^2+d\right)^{\frac{7}{2}}x^2}{c^2d}-\frac{2\left(c^2dx^2+d\right)^{\frac{7}{2}}}{c^4d}\right)ab\operatorname{arsinh}(cx)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima
")
```

```
[Out] 1/63*(7*(c^2*d*x^2 + d)^(7/2)*x^2/(c^2*d) - 2*(c^2*d*x^2 + d)^(7/2)/(c^4*d)
)*b^2*arcsinh(c*x)^2 + 2/63*(7*(c^2*d*x^2 + d)^(7/2)*x^2/(c^2*d) - 2*(c^2*d
*x^2 + d)^(7/2)/(c^4*d))*a*b*arcsinh(c*x) + 1/63*(7*(c^2*d*x^2 + d)^(7/2)*x
^2/(c^2*d) - 2*(c^2*d*x^2 + d)^(7/2)/(c^4*d))*a^2 + 2/250047*b^2*((343*sqrt
(c^2*x^2 + 1)*c^6*d^(5/2)*x^8 + 1147*sqrt(c^2*x^2 + 1)*c^4*d^(5/2)*x^6 + 10
05*sqrt(c^2*x^2 + 1)*c^2*d^(5/2)*x^4 - 899*sqrt(c^2*x^2 + 1)*d^(5/2)*x^2 -
6140*sqrt(c^2*x^2 + 1)*d^(5/2)/c^2)/c^2 - 63*(49*c^8*d^(5/2)*x^9 + 171*c^6*
d^(5/2)*x^7 + 189*c^4*d^(5/2)*x^5 + 21*c^2*d^(5/2)*x^3 - 126*d^(5/2)*x)*arc
sinh(c*x)/c^3) - 2/3969*(49*c^8*d^(5/2)*x^9 + 171*c^6*d^(5/2)*x^7 + 189*c^4
*d^(5/2)*x^5 + 21*c^2*d^(5/2)*x^3 - 126*d^(5/2)*x)*a*b/c^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(d\,c^2\,x^2+d\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int(x^3*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2,x)
[Out] Timed out
```

3.275 $\int x^2 \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=536

$$-\frac{5bd^2x^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{128c\sqrt{c^2x^2+1}}+\frac{5d^2x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{128c^2}-\frac{59bcd^2x^4\sqrt{c^2dx^2+d}\left(a+b\sinh\right.}{384\sqrt{c^2x^2+1}}$$

```
[Out] 5/48*d*x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2+1/8*x^3*(c^2*d*x^2+d)^(
5/2)*(a+b*arcsinh(c*x))^2-359/36864*b^2*d^2*x*(c^2*d*x^2+d)^(1/2)/c^2+1079/
55296*b^2*d^2*x^3*(c^2*d*x^2+d)^(1/2)+209/13824*b^2*c^2*d^2*x^5*(c^2*d*x^2+
d)^(1/2)+1/256*b^2*c^4*d^2*x^7*(c^2*d*x^2+d)^(1/2)+5/128*d^2*x*(a+b*arcsinh
(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^2+5/64*d^2*x^3*(a+b*arcsinh(c*x))^2*(c^2*d*x
^2+d)^(1/2)+359/36864*b^2*d^2*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2
+1)^(1/2)-5/128*b*d^2*x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2
+1)^(1/2)-59/384*b*c*d^2*x^4*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^
2+1)^(1/2)-17/144*b*c^3*d^2*x^6*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2
*x^2+1)^(1/2)-1/32*b*c^5*d^2*x^8*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^
2*x^2+1)^(1/2)-5/384*d^2*(a+b*arcsinh(c*x))^3*(c^2*d*x^2+d)^(1/2)/b/c^3/(c^
2*x^2+1)^(1/2)
```

Rubi [A] time = 1.05, antiderivative size = 536, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 14, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5744, 5742, 5758, 5675, 5661, 321, 215, 14, 5730, 12, 459, 266, 43, 1267}

$$-\frac{bc^5d^2x^8\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{32\sqrt{c^2x^2+1}}-\frac{17bc^3d^2x^6\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{144\sqrt{c^2x^2+1}}-\frac{59bcd^2x^4\sqrt{c^2dx^2+d}\left(a+b\right.}{384\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-359*b^2*d^2*x*Sqrt[d + c^2*d*x^2])/(36864*c^2) + (1079*b^2*d^2*x^3*Sqrt[d
 + c^2*d*x^2])/55296 + (209*b^2*c^2*d^2*x^5*Sqrt[d + c^2*d*x^2])/13824 + (b
^2*c^4*d^2*x^7*Sqrt[d + c^2*d*x^2])/256 + (359*b^2*d^2*Sqrt[d + c^2*d*x^2]*
ArcSinh[c*x])/(36864*c^3*Sqrt[1 + c^2*x^2]) - (5*b*d^2*x^2*Sqrt[d + c^2*d*x
^2]*(a + b*ArcSinh[c*x]))/(128*c*Sqrt[1 + c^2*x^2]) - (59*b*c*d^2*x^4*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(384*Sqrt[1 + c^2*x^2]) - (17*b*c^3*d^
2*x^6*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(144*Sqrt[1 + c^2*x^2]) - (
b*c^5*d^2*x^8*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(32*Sqrt[1 + c^2*x^
2]) + (5*d^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(128*c^2) + (5*d
^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/64 + (5*d*x^3*(d + c^2*d
*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/48 + (x^3*(d + c^2*d*x^2)^(5/2)*(a + b*
ArcSinh[c*x])^2)/8 - (5*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^3)/(38
4*b*c^3*Sqrt[1 + c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1267

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
```

```
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int x^2 \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right)^2 dx = \frac{1}{8}x^3 \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right)^2 + \frac{1}{8}(5d) \int x^2 \left(d+c^2dx^2\right)$$

$$= -\frac{bcd^2x^4\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{16\sqrt{1+c^2x^2}} - \frac{bc^3d^2x^6\sqrt{d+c^2dx^2}\left(a\right.}{12\sqrt{1+c}}$$

$$= -\frac{11bcd^2x^4\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{96\sqrt{1+c^2x^2}} - \frac{17bc^3d^2x^6\sqrt{d+c^2dx}}{144\sqrt{1}}$$

$$= \frac{1}{256}b^2c^4d^2x^7\sqrt{d+c^2dx^2} - \frac{59bcd^2x^4\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(c\right.}{384\sqrt{1+c^2x^2}}$$

$$= \frac{5}{512}b^2d^2x^3\sqrt{d+c^2dx^2} + \frac{209b^2c^2d^2x^5\sqrt{d+c^2dx^2}}{13824} + \frac{1}{256}b^2c^4d^2$$

$$= \frac{5b^2d^2x\sqrt{d+c^2dx^2}}{1024c^2} + \frac{1079b^2d^2x^3\sqrt{d+c^2dx^2}}{55296} + \frac{209b^2c^2d^2x^5\sqrt{d}}{13824}$$

$$= -\frac{359b^2d^2x\sqrt{d+c^2dx^2}}{36864c^2} + \frac{1079b^2d^2x^3\sqrt{d+c^2dx^2}}{55296} + \frac{209b^2c^2d^2x}{13}$$

$$= -\frac{359b^2d^2x\sqrt{d+c^2dx^2}}{36864c^2} + \frac{1079b^2d^2x^3\sqrt{d+c^2dx^2}}{55296} + \frac{209b^2c^2d^2x}{13}$$

Mathematica [A] time = 2.06, size = 619, normalized size = 1.15

$$\frac{d^2\left(34560a^2cx\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d} - 34560a^2\sqrt{d}\sqrt{c^2x^2+1}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right) + 110592a^2c^7x^7\sqrt{c}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^2*(34560*a^2*c*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 271872*a^2*c^3*
x^3*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 313344*a^2*c^5*x^5*Sqrt[1 + c^2
*x^2]*Sqrt[d + c^2*d*x^2] + 110592*a^2*c^7*x^7*Sqrt[1 + c^2*x^2]*Sqrt[d + c
^2*d*x^2] - 11520*b^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^3 + 13824*a*b*Sqrt[d
 + c^2*d*x^2]*Cosh[2*ArcSinh[c*x]] - 3456*a*b*Sqrt[d + c^2*d*x^2]*Cosh[4*Ar
cSinh[c*x]] - 1536*a*b*Sqrt[d + c^2*d*x^2]*Cosh[6*ArcSinh[c*x]] - 216*a*b*S
qrt[d + c^2*d*x^2]*Cosh[8*ArcSinh[c*x]] - 34560*a^2*Sqrt[d]*Sqrt[1 + c^2*x^
2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] - 6912*b^2*Sqrt[d + c^2*d*x^2]*
Sinh[2*ArcSinh[c*x]] + 864*b^2*Sqrt[d + c^2*d*x^2]*Sinh[4*ArcSinh[c*x]] + 2
56*b^2*Sqrt[d + c^2*d*x^2]*Sinh[6*ArcSinh[c*x]] + 27*b^2*Sqrt[d + c^2*d*x^2
]*Sinh[8*ArcSinh[c*x]] + 24*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*(576*b*Cosh[
2*ArcSinh[c*x]] - 144*b*Cosh[4*ArcSinh[c*x]] - 64*b*Cosh[6*ArcSinh[c*x]] -
9*b*Cosh[8*ArcSinh[c*x]] - 1152*a*Sinh[2*ArcSinh[c*x]] + 576*a*Sinh[4*ArcSi
nh[c*x]] + 384*a*Sinh[6*ArcSinh[c*x]] + 72*a*Sinh[8*ArcSinh[c*x]]) + 288*b*
Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^2*(-120*a - 48*b*Sinh[2*ArcSinh[c*x]] + 24
*b*Sinh[4*ArcSinh[c*x]] + 16*b*Sinh[6*ArcSinh[c*x]] + 3*b*Sinh[8*ArcSinh[c*
x]])))/(884736*c^3*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2c^4d^2x^6 + 2\,a^2c^2d^2x^4 + a^2d^2x^2 + \left(b^2c^4d^2x^6 + 2\,b^2c^2d^2x^4 + b^2d^2x^2\right)\text{arsinh}\,(cx)^2 + 2\left(abc^4d^2x^6 + 2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas
")
```

```
[Out] integral((a^2*c^4*d^2*x^6 + 2*a^2*c^2*d^2*x^4 + a^2*d^2*x^2 + (b^2*c^4*d^2*
x^6 + 2*b^2*c^2*d^2*x^4 + b^2*d^2*x^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^6
+ 2*a*b*c^2*d^2*x^4 + a*b*d^2*x^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c^2dx^2+d\right)^{\frac{5}{2}}(b\operatorname{arsinh}(cx)+a)^2x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^2*x^2, x)
```

maple [B] time = 0.58, size = 1204, normalized size = 2.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] -5/192*a^2/c^2*d*x*(c^2*d*x^2+d)^(3/2)-5/128*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2
*x^2+1)^(1/2)/c^3*arcsinh(c*x)^2*d^2+133/192*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/
(c^2*x^2+1)*arcsinh(c*x)*x^3-59/384*b^2*(d*(c^2*x^2+1))^(1/2)*d^2*c/(c^2*x^
2+1)^(1/2)*arcsinh(c*x)*x^4-5/128*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/c/(c^2*x^2+
1)^(1/2)*arcsinh(c*x)*x^2-1/32*b^2*(d*(c^2*x^2+1))^(1/2)*d^2*c^5/(c^2*x^2+1
)^(1/2)*arcsinh(c*x)*x^8-17/144*b^2*(d*(c^2*x^2+1))^(1/2)*d^2*c^3/(c^2*x^2+
1)^(1/2)*arcsinh(c*x)*x^6-1/32*a*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^5/(c^2*x^2+1
)^(1/2)*x^8-17/144*a*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^3/(c^2*x^2+1)^(1/2)*x^6-
59/384*a*b*(d*(c^2*x^2+1))^(1/2)*d^2*c/(c^2*x^2+1)^(1/2)*x^4-5/128*a*b*(d*(
c^2*x^2+1))^(1/2)*d^2/c/(c^2*x^2+1)^(1/2)*x^2+1/8*b^2*(d*(c^2*x^2+1))^(1/2)
*d^2*c^6/(c^2*x^2+1)*arcsinh(c*x)^2*x^9+23/48*b^2*(d*(c^2*x^2+1))^(1/2)*d^2
*c^4/(c^2*x^2+1)*arcsinh(c*x)^2*x^7+127/192*b^2*(d*(c^2*x^2+1))^(1/2)*d^2*c
^2/(c^2*x^2+1)*arcsinh(c*x)^2*x^5+5/128*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/c^2/(
c^2*x^2+1)*arcsinh(c*x)^2*x-1/48*a^2/c^2*x*(c^2*d*x^2+d)^(5/2)+1/4*a*b*(d*(
c^2*x^2+1))^(1/2)*d^2*c^6/(c^2*x^2+1)*arcsinh(c*x)*x^9+23/24*a*b*(d*(c^2*x^
2+1))^(1/2)*d^2*c^4/(c^2*x^2+1)*arcsinh(c*x)*x^7+127/96*a*b*(d*(c^2*x^2+1))
^(1/2)*d^2*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^5+5/64*a*b*(d*(c^2*x^2+1))^(1/2)*
d^2/c^2/(c^2*x^2+1)*arcsinh(c*x)*x+359/36864*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/
c^3/(c^2*x^2+1)^(1/2)-5/384*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3
*arcsinh(c*x)^3*d^2+133/384*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsi
nh(c*x)^2*x^3+359/36864*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/c^3/(c^2*x^2+1)^(1/2)
*arcsinh(c*x)+1/256*b^2*(d*(c^2*x^2+1))^(1/2)*d^2*c^6/(c^2*x^2+1)*x^9+263/1
3824*b^2*(d*(c^2*x^2+1))^(1/2)*d^2*c^4/(c^2*x^2+1)*x^7+1915/55296*b^2*(d*(c
^2*x^2+1))^(1/2)*d^2*c^2/(c^2*x^2+1)*x^5-359/36864*b^2*(d*(c^2*x^2+1))^(1/2
)*d^2/c^2/(c^2*x^2+1)*x+1081/110592*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+
1)*x^3-5/128*a^2/c^2*d^3*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2
*d)^(1/2)+1/8*a^2*x*(c^2*d*x^2+d)^(7/2)/c^2/d-5/128*a^2/c^2*d^2*x*(c^2*d*x^
2+d)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,(a+b\,\mathrm{asinh}\,(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int(x^2*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

3.276 $\int x\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=366

$$-\frac{2bd^2x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{7c\sqrt{c^2x^2+1}}-\frac{2bcd^2x^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{7\sqrt{c^2x^2+1}}+\frac{\left(c^2dx^2+d\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right.}{7c^2d}$$

```
[Out] 1/7*(c^2*d*x^2+d)^(7/2)*(a+b*arcsinh(c*x))^2/c^2/d+32/245*b^2*d^2*(c^2*d*x^
2+d)^(1/2)/c^2+16/735*b^2*d^2*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)/c^2+12/1225*b
^2*d^2*(c^2*x^2+1)^2*(c^2*d*x^2+d)^(1/2)/c^2+2/343*b^2*d^2*(c^2*x^2+1)^3*(c
^2*d*x^2+d)^(1/2)/c^2-2/7*b*d^2*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c/
(c^2*x^2+1)^(1/2)-2/7*b*c*d^2*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c
^2*x^2+1)^(1/2)-6/35*b*c^3*d^2*x^5*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(
c^2*x^2+1)^(1/2)-2/49*b*c^5*d^2*x^7*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/
(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5717, 194, 5679, 12, 1799, 1850}

$$-\frac{2bc^5d^2x^7\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{49\sqrt{c^2x^2+1}}-\frac{6bc^3d^2x^5\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{35\sqrt{c^2x^2+1}}-\frac{2bcd^2x^3\sqrt{c^2dx^2+d}\left(a+b\,\mathrm{s}\right.}{7\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (32*b^2*d^2*Sqrt[d + c^2*d*x^2])/(245*c^2) + (16*b^2*d^2*(1 + c^2*x^2)*Sqrt
[d + c^2*d*x^2])/(735*c^2) + (12*b^2*d^2*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2
])/(1225*c^2) + (2*b^2*d^2*(1 + c^2*x^2)^3*Sqrt[d + c^2*d*x^2])/(343*c^2) -
 (2*b*d^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(7*c*Sqrt[1 + c^2*x^2
]) - (2*b*c*d^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(7*Sqrt[1 + c
^2*x^2]) - (6*b*c^3*d^2*x^5*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(35*S
qrt[1 + c^2*x^2]) - (2*b*c^5*d^2*x^7*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x
]))/(49*Sqrt[1 + c^2*x^2]) + ((d + c^2*d*x^2)^(7/2)*(a + b*ArcSinh[c*x])^2)
/(7*c^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rule 5679

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\int x\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{\left(d+c^2dx^2\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)^2}{7c^2d}-\frac{\left(2bd^2\sqrt{d+c^2dx^2}\right)\int\left(1+c^2\right.}{7c\sqrt{1+}}$$

$$=-\frac{2bd^2x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{7c\sqrt{1+c^2x^2}}-\frac{2bcd^2x^3\sqrt{d+c^2dx^2}\left(a+\right.}{7\sqrt{1+c^2x^2}}$$

$$=-\frac{2bd^2x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{7c\sqrt{1+c^2x^2}}-\frac{2bcd^2x^3\sqrt{d+c^2dx^2}\left(a+\right.}{7\sqrt{1+c^2x^2}}$$

$$=-\frac{2bd^2x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{7c\sqrt{1+c^2x^2}}-\frac{2bcd^2x^3\sqrt{d+c^2dx^2}\left(a+\right.}{7\sqrt{1+c^2x^2}}$$

$$=-\frac{2bd^2x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{7c\sqrt{1+c^2x^2}}-\frac{2bcd^2x^3\sqrt{d+c^2dx^2}\left(a+\right.}{7\sqrt{1+c^2x^2}}$$

$$=\frac{32b^2d^2\sqrt{d+c^2dx^2}}{245c^2}+\frac{16b^2d^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}{735c^2}+\frac{12b^2d^2\left(1\right.}{}$$

Mathematica [A] time = 0.43, size = 224, normalized size = 0.61

$$\frac{d^2\sqrt{c^2dx^2+d}\left(3675a^2\left(c^2x^2+1\right)^4-210abcx\left(5c^6x^6+21c^4x^4+35c^2x^2+35\right)\sqrt{c^2x^2+1}+210b\sinh^{-1}(cx)\left(3\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^2*Sqrt[d + c^2*d*x^2]*(3675*a^2*(1 + c^2*x^2)^4 - 210*a*b*c*x*Sqrt[1 + c
^2*x^2]*(35 + 35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6) + 2*b^2*(2161 + 2918*c^2
*x^2 + 1108*c^4*x^4 + 426*c^6*x^6 + 75*c^8*x^8) + 210*b*(35*a*(1 + c^2*x^2)
^4 - b*c*x*Sqrt[1 + c^2*x^2]*(35 + 35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6))*Ar
cSinh[c*x] + 3675*b^2*(1 + c^2*x^2)^4*ArcSinh[c*x]^2))/(25725*c^2*(1 + c^2*
x^2))
```

fricas [A] time = 0.63, size = 446, normalized size = 1.22

$$\frac{3675\left(b^2c^8d^2x^8+4\,b^2c^6d^2x^6+6\,b^2c^4d^2x^4+4\,b^2c^2d^2x^2+b^2d^2\right)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2+210\left(35\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/25725*(3675*(b^2*c^8*d^2*x^8 + 4*b^2*c^6*d^2*x^6 + 6*b^2*c^4*d^2*x^4 + 4*
b^2*c^2*d^2*x^2 + b^2*d^2)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))
^2 + 210*(35*a*b*c^8*d^2*x^8 + 140*a*b*c^6*d^2*x^6 + 210*a*b*c^4*d^2*x^4 +
140*a*b*c^2*d^2*x^2 + 35*a*b*d^2 - (5*b^2*c^7*d^2*x^7 + 21*b^2*c^5*d^2*x^5
+ 35*b^2*c^3*d^2*x^3 + 35*b^2*c*d^2*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 +
d)*log(c*x + sqrt(c^2*x^2 + 1)) + (75*(49*a^2 + 2*b^2)*c^8*d^2*x^8 + 12*(12
25*a^2 + 71*b^2)*c^6*d^2*x^6 + 2*(11025*a^2 + 1108*b^2)*c^4*d^2*x^4 + 4*(36
75*a^2 + 1459*b^2)*c^2*d^2*x^2 + (3675*a^2 + 4322*b^2)*d^2 - 210*(5*a*b*c^7
*d^2*x^7 + 21*a*b*c^5*d^2*x^5 + 35*a*b*c^3*d^2*x^3 + 35*a*b*c*d^2*x)*sqrt(c
^2*x^2 + 1))*sqrt(c^2*d*x^2 + d))/(c^4*x^2 + c^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.41, size = 1773, normalized size = 4.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/7*a^2/c^2/d*(c^2*d*x^2+d)^(7/2)+b^2*(1/43904*(d*(c^2*x^2+1))^(1/2)*(64*c^
8*x^8+64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6+112*c^5*x^5*(c^2*x^2+1)^(1/2
)+104*c^4*x^4+56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2+7*c*x*(c^2*x^2+1)^(1/
2)+1)*(49*arcsinh(c*x)^2-14*arcsinh(c*x)+2)*d^2/c^2/(c^2*x^2+1)+1/3200*(d*(
c^2*x^2+1))^(1/2)*(16*c^6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^
3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(25*arcsinh(c
*x)^2-10*arcsinh(c*x)+2)*d^2/c^2/(c^2*x^2+1)+1/384*(d*(c^2*x^2+1))^(1/2)*(4
*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(
9*arcsinh(c*x)^2-6*arcsinh(c*x)+2)*d^2/c^2/(c^2*x^2+1)+5/128*(d*(c^2*x^2+1)
)^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2-2*arcsinh(c*x)+2)
*d^2/c^2/(c^2*x^2+1)+5/128*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(
1/2)+1)*(arcsinh(c*x)^2+2*arcsinh(c*x)+2)*d^2/c^2/(c^2*x^2+1)+1/384*(d*(c^2
*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*
x^2+1)^(1/2)+1)*(9*arcsinh(c*x)^2+6*arcsinh(c*x)+2)*d^2/c^2/(c^2*x^2+1)+1/3
200*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6-16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x
^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2-5*c*x*(c^2*x^2+1)^(1/2)+1)*(25*a
rcsinh(c*x)^2+10*arcsinh(c*x)+2)*d^2/c^2/(c^2*x^2+1)+1/43904*(d*(c^2*x^2+1)
)^(1/2)*(64*c^8*x^8-64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6-112*c^5*x^5*(c
^2*x^2+1)^(1/2)+104*c^4*x^4-56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2-7*c*x*(
c^2*x^2+1)^(1/2)+1)*(49*arcsinh(c*x)^2+14*arcsinh(c*x)+2)*d^2/c^2/(c^2*x^2+
1))+2*a*b*(1/6272*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8+64*c^7*x^7*(c^2*x^2+1)^
(1/2)+144*c^6*x^6+112*c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4+56*c^3*x^3*(c^2
*x^2+1)^(1/2)+25*c^2*x^2+7*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+7*arcsinh(c*x))*d^2
/c^2/(c^2*x^2+1)+1/640*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6+16*c^5*x^5*(c^2*x^
2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2+5*c*x*(c^2*x^
2+1)^(1/2)+1)*(-1+5*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+1/128*(d*(c^2*x^2+1))
^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(
```

```
1/2)+1)*(-1+3*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+5/128*(d*(c^2*x^2+1))^(1/2)
*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+5/
128*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(1+arcsinh(c*x)
)*d^2/c^2/(c^2*x^2+1)+1/128*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2
*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(1+3*arcsinh(c*x))*d^2/c
^2/(c^2*x^2+1)+1/640*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6-16*c^5*x^5*(c^2*x^2+
1)^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2-5*c*x*(c^2*x^2+
1)^(1/2)+1)*(1+5*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+1/6272*(d*(c^2*x^2+1))^(
1/2)*(64*c^8*x^8-64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6-112*c^5*x^5*(c^2*
x^2+1)^(1/2)+104*c^4*x^4-56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2-7*c*x*(c^2
*x^2+1)^(1/2)+1)*(1+7*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1))
```

maxima [A] time = 0.49, size = 274, normalized size = 0.75

$$\frac{\left(c^2dx^2+d\right)^{\frac{7}{2}}b^2\operatorname{arsinh}(cx)^2}{7\,c^2d}+\frac{2\left(c^2dx^2+d\right)^{\frac{7}{2}}ab\operatorname{arsinh}(cx)}{7\,c^2d}+\frac{2}{25725}\,b^2\left(\frac{75\,\sqrt{c^2x^2+1}\,c^4d^{\frac{7}{2}}x^6+351\,\sqrt{c^2x^2+1}\,c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/7*(c^2*d*x^2 + d)^(7/2)*b^2*arcsinh(c*x)^2/(c^2*d) + 2/7*(c^2*d*x^2 + d)^
(7/2)*a*b*arcsinh(c*x)/(c^2*d) + 2/25725*b^2*((75*sqrt(c^2*x^2 + 1)*c^4*d^(
7/2)*x^6 + 351*sqrt(c^2*x^2 + 1)*c^2*d^(7/2)*x^4 + 757*sqrt(c^2*x^2 + 1)*d^
(7/2)*x^2 + 2161*sqrt(c^2*x^2 + 1)*d^(7/2)/c^2)/d - 105*(5*c^6*d^(7/2)*x^7
+ 21*c^4*d^(7/2)*x^5 + 35*c^2*d^(7/2)*x^3 + 35*d^(7/2)*x)*arcsinh(c*x)/(c*d
)) + 1/7*(c^2*d*x^2 + d)^(7/2)*a^2/(c^2*d) - 2/245*(5*c^6*d^(7/2)*x^7 + 21*
c^4*d^(7/2)*x^5 + 35*c^2*d^(7/2)*x^3 + 35*d^(7/2)*x)*a*b/(c*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,(a+b\operatorname{asinh}(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int(x*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(d\left(c^2x^2+1\right)\right)^{\frac{5}{2}}(a+b\operatorname{asinh}(cx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x*(d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))**2, x)
```

3.277 $\int \left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=420

$$\frac{5d^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^3}{48bc\sqrt{c^2x^2+1}}+\frac{5}{16}d^2x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{bd^2\left(c^2x^2+1\right)^{5/2}\sqrt{c^2dx^2+d}\left(a+b\right.}{18c}$$

```
[Out] 5/24*d*x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2+1/6*x*(c^2*d*x^2+d)^(5/2)
*(a+b*arcsinh(c*x))^2+245/1152*b^2*d^2*x*(c^2*d*x^2+d)^(1/2)+65/1728*b^2*d^
2*x*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)+1/108*b^2*d^2*x*(c^2*x^2+1)^2*(c^2*d*x^
2+d)^(1/2)-5/48*b*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1
/2)/c-1/18*b*d^2*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c
+5/16*d^2*x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)-115/1152*b^2*d^2*arcsi
nh(c*x)*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-5/16*b*c*d^2*x^2*(a+b*arcsi
nh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+5/48*d^2*(a+b*arcsinh(c*x))^
3*(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 420, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {5684, 5682, 5675, 5661, 321, 215, 5717, 195}

$$\frac{5d^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^3}{48bc\sqrt{c^2x^2+1}}+\frac{5}{16}d^2x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{bd^2\left(c^2x^2+1\right)^{5/2}\sqrt{c^2dx^2+d}\left(a+b\right.}{18c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (245*b^2*d^2*x*Sqrt[d + c^2*d*x^2])/1152 + (65*b^2*d^2*x*(1 + c^2*x^2)*Sqrt
[d + c^2*d*x^2])/1728 + (b^2*d^2*x*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2])/108
 - (115*b^2*d^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/(1152*c*Sqrt[1 + c^2*x^2]
) - (5*b*c*d^2*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(16*Sqrt[1 + c
^2*x^2]) - (5*b*d^2*(1 + c^2*x^2)^(3/2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[
c*x]))/(48*c) - (b*d^2*(1 + c^2*x^2)^(5/2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSi
nh[c*x]))/(18*c) + (5*d^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/16
+ (5*d*x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/24 + (x*(d + c^2*d*x
^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/6 + (5*d^2*Sqrt[d + c^2*d*x^2]*(a + b*Arc
Sinh[c*x])^3)/(48*b*c*Sqrt[1 + c^2*x^2])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx = \frac{1}{6}x\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{6}(5d)\int \left(d+c^2dx^2\right)^{3/2}\left(a+b\right.$$

$$= -\frac{bd^2\left(1+c^2x^2\right)^{5/2}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{18c}+\frac{5}{24}dx\left(d+c^2dx^2\right)$$

$$= \frac{1}{108}b^2d^2x\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}-\frac{5bd^2\left(1+c^2x^2\right)^{3/2}\sqrt{d+c^2dx^2}\left(a\right.}{48c}$$

$$= \frac{65b^2d^2x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}{1728}+\frac{1}{108}b^2d^2x\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}-$$

$$= \frac{245b^2d^2x\sqrt{d+c^2dx^2}}{1152}+\frac{65b^2d^2x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}{1728}+\frac{1}{108}b^2d^2x\left(\right.$$

$$= \frac{245b^2d^2x\sqrt{d+c^2dx^2}}{1152}+\frac{65b^2d^2x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}{1728}+\frac{1}{108}b^2d^2x\left(\right.$$

Mathematica [A] time = 1.52, size = 499, normalized size = 1.19

$$\frac{d^2\left(9504a^2cx\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}+4320a^2\sqrt{d}\sqrt{c^2x^2+1}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)+2304a^2c^5x^5\sqrt{c^2x^2+1}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^2*(9504*a^2*c*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 7488*a^2*c^3*x^3
*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 2304*a^2*c^5*x^5*Sqrt[1 + c^2*x^2]
*Sqrt[d + c^2*d*x^2] + 1440*b^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^3 - 3240*a
*b*Sqrt[d + c^2*d*x^2]*Cosh[2*ArcSinh[c*x]] - 324*a*b*Sqrt[d + c^2*d*x^2]*C
osh[4*ArcSinh[c*x]] - 24*a*b*Sqrt[d + c^2*d*x^2]*Cosh[6*ArcSinh[c*x]] + 432
0*a^2*Sqrt[d]*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] +
1620*b^2*Sqrt[d + c^2*d*x^2]*Sinh[2*ArcSinh[c*x]] + 81*b^2*Sqrt[d + c^2*d*x
^2]*Sinh[4*ArcSinh[c*x]] + 4*b^2*Sqrt[d + c^2*d*x^2]*Sinh[6*ArcSinh[c*x]] -
 12*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*(270*b*Cosh[2*ArcSinh[c*x]] + 27*b*C
osh[4*ArcSinh[c*x]] + 2*b*Cosh[6*ArcSinh[c*x]] - 540*a*Sinh[2*ArcSinh[c*x]]
 - 108*a*Sinh[4*ArcSinh[c*x]] - 12*a*Sinh[6*ArcSinh[c*x]]) + 72*b*Sqrt[d +
c^2*d*x^2]*ArcSinh[c*x]^2*(60*a + 45*b*Sinh[2*ArcSinh[c*x]] + 9*b*Sinh[4*Ar
cSinh[c*x]] + b*Sinh[6*ArcSinh[c*x]])))/(13824*c*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(a^2c^4d^2x^4+2\,a^2c^2d^2x^2+a^2d^2+\left(b^2c^4d^2x^4+2\,b^2c^2d^2x^2+b^2d^2\right)\mathrm{arsinh}\left(cx\right)^2+2\left(abc^4d^2x^4+2\,abc^2d^2x\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
+ 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.37, size = 966, normalized size = 2.30

$$\frac{5a^2dx\left(c^2d\,x^2+d\right)^{\frac{3}{2}}}{24}+\frac{5a^2d^2x\sqrt{c^2d\,x^2+d}}{16}+\frac{5a^2d^3\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{16\sqrt{c^2d}}+\frac{ab\sqrt{d\left(c^2x^2+1\right)}\,d^2c^6\,\mathrm{arcsinh}\,(cx}{3c^2x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 5/24*a^2*d*x*(c^2*d*x^2+d)^(3/2)+5/16*a^2*d^2*x*(c^2*d*x^2+d)^(1/2)+5/16*a^
2*d^3*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/6*x*(c^
2*d*x^2+d)^(5/2)*a^2+1/3*a*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^6/(c^2*x^2+1)*arcs
inh(c*x)*x^7+17/12*a*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^4/(c^2*x^2+1)*arcsinh(c*
x)*x^5+59/24*a*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^3
+299/1152*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*x-299/1152*b^2*(d*(c^2*
x^2+1))^(1/2)*d^2/c/(c^2*x^2+1)^(1/2)*arcsinh(c*x)+1/108*b^2*(d*(c^2*x^2+1)
)^(1/2)*d^2*c^6/(c^2*x^2+1)*x^7+113/1728*b^2*(d*(c^2*x^2+1))^(1/2)*d^2*c^4/
(c^2*x^2+1)*x^5+1091/3456*b^2*(d*(c^2*x^2+1))^(1/2)*d^2*c^2/(c^2*x^2+1)*x^3
+5/48*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsinh(c*x)^3*d^2+11/1
6*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)^2*x-299/1152*a*b*(
d*(c^2*x^2+1))^(1/2)*d^2/c/(c^2*x^2+1)^(1/2)+1/6*b^2*(d*(c^2*x^2+1))^(1/2)*
d^2*c^6/(c^2*x^2+1)*arcsinh(c*x)^2*x^7+17/24*b^2*(d*(c^2*x^2+1))^(1/2)*d^2*
c^4/(c^2*x^2+1)*arcsinh(c*x)^2*x^5-1/18*b^2*(d*(c^2*x^2+1))^(1/2)*d^2*c^5/(
c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^6-13/48*b^2*(d*(c^2*x^2+1))^(1/2)*d^2*c^3/(
c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^4-11/16*b^2*(d*(c^2*x^2+1))^(1/2)*d^2*c/(c^
2*x^2+1)^(1/2)*arcsinh(c*x)*x^2-1/18*a*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^5/(c^2
*x^2+1)^(1/2)*x^6-13/48*a*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^3/(c^2*x^2+1)^(1/2)
*x^4-11/16*a*b*(d*(c^2*x^2+1))^(1/2)*d^2*c/(c^2*x^2+1)^(1/2)*x^2+59/48*b^2*
(d*(c^2*x^2+1))^(1/2)*d^2*c^2/(c^2*x^2+1)*arcsinh(c*x)^2*x^3+5/16*a*b*(d*(c
^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsinh(c*x)^2*d^2+11/8*a*b*(d*(c^2*x^
2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)*x
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{asinh}\,(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\left(c^2x^2+1\right)\right)^{\frac{5}{2}}(a+b\operatorname{asinh}(cx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))**2, x)
```

3.278 $\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x}\,dx$

Optimal. Leaf size=635

$$-\frac{2bd^2\sqrt{c^2dx^2+d}\,\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}+\frac{2bd^2\sqrt{c^2dx^2+d}\,\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}-\frac{2a}{}$$

```
[Out] 1/3*d*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2+1/5*(c^2*d*x^2+d)^(5/2)*(a+b
*arcsinh(c*x))^2+598/225*b^2*d^2*(c^2*d*x^2+d)^(1/2)+74/675*b^2*d^2*(c^2*x^
2+1)*(c^2*d*x^2+d)^(1/2)+2/125*b^2*d^2*(c^2*x^2+1)^2*(c^2*d*x^2+d)^(1/2)+d^
2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)-2*a*b*c*d^2*x*(c^2*d*x^2+d)^(1/2
)/(c^2*x^2+1)^(1/2)-2*b^2*c*d^2*x*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2
+1)^(1/2)-16/15*b*c*d^2*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1
)^(1/2)-22/45*b*c^3*d^2*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2
+1)^(1/2)-2/25*b*c^5*d^2*x^5*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^
2+1)^(1/2)-2*d^2*(a+b*arcsinh(c*x))^2*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*d
*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2*b*d^2*(a+b*arcsinh(c*x))*polylog(2,-c*x-(
c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+2*b*d^2*(a+b*arcsin
h(c*x))*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1
/2)+2*b^2*d^2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^
2+1)^(1/2)-2*b^2*d^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(
c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.91, antiderivative size = 635, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 16, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5744, 5742, 5760, 4182, 2531, 2282, 6589, 5653, 261, 5679, 444, 43, 194, 12, 1247, 698}

$$-\frac{2bd^2\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}+\frac{2bd^2\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)\left(a+b\right.}{\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] (598*b^2*d^2*Sqrt[d + c^2*d*x^2])/225 - (2*a*b*c*d^2*x*Sqrt[d + c^2*d*x^2])
/Sqrt[1 + c^2*x^2] + (74*b^2*d^2*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2])/675 + (
2*b^2*d^2*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2])/125 - (2*b^2*c*d^2*x*Sqrt[d
+ c^2*d*x^2]*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - (16*b*c*d^2*x*Sqrt[d + c^2*d
*x^2]*(a + b*ArcSinh[c*x]))/(15*Sqrt[1 + c^2*x^2]) - (22*b*c^3*d^2*x^3*Sqrt
[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(45*Sqrt[1 + c^2*x^2]) - (2*b*c^5*d^2
*x^5*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(25*Sqrt[1 + c^2*x^2]) + d^2
*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2 + (d*(d + c^2*d*x^2)^(3/2)*(a +
 b*ArcSinh[c*x])^2)/3 + ((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/5 -
(2*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/
Sqrt[1 + c^2*x^2] - (2*b*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*PolyL
og[2, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (2*b*d^2*Sqrt[d + c^2*d*x^2]*(a
 + b*ArcSinh[c*x])*PolyLog[2, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (2*b^2*d
^2*Sqrt[d + c^2*d*x^2]*PolyLog[3, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (2*
b^2*d^2*Sqrt[d + c^2*d*x^2]*PolyLog[3, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
```

```
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5679

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x}\,dx = \frac{1}{5}\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2 + d\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}\right.}{x}$$

$$= -\frac{2bcd^2x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{5\sqrt{1+c^2x^2}} - \frac{4bc^3d^2x^3\sqrt{d+c^2dx^2}\left(a+b\right.}{15\sqrt{1+c^2x^2}}$$

$$= -\frac{16bcd^2x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{15\sqrt{1+c^2x^2}} - \frac{22bc^3d^2x^3\sqrt{d+c^2dx^2}\left(a+\right.}{45\sqrt{1+c^2x^2}}$$

$$= -\frac{2abcd^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{16bcd^2x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{15\sqrt{1+c^2x^2}} - \frac{22b}{}$$

$$= -\frac{2abcd^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{2b^2cd^2x\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}} - \frac{16bcd^2x\sqrt{d}}{}$$

$$= \frac{598}{225}b^2d^2\sqrt{d+c^2dx^2} - \frac{2abcd^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \frac{74}{675}b^2d^2\left(1+c^2x^2\right)\sqrt{d}$$

$$= \frac{598}{225}b^2d^2\sqrt{d+c^2dx^2} - \frac{2abcd^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \frac{74}{675}b^2d^2\left(1+c^2x^2\right)\sqrt{d}$$

$$= \frac{598}{225}b^2d^2\sqrt{d+c^2dx^2} - \frac{2abcd^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \frac{74}{675}b^2d^2\left(1+c^2x^2\right)\sqrt{d}$$

Mathematica [A] time = 4.45, size = 710, normalized size = 1.12

$$\frac{d^2\left(54000a^2\sqrt{d}\sqrt{c^2x^2+1}\log(cx) - 54000a^2\sqrt{d}\sqrt{c^2x^2+1}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+d\right) + 3600a^2\sqrt{c^2x^2+1}\left(3c^4x^4\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] (d^2*(3600*a^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(23 + 11*c^2*x^2 + 3*c
^4*x^4) - 24000*a*b*Sqrt[d + c^2*d*x^2]*(3*c*x + c^3*x^3 - 3*(1 + c^2*x^2)^
(3/2)*ArcSinh[c*x]) - 480*a*b*Sqrt[d + c^2*d*x^2]*(c*x*(-30 + 5*c^2*x^2 + 9
*c^4*x^4) - 15*Sqrt[1 + c^2*x^2]*(-2 + c^2*x^2 + 3*c^4*x^4)*ArcSinh[c*x]) -
 b^2*Sqrt[d + c^2*d*x^2]*(480*c*x*(-30 + 5*c^2*x^2 + 9*c^4*x^4)*ArcSinh[c*x
] + 6750*Sqrt[1 + c^2*x^2]*(2 + ArcSinh[c*x]^2) + 125*(2 + 9*ArcSinh[c*x]^2
)*Cosh[3*ArcSinh[c*x]] - 27*(2 + 25*ArcSinh[c*x]^2)*Cosh[5*ArcSinh[c*x]]) +
 54000*a^2*Sqrt[d]*Sqrt[1 + c^2*x^2]*Log[c*x] - 54000*a^2*Sqrt[d]*Sqrt[1 +
c^2*x^2]*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] - 108000*a*b*Sqrt[d + c^2*d*x
^2]*(c*x - Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - ArcSinh[c*x]*Log[1 - E^(-ArcSin
h[c*x])] + ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] - PolyLog[2, -E^(-ArcSin
h[c*x])] + PolyLog[2, E^(-ArcSinh[c*x])]) + 54000*b^2*Sqrt[d + c^2*d*x^2]*(
2*Sqrt[1 + c^2*x^2] - 2*c*x*ArcSinh[c*x] + Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2
 + ArcSinh[c*x]^2*(Log[1 - E^(-ArcSinh[c*x])] - Log[1 + E^(-ArcSinh[c*x])])
 + 2*ArcSinh[c*x]*(PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[
c*x])]) + 2*(PolyLog[3, -E^(-ArcSinh[c*x])] - PolyLog[3, E^(-ArcSinh[c*x])]
)) + 1000*b^2*Sqrt[d + c^2*d*x^2]*(27*Sqrt[1 + c^2*x^2]*(2 + ArcSinh[c*x]^2
) + (2 + 9*ArcSinh[c*x]^2)*Cosh[3*ArcSinh[c*x]] - 6*ArcSinh[c*x]*(9*c*x + S
inh[3*ArcSinh[c*x]]))))/(54000*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2c^4d^2x^4 + 2\,a^2c^2d^2x^2 + a^2d^2 + \left(b^2c^4d^2x^4 + 2\,b^2c^2d^2x^2 + b^2d^2\right)\text{arsinh}\,(cx)^2 + 2\left(abc^4d^2x^4 + 2\,abc^2\right.}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
+ 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.46, size = 1321, normalized size = 2.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x,x)
```

```
[Out] -2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(3,c*x+(c^2*x^2+1)^(1
/2))*d^2+9394/3375*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)+2/5*a*b*(d*(c^
2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^6*c^6+28/15*a*b*(d*(c^2*x^2+
1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^4*c^4+68/15*a*b*(d*(c^2*x^2+1))^(1
/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^2*c^2-a^2*d^(5/2)*ln((2*d+2*d^(1/2)*(c^2
*d*x^2+d)^(1/2))/x)+a^2*(c^2*d*x^2+d)^(1/2)*d^2+1/3*a^2*d*(c^2*d*x^2+d)^(3/
2)+1/5*(c^2*d*x^2+d)^(5/2)*a^2+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2
)*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*d^2-b^2*(d*(c^2*x^2+1))^(1/
2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))*d^2+2/125*b
^2*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*c^6*x^6+532/3375*b^2*(d*(c^2*x^2+1
))^(1/2)*d^2/(c^2*x^2+1)*c^4*x^4+9872/3375*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(c
^2*x^2+1)*c^2*x^2+b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^
2*ln(1-c*x-(c^2*x^2+1)^(1/2))*d^2-2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(
1/2)*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*d^2+46/15*a*b*(d*(c^2*x
^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*
x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*d^2+2*a*b*(d*(c^2*x^2+1))^(1
/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*d^2+2*b^2*(d*(c^2*x^
2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*d^2+23/15*b
^2*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)^2-2*a*b*(d*(c^2*x^2+1
))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*d^2+2*a
*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1
)^(1/2))*d^2-46/15*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)*arcsinh(
c*x)*x*c-2/25*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)*c^5*x^5-22/45
*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)*c^3*x^3-2/25*b^2*(d*(c^2*x
^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^5*c^5-22/45*b^2*(d*(c^2*x
^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^3*c^3-46/15*a*b*(d*(c^2*x
^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)*c*x+1/5*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(c
^2*x^2+1)*arcsinh(c*x)^2*x^6*c^6+14/15*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x
```

```
^2+1)*arcsinh(c*x)^2*x^4*c^4+34/15*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1
)*arcsinh(c*x)^2*x^2*c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{15}\left(15\,d^{\frac{5}{2}}\,\mathrm{arsinh}\left(\frac{1}{c|x|}\right)-3\left(c^2dx^2+d\right)^{\frac{5}{2}}-5\left(c^2dx^2+d\right)^{\frac{3}{2}}d-15\,\sqrt{c^2dx^2+d}\,d^2\right)a^2+\int\frac{\left(c^2dx^2+d\right)^{\frac{5}{2}}b^2\log\left(cx+\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] -1/15*(15*d^(5/2)*arcsinh(1/(c*abs(x))) - 3*(c^2*d*x^2 + d)^(5/2) - 5*(c^2*
d*x^2 + d)^(3/2)*d - 15*sqrt(c^2*d*x^2 + d)*d^2)*a^2 + integrate((c^2*d*x^2
 + d)^(5/2)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/x + 2*(c^2*d*x^2 + d)^(5/2)*
a*b*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^{5/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2))/x,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\left(c^2x^2+1\right)\right)^{\frac{5}{2}}\,(a+b\,\mathrm{asinh}\,(cx))^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2/x,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))**2/x, x)
```

3.279 $\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^2}\,dx$

Optimal. Leaf size=530

$$\frac{15}{8}c^2d^2x\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2+\frac{5cd^2\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^3}{8b\sqrt{c^2x^2+1}}+\frac{cd^2\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(\right.}{\sqrt{c^2x^2+1}}$$

```
[Out] 5/4*c^2*d*x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2-(c^2*d*x^2+d)^(5/2)*(a
+b*arcsinh(c*x))^2/x+31/64*b^2*c^2*d^2*x*(c^2*d*x^2+d)^(1/2)+1/32*b^2*c^2*d
^2*x*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)-1/8*b*c*d^2*(c^2*x^2+1)^(3/2)*(a+b*arc
sinh(c*x))*(c^2*d*x^2+d)^(1/2)+15/8*c^2*d^2*x*(a+b*arcsinh(c*x))^2*(c^2*d*x
^2+d)^(1/2)-89/64*b^2*c*d^2*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1
/2)-15/8*b*c^3*d^2*x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(
1/2)+c*d^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+5/8*c
*d^2*(a+b*arcsinh(c*x))^3*(c^2*d*x^2+d)^(1/2)/b/(c^2*x^2+1)^(1/2)+2*b*c*d^2
*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*d*x^2+d)^(1/2)/(
c^2*x^2+1)^(1/2)-b^2*c*d^2*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*d*x^
2+d)^(1/2)/(c^2*x^2+1)^(1/2)+b*c*d^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)*(
c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.61, antiderivative size = 530, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 15, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.536, Rules used = {5739, 5684, 5682, 5675, 5661, 321, 215, 5717, 195, 5726, 5659, 3716, 2190, 2279, 2391}

$$\frac{b^2cd^2\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}-\frac{15bc^3d^2x^2\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{8\sqrt{c^2x^2+1}}+\frac{15}{8}c^2d^2x\sqrt{c^2dx^2+d}\,\left(a\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] (31*b^2*c^2*d^2*x*Sqrt[d + c^2*d*x^2])/64 + (b^2*c^2*d^2*x*(1 + c^2*x^2)*Sq
rt[d + c^2*d*x^2])/32 - (89*b^2*c*d^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/(64
*Sqrt[1 + c^2*x^2]) - (15*b*c^3*d^2*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[
c*x]))/(8*Sqrt[1 + c^2*x^2]) + b*c*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2
]*(a + b*ArcSinh[c*x]) - (b*c*d^2*(1 + c^2*x^2)^(3/2)*Sqrt[d + c^2*d*x^2]*(
a + b*ArcSinh[c*x]))/8 + (15*c^2*d^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c
*x])^2)/8 - (c*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/Sqrt[1 + c^2
*x^2] + (5*c^2*d*x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/4 - ((d +
c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x + (5*c*d^2*Sqrt[d + c^2*d*x^2]*(
a + b*ArcSinh[c*x])^3)/(8*b*Sqrt[1 + c^2*x^2]) + (2*b*c*d^2*Sqrt[d + c^2*d*
x^2]*(a + b*ArcSinh[c*x])*Log[1 - E^(2*ArcSinh[c*x])])/Sqrt[1 + c^2*x^2] +
(b^2*c*d^2*Sqrt[d + c^2*d*x^2]*PolyLog[2, E^(2*ArcSinh[c*x])])/Sqrt[1 + c^2
*x^2]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
```

```
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5726

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[((d + e*x^2)^p*(a + b*ArcSinh[c*x]))/(2*p), x] + (Dist[d
, Int[((d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]))/x, x], x] - Dist[(b*c*d^p)
/(2*p), Int[(1 + c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^2}\,dx = -\frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x} + \left(5c^2d\right)\int\left(d+c^2dx^2\right)^{3/2}\left(a+b\,\mathrm{s}\right.$$

$$= \frac{1}{2}bcd^2\left(1+c^2x^2\right)^{3/2}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{4}c^2dx\left(d+c^2dx^2\right.$$

$$= -\frac{1}{8}b^2c^2d^2x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}+bcd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}\left(a+b\right.$$

$$= -\frac{11}{16}b^2c^2d^2x\sqrt{d+c^2dx^2}+\frac{1}{32}b^2c^2d^2x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}-\frac{15bc^3d}{}$$

$$= \frac{31}{64}b^2c^2d^2x\sqrt{d+c^2dx^2}+\frac{1}{32}b^2c^2d^2x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}-\frac{11b^2cd^2}{}$$

$$= \frac{31}{64}b^2c^2d^2x\sqrt{d+c^2dx^2}+\frac{1}{32}b^2c^2d^2x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}-\frac{89b^2cd^2}{}$$

$$= \frac{31}{64}b^2c^2d^2x\sqrt{d+c^2dx^2}+\frac{1}{32}b^2c^2d^2x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}-\frac{89b^2cd^2}{}$$

$$= \frac{31}{64}b^2c^2d^2x\sqrt{d+c^2dx^2}+\frac{1}{32}b^2c^2d^2x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}-\frac{89b^2cd^2}{}$$

Mathematica [A] time = 2.18, size = 550, normalized size = 1.04

$$\frac{d^2\left(288a^2c^2x^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}-256a^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}+480a^2c\sqrt{d}\,x\sqrt{c^2x^2+1}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] (d^2*(-256*a^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 288*a^2*c^2*x^2*Sqrt
[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 64*a^2*c^4*x^4*Sqrt[1 + c^2*x^2]*Sqrt[d
 + c^2*d*x^2] + 160*b^2*c*x*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^3 - 128*a*b*c*
x*Sqrt[d + c^2*d*x^2]*Cosh[2*ArcSinh[c*x]] - 4*a*b*c*x*Sqrt[d + c^2*d*x^2]*
Cosh[4*ArcSinh[c*x]] + 512*a*b*c*x*Sqrt[d + c^2*d*x^2]*Log[c*x] + 480*a^2*c
*Sqrt[d]*x*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] - 256
*b^2*c*x*Sqrt[d + c^2*d*x^2]*PolyLog[2, E^(-2*ArcSinh[c*x])] + 64*b^2*c*x*S
qrt[d + c^2*d*x^2]*Sinh[2*ArcSinh[c*x]] + b^2*c*x*Sqrt[d + c^2*d*x^2]*Sinh[
4*ArcSinh[c*x]] - 4*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*(128*a*Sqrt[1 + c^2*
x^2] + 32*b*c*x*Cosh[2*ArcSinh[c*x]] + b*c*x*Cosh[4*ArcSinh[c*x]] - 128*b*c
*x*Log[1 - E^(-2*ArcSinh[c*x])] - 64*a*c*x*Sinh[2*ArcSinh[c*x]] - 4*a*c*x*S
inh[4*ArcSinh[c*x]]) + 8*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^2*(60*a*c*x + 3
2*b*c*x - 32*b*Sqrt[1 + c^2*x^2] + 16*b*c*x*Sinh[2*ArcSinh[c*x]] + b*c*x*Si
nh[4*ArcSinh[c*x]])))/(256*x*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(a^2c^4d^2x^4+2\,a^2c^2d^2x^2+a^2d^2+\left(b^2c^4d^2x^4+2\,b^2c^2d^2x^2+b^2d^2\right)\mathrm{arsinh}\left(cx\right)^2+2\left(abc^4d^2x^4+2\,abc^2d^2\right.\right.}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="fricas
")
```

```
[Out] integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
+ 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.45, size = 1223, normalized size = 2.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^2,x)
```

```
[Out] 15/8*a^2*c^2*d^3*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2
)-1/8*b^2*(d*(c^2*x^2+1))^(1/2)*d^2*c^5/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^4-
9/8*b^2*(d*(c^2*x^2+1))^(1/2)*d^2*c^3/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^2-1/
8*a*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^5/(c^2*x^2+1)^(1/2)*x^4-9/8*a*b*(d*(c^2*x
^2+1))^(1/2)*d^2*c^3/(c^2*x^2+1)^(1/2)*x^2-2*a*b*(d*(c^2*x^2+1))^(1/2)*d^2*
c/(c^2*x^2+1)^(1/2)*arcsinh(c*x)+1/4*b^2*(d*(c^2*x^2+1))^(1/2)*d^2*c^6/(c^2
*x^2+1)*arcsinh(c*x)^2*x^5+11/8*b^2*(d*(c^2*x^2+1))^(1/2)*d^2*c^4/(c^2*x^2+
1)*arcsinh(c*x)^2*x^3+1/8*b^2*(d*(c^2*x^2+1))^(1/2)*d^2*c^2/(c^2*x^2+1)*arc
sinh(c*x)^2*x+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln
(1-c*x-(c^2*x^2+1)^(1/2))*d^2*c+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/
2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*d^2*c+15/8*a*b*(d*(c^2*x^2+1))^
(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*d^2*c-2*a*b*(d*(c^2*x^2+1))^(1/2)*ar
csinh(c*x)*d^2/x/(c^2*x^2+1)+2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*
ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*d^2*c-a^2/d/x*(c^2*d*x^2+d)^(7/2)+a^2*c^2*x
*(c^2*d*x^2+d)^(5/2)+1/2*a*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^6/(c^2*x^2+1)*arcs
inh(c*x)*x^5+11/4*a*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^4/(c^2*x^2+1)*arcsinh(c*x
)*x^3+1/4*a*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^2/(c^2*x^2+1)*arcsinh(c*x)*x+1/32
*b^2*(d*(c^2*x^2+1))^(1/2)*d^2*c^6/(c^2*x^2+1)*x^5+35/64*b^2*(d*(c^2*x^2+1)
)^(1/2)*d^2*c^4/(c^2*x^2+1)*x^3+33/64*b^2*(d*(c^2*x^2+1))^(1/2)*d^2*c^2/(c^
2*x^2+1)*x-b^2*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2*d^2/x/(c^2*x^2+1)+2*b^2
*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*d
^2*c+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+
1)^(1/2))*d^2*c+5/8*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x
)^3*d^2*c-b^2*(d*(c^2*x^2+1))^(1/2)*d^2*c/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2-
33/64*b^2*(d*(c^2*x^2+1))^(1/2)*d^2*c/(c^2*x^2+1)^(1/2)*arcsinh(c*x)-33/64*
a*b*(d*(c^2*x^2+1))^(1/2)*d^2*c/(c^2*x^2+1)^(1/2)+5/4*a^2*(c^2*d*x^2+d)^(3/
2)*x*c^2*d+15/8*a^2*c^2*d^2*x*(c^2*d*x^2+d)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{asinh}\,(c\,x))^2 \left(d\,c^2\,x^2 + d\right)^{5/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2))/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\left(c^2x^2 + 1\right)\right)^{\frac{5}{2}} (a + b\,\mathrm{asinh}\,(cx))^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2/x**2,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))**2/x**2, x)
```

3.280 $\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^3}\,dx$

Optimal. Leaf size=687

$$-\frac{5bc^2d^2\sqrt{c^2dx^2+d}\,\mathrm{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}+\frac{5bc^2d^2\sqrt{c^2dx^2+d}\,\mathrm{Li}_2\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}$$

```
[Out] 5/6*c^2*d*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2-1/2*(c^2*d*x^2+d)^(5/2)*
(a+b*arcsinh(c*x))^2/x^2+40/9*b^2*c^2*d^2*(c^2*d*x^2+d)^(1/2)+2/27*b^2*c^2*
d^2*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)+5/2*c^2*d^2*(a+b*arcsinh(c*x))^2*(c^2*d
*x^2+d)^(1/2)-5*a*b*c^3*d^2*x*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-5*b^2*c
^3*d^2*x*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-b*c*d^2*(a+b*ar
csinh(c*x))*(c^2*d*x^2+d)^(1/2)/x/(c^2*x^2+1)^(1/2)+1/3*b*c^3*d^2*x*(a+b*ar
csinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2/9*b*c^5*d^2*x^3*(a+b*ar
csinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-5*c^2*d^2*(a+b*arcsinh(c*
x))^2*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-
b^2*c^2*d^2*arctanh((c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2
)-5*b*c^2*d^2*(a+b*arcsinh(c*x))*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x
^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+5*b*c^2*d^2*(a+b*arcsinh(c*x))*polylog(2,c*x+
(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+5*b^2*c^2*d^2*poly
log(3,-c*x-(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-5*b^2*c
^2*d^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/
2)
```

Rubi [A] time = 0.99, antiderivative size = 687, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 20, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5739, 5744, 5742, 5760, 4182, 2531, 2282, 6589, 5653, 261, 5679, 444, 43, 270, 5730, 12, 1251, 897, 1153, 208}

$$-\frac{5bc^2d^2\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}+\frac{5bc^2d^2\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)\left(a\right.}{\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] (40*b^2*c^2*d^2*Sqrt[d + c^2*d*x^2])/9 - (5*a*b*c^3*d^2*x*Sqrt[d + c^2*d*x^
2])/Sqrt[1 + c^2*x^2] + (2*b^2*c^2*d^2*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2])/2
7 - (5*b^2*c^3*d^2*x*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] -
(b*c*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(x*Sqrt[1 + c^2*x^2]) +
(b*c^3*d^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*Sqrt[1 + c^2*x^2]
) - (2*b*c^5*d^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(9*Sqrt[1 +
c^2*x^2]) + (5*c^2*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/2 + (5*c
^2*d*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/6 - ((d + c^2*d*x^2)^(5/
2)*(a + b*ArcSinh[c*x])^2)/(2*x^2) - (5*c^2*d^2*Sqrt[d + c^2*d*x^2]*(a + b*
ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (b^2*c^2*d^2*S
qrt[d + c^2*d*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]])/Sqrt[1 + c^2*x^2] - (5*b*c^2
*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcSinh[c*x]])/
Sqrt[1 + c^2*x^2] + (5*b*c^2*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*P
olyLog[2, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (5*b^2*c^2*d^2*Sqrt[d + c^2*
d*x^2]*PolyLog[3, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (5*b^2*c^2*d^2*Sqrt
[d + c^2*d*x^2]*PolyLog[3, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5679

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
```

```
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^3}\,dx = -\frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{2x^2}+\frac{1}{2}\left(5c^2d\right)\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a\right.}{}$$

$$= -\frac{bcd^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{1+c^2x^2}}+\frac{2bc^3d^2x\sqrt{d+c^2dx^2}\left(a+b\right.}{\sqrt{1+c^2x^2}}$$

$$= -\frac{bcd^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{1+c^2x^2}}+\frac{bc^3d^2x\sqrt{d+c^2dx^2}\left(a+b\,\mathrm{s}\right.}{3\sqrt{1+c^2x^2}}$$

$$= -\frac{5abc^3d^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}-\frac{bcd^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{1+c^2x^2}}+\frac{bc^3}{}$$

$$= -\frac{5abc^3d^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}-\frac{5b^2c^3d^2x\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}-\frac{bcd^2\sqrt{}}{}$$

$$= \frac{55}{9}b^2c^2d^2\sqrt{d+c^2dx^2}-\frac{5abc^3d^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}+\frac{5}{27}b^2c^2d^2\left(1+c^2x^2\right.$$

$$= \frac{40}{9}b^2c^2d^2\sqrt{d+c^2dx^2}-\frac{5abc^3d^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}+\frac{2}{27}b^2c^2d^2\left(1+c^2x^2\right.$$

$$= \frac{40}{9}b^2c^2d^2\sqrt{d+c^2dx^2}-\frac{5abc^3d^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}+\frac{2}{27}b^2c^2d^2\left(1+c^2x^2\right.$$

Mathematica [A] time = 8.02, size = 990, normalized size = 1.44

$$\frac{5}{2}a^2c^2\log(x)d^{5/2}-\frac{5}{2}a^2c^2\log\left(d+\sqrt{d\left(c^2x^2+1\right)}\sqrt{d}\right)d^{5/2}+2abc^2\left(\frac{1}{3}\left(c^2x^2+1\right)\sqrt{d\left(c^2x^2+1\right)}\sinh^{-1}(cx)-\frac{cx\sqrt{}}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] Sqrt[d*(1 + c^2*x^2)]*((7*a^2*c^2*d^2)/3 - (a^2*d^2)/(2*x^2) + (a^2*c^4*d^2
*x^2)/3) + 2*a*b*c^2*d^2*(-1/9*(c*x*Sqrt[d*(1 + c^2*x^2)]*(3 + c^2*x^2))/Sq
rt[1 + c^2*x^2] + ((1 + c^2*x^2)*Sqrt[d*(1 + c^2*x^2)]*ArcSinh[c*x])/3) + (
5*a^2*c^2*d^(5/2)*Log[x])/2 - (5*a^2*c^2*d^(5/2)*Log[d + Sqrt[d]*Sqrt[d*(1
+ c^2*x^2)]])/2 + (4*a*b*c^2*d^2*Sqrt[d*(1 + c^2*x^2)]*(-(c*x) + Sqrt[1 + c
^2*x^2]*ArcSinh[c*x] + ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*
x]*Log[1 + E^(-ArcSinh[c*x])] + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2,
 E^(-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2] + 2*b^2*c^2*d^2*Sqrt[d*(1 + c^2*x^2
)]*(2 - (2*c*x*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] + ArcSinh[c*x]^2 + (ArcSinh[
c*x]^2*(Log[1 - E^(-ArcSinh[c*x])] - Log[1 + E^(-ArcSinh[c*x])]))/Sqrt[1 +
c^2*x^2] + (2*ArcSinh[c*x]*(PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(
-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2] + (2*(PolyLog[3, -E^(-ArcSinh[c*x])] -
PolyLog[3, E^(-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2]) + (b^2*c^2*d^2*Sqrt[d*(1
 + c^2*x^2)]*(27*Sqrt[1 + c^2*x^2]*(2 + ArcSinh[c*x]^2) + (2 + 9*ArcSinh[c*
x]^2)*Cosh[3*ArcSinh[c*x]] - 6*ArcSinh[c*x]*(9*c*x + Sinh[3*ArcSinh[c*x]]))
```

```
)/(108*Sqrt[1 + c^2*x^2]) + (a*b*c^2*d^2*Sqrt[d*(1 + c^2*x^2)]*(-2*Coth[Arc
Sinh[c*x]/2] - ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*x]*Log[1 -
 E^(-ArcSinh[c*x])] - 4*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 4*PolyLog
[2, -E^(-ArcSinh[c*x])] - 4*PolyLog[2, E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Se
ch[ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSinh[c*x]/2]))/(4*Sqrt[1 + c^2*x^2]) + (b^
2*c^2*d^2*Sqrt[d*(1 + c^2*x^2)]*(-4*ArcSinh[c*x]*Coth[ArcSinh[c*x]/2] - Arc
Sinh[c*x]^2*Csch[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*x]^2*Log[1 - E^(-ArcSinh[c
*x])] - 4*ArcSinh[c*x]^2*Log[1 + E^(-ArcSinh[c*x])] + 8*Log[Tanh[ArcSinh[c*
x]/2]] + 8*ArcSinh[c*x]*PolyLog[2, -E^(-ArcSinh[c*x])] - 8*ArcSinh[c*x]*Pol
yLog[2, E^(-ArcSinh[c*x])] + 8*PolyLog[3, -E^(-ArcSinh[c*x])] - 8*PolyLog[3
, E^(-ArcSinh[c*x])] - ArcSinh[c*x]^2*Sech[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*
x]*Tanh[ArcSinh[c*x]/2]))/(8*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2c^4d^2x^4 + 2\,a^2c^2d^2x^2 + a^2d^2 + \left(b^2c^4d^2x^4 + 2\,b^2c^2d^2x^2 + b^2d^2\right)\text{arsinh}\,(cx)^2 + 2\left(abc^4d^2x^4 + 2\,abc^2d^2x\right.}{x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="fricas
")
```

```
[Out] integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
+ 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^3, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.57, size = 1404, normalized size = 2.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^3,x)
```

```
[Out] 1/2*a^2*c^2*(c^2*d*x^2+d)^(5/2)+11/6*b^2*(d*(c^2*x^2+1))^(1/2)*d^2*c^2/(c^2
*x^2+1)*arcsinh(c*x)^2-1/2*b^2*arcsinh(c*x)^2*(d*(c^2*x^2+1))^(1/2)*d^2/x^2
/(c^2*x^2+1)-2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arctanh(c*x+(c^2
*x^2+1)^(1/2))*d^2*c^2+5*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylo
g(3,-c*x-(c^2*x^2+1)^(1/2))*d^2*c^2-5*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)
^(1/2)*polylog(3,c*x+(c^2*x^2+1)^(1/2))*d^2*c^2+2/27*b^2*(d*(c^2*x^2+1))^(1
/2)*d^2*c^6/(c^2*x^2+1)*x^4+124/27*b^2*(d*(c^2*x^2+1))^(1/2)*d^2*c^4/(c^2*x
^2+1)*x^2+122/27*b^2*(d*(c^2*x^2+1))^(1/2)*d^2*c^2/(c^2*x^2+1)-1/2*a^2/d/x^
2*(c^2*d*x^2+d)^(7/2)+5/6*a^2*c^2*d*(c^2*d*x^2+d)^(3/2)-5/2*a^2*c^2*d^(5/2)
*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)+5/2*a^2*c^2*(c^2*d*x^2+d)^(1/2)*
d^2+16/3*a*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^4/(c^2*x^2+1)*arcsinh(c*x)*x^2+2/3
*a*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^6/(c^2*x^2+1)*arcsinh(c*x)*x^4+5*a*b*(d*(c
^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))
*d^2*c^2-5*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*
```

```
x+(c^2*x^2+1)^(1/2))*d^2*c^2-a*b*(d*(c^2*x^2+1))^(1/2)*d^2/x/(c^2*x^2+1)^(1
/2)*c-14/3*a*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^3/(c^2*x^2+1)^(1/2)*x-2/9*a*b*(d
*(c^2*x^2+1))^(1/2)*d^2*c^5/(c^2*x^2+1)^(1/2)*x^3+11/3*a*b*(d*(c^2*x^2+1))^
(1/2)*d^2*c^2/(c^2*x^2+1)*arcsinh(c*x)-a*b*arcsinh(c*x)*(d*(c^2*x^2+1))^(1/
2)*d^2/x^2/(c^2*x^2+1)-5*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylo
g(2,-c*x-(c^2*x^2+1)^(1/2))*d^2*c^2+5*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)
^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*d^2*c^2-2/9*b^2*(d*(c^2*x^2+1))^(1/
2)*d^2*c^5/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^3-14/3*b^2*(d*(c^2*x^2+1))^(1/2
)*d^2*c^3/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x-b^2*arcsinh(c*x)*(d*(c^2*x^2+1))
^(1/2)*d^2/x/(c^2*x^2+1)^(1/2)*c+5*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1
/2)*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*d^2*c^2+8/3*b^2*(d*(c^2*x
^2+1))^(1/2)*d^2*c^4/(c^2*x^2+1)*arcsinh(c*x)^2*x^2+1/3*b^2*(d*(c^2*x^2+1))
^(1/2)*d^2*c^6/(c^2*x^2+1)*arcsinh(c*x)^2*x^4-5/2*b^2*(d*(c^2*x^2+1))^(1/2)
/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))*d^2*c^2-5*b^2
*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*polylog(2,-c*x-(c^2*x
^2+1)^(1/2))*d^2*c^2+5/2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsin
h(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))*d^2*c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{6}\left(15\,c^2d^{\frac{5}{2}}\,\mathrm{arsinh}\left(\frac{1}{c|x|}\right)-3\left(c^2dx^2+d\right)^{\frac{5}{2}}c^2-5\left(c^2dx^2+d\right)^{\frac{3}{2}}c^2d-15\sqrt{c^2dx^2+d}\,c^2d^2+\frac{3\left(c^2dx^2+d\right)^{\frac{7}{2}}}{dx^2}\right)a^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="maxima
")
```

```
[Out] -1/6*(15*c^2*d^(5/2)*arcsinh(1/(c*abs(x))) - 3*(c^2*d*x^2 + d)^(5/2)*c^2 -
5*(c^2*d*x^2 + d)^(3/2)*c^2*d - 15*sqrt(c^2*d*x^2 + d)*c^2*d^2 + 3*(c^2*d*x
^2 + d)^(7/2)/(d*x^2))*a^2 + integrate((c^2*d*x^2 + d)^(5/2)*b^2*log(c*x +
sqrt(c^2*x^2 + 1))^2/x^3 + 2*(c^2*d*x^2 + d)^(5/2)*a*b*log(c*x + sqrt(c^2*x
^2 + 1))/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))^2\,\left(d\,c^2\,x^2+d\right)^{5/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2))/x^3,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\left(c^2x^2+1\right)\right)^{\frac{5}{2}}(a+b\,\mathrm{asinh}\,(cx))^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2/x**3,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))**2/x**3, x)
```

3.281 $\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^4}\,dx$

Optimal. Leaf size=561

$$-\frac{bcd^2\left(c^2x^2+1\right)^{3/2}\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{3x^2}-\frac{5c^2d\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3x}-\frac{\left(c^2dx^2+d\right)^{5/2}\left(a+b\right.}{3x^3}$$

```
[Out] -5/3*c^2*d*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x-1/3*(c^2*d*x^2+d)^(5/
2)*(a+b*arcsinh(c*x))^2/x^3+7/12*b^2*c^4*d^2*x*(c^2*d*x^2+d)^(1/2)-1/3*b^2*
c^2*d^2*(c^2*x^2+1)*(c^2*d*x^2+d)^(1/2)/x-1/3*b*c*d^2*(c^2*x^2+1)^(3/2)*(a+
b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^2+5/2*c^4*d^2*x*(a+b*arcsinh(c*x))^2*
(c^2*d*x^2+d)^(1/2)-23/12*b^2*c^3*d^2*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/(c^2
*x^2+1)^(1/2)-5/2*b*c^5*d^2*x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(c^2
*x^2+1)^(1/2)+7/3*c^3*d^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2
+1)^(1/2)+5/6*c^3*d^2*(a+b*arcsinh(c*x))^3*(c^2*d*x^2+d)^(1/2)/b/(c^2*x^2+1
)^(1/2)+14/3*b*c^3*d^2*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^2)
*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-7/3*b^2*c^3*d^2*polylog(2,1/(c*x+(c^
2*x^2+1)^(1/2))^2)*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+7/3*b*c^3*d^2*(a+b
*arcsinh(c*x))*(c^2*x^2+1)^(1/2)*(c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.85, antiderivative size = 561, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 15, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.536, Rules used = {5739, 5682, 5675, 5661, 321, 215, 5726, 5659, 3716, 2190, 2279, 2391, 195, 5728, 277}

$$\frac{7b^2c^3d^2\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{3\sqrt{c^2x^2+1}}-\frac{5bc^5d^2x^2\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{c^2x^2+1}}+\frac{5}{2}c^4d^2x\sqrt{c^2dx^2+d}\,\left(a+b\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] (7*b^2*c^4*d^2*x*Sqrt[d + c^2*d*x^2])/12 - (b^2*c^2*d^2*(1 + c^2*x^2)*Sqrt[
d + c^2*d*x^2])/(3*x) - (23*b^2*c^3*d^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/(
12*Sqrt[1 + c^2*x^2]) - (5*b*c^5*d^2*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh
[c*x]))/(2*Sqrt[1 + c^2*x^2]) + (7*b*c^3*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2
*d*x^2]*(a + b*ArcSinh[c*x]))/3 - (b*c*d^2*(1 + c^2*x^2)^(3/2)*Sqrt[d + c^2
*d*x^2]*(a + b*ArcSinh[c*x]))/(3*x^2) + (5*c^4*d^2*x*Sqrt[d + c^2*d*x^2]*(a
 + b*ArcSinh[c*x])^2)/2 - (7*c^3*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x
])^2)/(3*Sqrt[1 + c^2*x^2]) - (5*c^2*d*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh
[c*x])^2)/(3*x) - ((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(3*x^3) +
(5*c^3*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^3)/(6*b*Sqrt[1 + c^2*x^
2]) + (14*b*c^3*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*Log[1 - E^(2*A
rcSinh[c*x])])/(3*Sqrt[1 + c^2*x^2]) + (7*b^2*c^3*d^2*Sqrt[d + c^2*d*x^2]*P
olyLog[2, E^(2*ArcSinh[c*x])])/(3*Sqrt[1 + c^2*x^2])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
```

```
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5726

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[((d + e*x^2)^p*(a + b*ArcSinh[c*x]))/(2*p), x] + (Dist[d
, Int[((d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]))/x, x], x] - Dist[(b*c*d^p)
/(2*p), Int[(1 + c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5728

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*ArcSinh[c
*x]))/(f*(m + 1)), x] + (-Dist[(b*c*d^p)/(f*(m + 1)), Int[(f*x)^(m + 1)*(1
+ c^2*x^2)^(p - 1/2), x], x] - Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2
)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && ILtQ[(m + 1)/2, 0]
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^4}\,dx = -\frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3x^3} + \frac{1}{3}\left(5c^2d\right)\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a\right.}{}$$

$$= -\frac{bcd^2\left(1+c^2x^2\right)^{3/2}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2} - \frac{5c^2d\left(d+c^2d\right.}{}$$

$$= -\frac{b^2c^2d^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}{3x} + \frac{7}{3}bc^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}\left(a\right.$$

$$= -\frac{2}{3}b^2c^4d^2x\sqrt{d+c^2dx^2} - \frac{b^2c^2d^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}{3x} - \frac{5bc^5d^2x^2}{}$$

$$= \frac{7}{12}b^2c^4d^2x\sqrt{d+c^2dx^2} - \frac{b^2c^2d^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}{3x} - \frac{2b^2c^3d^2\sqrt{}}{}$$

$$= \frac{7}{12}b^2c^4d^2x\sqrt{d+c^2dx^2} - \frac{b^2c^2d^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}{3x} - \frac{23b^2c^3d^2}{}$$

$$= \frac{7}{12}b^2c^4d^2x\sqrt{d+c^2dx^2} - \frac{b^2c^2d^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}{3x} - \frac{23b^2c^3d^2}{}$$

$$= \frac{7}{12}b^2c^4d^2x\sqrt{d+c^2dx^2} - \frac{b^2c^2d^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}{3x} - \frac{23b^2c^3d^2}{}$$

Mathematica [A] time = 2.36, size = 616, normalized size = 1.10

$$\frac{d^2\left(-56a^2c^2x^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d} - 8a^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d} + 12a^2c^4x^4\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d} + 60a^2c^3\sqrt{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] (d^2*(-8*a*b*c*x*Sqrt[d + c^2*d*x^2] - 8*a^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2
*d*x^2] - 56*a^2*c^2*x^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] - 8*b^2*c^2*
x^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 12*a^2*c^4*x^4*Sqrt[1 + c^2*x^2
]*Sqrt[d + c^2*d*x^2] + 20*b^2*c^3*x^3*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^3 -
 6*a*b*c^3*x^3*Sqrt[d + c^2*d*x^2]*Cosh[2*ArcSinh[c*x]] + 112*a*b*c^3*x^3*S
qrt[d + c^2*d*x^2]*Log[c*x] + 60*a^2*c^3*Sqrt[d]*x^3*Sqrt[1 + c^2*x^2]*Log[
c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] - 56*b^2*c^3*x^3*Sqrt[d + c^2*d*x^2]*P
olyLog[2, E^(-2*ArcSinh[c*x])] + 3*b^2*c^3*x^3*Sqrt[d + c^2*d*x^2]*Sinh[2*A
rcSinh[c*x]] - 2*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*(4*b*c*x + 8*a*Sqrt[1 +
 c^2*x^2] + 56*a*c^2*x^2*Sqrt[1 + c^2*x^2] + 3*b*c^3*x^3*Cosh[2*ArcSinh[c*x
]] - 56*b*c^3*x^3*Log[1 - E^(-2*ArcSinh[c*x])] - 6*a*c^3*x^3*Sinh[2*ArcSinh
[c*x]]) + 2*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^2*(30*a*c^3*x^3 - 4*b*(-7*c^
3*x^3 + Sqrt[1 + c^2*x^2] + 7*c^2*x^2*Sqrt[1 + c^2*x^2]) + 3*b*c^3*x^3*Sinh
[2*ArcSinh[c*x]])))/(24*x^3*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2c^4d^2x^4 + 2\,a^2c^2d^2x^2 + a^2d^2 + \left(b^2c^4d^2x^4 + 2\,b^2c^2d^2x^2 + b^2d^2\right)\operatorname{arsinh}(cx)^2 + 2\left(abc^4d^2x^4 + 2\,abc^2\right.\right.}{x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="fricas
")
```

```
[Out] integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
+ 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^4, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.55, size = 3311, normalized size = 5.90

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^4,x)
```

```
[Out] -46/3*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)/x/(c^2*x^2+1)
*arcsinh(c*x)*c^2-294*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+
1)*x^5/(c^2*x^2+1)*arcsinh(c*x)*c^8+70*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^
4*x^4+15*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^5-406*a*b*(d*(c^2*
x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^3/(c^2*x^2+1)*arcsinh(c*x)*c^
6-380/3*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x/(c^2*x^2+
1)*arcsinh(c*x)*c^4-4/3*a^2*c^2/d/x*(c^2*d*x^2+d)^(7/2)-7/3*b^2*(d*(c^2*x^2
+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^3*c^6-1/4*a*b*(d*(c^2*x^2+1))^(1
/2)*d^2*c^3/(c^2*x^2+1)^(1/2)+5/6*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/
2)*arcsinh(c*x)^3*d^2*c^3+14/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*
polylog(2,-c*x-(c^2*x^2+1)^(1/2))*d^2*c^3-14/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c
^2*x^2+1)^(1/2)*arcsinh(c*x)^2*d^2*c^3+1/4*b^2*(d*(c^2*x^2+1))^(1/2)*d^2*c^
6/(c^2*x^2+1)*x^3+1/4*b^2*(d*(c^2*x^2+1))^(1/2)*d^2*c^4/(c^2*x^2+1)*x+14/3*
b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2)
)*d^2*c^3-1/4*b^2*(d*(c^2*x^2+1))^(1/2)*d^2*c^3/(c^2*x^2+1)^(1/2)*arcsinh(c
*x)-1/3*a^2/d/x^3*(c^2*d*x^2+d)^(7/2)+4/3*a^2*c^4*x*(c^2*d*x^2+d)^(5/2)-7/3
*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x/(c^2*x^2+1)*c^4-
2/3*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)/x^3/(c^2*x^2+1)
*arcsinh(c*x)+35*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^
2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^5-21*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63
*c^4*x^4+15*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^5-1/3*b^2*(d*(c
^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)/x^2/(c^2*x^2+1)^(1/2)*arcsin
h(c*x)*c+147*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^4/(c
^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^7-147*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^
4*x^4+15*c^2*x^2+1)*x^5/(c^2*x^2+1)*arcsinh(c*x)^2*c^8-49/3*b^2*(d*(c^2*x^2
+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^5/(c^2*x^2+1)*arcsinh(c*x)*c^8-2
03*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^3/(c^2*x^2+1)*
arcsinh(c*x)^2*c^6-56/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^
2+1)*x^3/(c^2*x^2+1)*arcsinh(c*x)*c^6-21*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*
c^4*x^4+15*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*c^5+14/3*a*b*(d*(c^2*x^2+1))^(1
/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^3-1/3*a*
b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)/x^2/(c^2*x^2+1)^(1/2)
*c+a*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^6/(c^2*x^2+1)*arcsinh(c*x)*x^3+a*b*(d*(c
```

```
^2*x^2+1))^(1/2)*d^2*c^4/(c^2*x^2+1)*arcsinh(c*x)*x-49/3*a*b*(d*(c^2*x^2+1)
)^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^5/(c^2*x^2+1)*c^8-56/3*a*b*(d*(c^2*
x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^3/(c^2*x^2+1)*c^6-190/3*b^2*(
d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x/(c^2*x^2+1)*arcsinh(c*
x)^2*c^4-7/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x/(c^2
*x^2+1)*arcsinh(c*x)*c^4-23/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*
c^2*x^2+1)/x/(c^2*x^2+1)*arcsinh(c*x)^2*c^2+294*a*b*(d*(c^2*x^2+1))^(1/2)*d
^2/(63*c^4*x^4+15*c^2*x^2+1)*x^4/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^7+49/3*a*
b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^3*c^6+7/3*a*b*(d*(c
^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x*c^4+1/3*b^2*(d*(c^2*x^2+1)
)^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)/(c^2*x^2+1)^(1/2)*c^3-1/2*a*b*(d*(c^2
*x^2+1))^(1/2)*d^2*c^5/(c^2*x^2+1)^(1/2)*x^2-5*a*b*(d*(c^2*x^2+1))^(1/2)*d^
2/(63*c^4*x^4+15*c^2*x^2+1)/(c^2*x^2+1)^(1/2)*c^3+49/3*b^2*(d*(c^2*x^2+1))^
(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^3*arcsinh(c*x)*c^6+7/3*b^2*(d*(c^2*x^
2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x*arcsinh(c*x)*c^4+7/3*b^2*(d*(c^
2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)/(c^2*x^2+1)^(1/2)*arcsinh(c*x
)^2*c^3-5*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)/(c^2*x^2+
1)^(1/2)*arcsinh(c*x)*c^3+21*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c
^2*x^2+1)*x^4/(c^2*x^2+1)^(1/2)*c^7-1/2*b^2*(d*(c^2*x^2+1))^(1/2)*d^2*c^5/(
c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^2+5*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x
^4+15*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*c^5-28/3*a*b*(d*(c^2*x^2+1))^(1/2)/(
c^2*x^2+1)^(1/2)*arcsinh(c*x)*d^2*c^3+5/2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^
2+1)^(1/2)*arcsinh(c*x)^2*d^2*c^3+14/3*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1
)^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*d^2*c^3+1/2*b^2*(d*(c^2*x^2+1))^(1/
2)*d^2*c^6/(c^2*x^2+1)*arcsinh(c*x)^2*x^3+1/2*b^2*(d*(c^2*x^2+1))^(1/2)*d^2
*c^4/(c^2*x^2+1)*arcsinh(c*x)^2*x-56/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^
4*x^4+15*c^2*x^2+1)*x^5/(c^2*x^2+1)*c^8-71/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/
(63*c^4*x^4+15*c^2*x^2+1)*x^3/(c^2*x^2+1)*c^6-16/3*b^2*(d*(c^2*x^2+1))^(1/2
)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x/(c^2*x^2+1)*c^4-1/3*b^2*(d*(c^2*x^2+1))^(
1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)/x/(c^2*x^2+1)*c^2-1/3*b^2*(d*(c^2*x^2+1)
)^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)/x^3/(c^2*x^2+1)*arcsinh(c*x)^2+14/3*b
^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1
)^(1/2))*d^2*c^3+14/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c
*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*d^2*c^3+5/2*a^2*c^4*d^2*x*(c^2*d*x^2+d)^(1/
2)+5/2*a^2*c^4*d^3*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1
/2)+5/3*a^2*c^4*d*x*(c^2*d*x^2+d)^(3/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{asinh}(c\,x))^2\,\left(d\,c^2\,x^2 + d\right)^{5/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2))/x^4,x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\left(c^2x^2+1\right)\right)^{\frac{5}{2}}\left(a+b\operatorname{asinh}\left(cx\right)\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2/x**4,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))**2/x**4, x)
```

3.282 $\int \frac{x^4 \sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=153

$$\frac{\sinh^{-1}(ax)^3}{8a^5}+\frac{15\sinh^{-1}(ax)}{64a^5}+\frac{3x^2\sinh^{-1}(ax)}{8a^3}+\frac{x^3\sqrt{a^2x^2+1}}{32a^2}+\frac{x^3\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2}{4a^2}-\frac{15x\sqrt{a^2x^2+1}}{64a^4}-3x\sqrt{}$$

```
[Out] 15/64*arcsinh(a*x)/a^5+3/8*x^2*arcsinh(a*x)/a^3-1/8*x^4*arcsinh(a*x)/a+1/8*
arcsinh(a*x)^3/a^5-15/64*x*(a^2*x^2+1)^(1/2)/a^4+1/32*x^3*(a^2*x^2+1)^(1/2)
/a^2-3/8*x*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/a^4+1/4*x^3*arcsinh(a*x)^2*(a^2
*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.29, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5758, 5675, 5661, 321, 215}

$$\frac{x^3\sqrt{a^2x^2+1}}{32a^2}-\frac{15x\sqrt{a^2x^2+1}}{64a^4}+\frac{x^3\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2}{4a^2}+\frac{3x^2\sinh^{-1}(ax)}{8a^3}-\frac{3x\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2}{8a^4}+\frac{\sinh^{-1}}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (-15*x*Sqrt[1 + a^2*x^2])/(64*a^4) + (x^3*Sqrt[1 + a^2*x^2])/(32*a^2) + (15
*ArcSinh[a*x])/(64*a^5) + (3*x^2*ArcSinh[a*x])/(8*a^3) - (x^4*ArcSinh[a*x])
/(8*a) - (3*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(8*a^4) + (x^3*Sqrt[1 + a^2
*x^2]*ArcSinh[a*x]^2)/(4*a^2) + ArcSinh[a*x]^3/(8*a^5)
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
```

```
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^4\sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \frac{x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{4a^2} - \frac{3\int \frac{x^2\sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx}{4a^2} - \frac{\int x^3\sinh^{-1}(ax)\,dx}{2a}$$

$$= -\frac{x^4\sinh^{-1}(ax)}{8a} - \frac{3x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{8a^4} + \frac{x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{4a^2} + \frac{1}{8}\int \frac{x^4}{\sqrt{1+a^2}}$$

$$= \frac{x^3\sqrt{1+a^2x^2}}{32a^2} + \frac{3x^2\sinh^{-1}(ax)}{8a^3} - \frac{x^4\sinh^{-1}(ax)}{8a} - \frac{3x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{8a^4} + x^3\sqrt{1+a}$$

$$= -\frac{15x\sqrt{1+a^2x^2}}{64a^4} + \frac{x^3\sqrt{1+a^2x^2}}{32a^2} + \frac{3x^2\sinh^{-1}(ax)}{8a^3} - \frac{x^4\sinh^{-1}(ax)}{8a} - \frac{3x\sqrt{1+a^2x^2}\,\sinh}{8a^4}$$

$$= -\frac{15x\sqrt{1+a^2x^2}}{64a^4} + \frac{x^3\sqrt{1+a^2x^2}}{32a^2} + \frac{15\sinh^{-1}(ax)}{64a^5} + \frac{3x^2\sinh^{-1}(ax)}{8a^3} - \frac{x^4\sinh^{-1}(ax)}{8a} - 3x$$

Mathematica [A] time = 0.08, size = 98, normalized size = 0.64

$$\frac{ax\sqrt{a^2x^2+1}\left(2a^2x^2-15\right)+8ax\sqrt{a^2x^2+1}\left(2a^2x^2-3\right)\sinh^{-1}(ax)^2+\left(-8a^4x^4+24a^2x^2+15\right)\sinh^{-1}(ax)+8\sin}{64a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (a*x*Sqrt[1 + a^2*x^2]*(-15 + 2*a^2*x^2) + (15 + 24*a^2*x^2 - 8*a^4*x^4)*Ar
cSinh[a*x] + 8*a*x*Sqrt[1 + a^2*x^2]*(-3 + 2*a^2*x^2)*ArcSinh[a*x]^2 + 8*Ar
cSinh[a*x]^3)/(64*a^5)
```

fricas [A] time = 0.64, size = 131, normalized size = 0.86

$$\frac{8\left(2a^3x^3-3ax\right)\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^2+8\log\left(ax+\sqrt{a^2x^2+1}\right)^3-\left(8a^4x^4-24a^2x^2-15\right)\log\left(ax\right.}{64a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/64*(8*(2*a^3*x^3 - 3*a*x)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^
2 + 8*log(a*x + sqrt(a^2*x^2 + 1))^3 - (8*a^4*x^4 - 24*a^2*x^2 - 15)*log(a*
x + sqrt(a^2*x^2 + 1)) + (2*a^3*x^3 - 15*a*x)*sqrt(a^2*x^2 + 1))/a^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\,\mathrm{arsinh}\left(ax\right)^2}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4*arcsinh(a*x)^2/sqrt(a^2*x^2 + 1), x)
```

maple [A] time = 0.10, size = 125, normalized size = 0.82

$$\frac{16\operatorname{arcsinh}(ax)^2\sqrt{a^2x^2+1}\,a^3x^3-8\operatorname{arcsinh}(ax)\,x^4a^4+2\sqrt{a^2x^2+1}\,x^3a^3-24\operatorname{arcsinh}(ax)^2\sqrt{a^2x^2+1}\,ax+}{64a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/64*(16*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)*a^3*x^3-8*arcsinh(a*x)*x^4*a^4+2*
(a^2*x^2+1)^(1/2)*x^3*a^3-24*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)*a*x+24*arcsin
h(a*x)*x^2*a^2+8*arcsinh(a*x)^3-15*(a^2*x^2+1)^(1/2)*x*a+15*arcsinh(a*x))/a
^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \operatorname{arsinh}(ax)^2}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4*arcsinh(a*x)^2/sqrt(a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4 \operatorname{asinh}(a\,x)^2}{\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*asinh(a*x)^2)/(a^2*x^2 + 1)^(1/2),x)
```

```
[Out] int((x^4*asinh(a*x)^2)/(a^2*x^2 + 1)^(1/2), x)
```

sympy [A] time = 3.68, size = 146, normalized size = 0.95

$$\begin{cases} -\frac{x^4\operatorname{asinh}(ax)}{8a}+\frac{x^3\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{4a^2}+\frac{x^3\sqrt{a^2x^2+1}}{32a^2}+\frac{3x^2\operatorname{asinh}(ax)}{8a^3}-\frac{3x\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{8a^4}-\frac{15x\sqrt{a^2x^2+1}}{64a^4}+\frac{\operatorname{asinh}^3(ax)}{8a^5}+ \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*asinh(a*x)**2/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-x**4*asinh(a*x)/(8*a) + x**3*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/
(4*a**2) + x**3*sqrt(a**2*x**2 + 1)/(32*a**2) + 3*x**2*asinh(a*x)/(8*a**3)
- 3*x*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/(8*a**4) - 15*x*sqrt(a**2*x**2 + 1)
/(64*a**4) + asinh(a*x)**3/(8*a**5) + 15*asinh(a*x)/(64*a**5), Ne(a, 0)), (
0, True))
```

3.283 $\int \frac{x^3 \sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}} \, dx$

Optimal. Leaf size=122

$$\frac{4x\sinh^{-1}(ax)}{3a^3}+\frac{x^2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2}{3a^2}+\frac{2\left(a^2x^2+1\right)^{3/2}}{27a^4}-\frac{14\sqrt{a^2x^2+1}}{9a^4}-\frac{2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2}{3a^4}-\frac{2x^3\sinh^{-1}(}{9a}$$

```
[Out] 2/27*(a^2*x^2+1)^(3/2)/a^4+4/3*x*arcsinh(a*x)/a^3-2/9*x^3*arcsinh(a*x)/a-14
/9*(a^2*x^2+1)^(1/2)/a^4-2/3*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/a^4+1/3*x^2*a
rcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.22, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5758, 5717, 5653, 261, 5661, 266, 43}

$$\frac{2\left(a^2x^2+1\right)^{3/2}}{27a^4}-\frac{14\sqrt{a^2x^2+1}}{9a^4}+\frac{x^2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2}{3a^2}-\frac{2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2}{3a^4}+\frac{4x\sinh^{-1}(ax)}{3a^3}-\frac{2x^3\sinh^{-1}(}{9a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (-14*Sqrt[1 + a^2*x^2])/(9*a^4) + (2*(1 + a^2*x^2)^(3/2))/(27*a^4) + (4*x*A
rcSinh[a*x])/(3*a^3) - (2*x^3*ArcSinh[a*x])/(9*a) - (2*Sqrt[1 + a^2*x^2]*Ar
cSinh[a*x]^2)/(3*a^4) + (x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(3*a^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^3 \sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \frac{x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{3a^2} - \frac{2\int \frac{x\sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx}{3a^2} - \frac{2\int x^2\sinh^{-1}(ax)\,dx}{3a}$$

$$= -\frac{2x^3\sinh^{-1}(ax)}{9a} - \frac{2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{3a^4} + \frac{x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{3a^2} + \frac{2}{9}\int \frac{x}{\sqrt{1+}}$$

$$= \frac{4x\sinh^{-1}(ax)}{3a^3} - \frac{2x^3\sinh^{-1}(ax)}{9a} - \frac{2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{3a^4} + \frac{x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax}{3a^2}$$

$$= -\frac{4\sqrt{1+a^2x^2}}{3a^4} + \frac{4x\sinh^{-1}(ax)}{3a^3} - \frac{2x^3\sinh^{-1}(ax)}{9a} - \frac{2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{3a^4} + \frac{x^2\sqrt{1-}}{}$$

$$= -\frac{14\sqrt{1+a^2x^2}}{9a^4} + \frac{2\left(1+a^2x^2\right)^{3/2}}{27a^4} + \frac{4x\sinh^{-1}(ax)}{3a^3} - \frac{2x^3\sinh^{-1}(ax)}{9a} - \frac{2\sqrt{1+a^2x^2}\,\sin}{3a^4}$$

Mathematica [A] time = 0.06, size = 79, normalized size = 0.65

$$\frac{2\left(a^2x^2-20\right)\sqrt{a^2x^2+1}+9\left(a^2x^2-2\right)\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2-6ax\left(a^2x^2-6\right)\sinh^{-1}(ax)}{27a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (2*(-20 + a^2*x^2)*Sqrt[1 + a^2*x^2] - 6*a*x*(-6 + a^2*x^2)*ArcSinh[a*x] +
9*(-2 + a^2*x^2)*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(27*a^4)
```

fricas [A] time = 0.45, size = 98, normalized size = 0.80

$$\frac{9\sqrt{a^2x^2+1}\left(a^2x^2-2\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^2-6\left(a^3x^3-6\,ax\right)\log\left(ax+\sqrt{a^2x^2+1}\right)+2\sqrt{a^2x^2+1}\left(a^2x^2-2\right.}{27\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/27*(9*sqrt(a^2*x^2 + 1)*(a^2*x^2 - 2)*log(a*x + sqrt(a^2*x^2 + 1))^2 - 6*
(a^3*x^3 - 6*a*x)*log(a*x + sqrt(a^2*x^2 + 1)) + 2*sqrt(a^2*x^2 + 1)*(a^2*x
^2 - 20))/a^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.09, size = 113, normalized size = 0.93

$$\frac{9 \operatorname{arcsinh}(ax)^2 x^4a^4 - 9 \operatorname{arcsinh}(ax)^2 a^2x^2 - 6 \operatorname{arcsinh}(ax) \sqrt{a^2x^2+1}\; a^3x^3 + 2x^4a^4 - 38a^2x^2 - 18 \operatorname{arcsinh}(ax)^2}{27a^4\sqrt{a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/27/a^4/(a^2*x^2+1)^(1/2)*(9*arcsinh(a*x)^2*x^4*a^4-9*arcsinh(a*x)^2*a^2*x
^2-6*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a^3*x^3+2*x^4*a^4-38*a^2*x^2-18*arcsinh
(a*x)^2+36*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a*x-40)
```

maxima [A] time = 0.46, size = 101, normalized size = 0.83

$$\frac{1}{3}\left(\frac{\sqrt{a^2x^2+1}\,x^2}{a^2} - \frac{2\sqrt{a^2x^2+1}}{a^4}\right)\operatorname{arsinh}(ax)^2 + \frac{2\left(\sqrt{a^2x^2+1}\,x^2 - \frac{20\sqrt{a^2x^2+1}}{a^2}\right)}{27\,a^2} - \frac{2\left(a^2x^3 - 6\,x\right)\operatorname{arsinh}(ax)}{9\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(sqrt(a^2*x^2 + 1)*x^2/a^2 - 2*sqrt(a^2*x^2 + 1)/a^4)*arcsinh(a*x)^2 +
2/27*(sqrt(a^2*x^2 + 1)*x^2 - 20*sqrt(a^2*x^2 + 1)/a^2)/a^2 - 2/9*(a^2*x^3
- 6*x)*arcsinh(a*x)/a^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \operatorname{asinh}(a\,x)^2}{\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*asinh(a*x)^2)/(a^2*x^2 + 1)^(1/2),x)
```

```
[Out] int((x^3*asinh(a*x)^2)/(a^2*x^2 + 1)^(1/2), x)
```

sympy [A] time = 2.17, size = 121, normalized size = 0.99

$$\begin{cases} -\frac{2x^3 \operatorname{asinh}(ax)}{9a} + \frac{x^2\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{3a^2} + \frac{2x^2\sqrt{a^2x^2+1}}{27a^2} + \frac{4x\operatorname{asinh}(ax)}{3a^3} - \frac{2\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{3a^4} - \frac{40\sqrt{a^2x^2+1}}{27a^4} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(a*x)**2/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-2*x**3*asinh(a*x)/(9*a) + x**2*sqrt(a**2*x**2 + 1)*asinh(a*x)**
2/(3*a**2) + 2*x**2*sqrt(a**2*x**2 + 1)/(27*a**2) + 4*x*asinh(a*x)/(3*a**3)
 - 2*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/(3*a**4) - 40*sqrt(a**2*x**2 + 1)/(2
7*a**4), Ne(a, 0)), (0, True))
```

3.284 $\int \frac{x^2 \sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}} dx$

Optimal. Leaf size=87

$$-\frac{\sinh^{-1}(ax)^3}{6a^3}-\frac{\sinh^{-1}(ax)}{4a^3}+\frac{x\sqrt{a^2x^2+1}}{4a^2}+\frac{x\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2}{2a^2}-\frac{x^2\sinh^{-1}(ax)}{2a}$$

```
[Out] -1/4*arcsinh(a*x)/a^3-1/2*x^2*arcsinh(a*x)/a-1/6*arcsinh(a*x)^3/a^3+1/4*x*(
a^2*x^2+1)^(1/2)/a^2+1/2*x*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.15, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5758, 5675, 5661, 321, 215}

$$\frac{x\sqrt{a^2x^2+1}}{4a^2}+\frac{x\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2}{2a^2}-\frac{\sinh^{-1}(ax)^3}{6a^3}-\frac{\sinh^{-1}(ax)}{4a^3}-\frac{x^2\sinh^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (x*Sqrt[1 + a^2*x^2])/(4*a^2) - ArcSinh[a*x]/(4*a^3) - (x^2*ArcSinh[a*x])/(
2*a) + (x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(2*a^2) - ArcSinh[a*x]^3/(6*a^3
)
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

```
&& GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2 \sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx &= \frac{x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{2a^2} - \frac{\int \frac{\sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx}{2a^2} - \frac{\int x\sinh^{-1}(ax)\,dx}{a}\\
&= -\frac{x^2\sinh^{-1}(ax)}{2a} + \frac{x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{2a^2} - \frac{\sinh^{-1}(ax)^3}{6a^3} + \frac{1}{2}\int \frac{x^2}{\sqrt{1+a^2x^2}}\,dx\\
&= \frac{x\sqrt{1+a^2x^2}}{4a^2} - \frac{x^2\sinh^{-1}(ax)}{2a} + \frac{x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{2a^2} - \frac{\sinh^{-1}(ax)^3}{6a^3} - \frac{\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{4a^2}\\
&= \frac{x\sqrt{1+a^2x^2}}{4a^2} - \frac{\sinh^{-1}(ax)}{4a^3} - \frac{x^2\sinh^{-1}(ax)}{2a} + \frac{x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{2a^2} - \frac{\sinh^{-1}(ax)^3}{6a^3}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 72, normalized size = 0.83

$$\frac{3ax\sqrt{a^2x^2+1} + 6ax\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2 - 3\left(2a^2x^2+1\right)\sinh^{-1}(ax) - 2\sinh^{-1}(ax)^3}{12a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (3*a*x*Sqrt[1 + a^2*x^2] - 3*(1 + 2*a^2*x^2)*ArcSinh[a*x] + 6*a*x*Sqrt[1 +
a^2*x^2]*ArcSinh[a*x]^2 - 2*ArcSinh[a*x]^3)/(12*a^3)
```

fricas [A] time = 0.53, size = 102, normalized size = 1.17

$$\frac{6\sqrt{a^2x^2+1}\,ax\log\left(ax+\sqrt{a^2x^2+1}\right)^2 - 2\log\left(ax+\sqrt{a^2x^2+1}\right)^3 + 3\sqrt{a^2x^2+1}\,ax - 3\left(2a^2x^2+1\right)\log\left(ax+\sqrt{a^2x^2+1}\right)}{12a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(6*sqrt(a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 + 1))^2 - 2*log(a*x +
sqrt(a^2*x^2 + 1))^3 + 3*sqrt(a^2*x^2 + 1)*a*x - 3*(2*a^2*x^2 + 1)*log(a*x
+ sqrt(a^2*x^2 + 1)))/a^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\operatorname{arsinh}(ax)^2}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arcsinh(a*x)^2/sqrt(a^2*x^2 + 1), x)
```

maple [A] time = 0.08, size = 69, normalized size = 0.79

$$-\frac{-6\operatorname{arcsinh}(ax)^2\sqrt{a^2x^2+1}\,ax + 6\operatorname{arcsinh}(ax)x^2a^2 + 2\operatorname{arcsinh}(ax)^3 - 3\sqrt{a^2x^2+1}\,xa + 3\operatorname{arcsinh}(ax)}{12a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/12*(-6*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)*a*x+6*arcsinh(a*x)*x^2*a^2+2*arc
sinh(a*x)^3-3*(a^2*x^2+1)^(1/2)*x*a+3*arcsinh(a*x))/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{arsinh}(ax)^2}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*arcsinh(a*x)^2/sqrt(a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 \operatorname{asinh}(a\,x)^2}{\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*asinh(a*x)^2)/(a^2*x^2 + 1)^(1/2),x)
```

```
[Out] int((x^2*asinh(a*x)^2)/(a^2*x^2 + 1)^(1/2), x)
```

sympy [A] time = 1.24, size = 78, normalized size = 0.90

$$\begin{cases} -\frac{x^2 \operatorname{asinh}(ax)}{2a} + \frac{x\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{2a^2} + \frac{x\sqrt{a^2x^2+1}}{4a^2} - \frac{\operatorname{asinh}^3(ax)}{6a^3} - \frac{\operatorname{asinh}(ax)}{4a^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(a*x)**2/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-x**2*asinh(a*x)/(2*a) + x*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/(2*
a**2) + x*sqrt(a**2*x**2 + 1)/(4*a**2) - asinh(a*x)**3/(6*a**3) - asinh(a*x
)/(4*a**3), Ne(a, 0)), (0, True))
```

3.285 $\int \frac{x \sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}} \, dx$

Optimal. Leaf size=52

$$\frac{2\sqrt{a^2x^2+1}}{a^2}+\frac{\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2}{a^2}-\frac{2x\sinh^{-1}(ax)}{a}$$

```
[Out] -2*x*arcsinh(a*x)/a+2*(a^2*x^2+1)^(1/2)/a^2+arcsinh(a*x)^2*(a^2*x^2+1)^(1/2
)/a^2
```

Rubi [A] time = 0.08, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5717, 5653, 261}

$$\frac{2\sqrt{a^2x^2+1}}{a^2}+\frac{\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2}{a^2}-\frac{2x\sinh^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (2*Sqrt[1 + a^2*x^2])/a^2 - (2*x*ArcSinh[a*x])/a + (Sqrt[1 + a^2*x^2]*ArcSi
nh[a*x]^2)/a^2
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx &= \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{a^2}-\frac{2\int \sinh^{-1}(ax)\,dx}{a}\\
&= -\frac{2x\sinh^{-1}(ax)}{a}+\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{a^2}+2\int \frac{x}{\sqrt{1+a^2x^2}}\,dx\\
&= \frac{2\sqrt{1+a^2x^2}}{a^2}-\frac{2x\sinh^{-1}(ax)}{a}+\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{a^2}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 48, normalized size = 0.92

$$\frac{2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2-2ax\sinh^{-1}(ax)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (2*Sqrt[1 + a^2*x^2] - 2*a*x*ArcSinh[a*x] + Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^
2)/a^2
```

fricas [A] time = 0.47, size = 70, normalized size = 1.35

$$-\frac{2\,ax\log\left(ax+\sqrt{a^2x^2+1}\right)-\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1}\right)^2-2\,\sqrt{a^2x^2+1}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*a*x*log(a*x + sqrt(a^2*x^2 + 1)) - sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2
*x^2 + 1))^2 - 2*sqrt(a^2*x^2 + 1))/a^2
```

giac [A] time = 0.38, size = 74, normalized size = 1.42

$$\frac{\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{a^2}-\frac{2\left(x\log\left(ax+\sqrt{a^2x^2+1}\right)-\frac{\sqrt{a^2x^2+1}}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2/a^2 - 2*(x*log(a*x + sqrt(
a^2*x^2 + 1)) - sqrt(a^2*x^2 + 1)/a)/a
```

maple [A] time = 0.08, size = 64, normalized size = 1.23

$$\frac{\operatorname{arcsinh}(ax)^2a^2x^2+\operatorname{arcsinh}(ax)^2-2\operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}\,ax+2a^2x^2+2}{a^2\sqrt{a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/a^2/(a^2*x^2+1)^(1/2)*(arcsinh(a*x)^2*a^2*x^2+arcsinh(a*x)^2-2*arcsinh(a*
x)*(a^2*x^2+1)^(1/2)*a*x+2*a^2*x^2+2)
```

maxima [A] time = 0.52, size = 48, normalized size = 0.92

$$\frac{\sqrt{a^2x^2+1}\operatorname{arsinh}(ax)^2}{a^2}-\frac{2\left(ax\operatorname{arsinh}(ax)-\sqrt{a^2x^2+1}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a^2*x^2 + 1)*arcsinh(a*x)^2/a^2 - 2*(a*x*arcsinh(a*x) - sqrt(a^2*x^2 +
 1))/a^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\mathrm{asinh}\left(a\,x\right)^2}{\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*asinh(a*x)^2)/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x*asinh(a*x)^2)/(a^2*x^2 + 1)^(1/2), x)
```

sympy [A] time = 0.72, size = 49, normalized size = 0.94

$$\begin{cases} -\frac{2x\,\mathrm{asinh}\,(ax)}{a} + \frac{\sqrt{a^2x^2+1}\,\mathrm{asinh}^2\,(ax)}{a^2} + \frac{2\sqrt{a^2x^2+1}}{a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(a*x)**2/(a**2*x**2+1)**(1/2),x)
[Out] Piecewise((-2*x*asinh(a*x)/a + sqrt(a**2*x**2 + 1)*asinh(a*x)**2/a**2 + 2*s
qrt(a**2*x**2 + 1)/a**2, Ne(a, 0)), (0, True))
```

3.286 $\int \frac{\sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=13

$$\frac{\sinh^{-1}(ax)^3}{3a}$$

```
[Out] 1/3*arcsinh(a*x)^3/a
```

Rubi [A] time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {5675}

$$\frac{\sinh^{-1}(ax)^3}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^2/Sqrt[1 + a^2*x^2],x]
```

```
[Out] ArcSinh[a*x]^3/(3*a)
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \frac{\sinh^{-1}(ax)^3}{3a}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{\sinh^{-1}(ax)^3}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^2/Sqrt[1 + a^2*x^2],x]
```

```
[Out] ArcSinh[a*x]^3/(3*a)
```

fricas [B] time = 0.44, size = 23, normalized size = 1.77

$$\frac{\log\left(ax+\sqrt{a^2x^2+1}\right)^3}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*log(a*x + sqrt(a^2*x^2 + 1))^3/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)^2}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^2/sqrt(a^2*x^2 + 1), x)
```

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$\frac{\operatorname{arcsinh}(ax)^3}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/3*arcsinh(a*x)^3/a
```

maxima [A] time = 0.46, size = 11, normalized size = 0.85

$$\frac{\operatorname{arsinh}(ax)^3}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*arcsinh(a*x)^3/a
```

mupad [B] time = 0.09, size = 11, normalized size = 0.85

$$\frac{\operatorname{asinh}(a\,x)^3}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^2/(a^2*x^2 + 1)^(1/2),x)
```

```
[Out] asinh(a*x)^3/(3*a)
```

sympy [A] time = 0.40, size = 10, normalized size = 0.77

$$\begin{cases} \frac{\operatorname{asinh}^3(ax)}{3a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**2/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((asinh(a*x)**3/(3*a), Ne(a, 0)), (0, True))
```

3.287 $\int \frac{\sinh^{-1}(ax)^2}{x\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=68

$$-2\sinh^{-1}(ax)\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right)+2\sinh^{-1}(ax)\text{Li}_2\left(e^{\sinh^{-1}(ax)}\right)+2\text{Li}_3\left(-e^{\sinh^{-1}(ax)}\right)-2\text{Li}_3\left(e^{\sinh^{-1}(ax)}\right)-2\sinh^{-1}(ax$$

```
[Out] -2*arcsinh(a*x)^2*arctanh(a*x+(a^2*x^2+1)^(1/2))-2*arcsinh(a*x)*polylog(2,-
a*x-(a^2*x^2+1)^(1/2))+2*arcsinh(a*x)*polylog(2,a*x+(a^2*x^2+1)^(1/2))+2*po
lylog(3,-a*x-(a^2*x^2+1)^(1/2))-2*polylog(3,a*x+(a^2*x^2+1)^(1/2))
```

Rubi [A] time = 0.15, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5760, 4182, 2531, 2282, 6589}

$$-2\sinh^{-1}(ax)\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)+2\sinh^{-1}(ax)\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)+2\text{PolyLog}\left(3,-e^{\sinh^{-1}(ax)}\right)-2\text{Pol}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^2/(x*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -2*ArcSinh[a*x]^2*ArcTanh[E^ArcSinh[a*x]] - 2*ArcSinh[a*x]*PolyLog[2, -E^Ar
cSinh[a*x]] + 2*ArcSinh[a*x]*PolyLog[2, E^ArcSinh[a*x]] + 2*PolyLog[3, -E^A
rcSinh[a*x]] - 2*PolyLog[3, E^ArcSinh[a*x]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^2}{x\sqrt{1+a^2x^2}}\,dx = \text{Subst}\left(\int x^2\text{csch}(x)\,dx, x, \sinh^{-1}(ax)\right)$$
$$= -2\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 2\,\text{Subst}\left(\int x\log\left(1-e^x\right)\,dx, x, \sinh^{-1}(ax)\right) + 2\,\text{Subst}$$
$$= -2\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 2\sinh^{-1}(ax)\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + 2\sinh^{-1}(ax)\text{Li}_2\left(e^{\sinh^{-}}\right.$$
$$= -2\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 2\sinh^{-1}(ax)\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + 2\sinh^{-1}(ax)\text{Li}_2\left(e^{\sinh^{-}}\right.$$
$$= -2\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 2\sinh^{-1}(ax)\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + 2\sinh^{-1}(ax)\text{Li}_2\left(e^{\sinh^{-}}\right.$$

Mathematica [A] time = 0.11, size = 100, normalized size = 1.47

$$2\sinh^{-1}(ax)\text{Li}_2\left(-e^{-\sinh^{-1}(ax)}\right) - 2\sinh^{-1}(ax)\text{Li}_2\left(e^{-\sinh^{-1}(ax)}\right) + 2\text{Li}_3\left(-e^{-\sinh^{-1}(ax)}\right) - 2\text{Li}_3\left(e^{-\sinh^{-1}(ax)}\right) + \sinh^{-1}(ax$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^2/(x*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] ArcSinh[a*x]^2*Log[1 - E^(-ArcSinh[a*x])] - ArcSinh[a*x]^2*Log[1 + E^(-ArcS
inh[a*x])] + 2*ArcSinh[a*x]*PolyLog[2, -E^(-ArcSinh[a*x])] - 2*ArcSinh[a*x]
*PolyLog[2, E^(-ArcSinh[a*x])] + 2*PolyLog[3, -E^(-ArcSinh[a*x])] - 2*PolyL
og[3, E^(-ArcSinh[a*x])]
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a^2x^2+1}\,\text{arsinh}\left(ax\right)^2}{a^2x^3+x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^2/(a^2*x^3 + x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arsinh}\left(ax\right)^2}{\sqrt{a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^2/(sqrt(a^2*x^2 + 1)*x), x)
```

maple [A] time = 0.09, size = 144, normalized size = 2.12

$$-\text{arcsinh}\left(ax\right)^2\ln\left(1+ax+\sqrt{a^2x^2+1}\right) - 2\,\text{arcsinh}\left(ax\right)\text{polylog}\left(2, -ax-\sqrt{a^2x^2+1}\right) + 2\,\text{polylog}\left(3, -ax-\sqrt{a^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^2/x/(a^2*x^2+1)^(1/2),x)
```

```
[Out] -arcsinh(a*x)^2*ln(1+a*x+(a^2*x^2+1)^(1/2))-2*arcsinh(a*x)*polylog(2,-a*x-(
a^2*x^2+1)^(1/2))+2*polylog(3,-a*x-(a^2*x^2+1)^(1/2))+arcsinh(a*x)^2*ln(1-a
*x-(a^2*x^2+1)^(1/2))+2*arcsinh(a*x)*polylog(2,a*x+(a^2*x^2+1)^(1/2))-2*pol
ylog(3,a*x+(a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)^2}{\sqrt{a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^2/(sqrt(a^2*x^2 + 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}(a\,x)^2}{x\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^2/(x*(a^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(asinh(a*x)^2/(x*(a^2*x^2 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^2(ax)}{x\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**2/x/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(asinh(a*x)**2/(x*sqrt(a**2*x**2 + 1)), x)
```

3.288 $\int \frac{\sinh^{-1}(ax)^2}{x^2\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=66

$$-\frac{\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2}{x}+a\mathrm{Li}_2\left(e^{2\sinh^{-1}(ax)}\right)-a\sinh^{-1}(ax)^2+2a\sinh^{-1}(ax)\log\left(1-e^{2\sinh^{-1}(ax)}\right)$$

```
[Out] -a*arcsinh(a*x)^2+2*a*arcsinh(a*x)*ln(1-(a*x+(a^2*x^2+1)^(1/2))^2)+a*polylo
g(2,(a*x+(a^2*x^2+1)^(1/2))^2)-arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.16, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5723, 5659, 3716, 2190, 2279, 2391}

$$a\text{PolyLog}\left(2,e^{2\sinh^{-1}(ax)}\right)-\frac{\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2}{x}-a\sinh^{-1}(ax)^2+2a\sinh^{-1}(ax)\log\left(1-e^{2\sinh^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^2/(x^2*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -(a*ArcSinh[a*x]^2) - (Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/x + 2*a*ArcSinh[a*
x]*Log[1 - E^(2*ArcSinh[a*x])] + a*PolyLog[2, E^(2*ArcSinh[a*x])]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{-1}(ax)^2}{x^2\sqrt{1+a^2x^2}}\,dx &= -\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{x}+(2a)\int \frac{\sinh^{-1}(ax)}{x}\,dx\\
&= -\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{x}+(2a)\,\text{Subst}\left(\int x\coth(x)\,dx,x,\sinh^{-1}(ax)\right)\\
&= -a\sinh^{-1}(ax)^2-\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{x}-(4a)\,\text{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx,x,\sinh^{-1}(ax)\right)\\
&= -a\sinh^{-1}(ax)^2-\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{x}+2a\sinh^{-1}(ax)\log\left(1-e^{2\sinh^{-1}(ax)}\right)-(2a)\,\text{Su}\\
&= -a\sinh^{-1}(ax)^2-\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{x}+2a\sinh^{-1}(ax)\log\left(1-e^{2\sinh^{-1}(ax)}\right)-a\,\text{Subs}\\
&= -a\sinh^{-1}(ax)^2-\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{x}+2a\sinh^{-1}(ax)\log\left(1-e^{2\sinh^{-1}(ax)}\right)+a\text{Li}_2\left(e\right.
\end{aligned}$$

Mathematica [A] time = 0.33, size = 65, normalized size = 0.98

$$a\left(\sinh^{-1}(ax)\left(-\frac{\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{ax}+\sinh^{-1}(ax)+2\log\left(1-e^{-2\sinh^{-1}(ax)}\right)\right)-\text{Li}_2\left(e^{-2\sinh^{-1}(ax)}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^2/(x^2*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] a*(ArcSinh[a*x]*(ArcSinh[a*x] - (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(a*x) + 2*
Log[1 - E^(-2*ArcSinh[a*x])]) - PolyLog[2, E^(-2*ArcSinh[a*x])])
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)^2}{a^2x^4+x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^2/(a^2*x^4 + x^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

maple [A] time = 0.15, size = 132, normalized size = 2.00

$$\frac{\left(ax-\sqrt{a^2x^2+1}\right)\operatorname{arcsinh}(ax)^2}{x}-2a\operatorname{arcsinh}(ax)^2+2a\operatorname{arcsinh}(ax)\ln\left(1+ax+\sqrt{a^2x^2+1}\right)+2a\operatorname{polylog}\left(2,-ax\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^2/x^2/(a^2*x^2+1)^(1/2),x)
```

```
[Out] (a*x-(a^2*x^2+1)^(1/2))/x*arcsinh(a*x)^2-2*a*arcsinh(a*x)^2+2*a*arcsinh(a*x
)*ln(1+a*x+(a^2*x^2+1)^(1/2))+2*a*polylog(2,-a*x-(a^2*x^2+1)^(1/2))+2*a*arc
sinh(a*x)*ln(1-a*x-(a^2*x^2+1)^(1/2))+2*a*polylog(2,a*x+(a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{x}+\int\frac{2\left(a^3x^2+\sqrt{a^2x^2+1}\,a^2x+a\right)\log\left(ax+\sqrt{a^2x^2+1}\right)}{\sqrt{a^2x^2+1}\,ax^2+\left(a^2x^2+1\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2/x + integrate(2*(a^3*x^2
+ sqrt(a^2*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))/(sqrt(a^2*x^2 +
 1)*a*x^2 + (a^2*x^2 + 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\operatorname{asinh}(a\,x)^2}{x^2\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^2/(x^2*(a^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(asinh(a*x)^2/(x^2*(a^2*x^2 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{asinh}^2(ax)}{x^2\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**2/x**2/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(asinh(a*x)**2/(x**2*sqrt(a**2*x**2 + 1)), x)
```

3.289 $\int \frac{\sinh^{-1}(ax)^2}{x^3\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=135

$$a^2\sinh^{-1}(ax)\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right)-a^2\sinh^{-1}(ax)\text{Li}_2\left(e^{\sinh^{-1}(ax)}\right)-a^2\text{Li}_3\left(-e^{\sinh^{-1}(ax)}\right)+a^2\text{Li}_3\left(e^{\sinh^{-1}(ax)}\right)-\frac{\sqrt{a^2x^2+}}{}$$

```
[Out] -a*arcsinh(a*x)/x+a^2*arcsinh(a*x)^2*arctanh(a*x+(a^2*x^2+1)^(1/2))-a^2*arc
tanh((a^2*x^2+1)^(1/2))+a^2*arcsinh(a*x)*polylog(2,-a*x-(a^2*x^2+1)^(1/2))-
a^2*arcsinh(a*x)*polylog(2,a*x+(a^2*x^2+1)^(1/2))-a^2*polylog(3,-a*x-(a^2*x
^2+1)^(1/2))+a^2*polylog(3,a*x+(a^2*x^2+1)^(1/2))-1/2*arcsinh(a*x)^2*(a^2*x
^2+1)^(1/2)/x^2
```

Rubi [A] time = 0.26, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {5747, 5760, 4182, 2531, 2282, 6589, 5661, 266, 63, 208}

$$a^2\sinh^{-1}(ax)\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)-a^2\sinh^{-1}(ax)\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)-a^2\text{PolyLog}\left(3,-e^{\sinh^{-1}(ax)}\right)+a^2\text{P}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^2/(x^3*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -((a*ArcSinh[a*x])/x) - (Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(2*x^2) + a^2*Ar
cSinh[a*x]^2*ArcTanh[E^ArcSinh[a*x]] - a^2*ArcTanh[Sqrt[1 + a^2*x^2]] + a^2
*ArcSinh[a*x]*PolyLog[2, -E^ArcSinh[a*x]] - a^2*ArcSinh[a*x]*PolyLog[2, E^A
rcSinh[a*x]] - a^2*PolyLog[3, -E^ArcSinh[a*x]] + a^2*PolyLog[3, E^ArcSinh[a
*x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^2}{x^3\sqrt{1+a^2x^2}}\,dx = -\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{2x^2} + a\int \frac{\sinh^{-1}(ax)}{x^2}\,dx - \frac{1}{2}a^2\int \frac{\sinh^{-1}(ax)^2}{x\sqrt{1+a^2x^2}}\,dx$$
$$= -\frac{a\sinh^{-1}(ax)}{x} - \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{2x^2} - \frac{1}{2}a^2\,\text{Subst}\left(\int x^2\text{csch}(x)\,dx, x, \sinh^{-1}(ax)\right) +$$
$$= -\frac{a\sinh^{-1}(ax)}{x} - \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{2x^2} + a^2\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) + \frac{1}{2}a^2\,\text{Sul}$$
$$= -\frac{a\sinh^{-1}(ax)}{x} - \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{2x^2} + a^2\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) + a^2\sinh$$
$$= -\frac{a\sinh^{-1}(ax)}{x} - \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{2x^2} + a^2\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - a^2\tanh$$
$$= -\frac{a\sinh^{-1}(ax)}{x} - \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{2x^2} + a^2\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - a^2\tanh$$

Mathematica [A] time = 1.20, size = 188, normalized size = 1.39

$$\frac{1}{8}a^2\left(-8\sinh^{-1}(ax)\text{Li}_2\left(-e^{-\sinh^{-1}(ax)}\right) + 8\sinh^{-1}(ax)\text{Li}_2\left(e^{-\sinh^{-1}(ax)}\right) - 8\text{Li}_3\left(-e^{-\sinh^{-1}(ax)}\right) + 8\text{Li}_3\left(e^{-\sinh^{-1}(ax}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^2/(x^3*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] (a^2*(-4*ArcSinh[a*x]*Coth[ArcSinh[a*x]/2] - ArcSinh[a*x]^2*Csch[ArcSinh[a*
x]/2]^2 - 4*ArcSinh[a*x]^2*Log[1 - E^(-ArcSinh[a*x])] + 4*ArcSinh[a*x]^2*Lo
g[1 + E^(-ArcSinh[a*x])] + 8*Log[Tanh[ArcSinh[a*x]/2]] - 8*ArcSinh[a*x]*Pol
yLog[2, -E^(-ArcSinh[a*x])] + 8*ArcSinh[a*x]*PolyLog[2, E^(-ArcSinh[a*x])]
- 8*PolyLog[3, -E^(-ArcSinh[a*x])] + 8*PolyLog[3, E^(-ArcSinh[a*x])] - ArcS
inh[a*x]^2*Sech[ArcSinh[a*x]/2]^2 + 4*ArcSinh[a*x]*Tanh[ArcSinh[a*x]/2]))/8
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)^2}{a^2x^5+x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^2/(a^2*x^5 + x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arsinh}\,(ax)^2}{\sqrt{a^2x^2+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^2/(sqrt(a^2*x^2 + 1)*x^3), x)
```

maple [A] time = 0.22, size = 233, normalized size = 1.73

$$-\frac{\text{arcsinh}\,(ax)\left(\text{arcsinh}\,(ax)\,x^2a^2 + 2\sqrt{a^2x^2+1}\,xa + \text{arcsinh}\,(ax)\right)}{2\sqrt{a^2x^2+1}\,x^2} + \frac{a^2\,\text{arcsinh}\,(ax)^2\ln\left(1+ax+\sqrt{a^2x^2+1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^2/x^3/(a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/2/(a^2*x^2+1)^(1/2)/x^2*arcsinh(a*x)*(arcsinh(a*x)*x^2*a^2+2*(a^2*x^2+1)
^(1/2)*x*a+arcsinh(a*x))+1/2*a^2*arcsinh(a*x)^2*ln(1+a*x+(a^2*x^2+1)^(1/2))
+a^2*arcsinh(a*x)*polylog(2,-a*x-(a^2*x^2+1)^(1/2))-a^2*polylog(3,-a*x-(a^2
*x^2+1)^(1/2))-1/2*a^2*arcsinh(a*x)^2*ln(1-a*x-(a^2*x^2+1)^(1/2))-a^2*arcsi
nh(a*x)*polylog(2,a*x+(a^2*x^2+1)^(1/2))+a^2*polylog(3,a*x+(a^2*x^2+1)^(1/2
))-2*a^2*arctanh(a*x+(a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)^2}{\sqrt{a^2x^2+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^2/(sqrt(a^2*x^2 + 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}(a\,x)^2}{x^3\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^2/(x^3*(a^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(asinh(a*x)^2/(x^3*(a^2*x^2 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^2(ax)}{x^3\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**2/x**3/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(asinh(a*x)**2/(x**3*sqrt(a**2*x**2 + 1)), x)
```

3.290 $\int \frac{x^5(a+b\sinh^{-1}(cx))^2}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=383

$$-\frac{2bx^5\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{25c\sqrt{c^2dx^2+d}}+\frac{x^4\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{5c^2d}+\frac{8\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{15c^6d}-\frac{1}{1}$$

```
[Out] 298/225*b^2*(c^2*x^2+1)/c^6/(c^2*d*x^2+d)^(1/2)-76/675*b^2*(c^2*x^2+1)^2/c^
6/(c^2*d*x^2+d)^(1/2)+2/125*b^2*(c^2*x^2+1)^3/c^6/(c^2*d*x^2+d)^(1/2)-16/15
*a*b*x*(c^2*x^2+1)^(1/2)/c^5/(c^2*d*x^2+d)^(1/2)-16/15*b^2*x*arcsinh(c*x)*(
c^2*x^2+1)^(1/2)/c^5/(c^2*d*x^2+d)^(1/2)+8/45*b*x^3*(a+b*arcsinh(c*x))*(c^2
*x^2+1)^(1/2)/c^3/(c^2*d*x^2+d)^(1/2)-2/25*b*x^5*(a+b*arcsinh(c*x))*(c^2*x^
2+1)^(1/2)/c/(c^2*d*x^2+d)^(1/2)+8/15*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1
/2)/c^6/d-4/15*x^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^4/d+1/5*x^4*(
a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^2/d
```

Rubi [A] time = 0.55, antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5758, 5717, 5653, 261, 5661, 266, 43}

$$-\frac{16abx\sqrt{c^2x^2+1}}{15c^5\sqrt{c^2dx^2+d}}-\frac{2bx^5\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{25c\sqrt{c^2dx^2+d}}+\frac{x^4\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{5c^2d}+\frac{8bx^3\sqrt{c^2x^2+1}\left(a\right.}{45c^3\sqrt{c^2d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (-16*a*b*x*Sqrt[1 + c^2*x^2])/(15*c^5*Sqrt[d + c^2*d*x^2]) + (298*b^2*(1 +
c^2*x^2))/(225*c^6*Sqrt[d + c^2*d*x^2]) - (76*b^2*(1 + c^2*x^2)^2)/(675*c^6
*Sqrt[d + c^2*d*x^2]) + (2*b^2*(1 + c^2*x^2)^3)/(125*c^6*Sqrt[d + c^2*d*x^2
]) - (16*b^2*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(15*c^5*Sqrt[d + c^2*d*x^2])
 + (8*b*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(45*c^3*Sqrt[d + c^2*d*
x^2]) - (2*b*x^5*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(25*c*Sqrt[d + c^2
*d*x^2]) + (8*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(15*c^6*d) - (4*x
^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(15*c^4*d) + (x^4*Sqrt[d + c
^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(5*c^2*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx = \frac{x^4\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{5c^2d} - \frac{4\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx}{5c^2} - \frac{\left(2b\sqrt{1+c^2x^2}\right)\int x}{5c\sqrt{d}}$$

$$= -\frac{2bx^5\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{25c\sqrt{d+c^2dx^2}} - \frac{4x^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{15c^4d} + \frac{x^4\sqrt{d}}{}$$

$$= \frac{8bx^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{45c^3\sqrt{d+c^2dx^2}} - \frac{2bx^5\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{25c\sqrt{d+c^2dx^2}} + \frac{8\sqrt{d+}}{}$$

$$= -\frac{16abx\sqrt{1+c^2x^2}}{15c^5\sqrt{d+c^2dx^2}} + \frac{8bx^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{45c^3\sqrt{d+c^2dx^2}} - \frac{2bx^5\sqrt{1+c^2x^2}\left(a+b\,\mathrm{s}\right.}{25c\sqrt{d+c^2dx}}$$

$$= -\frac{16abx\sqrt{1+c^2x^2}}{15c^5\sqrt{d+c^2dx^2}} + \frac{2b^2\left(1+c^2x^2\right)}{25c^6\sqrt{d+c^2dx^2}} - \frac{4b^2\left(1+c^2x^2\right)^2}{75c^6\sqrt{d+c^2dx^2}} + \frac{2b^2\left(1+c^2x^2\right)^3}{125c^6\sqrt{d+c^2dx^2}} - $$

$$= -\frac{16abx\sqrt{1+c^2x^2}}{15c^5\sqrt{d+c^2dx^2}} + \frac{298b^2\left(1+c^2x^2\right)}{225c^6\sqrt{d+c^2dx^2}} - \frac{76b^2\left(1+c^2x^2\right)^2}{675c^6\sqrt{d+c^2dx^2}} + \frac{2b^2\left(1+c^2x^2\right)^3}{125c^6\sqrt{d+c^2dx^2}}$$

Mathematica [A] time = 0.37, size = 230, normalized size = 0.60

$$\frac{225a^2\left(3c^6x^6-c^4x^4+4c^2x^2+8\right)-30abcx\sqrt{c^2x^2+1}\left(9c^4x^4-20c^2x^2+120\right)+30b\sinh^{-1}(cx)\left(15a\left(3c^6x^6-c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (-30*a*b*c*x*Sqrt[1 + c^2*x^2]*(120 - 20*c^2*x^2 + 9*c^4*x^4) + 225*a^2*(8
+ 4*c^2*x^2 - c^4*x^4 + 3*c^6*x^6) + 2*b^2*(2072 + 1936*c^2*x^2 - 109*c^4*x
^4 + 27*c^6*x^6) + 30*b*(b*c*x*Sqrt[1 + c^2*x^2]*(-120 + 20*c^2*x^2 - 9*c^4
*x^4) + 15*a*(8 + 4*c^2*x^2 - c^4*x^4 + 3*c^6*x^6))*ArcSinh[c*x] + 225*b^2*
(8 + 4*c^2*x^2 - c^4*x^4 + 3*c^6*x^6)*ArcSinh[c*x]^2)/(3375*c^6*Sqrt[d + c^
2*d*x^2])
```

fricas [A] time = 0.50, size = 319, normalized size = 0.83

$$\frac{225\left(3\,b^2c^6x^6-b^2c^4x^4+4\,b^2c^2x^2+8\,b^2\right)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2+30\left(45\,abc^6x^6-15\,abc^4x^4+60\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/3375*(225*(3*b^2*c^6*x^6 - b^2*c^4*x^4 + 4*b^2*c^2*x^2 + 8*b^2)*sqrt(c^2*
d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 30*(45*a*b*c^6*x^6 - 15*a*b*c^4
*x^4 + 60*a*b*c^2*x^2 + 120*a*b - (9*b^2*c^5*x^5 - 20*b^2*c^3*x^3 + 120*b^2
*c*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) +
 (27*(25*a^2 + 2*b^2)*c^6*x^6 - (225*a^2 + 218*b^2)*c^4*x^4 + 4*(225*a^2 +
968*b^2)*c^2*x^2 + 1800*a^2 + 4144*b^2 - 30*(9*a*b*c^5*x^5 - 20*a*b*c^3*x^3
 + 120*a*b*c*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d))/(c^8*d*x^2 + c^6*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.50, size = 1227, normalized size = 3.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a^2*(1/5*x^4/c^2/d*(c^2*d*x^2+d)^(1/2)-4/5/c^2*(1/3*x^2/c^2/d*(c^2*d*x^2+d)
^(1/2)-2/3/d/c^4*(c^2*d*x^2+d)^(1/2)))+b^2*(1/4000*(d*(c^2*x^2+1))^(1/2)*(1
6*c^6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1
/2)+13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(25*arcsinh(c*x)^2-10*arcsinh(c*x
)+2)/c^6/d/(c^2*x^2+1)-5/864*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^
2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arcsinh(c*x)^2-6*arc
```

```
sinh(c*x)+2)/c^6/d/(c^2*x^2+1)+5/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2
*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2-2*arcsinh(c*x)+2)/c^6/d/(c^2*x^2+1)+5/16*(
d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2+2*ar
csinh(c*x)+2)/c^6/d/(c^2*x^2+1)-5/864*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^
3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arcsinh(c*x
)^2+6*arcsinh(c*x)+2)/c^6/d/(c^2*x^2+1)+1/4000*(d*(c^2*x^2+1))^(1/2)*(16*c^
6*x^6-16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+
13*c^2*x^2-5*c*x*(c^2*x^2+1)^(1/2)+1)*(25*arcsinh(c*x)^2+10*arcsinh(c*x)+2)
/c^6/d/(c^2*x^2+1))+2*a*b*(1/800*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6+16*c^5*x
^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2+5*c
*x*(c^2*x^2+1)^(1/2)+1)*(-1+5*arcsinh(c*x))/c^6/d/(c^2*x^2+1)-5/288*(d*(c^2
*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*
x^2+1)^(1/2)+1)*(-1+3*arcsinh(c*x))/c^6/d/(c^2*x^2+1)+5/16*(d*(c^2*x^2+1))^
(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))/c^6/d/(c^2*x^2+1)
+5/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(1+arcsinh(c*
x))/c^6/d/(c^2*x^2+1)-5/288*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2
*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(1+3*arcsinh(c*x))/c^6/d
/(c^2*x^2+1)+1/800*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6-16*c^5*x^5*(c^2*x^2+1)
^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2-5*c*x*(c^2*x^2+1)
^(1/2)+1)*(1+5*arcsinh(c*x))/c^6/d/(c^2*x^2+1))
```

maxima [A] time = 0.52, size = 353, normalized size = 0.92

$$\frac{1}{15}\left(\frac{3\sqrt{c^2dx^2+d}\,x^4}{c^2d}-\frac{4\sqrt{c^2dx^2+d}\,x^2}{c^4d}+\frac{8\sqrt{c^2dx^2+d}}{c^6d}\right)b^2\operatorname{arsinh}(cx)^2+\frac{2}{15}\left(\frac{3\sqrt{c^2dx^2+d}\,x^4}{c^2d}-\frac{4\sqrt{c^2dx^2+d}\,x^2}{c^4d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/15*(3*sqrt(c^2*d*x^2 + d)*x^4/(c^2*d) - 4*sqrt(c^2*d*x^2 + d)*x^2/(c^4*d)
 + 8*sqrt(c^2*d*x^2 + d)/(c^6*d))*b^2*arcsinh(c*x)^2 + 2/15*(3*sqrt(c^2*d*x
^2 + d)*x^4/(c^2*d) - 4*sqrt(c^2*d*x^2 + d)*x^2/(c^4*d) + 8*sqrt(c^2*d*x^2
+ d)/(c^6*d))*a*b*arcsinh(c*x) + 1/15*(3*sqrt(c^2*d*x^2 + d)*x^4/(c^2*d) -
4*sqrt(c^2*d*x^2 + d)*x^2/(c^4*d) + 8*sqrt(c^2*d*x^2 + d)/(c^6*d))*a^2 + 2/
3375*b^2*((27*sqrt(c^2*x^2 + 1)*c^2*x^4 - 136*sqrt(c^2*x^2 + 1)*x^2 + 2072*
sqrt(c^2*x^2 + 1)/c^2)/(c^4*sqrt(d)) - 15*(9*c^4*x^5 - 20*c^2*x^3 + 120*x)*
arcsinh(c*x)/(c^5*sqrt(d))) - 2/225*(9*c^4*x^5 - 20*c^2*x^3 + 120*x)*a*b/(c
^5*sqrt(d))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,(a+b\,\mathrm{asinh}\,(c\,x))^2}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int((x^5*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\,(a+b\,\mathrm{asinh}\,(cx))^2}{\sqrt{d\left(c^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**5*(a + b*asinh(c*x))**2/sqrt(d*(c**2*x**2 + 1)), x)
```

3.291 $\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=323

$$-\frac{bx^4\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)}{8c\sqrt{c^2dx^2+d}}+\frac{x^3\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d}+\frac{\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)^3}{8bc^5\sqrt{c^2dx^2+d}}-\frac{3x\sqrt{c}}{}$$

```
[Out] -15/64*b^2*x*(c^2*x^2+1)/c^4/(c^2*d*x^2+d)^(1/2)+1/32*b^2*x^3*(c^2*x^2+1)/c
^2/(c^2*d*x^2+d)^(1/2)+15/64*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)/c^5/(c^2*d*
x^2+d)^(1/2)+3/8*b*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3/(c^2*d*x^2+
d)^(1/2)-1/8*b*x^4*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c/(c^2*d*x^2+d)^(1/
2)+1/8*(a+b*arcsinh(c*x))^3*(c^2*x^2+1)^(1/2)/b/c^5/(c^2*d*x^2+d)^(1/2)-3/8
*x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^4/d+1/4*x^3*(a+b*arcsinh(c*x)
)^2*(c^2*d*x^2+d)^(1/2)/c^2/d
```

Rubi [A] time = 0.48, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5758, 5677, 5675, 5661, 321, 215}

$$-\frac{bx^4\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)}{8c\sqrt{c^2dx^2+d}}+\frac{x^3\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d}+\frac{3bx^2\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)}{8c^3\sqrt{c^2dx^2+d}}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (-15*b^2*x*(1 + c^2*x^2))/(64*c^4*Sqrt[d + c^2*d*x^2]) + (b^2*x^3*(1 + c^2*
x^2))/(32*c^2*Sqrt[d + c^2*d*x^2]) + (15*b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]
)/(64*c^5*Sqrt[d + c^2*d*x^2]) + (3*b*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[
c*x]))/(8*c^3*Sqrt[d + c^2*d*x^2]) - (b*x^4*Sqrt[1 + c^2*x^2]*(a + b*ArcSin
h[c*x]))/(8*c*Sqrt[d + c^2*d*x^2]) - (3*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSin
h[c*x])^2)/(8*c^4*d) + (x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(4*
c^2*d) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(8*b*c^5*Sqrt[d + c^2*d
*x^2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] &&  !GtQ[d, 0]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx = \frac{x^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d} - \frac{3\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx}{4c^2} - \frac{\left(b\sqrt{1+c^2x^2}\right)\int x^3}{2c\sqrt{d}}$$

$$= -\frac{bx^4\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{8c\sqrt{d+c^2dx^2}} - \frac{3x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{8c^4d} + \frac{x^3\sqrt{d+}}{}$$

$$= \frac{b^2x^3\left(1+c^2x^2\right)}{32c^2\sqrt{d+c^2dx^2}} + \frac{3bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{8c^3\sqrt{d+c^2dx^2}} - \frac{bx^4\sqrt{1+c^2x^2}\left(a+b\sinh\right.}{8c\sqrt{d+c^2dx^2}}$$

$$= -\frac{15b^2x\left(1+c^2x^2\right)}{64c^4\sqrt{d+c^2dx^2}} + \frac{b^2x^3\left(1+c^2x^2\right)}{32c^2\sqrt{d+c^2dx^2}} + \frac{3bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{8c^3\sqrt{d+c^2dx^2}} - \frac{bx^4}{}$$

$$= -\frac{15b^2x\left(1+c^2x^2\right)}{64c^4\sqrt{d+c^2dx^2}} + \frac{b^2x^3\left(1+c^2x^2\right)}{32c^2\sqrt{d+c^2dx^2}} + \frac{15b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{64c^5\sqrt{d+c^2dx^2}} + \frac{3bx^2\sqrt{1+c}}{8c}$$

Mathematica [A] time = 0.82, size = 268, normalized size = 0.83

$$\frac{32a^2c\sqrt{d}\,x\left(c^2x^2+1\right)\left(2c^2x^2-3\right)+96a^2\sqrt{c^2dx^2+d}\,\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)+4ab\sqrt{d}\sqrt{c^2x^2+1}\left(4\sinh^{-1}(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (32*a^2*c*Sqrt[d]*x*(1 + c^2*x^2)*(-3 + 2*c^2*x^2) + 96*a^2*Sqrt[d + c^2*d*
x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + b^2*Sqrt[d]*Sqrt[1 + c^2*x^
2]*(32*ArcSinh[c*x]^3 - 4*ArcSinh[c*x]*(-16*Cosh[2*ArcSinh[c*x]] + Cosh[4*A
rcSinh[c*x]]) - 32*Sinh[2*ArcSinh[c*x]] + Sinh[4*ArcSinh[c*x]] + 8*ArcSinh[
c*x]^2*(-8*Sinh[2*ArcSinh[c*x]] + Sinh[4*ArcSinh[c*x]])) + 4*a*b*Sqrt[d]*Sq
rt[1 + c^2*x^2]*(16*Cosh[2*ArcSinh[c*x]] - Cosh[4*ArcSinh[c*x]] + 4*ArcSinh
```

```
[c*x]*(6*ArcSinh[c*x] - 8*Sinh[2*ArcSinh[c*x]] + Sinh[4*ArcSinh[c*x]])))/(2
56*c^5*Sqrt[d]*Sqrt[d + c^2*d*x^2])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2x^4 \operatorname{arsinh}(cx)^2 + 2\,abx^4 \operatorname{arsinh}(cx) + a^2x^4}{\sqrt{c^2dx^2 + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^4*arcsinh(c*x)^2 + 2*a*b*x^4*arcsinh(c*x) + a^2*x^4)/sqrt(c
^2*d*x^2 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2 x^4}{\sqrt{c^2dx^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^4/sqrt(c^2*d*x^2 + d), x)
```

maple [B] time = 0.54, size = 760, normalized size = 2.35

$$\frac{a^2x^3\sqrt{c^2d\,x^2+d}}{4c^2d} - \frac{3a^2x\sqrt{c^2d\,x^2+d}}{8c^4d} + \frac{3a^2\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}} + \sqrt{c^2d\,x^2+d}\right)}{8c^4\sqrt{c^2d}} + \frac{b^2\sqrt{d\left(c^2x^2+1\right)}\ \operatorname{arsinh}(cx)^3}{8\sqrt{c^2x^2+1}\ c^5d} + \frac{b^2\sqrt{d\,(c}}{32d\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] 1/4*a^2*x^3/c^2/d*(c^2*d*x^2+d)^(1/2)-3/8*a^2/c^4*x/d*(c^2*d*x^2+d)^(1/2)+3
/8*a^2/c^4*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/8*
b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d*arcsinh(c*x)^3+1/32*b^2*(
d*(c^2*x^2+1))^(1/2)/d/(c^2*x^2+1)*x^5-13/64*b^2*(d*(c^2*x^2+1))^(1/2)/c^2/
d/(c^2*x^2+1)*x^3-15/64*b^2*(d*(c^2*x^2+1))^(1/2)/c^4/d/(c^2*x^2+1)*x-1/8*b
^2*(d*(c^2*x^2+1))^(1/2)/c/d/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^4+3/8*b^2*(d*
(c^2*x^2+1))^(1/2)/c^3/d/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^2+1/4*b^2*(d*(c^2
*x^2+1))^(1/2)/d/(c^2*x^2+1)*arcsinh(c*x)^2*x^5-1/8*b^2*(d*(c^2*x^2+1))^(1/
2)/c^2/d/(c^2*x^2+1)*arcsinh(c*x)^2*x^3-3/8*b^2*(d*(c^2*x^2+1))^(1/2)/c^4/d
/(c^2*x^2+1)*arcsinh(c*x)^2*x+15/64*b^2*(d*(c^2*x^2+1))^(1/2)/c^5/d/(c^2*x^
2+1)^(1/2)*arcsinh(c*x)+3/8*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5
/d*arcsinh(c*x)^2+15/64*a*b*(d*(c^2*x^2+1))^(1/2)/c^5/d/(c^2*x^2+1)^(1/2)+1
/2*a*b*(d*(c^2*x^2+1))^(1/2)/d/(c^2*x^2+1)*arcsinh(c*x)*x^5-1/8*a*b*(d*(c^2
*x^2+1))^(1/2)/c/d/(c^2*x^2+1)^(1/2)*x^4-1/4*a*b*(d*(c^2*x^2+1))^(1/2)/c^2/
d/(c^2*x^2+1)*arcsinh(c*x)*x^3+3/8*a*b*(d*(c^2*x^2+1))^(1/2)/c^3/d/(c^2*x^2
+1)^(1/2)*x^2-3/4*a*b*(d*(c^2*x^2+1))^(1/2)/c^4/d/(c^2*x^2+1)*arcsinh(c*x)*
x
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,(a+b\,\mathrm{asinh}\,(c\,x))^2}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\,(a+b\,\mathrm{asinh}\,(cx))^2}{\sqrt{d\left(c^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**4*(a + b*asinh(c*x))**2/sqrt(d*(c**2*x**2 + 1)), x)
```

3.292 $\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=265

$$\frac{x^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d}-\frac{2bx^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{9c\sqrt{c^2dx^2+d}}-\frac{2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{3c^4d}+\frac{4ab}{3c^3}$$

```
[Out] -14/9*b^2*(c^2*x^2+1)/c^4/(c^2*d*x^2+d)^(1/2)+2/27*b^2*(c^2*x^2+1)^2/c^4/(c
^2*d*x^2+d)^(1/2)+4/3*a*b*x*(c^2*x^2+1)^(1/2)/c^3/(c^2*d*x^2+d)^(1/2)+4/3*b
^2*x*arcsinh(c*x)*(c^2*x^2+1)^(1/2)/c^3/(c^2*d*x^2+d)^(1/2)-2/9*b*x^3*(a+b*
arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c/(c^2*d*x^2+d)^(1/2)-2/3*(a+b*arcsinh(c*x)
)^2*(c^2*d*x^2+d)^(1/2)/c^4/d+1/3*x^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1
/2)/c^2/d
```

Rubi [A] time = 0.33, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5758, 5717, 5653, 261, 5661, 266, 43}

$$\frac{4abx\sqrt{c^2x^2+1}}{3c^3\sqrt{c^2dx^2+d}}-\frac{2bx^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{9c\sqrt{c^2dx^2+d}}+\frac{x^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d}-\frac{2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{3c^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (4*a*b*x*Sqrt[1 + c^2*x^2])/(3*c^3*Sqrt[d + c^2*d*x^2]) - (14*b^2*(1 + c^2*
x^2))/(9*c^4*Sqrt[d + c^2*d*x^2]) + (2*b^2*(1 + c^2*x^2)^2)/(27*c^4*Sqrt[d
+ c^2*d*x^2]) + (4*b^2*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(3*c^3*Sqrt[d + c^
2*d*x^2]) - (2*b*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(9*c*Sqrt[d +
c^2*d*x^2]) - (2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c^4*d) + (x
^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c^2*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx = \frac{x^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d} - \frac{2\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx}{3c^2} - \frac{\left(2b\sqrt{1+c^2x^2}\right)\int x^2 \ldots}{3c\sqrt{d \ldots}}$$

$$= -\frac{2bx^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{9c\sqrt{d+c^2dx^2}} - \frac{2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c^4d} + \frac{x^2\sqrt{d \ldots}}{\ldots}$$

$$= \frac{4abx\sqrt{1+c^2x^2}}{3c^3\sqrt{d+c^2dx^2}} - \frac{2bx^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{9c\sqrt{d+c^2dx^2}} - \frac{2\sqrt{d+c^2dx^2}\left(a+b\sinh \ldots\right)}{3c^4d}$$

$$= \frac{4abx\sqrt{1+c^2x^2}}{3c^3\sqrt{d+c^2dx^2}} + \frac{4b^2x\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{3c^3\sqrt{d+c^2dx^2}} - \frac{2bx^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{9c\sqrt{d+c^2dx^2}}$$

$$= \frac{4abx\sqrt{1+c^2x^2}}{3c^3\sqrt{d+c^2dx^2}} - \frac{14b^2\left(1+c^2x^2\right)}{9c^4\sqrt{d+c^2dx^2}} + \frac{2b^2\left(1+c^2x^2\right)^2}{27c^4\sqrt{d+c^2dx^2}} + \frac{4b^2x\sqrt{1+c^2x^2}\sinh^{-1}(c \ldots}{3c^3\sqrt{d+c^2dx^2}}$$

Mathematica [A] time = 0.28, size = 176, normalized size = 0.66

$$\frac{9a^2\left(c^4x^4-c^2x^2-2\right)-6abcx\left(c^2x^2-6\right)\sqrt{c^2x^2+1}-6b\sinh^{-1}(cx)\left(a\left(-3c^4x^4+3c^2x^2+6\right)+bcx\sqrt{c^2x^2+1}\left(c^2x \ldots\right.\right.}{27c^4\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (-6*a*b*c*x*(-6 + c^2*x^2)*Sqrt[1 + c^2*x^2] + 2*b^2*(-20 - 19*c^2*x^2 + c^
4*x^4) + 9*a^2*(-2 - c^2*x^2 + c^4*x^4) - 6*b*(b*c*x*(-6 + c^2*x^2)*Sqrt[1
+ c^2*x^2] + a*(6 + 3*c^2*x^2 - 3*c^4*x^4))*ArcSinh[c*x] + 9*b^2*(-2 - c^2*
x^2 + c^4*x^4)*ArcSinh[c*x]^2)/(27*c^4*Sqrt[d + c^2*d*x^2])
```

fricas [A] time = 0.46, size = 254, normalized size = 0.96

$$\frac{9\left(b^2c^4x^4 - b^2c^2x^2 - 2\,b^2\right)\sqrt{c^2dx^2 + d}\,\log\left(cx + \sqrt{c^2x^2 + 1}\right)^2 + 6\left(3\,abc^4x^4 - 3\,abc^2x^2 - 6\,ab - \left(b^2c^3x^3 - 6\,b^2c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/27*(9*(b^2*c^4*x^4 - b^2*c^2*x^2 - 2*b^2)*sqrt(c^2*d*x^2 + d)*log(c*x + s
qrt(c^2*x^2 + 1))^2 + 6*(3*a*b*c^4*x^4 - 3*a*b*c^2*x^2 - 6*a*b - (b^2*c^3*x
^3 - 6*b^2*c*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x
^2 + 1)) + ((9*a^2 + 2*b^2)*c^4*x^4 - (9*a^2 + 38*b^2)*c^2*x^2 - 18*a^2 - 4
0*b^2 - 6*(a*b*c^3*x^3 - 6*a*b*c*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d))
/(c^6*d*x^2 + c^4*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.38, size = 706, normalized size = 2.66

$$a^2\left(\frac{x^2\sqrt{c^2d\,x^2 + d}}{3c^2d} - \frac{2\sqrt{c^2d\,x^2 + d}}{3d\,c^4}\right) + b^2\left(\frac{\sqrt{d\left(c^2x^2 + 1\right)}\,\left(4c^4x^4 + 4c^3x^3\sqrt{c^2x^2 + 1} + 5c^2x^2 + 3cx\sqrt{c^2x^2 + 1} + \right.}{216c^4d\left(c^2x^2 + 1\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a^2*(1/3*x^2/c^2/d*(c^2*d*x^2+d)^(1/2)-2/3/d/c^4*(c^2*d*x^2+d)^(1/2))+b^2*(
1/216*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^
2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arcsinh(c*x)^2-6*arcsinh(c*x)+2)/c^4/d/(c^2
*x^2+1)-3/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(arcsin
h(c*x)^2-2*arcsinh(c*x)+2)/c^4/d/(c^2*x^2+1)-3/8*(d*(c^2*x^2+1))^(1/2)*(c^2
*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2+2*arcsinh(c*x)+2)/c^4/d/(c^2*
x^2+1)+1/216*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5
*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arcsinh(c*x)^2+6*arcsinh(c*x)+2)/c^4
/d/(c^2*x^2+1))+2*a*b*(1/72*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2
*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arcsinh(c*x))/c^4/
d/(c^2*x^2+1)-3/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(
-1+arcsinh(c*x))/c^4/d/(c^2*x^2+1)-3/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(
c^2*x^2+1)^(1/2)+1)*(1+arcsinh(c*x))/c^4/d/(c^2*x^2+1)+1/72*(d*(c^2*x^2+1))
^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(
1/2)+1)*(1+3*arcsinh(c*x))/c^4/d/(c^2*x^2+1))
```

maxima [A] time = 0.52, size = 243, normalized size = 0.92

$$\frac{1}{3}\,b^2\left(\frac{\sqrt{c^2dx^2 + d}\,x^2}{c^2d} - \frac{2\,\sqrt{c^2dx^2 + d}}{c^4d}\right)\operatorname{arsinh}(cx)^2 + \frac{2}{3}\,ab\left(\frac{\sqrt{c^2dx^2 + d}\,x^2}{c^2d} - \frac{2\,\sqrt{c^2dx^2 + d}}{c^4d}\right)\operatorname{arsinh}(cx) + \frac{1}{3}\,a^2\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/3*b^2*(sqrt(c^2*d*x^2 + d)*x^2/(c^2*d) - 2*sqrt(c^2*d*x^2 + d)/(c^4*d))*a
rcsinh(c*x)^2 + 2/3*a*b*(sqrt(c^2*d*x^2 + d)*x^2/(c^2*d) - 2*sqrt(c^2*d*x^2
 + d)/(c^4*d))*arcsinh(c*x) + 1/3*a^2*(sqrt(c^2*d*x^2 + d)*x^2/(c^2*d) - 2*
sqrt(c^2*d*x^2 + d)/(c^4*d)) + 2/27*b^2*((sqrt(c^2*x^2 + 1)*x^2 - 20*sqrt(c
^2*x^2 + 1)/c^2)/(c^2*sqrt(d)) - 3*(c^2*x^3 - 6*x)*arcsinh(c*x)/(c^3*sqrt(d
))) - 2/9*(c^2*x^3 - 6*x)*a*b/(c^3*sqrt(d))
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \left(a + b \operatorname{asinh}{\left(cx \right)}\right)^{2}}{\sqrt{d \left(c^{2} x^{2} + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*asinh(c*x))**2/sqrt(d*(c**2*x**2 + 1)), x)
```

3.293 $\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=204

$$\frac{x\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d}-\frac{bx^2\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)}{2c\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)^3}{6bc^3\sqrt{c^2dx^2+d}}+\frac{b^2x\left(c^2\right.}{4c^2\sqrt{c^2d}}$$

```
[Out] 1/4*b^2*x*(c^2*x^2+1)/c^2/(c^2*d*x^2+d)^(1/2)-1/4*b^2*arcsinh(c*x)*(c^2*x^2
+1)^(1/2)/c^3/(c^2*d*x^2+d)^(1/2)-1/2*b*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^
(1/2)/c/(c^2*d*x^2+d)^(1/2)-1/6*(a+b*arcsinh(c*x))^3*(c^2*x^2+1)^(1/2)/b/c^
3/(c^2*d*x^2+d)^(1/2)+1/2*x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^2/d
```

Rubi [A] time = 0.28, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5758, 5677, 5675, 5661, 321, 215}

$$-\frac{\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)^3}{6bc^3\sqrt{c^2dx^2+d}}+\frac{x\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d}-\frac{bx^2\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)}{2c\sqrt{c^2dx^2+d}}+\frac{b^2x\left(c\right.}{4c^2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (b^2*x*(1 + c^2*x^2))/(4*c^2*Sqrt[d + c^2*d*x^2]) - (b^2*Sqrt[1 + c^2*x^2]*
ArcSinh[c*x])/(4*c^3*Sqrt[d + c^2*d*x^2]) - (b*x^2*Sqrt[1 + c^2*x^2]*(a + b
*ArcSinh[c*x]))/(2*c*Sqrt[d + c^2*d*x^2]) + (x*Sqrt[d + c^2*d*x^2]*(a + b*A
rcSinh[c*x])^2)/(2*c^2*d) - (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(6*b
*c^3*Sqrt[d + c^2*d*x^2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] &&  !GtQ[d, 0]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx = \frac{x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d} - \frac{\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx}{2c^2} - \frac{\left(b\sqrt{1+c^2x^2}\right)\int x\left(a+\right.}{c\sqrt{d+c^2d}}$$

$$= -\frac{bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c\sqrt{d+c^2dx^2}} + \frac{x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d} + \frac{\left(b^2\sqrt{1+\right.}{2}$$

$$= \frac{b^2x\left(1+c^2x^2\right)}{4c^2\sqrt{d+c^2dx^2}} - \frac{bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c\sqrt{d+c^2dx^2}} + \frac{x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}\right.}{2c^2d}$$

$$= \frac{b^2x\left(1+c^2x^2\right)}{4c^2\sqrt{d+c^2dx^2}} - \frac{b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{4c^3\sqrt{d+c^2dx^2}} - \frac{bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c\sqrt{d+c^2dx^2}} +$$

Mathematica [A] time = 0.89, size = 198, normalized size = 0.97

$$\frac{12a^2cx\left(c^2dx^2+d\right) - 12a^2\sqrt{d}\sqrt{c^2dx^2+d}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right) - 6abd\sqrt{c^2x^2+1}\left(2\sinh^{-1}(cx)\left(\sinh^{-1}(cx\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (12*a^2*c*x*(d + c^2*d*x^2) - 12*a^2*Sqrt[d]*Sqrt[d + c^2*d*x^2]*Log[c*d*x
+ Sqrt[d]*Sqrt[d + c^2*d*x^2]] - 6*a*b*d*Sqrt[1 + c^2*x^2]*(Cosh[2*ArcSinh[
c*x]] + 2*ArcSinh[c*x]*(ArcSinh[c*x] - Sinh[2*ArcSinh[c*x]])) - b^2*d*Sqrt[
1 + c^2*x^2]*(4*ArcSinh[c*x]^3 + 6*ArcSinh[c*x]*Cosh[2*ArcSinh[c*x]] - 3*(1
 + 2*ArcSinh[c*x]^2)*Sinh[2*ArcSinh[c*x]]))/(24*c^3*d*Sqrt[d + c^2*d*x^2])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{b^2x^2\operatorname{arsinh}(cx)^2 + 2\,abx^2\operatorname{arsinh}(cx) + a^2x^2}{\sqrt{c^2dx^2+d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*x) + a^2*x^2)/sqrt(c
^2*d*x^2 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2 x^2}{\sqrt{c^2 d x^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^2/sqrt(c^2*d*x^2 + d), x)
```

maple [B] time = 0.40, size = 530, normalized size = 2.60

$$\frac{a^2 x\sqrt{c^2 d\, x^2+d}}{2c^2 d} - \frac{a^2 \ln\left(\frac{x c^2 d}{\sqrt{c^2 d}} + \sqrt{c^2 d\, x^2 + d}\right)}{2c^2\sqrt{c^2 d}} - \frac{b^2\sqrt{d\left(c^2x^2+1\right)}\, \operatorname{arcsinh}(cx)^3}{6\sqrt{c^2x^2+1}\, c^3 d} + \frac{b^2\sqrt{d\left(c^2x^2+1\right)}\, x^3}{4d\left(c^2x^2+1\right)} + \frac{b^2\sqrt{d\left(c^2\right.}}{4c^2 d\left(c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] 1/2*a^2*x/c^2/d*(c^2*d*x^2+d)^(1/2)-1/2*a^2/c^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c
^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)-1/6*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^
(1/2)/c^3/d*arcsinh(c*x)^3+1/4*b^2*(d*(c^2*x^2+1))^(1/2)/d/(c^2*x^2+1)*x^3+
1/4*b^2*(d*(c^2*x^2+1))^(1/2)/c^2/d/(c^2*x^2+1)*x-1/2*b^2*(d*(c^2*x^2+1))^(
1/2)/c/d/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^2-1/4*b^2*(d*(c^2*x^2+1))^(1/2)/c
^3/d/(c^2*x^2+1)^(1/2)*arcsinh(c*x)+1/2*b^2*(d*(c^2*x^2+1))^(1/2)/d/(c^2*x^
2+1)*arcsinh(c*x)^2*x^3+1/2*b^2*(d*(c^2*x^2+1))^(1/2)/c^2/d/(c^2*x^2+1)*arc
sinh(c*x)^2*x-1/2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3/d*arcsinh
(c*x)^2+a*b*(d*(c^2*x^2+1))^(1/2)/d/(c^2*x^2+1)*arcsinh(c*x)*x^3-1/2*a*b*(d
*(c^2*x^2+1))^(1/2)/c/d/(c^2*x^2+1)^(1/2)*x^2+a*b*(d*(c^2*x^2+1))^(1/2)/c^2
/d/(c^2*x^2+1)*arcsinh(c*x)*x-1/4*a*b*(d*(c^2*x^2+1))^(1/2)/c^3/d/(c^2*x^2+
1)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,(a + b\,\operatorname{asinh}(c\,x))^2}{\sqrt{d\,c^2\,x^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + b \operatorname{asinh}(cx))^2}{\sqrt{d\left(c^2x^2 + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*asinh(c*x))**2/sqrt(d*(c**2*x**2 + 1)), x)
```

3.294 $\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=138

$$-\frac{2abx\sqrt{c^2x^2+1}}{c\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d}+\frac{2b^2\left(c^2x^2+1\right)}{c^2\sqrt{c^2dx^2+d}}-\frac{2b^2x\sqrt{c^2x^2+1}\,\sinh^{-1}(cx)}{c\sqrt{c^2dx^2+d}}$$

```
[Out] 2*b^2*(c^2*x^2+1)/c^2/(c^2*d*x^2+d)^(1/2)-2*a*b*x*(c^2*x^2+1)^(1/2)/c/(c^2*
d*x^2+d)^(1/2)-2*b^2*x*arcsinh(c*x)*(c^2*x^2+1)^(1/2)/c/(c^2*d*x^2+d)^(1/2)
+(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^2/d
```

Rubi [A] time = 0.12, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {5717, 5653, 261}

$$-\frac{2abx\sqrt{c^2x^2+1}}{c\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d}+\frac{2b^2\left(c^2x^2+1\right)}{c^2\sqrt{c^2dx^2+d}}-\frac{2b^2x\sqrt{c^2x^2+1}\,\sinh^{-1}(cx)}{c\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (-2*a*b*x*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + c^2*d*x^2]) + (2*b^2*(1 + c^2*x^2)
)/(c^2*Sqrt[d + c^2*d*x^2]) - (2*b^2*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(c*S
qrt[d + c^2*d*x^2]) + (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(c^2*d)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx = \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d} - \frac{\left(2b\sqrt{1+c^2x^2}\right)\int\left(a+b\sinh^{-1}(cx)\right)dx}{c\sqrt{d+c^2dx^2}}$$

$$= -\frac{2abx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}} + \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d} - \frac{\left(2b^2\sqrt{1+c^2x^2}\right)\int\sinh^{-1}(c$$

$$= -\frac{2abx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}} - \frac{2b^2x\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{c\sqrt{d+c^2dx^2}} + \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d}$$

$$= -\frac{2abx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}} + \frac{2b^2\left(1+c^2x^2\right)}{c^2\sqrt{d+c^2dx^2}} - \frac{2b^2x\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{c\sqrt{d+c^2dx^2}} + \frac{\sqrt{d+c^2dx^2}\,(a}{c}$$

Mathematica [A] time = 0.28, size = 127, normalized size = 0.92

$$\frac{\sqrt{c^2dx^2+d}\left(a^2\sqrt{c^2x^2+1} - 2b\sinh^{-1}(cx)\left(bcx - a\sqrt{c^2x^2+1}\right) - 2abcx + 2b^2\sqrt{c^2x^2+1} + b^2\sqrt{c^2x^2+1}\sinh^{-1}(c\right.}{c^2d\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(-2*a*b*c*x + a^2*Sqrt[1 + c^2*x^2] + 2*b^2*Sqrt[1 + c
^2*x^2] - 2*b*(b*c*x - a*Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + b^2*Sqrt[1 + c^2
*x^2]*ArcSinh[c*x]^2))/(c^2*d*Sqrt[1 + c^2*x^2])
```

fricas [A] time = 0.46, size = 179, normalized size = 1.30

$$\frac{\left(b^2c^2x^2+b^2\right)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2 + 2\left(abc^2x^2 - \sqrt{c^2x^2+1}\,b^2cx + ab\right)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2}\right.}{c^4dx^2+c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] ((b^2*c^2*x^2 + b^2)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2
*(a*b*c^2*x^2 - sqrt(c^2*x^2 + 1)*b^2*c*x + a*b)*sqrt(c^2*d*x^2 + d)*log(c*
x + sqrt(c^2*x^2 + 1)) + ((a^2 + 2*b^2)*c^2*x^2 - 2*sqrt(c^2*x^2 + 1)*a*b*c
*x + a^2 + 2*b^2)*sqrt(c^2*d*x^2 + d))/(c^4*d*x^2 + c^2*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^2x}{\sqrt{c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x/sqrt(c^2*d*x^2 + d), x)
```

maple [B] time = 0.19, size = 296, normalized size = 2.14

$$\frac{a^2\sqrt{c^2d\,x^2+d}}{c^2d} + b^2\left(\frac{\sqrt{d\left(c^2x^2+1\right)}\left(c^2x^2+cx\sqrt{c^2x^2+1}+1\right)\left(\mathrm{arcsinh}\,(cx)^2 - 2\,\mathrm{arcsinh}\,(cx)+2\right)}{2c^2d\left(c^2x^2+1\right)} + \frac{\sqrt{d\left(c^2x^2+\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a^2/c^2/d*(c^2*d*x^2+d)^(1/2)+b^2*(1/2*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(
c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2-2*arcsinh(c*x)+2)/c^2/d/(c^2*x^2+1)+1/2
*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2+2*
arcsinh(c*x)+2)/c^2/d/(c^2*x^2+1))+2*a*b*(1/2*(d*(c^2*x^2+1))^(1/2)*(c^2*x^
2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))/c^2/d/(c^2*x^2+1)+1/2*(d*(c^2*
x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(1+arcsinh(c*x))/c^2/d/(c^2
*x^2+1))
```

maxima [A] time = 0.51, size = 125, normalized size = 0.91

$$-2\,b^{2}\left(\frac{x\operatorname{arsinh}\left(cx\right)}{c\sqrt{d}}-\frac{\sqrt{c^{2}x^{2}+1}}{c^{2}\sqrt{d}}\right)-\frac{2\,abx}{c\sqrt{d}}+\frac{\sqrt{c^{2}dx^{2}+d}\,b^{2}\operatorname{arsinh}\left(cx\right)^{2}}{c^{2}d}+\frac{2\,\sqrt{c^{2}dx^{2}+d}\,ab\operatorname{arsinh}\left(cx\right)}{c^{2}d}+\frac{\sqrt{c^{2}dx^{2}}}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*b^2*(x*arcsinh(c*x)/(c*sqrt(d)) - sqrt(c^2*x^2 + 1)/(c^2*sqrt(d))) - 2*a
*b*x/(c*sqrt(d)) + sqrt(c^2*d*x^2 + d)*b^2*arcsinh(c*x)^2/(c^2*d) + 2*sqrt(
c^2*d*x^2 + d)*a*b*arcsinh(c*x)/(c^2*d) + sqrt(c^2*d*x^2 + d)*a^2/(c^2*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(a+b\operatorname{asinh}\left(cx\right)\right)^{2}}{\sqrt{d\left(c^{2}x^{2}+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x*(a + b*asinh(c*x))**2/sqrt(d*(c**2*x**2 + 1)), x)
```

3.295 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=47

$$\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{c^2dx^2+d}}$$

```
[Out] 1/3*(a+b*arcsinh(c*x))^3*(c^2*x^2+1)^(1/2)/b/c/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {5677, 5675}

$$\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(3*b*c*Sqrt[d + c^2*d*x^2])
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] && !GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx &= \frac{\sqrt{1+c^2x^2}\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}\\ &= \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{d+c^2dx^2}}\end{aligned}$$

Mathematica [A] time = 0.13, size = 62, normalized size = 1.32

$$\frac{\sqrt{c^2x^2+1}\sinh^{-1}(cx)\left(3a^2+3ab\sinh^{-1}(cx)+b^2\sinh^{-1}(cx)^2\right)}{3c\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*(3*a^2 + 3*a*b*ArcSinh[c*x] + b^2*ArcSinh[c
*x]^2))/(3*c*Sqrt[d + c^2*d*x^2])
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\operatorname{arsinh}(cx)^2+2\,ab\operatorname{arsinh}(cx)+a^2}{\sqrt{c^2dx^2+d}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/sqrt(c^2*d*x^2 + d
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\operatorname{arsinh}(cx)+a)^2}{\sqrt{c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/sqrt(c^2*d*x^2 + d), x)
```

maple [B] time = 0.07, size = 120, normalized size = 2.55

$$\frac{a^2\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{\sqrt{c^2d}}+\frac{b^2\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsinh}(cx)^3}{3\sqrt{c^2x^2+1}\,cd}+\frac{ab\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsinh}(cx)^2}{\sqrt{c^2x^2+1}\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/3*b^2*(d*
(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d*arcsinh(c*x)^3+a*b*(d*(c^2*x^2+1))
^(1/2)/(c^2*x^2+1)^(1/2)/c/d*arcsinh(c*x)^2
```

maxima [A] time = 0.41, size = 47, normalized size = 1.00

$$\frac{b^2\operatorname{arsinh}(cx)^3}{3\,c\sqrt{d}}+\frac{ab\operatorname{arsinh}(cx)^2}{c\sqrt{d}}+\frac{a^2\operatorname{arsinh}(cx)}{c\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*b^2*arcsinh(c*x)^3/(c*sqrt(d)) + a*b*arcsinh(c*x)^2/(c*sqrt(d)) + a^2*a
rcsinh(c*x)/(c*sqrt(d))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{(a+b\operatorname{asinh}(c\,x))^2}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(a+b\operatorname{asinh}(cx))^2}{\sqrt{d\left(c^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/sqrt(d*(c**2*x**2 + 1)), x)
```

3.296 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=223

$$-\frac{2b\sqrt{c^2x^2+1}\,\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2dx^2+d}}+\frac{2b\sqrt{c^2x^2+1}\,\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2dx^2+d}}-\frac{2\sqrt{c^2x^2+}}{}$$

```
[Out] -2*(a+b*arcsinh(c*x))^2*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c
^2*d*x^2+d)^(1/2)-2*b*(a+b*arcsinh(c*x))*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*
(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+2*b*(a+b*arcsinh(c*x))*polylog(2,c*x+
(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+2*b^2*polylog(3,-c
*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-2*b^2*polylog(3
,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5764, 5760, 4182, 2531, 2282, 6589}

$$-\frac{2b\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2dx^2+d}}+\frac{2b\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}\right.}{\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] (-2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/Sqrt[
d + c^2*d*x^2] - (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, -E^
ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2] + (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[
c*x])*PolyLog[2, E^ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2] + (2*b^2*Sqrt[1 + c^2
*x^2]*PolyLog[3, -E^ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2] - (2*b^2*Sqrt[1 + c^
2*x^2]*PolyLog[3, E^ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 5764

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[
((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&  !GtQ[d, 0] && (Integer
Q[m] || EqQ[n, 1])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\sqrt{d+c^2dx^2}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}$$
$$= \frac{\sqrt{1+c^2x^2}\,\text{Subst}\left(\int (a+bx)^2\text{csch}(x)\,dx,x,\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}$$
$$= -\frac{2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{d+c^2dx^2}} - \frac{\left(2b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+\right.}{\sqrt{d}}$$
$$= -\frac{2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{d+c^2dx^2}} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(c\right.}{\sqrt{d+c^2dx}}$$
$$= -\frac{2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{d+c^2dx^2}} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(c\right.}{\sqrt{d+c^2dx}}$$
$$= -\frac{2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{d+c^2dx^2}} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(c\right.}{\sqrt{d+c^2dx}}$$

Mathematica [A] time = 0.87, size = 266, normalized size = 1.19

$$-\frac{a^2\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+d\right)}{\sqrt{d}}+\frac{a^2\log(cx)}{\sqrt{d}}+\frac{2ab\sqrt{c^2x^2+1}\left(\text{Li}_2\left(-e^{-\sinh^{-1}(cx)}\right)-\text{Li}_2\left(e^{-\sinh^{-1}(cx)}\right)+\sinh^{-1}(cx)\left(\text{lo}\right.\right.}{\sqrt{c^2dx^2+d}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] (a^2*Log[c*x])/Sqrt[d] - (a^2*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/Sqrt[d]
 + (2*a*b*Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(Log[1 - E^(-ArcSinh[c*x])] - Log
[1 + E^(-ArcSinh[c*x])]) + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-
ArcSinh[c*x])]))/Sqrt[d + c^2*d*x^2] + (b^2*Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]
^2*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]^2*Log[1 + E^(-ArcSinh[c*x])] +
 2*ArcSinh[c*x]*PolyLog[2, -E^(-ArcSinh[c*x])] - 2*ArcSinh[c*x]*PolyLog[2,
```

```
E^(-ArcSinh[c*x])] + 2*PolyLog[3, -E^(-ArcSinh[c*x])] - 2*PolyLog[3, E^(-Ar
cSinh[c*x])]))/Sqrt[d + c^2*d*x^2]
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(b^2\operatorname{arsinh}(cx)^2+2\,ab\operatorname{arsinh}(cx)+a^2\right)}{c^2dx^3+dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^2*d*x^3 + d*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\operatorname{arsinh}(cx)+a)^2}{\sqrt{c^2dx^2+d}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(sqrt(c^2*d*x^2 + d)*x), x)
```

maple [B] time = 0.30, size = 564, normalized size = 2.53

$$-\frac{a^2\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2d\,x^2+d}}{x}\right)}{\sqrt{d}}-\frac{b^2\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsinh}(cx)^2\ln\left(1+cx+\sqrt{c^2x^2+1}\right)}{\sqrt{c^2x^2+1}\,d}-\frac{2b^2\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsin}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -a^2/d^(1/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)-b^2*(d*(c^2*x^2+1))^
(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))-2*b^2*
(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*polylog(2,-c*x-(c^2*
x^2+1)^(1/2))+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(3,-c*
x-(c^2*x^2+1)^(1/2))+b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(
c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^
(1/2)/d*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))-2*b^2*(d*(c^2*x^2+1))
^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(3,c*x+(c^2*x^2+1)^(1/2))+2*a*b*(d*(c^2*x
^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+2
*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,c*x+(c^2*x^2+1)^(1
/2))-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1+c*x+
(c^2*x^2+1)^(1/2))-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(
2,-c*x-(c^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^2\operatorname{arsinh}\left(\frac{1}{c|x|}\right)}{\sqrt{d}}+\int\frac{b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{\sqrt{c^2dx^2+d}\,x}+\frac{2\,ab\log\left(cx+\sqrt{c^2x^2+1}\right)}{\sqrt{c^2dx^2+d}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] -a^2*arcsinh(1/(c*abs(x)))/sqrt(d) + integrate(b^2*log(c*x + sqrt(c^2*x^2 +
 1))^2/(sqrt(c^2*d*x^2 + d)*x) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(sqrt(c
^2*d*x^2 + d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{asinh}\,(c\,x))^2}{x\,\sqrt{d\,c^2\,x^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)^(1/2)),x)
[Out] int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{asinh}\,(cx))^2}{x\sqrt{d\left(c^2x^2 + 1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x/(c**2*d*x**2+d)**(1/2),x)
[Out] Integral((a + b*asinh(c*x))**2/(x*sqrt(d*(c**2*x**2 + 1))), x)
```

3.297 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=167

$$-\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{dx}+\frac{c\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{c^2dx^2+d}}+\frac{2bc\sqrt{c^2x^2+1}\log\left(1-e^{-2\sinh^{-1}(cx)}\right)\left(a+\right.}{\sqrt{c^2dx^2+d}}$$

```
[Out] c*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+2*b*c*(a+b*arc
sinh(c*x))*ln(1-1/(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d
)^(1/2)-b^2*c*polylog(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/(c^2
*d*x^2+d)^(1/2)-(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/d/x
```

Rubi [A] time = 0.23, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5723, 5659, 3716, 2190, 2279, 2391}

$$\frac{b^2c\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{dx}-\frac{c\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{c^2dx^2+d}}+\frac{2bc}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^2*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] -((c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2]) - (Sqrt
[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(d*x) + (2*b*c*Sqrt[1 + c^2*x^2]*(a
 + b*ArcSinh[c*x])*Log[1 - E^(2*ArcSinh[c*x])])/Sqrt[d + c^2*d*x^2] + (b^2*
c*Sqrt[1 + c^2*x^2]*PolyLog[2, E^(2*ArcSinh[c*x])])/Sqrt[d + c^2*d*x^2]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\sqrt{d+c^2dx^2}}\,dx = -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{dx} + \frac{\left(2bc\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{x}\,dx}{\sqrt{d+c^2dx^2}}$$

$$= -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{dx} + \frac{\left(2bc\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)\coth(x)\,dx, x,\right.}{\sqrt{d+c^2dx^2}}$$

$$= -\frac{c\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{dx} - \frac{\left(4bc\sqrt{1+c^2x\right.}{}$$

$$= -\frac{c\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{dx} + \frac{2bc\sqrt{1+c^2x^2}}{}$$

$$= -\frac{c\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{dx} + \frac{2bc\sqrt{1+c^2x^2}}{}$$

$$= -\frac{c\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{dx} + \frac{2bc\sqrt{1+c^2x^2}}{}$$

Mathematica [A] time = 0.46, size = 168, normalized size = 1.01

$$\frac{-a\left(ac^2x^2+a-2bcx\sqrt{c^2x^2+1}\log(cx)\right)-2b\sinh^{-1}(cx)\left(ac^2x^2+a-bcx\sqrt{c^2x^2+1}\log\left(1-e^{-2\sinh^{-1}(cx)}\right)\right)-b^2c}{x\sqrt{c^2dx^2+d}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^2*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] (b^2*(-1 - c^2*x^2 + c*x*Sqrt[1 + c^2*x^2])*ArcSinh[c*x]^2 - 2*b*ArcSinh[c*
x]*(a + a*c^2*x^2 - b*c*x*Sqrt[1 + c^2*x^2]*Log[1 - E^(-2*ArcSinh[c*x])]) -
 a*(a + a*c^2*x^2 - 2*b*c*x*Sqrt[1 + c^2*x^2]*Log[c*x]) - b^2*c*x*Sqrt[1 +
c^2*x^2]*PolyLog[2, E^(-2*ArcSinh[c*x])])/(x*Sqrt[d + c^2*d*x^2])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(b^2\,\text{arsinh}\,(cx)^2+2\,ab\,\text{arsinh}\,(cx)+a^2\right)}{c^2dx^4+dx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^2*d*x^4 + d*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2}{\sqrt{c^2dx^2 + d}\, x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(sqrt(c^2*d*x^2 + d)*x^2), x)
```

maple [B] time = 0.31, size = 526, normalized size = 3.15

$$-\frac{a^2\sqrt{c^2d\,x^2+d}}{dx}-\frac{b^2\sqrt{d\left(c^2x^2+1\right)}\ \operatorname{arcsinh}(cx)^2\,x\,c^2}{\left(c^2x^2+1\right)d}-\frac{b^2\sqrt{d\left(c^2x^2+1\right)}\ \operatorname{arcsinh}(cx)^2\,c}{\sqrt{c^2x^2+1}\ d}-\frac{b^2\sqrt{d\left(c^2x^2+1\right)}\ \operatorname{arc}}{\left(c^2x^2+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -a^2/d/x*(c^2*d*x^2+d)^(1/2)-b^2*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2/(c^2*
x^2+1)/d*x*c^2-b^2*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2/(c^2*x^2+1)^(1/2)/d
*c-b^2*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2/(c^2*x^2+1)/d/x+2*b^2*(d*(c^2*x
^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c
+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,-c*x-(c^2*x^2+1)
^(1/2))*c+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1
-c*x-(c^2*x^2+1)^(1/2))*c+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*p
olylog(2,c*x+(c^2*x^2+1)^(1/2))*c-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(
1/2)/d*arcsinh(c*x)*c-2*a*b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/(c^2*x^2+1)/
d*x*c^2-2*a*b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/(c^2*x^2+1)/d/x+2*a*b*(d*(
c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left((-1)^{2\,c^2dx^2+2\,d}\sqrt{d}\,\log\left(2\,c^2d+\frac{2\,d}{x^2}\right)-\sqrt{d}\,\log\left(x^2+\frac{1}{c^2}\right)\right)abc}{d}+b^2\int \frac{\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{\sqrt{c^2dx^2+d}\,x^2}\,dx-\frac{2\sqrt{c^2dx^2+d}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] -((-1)^(2*c^2*d*x^2 + 2*d)*sqrt(d)*log(2*c^2*d + 2*d/x^2) - sqrt(d)*log(x^2
 + 1/c^2))*a*b*c/d + b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/(sqrt(c^2
*d*x^2 + d)*x^2), x) - 2*sqrt(c^2*d*x^2 + d)*a*b*arcsinh(c*x)/(d*x) - sqrt(
c^2*d*x^2 + d)*a^2/(d*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\operatorname{asinh}(c\,x))^2}{x^2\,\sqrt{d\,c^2\,x^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)^(1/2)),x)
[Out] int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)^(1/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{asinh}(cx))^2}{x^2\sqrt{d\left(c^2x^2 + 1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**2/(c**2*d*x**2+d)**(1/2),x)
[Out] Integral((a + b*asinh(c*x))**2/(x**2*sqrt(d*(c**2*x**2 + 1))), x)
```

3.298 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^3\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=360

$$\frac{bc^2\sqrt{c^2x^2+1}\,\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2dx^2+d}}-\frac{bc^2\sqrt{c^2x^2+1}\,\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2dx^2+d}}-\frac{bc\sqrt{c^2x^2}}{x}$$

```
[Out] -b*c*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/x/(c^2*d*x^2+d)^(1/2)+c^2*(a+b*ar
csinh(c*x))^2*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d
)^(1/2)-b^2*c^2*arctanh((c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^
(1/2)+b*c^2*(a+b*arcsinh(c*x))*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1
)^(1/2)/(c^2*d*x^2+d)^(1/2)-b*c^2*(a+b*arcsinh(c*x))*polylog(2,c*x+(c^2*x^2
+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-b^2*c^2*polylog(3,-c*x-(c^
2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+b^2*c^2*polylog(3,c*x
+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-1/2*(a+b*arcsinh(
c*x))^2*(c^2*d*x^2+d)^(1/2)/d/x^2
```

Rubi [A] time = 0.57, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {5747, 5764, 5760, 4182, 2531, 2282, 6589, 5661, 266, 63, 208}

$$\frac{bc^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2dx^2+d}}-\frac{bc^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}\right.}{\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^3*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] -((b*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(x*Sqrt[d + c^2*d*x^2])) - (
Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*d*x^2) + (c^2*Sqrt[1 + c^2*x
^2]*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2] - (
b^2*c^2*Sqrt[1 + c^2*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]])/Sqrt[d + c^2*d*x^2] +
 (b*c^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcSinh[c*x]])
/Sqrt[d + c^2*d*x^2] - (b*c^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLo
g[2, E^ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2] - (b^2*c^2*Sqrt[1 + c^2*x^2]*Poly
Log[3, -E^ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2] + (b^2*c^2*Sqrt[1 + c^2*x^2]*P
olyLog[3, E^ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 5764

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[
((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&  !GtQ[d, 0] && (Integer
Q[m] || EqQ[n, 1])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^3\sqrt{d+c^2dx^2}}\,dx = -\frac{\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} - \frac{1}{2}c^2\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\sqrt{d+c^2dx^2}}\,dx + \frac{\left(bc\sqrt{1+c^2x\right.}}{\sqrt{\ }}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} - \frac{\left(c^2\sqrt{1+c^2\right.}}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} - \frac{\left(c^2\sqrt{1+c^2\right.}}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} + \frac{c^2\sqrt{1+c^2}}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} + \frac{c^2\sqrt{1+c^2}}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} + \frac{c^2\sqrt{1+c^2}}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} + \frac{c^2\sqrt{1+c^2}}{}$$

Mathematica [A] time = 5.47, size = 455, normalized size = 1.26

$$-\frac{4a^2\sqrt{c^2dx^2+d}}{x^2} + 4a^2c^2\sqrt{d}\,\log\left(\sqrt{d}\,\sqrt{c^2dx^2+d}\,+d\right) - 4a^2c^2\sqrt{d}\,\log(x) + \frac{2abc^2d^2\left(c^2x^2+1\right)^{3/2}\left(-4\text{Li}_2\left(-e^{-\sinh^{-1}(cx)}\right)+4\text{Li}_2\left(e\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^3*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] ((-4*a^2*Sqrt[d + c^2*d*x^2])/x^2 - 4*a^2*c^2*Sqrt[d]*Log[x] + 4*a^2*c^2*Sq
rt[d]*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (2*a*b*c^2*d^2*(1 + c^2*x^2)^(
3/2)*(-2*Coth[ArcSinh[c*x]/2] - ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - 4*Arc
Sinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] + 4*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c
*x])] - 4*PolyLog[2, -E^(-ArcSinh[c*x])] + 4*PolyLog[2, E^(-ArcSinh[c*x])]
- ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSinh[c*x]/2]))/(d + c^2*d
*x^2)^(3/2) + (b^2*c^2*d^2*(1 + c^2*x^2)^(3/2)*(-4*ArcSinh[c*x]*Coth[ArcSin
h[c*x]/2] - ArcSinh[c*x]^2*Csch[ArcSinh[c*x]/2]^2 - 4*ArcSinh[c*x]^2*Log[1
- E^(-ArcSinh[c*x])] + 4*ArcSinh[c*x]^2*Log[1 + E^(-ArcSinh[c*x])] + 8*Log[
Tanh[ArcSinh[c*x]/2]] - 8*ArcSinh[c*x]*PolyLog[2, -E^(-ArcSinh[c*x])] + 8*A
rcSinh[c*x]*PolyLog[2, E^(-ArcSinh[c*x])] - 8*PolyLog[3, -E^(-ArcSinh[c*x])
] + 8*PolyLog[3, E^(-ArcSinh[c*x])] - ArcSinh[c*x]^2*Sech[ArcSinh[c*x]/2]^2
 + 4*ArcSinh[c*x]*Tanh[ArcSinh[c*x]/2]))/(d + c^2*d*x^2)^(3/2))/(8*d)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(b^2\,\text{arsinh}\,(cx)^2+2\,ab\,\text{arsinh}\,(cx)+a^2\right)}{c^2dx^5+dx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^2*d*x^5 + d*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arsinh}(cx)+a)^2}{\sqrt{c^2dx^2+d}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(sqrt(c^2*d*x^2 + d)*x^3), x)
```

maple [B] time = 0.48, size = 901, normalized size = 2.50

$$-\frac{a^2\sqrt{c^2d\,x^2+d}}{2d\,x^2}+\frac{a^2c^2\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c^2d\,x^2+d}}{x}\right)}{2\sqrt{d}}-\frac{b^2\operatorname{arcsinh}(cx)^2\sqrt{d\left(c^2x^2+1\right)}\,c^2}{2d\left(c^2x^2+1\right)}-\frac{b^2\operatorname{arcsinh}(cx)\sqrt{d\left(c^2x^2+1\right)}\,c}{xd\sqrt{c^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -1/2*a^2/d/x^2*(c^2*d*x^2+d)^(1/2)+1/2*a^2*c^2/d^(1/2)*ln((2*d+2*d^(1/2)*(c
^2*d*x^2+d)^(1/2))/x)-1/2*b^2*arcsinh(c*x)^2*(d*(c^2*x^2+1))^(1/2)/d/(c^2*x
^2+1)*c^2-b^2*arcsinh(c*x)*(d*(c^2*x^2+1))^(1/2)/x/d/(c^2*x^2+1)^(1/2)*c-1/
2*b^2*arcsinh(c*x)^2*(d*(c^2*x^2+1))^(1/2)/x^2/d/(c^2*x^2+1)+1/2*b^2*(d*(c^
2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/
2))*c^2+b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*polylog(
2,-c*x-(c^2*x^2+1)^(1/2))*c^2-b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d
*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*c^2-1/2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x
^2+1)^(1/2)/d*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^2-b^2*(d*(c^2*x^
2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2
))*c^2+b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(3,c*x+(c^2*x^2
+1)^(1/2))*c^2-2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arctanh(c*x+
(c^2*x^2+1)^(1/2))*c^2-a*b*(d*(c^2*x^2+1))^(1/2)/d/(c^2*x^2+1)*arcsinh(c*x)
*c^2-a*b*(d*(c^2*x^2+1))^(1/2)/x/d/(c^2*x^2+1)^(1/2)*c-a*b*arcsinh(c*x)*(d*
(c^2*x^2+1))^(1/2)/x^2/d/(c^2*x^2+1)-a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^
(1/2)/d*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^2-a*b*(d*(c^2*x^2+1))^(1
/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c^2+a*b*(d*(c^2*x^
2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^
2+a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,-c*x-(c^2*x^2+1)^
(1/2))*c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(\frac{c^2\operatorname{arsinh}\left(\frac{1}{c|x|}\right)}{\sqrt{d}}-\frac{\sqrt{c^2dx^2+d}}{dx^2}\right)a^2+\int\frac{b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{\sqrt{c^2dx^2+d}\,x^3}+\frac{2\,ab\log\left(cx+\sqrt{c^2x^2+1}\right)}{\sqrt{c^2dx^2+d}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/2*(c^2*arcsinh(1/(c*abs(x)))/sqrt(d) - sqrt(c^2*d*x^2 + d)/(d*x^2))*a^2 +
 integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(sqrt(c^2*d*x^2 + d)*x^3) + 2
*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(sqrt(c^2*d*x^2 + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{asinh}\,(c\,x))^2}{x^3\,\sqrt{d\,c^2\,x^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{asinh}\,(cx))^2}{x^3\sqrt{d\left(c^2x^2 + 1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**3/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(x**3*sqrt(d*(c**2*x**2 + 1))), x)
```

3.299 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^4\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=299

$$\frac{2c^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{3dx}-\frac{bc\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{3x^2\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3}-\frac{2c^3\sqrt{c^2x^2}}{}$$

```
[Out] -1/3*b^2*c^2*(c^2*x^2+1)/x/(c^2*d*x^2+d)^(1/2)-1/3*b*c*(a+b*arcsinh(c*x))*(
c^2*x^2+1)^(1/2)/x^2/(c^2*d*x^2+d)^(1/2)-2/3*c^3*(a+b*arcsinh(c*x))^2*(c^2*
x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-4/3*b*c^3*(a+b*arcsinh(c*x))*ln(1-1/(c*x+(
c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+2/3*b^2*c^3*poly
log(2,1/(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-1/
3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/d/x^3+2/3*c^2*(a+b*arcsinh(c*x))
^2*(c^2*d*x^2+d)^(1/2)/d/x
```

Rubi [A] time = 0.43, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {5747, 5723, 5659, 3716, 2190, 2279, 2391, 5661, 264}

$$-\frac{2b^2c^3\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{3\sqrt{c^2dx^2+d}}+\frac{2c^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{3\sqrt{c^2dx^2+d}}+\frac{2c^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{3dx}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^4*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] -(b^2*c^2*(1 + c^2*x^2))/(3*x*Sqrt[d + c^2*d*x^2]) - (b*c*Sqrt[1 + c^2*x^2]
*(a + b*ArcSinh[c*x]))/(3*x^2*Sqrt[d + c^2*d*x^2]) + (2*c^3*Sqrt[1 + c^2*x^
2]*(a + b*ArcSinh[c*x])^2)/(3*Sqrt[d + c^2*d*x^2]) - (Sqrt[d + c^2*d*x^2]*(
a + b*ArcSinh[c*x])^2)/(3*d*x^3) + (2*c^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSin
h[c*x])^2)/(3*d*x) - (4*b*c^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1
- E^(2*ArcSinh[c*x])])/(3*Sqrt[d + c^2*d*x^2]) - (2*b^2*c^3*Sqrt[1 + c^2*x^
2]*PolyLog[2, E^(2*ArcSinh[c*x])])/(3*Sqrt[d + c^2*d*x^2])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^4\sqrt{d+c^2dx^2}}dx = -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3} - \frac{1}{3}\left(2c^2\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\sqrt{d+c^2dx^2}}dx + \frac{\left(2bc\sqrt{1+c^2\right.}{3}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3} + \frac{2c^2\sqrt{d+c^2dx}}{}$$

$$= -\frac{b^2c^2\left(1+c^2x^2\right)}{3x\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^3}$$

$$= -\frac{b^2c^2\left(1+c^2x^2\right)}{3x\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2\sqrt{d+c^2dx^2}} + \frac{2c^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx\right.}{3\sqrt{d+c^2dx^2}}$$

$$= -\frac{b^2c^2\left(1+c^2x^2\right)}{3x\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2\sqrt{d+c^2dx^2}} + \frac{2c^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx\right.}{3\sqrt{d+c^2dx^2}}$$

$$= -\frac{b^2c^2\left(1+c^2x^2\right)}{3x\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2\sqrt{d+c^2dx^2}} + \frac{2c^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx\right.}{3\sqrt{d+c^2dx^2}}$$

$$= -\frac{b^2c^2\left(1+c^2x^2\right)}{3x\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2\sqrt{d+c^2dx^2}} + \frac{2c^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx\right.}{3\sqrt{d+c^2dx^2}}$$

Mathematica [A] time = 0.65, size = 278, normalized size = 0.93

$$\frac{2a^2c^4x^4+a^2c^2x^2-a^2-abcx\sqrt{c^2x^2+1}-4abc^3x^3\sqrt{c^2x^2+1}\log(cx)-b\sinh^{-1}(cx)\left(-2a\left(2c^4x^4+c^2x^2-1\right)+bc\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^4*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] (-a^2 + a^2*c^2*x^2 - b^2*c^2*x^2 + 2*a^2*c^4*x^4 - b^2*c^4*x^4 - a*b*c*x*S
qrt[1 + c^2*x^2] + b^2*(-1 + c^2*x^2 + 2*c^4*x^4 - 2*c^3*x^3*Sqrt[1 + c^2*x
^2])*ArcSinh[c*x]^2 - b*ArcSinh[c*x]*(b*c*x*Sqrt[1 + c^2*x^2] - 2*a*(-1 + c
^2*x^2 + 2*c^4*x^4) + 4*b*c^3*x^3*Sqrt[1 + c^2*x^2]*Log[1 - E^(-2*ArcSinh[c
*x])]) - 4*a*b*c^3*x^3*Sqrt[1 + c^2*x^2]*Log[c*x] + 2*b^2*c^3*x^3*Sqrt[1 +
c^2*x^2]*PolyLog[2, E^(-2*ArcSinh[c*x])])/(3*x^3*Sqrt[d + c^2*d*x^2])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(b^2\,\text{arsinh}\,(cx)^2+2\,ab\,\text{arsinh}\,(cx)+a^2\right)}{c^2dx^6+dx^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^2*d*x^6 + d*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\text{arsinh}\,(cx)+a\right)^2}{\sqrt{c^2dx^2+d}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(sqrt(c^2*d*x^2 + d)*x^4), x)
```

maple [B] time = 0.49, size = 2147, normalized size = 7.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x*c^4+2/3*a^2*c^2/
d/x*(c^2*d*x^2+d)^(1/2)+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-
1)/d/x*c^2+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d/x^3*arcs
inh(c*x)^2+2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x^5*c^8-
1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x^3*c^6-4/3*b^2*(d*
(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c^3
-4/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,-c*x-(c^2*x^2+
1)^(1/2))*c^3-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*c^3*(
c^2*x^2+1)^(1/2)+4/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(
c*x)^2*c^3-4/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x^3*arcs
inh(c*x)*(c^2*x^2+1)*c^6-2*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1
)/d*x^2*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*c^5+2/3*b^2*(d*(c^2*x^2+1))^(1/2)/
(3*c^4*x^4+2*c^2*x^2-1)/d*x*arcsinh(c*x)*(c^2*x^2+1)*c^4+1/3*b^2*(d*(c^2*x^
2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d/x^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c-
4/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x^3*(c^2*x^2+1)*c^6
+4*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x^3*arcsinh(c*x)*c^6
+2/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x*(c^2*x^2+1)*c^4+
2/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x*arcsinh(c*x)*c^4+
4/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*arcsinh(c*x)*(c^2*x
^2+1)^(1/2)*c^3-8/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d/x*a
rcsinh(c*x)*c^2+1/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d/x^2
*c*(c^2*x^2+1)^(1/2)-1/3*a^2/d/x^3*(c^2*d*x^2+d)^(1/2)+2/3*a*b*(d*(c^2*x^2+
1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d/x^3*arcsinh(c*x)-4/3*a*b*(d*(c^2*x^2+1)
)^(1/2)/(c^2*x^2+1)^(1/2)/d*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c^3-4/3*b^2*(d*
(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1
/2))*c^3+b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x^2*(c^2*x^2+1
)^(1/2)*c^5+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x*arcsi
nh(c*x)^2*c^4-2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x*arc
sinh(c*x)*c^4-4/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d/x*arc
sinh(c*x)^2*c^2+4/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x^5
*arcsinh(c*x)*c^8+2*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x^3
*arcsinh(c*x)^2*c^6-2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d
*x^3*(c^2*x^2+1)*c^6-4*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*
x^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^5+2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4
*x^4+2*c^2*x^2-1)/d*x^3*arcsinh(c*x)*c^6-4/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2
*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^3+2/3*b^2*(d*(c^
2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*
c^3-b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*arcsinh(c*x)*(c^2*x
^2+1)^(1/2)*c^3+8/3*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c
*x)*c^3+4/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x^5*c^8+2/3
*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x^3*c^6-2/3*a*b*(d*(c^
2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x*c^4-a*b*(d*(c^2*x^2+1))^(1/2)/(
3*c^4*x^4+2*c^2*x^2-1)/d*c^3*(c^2*x^2+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}\left(\frac{4\,c^2\log(x)}{\sqrt{d}}+\frac{1}{\sqrt{d}\,x^2}\right)abc+\frac{2}{3}\,ab\left(\frac{2\,\sqrt{c^2dx^2+d}\,c^2}{dx}-\frac{\sqrt{c^2dx^2+d}}{dx^3}\right)\operatorname{arsinh}(cx)+\frac{1}{3}\,a^2\left(\frac{2\,\sqrt{c^2dx^2+d}\,c^2}{dx}-\frac{\sqrt{c^2}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/3*(4*c^2*log(x)/sqrt(d) + 1/(sqrt(d)*x^2))*a*b*c + 2/3*a*b*(2*sqrt(c^2*d
*x^2 + d)*c^2/(d*x) - sqrt(c^2*d*x^2 + d)/(d*x^3))*arcsinh(c*x) + 1/3*a^2*(
2*sqrt(c^2*d*x^2 + d)*c^2/(d*x) - sqrt(c^2*d*x^2 + d)/(d*x^3)) + b^2*integr
ate(log(c*x + sqrt(c^2*x^2 + 1))^2/(sqrt(c^2*d*x^2 + d)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))^2}{x^4\,\sqrt{d\,c^2\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(cx))^2}{x^4\sqrt{d\left(c^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**4/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(x**4*sqrt(d*(c**2*x**2 + 1))), x)
```

3.300 $\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=515

$$-\frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{c^2dx^2+d}}-\frac{8\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2}{3c^6d^2}+\frac{4b\sqrt{c^2x^2+1}\,\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^6d\sqrt{c^2dx^2+d}}$$

```
[Out] -32/9*b^2*(c^2*x^2+1)/c^6/d/(c^2*d*x^2+d)^(1/2)+2/27*b^2*(c^2*x^2+1)^2/c^6/
d/(c^2*d*x^2+d)^(1/2)-x^4*(a+b*arcsinh(c*x))^2/c^2/d/(c^2*d*x^2+d)^(1/2)+16
/3*a*b*x*(c^2*x^2+1)^(1/2)/c^5/d/(c^2*d*x^2+d)^(1/2)+16/3*b^2*x*arcsinh(c*x
)*(c^2*x^2+1)^(1/2)/c^5/d/(c^2*d*x^2+d)^(1/2)-2*b*x*(a+b*arcsinh(c*x))*(c^2
*x^2+1)^(1/2)/c^5/d/(c^2*d*x^2+d)^(1/2)-2/9*b*x^3*(a+b*arcsinh(c*x))*(c^2*x
^2+1)^(1/2)/c^3/d/(c^2*d*x^2+d)^(1/2)+4*b*(a+b*arcsinh(c*x))*arctan(c*x+(c^
2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/c^6/d/(c^2*d*x^2+d)^(1/2)-2*I*b^2*polylog
(2,-I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/c^6/d/(c^2*d*x^2+d)^(1/2)+
2*I*b^2*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/c^6/d/(c^2*d
*x^2+d)^(1/2)-8/3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^6/d^2+4/3*x^2*
(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^4/d^2
```

Rubi [A] time = 0.79, antiderivative size = 515, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {5751, 5758, 5717, 5653, 261, 5661, 266, 43, 5767, 5693, 4180, 2279, 2391}

$$-\frac{2ib^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{c^6d\sqrt{c^2dx^2+d}}+\frac{2ib^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{c^6d\sqrt{c^2dx^2+d}}+\frac{4x^2\sqrt{c^2dx^2+d}\,\left(a+b\sin\right.}{3c^4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (16*a*b*x*Sqrt[1 + c^2*x^2])/(3*c^5*d*Sqrt[d + c^2*d*x^2]) - (32*b^2*(1 + c
^2*x^2))/(9*c^6*d*Sqrt[d + c^2*d*x^2]) + (2*b^2*(1 + c^2*x^2)^2)/(27*c^6*d*
Sqrt[d + c^2*d*x^2]) + (16*b^2*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(3*c^5*d*S
qrt[d + c^2*d*x^2]) - (2*b*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(c^5*d
*Sqrt[d + c^2*d*x^2]) - (2*b*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(9
*c^3*d*Sqrt[d + c^2*d*x^2]) - (x^4*(a + b*ArcSinh[c*x])^2)/(c^2*d*Sqrt[d +
c^2*d*x^2]) - (8*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c^6*d^2) +
(4*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c^4*d^2) + (4*b*Sqrt[
1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c^6*d*Sqrt[d + c
^2*d*x^2]) - ((2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)*E^ArcSinh[c*x]])/
(c^6*d*Sqrt[d + c^2*d*x^2]) + ((2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I*E^A
rcSinh[c*x]])/(c^6*d*Sqrt[d + c^2*d*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
```

```
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5767

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(e*(m + 2*p + 1)), x] + (-Dist[(f^2*(m - 1))/(c^2*(m +
 2*p + 1)), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(c*(m + 2*p + 1)*(1 + c
^2*x^2)^FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcS
inh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2
*d] && GtQ[n, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx = -\frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{4\int\frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx}{c^2d} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int\frac{x^4\left(a+b\sin\ldots\right.}{1+c^2\ldots}}{cd\sqrt{d+c^2dx^2}}$$

$$= \frac{2bx^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3c^3d\sqrt{d+c^2dx^2}} - \frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{4x^2\sqrt{d+c^2dx^2}\ldots}{3\ldots}$$

$$= -\frac{2bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^5d\sqrt{d+c^2dx^2}} - \frac{2bx^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{9c^3d\sqrt{d+c^2dx^2}} - \frac{x^4\ldots}{\ldots}$$

$$= \frac{16abx\sqrt{1+c^2x^2}}{3c^5d\sqrt{d+c^2dx^2}} + \frac{2b^2\left(1+c^2x^2\right)}{c^6d\sqrt{d+c^2dx^2}} - \frac{2bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^5d\sqrt{d+c^2dx^2}} - \frac{2bx^3\ldots}{\ldots}$$

$$= \frac{16abx\sqrt{1+c^2x^2}}{3c^5d\sqrt{d+c^2dx^2}} + \frac{8b^2\left(1+c^2x^2\right)}{3c^6d\sqrt{d+c^2dx^2}} - \frac{2b^2\left(1+c^2x^2\right)^2}{9c^6d\sqrt{d+c^2dx^2}} + \frac{16b^2x\sqrt{1+c^2x^2}\,\sin\ldots}{3c^5d\sqrt{d+c^2d\ldots}}$$

$$= \frac{16abx\sqrt{1+c^2x^2}}{3c^5d\sqrt{d+c^2dx^2}} - \frac{32b^2\left(1+c^2x^2\right)}{9c^6d\sqrt{d+c^2dx^2}} + \frac{2b^2\left(1+c^2x^2\right)^2}{27c^6d\sqrt{d+c^2dx^2}} + \frac{16b^2x\sqrt{1+c^2x^2}\,\mathrm{si}\ldots}{3c^5d\sqrt{d+c^2\ldots}}$$

$$= \frac{16abx\sqrt{1+c^2x^2}}{3c^5d\sqrt{d+c^2dx^2}} - \frac{32b^2\left(1+c^2x^2\right)}{9c^6d\sqrt{d+c^2dx^2}} + \frac{2b^2\left(1+c^2x^2\right)^2}{27c^6d\sqrt{d+c^2dx^2}} + \frac{16b^2x\sqrt{1+c^2x^2}\,\mathrm{si}\ldots}{3c^5d\sqrt{d+c^2\ldots}}$$

Mathematica [A] time = 0.63, size = 427, normalized size = 0.83

$$9a^2c^4x^4 - 36a^2c^2x^2 - 72a^2 + 18abc^4x^4\sinh^{-1}(cx) + 90abcx\sqrt{c^2x^2+1} - 72abc^2x^2\sinh^{-1}(cx) + 108ab\sqrt{c^2x^2+\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^5*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (-72*a^2 - 94*b^2 - 36*a^2*c^2*x^2 - 92*b^2*c^2*x^2 + 9*a^2*c^4*x^4 + 2*b^2
*c^4*x^4 + 90*a*b*c*x*Sqrt[1 + c^2*x^2] - 6*a*b*c^3*x^3*Sqrt[1 + c^2*x^2] -
 144*a*b*ArcSinh[c*x] - 72*a*b*c^2*x^2*ArcSinh[c*x] + 18*a*b*c^4*x^4*ArcSin
h[c*x] + 90*b^2*c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - 6*b^2*c^3*x^3*Sqrt[1 +
 c^2*x^2]*ArcSinh[c*x] - 72*b^2*ArcSinh[c*x]^2 - 36*b^2*c^2*x^2*ArcSinh[c*x
]^2 + 9*b^2*c^4*x^4*ArcSinh[c*x]^2 + 108*a*b*Sqrt[1 + c^2*x^2]*ArcTan[Tanh[
ArcSinh[c*x]/2]] - (54*I)*b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 - I/E^Ar
cSinh[c*x]] + (54*I)*b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + I/E^ArcSinh
[c*x]] - (54*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)/E^ArcSinh[c*x]] + (54
*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I/E^ArcSinh[c*x]])/(27*c^6*d*Sqrt[d +
c^2*d*x^2])
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2x^5\operatorname{arsinh}(cx)^2 + 2\,abx^5\operatorname{arsinh}(cx) + a^2x^5\right)\sqrt{c^2dx^2 + d}}{c^4d^2x^4 + 2\,c^2d^2x^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^5*arcsinh(c*x)^2 + 2*a*b*x^5*arcsinh(c*x) + a^2*x^5)*sqrt(c
^2*d*x^2 + d)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.52, size = 933, normalized size = 1.81

$$\frac{a^2x^4}{3c^2d\sqrt{c^2d\,x^2+d}} - \frac{4a^2x^2}{3c^4d\sqrt{c^2d\,x^2+d}} - \frac{8a^2}{3c^6d\sqrt{c^2d\,x^2+d}} + \frac{2b^2\sqrt{d\left(c^2x^2+1\right)}\,x^4}{27c^2d^2\left(c^2x^2+1\right)} - \frac{92b^2\sqrt{d\left(c^2x^2+1\right)}\,x^2}{27c^4d^2\left(c^2x^2+1\right)} - \frac{2ib^2\sqrt{d\left(c\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] 1/3*a^2*x^4/c^2/d/(c^2*d*x^2+d)^(1/2)-4/3*a^2/c^4*x^2/d/(c^2*d*x^2+d)^(1/2)
-8/3*a^2/c^6/d/(c^2*d*x^2+d)^(1/2)+2/27*b^2*(d*(c^2*x^2+1))^(1/2)/c^2/d^2/(
c^2*x^2+1)*x^4-92/27*b^2*(d*(c^2*x^2+1))^(1/2)/c^4/d^2/(c^2*x^2+1)*x^2+2*I*
b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^6/d^2*arcsinh(c*x)*ln(1-I*(c*
x+(c^2*x^2+1)^(1/2)))-2/9*b^2*(d*(c^2*x^2+1))^(1/2)/c^3/d^2/(c^2*x^2+1)^(1/
2)*arcsinh(c*x)*x^3+10/3*b^2*(d*(c^2*x^2+1))^(1/2)/c^5/d^2/(c^2*x^2+1)^(1/2
)*arcsinh(c*x)*x-2*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^6/d^2*ar
csinh(c*x)*ln(1+I*(c*x+(c^2*x^2+1)^(1/2)))+2*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c
^2*x^2+1)^(1/2)/c^6/d^2*dilog(1-I*(c*x+(c^2*x^2+1)^(1/2)))-2*I*a*b*(d*(c^2*
x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^6/d^2*ln(c*x+(c^2*x^2+1)^(1/2)-I)-8/3*b^2
*(d*(c^2*x^2+1))^(1/2)/c^6/d^2/(c^2*x^2+1)*arcsinh(c*x)^2-94/27*b^2*(d*(c^2
```

```
*x^2+1))^(1/2)/c^6/d^2/(c^2*x^2+1)+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/c^2/d^2/(c
^2*x^2+1)*arcsinh(c*x)^2*x^4-4/3*b^2*(d*(c^2*x^2+1))^(1/2)/c^4/d^2/(c^2*x^2
+1)*arcsinh(c*x)^2*x^2-16/3*a*b*(d*(c^2*x^2+1))^(1/2)/c^6/d^2/(c^2*x^2+1)*a
rcsinh(c*x)-2*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^6/d^2*dilog(1
+I*(c*x+(c^2*x^2+1)^(1/2)))+2/3*a*b*(d*(c^2*x^2+1))^(1/2)/c^2/d^2/(c^2*x^2+
1)*arcsinh(c*x)*x^4-2/9*a*b*(d*(c^2*x^2+1))^(1/2)/c^3/d^2/(c^2*x^2+1)^(1/2)
*x^3-8/3*a*b*(d*(c^2*x^2+1))^(1/2)/c^4/d^2/(c^2*x^2+1)*arcsinh(c*x)*x^2+10/
3*a*b*(d*(c^2*x^2+1))^(1/2)/c^5/d^2/(c^2*x^2+1)^(1/2)*x+2*I*a*b*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^6/d^2*ln(c*x+(c^2*x^2+1)^(1/2)+I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}a^2\left(\frac{x^4}{\sqrt{c^2dx^2+d}\,c^2d}-\frac{4\,x^2}{\sqrt{c^2dx^2+d}\,c^4d}-\frac{8}{\sqrt{c^2dx^2+d}\,c^6d}\right)+\frac{\left(b^2c^4\sqrt{d}\,x^4-4\,b^2c^2\sqrt{d}\,x^2-8\,b^2\sqrt{d}\right)\sqrt{c^2x^2+1}\,\mathrm{lo}}{3\left(c^8d^2x^2+c^6d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
```

```
[Out] 1/3*a^2*(x^4/(sqrt(c^2*d*x^2 + d)*c^2*d) - 4*x^2/(sqrt(c^2*d*x^2 + d)*c^4*d
) - 8/(sqrt(c^2*d*x^2 + d)*c^6*d)) + 1/3*(b^2*c^4*sqrt(d)*x^4 - 4*b^2*c^2*s
qrt(d)*x^2 - 8*b^2*sqrt(d))*sqrt(c^2*x^2 + 1)*log(c*x + sqrt(c^2*x^2 + 1))^
2/(c^8*d^2*x^2 + c^6*d^2) + integrate(2/3*((4*b^2*c^3*x^3 + (3*a*b*c^5 - b^
2*c^5)*x^5 + 8*b^2*c*x)*(c^2*x^2 + 1) + (3*b^2*c^4*x^4 + (3*a*b*c^6 - b^2*c
^6)*x^6 + 12*b^2*c^2*x^2 + 8*b^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2
 + 1))/(c^10*d^(3/2)*x^5 + 2*c^8*d^(3/2)*x^3 + c^6*d^(3/2)*x + (c^9*d^(3/2)
*x^4 + 2*c^7*d^(3/2)*x^2 + c^5*d^(3/2))*sqrt(c^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d\,c^2\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int((x^5*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5} \left(a + b \operatorname{asinh}{\left(c x \right)}\right)^{2}}{\left(d \left(c^{2} x^{2} + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**5*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(3/2), x)
```

3.301 $\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=400

$$-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{2bc^5d\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{c^5d\sqrt{c^2dx^2+d}}-\frac{2b\sqrt{c^2x^2+1}\log\left(e^{2\sinh^{-1}(cx)}\right.}{c^5d\sqrt{}}$$

```
[Out] 1/4*b^2*x*(c^2*x^2+1)/c^4/d/(c^2*d*x^2+d)^(1/2)-x^3*(a+b*arcsinh(c*x))^2/c^
2/d/(c^2*d*x^2+d)^(1/2)-1/4*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)/c^5/d/(c^2*d
*x^2+d)^(1/2)-1/2*b*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3/d/(c^2*d*x
^2+d)^(1/2)+(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/c^5/d/(c^2*d*x^2+d)^(1/2
)-1/2*(a+b*arcsinh(c*x))^3*(c^2*x^2+1)^(1/2)/b/c^5/d/(c^2*d*x^2+d)^(1/2)-2*
b*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/c^5/
d/(c^2*d*x^2+d)^(1/2)-b^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)
^(1/2)/c^5/d/(c^2*d*x^2+d)^(1/2)+3/2*x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(
1/2)/c^4/d^2
```

Rubi [A] time = 0.66, antiderivative size = 400, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {5751, 5758, 5677, 5675, 5661, 321, 215, 5767, 5714, 3718, 2190, 2279, 2391}

$$-\frac{b^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{c^5d\sqrt{c^2dx^2+d}}+\frac{3x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{2c^4d^2}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{c^2dx^2+d}}-\frac{bx^2\sqrt{c^2x^2}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (b^2*x*(1 + c^2*x^2))/(4*c^4*d*Sqrt[d + c^2*d*x^2]) - (b^2*Sqrt[1 + c^2*x^2
]*ArcSinh[c*x])/(4*c^5*d*Sqrt[d + c^2*d*x^2]) - (b*x^2*Sqrt[1 + c^2*x^2]*(a
 + b*ArcSinh[c*x]))/(2*c^3*d*Sqrt[d + c^2*d*x^2]) - (x^3*(a + b*ArcSinh[c*x
])^2)/(c^2*d*Sqrt[d + c^2*d*x^2]) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])
^2)/(c^5*d*Sqrt[d + c^2*d*x^2]) + (3*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c
*x])^2)/(2*c^4*d^2) - (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(2*b*c^5*d
*Sqrt[d + c^2*d*x^2]) - (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 +
 E^(2*ArcSinh[c*x])])/(c^5*d*Sqrt[d + c^2*d*x^2]) - (b^2*Sqrt[1 + c^2*x^2]*
PolyLog[2, -E^(2*ArcSinh[c*x])])/(c^5*d*Sqrt[d + c^2*d*x^2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
```

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] &&  !GtQ[d, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5767

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(e*(m + 2*p + 1)), x] + (-Dist[(f^2*(m - 1))/(c^2*(m +
 2*p + 1)), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(c*(m + 2*p + 1)*(1 + c
^2*x^2)^FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcS
inh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2
*d] && GtQ[n, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx = -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{3\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx}{c^2d} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{x^3\left(a+b\sinh^{-1}\right.}{1+c^2x^2}}{cd\sqrt{d+c^2dx^2}}$$

$$= \frac{bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^3d\sqrt{d+c^2dx^2}} - \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{3x\sqrt{d+c^2dx^2}\left(a+b\right.}{2c^4d^2}$$

$$= -\frac{b^2x\left(1+c^2x^2\right)}{2c^4d\sqrt{d+c^2dx^2}} - \frac{bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^3d\sqrt{d+c^2dx^2}} - \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}} +$$

$$= \frac{b^2x\left(1+c^2x^2\right)}{4c^4d\sqrt{d+c^2dx^2}} + \frac{b^2\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)}{2c^5d\sqrt{d+c^2dx^2}} - \frac{bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^3d\sqrt{d+c^2dx^2}} -$$

$$= \frac{b^2x\left(1+c^2x^2\right)}{4c^4d\sqrt{d+c^2dx^2}} - \frac{b^2\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)}{4c^5d\sqrt{d+c^2dx^2}} - \frac{bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^3d\sqrt{d+c^2dx^2}} -$$

$$= \frac{b^2x\left(1+c^2x^2\right)}{4c^4d\sqrt{d+c^2dx^2}} - \frac{b^2\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)}{4c^5d\sqrt{d+c^2dx^2}} - \frac{bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^3d\sqrt{d+c^2dx^2}} -$$

$$= \frac{b^2x\left(1+c^2x^2\right)}{4c^4d\sqrt{d+c^2dx^2}} - \frac{b^2\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)}{4c^5d\sqrt{d+c^2dx^2}} - \frac{bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^3d\sqrt{d+c^2dx^2}} -$$

Mathematica [A] time = 1.90, size = 288, normalized size = 0.72

$$4a^2c\sqrt{d}\,x\left(c^2x^2+3\right) - 12a^2\sqrt{c^2dx^2+d}\,\log\left(\sqrt{d}\,\sqrt{c^2dx^2+d}+cdx\right) + 2ab\sqrt{d}\left(8cx\sinh^{-1}(cx) - \sqrt{c^2x^2+1}\left(4\log\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (4*a^2*c*Sqrt[d]*x*(3 + c^2*x^2) - 12*a^2*Sqrt[d + c^2*d*x^2]*Log[c*d*x + S
qrt[d]*Sqrt[d + c^2*d*x^2]] + b^2*Sqrt[d]*(8*c*x*ArcSinh[c*x]^2 + 8*Sqrt[1
```

```
+ c^2*x^2]*PolyLog[2, -E^(-2*ArcSinh[c*x])] + Sqrt[1 + c^2*x^2]*(-4*ArcSinh
[c*x]^3 - 2*ArcSinh[c*x]*(Cosh[2*ArcSinh[c*x]] + 8*Log[1 + E^(-2*ArcSinh[c*
x])]) + 2*ArcSinh[c*x]^2*(-4 + Sinh[2*ArcSinh[c*x]]) + Sinh[2*ArcSinh[c*x]]
)) + 2*a*b*Sqrt[d]*(8*c*x*ArcSinh[c*x] - Sqrt[1 + c^2*x^2]*(6*ArcSinh[c*x]^
2 + Cosh[2*ArcSinh[c*x]] + 4*Log[1 + c^2*x^2] - 2*ArcSinh[c*x]*Sinh[2*ArcSi
nh[c*x]])))/(8*c^5*d^(3/2)*Sqrt[d + c^2*d*x^2])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2x^4\operatorname{arsinh}(cx)^2+2\,abx^4\operatorname{arsinh}(cx)+a^2x^4\right)\sqrt{c^2dx^2+d}}{c^4d^2x^4+2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^4*arcsinh(c*x)^2 + 2*a*b*x^4*arcsinh(c*x) + a^2*x^4)*sqrt(c
^2*d*x^2 + d)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.54, size = 816, normalized size = 2.04

$$\frac{a^2x^3}{2c^2d\sqrt{c^2d\,x^2+d}}+\frac{3a^2x}{2c^4d\sqrt{c^2d\,x^2+d}}-\frac{3a^2\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2c^4d\sqrt{c^2d}}-\frac{2b^2\sqrt{d\left(c^2x^2+1\right)}\ \operatorname{arcsinh}(cx)\ln\left(1+\left(c\right.\right.}{\sqrt{c^2x^2+1}\ c^5d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] 1/2*a^2*x^3/c^2/d/(c^2*d*x^2+d)^(1/2)+3/2*a^2/c^4*x/d/(c^2*d*x^2+d)^(1/2)-3
/2*a^2/c^4/d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)-2*
b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d^2*arcsinh(c*x)*ln(1+(c*x+
(c^2*x^2+1)^(1/2))^2)+1/2*b^2*(d*(c^2*x^2+1))^(1/2)/c^2/d^2/(c^2*x^2+1)*arc
sinh(c*x)^2*x^3+3/2*b^2*(d*(c^2*x^2+1))^(1/2)/c^4/d^2/(c^2*x^2+1)*arcsinh(c
*x)^2*x+b^2*(d*(c^2*x^2+1))^(1/2)/c^5/d^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2-
1/4*b^2*(d*(c^2*x^2+1))^(1/2)/c^5/d^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)-1/2*b^
2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d^2*arcsinh(c*x)^3+1/4*b^2*(d
*(c^2*x^2+1))^(1/2)/c^2/d^2/(c^2*x^2+1)*x^3+1/4*b^2*(d*(c^2*x^2+1))^(1/2)/c
^4/d^2/(c^2*x^2+1)*x-b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d^2*po
lylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)-1/2*b^2*(d*(c^2*x^2+1))^(1/2)/c^3/d^2/(
c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^2-3/2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)
^(1/2)/c^5/d^2*arcsinh(c*x)^2+a*b*(d*(c^2*x^2+1))^(1/2)/c^2/d^2/(c^2*x^2+1)
*arcsinh(c*x)*x^3-1/2*a*b*(d*(c^2*x^2+1))^(1/2)/c^3/d^2/(c^2*x^2+1)^(1/2)*x
^2+3*a*b*(d*(c^2*x^2+1))^(1/2)/c^4/d^2/(c^2*x^2+1)*arcsinh(c*x)*x+2*a*b*(d*
(c^2*x^2+1))^(1/2)/c^5/d^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)-1/4*a*b*(d*(c^2*x
^2+1))^(1/2)/c^5/d^2/(c^2*x^2+1)^(1/2)-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2
+1)^(1/2)/c^5/d^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,a^2\left(\frac{x^3}{\sqrt{c^2dx^2+d}\,c^2d}+\frac{3\,x}{\sqrt{c^2dx^2+d}\,c^4d}-\frac{3\,\operatorname{arsinh}(cx)}{c^5d^{\frac{3}{2}}}\right)+\int\frac{b^2x^4\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}}+\frac{2\,abx^4\log\left(cx+\sqrt{c^2x\right.}}{\left(c^2dx^2+d\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
```

```
[Out] 1/2*a^2*(x^3/(sqrt(c^2*d*x^2 + d)*c^2*d) + 3*x/(sqrt(c^2*d*x^2 + d)*c^4*d)
- 3*arcsinh(c*x)/(c^5*d^(3/2))) + integrate(b^2*x^4*log(c*x + sqrt(c^2*x^2
+ 1))^2/(c^2*d*x^2 + d)^(3/2) + 2*a*b*x^4*log(c*x + sqrt(c^2*x^2 + 1))/(c^2
*d*x^2 + d)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x^4\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2}{{\left(d\,c^2\,x^2+d\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^4\left(a+b\operatorname{asinh}(cx)\right)^2}{\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**4*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(3/2), x)
```

3.302 $\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=383

$$-\frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{c^4d^2}-\frac{4b\sqrt{c^2x^2+1}\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^4d\sqrt{c^2dx^2+d}}$$

```
[Out] 2*b^2*(c^2*x^2+1)/c^4/d/(c^2*d*x^2+d)^(1/2)-x^2*(a+b*arcsinh(c*x))^2/c^2/d/
(c^2*d*x^2+d)^(1/2)-4*a*b*x*(c^2*x^2+1)^(1/2)/c^3/d/(c^2*d*x^2+d)^(1/2)-4*b
^2*x*arcsinh(c*x)*(c^2*x^2+1)^(1/2)/c^3/d/(c^2*d*x^2+d)^(1/2)+2*b*x*(a+b*ar
csinh(c*x))*(c^2*x^2+1)^(1/2)/c^3/d/(c^2*d*x^2+d)^(1/2)-4*b*(a+b*arcsinh(c*
x))*arctan(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/c^4/d/(c^2*d*x^2+d)^(1/
2)+2*I*b^2*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/c^4/d/(c
^2*d*x^2+d)^(1/2)-2*I*b^2*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^
(1/2)/c^4/d/(c^2*d*x^2+d)^(1/2)+2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/
c^4/d^2
```

Rubi [A] time = 0.46, antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {5751, 5717, 5653, 261, 5767, 5693, 4180, 2279, 2391}

$$\frac{2ib^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{c^4d\sqrt{c^2dx^2+d}}-\frac{2ib^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{c^4d\sqrt{c^2dx^2+d}}+\frac{2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(\right.}{c^4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (-4*a*b*x*Sqrt[1 + c^2*x^2])/(c^3*d*Sqrt[d + c^2*d*x^2]) + (2*b^2*(1 + c^2*
x^2))/(c^4*d*Sqrt[d + c^2*d*x^2]) - (4*b^2*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]
)/(c^3*d*Sqrt[d + c^2*d*x^2]) + (2*b*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x
]))/(c^3*d*Sqrt[d + c^2*d*x^2]) - (x^2*(a + b*ArcSinh[c*x])^2)/(c^2*d*Sqrt[
d + c^2*d*x^2]) + (2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(c^4*d^2)
- (4*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c^4*
d*Sqrt[d + c^2*d*x^2]) + ((2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)*E^Arc
Sinh[c*x]])/(c^4*d*Sqrt[d + c^2*d*x^2]) - ((2*I)*b^2*Sqrt[1 + c^2*x^2]*Poly
Log[2, I*E^ArcSinh[c*x]])/(c^4*d*Sqrt[d + c^2*d*x^2])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5767

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(e*(m + 2*p + 1)), x] + (-Dist[(f^2*(m - 1))/(c^2*(m +
 2*p + 1)), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(c*(m + 2*p + 1)*(1 + c
^2*x^2)^FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcS
inh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2
*d] && GtQ[n, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx = -\frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{2\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}dx}{c^2d} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{x^2(a+b\sinh}{1+c^2x}}{cd\sqrt{d+c^2dx^2}}$$

$$= \frac{2bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^3d\sqrt{d+c^2dx^2}} - \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{2\sqrt{d+c^2dx^2}\left(a+\right.}{c^4d}$$

$$= -\frac{4abx\sqrt{1+c^2x^2}}{c^3d\sqrt{d+c^2dx^2}} - \frac{2b^2\left(1+c^2x^2\right)}{c^4d\sqrt{d+c^2dx^2}} + \frac{2bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^3d\sqrt{d+c^2dx^2}} - \frac{x^2\left(\right.}{c}$$

$$= -\frac{4abx\sqrt{1+c^2x^2}}{c^3d\sqrt{d+c^2dx^2}} - \frac{2b^2\left(1+c^2x^2\right)}{c^4d\sqrt{d+c^2dx^2}} - \frac{4b^2x\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{c^3d\sqrt{d+c^2dx^2}} + \frac{2bx\sqrt{1+c}}{c^3}$$

$$= -\frac{4abx\sqrt{1+c^2x^2}}{c^3d\sqrt{d+c^2dx^2}} + \frac{2b^2\left(1+c^2x^2\right)}{c^4d\sqrt{d+c^2dx^2}} - \frac{4b^2x\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{c^3d\sqrt{d+c^2dx^2}} + \frac{2bx\sqrt{1+c}}{c^3}$$

$$= -\frac{4abx\sqrt{1+c^2x^2}}{c^3d\sqrt{d+c^2dx^2}} + \frac{2b^2\left(1+c^2x^2\right)}{c^4d\sqrt{d+c^2dx^2}} - \frac{4b^2x\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{c^3d\sqrt{d+c^2dx^2}} + \frac{2bx\sqrt{1+c}}{c^3}$$

Mathematica [A] time = 0.47, size = 318, normalized size = 0.83

$$\frac{a^2c^2x^2 + 2a^2 - 2abcx\sqrt{c^2x^2+1} + 2abc^2x^2\sinh^{-1}(cx) - 4ab\sqrt{c^2x^2+1}\tan^{-1}\left(\tanh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right) + 4ab\sinh}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (2*a^2 + 2*b^2 + a^2*c^2*x^2 + 2*b^2*c^2*x^2 - 2*a*b*c*x*Sqrt[1 + c^2*x^2]
+ 4*a*b*ArcSinh[c*x] + 2*a*b*c^2*x^2*ArcSinh[c*x] - 2*b^2*c*x*Sqrt[1 + c^2*
x^2]*ArcSinh[c*x] + 2*b^2*ArcSinh[c*x]^2 + b^2*c^2*x^2*ArcSinh[c*x]^2 - 4*a
*b*Sqrt[1 + c^2*x^2]*ArcTan[Tanh[ArcSinh[c*x]/2]] + (2*I)*b^2*Sqrt[1 + c^2*
x^2]*ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]] - (2*I)*b^2*Sqrt[1 + c^2*x^2]*A
rcSinh[c*x]*Log[1 + I/E^ArcSinh[c*x]] + (2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog
[2, (-I)/E^ArcSinh[c*x]] - (2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I/E^ArcSi
nh[c*x]])/(c^4*d*Sqrt[d + c^2*d*x^2])
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2x^3\,\text{arsinh}\,(cx)^2 + 2\,abx^3\,\text{arsinh}\,(cx) + a^2x^3\right)\sqrt{c^2dx^2+d}}{c^4d^2x^4 + 2\,c^2d^2x^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^3*arcsinh(c*x)^2 + 2*a*b*x^3*arcsinh(c*x) + a^2*x^3)*sqrt(c
^2*d*x^2 + d)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.40, size = 703, normalized size = 1.84

$$\frac{a^2x^2}{c^2d\sqrt{c^2d\,x^2+d}}+\frac{2a^2}{d\,c^4\sqrt{c^2d\,x^2+d}}+\frac{b^2\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsinh}(cx)^2\,x^2}{c^2d^2\left(c^2x^2+1\right)}-\frac{2b^2\sqrt{d\left(c^2x^2+1\right)}\,\operatorname{arcsinh}(cx)\,x}{c^3d^2\sqrt{c^2x^2+1}}+\frac{2b^2\sqrt{a}}{c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] a^2*x^2/c^2/d/(c^2*d*x^2+d)^(1/2)+2*a^2/d/c^4/(c^2*d*x^2+d)^(1/2)+b^2*(d*(c
^2*x^2+1))^(1/2)/c^2/d^2/(c^2*x^2+1)*arcsinh(c*x)^2*x^2-2*b^2*(d*(c^2*x^2+1
))^(1/2)/c^3/d^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x+2*b^2*(d*(c^2*x^2+1))^(1/
2)/c^2/d^2/(c^2*x^2+1)*x^2+2*b^2*(d*(c^2*x^2+1))^(1/2)/c^4/d^2/(c^2*x^2+1)*
arcsinh(c*x)^2+2*b^2*(d*(c^2*x^2+1))^(1/2)/c^4/d^2/(c^2*x^2+1)+2*I*b^2*(d*(
c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d^2*arcsinh(c*x)*ln(1+I*(c*x+(c^2*x
^2+1)^(1/2)))-2*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d^2*dilog
(1-I*(c*x+(c^2*x^2+1)^(1/2)))-2*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/
2)/c^4/d^2*arcsinh(c*x)*ln(1-I*(c*x+(c^2*x^2+1)^(1/2)))+2*I*b^2*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d^2*dilog(1+I*(c*x+(c^2*x^2+1)^(1/2)))+2*a
*b*(d*(c^2*x^2+1))^(1/2)/c^2/d^2/(c^2*x^2+1)*arcsinh(c*x)*x^2-2*a*b*(d*(c^2
*x^2+1))^(1/2)/c^3/d^2/(c^2*x^2+1)^(1/2)*x+4*a*b*(d*(c^2*x^2+1))^(1/2)/c^4/
d^2/(c^2*x^2+1)*arcsinh(c*x)+2*I*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2
)/c^4/d^2*ln(c*x+(c^2*x^2+1)^(1/2)-I)-2*I*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^
2+1)^(1/2)/c^4/d^2*ln(c*x+(c^2*x^2+1)^(1/2)+I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-2\,abc\left(\frac{x}{c^4d^{\frac{3}{2}}}+\frac{\arctan(cx)}{c^5d^{\frac{3}{2}}}\right)+2\,ab\left(\frac{x^2}{\sqrt{c^2dx^2+d}\,c^2d}+\frac{2}{\sqrt{c^2dx^2+d}\,c^4d}\right)\operatorname{arsinh}(cx)+a^2\left(\frac{x^2}{\sqrt{c^2dx^2+d}\,c^2d}+\frac{2}{\sqrt{c^2dx^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
```

```
[Out] -2*a*b*c*(x/(c^4*d^(3/2)) + arctan(c*x)/(c^5*d^(3/2))) + 2*a*b*(x^2/(sqrt(c
^2*d*x^2 + d)*c^2*d) + 2/(sqrt(c^2*d*x^2 + d)*c^4*d))*arcsinh(c*x) + a^2*(x
^2/(sqrt(c^2*d*x^2 + d)*c^2*d) + 2/(sqrt(c^2*d*x^2 + d)*c^4*d)) + b^2*((c^2
*x^2 + 2)*log(c*x + sqrt(c^2*x^2 + 1))^2/(sqrt(c^2*x^2 + 1)*c^4*d^(3/2)) -
integrate(2*(c^4*x^4 + 3*c^2*x^2 + (c^3*x^3 + 2*c*x)*sqrt(c^2*x^2 + 1) + 2)
*log(c*x + sqrt(c^2*x^2 + 1))/((c^5*d^(3/2)*x^2 + c^3*d^(3/2))*(c^2*x^2 + 1
) + (c^6*d^(3/2)*x^3 + c^4*d^(3/2)*x)*sqrt(c^2*x^2 + 1)), x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d\,c^2\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 (a + b \operatorname{asinh}(cx))^2}{(d (c^2x^2 + 1))^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(3/2),x)
[Out] Integral(x**3*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(3/2), x)
```

3.303 $\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=233

$$-\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)^3}{3bc^3d\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)^2}{c^3d\sqrt{c^2dx^2+d}}+\frac{2b\sqrt{c^2x^2+1}\,\log\left(e^{2\sinh\cdots}\right.}{c^3d\sqrt{\cdots}}$$

```
[Out] -x*(a+b*arcsinh(c*x))^2/c^2/d/(c^2*d*x^2+d)^(1/2)-(a+b*arcsinh(c*x))^2*(c^2
*x^2+1)^(1/2)/c^3/d/(c^2*d*x^2+d)^(1/2)+1/3*(a+b*arcsinh(c*x))^3*(c^2*x^2+1
)^(1/2)/b/c^3/d/(c^2*d*x^2+d)^(1/2)+2*b*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x
^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/c^3/d/(c^2*d*x^2+d)^(1/2)+b^2*polylog(2,-
(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/c^3/d/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5751, 5677, 5675, 5714, 3718, 2190, 2279, 2391}

$$\frac{b^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{c^3d\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)^3}{3bc^3d\sqrt{c^2dx^2+d}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2x^2+1}\,\left(a+b\cdots\right.}{c^3d\sqrt{c^2dx\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] -((x*(a + b*ArcSinh[c*x])^2)/(c^2*d*Sqrt[d + c^2*d*x^2])) - (Sqrt[1 + c^2*x
^2]*(a + b*ArcSinh[c*x])^2)/(c^3*d*Sqrt[d + c^2*d*x^2]) + (Sqrt[1 + c^2*x^2
]*(a + b*ArcSinh[c*x])^3)/(3*b*c^3*d*Sqrt[d + c^2*d*x^2]) + (2*b*Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(c^3*d*Sqrt[d + c
^2*d*x^2]) + (b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c^3*d
*Sqrt[d + c^2*d*x^2])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] &&  !GtQ[d, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rubi steps

$$\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx = -\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx}{c^2d} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx\right.}{1+c^2x^2}}{cd\sqrt{d+c^2dx^2}}$$

$$= -\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int(a+bx)\tanh(x)\,dx,x,\sinh^{-}\right.}{c^3d\sqrt{d+c^2dx^2}}$$

$$= -\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}} - \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{c^3d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}\left(a+b\,\mathrm{si}\right.}{3bc^3d\sqrt{d+c^2}}$$

$$= -\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}} - \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{c^3d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}\left(a+b\,\mathrm{si}\right.}{3bc^3d\sqrt{d+c^2}}$$

$$= -\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}} - \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{c^3d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}\left(a+b\,\mathrm{si}\right.}{3bc^3d\sqrt{d+c^2}}$$

$$= -\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}} - \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{c^3d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}\left(a+b\,\mathrm{si}\right.}{3bc^3d\sqrt{d+c^2}}$$

Mathematica [A] time = 1.01, size = 215, normalized size = 0.92

$$\frac{3a^2\sqrt{d}\sqrt{c^2dx^2+d}\,\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)-3a^2cdx-3abd\left(2cx\sinh^{-1}(cx)-\sqrt{c^2x^2+1}\left(\log\left(c^2x^2+1\right)\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (-3*a^2*c*d*x - 3*a*b*d*(2*c*x*ArcSinh[c*x] - Sqrt[1 + c^2*x^2]*(ArcSinh[c*
x]^2 + Log[1 + c^2*x^2])) + 3*a^2*Sqrt[d]*Sqrt[d + c^2*d*x^2]*Log[c*d*x + S
qrt[d]*Sqrt[d + c^2*d*x^2]] + b^2*d*(ArcSinh[c*x]*(-3*c*x*ArcSinh[c*x] + Sq
rt[1 + c^2*x^2]*(ArcSinh[c*x]*(3 + ArcSinh[c*x]) + 6*Log[1 + E^(-2*ArcSinh[
c*x])])) - 3*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-2*ArcSinh[c*x])]))/(3*c^3*d^
2*Sqrt[d + c^2*d*x^2])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2x^2\,\text{arsinh}\,(cx)^2+2\,abx^2\,\text{arsinh}\,(cx)+a^2x^2\right)\sqrt{c^2dx^2+d}}{c^4d^2x^4+2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*x) + a^2*x^2)*sqrt(c
^2*d*x^2 + d)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\,\text{arsinh}\,(cx)+a\right)^2x^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^2/(c^2*d*x^2 + d)^(3/2), x)
```

maple [B] time = 0.41, size = 478, normalized size = 2.05

$$-\frac{a^2x}{c^2d\sqrt{c^2d\,x^2+d}}+\frac{a^2\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{c^2d\sqrt{c^2d}}+\frac{b^2\sqrt{d\left(c^2x^2+1\right)}\,\text{arcsinh}\,(cx)^3}{3\sqrt{c^2x^2+1}\,c^3d^2}-\frac{b^2\sqrt{d\left(c^2x^2+1\right)}\,\text{arcsinh}\,(cx)^2}{c^2d^2\left(c^2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] -a^2*x/c^2/d/(c^2*d*x^2+d)^(1/2)+a^2/c^2/d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*
x^2+d)^(1/2))/(c^2*d)^(1/2)+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)
/c^3/d^2*arcsinh(c*x)^3-b^2*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2/c^2/d^2/(c
^2*x^2+1)*x-b^2*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2/c^3/d^2/(c^2*x^2+1)^(1
/2)+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3/d^2*arcsinh(c*x)*ln(1
+(c*x+(c^2*x^2+1)^(1/2))^2)+b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3
/d^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)+a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x
^2+1)^(1/2)/c^3/d^2*arcsinh(c*x)^2-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^
(1/2)/c^3/d^2*arcsinh(c*x)-2*a*b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/c^2/d^2
/(c^2*x^2+1)*x+2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3/d^2*ln(1+(
c*x+(c^2*x^2+1)^(1/2))^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\frac{x}{\sqrt{c^2dx^2+d}\,c^2d}-\frac{\text{arsinh}\,(cx)}{c^3d^{\frac{3}{2}}}\right)+\int\frac{b^2x^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}}+\frac{2\,abx^2\log\left(cx+\sqrt{c^2x^2+1}\right)}{\left(c^2dx^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
```

```
[Out] -a^2*(x/(sqrt(c^2*d*x^2 + d)*c^2*d) - arcsinh(c*x)/(c^3*d^(3/2))) + integra
te(b^2*x^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d*x^2 + d)^(3/2) + 2*a*b*x^2
*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d\,c^2\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(a + b \operatorname{asinh}{\left(c x \right)}\right)^{2}}{\left(d \left(c^{2} x^{2} + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(3/2), x)
```

3.304 $\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=188

$$-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{4b\sqrt{c^2x^2+1}\,\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{c^2dx^2+d}}-\frac{2ib^2\sqrt{c^2x^2+1}\,\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{c^2d\sqrt{c^2dx^2+d}}+\frac{2ib^2\cdots}{\cdots}$$

```
[Out] -(a+b*arcsinh(c*x))^2/c^2/d/(c^2*d*x^2+d)^(1/2)+4*b*(a+b*arcsinh(c*x))*arct
an(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/c^2/d/(c^2*d*x^2+d)^(1/2)-2*I*b
^2*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/c^2/d/(c^2*d*x^2
+d)^(1/2)+2*I*b^2*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/c^
2/d/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5717, 5693, 4180, 2279, 2391}

$$-\frac{2ib^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{c^2d\sqrt{c^2dx^2+d}}+\frac{2ib^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{c^2d\sqrt{c^2dx^2+d}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{4b\sqrt{c^2x\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] -((a + b*ArcSinh[c*x])^2/(c^2*d*Sqrt[d + c^2*d*x^2])) + (4*b*Sqrt[1 + c^2*x
^2]*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c^2*d*Sqrt[d + c^2*d*x^2]
) - ((2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^2*d*Sq
rt[d + c^2*d*x^2]) + ((2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I*E^ArcSinh[c*
x]])/(c^2*d*Sqrt[d + c^2*d*x^2])
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int\frac{a+b\sinh^{-1}(cx)}{1+c^2x^2}\,dx}{cd\sqrt{d+c^2dx^2}}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int(a+bx)\text{sech}(x)\,dx,x,\sinh^{-1}(c\right.}{c^2d\sqrt{d+c^2dx^2}}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{4b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{c^2d\sqrt{d+c^2dx^2}} - \frac{(2i}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{4b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{c^2d\sqrt{d+c^2dx^2}} - \frac{(2i}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{4b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{c^2d\sqrt{d+c^2dx^2}} - \frac{2ib}{}$$

Mathematica [A] time = 0.46, size = 217, normalized size = 1.15

$$-\frac{a^2-4ab\sqrt{c^2x^2+1}\tan^{-1}\left(\tanh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)+2ab\sinh^{-1}(cx)+2ib^2\sqrt{c^2x^2+1}\,\text{Li}_2\left(-ie^{-\sinh^{-1}(cx)}\right)-2ib^2}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] -((a^2 + 2*a*b*ArcSinh[c*x] + b^2*ArcSinh[c*x]^2 - 4*a*b*Sqrt[1 + c^2*x^2]*
ArcTan[Tanh[ArcSinh[c*x]/2]] + (2*I)*b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log
[1 - I/E^ArcSinh[c*x]] - (2*I)*b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + I
/E^ArcSinh[c*x]] + (2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)/E^ArcSinh[c*
x]] - (2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I/E^ArcSinh[c*x]])/(c^2*d*Sqrt
[d + c^2*d*x^2]))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(b^2x\,\text{arsinh}\left(cx\right)^2+2\,abx\,\text{arsinh}\left(cx\right)+a^2x\right)}{c^4d^2x^4+2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*x*arcsinh(c*x)^2 + 2*a*b*x*arcsinh(c*x) +
 a^2*x)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \operatorname{arsinh}(cx) + a\right)^2 x}{\left(c^2 dx^2 + d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x/(c^2*d*x^2 + d)^(3/2), x)
```

maple [B] time = 0.22, size = 446, normalized size = 2.37

$$-\frac{a^2}{c^2 d\sqrt{c^2 d\, x^2 + d}} - \frac{b^2\sqrt{d\left(c^2x^2+1\right)}\, \operatorname{arcsinh}(cx)^2}{c^2 d^2\left(c^2x^2+1\right)} - \frac{2ib^2\sqrt{d\left(c^2x^2+1\right)}\, \operatorname{arcsinh}(cx)\ln\left(1+i\left(cx+\sqrt{c^2x^2+1}\right)\right)}{\sqrt{c^2x^2+1}\, c^2 d^2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] -a^2/c^2/d/(c^2*d*x^2+d)^(1/2)-b^2*(d*(c^2*x^2+1))^(1/2)/c^2/d^2/(c^2*x^2+1
)*arcsinh(c*x)^2-2*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^2*ar
csinh(c*x)*ln(1+I*(c*x+(c^2*x^2+1)^(1/2)))+2*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c
^2*x^2+1)^(1/2)/c^2/d^2*arcsinh(c*x)*ln(1-I*(c*x+(c^2*x^2+1)^(1/2)))-2*I*b^
2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^2*dilog(1+I*(c*x+(c^2*x^2+1
)^(1/2)))+2*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^2*dilog(1-I
*(c*x+(c^2*x^2+1)^(1/2)))-2*a*b*(d*(c^2*x^2+1))^(1/2)/c^2/d^2/(c^2*x^2+1)*a
rcsinh(c*x)+2*I*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^2*ln(c*x+
(c^2*x^2+1)^(1/2)+I)-2*I*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^
2*ln(c*x+(c^2*x^2+1)^(1/2)-I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^2}{\sqrt{c^2dx^2+d}\, c^2 d} + \int \frac{b^2 x \log\left(cx+\sqrt{c^2x^2+1}\right)^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}} + \frac{2\, abx \log\left(cx+\sqrt{c^2x^2+1}\right)}{\left(c^2dx^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] -a^2/(sqrt(c^2*d*x^2 + d)*c^2*d) + integrate(b^2*x*log(c*x + sqrt(c^2*x^2 +
 1))^2/(c^2*d*x^2 + d)^(3/2) + 2*a*b*x*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*
x^2 + d)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\left(a + b \operatorname{asinh}(c\,x)\right)^2}{\left(d\,c^2\,x^2 + d\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(a + b \operatorname{asinh}(cx)\right)^2}{\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(3/2), x)
```

3.305 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=179

$$\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{cd\sqrt{c^2dx^2+d}}-\frac{2b\sqrt{c^2x^2+1}\log\left(e^{2\sinh^{-1}(cx)}+1\right)\left(a+b\sinh^{-1}(cx)\right)}{cd\sqrt{c^2dx^2+d}}-\frac{b^2\sqrt{}}{}$$

```
[Out] x*(a+b*arcsinh(c*x))^2/d/(c^2*d*x^2+d)^(1/2)+(a+b*arcsinh(c*x))^2*(c^2*x^2+
1)^(1/2)/c/d/(c^2*d*x^2+d)^(1/2)-2*b*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+
1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/c/d/(c^2*d*x^2+d)^(1/2)-b^2*polylog(2,-(c*x+
(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/c/d/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {5687, 5714, 3718, 2190, 2279, 2391}

$$-\frac{b^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{cd\sqrt{c^2dx^2+d}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{cd\sqrt{c^2dx^2+d}}-\frac{2b\sqrt{c^2x^2+1}\,\text{lo}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcSinh[c*x])^2)/(d*Sqrt[d + c^2*d*x^2]) + (Sqrt[1 + c^2*x^2]*(a
+ b*ArcSinh[c*x])^2)/(c*d*Sqrt[d + c^2*d*x^2]) - (2*b*Sqrt[1 + c^2*x^2]*(a
+ b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(c*d*Sqrt[d + c^2*d*x^2]) -
(b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*d*Sqrt[d + c^2*d
*x^2])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx = \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} - \frac{\left(2bc\sqrt{1+c^2x^2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{1+c^2x^2}\,dx}{d\sqrt{d+c^2dx^2}}$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} - \frac{\left(2b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)\tanh(x)\,dx, x, \sinh^{-1}(cx\right.}{cd\sqrt{d+c^2dx^2}}$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{cd\sqrt{d+c^2dx^2}} - \frac{\left(4b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\right.}{cd\sqrt{d}}$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{cd\sqrt{d+c^2dx^2}} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh\right.}{cd\sqrt{}}$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{cd\sqrt{d+c^2dx^2}} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh\right.}{cd\sqrt{}}$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{cd\sqrt{d+c^2dx^2}} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh\right.}{cd\sqrt{}}$$

Mathematica [A] time = 0.44, size = 152, normalized size = 0.85

$$\frac{a\left(acx-b\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)\right)+2b\sinh^{-1}(cx)\left(acx-b\sqrt{c^2x^2+1}\log\left(e^{-2\sinh^{-1}(cx)}+1\right)\right)+b^2\sqrt{c^2x^2+1}}{cd\sqrt{c^2dx^2+d}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (-(b^2*(-(c*x) + Sqrt[1 + c^2*x^2])*ArcSinh[c*x]^2) + 2*b*ArcSinh[c*x]*(a*c
*x - b*Sqrt[1 + c^2*x^2]*Log[1 + E^(-2*ArcSinh[c*x])]) + a*(a*c*x - b*Sqrt[
1 + c^2*x^2]*Log[1 + c^2*x^2]) + b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-2*Ar
cSinh[c*x])])/(c*d*Sqrt[d + c^2*d*x^2])
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(b^2\,\text{arsinh}\,(cx)^2+2\,ab\,\text{arsinh}\,(cx)+a^2\right)}{c^4d^2x^4+2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(c^2*d*x^2 + d)^(3/2), x)
```

maple [A] time = 0.18, size = 343, normalized size = 1.92

$$\frac{a^2x}{d\sqrt{c^2d\,x^2+d}}+\frac{b^2\sqrt{d\left(c^2x^2+1\right)}\,\mathrm{arcsinh}\,(cx)^2\,x}{d^2\left(c^2x^2+1\right)}+\frac{b^2\sqrt{d\left(c^2x^2+1\right)}\,\mathrm{arcsinh}\,(cx)^2}{c\,d^2\sqrt{c^2x^2+1}}-\frac{2b^2\sqrt{d\left(c^2x^2+1\right)}\,\mathrm{arcsinh}\,(c\cdots}{\sqrt{c^2x^2\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] a^2*x/d/(c^2*d*x^2+d)^(1/2)+b^2*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2/d^2/(c
^2*x^2+1)*x+b^2*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2/c/d^2/(c^2*x^2+1)^(1/2
)-2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d^2*arcsinh(c*x)*ln(1+(c*
x+(c^2*x^2+1)^(1/2))^2)-b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d^2*p
olylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)+2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1
)^(1/2)/c/d^2*arcsinh(c*x)+2*a*b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/d^2/(c^
2*x^2+1)*x-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d^2*ln(1+(c*x+(c
^2*x^2+1)^(1/2))^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^2\int \frac{\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}}\,dx+\frac{2\,abx\,\mathrm{arsinh}\,(cx)}{\sqrt{c^2dx^2+d}\,d}+\frac{a^2x}{\sqrt{c^2dx^2+d}\,d}-\frac{ab\log\left(x^2+\frac{1}{c^2}\right)}{cd^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d*x^2 + d)^(3/2), x) + 2*
a*b*x*arcsinh(c*x)/(sqrt(c^2*d*x^2 + d)*d) + a^2*x/(sqrt(c^2*d*x^2 + d)*d)
- a*b*log(x^2 + 1/c^2)/(c*d^(3/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))^2}{\left(d\,c^2\,x^2+d\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asinh}(cx))^2}{\left(d\left(c^2x^2 + 1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(3/2),x)
[Out] Integral((a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(3/2), x)
```

3.306 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=412

$$-\frac{2b\sqrt{c^2x^2+1}\,\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{c^2dx^2+d}}+\frac{2b\sqrt{c^2x^2+1}\,\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{c^2dx^2+d}}+\frac{\left(a+b\sinh^{-}\right.}{d\sqrt{c^2dx^2}}$$

```
[Out] (a+b*arcsinh(c*x))^2/d/(c^2*d*x^2+d)^(1/2)-4*b*(a+b*arcsinh(c*x))*arctan(c*
x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)-2*(a+b*arcsinh
(c*x))^2*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(
1/2)-2*b*(a+b*arcsinh(c*x))*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(
1/2)/d/(c^2*d*x^2+d)^(1/2)+2*I*b^2*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))*(c
^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)-2*I*b^2*polylog(2,I*(c*x+(c^2*x^2+1)^
(1/2)))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)+2*b*(a+b*arcsinh(c*x))*poly
log(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)+2*b^2*
polylog(3,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)-2
*b^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/
2)
```

Rubi [A] time = 0.59, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {5755, 5764, 5760, 4182, 2531, 2282, 6589, 5693, 4180, 2279, 2391}

$$-\frac{2b\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{c^2dx^2+d}}+\frac{2b\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx\right.}{d\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] (a + b*ArcSinh[c*x])^2/(d*Sqrt[d + c^2*d*x^2]) - (4*b*Sqrt[1 + c^2*x^2]*(a
+ b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) - (2*Sqrt
[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/(d*Sqrt[d + c
^2*d*x^2]) - (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcS
inh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) + ((2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2
, (-I)*E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) - ((2*I)*b^2*Sqrt[1 + c^2*x
^2]*PolyLog[2, I*E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) + (2*b*Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*
x^2]) + (2*b^2*Sqrt[1 + c^2*x^2]*PolyLog[3, -E^ArcSinh[c*x]])/(d*Sqrt[d + c
^2*d*x^2]) - (2*b^2*Sqrt[1 + c^2*x^2]*PolyLog[3, E^ArcSinh[c*x]])/(d*Sqrt[d
 + c^2*d*x^2])
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 5764

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[
((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&  !GtQ[d, 0] && (Integer
Q[m] || EqQ[n, 1])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)^{3/2}}\,dx = \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} + \frac{\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\sqrt{d+c^2dx^2}}\,dx}{d} - \frac{\left(2bc\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{1+c^2x^2}\,dx}{d\sqrt{d+c^2dx^2}}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\sqrt{1+c^2x^2}}\,dx}{d\sqrt{d+c^2dx^2}} - \frac{\left(2b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+\right.}{d\sqrt{d-}}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} - \frac{4b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x}}{}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} - \frac{4b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{d\sqrt{d+c^2dx^2}} - \frac{2\sqrt{1+c^2}}{}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} - \frac{4b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{d\sqrt{d+c^2dx^2}} - \frac{2\sqrt{1+c^2}}{}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} - \frac{4b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{d\sqrt{d+c^2dx^2}} - \frac{2\sqrt{1+c^2}}{}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} - \frac{4b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{d\sqrt{d+c^2dx^2}} - \frac{2\sqrt{1+c^2}}{}$$

Mathematica [A] time = 1.54, size = 568, normalized size = 1.38

$$\frac{a^2\sqrt{d}\sqrt{c^2dx^2+d}\log(cx) - a^2\sqrt{d}\sqrt{c^2dx^2+d}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+d\right) + a^2d + 2abd\left(\sqrt{c^2x^2+1}\,\text{Li}_2\left(-e^{-\sinh^{-1}(c}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] (a^2*d + a^2*Sqrt[d]*Sqrt[d + c^2*d*x^2]*Log[c*x] - a^2*Sqrt[d]*Sqrt[d + c^
2*d*x^2]*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + 2*a*b*d*(ArcSinh[c*x] - 2*S
qrt[1 + c^2*x^2]*ArcTan[Tanh[ArcSinh[c*x]/2]] + Sqrt[1 + c^2*x^2]*ArcSinh[c
*x]*Log[1 - E^(-ArcSinh[c*x])] - Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + E^(
-ArcSinh[c*x])] + Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-ArcSinh[c*x])] - Sqrt[1
 + c^2*x^2]*PolyLog[2, E^(-ArcSinh[c*x])]) + b^2*d*(ArcSinh[c*x]^2 + Sqrt[1
 + c^2*x^2]*ArcSinh[c*x]^2*Log[1 - E^(-ArcSinh[c*x])] + (2*I)*Sqrt[1 + c^2*
x^2]*ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]] - (2*I)*Sqrt[1 + c^2*x^2]*ArcSi
nh[c*x]*Log[1 + I/E^ArcSinh[c*x]] - Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2*Log[1
+ E^(-ArcSinh[c*x])] + 2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*PolyLog[2, -E^(-Arc
Sinh[c*x])] + (2*I)*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)/E^ArcSinh[c*x]] - (2*
I)*Sqrt[1 + c^2*x^2]*PolyLog[2, I/E^ArcSinh[c*x]] - 2*Sqrt[1 + c^2*x^2]*Arc
```

```
Sinh[c*x]*PolyLog[2, E^(-ArcSinh[c*x])] + 2*Sqrt[1 + c^2*x^2]*PolyLog[3, -E
^(-ArcSinh[c*x])] - 2*Sqrt[1 + c^2*x^2]*PolyLog[3, E^(-ArcSinh[c*x])]))/(d^
2*Sqrt[d + c^2*d*x^2])
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(b^2\,\text{arsinh}\,(cx)^2+2\,ab\,\text{arsinh}\,(cx)+a^2\right)}{c^4d^2x^5+2\,c^2d^2x^3+d^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\,\text{arsinh}\,(cx)+a)^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^(3/2)*x), x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int\frac{(a+b\,\text{arcsinh}\,(cx))^2}{x\left(c^2d\,x^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\frac{\text{arsinh}\left(\frac{1}{c|x|}\right)}{d^{\frac{3}{2}}}-\frac{1}{\sqrt{c^2dx^2+d}\,d}\right)+\int\frac{b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}x}+\frac{2\,ab\log\left(cx+\sqrt{c^2x^2+1}\right)}{\left(c^2dx^2+d\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] -a^2*(arcsinh(1/(c*abs(x)))/d^(3/2) - 1/(sqrt(c^2*d*x^2 + d)*d)) + integrat
e(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/((c^2*d*x^2 + d)^(3/2)*x) + 2*a*b*log(
c*x + sqrt(c^2*x^2 + 1))/((c^2*d*x^2 + d)^(3/2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{(a+b\,\text{asinh}\,(c\,x))^2}{x\left(d\,c^2\,x^2+d\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)^(3/2)),x)
[Out] int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{asinh}(cx))^2}{x\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x/(c**2*d*x**2+d)**(3/2),x)
[Out] Integral((a + b*asinh(c*x))**2/(x*(d*(c**2*x**2 + 1))**(3/2)), x)
```

3.307 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=305

$$-\frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{c^2dx^2+d}}-\frac{2c\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{c^2dx^2+d}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\sqrt{c^2dx^2+d}}+\frac{4bc\sqrt{c^2x^2+1}\log\left(e^{2\sinh^{-1}}\right.}{d\sqrt{c^2d}}$$

```
[Out] -(a+b*arcsinh(c*x))^2/d/x/(c^2*d*x^2+d)^(1/2)-2*c^2*x*(a+b*arcsinh(c*x))^2/
d/(c^2*d*x^2+d)^(1/2)-2*c*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/d/(c^2*d*x
^2+d)^(1/2)-4*b*c*(a+b*arcsinh(c*x))*arctanh((c*x+(c^2*x^2+1)^(1/2))^2)*(c^
2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)+4*b*c*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^
2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)+b^2*c*polylog(2,
-(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)+b^2*c*p
olylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5747, 5687, 5714, 3718, 2190, 2279, 2391, 5720, 5461, 4182}

$$\frac{b^2c\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{d\sqrt{c^2dx^2+d}}+\frac{b^2c\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{d\sqrt{c^2dx^2+d}}-\frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{c^2dx^2+d}}-\frac{2c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] -((a + b*ArcSinh[c*x])^2/(d*x*Sqrt[d + c^2*d*x^2])) - (2*c^2*x*(a + b*ArcSi
nh[c*x])^2)/(d*Sqrt[d + c^2*d*x^2]) - (2*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh
[c*x])^2)/(d*Sqrt[d + c^2*d*x^2]) - (4*b*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh
[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/(d*Sqrt[d + c^2*d*x^2]) + (4*b*c*Sqrt[1
 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(d*Sqrt[d + c
^2*d*x^2]) + (b^2*c*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(2*ArcSinh[c*x])])/(d*S
qrt[d + c^2*d*x^2]) + (b^2*c*Sqrt[1 + c^2*x^2]*PolyLog[2, E^(2*ArcSinh[c*x]
)])/(d*Sqrt[d + c^2*d*x^2])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5720

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\sqrt{d+c^2dx^2}} - \left(2c^2\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx + \frac{\left(2bc\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{x(1}}{d\sqrt{d+c^2dx^2}}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\sqrt{d+c^2dx^2}} - \frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} + \frac{\left(2bc\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+}{d\sqrt{}}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\sqrt{d+c^2dx^2}} - \frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} + \frac{\left(4bc\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+}{d\sqrt{d+}}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\sqrt{d+c^2dx^2}} - \frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} - \frac{2c\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx\right.}{d\sqrt{d+c^2dx^2}}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\sqrt{d+c^2dx^2}} - \frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} - \frac{2c\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx\right.}{d\sqrt{d+c^2dx^2}}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\sqrt{d+c^2dx^2}} - \frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} - \frac{2c\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx\right.}{d\sqrt{d+c^2dx^2}}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\sqrt{d+c^2dx^2}} - \frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} - \frac{2c\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx\right.}{d\sqrt{d+c^2dx^2}}$$

Mathematica [A] time = 0.93, size = 296, normalized size = 0.97

$$-\frac{2a^2c^2x^2+a^2-2abcx\sqrt{c^2x^2+1}\,\log(cx)-abcx\sqrt{c^2x^2+1}\,\log\left(c^2x^2+1\right)+4abc^2x^2\sinh^{-1}(cx)+2ab\sinh^{-1}(}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] -((a^2 + 2*a^2*c^2*x^2 + 2*a*b*ArcSinh[c*x] + 4*a*b*c^2*x^2*ArcSinh[c*x] +
b^2*ArcSinh[c*x]^2 + 2*b^2*c^2*x^2*ArcSinh[c*x]^2 - 2*b^2*c*x*Sqrt[1 + c^2*
x^2]*ArcSinh[c*x]^2 - 2*b^2*c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 - E^(-
2*ArcSinh[c*x])] - 2*b^2*c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + E^(-2*A
rcSinh[c*x])] - 2*a*b*c*x*Sqrt[1 + c^2*x^2]*Log[c*x] - a*b*c*x*Sqrt[1 + c^2
*x^2]*Log[1 + c^2*x^2] + b^2*c*x*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-2*ArcSin
h[c*x])] + b^2*c*x*Sqrt[1 + c^2*x^2]*PolyLog[2, E^(-2*ArcSinh[c*x])])/(d*x*
Sqrt[d + c^2*d*x^2]))
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(b^2\,\text{arsinh}\,(cx)^2+2\,ab\,\text{arsinh}\,(cx)+a^2\right)}{c^4d^2x^6+2\,c^2d^2x^4+d^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arsinh}(cx) + a)^2}{(c^2dx^2 + d)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^(3/2)*x^2), x)
```

maple [B] time = 0.31, size = 660, normalized size = 2.16

$$-\frac{a^2}{dx\sqrt{c^2d\,x^2+d}}-\frac{2a^2c^2x}{d\sqrt{c^2d\,x^2+d}}-\frac{2b^2\sqrt{d\left(c^2x^2+1\right)}\ \operatorname{arcsinh}(cx)^2\,x\,c^2}{\left(c^2x^2+1\right)d^2}-\frac{2b^2\sqrt{d\left(c^2x^2+1\right)}\ \operatorname{arcsinh}(cx)^2\,c}{\sqrt{c^2x^2+1}\ d^2}-\frac{b^2\sqrt{d}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] -a^2/d/x/(c^2*d*x^2+d)^(1/2)-2*a^2*c^2/d*x/(c^2*d*x^2+d)^(1/2)-2*b^2*(d*(c^
2*x^2+1))^(1/2)*arcsinh(c*x)^2/(c^2*x^2+1)/d^2*x*c^2-2*b^2*(d*(c^2*x^2+1))^
(1/2)*arcsinh(c*x)^2/(c^2*x^2+1)^(1/2)/d^2*c-b^2*(d*(c^2*x^2+1))^(1/2)*arcs
inh(c*x)^2/(c^2*x^2+1)/d^2/x+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/
d^2*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c+2*b^2*(d*(c^2*x^2+1))^(1/2)/
(c^2*x^2+1)^(1/2)/d^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c+2*b^2*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^
2)*c+b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*polylog(2,-(c*x+(c^2*x
^2+1)^(1/2))^2)*c+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*arcsinh
(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)
^(1/2)/d^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c-4*a*b*(d*(c^2*x^2+1))^(1/2)/(
c^2*x^2+1)^(1/2)/d^2*arcsinh(c*x)*c-4*a*b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x
)/(c^2*x^2+1)/d^2*x*c^2-2*a*b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/(c^2*x^2+1
)/d^2/x+2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*ln((c*x+(c^2*x^2+
1)^(1/2))^4-1)*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$abc\left(\frac{\log\left(c^2x^2+1\right)}{d^{\frac{3}{2}}}+\frac{2\,\log(x)}{d^{\frac{3}{2}}}\right)-2\left(\frac{2\,c^2x}{\sqrt{c^2dx^2+d}\,d}+\frac{1}{\sqrt{c^2dx^2+d}\,dx}\right)ab\operatorname{arsinh}(cx)-\left(\frac{2\,c^2x}{\sqrt{c^2dx^2+d}\,d}+\frac{1}{\sqrt{c^2dx^2+d}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
```

```
[Out] a*b*c*(log(c^2*x^2 + 1)/d^(3/2) + 2*log(x)/d^(3/2)) - 2*(2*c^2*x/(sqrt(c^2*
d*x^2 + d)*d) + 1/(sqrt(c^2*d*x^2 + d)*d*x))*a*b*arcsinh(c*x) - (2*c^2*x/(s
qrt(c^2*d*x^2 + d)*d) + 1/(sqrt(c^2*d*x^2 + d)*d*x))*a^2 + b^2*integrate(lo
g(c*x + sqrt(c^2*x^2 + 1))^2/((c^2*d*x^2 + d)^(3/2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{asinh}(c\,x))^2}{x^2\,\left(d\,c^2\,x^2 + d\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)^(3/2)),x)
[Out] int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{asinh}(cx))^2}{x^2 \left(d\left(c^2x^2 + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**2/(c**2*d*x**2+d)**(3/2),x)
[Out] Integral((a + b*asinh(c*x))**2/(x**2*(d*(c**2*x**2 + 1))**(3/2)), x)
```

3.308 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^3\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=573

$$\frac{3bc^2\sqrt{c^2x^2+1}\,\mathrm{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{c^2dx^2+d}}-\frac{3bc^2\sqrt{c^2x^2+1}\,\mathrm{Li}_2\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{c^2dx^2+d}}-\frac{3c^2\left(a+b\right.}{2d\sqrt{c^2}}$$

```
[Out] -3/2*c^2*(a+b*arcsinh(c*x))^2/d/(c^2*d*x^2+d)^(1/2)-1/2*(a+b*arcsinh(c*x))^
2/d/x^2/(c^2*d*x^2+d)^(1/2)-b*c*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/d/x/(c
^2*d*x^2+d)^(1/2)+4*b*c^2*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))*
(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)+3*c^2*(a+b*arcsinh(c*x))^2*arctanh(
c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)-b^2*c^2*arct
anh((c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)+3*b*c^2*(a+b
*arcsinh(c*x))*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d
*x^2+d)^(1/2)-2*I*b^2*c^2*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)
^(1/2)/d/(c^2*d*x^2+d)^(1/2)+2*I*b^2*c^2*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)
))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)-3*b*c^2*(a+b*arcsinh(c*x))*polyl
og(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)-3*b^2*c
^2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2
)+3*b^2*c^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2
+d)^(1/2)
```

Rubi [A] time = 0.93, antiderivative size = 573, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 15, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.536, Rules used = {5747, 5755, 5764, 5760, 4182, 2531, 2282, 6589, 5693, 4180, 2279, 2391, 266, 63, 208}

$$\frac{3bc^2\sqrt{c^2x^2+1}\,\mathrm{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{c^2dx^2+d}}-\frac{3bc^2\sqrt{c^2x^2+1}\,\mathrm{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(\right.}{d\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] -((b*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(d*x*Sqrt[d + c^2*d*x^2])) -
 (3*c^2*(a + b*ArcSinh[c*x])^2)/(2*d*Sqrt[d + c^2*d*x^2]) - (a + b*ArcSinh[
c*x])^2/(2*d*x^2*Sqrt[d + c^2*d*x^2]) + (4*b*c^2*Sqrt[1 + c^2*x^2]*(a + b*A
rcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) + (3*c^2*Sqrt[
1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/(d*Sqrt[d + c^
2*d*x^2]) - (b^2*c^2*Sqrt[1 + c^2*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]])/(d*Sqrt[
d + c^2*d*x^2]) + (3*b*c^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2
, -E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) - ((2*I)*b^2*c^2*Sqrt[1 + c^2*x
^2]*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) + ((2*I)*b^2*c
^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I*E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2])
- (3*b*c^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, E^ArcSinh[c*x]
])/(d*Sqrt[d + c^2*d*x^2]) - (3*b^2*c^2*Sqrt[1 + c^2*x^2]*PolyLog[3, -E^Arc
Sinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) + (3*b^2*c^2*Sqrt[1 + c^2*x^2]*PolyLog[
3, E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
```

```
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 5764

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[
((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&  !GtQ[d, 0] && (Integer
Q[m] || EqQ[n, 1])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^3\left(d+c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2\sqrt{d+c^2dx^2}} - \frac{1}{2}\left(3c^2\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)^{3/2}}\,dx + \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{a+b}{x^2}}{d\sqrt{d+c^2dx}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{dx\sqrt{d+c^2dx^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)^2}{2d\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{dx\sqrt{d+c^2dx^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)^2}{2d\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{dx\sqrt{d+c^2dx^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)^2}{2d\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{dx\sqrt{d+c^2dx^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)^2}{2d\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{dx\sqrt{d+c^2dx^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)^2}{2d\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{dx\sqrt{d+c^2dx^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)^2}{2d\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{dx\sqrt{d+c^2dx^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)^2}{2d\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2\sqrt{d+c^2dx^2}}$$

Mathematica [A] time = 7.53, size = 884, normalized size = 1.54

$$-\frac{3a^2\log(x)c^2}{2d^{3/2}} + \frac{3a^2\log\left(d+\sqrt{d\left(c^2x^2+1\right)}\sqrt{d}\right)c^2}{2d^{3/2}} + \frac{ab\left(-\sqrt{c^2x^2+1}\,\sinh^{-1}(cx)\operatorname{csch}^2\left(\frac{1}{2}\sinh^{-1}(cx)\right) - \sqrt{c^2x^2}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] Sqrt[d*(1 + c^2*x^2)]*(-1/2*a^2/(d^2*x^2) - (a^2*c^2)/(d^2*(1 + c^2*x^2)))
- (3*a^2*c^2*Log[x])/(2*d^(3/2)) + (3*a^2*c^2*Log[d + Sqrt[d]*Sqrt[d*(1 + c
^2*x^2)]])/(2*d^(3/2)) + (a*b*c^2*(-8*ArcSinh[c*x] + 16*Sqrt[1 + c^2*x^2]*A
rcTan[Tanh[ArcSinh[c*x]/2]] - 2*Sqrt[1 + c^2*x^2]*Coth[ArcSinh[c*x]/2] - Sq
rt[1 + c^2*x^2]*ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - 12*Sqrt[1 + c^2*x^2]*
ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] + 12*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]
*Log[1 + E^(-ArcSinh[c*x])] - 12*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-ArcSinh[
c*x])] + 12*Sqrt[1 + c^2*x^2]*PolyLog[2, E^(-ArcSinh[c*x])] - Sqrt[1 + c^2*
x^2]*ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Sqrt[1 + c^2*x^2]*Tanh[ArcSinh
[c*x]/2]))/(4*d*Sqrt[d*(1 + c^2*x^2)]) + (b^2*c^2*(-8*ArcSinh[c*x]^2 - 4*Sq
rt[1 + c^2*x^2]*ArcSinh[c*x]*Coth[ArcSinh[c*x]/2] - Sqrt[1 + c^2*x^2]*ArcSi
nh[c*x]^2*Csch[ArcSinh[c*x]/2]^2 - 12*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2*Log[
1 - E^(-ArcSinh[c*x])] - (16*I)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 - I/E^
ArcSinh[c*x]] + (16*I)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + I/E^ArcSinh[c
*x]] + 12*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2*Log[1 + E^(-ArcSinh[c*x])] + 8*S
qrt[1 + c^2*x^2]*Log[Tanh[ArcSinh[c*x]/2]] - 24*Sqrt[1 + c^2*x^2]*ArcSinh[c
*x]*PolyLog[2, -E^(-ArcSinh[c*x])] - (16*I)*Sqrt[1 + c^2*x^2]*PolyLog[2, (-
I)/E^ArcSinh[c*x]] + (16*I)*Sqrt[1 + c^2*x^2]*PolyLog[2, I/E^ArcSinh[c*x]]
```

```
+ 24*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*PolyLog[2, E^(-ArcSinh[c*x])] - 24*Sqrt
[1 + c^2*x^2]*PolyLog[3, -E^(-ArcSinh[c*x])] + 24*Sqrt[1 + c^2*x^2]*PolyLog
[3, E^(-ArcSinh[c*x])] - Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2*Sech[ArcSinh[c*x]
/2]^2 + 4*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Tanh[ArcSinh[c*x]/2]))/(8*d*Sqrt[d
*(1 + c^2*x^2)])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(b^2\operatorname{arsinh}(cx)^2+2\,ab\operatorname{arsinh}(cx)+a^2\right)}{c^4d^2x^7+2\,c^2d^2x^5+d^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^4*d^2*x^7 + 2*c^2*d^2*x^5 + d^2*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\operatorname{arsinh}(cx)+a)^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^(3/2)*x^3), x)
```

maple [F] time = 0.54, size = 0, normalized size = 0.00

$$\int\frac{(a+b\operatorname{arcsinh}(cx))^2}{x^3\left(c^2d\,x^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(\frac{3\,c^2\operatorname{arsinh}\left(\frac{1}{c|x|}\right)}{d^{\frac{3}{2}}}-\frac{3\,c^2}{\sqrt{c^2dx^2+d}\,d}-\frac{1}{\sqrt{c^2dx^2+d}\,dx^2}\right)a^2+\int\frac{b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}x^3}+\frac{2\,ab\log\left(cx+\sqrt{c^2x^2+\right.}{\left(c^2dx^2+d\right)^{\frac{3}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
```

```
[Out] 1/2*(3*c^2*arcsinh(1/(c*abs(x)))/d^(3/2) - 3*c^2/(sqrt(c^2*d*x^2 + d)*d) -
1/(sqrt(c^2*d*x^2 + d)*d*x^2))*a^2 + integrate(b^2*log(c*x + sqrt(c^2*x^2 +
 1))^2/((c^2*d*x^2 + d)^(3/2)*x^3) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/((c
^2*d*x^2 + d)^(3/2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{(a+b\operatorname{asinh}(c\,x))^2}{x^3\left(d\,c^2\,x^2+d\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)^(3/2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asinh}(cx))^2}{x^3 \left(d \left(c^2 x^2 + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**3/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(x**3*(d*(c**2*x**2 + 1))**(3/2)), x)
```

3.309 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^4\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=452

$$\frac{4c^2\left(a+b\sinh^{-1}(cx)\right)^2}{3dx\sqrt{c^2dx^2+d}}-\frac{bc\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2\sqrt{c^2dx^2+d}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\sqrt{c^2dx^2+d}}+\frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)^2}{3d\sqrt{c^2dx^2+d}}+\frac{8c^3\sqrt{c^2}}{}$$

```
[Out] -1/3*b^2*c^2*(c^2*x^2+1)/d/x/(c^2*d*x^2+d)^(1/2)-1/3*(a+b*arcsinh(c*x))^2/d
/x^3/(c^2*d*x^2+d)^(1/2)+4/3*c^2*(a+b*arcsinh(c*x))^2/d/x/(c^2*d*x^2+d)^(1/
2)+8/3*c^4*x*(a+b*arcsinh(c*x))^2/d/(c^2*d*x^2+d)^(1/2)-1/3*b*c*(a+b*arcsin
h(c*x))*(c^2*x^2+1)^(1/2)/d/x^2/(c^2*d*x^2+d)^(1/2)+8/3*c^3*(a+b*arcsinh(c*
x))^2*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)+20/3*b*c^3*(a+b*arcsinh(c*x))
*arctanh((c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)
-16/3*b*c^3*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^
(1/2)/d/(c^2*d*x^2+d)^(1/2)-b^2*c^3*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*(
c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)-5/3*b^2*c^3*polylog(2,(c*x+(c^2*x^2+
1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/d/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.84, antiderivative size = 452, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {5747, 5687, 5714, 3718, 2190, 2279, 2391, 5720, 5461, 4182, 264}

$$-\frac{b^2c^3\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{d\sqrt{c^2dx^2+d}}-\frac{5b^2c^3\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{3d\sqrt{c^2dx^2+d}}+\frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)^2}{3d\sqrt{c^2dx^2+d}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] -(b^2*c^2*(1 + c^2*x^2))/(3*d*x*Sqrt[d + c^2*d*x^2]) - (b*c*Sqrt[1 + c^2*x^
2]*(a + b*ArcSinh[c*x]))/(3*d*x^2*Sqrt[d + c^2*d*x^2]) - (a + b*ArcSinh[c*x
])^2/(3*d*x^3*Sqrt[d + c^2*d*x^2]) + (4*c^2*(a + b*ArcSinh[c*x])^2)/(3*d*x*
Sqrt[d + c^2*d*x^2]) + (8*c^4*x*(a + b*ArcSinh[c*x])^2)/(3*d*Sqrt[d + c^2*d
*x^2]) + (8*c^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(3*d*Sqrt[d + c^2
*d*x^2]) + (20*b*c^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^(2*Ar
cSinh[c*x])])/(3*d*Sqrt[d + c^2*d*x^2]) - (16*b*c^3*Sqrt[1 + c^2*x^2]*(a +
b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(3*d*Sqrt[d + c^2*d*x^2]) - (b
^2*c^3*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(2*ArcSinh[c*x])])/(d*Sqrt[d + c^2*d
*x^2]) - (5*b^2*c^3*Sqrt[1 + c^2*x^2]*PolyLog[2, E^(2*ArcSinh[c*x])])/(3*d*
Sqrt[d + c^2*d*x^2])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5720

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
```

```
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^4\left(d+c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\sqrt{d+c^2dx^2}} - \frac{1}{3}\left(4c^2\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)^{3/2}}\,dx + \frac{\left(2bc\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{x^3(1+\cdots}}{3d\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\sqrt{d+c^2dx^2}} + \frac{4c^2\left(a+b\sinh^{-1}(cx)\right)^2}{3dx\sqrt{d+c^2dx^2}} + \cdots$$

$$= -\frac{b^2c^2\left(1+c^2x^2\right)}{3dx\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\sqrt{d+c^2dx^2}} + \frac{4c^2\left(a\cdots\right.}{3d\cdots}$$

$$= -\frac{b^2c^2\left(1+c^2x^2\right)}{3dx\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\sqrt{d+c^2dx^2}} + \frac{4c^2\left(a\cdots\right.}{3d\cdots}$$

$$= -\frac{b^2c^2\left(1+c^2x^2\right)}{3dx\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\sqrt{d+c^2dx^2}} + \frac{4c^2\left(a\cdots\right.}{3d\cdots}$$

$$= -\frac{b^2c^2\left(1+c^2x^2\right)}{3dx\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\sqrt{d+c^2dx^2}} + \frac{4c^2\left(a\cdots\right.}{3d\cdots}$$

$$= -\frac{b^2c^2\left(1+c^2x^2\right)}{3dx\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\sqrt{d+c^2dx^2}} + \frac{4c^2\left(a\cdots\right.}{3d\cdots}$$

$$= -\frac{b^2c^2\left(1+c^2x^2\right)}{3dx\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\sqrt{d+c^2dx^2}} + \frac{4c^2\left(a\cdots\right.}{3d\cdots}$$

Mathematica [A] time = 0.89, size = 438, normalized size = 0.97

$$\frac{8a^2c^4x^4+4a^2c^2x^2-a^2+16abc^4x^4\sinh^{-1}(cx)-abcx\sqrt{c^2x^2+1}+8abc^2x^2\sinh^{-1}(cx)-10abc^3x^3\sqrt{c^2x^2+1}\log(\cdots}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] (-a^2 + 4*a^2*c^2*x^2 - b^2*c^2*x^2 + 8*a^2*c^4*x^4 - b^2*c^4*x^4 - a*b*c*x
*Sqrt[1 + c^2*x^2] - 2*a*b*ArcSinh[c*x] + 8*a*b*c^2*x^2*ArcSinh[c*x] + 16*a
*b*c^4*x^4*ArcSinh[c*x] - b^2*c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - b^2*ArcS
inh[c*x]^2 + 4*b^2*c^2*x^2*ArcSinh[c*x]^2 + 8*b^2*c^4*x^4*ArcSinh[c*x]^2 -
8*b^2*c^3*x^3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2 - 10*b^2*c^3*x^3*Sqrt[1 + c^
2*x^2]*ArcSinh[c*x]*Log[1 - E^(-2*ArcSinh[c*x])] - 6*b^2*c^3*x^3*Sqrt[1 + c
^2*x^2]*ArcSinh[c*x]*Log[1 + E^(-2*ArcSinh[c*x])] - 10*a*b*c^3*x^3*Sqrt[1 +
 c^2*x^2]*Log[c*x] - 3*a*b*c^3*x^3*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2] + 3*b
^2*c^3*x^3*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-2*ArcSinh[c*x])] + 5*b^2*c^3*x
^3*Sqrt[1 + c^2*x^2]*PolyLog[2, E^(-2*ArcSinh[c*x])])/(3*d*x^3*Sqrt[d + c^2
*d*x^2])
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(b^2\operatorname{arsinh}(cx)^2+2\,ab\operatorname{arsinh}(cx)+a^2\right)}{c^4d^2x^8+2\,c^2d^2x^6+d^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^4*d^2*x^8 + 2*c^2*d^2*x^6 + d^2*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2}{(c^2dx^2 + d)^{\frac{3}{2}} x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^(3/2)*x^4), x)
```

maple [B] time = 0.50, size = 2609, normalized size = 5.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] 1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2/x*c^2+4/3*a^2*c^2
/d/x/(c^2*d*x^2+d)^(1/2)-1/3*a^2/d/x^3/(c^2*d*x^2+d)^(1/2)-128/3*a*b*(d*(c^
2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^2*arcsinh(c*x)*(c^2*x^2+1)^(1
/2)*c^5+8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^2*c^5*(
c^2*x^2+1)^(1/2)+8*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x*
arcsinh(c*x)^2*c^4-8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^
2*x*arcsinh(c*x)*c^4+128/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1
)/d^2*x^3*arcsinh(c*x)*c^6+8/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x
^2-1)/d^2*x*(c^2*x^2+1)*c^4+16*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x
^2-1)/d^2*x*arcsinh(c*x)*c^4+16/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^
2*x^2-1)/d^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^3-8*a*b*(d*(c^2*x^2+1))^(1/2)
/(8*c^4*x^4+7*c^2*x^2-1)/d^2/x*arcsinh(c*x)*c^2+1/3*a*b*(d*(c^2*x^2+1))^(1/
2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2/x^2*c*(c^2*x^2+1)^(1/2)-64/3*a*b*(d*(c^2*x^2
+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^5*(c^2*x^2+1)*c^8-32/3*a*b*(d*(c^2
*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^3*(c^2*x^2+1)*c^6-64/3*b^2*(d*
(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^2*arcsinh(c*x)^2*(c^2*x^2+
1)^(1/2)*c^5+8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x*ar
csinh(c*x)*(c^2*x^2+1)*c^4+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x
^2-1)/d^2/x^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c-64/3*b^2*(d*(c^2*x^2+1))^(1/
2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^5*arcsinh(c*x)*(c^2*x^2+1)*c^8-32/3*b^2*(d
*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^3*arcsinh(c*x)*(c^2*x^2+1
)*c^6+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2/x^3*arcsinh
(c*x)^2-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*c^3*(c^2*
x^2+1)^(1/2)-10/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*polylog(2
,-c*x-(c^2*x^2+1)^(1/2))*c^3-10/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/
2)/d^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c^3+16/3*b^2*(d*(c^2*x^2+1))^(1/2)/
(c^2*x^2+1)^(1/2)/d^2*arcsinh(c*x)^2*c^3-b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2
+1)^(1/2)/d^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*c^3+32/3*b^2*(d*(c^2*x^
2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^7*c^10+40/3*b^2*(d*(c^2*x^2+1))^(
1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^5*c^8-7/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c
^4*x^4+7*c^2*x^2-1)/d^2*x*c^4+8/3*a^2*c^4/d*x/(c^2*d*x^2+d)^(1/2)-4*b^2*(d*
(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2/x*arcsinh(c*x)^2*c^2+8/3*b^2
*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*arcsinh(c*x)^2*(c^2*x^2+
1)^(1/2)*c^3-8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*arcs
```

```
inh(c*x)*(c^2*x^2+1)^(1/2)*c^3-10/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(
1/2)/d^2*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^3-10/3*b^2*(d*(c^2*x^2+
1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^
3+32/3*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*arcsinh(c*x)*c^3+64/
3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^7*c^10+32*a*b*(d*
(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^5*c^8+8*a*b*(d*(c^2*x^2+1)
)^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^3*c^6-8/3*a*b*(d*(c^2*x^2+1))^(1/2)/(
8*c^4*x^4+7*c^2*x^2-1)/d^2*x*c^4-8/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7
*c^2*x^2-1)/d^2*c^3*(c^2*x^2+1)^(1/2)+2/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*
x^4+7*c^2*x^2-1)/d^2/x^3*arcsinh(c*x)-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+
1)^(1/2)/d^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*c^3-10/3*a*b*(d*(c^2*x^2+1))^(
1/2)/(c^2*x^2+1)^(1/2)/d^2*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c^3-2*b^2*(d*(c^
2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1
/2))^2)*c^3+64/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^7*
arcsinh(c*x)*c^10-32/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^
2*x^5*(c^2*x^2+1)*c^8+32*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/
d^2*x^5*arcsinh(c*x)*c^8+64/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^
2-1)/d^2*x^3*arcsinh(c*x)^2*c^6-8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*
c^2*x^2-1)/d^2*x^3*(c^2*x^2+1)*c^6+8*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7
*c^2*x^2-1)/d^2*x^3*arcsinh(c*x)*c^6
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\left(\frac{8\,c^4x}{\sqrt{c^2dx^2+d}\,d}+\frac{4\,c^2}{\sqrt{c^2dx^2+d}\,dx}-\frac{1}{\sqrt{c^2dx^2+d}\,dx^3}\right)a^2+\int\frac{b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}x^4}+\frac{2\,ab\log\left(cx+\sqrt{c^2x^2+1}\right)}{\left(c^2dx^2+d\right)^{\frac{3}{2}}x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
```

```
[Out] 1/3*(8*c^4*x/(sqrt(c^2*d*x^2 + d)*d) + 4*c^2/(sqrt(c^2*d*x^2 + d)*d*x) - 1/
(sqrt(c^2*d*x^2 + d)*d*x^3))*a^2 + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1
))^2/((c^2*d*x^2 + d)^(3/2)*x^4) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/((c^2
*d*x^2 + d)^(3/2)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{x^4\,{\left(d\,c^2\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)^(3/2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{asinh}{\left(cx \right)}\right)^{2}}{x^{4} \left(d \left(c^{2} x^{2} + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**4/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(x**4*(d*(c**2*x**2 + 1))**(3/2)), x)
```

3.310 $\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=512

$$-\frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(c^2dx^2+d\right)^{3/2}}+\frac{8\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{3c^6d^3}-\frac{22b\sqrt{c^2x^2+1}\,\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx\right.}{3c^6d^2\sqrt{c^2dx^2+d}}$$

```
[Out] -1/3*x^4*(a+b*arcsinh(c*x))^2/c^2/d/(c^2*d*x^2+d)^(3/2)+1/3*b^2/c^6/d^2/(c^
2*d*x^2+d)^(1/2)+2*b^2*(c^2*x^2+1)/c^6/d^2/(c^2*d*x^2+d)^(1/2)-4/3*x^2*(a+b
*arcsinh(c*x))^2/c^4/d^2/(c^2*d*x^2+d)^(1/2)-1/3*b*x^3*(a+b*arcsinh(c*x))/c
^3/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-16/3*a*b*x*(c^2*x^2+1)^(1/2)/c
^5/d^2/(c^2*d*x^2+d)^(1/2)-16/3*b^2*x*arcsinh(c*x)*(c^2*x^2+1)^(1/2)/c^5/d^
2/(c^2*d*x^2+d)^(1/2)+11/3*b*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^5/d^2
/(c^2*d*x^2+d)^(1/2)-22/3*b*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2)
)*(c^2*x^2+1)^(1/2)/c^6/d^2/(c^2*d*x^2+d)^(1/2)+11/3*I*b^2*polylog(2,-I*(c*
x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/c^6/d^2/(c^2*d*x^2+d)^(1/2)-11/3*I*
b^2*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/c^6/d^2/(c^2*d*x
^2+d)^(1/2)+8/3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/c^6/d^3
```

Rubi [A] time = 0.88, antiderivative size = 512, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {5751, 5717, 5653, 261, 5767, 5693, 4180, 2279, 2391, 266, 43}

$$\frac{11ib^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{3c^6d^2\sqrt{c^2dx^2+d}}-\frac{11ib^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{3c^6d^2\sqrt{c^2dx^2+d}}-\frac{16abx\sqrt{c^2x^2+1}}{3c^5d^2\sqrt{c^2dx^2+d}}-\frac{bx^3}{3c^3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] b^2/(3*c^6*d^2*Sqrt[d + c^2*d*x^2]) - (16*a*b*x*Sqrt[1 + c^2*x^2])/(3*c^5*d
^2*Sqrt[d + c^2*d*x^2]) + (2*b^2*(1 + c^2*x^2))/(c^6*d^2*Sqrt[d + c^2*d*x^2
]) - (16*b^2*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(3*c^5*d^2*Sqrt[d + c^2*d*x^
2]) - (b*x^3*(a + b*ArcSinh[c*x]))/(3*c^3*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^
2*d*x^2]) + (11*b*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(3*c^5*d^2*Sqrt
[d + c^2*d*x^2]) - (x^4*(a + b*ArcSinh[c*x])^2)/(3*c^2*d*(d + c^2*d*x^2)^(3
/2)) - (4*x^2*(a + b*ArcSinh[c*x])^2)/(3*c^4*d^2*Sqrt[d + c^2*d*x^2]) + (8*
Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c^6*d^3) - (22*b*Sqrt[1 + c^
2*x^2]*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(3*c^6*d^2*Sqrt[d + c^2
*d*x^2]) + (((11*I)/3)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)*E^ArcSinh[c*x]
])/(c^6*d^2*Sqrt[d + c^2*d*x^2]) - (((11*I)/3)*b^2*Sqrt[1 + c^2*x^2]*PolyLo
g[2, I*E^ArcSinh[c*x]])/(c^6*d^2*Sqrt[d + c^2*d*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
```

```
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5767

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(e*(m + 2*p + 1)), x] + (-Dist[(f^2*(m - 1))/(c^2*(m +
 2*p + 1)), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(c*(m + 2*p + 1)*(1 + c
^2*x^2)^FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcS
inh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2
*d] && GtQ[n, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx = -\frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} + \frac{4\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx}{3c^2d} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{x^4\left(a+b\sin\ldots\right.}{\left(1+c^2\ldots\right.}}{3cd^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bx^3\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \frac{4x^2\left(a+b\sinh^{-1}(cx\ldots\right.}{3c^4d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bx^3\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{11bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3c^5d^2\sqrt{d+c^2dx^2}} - \frac{x^4\left(a+b\ldots\right.}{3c^2d\left(d\ldots\right.}$$

$$= -\frac{16abx\sqrt{1+c^2x^2}}{3c^5d^2\sqrt{d+c^2dx^2}} - \frac{11b^2\left(1+c^2x^2\right)}{3c^6d^2\sqrt{d+c^2dx^2}} - \frac{bx^3\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{11bx\ldots}{\ldots}$$

$$= \frac{b^2}{3c^6d^2\sqrt{d+c^2dx^2}} - \frac{16abx\sqrt{1+c^2x^2}}{3c^5d^2\sqrt{d+c^2dx^2}} - \frac{10b^2\left(1+c^2x^2\right)}{3c^6d^2\sqrt{d+c^2dx^2}} - \frac{16b^2x\sqrt{1+c^2x^2}}{3c^5d^2\sqrt{d+\ldots}}$$

$$= \frac{b^2}{3c^6d^2\sqrt{d+c^2dx^2}} - \frac{16abx\sqrt{1+c^2x^2}}{3c^5d^2\sqrt{d+c^2dx^2}} + \frac{2b^2\left(1+c^2x^2\right)}{c^6d^2\sqrt{d+c^2dx^2}} - \frac{16b^2x\sqrt{1+c^2x^2}\ \mathrm{s}\ldots}{3c^5d^2\sqrt{d+c^2\ldots}}$$

$$= \frac{b^2}{3c^6d^2\sqrt{d+c^2dx^2}} - \frac{16abx\sqrt{1+c^2x^2}}{3c^5d^2\sqrt{d+c^2dx^2}} + \frac{2b^2\left(1+c^2x^2\right)}{c^6d^2\sqrt{d+c^2dx^2}} - \frac{16b^2x\sqrt{1+c^2x^2}\ \mathrm{s}\ldots}{3c^5d^2\sqrt{d+c^2\ldots}}$$

Mathematica [A] time = 1.77, size = 333, normalized size = 0.65

$$\frac{\sqrt{c^2dx^2+d}\left(a^2\left(3c^4x^4+12c^2x^2+8\right)+ab\left(2\left(3c^4x^4+12c^2x^2+8\right)\sinh^{-1}(cx)-\sqrt{c^2x^2+1}\left(cx\left(6c^2x^2+5\right)+2\ldots\right.\right.\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^5*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(a^2*(8 + 12*c^2*x^2 + 3*c^4*x^4) + a*b*(2*(8 + 12*c^2
*x^2 + 3*c^4*x^4)*ArcSinh[c*x] - Sqrt[1 + c^2*x^2]*(c*x*(5 + 6*c^2*x^2) + 2
2*(1 + c^2*x^2)*ArcTan[Tanh[ArcSinh[c*x]/2]])) + b^2*(c*x*Sqrt[1 + c^2*x^2]
```

```
*ArcSinh[c*x] - 6*c*x*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x] - ArcSinh[c*x]^2 + 3
*(1 + c^2*x^2)^2*(2 + ArcSinh[c*x]^2) + (1 + c^2*x^2)*(1 + 6*ArcSinh[c*x]^2
) + (11*I)*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*(Log[1 - I/E^ArcSinh[c*x]] - Lo
g[1 + I/E^ArcSinh[c*x]]) + (11*I)*(1 + c^2*x^2)^(3/2)*(PolyLog[2, (-I)/E^Ar
cSinh[c*x]] - PolyLog[2, I/E^ArcSinh[c*x]]))))/(3*c^6*d^3*(1 + c^2*x^2)^2)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2x^5 \operatorname{arsinh}(cx)^2 + 2\,abx^5 \operatorname{arsinh}(cx) + a^2x^5\right)\sqrt{c^2dx^2 + d}}{c^6d^3x^6 + 3\,c^4d^3x^4 + 3\,c^2d^3x^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^5*arcsinh(c*x)^2 + 2*a*b*x^5*arcsinh(c*x) + a^2*x^5)*sqrt(c
^2*d*x^2 + d)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.50, size = 1040, normalized size = 2.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] 1/3*b^2*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2/c^6+2*b^2*(d*(c^2*x^2+1))^(
1/2)/c^4/d^3/(c^2*x^2+1)*x^2+b^2*(d*(c^2*x^2+1))^(1/2)/c^6/d^3/(c^2*x^2+1)*
arcsinh(c*x)^2+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2/c^4*x^2+5/3*
b^2*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2/c^6*arcsinh(c*x)^2+2*b^2*(d*(c^
2*x^2+1))^(1/2)/c^6/d^3/(c^2*x^2+1)+a^2*x^4/c^2/d/(c^2*d*x^2+d)^(3/2)+4*a^2
/c^4*x^2/d/(c^2*d*x^2+d)^(3/2)-2*b^2*(d*(c^2*x^2+1))^(1/2)/c^5/d^3/(c^2*x^2
+1)^(1/2)*arcsinh(c*x)*x+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^(3/2
)/c^5*arcsinh(c*x)*x+b^2*(d*(c^2*x^2+1))^(1/2)/c^4/d^3/(c^2*x^2+1)*arcsinh(
c*x)^2*x^2+2*b^2*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2/c^4*arcsinh(c*x)^2
*x^2+11/3*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^6/d^3*dilog(1+I*(
c*x+(c^2*x^2+1)^(1/2)))-11/3*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/
c^6/d^3*dilog(1-I*(c*x+(c^2*x^2+1)^(1/2)))-2*a*b*(d*(c^2*x^2+1))^(1/2)/c^5/
d^3/(c^2*x^2+1)^(1/2)*x+1/3*a*b*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^(3/2)
/c^5*x+2*a*b*(d*(c^2*x^2+1))^(1/2)/c^6/d^3/(c^2*x^2+1)*arcsinh(c*x)+10/3*a*
b*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2/c^6*arcsinh(c*x)+8/3*a^2/c^6/d/(c
^2*d*x^2+d)^(3/2)+11/3*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^6/d^
3*arcsinh(c*x)*ln(1+I*(c*x+(c^2*x^2+1)^(1/2)))-11/3*I*b^2*(d*(c^2*x^2+1))^(
1/2)/(c^2*x^2+1)^(1/2)/c^6/d^3*arcsinh(c*x)*ln(1-I*(c*x+(c^2*x^2+1)^(1/2)))
+2*a*b*(d*(c^2*x^2+1))^(1/2)/c^4/d^3/(c^2*x^2+1)*arcsinh(c*x)*x^2+4*a*b*(d*
(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2/c^4*arcsinh(c*x)*x^2+11/3*I*a*b*(d*(c^
2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^6/d^3*ln(c*x+(c^2*x^2+1)^(1/2)-I)-11/3*
I*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^6/d^3*ln(c*x+(c^2*x^2+1)^(1
/2)+I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}a^2\left(\frac{3x^4}{\left(c^2dx^2+d\right)^{\frac{3}{2}}c^2d}+\frac{12x^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}c^4d}+\frac{8}{\left(c^2dx^2+d\right)^{\frac{3}{2}}c^6d}\right)+\frac{\left(3b^2c^4\sqrt{d}x^4+12b^2c^2\sqrt{d}x^2+8b^2\sqrt{d}\right)\sqrt{c^2x^2}}{3\left(c^{10}d^3x^4+2c^8d^3x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima
")
```

```
[Out] 1/3*a^2*(3*x^4/((c^2*d*x^2 + d)^(3/2)*c^2*d) + 12*x^2/((c^2*d*x^2 + d)^(3/2
)*c^4*d) + 8/((c^2*d*x^2 + d)^(3/2)*c^6*d)) + 1/3*(3*b^2*c^4*sqrt(d)*x^4 +
12*b^2*c^2*sqrt(d)*x^2 + 8*b^2*sqrt(d))*sqrt(c^2*x^2 + 1)*log(c*x + sqrt(c^
2*x^2 + 1))^2/(c^10*d^3*x^4 + 2*c^8*d^3*x^2 + c^6*d^3) + integrate(-2/3*((1
2*b^2*c^3*x^3 - 3*(a*b*c^5 - b^2*c^5)*x^5 + 8*b^2*c*x)*(c^2*x^2 + 1) + (15*
b^2*c^4*x^4 - 3*(a*b*c^6 - b^2*c^6)*x^6 + 20*b^2*c^2*x^2 + 8*b^2)*sqrt(c^2*
x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))/(c^12*d^(5/2)*x^7 + 3*c^10*d^(5/2)*x
^5 + 3*c^8*d^(5/2)*x^3 + c^6*d^(5/2)*x + (c^11*d^(5/2)*x^6 + 3*c^9*d^(5/2)*
x^4 + 3*c^7*d^(5/2)*x^2 + c^5*d^(5/2))*sqrt(c^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2}{\left(d\,c^2\,x^2+d\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int((x^5*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\left(a+b\,\mathrm{asinh}\left(cx\right)\right)^2}{\left(d\left(c^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**5*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(5/2), x)
```

3.311 $\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=398

$$-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(c^2dx^2+d\right)^{3/2}}+\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc^5d^2\sqrt{c^2dx^2+d}}-\frac{4\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{3c^5d^2\sqrt{c^2dx^2+d}}+\frac{8b\sqrt{c^2x^2+1}\log\left(e^{2\sinh^{-1}(cx)}\right.}{3c^5d}$$

```
[Out] -1/3*x^3*(a+b*arcsinh(c*x))^2/c^2/d/(c^2*d*x^2+d)^(3/2)-1/3*b^2*x/c^4/d^2/(
c^2*d*x^2+d)^(1/2)-x*(a+b*arcsinh(c*x))^2/c^4/d^2/(c^2*d*x^2+d)^(1/2)-1/3*b
*x^2*(a+b*arcsinh(c*x))/c^3/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+1/3*b
^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)/c^5/d^2/(c^2*d*x^2+d)^(1/2)-4/3*(a+b*arcs
inh(c*x))^2*(c^2*x^2+1)^(1/2)/c^5/d^2/(c^2*d*x^2+d)^(1/2)+1/3*(a+b*arcsinh(
c*x))^3*(c^2*x^2+1)^(1/2)/b/c^5/d^2/(c^2*d*x^2+d)^(1/2)+8/3*b*(a+b*arcsinh(
c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/c^5/d^2/(c^2*d*x^2+
d)^(1/2)+4/3*b^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/c^
5/d^2/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.76, antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5751, 5677, 5675, 5714, 3718, 2190, 2279, 2391, 288, 215}

$$\frac{4b^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{3c^5d^2\sqrt{c^2dx^2+d}}-\frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^4d^2\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2x^2+1}\left(a+b\text{s}\right.}{3bc^5d^2\sqrt{c^2d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] -(b^2*x)/(3*c^4*d^2*Sqrt[d + c^2*d*x^2]) + (b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c
*x])/(3*c^5*d^2*Sqrt[d + c^2*d*x^2]) - (b*x^2*(a + b*ArcSinh[c*x]))/(3*c^3*
d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (x^3*(a + b*ArcSinh[c*x])^2)/(
3*c^2*d*(d + c^2*d*x^2)^(3/2)) - (x*(a + b*ArcSinh[c*x])^2)/(c^4*d^2*Sqrt[d
 + c^2*d*x^2]) - (4*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c^5*d^2*Sq
rt[d + c^2*d*x^2]) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(3*b*c^5*d^
2*Sqrt[d + c^2*d*x^2]) + (8*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1
+ E^(2*ArcSinh[c*x])])/(3*c^5*d^2*Sqrt[d + c^2*d*x^2]) + (4*b^2*Sqrt[1 + c^
2*x^2]*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*c^5*d^2*Sqrt[d + c^2*d*x^2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] &&  !GtQ[d, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rubi steps

$$\int \frac{x^4 \left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx = -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} + \frac{\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}dx}{c^2d} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{x^3\left(a+b\sinh^{-1}(c\right.}{\left(1+c^2x^2\right)^2}}{3cd^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^4d^2\sqrt{d+c^2dx^2}} +$$

$$= -\frac{b^2x}{3c^4d^2\sqrt{d+c^2dx^2}} - \frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \frac{x(a}{c}$$

$$= -\frac{b^2x}{3c^4d^2\sqrt{d+c^2dx^2}} + \frac{b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{3c^5d^2\sqrt{d+c^2dx^2}} - \frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{x}{}$$

$$= -\frac{b^2x}{3c^4d^2\sqrt{d+c^2dx^2}} + \frac{b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{3c^5d^2\sqrt{d+c^2dx^2}} - \frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{x}{}$$

$$= -\frac{b^2x}{3c^4d^2\sqrt{d+c^2dx^2}} + \frac{b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{3c^5d^2\sqrt{d+c^2dx^2}} - \frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{x}{}$$

$$= -\frac{b^2x}{3c^4d^2\sqrt{d+c^2dx^2}} + \frac{b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{3c^5d^2\sqrt{d+c^2dx^2}} - \frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{x}{}$$

Mathematica [A] time = 1.25, size = 359, normalized size = 0.90

$$\frac{a^2(-c)\sqrt{d}\,x\left(4c^2x^2+3\right)+3a^2\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}\,\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)+ab\sqrt{d}\left(\sqrt{c^2x^2+1}-8cx\left(c^2x^2\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (-(a^2*c*Sqrt[d]*x*(3 + 4*c^2*x^2)) + a*b*Sqrt[d]*(Sqrt[1 + c^2*x^2] + 2*c*
x*ArcSinh[c*x] - 8*c*x*(1 + c^2*x^2)*ArcSinh[c*x] + (1 + c^2*x^2)^(3/2)*(3*
ArcSinh[c*x]^2 + 4*Log[1 + c^2*x^2])) + 3*a^2*(1 + c^2*x^2)*Sqrt[d + c^2*d*
x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] - b^2*Sqrt[d]*(c*x + c^3*x^3
- Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + 3*c*x*ArcSinh[c*x]^2 + 4*c^3*x^3*ArcSinh
[c*x]^2 - 4*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]^2 - (1 + c^2*x^2)^(3/2)*ArcSin
h[c*x]^3 - 8*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*Log[1 + E^(-2*ArcSinh[c*x])]
+ 4*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(-2*ArcSinh[c*x])]))/(3*c^5*d^(5/2)*(
1 + c^2*x^2)*Sqrt[d + c^2*d*x^2])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2x^4\operatorname{arsinh}(cx)^2 + 2\,abx^4\operatorname{arsinh}(cx) + a^2x^4\right)\sqrt{c^2dx^2+d}}{c^6d^3x^6 + 3\,c^4d^3x^4 + 3\,c^2d^3x^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^4*arcsinh(c*x)^2 + 2*a*b*x^4*arcsinh(c*x) + a^2*x^4)*sqrt(c
^2*d*x^2 + d)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2 x^4}{(c^2dx^2 + d)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^4/(c^2*d*x^2 + d)^(5/2), x)
```

maple [B] time = 0.56, size = 3705, normalized size = 9.31

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] 64*a*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+
16)*c/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^6+28/3*a*b*(d*(c^2*x^2+1))^(1/2)
/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^2/d^3*(c^2*x^2+1)*x^3-
362/3*a*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x
^2+16)/c^2/d^3*arcsinh(c*x)*x^3+13*a*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87
*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^3/d^3*x^2*(c^2*x^2+1)^(1/2)+4*a*b*(d*
(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^4/d^
3*(c^2*x^2+1)*x-32*a*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4
*x^4+71*c^2*x^2+16)/c^4/d^3*arcsinh(c*x)*x+128/3*a*b*(d*(c^2*x^2+1))^(1/2)/
(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^5/d^3*arcsinh(c*x)*(c^2
*x^2+1)^(1/2)+440/3*a*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^
4*x^4+71*c^2*x^2+16)/c^3/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2-4*b^2*(d*(c
^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^4/d^3*
arcsinh(c*x)*x+8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d^3*arcs
inh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+16/3*b^2*(d*(c^2*x^2+1))^(1/2)/(24
*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/d^3*arcsinh(c*x)*(c^2*x^2+1)
*x^5-4*a*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*
x^2+16)/c^4/d^3*x+16/3*a*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118
*c^4*x^4+71*c^2*x^2+16)/c^5/d^3*(c^2*x^2+1)^(1/2)+8/3*a*b*(d*(c^2*x^2+1))^(
1/2)/(c^2*x^2+1)^(1/2)/c^5/d^3*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+a*b*(d*(c^2*
x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d^3*arcsinh(c*x)^2-16/3*a*b*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d^3*arcsinh(c*x)-16/3*a*b*(d*(c^2*x^2+1))^
(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)*c^2/d^3*x^7+16/3*a*
b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/d
^3*(c^2*x^2+1)*x^5-152*a*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118
*c^4*x^4+71*c^2*x^2+16)/d^3*arcsinh(c*x)*x^5-40/3*a*b*(d*(c^2*x^2+1))^(1/2)
/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^2/d^3*x^3+16/3*b^2*(d*
(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^5/d^
3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-32*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+8
7*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)*c^2/d^3*arcsinh(c*x)^2*x^7-1/3*a^2*x^3
/c^2/d/(c^2*d*x^2+d)^(3/2)+168*a*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6
*x^6+118*c^4*x^4+71*c^2*x^2+16)/c/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^4+13
*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16
)/c^3/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2+4*b^2*(d*(c^2*x^2+1))^(1/2)/(2
4*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^4/d^3*arcsinh(c*x)*(c^2*x
^2+1)*x+32*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*
c^2*x^2+16)*c/d^3*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*x^6+84*b^2*(d*(c^2*x^2+1
))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c/d^3*arcsinh(c*
```

```
x)^2*(c^2*x^2+1)^(1/2)*x^4+8*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x
^6+118*c^4*x^4+71*c^2*x^2+16)/c/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^4+28/3
*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16
)/c^2/d^3*arcsinh(c*x)*(c^2*x^2+1)*x^3+220/3*b^2*(d*(c^2*x^2+1))^(1/2)/(24*
c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^3/d^3*arcsinh(c*x)^2*(c^2*x
^2+1)^(1/2)*x^2-64*a*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4
*x^4+71*c^2*x^2+16)*c^2/d^3*arcsinh(c*x)*x^7+8*a*b*(d*(c^2*x^2+1))^(1/2)/(2
4*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c/d^3*(c^2*x^2+1)^(1/2)*x^4
-16/3*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x
^2+16)*c^2/d^3*arcsinh(c*x)*x^7+8*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*
c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)*c/d^3*(c^2*x^2+1)^(1/2)*x^6+21*b^2*(d*(c
^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c/d^3*(c
^2*x^2+1)^(1/2)*x^4-181/3*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+
118*c^4*x^4+71*c^2*x^2+16)/c^2/d^3*arcsinh(c*x)^2*x^3-8/3*b^2*(d*(c^2*x^2+1
))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^2/d^3*(c^2*x^2
+1)*x^3-40/3*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+7
1*c^2*x^2+16)/c^2/d^3*arcsinh(c*x)*x^3+55/3*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c
^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^3/d^3*x^2*(c^2*x^2+1)^(1/2)-
16*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+
16)/c^4/d^3*arcsinh(c*x)^2*x-4/3*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c
^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^4/d^3*(c^2*x^2+1)*x+64/3*b^2*(d*(c^2*x^
2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^5/d^3*arcsi
nh(c*x)^2*(c^2*x^2+1)^(1/2)-a^2/c^4/d^2*x/(c^2*d*x^2+d)^(1/2)+a^2/c^4/d^2*l
n(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)-17*b^2*(d*(c^2*x
^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/d^3*x^5-8/3*
b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d^3*arcsinh(c*x)^2+4/3*b^2*
(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d^3*polylog(2,-(c*x+(c^2*x^2+1)
^(1/2))^2)-76*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+
71*c^2*x^2+16)/d^3*arcsinh(c*x)^2*x^5-4/3*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8
*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/d^3*(c^2*x^2+1)*x^5-44/3*b^2*(d*
(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/d^3*ar
csinh(c*x)*x^5-20/3*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^
4*x^4+71*c^2*x^2+16)*c^2/d^3*x^7-43/3*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8
+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^2/d^3*x^3-4*b^2*(d*(c^2*x^2+1))^(1
/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^4/d^3*x+16/3*b^2*(d
*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^5/d
^3*(c^2*x^2+1)^(1/2)+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d^
3*arcsinh(c*x)^3-44/3*a*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*
c^4*x^4+71*c^2*x^2+16)/d^3*x^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}\left(x\left(\frac{3\,x^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}c^2d}+\frac{2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}c^4d}\right)+\frac{x}{\sqrt{c^2dx^2+d}\,c^4d^2}-\frac{3\,\operatorname{arsinh}(cx)}{c^5d^{\frac{5}{2}}}\right)a^2+\int\frac{b^2x^4\log\left(cx+\sqrt{c^2x^2+1}\right.}{\left(c^2dx^2+d\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima
")
```

```
[Out] -1/3*(x*(3*x^2/((c^2*d*x^2 + d)^(3/2)*c^2*d) + 2/((c^2*d*x^2 + d)^(3/2)*c^4
*d)) + x/(sqrt(c^2*d*x^2 + d)*c^4*d^2) - 3*arcsinh(c*x)/(c^5*d^(5/2)))*a^2
+ integrate(b^2*x^4*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d*x^2 + d)^(5/2) +
2*a*b*x^4*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x^4\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d\,c^2\,x^2+d\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int((x^4*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 (a + b \operatorname{asinh}(cx))^2}{\left(d \left(c^2 x^2 + 1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**4*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(5/2), x)
```

3.312 $\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=307

$$-\frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(c^2dx^2+d\right)^{3/2}}-\frac{2\left(a+b\sinh^{-1}(cx)\right)^2}{3c^4d^2\sqrt{c^2dx^2+d}}+\frac{10b\sqrt{c^2x^2+1}\,\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{3c^4d^2\sqrt{c^2dx^2+d}}-\frac{bx\left(a+b\right.}{3c^3d^2\sqrt{c^2x^2}}$$

```
[Out] -1/3*x^2*(a+b*arcsinh(c*x))^2/c^2/d/(c^2*d*x^2+d)^(3/2)-1/3*b^2/c^4/d^2/(c^
2*d*x^2+d)^(1/2)-2/3*(a+b*arcsinh(c*x))^2/c^4/d^2/(c^2*d*x^2+d)^(1/2)-1/3*b
*x*(a+b*arcsinh(c*x))/c^3/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+10/3*b*
(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/c^4/d^2/
(c^2*d*x^2+d)^(1/2)-5/3*I*b^2*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^
2+1)^(1/2)/c^4/d^2/(c^2*d*x^2+d)^(1/2)+5/3*I*b^2*polylog(2,I*(c*x+(c^2*x^2+
1)^(1/2)))*(c^2*x^2+1)^(1/2)/c^4/d^2/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.50, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5751, 5717, 5693, 4180, 2279, 2391, 261}

$$-\frac{5ib^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{3c^4d^2\sqrt{c^2dx^2+d}}+\frac{5ib^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{3c^4d^2\sqrt{c^2dx^2+d}}-\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}-$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] -b^2/(3*c^4*d^2*Sqrt[d + c^2*d*x^2]) - (b*x*(a + b*ArcSinh[c*x]))/(3*c^3*d^
2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (x^2*(a + b*ArcSinh[c*x])^2)/(3*
c^2*d*(d + c^2*d*x^2)^(3/2)) - (2*(a + b*ArcSinh[c*x])^2)/(3*c^4*d^2*Sqrt[d
 + c^2*d*x^2]) + (10*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTan[E^ArcS
inh[c*x]])/(3*c^4*d^2*Sqrt[d + c^2*d*x^2]) - (((5*I)/3)*b^2*Sqrt[1 + c^2*x^
2]*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^4*d^2*Sqrt[d + c^2*d*x^2]) + (((5*I)
/3)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I*E^ArcSinh[c*x]])/(c^4*d^2*Sqrt[d + c
^2*d*x^2])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
```

```
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rubi steps

$$\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx = -\frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} + \frac{2\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx}{3c^2d} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{x^2\left(a+b\sinh \cdots\right.}{\left(1+c^2x \cdots\right.}}{3cd^2\sqrt{d+c^2dx^2}} \cdots$$

$$= -\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}} - \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \frac{2\left(a+b\sinh^{-1}(cx)\right)^2}{3c^4d^2\sqrt{d+c^2dx^2}} \cdots$$

$$= -\frac{b^2}{3c^4d^2\sqrt{d+c^2dx^2}} - \frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}} - \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \cdots$$

$$= -\frac{b^2}{3c^4d^2\sqrt{d+c^2dx^2}} - \frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}} - \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \cdots$$

$$= -\frac{b^2}{3c^4d^2\sqrt{d+c^2dx^2}} - \frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}} - \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \cdots$$

$$= -\frac{b^2}{3c^4d^2\sqrt{d+c^2dx^2}} - \frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}} - \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \cdots$$

Mathematica [A] time = 1.11, size = 301, normalized size = 0.98

$$-\left(a^2\left(3c^2x^2+2\right)\right)+ab\left(\sqrt{c^2x^2+1}\left(10\left(c^2x^2+1\right)\tan^{-1}\left(\tanh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)-cx\right)-2\left(3c^2x^2+2\right)\sinh^{-1}(cx)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (-(a^2*(2 + 3*c^2*x^2)) + a*b*(-2*(2 + 3*c^2*x^2)*ArcSinh[c*x] + Sqrt[1 + c
^2*x^2]*(-(c*x) + 10*(1 + c^2*x^2)*ArcTan[Tanh[ArcSinh[c*x]/2]])) - b^2*(1
+ c^2*x^2 + c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + 2*ArcSinh[c*x]^2 + 3*c^2*x
^2*ArcSinh[c*x]^2 + (5*I)*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*Log[1 - I/E^ArcS
inh[c*x]] - (5*I)*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*Log[1 + I/E^ArcSinh[c*x]
] + (5*I)*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-I)/E^ArcSinh[c*x]] - (5*I)*(1 +
c^2*x^2)^(3/2)*PolyLog[2, I/E^ArcSinh[c*x]]))/(3*c^4*d^2*(1 + c^2*x^2)*Sqrt
[d + c^2*d*x^2])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2x^3\operatorname{arsinh}(cx)^2+2\,abx^3\operatorname{arsinh}(cx)+a^2x^3\right)\sqrt{c^2dx^2+d}}{c^6d^3x^6+3\,c^4d^3x^4+3\,c^2d^3x^2+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^3*arcsinh(c*x)^2 + 2*a*b*x^3*arcsinh(c*x) + a^2*x^3)*sqrt(c
^2*d*x^2 + d)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.39, size = 705, normalized size = 2.30

$$-\frac{a^2x^2}{c^2d\left(c^2d\,x^2+d\right)^{\frac{3}{2}}}-\frac{2a^2}{3d\,c^4\left(c^2d\,x^2+d\right)^{\frac{3}{2}}}-\frac{b^2\sqrt{d\left(c^2x^2+1\right)}\operatorname{arcsinh}(cx)^2x^2}{d^3\left(c^2x^2+1\right)^2c^2}-\frac{b^2\sqrt{d\left(c^2x^2+1\right)}\operatorname{arcsinh}(cx)\,x}{3d^3\left(c^2x^2+1\right)^{\frac{3}{2}}c^3}-\frac{b^2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] -a^2*x^2/c^2/d/(c^2*d*x^2+d)^(3/2)-2/3*a^2/d/c^4/(c^2*d*x^2+d)^(3/2)-b^2*(d
*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2/c^2*arcsinh(c*x)^2*x^2-1/3*b^2*(d*(c^
2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^(3/2)/c^3*arcsinh(c*x)*x-1/3*b^2*(d*(c^2*x^
2+1))^(1/2)/d^3/(c^2*x^2+1)^2/c^2*x^2-2/3*b^2*(d*(c^2*x^2+1))^(1/2)/d^3/(c^
2*x^2+1)^2/c^4*arcsinh(c*x)^2-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)
^2/c^4+5/3*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d^3*dilog(1-I*
(c*x+(c^2*x^2+1)^(1/2)))-5/3*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/
```

```
c^4/d^3*arcsinh(c*x)*ln(1+I*(c*x+(c^2*x^2+1)^(1/2)))+5/3*I*b^2*(d*(c^2*x^2+
1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d^3*arcsinh(c*x)*ln(1-I*(c*x+(c^2*x^2+1)^(1
/2)))-5/3*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d^3*dilog(1+I*(
c*x+(c^2*x^2+1)^(1/2)))-2*a*b*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2/c^2*a
rcsinh(c*x)*x^2-1/3*a*b*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^(3/2)/c^3*x-4
/3*a*b*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2/c^4*arcsinh(c*x)+5/3*I*a*b*(
d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d^3*ln(c*x+(c^2*x^2+1)^(1/2)+I)-
5/3*I*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d^3*ln(c*x+(c^2*x^2+1
)^(1/2)-I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}abc\left(\frac{x}{c^6d^{\frac{5}{2}}x^2+c^4d^{\frac{5}{2}}}-\frac{5\arctan(cx)}{c^5d^{\frac{5}{2}}}\right)-\frac{2}{3}ab\left(\frac{3x^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}c^2d}+\frac{2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}c^4d}\right)\text{arsinh}(cx)-\frac{1}{3}a^2\left(\frac{3}{\left(c^2dx^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima
")
```

```
[Out] -1/3*a*b*c*(x/(c^6*d^(5/2)*x^2 + c^4*d^(5/2)) - 5*arctan(c*x)/(c^5*d^(5/2))
) - 2/3*a*b*(3*x^2/((c^2*d*x^2 + d)^(3/2)*c^2*d) + 2/((c^2*d*x^2 + d)^(3/2)
*c^4*d))*arcsinh(c*x) - 1/3*a^2*(3*x^2/((c^2*d*x^2 + d)^(3/2)*c^2*d) + 2/((
c^2*d*x^2 + d)^(3/2)*c^4*d)) + b^2*integrate(x^3*log(c*x + sqrt(c^2*x^2 + 1
))^2/(c^2*d*x^2 + d)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d\,c^2\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int((x^3*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \left(a + b \operatorname{asinh}{\left(cx \right)}\right)^{2}}{\left(d \left(c^{2} x^{2} + 1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**3*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(5/2), x)
```

3.313 $\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=312

$$\frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{c^2x^2+1}\,\sqrt{c^2dx^2+d}}+\frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(c^2dx^2+d\right)^{3/2}}+\frac{\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)^2}{3c^3d^2\sqrt{c^2dx^2+d}}-\frac{2b\sqrt{c^2x^2+1}\,\log\left(e^{2\sinh^{-1}(c}\right.}{3c^3d^2\sqrt{c^2}}$$

```
[Out] 1/3*x^3*(a+b*arcsinh(c*x))^2/d/(c^2*d*x^2+d)^(3/2)+1/3*b^2*x/c^2/d^2/(c^2*d
*x^2+d)^(1/2)+1/3*b*x^2*(a+b*arcsinh(c*x))/c/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x
^2+d)^(1/2)-1/3*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)/c^3/d^2/(c^2*d*x^2+d)^(1
/2)+1/3*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/c^3/d^2/(c^2*d*x^2+d)^(1/2)-
2/3*b*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/
c^3/d^2/(c^2*d*x^2+d)^(1/2)-1/3*b^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*(
c^2*x^2+1)^(1/2)/c^3/d^2/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {5723, 5751, 5714, 3718, 2190, 2279, 2391, 288, 215}

$$-\frac{b^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{3c^3d^2\sqrt{c^2dx^2+d}}+\frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{c^2x^2+1}\,\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)^2}{3c^3d^2\sqrt{c^2dx^2+d}}-\frac{2b\sqrt{c^2x^2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (b^2*x)/(3*c^2*d^2*Sqrt[d + c^2*d*x^2]) - (b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*
x])/(3*c^3*d^2*Sqrt[d + c^2*d*x^2]) + (b*x^2*(a + b*ArcSinh[c*x]))/(3*c*d^2
*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) + (x^3*(a + b*ArcSinh[c*x])^2)/(3*d
*(d + c^2*d*x^2)^(3/2)) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c^3
*d^2*Sqrt[d + c^2*d*x^2]) - (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log
[1 + E^(2*ArcSinh[c*x])])/(3*c^3*d^2*Sqrt[d + c^2*d*x^2]) - (b^2*Sqrt[1 + c
^2*x^2]*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*c^3*d^2*Sqrt[d + c^2*d*x^2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rubi steps

$$\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx = \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} - \frac{\left(2bc\sqrt{1+c^2x^2}\right)\int\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^2}dx}{3d^2\sqrt{d+c^2dx^2}}$$

$$= \frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} - \frac{\left(b^2\sqrt{1+c^2x^2}\right)\int\frac{x^2}{\left(1+c^2x^2\right)^{3/2}}}{3d^2\sqrt{d+c^2dx^2}}$$

$$= \frac{b^2x}{3c^2d^2\sqrt{d+c^2dx^2}} + \frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} - \frac{\left(2b\sqrt{\ }\right.}{}$$

$$= \frac{b^2x}{3c^2d^2\sqrt{d+c^2dx^2}} - \frac{b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{3c^3d^2\sqrt{d+c^2dx^2}} + \frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x^3\left(\right.}{3}$$

$$= \frac{b^2x}{3c^2d^2\sqrt{d+c^2dx^2}} - \frac{b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{3c^3d^2\sqrt{d+c^2dx^2}} + \frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x^3\left(\right.}{3}$$

$$= \frac{b^2x}{3c^2d^2\sqrt{d+c^2dx^2}} - \frac{b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{3c^3d^2\sqrt{d+c^2dx^2}} + \frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x^3\left(\right.}{3}$$

$$= \frac{b^2x}{3c^2d^2\sqrt{d+c^2dx^2}} - \frac{b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{3c^3d^2\sqrt{d+c^2dx^2}} + \frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x^3\left(\right.}{3}$$

Mathematica [A] time = 0.96, size = 280, normalized size = 0.90

$$\frac{a^2c^3x^3 - ab\sqrt{c^2x^2+1} - abc^2x^2\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right) - ab\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right) - b\sinh^{-1}(cx)\left(-2ac^3x^3+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (b^2*c*x + a^2*c^3*x^3 + b^2*c^3*x^3 - a*b*Sqrt[1 + c^2*x^2] - b^2*(-(c^3*x
^3) + Sqrt[1 + c^2*x^2] + c^2*x^2*Sqrt[1 + c^2*x^2])*ArcSinh[c*x]^2 - b*Arc
Sinh[c*x]*(-2*a*c^3*x^3 + b*Sqrt[1 + c^2*x^2] + 2*b*(1 + c^2*x^2)^(3/2)*Log
[1 + E^(-2*ArcSinh[c*x])]) - a*b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2] - a*b*c
^2*x^2*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2] + b^2*(1 + c^2*x^2)^(3/2)*PolyLog
[2, -E^(-2*ArcSinh[c*x])])/(3*c^3*d^2*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2])
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2x^2\operatorname{arsinh}(cx)^2 + 2\,abx^2\operatorname{arsinh}(cx) + a^2x^2\right)\sqrt{c^2dx^2+d}}{c^6d^3x^6 + 3\,c^4d^3x^4 + 3\,c^2d^3x^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*x) + a^2*x^2)*sqrt(c
^2*d*x^2 + d)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^2x^2}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2*x^2/(c^2*d*x^2 + d)^(5/2), x)
```

maple [B] time = 0.46, size = 3112, normalized size = 9.97

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x)
[Out] -1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)
/c^3/d^3*(c^2*x^2+1)^(1/2)-1/3*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/
c^3/d^3*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)+1/3*b^2*(d*(c^2*x^2+1))^(1/2)
/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/d^3*arcsinh(c*x)^2*x^3+2/3*b^
2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/d^3*(c
^2*x^2+1)*x^3+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4
+5*c^2*x^2+1)/d^3*arcsinh(c*x)*x^3+2/3*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1)
)^(1/2)/c^3/d^3*arcsinh(c*x)^2+1/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c
^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/d^3*x^3+2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8
*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^4/d^3*x^7+b^2*(d*(c^2*x^2+1))^(1/2
)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^2/d^3*x^5+1/3*b^2*(d*(c^2*
x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^4/d^3*arcsinh(
c*x)*x^7-b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^
2+1)*c^3/d^3*(c^2*x^2+1)^(1/2)*x^6+b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c
^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^2/d^3*arcsinh(c*x)^2*x^5+1/3*b^2*(d*(c^2*x
^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^2/d^3*(c^2*x^2+
1)*x^5+2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*
x^2+1)*c^2/d^3*arcsinh(c*x)*x^5-2*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^
6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c/d^3*(c^2*x^2+1)^(1/2)*x^4-4/3*b^2*(d*(c^2*x
^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/c/d^3*x^2*(c^2*x^
2+1)^(1/2)+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*
c^2*x^2+1)/c^2/d^3*(c^2*x^2+1)*x-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9
*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/c^3/d^3*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)-1
/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/c
^3/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+4/3*a*b/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2
+1))^(1/2)/c^3/d^3*arcsinh(c*x)+1/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*
c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^4/d^3*x^7+2/3*a*b*(d*(c^2*x^2+1))^(1/2)/(
3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^2/d^3*x^5-1/3*a*b*(d*(c^2*x^2
+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/d^3*(c^2*x^2+1)*x^3
+2/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)
/d^3*arcsinh(c*x)*x^3-1/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10
*c^4*x^4+5*c^2*x^2+1)/c^3/d^3*(c^2*x^2+1)^(1/2)-2/3*a*b/(c^2*x^2+1)^(1/2)*(
d*(c^2*x^2+1))^(1/2)/c^3/d^3*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+1/3*b^2*(d*(c^
2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/d^3*x^3-2*a*b*
(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^3/d^3*
arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^6-4*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9
*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^4-8
/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/c
/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2+2*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*
x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^2/d^3*arcsinh(c*x)*x^5-a*b*(d*(c^2*
x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c/d^3*(c^2*x^2+1
```

```
)^(1/2)*x^4-b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2
*x^2+1)*c/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^4-4/3*b^2*(d*(c^2*x^2+1))^(1
/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/c/d^3*arcsinh(c*x)^2*(c^2*
x^2+1)^(1/2)*x^2-b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+
5*c^2*x^2+1)/c/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2+2*a*b*(d*(c^2*x^2+1))
^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^4/d^3*arcsinh(c*x)*x^
7-1/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1
)*c^2/d^3*(c^2*x^2+1)*x^5-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^
6+10*c^4*x^4+5*c^2*x^2+1)*c^2/d^3*arcsinh(c*x)*(c^2*x^2+1)*x^5-2*b^2*(d*(c^
2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c/d^3*arcsinh(
c*x)^2*(c^2*x^2+1)^(1/2)*x^4-b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6
+10*c^4*x^4+5*c^2*x^2+1)*c^3/d^3*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*x^6-a*b*(
d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/c/d^3*x^2
*(c^2*x^2+1)^(1/2)-2/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^
4*x^4+5*c^2*x^2+1)/c^3/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-1/3*a^2/c^2*x/d/(
c^2*d*x^2+d)^(3/2)+1/3*a^2/c^2/d^2*x/(c^2*d*x^2+d)^(1/2)-2/3*b^2/(c^2*x^2+1
)^(1/2)*(d*(c^2*x^2+1))^(1/2)/c^3/d^3*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1
/2))^2)-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2
*x^2+1)/d^3*arcsinh(c*x)*(c^2*x^2+1)*x^3+b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x
^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^4/d^3*arcsinh(c*x)^2*x^7
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}\,abc\left(\frac{1}{c^6d^{\frac{5}{2}}x^2+c^4d^{\frac{5}{2}}}+\frac{\log\left(c^2x^2+1\right)}{c^4d^{\frac{5}{2}}}\right)+\frac{2}{3}\,ab\left(\frac{x}{\sqrt{c^2dx^2+d}\,c^2d^2}-\frac{x}{\left(c^2dx^2+d\right)^{\frac{3}{2}}c^2d}\right)\operatorname{arsinh}\left(cx\right)+\frac{1}{3}\,a^2\left(\frac{x}{\sqrt{c^2dx^2\ldots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima
")
```

```
[Out] -1/3*a*b*c*(1/(c^6*d^(5/2)*x^2 + c^4*d^(5/2)) + log(c^2*x^2 + 1)/(c^4*d^(5/
2))) + 2/3*a*b*(x/(sqrt(c^2*d*x^2 + d)*c^2*d^2) - x/((c^2*d*x^2 + d)^(3/2)*
c^2*d))*arcsinh(c*x) + 1/3*a^2*(x/(sqrt(c^2*d*x^2 + d)*c^2*d^2) - x/((c^2*d
*x^2 + d)^(3/2)*c^2*d)) + b^2*integrate(x^2*log(c*x + sqrt(c^2*x^2 + 1))^2/
(c^2*d*x^2 + d)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d\,c^2\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int((x^2*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(a + b \operatorname{asinh}{\left(cx \right)}\right)^{2}}{\left(d \left(c^{2} x^{2} + 1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**2*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(5/2), x)
```

3.314 $\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=270

$$\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}+\frac{2b\sqrt{c^2x^2+1}\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{3c^2d^2\sqrt{c^2dx^2+d}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(c^2dx^2+d\right)^{3/2}}-\frac{ib^2\sqrt{c^2x}}{3c}$$

```
[Out] -1/3*(a+b*arcsinh(c*x))^2/c^2/d/(c^2*d*x^2+d)^(3/2)+1/3*b^2/c^2/d^2/(c^2*d*
x^2+d)^(1/2)+1/3*b*x*(a+b*arcsinh(c*x))/c/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+
d)^(1/2)+2/3*b*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)
^(1/2)/c^2/d^2/(c^2*d*x^2+d)^(1/2)-1/3*I*b^2*polylog(2,-I*(c*x+(c^2*x^2+1)^
(1/2)))*(c^2*x^2+1)^(1/2)/c^2/d^2/(c^2*d*x^2+d)^(1/2)+1/3*I*b^2*polylog(2,I
*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/c^2/d^2/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5717, 5690, 5693, 4180, 2279, 2391, 261}

$$-\frac{ib^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{3c^2d^2\sqrt{c^2dx^2+d}}+\frac{ib^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{3c^2d^2\sqrt{c^2dx^2+d}}+\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}+$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] b^2/(3*c^2*d^2*Sqrt[d + c^2*d*x^2]) + (b*x*(a + b*ArcSinh[c*x]))/(3*c*d^2*S
qrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (a + b*ArcSinh[c*x])^2/(3*c^2*d*(d
+ c^2*d*x^2)^(3/2)) + (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTan[E^
ArcSinh[c*x]])/(3*c^2*d^2*Sqrt[d + c^2*d*x^2]) - ((I/3)*b^2*Sqrt[1 + c^2*x^
2]*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^2*d^2*Sqrt[d + c^2*d*x^2]) + ((I/3)*
b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I*E^ArcSinh[c*x]])/(c^2*d^2*Sqrt[d + c^2*d
*x^2])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
```

```
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{\left(1+c^2x^2\right)^2}\,dx}{3cd^2\sqrt{d+c^2dx^2}}$$

$$= \frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \frac{\left(b^2\sqrt{1+c^2x^2}\right)\int \frac{x}{\left(1+c^2x^2\right)^{3/2}}\,dx}{3d^2\sqrt{d+c^2dx^2}}$$

$$= \frac{b^2}{3c^2d^2\sqrt{d+c^2dx^2}} + \frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} + \frac{\left(b\sqrt{1+c\right.}{}$$

$$= \frac{b^2}{3c^2d^2\sqrt{d+c^2dx^2}} + \frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} + \frac{2b\sqrt{1+c}}{}$$

$$= \frac{b^2}{3c^2d^2\sqrt{d+c^2dx^2}} + \frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} + \frac{2b\sqrt{1+c}}{}$$

$$= \frac{b^2}{3c^2d^2\sqrt{d+c^2dx^2}} + \frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} + \frac{2b\sqrt{1+c}}{}$$

Mathematica [A] time = 1.02, size = 254, normalized size = 0.94

$$-a^2+ab\left(\sqrt{c^2x^2+1}\left(2\left(c^2x^2+1\right)\tan^{-1}\left(\tanh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)+cx\right)-2\sinh^{-1}(cx)\right)+b^2\left(-i\left(c^2x^2+1\right)^{3/2}\text{Li}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (-a^2 + a*b*(-2*ArcSinh[c*x] + Sqrt[1 + c^2*x^2]*(c*x + 2*(1 + c^2*x^2)*Arc
Tan[Tanh[ArcSinh[c*x]/2]])) + b^2*(1 + c^2*x^2 + c*x*Sqrt[1 + c^2*x^2]*ArcS
inh[c*x] - ArcSinh[c*x]^2 - I*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*Log[1 - I/E^
ArcSinh[c*x]] + I*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*Log[1 + I/E^ArcSinh[c*x]
] - I*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-I)/E^ArcSinh[c*x]] + I*(1 + c^2*x^2)
^(3/2)*PolyLog[2, I/E^ArcSinh[c*x]]))/(3*c^2*d*(d + c^2*d*x^2)^(3/2))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(b^2x\,\text{arsinh}\,(cx)^2+2\,abx\,\text{arsinh}\,(cx)+a^2x\right)}{c^6d^3x^6+3\,c^4d^3x^4+3\,c^2d^3x^2+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*x*arcsinh(c*x)^2 + 2*a*b*x*arcsinh(c*x) +
 a^2*x)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\,\text{arsinh}\,(cx)+a\right)^2x}{\left(c^2dx^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x/(c^2*d*x^2 + d)^(5/2), x)
```

maple [B] time = 0.25, size = 591, normalized size = 2.19

$$-\frac{a^2}{3c^2d\left(c^2d\,x^2+d\right)^{\frac{3}{2}}}+\frac{b^2\sqrt{d\left(c^2x^2+1\right)}\;\text{arcsinh}\,(cx)\,x}{3d^3\left(c^2x^2+1\right)^{\frac{3}{2}}c}+\frac{b^2\sqrt{d\left(c^2x^2+1\right)}\;x^2}{3d^3\left(c^2x^2+1\right)^2}-\frac{b^2\sqrt{d\left(c^2x^2+1\right)}\;\text{arcsinh}\,(cx)^2}{3d^3\left(c^2x^2+1\right)^2c^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] -1/3*a^2/c^2/d/(c^2*d*x^2+d)^(3/2)+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x
^2+1)^(3/2)/c*arcsinh(c*x)*x+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^
2*x^2-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2/c^2*arcsinh(c*x)^2+1/
3*b^2*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2/c^2-1/3*I*b^2*(d*(c^2*x^2+1))
^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^3*arcsinh(c*x)*ln(1+I*(c*x+(c^2*x^2+1)^(1/2)
))+1/3*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^3*arcsinh(c*x)*l
n(1-I*(c*x+(c^2*x^2+1)^(1/2)))-1/3*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^
(1/2)/c^2/d^3*dilog(1+I*(c*x+(c^2*x^2+1)^(1/2)))+1/3*I*b^2*(d*(c^2*x^2+1))^
(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^3*dilog(1-I*(c*x+(c^2*x^2+1)^(1/2)))+1/3*a*b*
(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^(3/2)/c*x-2/3*a*b*(d*(c^2*x^2+1))^(1/
```

```
2)/d^3/(c^2*x^2+1)^2/c^2*arcsinh(c*x)+1/3*I*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*
x^2+1)^(1/2)/c^2/d^3*ln(c*x+(c^2*x^2+1)^(1/2)+I)-1/3*I*a*b*(d*(c^2*x^2+1))^
(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^3*ln(c*x+(c^2*x^2+1)^(1/2)-I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^2}{3\left(c^2dx^2+d\right)^{\frac{3}{2}}c^2d}+\int\frac{b^2x\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{\left(c^2dx^2+d\right)^{\frac{5}{2}}}+\frac{2\,abx\log\left(cx+\sqrt{c^2x^2+1}\right)}{\left(c^2dx^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*a^2/((c^2*d*x^2 + d)^(3/2)*c^2*d) + integrate(b^2*x*log(c*x + sqrt(c^2
*x^2 + 1))^2/(c^2*d*x^2 + d)^(5/2) + 2*a*b*x*log(c*x + sqrt(c^2*x^2 + 1))/(
c^2*d*x^2 + d)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d\,c^2\,x^2+d\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int((x*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x\left(a+b\operatorname{asinh}\left(cx\right)\right)^{2}}{\left(d\left(c^{2}x^{2}+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(5/2), x)
```

3.315 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=292

$$\frac{b\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}+\frac{2x\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{3cd^2\sqrt{c^2dx^2+d}}-\frac{4b\sqrt{c^2x^2+1}\log\left(e^{2\sin}\right.}{3cd^2}$$

```
[Out] 1/3*x*(a+b*arcsinh(c*x))^2/d/(c^2*d*x^2+d)^(3/2)-1/3*b^2*x/d^2/(c^2*d*x^2+d
)^(1/2)+2/3*x*(a+b*arcsinh(c*x))^2/d^2/(c^2*d*x^2+d)^(1/2)+1/3*b*(a+b*arcsi
nh(c*x))/c/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+2/3*(a+b*arcsinh(c*x))
^2*(c^2*x^2+1)^(1/2)/c/d^2/(c^2*d*x^2+d)^(1/2)-4/3*b*(a+b*arcsinh(c*x))*ln(
1+(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/c/d^2/(c^2*d*x^2+d)^(1/2)-2/
3*b^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/c/d^2/(c^2*d*
x^2+d)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {5690, 5687, 5714, 3718, 2190, 2279, 2391, 5717, 191}

$$-\frac{2b^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{3cd^2\sqrt{c^2dx^2+d}}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}+\frac{2x\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\sqrt{c^2x^2+1}\left(\right.}{3cd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] -(b^2*x)/(3*d^2*Sqrt[d + c^2*d*x^2]) + (b*(a + b*ArcSinh[c*x]))/(3*c*d^2*Sq
rt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) + (x*(a + b*ArcSinh[c*x])^2)/(3*d*(d +
 c^2*d*x^2)^(3/2)) + (2*x*(a + b*ArcSinh[c*x])^2)/(3*d^2*Sqrt[d + c^2*d*x^2
]) + (2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c*d^2*Sqrt[d + c^2*d*x
^2]) - (4*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x
])])/(3*c*d^2*Sqrt[d + c^2*d*x^2]) - (2*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, -E
^(2*ArcSinh[c*x])])/(3*c*d^2*Sqrt[d + c^2*d*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx = \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{2\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}dx}{3d} - \frac{\left(2bc\sqrt{1+c^2x^2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^2}}{3d^2\sqrt{d+c^2dx^2}}$$

$$= \frac{b\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{2x\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2\sqrt{d+c^2dx^2}} - $$

$$= -\frac{b^2x}{3d^2\sqrt{d+c^2dx^2}} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{2x\left(a-\right.}{3d^2}$$

$$= -\frac{b^2x}{3d^2\sqrt{d+c^2dx^2}} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{2x\left(a-\right.}{3d^2}$$

$$= -\frac{b^2x}{3d^2\sqrt{d+c^2dx^2}} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{2x\left(a-\right.}{3d^2}$$

$$= -\frac{b^2x}{3d^2\sqrt{d+c^2dx^2}} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{2x\left(a-\right.}{3d^2}$$

$$= -\frac{b^2x}{3d^2\sqrt{d+c^2dx^2}} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{2x\left(a-\right.}{3d^2}$$

Mathematica [A] time = 1.27, size = 236, normalized size = 0.81

$$\frac{a^2cx\left(2c^2x^2+3\right)+ab\left(\left(4c^3x^3+6cx\right)\sinh^{-1}(cx)+\sqrt{c^2x^2+1}\left(1-2\left(c^2x^2+1\right)\log\left(c^2x^2+1\right)\right)\right)-b^2\left(c^3x^3-2\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (a^2*c*x*(3 + 2*c^2*x^2) + a*b*((6*c*x + 4*c^3*x^3)*ArcSinh[c*x] + Sqrt[1 +
 c^2*x^2]*(1 - 2*(1 + c^2*x^2)*Log[1 + c^2*x^2])) - b^2*(c*x + c^3*x^3 - Sq
rt[1 + c^2*x^2]*ArcSinh[c*x] - c*x*ArcSinh[c*x]^2 - 2*c*x*(1 + c^2*x^2)*Arc
Sinh[c*x]^2 + 2*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*(ArcSinh[c*x] + 2*Log[1 +
E^(-2*ArcSinh[c*x])]) - 2*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(-2*ArcSinh[c*x
])]))/(3*d^2*(c + c^3*x^2)*Sqrt[d + c^2*d*x^2])
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(b^2\operatorname{arsinh}(cx)^2+2\,ab\operatorname{arsinh}(cx)+a^2\right)}{c^6d^3x^6+3\,c^4d^3x^4+3\,c^2d^3x^2+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/(c^2*d*x^2 + d)^(5/2), x)
```

maple [B] time = 0.25, size = 2729, normalized size = 9.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x)
[Out] -2*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/d^3*x-2/3*
b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/c/d^3*polylog(2,-(c*x+(c^2*x^2+
1)^(1/2))^2)+4*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4
)/d^3*arcsinh(c*x)^2*x+2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+
11*c^2*x^2+4)/d^3*(c^2*x^2+1)*x-2*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c
^4*x^4+11*c^2*x^2+4)/d^3*arcsinh(c*x)*x+4/3*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x
^2+1))^(1/2)/c/d^3*arcsinh(c*x)^2-2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+
10*c^4*x^4+11*c^2*x^2+4)*c^6/d^3*x^7-3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6
+10*c^4*x^4+11*c^2*x^2+4)*c^4/d^3*x^5-13/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6
*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^2/d^3*x^3+4/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*
c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/c/d^3*(c^2*x^2+1)^(1/2)+1/3*a^2*x/d/(c^2*d
*x^2+d)^(3/2)+2/3*a^2/d^2*x/(c^2*d*x^2+d)^(1/2)-28/3*a*b*(d*(c^2*x^2+1))^(1
/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2
)*x^2-4*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^3/d
^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^4+10/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6
*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^2/d^3*(c^2*x^2+1)*x^3+34/3*a*b*(d*(c^2*x^2+
1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^2/d^3*arcsinh(c*x)*x^3+a*b*
(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c/d^3*x^2*(c^2*x^
2+1)^(1/2)-16/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+
4)/c/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-2*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*
x^6+10*c^4*x^4+11*c^2*x^2+4)*c^3/d^3*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*x^4+1
0/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^2/d^3*a
rcsinh(c*x)*(c^2*x^2+1)*x^3-14/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^
4*x^4+11*c^2*x^2+4)*c/d^3*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*x^2+b^2*(d*(c^2*
x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c/d^3*arcsinh(c*x)*(c^2*x
^2+1)^(1/2)*x^2+4/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*
x^2+4)*c^4/d^3*(c^2*x^2+1)*x^5+4*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^
4*x^4+11*c^2*x^2+4)*c^4/d^3*arcsinh(c*x)*x^5+4/3*b^2*(d*(c^2*x^2+1))^(1/2)/
(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^4/d^3*arcsinh(c*x)*(c^2*x^2+1)*x^5-2*
b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/d^3*x+4/3*b^2
*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^2/d^3*(c^2*x^2
+1)*x^3-16/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*
c^2/d^3*arcsinh(c*x)*x^3+7/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^
4+11*c^2*x^2+4)*c/d^3*x^2*(c^2*x^2+1)^(1/2)-8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(
3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/c/d^3*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)+4
/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/c/d^3*arcs
inh(c*x)*(c^2*x^2+1)^(1/2)+2*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^
4+11*c^2*x^2+4)/d^3*arcsinh(c*x)*(c^2*x^2+1)*x-4/3*b^2/(c^2*x^2+1)^(1/2)*(d
*(c^2*x^2+1))^(1/2)/c/d^3*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-4/3*
b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^6/d^3*arcsi
nh(c*x)*x^7+2*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)
```

```
*c^4/d^3*arcsinh(c*x)^2*x^5+2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4
*x^4+11*c^2*x^2+4)*c^4/d^3*(c^2*x^2+1)*x^5-14/3*b^2*(d*(c^2*x^2+1))^(1/2)/(
3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^4/d^3*arcsinh(c*x)*x^5+8/3*a*b/(c^2*x^
2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/c/d^3*arcsinh(c*x)-4/3*a*b*(d*(c^2*x^2+1))
^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^6/d^3*x^7-14/3*a*b*(d*(c^2*x^2
+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^4/d^3*x^5-16/3*a*b*(d*(c^2
*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^2/d^3*x^3+2*a*b*(d*(c^
2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/d^3*(c^2*x^2+1)*x+8*a*b
*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/d^3*arcsinh(c*x)
*x+4/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/c/d^3*
(c^2*x^2+1)^(1/2)-4/3*a*b/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/c/d^3*ln(
1+(c*x+(c^2*x^2+1)^(1/2))^2)+b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^
4+11*c^2*x^2+4)*c^3/d^3*(c^2*x^2+1)^(1/2)*x^4+17/3*b^2*(d*(c^2*x^2+1))^(1/2
)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^2/d^3*arcsinh(c*x)^2*x^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}abc\left(\frac{1}{c^4d^{\frac{5}{2}}x^2+c^2d^{\frac{5}{2}}}-\frac{2\log\left(c^2x^2+1\right)}{c^2d^{\frac{5}{2}}}\right)+\frac{2}{3}ab\left(\frac{2x}{\sqrt{c^2dx^2+d}\,d^2}+\frac{x}{\left(c^2dx^2+d\right)^{\frac{3}{2}}d}\right)\operatorname{arsinh}(cx)+\frac{1}{3}a^2\left(\frac{2x}{\sqrt{c^2dx^2+\cdots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*b*c*(1/(c^4*d^(5/2)*x^2 + c^2*d^(5/2)) - 2*log(c^2*x^2 + 1)/(c^2*d^(5
/2))) + 2/3*a*b*(2*x/(sqrt(c^2*d*x^2 + d)*d^2) + x/((c^2*d*x^2 + d)^(3/2)*d
))*arcsinh(c*x) + 1/3*a^2*(2*x/(sqrt(c^2*d*x^2 + d)*d^2) + x/((c^2*d*x^2 +
d)^(3/2)*d)) + b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d*x^2 + d)
^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d\,c^2\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(d + c^2*d*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{asinh}{\left(cx \right)}\right)^{2}}{\left(d \left(c^{2} x^{2} + 1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(d*(c**2*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(5/2), x)
```

3.316 $\int \frac{(a+b\sinh^{-1}(cx))^2}{x(d+c^2dx^2)^{5/2}}\,dx$

Optimal. Leaf size=518

$$-\frac{2b\sqrt{c^2x^2+1}\,\text{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{c^2dx^2+d}}+\frac{2b\sqrt{c^2x^2+1}\,\text{Li}_2\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{c^2dx^2+d}}-\frac{bcx\left(a+b\right.}{3d^2\sqrt{c^2x^2+}}$$

```
[Out] 1/3*(a+b*arcsinh(c*x))^2/d/(c^2*d*x^2+d)^(3/2)-1/3*b^2/d^2/(c^2*d*x^2+d)^(1
/2)+(a+b*arcsinh(c*x))^2/d^2/(c^2*d*x^2+d)^(1/2)-1/3*b*c*x*(a+b*arcsinh(c*x
))/d^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-14/3*b*(a+b*arcsinh(c*x))*arct
an(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)-2*(a+b*
arcsinh(c*x))^2*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d
*x^2+d)^(1/2)-2*b*(a+b*arcsinh(c*x))*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^2
*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)+7/3*I*b^2*polylog(2,-I*(c*x+(c^2*x^2+
1)^(1/2)))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)-7/3*I*b^2*polylog(2,I*
(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)+2*b*(a+b
*arcsinh(c*x))*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2*
d*x^2+d)^(1/2)+2*b^2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^
2/(c^2*d*x^2+d)^(1/2)-2*b^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1
/2)/d^2/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.86, antiderivative size = 518, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {5755, 5764, 5760, 4182, 2531, 2282, 6589, 5693, 4180, 2279, 2391, 5690, 261}

$$-\frac{2b\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{c^2dx^2+d}}+\frac{2b\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx\right.}{d^2\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] -b^2/(3*d^2*Sqrt[d + c^2*d*x^2]) - (b*c*x*(a + b*ArcSinh[c*x]))/(3*d^2*Sqrt
[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) + (a + b*ArcSinh[c*x])^2/(3*d*(d + c^2*d
*x^2)^(3/2)) + (a + b*ArcSinh[c*x])^2/(d^2*Sqrt[d + c^2*d*x^2]) - (14*b*Sqr
t[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(3*d^2*Sqrt[d +
 c^2*d*x^2]) - (2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSin
h[c*x]])/(d^2*Sqrt[d + c^2*d*x^2]) - (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[
c*x])*PolyLog[2, -E^ArcSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2]) + (((7*I)/3)*b
^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x
^2]) - (((7*I)/3)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I*E^ArcSinh[c*x]])/(d^2*
Sqrt[d + c^2*d*x^2]) + (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[
2, E^ArcSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2]) + (2*b^2*Sqrt[1 + c^2*x^2]*Po
lyLog[3, -E^ArcSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2]) - (2*b^2*Sqrt[1 + c^2*
x^2]*PolyLog[3, E^ArcSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
```

```
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 5764

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[
((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&  !GtQ[d, 0] && (Integer
Q[m] || EqQ[n, 1])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)^{5/2}}dx = \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)^{3/2}}dx}{d} - \frac{\left(2bc\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{\left(1+c^2x^2\right)^2}dx}{3d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^2\sqrt{d+c^2dx^2}} + \int \frac{(a+}{x}$$

$$= -\frac{b^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{(a+b\sin}{d^2\sqrt{d-}}$$

$$= -\frac{b^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{(a+b\sin}{d^2\sqrt{d-}}$$

$$= -\frac{b^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{(a+b\sin}{d^2\sqrt{d-}}$$

$$= -\frac{b^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{(a+b\sin}{d^2\sqrt{d-}}$$

$$= -\frac{b^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{(a+b\sin}{d^2\sqrt{d-}}$$

$$= -\frac{b^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{(a+b\sin}{d^2\sqrt{d-}}$$

Mathematica [A] time = 3.81, size = 547, normalized size = 1.06

$$\frac{a^2\left(3c^2x^2+4\right)\sqrt{c^2dx^2+d}}{\left(c^2x^2+1\right)^2} - 3a^2\sqrt{d}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+d\right) + 3a^2\sqrt{d}\log(cx) + \frac{abd^2\left(c^2x^2+1\right)^{3/2}\left(-\frac{cx}{c^2x^2+1}+\frac{6\sinh^{-1}(cx)}{\sqrt{c^2x^2+1}}+\frac{2\sinh^{-}}{(c^2x^2+}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] ((a^2*(4 + 3*c^2*x^2)*Sqrt[d + c^2*d*x^2])/(1 + c^2*x^2)^2 + 3*a^2*Sqrt[d]*
Log[c*x] - 3*a^2*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (a*b*d^2*(1
 + c^2*x^2)^(3/2)*(-((c*x)/(1 + c^2*x^2)) + (2*ArcSinh[c*x])/(1 + c^2*x^2)^
(3/2) + (6*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - 14*ArcTan[Tanh[ArcSinh[c*x]/2]
] + 6*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 6*ArcSinh[c*x]*Log[1 + E^(-
ArcSinh[c*x])] + 6*PolyLog[2, -E^(-ArcSinh[c*x])] - 6*PolyLog[2, E^(-ArcSin
h[c*x])]))/(d + c^2*d*x^2)^(3/2) + (b^2*d^2*(1 + c^2*x^2)^(3/2)*(-(1/Sqrt[1
 + c^2*x^2]) - (c*x*ArcSinh[c*x])/(1 + c^2*x^2) + ArcSinh[c*x]^2/(1 + c^2*x
^2)^(3/2) + (3*ArcSinh[c*x]^2)/Sqrt[1 + c^2*x^2] + 3*ArcSinh[c*x]^2*Log[1 -
 E^(-ArcSinh[c*x])] + (7*I)*ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]] - (7*I)*
ArcSinh[c*x]*Log[1 + I/E^ArcSinh[c*x]] - 3*ArcSinh[c*x]^2*Log[1 + E^(-ArcSi
nh[c*x])] + 6*ArcSinh[c*x]*PolyLog[2, -E^(-ArcSinh[c*x])] + (7*I)*PolyLog[2
, (-I)/E^ArcSinh[c*x]] - (7*I)*PolyLog[2, I/E^ArcSinh[c*x]] - 6*ArcSinh[c*x
]*PolyLog[2, E^(-ArcSinh[c*x])] + 6*PolyLog[3, -E^(-ArcSinh[c*x])] - 6*Poly
Log[3, E^(-ArcSinh[c*x])]))/(d + c^2*d*x^2)^(3/2))/(3*d^3)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(b^2\operatorname{arsinh}(cx)^2+2\,ab\operatorname{arsinh}(cx)+a^2\right)}{c^6d^3x^7+3\,c^4d^3x^5+3\,c^2d^3x^3+d^3x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^6*d^3*x^7 + 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 + d^3*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\operatorname{arsinh}(cx)+a)^2}{\left(c^2dx^2+d\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^(5/2)*x), x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int\frac{(a+b\operatorname{arcsinh}(cx))^2}{x\left(c^2d\,x^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}a^2\left(\frac{3\operatorname{arsinh}\left(\frac{1}{c|x|}\right)}{d^{\frac{5}{2}}}-\frac{3}{\sqrt{c^2dx^2+d}\,d^2}-\frac{1}{\left(c^2dx^2+d\right)^{\frac{3}{2}}d}\right)+\int\frac{b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{\left(c^2dx^2+d\right)^{\frac{5}{2}}x}+\frac{2\,ab\log\left(cx+\sqrt{c^2x^2+\right.}}{\left(c^2dx^2+d\right)^{\frac{5}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*a^2*(3*arcsinh(1/(c*abs(x)))/d^(5/2) - 3/(sqrt(c^2*d*x^2 + d)*d^2) - 1
/((c^2*d*x^2 + d)^(3/2)*d)) + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/
((c^2*d*x^2 + d)^(5/2)*x) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/((c^2*d*x^2
+ d)^(5/2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{(a+b\operatorname{asinh}(c\,x))^2}{x\left(d\,c^2\,x^2+d\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)^(5/2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(x*(d + c^2*d*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{asinh}\,(cx))^2}{x\left(d\left(c^2x^2 + 1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x/(c**2*d*x**2+d)**(5/2),x)
[Out] Integral((a + b*asinh(c*x))**2/(x*(d*(c**2*x**2 + 1))**(5/2)), x)
```

3.317 $\int \frac{(a+b\sinh^{-1}(cx))^2}{x^2(d+c^2dx^2)^{5/2}}\,dx$

Optimal. Leaf size=421

$$-\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}-\frac{8c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2\sqrt{c^2dx^2+d}}-\frac{8c\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2\sqrt{c^2dx^2+d}}+\frac{16bc\sqrt{c^2x^2+1}\log\left(e^2\right.}{3d^2}$$

```
[Out] -(a+b*arcsinh(c*x))^2/d/x/(c^2*d*x^2+d)^(3/2)-4/3*c^2*x*(a+b*arcsinh(c*x))^
2/d/(c^2*d*x^2+d)^(3/2)+1/3*b^2*c^2*x/d^2/(c^2*d*x^2+d)^(1/2)-8/3*c^2*x*(a+
b*arcsinh(c*x))^2/d^2/(c^2*d*x^2+d)^(1/2)-1/3*b*c*(a+b*arcsinh(c*x))/d^2/(c
^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-8/3*c*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^
(1/2)/d^2/(c^2*d*x^2+d)^(1/2)-4*b*c*(a+b*arcsinh(c*x))*arctanh((c*x+(c^2*x^
2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)+16/3*b*c*(a+b*arcs
inh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+
d)^(1/2)+5/3*b^2*c*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/
d^2/(c^2*d*x^2+d)^(1/2)+b^2*c*polylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2
+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.66, antiderivative size = 421, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 14, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5747, 5690, 5687, 5714, 3718, 2190, 2279, 2391, 5717, 191, 5755, 5720, 5461, 4182}

$$\frac{5b^2c\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{b^2c\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{d^2\sqrt{c^2dx^2+d}}-\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}-\frac{8c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] (b^2*c^2*x)/(3*d^2*Sqrt[d + c^2*d*x^2]) - (b*c*(a + b*ArcSinh[c*x]))/(3*d^2
*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (a + b*ArcSinh[c*x])^2/(d*x*(d +
c^2*d*x^2)^(3/2)) - (4*c^2*x*(a + b*ArcSinh[c*x])^2)/(3*d*(d + c^2*d*x^2)^(
3/2)) - (8*c^2*x*(a + b*ArcSinh[c*x])^2)/(3*d^2*Sqrt[d + c^2*d*x^2]) - (8*c
*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(3*d^2*Sqrt[d + c^2*d*x^2]) - (4
*b*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/(d
^2*Sqrt[d + c^2*d*x^2]) + (16*b*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Lo
g[1 + E^(2*ArcSinh[c*x])])/(3*d^2*Sqrt[d + c^2*d*x^2]) + (5*b^2*c*Sqrt[1 +
c^2*x^2]*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*d^2*Sqrt[d + c^2*d*x^2]) + (b^
2*c*Sqrt[1 + c^2*x^2]*PolyLog[2, E^(2*ArcSinh[c*x])])/(d^2*Sqrt[d + c^2*d*x
^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
```

```
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5720

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)^{5/2}}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\left(d+c^2dx^2\right)^{3/2}} - \left(4c^2\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx + \frac{\left(2bc\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{x(1+\ldots}}{d^2\sqrt{d+c^2dx^2}}$$

$$= \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\left(d+c^2dx^2\right)^{3/2}} - \frac{4c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} - \frac{\left(8c\ldots\right.}{\ldots}$$

$$= -\frac{b^2c^2x}{d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\left(d+c^2dx^2\right)^{3/2}} - \frac{4c^2x\left(a+b\ldots\right.}{3d\left(d+\ldots\right.}$$

$$= \frac{b^2c^2x}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\left(d+c^2dx^2\right)^{3/2}} - \frac{4c^2x\left(a+b\ldots\right.}{3d\left(d+\ldots\right.}$$

$$= \frac{b^2c^2x}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\left(d+c^2dx^2\right)^{3/2}} - \frac{4c^2x\left(a+b\ldots\right.}{3d\left(d+\ldots\right.}$$

$$= \frac{b^2c^2x}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\left(d+c^2dx^2\right)^{3/2}} - \frac{4c^2x\left(a+b\ldots\right.}{3d\left(d+\ldots\right.}$$

$$= \frac{b^2c^2x}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\left(d+c^2dx^2\right)^{3/2}} - \frac{4c^2x\left(a+b\ldots\right.}{3d\left(d+\ldots\right.}$$

$$= \frac{b^2c^2x}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\left(d+c^2dx^2\right)^{3/2}} - \frac{4c^2x\left(a+b\ldots\right.}{3d\left(d+\ldots\right.}$$

Mathematica [A] time = 1.88, size = 408, normalized size = 0.97

$$-\frac{8a^2c^4x^4+12a^2c^2x^2+3a^2+16abc^4x^4\sinh^{-1}(cx)+abcx\sqrt{c^2x^2+1}-6abcx\left(c^2x^2+1\right)^{3/2}\log(cx)-5abcx\left(c^2\ldots\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] -1/3*(3*a^2 + 12*a^2*c^2*x^2 - b^2*c^2*x^2 + 8*a^2*c^4*x^4 - b^2*c^4*x^4 +
a*b*c*x*Sqrt[1 + c^2*x^2] + 6*a*b*ArcSinh[c*x] + 24*a*b*c^2*x^2*ArcSinh[c*x
] + 16*a*b*c^4*x^4*ArcSinh[c*x] + b^2*c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] +
3*b^2*ArcSinh[c*x]^2 + 12*b^2*c^2*x^2*ArcSinh[c*x]^2 + 8*b^2*c^4*x^4*ArcSin
h[c*x]^2 - 8*b^2*c*x*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]^2 - 6*b^2*c*x*(1 + c^
2*x^2)^(3/2)*ArcSinh[c*x]*Log[1 - E^(-2*ArcSinh[c*x])] - 10*b^2*c*x*(1 + c^
2*x^2)^(3/2)*ArcSinh[c*x]*Log[1 + E^(-2*ArcSinh[c*x])] - 6*a*b*c*x*(1 + c^2
*x^2)^(3/2)*Log[c*x] - 5*a*b*c*x*(1 + c^2*x^2)^(3/2)*Log[1 + c^2*x^2] + 5*b
^2*c*x*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(-2*ArcSinh[c*x])] + 3*b^2*c*x*(1
+ c^2*x^2)^(3/2)*PolyLog[2, E^(-2*ArcSinh[c*x])])/(d*x*(d + c^2*d*x^2)^(3/2
))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(b^2\operatorname{arsinh}(cx)^2+2\,ab\operatorname{arsinh}(cx)+a^2\right)}{c^6d^3x^8+3\,c^4d^3x^6+3\,c^2d^3x^4+d^3x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^6*d^3*x^8 + 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4 + d^3*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2}{(c^2dx^2 + d)^{\frac{5}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^(5/2)*x^2), x)
```

maple [B] time = 0.42, size = 3517, normalized size = 8.35

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] -40*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^7*c
^8-160/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*
x^5*c^6-29*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^
3*x^3*c^4-16/3*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/d^3*arcsinh(c*x)
^2*c+2*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/d^3*polylog(2,c*x+(c^2*x
^2+1)^(1/2))*c+5/3*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/d^3*polylog(
2,-(c*x+(c^2*x^2+1)^(1/2))^2)*c+2*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/
2)/d^3*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c-5*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c
^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x*c^2-9*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^
6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3/x*arcsinh(c*x)^2-3*b^2*(d*(c^2*x^2+1))^(
1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*c*(c^2*x^2+1)^(1/2)-32/3*b^2*(
d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^9*c^10-a^2/d
/x/(c^2*d*x^2+d)^(3/2)+128/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^
4+26*c^2*x^2+9)/d^3*x^4*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^5+272/3*a*b*(d*(c^
2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^2*(c^2*x^2+1)^(1/
2)*arcsinh(c*x)*c^3+64/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26
*c^2*x^2+9)/d^3*x^7*(c^2*x^2+1)*c^8+160/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*
x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^5*(c^2*x^2+1)*c^6-128/3*a*b*(d*(c^2*x^2+
1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^5*arcsinh(c*x)*c^6+40*a
*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^3*(c^2*x
^2+1)*c^4-112*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)
/d^3*x^3*arcsinh(c*x)*c^4-8/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x
^4+26*c^2*x^2+9)/d^3*x^2*c^3*(c^2*x^2+1)^(1/2)+8*a*b*(d*(c^2*x^2+1))^(1/2)/
(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x*(c^2*x^2+1)*c^2-88*a*b*(d*(c^2*x^
2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x*arcsinh(c*x)*c^2+48*a
*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*(c^2*x^2+1
)^(1/2)*arcsinh(c*x)*c+64/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4
+26*c^2*x^2+9)/d^3*x^7*(c^2*x^2+1)*arcsinh(c*x)*c^8+160/3*b^2*(d*(c^2*x^2+1
))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^5*(c^2*x^2+1)*arcsinh(c*
x)*c^6+64/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d
^3*x^4*(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^5+40*b^2*(d*(c^2*x^2+1))^(1/2)/(8
*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^3*(c^2*x^2+1)*arcsinh(c*x)*c^4+136/
3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^2*(c^
2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^3-8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+
```

```
25*c^4*x^4+26*c^2*x^2+9)/d^3*x^2*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^3+8*b^2*(
d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x*(c^2*x^2+1)*
arcsinh(c*x)*c^2-4/3*a^2*c^2*x/d/(c^2*d*x^2+d)^(3/2)+80/3*b^2*(d*(c^2*x^2+1
))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^3*(c^2*x^2+1)*c^4-56*b^2
*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^3*arcsinh(
c*x)^2*c^4-48*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)
/d^3*x^3*arcsinh(c*x)*c^4-280/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4
*x^4+26*c^2*x^2+9)/d^3*x^5*arcsinh(c*x)*c^6-64/3*b^2*(d*(c^2*x^2+1))^(1/2)/
(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^5*arcsinh(c*x)^2*c^6-32/3*a*b/(c^
2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/d^3*arcsinh(c*x)*c-64/3*a*b*(d*(c^2*x^
2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^9*c^10-224/3*a*b*(d*(
c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^7*c^8+24*b^2*(d
*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*(c^2*x^2+1)^(1/
2)*arcsinh(c*x)^2*c-280/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+2
6*c^2*x^2+9)/d^3*x^5*c^6-48*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4
+26*c^2*x^2+9)/d^3*x^3*c^4-8*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^
4+26*c^2*x^2+9)/d^3*x*c^2-3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4
+26*c^2*x^2+9)/d^3*c*(c^2*x^2+1)^(1/2)-18*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*
x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3/x*arcsinh(c*x)+10/3*a*b/(c^2*x^2+1)^(1/2)*
(d*(c^2*x^2+1))^(1/2)/d^3*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*c+2*a*b/(c^2*x^2+
1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/d^3*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c-3*b^2*
(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*(c^2*x^2+1)^(
1/2)*arcsinh(c*x)*c+88/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26
*c^2*x^2+9)/d^3*x^5*(c^2*x^2+1)*c^6-17/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x
^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^2*c^3*(c^2*x^2+1)^(1/2)+8*b^2*(d*(c^2*x^2
+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x*(c^2*x^2+1)*c^2-44*b^2
*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x*arcsinh(c*
x)^2*c^2-8*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^
3*x*arcsinh(c*x)*c^2+2*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/d^3*arcs
inh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c+2*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+
1))^(1/2)/d^3*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c+10/3*b^2/(c^2*x^2+
1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/d^3*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2)
)^2)*c-8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^
3*x^4*c^5*(c^2*x^2+1)^(1/2)-64/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^
4*x^4+26*c^2*x^2+9)/d^3*x^9*arcsinh(c*x)*c^10+32/3*b^2*(d*(c^2*x^2+1))^(1/2
)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^7*(c^2*x^2+1)*c^8-224/3*b^2*(d*
(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^7*arcsinh(c*x)
*c^8-8/3*a^2*c^2/d^2*x/(c^2*d*x^2+d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}a^2\left(\frac{8c^2x}{\sqrt{c^2dx^2+d}\,d^2}+\frac{4c^2x}{\left(c^2dx^2+d\right)^{\frac{3}{2}}d}+\frac{3}{\left(c^2dx^2+d\right)^{\frac{3}{2}}dx}\right)+\int\frac{b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{\left(c^2dx^2+d\right)^{\frac{5}{2}}x^2}+\frac{2ab\log\left(cx+\sqrt{c^2x^2+1}\right)}{\left(c^2dx^2+d\right)^{\frac{5}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima
")
```

```
[Out] -1/3*a^2*(8*c^2*x/(sqrt(c^2*d*x^2 + d)*d^2) + 4*c^2*x/((c^2*d*x^2 + d)^(3/2
)*d) + 3/((c^2*d*x^2 + d)^(3/2)*d*x)) + integrate(b^2*log(c*x + sqrt(c^2*x^
2 + 1))^2/((c^2*d*x^2 + d)^(5/2)*x^2) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/
((c^2*d*x^2 + d)^(5/2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{x^2\,{\left(d\,c^2\,x^2+d\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)^(5/2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(x^2*(d + c^2*d*x^2)^(5/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asinh}(cx))^2}{x^2 \left(d \left(c^2 x^2 + 1\right)\right)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**2/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(x**2*(d*(c**2*x**2 + 1))**(5/2)), x)
```

3.318 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^3\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=687

$$\frac{5bc^2\sqrt{c^2x^2+1}\,\mathrm{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{c^2dx^2+d}}-\frac{5bc^2\sqrt{c^2x^2+1}\,\mathrm{Li}_2\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{c^2dx^2+d}}-\frac{5c^2\left(a\right.}{2d^2}$$

```
[Out] -5/6*c^2*(a+b*arcsinh(c*x))^2/d/(c^2*d*x^2+d)^(3/2)-1/2*(a+b*arcsinh(c*x))^
2/d/x^2/(c^2*d*x^2+d)^(3/2)+1/3*b^2*c^2/d^2/(c^2*d*x^2+d)^(1/2)-5/2*c^2*(a+
b*arcsinh(c*x))^2/d^2/(c^2*d*x^2+d)^(1/2)-b*c*(a+b*arcsinh(c*x))/d^2/x/(c^2
*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)-2/3*b*c^3*x*(a+b*arcsinh(c*x))/d^2/(c^2*x
^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+26/3*b*c^2*(a+b*arcsinh(c*x))*arctan(c*x+(c
^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)+5*c^2*(a+b*arcsi
nh(c*x))^2*arctanh(c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+
d)^(1/2)-b^2*c^2*arctanh((c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^
2+d)^(1/2)+5*b*c^2*(a+b*arcsinh(c*x))*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*(c^
2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)-13/3*I*b^2*c^2*polylog(2,-I*(c*x+(c^
2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)+13/3*I*b^2*c^2*p
olylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/
2)-5*b*c^2*(a+b*arcsinh(c*x))*polylog(2,c*x+(c^2*x^2+1)^(1/2))*(c^2*x^2+1)^
(1/2)/d^2/(c^2*d*x^2+d)^(1/2)-5*b^2*c^2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*(
c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)+5*b^2*c^2*polylog(3,c*x+(c^2*x^2+1
)^(1/2))*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 1.26, antiderivative size = 687, normalized size of antiderivative = 1.00, number of steps used = 39, number of rules used = 18, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5747, 5755, 5764, 5760, 4182, 2531, 2282, 6589, 5693, 4180, 2279, 2391, 5690, 261, 266, 51, 63, 208}

$$\frac{5bc^2\sqrt{c^2x^2+1}\,\mathrm{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{c^2dx^2+d}}-\frac{5bc^2\sqrt{c^2x^2+1}\,\mathrm{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh\right.}{d^2\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] (b^2*c^2)/(3*d^2*Sqrt[d + c^2*d*x^2]) - (b*c*(a + b*ArcSinh[c*x]))/(d^2*x*S
qrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (2*b*c^3*x*(a + b*ArcSinh[c*x]))/(3
*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (5*c^2*(a + b*ArcSinh[c*x])^2
)/(6*d*(d + c^2*d*x^2)^(3/2)) - (a + b*ArcSinh[c*x])^2/(2*d*x^2*(d + c^2*d*
x^2)^(3/2)) - (5*c^2*(a + b*ArcSinh[c*x])^2)/(2*d^2*Sqrt[d + c^2*d*x^2]) +
(26*b*c^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(3
*d^2*Sqrt[d + c^2*d*x^2]) + (5*c^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2
*ArcTanh[E^ArcSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2]) - (b^2*c^2*Sqrt[1 + c^2
*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]])/(d^2*Sqrt[d + c^2*d*x^2]) + (5*b*c^2*Sqrt
[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcSinh[c*x]])/(d^2*Sqrt[d
 + c^2*d*x^2]) - (((13*I)/3)*b^2*c^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)*E^Ar
cSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2]) + (((13*I)/3)*b^2*c^2*Sqrt[1 + c^2*x
^2]*PolyLog[2, I*E^ArcSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2]) - (5*b*c^2*Sqrt
[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, E^ArcSinh[c*x]])/(d^2*Sqrt[d
+ c^2*d*x^2]) - (5*b^2*c^2*Sqrt[1 + c^2*x^2]*PolyLog[3, -E^ArcSinh[c*x]])/(
d^2*Sqrt[d + c^2*d*x^2]) + (5*b^2*c^2*Sqrt[1 + c^2*x^2]*PolyLog[3, E^ArcSin
h[c*x]])/(d^2*Sqrt[d + c^2*d*x^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
```

```
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 5764

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[
((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&  !GtQ[d, 0] && (Integer
Q[m] || EqQ[n, 1])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^3\left(d+c^2dx^2\right)^{5/2}}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2\left(d+c^2dx^2\right)^{3/2}} - \frac{1}{2}\left(5c^2\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)^{5/2}}\,dx + \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh}{x^2(1+c}}{d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{5c^2\left(a+b\sinh^{-1}(cx)\right)^2}{6d\left(d+c^2dx^2\right)^{3/2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2\left(d+c^2dx^2\right)^{3/2}} - \frac{\left(5c^2\right)}{}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{2bc^3x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{5c^2\left(a+b\sinh^{-1}(cx)\right)^2}{6d\left(d+c^2dx^2\right)^{3/2}} -$$

$$= \frac{b^2c^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{2bc^3x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{5c^2\left(a+\right.}{6d\left(\right.}$$

$$= \frac{b^2c^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{2bc^3x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{5c^2\left(a+\right.}{6d\left(\right.}$$

$$= \frac{b^2c^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{2bc^3x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{5c^2\left(a+\right.}{6d\left(\right.}$$

$$= \frac{b^2c^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{2bc^3x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{5c^2\left(a+\right.}{6d\left(\right.}$$

$$= \frac{b^2c^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{2bc^3x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{5c^2\left(a+\right.}{6d\left(\right.}$$

$$= \frac{b^2c^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{2bc^3x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{5c^2\left(a+\right.}{6d\left(\right.}$$

Mathematica [A] time = 8.02, size = 983, normalized size = 1.43

$$-\frac{5a^2\log(x)c^2}{2d^{5/2}}+\frac{5a^2\log\left(d+\sqrt{d\left(c^2x^2+1\right)}\sqrt{d}\right)c^2}{2d^{5/2}}+\frac{ab\left(-3\sqrt{c^2x^2+1}\,\sinh^{-1}(cx)\mathrm{csch}^2\left(\frac{1}{2}\sinh^{-1}(cx)\right)-3\sqrt{c^2\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] Sqrt[d*(1 + c^2*x^2)]*(-1/2*a^2/(d^3*x^2) - (a^2*c^2)/(3*d^3*(1 + c^2*x^2)^
2) - (2*a^2*c^2)/(d^3*(1 + c^2*x^2))) - (5*a^2*c^2*Log[x])/(2*d^(5/2)) + (5
*a^2*c^2*Log[d + Sqrt[d]*Sqrt[d*(1 + c^2*x^2)]])/(2*d^(5/2)) + (a*b*c^2*((4
*c*x)/Sqrt[1 + c^2*x^2] - 48*ArcSinh[c*x] - (8*ArcSinh[c*x])/(1 + c^2*x^2)
+ 104*Sqrt[1 + c^2*x^2]*ArcTan[Tanh[ArcSinh[c*x]/2]] - 6*Sqrt[1 + c^2*x^2]*
Coth[ArcSinh[c*x]/2] - 3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Csch[ArcSinh[c*x]/2
]^2 - 60*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] + 60*Sqr
t[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] - 60*Sqrt[1 + c^2*x^
2]*PolyLog[2, -E^(-ArcSinh[c*x])] + 60*Sqrt[1 + c^2*x^2]*PolyLog[2, E^(-Arc
Sinh[c*x])] - 3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 6*S
qrt[1 + c^2*x^2]*Tanh[ArcSinh[c*x]/2]))/(12*d^2*Sqrt[d*(1 + c^2*x^2)]) + (b
^2*c^2*(8 + (8*c*x*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - 48*ArcSinh[c*x]^2 - (8
*ArcSinh[c*x]^2)/(1 + c^2*x^2) - 12*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Coth[Arc
Sinh[c*x]/2] - 3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2*Csch[ArcSinh[c*x]/2]^2 -
60*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2*Log[1 - E^(-ArcSinh[c*x])] - (104*I)*Sq
rt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]] + (104*I)*Sqrt[1 + c
^2*x^2]*ArcSinh[c*x]*Log[1 + I/E^ArcSinh[c*x]] + 60*Sqrt[1 + c^2*x^2]*ArcSi
nh[c*x]^2*Log[1 + E^(-ArcSinh[c*x])] + 24*Sqrt[1 + c^2*x^2]*Log[Tanh[ArcSin
h[c*x]/2]] - 120*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*PolyLog[2, -E^(-ArcSinh[c*x
])] - (104*I)*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)/E^ArcSinh[c*x]] + (104*I)*S
qrt[1 + c^2*x^2]*PolyLog[2, I/E^ArcSinh[c*x]] + 120*Sqrt[1 + c^2*x^2]*ArcSi
nh[c*x]*PolyLog[2, E^(-ArcSinh[c*x])] - 120*Sqrt[1 + c^2*x^2]*PolyLog[3, -E
^(-ArcSinh[c*x])] + 120*Sqrt[1 + c^2*x^2]*PolyLog[3, E^(-ArcSinh[c*x])] - 3
*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2*Sech[ArcSinh[c*x]/2]^2 + 12*Sqrt[1 + c^2*
x^2]*ArcSinh[c*x]*Tanh[ArcSinh[c*x]/2]))/(24*d^2*Sqrt[d*(1 + c^2*x^2)])
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(b^2\,\mathrm{arsinh}\,(cx)^2+2\,ab\,\mathrm{arsinh}\,(cx)+a^2\right)}{c^6d^3x^9+3\,c^4d^3x^7+3\,c^2d^3x^5+d^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^6*d^3*x^9 + 3*c^4*d^3*x^7 + 3*c^2*d^3*x^5 + d^3*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\,\mathrm{arsinh}\,(cx)+a\right)^2}{\left(c^2dx^2+d\right)^{\frac{5}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^(5/2)*x^3), x)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^{2}}{x^{3}\left(c^{2}d\,x^{2}+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}\,a^{2}\left(\frac{15\,c^{2}\,\mathrm{arsinh}\left(\frac{1}{c|x|}\right)}{d^{\frac{5}{2}}}-\frac{15\,c^{2}}{\sqrt{c^{2}dx^{2}+d}\,d^{2}}-\frac{5\,c^{2}}{\left(c^{2}dx^{2}+d\right)^{\frac{3}{2}}d}-\frac{3}{\left(c^{2}dx^{2}+d\right)^{\frac{3}{2}}dx^{2}}\right)+\int\frac{b^{2}\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)^{2}}{\left(c^{2}dx^{2}+d\right)^{\frac{5}{2}}x^{3}}+\frac{2\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima
")
```

```
[Out] 1/6*a^2*(15*c^2*arcsinh(1/(c*abs(x)))/d^(5/2) - 15*c^2/(sqrt(c^2*d*x^2 + d)
*d^2) - 5*c^2/((c^2*d*x^2 + d)^(3/2)*d) - 3/((c^2*d*x^2 + d)^(3/2)*d*x^2))
+ integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/((c^2*d*x^2 + d)^(5/2)*x^3)
+ 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/((c^2*d*x^2 + d)^(5/2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{x^3\,{\left(d\,c^2\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)^(5/2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(x^3*(d + c^2*d*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{asinh}{\left(cx \right)}\right)^{2}}{x^{3} \left(d \left(c^{2} x^{2} + 1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**3/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(x**3*(d*(c**2*x**2 + 1))**(5/2)), x)
```

3.319 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^4\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=506

$$-\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}+\frac{2c^2\left(a+b\sinh^{-1}(cx)\right)^2}{dx\left(c^2dx^2+d\right)^{3/2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\left(c^2dx^2+d\right)^{3/2}}+\frac{16c^4x\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2\sqrt{c^2dx^2+d}}+\frac{8c^4x}{3}$$

```
[Out] -1/3*(a+b*arcsinh(c*x))^2/d/x^3/(c^2*d*x^2+d)^(3/2)+2*c^2*(a+b*arcsinh(c*x)
)^2/d/x/(c^2*d*x^2+d)^(3/2)+8/3*c^4*x*(a+b*arcsinh(c*x))^2/d/(c^2*d*x^2+d)^
(3/2)-1/3*b^2*c^2/d^2/x/(c^2*d*x^2+d)^(1/2)-2/3*b^2*c^4*x/d^2/(c^2*d*x^2+d)
^(1/2)+16/3*c^4*x*(a+b*arcsinh(c*x))^2/d^2/(c^2*d*x^2+d)^(1/2)-1/3*b*c*(a+b
*arcsinh(c*x))/d^2/x^2/(c^2*x^2+1)^(1/2)/(c^2*d*x^2+d)^(1/2)+16/3*c^3*(a+b*
arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)+32/3*b*c^3*(a+b*a
rcsinh(c*x))*arctanh((c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/d^2/(c^2*
d*x^2+d)^(1/2)-32/3*b*c^3*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2
)*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)-8/3*b^2*c^3*polylog(2,-(c*x+(c^
2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2)-8/3*b^2*c^3*po
lylog(2,(c*x+(c^2*x^2+1)^(1/2))^2)*(c^2*x^2+1)^(1/2)/d^2/(c^2*d*x^2+d)^(1/2
)
```

Rubi [A] time = 1.09, antiderivative size = 506, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 15, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.536, Rules used = {5747, 5690, 5687, 5714, 3718, 2190, 2279, 2391, 5717, 191, 5755, 5720, 5461, 4182, 271}

$$-\frac{8b^2c^3\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{3d^2\sqrt{c^2dx^2+d}}-\frac{8b^2c^3\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{16c^4x\left(a+b\sinh^{-1}(cx\right.}{3d^2\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] -(b^2*c^2)/(3*d^2*x*Sqrt[d + c^2*d*x^2]) - (2*b^2*c^4*x)/(3*d^2*Sqrt[d + c^
2*d*x^2]) - (b*c*(a + b*ArcSinh[c*x]))/(3*d^2*x^2*Sqrt[1 + c^2*x^2]*Sqrt[d
+ c^2*d*x^2]) - (a + b*ArcSinh[c*x])^2/(3*d*x^3*(d + c^2*d*x^2)^(3/2)) + (2
*c^2*(a + b*ArcSinh[c*x])^2)/(d*x*(d + c^2*d*x^2)^(3/2)) + (8*c^4*x*(a + b*
ArcSinh[c*x])^2)/(3*d*(d + c^2*d*x^2)^(3/2)) + (16*c^4*x*(a + b*ArcSinh[c*x
])^2)/(3*d^2*Sqrt[d + c^2*d*x^2]) + (16*c^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSin
h[c*x])^2)/(3*d^2*Sqrt[d + c^2*d*x^2]) + (32*b*c^3*Sqrt[1 + c^2*x^2]*(a + b
*ArcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/(3*d^2*Sqrt[d + c^2*d*x^2]) - (
32*b*c^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])]
)/(3*d^2*Sqrt[d + c^2*d*x^2]) - (8*b^2*c^3*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^
(2*ArcSinh[c*x])])/(3*d^2*Sqrt[d + c^2*d*x^2]) - (8*b^2*c^3*Sqrt[1 + c^2*x^
2]*PolyLog[2, E^(2*ArcSinh[c*x])])/(3*d^2*Sqrt[d + c^2*d*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
```

```
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

```
&& GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5720

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^4\left(d+c^2dx^2\right)^{5/2}}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\left(d+c^2dx^2\right)^{3/2}} - \left(2c^2\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)^{5/2}}\,dx + \frac{\left(2bc\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh}{x^3(1+c^2}}{3d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\left(d+c^2dx^2\right)^{3/2}} + \frac{2c^2\left(a+b\sinh^{-1}(cx)\right)^2}{dx\left(d+c^2dx^2\right)^{3/2}} + (8c$$

$$= -\frac{b^2c^2}{3d^2x\sqrt{d+c^2dx^2}} - \frac{8bc^3\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{(a}{3d}$$

$$= -\frac{b^2c^2}{3d^2x\sqrt{d+c^2dx^2}} + \frac{2b^2c^4x}{d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(\right.}{3dx^3\left(d+c^2dx\right.}$$

$$= -\frac{b^2c^2}{3d^2x\sqrt{d+c^2dx^2}} - \frac{2b^2c^4x}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}\right.}{3dx^3\left(d+c^2d\right.}$$

$$= -\frac{b^2c^2}{3d^2x\sqrt{d+c^2dx^2}} - \frac{2b^2c^4x}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}\right.}{3dx^3\left(d+c^2d\right.}$$

$$= -\frac{b^2c^2}{3d^2x\sqrt{d+c^2dx^2}} - \frac{2b^2c^4x}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}\right.}{3dx^3\left(d+c^2d\right.}$$

$$= -\frac{b^2c^2}{3d^2x\sqrt{d+c^2dx^2}} - \frac{2b^2c^4x}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}\right.}{3dx^3\left(d+c^2d\right.}$$

$$= -\frac{b^2c^2}{3d^2x\sqrt{d+c^2dx^2}} - \frac{2b^2c^4x}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}\right.}{3dx^3\left(d+c^2d\right.}$$

Mathematica [A] time = 3.05, size = 417, normalized size = 0.82

$$\frac{\frac{a^2\left(16c^6x^6+24c^4x^4+6c^2x^2-1\right)}{x^3} - \frac{ab\left(cx\sqrt{c^2x^2+1}\left(16\left(c^4x^4+c^2x^2\right)\log(cx)+8\left(c^4x^4+c^2x^2\right)\log\left(c^2x^2+1\right)+1\right)-2\left(16c^6x^6+24c^4x^4+6c^2x^2-1\right)\sinh^{-1}(cx)\right)}{x^3} + }{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] ((a^2*(-1 + 6*c^2*x^2 + 24*c^4*x^4 + 16*c^6*x^6))/x^3 - (a*b*(-2*(-1 + 6*c^
2*x^2 + 24*c^4*x^4 + 16*c^6*x^6)*ArcSinh[c*x] + c*x*Sqrt[1 + c^2*x^2]*(1 +
16*(c^2*x^2 + c^4*x^4)*Log[c*x] + 8*(c^2*x^2 + c^4*x^4)*Log[1 + c^2*x^2])))
/x^3 + b^2*c^3*(1 + c^2*x^2)^(3/2)*(-((c*x)/Sqrt[1 + c^2*x^2]) - Sqrt[1 + c
^2*x^2]/(c*x) - ArcSinh[c*x]/(c^2*x^2) + ArcSinh[c*x]/(1 + c^2*x^2) - 16*Ar
cSinh[c*x]^2 + (c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2)^(3/2) + (8*c*x*ArcSinh[c*
x]^2)/Sqrt[1 + c^2*x^2] - (Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2)/(c^3*x^3) + (8
*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2)/(c*x) - 16*ArcSinh[c*x]*Log[1 - E^(-2*Ar
cSinh[c*x])] - 16*ArcSinh[c*x]*Log[1 + E^(-2*ArcSinh[c*x])] + 8*PolyLog[2,
-E^(-2*ArcSinh[c*x])] + 8*PolyLog[2, E^(-2*ArcSinh[c*x])]))/(3*d*(d + c^2*d
*x^2)^(3/2))
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(b^2\,\text{arsinh}\left(cx\right)^2+2\,ab\,\text{arsinh}\left(cx\right)+a^2\right)}{c^6d^3x^{10}+3\,c^4d^3x^8+3\,c^2d^3x^6+d^3x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^6*d^3*x^10 + 3*c^4*d^3*x^8 + 3*c^2*d^3*x^6 + d^3*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\text{arsinh}\left(cx\right)+a\right)^2}{\left(c^2dx^2+d\right)^{\frac{5}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^(5/2)*x^4), x)
```

maple [B] time = 0.53, size = 4955, normalized size = 9.79

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] -16/3*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/d^3*polylog(2,-c*x-(c^2*x
^2+1)^(1/2))*c^3+2*a^2*c^2/d/x/(c^2*d*x^2+d)^(3/2)-256/3*a*b*(d*(c^2*x^2+1)
)^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^9*(c^2*x^2+1)
*c^12-640/3*a*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*
c^2*x^2-1)/d^3*x^7*(c^2*x^2+1)*c^10+128*a*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x
^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^7*arcsinh(c*x)*c^10-1/3*a^2/d/
x^3/(c^2*d*x^2+d)^(3/2)+8*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+
35*c^4*x^4+10*c^2*x^2-1)/d^3*x^6*(c^2*x^2+1)^(1/2)*c^9+160*b^2*(d*(c^2*x^2+
1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^5*arcsinh(c
*x)^2*c^8-256*a*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+1
0*c^2*x^2-1)/d^3*x^4*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^7-352/3*a*b*(d*(c^2*x
^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^2*arcsin
h(c*x)*(c^2*x^2+1)^(1/2)*c^5-128*a*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c
^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^6*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^9+
32/3*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/d^3*arcsinh(c*x)^2*c^3-16/
3*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/d^3*polylog(2,c*x+(c^2*x^2+1)
^(1/2))*c^3+64/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^
4+10*c^2*x^2-1)/d^3*x^11*c^14+224/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+3
6*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^9*c^12+88*b^2*(d*(c^2*x^2+1))^(1/2
)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^7*c^10+100/3*b^2*(d
*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^5
*c^8-14/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^
2*x^2-1)/d^3*x^3*c^6-3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*
c^4*x^4+10*c^2*x^2-1)/d^3*x*c^4+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+3
6*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3/x*c^2+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/
(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3/x^3*arcsinh(c*x)^2-8/3*
b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/d^3*polylog(2,-(c*x+(c^2*x^2+1)
^(1/2))^2)*c^3-2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*
```

x^4+10*c^2*x^2-1)/d^3*c^3*(c^2*x^2+1)^(1/2)-80/3*b^2*(d*(c^2*x^2+1))^(1/2)/
(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^3*arcsinh(c*x)*(c^2*x
^2+1)*c^6-176/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4
+10*c^2*x^2-1)/d^3*x^2*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*c^5-4*b^2*(d*(c^2*x
^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^2*arcsin
h(c*x)*(c^2*x^2+1)^(1/2)*c^5+16/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*
c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x*arcsinh(c*x)*(c^2*x^2+1)*c^4+1/3*b^2
(d(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3/
x^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c-64/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8
*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^9*(c^2*x^2+1)*c^12+896/3*b^2
(d(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*
x^9*arcsinh(c*x)*c^12+32/3*a*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6
+35*c^4*x^4+10*c^2*x^2-1)/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^3-160*a*b*(d
*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^5
*(c^2*x^2+1)*c^8+320*a*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^
4*x^4+10*c^2*x^2-1)/d^3*x^5*arcsinh(c*x)*c^8-80/3*a*b*(d*(c^2*x^2+1))^(1/2)
/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^3*(c^2*x^2+1)*c^6+68
8/3*a*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-
1)/d^3*x^3*arcsinh(c*x)*c^6-4*a*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*
x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^2*c^5*(c^2*x^2+1)^(1/2)+24*a*b*(d*(c^2*x
^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x*arcsinh(
c*x)*c^4+16/3*a*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+1
0*c^2*x^2-1)/d^3*x*(c^2*x^2+1)*c^4-12*a*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8
+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3/x*arcsinh(c*x)*c^2+1/3*a*b*(d*(c^2
*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3/x^2*c*(c
^2*x^2+1)^(1/2)-64*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*
x^4+10*c^2*x^2-1)/d^3*x^6*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*c^9-160*b^2*(d*(
c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^5*a
rcsinh(c*x)*(c^2*x^2+1)*c^8-128*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^
6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^4*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*c^7
-640/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x
^2-1)/d^3*x^7*arcsinh(c*x)*(c^2*x^2+1)*c^10-256/3*b^2*(d*(c^2*x^2+1))^(1/2)
/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^9*arcsinh(c*x)*(c^2*
x^2+1)*c^12-32/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^
4+10*c^2*x^2-1)/d^3*x^3*(c^2*x^2+1)*c^6+64/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*
c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^3*arcsinh(c*x)*c^6+22/3*b
^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^
3*x^2*c^5*(c^2*x^2+1)^(1/2)+12*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6
*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x*arcsinh(c*x)^2*c^4+64*b^2*(d*(c^2*x^2+1
))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^7*arcsinh(c*
x)^2*c^10-160/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4
+10*c^2*x^2-1)/d^3*x^7*(c^2*x^2+1)*c^10+1120/3*b^2*(d*(c^2*x^2+1))^(1/2)/(1
2*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^7*arcsinh(c*x)*c^10+560
/3*a*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1
)/d^3*x^5*c^8+16/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*
x^4+10*c^2*x^2-1)/d^3*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*c^3-4*b^2*(d*(c^2*x^
2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*arcsinh(c*x
)*(c^2*x^2+1)^(1/2)*c^3-16/3*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/d^
3*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^3+2/3*a*b*(d*(c^2*x^2+1))^(1/2
)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3/x^3*arcsinh(c*x)-128/
3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)
/d^3*x^5*(c^2*x^2+1)*c^8+560/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6
*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^5*arcsinh(c*x)*c^8+16*b^2*(d*(c^2*x^2+1
))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^4*c^7*(c^2*x
^2+1)^(1/2)+344/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x
^4+10*c^2*x^2-1)/d^3*x^3*arcsinh(c*x)^2*c^6+8/3*a^2*c^4*x/d/(c^2*d*x^2+d)^(
3/2)+16/3*a^2*c^4/d^2*x/(c^2*d*x^2+d)^(1/2)-16/3*a*b/(c^2*x^2+1)^(1/2)*(d*(
c^2*x^2+1))^(1/2)/d^3*ln((c*x+(c^2*x^2+1)^(1/2))^4-1)*c^3+64/3*a*b*(d*(c^2*
x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^3*c^6-1

```
6/3*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/d^3*arcsinh(c*x)*ln(1+(c*x+
(c^2*x^2+1)^(1/2))^2)*c^3+256/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^
6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^11*arcsinh(c*x)*c^14+64/3*a*b/(c^2*x^2
+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/d^3*arcsinh(c*x)*c^3-16/3*b^2*(d*(c^2*x^2+1
))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x*arcsinh(c*x)
*c^4-6*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x
^2-1)/d^3/x*arcsinh(c*x)^2*c^2-16/3*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(
1/2)/d^3*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^3+256/3*a*b*(d*(c^2*x^2
+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^11*c^14+89
6/3*a*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-
1)/d^3*x^9*c^12+1120/3*a*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*
c^4*x^4+10*c^2*x^2-1)/d^3*x^7*c^10-16/3*a*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x
^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x*c^4-4*a*b*(d*(c^2*x^2+1))^(1/2
)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*c^3*(c^2*x^2+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}\,abc\left(\frac{8\,c^2\log\left(c^2x^2+1\right)}{d^{\frac{5}{2}}}+\frac{16\,c^2\log(x)}{d^{\frac{5}{2}}}+\frac{1}{c^2d^{\frac{5}{2}}x^4+d^{\frac{5}{2}}x^2}\right)+\frac{2}{3}\left(\frac{16\,c^4x}{\sqrt{c^2dx^2+d}\,d^2}+\frac{8\,c^4x}{\left(c^2dx^2+d\right)^{\frac{3}{2}}d}+\frac{6\,c^2}{\left(c^2dx^2+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima
")
```

```
[Out] -1/3*a*b*c*(8*c^2*log(c^2*x^2 + 1)/d^(5/2) + 16*c^2*log(x)/d^(5/2) + 1/(c^2
*d^(5/2)*x^4 + d^(5/2)*x^2)) + 2/3*(16*c^4*x/(sqrt(c^2*d*x^2 + d)*d^2) + 8*
c^4*x/((c^2*d*x^2 + d)^(3/2)*d) + 6*c^2/((c^2*d*x^2 + d)^(3/2)*d*x) - 1/((c
^2*d*x^2 + d)^(3/2)*d*x^3))*a*b*arcsinh(c*x) + 1/3*(16*c^4*x/(sqrt(c^2*d*x^
2 + d)*d^2) + 8*c^4*x/((c^2*d*x^2 + d)^(3/2)*d) + 6*c^2/((c^2*d*x^2 + d)^(3
/2)*d*x) - 1/((c^2*d*x^2 + d)^(3/2)*d*x^3))*a^2 + b^2*integrate(log(c*x + s
qrt(c^2*x^2 + 1))^2/((c^2*d*x^2 + d)^(5/2)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{x^4\,{\left(d\,c^2\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)^(5/2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(x^4*(d + c^2*d*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{asinh}{\left(cx \right)}\right)^{2}}{x^{4} \left(d \left(c^{2} x^{2} + 1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**4/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(x**4*(d*(c**2*x**2 + 1))**(5/2)), x)
```

3.320 $\int \frac{\sinh^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=366

$$-\frac{8\sqrt{a^2x^2+1}\,\text{Li}_2\left(-e^{2\sinh^{-1}(ax)}\right)}{15ac^3\sqrt{a^2cx^2+c}}-\frac{x}{3c^3\sqrt{a^2cx^2+c}}-\frac{x}{30c^3\left(a^2x^2+1\right)\sqrt{a^2cx^2+c}}+\frac{8x\sinh^{-1}(ax)^2}{15c^3\sqrt{a^2cx^2+c}}+\frac{8\sqrt{a^2x^2+1}\,\sin}{15ac^3\sqrt{a^2cx}}$$

```
[Out] 1/5*x*arcsinh(a*x)^2/c/(a^2*c*x^2+c)^(5/2)+4/15*x*arcsinh(a*x)^2/c^2/(a^2*c
*x^2+c)^(3/2)-1/3*x/c^3/(a^2*c*x^2+c)^(1/2)-1/30*x/c^3/(a^2*x^2+1)/(a^2*c*x
^2+c)^(1/2)+1/10*arcsinh(a*x)/a/c^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2)+8
/15*x*arcsinh(a*x)^2/c^3/(a^2*c*x^2+c)^(1/2)+4/15*arcsinh(a*x)/a/c^3/(a^2*x
^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+8/15*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/a/c^3
/(a^2*c*x^2+c)^(1/2)-16/15*arcsinh(a*x)*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2)*(a^
2*x^2+1)^(1/2)/a/c^3/(a^2*c*x^2+c)^(1/2)-8/15*polylog(2,-(a*x+(a^2*x^2+1)^(
1/2))^2)*(a^2*x^2+1)^(1/2)/a/c^3/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {5690, 5687, 5714, 3718, 2190, 2279, 2391, 5717, 191, 192}

$$-\frac{8\sqrt{a^2x^2+1}\,\text{PolyLog}\left(2,-e^{2\sinh^{-1}(ax)}\right)}{15ac^3\sqrt{a^2cx^2+c}}-\frac{x}{3c^3\sqrt{a^2cx^2+c}}-\frac{x}{30c^3\left(a^2x^2+1\right)\sqrt{a^2cx^2+c}}+\frac{8x\sinh^{-1}(ax)^2}{15c^3\sqrt{a^2cx^2+c}}+\frac{8\sqrt{a^2x^2}}{15a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^2/(c + a^2*c*x^2)^(7/2),x]
```

```
[Out] -x/(3*c^3*Sqrt[c + a^2*c*x^2]) - x/(30*c^3*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2
]) + ArcSinh[a*x]/(10*a*c^3*(1 + a^2*x^2)^(3/2)*Sqrt[c + a^2*c*x^2]) + (4*A
rcSinh[a*x])/(15*a*c^3*Sqrt[1 + a^2*x^2]*Sqrt[c + a^2*c*x^2]) + (x*ArcSinh[
a*x]^2)/(5*c*(c + a^2*c*x^2)^(5/2)) + (4*x*ArcSinh[a*x]^2)/(15*c^2*(c + a^2
*c*x^2)^(3/2)) + (8*x*ArcSinh[a*x]^2)/(15*c^3*Sqrt[c + a^2*c*x^2]) + (8*Sqr
t[1 + a^2*x^2]*ArcSinh[a*x]^2)/(15*a*c^3*Sqrt[c + a^2*c*x^2]) - (16*Sqrt[1
+ a^2*x^2]*ArcSinh[a*x]*Log[1 + E^(2*ArcSinh[a*x])])/(15*a*c^3*Sqrt[c + a^2
*c*x^2]) - (8*Sqrt[1 + a^2*x^2]*PolyLog[2, -E^(2*ArcSinh[a*x])])/(15*a*c^3*
Sqrt[c + a^2*c*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{7/2}}\,dx = \frac{x\sinh^{-1}(ax)^2}{5c\left(c+a^2cx^2\right)^{5/2}} + \frac{4\int \frac{\sinh^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{5/2}}\,dx}{5c} - \frac{\left(2a\sqrt{1+a^2x^2}\right)\int \frac{x\sinh^{-1}(ax)}{\left(1+a^2x^2\right)^3}\,dx}{5c^3\sqrt{c+a^2cx^2}}$$

$$= \frac{\sinh^{-1}(ax)}{10ac^3\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)^2}{5c\left(c+a^2cx^2\right)^{5/2}} + \frac{4x\sinh^{-1}(ax)^2}{15c^2\left(c+a^2cx^2\right)^{3/2}} + \frac{8\int \frac{\sinh^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{3/2}}}{15c^2}$$

$$= -\frac{x}{30c^3\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}} + \frac{\sinh^{-1}(ax)}{10ac^3\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}} + \frac{4\sinh^{-1}(ax)}{15ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2}}$$

$$= -\frac{x}{3c^3\sqrt{c+a^2cx^2}} - \frac{x}{30c^3\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}} + \frac{\sinh^{-1}(ax)}{10ac^3\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}} + \frac{}{15ac^3}$$

$$= -\frac{x}{3c^3\sqrt{c+a^2cx^2}} - \frac{x}{30c^3\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}} + \frac{\sinh^{-1}(ax)}{10ac^3\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}} + \frac{}{15ac^3}$$

$$= -\frac{x}{3c^3\sqrt{c+a^2cx^2}} - \frac{x}{30c^3\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}} + \frac{\sinh^{-1}(ax)}{10ac^3\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}} + \frac{}{15ac^3}$$

$$= -\frac{x}{3c^3\sqrt{c+a^2cx^2}} - \frac{x}{30c^3\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}} + \frac{\sinh^{-1}(ax)}{10ac^3\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}} + \frac{}{15ac^3}$$

$$= -\frac{x}{3c^3\sqrt{c+a^2cx^2}} - \frac{x}{30c^3\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}} + \frac{\sinh^{-1}(ax)}{10ac^3\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}} + \frac{}{15ac^3}$$

Mathematica [A] time = 0.80, size = 178, normalized size = 0.49

$$\frac{16\sqrt{a^2x^2+1}\,\mathrm{Li}_2\left(-e^{-2\sinh^{-1}(ax)}\right) + ax\left(-\frac{1}{a^2x^2+1}-10\right) + \frac{\sinh^{-1}(ax)\left(8a^2x^2-32\left(a^2x^2+1\right)^2\log\left(e^{-2\sinh^{-1}(ax)}+1\right)+11\right)}{\left(a^2x^2+1\right)^{3/2}} + \left(\frac{2ax\left(8a^4x^4+\right.}{\left(a^2x^2\right.}\right.}{30ac^3\sqrt{a^2cx^2+c}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^2/(c + a^2*c*x^2)^(7/2),x]
```

```
[Out] (a*x*(-10 - (1 + a^2*x^2)^(-1)) + (-16*Sqrt[1 + a^2*x^2] + (2*a*x*(15 + 20*
a^2*x^2 + 8*a^4*x^4))/(1 + a^2*x^2)^2)*ArcSinh[a*x]^2 + (ArcSinh[a*x]*(11 +
 8*a^2*x^2 - 32*(1 + a^2*x^2)^2*Log[1 + E^(-2*ArcSinh[a*x])]))/(1 + a^2*x^2
)^(3/2) + 16*Sqrt[1 + a^2*x^2]*PolyLog[2, -E^(-2*ArcSinh[a*x])])/(30*a*c^3*
Sqrt[c + a^2*c*x^2])
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{arsinh}\left(ax\right)^2}{a^8c^4x^8+4\,a^6c^4x^6+6\,a^4c^4x^4+4\,a^2c^4x^2+c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/(a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^2/(a^8*c^4*x^8 + 4*a^6*c^4*x^6 +
6*a^4*c^4*x^4 + 4*a^2*c^4*x^2 + c^4), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/(a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, choosing root of [1,0,%%%{-2,[2,1,2
]%%%}+%%%{-2,[2,0,2]%%%}+%%%{-2,[0,1,0]%%%}+%%%{-2,[0,0,0]%%%},0,%%%{1,[4,2
,4]%%%}+%%%{-2,[4,1,4]%%%}+%%%{1,[4,0,4]%%%}+%%%{2,[2,2,2]%%%}+%%%{-4,[2,1,
2]%%%}+%%%{2,[2,0,2]%%%}+%%%{1,[0,2,0]%%%}+%%%{-2,[0,1,0]%%%}+%%%{1,[0,0,0]
%%%}] at parameters values [86,-97,-82]sym2poly/r2sym(const gen & e,const i
ndex_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.25, size = 570, normalized size = 1.56

$$\sqrt{c\left(a^2x^2+1\right)}\left(8x^5a^5-8\sqrt{a^2x^2+1}\,x^4a^4+20x^3a^3-16\sqrt{a^2x^2+1}\,x^2a^2+15ax-8\sqrt{a^2x^2+1}\right)\left(-64\operatorname{arcsinh}(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^2/(a^2*c*x^2+c)^(7/2),x)
```

```
[Out] 1/30*(c*(a^2*x^2+1))^(1/2)*(8*x^5*a^5-8*(a^2*x^2+1)^(1/2)*x^4*a^4+20*x^3*a^
3-16*(a^2*x^2+1)^(1/2)*x^2*a^2+15*a*x-8*(a^2*x^2+1)^(1/2))*(-64*arcsinh(a*x
)*x^8*a^8-64*(a^2*x^2+1)^(1/2)*arcsinh(a*x)*x^7*a^7-32*x^8*a^8-32*(a^2*x^2+
1)^(1/2)*x^7*a^7-280*arcsinh(a*x)*x^6*a^6-248*(a^2*x^2+1)^(1/2)*arcsinh(a*x
)*x^5*a^5-142*x^6*a^6-126*(a^2*x^2+1)^(1/2)*x^5*a^5+80*arcsinh(a*x)^2*x^4*a
^4-456*arcsinh(a*x)*x^4*a^4-340*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a^3*x^3-265*
x^4*a^4-156*(a^2*x^2+1)^(1/2)*x^3*a^3+190*arcsinh(a*x)^2*a^2*x^2-328*arcsin
h(a*x)*x^2*a^2-165*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a*x-235*a^2*x^2-62*(a^2*x
^2+1)^(1/2)*x*a+128*arcsinh(a*x)^2-88*arcsinh(a*x)-80)/(40*a^10*x^10+215*a^
8*x^8+469*a^6*x^6+517*a^4*x^4+287*a^2*x^2+64)/a/c^4+16/15/(a^2*x^2+1)^(1/2)
*(c*(a^2*x^2+1))^(1/2)/a/c^4*arcsinh(a*x)^2-16/15/(a^2*x^2+1)^(1/2)*(c*(a^2
*x^2+1))^(1/2)/a/c^4*arcsinh(a*x)*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2)-8/15/(a^2
*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/a/c^4*polylog(2,-(a*x+(a^2*x^2+1)^(1/2)
)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)^2}{\left(a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/(a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^2/(a^2*c*x^2 + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{asinh}(a\,x)^2}{{\left(c\,a^2\,x^2+c\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^2/(c + a^2*c*x^2)^(7/2),x)
```

```
[Out] int(asinh(a*x)^2/(c + a^2*c*x^2)^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^2(ax)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**2/(a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Integral(asinh(a*x)**2/(c*(a**2*x**2 + 1))**(7/2), x)
```

3.321 $\int x^m \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=936

$$\frac{\left(c^2dx^2+d\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2 x^{m+1}}{m+6}+\frac{5d\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2 x^{m+1}}{(m+4)(m+6)}+\frac{15d^2\sqrt{c^2dx^2+d}\,\left(a+b\,\mathrm{s}\right.}{(m+6)\left(m^2+\right.}$$

```
[Out] 5*d*x^(1+m)*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/(4+m)/(6+m)+x^(1+m)*(c
^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/(6+m)+10*b^2*c^2*d^2*x^(3+m)*(c^2*d*
x^2+d)^(1/2)/(4+m)^3/(6+m)+2*b^2*c^2*d^2*(m^2+15*m+52)*x^(3+m)*(c^2*d*x^2+d
)^(1/2)/(4+m)^2/(6+m)^3+2*b^2*c^4*d^2*x^(5+m)*(c^2*d*x^2+d)^(1/2)/(6+m)^3+1
5*d^2*x^(1+m)*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/(6+m)/(m^2+6*m+8)-30
*b*c*d^2*x^(2+m)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(2+m)^2/(4+m)/(6+m)
/(c^2*x^2+1)^(1/2)-10*b*c*d^2*x^(2+m)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2
)/(6+m)/(m^2+6*m+8)/(c^2*x^2+1)^(1/2)-2*b*c*d^2*x^(2+m)*(a+b*arcsinh(c*x))*
(c^2*d*x^2+d)^(1/2)/(m^2+8*m+12)/(c^2*x^2+1)^(1/2)-10*b*c^3*d^2*x^(4+m)*(a+
b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(4+m)^2/(6+m)/(c^2*x^2+1)^(1/2)-4*b*c^3
*d^2*x^(4+m)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(4+m)/(6+m)/(c^2*x^2+1)
^(1/2)-2*b*c^5*d^2*x^(6+m)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(6+m)^2/(
c^2*x^2+1)^(1/2)+10*b^2*c^2*d^2*(10+3*m)*x^(3+m)*hypergeom([1/2, 3/2+1/2*m]
,[5/2+1/2*m],-c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(4+m)^3/(6+m)/(m^2+5*m+6)/(c^2*x
^2+1)^(1/2)+30*b^2*c^2*d^2*x^(3+m)*hypergeom([1/2, 3/2+1/2*m],[5/2+1/2*m],-
c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(2+m)^2/(6+m)/(m^2+7*m+12)/(c^2*x^2+1)^(1/2)+2
*b^2*c^2*d^2*(15*m^2+130*m+264)*x^(3+m)*hypergeom([1/2, 3/2+1/2*m],[5/2+1/2
*m],-c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(4+m)^2/(6+m)^3/(m^2+5*m+6)/(c^2*x^2+1)^(
1/2)+15*d^3*Unintegrable(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x)/(6
+m)/(m^2+6*m+8)
```

Rubi [A] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
[Out] Defer[Int][x^m*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2, x]
```

Rubi steps

$$\int x^m \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx=\int x^m \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$$

Mathematica [A] time = 2.80, size = 0, normalized size = 0.00

$$\int x^m \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
[Out] Integrate[x^m*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2, x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2c^4d^2x^4 + 2\,a^2c^2d^2x^2 + a^2d^2 + \left(b^2c^4d^2x^4 + 2\,b^2c^2d^2x^2 + b^2d^2\right)\operatorname{arsinh}\left(cx\right)^2 + 2\,\left(abc^4d^2x^4 + 2\,abc^2d^2x\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas
")
```

```
[Out] integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
+ 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)*x^m, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 1.87, size = 0, normalized size = 0.00

$$\int x^{m}\left(c^{2}d\,x^{2}+d\right)^{\frac{5}{2}}\left(a+b\,\operatorname{arcsinh}\left(cx\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c^{2}dx^{2}+d\right)^{\frac{5}{2}}\left(b\operatorname{arsinh}\left(cx\right)+a\right)^{2}x^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima
")
```

```
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^2*x^m, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{m}\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^{2}\,\left(d\,c^{2}\,x^{2}+d\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int(x^m*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

3.322 $\int x^m \left(d+c^2 d x^2\right)^{3/2} \left(a+b \sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=488

$$\frac{3d^2 \text{Int}\left(\frac{x^m \left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{c^2dx^2+d}}, x\right)}{m^2+6m+8}+\frac{3dx^{m+1}\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2}{m^2+6m+8}-\frac{2bcdx^{m+2}\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)}{\left(m^2+6m+8\right)\sqrt{c^2x^2+1}}$$

```
[Out] x^(1+m)*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/(4+m)+2*b^2*c^2*d*x^(3+m)*
(c^2*d*x^2+d)^(1/2)/(4+m)^3+3*d*x^(1+m)*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^
(1/2)/(m^2+6*m+8)-6*b*c*d*x^(2+m)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(2
+m)^2/(4+m)/(c^2*x^2+1)^(1/2)-2*b*c*d*x^(2+m)*(a+b*arcsinh(c*x))*(c^2*d*x^2
+d)^(1/2)/(m^2+6*m+8)/(c^2*x^2+1)^(1/2)-2*b*c^3*d*x^(4+m)*(a+b*arcsinh(c*x)
)*(c^2*d*x^2+d)^(1/2)/(4+m)^2/(c^2*x^2+1)^(1/2)+2*b^2*c^2*d*(10+3*m)*x^(3+m
)*hypergeom([1/2, 3/2+1/2*m],[5/2+1/2*m],-c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(4+m
)^3/(m^2+5*m+6)/(c^2*x^2+1)^(1/2)+6*b^2*c^2*d*x^(3+m)*hypergeom([1/2, 3/2+1
/2*m],[5/2+1/2*m],-c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(2+m)^2/(m^2+7*m+12)/(c^2*x
^2+1)^(1/2)+3*d^2*Unintegrable(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2)
,x)/(m^2+6*m+8)
```

Rubi [A] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m \left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
[Out] Defer[Int][x^m*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2, x]
```

Rubi steps

$$\int x^m \left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx = \int x^m \left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$$

Mathematica [A] time = 0.43, size = 0, normalized size = 0.00

$$\int x^m \left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
[Out] Integrate[x^m*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2, x]
```

fricas [A] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2c^2dx^2+a^2d+\left(b^2c^2dx^2+b^2d\right)\text{arsinh}\,(cx)^2+2\left(abc^2dx^2+abd\right)\text{arsinh}\,(cx)\right)\sqrt{c^2dx^2+d}\,x^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas
")
```

```
[Out] integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)*x^m, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 1.42, size = 0, normalized size = 0.00

$$\int x^{m} \left(c^{2} d\,x^{2}+d\right)^{\frac{3}{2}} \left(a +b \,\mathrm{arcsinh}\left(c x \right)\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int {\left(c^{2} d x^{2} + d\right)}^{\frac{3}{2}} {\left(b \operatorname{arsinh}\left(c x\right) + a\right)}^{2} x^{m}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima
")
```

```
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^2*x^m, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int x^m\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(d\,c^2\,x^2+d\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int(x^m*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

3.323 $\int x^m \sqrt{d+c^2dx^2} \left(a+b\sinh^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=199

$$\frac{d\text{Int}\left(\frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{c^2dx^2+d}},x\right)}{m+2}+\frac{x^{m+1}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{m+2}-\frac{2bcx^{m+2}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{(m+2)^2\sqrt{c^2x^2+1}}+\frac{2b^2 \cdots}{\cdots}$$

```
[Out] x^(1+m)*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/(2+m)-2*b*c*x^(2+m)*(a+b*a
rcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(2+m)^2/(c^2*x^2+1)^(1/2)+2*b^2*c^2*x^(3+m
)*hypergeom([1/2, 3/2+1/2*m],[5/2+1/2*m],-c^2*x^2)*(c^2*d*x^2+d)^(1/2)/(2+m
)^2/(3+m)/(c^2*x^2+1)^(1/2)+d*Unintegrable(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*
x^2+d)^(1/2),x)/(2+m)
```

Rubi [A] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m \sqrt{d+c^2dx^2} \left(a+b\sinh^{-1}(cx)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] Defer[Int][x^m*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2, x]
```

Rubi steps

$$\int x^m \sqrt{d+c^2dx^2} \left(a+b\sinh^{-1}(cx)\right)^2 dx = \int x^m \sqrt{d+c^2dx^2} \left(a+b\sinh^{-1}(cx)\right)^2 dx$$

Mathematica [A] time = 0.29, size = 0, normalized size = 0.00

$$\int x^m \sqrt{d+c^2dx^2} \left(a+b\sinh^{-1}(cx)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] Integrate[x^m*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2, x]
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{c^2dx^2+d}\left(b^2\operatorname{arsinh}(cx)^2+2\,ab\operatorname{arsinh}(cx)+a^2\right)x^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)*x^m, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 1.22, size = 0, normalized size = 0.00

$$\int x^{m}\sqrt{c^{2}d\,x^{2}+d}\;\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c^{2}dx^{2}+d}\,\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)^{2}x^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^2*x^m, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m}\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^{2}\,\sqrt{d\,c^{2}\,x^{2}+d}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int(x^m*(a + b*asinh(c*x))^2*(d + c^2*d*x^2)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m}\sqrt{d\left(c^{2}x^{2}+1\right)}\;\left(a+b\,\mathrm{asinh}\left(cx\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*d*x**2+d)**(1/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x**m*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2, x)
```

3.324 $\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{c^2dx^2+d}},x\right)$$

```
[Out] Unintegrable(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x)
```

Rubi [A] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] Defer[Int][(x^m*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2], x]
```

Rubi steps

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx$$

Mathematica [A] time = 2.86, size = 0, normalized size = 0.00

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] Integrate[(x^m*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2], x]
```

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2\,\text{arsinh}\,(cx)^2+2\,ab\,\text{arsinh}\,(cx)+a^2\right)x^m}{\sqrt{c^2dx^2+d}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)*x^m/sqrt(c^2*d*x^2
 + d), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\text{arsinh}\,(cx)+a\right)^2x^m}{\sqrt{c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^m/sqrt(c^2*d*x^2 + d), x)
```

maple [A] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{x^{m} \left(a+b \,\mathrm{arcsinh}\left(c x\right)\right)^{2}}{\sqrt{c^{2} d \,x^{2}+d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{{\left(b \operatorname{arsinh}\left(c x\right) + a\right)}^{2} x^{m}}{\sqrt{c^{2} d x^{2} + d}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^m/sqrt(c^2*d*x^2 + d), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{\sqrt{d\,c^2\,x^2+d}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int((x^m*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m} \left(a + b \operatorname{asinh}{\left(c x \right)}\right)^{2}}{\sqrt{d \left(c^{2} x^{2} + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**m*(a + b*asinh(c*x))**2/sqrt(d*(c**2*x**2 + 1)), x)
```

3.325 $\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\left(c^2dx^2+d\right)^{3/2}},x\right)$$

```
[Out] Unintegrable(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x)
```

Rubi [A] time = 0.16, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] Defer[Int][(x^m*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx=\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx$$

Mathematica [A] time = 4.35, size = 0, normalized size = 0.00

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] Integrate[(x^m*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2), x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(b^2\operatorname{arsinh}(cx)^2+2\,ab\operatorname{arsinh}(cx)+a^2\right)x^m}{c^4d^2x^4+2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)*x^m/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\operatorname{arsinh}(cx)+a\right)^2x^m}{\left(c^2dx^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^m/(c^2*d*x^2 + d)^(3/2), x)
```

maple [A] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{x^{m} \left(a + b \operatorname{arcsinh}\left(c x\right)\right)^{2}}{\left(c^{2} d \,x^{2}+d \right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] int(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{{\left(b \operatorname{arsinh}\left(c x\right) + a\right)}^{2} x^{m}}{{\left(c^{2} d x^{2} + d\right)}^{\frac{3}{2}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^m/(c^2*d*x^2 + d)^(3/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d\,c^2\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int((x^m*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(3/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m} \left(a + b \operatorname{asinh}{\left(c x \right)}\right)^{2}}{\left(d \left(c^{2} x^{2} + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**m*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(3/2), x)
```

3.326 $\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\left(c^2dx^2+d\right)^{5/2}},x\right)$$

```
[Out] Unintegrable(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x)
```

Rubi [A] time = 0.16, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] Defer[Int][(x^m*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2), x]
```

Rubi steps

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx=\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx$$

Mathematica [A] time = 4.47, size = 0, normalized size = 0.00

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] Integrate[(x^m*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2), x]
```

fricas [A] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(b^2\operatorname{arsinh}(cx)^2+2\,ab\operatorname{arsinh}(cx)+a^2\right)x^m}{c^6d^3x^6+3\,c^4d^3x^4+3\,c^2d^3x^2+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)*x^m/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\operatorname{arsinh}(cx)+a\right)^2x^m}{\left(c^2dx^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^m/(c^2*d*x^2 + d)^(5/2), x)
```

maple [A] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{x^{m} \left(a+b \,\mathrm{arcsinh}\left(c x\right)\right)^{2}}{\left(c^{2} d \,x^{2}+d \right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] int(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{{\left(b \operatorname{arsinh}\left(c x\right) + a\right)}^{2} x^{m}}{{\left(c^{2} d x^{2} + d\right)}^{\frac{5}{2}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima
")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^m/(c^2*d*x^2 + d)^(5/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{{\left(d\,c^2\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int((x^m*(a + b*asinh(c*x))^2)/(d + c^2*d*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

3.327 $\int \frac{x^m \sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}} dx$

Optimal. Leaf size=26

$$\mathrm{Int}\left(\frac{x^m \sinh^{-1}(ax)^2}{\sqrt{a^2x^2+1}}, x\right)$$

```
[Out] Unintegrable(x^m*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x)
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m \sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}} dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
[Out] Defer[Int][(x^m*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2], x]
```

Rubi steps

$$\int \frac{x^m \sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}} dx = \int \frac{x^m \sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}} dx$$

Mathematica [A] time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{x^m \sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
[Out] Integrate[(x^m*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2], x]
```

fricas [A] time = 0.52, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{x^m \operatorname{arsinh}(ax)^2}{\sqrt{a^2x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(x^m*arcsinh(a*x)^2/sqrt(a^2*x^2 + 1), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \operatorname{arsinh}(ax)^2}{\sqrt{a^2x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^m*arcsinh(a*x)^2/sqrt(a^2*x^2 + 1), x)
```

maple [A] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{x^{m} \operatorname{arcsinh}\left(a x\right)^{2}}{\sqrt{a^{2} x^{2}+1}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x)
[Out] int(x^m*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m} \operatorname{arsinh}\left(a x\right)^{2}}{\sqrt{a^{2} x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^m*arcsinh(a*x)^2/sqrt(a^2*x^2 + 1), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m\,{\mathrm{asinh}\left(a\,x\right)}^2}{\sqrt{a^2\,x^2+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*asinh(a*x)^2)/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x^m*asinh(a*x)^2)/(a^2*x^2 + 1)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m} \operatorname{asinh}^{2}{\left(a x \right)}}{\sqrt{a^{2} x^{2} + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*asinh(a*x)**2/(a**2*x**2+1)**(1/2),x)
[Out] Integral(x**m*asinh(a*x)**2/sqrt(a**2*x**2 + 1), x)
```

3.328 $\int \left(c+a^2cx^2\right)^3 \sinh^{-1}(ax)^3\,dx$

Optimal. Leaf size=359

$$\frac{6}{343}a^6c^3x^7\sinh^{-1}(ax)+\frac{702a^4c^3x^5\sinh^{-1}(ax)}{6125}+\frac{1514a^2c^3x^3\sinh^{-1}(ax)}{3675}-\frac{6c^3\left(a^2x^2+1\right)^{7/2}}{2401a}-\frac{2664c^3\left(a^2x^2+1\right)^{5/}}{214375a}$$

```
[Out] -30256/385875*c^3*(a^2*x^2+1)^(3/2)/a-2664/214375*c^3*(a^2*x^2+1)^(5/2)/a-6
/2401*c^3*(a^2*x^2+1)^(7/2)/a+4322/1225*c^3*x*arcsinh(a*x)+1514/3675*a^2*c^
3*x^3*arcsinh(a*x)+702/6125*a^4*c^3*x^5*arcsinh(a*x)+6/343*a^6*c^3*x^7*arcs
inh(a*x)-8/35*c^3*(a^2*x^2+1)^(3/2)*arcsinh(a*x)^2/a-18/175*c^3*(a^2*x^2+1)
^(5/2)*arcsinh(a*x)^2/a-3/49*c^3*(a^2*x^2+1)^(7/2)*arcsinh(a*x)^2/a+16/35*c
^3*x*arcsinh(a*x)^3+8/35*c^3*x*(a^2*x^2+1)*arcsinh(a*x)^3+6/35*c^3*x*(a^2*x
^2+1)^2*arcsinh(a*x)^3+1/7*c^3*x*(a^2*x^2+1)^3*arcsinh(a*x)^3-413312/128625
*c^3*(a^2*x^2+1)^(1/2)/a-48/35*c^3*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.73, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 13, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.684, Rules used = {5684, 5653, 5717, 261, 5679, 444, 43, 194, 12, 1247, 698, 1799, 1850}

$$-\frac{6c^3\left(a^2x^2+1\right)^{7/2}}{2401a}-\frac{2664c^3\left(a^2x^2+1\right)^{5/2}}{214375a}-\frac{30256c^3\left(a^2x^2+1\right)^{3/2}}{385875a}-\frac{413312c^3\sqrt{a^2x^2+1}}{128625a}+\frac{6}{343}a^6c^3x^7\sinh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^3*ArcSinh[a*x]^3,x]
```

```
[Out] (-413312*c^3*Sqrt[1 + a^2*x^2])/(128625*a) - (30256*c^3*(1 + a^2*x^2)^(3/2)
)/(385875*a) - (2664*c^3*(1 + a^2*x^2)^(5/2))/(214375*a) - (6*c^3*(1 + a^2*
x^2)^(7/2))/(2401*a) + (4322*c^3*x*ArcSinh[a*x])/1225 + (1514*a^2*c^3*x^3*A
rcSinh[a*x])/3675 + (702*a^4*c^3*x^5*ArcSinh[a*x])/6125 + (6*a^6*c^3*x^7*Ar
cSinh[a*x])/343 - (48*c^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(35*a) - (8*c^3
*(1 + a^2*x^2)^(3/2)*ArcSinh[a*x]^2)/(35*a) - (18*c^3*(1 + a^2*x^2)^(5/2)*A
rcSinh[a*x]^2)/(175*a) - (3*c^3*(1 + a^2*x^2)^(7/2)*ArcSinh[a*x]^2)/(49*a)
+ (16*c^3*x*ArcSinh[a*x]^3)/35 + (8*c^3*x*(1 + a^2*x^2)*ArcSinh[a*x]^3)/35
+ (6*c^3*x*(1 + a^2*x^2)^2*ArcSinh[a*x]^3)/35 + (c^3*x*(1 + a^2*x^2)^3*ArcS
inh[a*x]^3)/7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5679

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^3 \sinh^{-1}(ax)^3\,dx = \frac{1}{7}c^3x\left(1+a^2x^2\right)^3\sinh^{-1}(ax)^3+\frac{1}{7}(6c)\int\left(c+a^2cx^2\right)^2\sinh^{-1}(ax)^3\,dx-\frac{1}{7}\left(3a\right.$$

$$= -\frac{3c^3\left(1+a^2x^2\right)^{7/2}\sinh^{-1}(ax)^2}{49a}+\frac{6}{35}c^3x\left(1+a^2x^2\right)^2\sinh^{-1}(ax)^3+\frac{1}{7}c^3x\left(1+\right.$$

$$= \frac{6}{49}c^3x\sinh^{-1}(ax)+\frac{6}{49}a^2c^3x^3\sinh^{-1}(ax)+\frac{18}{245}a^4c^3x^5\sinh^{-1}(ax)+\frac{6}{343}a^6c$$

$$= \frac{402c^3x\sinh^{-1}(ax)}{1225}+\frac{318a^2c^3x^3\sinh^{-1}(ax)}{1225}+\frac{702a^4c^3x^5\sinh^{-1}(ax)}{6125}+\frac{6}{343}a$$

$$= \frac{962c^3x\sinh^{-1}(ax)}{1225}+\frac{1514a^2c^3x^3\sinh^{-1}(ax)}{3675}+\frac{702a^4c^3x^5\sinh^{-1}(ax)}{6125}+\frac{6}{343}$$

$$= \frac{4322c^3x\sinh^{-1}(ax)}{1225}+\frac{1514a^2c^3x^3\sinh^{-1}(ax)}{3675}+\frac{702a^4c^3x^5\sinh^{-1}(ax)}{6125}+\frac{6}{34}$$

$$= -\frac{960c^3\sqrt{1+a^2x^2}}{343a}-\frac{16c^3\left(1+a^2x^2\right)^{3/2}}{1715a}-\frac{36c^3\left(1+a^2x^2\right)^{5/2}}{8575a}-\frac{6c^3\left(1+a^2x^2\right.}{2401a}$$

$$= -\frac{413312c^3\sqrt{1+a^2x^2}}{128625a}-\frac{30256c^3\left(1+a^2x^2\right)^{3/2}}{385875a}-\frac{2664c^3\left(1+a^2x^2\right)^{5/2}}{214375a}-\frac{6c^3}{}$$

Mathematica [A] time = 0.26, size = 169, normalized size = 0.47

$$\frac{c^3\left(-2\sqrt{a^2x^2+1}\left(16875a^6x^6+134541a^4x^4+747937a^2x^2+22329151\right)+385875ax\left(5a^6x^6+21a^4x^4+35a^2x\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^3*ArcSinh[a*x]^3,x]
```

```
[Out] (c^3*(-2*Sqrt[1 + a^2*x^2]*(22329151 + 747937*a^2*x^2 + 134541*a^4*x^4 + 16
875*a^6*x^6) + 210*a*x*(226905 + 26495*a^2*x^2 + 7371*a^4*x^4 + 1125*a^6*x^
6)*ArcSinh[a*x] - 11025*Sqrt[1 + a^2*x^2]*(2161 + 757*a^2*x^2 + 351*a^4*x^4
 + 75*a^6*x^6)*ArcSinh[a*x]^2 + 385875*a*x*(35 + 35*a^2*x^2 + 21*a^4*x^4 +
5*a^6*x^6)*ArcSinh[a*x]^3))/(13505625*a)
```

fricas [A] time = 0.49, size = 248, normalized size = 0.69

$$\frac{385875\left(5\,a^7c^3x^7+21\,a^5c^3x^5+35\,a^3c^3x^3+35\,ac^3x\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^3-11025\left(75\,a^6c^3x^6+351\,a^4c^3x^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/13505625*(385875*(5*a^7*c^3*x^7 + 21*a^5*c^3*x^5 + 35*a^3*c^3*x^3 + 35*a*
c^3*x)*log(a*x + sqrt(a^2*x^2 + 1))^3 - 11025*(75*a^6*c^3*x^6 + 351*a^4*c^3
*x^4 + 757*a^2*c^3*x^2 + 2161*c^3)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2
 + 1))^2 + 210*(1125*a^7*c^3*x^7 + 7371*a^5*c^3*x^5 + 26495*a^3*c^3*x^3 + 2
26905*a*c^3*x)*log(a*x + sqrt(a^2*x^2 + 1)) - 2*(16875*a^6*c^3*x^6 + 134541
*a^4*c^3*x^4 + 747937*a^2*c^3*x^2 + 22329151*c^3)*sqrt(a^2*x^2 + 1))/a
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.11, size = 270, normalized size = 0.75

$$\frac{c^3 \left(1929375 \operatorname{arcsinh}(ax)^3 a^7 x^7 - 826875 \operatorname{arcsinh}(ax)^2 \sqrt{a^2x^2+1}\, a^6 x^6 + 8103375 \operatorname{arcsinh}(ax)^3 a^5 x^5 + 236250\,\mathrm{a}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arcsinh(a*x)^3,x)
```

```
[Out] 1/13505625/a*c^3*(1929375*arcsinh(a*x)^3*a^7*x^7-826875*arcsinh(a*x)^2*(a^2
*x^2+1)^(1/2)*a^6*x^6+8103375*arcsinh(a*x)^3*a^5*x^5+236250*arcsinh(a*x)*a^
7*x^7-3869775*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)*a^4*x^4-33750*a^6*x^6*(a^2*x
^2+1)^(1/2)+13505625*arcsinh(a*x)^3*a^3*x^3+1547910*arcsinh(a*x)*a^5*x^5-83
45925*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)*a^2*x^2-269082*(a^2*x^2+1)^(1/2)*x^4
*a^4+13505625*arcsinh(a*x)^3*a*x+5563950*arcsinh(a*x)*a^3*x^3-23825025*arcs
inh(a*x)^2*(a^2*x^2+1)^(1/2)-1495874*(a^2*x^2+1)^(1/2)*x^2*a^2+47650050*a*x
*arcsinh(a*x)-44658302*(a^2*x^2+1)^(1/2))
```

maxima [A] time = 0.32, size = 276, normalized size = 0.77

$$-\frac{1}{1225}\left(75\sqrt{a^2x^2+1}\,a^4c^3x^6 + 351\sqrt{a^2x^2+1}\,a^2c^3x^4 + 757\sqrt{a^2x^2+1}\,c^3x^2 + \frac{2161\sqrt{a^2x^2+1}\,c^3}{a^2}\right)a \operatorname{arsinh}(ax)^2 + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/1225*(75*sqrt(a^2*x^2 + 1)*a^4*c^3*x^6 + 351*sqrt(a^2*x^2 + 1)*a^2*c^3*x
^4 + 757*sqrt(a^2*x^2 + 1)*c^3*x^2 + 2161*sqrt(a^2*x^2 + 1)*c^3/a^2)*a*arcs
inh(a*x)^2 + 1/35*(5*a^6*c^3*x^7 + 21*a^4*c^3*x^5 + 35*a^2*c^3*x^3 + 35*c^3
*x)*arcsinh(a*x)^3 - 2/13505625*(16875*sqrt(a^2*x^2 + 1)*a^4*c^3*x^6 + 1345
41*sqrt(a^2*x^2 + 1)*a^2*c^3*x^4 + 747937*sqrt(a^2*x^2 + 1)*c^3*x^2 + 22329
151*sqrt(a^2*x^2 + 1)*c^3/a^2 - 105*(1125*a^6*c^3*x^7 + 7371*a^4*c^3*x^5 +
26495*a^2*c^3*x^3 + 226905*c^3*x)*arcsinh(a*x)/a)*a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{asinh}(a\,x)^3\,{\left(c\,a^2\,x^2+c\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^3*(c + a^2*c*x^2)^3,x)
```

[Out] int(asinh(a*x)^3*(c + a^2*c*x^2)^3, x)

sympy [A] time = 18.41, size = 355, normalized size = 0.99

$$\begin{cases} \frac{a^6c^3x^7\operatorname{asinh}^3(ax)}{7} + \frac{6a^6c^3x^7\operatorname{asinh}(ax)}{343} - \frac{3a^5c^3x^6\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{49} - \frac{6a^5c^3x^6\sqrt{a^2x^2+1}}{2401} + \frac{3a^4c^3x^5\operatorname{asinh}^3(ax)}{5} + \frac{702a^4c^3x^5\operatorname{asinh}(a
\\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*asinh(a*x)**3,x)
```

```
[Out] Piecewise((a**6*c**3*x**7*asinh(a*x)**3/7 + 6*a**6*c**3*x**7*asinh(a*x)/343
 - 3*a**5*c**3*x**6*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/49 - 6*a**5*c**3*x**6
*sqrt(a**2*x**2 + 1)/2401 + 3*a**4*c**3*x**5*asinh(a*x)**3/5 + 702*a**4*c**
3*x**5*asinh(a*x)/6125 - 351*a**3*c**3*x**4*sqrt(a**2*x**2 + 1)*asinh(a*x)*
*2/1225 - 29898*a**3*c**3*x**4*sqrt(a**2*x**2 + 1)/1500625 + a**2*c**3*x**3
*asinh(a*x)**3 + 1514*a**2*c**3*x**3*asinh(a*x)/3675 - 757*a*c**3*x**2*sqrt
(a**2*x**2 + 1)*asinh(a*x)**2/1225 - 1495874*a*c**3*x**2*sqrt(a**2*x**2 + 1
)/13505625 + c**3*x*asinh(a*x)**3 + 4322*c**3*x*asinh(a*x)/1225 - 2161*c**3
*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/(1225*a) - 44658302*c**3*sqrt(a**2*x**2
+ 1)/(13505625*a), Ne(a, 0)), (0, True))
```

3.329 $\int \left(c + a^2cx^2\right)^2 \sinh^{-1}(ax)^3 \, dx$

Optimal. Leaf size=265

$$\frac{6}{125}a^4c^2x^5\sinh^{-1}(ax)+\frac{76}{225}a^2c^2x^3\sinh^{-1}(ax)-\frac{6c^2\left(a^2x^2+1\right)^{5/2}}{625a}-\frac{272c^2\left(a^2x^2+1\right)^{3/2}}{3375a}-\frac{4144c^2\sqrt{a^2x^2+1}}{1125a}+\frac{1}{5}c^2x\left(\right.$$

```
[Out] -272/3375*c^2*(a^2*x^2+1)^(3/2)/a-6/625*c^2*(a^2*x^2+1)^(5/2)/a+298/75*c^2*
x*arcsinh(a*x)+76/225*a^2*c^2*x^3*arcsinh(a*x)+6/125*a^4*c^2*x^5*arcsinh(a*
x)-4/15*c^2*(a^2*x^2+1)^(3/2)*arcsinh(a*x)^2/a-3/25*c^2*(a^2*x^2+1)^(5/2)*a
rcsinh(a*x)^2/a+8/15*c^2*x*arcsinh(a*x)^3+4/15*c^2*x*(a^2*x^2+1)*arcsinh(a*
x)^3+1/5*c^2*x*(a^2*x^2+1)^2*arcsinh(a*x)^3-4144/1125*c^2*(a^2*x^2+1)^(1/2)
/a-8/5*c^2*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.42, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {5684, 5653, 5717, 261, 5679, 444, 43, 194, 12, 1247, 698}

$$-\frac{6c^2\left(a^2x^2+1\right)^{5/2}}{625a}-\frac{272c^2\left(a^2x^2+1\right)^{3/2}}{3375a}-\frac{4144c^2\sqrt{a^2x^2+1}}{1125a}+\frac{6}{125}a^4c^2x^5\sinh^{-1}(ax)+\frac{76}{225}a^2c^2x^3\sinh^{-1}(ax)+\frac{1}{5}c^2x$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^2*ArcSinh[a*x]^3,x]
```

```
[Out] (-4144*c^2*Sqrt[1 + a^2*x^2])/(1125*a) - (272*c^2*(1 + a^2*x^2)^(3/2))/(337
5*a) - (6*c^2*(1 + a^2*x^2)^(5/2))/(625*a) + (298*c^2*x*ArcSinh[a*x])/75 +
(76*a^2*c^2*x^3*ArcSinh[a*x])/225 + (6*a^4*c^2*x^5*ArcSinh[a*x])/125 - (8*c
^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(5*a) - (4*c^2*(1 + a^2*x^2)^(3/2)*Arc
Sinh[a*x]^2)/(15*a) - (3*c^2*(1 + a^2*x^2)^(5/2)*ArcSinh[a*x]^2)/(25*a) + (
8*c^2*x*ArcSinh[a*x]^3)/15 + (4*c^2*x*(1 + a^2*x^2)*ArcSinh[a*x]^3)/15 + (c
^2*x*(1 + a^2*x^2)^2*ArcSinh[a*x]^3)/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5679

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^2 \sinh^{-1}(ax)^3\,dx = \frac{1}{5}c^2x\left(1+a^2x^2\right)^2\sinh^{-1}(ax)^3 + \frac{1}{5}(4c)\int \left(c+a^2cx^2\right)\sinh^{-1}(ax)^3\,dx - \frac{1}{5}\left(3ac^2\right)$$

$$= -\frac{3c^2\left(1+a^2x^2\right)^{5/2}\sinh^{-1}(ax)^2}{25a} + \frac{4}{15}c^2x\left(1+a^2x^2\right)\sinh^{-1}(ax)^3 + \frac{1}{5}c^2x\left(1+a^2x\right.$$

$$= \frac{6}{25}c^2x\sinh^{-1}(ax) + \frac{4}{25}a^2c^2x^3\sinh^{-1}(ax) + \frac{6}{125}a^4c^2x^5\sinh^{-1}(ax) - \frac{4c^2\left(1+a^2\right.}{}$$

$$= \frac{58}{75}c^2x\sinh^{-1}(ax) + \frac{76}{225}a^2c^2x^3\sinh^{-1}(ax) + \frac{6}{125}a^4c^2x^5\sinh^{-1}(ax) - \frac{8c^2\sqrt{1+}}{}$$

$$= \frac{298}{75}c^2x\sinh^{-1}(ax) + \frac{76}{225}a^2c^2x^3\sinh^{-1}(ax) + \frac{6}{125}a^4c^2x^5\sinh^{-1}(ax) - \frac{8c^2\sqrt{1+}}{}$$

$$= -\frac{16c^2\sqrt{1+a^2x^2}}{5a} + \frac{298}{75}c^2x\sinh^{-1}(ax) + \frac{76}{225}a^2c^2x^3\sinh^{-1}(ax) + \frac{6}{125}a^4c^2x^5\sin$$

$$= -\frac{4144c^2\sqrt{1+a^2x^2}}{1125a} - \frac{272c^2\left(1+a^2x^2\right)^{3/2}}{3375a} - \frac{6c^2\left(1+a^2x^2\right)^{5/2}}{625a} + \frac{298}{75}c^2x\sinh^{-1}($$

Mathematica [A] time = 0.15, size = 137, normalized size = 0.52

$$\frac{c^2\left(-2\sqrt{a^2x^2+1}\left(81a^4x^4+842a^2x^2+31841\right)+1125ax\left(3a^4x^4+10a^2x^2+15\right)\sinh^{-1}(ax)^3-225\sqrt{a^2x^2+1}\left(9a^4\right.\right.}{16875a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^2*ArcSinh[a*x]^3,x]
```

```
[Out] (c^2*(-2*Sqrt[1 + a^2*x^2]*(31841 + 842*a^2*x^2 + 81*a^4*x^4) + 30*a*x*(223
5 + 190*a^2*x^2 + 27*a^4*x^4)*ArcSinh[a*x] - 225*Sqrt[1 + a^2*x^2]*(149 + 3
8*a^2*x^2 + 9*a^4*x^4)*ArcSinh[a*x]^2 + 1125*a*x*(15 + 10*a^2*x^2 + 3*a^4*x
^4)*ArcSinh[a*x]^3))/(16875*a)
```

fricas [A] time = 0.55, size = 204, normalized size = 0.77

$$\frac{1125\left(3\,a^5c^2x^5+10\,a^3c^2x^3+15\,ac^2x\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^3-225\left(9\,a^4c^2x^4+38\,a^2c^2x^2+149\,c^2\right)\sqrt{a^2x^2+1}\,\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/16875*(1125*(3*a^5*c^2*x^5 + 10*a^3*c^2*x^3 + 15*a*c^2*x)*log(a*x + sqrt(
a^2*x^2 + 1))^3 - 225*(9*a^4*c^2*x^4 + 38*a^2*c^2*x^2 + 149*c^2)*sqrt(a^2*x
^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2 + 30*(27*a^5*c^2*x^5 + 190*a^3*c^2*x
^3 + 2235*a*c^2*x)*log(a*x + sqrt(a^2*x^2 + 1)) - 2*(81*a^4*c^2*x^4 + 842*a
^2*c^2*x^2 + 31841*c^2)*sqrt(a^2*x^2 + 1))/a
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arcsinh(a*x)^3,x, algorithm="giac")
```

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

maple [A] time = 0.06, size = 200, normalized size = 0.75

$$c^2\left(3375\operatorname{arcsinh}(ax)^3 a^5x^5 - 2025\operatorname{arcsinh}(ax)^2\sqrt{a^2x^2+1}\,a^4x^4 + 11250\operatorname{arcsinh}(ax)^3 a^3x^3 + 810\operatorname{arcsinh}(a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2*c*x^2+c)^2*arcsinh(a*x)^3,x)

[Out] 1/16875/a*c^2*(3375*arcsinh(a*x)^3*a^5*x^5-2025*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)*a^4*x^4+11250*arcsinh(a*x)^3*a^3*x^3+810*arcsinh(a*x)*a^5*x^5-8550*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)*a^2*x^2-162*(a^2*x^2+1)^(1/2)*x^4*a^4+16875*arcsinh(a*x)^3*a*x+5700*arcsinh(a*x)*a^3*x^3-33525*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)-1684*(a^2*x^2+1)^(1/2)*x^2*a^2+67050*a*x*arcsinh(a*x)-63682*(a^2*x^2+1)^(1/2))

maxima [A] time = 0.32, size = 210, normalized size = 0.79

$$-\frac{1}{75}\left(9\sqrt{a^2x^2+1}\,a^2c^2x^4 + 38\sqrt{a^2x^2+1}\,c^2x^2 + \frac{149\sqrt{a^2x^2+1}\,c^2}{a^2}\right)a\operatorname{arsinh}(ax)^2 + \frac{1}{15}\left(3a^4c^2x^5 + 10a^2c^2x^3 + 1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2*c*x^2+c)^2*arcsinh(a*x)^3,x, algorithm="maxima")

[Out] -1/75*(9*sqrt(a^2*x^2 + 1)*a^2*c^2*x^4 + 38*sqrt(a^2*x^2 + 1)*c^2*x^2 + 149*sqrt(a^2*x^2 + 1)*c^2/a^2)*a*arcsinh(a*x)^2 + 1/15*(3*a^4*c^2*x^5 + 10*a^2*c^2*x^3 + 15*c^2*x)*arcsinh(a*x)^3 - 2/16875*(81*sqrt(a^2*x^2 + 1)*a^2*c^2*x^4 + 842*sqrt(a^2*x^2 + 1)*c^2*x^2 - 15*(27*a^4*c^2*x^5 + 190*a^2*c^2*x^3 + 2235*c^2*x)*arcsinh(a*x)/a + 31841*sqrt(a^2*x^2 + 1)*c^2/a^2)*a

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{asinh}(a\,x)^3\left(c\,a^2\,x^2+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(asinh(a*x)^3*(c + a^2*c*x^2)^2,x)

[Out] int(asinh(a*x)^3*(c + a^2*c*x^2)^2, x)

sympy [A] time = 6.63, size = 262, normalized size = 0.99

$$\begin{cases} \frac{a^4c^2x^5\operatorname{asinh}^3(ax)}{5} + \frac{6a^4c^2x^5\operatorname{asinh}(ax)}{125} - \frac{3a^3c^2x^4\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{25} - \frac{6a^3c^2x^4\sqrt{a^2x^2+1}}{625} + \frac{2a^2c^2x^3\operatorname{asinh}^3(ax)}{3} + \frac{76a^2c^2x^3\operatorname{asinh}(ax)}{225} \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a**2*c*x**2+c)**2*asinh(a*x)**3,x)

[Out] Piecewise((a**4*c**2*x**5*asinh(a*x)**3/5 + 6*a**4*c**2*x**5*asinh(a*x)/125 - 3*a**3*c**2*x**4*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/25 - 6*a**3*c**2*x**4*sqrt(a**2*x**2 + 1)/625 + 2*a**2*c**2*x**3*asinh(a*x)**3/3 + 76*a**2*c**2*x**3*asinh(a*x)/225 - 38*a*c**2*x**2*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/75 - 1684*a*c**2*x**2*sqrt(a**2*x**2 + 1)/16875 + c**2*x*asinh(a*x)**3 + 298*c**2*x*asinh(a*x)/75 - 149*c**2*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/(75*a) - 63682*c**2*sqrt(a**2*x**2 + 1)/(16875*a), Ne(a, 0)), (0, True))

3.330 $\int \left(c + a^2cx^2\right) \sinh^{-1}(ax)^3\, dx$

Optimal. Leaf size=153

$$\frac{2}{9}a^2cx^3\sinh^{-1}(ax) - \frac{2c\left(a^2x^2+1\right)^{3/2}}{27a} - \frac{40c\sqrt{a^2x^2+1}}{9a} + \frac{1}{3}cx\left(a^2x^2+1\right)\sinh^{-1}(ax)^3 - \frac{c\left(a^2x^2+1\right)^{3/2}\sinh^{-1}(ax)^2}{3a} - \frac{2c}{}$$

```
[Out] -2/27*c*(a^2*x^2+1)^(3/2)/a+14/3*c*x*arcsinh(a*x)+2/9*a^2*c*x^3*arcsinh(a*x
)-1/3*c*(a^2*x^2+1)^(3/2)*arcsinh(a*x)^2/a+2/3*c*x*arcsinh(a*x)^3+1/3*c*x*(
a^2*x^2+1)*arcsinh(a*x)^3-40/9*c*(a^2*x^2+1)^(1/2)/a-2*c*arcsinh(a*x)^2*(a^
2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.22, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {5684, 5653, 5717, 261, 5679, 444, 43}

$$-\frac{2c\left(a^2x^2+1\right)^{3/2}}{27a} - \frac{40c\sqrt{a^2x^2+1}}{9a} + \frac{2}{9}a^2cx^3\sinh^{-1}(ax) + \frac{1}{3}cx\left(a^2x^2+1\right)\sinh^{-1}(ax)^3 - \frac{c\left(a^2x^2+1\right)^{3/2}\sinh^{-1}(ax)^2}{3a} -$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)*ArcSinh[a*x]^3,x]
```

```
[Out] (-40*c*Sqrt[1 + a^2*x^2])/(9*a) - (2*c*(1 + a^2*x^2)^(3/2))/(27*a) + (14*c*
x*ArcSinh[a*x])/3 + (2*a^2*c*x^3*ArcSinh[a*x])/9 - (2*c*Sqrt[1 + a^2*x^2]*A
rcSinh[a*x]^2)/a - (c*(1 + a^2*x^2)^(3/2)*ArcSinh[a*x]^2)/(3*a) + (2*c*x*Ar
cSinh[a*x]^3)/3 + (c*x*(1 + a^2*x^2)*ArcSinh[a*x]^3)/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5679

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
```

```
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \left(c+a^2cx^2\right)\sinh^{-1}(ax)^3\,dx &= \frac{1}{3}cx\left(1+a^2x^2\right)\sinh^{-1}(ax)^3+\frac{1}{3}(2c)\int \sinh^{-1}(ax)^3\,dx-(ac)\int x\sqrt{1+a^2x^2}\ \mathrm{si}\\
&= -\frac{c\left(1+a^2x^2\right)^{3/2}\sinh^{-1}(ax)^2}{3a}+\frac{2}{3}cx\sinh^{-1}(ax)^3+\frac{1}{3}cx\left(1+a^2x^2\right)\sinh^{-1}(ax)^3\\
&= \frac{2}{3}cx\sinh^{-1}(ax)+\frac{2}{9}a^2cx^3\sinh^{-1}(ax)-\frac{2c\sqrt{1+a^2x^2}\ \sinh^{-1}(ax)^2}{a}-\frac{c\left(1+a^2x\right.}{}\\
&= \frac{14}{3}cx\sinh^{-1}(ax)+\frac{2}{9}a^2cx^3\sinh^{-1}(ax)-\frac{2c\sqrt{1+a^2x^2}\ \sinh^{-1}(ax)^2}{a}-\frac{c\left(1+a^2\right.}{}\\
&= -\frac{4c\sqrt{1+a^2x^2}}{a}+\frac{14}{3}cx\sinh^{-1}(ax)+\frac{2}{9}a^2cx^3\sinh^{-1}(ax)-\frac{2c\sqrt{1+a^2x^2}\ \sinh^{-}}{a}\\
&= -\frac{40c\sqrt{1+a^2x^2}}{9a}-\frac{2c\left(1+a^2x^2\right)^{3/2}}{27a}+\frac{14}{3}cx\sinh^{-1}(ax)+\frac{2}{9}a^2cx^3\sinh^{-1}(ax)-
\end{aligned}$$

Mathematica [A] time = 0.08, size = 99, normalized size = 0.65

$$\frac{c\left(-2\sqrt{a^2x^2+1}\left(a^2x^2+61\right)+9ax\left(a^2x^2+3\right)\sinh^{-1}(ax)^3-9\sqrt{a^2x^2+1}\left(a^2x^2+7\right)\sinh^{-1}(ax)^2+6ax\left(a^2x^2+\right.\right.}{27a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)*ArcSinh[a*x]^3,x]
```

```
[Out] (c*(-2*Sqrt[1 + a^2*x^2]*(61 + a^2*x^2) + 6*a*x*(21 + a^2*x^2)*ArcSinh[a*x]
 - 9*Sqrt[1 + a^2*x^2]*(7 + a^2*x^2)*ArcSinh[a*x]^2 + 9*a*x*(3 + a^2*x^2)*A
rcSinh[a*x]^3))/(27*a)
```

fricas [A] time = 0.58, size = 140, normalized size = 0.92

$$\frac{9\left(a^3cx^3+3\,acx\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^3-9\left(a^2cx^2+7\,c\right)\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1}\right)^2+6\left(a^3cx^3+21\,a\right.}{27\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/27*(9*(a^3*c*x^3 + 3*a*c*x)*log(a*x + sqrt(a^2*x^2 + 1))^3 - 9*(a^2*c*x^2
 + 7*c)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2 + 6*(a^3*c*x^3 + 2
1*a*c*x)*log(a*x + sqrt(a^2*x^2 + 1)) - 2*(a^2*c*x^2 + 61*c)*sqrt(a^2*x^2 +
 1))/a
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.06, size = 128, normalized size = 0.84

$$\frac{c\left(9\operatorname{arcsinh}(ax)^3 a^3x^3 - 9\operatorname{arcsinh}(ax)^2\sqrt{a^2x^2+1}\,a^2x^2 + 27\operatorname{arcsinh}(ax)^3 ax + 6\operatorname{arcsinh}(ax)\,a^3x^3 - 63\operatorname{arcsin}\right.}{27a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arcsinh(a*x)^3,x)
```

```
[Out] 1/27/a*c*(9*arcsinh(a*x)^3*a^3*x^3-9*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)*a^2*x
^2+27*arcsinh(a*x)^3*a*x+6*arcsinh(a*x)*a^3*x^3-63*arcsinh(a*x)^2*(a^2*x^2+
1)^(1/2)-2*(a^2*x^2+1)^(1/2)*x^2*a^2+126*a*x*arcsinh(a*x)-122*(a^2*x^2+1)^(
1/2))
```

maxima [A] time = 0.56, size = 124, normalized size = 0.81

$$-\frac{1}{3}\left(\sqrt{a^2x^2+1}\,cx^2 + \frac{7\sqrt{a^2x^2+1}\,c}{a^2}\right)a\operatorname{arsinh}(ax)^2 + \frac{1}{3}\left(a^2cx^3 + 3\,cx\right)\operatorname{arsinh}(ax)^3 - \frac{2}{27}\left(\sqrt{a^2x^2+1}\,cx^2 - \frac{3\left(a^2cx^3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/3*(sqrt(a^2*x^2 + 1)*c*x^2 + 7*sqrt(a^2*x^2 + 1)*c/a^2)*a*arcsinh(a*x)^2
 + 1/3*(a^2*c*x^3 + 3*c*x)*arcsinh(a*x)^3 - 2/27*(sqrt(a^2*x^2 + 1)*c*x^2 -
 3*(a^2*c*x^3 + 21*c*x)*arcsinh(a*x)/a + 61*sqrt(a^2*x^2 + 1)*c/a^2)*a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{asinh}(a\,x)^3\,\left(c\,a^2\,x^2 + c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^3*(c + a^2*c*x^2),x)
```

```
[Out] int(asinh(a*x)^3*(c + a^2*c*x^2), x)
```

sympy [A] time = 2.08, size = 150, normalized size = 0.98

$$\begin{cases} \frac{a^2cx^3\operatorname{asinh}^3(ax)}{3} + \frac{2a^2cx^3\operatorname{asinh}(ax)}{9} - \frac{acx^2\sqrt{a^2x^2+1}\,\operatorname{asinh}^2(ax)}{3} - \frac{2acx^2\sqrt{a^2x^2+1}}{27} + cx\operatorname{asinh}^3(ax) + \frac{14cx\operatorname{asinh}(ax)}{3} - \frac{7c\sqrt{a^2x^2+1}\,a}{3a} \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*asinh(a*x)**3,x)
```

```
[Out] Piecewise((a**2*c*x**3*asinh(a*x)**3/3 + 2*a**2*c*x**3*asinh(a*x)/9 - a*c*x
**2*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/3 - 2*a*c*x**2*sqrt(a**2*x**2 + 1)/27
 + c*x*asinh(a*x)**3 + 14*c*x*asinh(a*x)/3 - 7*c*sqrt(a**2*x**2 + 1)*asinh(
a*x)**2/(3*a) - 122*c*sqrt(a**2*x**2 + 1)/(27*a), Ne(a, 0)), (0, True))
```

3.331 $\int \frac{\sinh^{-1}(ax)^3}{c+a^2cx^2} \, dx$

Optimal. Leaf size=174

$$-\frac{3i\sinh^{-1}(ax)^2\text{Li}_2\left(-ie^{\sinh^{-1}(ax)}\right)}{ac}+\frac{3i\sinh^{-1}(ax)^2\text{Li}_2\left(ie^{\sinh^{-1}(ax)}\right)}{ac}+\frac{6i\sinh^{-1}(ax)\text{Li}_3\left(-ie^{\sinh^{-1}(ax)}\right)}{ac}-\frac{6i\sinh^{-1}(ax)}{}$$

```
[Out] 2*arcsinh(a*x)^3*arctan(a*x+(a^2*x^2+1)^(1/2))/a/c-3*I*arcsinh(a*x)^2*polyl
og(2,-I*(a*x+(a^2*x^2+1)^(1/2)))/a/c+3*I*arcsinh(a*x)^2*polylog(2,I*(a*x+(a
^2*x^2+1)^(1/2)))/a/c+6*I*arcsinh(a*x)*polylog(3,-I*(a*x+(a^2*x^2+1)^(1/2))
)/a/c-6*I*arcsinh(a*x)*polylog(3,I*(a*x+(a^2*x^2+1)^(1/2)))/a/c-6*I*polylog
(4,-I*(a*x+(a^2*x^2+1)^(1/2)))/a/c+6*I*polylog(4,I*(a*x+(a^2*x^2+1)^(1/2)))
/a/c
```

Rubi [A] time = 0.13, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5693, 4180, 2531, 6609, 2282, 6589}

$$-\frac{3i\sinh^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{\sinh^{-1}(ax)}\right)}{ac}+\frac{3i\sinh^{-1}(ax)^2\text{PolyLog}\left(2,ie^{\sinh^{-1}(ax)}\right)}{ac}+\frac{6i\sinh^{-1}(ax)\text{PolyLog}\left(3,-ie\right.}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/(c + a^2*c*x^2),x]
```

```
[Out] (2*ArcSinh[a*x]^3*ArcTan[E^ArcSinh[a*x]])/(a*c) - ((3*I)*ArcSinh[a*x]^2*Pol
yLog[2, (-I)*E^ArcSinh[a*x]])/(a*c) + ((3*I)*ArcSinh[a*x]^2*PolyLog[2, I*E^
ArcSinh[a*x]])/(a*c) + ((6*I)*ArcSinh[a*x]*PolyLog[3, (-I)*E^ArcSinh[a*x]])
/(a*c) - ((6*I)*ArcSinh[a*x]*PolyLog[3, I*E^ArcSinh[a*x]])/(a*c) - ((6*I)*P
olyLog[4, (-I)*E^ArcSinh[a*x]])/(a*c) + ((6*I)*PolyLog[4, I*E^ArcSinh[a*x]]
)/(a*c)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^3}{c+a^2cx^2}\,dx = \frac{\text{Subst}\left(\int x^3\text{sech}(x)\,dx, x, \sinh^{-1}(ax)\right)}{ac}$$

$$= \frac{2\sinh^{-1}(ax)^3\tan^{-1}\left(e^{\sinh^{-1}(ax)}\right)}{ac} - \frac{(3i)\,\text{Subst}\left(\int x^2\log\left(1-ie^x\right)\,dx, x, \sinh^{-1}(ax)\right)}{ac} + \frac{(3i)}{}$$

$$= \frac{2\sinh^{-1}(ax)^3\tan^{-1}\left(e^{\sinh^{-1}(ax)}\right)}{ac} - \frac{3i\sinh^{-1}(ax)^2\text{Li}_2\left(-ie^{\sinh^{-1}(ax)}\right)}{ac} + \frac{3i\sinh^{-1}(ax)^2\text{Li}_2\left(\right.}{ac}$$

$$= \frac{2\sinh^{-1}(ax)^3\tan^{-1}\left(e^{\sinh^{-1}(ax)}\right)}{ac} - \frac{3i\sinh^{-1}(ax)^2\text{Li}_2\left(-ie^{\sinh^{-1}(ax)}\right)}{ac} + \frac{3i\sinh^{-1}(ax)^2\text{Li}_2\left(\right.}{ac}$$

$$= \frac{2\sinh^{-1}(ax)^3\tan^{-1}\left(e^{\sinh^{-1}(ax)}\right)}{ac} - \frac{3i\sinh^{-1}(ax)^2\text{Li}_2\left(-ie^{\sinh^{-1}(ax)}\right)}{ac} + \frac{3i\sinh^{-1}(ax)^2\text{Li}_2\left(\right.}{ac}$$

$$= \frac{2\sinh^{-1}(ax)^3\tan^{-1}\left(e^{\sinh^{-1}(ax)}\right)}{ac} - \frac{3i\sinh^{-1}(ax)^2\text{Li}_2\left(-ie^{\sinh^{-1}(ax)}\right)}{ac} + \frac{3i\sinh^{-1}(ax)^2\text{Li}_2\left(\right.}{ac}$$

Mathematica [B] time = 0.23, size = 454, normalized size = 2.61

$$-\frac{i\left(192\sinh^{-1}(ax)^2\text{Li}_2\left(-ie^{\sinh^{-1}(ax)}\right) + 192i\pi\sinh^{-1}(ax)\text{Li}_2\left(ie^{\sinh^{-1}(ax)}\right) + 384\sinh^{-1}(ax)\text{Li}_3\left(-ie^{-\sinh^{-1}(ax)}\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^3/(c + a^2*c*x^2),x]
```

```
[Out] ((-1/64*I)*(7*Pi^4 + (8*I)*Pi^3*ArcSinh[a*x] + 24*Pi^2*ArcSinh[a*x]^2 - (32
*I)*Pi*ArcSinh[a*x]^3 - 16*ArcSinh[a*x]^4 + (8*I)*Pi^3*Log[1 + I/E^ArcSinh[
a*x]] + 48*Pi^2*ArcSinh[a*x]*Log[1 + I/E^ArcSinh[a*x]] - (96*I)*Pi*ArcSinh[
a*x]^2*Log[1 + I/E^ArcSinh[a*x]] - 64*ArcSinh[a*x]^3*Log[1 + I/E^ArcSinh[a*
x]] - 48*Pi^2*ArcSinh[a*x]*Log[1 - I*E^ArcSinh[a*x]] + (96*I)*Pi*ArcSinh[a*
x]^2*Log[1 - I*E^ArcSinh[a*x]] - (8*I)*Pi^3*Log[1 + I*E^ArcSinh[a*x]] + 64*
ArcSinh[a*x]^3*Log[1 + I*E^ArcSinh[a*x]] + (8*I)*Pi^3*Log[Tan[(Pi + (2*I)*A
rcSinh[a*x])/4]] - 48*(Pi - (2*I)*ArcSinh[a*x])^2*PolyLog[2, (-I)/E^ArcSinh
[a*x]] + 192*ArcSinh[a*x]^2*PolyLog[2, (-I)*E^ArcSinh[a*x]] - 48*Pi^2*PolyL
og[2, I*E^ArcSinh[a*x]] + (192*I)*Pi*ArcSinh[a*x]*PolyLog[2, I*E^ArcSinh[a*
```

```
x]] + (192*I)*Pi*PolyLog[3, (-I)/E^ArcSinh[a*x]] + 384*ArcSinh[a*x]*PolyLog
[3, (-I)/E^ArcSinh[a*x]] - 384*ArcSinh[a*x]*PolyLog[3, (-I)*E^ArcSinh[a*x]]
 - (192*I)*Pi*PolyLog[3, I*E^ArcSinh[a*x]] + 384*PolyLog[4, (-I)/E^ArcSinh[
a*x]] + 384*PolyLog[4, (-I)*E^ArcSinh[a*x]]))/(a*c)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{arsinh}(ax)^3}{a^2cx^2+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c),x, algorithm="fricas")
[Out] integral(arcsinh(a*x)^3/(a^2*c*x^2 + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)^3}{a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c),x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsinh}(ax)^3}{a^2c\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/(a^2*c*x^2+c),x)
[Out] int(arcsinh(a*x)^3/(a^2*c*x^2+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)^3}{a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c),x, algorithm="maxima")
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}(a\,x)^3}{c\,a^2\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^3/(c + a^2*c*x^2),x)
[Out] int(asinh(a*x)^3/(c + a^2*c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{asinh}^3(ax)}{a^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/(a**2*c*x**2+c),x)
```

```
[Out] Integral(asinh(a*x)**3/(a**2*x**2 + 1), x)/c
```

3.332 $\int \frac{\sinh^{-1}(ax)^3}{\left(c+a^2cx^2\right)^2}\,dx$

Optimal. Leaf size=294

$$\frac{x\sinh^{-1}(ax)^3}{2c^2\left(a^2x^2+1\right)}+\frac{3\sinh^{-1}(ax)^2}{2ac^2\sqrt{a^2x^2+1}}-\frac{3i\sinh^{-1}(ax)^2\text{Li}_2\left(-ie^{\sinh^{-1}(ax)}\right)}{2ac^2}+\frac{3i\sinh^{-1}(ax)^2\text{Li}_2\left(ie^{\sinh^{-1}(ax)}\right)}{2ac^2}+\frac{3i\sinh^{-1}(ax)}{}$$

```
[Out] 1/2*x*arcsinh(a*x)^3/c^2/(a^2*x^2+1)-6*arcsinh(a*x)*arctan(a*x+(a^2*x^2+1)^
(1/2))/a/c^2+arcsinh(a*x)^3*arctan(a*x+(a^2*x^2+1)^(1/2))/a/c^2+3*I*polylog
(2,-I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^2-3/2*I*arcsinh(a*x)^2*polylog(2,-I*(a*x
+(a^2*x^2+1)^(1/2)))/a/c^2-3*I*polylog(2,I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^2+3
/2*I*arcsinh(a*x)^2*polylog(2,I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^2+3*I*arcsinh(
a*x)*polylog(3,-I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^2-3*I*arcsinh(a*x)*polylog(3
,I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^2-3*I*polylog(4,-I*(a*x+(a^2*x^2+1)^(1/2)))
/a/c^2+3*I*polylog(4,I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^2+3/2*arcsinh(a*x)^2/a/
c^2/(a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {5690, 5693, 4180, 2531, 6609, 2282, 6589, 5717, 2279, 2391}

$$-\frac{3i\sinh^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{\sinh^{-1}(ax)}\right)}{2ac^2}+\frac{3i\sinh^{-1}(ax)^2\text{PolyLog}\left(2,ie^{\sinh^{-1}(ax)}\right)}{2ac^2}+\frac{3i\sinh^{-1}(ax)\text{PolyLog}\left(3,-ie\right.}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/(c + a^2*c*x^2)^2,x]
```

```
[Out] (3*ArcSinh[a*x]^2)/(2*a*c^2*Sqrt[1 + a^2*x^2]) + (x*ArcSinh[a*x]^3)/(2*c^2*
(1 + a^2*x^2)) - (6*ArcSinh[a*x]*ArcTan[E^ArcSinh[a*x]])/(a*c^2) + (ArcSinh
[a*x]^3*ArcTan[E^ArcSinh[a*x]])/(a*c^2) + ((3*I)*PolyLog[2, (-I)*E^ArcSinh[
a*x]])/(a*c^2) - (((3*I)/2)*ArcSinh[a*x]^2*PolyLog[2, (-I)*E^ArcSinh[a*x]])
/(a*c^2) - ((3*I)*PolyLog[2, I*E^ArcSinh[a*x]])/(a*c^2) + (((3*I)/2)*ArcSin
h[a*x]^2*PolyLog[2, I*E^ArcSinh[a*x]])/(a*c^2) + ((3*I)*ArcSinh[a*x]*PolyLo
g[3, (-I)*E^ArcSinh[a*x]])/(a*c^2) - ((3*I)*ArcSinh[a*x]*PolyLog[3, I*E^Arc
Sinh[a*x]])/(a*c^2) - ((3*I)*PolyLog[4, (-I)*E^ArcSinh[a*x]])/(a*c^2) + ((3
*I)*PolyLog[4, I*E^ArcSinh[a*x]])/(a*c^2)
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^3}{\left(c+a^2cx^2\right)^2}\,dx = \frac{x\sinh^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)} - \frac{(3a)\int \frac{x\sinh^{-1}(ax)^2}{\left(1+a^2x^2\right)^{3/2}}\,dx}{2c^2} + \frac{\int \frac{\sinh^{-1}(ax)^3}{c+a^2cx^2}\,dx}{2c}$$

$$= \frac{3\sinh^{-1}(ax)^2}{2ac^2\sqrt{1+a^2x^2}} + \frac{x\sinh^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)} - \frac{3\int \frac{\sinh^{-1}(ax)}{1+a^2x^2}\,dx}{c^2} + \frac{\text{Subst}\left(\int x^3\text{sech}(x)\,dx, x, \sinh^{-1}(ax)\right.}{2ac^2}$$

$$= \frac{3\sinh^{-1}(ax)^2}{2ac^2\sqrt{1+a^2x^2}} + \frac{x\sinh^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)} + \frac{\sinh^{-1}(ax)^3\tan^{-1}\left(e^{\sinh^{-1}(ax)}\right)}{ac^2} - \frac{(3i)\,\text{Subst}\left(\int x^2\log(1\right.}{2}$$

$$= \frac{3\sinh^{-1}(ax)^2}{2ac^2\sqrt{1+a^2x^2}} + \frac{x\sinh^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)} - \frac{6\sinh^{-1}(ax)\tan^{-1}\left(e^{\sinh^{-1}(ax)}\right)}{ac^2} + \frac{\sinh^{-1}(ax)^3\tan^{-1}\left(e^{\mathrm{si}}\right.}{ac^2}$$

$$= \frac{3\sinh^{-1}(ax)^2}{2ac^2\sqrt{1+a^2x^2}} + \frac{x\sinh^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)} - \frac{6\sinh^{-1}(ax)\tan^{-1}\left(e^{\sinh^{-1}(ax)}\right)}{ac^2} + \frac{\sinh^{-1}(ax)^3\tan^{-1}\left(e^{\mathrm{si}}\right.}{ac^2}$$

$$= \frac{3\sinh^{-1}(ax)^2}{2ac^2\sqrt{1+a^2x^2}} + \frac{x\sinh^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)} - \frac{6\sinh^{-1}(ax)\tan^{-1}\left(e^{\sinh^{-1}(ax)}\right)}{ac^2} + \frac{\sinh^{-1}(ax)^3\tan^{-1}\left(e^{\mathrm{si}}\right.}{ac^2}$$

$$= \frac{3\sinh^{-1}(ax)^2}{2ac^2\sqrt{1+a^2x^2}} + \frac{x\sinh^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)} - \frac{6\sinh^{-1}(ax)\tan^{-1}\left(e^{\sinh^{-1}(ax)}\right)}{ac^2} + \frac{\sinh^{-1}(ax)^3\tan^{-1}\left(e^{\mathrm{si}}\right.}{ac^2}$$

Mathematica [A] time = 2.60, size = 568, normalized size = 1.93

$$-\frac{i\left(\frac{64iax\sinh^{-1}(ax)^3}{a^2x^2+1} + \frac{192i\sinh^{-1}(ax)^2}{\sqrt{a^2x^2+1}} + 192\sinh^{-1}(ax)^2\text{Li}_2\left(-ie^{\sinh^{-1}(ax)}\right) + 192i\pi\sinh^{-1}(ax)\text{Li}_2\left(ie^{\sinh^{-1}(ax)}\right) + 384\,\mathrm{s}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^3/(c + a^2*c*x^2)^2,x]
```

```
[Out] ((-1/128*I)*(7*Pi^4 + (8*I)*Pi^3*ArcSinh[a*x] + 24*Pi^2*ArcSinh[a*x]^2 + ((
192*I)*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2] - (32*I)*Pi*ArcSinh[a*x]^3 + ((64*
I)*a*x*ArcSinh[a*x]^3)/(1 + a^2*x^2) - 16*ArcSinh[a*x]^4 - 384*ArcSinh[a*x]
*Log[1 - I/E^ArcSinh[a*x]] + (8*I)*Pi^3*Log[1 + I/E^ArcSinh[a*x]] + 384*Arc
Sinh[a*x]*Log[1 + I/E^ArcSinh[a*x]] + 48*Pi^2*ArcSinh[a*x]*Log[1 + I/E^ArcS
inh[a*x]] - (96*I)*Pi*ArcSinh[a*x]^2*Log[1 + I/E^ArcSinh[a*x]] - 64*ArcSinh
[a*x]^3*Log[1 + I/E^ArcSinh[a*x]] - 48*Pi^2*ArcSinh[a*x]*Log[1 - I*E^ArcSin
h[a*x]] + (96*I)*Pi*ArcSinh[a*x]^2*Log[1 - I*E^ArcSinh[a*x]] - (8*I)*Pi^3*L
og[1 + I*E^ArcSinh[a*x]] + 64*ArcSinh[a*x]^3*Log[1 + I*E^ArcSinh[a*x]] + (8
*I)*Pi^3*Log[Tan[(Pi + (2*I)*ArcSinh[a*x])/4]] - 48*(8 + Pi^2 - (4*I)*Pi*Ar
cSinh[a*x] - 4*ArcSinh[a*x]^2)*PolyLog[2, (-I)/E^ArcSinh[a*x]] + 384*PolyLo
g[2, I/E^ArcSinh[a*x]] + 192*ArcSinh[a*x]^2*PolyLog[2, (-I)*E^ArcSinh[a*x]]
 - 48*Pi^2*PolyLog[2, I*E^ArcSinh[a*x]] + (192*I)*Pi*ArcSinh[a*x]*PolyLog[2
, I*E^ArcSinh[a*x]] + (192*I)*Pi*PolyLog[3, (-I)/E^ArcSinh[a*x]] + 384*ArcS
inh[a*x]*PolyLog[3, (-I)/E^ArcSinh[a*x]] - 384*ArcSinh[a*x]*PolyLog[3, (-I)
*E^ArcSinh[a*x]] - (192*I)*Pi*PolyLog[3, I*E^ArcSinh[a*x]] + 384*PolyLog[4,
 (-I)/E^ArcSinh[a*x]] + 384*PolyLog[4, (-I)*E^ArcSinh[a*x]]))/(a*c^2)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arsinh}\,(ax)^3}{a^4c^2x^4 + 2\,a^2c^2x^2 + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="fricas")
[Out] integral(arcsinh(a*x)^3/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)^3}{(a^2cx^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^2, x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(a^2c\,x^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/(a^2*c*x^2+c)^2,x)
[Out] int(arcsinh(a*x)^3/(a^2*c*x^2+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)^3}{(a^2cx^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="maxima")
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{asinh}(a\,x)^3}{(c\,a^2\,x^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^3/(c + a^2*c*x^2)^2,x)
[Out] int(asinh(a*x)^3/(c + a^2*c*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{asinh}^3(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/(a**2*c*x**2+c)**2,x)
[Out] Integral(asinh(a*x)**3/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

3.333 $\int \frac{\sinh^{-1}(ax)^3}{\left(c+a^2cx^2\right)^3}\,dx$

Optimal. Leaf size=409

$$-\frac{1}{4ac^3\sqrt{a^2x^2+1}}+\frac{3x\sinh^{-1}(ax)^3}{8c^3\left(a^2x^2+1\right)}+\frac{x\sinh^{-1}(ax)^3}{4c^3\left(a^2x^2+1\right)^2}+\frac{9\sinh^{-1}(ax)^2}{8ac^3\sqrt{a^2x^2+1}}+\frac{\sinh^{-1}(ax)^2}{4ac^3\left(a^2x^2+1\right)^{3/2}}-\frac{x\sinh^{-1}(ax)}{4c^3\left(a^2x^2+1\right)}-\frac{9i\sinh}{}$$

```
[Out] -1/4*x*arcsinh(a*x)/c^3/(a^2*x^2+1)+1/4*arcsinh(a*x)^2/a/c^3/(a^2*x^2+1)^(3
/2)+1/4*x*arcsinh(a*x)^3/c^3/(a^2*x^2+1)^2+3/8*x*arcsinh(a*x)^3/c^3/(a^2*x^
2+1)-5*arcsinh(a*x)*arctan(a*x+(a^2*x^2+1)^(1/2))/a/c^3+3/4*arcsinh(a*x)^3*
arctan(a*x+(a^2*x^2+1)^(1/2))/a/c^3+9/4*I*arcsinh(a*x)*polylog(3,-I*(a*x+(a
^2*x^2+1)^(1/2)))/a/c^3-9/8*I*arcsinh(a*x)^2*polylog(2,-I*(a*x+(a^2*x^2+1)^
(1/2)))/a/c^3-9/4*I*arcsinh(a*x)*polylog(3,I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^3
+9/8*I*arcsinh(a*x)^2*polylog(2,I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^3+5/2*I*poly
log(2,-I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^3-5/2*I*polylog(2,I*(a*x+(a^2*x^2+1)^
(1/2)))/a/c^3-9/4*I*polylog(4,-I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^3+9/4*I*polyl
og(4,I*(a*x+(a^2*x^2+1)^(1/2)))/a/c^3-1/4/a/c^3/(a^2*x^2+1)^(1/2)+9/8*arcsi
nh(a*x)^2/a/c^3/(a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 409, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {5690, 5693, 4180, 2531, 6609, 2282, 6589, 5717, 2279, 2391, 261}

$$-\frac{9i\sinh^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{\sinh^{-1}(ax)}\right)}{8ac^3}+\frac{9i\sinh^{-1}(ax)^2\text{PolyLog}\left(2,ie^{\sinh^{-1}(ax)}\right)}{8ac^3}+\frac{9i\sinh^{-1}(ax)\text{PolyLog}\left(3,-ie\right.}{4ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/(c + a^2*c*x^2)^3,x]
```

```
[Out] -1/(4*a*c^3*Sqrt[1 + a^2*x^2]) - (x*ArcSinh[a*x])/(4*c^3*(1 + a^2*x^2)) + A
rcSinh[a*x]^2/(4*a*c^3*(1 + a^2*x^2)^(3/2)) + (9*ArcSinh[a*x]^2)/(8*a*c^3*S
qrt[1 + a^2*x^2]) + (x*ArcSinh[a*x]^3)/(4*c^3*(1 + a^2*x^2)^2) + (3*x*ArcSi
nh[a*x]^3)/(8*c^3*(1 + a^2*x^2)) - (5*ArcSinh[a*x]*ArcTan[E^ArcSinh[a*x]])/
(a*c^3) + (3*ArcSinh[a*x]^3*ArcTan[E^ArcSinh[a*x]])/(4*a*c^3) + (((5*I)/2)*
PolyLog[2, (-I)*E^ArcSinh[a*x]])/(a*c^3) - (((9*I)/8)*ArcSinh[a*x]^2*PolyLo
g[2, (-I)*E^ArcSinh[a*x]])/(a*c^3) - (((5*I)/2)*PolyLog[2, I*E^ArcSinh[a*x]
])/(a*c^3) + (((9*I)/8)*ArcSinh[a*x]^2*PolyLog[2, I*E^ArcSinh[a*x]])/(a*c^3
) + (((9*I)/4)*ArcSinh[a*x]*PolyLog[3, (-I)*E^ArcSinh[a*x]])/(a*c^3) - (((9
*I)/4)*ArcSinh[a*x]*PolyLog[3, I*E^ArcSinh[a*x]])/(a*c^3) - (((9*I)/4)*Poly
Log[4, (-I)*E^ArcSinh[a*x]])/(a*c^3) + (((9*I)/4)*PolyLog[4, I*E^ArcSinh[a*
x]])/(a*c^3)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^3}{(c+a^2cx^2)^3}\,dx = \frac{x\sinh^{-1}(ax)^3}{4c^3(1+a^2x^2)^2} - \frac{(3a)\int \frac{x\sinh^{-1}(ax)^2}{(1+a^2x^2)^{5/2}}\,dx}{4c^3} + \frac{3\int \frac{\sinh^{-1}(ax)^3}{(c+a^2cx^2)^2}\,dx}{4c}$$

$$= \frac{\sinh^{-1}(ax)^2}{4ac^3(1+a^2x^2)^{3/2}} + \frac{x\sinh^{-1}(ax)^3}{4c^3(1+a^2x^2)^2} + \frac{3x\sinh^{-1}(ax)^3}{8c^3(1+a^2x^2)} - \frac{\int \frac{\sinh^{-1}(ax)}{(1+a^2x^2)^2}\,dx}{2c^3} - \frac{(9a)\int \frac{x\sinh^{-1}(ax)^2}{(1+a^2x^2)^{3/2}}}{8c^3}$$

$$= -\frac{x\sinh^{-1}(ax)}{4c^3(1+a^2x^2)} + \frac{\sinh^{-1}(ax)^2}{4ac^3(1+a^2x^2)^{3/2}} + \frac{9\sinh^{-1}(ax)^2}{8ac^3\sqrt{1+a^2x^2}} + \frac{x\sinh^{-1}(ax)^3}{4c^3(1+a^2x^2)^2} + \frac{3x\sinh^{-1}(ax)^3}{8c^3(1+a^2x^2)}$$

$$= -\frac{1}{4ac^3\sqrt{1+a^2x^2}} - \frac{x\sinh^{-1}(ax)}{4c^3(1+a^2x^2)} + \frac{\sinh^{-1}(ax)^2}{4ac^3(1+a^2x^2)^{3/2}} + \frac{9\sinh^{-1}(ax)^2}{8ac^3\sqrt{1+a^2x^2}} + \frac{x\sinh^{-1}(ax)^3}{4c^3(1+a^2x^2)^2}$$

$$= -\frac{1}{4ac^3\sqrt{1+a^2x^2}} - \frac{x\sinh^{-1}(ax)}{4c^3(1+a^2x^2)} + \frac{\sinh^{-1}(ax)^2}{4ac^3(1+a^2x^2)^{3/2}} + \frac{9\sinh^{-1}(ax)^2}{8ac^3\sqrt{1+a^2x^2}} + \frac{x\sinh^{-1}(ax)^3}{4c^3(1+a^2x^2)^2}$$

$$= -\frac{1}{4ac^3\sqrt{1+a^2x^2}} - \frac{x\sinh^{-1}(ax)}{4c^3(1+a^2x^2)} + \frac{\sinh^{-1}(ax)^2}{4ac^3(1+a^2x^2)^{3/2}} + \frac{9\sinh^{-1}(ax)^2}{8ac^3\sqrt{1+a^2x^2}} + \frac{x\sinh^{-1}(ax)^3}{4c^3(1+a^2x^2)^2}$$

$$= -\frac{1}{4ac^3\sqrt{1+a^2x^2}} - \frac{x\sinh^{-1}(ax)}{4c^3(1+a^2x^2)} + \frac{\sinh^{-1}(ax)^2}{4ac^3(1+a^2x^2)^{3/2}} + \frac{9\sinh^{-1}(ax)^2}{8ac^3\sqrt{1+a^2x^2}} + \frac{x\sinh^{-1}(ax)^3}{4c^3(1+a^2x^2)^2}$$

$$= -\frac{1}{4ac^3\sqrt{1+a^2x^2}} - \frac{x\sinh^{-1}(ax)}{4c^3(1+a^2x^2)} + \frac{\sinh^{-1}(ax)^2}{4ac^3(1+a^2x^2)^{3/2}} + \frac{9\sinh^{-1}(ax)^2}{8ac^3\sqrt{1+a^2x^2}} + \frac{x\sinh^{-1}(ax)^3}{4c^3(1+a^2x^2)^2}$$

Mathematica [A] time = 5.47, size = 654, normalized size = 1.60

$$-\frac{i\left(-\frac{128i}{\sqrt{a^2x^2+1}} + \frac{192iax\sinh^{-1}(ax)^3}{a^2x^2+1} + \frac{128iax\sinh^{-1}(ax)^3}{(a^2x^2+1)^2} + \frac{576i\sinh^{-1}(ax)^2}{\sqrt{a^2x^2+1}} + \frac{128i\sinh^{-1}(ax)^2}{(a^2x^2+1)^{3/2}} - \frac{128iax\sinh^{-1}(ax)}{a^2x^2+1} + 576\sinh^{-1}(a\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^3/(c + a^2*c*x^2)^3,x]
```

```
[Out] ((-1/512*I)*(21*Pi^4 - (128*I)/Sqrt[1 + a^2*x^2] + (24*I)*Pi^3*ArcSinh[a*x]
 - ((128*I)*a*x*ArcSinh[a*x])/(1 + a^2*x^2) + 72*Pi^2*ArcSinh[a*x]^2 + ((12
8*I)*ArcSinh[a*x]^2)/(1 + a^2*x^2)^(3/2) + ((576*I)*ArcSinh[a*x]^2)/Sqrt[1
+ a^2*x^2] - (96*I)*Pi*ArcSinh[a*x]^3 + ((128*I)*a*x*ArcSinh[a*x]^3)/(1 + a
```

```
^2*x^2)^2 + ((192*I)*a*x*ArcSinh[a*x]^3)/(1 + a^2*x^2) - 48*ArcSinh[a*x]^4
- 1280*ArcSinh[a*x]*Log[1 - I/E^ArcSinh[a*x]] + (24*I)*Pi^3*Log[1 + I/E^Arc
Sinh[a*x]] + 1280*ArcSinh[a*x]*Log[1 + I/E^ArcSinh[a*x]] + 144*Pi^2*ArcSinh
[a*x]*Log[1 + I/E^ArcSinh[a*x]] - (288*I)*Pi*ArcSinh[a*x]^2*Log[1 + I/E^Arc
Sinh[a*x]] - 192*ArcSinh[a*x]^3*Log[1 + I/E^ArcSinh[a*x]] - 144*Pi^2*ArcSin
h[a*x]*Log[1 - I*E^ArcSinh[a*x]] + (288*I)*Pi*ArcSinh[a*x]^2*Log[1 - I*E^Ar
cSinh[a*x]] - (24*I)*Pi^3*Log[1 + I*E^ArcSinh[a*x]] + 192*ArcSinh[a*x]^3*Lo
g[1 + I*E^ArcSinh[a*x]] + (24*I)*Pi^3*Log[Tan[(Pi + (2*I)*ArcSinh[a*x])/4]]
 - 16*(80 + 9*Pi^2 - (36*I)*Pi*ArcSinh[a*x] - 36*ArcSinh[a*x]^2)*PolyLog[2,
 (-I)/E^ArcSinh[a*x]] + 1280*PolyLog[2, I/E^ArcSinh[a*x]] + 576*ArcSinh[a*x
]^2*PolyLog[2, (-I)*E^ArcSinh[a*x]] - 144*Pi^2*PolyLog[2, I*E^ArcSinh[a*x]]
 + (576*I)*Pi*ArcSinh[a*x]*PolyLog[2, I*E^ArcSinh[a*x]] + (576*I)*Pi*PolyLo
g[3, (-I)/E^ArcSinh[a*x]] + 1152*ArcSinh[a*x]*PolyLog[3, (-I)/E^ArcSinh[a*x
]] - 1152*ArcSinh[a*x]*PolyLog[3, (-I)*E^ArcSinh[a*x]] - (576*I)*Pi*PolyLog
[3, I*E^ArcSinh[a*x]] + 1152*PolyLog[4, (-I)/E^ArcSinh[a*x]] + 1152*PolyLog
[4, (-I)*E^ArcSinh[a*x]]))/(a*c^3)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{arsinh}(ax)^3}{a^6c^3x^6 + 3\,a^4c^3x^4 + 3\,a^2c^3x^2 + c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^3/(a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)^3}{(a^2cx^2 + c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^3, x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsinh}(ax)^3}{(a^2c\,x^2 + c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/(a^2*c*x^2+c)^3,x)
```

```
[Out] int(arcsinh(a*x)^3/(a^2*c*x^2+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)^3}{(a^2cx^2 + c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{asinh}(a\,x)^3}{\left(c\,a^2\,x^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^3/(c + a^2*c*x^2)^3,x)
```

```
[Out] int(asinh(a*x)^3/(c + a^2*c*x^2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{asinh}^3(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(asinh(a*x)**3/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/c**3
```

3.334 $\int \left(c+a^2cx^2\right)^{5/2}\sinh^{-1}(ax)^3\,dx$

Optimal. Leaf size=509

$$-\frac{865ac^2x^2\sqrt{a^2cx^2+c}}{2304\sqrt{a^2x^2+1}}-\frac{c^2\left(a^2x^2+1\right)^{5/2}\sqrt{a^2cx^2+c}}{216a}-\frac{15ac^2x^2\sqrt{a^2cx^2+c}\sinh^{-1}(ax)^2}{32\sqrt{a^2x^2+1}}+\frac{5}{16}c^2x\sqrt{a^2cx^2+c}\,\text{sinh}$$

```
[Out] 5/24*c*x*(a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^3+1/6*x*(a^2*c*x^2+c)^(5/2)*arcsi
nh(a*x)^3-1/216*c^2*(a^2*x^2+1)^(5/2)*(a^2*c*x^2+c)^(1/2)/a+245/384*c^2*x*a
rcsinh(a*x)*(a^2*c*x^2+c)^(1/2)+65/576*c^2*x*(a^2*x^2+1)*arcsinh(a*x)*(a^2*
c*x^2+c)^(1/2)+1/36*c^2*x*(a^2*x^2+1)^2*arcsinh(a*x)*(a^2*c*x^2+c)^(1/2)-5/
32*c^2*(a^2*x^2+1)^(3/2)*arcsinh(a*x)^2*(a^2*c*x^2+c)^(1/2)/a-1/12*c^2*(a^2
*x^2+1)^(5/2)*arcsinh(a*x)^2*(a^2*c*x^2+c)^(1/2)/a+5/16*c^2*x*arcsinh(a*x)^
3*(a^2*c*x^2+c)^(1/2)-865/2304*a*c^2*x^2*(a^2*c*x^2+c)^(1/2)/(a^2*x^2+1)^(1
/2)-65/2304*a^3*c^2*x^4*(a^2*c*x^2+c)^(1/2)/(a^2*x^2+1)^(1/2)-115/768*c^2*a
rcsinh(a*x)^2*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-15/32*a*c^2*x^2*arcsi
nh(a*x)^2*(a^2*c*x^2+c)^(1/2)/(a^2*x^2+1)^(1/2)+5/64*c^2*arcsinh(a*x)^4*(a^
2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.59, antiderivative size = 509, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5684, 5682, 5675, 5661, 5758, 30, 5717, 14, 261}

$$-\frac{65a^3c^2x^4\sqrt{a^2cx^2+c}}{2304\sqrt{a^2x^2+1}}-\frac{865ac^2x^2\sqrt{a^2cx^2+c}}{2304\sqrt{a^2x^2+1}}-\frac{c^2\left(a^2x^2+1\right)^{5/2}\sqrt{a^2cx^2+c}}{216a}-\frac{15ac^2x^2\sqrt{a^2cx^2+c}\sinh^{-1}(ax)^2}{32\sqrt{a^2x^2+1}}+$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^3,x]
```

```
[Out] (-865*a*c^2*x^2*Sqrt[c + a^2*c*x^2])/(2304*Sqrt[1 + a^2*x^2]) - (65*a^3*c^2
*x^4*Sqrt[c + a^2*c*x^2])/(2304*Sqrt[1 + a^2*x^2]) - (c^2*(1 + a^2*x^2)^(5/
2)*Sqrt[c + a^2*c*x^2])/(216*a) + (245*c^2*x*Sqrt[c + a^2*c*x^2]*ArcSinh[a*
x])/384 + (65*c^2*x*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x])/576 + (
c^2*x*(1 + a^2*x^2)^2*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x])/36 - (115*c^2*Sqrt[
c + a^2*c*x^2]*ArcSinh[a*x]^2)/(768*a*Sqrt[1 + a^2*x^2]) - (15*a*c^2*x^2*Sq
rt[c + a^2*c*x^2]*ArcSinh[a*x]^2)/(32*Sqrt[1 + a^2*x^2]) - (5*c^2*(1 + a^2*
x^2)^(3/2)*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^2)/(32*a) - (c^2*(1 + a^2*x^2)^
(5/2)*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^2)/(12*a) + (5*c^2*x*Sqrt[c + a^2*c*
x^2]*ArcSinh[a*x]^3)/16 + (5*c*x*(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^3)/24 +
 (x*(c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^3)/6 + (5*c^2*Sqrt[c + a^2*c*x^2]*Ar
cSinh[a*x]^4)/(64*a*Sqrt[1 + a^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^{5/2} \sinh^{-1}(ax)^3\,dx = \frac{1}{6}x\left(c+a^2cx^2\right)^{5/2}\sinh^{-1}(ax)^3 + \frac{1}{6}(5c)\int\left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^3\,dx - \frac{(a}{}$$

$$= -\frac{c^2\left(1+a^2x^2\right)^{5/2}\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^2}{12a} + \frac{5}{24}cx\left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)$$

$$= \frac{1}{36}c^2x\left(1+a^2x^2\right)^2\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax) - \frac{5c^2\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}\,\text{si}}{32a}$$

$$= -\frac{c^2\left(1+a^2x^2\right)^{5/2}\sqrt{c+a^2cx^2}}{216a} + \frac{65}{576}c^2x\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax) -$$

$$= -\frac{c^2\left(1+a^2x^2\right)^{5/2}\sqrt{c+a^2cx^2}}{216a} + \frac{245}{384}c^2x\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax) + \frac{65}{576}c^2x\left(1\right.$$

$$= -\frac{865ac^2x^2\sqrt{c+a^2cx^2}}{2304\sqrt{1+a^2x^2}} - \frac{65a^3c^2x^4\sqrt{c+a^2cx^2}}{2304\sqrt{1+a^2x^2}} - \frac{c^2\left(1+a^2x^2\right)^{5/2}\sqrt{c+a^2cx}}{216a}$$

Mathematica [A] time = 0.69, size = 177, normalized size = 0.35

$$\frac{c^2\sqrt{a^2cx^2+c}\,\left(4320\sinh^{-1}(ax)^4 + 288\left(45\sinh\left(2\sinh^{-1}(ax)\right) + 9\sinh\left(4\sinh^{-1}(ax)\right) + \sinh\left(6\sinh^{-1}(ax)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^3,x]
```

```
[Out] (c^2*Sqrt[c + a^2*c*x^2]*(4320*ArcSinh[a*x]^4 - 9720*Cosh[2*ArcSinh[a*x]] -
 243*Cosh[4*ArcSinh[a*x]] - 8*Cosh[6*ArcSinh[a*x]] - 72*ArcSinh[a*x]^2*(270
*Cosh[2*ArcSinh[a*x]] + 27*Cosh[4*ArcSinh[a*x]] + 2*Cosh[6*ArcSinh[a*x]]) +
 288*ArcSinh[a*x]^3*(45*Sinh[2*ArcSinh[a*x]] + 9*Sinh[4*ArcSinh[a*x]] + Sin
h[6*ArcSinh[a*x]]) + 12*ArcSinh[a*x]*(1620*Sinh[2*ArcSinh[a*x]] + 81*Sinh[4
*ArcSinh[a*x]] + 4*Sinh[6*ArcSinh[a*x]])))/(55296*a*Sqrt[1 + a^2*x^2])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^4c^2x^4 + 2\,a^2c^2x^2 + c^2\right)\sqrt{a^2cx^2+c}\;\text{arsinh}\left(ax\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arcsinh(a*
x)^3, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.26, size = 802, normalized size = 1.58

$$\frac{5\sqrt{c\left(a^{2}x^{2}+1\right)}\ \operatorname{arcsinh}\left(ax\right)^{4}c^{2}}{64\sqrt{a^{2}x^{2}+1}\ a}+\frac{\sqrt{c\left(a^{2}x^{2}+1\right)}\ \left(32x^{7}a^{7}+32a^{6}x^{6}\sqrt{a^{2}x^{2}+1}\ +64x^{5}a^{5}+48\sqrt{a^{2}x^{2}+1}\ x^{4}a^{4}+3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)*arcsinh(a*x)^3,x)
```

```
[Out] 5/64*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a*arcsinh(a*x)^4*c^2+1/13824*(
c*(a^2*x^2+1))^(1/2)*(32*x^7*a^7+32*a^6*x^6*(a^2*x^2+1)^(1/2)+64*x^5*a^5+48
*(a^2*x^2+1)^(1/2)*x^4*a^4+38*x^3*a^3+18*(a^2*x^2+1)^(1/2)*x^2*a^2+6*a*x+(a
^2*x^2+1)^(1/2))*(36*arcsinh(a*x)^3-18*arcsinh(a*x)^2+6*arcsinh(a*x)-1)*c^2
/a/(a^2*x^2+1)+3/4096*(c*(a^2*x^2+1))^(1/2)*(8*x^5*a^5+8*(a^2*x^2+1)^(1/2)*
x^4*a^4+12*x^3*a^3+8*(a^2*x^2+1)^(1/2)*x^2*a^2+4*a*x+(a^2*x^2+1)^(1/2))*(32
*arcsinh(a*x)^3-24*arcsinh(a*x)^2+12*arcsinh(a*x)-3)*c^2/a/(a^2*x^2+1)+15/5
12*(c*(a^2*x^2+1))^(1/2)*(2*x^3*a^3+2*(a^2*x^2+1)^(1/2)*x^2*a^2+2*a*x+(a^2*
x^2+1)^(1/2))*(4*arcsinh(a*x)^3-6*arcsinh(a*x)^2+6*arcsinh(a*x)-3)*c^2/a/(a
^2*x^2+1)+15/512*(c*(a^2*x^2+1))^(1/2)*(2*x^3*a^3-2*(a^2*x^2+1)^(1/2)*x^2*a
^2+2*a*x-(a^2*x^2+1)^(1/2))*(4*arcsinh(a*x)^3+6*arcsinh(a*x)^2+6*arcsinh(a*
x)+3)*c^2/a/(a^2*x^2+1)+3/4096*(c*(a^2*x^2+1))^(1/2)*(8*x^5*a^5-8*(a^2*x^2+
1)^(1/2)*x^4*a^4+12*x^3*a^3-8*(a^2*x^2+1)^(1/2)*x^2*a^2+4*a*x-(a^2*x^2+1)^(
1/2))*(32*arcsinh(a*x)^3+24*arcsinh(a*x)^2+12*arcsinh(a*x)+3)*c^2/a/(a^2*x^
2+1)+1/13824*(c*(a^2*x^2+1))^(1/2)*(32*x^7*a^7-32*a^6*x^6*(a^2*x^2+1)^(1/2)
+64*x^5*a^5-48*(a^2*x^2+1)^(1/2)*x^4*a^4+38*x^3*a^3-18*(a^2*x^2+1)^(1/2)*x^
2*a^2+6*a*x-(a^2*x^2+1)^(1/2))*(36*arcsinh(a*x)^3+18*arcsinh(a*x)^2+6*arcsi
nh(a*x)+1)*c^2/a/(a^2*x^2+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{asinh}\left(a\,x\right)^3\,{\left(c\,a^2\,x^2+c\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^3*(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] int(asinh(a*x)^3*(c + a^2*c*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*asinh(a*x)**3,x)
```

```
[Out] Timed out
```

3.335 $\int \left(c+a^2cx^2\right)^{3/2} \sinh^{-1}(ax)^3 \, dx$

Optimal. Leaf size=348

$$-\frac{51acx^2\sqrt{a^2cx^2+c}}{128\sqrt{a^2x^2+1}}-\frac{9acx^2\sqrt{a^2cx^2+c}\,\sinh^{-1}(ax)^2}{16\sqrt{a^2x^2+1}}+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\sinh^{-1}(ax)^3+\frac{3}{8}cx\sqrt{a^2cx^2+c}\,\sinh^{-1}(a$$

```
[Out] 1/4*x*(a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^3+45/64*c*x*arcsinh(a*x)*(a^2*c*x^2+
c)^(1/2)+3/32*c*x*(a^2*x^2+1)*arcsinh(a*x)*(a^2*c*x^2+c)^(1/2)-3/16*c*(a^2*
x^2+1)^(3/2)*arcsinh(a*x)^2*(a^2*c*x^2+c)^(1/2)/a+3/8*c*x*arcsinh(a*x)^3*(a
^2*c*x^2+c)^(1/2)-51/128*a*c*x^2*(a^2*c*x^2+c)^(1/2)/(a^2*x^2+1)^(1/2)-3/12
8*a^3*c*x^4*(a^2*c*x^2+c)^(1/2)/(a^2*x^2+1)^(1/2)-27/128*c*arcsinh(a*x)^2*(
a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-9/16*a*c*x^2*arcsinh(a*x)^2*(a^2*c*x
^2+c)^(1/2)/(a^2*x^2+1)^(1/2)+3/32*c*arcsinh(a*x)^4*(a^2*c*x^2+c)^(1/2)/a/(
a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {5684, 5682, 5675, 5661, 5758, 30, 5717, 14}

$$-\frac{3a^3cx^4\sqrt{a^2cx^2+c}}{128\sqrt{a^2x^2+1}}-\frac{51acx^2\sqrt{a^2cx^2+c}}{128\sqrt{a^2x^2+1}}-\frac{9acx^2\sqrt{a^2cx^2+c}\,\sinh^{-1}(ax)^2}{16\sqrt{a^2x^2+1}}+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\sinh^{-1}(ax)^3+\frac{3}{8}cx$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^3,x]
```

```
[Out] (-51*a*c*x^2*Sqrt[c + a^2*c*x^2])/(128*Sqrt[1 + a^2*x^2]) - (3*a^3*c*x^4*Sq
rt[c + a^2*c*x^2])/(128*Sqrt[1 + a^2*x^2]) + (45*c*x*Sqrt[c + a^2*c*x^2]*Ar
cSinh[a*x])/64 + (3*c*x*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x])/32
- (27*c*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^2)/(128*a*Sqrt[1 + a^2*x^2]) - (9*
a*c*x^2*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^2)/(16*Sqrt[1 + a^2*x^2]) - (3*c*(
1 + a^2*x^2)^(3/2)*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^2)/(16*a) + (3*c*x*Sqrt
[c + a^2*c*x^2]*ArcSinh[a*x]^3)/8 + (x*(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^3
)/4 + (3*c*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^4)/(32*a*Sqrt[1 + a^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
```

```
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^{3/2} \sinh^{-1}(ax)^3\,dx = \frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^3 + \frac{1}{4}(3c)\int \sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^3\,dx - \frac{\left(3ac\right.}{}$$

$$= -\frac{3c\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^2}{16a} + \frac{3}{8}cx\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^3 +$$

$$= \frac{3}{32}cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax) - \frac{9acx^2\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^2}{16\sqrt{1+a^2x^2}} -$$

$$= \frac{45}{64}cx\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax) + \frac{3}{32}cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax) - \frac{9a}{}$$

$$= -\frac{51acx^2\sqrt{c+a^2cx^2}}{128\sqrt{1+a^2x^2}} - \frac{3a^3cx^4\sqrt{c+a^2cx^2}}{128\sqrt{1+a^2x^2}} + \frac{45}{64}cx\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax) +$$

Mathematica [A] time = 0.29, size = 136, normalized size = 0.39

$$\frac{c\sqrt{a^2cx^2+c}\left(96\sinh^{-1}(ax)^4 + 32\left(8\sinh\left(2\sinh^{-1}(ax)\right)+\sinh\left(4\sinh^{-1}(ax)\right)\right)\sinh^{-1}(ax)^3 + 12\left(32\sinh\left(2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^3,x]
```

```
[Out] (c*Sqrt[c + a^2*c*x^2]*(96*ArcSinh[a*x]^4 - 24*ArcSinh[a*x]^2*(16*Cosh[2*Ar
cSinh[a*x]] + Cosh[4*ArcSinh[a*x]]) - 3*(64*Cosh[2*ArcSinh[a*x]] + Cosh[4*A
rcSinh[a*x]]) + 32*ArcSinh[a*x]^3*(8*Sinh[2*ArcSinh[a*x]] + Sinh[4*ArcSinh[
a*x]]) + 12*ArcSinh[a*x]*(32*Sinh[2*ArcSinh[a*x]] + Sinh[4*ArcSinh[a*x]])))
/(1024*a*Sqrt[1 + a^2*x^2])
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(a^2cx^2+c\right)^{\frac{3}{2}}\operatorname{arsinh}(ax)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*arcsinh(a*x)^3, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.20, size = 484, normalized size = 1.39

$$\frac{3\sqrt{c\left(a^2x^2+1\right)}\,\operatorname{arcsinh}(ax)^4 c}{32\sqrt{a^2x^2+1}\,a} + \frac{\sqrt{c\left(a^2x^2+1\right)}\left(8x^5a^5 + 8\sqrt{a^2x^2+1}\,x^4a^4 + 12x^3a^3 + 8\sqrt{a^2x^2+1}\,x^2a^2 + 4a\right.}{2048a\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^3,x)
```

```
[Out] 3/32*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a*arcsinh(a*x)^4*c+1/2048*(c*(
a^2*x^2+1))^(1/2)*(8*x^5*a^5+8*(a^2*x^2+1)^(1/2)*x^4*a^4+12*x^3*a^3+8*(a^2*
x^2+1)^(1/2)*x^2*a^2+4*a*x+(a^2*x^2+1)^(1/2))*(32*arcsinh(a*x)^3-24*arcsinh
(a*x)^2+12*arcsinh(a*x)-3)*c/a/(a^2*x^2+1)+1/32*(c*(a^2*x^2+1))^(1/2)*(2*x^
3*a^3+2*(a^2*x^2+1)^(1/2)*x^2*a^2+2*a*x+(a^2*x^2+1)^(1/2))*(4*arcsinh(a*x)^
3-6*arcsinh(a*x)^2+6*arcsinh(a*x)-3)*c/a/(a^2*x^2+1)+1/32*(c*(a^2*x^2+1))^(
1/2)*(2*x^3*a^3-2*(a^2*x^2+1)^(1/2)*x^2*a^2+2*a*x-(a^2*x^2+1)^(1/2))*(4*arc
sinh(a*x)^3+6*arcsinh(a*x)^2+6*arcsinh(a*x)+3)*c/a/(a^2*x^2+1)+1/2048*(c*(a
^2*x^2+1))^(1/2)*(8*x^5*a^5-8*(a^2*x^2+1)^(1/2)*x^4*a^4+12*x^3*a^3-8*(a^2*x
^2+1)^(1/2)*x^2*a^2+4*a*x-(a^2*x^2+1)^(1/2))*(32*arcsinh(a*x)^3+24*arcsinh(
a*x)^2+12*arcsinh(a*x)+3)*c/a/(a^2*x^2+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{asinh}(a\,x)^3\,{\left(c\,a^2\,x^2+c\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^3*(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int(asinh(a*x)^3*(c + a^2*c*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c \left(a^{2} x^{2} + 1\right)\right)^{\frac{3}{2}} \operatorname{asinh}^{3}{\left(a x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*asinh(a*x)**3,x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*asinh(a*x)**3, x)
```

3.336 $\int \sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^3\,dx$

Optimal. Leaf size=205

$$-\frac{3ax^2\sqrt{a^2cx^2+c}}{8\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\,\sinh^{-1}(ax)^4}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\sqrt{a^2cx^2+c}\,\sinh^{-1}(ax)^3-\frac{3ax^2\sqrt{a^2cx^2+c}\,\sinh^{-1}(ax)^2}{4\sqrt{a^2x^2+1}}-\frac{3\sqrt{a}}{}$$

```
[Out] 3/4*x*arcsinh(a*x)*(a^2*c*x^2+c)^(1/2)+1/2*x*arcsinh(a*x)^3*(a^2*c*x^2+c)^(
1/2)-3/8*a*x^2*(a^2*c*x^2+c)^(1/2)/(a^2*x^2+1)^(1/2)-3/8*arcsinh(a*x)^2*(a^
2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-3/4*a*x^2*arcsinh(a*x)^2*(a^2*c*x^2+c)
^(1/2)/(a^2*x^2+1)^(1/2)+1/8*arcsinh(a*x)^4*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+
1)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5682, 5675, 5661, 5758, 30}

$$-\frac{3ax^2\sqrt{a^2cx^2+c}}{8\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\,\sinh^{-1}(ax)^4}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\sqrt{a^2cx^2+c}\,\sinh^{-1}(ax)^3-\frac{3ax^2\sqrt{a^2cx^2+c}\,\sinh^{-1}(ax)^2}{4\sqrt{a^2x^2+1}}-\frac{3\sqrt{a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^3,x]
```

```
[Out] (-3*a*x^2*Sqrt[c + a^2*c*x^2])/(8*Sqrt[1 + a^2*x^2]) + (3*x*Sqrt[c + a^2*c*
x^2]*ArcSinh[a*x])/4 - (3*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^2)/(8*a*Sqrt[1 +
 a^2*x^2]) - (3*a*x^2*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^2)/(4*Sqrt[1 + a^2*x
^2]) + (x*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^3)/2 + (Sqrt[c + a^2*c*x^2]*ArcS
inh[a*x]^4)/(8*a*Sqrt[1 + a^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int \sqrt{c+a^2cx^2}\sinh^{-1}(ax)^3\,dx = \frac{1}{2}x\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^3 + \frac{\sqrt{c+a^2cx^2}\int\frac{\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{2\sqrt{1+a^2x^2}} - \frac{\left(3a\sqrt{c+a^2cx^2}\right)\int x$$

$$= -\frac{3ax^2\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^2}{4\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^3 + \frac{\sqrt{c+a^2cx^2}\,\sinh}{8a\sqrt{1+a^2x}}$$

$$= \frac{3}{4}x\sqrt{c+a^2cx^2}\sinh^{-1}(ax) - \frac{3ax^2\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^2}{4\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2cx^2}\,\sinh$$

$$= -\frac{3ax^2\sqrt{c+a^2cx^2}}{8\sqrt{1+a^2x^2}} + \frac{3}{4}x\sqrt{c+a^2cx^2}\sinh^{-1}(ax) - \frac{3\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^2}{8a\sqrt{1+a^2x^2}} - \frac{3a}{}$$

Mathematica [A] time = 0.15, size = 86, normalized size = 0.42

$$\frac{\sqrt{c\left(a^2x^2+1\right)}\left(2\sinh^{-1}(ax)\left(\sinh^{-1}(ax)^3+\left(2\sinh^{-1}(ax)^2+3\right)\sinh\left(2\sinh^{-1}(ax)\right)\right)-3\left(2\sinh^{-1}(ax)^2+1\right)\cos}{16a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^3,x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*(-3*(1 + 2*ArcSinh[a*x]^2)*Cosh[2*ArcSinh[a*x]] + 2*
ArcSinh[a*x]*(ArcSinh[a*x]^3 + (3 + 2*ArcSinh[a*x]^2)*Sinh[2*ArcSinh[a*x]])
))/(16*a*Sqrt[1 + a^2*x^2])
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\sqrt{a^2cx^2+c}\,\operatorname{arsinh}(ax)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^3, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.16, size = 231, normalized size = 1.13

$$\frac{\sqrt{c\left(a^2x^2+1\right)}\ \operatorname{arcsinh}(ax)^4}{8\sqrt{a^2x^2+1}\ a}+\frac{\sqrt{c\left(a^2x^2+1\right)}\ \left(2x^3a^3+2\sqrt{a^2x^2+1}\ x^2a^2+2ax+\sqrt{a^2x^2+1}\right)\left(4\operatorname{arcsinh}(ax)^3\right.}{32a\left(a^2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3*(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/8*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a*arcsinh(a*x)^4+1/32*(c*(a^2*x
^2+1))^(1/2)*(2*x^3*a^3+2*(a^2*x^2+1)^(1/2)*x^2*a^2+2*a*x+(a^2*x^2+1)^(1/2)
)*(4*arcsinh(a*x)^3-6*arcsinh(a*x)^2+6*arcsinh(a*x)-3)/a/(a^2*x^2+1)+1/32*(
c*(a^2*x^2+1))^(1/2)*(2*x^3*a^3-2*(a^2*x^2+1)^(1/2)*x^2*a^2+2*a*x-(a^2*x^2+
1)^(1/2))*(4*arcsinh(a*x)^3+6*arcsinh(a*x)^2+6*arcsinh(a*x)+3)/a/(a^2*x^2+1
)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{asinh}(a\,x)^3\,\sqrt{c\,a^2\,x^2+c}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^3*(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(asinh(a*x)^3*(c + a^2*c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c\left(a^2x^2+1\right)}\ \operatorname{asinh}^3(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*asinh(a*x)**3, x)
```

3.337 $\int \frac{\sinh^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=40

$$\frac{\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^4}{4a\sqrt{a^2cx^2+c}}$$

```
[Out] 1/4*arcsinh(a*x)^4*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5677, 5675}

$$\frac{\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^4}{4a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^4)/(4*a*Sqrt[c + a^2*c*x^2])
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] && !GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx &= \frac{\sqrt{1+a^2x^2}\int \frac{\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}\\ &= \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^4}{4a\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 40, normalized size = 1.00

$$\frac{\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^4}{4a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^3/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^4)/(4*a*Sqrt[c + a^2*c*x^2])
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{arsinh}(ax)^3}{\sqrt{a^2cx^2+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)^3}{\sqrt{a^2cx^2 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

maple [A] time = 0.04, size = 39, normalized size = 0.98

$$\frac{\sqrt{c\left(a^2x^2+1\right)}\ \operatorname{arcsinh}(ax)^4}{4\sqrt{a^2x^2+1}\ ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/4*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a/c*arcsinh(a*x)^4
```

maxima [A] time = 0.37, size = 14, normalized size = 0.35

$$\frac{\operatorname{arsinh}(ax)^4}{4\,a\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*arcsinh(a*x)^4/(a*sqrt(c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{asinh}(a\,x)^3}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^3/(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(asinh(a*x)^3/(c + a^2*c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^3(ax)}{\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(asinh(a*x)**3/sqrt(c*(a**2*x**2 + 1)), x)
```

3.338 $\int \frac{\sinh^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=218

$$-\frac{3\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)\text{Li}_2\left(-e^{2\sinh^{-1}(ax)}\right)}{ac\sqrt{a^2cx^2+c}}+\frac{3\sqrt{a^2x^2+1}\,\text{Li}_3\left(-e^{2\sinh^{-1}(ax)}\right)}{2ac\sqrt{a^2cx^2+c}}+\frac{x\sinh^{-1}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\,\sinh^{-1}(a$$

```
[Out] x*arcsinh(a*x)^3/c/(a^2*c*x^2+c)^(1/2)+arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)/a/c
/(a^2*c*x^2+c)^(1/2)-3*arcsinh(a*x)^2*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2)*(a^2*
x^2+1)^(1/2)/a/c/(a^2*c*x^2+c)^(1/2)-3*arcsinh(a*x)*polylog(2,-(a*x+(a^2*x^
2+1)^(1/2))^2)*(a^2*x^2+1)^(1/2)/a/c/(a^2*c*x^2+c)^(1/2)+3/2*polylog(3,-(a*
x+(a^2*x^2+1)^(1/2))^2)*(a^2*x^2+1)^(1/2)/a/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5687, 5714, 3718, 2190, 2531, 2282, 6589}

$$-\frac{3\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)\text{PolyLog}\left(2,-e^{2\sinh^{-1}(ax)}\right)}{ac\sqrt{a^2cx^2+c}}+\frac{3\sqrt{a^2x^2+1}\,\text{PolyLog}\left(3,-e^{2\sinh^{-1}(ax)}\right)}{2ac\sqrt{a^2cx^2+c}}+\frac{x\sinh^{-1}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (x*ArcSinh[a*x]^3)/(c*Sqrt[c + a^2*c*x^2]) + (Sqrt[1 + a^2*x^2]*ArcSinh[a*x
]^3)/(a*c*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2*Log[1
+ E^(2*ArcSinh[a*x])])/(a*c*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[1 + a^2*x^2]*Arc
Sinh[a*x]*PolyLog[2, -E^(2*ArcSinh[a*x])])/(a*c*Sqrt[c + a^2*c*x^2]) + (3*S
qrt[1 + a^2*x^2]*PolyLog[3, -E^(2*ArcSinh[a*x])])/(2*a*c*Sqrt[c + a^2*c*x^2
])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{3/2}}\,dx = \frac{x\sinh^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} - \frac{\left(3a\sqrt{1+a^2x^2}\right)\int \frac{x\sinh^{-1}(ax)^2}{1+a^2x^2}\,dx}{c\sqrt{c+a^2cx^2}}$$

$$= \frac{x\sinh^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} - \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int x^2\tanh(x)\,dx, x, \sinh^{-1}(ax)\right)}{ac\sqrt{c+a^2cx^2}}$$

$$= \frac{x\sinh^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{ac\sqrt{c+a^2cx^2}} - \frac{\left(6\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{e^{2x}x^2}{1+e^{2x}}\,dx, x, \sinh^{-1}(a\right.}{ac\sqrt{c+a^2cx^2}}$$

$$= \frac{x\sinh^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{ac\sqrt{c+a^2cx^2}} - \frac{3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2\log\left(1+e^{2\sinh^{-1}(ax}\right.}{ac\sqrt{c+a^2cx^2}}$$

$$= \frac{x\sinh^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{ac\sqrt{c+a^2cx^2}} - \frac{3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2\log\left(1+e^{2\sinh^{-1}(ax}\right.}{ac\sqrt{c+a^2cx^2}}$$

$$= \frac{x\sinh^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{ac\sqrt{c+a^2cx^2}} - \frac{3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2\log\left(1+e^{2\sinh^{-1}(ax}\right.}{ac\sqrt{c+a^2cx^2}}$$

$$= \frac{x\sinh^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{ac\sqrt{c+a^2cx^2}} - \frac{3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2\log\left(1+e^{2\sinh^{-1}(ax}\right.}{ac\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.26, size = 133, normalized size = 0.61

$$\frac{6\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)\text{Li}_2\left(-e^{-2\sinh^{-1}(ax)}\right)+3\sqrt{a^2x^2+1}\,\text{Li}_3\left(-e^{-2\sinh^{-1}(ax)}\right)-2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2\left(\sinh^{-}\right.}{2ac\sqrt{c\left(a^2x^2+1\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^3/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (2*a*x*ArcSinh[a*x]^3 - 2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2*(ArcSinh[a*x] +
3*Log[1 + E^(-2*ArcSinh[a*x])]) + 6*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]*PolyLog[
2, -E^(-2*ArcSinh[a*x])] + 3*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(-2*ArcSinh[a*
x])])/(2*a*c*Sqrt[c*(1 + a^2*x^2)])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\,\operatorname{arsinh}(ax)^3}{a^4c^2x^4+2\,a^2c^2x^2+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^3/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 +
c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)^3}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^(3/2), x)
```

maple [A] time = 0.19, size = 262, normalized size = 1.20

$$\frac{\sqrt{c\left(a^2x^2+1\right)}\,\left(ax-\sqrt{a^2x^2+1}\right)\operatorname{arsinh}(ax)^3}{a\,c^2\left(a^2x^2+1\right)}+\frac{2\sqrt{c\left(a^2x^2+1\right)}\,\operatorname{arsinh}(ax)^3}{\sqrt{a^2x^2+1}\,a\,c^2}-\frac{3\sqrt{c\left(a^2x^2+1\right)}\,\operatorname{arsinh}(ax)^2\,\mathrm{l}}{\sqrt{a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] (c*(a^2*x^2+1))^(1/2)*(a*x-(a^2*x^2+1)^(1/2))*arcsinh(a*x)^3/a/c^2/(a^2*x^2
+1)+2*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a/c^2*arcsinh(a*x)^3-3*(c*(a^
2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a/c^2*arcsinh(a*x)^2*ln(1+(a*x+(a^2*x^2+1
)^(1/2))^2)-3*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a/c^2*arcsinh(a*x)*po
lylog(2,-(a*x+(a^2*x^2+1)^(1/2))^2)+3/2*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(
1/2)/a/c^2*polylog(3,-(a*x+(a^2*x^2+1)^(1/2))^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)^3}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{asinh}(a\,x)^3}{\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^3/(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int(asinh(a*x)^3/(c + a^2*c*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^3(ax)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(asinh(a*x)**3/(c*(a**2*x**2 + 1))**(3/2), x)
```

3.339 $\int \frac{\sinh^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{5/2}} dx$

Optimal. Leaf size=363

$$-\frac{2\sqrt{a^2x^2+1}\sinh^{-1}(ax)\text{Li}_2\left(-e^{2\sinh^{-1}(ax)}\right)}{ac^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\text{Li}_3\left(-e^{2\sinh^{-1}(ax)}\right)}{ac^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\log\left(a^2x^2+1\right)}{2ac^2\sqrt{a^2cx^2+c}}+\frac{2x\sinh^{-1}}{3c^2\sqrt{a^2cx^2}}$$

```
[Out] 1/3*x*arcsinh(a*x)^3/c/(a^2*c*x^2+c)^(3/2)-x*arcsinh(a*x)/c^2/(a^2*c*x^2+c)
^(1/2)+2/3*x*arcsinh(a*x)^3/c^2/(a^2*c*x^2+c)^(1/2)+1/2*arcsinh(a*x)^2/a/c^
2/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+2/3*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2
)/a/c^2/(a^2*c*x^2+c)^(1/2)-2*arcsinh(a*x)^2*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2
)*(a^2*x^2+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)+1/2*ln(a^2*x^2+1)*(a^2*x^2+1)
^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)-2*arcsinh(a*x)*polylog(2,-(a*x+(a^2*x^2+1)
^(1/2))^2)*(a^2*x^2+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)+polylog(3,-(a*x+(a^2
*x^2+1)^(1/2))^2)*(a^2*x^2+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {5690, 5687, 5714, 3718, 2190, 2531, 2282, 6589, 5717, 260}

$$-\frac{2\sqrt{a^2x^2+1}\sinh^{-1}(ax)\text{PolyLog}\left(2,-e^{2\sinh^{-1}(ax)}\right)}{ac^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\text{PolyLog}\left(3,-e^{2\sinh^{-1}(ax)}\right)}{ac^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\log\left(a^2x^2\right.}{2ac^2\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] -((x*ArcSinh[a*x])/(c^2*Sqrt[c + a^2*c*x^2])) + ArcSinh[a*x]^2/(2*a*c^2*Sqr
t[1 + a^2*x^2]*Sqrt[c + a^2*c*x^2]) + (x*ArcSinh[a*x]^3)/(3*c*(c + a^2*c*x^
2)^(3/2)) + (2*x*ArcSinh[a*x]^3)/(3*c^2*Sqrt[c + a^2*c*x^2]) + (2*Sqrt[1 +
a^2*x^2]*ArcSinh[a*x]^3)/(3*a*c^2*Sqrt[c + a^2*c*x^2]) - (2*Sqrt[1 + a^2*x^
2]*ArcSinh[a*x]^2*Log[1 + E^(2*ArcSinh[a*x])])/(a*c^2*Sqrt[c + a^2*c*x^2])
+ (Sqrt[1 + a^2*x^2]*Log[1 + a^2*x^2])/(2*a*c^2*Sqrt[c + a^2*c*x^2]) - (2*S
qrt[1 + a^2*x^2]*ArcSinh[a*x]*PolyLog[2, -E^(2*ArcSinh[a*x])])/(a*c^2*Sqrt[
c + a^2*c*x^2]) + (Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(2*ArcSinh[a*x])])/(a*c^
2*Sqrt[c + a^2*c*x^2])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{5/2}}\,dx = \frac{x\sinh^{-1}(ax)^3}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{2\int \frac{\sinh^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{3c} - \frac{\left(a\sqrt{1+a^2x^2}\right)\int \frac{x\sinh^{-1}(ax)^2}{\left(1+a^2x^2\right)^2}\,dx}{c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{\sinh^{-1}(ax)^2}{2ac^2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)^3}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{2x\sinh^{-1}(ax)^3}{3c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a^2x^2}\int \frac{\sinh^{-1}(ax)}{\left(1+a^2x^2\right)^{3/2}}}{c^2\sqrt{c+a^2cx^2}}$$

$$= -\frac{x\sinh^{-1}(ax)}{c^2\sqrt{c+a^2cx^2}} + \frac{\sinh^{-1}(ax)^2}{2ac^2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)^3}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{2x\sinh^{-1}(ax)^3}{3c^2\sqrt{c+a^2cx^2}} - \frac{(2\sqrt{}}{}$$

$$= -\frac{x\sinh^{-1}(ax)}{c^2\sqrt{c+a^2cx^2}} + \frac{\sinh^{-1}(ax)^2}{2ac^2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)^3}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{2x\sinh^{-1}(ax)^3}{3c^2\sqrt{c+a^2cx^2}} + \frac{2\sqrt{}}{}$$

$$= -\frac{x\sinh^{-1}(ax)}{c^2\sqrt{c+a^2cx^2}} + \frac{\sinh^{-1}(ax)^2}{2ac^2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)^3}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{2x\sinh^{-1}(ax)^3}{3c^2\sqrt{c+a^2cx^2}} + \frac{2\sqrt{}}{}$$

$$= -\frac{x\sinh^{-1}(ax)}{c^2\sqrt{c+a^2cx^2}} + \frac{\sinh^{-1}(ax)^2}{2ac^2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)^3}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{2x\sinh^{-1}(ax)^3}{3c^2\sqrt{c+a^2cx^2}} + \frac{2\sqrt{}}{}$$

$$= -\frac{x\sinh^{-1}(ax)}{c^2\sqrt{c+a^2cx^2}} + \frac{\sinh^{-1}(ax)^2}{2ac^2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)^3}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{2x\sinh^{-1}(ax)^3}{3c^2\sqrt{c+a^2cx^2}} + \frac{2\sqrt{}}{}$$

$$= -\frac{x\sinh^{-1}(ax)}{c^2\sqrt{c+a^2cx^2}} + \frac{\sinh^{-1}(ax)^2}{2ac^2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)^3}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{2x\sinh^{-1}(ax)^3}{3c^2\sqrt{c+a^2cx^2}} + \frac{2\sqrt{}}{}$$

Mathematica [A] time = 0.55, size = 195, normalized size = 0.54

$$\frac{\left(a^2x^2+1\right)^{3/2}\left(3\log\left(a^2x^2+1\right) + \frac{4ax\sinh^{-1}(ax)^3}{\sqrt{a^2x^2+1}} + \frac{2ax\sinh^{-1}(ax)^3}{\left(a^2x^2+1\right)^{3/2}} + \frac{3\sinh^{-1}(ax)^2}{a^2x^2+1} - \frac{6ax\sinh^{-1}(ax)}{\sqrt{a^2x^2+1}} + 12\sinh^{-1}(ax)\mathrm{Li}_2\left(-e^{\cdots}\right.\right.}{6ac\left(a^2cx^2+c\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^3/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] ((1 + a^2*x^2)^(3/2)*((-6*a*x*ArcSinh[a*x])/Sqrt[1 + a^2*x^2] + (3*ArcSinh[
a*x]^2)/(1 + a^2*x^2) - 4*ArcSinh[a*x]^3 + (2*a*x*ArcSinh[a*x]^3)/(1 + a^2*
x^2)^(3/2) + (4*a*x*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2] - 12*ArcSinh[a*x]^2*L
og[1 + E^(-2*ArcSinh[a*x])] + 3*Log[1 + a^2*x^2] + 12*ArcSinh[a*x]*PolyLog[
2, -E^(-2*ArcSinh[a*x])] + 6*PolyLog[3, -E^(-2*ArcSinh[a*x])]))/(6*a*c*(c +
 a^2*c*x^2)^(3/2))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{arsinh}\left(ax\right)^3}{a^6c^3x^6+3\,a^4c^3x^4+3\,a^2c^3x^2+c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^3/(a^6*c^3*x^6 + 3*a^4*c^3*x^4 +
3*a^2*c^3*x^2 + c^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, choosing root of [1,0,%%%{-2,[2,1,2
]%%%}+%%%{-2,[2,0,2]%%%}+%%%{-2,[0,1,0]%%%}+%%%{-2,[0,0,0]%%%},0,%%%{1,[4,2
,4]%%%}+%%%{-2,[4,1,4]%%%}+%%%{1,[4,0,4]%%%}+%%%{2,[2,2,2]%%%}+%%%{-4,[2,1,
2]%%%}+%%%{2,[2,0,2]%%%}+%%%{1,[0,2,0]%%%}+%%%{-2,[0,1,0]%%%}+%%%{1,[0,0,0]
%%%}] at parameters values [86,-97,-82]sym2poly/r2sym(const gen & e,const i
ndex_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.27, size = 550, normalized size = 1.52

$$\frac{\sqrt{c\left(a^{2}x^{2}+1\right)}\,\left(2x^{3}a^{3}-2\sqrt{a^{2}x^{2}+1}\,x^{2}a^{2}+3ax-2\sqrt{a^{2}x^{2}+1}\right)\operatorname{arcsinh}\left(ax\right)\left(-6\operatorname{arcsinh}\left(ax\right)x^{4}a^{4}-6\operatorname{arcsin}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/6*(c*(a^2*x^2+1))^(1/2)*(2*x^3*a^3-2*(a^2*x^2+1)^(1/2)*x^2*a^2+3*a*x-2*(a
^2*x^2+1)^(1/2))*arcsinh(a*x)*(-6*arcsinh(a*x)*x^4*a^4-6*arcsinh(a*x)*(a^2*
x^2+1)^(1/2)*a^3*x^3-6*x^4*a^4-6*(a^2*x^2+1)^(1/2)*x^3*a^3+6*arcsinh(a*x)^2
*a^2*x^2-12*arcsinh(a*x)*x^2*a^2-9*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a*x-18*a^
2*x^2-6*(a^2*x^2+1)^(1/2)*x*a+8*arcsinh(a*x)^2-6*arcsinh(a*x)-12)/(3*a^6*x^
6+10*a^4*x^4+11*a^2*x^2+4)/a/c^3-2*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/
a/c^3*ln(a*x+(a^2*x^2+1)^(1/2))+(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a/c
^3*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2)+4/3*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1
/2)/a/c^3*arcsinh(a*x)^3-2*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a/c^3*ar
csinh(a*x)^2*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2)-2*(c*(a^2*x^2+1))^(1/2)/(a^2*x
^2+1)^(1/2)/a/c^3*arcsinh(a*x)*polylog(2,-(a*x+(a^2*x^2+1)^(1/2))^2)+(c*(a^
2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a/c^3*polylog(3,-(a*x+(a^2*x^2+1)^(1/2))^
2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}\left(ax\right)^{3}}{\left(a^{2}cx^{2}+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{asinh}\left(a\,x\right)^{3}}{{\left(c\,a^{2}\,x^{2}+c\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^3/(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] int(asinh(a*x)^3/(c + a^2*c*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^{3}(ax)}{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(asinh(a*x)**3/(c*(a**2*x**2 + 1))**(5/2), x)
```

3.340 $\int \frac{\sinh^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=515

$$-\frac{8\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)\text{Li}_2\left(-e^{2\sinh^{-1}(ax)}\right)}{5ac^3\sqrt{a^2cx^2+c}}+\frac{4\sqrt{a^2x^2+1}\,\text{Li}_3\left(-e^{2\sinh^{-1}(ax)}\right)}{5ac^3\sqrt{a^2cx^2+c}}-\frac{1}{20ac^3\sqrt{a^2x^2+1}\,\sqrt{a^2cx^2+c}}+\frac{\sqrt{a}}{}$$

```
[Out] 1/5*x*arcsinh(a*x)^3/c/(a^2*c*x^2+c)^(5/2)+4/15*x*arcsinh(a*x)^3/c^2/(a^2*c
*x^2+c)^(3/2)-x*arcsinh(a*x)/c^3/(a^2*c*x^2+c)^(1/2)-1/10*x*arcsinh(a*x)/c^
3/(a^2*x^2+1)/(a^2*c*x^2+c)^(1/2)+3/20*arcsinh(a*x)^2/a/c^3/(a^2*x^2+1)^(3/
2)/(a^2*c*x^2+c)^(1/2)+8/15*x*arcsinh(a*x)^3/c^3/(a^2*c*x^2+c)^(1/2)-1/20/a
/c^3/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+2/5*arcsinh(a*x)^2/a/c^3/(a^2*x^
2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+8/15*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)/a/c^3/
(a^2*c*x^2+c)^(1/2)-8/5*arcsinh(a*x)^2*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2)*(a^2
*x^2+1)^(1/2)/a/c^3/(a^2*c*x^2+c)^(1/2)+1/2*ln(a^2*x^2+1)*(a^2*x^2+1)^(1/2)
/a/c^3/(a^2*c*x^2+c)^(1/2)-8/5*arcsinh(a*x)*polylog(2,-(a*x+(a^2*x^2+1)^(1/
2))^2)*(a^2*x^2+1)^(1/2)/a/c^3/(a^2*c*x^2+c)^(1/2)+4/5*polylog(3,-(a*x+(a^2
*x^2+1)^(1/2))^2)*(a^2*x^2+1)^(1/2)/a/c^3/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 515, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {5690, 5687, 5714, 3718, 2190, 2531, 2282, 6589, 5717, 260, 261}

$$-\frac{8\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)\text{PolyLog}\left(2,-e^{2\sinh^{-1}(ax)}\right)}{5ac^3\sqrt{a^2cx^2+c}}+\frac{4\sqrt{a^2x^2+1}\,\text{PolyLog}\left(3,-e^{2\sinh^{-1}(ax)}\right)}{5ac^3\sqrt{a^2cx^2+c}}-\frac{1}{20ac^3\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/(c + a^2*c*x^2)^(7/2),x]
```

```
[Out] -1/(20*a*c^3*Sqrt[1 + a^2*x^2]*Sqrt[c + a^2*c*x^2]) - (x*ArcSinh[a*x])/(c^3
*Sqrt[c + a^2*c*x^2]) - (x*ArcSinh[a*x])/(10*c^3*(1 + a^2*x^2)*Sqrt[c + a^2
*c*x^2]) + (3*ArcSinh[a*x]^2)/(20*a*c^3*(1 + a^2*x^2)^(3/2)*Sqrt[c + a^2*c*
x^2]) + (2*ArcSinh[a*x]^2)/(5*a*c^3*Sqrt[1 + a^2*x^2]*Sqrt[c + a^2*c*x^2])
+ (x*ArcSinh[a*x]^3)/(5*c*(c + a^2*c*x^2)^(5/2)) + (4*x*ArcSinh[a*x]^3)/(15
*c^2*(c + a^2*c*x^2)^(3/2)) + (8*x*ArcSinh[a*x]^3)/(15*c^3*Sqrt[c + a^2*c*x
^2]) + (8*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(15*a*c^3*Sqrt[c + a^2*c*x^2])
- (8*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2*Log[1 + E^(2*ArcSinh[a*x])])/(5*a*c^3
*Sqrt[c + a^2*c*x^2]) + (Sqrt[1 + a^2*x^2]*Log[1 + a^2*x^2])/(2*a*c^3*Sqrt[
c + a^2*c*x^2]) - (8*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]*PolyLog[2, -E^(2*ArcSin
h[a*x])])/(5*a*c^3*Sqrt[c + a^2*c*x^2]) + (4*Sqrt[1 + a^2*x^2]*PolyLog[3, -
E^(2*ArcSinh[a*x])])/(5*a*c^3*Sqrt[c + a^2*c*x^2])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
```

```
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{7/2}}\,dx = \frac{x\sinh^{-1}(ax)^3}{5c\left(c+a^2cx^2\right)^{5/2}} + \frac{4\int \frac{\sinh^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{5/2}}\,dx}{5c} - \frac{\left(3a\sqrt{1+a^2x^2}\right)\int \frac{x\sinh^{-1}(ax)^2}{\left(1+a^2x^2\right)^3}\,dx}{5c^3\sqrt{c+a^2cx^2}}$$

$$= \frac{3\sinh^{-1}(ax)^2}{20ac^3\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)^3}{5c\left(c+a^2cx^2\right)^{5/2}} + \frac{4x\sinh^{-1}(ax)^3}{15c^2\left(c+a^2cx^2\right)^{3/2}} + \frac{8\int \frac{\sinh^{-1}(a}{(c+a^2cx^2}}{15c^2}$$

$$= -\frac{x\sinh^{-1}(ax)}{10c^3\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}} + \frac{3\sinh^{-1}(ax)^2}{20ac^3\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}} + \frac{2\sinh^{-1}(ax)^2}{5ac^3\sqrt{1+a^2x^2}\sqrt{c+}}$$

$$= -\frac{1}{20ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} - \frac{x\sinh^{-1}(ax)}{c^3\sqrt{c+a^2cx^2}} - \frac{x\sinh^{-1}(ax)}{10c^3\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}} + \frac{}{20ac^3(}$$

$$= -\frac{1}{20ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} - \frac{x\sinh^{-1}(ax)}{c^3\sqrt{c+a^2cx^2}} - \frac{x\sinh^{-1}(ax)}{10c^3\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}} + \frac{}{20ac^3(}$$

$$= -\frac{1}{20ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} - \frac{x\sinh^{-1}(ax)}{c^3\sqrt{c+a^2cx^2}} - \frac{x\sinh^{-1}(ax)}{10c^3\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}} + \frac{}{20ac^3(}$$

$$= -\frac{1}{20ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} - \frac{x\sinh^{-1}(ax)}{c^3\sqrt{c+a^2cx^2}} - \frac{x\sinh^{-1}(ax)}{10c^3\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}} + \frac{}{20ac^3(}$$

$$= -\frac{1}{20ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} - \frac{x\sinh^{-1}(ax)}{c^3\sqrt{c+a^2cx^2}} - \frac{x\sinh^{-1}(ax)}{10c^3\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}} + \frac{}{20ac^3(}$$

$$= -\frac{1}{20ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} - \frac{x\sinh^{-1}(ax)}{c^3\sqrt{c+a^2cx^2}} - \frac{x\sinh^{-1}(ax)}{10c^3\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}} + \frac{}{20ac^3(}$$

Mathematica [A] time = 0.66, size = 297, normalized size = 0.58

$$96\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)\mathrm{Li}_2\left(-e^{-2\sinh^{-1}(ax)}\right) + 48\sqrt{a^2x^2+1}\,\mathrm{Li}_3\left(-e^{-2\sinh^{-1}(ax)}\right) - \frac{3}{\sqrt{a^2x^2+1}} + 30\sqrt{a^2x^2+1}\,\log\Big($$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^3/(c + a^2*c*x^2)^(7/2),x]
```

```
[Out] (-3/Sqrt[1 + a^2*x^2] - 60*a*x*ArcSinh[a*x] - (6*a*x*ArcSinh[a*x])/(1 + a^2
*x^2) + (9*ArcSinh[a*x]^2)/(1 + a^2*x^2)^(3/2) + (24*ArcSinh[a*x]^2)/Sqrt[1
 + a^2*x^2] + 32*a*x*ArcSinh[a*x]^3 + (12*a*x*ArcSinh[a*x]^3)/(1 + a^2*x^2)
^2 + (16*a*x*ArcSinh[a*x]^3)/(1 + a^2*x^2) - 32*Sqrt[1 + a^2*x^2]*ArcSinh[a
*x]^3 - 96*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2*Log[1 + E^(-2*ArcSinh[a*x])] +
30*Sqrt[1 + a^2*x^2]*Log[1 + a^2*x^2] + 96*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]*P
olyLog[2, -E^(-2*ArcSinh[a*x])] + 48*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(-2*Ar
cSinh[a*x])])/(60*a*c^3*Sqrt[c + a^2*c*x^2])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\,\operatorname{arsinh}(ax)^3}{a^8c^4x^8+4\,a^6c^4x^6+6\,a^4c^4x^4+4\,a^2c^4x^2+c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^3/(a^8*c^4*x^8 + 4*a^6*c^4*x^6 +
6*a^4*c^4*x^4 + 4*a^2*c^4*x^2 + c^4), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, choosing root of [1,0,%%%{-2,[2,1,2
]%%%}+%%%{-2,[2,0,2]%%%}+%%%{-2,[0,1,0]%%%}+%%%{-2,[0,0,0]%%%},0,%%%{1,[4,2
,4]%%%}+%%%{-2,[4,1,4]%%%}+%%%{1,[4,0,4]%%%}+%%%{2,[2,2,2]%%%}+%%%{-4,[2,1,
2]%%%}+%%%{2,[2,0,2]%%%}+%%%{1,[0,2,0]%%%}+%%%{-2,[0,1,0]%%%}+%%%{1,[0,0,0]
%%%}] at parameters values [86,-97,-82]sym2poly/r2sym(const gen & e,const i
ndex_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.35, size = 888, normalized size = 1.72

$$\frac{\sqrt{c\left(a^2x^2+1\right)}\,\left(8x^5a^5-8\sqrt{a^2x^2+1}\,x^4a^4+20x^3a^3-16\sqrt{a^2x^2+1}\,x^2a^2+15ax-8\sqrt{a^2x^2+1}\right)\left(24-1020\operatorname{arcsin}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/(a^2*c*x^2+c)^(7/2),x)
```

```
[Out] 1/60*(c*(a^2*x^2+1))^(1/2)*(8*x^5*a^5-8*(a^2*x^2+1)^(1/2)*x^4*a^4+20*x^3*a^
3-16*(a^2*x^2+1)^(1/2)*x^2*a^2+15*a*x-8*(a^2*x^2+1)^(1/2))*(24-1020*arcsinh
(a*x)^2*(a^2*x^2+1)^(1/2)*a^3*x^3-495*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)*a*x-
936*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a^3*x^3-372*arcsinh(a*x)*(a^2*x^2+1)^(1/
2)*a*x+24*x^8*a^8+96*x^6*a^6+144*x^4*a^4-192*(a^2*x^2+1)^(1/2)*arcsinh(a*x)
*x^7*a^7-756*(a^2*x^2+1)^(1/2)*arcsinh(a*x)*x^5*a^5-480*arcsinh(a*x)+96*a^2
*x^2-192*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)*x^7*a^7-744*arcsinh(a*x)^2*(a^2*x
^2+1)^(1/2)*x^5*a^5+256*arcsinh(a*x)^3-264*arcsinh(a*x)^2-1368*arcsinh(a*x)
^2*x^4*a^4-984*arcsinh(a*x)^2*a^2*x^2-192*arcsinh(a*x)*x^8*a^8-852*arcsinh(
a*x)*x^6*a^6+24*(a^2*x^2+1)^(1/2)*x^7*a^7+84*(a^2*x^2+1)^(1/2)*x^5*a^5-1590
*arcsinh(a*x)*x^4*a^4+105*(a^2*x^2+1)^(1/2)*x^3*a^3-1410*arcsinh(a*x)*x^2*a
^2+45*(a^2*x^2+1)^(1/2)*x*a+380*arcsinh(a*x)^3*x^2*a^2-192*arcsinh(a*x)^2*x
^8*a^8-840*arcsinh(a*x)^2*x^6*a^6+160*arcsinh(a*x)^3*x^4*a^4)/(40*a^10*x^10
+215*a^8*x^8+469*a^6*x^6+517*a^4*x^4+287*a^2*x^2+64)/a/c^4-2*(c*(a^2*x^2+1)
```

```
)^(1/2)/(a^2*x^2+1)^(1/2)/a/c^4*ln(a*x+(a^2*x^2+1)^(1/2))+(c*(a^2*x^2+1))^(
1/2)/(a^2*x^2+1)^(1/2)/a/c^4*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2)+16/15*(c*(a^2*
x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a/c^4*arcsinh(a*x)^3-8/5*(c*(a^2*x^2+1))^(1
/2)/(a^2*x^2+1)^(1/2)/a/c^4*arcsinh(a*x)^2*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2)-
8/5*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a/c^4*arcsinh(a*x)*polylog(2,-(
a*x+(a^2*x^2+1)^(1/2))^2)+4/5*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a/c^4
*polylog(3,-(a*x+(a^2*x^2+1)^(1/2))^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)^3}{(a^2cx^2+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{asinh}(a\,x)^3}{{\left(c\,a^2\,x^2+c\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^3/(c + a^2*c*x^2)^(7/2),x)
[Out] int(asinh(a*x)^3/(c + a^2*c*x^2)^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^3(ax)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/(a**2*c*x**2+c)**(7/2),x)
[Out] Integral(asinh(a*x)**3/(c*(a**2*x**2 + 1))**(7/2), x)
```

3.341 $\int \frac{x^m \sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{x^m \sinh^{-1}(ax)^3}{\sqrt{a^2x^2+1}},x\right)$$

```
[Out] Unintegrable(x^m*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x)
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m \sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
[Out] Defer[Int][(x^m*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2], x]
```

Rubi steps

$$\int \frac{x^m \sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$$

Mathematica [A] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{x^m \sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
[Out] Integrate[(x^m*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2], x]
```

fricas [A] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m \operatorname{arsinh}(ax)^3}{\sqrt{a^2x^2+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(x^m*arcsinh(a*x)^3/sqrt(a^2*x^2 + 1), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \operatorname{arsinh}(ax)^3}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^m*arcsinh(a*x)^3/sqrt(a^2*x^2 + 1), x)
```

maple [A] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{x^{m} \operatorname{arcsinh}\left(a x\right)^{3}}{\sqrt{a^{2} x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^m*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m} \operatorname{arsinh}\left(a x\right)^{3}}{\sqrt{a^{2} x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*arcsinh(a*x)^3/sqrt(a^2*x^2 + 1), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^{m} \operatorname{asinh}\left(a\, x\right)^{3}}{\sqrt{a^{2}\, x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*asinh(a*x)^3)/(a^2*x^2 + 1)^(1/2),x)
```

```
[Out] int((x^m*asinh(a*x)^3)/(a^2*x^2 + 1)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m} \operatorname{asinh}^{3}\left(a x\right)}{\sqrt{a^{2} x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*asinh(a*x)**3/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**m*asinh(a*x)**3/sqrt(a**2*x**2 + 1), x)
```

3.342 $\int \frac{x^4 \sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=187

$$\frac{3\sinh^{-1}(ax)^4}{32a^5}+\frac{45\sinh^{-1}(ax)^2}{128a^5}+\frac{45x^2}{128a^3}+\frac{9x^2\sinh^{-1}(ax)^2}{16a^3}+\frac{x^3\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3}{4a^2}+\frac{3x^3\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{32a^2}-$$

```
[Out] 45/128*x^2/a^3-3/128*x^4/a+45/128*arcsinh(a*x)^2/a^5+9/16*x^2*arcsinh(a*x)^
2/a^3-3/16*x^4*arcsinh(a*x)^2/a+3/32*arcsinh(a*x)^4/a^5-45/64*x*arcsinh(a*x
)*(a^2*x^2+1)^(1/2)/a^4+3/32*x^3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^2-3/8*x*a
rcsinh(a*x)^3*(a^2*x^2+1)^(1/2)/a^4+1/4*x^3*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2
)/a^2
```

Rubi **[A]** time = 0.50, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {5758, 5675, 5661, 30}

$$\frac{45x^2}{128a^3}+\frac{x^3\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3}{4a^2}+\frac{3x^3\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{32a^2}+\frac{9x^2\sinh^{-1}(ax)^2}{16a^3}-\frac{3x\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3}{8a^4}-\frac{45x}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (45*x^2)/(128*a^3) - (3*x^4)/(128*a) - (45*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]
)/(64*a^4) + (3*x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(32*a^2) + (45*ArcSinh[
a*x]^2)/(128*a^5) + (9*x^2*ArcSinh[a*x]^2)/(16*a^3) - (3*x^4*ArcSinh[a*x]^2
)/(16*a) - (3*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(8*a^4) + (x^3*Sqrt[1 + a
^2*x^2]*ArcSinh[a*x]^3)/(4*a^2) + (3*ArcSinh[a*x]^4)/(32*a^5)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^4 \sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \frac{x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{4a^2} - \frac{3\int \frac{x^2\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{4a^2} - \frac{3\int x^3\sinh^{-1}(ax)^2\,dx}{4a}$$

$$= -\frac{3x^4\sinh^{-1}(ax)^2}{16a} - \frac{3x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{8a^4} + \frac{x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{4a^2} + \frac{3}{8}\int \frac{x^4}{\sqrt{}}$$

$$= \frac{3x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{32a^2} + \frac{9x^2\sinh^{-1}(ax)^2}{16a^3} - \frac{3x^4\sinh^{-1}(ax)^2}{16a} - \frac{3x\sqrt{1+a^2x^2}\,\sinh}{8a^4}$$

$$= -\frac{3x^4}{128a} - \frac{45x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{64a^4} + \frac{3x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{32a^2} + \frac{9x^2\sinh^{-1}(ax)^2}{16a^3} -$$

$$= \frac{45x^2}{128a^3} - \frac{3x^4}{128a} - \frac{45x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{64a^4} + \frac{3x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{32a^2} + \frac{45\sinh^{-1}(}{128a^5}$$

Mathematica [A] time = 0.08, size = 121, normalized size = 0.65

$$\frac{-3a^4x^4 + 45a^2x^2 + 16ax\sqrt{a^2x^2+1}\left(2a^2x^2-3\right)\sinh^{-1}(ax)^3 + 6ax\sqrt{a^2x^2+1}\left(2a^2x^2-15\right)\sinh^{-1}(ax) + \left(-24a\right.}{128a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (45*a^2*x^2 - 3*a^4*x^4 + 6*a*x*Sqrt[1 + a^2*x^2]*(-15 + 2*a^2*x^2)*ArcSinh
[a*x] + (45 + 72*a^2*x^2 - 24*a^4*x^4)*ArcSinh[a*x]^2 + 16*a*x*Sqrt[1 + a^2
*x^2]*(-3 + 2*a^2*x^2)*ArcSinh[a*x]^3 + 12*ArcSinh[a*x]^4)/(128*a^5)
```

fricas [A] time = 0.47, size = 166, normalized size = 0.89

$$-\frac{3\,a^4x^4 - 16\left(2\,a^3x^3 - 3\,ax\right)\sqrt{a^2x^2+1}\,\log\left(ax + \sqrt{a^2x^2+1}\right)^3 - 45\,a^2x^2 - 12\,\log\left(ax + \sqrt{a^2x^2+1}\right)^4 + 3\left(8\,a^4\right.}{128\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/128*(3*a^4*x^4 - 16*(2*a^3*x^3 - 3*a*x)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt
(a^2*x^2 + 1))^3 - 45*a^2*x^2 - 12*log(a*x + sqrt(a^2*x^2 + 1))^4 + 3*(8*a^
4*x^4 - 24*a^2*x^2 - 15)*log(a*x + sqrt(a^2*x^2 + 1))^2 - 6*(2*a^3*x^3 - 15
*a*x)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/a^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\,\operatorname{arsinh}(ax)^3}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4*arcsinh(a*x)^3/sqrt(a^2*x^2 + 1), x)
```

maple [A] time = 0.12, size = 156, normalized size = 0.83

$$32\operatorname{arcsinh}(ax)^3\sqrt{a^2x^2+1}\;a^3x^3 - 24\operatorname{arcsinh}(ax)^2x^4a^4 + 12\operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}\;a^3x^3 - 3x^4a^4 - 48\operatorname{arcsi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/128*(32*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)*a^3*x^3-24*arcsinh(a*x)^2*x^4*a^
4+12*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a^3*x^3-3*x^4*a^4-48*arcsinh(a*x)^3*(a^
2*x^2+1)^(1/2)*a*x+72*arcsinh(a*x)^2*a^2*x^2+12*arcsinh(a*x)^4-90*arcsinh(a
*x)*(a^2*x^2+1)^(1/2)*a*x+45*a^2*x^2+45*arcsinh(a*x)^2+45)/a^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \operatorname{arsinh}(ax)^3}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4*arcsinh(a*x)^3/sqrt(a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4 \operatorname{asinh}(a\,x)^3}{\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*asinh(a*x)^3)/(a^2*x^2 + 1)^(1/2),x)
```

```
[Out] int((x^4*asinh(a*x)^3)/(a^2*x^2 + 1)^(1/2), x)
```

sympy [A] time = 6.47, size = 185, normalized size = 0.99

$$\begin{cases} -\frac{3x^4 \operatorname{asinh}^2(ax)}{16a} - \frac{3x^4}{128a} + \frac{x^3\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{4a^2} + \frac{3x^3\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{32a^2} + \frac{9x^2\operatorname{asinh}^2(ax)}{16a^3} + \frac{45x^2}{128a^3} - \frac{3x\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{8a^4} - \frac{45x}{} \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*asinh(a*x)**3/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-3*x**4*asinh(a*x)**2/(16*a) - 3*x**4/(128*a) + x**3*sqrt(a**2*x
**2 + 1)*asinh(a*x)**3/(4*a**2) + 3*x**3*sqrt(a**2*x**2 + 1)*asinh(a*x)/(32
*a**2) + 9*x**2*asinh(a*x)**2/(16*a**3) + 45*x**2/(128*a**3) - 3*x*sqrt(a**
2*x**2 + 1)*asinh(a*x)**3/(8*a**4) - 45*x*sqrt(a**2*x**2 + 1)*asinh(a*x)/(6
4*a**4) + 3*asinh(a*x)**4/(32*a**5) + 45*asinh(a*x)**2/(128*a**5), Ne(a, 0)
), (0, True))
```

3.343 $\int \frac{x^3 \sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}} \, dx$

Optimal. Leaf size=153

$$\frac{40x}{9a^3}+\frac{2x\sinh^{-1}(ax)^2}{a^3}+\frac{x^2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3}{3a^2}+\frac{2x^2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{9a^2}-\frac{2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3}{3a^4}-\frac{40\sqrt{a\cdots}}{\cdots}$$

```
[Out] 40/9*x/a^3-2/27*x^3/a+2*x*arcsinh(a*x)^2/a^3-1/3*x^3*arcsinh(a*x)^2/a-40/9*
arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^4+2/9*x^2*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a
^2-2/3*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)/a^4+1/3*x^2*arcsinh(a*x)^3*(a^2*x^2
+1)^(1/2)/a^2
```

Rubi [A] time = 0.34, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5758, 5717, 5653, 8, 5661, 30}

$$\frac{x^2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3}{3a^2}+\frac{2x^2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{9a^2}-\frac{2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3}{3a^4}-\frac{40\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{9a^4}+\frac{4\cdots}{9\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (40*x)/(9*a^3) - (2*x^3)/(27*a) - (40*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(9*a^
4) + (2*x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(9*a^2) + (2*x*ArcSinh[a*x]^2)/
a^3 - (x^3*ArcSinh[a*x]^2)/(3*a) - (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(3*
a^4) + (x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(3*a^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^3\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \frac{x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{3a^2} - \frac{2\int \frac{x\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{3a^2} - \frac{\int x^2\sinh^{-1}(ax)^2\,dx}{a}$$

$$= -\frac{x^3\sinh^{-1}(ax)^2}{3a} - \frac{2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{3a^4} + \frac{x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{3a^2} + \frac{2}{3}\int \frac{x^3\sinh^{-}}{\sqrt{1+a}}$$

$$= \frac{2x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{9a^2} + \frac{2x\sinh^{-1}(ax)^2}{a^3} - \frac{x^3\sinh^{-1}(ax)^2}{3a} - \frac{2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{3a^4}$$

$$= -\frac{2x^3}{27a} - \frac{40\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{9a^4} + \frac{2x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{9a^2} + \frac{2x\sinh^{-1}(ax)^2}{a^3} - \frac{x^3\sin}{}$$

$$= \frac{40x}{9a^3} - \frac{2x^3}{27a} - \frac{40\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{9a^4} + \frac{2x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{9a^2} + \frac{2x\sinh^{-1}(ax)^2}{a^3} -$$

Mathematica [A] time = 0.07, size = 98, normalized size = 0.64

$$\frac{-2ax\left(a^2x^2-60\right)+9\left(a^2x^2-2\right)\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3-9ax\left(a^2x^2-6\right)\sinh^{-1}(ax)^2+6\left(a^2x^2-20\right)\sqrt{a^2x^2+1}\,s}{27a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (-2*a*x*(-60 + a^2*x^2) + 6*(-20 + a^2*x^2)*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]
- 9*a*x*(-6 + a^2*x^2)*ArcSinh[a*x]^2 + 9*(-2 + a^2*x^2)*Sqrt[1 + a^2*x^2]*
ArcSinh[a*x]^3)/(27*a^4)
```

fricas [A] time = 0.52, size = 128, normalized size = 0.84

$$-\frac{2\,a^3x^3-9\,\sqrt{a^2x^2+1}\left(a^2x^2-2\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^3+9\left(a^3x^3-6\,ax\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^2-6\,\sqrt{a^2x^2+1}\,(}{27\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/27*(2*a^3*x^3 - 9*sqrt(a^2*x^2 + 1)*(a^2*x^2 - 2)*log(a*x + sqrt(a^2*x^2
 + 1))^3 + 9*(a^3*x^3 - 6*a*x)*log(a*x + sqrt(a^2*x^2 + 1))^2 - 6*sqrt(a^2*
x^2 + 1)*(a^2*x^2 - 20)*log(a*x + sqrt(a^2*x^2 + 1)) - 120*a*x)/a^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.10, size = 164, normalized size = 1.07

$$\frac{9 \operatorname{arcsinh}(ax)^3 x^4 a^4 - 9 \operatorname{arcsinh}(ax)^3 x^2 a^2 - 9 \operatorname{arcsinh}(ax)^2 \sqrt{a^2x^2+1}\, a^3 x^3 + 6 \operatorname{arcsinh}(ax) x^4 a^4 - 114 \operatorname{arcs}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/27/a^4/(a^2*x^2+1)^(1/2)*(9*arcsinh(a*x)^3*x^4*a^4-9*arcsinh(a*x)^3*x^2*a
^2-9*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)*a^3*x^3+6*arcsinh(a*x)*x^4*a^4-114*ar
csinh(a*x)*x^2*a^2-2*(a^2*x^2+1)^(1/2)*x^3*a^3-18*arcsinh(a*x)^3+54*arcsinh
(a*x)^2*(a^2*x^2+1)^(1/2)*a*x-120*arcsinh(a*x)+120*(a^2*x^2+1)^(1/2)*x*a)
```

maxima [A] time = 0.41, size = 127, normalized size = 0.83

$$\frac{1}{3}\left(\frac{\sqrt{a^2x^2+1}\,x^2}{a^2} - \frac{2\sqrt{a^2x^2+1}}{a^4}\right)\operatorname{arsinh}(ax)^3 + \frac{2}{27}a\left(\frac{3\left(\sqrt{a^2x^2+1}\,x^2 - \frac{20\sqrt{a^2x^2+1}}{a^2}\right)\operatorname{arsinh}(ax)}{a^3} - \frac{a^2x^3 - 60\,x}{a^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(sqrt(a^2*x^2 + 1)*x^2/a^2 - 2*sqrt(a^2*x^2 + 1)/a^4)*arcsinh(a*x)^3 +
2/27*a*(3*(sqrt(a^2*x^2 + 1)*x^2 - 20*sqrt(a^2*x^2 + 1)/a^2)*arcsinh(a*x)/a
^3 - (a^2*x^3 - 60*x)/a^4) - 1/3*(a^2*x^3 - 6*x)*arcsinh(a*x)^2/a^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \operatorname{asinh}(a\,x)^3}{\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*asinh(a*x)^3)/(a^2*x^2 + 1)^(1/2),x)
```

```
[Out] int((x^3*asinh(a*x)^3)/(a^2*x^2 + 1)^(1/2), x)
```

sympy [A] time = 3.57, size = 148, normalized size = 0.97

$$\begin{cases} -\frac{x^3 \operatorname{asinh}^2(ax)}{3a} - \frac{2x^3}{27a} + \frac{x^2\sqrt{a^2x^2+1}\,\operatorname{asinh}^3(ax)}{3a^2} + \frac{2x^2\sqrt{a^2x^2+1}\,\operatorname{asinh}(ax)}{9a^2} + \frac{2x\operatorname{asinh}^2(ax)}{a^3} + \frac{40x}{9a^3} - \frac{2\sqrt{a^2x^2+1}\,\operatorname{asinh}^3(ax)}{3a^4} - \frac{40\sqrt{a^2}}{} \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(a*x)**3/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-x**3*asinh(a*x)**2/(3*a) - 2*x**3/(27*a) + x**2*sqrt(a**2*x**2
+ 1)*asinh(a*x)**3/(3*a**2) + 2*x**2*sqrt(a**2*x**2 + 1)*asinh(a*x)/(9*a**2
) + 2*x*asinh(a*x)**2/a**3 + 40*x/(9*a**3) - 2*sqrt(a**2*x**2 + 1)*asinh(a*
x)**3/(3*a**4) - 40*sqrt(a**2*x**2 + 1)*asinh(a*x)/(9*a**4), Ne(a, 0)), (0,
 True))
```

3.344 $\int \frac{x^2 \sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}} \, dx$

Optimal. Leaf size=105

$$-\frac{\sinh^{-1}(ax)^4}{8a^3}-\frac{3\sinh^{-1}(ax)^2}{8a^3}+\frac{x\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3}{2a^2}+\frac{3x\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{4a^2}-\frac{3x^2}{8a}-\frac{3x^2\sinh^{-1}(ax)^2}{4a}$$

```
[Out] -3/8*x^2/a-3/8*arcsinh(a*x)^2/a^3-3/4*x^2*arcsinh(a*x)^2/a-1/8*arcsinh(a*x)
^4/a^3+3/4*x*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^2+1/2*x*arcsinh(a*x)^3*(a^2*x
^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.22, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {5758, 5675, 5661, 30}

$$\frac{x\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3}{2a^2}+\frac{3x\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{4a^2}-\frac{\sinh^{-1}(ax)^4}{8a^3}-\frac{3\sinh^{-1}(ax)^2}{8a^3}-\frac{3x^2}{8a}-\frac{3x^2\sinh^{-1}(ax)^2}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (-3*x^2)/(8*a) + (3*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(4*a^2) - (3*ArcSinh[
a*x]^2)/(8*a^3) - (3*x^2*ArcSinh[a*x]^2)/(4*a) + (x*Sqrt[1 + a^2*x^2]*ArcSi
nh[a*x]^3)/(2*a^2) - ArcSinh[a*x]^4/(8*a^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^2 \sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \frac{x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{2a^2} - \frac{\int \frac{\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{2a^2} - \frac{3\int x\sinh^{-1}(ax)^2\,dx}{2a}$$
$$= -\frac{3x^2\sinh^{-1}(ax)^2}{4a} + \frac{x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{2a^2} - \frac{\sinh^{-1}(ax)^4}{8a^3} + \frac{3}{2}\int \frac{x^2\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx$$
$$= \frac{3x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{4a^2} - \frac{3x^2\sinh^{-1}(ax)^2}{4a} + \frac{x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{2a^2} - \frac{\sinh^{-1}(ax)^4}{8a^3}$$
$$= -\frac{3x^2}{8a} + \frac{3x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{4a^2} - \frac{3\sinh^{-1}(ax)^2}{8a^3} - \frac{3x^2\sinh^{-1}(ax)^2}{4a} + \frac{x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{2a^2}$$

Mathematica [A] time = 0.06, size = 83, normalized size = 0.79

$$-\frac{3a^2x^2 - 4ax\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3 + \left(6a^2x^2+3\right)\sinh^{-1}(ax)^2 - 6ax\sqrt{a^2x^2+1}\,\sinh^{-1}(ax) + \sinh^{-1}(ax)^4}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] -1/8*(3*a^2*x^2 - 6*a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] + (3 + 6*a^2*x^2)*Ar
cSinh[a*x]^2 - 4*a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3 + ArcSinh[a*x]^4)/a^3
```

fricas [A] time = 0.53, size = 128, normalized size = 1.22

$$\frac{4\sqrt{a^2x^2+1}\,ax\log\left(ax+\sqrt{a^2x^2+1}\right)^3 - 3a^2x^2 - \log\left(ax+\sqrt{a^2x^2+1}\right)^4 + 6\sqrt{a^2x^2+1}\,ax\log\left(ax+\sqrt{a^2x^2+1}\right)}{8a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(4*sqrt(a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 + 1))^3 - 3*a^2*x^2 - l
og(a*x + sqrt(a^2*x^2 + 1))^4 + 6*sqrt(a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*
x^2 + 1)) - 3*(2*a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2)/a^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\,\mathrm{arsinh}\left(ax\right)^3}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arcsinh(a*x)^3/sqrt(a^2*x^2 + 1), x)
```

maple [A] time = 0.09, size = 84, normalized size = 0.80

$$-\frac{-4\,\mathrm{arcsinh}\left(ax\right)^3\sqrt{a^2x^2+1}\;ax + 6\,\mathrm{arcsinh}\left(ax\right)^2a^2x^2 + \mathrm{arcsinh}\left(ax\right)^4 - 6\,\mathrm{arcsinh}\left(ax\right)\sqrt{a^2x^2+1}\;ax + 3a^2x}{8a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/8*(-4*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)*a*x+6*arcsinh(a*x)^2*a^2*x^2+arcs
inh(a*x)^4-6*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a*x+3*a^2*x^2+3*arcsinh(a*x)^2+
3)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{arsinh}(ax)^3}{\sqrt{a^2x^2+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*arcsinh(a*x)^3/sqrt(a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 \operatorname{asinh}(a\,x)^3}{\sqrt{a^2\,x^2+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*asinh(a*x)^3)/(a^2*x^2 + 1)^(1/2),x)
```

```
[Out] int((x^2*asinh(a*x)^3)/(a^2*x^2 + 1)^(1/2), x)
```

sympy [A] time = 2.20, size = 100, normalized size = 0.95

$$\begin{cases} -\frac{3x^2 \operatorname{asinh}^2(ax)}{4a} - \frac{3x^2}{8a} + \frac{x\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{2a^2} + \frac{3x\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{4a^2} - \frac{\operatorname{asinh}^4(ax)}{8a^3} - \frac{3\operatorname{asinh}^2(ax)}{8a^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(a*x)**3/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-3*x**2*asinh(a*x)**2/(4*a) - 3*x**2/(8*a) + x*sqrt(a**2*x**2 +
1)*asinh(a*x)**3/(2*a**2) + 3*x*sqrt(a**2*x**2 + 1)*asinh(a*x)/(4*a**2) - a
sinh(a*x)**4/(8*a**3) - 3*asinh(a*x)**2/(8*a**3), Ne(a, 0)), (0, True))
```

3.345 $\int \frac{x\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=64

$$\frac{\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3}{a^2}+\frac{6\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{a^2}-\frac{6x}{a}-\frac{3x\sinh^{-1}(ax)^2}{a}$$

```
[Out] -6*x/a-3*x*arcsinh(a*x)^2/a+6*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^2+arcsinh(a*
x)^3*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.11, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5717, 5653, 8}

$$\frac{\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3}{a^2}+\frac{6\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{a^2}-\frac{6x}{a}-\frac{3x\sinh^{-1}(ax)^2}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (-6*x)/a + (6*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^2 - (3*x*ArcSinh[a*x]^2)/a
+ (Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/a^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx &= \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{a^2}-\frac{3\int \sinh^{-1}(ax)^2\,dx}{a}\\
&= -\frac{3x\sinh^{-1}(ax)^2}{a}+\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{a^2}+6\int \frac{x\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx\\
&= \frac{6\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{a^2}-\frac{3x\sinh^{-1}(ax)^2}{a}+\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{a^2}-\frac{6\int 1\,dx}{a}\\
&= -\frac{6x}{a}+\frac{6\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{a^2}-\frac{3x\sinh^{-1}(ax)^2}{a}+\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{a^2}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 58, normalized size = 0.91

$$\frac{\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3+6\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)-6ax-3ax\sinh^{-1}(ax)^2}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (-6*a*x + 6*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] - 3*a*x*ArcSinh[a*x]^2 + Sqrt[1
+ a^2*x^2]*ArcSinh[a*x]^3)/a^2
```

fricas [A] time = 0.49, size = 92, normalized size = 1.44

$$-\frac{3\,ax\log\left(ax+\sqrt{a^2x^2+1}\right)^2-\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1}\right)^3+6\,ax-6\,\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(3*a*x*log(a*x + sqrt(a^2*x^2 + 1))^2 - sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a
^2*x^2 + 1))^3 + 6*a*x - 6*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/
a^2
```

giac [A] time = 0.42, size = 101, normalized size = 1.58

$$\frac{\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1}\right)^3}{a^2}-\frac{3\left(x\log\left(ax+\sqrt{a^2x^2+1}\right)^2+2\,a\left(\frac{x}{a}-\frac{\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1}\right)}{a^2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^3/a^2 - 3*(x*log(a*x + sqrt(
a^2*x^2 + 1))^2 + 2*a*(x/a - sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))
/a^2))/a
```

maple [A] time = 0.08, size = 90, normalized size = 1.41

$$\frac{\operatorname{arcsinh}(ax)^3x^2a^2+\operatorname{arcsinh}(ax)^3-3\operatorname{arcsinh}(ax)^2\sqrt{a^2x^2+1}\,ax+6\operatorname{arcsinh}(ax)x^2a^2+6\operatorname{arcsinh}(ax)-6\sqrt{a\cdots}}{a^2\sqrt{a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/a^2/(a^2*x^2+1)^(1/2)*(arcsinh(a*x)^3*x^2*a^2+arcsinh(a*x)^3-3*arcsinh(a*
x)^2*(a^2*x^2+1)^(1/2)*a*x+6*arcsinh(a*x)*x^2*a^2+6*arcsinh(a*x)-6*(a^2*x^2
+1)^(1/2)*x*a)
```

maxima [A] time = 0.32, size = 61, normalized size = 0.95

$$-\frac{3\,x\operatorname{arsinh}(ax)^2}{a}+\frac{\sqrt{a^2x^2+1}\,\operatorname{arsinh}(ax)^3}{a^2}-\frac{6\left(x-\frac{\sqrt{a^2x^2+1}\,\operatorname{arsinh}(ax)}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -3*x*arcsinh(a*x)^2/a + sqrt(a^2*x^2 + 1)*arcsinh(a*x)^3/a^2 - 6*(x - sqrt(
a^2*x^2 + 1)*arcsinh(a*x)/a)/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x \operatorname{asinh}(a x)^3}{\sqrt{a^2 x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*asinh(a*x)^3)/(a^2*x^2 + 1)^(1/2),x)
```

```
[Out] int((x*asinh(a*x)^3)/(a^2*x^2 + 1)^(1/2), x)
```

sympy [A] time = 1.19, size = 61, normalized size = 0.95

$$\begin{cases} -\frac{3x \operatorname{asinh}^2(ax)}{a} - \frac{6x}{a} + \frac{\sqrt{a^2x^2+1} \operatorname{asinh}^3(ax)}{a^2} + \frac{6\sqrt{a^2x^2+1} \operatorname{asinh}(ax)}{a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(a*x)**3/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-3*x*asinh(a*x)**2/a - 6*x/a + sqrt(a**2*x**2 + 1)*asinh(a*x)**3
/a**2 + 6*sqrt(a**2*x**2 + 1)*asinh(a*x)/a**2, Ne(a, 0)), (0, True))
```

3.346 $\int \frac{\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\, dx$

Optimal. Leaf size=13

$$\frac{\sinh^{-1}(ax)^4}{4a}$$

```
[Out] 1/4*arcsinh(a*x)^4/a
```

Rubi [A] time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {5675}

$$\frac{\sinh^{-1}(ax)^4}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/Sqrt[1 + a^2*x^2],x]
```

```
[Out] ArcSinh[a*x]^4/(4*a)
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\, dx = \frac{\sinh^{-1}(ax)^4}{4a}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{\sinh^{-1}(ax)^4}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^3/Sqrt[1 + a^2*x^2],x]
```

```
[Out] ArcSinh[a*x]^4/(4*a)
```

fricas [B] time = 0.42, size = 23, normalized size = 1.77

$$\frac{\log\left(ax+\sqrt{a^2x^2+1}\right)^4}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*log(a*x + sqrt(a^2*x^2 + 1))^4/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)^3}{\sqrt{a^2x^2+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^3/sqrt(a^2*x^2 + 1), x)
```

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$\frac{\operatorname{arcsinh}(ax)^4}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x)
[Out] 1/4*arcsinh(a*x)^4/a
```

maxima [A] time = 0.35, size = 11, normalized size = 0.85

$$\frac{\operatorname{arsinh}(ax)^4}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/4*arcsinh(a*x)^4/a
```

mupad [B] time = 0.10, size = 11, normalized size = 0.85

$$\frac{\operatorname{asinh}(a\,x)^4}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^3/(a^2*x^2 + 1)^(1/2),x)
[Out] asinh(a*x)^4/(4*a)
```

sympy [A] time = 0.68, size = 10, normalized size = 0.77

$$\begin{cases} \frac{\operatorname{asinh}^4(ax)}{4a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/(a**2*x**2+1)**(1/2),x)
[Out] Piecewise((asinh(a*x)**4/(4*a), Ne(a, 0)), (0, True))
```

3.347 $\int \frac{\sinh^{-1}(ax)^3}{x\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=102

$$-3\sinh^{-1}(ax)^2\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right)+3\sinh^{-1}(ax)^2\text{Li}_2\left(e^{\sinh^{-1}(ax)}\right)+6\sinh^{-1}(ax)\text{Li}_3\left(-e^{\sinh^{-1}(ax)}\right)-6\sinh^{-1}(ax)\text{Li}_3\left(e^{\sinh^{-1}(ax)}\right)$$

```
[Out] -2*arcsinh(a*x)^3*arctanh(a*x+(a^2*x^2+1)^(1/2))-3*arcsinh(a*x)^2*polylog(2
,-a*x-(a^2*x^2+1)^(1/2))+3*arcsinh(a*x)^2*polylog(2,a*x+(a^2*x^2+1)^(1/2))+
6*arcsinh(a*x)*polylog(3,-a*x-(a^2*x^2+1)^(1/2))-6*arcsinh(a*x)*polylog(3,a
*x+(a^2*x^2+1)^(1/2))-6*polylog(4,-a*x-(a^2*x^2+1)^(1/2))+6*polylog(4,a*x+(
a^2*x^2+1)^(1/2))
```

Rubi [A] time = 0.16, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5760, 4182, 2531, 6609, 2282, 6589}

$$-3\sinh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)+3\sinh^{-1}(ax)^2\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)+6\sinh^{-1}(ax)\text{PolyLog}\left(3,-e^{\sinh^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/(x*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -2*ArcSinh[a*x]^3*ArcTanh[E^ArcSinh[a*x]] - 3*ArcSinh[a*x]^2*PolyLog[2, -E^
ArcSinh[a*x]] + 3*ArcSinh[a*x]^2*PolyLog[2, E^ArcSinh[a*x]] + 6*ArcSinh[a*x
]*PolyLog[3, -E^ArcSinh[a*x]] - 6*ArcSinh[a*x]*PolyLog[3, E^ArcSinh[a*x]] -
 6*PolyLog[4, -E^ArcSinh[a*x]] + 6*PolyLog[4, E^ArcSinh[a*x]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^3}{x\sqrt{1+a^2x^2}}\,dx = \text{Subst}\left(\int x^3\text{csch}(x)\,dx, x, \sinh^{-1}(ax)\right)$$
$$= -2\sinh^{-1}(ax)^3\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 3\,\text{Subst}\left(\int x^2\log\left(1-e^x\right)\,dx, x, \sinh^{-1}(ax)\right) + 3\,\text{Su}$$
$$= -2\sinh^{-1}(ax)^3\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 3\sinh^{-1}(ax)^2\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + 3\sinh^{-1}(ax)^2\text{Li}_2\left(e\right.$$
$$= -2\sinh^{-1}(ax)^3\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 3\sinh^{-1}(ax)^2\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + 3\sinh^{-1}(ax)^2\text{Li}_2\left(e\right.$$
$$= -2\sinh^{-1}(ax)^3\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 3\sinh^{-1}(ax)^2\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + 3\sinh^{-1}(ax)^2\text{Li}_2\left(e\right.$$
$$= -2\sinh^{-1}(ax)^3\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 3\sinh^{-1}(ax)^2\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + 3\sinh^{-1}(ax)^2\text{Li}_2\left(e\right.$$

Mathematica [A] time = 0.14, size = 146, normalized size = 1.43

$$\frac{1}{8}\left(24\sinh^{-1}(ax)^2\text{Li}_2\left(-e^{-\sinh^{-1}(ax)}\right) + 24\sinh^{-1}(ax)^2\text{Li}_2\left(e^{\sinh^{-1}(ax)}\right) + 48\sinh^{-1}(ax)\text{Li}_3\left(-e^{-\sinh^{-1}(ax)}\right) - 48\,\text{s}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^3/(x*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] (Pi^4 - 2*ArcSinh[a*x]^4 - 8*ArcSinh[a*x]^3*Log[1 + E^(-ArcSinh[a*x])] + 8*
ArcSinh[a*x]^3*Log[1 - E^ArcSinh[a*x]] + 24*ArcSinh[a*x]^2*PolyLog[2, -E^(-
ArcSinh[a*x])] + 24*ArcSinh[a*x]^2*PolyLog[2, E^ArcSinh[a*x]] + 48*ArcSinh[
a*x]*PolyLog[3, -E^(-ArcSinh[a*x])] - 48*ArcSinh[a*x]*PolyLog[3, E^ArcSinh[
a*x]] + 48*PolyLog[4, -E^(-ArcSinh[a*x])] + 48*PolyLog[4, E^ArcSinh[a*x]])/
8
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)^3}{a^2x^3+x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^3/(a^2*x^3 + x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arsinh}\,(ax)^3}{\sqrt{a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^3/(sqrt(a^2*x^2 + 1)*x), x)
```

maple [A] time = 0.10, size = 197, normalized size = 1.93

$$-\operatorname{arcsinh}(ax)^3 \ln\left(1+ax+\sqrt{a^2x^2+1}\right)-3\operatorname{arcsinh}(ax)^2 \operatorname{polylog}\left(2,-ax-\sqrt{a^2x^2+1}\right)+6\operatorname{arcsinh}(ax)\operatorname{polylog}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/x/(a^2*x^2+1)^(1/2),x)
```

```
[Out] -arcsinh(a*x)^3*ln(1+a*x+(a^2*x^2+1)^(1/2))-3*arcsinh(a*x)^2*polylog(2,-a*x
-(a^2*x^2+1)^(1/2))+6*arcsinh(a*x)*polylog(3,-a*x-(a^2*x^2+1)^(1/2))-6*poly
log(4,-a*x-(a^2*x^2+1)^(1/2))+arcsinh(a*x)^3*ln(1-a*x-(a^2*x^2+1)^(1/2))+3*
arcsinh(a*x)^2*polylog(2,a*x+(a^2*x^2+1)^(1/2))-6*arcsinh(a*x)*polylog(3,a*
x+(a^2*x^2+1)^(1/2))+6*polylog(4,a*x+(a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)^3}{\sqrt{a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^3/(sqrt(a^2*x^2 + 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}(a\,x)^3}{x\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^3/(x*(a^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(asinh(a*x)^3/(x*(a^2*x^2 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^3(ax)}{x\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/x/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(asinh(a*x)**3/(x*sqrt(a**2*x**2 + 1)), x)
```

3.348 $\int \frac{\sinh^{-1}(ax)^3}{x^2\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=88

$$-\frac{\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3}{x}+3a\sinh^{-1}(ax)\text{Li}_2\left(e^{2\sinh^{-1}(ax)}\right)-\frac{3}{2}a\text{Li}_3\left(e^{2\sinh^{-1}(ax)}\right)-a\sinh^{-1}(ax)^3+3a\sinh^{-1}(ax)^2 \text{l}$$

```
[Out] -a*arcsinh(a*x)^3+3*a*arcsinh(a*x)^2*ln(1-(a*x+(a^2*x^2+1)^(1/2))^2)+3*a*ar
csinh(a*x)*polylog(2,(a*x+(a^2*x^2+1)^(1/2))^2)-3/2*a*polylog(3,(a*x+(a^2*x
^2+1)^(1/2))^2)-arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.19, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5723, 5659, 3716, 2190, 2531, 2282, 6589}

$$3a\sinh^{-1}(ax)\text{PolyLog}\left(2,e^{2\sinh^{-1}(ax)}\right)-\frac{3}{2}a\text{PolyLog}\left(3,e^{2\sinh^{-1}(ax)}\right)-\frac{\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3}{x}-a\sinh^{-1}(ax)^3+3$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/(x^2*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -(a*ArcSinh[a*x]^3) - (Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/x + 3*a*ArcSinh[a*
x]^2*Log[1 - E^(2*ArcSinh[a*x])] + 3*a*ArcSinh[a*x]*PolyLog[2, E^(2*ArcSinh
[a*x])] - (3*a*PolyLog[3, E^(2*ArcSinh[a*x])])/2
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{-1}(ax)^3}{x^2\sqrt{1+a^2x^2}}\,dx &= -\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{x}+(3a)\int \frac{\sinh^{-1}(ax)^2}{x}\,dx\\
&= -\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{x}+(3a)\,\text{Subst}\left(\int x^2\coth(x)\,dx,x,\sinh^{-1}(ax)\right)\\
&= -a\sinh^{-1}(ax)^3-\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{x}-(6a)\,\text{Subst}\left(\int \frac{e^{2x}x^2}{1-e^{2x}}\,dx,x,\sinh^{-1}(ax)\right)\\
&= -a\sinh^{-1}(ax)^3-\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{x}+3a\sinh^{-1}(ax)^2\log\left(1-e^{2\sinh^{-1}(ax)}\right)-(6a)\,\text{Sub}\\
&= -a\sinh^{-1}(ax)^3-\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{x}+3a\sinh^{-1}(ax)^2\log\left(1-e^{2\sinh^{-1}(ax)}\right)+3a\sinh^{-}\\
&= -a\sinh^{-1}(ax)^3-\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{x}+3a\sinh^{-1}(ax)^2\log\left(1-e^{2\sinh^{-1}(ax)}\right)+3a\sinh^{-}\\
&= -a\sinh^{-1}(ax)^3-\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{x}+3a\sinh^{-1}(ax)^2\log\left(1-e^{2\sinh^{-1}(ax)}\right)+3a\sinh^{-}
\end{aligned}$$

Mathematica [C] time = 0.21, size = 97, normalized size = 1.10

$$\frac{1}{8}a\left(-\frac{8\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3}{ax}+24\sinh^{-1}(ax)\text{Li}_2\left(e^{2\sinh^{-1}(ax)}\right)-12\text{Li}_3\left(e^{2\sinh^{-1}(ax)}\right)-8\sinh^{-1}(ax)^3+24\sinh^{-1}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^3/(x^2*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] (a*(I*Pi^3 - 8*ArcSinh[a*x]^3 - (8*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(a*x)
+ 24*ArcSinh[a*x]^2*Log[1 - E^(2*ArcSinh[a*x])] + 24*ArcSinh[a*x]*PolyLog[2
, E^(2*ArcSinh[a*x])] - 12*PolyLog[3, E^(2*ArcSinh[a*x])]))/8
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a^2x^2+1}\,\operatorname{arsinh}(ax)^3}{a^2x^4+x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^3/(a^2*x^4 + x^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.18, size = 187, normalized size = 2.12

$$\frac{\left(ax-\sqrt{a^2x^2+1}\right)\operatorname{arcsinh}(ax)^3}{x}-2a\operatorname{arcsinh}(ax)^3+3a\operatorname{arcsinh}(ax)^2\ln\left(1+ax+\sqrt{a^2x^2+1}\right)+6a\operatorname{arcsinh}(ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/x^2/(a^2*x^2+1)^(1/2),x)
```

```
[Out] (a*x-(a^2*x^2+1)^(1/2))/x*arcsinh(a*x)^3-2*a*arcsinh(a*x)^3+3*a*arcsinh(a*x
)^2*ln(1+a*x+(a^2*x^2+1)^(1/2))+6*a*arcsinh(a*x)*polylog(2,-a*x-(a^2*x^2+1)
^(1/2))-6*a*polylog(3,-a*x-(a^2*x^2+1)^(1/2))+3*a*arcsinh(a*x)^2*ln(1-a*x-(
a^2*x^2+1)^(1/2))+6*a*arcsinh(a*x)*polylog(2,a*x+(a^2*x^2+1)^(1/2))-6*a*pol
ylog(3,a*x+(a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^3}{x}+\int\frac{3\left(a^3x^2+\sqrt{a^2x^2+1}\,a^2x+a\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{\sqrt{a^2x^2+1}\,ax^2+\left(a^2x^2+1\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^3/x + integrate(3*(a^3*x^2
+ sqrt(a^2*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))^2/(sqrt(a^2*x^2
 + 1)*a*x^2 + (a^2*x^2 + 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\operatorname{asinh}(a\,x)^3}{x^2\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^3/(x^2*(a^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(asinh(a*x)^3/(x^2*(a^2*x^2 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{asinh}^3(ax)}{x^2\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/x**2/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(asinh(a*x)**3/(x**2*sqrt(a**2*x**2 + 1)), x)
```

3.349 $\int \frac{\sinh^{-1}(ax)^3}{x^3\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=210

$$\frac{3}{2}a^2\sinh^{-1}(ax)^2\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right)-\frac{3}{2}a^2\sinh^{-1}(ax)^2\text{Li}_2\left(e^{\sinh^{-1}(ax)}\right)-3a^2\sinh^{-1}(ax)\text{Li}_3\left(-e^{\sinh^{-1}(ax)}\right)+3a^2\sinh^{-}$$

```
[Out] -3/2*a*arcsinh(a*x)^2/x-6*a^2*arcsinh(a*x)*arctanh(a*x+(a^2*x^2+1)^(1/2))+a
^2*arcsinh(a*x)^3*arctanh(a*x+(a^2*x^2+1)^(1/2))-3*a^2*polylog(2,-a*x-(a^2*
x^2+1)^(1/2))+3/2*a^2*arcsinh(a*x)^2*polylog(2,-a*x-(a^2*x^2+1)^(1/2))+3*a^
2*polylog(2,a*x+(a^2*x^2+1)^(1/2))-3/2*a^2*arcsinh(a*x)^2*polylog(2,a*x+(a^
2*x^2+1)^(1/2))-3*a^2*arcsinh(a*x)*polylog(3,-a*x-(a^2*x^2+1)^(1/2))+3*a^2*
arcsinh(a*x)*polylog(3,a*x+(a^2*x^2+1)^(1/2))+3*a^2*polylog(4,-a*x-(a^2*x^2
+1)^(1/2))-3*a^2*polylog(4,a*x+(a^2*x^2+1)^(1/2))-1/2*arcsinh(a*x)^3*(a^2*x
^2+1)^(1/2)/x^2
```

Rubi [A] time = 0.36, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {5747, 5760, 4182, 2531, 6609, 2282, 6589, 5661, 2279, 2391}

$$\frac{3}{2}a^2\sinh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)-\frac{3}{2}a^2\sinh^{-1}(ax)^2\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)-3a^2\sinh^{-1}(ax)\text{PolyLog}\left(3\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/(x^3*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] (-3*a*ArcSinh[a*x]^2)/(2*x) - (Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(2*x^2) -
6*a^2*ArcSinh[a*x]*ArcTanh[E^ArcSinh[a*x]] + a^2*ArcSinh[a*x]^3*ArcTanh[E^A
rcSinh[a*x]] - 3*a^2*PolyLog[2, -E^ArcSinh[a*x]] + (3*a^2*ArcSinh[a*x]^2*Po
lyLog[2, -E^ArcSinh[a*x]])/2 + 3*a^2*PolyLog[2, E^ArcSinh[a*x]] - (3*a^2*Ar
cSinh[a*x]^2*PolyLog[2, E^ArcSinh[a*x]])/2 - 3*a^2*ArcSinh[a*x]*PolyLog[3,
-E^ArcSinh[a*x]] + 3*a^2*ArcSinh[a*x]*PolyLog[3, E^ArcSinh[a*x]] + 3*a^2*Po
lyLog[4, -E^ArcSinh[a*x]] - 3*a^2*PolyLog[4, E^ArcSinh[a*x]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
```

```
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^3}{x^3\sqrt{1+a^2x^2}}\,dx = -\frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{2x^2} + \frac{1}{2}(3a)\int \frac{\sinh^{-1}(ax)^2}{x^2}\,dx - \frac{1}{2}a^2\int \frac{\sinh^{-1}(ax)^3}{x\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{3a\sinh^{-1}(ax)^2}{2x} - \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{2x^2} - \frac{1}{2}a^2\,\text{Subst}\left(\int x^3\text{csch}(x)\,dx, x, \sinh^{-1}(ax)\right.$$

$$= -\frac{3a\sinh^{-1}(ax)^2}{2x} - \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{2x^2} + a^2\sinh^{-1}(ax)^3\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) + \frac{1}{2}\left(3a\right.$$

$$= -\frac{3a\sinh^{-1}(ax)^2}{2x} - \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{2x^2} - 6a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) + a^2\sin$$

$$= -\frac{3a\sinh^{-1}(ax)^2}{2x} - \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{2x^2} - 6a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) + a^2\sin$$

$$= -\frac{3a\sinh^{-1}(ax)^2}{2x} - \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{2x^2} - 6a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) + a^2\sin$$

$$= -\frac{3a\sinh^{-1}(ax)^2}{2x} - \frac{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{2x^2} - 6a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) + a^2\sin$$

Mathematica [A] time = 4.62, size = 304, normalized size = 1.45

$$\frac{a\left(-24ax\sinh^{-1}(ax)^2\text{Li}_2\left(e^{\sinh^{-1}(ax)}\right) - 48ax\sinh^{-1}(ax)\text{Li}_3\left(-e^{-\sinh^{-1}(ax)}\right) + 48ax\sinh^{-1}(ax)\text{Li}_3\left(e^{\sinh^{-1}(ax)}\right) - \right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^3/(x^3*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] (a*(-(a*Pi^4*x) + 2*a*x*ArcSinh[a*x]^4 - 12*a*x*ArcSinh[a*x]^2*Coth[ArcSinh
[a*x]/2] - 2*a*x*ArcSinh[a*x]^3*Csch[ArcSinh[a*x]/2]^2 + 48*a*x*ArcSinh[a*x
]*Log[1 - E^(-ArcSinh[a*x])] - 48*a*x*ArcSinh[a*x]*Log[1 + E^(-ArcSinh[a*x]
)] + 8*a*x*ArcSinh[a*x]^3*Log[1 + E^(-ArcSinh[a*x])] - 8*a*x*ArcSinh[a*x]^3
*Log[1 - E^ArcSinh[a*x]] - 24*a*x*(-2 + ArcSinh[a*x]^2)*PolyLog[2, -E^(-Arc
Sinh[a*x])] - 48*a*x*PolyLog[2, E^(-ArcSinh[a*x])] - 24*a*x*ArcSinh[a*x]^2*
PolyLog[2, E^ArcSinh[a*x]] - 48*a*x*ArcSinh[a*x]*PolyLog[3, -E^(-ArcSinh[a*
x])] + 48*a*x*ArcSinh[a*x]*PolyLog[3, E^ArcSinh[a*x]] - 48*a*x*PolyLog[4, -
E^(-ArcSinh[a*x])] - 48*a*x*PolyLog[4, E^ArcSinh[a*x]] + 12*a*x*ArcSinh[a*x
]^2*Tanh[ArcSinh[a*x]/2] - 4*ArcSinh[a*x]^3*Tanh[ArcSinh[a*x]/2]))/(16*x)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)^3}{a^2x^5+x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^3/(a^2*x^5 + x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arsinh}\,(ax)^3}{\sqrt{a^2x^2+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^3/(sqrt(a^2*x^2 + 1)*x^3), x)
```

maple [A] time = 0.26, size = 377, normalized size = 1.80

$$-\frac{\operatorname{arcsinh}(ax)^2\left(\operatorname{arcsinh}(ax)\,x^2a^2+3\sqrt{a^2x^2+1}\;xa+\operatorname{arcsinh}(ax)\right)}{2\sqrt{a^2x^2+1}\;x^2}+\frac{a^2\operatorname{arcsinh}(ax)^3\ln\left(1+ax+\sqrt{a^2x^2+1}\right)}{2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/x^3/(a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/2/(a^2*x^2+1)^(1/2)/x^2*arcsinh(a*x)^2*(arcsinh(a*x)*x^2*a^2+3*(a^2*x^2+
1)^(1/2)*x*a+arcsinh(a*x))+1/2*a^2*arcsinh(a*x)^3*ln(1+a*x+(a^2*x^2+1)^(1/2
))+3/2*a^2*arcsinh(a*x)^2*polylog(2,-a*x-(a^2*x^2+1)^(1/2))-3*a^2*arcsinh(a
*x)*polylog(3,-a*x-(a^2*x^2+1)^(1/2))+3*a^2*polylog(4,-a*x-(a^2*x^2+1)^(1/2
))-1/2*a^2*arcsinh(a*x)^3*ln(1-a*x-(a^2*x^2+1)^(1/2))-3/2*a^2*arcsinh(a*x)^
2*polylog(2,a*x+(a^2*x^2+1)^(1/2))+3*a^2*arcsinh(a*x)*polylog(3,a*x+(a^2*x^
2+1)^(1/2))-3*a^2*polylog(4,a*x+(a^2*x^2+1)^(1/2))-3*a^2*arcsinh(a*x)*ln(1+
a*x+(a^2*x^2+1)^(1/2))-3*a^2*polylog(2,-a*x-(a^2*x^2+1)^(1/2))+3*a^2*arcsin
h(a*x)*ln(1-a*x-(a^2*x^2+1)^(1/2))+3*a^2*polylog(2,a*x+(a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{arsinh}(ax)^3}{\sqrt{a^2x^2+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^3/(sqrt(a^2*x^2 + 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\operatorname{asinh}(a\,x)^3}{x^3\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^3/(x^3*(a^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(asinh(a*x)^3/(x^3*(a^2*x^2 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{asinh}^3(ax)}{x^3\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/x**3/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(asinh(a*x)**3/(x**3*sqrt(a**2*x**2 + 1)), x)
```

3.350 $\int \frac{(c+a^2cx^2)^3}{\sinh^{-1}(ax)} \, dx$

Optimal. Leaf size=67

$$\frac{35c^3\text{Chi}\left(\sinh^{-1}(ax)\right)}{64a}+\frac{21c^3\text{Chi}\left(3\sinh^{-1}(ax)\right)}{64a}+\frac{7c^3\text{Chi}\left(5\sinh^{-1}(ax)\right)}{64a}+\frac{c^3\text{Chi}\left(7\sinh^{-1}(ax)\right)}{64a}$$

```
[Out] 35/64*c^3*Chi(arcsinh(a*x))/a+21/64*c^3*Chi(3*arcsinh(a*x))/a+7/64*c^3*Chi(
5*arcsinh(a*x))/a+1/64*c^3*Chi(7*arcsinh(a*x))/a
```

Rubi [A] time = 0.11, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {5699, 3312, 3301}

$$\frac{35c^3\text{Chi}\left(\sinh^{-1}(ax)\right)}{64a}+\frac{21c^3\text{Chi}\left(3\sinh^{-1}(ax)\right)}{64a}+\frac{7c^3\text{Chi}\left(5\sinh^{-1}(ax)\right)}{64a}+\frac{c^3\text{Chi}\left(7\sinh^{-1}(ax)\right)}{64a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^3/ArcSinh[a*x],x]
```

```
[Out] (35*c^3*CoshIntegral[ArcSinh[a*x]])/(64*a) + (21*c^3*CoshIntegral[3*ArcSinh
[a*x]])/(64*a) + (7*c^3*CoshIntegral[5*ArcSinh[a*x]])/(64*a) + (c^3*CoshInt
egral[7*ArcSinh[a*x]])/(64*a)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c+a^2cx^2\right)^3}{\sinh^{-1}(ax)} \, dx &= \frac{c^3 \text{Subst}\left(\int \frac{\cosh^7(x)}{x} \, dx, x, \sinh^{-1}(ax)\right)}{a}\\
&= \frac{c^3 \text{Subst}\left(\int \left(\frac{35\cosh(x)}{64x}+\frac{21\cosh(3x)}{64x}+\frac{7\cosh(5x)}{64x}+\frac{\cosh(7x)}{64x}\right) dx, x, \sinh^{-1}(ax)\right)}{a}\\
&= \frac{c^3 \text{Subst}\left(\int \frac{\cosh(7x)}{x} \, dx, x, \sinh^{-1}(ax)\right)}{64a}+\frac{\left(7c^3\right) \text{Subst}\left(\int \frac{\cosh(5x)}{x} \, dx, x, \sinh^{-1}(ax)\right)}{64a}+\frac{(21}{}\\
&= \frac{35c^3\text{Chi}\left(\sinh^{-1}(ax)\right)}{64a}+\frac{21c^3\text{Chi}\left(3\sinh^{-1}(ax)\right)}{64a}+\frac{7c^3\text{Chi}\left(5\sinh^{-1}(ax)\right)}{64a}+\frac{c^3\text{Chi}\left(7\,\text{si}\right.}{64}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 43, normalized size = 0.64

$$\frac{c^3\left(35\text{Chi}\left(\sinh^{-1}(ax)\right)+21\text{Chi}\left(3\sinh^{-1}(ax)\right)+7\text{Chi}\left(5\sinh^{-1}(ax)\right)+\text{Chi}\left(7\sinh^{-1}(ax)\right)\right)}{64a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^3/ArcSinh[a*x],x]
```

```
[Out] (c^3*(35*CoshIntegral[ArcSinh[a*x]] + 21*CoshIntegral[3*ArcSinh[a*x]] + 7*C
oshIntegral[5*ArcSinh[a*x]] + CoshIntegral[7*ArcSinh[a*x]]))/(64*a)
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^6c^3x^6+3\,a^4c^3x^4+3\,a^2c^3x^2+c^3}{\text{arsinh}\,(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)/arcsinh(a*x),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a^2cx^2+c\right)^3}{\text{arsinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arcsinh(a*x),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3/arcsinh(a*x), x)
```

maple [A] time = 0.08, size = 42, normalized size = 0.63

$$\frac{c^3\,(35\text{X}\,(\text{arcsinh}\,(ax))+21\text{X}\,(3\,\text{arcsinh}\,(ax))+7\text{X}\,(5\,\text{arcsinh}\,(ax))+\text{X}\,(7\,\text{arcsinh}\,(ax)))}{64a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3/arcsinh(a*x),x)
```

```
[Out] 1/64/a*c^3*(35*Chi(arcsinh(a*x))+21*Chi(3*arcsinh(a*x))+7*Chi(5*arcsinh(a*x
))+Chi(7*arcsinh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a^2cx^2+c\right)^3}{\text{arsinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^3/arcsinh(a*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(c\,a^2\,x^2+c\right)^3}{\text{asinh}\,(a\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^3/asinh(a*x),x)
```

```
[Out] int((c + a^2*c*x^2)^3/asinh(a*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^{3}\left(\int \frac{3a^{2}x^{2}}{\operatorname{asinh}(ax)}\,dx + \int \frac{3a^{4}x^{4}}{\operatorname{asinh}(ax)}\,dx + \int \frac{a^{6}x^{6}}{\operatorname{asinh}(ax)}\,dx + \int \frac{1}{\operatorname{asinh}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3/asinh(a*x),x)
```

```
[Out] c**3*(Integral(3*a**2*x**2/asinh(a*x), x) + Integral(3*a**4*x**4/asinh(a*x)
, x) + Integral(a**6*x**6/asinh(a*x), x) + Integral(1/asinh(a*x), x))
```

3.351 $\int \frac{(c+a^2cx^2)^2}{\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=50

$$\frac{5c^2\text{Chi}\left(\sinh^{-1}(ax)\right)}{8a}+\frac{5c^2\text{Chi}\left(3\sinh^{-1}(ax)\right)}{16a}+\frac{c^2\text{Chi}\left(5\sinh^{-1}(ax)\right)}{16a}$$

```
[Out] 5/8*c^2*Chi(arcsinh(a*x))/a+5/16*c^2*Chi(3*arcsinh(a*x))/a+1/16*c^2*Chi(5*a
rcsinh(a*x))/a
```

Rubi [A] time = 0.10, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {5699, 3312, 3301}

$$\frac{5c^2\text{Chi}\left(\sinh^{-1}(ax)\right)}{8a}+\frac{5c^2\text{Chi}\left(3\sinh^{-1}(ax)\right)}{16a}+\frac{c^2\text{Chi}\left(5\sinh^{-1}(ax)\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^2/ArcSinh[a*x],x]
```

```
[Out] (5*c^2*CoshIntegral[ArcSinh[a*x]])/(8*a) + (5*c^2*CoshIntegral[3*ArcSinh[a*
x]])/(16*a) + (c^2*CoshIntegral[5*ArcSinh[a*x]])/(16*a)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+a^2cx^2)^2}{\sinh^{-1}(ax)}\,dx &= \frac{c^2\,\text{Subst}\left(\int \frac{\cosh^5(x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{a}\\ &= \frac{c^2\,\text{Subst}\left(\int \left(\frac{5\cosh(x)}{8x}+\frac{5\cosh(3x)}{16x}+\frac{\cosh(5x)}{16x}\right)dx,x,\sinh^{-1}(ax)\right)}{a}\\ &= \frac{c^2\,\text{Subst}\left(\int \frac{\cosh(5x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{16a}+\frac{\left(5c^2\right)\text{Subst}\left(\int \frac{\cosh(3x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{16a}+\frac{\left(5c^2\right)\text{S}}{}\\ &= \frac{5c^2\text{Chi}\left(\sinh^{-1}(ax)\right)}{8a}+\frac{5c^2\text{Chi}\left(3\sinh^{-1}(ax)\right)}{16a}+\frac{c^2\text{Chi}\left(5\sinh^{-1}(ax)\right)}{16a}\end{aligned}$$

Mathematica [A] time = 0.08, size = 34, normalized size = 0.68

$$\frac{c^2\left(10\mathrm{Chi}\left(\sinh^{-1}(ax)\right)+5\mathrm{Chi}\left(3\sinh^{-1}(ax)\right)+\mathrm{Chi}\left(5\sinh^{-1}(ax)\right)\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^2/ArcSinh[a*x],x]
```

```
[Out] (c^2*(10*CoshIntegral[ArcSinh[a*x]] + 5*CoshIntegral[3*ArcSinh[a*x]] + Cosh
Integral[5*ArcSinh[a*x]]))/(16*a)
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{a^4c^2x^4+2\,a^2c^2x^2+c^2}{\mathrm{arsinh}\left(ax\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)/arcsinh(a*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a^2cx^2+c\right)^2}{\mathrm{arsinh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arcsinh(a*x),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2/arcsinh(a*x), x)
```

maple [A] time = 0.04, size = 33, normalized size = 0.66

$$\frac{c^2\left(10\mathrm{X}\left(\mathrm{arcsinh}\left(ax\right)\right)+5\mathrm{X}\left(3\,\mathrm{arcsinh}\left(ax\right)\right)+\mathrm{X}\left(5\,\mathrm{arcsinh}\left(ax\right)\right)\right)}{16a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2/arcsinh(a*x),x)
```

```
[Out] 1/16/a*c^2*(10*Chi(arcsinh(a*x))+5*Chi(3*arcsinh(a*x))+Chi(5*arcsinh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a^2cx^2+c\right)^2}{\mathrm{arsinh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^2/arcsinh(a*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\left(c\,a^2\,x^2+c\right)^2}{\mathrm{asinh}\left(a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^2/asinh(a*x),x)
```

```
[Out] int((c + a^2*c*x^2)^2/asinh(a*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int \frac{2a^2x^2}{\operatorname{asinh}(ax)}\,dx + \int \frac{a^4x^4}{\operatorname{asinh}(ax)}\,dx + \int \frac{1}{\operatorname{asinh}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2/asinh(a*x),x)
```

```
[Out] c**2*(Integral(2*a**2*x**2/asinh(a*x), x) + Integral(a**4*x**4/asinh(a*x),
x) + Integral(1/asinh(a*x), x))
```

3.352 $\int \frac{c+a^2cx^2}{\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=29

$$\frac{3c\text{Chi}\left(\sinh^{-1}(ax)\right)}{4a}+\frac{c\text{Chi}\left(3\sinh^{-1}(ax)\right)}{4a}$$

```
[Out] 3/4*c*Chi(arcsinh(a*x))/a+1/4*c*Chi(3*arcsinh(a*x))/a
```

Rubi [A] time = 0.07, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {5699, 3312, 3301}

$$\frac{3c\text{Chi}\left(\sinh^{-1}(ax)\right)}{4a}+\frac{c\text{Chi}\left(3\sinh^{-1}(ax)\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)/ArcSinh[a*x],x]
```

```
[Out] (3*c*CoshIntegral[ArcSinh[a*x]])/(4*a) + (c*CoshIntegral[3*ArcSinh[a*x]])/(
4*a)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{c+a^2cx^2}{\sinh^{-1}(ax)}\,dx &= \frac{c\,\text{Subst}\left(\int \frac{\cosh^3(x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{a}\\
&= \frac{c\,\text{Subst}\left(\int \left(\frac{3\cosh(x)}{4x}+\frac{\cosh(3x)}{4x}\right)dx,x,\sinh^{-1}(ax)\right)}{a}\\
&= \frac{c\,\text{Subst}\left(\int \frac{\cosh(3x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{4a}+\frac{(3c)\,\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{4a}\\
&= \frac{3c\text{Chi}\left(\sinh^{-1}(ax)\right)}{4a}+\frac{c\text{Chi}\left(3\sinh^{-1}(ax)\right)}{4a}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 23, normalized size = 0.79

$$\frac{c\left(3\text{Chi}\left(\sinh^{-1}(ax)\right)+\text{Chi}\left(3\sinh^{-1}(ax)\right)\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)/ArcSinh[a*x],x]
[Out] (c*(3*CoshIntegral[ArcSinh[a*x]] + CoshIntegral[3*ArcSinh[a*x]]))/(4*a)
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^2cx^2+c}{\operatorname{arsinh}(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arcsinh(a*x),x, algorithm="fricas")
[Out] integral((a^2*c*x^2 + c)/arcsinh(a*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a^2cx^2+c}{\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arcsinh(a*x),x, algorithm="giac")
[Out] integrate((a^2*c*x^2 + c)/arcsinh(a*x), x)
```

maple [A] time = 0.04, size = 22, normalized size = 0.76

$$\frac{c\left(3\text{X}(\operatorname{arcsinh}(ax))+\text{X}(3\operatorname{arcsinh}(ax))\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)/arcsinh(a*x),x)
[Out] 1/4/a*c*(3*Chi(arcsinh(a*x))+Chi(3*arcsinh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a^2cx^2+c}{\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arcsinh(a*x),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)/arcsinh(a*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{c\,a^2\,x^2+c}{\operatorname{asinh}(a\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)/asinh(a*x),x)
```

```
[Out] int((c + a^2*c*x^2)/asinh(a*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int \frac{a^2x^2}{\operatorname{asinh}(ax)}\,dx + \int \frac{1}{\operatorname{asinh}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)/asinh(a*x),x)
[Out] c*(Integral(a**2*x**2/asinh(a*x), x) + Integral(1/asinh(a*x), x))
```

3.353 $\int \frac{1}{(c+a^2cx^2)\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=22

$$\mathrm{Int}\left(\frac{1}{(a^2cx^2+c)\sinh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable(1/(a^2*c*x^2+c)/arcsinh(a*x),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(c+a^2cx^2)\sinh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + a^2*c*x^2)*ArcSinh[a*x]),x]
[Out] Defer[Int][1/((c + a^2*c*x^2)*ArcSinh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{(c+a^2cx^2)\sinh^{-1}(ax)}\,dx=\int \frac{1}{(c+a^2cx^2)\sinh^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+a^2cx^2)\sinh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + a^2*c*x^2)*ArcSinh[a*x]),x]
[Out] Integrate[1/((c + a^2*c*x^2)*ArcSinh[a*x]), x]
```

fricas [A] time = 0.58, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{(a^2cx^2+c)\,\mathrm{arsinh}\,(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arcsinh(a*x),x, algorithm="fricas")
[Out] integral(1/((a^2*c*x^2 + c)*arcsinh(a*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a^2cx^2+c)\,\mathrm{arsinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arcsinh(a*x),x, algorithm="giac")
[Out] integrate(1/((a^2*c*x^2 + c)*arcsinh(a*x)), x)
```

maple [A] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{2} c\, x^{2}+c\right) \operatorname{arcsinh}\left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)/arcsinh(a*x),x)
[Out] int(1/(a^2*c*x^2+c)/arcsinh(a*x),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{2} c x^{2}+c\right) \operatorname{arsinh}\left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arcsinh(a*x),x, algorithm="maxima")
[Out] integrate(1/((a^2*c*x^2 + c)*arcsinh(a*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\operatorname{asinh}\left(a\, x\right)\, \left(c\, a^2\, x^2+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(asinh(a*x)*(c + a^2*c*x^2)),x)
[Out] int(1/(asinh(a*x)*(c + a^2*c*x^2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^{2} x^{2} \operatorname{asinh}{\left(a x \right)} + \operatorname{asinh}{\left(a x \right)}}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)/asinh(a*x),x)
[Out] Integral(1/(a**2*x**2*asinh(a*x) + asinh(a*x)), x)/c
```

3.354 $\int \frac{1}{\left(c+a^2cx^2\right)^2 \sinh^{-1}(ax)} \, dx$

Optimal. Leaf size=22

$$\text{Int}\left(\frac{1}{\left(a^2cx^2+c\right)^2 \sinh^{-1}(ax)},x\right)$$

[Out] Unintegrable(1/(a^2*c*x^2+c)^2/arcsinh(a*x),x)

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(c+a^2cx^2\right)^2 \sinh^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

[In] Int[1/((c + a^2*c*x^2)^2*ArcSinh[a*x]),x]

[Out] Defer[Int][1/((c + a^2*c*x^2)^2*ArcSinh[a*x]), x]

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)^2 \sinh^{-1}(ax)} \, dx = \int \frac{1}{\left(c+a^2cx^2\right)^2 \sinh^{-1}(ax)} \, dx$$

Mathematica [A] time = 1.61, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c+a^2cx^2\right)^2 \sinh^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((c + a^2*c*x^2)^2*ArcSinh[a*x]),x]

[Out] Integrate[1/((c + a^2*c*x^2)^2*ArcSinh[a*x]), x]

fricas [A] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\left(a^4c^2x^4+2\,a^2c^2x^2+c^2\right)\text{arsinh}\,(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2*c*x^2+c)^2/arcsinh(a*x),x, algorithm="fricas")

[Out] integral(1/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arcsinh(a*x)), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2cx^2+c\right)^2 \text{arsinh}\,(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2*c*x^2+c)^2/arcsinh(a*x),x, algorithm="giac")

```
[Out] integrate(1/((a^2*c*x^2 + c)^2*arcsinh(a*x)), x)
```

maple [A] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{2} c\,x^{2}+c\right)^{2} \operatorname{arcsinh}\left(a x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^2/arcsinh(a*x),x)
[Out] int(1/(a^2*c*x^2+c)^2/arcsinh(a*x),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{2} c x^{2}+c\right)^{2} \operatorname{arsinh}\left(a x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arcsinh(a*x),x, algorithm="maxima")
[Out] integrate(1/((a^2*c*x^2 + c)^2*arcsinh(a*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\operatorname{asinh}\left(a\,x\right)\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(asinh(a*x)*(c + a^2*c*x^2)^2),x)
[Out] int(1/(asinh(a*x)*(c + a^2*c*x^2)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^{4} x^{4} \operatorname{asinh}{\left(a x \right)} + 2 a^{2} x^{2} \operatorname{asinh}{\left(a x \right)} + \operatorname{asinh}{\left(a x \right)}}\, dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**2/asinh(a*x),x)
[Out] Integral(1/(a**4*x**4*asinh(a*x) + 2*a**2*x**2*asinh(a*x) + asinh(a*x)), x)
/c**2
```

3.355 $\int \frac{x^4\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=206

$$-\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{32bc^5}-\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16bc^5}+\frac{\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{32bc^5}+\frac{\sinh\left(\frac{2a}{b}\right)\text{Sh}}{}$$

```
[Out] -1/32*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b/c^5-1/16*Chi(4*(a+b*arcsinh
(c*x))/b)*cosh(4*a/b)/b/c^5+1/32*Chi(6*(a+b*arcsinh(c*x))/b)*cosh(6*a/b)/b/
c^5+1/16*ln(a+b*arcsinh(c*x))/b/c^5+1/32*Shi(2*(a+b*arcsinh(c*x))/b)*sinh(2
*a/b)/b/c^5+1/16*Shi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b/c^5-1/32*Shi(6*(
a+b*arcsinh(c*x))/b)*sinh(6*a/b)/b/c^5
```

Rubi [A] time = 0.51, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {5779, 5448, 3303, 3298, 3301}

$$-\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{32bc^5}-\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{16bc^5}+\frac{\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6a}{b}+6\sinh^{-1}(cx)\right)}{32bc^5}+\frac{\text{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(32*b*c^5) - (Cosh[
(4*a)/b]*CoshIntegral[(4*a)/b + 4*ArcSinh[c*x]])/(16*b*c^5) + (Cosh[(6*a)/b
]*CoshIntegral[(6*a)/b + 6*ArcSinh[c*x]])/(32*b*c^5) + Log[a + b*ArcSinh[c*
x]]/(16*b*c^5) + (Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(32
*b*c^5) + (Sinh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcSinh[c*x]])/(16*b*c^5)
 - (Sinh[(6*a)/b]*SinhIntegral[(6*a)/b + 6*ArcSinh[c*x]])/(32*b*c^5)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^4\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^2(x)\sinh^4(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^5}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{16(a+bx)} - \frac{\cosh(2x)}{32(a+bx)} - \frac{\cosh(4x)}{16(a+bx)} + \frac{\cosh(6x)}{32(a+bx)}\right) dx, x, \sinh^{-1}(cx)\right)}{c^5}$$

$$= \frac{\log\left(a+b\sinh^{-1}(cx)\right)}{16bc^5} - \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{32c^5} + \frac{\text{Subst}\left(\int \frac{\cosh(6x)}{a+bx}\,dx,\right.}{32c^5}$$

$$= \frac{\log\left(a+b\sinh^{-1}(cx)\right)}{16bc^5} - \frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{32c^5} - \frac{\cosh\left(\right.}{}$$

$$= -\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{32bc^5} - \frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{16bc^5} + \frac{\cosh\left(\frac{6}{}\right.}{}$$

Mathematica [A] time = 0.36, size = 152, normalized size = 0.74

$$\frac{-\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) - 2\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) + \cosh\left(\frac{6a}{b}\right)\text{Chi}\left(6\left(\frac{a}{b}+\sinh^{-1}(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-(Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x])]) - 2*Cosh[(4*a)/b]*Co
shIntegral[4*(a/b + ArcSinh[c*x])] + Cosh[(6*a)/b]*CoshIntegral[6*(a/b + Ar
cSinh[c*x])] + 2*Log[a + b*ArcSinh[c*x]] + Sinh[(2*a)/b]*SinhIntegral[2*(a/
b + ArcSinh[c*x])] + 2*Sinh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])] -
 Sinh[(6*a)/b]*SinhIntegral[6*(a/b + ArcSinh[c*x])])/(32*b*c^5)
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}\,x^4}{b\,\text{arsinh}\,(cx)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^4/(b*arcsinh(c*x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}\,x^4}{b\,\text{arsinh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate(sqrt(c^2*x^2 + 1)*x^4/(b*arcsinh(c*x) + a), x)
```

maple [A] time = 0.51, size = 199, normalized size = 0.97

$$\frac{\ln\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)}{16b\,c^{5}}-\frac{e^{\frac{6a}{b}}\,\mathrm{Ei}\left(1,6\,\mathrm{arcsinh}\left(cx\right)+\frac{6a}{b}\right)}{64c^{5}b}+\frac{e^{\frac{4a}{b}}\,\mathrm{Ei}\left(1,4\,\mathrm{arcsinh}\left(cx\right)+\frac{4a}{b}\right)}{32c^{5}b}+\frac{e^{\frac{2a}{b}}\,\mathrm{Ei}\left(1,2\,\mathrm{arcsinh}\left(cx\right)+\dots\right.}{64c^{5}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x)
[Out] 1/16*ln(a+b*arcsinh(c*x))/b/c^5-1/64/c^5/b*exp(6*a/b)*Ei(1,6*arcsinh(c*x)+6
*a/b)+1/32/c^5/b*exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)+1/64/c^5/b*exp(2*a/b
)*Ei(1,2*arcsinh(c*x)+2*a/b)+1/64/c^5/b*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*
a/b)+1/32/c^5/b*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)-1/64/c^5/b*exp(-6*a
/b)*Ei(1,-6*arcsinh(c*x)-6*a/b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^{2}x^{2}+1}\,x^{4}}{b\,\mathrm{arsinh}\left(cx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(sqrt(c^2*x^2 + 1)*x^4/(b*arcsinh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{4}\,\sqrt{c^{2}\,x^{2}+1}}{a+b\,\mathrm{asinh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x)),x)
[Out] int((x^4*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}\sqrt{c^{2}x^{2}+1}}{a+b\,\mathrm{asinh}\left(cx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x)),x)
[Out] Integral(x**4*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x)), x)
```

3.356 $\int \frac{x^3\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=183

$$\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8bc^4}+\frac{\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16bc^4}-\frac{\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16bc^4}-\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{}\right.}{8bc^4}$$

```
[Out] -1/8*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b/c^4-1/16*cosh(3*a/b)*Shi(3*(a+b*
arcsinh(c*x))/b)/b/c^4+1/16*cosh(5*a/b)*Shi(5*(a+b*arcsinh(c*x))/b)/b/c^4+1
/8*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b/c^4+1/16*Chi(3*(a+b*arcsinh(c*x))/
b)*sinh(3*a/b)/b/c^4-1/16*Chi(5*(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b/c^4
```

Rubi [A] time = 0.53, antiderivative size = 179, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5779, 5448, 3303, 3298, 3301}

$$\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{8bc^4}+\frac{\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{16bc^4}-\frac{\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5a}{b}+5\sinh^{-1}(cx)\right)}{16bc^4}-\cosh$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b])/(8*b*c^4) + (CoshIntegral[(3*a
)/b + 3*ArcSinh[c*x]]*Sinh[(3*a)/b])/(16*b*c^4) - (CoshIntegral[(5*a)/b + 5
*ArcSinh[c*x]]*Sinh[(5*a)/b])/(16*b*c^4) - (Cosh[a/b]*SinhIntegral[a/b + Ar
cSinh[c*x]])/(8*b*c^4) - (Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*
x]])/(16*b*c^4) + (Cosh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcSinh[c*x]])/(1
6*b*c^4)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^3\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^2(x)\sinh^3(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^4}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{\sinh(x)}{8(a+bx)} - \frac{\sinh(3x)}{16(a+bx)} + \frac{\sinh(5x)}{16(a+bx)}\right) dx, x, \sinh^{-1}(cx)\right)}{c^4}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sinh(3x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{16c^4} + \frac{\text{Subst}\left(\int \frac{\sinh(5x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{16c^4} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{8c^4} - \frac{\cosh\left(\frac{3a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}+3x\right)}{a+bx}\,dx, x,\right.}{16c^4}$$

$$= \frac{\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{8bc^4} + \frac{\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)\sinh\left(\frac{3a}{b}\right)}{16bc^4} - \frac{\text{Chi}\left(\frac{5a}{b}+5\sinh\right.}{16}$$

Mathematica [A] time = 0.29, size = 135, normalized size = 0.74

$$\frac{2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-2\,\text{c}}{16bc^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (2*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] + CoshIntegral[3*(a/b + ArcSi
nh[c*x])]*Sinh[(3*a)/b] - CoshIntegral[5*(a/b + ArcSinh[c*x])]*Sinh[(5*a)/b
] - 2*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - Cosh[(3*a)/b]*SinhIntegr
al[3*(a/b + ArcSinh[c*x])] + Cosh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[c*
x])])/(16*b*c^4)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}\,x^3}{b\,\text{arsinh}\,(cx)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^3/(b*arcsinh(c*x) + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.35, size = 178, normalized size = 0.97

$$\frac{e^{\frac{5a}{b}}\,\mathrm{Ei}\left(1,5\,\mathrm{arcsinh}\left(cx\right)+\frac{5a}{b}\right)}{32c^4b}-\frac{e^{\frac{3a}{b}}\,\mathrm{Ei}\left(1,3\,\mathrm{arcsinh}\left(cx\right)+\frac{3a}{b}\right)}{32c^4b}-\frac{e^{\frac{a}{b}}\,\mathrm{Ei}\left(1,\mathrm{arcsinh}\left(cx\right)+\frac{a}{b}\right)}{16c^4b}+\frac{e^{-\frac{a}{b}}\,\mathrm{Ei}\left(1,-\mathrm{arcsir}\right.}{16c^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/32/c^4/b*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)-1/32/c^4/b*exp(3*a/b)*Ei(1
,3*arcsinh(c*x)+3*a/b)-1/16/c^4/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)+1/16/c^4/
b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)+1/32/c^4/b*exp(-3*a/b)*Ei(1,-3*arcsinh(
c*x)-3*a/b)-1/32/c^4/b*exp(-5*a/b)*Ei(1,-5*arcsinh(c*x)-5*a/b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}\,x^3}{b\,\mathrm{arsinh}\left(cx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)*x^3/(b*arcsinh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\sqrt{c^2\,x^2+1}}{a+b\,\mathrm{asinh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x)),x)
```

```
[Out] int((x^3*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{c^2x^2+1}}{a+b\,\mathrm{asinh}\left(cx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x**3*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x)), x)
```

3.357 $\int \frac{x^2\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=82

$$\frac{\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{8bc^3}-\frac{\sinh\left(\frac{4a}{b}\right)\mathrm{Shi}\left(\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{8bc^3}-\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{8bc^3}$$

```
[Out] 1/8*Chi(4*(a+b*arcsinh(c*x))/b)*cosh(4*a/b)/b/c^3-1/8*ln(a+b*arcsinh(c*x))/
b/c^3-1/8*Shi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b/c^3
```

Rubi [A] time = 0.27, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5779, 5448, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{8bc^3}-\frac{\sinh\left(\frac{4a}{b}\right)\mathrm{Shi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{8bc^3}-\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{8bc^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Cosh[(4*a)/b]*CoshIntegral[(4*a)/b + 4*ArcSinh[c*x]])/(8*b*c^3) - Log[a +
b*ArcSinh[c*x]]/(8*b*c^3) - (Sinh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcSinh
[c*x]])/(8*b*c^3)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
```

```
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^2(x)\sinh^2(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^3}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{8(a+bx)}+\frac{\cosh(4x)}{8(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{c^3}$$
$$= -\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{8bc^3}+\frac{\text{Subst}\left(\int \frac{\cosh(4x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{8c^3}$$
$$= -\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{8bc^3}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{4a}{b}+4x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{8c^3}-\frac{\sinh}{}$$
$$= \frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{8bc^3}-\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{8bc^3}-\frac{\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4a}{b}+4\right.}{8bc^3}$$

Mathematica [A] time = 0.18, size = 65, normalized size = 0.79

$$\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-\log\left(a+b\sinh^{-1}(cx)\right)}{8bc^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Cosh[(4*a)/b]*CoshIntegral[4*(a/b + ArcSinh[c*x])] - Log[a + b*ArcSinh[c*x
]] - Sinh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])])/(8*b*c^3)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}\,x^2}{b\,\text{arsinh}\,(cx)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^2/(b*arcsinh(c*x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}\,x^2}{b\,\text{arsinh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)*x^2/(b*arcsinh(c*x) + a), x)
```

maple [A] time = 0.21, size = 79, normalized size = 0.96

$$-\frac{\ln\left(a+b\,\text{arcsinh}\,(cx)\right)}{8b\,c^3}-\frac{\mathrm{e}^{\frac{4a}{b}}\,\text{Ei}\left(1, 4\,\text{arcsinh}\,(cx)+\frac{4a}{b}\right)}{16c^3b}-\frac{\mathrm{e}^{-\frac{4a}{b}}\,\text{Ei}\left(1, -4\,\text{arcsinh}\,(cx)-\frac{4a}{b}\right)}{16c^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] -1/8*ln(a+b*arcsinh(c*x))/b/c^3-1/16/c^3/b*exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4
*a/b)-1/16/c^3/b*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}\,x^2}{b\operatorname{arsinh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)*x^2/(b*arcsinh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{c^2\,x^2+1}}{a+b\,\operatorname{asinh}(c\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x)),x)
```

```
[Out] int((x^2*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{c^2x^2+1}}{a+b\operatorname{asinh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x**2*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x)), x)
```

3.358 $\int \frac{x\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=121

$$-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4bc^2}-\frac{\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{4bc^2}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4bc^2}+\frac{\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(\cdots}{\cdots}\right)}{4bc^2}$$

```
[Out] 1/4*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b/c^2+1/4*cosh(3*a/b)*Shi(3*(a+b*ar
csinh(c*x))/b)/b/c^2-1/4*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b/c^2-1/4*Chi(
3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b/c^2
```

Rubi [A] time = 0.31, antiderivative size = 117, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5779, 5448, 3303, 3298, 3301}

$$-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4bc^2}-\frac{\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{4bc^2}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4bc^2}+\frac{\cosh\left(\frac{3a}{b}\cdots\right)}{4bc^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b])/(4*b*c^2) - (CoshIntegral[(3*
a)/b + 3*ArcSinh[c*x]]*Sinh[(3*a)/b])/(4*b*c^2) + (Cosh[a/b]*SinhIntegral[a
/b + ArcSinh[c*x]])/(4*b*c^2) + (Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*Arc
Sinh[c*x]])/(4*b*c^2)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
```

```
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^2}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{\sinh(x)}{4(a+bx)}+\frac{\sinh(3x)}{4(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{c^2}$$
$$= \frac{\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{4c^2}+\frac{\text{Subst}\left(\int \frac{\sinh(3x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{4c^2}$$
$$= \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{4c^2}+\frac{\cosh\left(\frac{3a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}+3x\right)}{a+bx}\,dx, x, \text{si}\right.}{4c^2}$$
$$= -\frac{\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{4bc^2}-\frac{\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)\sinh\left(\frac{3a}{b}\right)}{4bc^2}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}\right.}{4b}$$

Mathematica [A] time = 0.20, size = 91, normalized size = 0.75

$$\frac{\sinh\left(\frac{a}{b}\right)\left(-\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+\cosh\left(\right.}{4bc^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-(CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b]) - CoshIntegral[3*(a/b + ArcS
inh[c*x])]*Sinh[(3*a)/b] + Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + Cos
h[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])])/(4*b*c^2)
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}\,x}{b\,\text{arsinh}\,(cx)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x/(b*arcsinh(c*x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}\,x}{b\,\text{arsinh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)*x/(b*arcsinh(c*x) + a), x)
```

maple [A] time = 0.20, size = 118, normalized size = 0.98

$$\frac{e^{\frac{3a}{b}} \operatorname{Ei}\left(1, 3\,\operatorname{arcsinh}(cx) + \frac{3a}{b}\right)}{8c^2b} + \frac{e^{\frac{a}{b}} \operatorname{Ei}\left(1, \operatorname{arcsinh}(cx) + \frac{a}{b}\right)}{8c^2b} - \frac{e^{-\frac{a}{b}} \operatorname{Ei}\left(1, -\operatorname{arcsinh}(cx) - \frac{a}{b}\right)}{8c^2b} - \frac{e^{-\frac{3a}{b}} \operatorname{Ei}\left(1, -3\,\operatorname{arcs}\right.}{8c^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/8/c^2/b*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)+1/8/c^2/b*exp(a/b)*Ei(1,arc
sinh(c*x)+a/b)-1/8/c^2/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)-1/8/c^2/b*exp(-3
*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}\,x}{b\,\operatorname{arsinh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)*x/(b*arcsinh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{c^2\,x^2+1}}{a+b\,\operatorname{asinh}(c\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x)),x)
```

```
[Out] int((x*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{c^2x^2+1}}{a+b\,\operatorname{asinh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x)), x)
```

3.359 $\int \frac{\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=82

$$\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{2bc}-\frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{2bc}+\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{2bc}$$

```
[Out] 1/2*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b/c+1/2*ln(a+b*arcsinh(c*x))/b/
c-1/2*Shi(2*(a+b*arcsinh(c*x))/b)*sinh(2*a/b)/b/c
```

Rubi [A] time = 0.18, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5699, 3312, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc}-\frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc}+\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{2bc}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + c^2*x^2]/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(2*b*c) + Log[a + b*
ArcSinh[c*x]]/(2*b*c) - (Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x
]])/(2*b*c)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^2(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{2(a+bx)}+\frac{\cosh(2x)}{2(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{c}$$
$$= \frac{\log\left(a+b\sinh^{-1}(cx)\right)}{2bc} + \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{2c}$$
$$= \frac{\log\left(a+b\sinh^{-1}(cx)\right)}{2bc} + \frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{2c} - \frac{\sinh\left(\right.}{}$$
$$= \frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc} + \frac{\log\left(a+b\sinh^{-1}(cx)\right)}{2bc} - \frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\right.}{2bc}$$

Mathematica [A] time = 0.13, size = 63, normalized size = 0.77

$$\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+\log\left(a+b\sinh^{-1}(cx)\right)}{2bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + c^2*x^2]/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x])] + Log[a + b*ArcSinh[c*x
]] - Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])])/(2*b*c)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}}{b\,\text{arsinh}\,(cx)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{b\,\text{arsinh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a), x)
```

maple [A] time = 0.15, size = 79, normalized size = 0.96

$$\frac{\ln\left(a+b\,\text{arcsinh}\,(cx)\right)}{2bc} - \frac{e^{\frac{2a}{b}}\,\text{Ei}\left(1, 2\,\text{arcsinh}\,(cx)+\frac{2a}{b}\right)}{4cb} - \frac{e^{-\frac{2a}{b}}\,\text{Ei}\left(1, -2\,\text{arcsinh}\,(cx)-\frac{2a}{b}\right)}{4cb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/2*ln(a+b*arcsinh(c*x))/b/c-1/4/c/b*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)-
1/4/c/b*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{b\operatorname{arsinh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c^2\,x^2+1}}{a+b\,\operatorname{asinh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(1/2)/(a + b*asinh(c*x)),x)
```

```
[Out] int((c^2*x^2 + 1)^(1/2)/(a + b*asinh(c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{a+b\operatorname{asinh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(1/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x)), x)
```

3.360 $\int \frac{\sqrt{1+c^2x^2}}{x\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=78

$$\text{Int}\left(\frac{1}{x\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)},x\right)-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b}$$

```
[Out] cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b-Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b
+Unintegrable(1/x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

Rubi [A] time = 0.43, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1+c^2x^2}}{x\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[1 + c^2*x^2]/(x*(a + b*ArcSinh[c*x])),x]
```

```
[Out] -((CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b])/b) + (Cosh[a/b]*SinhIntegral
[a/b + ArcSinh[c*x]])/b + Defer[Int][1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[
c*x])), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+c^2x^2}}{x\left(a+b\sinh^{-1}(cx)\right)}\,dx &= \int \left(\frac{1}{x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}+\frac{c^2x}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\right)dx\\
&= c^2\int \frac{x}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx+\int \frac{1}{x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx\\
&= \int \frac{1}{x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx+\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)\\
&= \cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)-\sinh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\right.}{a+}\right.\\
&= -\frac{\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{b}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{b}+\int \frac{}{x\sqrt{1+c^2}}
\end{aligned}$$

Mathematica [A] time = 1.96, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+c^2x^2}}{x\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[1 + c^2*x^2]/(x*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Integrate[Sqrt[1 + c^2*x^2]/(x*(a + b*ArcSinh[c*x])), x]
```

fricas [A] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}}{bx\,\text{arsinh}(cx)+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(b*x*arcsinh(c*x) + a*x), x)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [**A**] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{x\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x)),x)
```

```
[Out] int((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x)),x)
```

maxima [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{(b\,\mathrm{arsinh}\left(cx\right)+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)*x), x)
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c^2\,x^2+1}}{x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(1/2)/(x*(a + b*asinh(c*x))),x)
```

```
[Out] int((c^2*x^2 + 1)^(1/2)/(x*(a + b*asinh(c*x))), x)
```

sympy [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{x\left(a+b\,\mathrm{asinh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(1/2)/x/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(sqrt(c**2*x**2 + 1)/(x*(a + b*asinh(c*x))), x)
```

3.361 $\int \frac{\sqrt{1+c^2x^2}}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=45

$$\text{Int}\left(\frac{1}{x^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)},x\right)+\frac{c\log\left(a+b\sinh^{-1}(cx)\right)}{b}$$

```
[Out] c*ln(a+b*arcsinh(c*x))/b+Unintegrable(1/x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^
(1/2),x)
```

Rubi [A] time = 0.30, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1+c^2x^2}}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[1 + c^2*x^2]/(x^2*(a + b*ArcSinh[c*x])),x]
```

```
[Out] (c*Log[a + b*ArcSinh[c*x]])/b + Defer[Int][1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*
ArcSinh[c*x])), x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+c^2x^2}}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx &= \int \left(\frac{c^2}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}+\frac{1}{x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\right)dx\\ &= c^2\int \frac{1}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx+\int \frac{1}{x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx\\ &= \frac{c\log\left(a+b\sinh^{-1}(cx)\right)}{b}+\int \frac{1}{x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx\end{aligned}$$

Mathematica [A] time = 1.15, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+c^2x^2}}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[1 + c^2*x^2]/(x^2*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Integrate[Sqrt[1 + c^2*x^2]/(x^2*(a + b*ArcSinh[c*x])), x]
```

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}}{bx^2\,\text{arsinh}\,(cx)+ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(b*x^2*arcsinh(c*x) + a*x^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{(b\,\mathrm{arsinh}\,(cx)+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)*x^2), x)
```

maple [A] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{x^2\,(a+b\,\mathrm{arcsinh}\,(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x)),x)
[Out] int((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{(b\,\mathrm{arsinh}\,(cx)+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)*x^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c^2\,x^2+1}}{x^2\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(1/2)/(x^2*(a + b*asinh(c*x))),x)
[Out] int((c^2*x^2 + 1)^(1/2)/(x^2*(a + b*asinh(c*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{x^2\,(a+b\,\mathrm{asinh}\,(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(1/2)/x**2/(a+b*asinh(c*x)),x)
[Out] Integral(sqrt(c**2*x**2 + 1)/(x**2*(a + b*asinh(c*x))), x)
```

3.362 $\int \frac{\sqrt{1+c^2x^2}}{x^3\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{\sqrt{c^2x^2+1}}{x^3\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

[Out] Unintegrable((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x)),x)

Rubi [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1+c^2x^2}}{x^3\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

[In] Int[Sqrt[1 + c^2*x^2]/(x^3*(a + b*ArcSinh[c*x])),x]

[Out] Defer[Int][Sqrt[1 + c^2*x^2]/(x^3*(a + b*ArcSinh[c*x])), x]

Rubi steps

$$\int \frac{\sqrt{1+c^2x^2}}{x^3\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{\sqrt{1+c^2x^2}}{x^3\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 4.73, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+c^2x^2}}{x^3\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[Sqrt[1 + c^2*x^2]/(x^3*(a + b*ArcSinh[c*x])),x]

[Out] Integrate[Sqrt[1 + c^2*x^2]/(x^3*(a + b*ArcSinh[c*x])), x]

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}}{bx^3\,\text{arsinh}\,(cx)+ax^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="fricas")

[Out] integral(sqrt(c^2*x^2 + 1)/(b*x^3*arcsinh(c*x) + a*x^3), x)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="giac")

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{x^3\,(a+b\operatorname{arcsinh}(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x)),x)
[Out] int((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{(b\operatorname{arsinh}(cx)+a)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)*x^3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{c^2\,x^2+1}}{x^3\,\left(a+b\operatorname{asinh}\left(c\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(1/2)/(x^3*(a + b*asinh(c*x))),x)
[Out] int((c^2*x^2 + 1)^(1/2)/(x^3*(a + b*asinh(c*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{x^3\,(a+b\operatorname{asinh}(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(1/2)/x**3/(a+b*asinh(c*x)),x)
[Out] Integral(sqrt(c**2*x**2 + 1)/(x**3*(a + b*asinh(c*x))), x)
```

3.363 $\int \frac{\sqrt{1+c^2x^2}}{x^4\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=30

$$\operatorname{Int}\left(\frac{\sqrt{c^2x^2+1}}{x^4\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x)),x)
```

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1+c^2x^2}}{x^4\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[1 + c^2*x^2]/(x^4*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][Sqrt[1 + c^2*x^2]/(x^4*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{\sqrt{1+c^2x^2}}{x^4\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{\sqrt{1+c^2x^2}}{x^4\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 0.83, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+c^2x^2}}{x^4\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[1 + c^2*x^2]/(x^4*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[Sqrt[1 + c^2*x^2]/(x^4*(a + b*ArcSinh[c*x])), x]
```

fricas [A] time = 0.52, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{c^2x^2+1}}{bx^4\operatorname{arsinh}(cx)+ax^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)/(b*x^4*arcsinh(c*x) + a*x^4), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{(b\operatorname{arsinh}(cx)+a)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)*x^4), x)
```

maple [A] time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{x^4\,(a+b\,\mathrm{arcsinh}\,(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x)),x)
[Out] int((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{(b\,\mathrm{arsinh}\,(cx)+a)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)*x^4), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{c^2\,x^2+1}}{x^4\,\left(a+b\,\mathrm{asinh}\,(c\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(1/2)/(x^4*(a + b*asinh(c*x))),x)
[Out] int((c^2*x^2 + 1)^(1/2)/(x^4*(a + b*asinh(c*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{x^4\,(a+b\,\mathrm{asinh}\,(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(1/2)/x**4/(a+b*asinh(c*x)),x)
[Out] Integral(sqrt(c**2*x**2 + 1)/(x**4*(a + b*asinh(c*x))), x)
```

3.364 $\int \frac{x^3(1+c^2x^2)^{3/2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=245

$$\frac{3\sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{64bc^4}+\frac{3\sinh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{64bc^4}-\frac{\sinh\left(\frac{5a}{b}\right)\mathrm{Chi}\left(\frac{5\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{64bc^4}-\frac{\sinh\left(\frac{7a}{b}\right)\mathrm{C}}{}$$

```
[Out] -3/64*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b/c^4-3/64*cosh(3*a/b)*Shi(3*(a+b
*arcsinh(c*x))/b)/b/c^4+1/64*cosh(5*a/b)*Shi(5*(a+b*arcsinh(c*x))/b)/b/c^4+
1/64*cosh(7*a/b)*Shi(7*(a+b*arcsinh(c*x))/b)/b/c^4+3/64*Chi((a+b*arcsinh(c*
x))/b)*sinh(a/b)/b/c^4+3/64*Chi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b/c^4-1
/64*Chi(5*(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b/c^4-1/64*Chi(7*(a+b*arcsinh(c
*x))/b)*sinh(7*a/b)/b/c^4
```

Rubi [A] time = 0.62, antiderivative size = 241, normalized size of antiderivative = 0.98, number of steps used = 15, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5779, 5448, 3303, 3298, 3301}

$$\frac{3\sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{64bc^4}+\frac{3\sinh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{64bc^4}-\frac{\sinh\left(\frac{5a}{b}\right)\mathrm{Chi}\left(\frac{5a}{b}+5\sinh^{-1}(cx)\right)}{64bc^4}-\frac{\mathrm{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (3*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b])/(64*b*c^4) + (3*CoshIntegral
[(3*a)/b + 3*ArcSinh[c*x]]*Sinh[(3*a)/b])/(64*b*c^4) - (CoshIntegral[(5*a)/
b + 5*ArcSinh[c*x]]*Sinh[(5*a)/b])/(64*b*c^4) - (CoshIntegral[(7*a)/b + 7*A
rcSinh[c*x]]*Sinh[(7*a)/b])/(64*b*c^4) - (3*Cosh[a/b]*SinhIntegral[a/b + Ar
cSinh[c*x]])/(64*b*c^4) - (3*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh
[c*x]])/(64*b*c^4) + (Cosh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcSinh[c*x]])
/(64*b*c^4) + (Cosh[(7*a)/b]*SinhIntegral[(7*a)/b + 7*ArcSinh[c*x]])/(64*b*
c^4)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
```

```
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^3\left(1+c^2x^2\right)^{3/2}}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^4(x)\sinh^3(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^4}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{3\sinh(x)}{64(a+bx)}-\frac{3\sinh(3x)}{64(a+bx)}+\frac{\sinh(5x)}{64(a+bx)}+\frac{\sinh(7x)}{64(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{c^4}$$

$$= \frac{\text{Subst}\left(\int \frac{\sinh(5x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{64c^4}+\frac{\text{Subst}\left(\int \frac{\sinh(7x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{64c^4}-\frac{3\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\left(3\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{64c^4}-\frac{\left(3\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}+3x\right.}{a+bx}\right.}{64c^4}$$

$$= \frac{3\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{64bc^4}+\frac{3\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)\sinh\left(\frac{3a}{b}\right)}{64bc^4}-\frac{\text{Chi}\left(\frac{5a}{b}+5\,\text{si}\right.}{}$$

Mathematica [A] time = 0.66, size = 179, normalized size = 0.73

$$\frac{3\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+3\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-\text{si}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (3*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] + 3*CoshIntegral[3*(a/b + Arc
Sinh[c*x])]*Sinh[(3*a)/b] - CoshIntegral[5*(a/b + ArcSinh[c*x])]*Sinh[(5*a)
/b] - CoshIntegral[7*(a/b + ArcSinh[c*x])]*Sinh[(7*a)/b] - 3*Cosh[a/b]*Sinh
Integral[a/b + ArcSinh[c*x]] - 3*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSin
h[c*x])] + Cosh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[c*x])] + Cosh[(7*a)/
b]*SinhIntegral[7*(a/b + ArcSinh[c*x])])/(64*b*c^4)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^2x^5+x^3\right)\sqrt{c^2x^2+1}}{b\,\text{arsinh}\,(cx)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.45, size = 238, normalized size = 0.97

$$\frac{e^{\frac{7a}{b}}\operatorname{Ei}\left(1,7\operatorname{arcsinh}(cx)+\frac{7a}{b}\right)}{128c^4b}+\frac{e^{\frac{5a}{b}}\operatorname{Ei}\left(1,5\operatorname{arcsinh}(cx)+\frac{5a}{b}\right)}{128c^4b}-\frac{3\,e^{\frac{3a}{b}}\operatorname{Ei}\left(1,3\operatorname{arcsinh}(cx)+\frac{3a}{b}\right)}{128c^4b}-\frac{3\,e^{\frac{a}{b}}\operatorname{Ei}\left(1,\operatorname{ar}\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/128/c^4/b*exp(7*a/b)*Ei(1,7*arcsinh(c*x)+7*a/b)+1/128/c^4/b*exp(5*a/b)*Ei
(1,5*arcsinh(c*x)+5*a/b)-3/128/c^4/b*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-
3/128/c^4/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)+3/128/c^4/b*exp(-a/b)*Ei(1,-arc
sinh(c*x)-a/b)+3/128/c^4/b*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)-1/128/c^
4/b*exp(-5*a/b)*Ei(1,-5*arcsinh(c*x)-5*a/b)-1/128/c^4/b*exp(-7*a/b)*Ei(1,-7
*arcsinh(c*x)-7*a/b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}x^3}{b\operatorname{arsinh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + 1)^(3/2)*x^3/(b*arcsinh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\left(c^2\,x^2+1\right)^{3/2}}{a+b\operatorname{asinh}(c\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x)),x)
```

```
[Out] int((x^3*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(c^2x^2+1\right)^{\frac{3}{2}}}{a+b\operatorname{asinh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x**3*(c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x)), x)
```

3.365 $\int \frac{x^2(1+c^2x^2)^{3/2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=206

$$-\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{32bc^3}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\sinh^{-1}(cx))}{b}\right)}{16bc^3}+\frac{\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\sinh^{-1}(cx))}{b}\right)}{32bc^3}+\frac{\sinh\left(\frac{2a}{b}\right)\text{Sh}}{}$$

```
[Out] -1/32*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b/c^3+1/16*Chi(4*(a+b*arcsinh
(c*x))/b)*cosh(4*a/b)/b/c^3+1/32*Chi(6*(a+b*arcsinh(c*x))/b)*cosh(6*a/b)/b/
c^3-1/16*ln(a+b*arcsinh(c*x))/b/c^3+1/32*Shi(2*(a+b*arcsinh(c*x))/b)*sinh(2
*a/b)/b/c^3-1/16*Shi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b/c^3-1/32*Shi(6*(
a+b*arcsinh(c*x))/b)*sinh(6*a/b)/b/c^3
```

Rubi [A] time = 0.49, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5779, 5448, 3303, 3298, 3301}

$$-\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{32bc^3}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{16bc^3}+\frac{\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6a}{b}+6\sinh^{-1}(cx)\right)}{32bc^3}+\text{si}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(32*b*c^3) + (Cosh[
(4*a)/b]*CoshIntegral[(4*a)/b + 4*ArcSinh[c*x]])/(16*b*c^3) + (Cosh[(6*a)/b
]*CoshIntegral[(6*a)/b + 6*ArcSinh[c*x]])/(32*b*c^3) - Log[a + b*ArcSinh[c*
x]]/(16*b*c^3) + (Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(32
*b*c^3) - (Sinh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcSinh[c*x]])/(16*b*c^3)
 - (Sinh[(6*a)/b]*SinhIntegral[(6*a)/b + 6*ArcSinh[c*x]])/(32*b*c^3)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2\left(1+c^2x^2\right)^{3/2}}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^4(x)\sinh^2(x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{c^3}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{16(a+bx)}-\frac{\cosh(2x)}{32(a+bx)}+\frac{\cosh(4x)}{16(a+bx)}+\frac{\cosh(6x)}{32(a+bx)}\right)dx,x,\sinh^{-1}(cx)\right)}{c^3}$$

$$= -\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{16bc^3}-\frac{\text{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{32c^3}+\frac{\text{Subst}\left(\int \frac{\cosh(6x)}{a+bx}\,dx\right.}{32c^3}$$

$$= -\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{16bc^3}-\frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{32c^3}+\frac{\cosh}{}$$

$$= -\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{32bc^3}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{16bc^3}+\frac{\cosh\left(\right.}{}$$

Mathematica [A] time = 0.48, size = 152, normalized size = 0.74

$$\frac{-\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+2\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(6\left(\frac{a}{b}+\sinh^{-1}(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-(Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x])]) + 2*Cosh[(4*a)/b]*Co
shIntegral[4*(a/b + ArcSinh[c*x])] + Cosh[(6*a)/b]*CoshIntegral[6*(a/b + Ar
cSinh[c*x])] - 2*Log[a + b*ArcSinh[c*x]] + Sinh[(2*a)/b]*SinhIntegral[2*(a/
b + ArcSinh[c*x])] - 2*Sinh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])] -
 Sinh[(6*a)/b]*SinhIntegral[6*(a/b + ArcSinh[c*x])])/(32*b*c^3)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^2x^4+x^2\right)\sqrt{c^2x^2+1}}{b\,\text{arsinh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + x^2)*sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}x^2}{b\,\text{arsinh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(3/2)*x^2/(b*arcsinh(c*x) + a), x)
```

maple [A] time = 0.42, size = 199, normalized size = 0.97

$$-\frac{\ln\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)}{16b\,c^{3}}-\frac{\mathrm{e}^{\frac{6a}{b}}\,\mathrm{Ei}\left(1,6\,\mathrm{arcsinh}\left(cx\right)+\frac{6a}{b}\right)}{64c^{3}b}-\frac{\mathrm{e}^{\frac{4a}{b}}\,\mathrm{Ei}\left(1,4\,\mathrm{arcsinh}\left(cx\right)+\frac{4a}{b}\right)}{32c^{3}b}+\frac{\mathrm{e}^{\frac{2a}{b}}\,\mathrm{Ei}\left(1,2\,\mathrm{arcsinh}\left(cx\right)\right.}{64c^{3}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] -1/16*ln(a+b*arcsinh(c*x))/b/c^3-1/64/c^3/b*exp(6*a/b)*Ei(1,6*arcsinh(c*x)+
6*a/b)-1/32/c^3/b*exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)+1/64/c^3/b*exp(2*a/
b)*Ei(1,2*arcsinh(c*x)+2*a/b)+1/64/c^3/b*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2
*a/b)-1/32/c^3/b*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)-1/64/c^3/b*exp(-6*
a/b)*Ei(1,-6*arcsinh(c*x)-6*a/b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}x^{2}}{b\,\mathrm{arsinh}\left(cx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + 1)^(3/2)*x^2/(b*arcsinh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{2}\,{\left(c^{2}\,x^{2}+1\right)}^{3/2}}{a+b\,\mathrm{asinh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x)),x)
```

```
[Out] int((x^2*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}}{a+b\,\mathrm{asinh}\left(cx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x**2*(c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x)), x)
```

3.366 $\int \frac{x\left(1+c^2x^2\right)^{3/2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=183

$$-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8bc^2}-\frac{3\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16bc^2}-\frac{\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16bc^2}+\frac{\cosh\left(\frac{a}{b}\right)\text{Sh}}{8}$$

```
[Out] 1/8*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b/c^2+3/16*cosh(3*a/b)*Shi(3*(a+b*a
rcsinh(c*x))/b)/b/c^2+1/16*cosh(5*a/b)*Shi(5*(a+b*arcsinh(c*x))/b)/b/c^2-1/
8*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b/c^2-3/16*Chi(3*(a+b*arcsinh(c*x))/b
)*sinh(3*a/b)/b/c^2-1/16*Chi(5*(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b/c^2
```

Rubi [A] time = 0.42, antiderivative size = 179, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5779, 5448, 3303, 3298, 3301}

$$-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{8bc^2}-\frac{3\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{16bc^2}-\frac{\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5a}{b}+5\sinh^{-1}(cx)\right)}{16bc^2}+\text{co}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b])/(8*b*c^2) - (3*CoshIntegral[(
3*a)/b + 3*ArcSinh[c*x]]*Sinh[(3*a)/b])/(16*b*c^2) - (CoshIntegral[(5*a)/b
+ 5*ArcSinh[c*x]]*Sinh[(5*a)/b])/(16*b*c^2) + (Cosh[a/b]*SinhIntegral[a/b +
 ArcSinh[c*x]])/(8*b*c^2) + (3*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSi
nh[c*x]])/(16*b*c^2) + (Cosh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcSinh[c*x]
])/(16*b*c^2)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x\left(1+c^2x^2\right)^{3/2}}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^4(x)\sinh(x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{c^2}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{\sinh(x)}{8(a+bx)}+\frac{3\sinh(3x)}{16(a+bx)}+\frac{\sinh(5x)}{16(a+bx)}\right)dx,x,\sinh^{-1}(cx)\right)}{c^2}$$

$$= \frac{\text{Subst}\left(\int \frac{\sinh(5x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{16c^2}+\frac{\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{8c^2}+\frac{3\,\text{Subst}\left(\int \frac{s}{}\right.}{}$$

$$= \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{8c^2}+\frac{\left(3\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}+3x\right)}{a+bx}\,dx,\right.}{16c^2}$$

$$= -\frac{\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{8bc^2}-\frac{3\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)\sinh\left(\frac{3a}{b}\right)}{16bc^2}-\frac{\text{Chi}\left(\frac{5a}{b}+5\,\text{si}\right.}{}$$

Mathematica [A] time = 0.44, size = 136, normalized size = 0.74

$$\frac{-2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-3\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+}{16bc^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-2*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] - 3*CoshIntegral[3*(a/b + Ar
cSinh[c*x])]*Sinh[(3*a)/b] - CoshIntegral[5*(a/b + ArcSinh[c*x])]*Sinh[(5*a
)/b] + 2*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 3*Cosh[(3*a)/b]*SinhI
ntegral[3*(a/b + ArcSinh[c*x])] + Cosh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSi
nh[c*x])])/(16*b*c^2)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^2x^3+x\right)\sqrt{c^2x^2+1}}{b\,\text{arsinh}\left(cx\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^3 + x)*sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value

maple [A] time = 0.28, size = 178, normalized size = 0.97

$$\frac{e^{\frac{5a}{b}}\operatorname{Ei}\left(1,5\,\mathrm{arcsinh}\,(cx)+\frac{5a}{b}\right)}{32c^2b}+\frac{3\,e^{\frac{3a}{b}}\operatorname{Ei}\left(1,3\,\mathrm{arcsinh}\,(cx)+\frac{3a}{b}\right)}{32c^2b}+\frac{e^{\frac{a}{b}}\operatorname{Ei}\left(1,\mathrm{arcsinh}\,(cx)+\frac{a}{b}\right)}{16c^2b}-\frac{e^{-\frac{a}{b}}\operatorname{Ei}\left(1,-\mathrm{arcs}\right.}{16c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)

[Out] 1/32/c^2/b*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)+3/32/c^2/b*exp(3*a/b)*Ei(1
,3*arcsinh(c*x)+3*a/b)+1/16/c^2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/16/c^2/
b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)-3/32/c^2/b*exp(-3*a/b)*Ei(1,-3*arcsinh(
c*x)-3*a/b)-1/32/c^2/b*exp(-5*a/b)*Ei(1,-5*arcsinh(c*x)-5*a/b)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}x}{b\,\mathrm{arsinh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")

[Out] integrate((c^2*x^2 + 1)^(3/2)*x/(b*arcsinh(c*x) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\left(c^2\,x^2+1\right)^{3/2}}{a+b\,\mathrm{asinh}\,(c\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x)),x)

[Out] int((x*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(c^2x^2+1\right)^{\frac{3}{2}}}{a+b\,\mathrm{asinh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)

[Out] Integral(x*(c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x)), x)

3.367 $\int \frac{\left(1+c^2x^2\right)^{3/2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=144

$$\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{2bc}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{8bc}-\frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{2bc}-\frac{\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\right.}{8}$$

```
[Out] 1/2*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b/c+1/8*Chi(4*(a+b*arcsinh(c*x)
)/b)*cosh(4*a/b)/b/c+3/8*ln(a+b*arcsinh(c*x))/b/c-1/2*Shi(2*(a+b*arcsinh(c*
x))/b)*sinh(2*a/b)/b/c-1/8*Shi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b/c
```

Rubi [A] time = 0.29, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5699, 3312, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{8bc}-\frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc}-\text{sinh}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + c^2*x^2)^(3/2)/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(2*b*c) + (Cosh[(4*a
)/b]*CoshIntegral[(4*a)/b + 4*ArcSinh[c*x]])/(8*b*c) + (3*Log[a + b*ArcSinh
[c*x]])/(8*b*c) - (Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(2
*b*c) - (Sinh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcSinh[c*x]])/(8*b*c)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
```

```
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{\left(1+c^2x^2\right)^{3/2}}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^4(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{3}{8(a+bx)}+\frac{\cosh(2x)}{2(a+bx)}+\frac{\cosh(4x)}{8(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{c}$$
$$= \frac{3\log\left(a+b\sinh^{-1}(cx)\right)}{8bc}+\frac{\text{Subst}\left(\int \frac{\cosh(4x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{8c}+\frac{\text{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\,d\right.}{2c}$$
$$= \frac{3\log\left(a+b\sinh^{-1}(cx)\right)}{8bc}+\frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{2c}+\frac{\cosh}{}$$
$$= \frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{8bc}+\frac{3\log\left(a\right.}{}$$

Mathematica [A] time = 0.25, size = 109, normalized size = 0.76

$$\frac{4\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-4\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(c\right.\right.}{8bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + c^2*x^2)^(3/2)/(a + b*ArcSinh[c*x]),x]
[Out] (4*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x])] + Cosh[(4*a)/b]*CoshI
ntegral[4*(a/b + ArcSinh[c*x])] + 3*Log[a + b*ArcSinh[c*x]] - 4*Sinh[(2*a)/
b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] - Sinh[(4*a)/b]*SinhIntegral[4*(a/b
 + ArcSinh[c*x])])/(8*b*c)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{b\,\text{arsinh}\,(cx)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((c^2*x^2 + 1)^(3/2)/(b*arcsinh(c*x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{b\,\text{arsinh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate((c^2*x^2 + 1)^(3/2)/(b*arcsinh(c*x) + a), x)
```

maple [A] time = 0.31, size = 139, normalized size = 0.97

$$\frac{3\ln\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)}{8bc}-\frac{\mathrm{e}^{\frac{4a}{b}}\,\mathrm{Ei}\left(1,4\,\mathrm{arcsinh}\left(cx\right)+\frac{4a}{b}\right)}{16cb}-\frac{\mathrm{e}^{\frac{2a}{b}}\,\mathrm{Ei}\left(1,2\,\mathrm{arcsinh}\left(cx\right)+\frac{2a}{b}\right)}{4cb}-\frac{\mathrm{e}^{-\frac{2a}{b}}\,\mathrm{Ei}\left(1,-2\,\mathrm{arcsinh}\left(\right.\right.}{4cb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] 3/8*ln(a+b*arcsinh(c*x))/b/c-1/16/c/b*exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)
-1/4/c/b*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)-1/4/c/b*exp(-2*a/b)*Ei(1,-2*
arcsinh(c*x)-2*a/b)-1/16/c/b*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{b\,\mathrm{arsinh}\left(cx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + 1)^(3/2)/(b*arcsinh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c^2\,x^2+1\right)^{3/2}}{a+b\,\mathrm{asinh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(3/2)/(a + b*asinh(c*x)),x)
```

```
[Out] int((c^2*x^2 + 1)^(3/2)/(a + b*asinh(c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{a+b\,\mathrm{asinh}\left(cx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral((c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x)), x)
```

3.368 $\int \frac{\left(1+c^2x^2\right)^{3/2}}{x\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=139

$$\text{Int}\left(\frac{1}{x\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)},x\right)-\frac{5\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4b}-\frac{\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{4b}+\frac{5\cosh}{}$$

```
[Out] 5/4*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b+1/4*cosh(3*a/b)*Shi(3*(a+b*arcsin
h(c*x))/b)/b-5/4*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b-1/4*Chi(3*(a+b*arcsi
nh(c*x))/b)*sinh(3*a/b)/b+Unintegrable(1/x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(
1/2),x)
```

Rubi [A] time = 0.85, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1+c^2x^2\right)^{3/2}}{x\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(3/2)/(x*(a + b*ArcSinh[c*x])),x]
```

```
[Out] (-5*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b])/(4*b) - (CoshIntegral[(3*a)
/b + 3*ArcSinh[c*x]]*Sinh[(3*a)/b])/(4*b) + (5*Cosh[a/b]*SinhIntegral[a/b +
 ArcSinh[c*x]])/(4*b) + (Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x
]])/(4*b) + Defer[Int][1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{\left(1+c^2x^2\right)^{3/2}}{x\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \left(\frac{1}{x\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}+\frac{2c^2x}{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}+\frac{}{\sqrt{1+c^2x^2}}\right.$$

$$=\left(2c^2\right)\int \frac{x}{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}\,dx+c^4\int \frac{x^3}{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}\,d$$

$$=2\,\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)+\int \frac{1}{x\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}\,dx+$$

$$=i\,\text{Subst}\left(\int \left(\frac{3i\sinh(x)}{4(a+bx)}-\frac{i\sinh(3x)}{4(a+bx)}\right)dx,x,\sinh^{-1}(cx)\right)+\left(2\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\right.$$

$$=-\frac{2\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{b}+\frac{2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{b}+\frac{1}{4}\text{Subst}\left(\right.$$

$$=-\frac{2\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{b}+\frac{2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{b}-\frac{1}{4}\left(3\cosh\right.$$

$$=-\frac{5\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{4b}-\frac{\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)\sinh\left(\frac{3a}{b}\right)}{4b}+\frac{5\cosh}{}$$

Mathematica [A] time = 1.99, size = 0, normalized size = 0.00

$$\int \frac{\left(1+c^2x^2\right)^{3/2}}{x\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(3/2)/(x*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[(1 + c^2*x^2)^(3/2)/(x*(a + b*ArcSinh[c*x])), x]
```

fricas [A] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{bx\,\text{arsinh}\,(cx)+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((c^2*x^2 + 1)^(3/2)/(b*x*arcsinh(c*x) + a*x), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{x\left(a+b\,\text{arcsinh}\,(cx)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x)),x)
[Out] int((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{\left(b\,\text{arsinh}\,(cx)+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c^2\,x^2+1\right)^{3/2}}{x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(3/2)/(x*(a + b*asinh(c*x))),x)
[Out] int((c^2*x^2 + 1)^(3/2)/(x*(a + b*asinh(c*x))), x)
```

sympy [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{x\left(a+b\,\mathrm{asinh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(3/2)/x/(a+b*asinh(c*x)),x)
[Out] Integral((c**2*x**2 + 1)**(3/2)/(x*(a + b*asinh(c*x))), x)
```

3.369 $\int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=106

$$\text{Int}\left(\frac{1}{x^2\sqrt{c^2x^2+1}\,(a+b\sinh^{-1}(cx))},x\right)+\frac{c\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{2b}-\frac{c\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{2b}+\frac{3c\,\text{l}}{}$$

[Out] 1/2*c*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b+3/2*c*ln(a+b*arcsinh(c*x))/b-1/2*c*Shi(2*(a+b*arcsinh(c*x))/b)*sinh(2*a/b)/b+Unintegrable(1/x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)

Rubi [A] time = 0.60, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1+c^2x^2)^{3/2}}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

[In] Int[(1 + c^2*x^2)^(3/2)/(x^2*(a + b*ArcSinh[c*x])),x]

[Out] (c*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(2*b) + (3*c*Log[a + b*ArcSinh[c*x]])/(2*b) - (c*Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(2*b) + Defer[Int][1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]

Rubi steps

$$\begin{aligned}
\int \frac{(1+c^2x^2)^{3/2}}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx &= \int \left(\frac{2c^2}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}+\frac{1}{x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}+\frac{}{\sqrt{1+c^2x^2}\,(}\right.\\
&= \left(2c^2\right)\int \frac{1}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx+c^4\int \frac{x^2}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx\\
&= \frac{2c\log\left(a+b\sinh^{-1}(cx)\right)}{b}+c\,\text{Subst}\left(\int \frac{\sinh^2(x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)+\int \frac{}{x^2\sqrt{1+c^2}}\\
&= \frac{2c\log\left(a+b\sinh^{-1}(cx)\right)}{b}-c\,\text{Subst}\left(\int \left(\frac{1}{2(a+bx)}-\frac{\cosh(2x)}{2(a+bx)}\right)dx,x,\sinh^{-1}(cx)\right)\\
&= \frac{3c\log\left(a+b\sinh^{-1}(cx)\right)}{2b}+\frac{1}{2}c\,\text{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)+\int \frac{}{x^2\sqrt{1+}}\\
&= \frac{3c\log\left(a+b\sinh^{-1}(cx)\right)}{2b}+\frac{1}{2}\left(c\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx,x,\sinh\right.\\
&= \frac{c\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2b}+\frac{3c\log\left(a+b\sinh^{-1}(cx)\right)}{2b}-\frac{c\sinh\left(\frac{2a}{b}\right)\text{Shi}}{}
\end{aligned}$$

Mathematica [A] time = 1.44, size = 0, normalized size = 0.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(3/2)/(x^2*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[(1 + c^2*x^2)^(3/2)/(x^2*(a + b*ArcSinh[c*x])), x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{bx^2\operatorname{arsinh}\left(cx\right)+ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((c^2*x^2 + 1)^(3/2)/(b*x^2*arcsinh(c*x) + a*x^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)*x^2), x)
```

maple [A] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{x^2\left(a+b\operatorname{arcsinh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x)),x)
[Out] int((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)*x^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c^2\,x^2+1\right)}^{3/2}}{x^2\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(3/2)/(x^2*(a + b*asinh(c*x))),x)
[Out] int((c^2*x^2 + 1)^(3/2)/(x^2*(a + b*asinh(c*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{x^2\left(a+b\operatorname{asinh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(3/2)/x**2/(a+b*asinh(c*x)),x)
```

```
[Out] Integral((c**2*x**2 + 1)**(3/2)/(x**2*(a + b*asinh(c*x))), x)
```

3.370 $\int \frac{\left(1+c^2x^2\right)^{3/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{\left(c^2x^2+1\right)^{3/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x)),x)
```

Rubi [A] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1+c^2x^2\right)^{3/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(3/2)/(x^3*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][(1 + c^2*x^2)^(3/2)/(x^3*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{\left(1+c^2x^2\right)^{3/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{\left(1+c^2x^2\right)^{3/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 4.69, size = 0, normalized size = 0.00

$$\int \frac{\left(1+c^2x^2\right)^{3/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(3/2)/(x^3*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[(1 + c^2*x^2)^(3/2)/(x^3*(a + b*ArcSinh[c*x])), x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{bx^3\,\text{arsinh}\,(cx)+ax^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((c^2*x^2 + 1)^(3/2)/(b*x^3*arcsinh(c*x) + a*x^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.83, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{x^3\left(a+b\operatorname{arcsinh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x)),x)
```

```
[Out] int((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)*x^3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(c^2\,x^2+1\right)}^{3/2}}{x^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(3/2)/(x^3*(a + b*asinh(c*x))),x)
```

```
[Out] int((c^2*x^2 + 1)^(3/2)/(x^3*(a + b*asinh(c*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{x^3\left(a+b\operatorname{asinh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(3/2)/x**3/(a+b*asinh(c*x)),x)
```

```
[Out] Integral((c**2*x**2 + 1)**(3/2)/(x**3*(a + b*asinh(c*x))), x)
```

3.371 $\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=30

$$\operatorname{Int}\left(\frac{(c^2x^2+1)^{3/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x)),x)
```

Rubi [A] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(3/2)/(x^4*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][(1 + c^2*x^2)^(3/2)/(x^4*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{(1+c^2x^2)^{3/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 0.86, size = 0, normalized size = 0.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(3/2)/(x^4*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[(1 + c^2*x^2)^(3/2)/(x^4*(a + b*ArcSinh[c*x])), x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{(c^2x^2+1)^{\frac{3}{2}}}{bx^4\operatorname{arsinh}(cx)+ax^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((c^2*x^2 + 1)^(3/2)/(b*x^4*arcsinh(c*x) + a*x^4), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c^2x^2+1)^{\frac{3}{2}}}{(b\operatorname{arsinh}(cx)+a)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)*x^4), x)
```

maple [A] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{\left(c^{2} x^{2}+1\right)^{\frac{3}{2}}}{x^{4} \left(a+b \operatorname{arcsinh}\left(c x\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x)),x)
[Out] int((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^{2} x^{2}+1\right)^{\frac{3}{2}}}{\left(b \operatorname{arsinh}\left(c x\right)+a\right) x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)*x^4), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(c^2\,x^2+1\right)}^{3/2}}{x^4\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(3/2)/(x^4*(a + b*asinh(c*x))),x)
[Out] int((c^2*x^2 + 1)^(3/2)/(x^4*(a + b*asinh(c*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^{2} x^{2} + 1\right)^{\frac{3}{2}}}{x^{4} \left(a + b \operatorname{asinh}{\left(c x \right)}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(3/2)/x**4/(a+b*asinh(c*x)),x)
[Out] Integral((c**2*x**2 + 1)**(3/2)/(x**4*(a + b*asinh(c*x))), x)
```

3.372 $\int \frac{x^3(1+c^2x^2)^{5/2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=245

$$\frac{3\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{128bc^4}+\frac{\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{32bc^4}-\frac{3\sinh\left(\frac{7a}{b}\right)\text{Chi}\left(\frac{7\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{256bc^4}-\frac{\sinh\left(\frac{9a}{b}\right)\text{C}$$

```
[Out] -3/128*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b/c^4-1/32*cosh(3*a/b)*Shi(3*(a+
b*arcsinh(c*x))/b)/b/c^4+3/256*cosh(7*a/b)*Shi(7*(a+b*arcsinh(c*x))/b)/b/c^
4+1/256*cosh(9*a/b)*Shi(9*(a+b*arcsinh(c*x))/b)/b/c^4+3/128*Chi((a+b*arcsin
h(c*x))/b)*sinh(a/b)/b/c^4+1/32*Chi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b/c
^4-3/256*Chi(7*(a+b*arcsinh(c*x))/b)*sinh(7*a/b)/b/c^4-1/256*Chi(9*(a+b*arc
sinh(c*x))/b)*sinh(9*a/b)/b/c^4
```

Rubi [A] time = 0.58, antiderivative size = 241, normalized size of antiderivative = 0.98, number of steps used = 15, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5779, 5448, 3303, 3298, 3301}

$$\frac{3\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{128bc^4}+\frac{\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{32bc^4}-\frac{3\sinh\left(\frac{7a}{b}\right)\text{Chi}\left(\frac{7a}{b}+7\sinh^{-1}(cx)\right)}{256bc^4}-\text{si}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (3*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b])/(128*b*c^4) + (CoshIntegral[
(3*a)/b + 3*ArcSinh[c*x]]*Sinh[(3*a)/b])/(32*b*c^4) - (3*CoshIntegral[(7*a)
/b + 7*ArcSinh[c*x]]*Sinh[(7*a)/b])/(256*b*c^4) - (CoshIntegral[(9*a)/b + 9
*ArcSinh[c*x]]*Sinh[(9*a)/b])/(256*b*c^4) - (3*Cosh[a/b]*SinhIntegral[a/b +
 ArcSinh[c*x]])/(128*b*c^4) - (Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSi
nh[c*x]])/(32*b*c^4) + (3*Cosh[(7*a)/b]*SinhIntegral[(7*a)/b + 7*ArcSinh[c*
x]])/(256*b*c^4) + (Cosh[(9*a)/b]*SinhIntegral[(9*a)/b + 9*ArcSinh[c*x]])/(
256*b*c^4)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
```

```
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^3\left(1+c^2x^2\right)^{5/2}}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^6(x)\sinh^3(x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{c^4}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{3\sinh(x)}{128(a+bx)}-\frac{\sinh(3x)}{32(a+bx)}+\frac{3\sinh(7x)}{256(a+bx)}+\frac{\sinh(9x)}{256(a+bx)}\right)dx,x,\sinh^{-1}(cx)\right)}{c^4}$$
$$= \frac{\text{Subst}\left(\int \frac{\sinh(9x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{256c^4}+\frac{3\,\text{Subst}\left(\int \frac{\sinh(7x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{256c^4}-\frac{3\,\text{Subst}\left(\right.}{}$$
$$= -\frac{\left(3\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{128c^4}-\frac{\cosh\left(\frac{3a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}+3x\right)}{a+bx}\,dx\right.}{32c^4}$$
$$= \frac{3\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{128bc^4}+\frac{\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)\sinh\left(\frac{3a}{b}\right)}{32bc^4}-\frac{3\text{Chi}\left(\frac{7a}{b}+7\,\text{si}\right.}{2}$$

Mathematica [A] time = 0.99, size = 180, normalized size = 0.73

$$\frac{6\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+8\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-3\sinh\left(\frac{7a}{b}\right)\text{Chi}\left(7\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (6*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] + 8*CoshIntegral[3*(a/b + Arc
Sinh[c*x])]*Sinh[(3*a)/b] - 3*CoshIntegral[7*(a/b + ArcSinh[c*x])]*Sinh[(7*
a)/b] - CoshIntegral[9*(a/b + ArcSinh[c*x])]*Sinh[(9*a)/b] - 6*Cosh[a/b]*Si
nhIntegral[a/b + ArcSinh[c*x]] - 8*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcS
inh[c*x])] + 3*Cosh[(7*a)/b]*SinhIntegral[7*(a/b + ArcSinh[c*x])] + Cosh[(9
*a)/b]*SinhIntegral[9*(a/b + ArcSinh[c*x])])/(256*b*c^4)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^4x^7+2\,c^2x^5+x^3\right)\sqrt{c^2x^2+1}}{b\,\text{arsinh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^4*x^7 + 2*c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a)
, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.54, size = 238, normalized size = 0.97

$$\frac{e^{\frac{9a}{b}} \operatorname{Ei}\left(1, 9 \operatorname{arcsinh}(cx) + \frac{9a}{b}\right)}{512c^4b} + \frac{3\, e^{\frac{7a}{b}} \operatorname{Ei}\left(1, 7 \operatorname{arcsinh}(cx) + \frac{7a}{b}\right)}{512c^4b} - \frac{e^{\frac{3a}{b}} \operatorname{Ei}\left(1, 3 \operatorname{arcsinh}(cx) + \frac{3a}{b}\right)}{64c^4b} - \frac{3\, e^{\frac{a}{b}} \operatorname{Ei}\left(1, \text{ar}\right.}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/512/c^4/b*exp(9*a/b)*Ei(1,9*arcsinh(c*x)+9*a/b)+3/512/c^4/b*exp(7*a/b)*Ei
(1,7*arcsinh(c*x)+7*a/b)-1/64/c^4/b*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-3
/256/c^4/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)+3/256/c^4/b*exp(-a/b)*Ei(1,-arcs
inh(c*x)-a/b)+1/64/c^4/b*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)-3/512/c^4/
b*exp(-7*a/b)*Ei(1,-7*arcsinh(c*x)-7*a/b)-1/512/c^4/b*exp(-9*a/b)*Ei(1,-9*a
rcsinh(c*x)-9*a/b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2+1\right)^{\frac{5}{2}} x^3}{b \operatorname{arsinh}(cx) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)*x^3/(b*arcsinh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \left(c^2\, x^2+1\right)^{5/2}}{a + b \operatorname{asinh}(c\, x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x)),x)
```

```
[Out] int((x^3*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \left(c^2x^2+1\right)^{\frac{5}{2}}}{a + b \operatorname{asinh}(cx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x**3*(c**2*x**2 + 1)**(5/2)/(a + b*asinh(c*x)), x)
```

3.373 $\int \frac{x^2\left(1+c^2x^2\right)^{5/2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=268

$$-\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{32bc^3}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{32bc^3}+\frac{\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{32bc^3}+\frac{\cosh\left(\frac{8a}{b}\right)\text{C}}{}$$

```
[Out] -1/32*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b/c^3+1/32*Chi(4*(a+b*arcsinh
(c*x))/b)*cosh(4*a/b)/b/c^3+1/32*Chi(6*(a+b*arcsinh(c*x))/b)*cosh(6*a/b)/b/
c^3+1/128*Chi(8*(a+b*arcsinh(c*x))/b)*cosh(8*a/b)/b/c^3-5/128*ln(a+b*arcsin
h(c*x))/b/c^3+1/32*Shi(2*(a+b*arcsinh(c*x))/b)*sinh(2*a/b)/b/c^3-1/32*Shi(4
*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b/c^3-1/32*Shi(6*(a+b*arcsinh(c*x))/b)*s
inh(6*a/b)/b/c^3-1/128*Shi(8*(a+b*arcsinh(c*x))/b)*sinh(8*a/b)/b/c^3
```

Rubi [A] time = 0.62, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5779, 5448, 3303, 3298, 3301}

$$-\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{32bc^3}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{32bc^3}+\frac{\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6a}{b}+6\sinh^{-1}(cx)\right)}{32bc^3}+\frac{\text{cc}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(32*b*c^3) + (Cosh[
(4*a)/b]*CoshIntegral[(4*a)/b + 4*ArcSinh[c*x]])/(32*b*c^3) + (Cosh[(6*a)/b
]*CoshIntegral[(6*a)/b + 6*ArcSinh[c*x]])/(32*b*c^3) + (Cosh[(8*a)/b]*CoshI
ntegral[(8*a)/b + 8*ArcSinh[c*x]])/(128*b*c^3) - (5*Log[a + b*ArcSinh[c*x]]
)/(128*b*c^3) + (Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(32*
b*c^3) - (Sinh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcSinh[c*x]])/(32*b*c^3)
- (Sinh[(6*a)/b]*SinhIntegral[(6*a)/b + 6*ArcSinh[c*x]])/(32*b*c^3) - (Sinh
[(8*a)/b]*SinhIntegral[(8*a)/b + 8*ArcSinh[c*x]])/(128*b*c^3)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
```

```
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2\left(1+c^2x^2\right)^{5/2}}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^6(x)\sinh^2(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^3}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{5}{128(a+bx)}-\frac{\cosh(2x)}{32(a+bx)}+\frac{\cosh(4x)}{32(a+bx)}+\frac{\cosh(6x)}{32(a+bx)}+\frac{\cosh(8x)}{128(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{c^3}$$

$$= -\frac{5\log\left(a+b\sinh^{-1}(cx)\right)}{128bc^3}+\frac{\text{Subst}\left(\int \frac{\cosh(8x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{128c^3}-\frac{\text{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\right.}{32}$$

$$= -\frac{5\log\left(a+b\sinh^{-1}(cx)\right)}{128bc^3}-\frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{32c^3}+\frac{\cos}{}$$

$$= -\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{32bc^3}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{32bc^3}+\frac{\cosh\left(\frac{6}{l}\right.}{}$$

Mathematica [A] time = 0.88, size = 197, normalized size = 0.74

$$\frac{-4\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+4\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+4\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(6\left(\frac{a}{b}+\sinh\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-4*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x])] + 4*Cosh[(4*a)/b]*Co
shIntegral[4*(a/b + ArcSinh[c*x])] + 4*Cosh[(6*a)/b]*CoshIntegral[6*(a/b +
ArcSinh[c*x])] + Cosh[(8*a)/b]*CoshIntegral[8*(a/b + ArcSinh[c*x])] - 5*Log
[a + b*ArcSinh[c*x]] + 4*Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])]
 - 4*Sinh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])] - 4*Sinh[(6*a)/b]*S
inhIntegral[6*(a/b + ArcSinh[c*x])] - Sinh[(8*a)/b]*SinhIntegral[8*(a/b + A
rcSinh[c*x])])/(128*b*c^3)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^4x^6+2\,c^2x^4+x^2\right)\sqrt{c^2x^2+1}}{b\,\text{arsinh}\left(cx\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^4*x^6 + 2*c^2*x^4 + x^2)*sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2+1\right)^{\frac{5}{2}}x^2}{b\operatorname{arsinh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate((c^2*x^2 + 1)^(5/2)*x^2/(b*arcsinh(c*x) + a), x)
```

maple [A] time = 0.60, size = 259, normalized size = 0.97

$$-\frac{5\ln\left(a+b\operatorname{arcsinh}(cx)\right)}{128b\,c^3}-\frac{e^{\frac{8a}{b}}\operatorname{Ei}\left(1,8\operatorname{arcsinh}(cx)+\frac{8a}{b}\right)}{256c^3b}-\frac{e^{\frac{6a}{b}}\operatorname{Ei}\left(1,6\operatorname{arcsinh}(cx)+\frac{6a}{b}\right)}{64c^3b}-\frac{e^{\frac{4a}{b}}\operatorname{Ei}\left(1,4\operatorname{arcsinh}(c\right.}{64c^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x)
[Out] -5/128*ln(a+b*arcsinh(c*x))/b/c^3-1/256/c^3/b*exp(8*a/b)*Ei(1,8*arcsinh(c*x
)+8*a/b)-1/64/c^3/b*exp(6*a/b)*Ei(1,6*arcsinh(c*x)+6*a/b)-1/64/c^3/b*exp(4*
a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)+1/64/c^3/b*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2
*a/b)+1/64/c^3/b*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)-1/64/c^3/b*exp(-4*
a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)-1/64/c^3/b*exp(-6*a/b)*Ei(1,-6*arcsinh(c*x
)-6*a/b)-1/256/c^3/b*exp(-8*a/b)*Ei(1,-8*arcsinh(c*x)-8*a/b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2+1\right)^{\frac{5}{2}}x^2}{b\operatorname{arsinh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate((c^2*x^2 + 1)^(5/2)*x^2/(b*arcsinh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(c^2\,x^2+1\right)}^{5/2}}{a+b\,\mathrm{asinh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x)),x)
[Out] int((x^2*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(c^2x^2+1\right)^{\frac{5}{2}}}{a+b\operatorname{asinh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x)),x)
[Out] Integral(x**2*(c**2*x**2 + 1)**(5/2)/(a + b*asinh(c*x)), x)
```

3.374 $\int \frac{x(1+c^2x^2)^{5/2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=245

$$-\frac{5\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{64bc^2}-\frac{9\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{64bc^2}-\frac{5\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\sinh^{-1}(cx))}{b}\right)}{64bc^2}-\frac{\sinh\left(\frac{7a}{b}\right)\dots}{\dots}$$

```
[Out] 5/64*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b/c^2+9/64*cosh(3*a/b)*Shi(3*(a+b*
arcsinh(c*x))/b)/b/c^2+5/64*cosh(5*a/b)*Shi(5*(a+b*arcsinh(c*x))/b)/b/c^2+1
/64*cosh(7*a/b)*Shi(7*(a+b*arcsinh(c*x))/b)/b/c^2-5/64*Chi((a+b*arcsinh(c*x
))/b)*sinh(a/b)/b/c^2-9/64*Chi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b/c^2-5/
64*Chi(5*(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b/c^2-1/64*Chi(7*(a+b*arcsinh(c*
x))/b)*sinh(7*a/b)/b/c^2
```

Rubi [A] time = 0.49, antiderivative size = 241, normalized size of antiderivative = 0.98, number of steps used = 15, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5779, 5448, 3303, 3298, 3301}

$$-\frac{5\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{64bc^2}-\frac{9\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{64bc^2}-\frac{5\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5a}{b}+5\sinh^{-1}(cx)\right)}{64bc^2}-\dots$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-5*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b])/(64*b*c^2) - (9*CoshIntegra
l[(3*a)/b + 3*ArcSinh[c*x]]*Sinh[(3*a)/b])/(64*b*c^2) - (5*CoshIntegral[(5*
a)/b + 5*ArcSinh[c*x]]*Sinh[(5*a)/b])/(64*b*c^2) - (CoshIntegral[(7*a)/b +
7*ArcSinh[c*x]]*Sinh[(7*a)/b])/(64*b*c^2) + (5*Cosh[a/b]*SinhIntegral[a/b +
 ArcSinh[c*x]])/(64*b*c^2) + (9*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcS
inh[c*x]])/(64*b*c^2) + (5*Cosh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcSinh[c
*x]])/(64*b*c^2) + (Cosh[(7*a)/b]*SinhIntegral[(7*a)/b + 7*ArcSinh[c*x]])/(
64*b*c^2)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
```

```
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x\left(1+c^2x^2\right)^{5/2}}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^6(x)\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^2}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{5\sinh(x)}{64(a+bx)}+\frac{9\sinh(3x)}{64(a+bx)}+\frac{5\sinh(5x)}{64(a+bx)}+\frac{\sinh(7x)}{64(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{c^2}$$
$$= \frac{\text{Subst}\left(\int \frac{\sinh(7x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{64c^2} + \frac{5\,\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{64c^2} + \frac{5\,\text{Subst}\left(\int\right.}{}$$
$$= \frac{\left(5\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{64c^2} + \frac{\left(9\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}+3x\right)}{a+bx}\right.}{64c^2}$$
$$= -\frac{5\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{64bc^2} - \frac{9\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)\sinh\left(\frac{3a}{b}\right)}{64bc^2} - \frac{5\text{Chi}\left(\frac{5a}{b}+5\right.}{}$$

Mathematica [A] time = 0.80, size = 180, normalized size = 0.73

$$\frac{-5\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-9\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-5\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-5*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] - 9*CoshIntegral[3*(a/b + Ar
cSinh[c*x])]*Sinh[(3*a)/b] - 5*CoshIntegral[5*(a/b + ArcSinh[c*x])]*Sinh[(5
*a)/b] - CoshIntegral[7*(a/b + ArcSinh[c*x])]*Sinh[(7*a)/b] + 5*Cosh[a/b]*S
inhIntegral[a/b + ArcSinh[c*x]] + 9*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + Arc
Sinh[c*x])] + 5*Cosh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[c*x])] + Cosh[(
7*a)/b]*SinhIntegral[7*(a/b + ArcSinh[c*x])])/(64*b*c^2)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^4x^5+2\,c^2x^3+x\right)\sqrt{c^2x^2+1}}{b\,\text{arsinh}\,(cx)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^4*x^5 + 2*c^2*x^3 + x)*sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a),
x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.42, size = 238, normalized size = 0.97

$$\frac{e^{\frac{7a}{b}} \operatorname{Ei}\left(1, 7 \operatorname{arcsinh}(cx) + \frac{7a}{b}\right)}{128c^2b} + \frac{5\, e^{\frac{5a}{b}} \operatorname{Ei}\left(1, 5 \operatorname{arcsinh}(cx) + \frac{5a}{b}\right)}{128c^2b} + \frac{9\, e^{\frac{3a}{b}} \operatorname{Ei}\left(1, 3 \operatorname{arcsinh}(cx) + \frac{3a}{b}\right)}{128c^2b} + \frac{5\, e^{\frac{a}{b}} \operatorname{Ei}\left(1,\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/128/c^2/b*exp(7*a/b)*Ei(1,7*arcsinh(c*x)+7*a/b)+5/128/c^2/b*exp(5*a/b)*Ei
(1,5*arcsinh(c*x)+5*a/b)+9/128/c^2/b*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)+
5/128/c^2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-5/128/c^2/b*exp(-a/b)*Ei(1,-arc
sinh(c*x)-a/b)-9/128/c^2/b*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)-5/128/c^
2/b*exp(-5*a/b)*Ei(1,-5*arcsinh(c*x)-5*a/b)-1/128/c^2/b*exp(-7*a/b)*Ei(1,-7
*arcsinh(c*x)-7*a/b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2 + 1\right)^{\frac{5}{2}} x}{b \operatorname{arsinh}(cx) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)*x/(b*arcsinh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x \left(c^2 x^2 + 1\right)^{5/2}}{a + b \operatorname{asinh}(c x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x)),x)
```

```
[Out] int((x*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \left(c^2x^2 + 1\right)^{\frac{5}{2}}}{a + b \operatorname{asinh}(cx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x*(c**2*x**2 + 1)**(5/2)/(a + b*asinh(c*x)), x)
```

3.375 $\int \frac{(1+c^2x^2)^{5/2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=206

$$\frac{15\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{32bc}+\frac{3\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\sinh^{-1}(cx))}{b}\right)}{16bc}+\frac{\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\sinh^{-1}(cx))}{b}\right)}{32bc}-\frac{15\sinh(}{}$$

```
[Out] 15/32*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b/c+3/16*Chi(4*(a+b*arcsinh(c
*x))/b)*cosh(4*a/b)/b/c+1/32*Chi(6*(a+b*arcsinh(c*x))/b)*cosh(6*a/b)/b/c+5/
16*ln(a+b*arcsinh(c*x))/b/c-15/32*Shi(2*(a+b*arcsinh(c*x))/b)*sinh(2*a/b)/b
/c-3/16*Shi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b/c-1/32*Shi(6*(a+b*arcsinh
(c*x))/b)*sinh(6*a/b)/b/c
```

Rubi [A] time = 0.36, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5699, 3312, 3303, 3298, 3301}

$$\frac{15\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{32bc}+\frac{3\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{16bc}+\frac{\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6a}{b}+6\sinh^{-1}(cx)\right)}{32bc}-$$

Antiderivative was successfully verified.

```
[In] Int[(1 + c^2*x^2)^(5/2)/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (15*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(32*b*c) + (3*Cos
h[(4*a)/b]*CoshIntegral[(4*a)/b + 4*ArcSinh[c*x]])/(16*b*c) + (Cosh[(6*a)/b
]*CoshIntegral[(6*a)/b + 6*ArcSinh[c*x]])/(32*b*c) + (5*Log[a + b*ArcSinh[c
*x]])/(16*b*c) - (15*Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]])/
(32*b*c) - (3*Sinh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcSinh[c*x]])/(16*b*c
) - (Sinh[(6*a)/b]*SinhIntegral[(6*a)/b + 6*ArcSinh[c*x]])/(32*b*c)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{\left(1+c^2x^2\right)^{5/2}}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^6(x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{c}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{5}{16(a+bx)}+\frac{15\cosh(2x)}{32(a+bx)}+\frac{3\cosh(4x)}{16(a+bx)}+\frac{\cosh(6x)}{32(a+bx)}\right)dx,x,\sinh^{-1}(cx)\right)}{c}$$
$$= \frac{5\log\left(a+b\sinh^{-1}(cx)\right)}{16bc}+\frac{\text{Subst}\left(\int \frac{\cosh(6x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{32c}+\frac{3\,\text{Subst}\left(\int \frac{\cosh(4x)}{a+bx}\right.}{16}$$
$$= \frac{5\log\left(a+b\sinh^{-1}(cx)\right)}{16bc}+\frac{\left(15\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{32c}+$$
$$= \frac{15\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{32bc}+\frac{3\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{16bc}+\frac{\cosh}{}$$

Mathematica [A] time = 0.55, size = 153, normalized size = 0.74

$$\frac{15\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+6\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(6\left(\frac{a}{b}+\sinh^{-1}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + c^2*x^2)^(5/2)/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (15*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x])] + 6*Cosh[(4*a)/b]*Co
shIntegral[4*(a/b + ArcSinh[c*x])] + Cosh[(6*a)/b]*CoshIntegral[6*(a/b + Ar
cSinh[c*x])] + 10*Log[a + b*ArcSinh[c*x]] - 15*Sinh[(2*a)/b]*SinhIntegral[2
*(a/b + ArcSinh[c*x])] - 6*Sinh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x]
)] - Sinh[(6*a)/b]*SinhIntegral[6*(a/b + ArcSinh[c*x])])/(32*b*c)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^4x^4+2\,c^2x^2+1\right)\sqrt{c^2x^2+1}}{b\,\text{arsinh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2+1\right)^{\frac{5}{2}}}{b\,\text{arsinh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)/(b*arcsinh(c*x) + a), x)
```

maple [A] time = 0.51, size = 199, normalized size = 0.97

$$\frac{5\ln\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)}{16bc}-\frac{\mathrm{e}^{\frac{6a}{b}}\,\mathrm{Ei}\left(1,6\,\mathrm{arcsinh}\left(cx\right)+\frac{6a}{b}\right)}{64cb}-\frac{3\,\mathrm{e}^{\frac{4a}{b}}\,\mathrm{Ei}\left(1,4\,\mathrm{arcsinh}\left(cx\right)+\frac{4a}{b}\right)}{32cb}-\frac{15\,\mathrm{e}^{\frac{2a}{b}}\,\mathrm{Ei}\left(1,2\,\mathrm{arcsinh}\right.}{64cb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] 5/16*ln(a+b*arcsinh(c*x))/b/c-1/64/c/b*exp(6*a/b)*Ei(1,6*arcsinh(c*x)+6*a/b
)-3/32/c/b*exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)-15/64/c/b*exp(2*a/b)*Ei(1,
2*arcsinh(c*x)+2*a/b)-15/64/c/b*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)-3/3
2/c/b*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)-1/64/c/b*exp(-6*a/b)*Ei(1,-6*
arcsinh(c*x)-6*a/b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2+1\right)^{\frac{5}{2}}}{b\,\mathrm{arsinh}\left(cx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)/(b*arcsinh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c^2\,x^2+1\right)}^{5/2}}{a+b\,\mathrm{asinh}\left(c\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(5/2)/(a + b*asinh(c*x)),x)
```

```
[Out] int((c^2*x^2 + 1)^(5/2)/(a + b*asinh(c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^{2} x^{2} + 1\right)^{\frac{5}{2}}}{a + b \operatorname{asinh}{\left(c x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(5/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral((c**2*x**2 + 1)**(5/2)/(a + b*asinh(c*x)), x)
```

3.376 $\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\sinh^{-1}(cx))} dx$

Optimal. Leaf size=195

$$\text{Int}\left(\frac{1}{x\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)},x\right)-\frac{11\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8b}-\frac{7\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{16b}-\frac{\sin}{}$$

```
[Out] 11/8*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b+7/16*cosh(3*a/b)*Shi(3*(a+b*arcs
inh(c*x))/b)/b+1/16*cosh(5*a/b)*Shi(5*(a+b*arcsinh(c*x))/b)/b-11/8*Chi((a+b
*arcsinh(c*x))/b)*sinh(a/b)/b-7/16*Chi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/
b-1/16*Chi(5*(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b+Unintegrable(1/x/(a+b*arcs
inh(c*x))/(c^2*x^2+1)^(1/2),x)
```

Rubi [A] time = 1.28, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1+c^2x^2\right)^{5/2}}{x\left(a+b\sinh^{-1}(cx)\right)} dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(5/2)/(x*(a + b*ArcSinh[c*x])),x]
```

```
[Out] (-11*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b])/(8*b) - (7*CoshIntegral[(3
*a)/b + 3*ArcSinh[c*x]]*Sinh[(3*a)/b])/(16*b) - (CoshIntegral[(5*a)/b + 5*A
rcSinh[c*x]]*Sinh[(5*a)/b])/(16*b) + (11*Cosh[a/b]*SinhIntegral[a/b + ArcSi
nh[c*x]])/(8*b) + (7*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])/
(16*b) + (Cosh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcSinh[c*x]])/(16*b) + De
fer[Int][1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(1+c^2x^2\right)^{5/2}}{x\left(a+b\sinh^{-1}(cx)\right)} dx &= \int \left(\frac{1}{x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}+\frac{3c^2x}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}+\frac{}{\sqrt{1+c^2x^2}}\right.\\
&= \left(3c^2\right)\int \frac{x}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)} dx+\left(3c^4\right)\int \frac{x^3}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx\right.}\\
&= 3\,\text{Subst}\left(\int \frac{\sinh(x)}{a+bx} dx, x, \sinh^{-1}(cx)\right)+3\,\text{Subst}\left(\int \frac{\sinh^3(x)}{a+bx} dx, x, \sinh^{-1}(cx)\right)\\
&= -\left(i\,\text{Subst}\left(\int \left(\frac{5i\sinh(x)}{8(a+bx)}-\frac{5i\sinh(3x)}{16(a+bx)}+\frac{i\sinh(5x)}{16(a+bx)}\right) dx, x, \sinh^{-1}(cx)\right)\right)+3i\,\text{S}\\
&= -\frac{3\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{b}+\frac{3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{b}+\frac{1}{16}\text{Subst}\\
&= -\frac{3\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{b}+\frac{3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{b}+\frac{1}{8}\left(5\cosh\right.\\
&= -\frac{11\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{8b}-\frac{7\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)\sinh\left(\frac{3a}{b}\right)}{16b}-\frac{\text{Chi}\left(\right.}{}
\end{aligned}$$

Mathematica [A] time = 2.01, size = 0, normalized size = 0.00

$$\int \frac{\left(1+c^2x^2\right)^{5/2}}{x\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(5/2)/(x*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Integrate[(1 + c^2*x^2)^(5/2)/(x*(a + b*ArcSinh[c*x])), x]
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^4x^4+2\,c^2x^2+1\right)\sqrt{c^2x^2+1}}{bx\,\text{arsinh}\,(cx)+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b*x*arcsinh(c*x) + a*
x), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2+1\right)^{\frac{5}{2}}}{x\left(a+b\,\text{arcsinh}\,(cx)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x)),x)
```

```
[Out] int((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2+1\right)^{\frac{5}{2}}}{\left(b\,\text{arsinh}\,(cx)+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c^2\,x^2+1\right)^{5/2}}{x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(5/2)/(x*(a + b*asinh(c*x))),x)
```

```
[Out] int((c^2*x^2 + 1)^(5/2)/(x*(a + b*asinh(c*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2+1\right)^{\frac{5}{2}}}{x\left(a+b\operatorname{asinh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(5/2)/x/(a+b*asinh(c*x)),x)
```

```
[Out] Integral((c**2*x**2 + 1)**(5/2)/(x*(a + b*asinh(c*x))), x)
```

3.377 $\int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=159

$$\text{Int}\left(\frac{1}{x^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)},x\right)+\frac{c\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{b}+\frac{c\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{8b}-\frac{c\,\text{s}}{}$$

```
[Out] c*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b+1/8*c*Chi(4*(a+b*arcsinh(c*x))/
b)*cosh(4*a/b)/b+15/8*c*ln(a+b*arcsinh(c*x))/b-c*Shi(2*(a+b*arcsinh(c*x))/b
)*sinh(2*a/b)/b-1/8*c*Shi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b+Unintegrabl
e(1/x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

Rubi [A] time = 0.97, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(5/2)/(x^2*(a + b*ArcSinh[c*x])),x]
```

```
[Out] (c*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcSinh[c*x]])/b + (c*Cosh[(4*a)/
b]*CoshIntegral[(4*a)/b + 4*ArcSinh[c*x]])/(8*b) + (15*c*Log[a + b*ArcSinh[
c*x]])/(8*b) - (c*Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]])/b -
 (c*Sinh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcSinh[c*x]])/(8*b) + Defer[Int
][1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx &= \int \left(\frac{3c^2}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}+\frac{1}{x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}+\frac{}{\sqrt{1+c^2x^2}\,(}\right.\\
&= \left(3c^2\right)\int \frac{1}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx+\left(3c^4\right)\int \frac{x^2}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\\
&= \frac{3c\log\left(a+b\sinh^{-1}(cx)\right)}{b}+c\,\text{Subst}\left(\int \frac{\sinh^4(x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)+(3c)\,\text{Subst}\left(\int\right.\\
&= \frac{3c\log\left(a+b\sinh^{-1}(cx)\right)}{b}+c\,\text{Subst}\left(\int \left(\frac{3}{8(a+bx)}-\frac{\cosh(2x)}{2(a+bx)}+\frac{\cosh(4x)}{8(a+bx)}\right)dx,x\right.\\
&= \frac{15c\log\left(a+b\sinh^{-1}(cx)\right)}{8b}+\frac{1}{8}c\,\text{Subst}\left(\int \frac{\cosh(4x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)-\frac{1}{2}c\,\text{Subst}\\
&= \frac{15c\log\left(a+b\sinh^{-1}(cx)\right)}{8b}-\frac{1}{2}\left(c\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx,x,\text{sinh}\right.\\
&= \frac{c\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{b}+\frac{c\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{8b}+\frac{15c\,\text{l}}{}
\end{aligned}$$

Mathematica [A] time = 1.37, size = 0, normalized size = 0.00

$$\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(5/2)/(x^2*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[(1 + c^2*x^2)^(5/2)/(x^2*(a + b*ArcSinh[c*x])), x]
```

fricas [A] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^4x^4+2\,c^2x^2+1\right)\sqrt{c^2x^2+1}}{bx^2\,\text{arsinh}\,(cx)+ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b*x^2*arcsinh(c*x) +
a*x^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2+1\right)^{\frac{5}{2}}}{(b\,\text{arsinh}\,(cx)+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)*x^2), x)
```

maple [A] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2+1\right)^{\frac{5}{2}}}{x^2\,(a+b\,\text{arcsinh}\,(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x)),x)
[Out] int((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2+1\right)^{\frac{5}{2}}}{(b\,\text{arsinh}\,(cx)+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)*x^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c^2\,x^2+1\right)^{5/2}}{x^2\,\left(a+b\,\text{asinh}\left(c\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(5/2)/(x^2*(a + b*asinh(c*x))),x)
```

```
[Out] int((c^2*x^2 + 1)^(5/2)/(x^2*(a + b*asinh(c*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2+1\right)^{\frac{5}{2}}}{x^2\left(a+b\operatorname{asinh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(5/2)/x**2/(a+b*asinh(c*x)),x)
```

```
[Out] Integral((c**2*x**2 + 1)**(5/2)/(x**2*(a + b*asinh(c*x))), x)
```

3.378 $\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\sinh^{-1}(cx))} dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{(c^2x^2+1)^{5/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x)),x)
```

Rubi [A] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)} dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(5/2)/(x^3*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][(1 + c^2*x^2)^(5/2)/(x^3*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)} dx = \int \frac{(1+c^2x^2)^{5/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)} dx$$

Mathematica [A] time = 4.68, size = 0, normalized size = 0.00

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(5/2)/(x^3*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[(1 + c^2*x^2)^(5/2)/(x^3*(a + b*ArcSinh[c*x])), x]
```

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(c^4x^4+2\,c^2x^2+1)\sqrt{c^2x^2+1}}{bx^3\,\text{arsinh}\,(cx)+ax^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b*x^3*arcsinh(c*x) + a*x^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.95, size = 0, normalized size = 0.00

$$\int \frac{\left(c^{2}x^{2}+1\right)^{\frac{5}{2}}}{x^{3}\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x)),x)
```

```
[Out] int((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^{2}x^{2}+1\right)^{\frac{5}{2}}}{\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)*x^3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(c^2\,x^2+1\right)}^{5/2}}{x^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(5/2)/(x^3*(a + b*asinh(c*x))),x)
```

```
[Out] int((c^2*x^2 + 1)^(5/2)/(x^3*(a + b*asinh(c*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^{2}x^{2}+1\right)^{\frac{5}{2}}}{x^{3}\left(a+b\,\mathrm{asinh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(5/2)/x**3/(a+b*asinh(c*x)),x)
```

```
[Out] Integral((c**2*x**2 + 1)**(5/2)/(x**3*(a + b*asinh(c*x))), x)
```

3.379 $\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{\left(c^2x^2+1\right)^{5/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x)),x)
```

Rubi [A] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(5/2)/(x^4*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][(1 + c^2*x^2)^(5/2)/(x^4*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 0.93, size = 0, normalized size = 0.00

$$\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(5/2)/(x^4*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[(1 + c^2*x^2)^(5/2)/(x^4*(a + b*ArcSinh[c*x])), x]
```

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^4x^4+2\,c^2x^2+1\right)\sqrt{c^2x^2+1}}{bx^4\,\text{arsinh}\left(cx\right)+ax^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b*x^4*arcsinh(c*x) +
a*x^4), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2+1\right)^{\frac{5}{2}}}{\left(b\,\text{arsinh}\left(cx\right)+a\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)*x^4), x)
```

maple [A] time = 0.69, size = 0, normalized size = 0.00

$$\int \frac{\left(c^{2} x^{2}+1\right)^{\frac{5}{2}}}{x^{4} \left(a +b \,\mathrm{arcsinh}\left(c x \right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x)),x)
[Out] int((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{{\left(c^{2} x^{2} + 1\right)}^{\frac{5}{2}}}{{\left(b \operatorname{arsinh}\left(c x\right) + a\right)} x^{4}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)*x^4), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(c^2\,x^2+1\right)}^{5/2}}{x^4\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(5/2)/(x^4*(a + b*asinh(c*x))),x)
[Out] int((c^2*x^2 + 1)^(5/2)/(x^4*(a + b*asinh(c*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^{2} x^{2} + 1\right)^{\frac{5}{2}}}{x^{4} \left(a + b \operatorname{asinh}{\left(c x \right)}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(5/2)/x**4/(a+b*asinh(c*x)),x)
[Out] Integral((c**2*x**2 + 1)**(5/2)/(x**4*(a + b*asinh(c*x))), x)
```

3.380 $\int \frac{x^4}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=41

$$-\frac{\text{Chi}\left(2\sinh^{-1}(ax)\right)}{2a^5}+\frac{\text{Chi}\left(4\sinh^{-1}(ax)\right)}{8a^5}+\frac{3\log\left(\sinh^{-1}(ax)\right)}{8a^5}$$

```
[Out] -1/2*Chi(2*arcsinh(a*x))/a^5+1/8*Chi(4*arcsinh(a*x))/a^5+3/8*ln(arcsinh(a*x
))/a^5
```

Rubi [A] time = 0.16, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5779, 3312, 3301}

$$-\frac{\text{Chi}\left(2\sinh^{-1}(ax)\right)}{2a^5}+\frac{\text{Chi}\left(4\sinh^{-1}(ax)\right)}{8a^5}+\frac{3\log\left(\sinh^{-1}(ax)\right)}{8a^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

```
[Out] -CoshIntegral[2*ArcSinh[a*x]]/(2*a^5) + CoshIntegral[4*ArcSinh[a*x]]/(8*a^5
) + (3*Log[ArcSinh[a*x]])/(8*a^5)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx &= \frac{\text{Subst}\left(\int \frac{\sinh^4(x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{a^5}\\
&= \frac{\text{Subst}\left(\int \left(\frac{3}{8x}-\frac{\cosh(2x)}{2x}+\frac{\cosh(4x)}{8x}\right)dx,x,\sinh^{-1}(ax)\right)}{a^5}\\
&= \frac{3\log\left(\sinh^{-1}(ax)\right)}{8a^5}+\frac{\text{Subst}\left(\int \frac{\cosh(4x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{8a^5}-\frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,d\right.}{2a}\\
&= -\frac{\text{Chi}\left(2\sinh^{-1}(ax)\right)}{2a^5}+\frac{\text{Chi}\left(4\sinh^{-1}(ax)\right)}{8a^5}+\frac{3\log\left(\sinh^{-1}(ax)\right)}{8a^5}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 31, normalized size = 0.76

$$\frac{-4\text{Chi}\left(2\sinh^{-1}(ax)\right)+\text{Chi}\left(4\sinh^{-1}(ax)\right)+3\log\left(\sinh^{-1}(ax)\right)}{8a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

```
[Out] (-4*CoshIntegral[2*ArcSinh[a*x]] + CoshIntegral[4*ArcSinh[a*x]] + 3*Log[Arc
Sinh[a*x]])/(8*a^5)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^4}{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^4/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^4}{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

maple [A] time = 0.11, size = 36, normalized size = 0.88

$$-\frac{\text{X}\,(2\,\text{arcsinh}\,(ax))}{2a^5}+\frac{\text{X}\,(4\,\text{arcsinh}\,(ax))}{8a^5}+\frac{3\ln\,(\text{arcsinh}\,(ax))}{8a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/2*Chi(2*arcsinh(a*x))/a^5+1/8*Chi(4*arcsinh(a*x))/a^5+3/8*ln(arcsinh(a*x
))/a^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^4}{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{x^4}{\text{asinh}\,(a\,x)\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)),x)
[Out] int(x^4/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{a^2x^2+1}\ \operatorname{asinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/asinh(a*x)/(a**2*x**2+1)**(1/2),x)
[Out] Integral(x**4/(sqrt(a**2*x**2 + 1)*asinh(a*x)), x)
```

3.381 $\int \frac{x^3}{\sqrt{1+a^2x^2} \sinh^{-1}(ax)} dx$

Optimal. Leaf size=27

$$\frac{\text{Shi}\left(3\sinh^{-1}(ax)\right)}{4a^4}-\frac{3\text{Shi}\left(\sinh^{-1}(ax)\right)}{4a^4}$$

```
[Out] -3/4*Shi(arcsinh(a*x))/a^4+1/4*Shi(3*arcsinh(a*x))/a^4
```

Rubi [A] time = 0.16, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5779, 3312, 3298}

$$\frac{\text{Shi}\left(3\sinh^{-1}(ax)\right)}{4a^4}-\frac{3\text{Shi}\left(\sinh^{-1}(ax)\right)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

```
[Out] (-3*SinhIntegral[ArcSinh[a*x]])/(4*a^4) + SinhIntegral[3*ArcSinh[a*x]]/(4*a
^4)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx &= \frac{\text{Subst}\left(\int \frac{\sinh^3(x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{a^4}\\
&= \frac{i\,\text{Subst}\left(\int \left(\frac{3i\sinh(x)}{4x}-\frac{i\sinh(3x)}{4x}\right)dx,x,\sinh^{-1}(ax)\right)}{a^4}\\
&= \frac{\text{Subst}\left(\int \frac{\sinh(3x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{4a^4}-\frac{3\,\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{4a^4}\\
&= -\frac{3\text{Shi}\left(\sinh^{-1}(ax)\right)}{4a^4}+\frac{\text{Shi}\left(3\sinh^{-1}(ax)\right)}{4a^4}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 22, normalized size = 0.81

$$\frac{\mathrm{Shi}\left(3\sinh^{-1}(ax)\right)-3\mathrm{Shi}\left(\sinh^{-1}(ax)\right)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
[Out] (-3*SinhIntegral[ArcSinh[a*x]] + SinhIntegral[3*ArcSinh[a*x]])/(4*a^4)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{x^3}{\sqrt{a^2x^2+1}\ \mathrm{arsinh}\,(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(x^3/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.10, size = 23, normalized size = 0.85

$$-\frac{3\,\mathrm{Shi}\,(\mathrm{arcsinh}\,(ax))-\mathrm{Shi}\,(3\,\mathrm{arcsinh}\,(ax))}{4a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
[Out] -1/4*(3*Shi(arcsinh(a*x))-Shi(3*arcsinh(a*x)))/a^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\sqrt{a^2x^2+1}\ \mathrm{arsinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int\frac{x^3}{\mathrm{asinh}\,(a\,x)\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)),x)
[Out] int(x^3/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a^2x^2+1}\,\operatorname{asinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/asinh(a*x)/(a**2*x**2+1)**(1/2),x)
[Out] Integral(x**3/(sqrt(a**2*x**2 + 1)*asinh(a*x)), x)
```

3.382 $\int \frac{x^2}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=27

$$\frac{\text{Chi}\left(2\sinh^{-1}(ax)\right)}{2a^3}-\frac{\log\left(\sinh^{-1}(ax)\right)}{2a^3}$$

```
[Out] 1/2*Chi(2*arcsinh(a*x))/a^3-1/2*ln(arcsinh(a*x))/a^3
```

Rubi [A] time = 0.14, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5779, 3312, 3301}

$$\frac{\text{Chi}\left(2\sinh^{-1}(ax)\right)}{2a^3}-\frac{\log\left(\sinh^{-1}(ax)\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

```
[Out] CoshIntegral[2*ArcSinh[a*x]]/(2*a^3) - Log[ArcSinh[a*x]]/(2*a^3)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx &= \frac{\text{Subst}\left(\int \frac{\sinh^2(x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{a^3}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{2x}-\frac{\cosh(2x)}{2x}\right)dx,x,\sinh^{-1}(ax)\right)}{a^3}\\
&= -\frac{\log\left(\sinh^{-1}(ax)\right)}{2a^3}+\frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{2a^3}\\
&= \frac{\text{Chi}\left(2\sinh^{-1}(ax)\right)}{2a^3}-\frac{\log\left(\sinh^{-1}(ax)\right)}{2a^3}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 22, normalized size = 0.81

$$\frac{\operatorname{Chi}\left(2\sinh^{-1}(ax)\right)-\log\left(\sinh^{-1}(ax)\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
[Out] (CoshIntegral[2*ArcSinh[a*x]] - Log[ArcSinh[a*x]])/(2*a^3)
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2}{\sqrt{a^2x^2+1}\ \operatorname{arsinh}(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(x^2/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\sqrt{a^2x^2+1}\ \operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

maple [A] time = 0.08, size = 24, normalized size = 0.89

$$\frac{\mathrm{X}\left(2\operatorname{arcsinh}(ax)\right)}{2a^3}-\frac{\ln\left(\operatorname{arcsinh}(ax)\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
[Out] 1/2*Chi(2*arcsinh(a*x))/a^3-1/2*ln(arcsinh(a*x))/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\sqrt{a^2x^2+1}\ \operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int\frac{x^2}{\operatorname{asinh}(a\,x)\ \sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(x^2/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a^2x^2+1}\ \operatorname{asinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/asinh(a*x)/(a**2*x**2+1)**(1/2),x)
[Out] Integral(x**2/(sqrt(a**2*x**2 + 1)*asinh(a*x)), x)
```

3.383 $\int \frac{x^2}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=27

$$\frac{\text{Chi}\left(2\sinh^{-1}(ax)\right)}{2a^3}-\frac{\log\left(\sinh^{-1}(ax)\right)}{2a^3}$$

[Out] 1/2*Chi(2*arcsinh(a*x))/a^3-1/2*ln(arcsinh(a*x))/a^3

Rubi [A] time = 0.14, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5779, 3312, 3301}

$$\frac{\text{Chi}\left(2\sinh^{-1}(ax)\right)}{2a^3}-\frac{\log\left(\sinh^{-1}(ax)\right)}{2a^3}$$

Antiderivative was successfully verified.

[In] Int[x^2/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]

[Out] CoshIntegral[2*ArcSinh[a*x]]/(2*a^3) - Log[ArcSinh[a*x]]/(2*a^3)

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx &= \frac{\text{Subst}\left(\int \frac{\sinh^2(x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{a^3}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{2x}-\frac{\cosh(2x)}{2x}\right)dx,x,\sinh^{-1}(ax)\right)}{a^3}\\
&= -\frac{\log\left(\sinh^{-1}(ax)\right)}{2a^3}+\frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{2a^3}\\
&= \frac{\text{Chi}\left(2\sinh^{-1}(ax)\right)}{2a^3}-\frac{\log\left(\sinh^{-1}(ax)\right)}{2a^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 0.81

$$\frac{\mathrm{Chi}\left(2\sinh^{-1}(ax)\right)-\log\left(\sinh^{-1}(ax)\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
[Out] (CoshIntegral[2*ArcSinh[a*x]] - Log[ArcSinh[a*x]])/(2*a^3)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{x^2}{\sqrt{a^2x^2+1}\ \mathrm{arsinh}\,(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(x^2/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\sqrt{a^2x^2+1}\ \mathrm{arsinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

maple [A] time = 0.00, size = 24, normalized size = 0.89

$$\frac{\mathrm{X}\,(2\,\mathrm{arcsinh}\,(ax))}{2a^3}-\frac{\ln\,(\mathrm{arcsinh}\,(ax))}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
[Out] 1/2*Chi(2*arcsinh(a*x))/a^3-1/2*ln(arcsinh(a*x))/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\sqrt{a^2x^2+1}\ \mathrm{arsinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int\frac{x^2}{\mathrm{asinh}\,(a\,x)\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(x^2/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a^2x^2+1}\,\operatorname{asinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/asinh(a*x)/(a**2*x**2+1)**(1/2),x)
[Out] Integral(x**2/(sqrt(a**2*x**2 + 1)*asinh(a*x)), x)
```

3.384 $\int \frac{x}{\sqrt{1+a^2x^2} \sinh^{-1}(ax)} \, dx$

Optimal. Leaf size=9

$$\frac{\mathrm{Shi}\left(\sinh^{-1}(ax)\right)}{a^2}$$

```
[Out] Shi(arcsinh(a*x))/a^2
```

Rubi [A] time = 0.08, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5779, 3298}

$$\frac{\mathrm{Shi}\left(\sinh^{-1}(ax)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

```
[Out] SinhIntegral[ArcSinh[a*x]]/a^2
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{1+a^2x^2} \sinh^{-1}(ax)} \, dx &= \frac{\mathrm{Subst}\left(\int \frac{\sinh(x)}{x} \, dx, x, \sinh^{-1}(ax)\right)}{a^2}\\
&= \frac{\mathrm{Shi}\left(\sinh^{-1}(ax)\right)}{a^2}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 9, normalized size = 1.00

$$\frac{\mathrm{Shi}\left(\sinh^{-1}(ax)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

```
[Out] SinhIntegral[ArcSinh[a*x]]/a^2
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{x}{\sqrt{a^2x^2+1}\,\mathrm{arsinh}\,(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

maple [A] time = 0.06, size = 10, normalized size = 1.11

$$\frac{\text{Shi}\,(\text{arcsinh}\,(ax))}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

```
[Out] Shi(arcsinh(a*x))/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.11

$$\int \frac{x}{\text{asinh}\,(a\,x)\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(x/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a^2x^2+1}\,\text{asinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a**2*x**2 + 1)*asinh(a*x)), x)
```

3.385 $\int \frac{1}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=9

$$\frac{\log\left(\sinh^{-1}(ax)\right)}{a}$$

[Out] ln(arcsinh(a*x))/a

Rubi [A] time = 0.04, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {5673}

$$\frac{\log\left(\sinh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]

[Out] Log[ArcSinh[a*x]]/a

Rule 5673

```
Int[1/(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*Sqrt[(d_) + (e_.)*(x_)^2]), x_Sy
mbol] :> Simp[Log[a + b*ArcSinh[c*x]]/(b*c*Sqrt[d]), x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx = \frac{\log\left(\sinh^{-1}(ax)\right)}{a}$$

Mathematica [A] time = 0.02, size = 9, normalized size = 1.00

$$\frac{\log\left(\sinh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]

[Out] Log[ArcSinh[a*x]]/a

fricas [B] time = 0.45, size = 21, normalized size = 2.33

$$\frac{\log\left(\log\left(ax+\sqrt{a^2x^2+1}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")

[Out] log(log(a*x + sqrt(a^2*x^2 + 1)))/a

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2x^2+1}\,\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

maple [A] time = 0.01, size = 10, normalized size = 1.11

$$\frac{\ln(\operatorname{arcsinh}(ax))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
[Out] ln(arcsinh(a*x))/a
```

maxima [A] time = 0.42, size = 9, normalized size = 1.00

$$\frac{\log(\operatorname{arsinh}(ax))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] log(arcsinh(a*x))/a
```

mupad [B] time = 0.10, size = 9, normalized size = 1.00

$$\frac{\ln(\operatorname{asinh}(a\,x))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(asinh(a*x)*(a^2*x^2 + 1)^(1/2)),x)
[Out] log(asinh(a*x))/a
```

sympy [A] time = 0.48, size = 7, normalized size = 0.78

$$\frac{\log(\operatorname{asinh}(ax))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(a*x)/(a**2*x**2+1)**(1/2),x)
[Out] log(asinh(a*x))/a
```

3.386 $\int \frac{1}{x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{1}{x\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable(1/x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
[Out] Defer[Int][1/(x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}\,dx=\int \frac{1}{x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}\,dx$$

Mathematica [A] time = 1.12, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
[Out] Integrate[1/(x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]), x]
```

fricas [A] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a^2x^2+1}}{\left(a^2x^3+x\right)\text{arsinh}\left(ax\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*x^2 + 1)/((a^2*x^3 + x)*arcsinh(a*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2x^2+1}\,x\,\text{arsinh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(a^2*x^2 + 1)*x*arcsinh(a*x)), x)
```

maple [A] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
[Out] int(1/x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2x^2+1}\,x \operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(a^2*x^2 + 1)*x*arcsinh(a*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x \operatorname{asinh}(a\,x)\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*asinh(a*x)*(a^2*x^2 + 1)^(1/2)),x)
[Out] int(1/(x*asinh(a*x)*(a^2*x^2 + 1)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a^2x^2+1}\,\operatorname{asinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/asinh(a*x)/(a**2*x**2+1)**(1/2),x)
[Out] Integral(1/(x*sqrt(a**2*x**2 + 1)*asinh(a*x)), x)
```

3.387 $\int \frac{1}{x^2 \sqrt{1+a^2x^2} \sinh^{-1}(ax)} dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{1}{x^2\sqrt{a^2x^2+1}\sinh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable(1/x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
[Out] Defer[Int][1/(x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx=\int \frac{1}{x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.75, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
[Out] Integrate[1/(x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]), x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a^2x^2+1}}{\left(a^2x^4+x^2\right)\text{arsinh}\left(ax\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*x^2 + 1)/((a^2*x^4 + x^2)*arcsinh(a*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2x^2+1}\,x^2\,\text{arsinh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(a^2*x^2 + 1)*x^2*arcsinh(a*x)), x)
```

maple [A] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
[Out] int(1/x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2x^2+1}\,x^2 \operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(a^2*x^2 + 1)*x^2*arcsinh(a*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^2 \operatorname{asinh}(a\,x)\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*asinh(a*x)*(a^2*x^2 + 1)^(1/2)),x)
[Out] int(1/(x^2*asinh(a*x)*(a^2*x^2 + 1)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{a^2x^2+1}\,\operatorname{asinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/asinh(a*x)/(a**2*x**2+1)**(1/2),x)
[Out] Integral(1/(x**2*sqrt(a**2*x**2 + 1)*asinh(a*x)), x)
```

3.388 $\int \frac{x^5}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=183

$$-\frac{5\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8bc^6}+\frac{5\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16bc^6}-\frac{\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16bc^6}+\frac{5\cosh\left(\frac{a}{b}\right)}{}$$

```
[Out] 5/8*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b/c^6-5/16*cosh(3*a/b)*Shi(3*(a+b*a
rcsinh(c*x))/b)/b/c^6+1/16*cosh(5*a/b)*Shi(5*(a+b*arcsinh(c*x))/b)/b/c^6-5/
8*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b/c^6+5/16*Chi(3*(a+b*arcsinh(c*x))/b
)*sinh(3*a/b)/b/c^6-1/16*Chi(5*(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b/c^6
```

Rubi [A] time = 0.46, antiderivative size = 179, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5779, 3312, 3303, 3298, 3301}

$$-\frac{5\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{8bc^6}+\frac{5\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{16bc^6}-\frac{\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5a}{b}+5\sinh^{-1}(cx)\right)}{16bc^6}+\frac{5}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] (-5*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b])/(8*b*c^6) + (5*CoshIntegral
[(3*a)/b + 3*ArcSinh[c*x]]*Sinh[(3*a)/b])/(16*b*c^6) - (CoshIntegral[(5*a)/
b + 5*ArcSinh[c*x]]*Sinh[(5*a)/b])/(16*b*c^6) + (5*Cosh[a/b]*SinhIntegral[a
/b + ArcSinh[c*x]])/(8*b*c^6) - (5*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*A
rcSinh[c*x]])/(16*b*c^6) + (Cosh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcSinh[
c*x]])/(16*b*c^6)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^5}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^5(x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{c^6}$$

$$= -\frac{i\,\text{Subst}\left(\int \left(\frac{5i\sinh(x)}{8(a+bx)}-\frac{5i\sinh(3x)}{16(a+bx)}+\frac{i\sinh(5x)}{16(a+bx)}\right)dx,x,\sinh^{-1}(cx)\right)}{c^6}$$

$$= \frac{\text{Subst}\left(\int \frac{\sinh(5x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{16c^6} - \frac{5\,\text{Subst}\left(\int \frac{\sinh(3x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{16c^6}$$

$$= \frac{\left(5\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{8c^6} - \frac{\left(5\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\right.}{}$$

$$= -\frac{5\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{8bc^6} + \frac{5\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)\sinh\left(\frac{3a}{b}\right)}{16bc^6} -$$

Mathematica [A] time = 0.34, size = 136, normalized size = 0.74

$$-\frac{10\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-5\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] -1/16*(10*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] - 5*CoshIntegral[3*(a/
b + ArcSinh[c*x])]*Sinh[(3*a)/b] + CoshIntegral[5*(a/b + ArcSinh[c*x])]*Sin
h[(5*a)/b] - 10*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 5*Cosh[(3*a)/b
]*SinhIntegral[3*(a/b + ArcSinh[c*x])] - Cosh[(5*a)/b]*SinhIntegral[5*(a/b
+ ArcSinh[c*x])])/(b*c^6)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}\,x^5}{ac^2x^2+\left(bc^2x^2+b\right)\text{arsinh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^5/(a*c^2*x^2 + (b*c^2*x^2 + b)*arcsinh(c*x) +
a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

maple [A] time = 0.33, size = 178, normalized size = 0.97

$$\frac{e^{\frac{5a}{b}}\operatorname{Ei}\left(1,5\operatorname{arcsinh}(cx)+\frac{5a}{b}\right)}{32c^6b}-\frac{5\,e^{\frac{3a}{b}}\operatorname{Ei}\left(1,3\operatorname{arcsinh}(cx)+\frac{3a}{b}\right)}{32c^6b}+\frac{5\,e^{\frac{a}{b}}\operatorname{Ei}\left(1,\operatorname{arcsinh}(cx)+\frac{a}{b}\right)}{16c^6b}-\frac{5\,e^{-\frac{a}{b}}\operatorname{Ei}\left(1,-a\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)

[Out] 1/32/c^6/b*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)-5/32/c^6/b*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)+5/16/c^6/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-5/16/c^6/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)+5/32/c^6/b*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)-1/32/c^6/b*exp(-5*a/b)*Ei(1,-5*arcsinh(c*x)-5*a/b)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{c^2x^2+1}\,(b\operatorname{arsinh}(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="maxima")

[Out] integrate(x^5/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{(a+b\operatorname{asinh}(c\,x))\,\sqrt{c^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)),x)

[Out] int(x^5/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a+b\operatorname{asinh}(cx))\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)

[Out] Integral(x**5/((a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)

3.389 $\int \frac{x^4}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=144

$$-\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{2bc^5}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{8bc^5}+\frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{2bc^5}-\frac{\sinh\left(\frac{4a}{b}\right)\text{Sh}}{}$$

```
[Out] -1/2*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b/c^5+1/8*Chi(4*(a+b*arcsinh(c
*x))/b)*cosh(4*a/b)/b/c^5+3/8*ln(a+b*arcsinh(c*x))/b/c^5+1/2*Shi(2*(a+b*arc
sinh(c*x))/b)*sinh(2*a/b)/b/c^5-1/8*Shi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)
/b/c^5
```

Rubi [A] time = 0.40, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5779, 3312, 3303, 3298, 3301}

$$-\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc^5}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{8bc^5}+\frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc^5}-\frac{\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] -(Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(2*b*c^5) + (Cosh[(
4*a)/b]*CoshIntegral[(4*a)/b + 4*ArcSinh[c*x]])/(8*b*c^5) + (3*Log[a + b*Ar
cSinh[c*x]])/(8*b*c^5) + (Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*
x]])/(2*b*c^5) - (Sinh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcSinh[c*x]])/(8*
b*c^5)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
```

```
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^4}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}dx = \frac{\text{Subst}\left(\int \frac{\sinh^4(x)}{a+bx}dx, x, \sinh^{-1}(cx)\right)}{c^5}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{3}{8(a+bx)}-\frac{\cosh(2x)}{2(a+bx)}+\frac{\cosh(4x)}{8(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{c^5}$$
$$= \frac{3\log\left(a+b\sinh^{-1}(cx)\right)}{8bc^5}+\frac{\text{Subst}\left(\int \frac{\cosh(4x)}{a+bx}dx, x, \sinh^{-1}(cx)\right)}{8c^5}-\frac{\text{Subst}}{}$$
$$= \frac{3\log\left(a+b\sinh^{-1}(cx)\right)}{8bc^5}-\frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}dx, x, \sinh^{-1}(\right.}{2c^5}$$
$$= -\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc^5}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx\right.}{8bc^5}$$

Mathematica [A] time = 0.22, size = 109, normalized size = 0.76

$$-\frac{4\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-4\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(\right.\right.}{8bc^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] -1/8*(4*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x])] - Cosh[(4*a)/b]*
CoshIntegral[4*(a/b + ArcSinh[c*x])] - 3*Log[a + b*ArcSinh[c*x]] - 4*Sinh[(
2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] + Sinh[(4*a)/b]*SinhIntegral[4
*(a/b + ArcSinh[c*x])])/(b*c^5)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}\,x^4}{ac^2x^2+\left(bc^2x^2+b\right)\text{arsinh}\,(cx)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^4/(a*c^2*x^2 + (b*c^2*x^2 + b)*arcsinh(c*x) +
a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{c^2x^2+1}\,(b\,\text{arsinh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

maple [A] time = 0.40, size = 139, normalized size = 0.97

$$\frac{3\ln\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)}{8b\,c^{5}}-\frac{\mathrm{e}^{\frac{4a}{b}}\,\mathrm{Ei}\left(1,4\,\mathrm{arcsinh}\left(cx\right)+\frac{4a}{b}\right)}{16c^{5}b}+\frac{\mathrm{e}^{\frac{2a}{b}}\,\mathrm{Ei}\left(1,2\,\mathrm{arcsinh}\left(cx\right)+\frac{2a}{b}\right)}{4c^{5}b}+\frac{\mathrm{e}^{-\frac{2a}{b}}\,\mathrm{Ei}\left(1,-2\,\mathrm{arcsinh}\left(\right.\right.}{4c^{5}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

```
[Out] 3/8*ln(a+b*arcsinh(c*x))/b/c^5-1/16/c^5/b*exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*
a/b)+1/4/c^5/b*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)+1/4/c^5/b*exp(-2*a/b)*
Ei(1,-2*arcsinh(c*x)-2*a/b)-1/16/c^5/b*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a
/b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}}{\sqrt{c^{2}x^{2}+1}\,(b\,\mathrm{arsinh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{4}}{(a+b\,\mathrm{asinh}\,(c\,x))\,\sqrt{c^{2}\,x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(x^4/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}}{(a+b\,\mathrm{asinh}\,(cx))\,\sqrt{c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**4/((a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

3.390 $\int \frac{x^3}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=121

$$\frac{3\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4bc^4}-\frac{\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{4bc^4}-\frac{3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4bc^4}+\frac{\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\right.}{4b}$$

```
[Out] -3/4*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b/c^4+1/4*cosh(3*a/b)*Shi(3*(a+b*a
rcsinh(c*x))/b)/b/c^4+3/4*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b/c^4-1/4*Chi
(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b/c^4
```

Rubi [A] time = 0.39, antiderivative size = 117, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5779, 3312, 3303, 3298, 3301}

$$\frac{3\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4bc^4}-\frac{\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{4bc^4}-\frac{3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4bc^4}+\frac{\cosh\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] (3*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b])/(4*b*c^4) - (CoshIntegral[(3
*a)/b + 3*ArcSinh[c*x]]*Sinh[(3*a)/b])/(4*b*c^4) - (3*Cosh[a/b]*SinhIntegra
l[a/b + ArcSinh[c*x]])/(4*b*c^4) + (Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*
ArcSinh[c*x]])/(4*b*c^4)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
```

```
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^3}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}dx = \frac{\text{Subst}\left(\int \frac{\sinh^3(x)}{a+bx}dx,x,\sinh^{-1}(cx)\right)}{c^4}$$
$$= \frac{i\,\text{Subst}\left(\int\left(\frac{3i\sinh(x)}{4(a+bx)}-\frac{i\sinh(3x)}{4(a+bx)}\right)dx,x,\sinh^{-1}(cx)\right)}{c^4}$$
$$= \frac{\text{Subst}\left(\int \frac{\sinh(3x)}{a+bx}dx,x,\sinh^{-1}(cx)\right)}{4c^4}-\frac{3\,\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}dx,x,\sinh^{-1}(cx)\right)}{4c^4}$$
$$= -\frac{\left(3\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}dx,x,\sinh^{-1}(cx)\right)}{4c^4}+\frac{\cosh\left(\frac{3a}{b}\right)\text{Subst}\left(\int\right.}{}$$
$$= \frac{3\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{4bc^4}-\frac{\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)\sinh\left(\frac{3a}{b}\right)}{4bc^4}-\frac{3\,\text{c}}{}$$

Mathematica [A] time = 0.21, size = 92, normalized size = 0.76

$$\frac{3\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+\cosh\left(\right.}{4bc^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] (3*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] - CoshIntegral[3*(a/b + ArcSi
nh[c*x])]*Sinh[(3*a)/b] - 3*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + Co
sh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])])/(4*b*c^4)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}\,x^3}{ac^2x^2+\left(bc^2x^2+b\right)\text{arsinh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^3/(a*c^2*x^2 + (b*c^2*x^2 + b)*arcsinh(c*x) +
a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.23, size = 118, normalized size = 0.98

$$\frac{e^{\frac{3a}{b}} \operatorname{Ei}\left(1, 3\operatorname{arcsinh}(cx) + \frac{3a}{b}\right)}{8c^4b} - \frac{3\, e^{\frac{a}{b}} \operatorname{Ei}\left(1, \operatorname{arcsinh}(cx) + \frac{a}{b}\right)}{8c^4b} + \frac{3\, e^{-\frac{a}{b}} \operatorname{Ei}\left(1, -\operatorname{arcsinh}(cx) - \frac{a}{b}\right)}{8c^4b} - \frac{e^{-\frac{3a}{b}} \operatorname{Ei}\left(1, -3\,\mathrm{a}\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

```
[Out] 1/8/c^4/b*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-3/8/c^4/b*exp(a/b)*Ei(1,arc
sinh(c*x)+a/b)+3/8/c^4/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)-1/8/c^4/b*exp(-3
*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{c^2x^2+1}\,(b\operatorname{arsinh}(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{(a+b\operatorname{asinh}(c\,x))\,\sqrt{c^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(x^3/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a+b\operatorname{asinh}(cx))\,\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3/((a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

3.391 $\int \frac{x^2}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=82

$$\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{2bc^3}-\frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{2bc^3}-\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{2bc^3}$$

```
[Out] 1/2*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b/c^3-1/2*ln(a+b*arcsinh(c*x))/
b/c^3-1/2*Shi(2*(a+b*arcsinh(c*x))/b)*sinh(2*a/b)/b/c^3
```

Rubi [A] time = 0.29, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5779, 3312, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc^3}-\frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc^3}-\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{2bc^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] (Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(2*b*c^3) - Log[a +
b*ArcSinh[c*x]]/(2*b*c^3) - (Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh
[c*x]])/(2*b*c^3)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}dx = \frac{\text{Subst}\left(\int \frac{\sinh^2(x)}{a+bx}dx, x, \sinh^{-1}(cx)\right)}{c^3}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{2(a+bx)} - \frac{\cosh(2x)}{2(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{c^3}$$
$$= -\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{2bc^3} + \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{a+bx}dx, x, \sinh^{-1}(cx)\right)}{2c^3}$$
$$= -\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{2bc^3} + \frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}dx, x, \sinh^{-1}(\right.}{2c^3}$$
$$= \frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc^3} - \frac{\log\left(a+b\sinh^{-1}(cx)\right)}{2bc^3} - \frac{\sinh\left(\frac{2a}{b}\right.}{}$$

Mathematica [A] time = 0.18, size = 65, normalized size = 0.79

$$\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) - \sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) - \log\left(a+b\sinh^{-1}(cx)\right)}{2bc^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] (Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x])] - Log[a + b*ArcSinh[c*x
]] - Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])])/(2*b*c^3)
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}\,x^2}{ac^2x^2+\left(bc^2x^2+b\right)\text{arsinh}(cx)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^2/(a*c^2*x^2 + (b*c^2*x^2 + b)*arcsinh(c*x) +
a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{c^2x^2+1}\,(b\,\text{arsinh}(cx)+a)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

maple [A] time = 0.20, size = 79, normalized size = 0.96

$$-\frac{\ln(a+b\,\text{arcsinh}(cx))}{2b\,c^3} - \frac{e^{\frac{2a}{b}}\,\text{Ei}\left(1, 2\,\text{arcsinh}(cx)+\frac{2a}{b}\right)}{4c^3b} - \frac{e^{-\frac{2a}{b}}\,\text{Ei}\left(1, -2\,\text{arcsinh}(cx)-\frac{2a}{b}\right)}{4c^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

```
[Out] -1/2*ln(a+b*arcsinh(c*x))/b/c^3-1/4/c^3/b*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*
a/b)-1/4/c^3/b*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{c^2x^2+1}\,(b\,\text{arsinh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{(a+b\,\text{asinh}\,(c\,x))\,\sqrt{c^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(x^2/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+b\,\text{asinh}\,(cx))\,\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2/((a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

3.392 $\int \frac{x}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=54

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc^2}-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc^2}$$

```
[Out] cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b/c^2-Chi((a+b*arcsinh(c*x))/b)*sinh(a/
b)/b/c^2
```

Rubi [A] time = 0.19, antiderivative size = 50, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {5779, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc^2}-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] -((CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b])/(b*c^2)) + (Cosh[a/b]*SinhIn
tegral[a/b + ArcSinh[c*x]])/(b*c^2)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^2}$$
$$= \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^2} - \frac{\sinh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+\right.}{a+bx}\right.}{c^2}$$
$$= -\frac{\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{bc^2} + \frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc^2}$$

Mathematica [A] time = 0.11, size = 46, normalized size = 0.85

$$-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] -((CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] - Cosh[a/b]*SinhIntegral[a/b
+ ArcSinh[c*x]])/(b*c^2))
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}\,x}{ac^2x^2+\left(bc^2x^2+b\right)\text{arsinh}\left(cx\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x/(a*c^2*x^2 + (b*c^2*x^2 + b)*arcsinh(c*x) + a)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{c^2x^2+1}\left(b\,\text{arsinh}\left(cx\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

maple [A] time = 0.08, size = 58, normalized size = 1.07

$$\frac{e^{\frac{a}{b}}\,\text{Ei}\left(1, \text{arcsinh}\left(cx\right)+\frac{a}{b}\right)}{2c^2b} - \frac{e^{-\frac{a}{b}}\,\text{Ei}\left(1, -\text{arcsinh}\left(cx\right)-\frac{a}{b}\right)}{2c^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

```
[Out] 1/2/c^2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/2/c^2/b*exp(-a/b)*Ei(1,-arcsinh
(c*x)-a/b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{c^2x^2+1}\left(b\,\text{arsinh}\left(cx\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{(a+b\,\mathrm{asinh}\,(c\,x))\,\sqrt{c^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)),x)
[Out] int(x/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a+b\,\mathrm{asinh}\,(cx))\,\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)
[Out] Integral(x/((a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

3.393 $\int \frac{1}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=16

$$\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{bc}$$

```
[Out] ln(a+b*arcsinh(c*x))/b/c
```

Rubi [A] time = 0.05, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {5673}

$$\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Log[a + b*ArcSinh[c*x]]/(b*c)
```

Rule 5673

```
Int[1/(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*Sqrt[(d_) + (e_.)*(x_)^2]), x_Sy
mbol] :> Simp[Log[a + b*ArcSinh[c*x]]/(b*c*Sqrt[d]), x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx=\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{bc}$$

Mathematica [A] time = 0.03, size = 16, normalized size = 1.00

$$\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Log[a + b*ArcSinh[c*x]]/(b*c)
```

fricas [A] time = 0.40, size = 28, normalized size = 1.75

$$\frac{\log\left(b\log\left(cx+\sqrt{c^2x^2+1}\right)+a\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] log(b*log(c*x + sqrt(c^2*x^2 + 1)) + a)/(b*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c^2x^2+1}\,(b\,\mathrm{arsinh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

maple [A] time = 0.01, size = 17, normalized size = 1.06

$$\frac{\ln\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

```
[Out] ln(a+b*arcsinh(c*x))/b/c
```

maxima [A] time = 0.41, size = 16, normalized size = 1.00

$$\frac{\log\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] log(b*arcsinh(c*x) + a)/(b*c)
```

mupad [B] time = 0.14, size = 16, normalized size = 1.00

$$\frac{\ln\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)),x)
```

```
[Out] log(a + b*asinh(c*x))/(b*c)
```

sympy [A] time = 1.51, size = 26, normalized size = 1.62

$$\begin{cases} \frac{x}{a} & \text{for}\: c = 0 \wedge \left(b = 0 \vee c = 0\right) \\ \frac{\operatorname{asinh}{\left(cx \right)}}{ac} & \text{for}\: b = 0 \\ \frac{\log{\left(\frac{a}{b} + \operatorname{asinh}{\left(cx \right)} \right)}}{bc} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((x/a, Eq(c, 0) & (Eq(b, 0) | Eq(c, 0))), (asinh(c*x)/(a*c), Eq(b,
 0)), (log(a/b + asinh(c*x))/(b*c), True))
```

3.394 $\int \frac{1}{x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{1}{x\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

Rubi [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{1}{x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 1.76, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}}{ac^2x^3+ax+\left(bc^2x^3+bx\right)\text{arsinh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)/(a*c^2*x^3 + a*x + (b*c^2*x^3 + b*x)*arcsinh(c*x
)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)\sqrt{c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

```
[Out] int(1/x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c^{2}x^{2}+1}\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\sqrt{c^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(1/(x*(a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\,\mathrm{asinh}\left(cx\right)\right)\sqrt{c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(x*(a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

3.395 $\int \frac{1}{x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{1}{x^2\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

Rubi [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{1}{x^2\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 1.20, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}}{ac^2x^4+ax^2+\left(bc^2x^4+bx^2\right)\operatorname{arsinh}(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)/(a*c^2*x^4 + a*x^2 + (b*c^2*x^4 + b*x^2)*arcsinh
(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c^2x^2+1}\,(b\operatorname{arsinh}(cx)+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)*x^2), x)
```

maple [A] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(a+b \,\mathrm{arcsinh}\left(c x\right)\right) \sqrt{c^{2} x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
[Out] int(1/x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c^{2} x^{2}+1} \left(b \,\mathrm{arsinh}\left(c x\right)+a\right) x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)*x^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x^2\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\sqrt{c^2\,x^2+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)),x)
[Out] int(1/(x^2*(a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(a + b \operatorname{asinh}{\left(c x \right)}\right) \sqrt{c^{2} x^{2} + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)
[Out] Integral(1/(x**2*(a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

3.396 $\int \frac{x^2}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=30

$$\operatorname{Int}\left(\frac{x^2}{\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Rubi [A] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][x^2/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{x^2}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{x^2}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 2.11, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[x^2/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{c^2x^2+1}\,x^2}{ac^4x^4+2\,ac^2x^2+\left(bc^4x^4+2\,bc^2x^2+b\right)\operatorname{arsinh}(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)*x^2/(a*c^4*x^4 + 2*a*c^2*x^2 + (b*c^4*x^4 + 2*b*
c^2*x^2 + b)*arcsinh(c*x) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(c^2x^2+1\right)^{\frac{3}{2}}\left(b\operatorname{arsinh}(cx)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate(x^2/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

maple [A] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(c^{2} x^{2}+1\right)^{\frac{3}{2}} \left(a +b \,\mathrm{arcsinh}\left(c x \right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
[Out] int(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{{\left(c^{2} x^{2} + 1\right)}^{\frac{3}{2}} {\left(b \operatorname{arsinh}\left(c x\right) + a\right)}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(x^2/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(c^2\,x^2+1\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)),x)
[Out] int(x^2/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(a + b \operatorname{asinh}{\left(c x \right)}\right) \left(c^{2} x^{2} + 1\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
[Out] Integral(x**2/((a + b*asinh(c*x))*(c**2*x**2 + 1)**(3/2)), x)
```

3.397 $\int \frac{x}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=28

$$\mathrm{Int}\left(\frac{x}{\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][x/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{x}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{x}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 1.08, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[x/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{c^2x^2+1}\,x}{ac^4x^4+2\,ac^2x^2+\left(bc^4x^4+2\,bc^2x^2+b\right)\operatorname{arsinh}(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)*x/(a*c^4*x^4 + 2*a*c^2*x^2 + (b*c^4*x^4 + 2*b*c^
2*x^2 + b)*arcsinh(c*x) + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(x/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(c^2\,x^2+1\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)),x)
```

```
[Out] int(x/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+b\,\mathrm{asinh}\left(cx\right)\right)\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x/((a + b*asinh(c*x))*(c**2*x**2 + 1)**(3/2)), x)
```

3.398 $\int \frac{1}{(1+c^2x^2)^{3/2}(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{1}{\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][1/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{1}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[1/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}}{ac^4x^4+2\,ac^2x^2+\left(bc^4x^4+2\,bc^2x^2+b\right)\text{arsinh}\left(cx\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)/(a*c^4*x^4 + 2*a*c^2*x^2 + (b*c^4*x^4 + 2*b*c^2*
x^2 + b)*arcsinh(c*x) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c^2x^2+1\right)^{\frac{3}{2}}\left(b\,\text{arsinh}\left(cx\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

maple [A] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] int(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(c^2\,x^2+1\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)),x)
```

```
[Out] int(1/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(cx\right)\right)\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(1/((a + b*asinh(c*x))*(c**2*x**2 + 1)**(3/2)), x)
```

3.399 $\int \frac{1}{x(1+c^2x^2)^{3/2}(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=30

$$\mathrm{Int}\left(\frac{1}{x\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Rubi [A] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][1/(x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 1.50, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[1/(x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{c^2x^2+1}}{ac^4x^5+2\,ac^2x^3+ax+\left(bc^4x^5+2\,bc^2x^3+bx\right)\operatorname{arsinh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)/(a*c^4*x^5 + 2*a*c^2*x^3 + a*x + (b*c^4*x^5 + 2*
b*c^2*x^3 + b*x)*arcsinh(c*x)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] int(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(c^2\,x^2+1\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)),x)
```

```
[Out] int(1/(x*(a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\,\mathrm{asinh}\left(cx\right)\right)\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(1/(x*(a + b*asinh(c*x))*(c**2*x**2 + 1)**(3/2)), x)
```

3.400 $\int \frac{1}{x^2(1+c^2x^2)^{3/2}(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{1}{x^2\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Rubi [A] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][1/(x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{x^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{1}{x^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 1.12, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[1/(x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}}{ac^4x^6+2\,ac^2x^4+ax^2+\left(bc^4x^6+2\,bc^2x^4+bx^2\right)\operatorname{arsinh}(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)/(a*c^4*x^6 + 2*a*c^2*x^4 + a*x^2 + (b*c^4*x^6 +
2*b*c^2*x^4 + b*x^2)*arcsinh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c^2x^2+1\right)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)*x^2), x)
```

maple [A] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(c^{2} x^{2}+1\right)^{\frac{3}{2}} \left(a +b \,\mathrm{arcsinh}\left(c x \right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] int(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{{\left(c^{2} x^{2} + 1\right)}^{\frac{3}{2}} {\left(b \operatorname{arsinh}\left(c x\right) + a\right)} x^{2}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)*x^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x^2\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(c^2\,x^2+1\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)),x)
```

```
[Out] int(1/(x^2*(a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(a + b \operatorname{asinh}{\left(c x \right)}\right) \left(c^{2} x^{2} + 1\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(1/(x**2*(a + b*asinh(c*x))*(c**2*x**2 + 1)**(3/2)), x)
```

3.401 $\int \frac{x^m\left(1+c^2x^2\right)^{5/2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{\left(c^2x^2+1\right)^{5/2}x^m}{a+b\sinh^{-1}(cx)},x\right)$$

```
[Out] Unintegrable(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x)
```

Rubi [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m\left(1+c^2x^2\right)^{5/2}}{a+b\sinh^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]),x]
[Out] Defer[Int][(x^m*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]), x]
```

Rubi steps

$$\int \frac{x^m\left(1+c^2x^2\right)^{5/2}}{a+b\sinh^{-1}(cx)}\,dx=\int \frac{x^m\left(1+c^2x^2\right)^{5/2}}{a+b\sinh^{-1}(cx)}\,dx$$

Mathematica [A] time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{x^m\left(1+c^2x^2\right)^{5/2}}{a+b\sinh^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]),x]
[Out] Integrate[(x^m*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]), x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^4x^4+2\,c^2x^2+1\right)\sqrt{c^2x^2+1}\,x^m}{b\,\text{arsinh}\left(cx\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)*x^m/(b*arcsinh(c*x) + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [**F(-2)**] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \left(c^2 x^2+1\right)^{\frac{5}{2}}}{a+b \operatorname{arcsinh}\left(c x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x)
```

maxima [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2 x^2+1\right)^{\frac{5}{2}} x^m}{b \operatorname{arsinh}\left(c x\right)+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)*x^m/(b*arcsinh(c*x) + a), x)
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m \left(c^2\, x^2+1\right)^{5/2}}{a+b \operatorname{asinh}\left(c\, x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x)),x)
```

```
[Out] int((x^m*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

3.402 $\int \frac{x^m(1+c^2x^2)^{3/2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{\left(c^2x^2+1\right)^{3/2}x^m}{a+b\sinh^{-1}(cx)},x\right)$$

```
[Out] Unintegrable(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Rubi [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m\left(1+c^2x^2\right)^{3/2}}{a+b\sinh^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]),x]
[Out] Defer[Int][(x^m*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]), x]
```

Rubi steps

$$\int \frac{x^m\left(1+c^2x^2\right)^{3/2}}{a+b\sinh^{-1}(cx)}\,dx = \int \frac{x^m\left(1+c^2x^2\right)^{3/2}}{a+b\sinh^{-1}(cx)}\,dx$$

Mathematica [A] time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{x^m\left(1+c^2x^2\right)^{3/2}}{a+b\sinh^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]),x]
[Out] Integrate[(x^m*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]), x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^2x^2+1\right)^{\frac{3}{2}}x^m}{b\,\text{arsinh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((c^2*x^2 + 1)^(3/2)*x^m/(b*arcsinh(c*x) + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^m (c^2x^2+1)^{\frac{3}{2}}}{a+b\operatorname{arcsinh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c^2x^2+1)^{\frac{3}{2}} x^m}{b\operatorname{arsinh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + 1)^(3/2)*x^m/(b*arcsinh(c*x) + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m (c^2\,x^2+1)^{3/2}}{a+b\operatorname{asinh}(c\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x)),x)
```

```
[Out] int((x^m*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m (c^2x^2+1)^{\frac{3}{2}}}{a+b\operatorname{asinh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x**m*(c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x)), x)
```

3.403 $\int \frac{x^m \sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=30

$$\mathrm{Int}\left(\frac{\sqrt{c^2x^2+1}\,x^m}{a+b\sinh^{-1}(cx)},x\right)$$

```
[Out] Unintegrable(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x)
```

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
[Out] Defer[Int][(x^m*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]), x]
```

Rubi steps

$$\int \frac{x^m\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx = \int \frac{x^m\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx$$

Mathematica [A] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{x^m\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
[Out] Integrate[(x^m*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]), x]
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{c^2x^2+1}\,x^m}{b\,\mathrm{arsinh}(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)*x^m/(b*arcsinh(c*x) + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [**F(-2)**] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m}\sqrt{c^{2}x^{2}+1}}{a+b\,\mathrm{arcsinh}\left(cx\right)}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x)
[Out] int(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x)
```

maxima [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^{2}x^{2}+1}\,x^{m}}{b\,\mathrm{arsinh}\left(cx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(sqrt(c^2*x^2 + 1)*x^m/(b*arcsinh(c*x) + a), x)
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m\,\sqrt{c^2\,x^2+1}}{a+b\,\mathrm{asinh}\left(c\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x)),x)
[Out] int((x^m*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x)), x)
```

sympy [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m}\sqrt{c^{2}x^{2}+1}}{a+b\operatorname{asinh}{\left(cx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x)),x)
[Out] Integral(x**m*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x)), x)
```

3.404 $\int \frac{x^m}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{x^m}{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x)
```

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][x^m/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{x^m}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx = \int \frac{x^m}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[x^m/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

fricas [A] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}\,x^m}{ac^2x^2+\left(bc^2x^2+b\right)\text{arsinh}\left(cx\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)*x^m/(a*c^2*x^2 + (b*c^2*x^2 + b)*arcsinh(c*x) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{c^2x^2+1}\left(b\,\text{arsinh}\left(cx\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(x^m/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

maple [A] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{c^2x^2+1}\ \left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{c^2x^2+1}\,\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\sqrt{c^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(x^m/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(a+b\,\mathrm{asinh}\left(cx\right)\right)\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x**m/((a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

3.405 $\int \frac{x^m}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{x^m}{\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Rubi [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][x^m/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{x^m}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{x^m}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[x^m/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

fricas [A] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}\,x^m}{ac^4x^4+2\,ac^2x^2+\left(bc^4x^4+2\,bc^2x^2+b\right)\text{arsinh}\left(cx\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)*x^m/(a*c^4*x^4 + 2*a*c^2*x^2 + (b*c^4*x^4 + 2*b*
c^2*x^2 + b)*arcsinh(c*x) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(c^2x^2+1\right)^{\frac{3}{2}}\left(b\,\text{arsinh}\left(cx\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate(x^m/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

maple [A] time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(c^2x^2+1\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
[Out] int(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(c^2x^2+1\right)^{\frac{3}{2}}\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(x^m/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(c^2\,x^2+1\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)),x)
[Out] int(x^m/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(a+b\,\mathrm{asinh}\left(cx\right)\right)\left(c^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
[Out] Integral(x**m/((a + b*asinh(c*x))*(c**2*x**2 + 1)**(3/2)), x)
```

3.406 $\int \frac{(c+a^2cx^2)^3}{\sinh^{-1}(ax)^2} dx$

Optimal. Leaf size=94

$$-\frac{c^3(a^2x^2+1)^{7/2}}{a\sinh^{-1}(ax)}+\frac{35c^3\text{Shi}(\sinh^{-1}(ax))}{64a}+\frac{63c^3\text{Shi}(3\sinh^{-1}(ax))}{64a}+\frac{35c^3\text{Shi}(5\sinh^{-1}(ax))}{64a}+\frac{7c^3\text{Shi}(7\sinh^{-1}(ax$$

```
[Out] -c^3*(a^2*x^2+1)^(7/2)/a/arcsinh(a*x)+35/64*c^3*Shi(arcsinh(a*x))/a+63/64*c
^3*Shi(3*arcsinh(a*x))/a+35/64*c^3*Shi(5*arcsinh(a*x))/a+7/64*c^3*Shi(7*arc
sinh(a*x))/a
```

Rubi [A] time = 0.18, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {5696, 5779, 5448, 3298}

$$-\frac{c^3(a^2x^2+1)^{7/2}}{a\sinh^{-1}(ax)}+\frac{35c^3\text{Shi}(\sinh^{-1}(ax))}{64a}+\frac{63c^3\text{Shi}(3\sinh^{-1}(ax))}{64a}+\frac{35c^3\text{Shi}(5\sinh^{-1}(ax))}{64a}+\frac{7c^3\text{Shi}(7\sinh^{-1}(ax$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^3/ArcSinh[a*x]^2,x]
```

```
[Out] -((c^3*(1 + a^2*x^2)^(7/2))/(a*ArcSinh[a*x])) + (35*c^3*SinhIntegral[ArcSin
h[a*x]])/(64*a) + (63*c^3*SinhIntegral[3*ArcSinh[a*x]])/(64*a) + (35*c^3*Si
nhIntegral[5*ArcSinh[a*x]])/(64*a) + (7*c^3*SinhIntegral[7*ArcSinh[a*x]])/(
64*a)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5696

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(Sqrt[1 + c^2*x^2]*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^(n +
 1))/(b*c*(n + 1)), x] - Dist[(c*(2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(b*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ
[e, c^2*d] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^3}{\sinh^{-1}(ax)^2}\,dx = -\frac{c^3\left(1+a^2x^2\right)^{7/2}}{a\sinh^{-1}(ax)} + \left(7ac^3\right)\int \frac{x\left(1+a^2x^2\right)^{5/2}}{\sinh^{-1}(ax)}\,dx$$

$$= -\frac{c^3\left(1+a^2x^2\right)^{7/2}}{a\sinh^{-1}(ax)} + \frac{\left(7c^3\right)\text{Subst}\left(\int \frac{\cosh^6(x)\sinh(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{a}$$

$$= -\frac{c^3\left(1+a^2x^2\right)^{7/2}}{a\sinh^{-1}(ax)} + \frac{\left(7c^3\right)\text{Subst}\left(\int \left(\frac{5\sinh(x)}{64x}+\frac{9\sinh(3x)}{64x}+\frac{5\sinh(5x)}{64x}+\frac{\sinh(7x)}{64x}\right)dx, x, \sinh^{-1}\right.}{a}$$

$$= -\frac{c^3\left(1+a^2x^2\right)^{7/2}}{a\sinh^{-1}(ax)} + \frac{\left(7c^3\right)\text{Subst}\left(\int \frac{\sinh(7x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{64a} + \frac{\left(35c^3\right)\text{Subst}\left(\int \frac{\sinh(x)}{x}\,d\right.}{64a}$$

$$= -\frac{c^3\left(1+a^2x^2\right)^{7/2}}{a\sinh^{-1}(ax)} + \frac{35c^3\text{Shi}\left(\sinh^{-1}(ax)\right)}{64a} + \frac{63c^3\text{Shi}\left(3\sinh^{-1}(ax)\right)}{64a} + \frac{35c^3\text{Shi}\left(5\sinh^{-}\right.}{64a}$$

Mathematica [A] time = 0.50, size = 82, normalized size = 0.87

$$\frac{c^3\left(-64\left(a^2x^2+1\right)^{7/2}+35\sinh^{-1}(ax)\text{Shi}\left(\sinh^{-1}(ax)\right)+63\sinh^{-1}(ax)\text{Shi}\left(3\sinh^{-1}(ax)\right)+35\sinh^{-1}(ax)\text{Shi}\right.}{64a\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^3/ArcSinh[a*x]^2,x]
```

```
[Out] (c^3*(-64*(1 + a^2*x^2)^(7/2) + 35*ArcSinh[a*x]*SinhIntegral[ArcSinh[a*x]]
+ 63*ArcSinh[a*x]*SinhIntegral[3*ArcSinh[a*x]] + 35*ArcSinh[a*x]*SinhIntegr
al[5*ArcSinh[a*x]] + 7*ArcSinh[a*x]*SinhIntegral[7*ArcSinh[a*x]]))/(64*a*Ar
cSinh[a*x])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^6c^3x^6+3\,a^4c^3x^4+3\,a^2c^3x^2+c^3}{\text{arsinh}\,(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)/arcsinh(a*x)^2
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2cx^2+c\right)^3}{\text{arsinh}\,(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3/arcsinh(a*x)^2, x)
```

maple [A] time = 0.13, size = 106, normalized size = 1.13

$$c^3\left(35\,\text{Shi}\,(5\,\text{arcsinh}\,(ax))\,\text{arcsinh}\,(ax)+7\,\text{Shi}\,(7\,\text{arcsinh}\,(ax))\,\text{arcsinh}\,(ax)+35\,\text{Shi}\,(\text{arcsinh}\,(ax))\,\text{arcsinh}\,(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3/arcsinh(a*x)^2,x)
```

```
[Out] 1/64/a*c^3*(35*Shi(5*arcsinh(a*x))*arcsinh(a*x)+7*Shi(7*arcsinh(a*x))*arcsi
nh(a*x)+35*Shi(arcsinh(a*x))*arcsinh(a*x)+63*Shi(3*arcsinh(a*x))*arcsinh(a*
x)-35*(a^2*x^2+1)^(1/2)-cosh(7*arcsinh(a*x))-21*cosh(3*arcsinh(a*x))-7*cosh
(5*arcsinh(a*x)))/arcsinh(a*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^9c^3x^9+4\,a^7c^3x^7+6\,a^5c^3x^5+4\,a^3c^3x^3+ac^3x+\left(a^8c^3x^8+4\,a^6c^3x^6+6\,a^4c^3x^4+4\,a^2c^3x^2+c^3\right)\sqrt{a^2x^2+1}}{\left(a^3x^2+\sqrt{a^2x^2+1}\,a^2x+a\right)\log\left(ax+\sqrt{a^2x^2+1}\right)}+\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^9*c^3*x^9 + 4*a^7*c^3*x^7 + 6*a^5*c^3*x^5 + 4*a^3*c^3*x^3 + a*c^3*x + (
a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*x^2 + c^3)*sqrt(a^2
*x^2 + 1))/((a^3*x^2 + sqrt(a^2*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2
+ 1))) + integrate((7*a^10*c^3*x^10 + 29*a^8*c^3*x^8 + 46*a^6*c^3*x^6 + 34*
a^4*c^3*x^4 + 11*a^2*c^3*x^2 + c^3 + (7*a^8*c^3*x^8 + 20*a^6*c^3*x^6 + 18*a
^4*c^3*x^4 + 4*a^2*c^3*x^2 - c^3)*(a^2*x^2 + 1) + 7*(2*a^9*c^3*x^9 + 7*a^7*
c^3*x^7 + 9*a^5*c^3*x^5 + 5*a^3*c^3*x^3 + a*c^3*x)*sqrt(a^2*x^2 + 1))/((a^4
*x^4 + (a^2*x^2 + 1)*a^2*x^2 + 2*a^2*x^2 + 2*(a^3*x^3 + a*x)*sqrt(a^2*x^2 +
 1) + 1)*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,a^2\,x^2+c\right)}^3}{{\mathrm{asinh}\left(a\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^3/asinh(a*x)^2,x)
```

```
[Out] int((c + a^2*c*x^2)^3/asinh(a*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^3\left(\int \frac{3a^2x^2}{\operatorname{asinh}^2(ax)}\,dx+\int \frac{3a^4x^4}{\operatorname{asinh}^2(ax)}\,dx+\int \frac{a^6x^6}{\operatorname{asinh}^2(ax)}\,dx+\int \frac{1}{\operatorname{asinh}^2(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3/asinh(a*x)**2,x)
```

```
[Out] c**3*(Integral(3*a**2*x**2/asinh(a*x)**2, x) + Integral(3*a**4*x**4/asinh(a
*x)**2, x) + Integral(a**6*x**6/asinh(a*x)**2, x) + Integral(asinh(a*x)**(-
2), x))
```

3.407 $\int \frac{(c+a^2cx^2)^2}{\sinh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=77

$$-\frac{c^2\left(a^2x^2+1\right)^{5/2}}{a\sinh^{-1}(ax)}+\frac{5c^2\text{Shi}\left(\sinh^{-1}(ax)\right)}{8a}+\frac{15c^2\text{Shi}\left(3\sinh^{-1}(ax)\right)}{16a}+\frac{5c^2\text{Shi}\left(5\sinh^{-1}(ax)\right)}{16a}$$

```
[Out] -c^2*(a^2*x^2+1)^(5/2)/a/arcsinh(a*x)+5/8*c^2*Shi(arcsinh(a*x))/a+15/16*c^2
*Shi(3*arcsinh(a*x))/a+5/16*c^2*Shi(5*arcsinh(a*x))/a
```

Rubi [A] time = 0.17, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {5696, 5779, 5448, 3298}

$$-\frac{c^2\left(a^2x^2+1\right)^{5/2}}{a\sinh^{-1}(ax)}+\frac{5c^2\text{Shi}\left(\sinh^{-1}(ax)\right)}{8a}+\frac{15c^2\text{Shi}\left(3\sinh^{-1}(ax)\right)}{16a}+\frac{5c^2\text{Shi}\left(5\sinh^{-1}(ax)\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^2/ArcSinh[a*x]^2,x]
```

```
[Out] -((c^2*(1 + a^2*x^2)^(5/2))/(a*ArcSinh[a*x])) + (5*c^2*SinhIntegral[ArcSinh
[a*x]])/(8*a) + (15*c^2*SinhIntegral[3*ArcSinh[a*x]])/(16*a) + (5*c^2*SinhI
ntegral[5*ArcSinh[a*x]])/(16*a)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5696

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(Sqrt[1 + c^2*x^2]*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^(n +
 1))/(b*c*(n + 1)), x] - Dist[(c*(2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(b*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ
[e, c^2*d] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^2}{\sinh^{-1}(ax)^2}\,dx = -\frac{c^2\left(1+a^2x^2\right)^{5/2}}{a\sinh^{-1}(ax)} + \left(5ac^2\right)\int \frac{x\left(1+a^2x^2\right)^{3/2}}{\sinh^{-1}(ax)}\,dx$$

$$= -\frac{c^2\left(1+a^2x^2\right)^{5/2}}{a\sinh^{-1}(ax)} + \frac{\left(5c^2\right)\text{Subst}\left(\int \frac{\cosh^4(x)\sinh(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{a}$$

$$= -\frac{c^2\left(1+a^2x^2\right)^{5/2}}{a\sinh^{-1}(ax)} + \frac{\left(5c^2\right)\text{Subst}\left(\int \left(\frac{\sinh(x)}{8x}+\frac{3\sinh(3x)}{16x}+\frac{\sinh(5x)}{16x}\right)dx, x, \sinh^{-1}(ax)\right)}{a}$$

$$= -\frac{c^2\left(1+a^2x^2\right)^{5/2}}{a\sinh^{-1}(ax)} + \frac{\left(5c^2\right)\text{Subst}\left(\int \frac{\sinh(5x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{16a} + \frac{\left(5c^2\right)\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x,\right.}{8a}$$

$$= -\frac{c^2\left(1+a^2x^2\right)^{5/2}}{a\sinh^{-1}(ax)} + \frac{5c^2\text{Shi}\left(\sinh^{-1}(ax)\right)}{8a} + \frac{15c^2\text{Shi}\left(3\sinh^{-1}(ax)\right)}{16a} + \frac{5c^2\text{Shi}\left(5\sinh^{-1}(ax)\right)}{16a}$$

Mathematica [A] time = 0.40, size = 69, normalized size = 0.90

$$\frac{c^2\left(-16\left(a^2x^2+1\right)^{5/2}+10\sinh^{-1}(ax)\text{Shi}\left(\sinh^{-1}(ax)\right)+15\sinh^{-1}(ax)\text{Shi}\left(3\sinh^{-1}(ax)\right)+5\sinh^{-1}(ax)\text{Shi}\left(5\,\text{si}\right.\right.}{16a\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^2/ArcSinh[a*x]^2,x]
```

```
[Out] (c^2*(-16*(1 + a^2*x^2)^(5/2) + 10*ArcSinh[a*x]*SinhIntegral[ArcSinh[a*x]]
+ 15*ArcSinh[a*x]*SinhIntegral[3*ArcSinh[a*x]] + 5*ArcSinh[a*x]*SinhIntegra
l[5*ArcSinh[a*x]]))/(16*a*ArcSinh[a*x])
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^4c^2x^4+2\,a^2c^2x^2+c^2}{\text{arsinh}\,(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)/arcsinh(a*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2cx^2+c\right)^2}{\text{arsinh}\,(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2/arcsinh(a*x)^2, x)
```

maple [A] time = 0.07, size = 84, normalized size = 1.09

$$\frac{c^2\left(10\,\text{Shi}\,(\text{arcsinh}\,(ax))\,\text{arcsinh}\,(ax)+15\,\text{Shi}\,(3\,\text{arcsinh}\,(ax))\,\text{arcsinh}\,(ax)+5\,\text{Shi}\,(5\,\text{arcsinh}\,(ax))\,\text{arcsinh}\,(ax)\right.}{16a\,\text{arcsinh}\,(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2/arcsinh(a*x)^2,x)
```

```
[Out] 1/16/a*c^2*(10*Shi(arcsinh(a*x))*arcsinh(a*x)+15*Shi(3*arcsinh(a*x))*arcsin
h(a*x)+5*Shi(5*arcsinh(a*x))*arcsinh(a*x)-10*(a^2*x^2+1)^(1/2)-5*cosh(3*arc
sinh(a*x))-cosh(5*arcsinh(a*x)))/arcsinh(a*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^7c^2x^7+3\,a^5c^2x^5+3\,a^3c^2x^3+ac^2x+\left(a^6c^2x^6+3\,a^4c^2x^4+3\,a^2c^2x^2+c^2\right)\sqrt{a^2x^2+1}}{\left(a^3x^2+\sqrt{a^2x^2+1}\,a^2x+a\right)\log\left(ax+\sqrt{a^2x^2+1}\right)}+\int\frac{5\,a^8c^2x^8+16\,a^6c^2x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^7*c^2*x^7 + 3*a^5*c^2*x^5 + 3*a^3*c^2*x^3 + a*c^2*x + (a^6*c^2*x^6 + 3*
a^4*c^2*x^4 + 3*a^2*c^2*x^2 + c^2)*sqrt(a^2*x^2 + 1))/((a^3*x^2 + sqrt(a^2*
x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))) + integrate((5*a^8*c^2*x^
8 + 16*a^6*c^2*x^6 + 18*a^4*c^2*x^4 + 8*a^2*c^2*x^2 + (5*a^6*c^2*x^6 + 9*a^
4*c^2*x^4 + 3*a^2*c^2*x^2 - c^2)*(a^2*x^2 + 1) + c^2 + 5*(2*a^7*c^2*x^7 + 5
*a^5*c^2*x^5 + 4*a^3*c^2*x^3 + a*c^2*x)*sqrt(a^2*x^2 + 1))/((a^4*x^4 + (a^2
*x^2 + 1)*a^2*x^2 + 2*a^2*x^2 + 2*(a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1) + 1)*lo
g(a*x + sqrt(a^2*x^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,a^2\,x^2+c\right)}^2}{{\mathrm{asinh}\left(a\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^2/asinh(a*x)^2,x)
```

```
[Out] int((c + a^2*c*x^2)^2/asinh(a*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int\frac{2a^2x^2}{\operatorname{asinh}^2(ax)}\,dx+\int\frac{a^4x^4}{\operatorname{asinh}^2(ax)}\,dx+\int\frac{1}{\operatorname{asinh}^2(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2/asinh(a*x)**2,x)
```

```
[Out] c**2*(Integral(2*a**2*x**2/asinh(a*x)**2, x) + Integral(a**4*x**4/asinh(a*x
)**2, x) + Integral(asinh(a*x)**(-2), x))
```

3.408 $\int \frac{c+a^2cx^2}{\sinh^{-1}(ax)^2} \, dx$

Optimal. Leaf size=54

$$-\frac{c\left(a^2x^2+1\right)^{3/2}}{a\sinh^{-1}(ax)}+\frac{3c\text{Shi}\left(\sinh^{-1}(ax)\right)}{4a}+\frac{3c\text{Shi}\left(3\sinh^{-1}(ax)\right)}{4a}$$

```
[Out] -c*(a^2*x^2+1)^(3/2)/a/arcsinh(a*x)+3/4*c*Shi(arcsinh(a*x))/a+3/4*c*Shi(3*a
rcsinh(a*x))/a
```

Rubi [A] time = 0.14, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {5696, 5779, 5448, 3298}

$$-\frac{c\left(a^2x^2+1\right)^{3/2}}{a\sinh^{-1}(ax)}+\frac{3c\text{Shi}\left(\sinh^{-1}(ax)\right)}{4a}+\frac{3c\text{Shi}\left(3\sinh^{-1}(ax)\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)/ArcSinh[a*x]^2,x]
```

```
[Out] -((c*(1 + a^2*x^2)^(3/2))/(a*ArcSinh[a*x])) + (3*c*SinhIntegral[ArcSinh[a*x
]])/(4*a) + (3*c*SinhIntegral[3*ArcSinh[a*x]])/(4*a)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5696

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(Sqrt[1 + c^2*x^2]*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^(n +
 1))/(b*c*(n + 1)), x] - Dist[(c*(2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(b*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ
[e, c^2*d] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{c+a^2cx^2}{\sinh^{-1}(ax)^2}\,dx = -\frac{c\left(1+a^2x^2\right)^{3/2}}{a\sinh^{-1}(ax)} + (3ac)\int \frac{x\sqrt{1+a^2x^2}}{\sinh^{-1}(ax)}\,dx$$
$$= -\frac{c\left(1+a^2x^2\right)^{3/2}}{a\sinh^{-1}(ax)} + \frac{(3c)\,\mathrm{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{a}$$
$$= -\frac{c\left(1+a^2x^2\right)^{3/2}}{a\sinh^{-1}(ax)} + \frac{(3c)\,\mathrm{Subst}\left(\int \left(\frac{\sinh(x)}{4x}+\frac{\sinh(3x)}{4x}\right)dx, x, \sinh^{-1}(ax)\right)}{a}$$
$$= -\frac{c\left(1+a^2x^2\right)^{3/2}}{a\sinh^{-1}(ax)} + \frac{(3c)\,\mathrm{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{4a} + \frac{(3c)\,\mathrm{Subst}\left(\int \frac{\sinh(3x)}{x}\,dx, x, s\right.}{4a}$$
$$= -\frac{c\left(1+a^2x^2\right)^{3/2}}{a\sinh^{-1}(ax)} + \frac{3c\mathrm{Shi}\left(\sinh^{-1}(ax)\right)}{4a} + \frac{3c\mathrm{Shi}\left(3\sinh^{-1}(ax)\right)}{4a}$$

Mathematica [A] time = 0.17, size = 54, normalized size = 1.00

$$\frac{c\left(-4\left(a^2x^2+1\right)^{3/2}+3\sinh^{-1}(ax)\mathrm{Shi}\left(\sinh^{-1}(ax)\right)+3\sinh^{-1}(ax)\mathrm{Shi}\left(3\sinh^{-1}(ax)\right)\right)}{4a\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)/ArcSinh[a*x]^2,x]
```

```
[Out] (c*(-4*(1 + a^2*x^2)^(3/2) + 3*ArcSinh[a*x]*SinhIntegral[ArcSinh[a*x]] + 3*
ArcSinh[a*x]*SinhIntegral[3*ArcSinh[a*x]]))/(4*a*ArcSinh[a*x])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{a^2cx^2+c}{\operatorname{arsinh}(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)/arcsinh(a*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^2cx^2+c}{\operatorname{arsinh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)/arcsinh(a*x)^2, x)
```

maple [A] time = 0.06, size = 60, normalized size = 1.11

$$\frac{c\left(3\,\mathrm{Shi}\left(\operatorname{arcsinh}(ax)\right)\operatorname{arcsinh}(ax)+3\,\mathrm{Shi}\left(3\operatorname{arcsinh}(ax)\right)\operatorname{arcsinh}(ax)-3\sqrt{a^2x^2+1}-\cosh\left(3\operatorname{arcsinh}(ax\right.\right.}{4a\operatorname{arcsinh}(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)/arcsinh(a*x)^2,x)
```

```
[Out] 1/4/a*c*(3*Shi(arcsinh(a*x))*arcsinh(a*x)+3*Shi(3*arcsinh(a*x))*arcsinh(a*x
)-3*(a^2*x^2+1)^(1/2)-cosh(3*arcsinh(a*x)))/arcsinh(a*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^5cx^5 + 2\,a^3cx^3 + acx + \left(a^4cx^4 + 2\,a^2cx^2 + c\right)\sqrt{a^2x^2+1}}{\left(a^3x^2 + \sqrt{a^2x^2+1}\,a^2x + a\right)\log\left(ax + \sqrt{a^2x^2+1}\right)} + \int \frac{3\,a^6cx^6 + 7\,a^4cx^4 + 5\,a^2cx^2 + \left(3\,a^4cx^4 + 2\,a^2cx^2 - \right.}{\left(a^4x^4 + \left(a^2x^2+1\right)a^2x^2 + 2\,a^2x^2 + 2\left(a^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^5*c*x^5 + 2*a^3*c*x^3 + a*c*x + (a^4*c*x^4 + 2*a^2*c*x^2 + c)*sqrt(a^2*
x^2 + 1))/((a^3*x^2 + sqrt(a^2*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 +
 1))) + integrate((3*a^6*c*x^6 + 7*a^4*c*x^4 + 5*a^2*c*x^2 + (3*a^4*c*x^4 +
 2*a^2*c*x^2 - c)*(a^2*x^2 + 1) + 3*(2*a^5*c*x^5 + 3*a^3*c*x^3 + a*c*x)*sqr
t(a^2*x^2 + 1) + c)/((a^4*x^4 + (a^2*x^2 + 1)*a^2*x^2 + 2*a^2*x^2 + 2*(a^3*
x^3 + a*x)*sqrt(a^2*x^2 + 1) + 1)*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{c\,a^2\,x^2 + c}{{\mathrm{asinh}\left(a\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)/asinh(a*x)^2,x)
```

```
[Out] int((c + a^2*c*x^2)/asinh(a*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int \frac{a^2x^2}{\mathrm{asinh}^2\left(ax\right)}\,dx + \int \frac{1}{\mathrm{asinh}^2\left(ax\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)/asinh(a*x)**2,x)
```

```
[Out] c*(Integral(a**2*x**2/asinh(a*x)**2, x) + Integral(asinh(a*x)**(-2), x))
```

3.409 $\int \frac{1}{\left(c+a^2cx^2\right)\sinh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=58

$$-\frac{a\text{Int}\left(\frac{x}{\left(a^2x^2+1\right)^{3/2}\sinh^{-1}(ax)},x\right)}{c}-\frac{1}{ac\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}$$

```
[Out] -1/a/c/arcsinh(a*x)/(a^2*x^2+1)^(1/2)-a*Unintegrable(x/(a^2*x^2+1)^(3/2)/ar
csinh(a*x),x)/c
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(c+a^2cx^2\right)\sinh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + a^2*c*x^2)*ArcSinh[a*x]^2),x]
```

```
[Out] -(1/(a*c*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])) - (a*Defer[Int][x/((1 + a^2*x^2)^
(3/2)*ArcSinh[a*x]), x])/c
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)\sinh^{-1}(ax)^2}\,dx=-\frac{1}{ac\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}-\frac{a\int \frac{x}{\left(1+a^2x^2\right)^{3/2}\sinh^{-1}(ax)}\,dx}{c}$$

Mathematica [A] time = 1.53, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c+a^2cx^2\right)\sinh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + a^2*c*x^2)*ArcSinh[a*x]^2),x]
```

```
[Out] Integrate[1/((c + a^2*c*x^2)*ArcSinh[a*x]^2), x]
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\left(a^2cx^2+c\right)\operatorname{arsinh}(ax)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((a^2*c*x^2 + c)*arcsinh(a*x)^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2cx^2+c\right)\operatorname{arsinh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)*arcsinh(a*x)^2), x)
```

maple [A] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{2} c\, x^{2}+c\right) \operatorname{arcsinh}\left(a x\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)/arcsinh(a*x)^2,x)
```

```
[Out] int(1/(a^2*c*x^2+c)/arcsinh(a*x)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{ax+\sqrt{a^2x^2+1}}{\left(a^3cx^2+\sqrt{a^2x^2+1}\,a^2cx+ac\right)\log\left(ax+\sqrt{a^2x^2+1}\right)}-\int\frac{a^4x^4+\left(a^2x^2+1\right)}{\left(a^6cx^6+3\,a^4cx^4+3\,a^2cx^2+\left(a^4cx^4+a^2cx^2\right)\left(a^2x^2+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a*x + sqrt(a^2*x^2 + 1))/((a^3*c*x^2 + sqrt(a^2*x^2 + 1)*a^2*c*x + a*c)*l
og(a*x + sqrt(a^2*x^2 + 1))) - integrate((a^4*x^4 + (a^2*x^2 + 1)^2 + (2*a^
3*x^3 + a*x)*sqrt(a^2*x^2 + 1) - 1)/((a^6*c*x^6 + 3*a^4*c*x^4 + 3*a^2*c*x^2
 + (a^4*c*x^4 + a^2*c*x^2)*(a^2*x^2 + 1) + 2*(a^5*c*x^5 + 2*a^3*c*x^3 + a*c
*x)*sqrt(a^2*x^2 + 1) + c)*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\mathrm{asinh}\left(a\,x\right)^2\,\left(c\,a^2\,x^2+c\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(asinh(a*x)^2*(c + a^2*c*x^2)),x)
```

```
[Out] int(1/(asinh(a*x)^2*(c + a^2*c*x^2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^{2} x^{2} \operatorname{asinh}^{2}{\left(a x \right)} + \operatorname{asinh}^{2}{\left(a x \right)}}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)/asinh(a*x)**2,x)
```

```
[Out] Integral(1/(a**2*x**2*asinh(a*x)**2 + asinh(a*x)**2), x)/c
```

3.410 $\int \frac{1}{\left(c+a^2cx^2\right)^2 \sinh^{-1}(ax)^2} dx$

Optimal. Leaf size=58

$$-\frac{3a\text{Int}\left(\frac{x}{\left(a^2x^2+1\right)^{5/2}\sinh^{-1}(ax)},x\right)}{c^2}-\frac{1}{ac^2\left(a^2x^2+1\right)^{3/2}\sinh^{-1}(ax)}$$

```
[Out] -1/a/c^2/(a^2*x^2+1)^(3/2)/arcsinh(a*x)-3*a*Unintegrable(x/(a^2*x^2+1)^(5/2
)/arcsinh(a*x),x)/c^2
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(c+a^2cx^2\right)^2 \sinh^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + a^2*c*x^2)^2*ArcSinh[a*x]^2),x]
```

```
[Out] -(1/(a*c^2*(1 + a^2*x^2)^(3/2)*ArcSinh[a*x])) - (3*a*Defer[Int][x/((1 + a^2
*x^2)^(5/2)*ArcSinh[a*x]), x])/c^2
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)^2 \sinh^{-1}(ax)^2} dx=-\frac{1}{ac^2\left(1+a^2x^2\right)^{3/2}\sinh^{-1}(ax)}-\frac{(3a)\int \frac{x}{\left(1+a^2x^2\right)^{5/2}\sinh^{-1}(ax)} dx}{c^2}$$

Mathematica [A] time = 3.77, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c+a^2cx^2\right)^2 \sinh^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + a^2*c*x^2)^2*ArcSinh[a*x]^2),x]
```

```
[Out] Integrate[1/((c + a^2*c*x^2)^2*ArcSinh[a*x]^2), x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\left(a^4c^2x^4+2\,a^2c^2x^2+c^2\right)\text{arsinh}\left(ax\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arcsinh(a*x)^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2cx^2+c\right)^2 \text{arsinh}\left(ax\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^2*arcsinh(a*x)^2), x)
```

maple [A] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2 c\, x^2 + c\right)^2 \operatorname{arcsinh}(ax)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^2/arcsinh(a*x)^2,x)
```

```
[Out] int(1/(a^2*c*x^2+c)^2/arcsinh(a*x)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{ax + \sqrt{a^2x^2+1}}{\left(a^5c^2x^4 + 2\,a^3c^2x^2 + ac^2 + \left(a^4c^2x^3 + a^2c^2x\right)\sqrt{a^2x^2+1}\right)\log\left(ax + \sqrt{a^2x^2+1}\right)} - \int \frac{}{\left(a^8c^2x^8 + 4\,a^6c^2x^6 + 6\,a^4c^2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a*x + sqrt(a^2*x^2 + 1))/((a^5*c^2*x^4 + 2*a^3*c^2*x^2 + a*c^2 + (a^4*c^2
*x^3 + a^2*c^2*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))) - integr
ate((3*a^4*x^4 + 2*a^2*x^2 + (3*a^2*x^2 + 1)*(a^2*x^2 + 1) + 3*(2*a^3*x^3 +
 a*x)*sqrt(a^2*x^2 + 1) - 1)/((a^8*c^2*x^8 + 4*a^6*c^2*x^6 + 6*a^4*c^2*x^4
+ 4*a^2*c^2*x^2 + (a^6*c^2*x^6 + 2*a^4*c^2*x^4 + a^2*c^2*x^2)*(a^2*x^2 + 1)
 + c^2 + 2*(a^7*c^2*x^7 + 3*a^5*c^2*x^5 + 3*a^3*c^2*x^3 + a*c^2*x)*sqrt(a^2
*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\operatorname{asinh}(a\,x)^2 \left(c\,a^2\,x^2 + c\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(asinh(a*x)^2*(c + a^2*c*x^2)^2),x)
```

```
[Out] int(1/(asinh(a*x)^2*(c + a^2*c*x^2)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^4x^4\operatorname{asinh}^2(ax) + 2a^2x^2\operatorname{asinh}^2(ax) + \operatorname{asinh}^2(ax)}\, dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**2/asinh(a*x)**2,x)
```

```
[Out] Integral(1/(a**4*x**4*asinh(a*x)**2 + 2*a**2*x**2*asinh(a*x)**2 + asinh(a*x
)**2), x)/c**2
```

3.411 $\int \frac{x^3\sqrt{1+c^2x^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=213

$$-\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8b^2c^4}-\frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16b^2c^4}+\frac{5\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16b^2c^4}+\frac{\sinh\left(\frac{a}{b}\right)\text{S}}{}$$

```
[Out] -x^3*(c^2*x^2+1)/b/c/(a+b*arcsinh(c*x))-1/8*Chi((a+b*arcsinh(c*x))/b)*cosh(
a/b)/b^2/c^4-3/16*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b^2/c^4+5/16*Chi(
5*(a+b*arcsinh(c*x))/b)*cosh(5*a/b)/b^2/c^4+1/8*Shi((a+b*arcsinh(c*x))/b)*s
inh(a/b)/b^2/c^4+3/16*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^4-5/16*
Shi(5*(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b^2/c^4
```

Rubi [A] time = 0.71, antiderivative size = 209, normalized size of antiderivative = 0.98, number of steps used = 22, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5777, 5669, 5448, 3303, 3298, 3301}

$$-\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{8b^2c^4}-\frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{16b^2c^4}+\frac{5\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5a}{b}+5\sinh^{-1}(cx)\right)}{16b^2c^4}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((x^3*(1 + c^2*x^2))/(b*c*(a + b*ArcSinh[c*x]))) - (Cosh[a/b]*CoshIntegral
[a/b + ArcSinh[c*x]])/(8*b^2*c^4) - (3*Cosh[(3*a)/b]*CoshIntegral[(3*a)/b +
 3*ArcSinh[c*x]])/(16*b^2*c^4) + (5*Cosh[(5*a)/b]*CoshIntegral[(5*a)/b + 5*
ArcSinh[c*x]])/(16*b^2*c^4) + (Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/
(8*b^2*c^4) + (3*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(16*
b^2*c^4) - (5*Sinh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcSinh[c*x]])/(16*b^2
*c^4)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
```

```
& IGtQ[p, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^
p*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-Dist[(f*m*d^IntPart[p
]*(d + e*x^2)^FracPart[p])/(b*c*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*
x)^(m - 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - D
ist[(c*(m + 2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(b*f*(n + 1)*(1
+ c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*A
rcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[2*p, 0]
```

Rubi steps

$$\int \frac{x^3\sqrt{1+c^2x^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{x^3\left(1+c^2x^2\right)}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{3\int \frac{x^2}{a+b\sinh^{-1}(cx)}\,dx}{bc} + \frac{(5c)\int \frac{x^4}{a+b\sinh^{-1}(cx)}\,dx}{b}$$

$$= -\frac{x^3\left(1+c^2x^2\right)}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{3\,\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{bc^4} + \frac{5\,\text{Subst}\left(\int \frac{\text{c}}{}\right.}{}$$

$$= -\frac{x^3\left(1+c^2x^2\right)}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{3\,\text{Subst}\left(\int\left(-\frac{\cosh(x)}{4(a+bx)}+\frac{\cosh(3x)}{4(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{bc^4} + \frac{5\,\text{Sub}}{}$$

$$= -\frac{x^3\left(1+c^2x^2\right)}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{5\,\text{Subst}\left(\int \frac{\cosh(5x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{16bc^4} + \frac{5\,\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\right.}{8bc}$$

$$= -\frac{x^3\left(1+c^2x^2\right)}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\left(5\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{8bc^4} - \frac{(3\cos}{}$$

$$= -\frac{x^3\left(1+c^2x^2\right)}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{8b^2c^4} - \frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\,\text{s}\right.}{16b^2c^4}$$

Mathematica [A] time = 0.67, size = 175, normalized size = 0.82

$$-\frac{\frac{16bc^5x^5}{a+b\sinh^{-1}(cx)} + \frac{16bc^3x^3}{a+b\sinh^{-1}(cx)} + 2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right) + 3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) - 5\cosh\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -1/16*((16*b*c^3*x^3)/(a + b*ArcSinh[c*x]) + (16*b*c^5*x^5)/(a + b*ArcSinh[
c*x]) + 2*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] + 3*Cosh[(3*a)/b]*Cosh
Integral[3*(a/b + ArcSinh[c*x])] - 5*Cosh[(5*a)/b]*CoshIntegral[5*(a/b + Ar
```

```
cSinh[c*x])] - 2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - 3*Sinh[(3*a)/
b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] + 5*Sinh[(5*a)/b]*SinhIntegral[5*(a
/b + ArcSinh[c*x])])/(b^2*c^4)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}\,x^3}{b^2\operatorname{arsinh}(cx)^2+2\,ab\operatorname{arsinh}(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^3/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a
^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.50, size = 633, normalized size = 2.97

$$-\frac{16c^5x^5-16c^4x^4\sqrt{c^2x^2+1}+20c^3x^3-12c^2x^2\sqrt{c^2x^2+1}+5cx-\sqrt{c^2x^2+1}}{32c^4b\,(a+b\operatorname{arcsinh}(cx))}-\frac{5\,e^{\frac{5a}{b}}\operatorname{Ei}\left(1,5\operatorname{arcsinh}(cx)+\frac{5a}{b}\right)}{32c^4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] -1/32*(16*c^5*x^5-16*c^4*x^4*(c^2*x^2+1)^(1/2)+20*c^3*x^3-12*c^2*x^2*(c^2*x
^2+1)^(1/2)+5*c*x-(c^2*x^2+1)^(1/2))/c^4/b/(a+b*arcsinh(c*x))-5/32/c^4/b^2*
exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)+1/32*(4*c^3*x^3-4*c^2*x^2*(c^2*x^2+1)
^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))/c^4/b/(a+b*arcsinh(c*x))+3/32/c^4/b^2*exp(3
*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)+1/16*(c*x-(c^2*x^2+1)^(1/2))/c^4/b/(a+b*ar
csinh(c*x))+1/16/c^4/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)+1/16/c^4/b^2*(arcs
inh(c*x)*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*b+exp(-a/b)*Ei(1,-arcsinh(c*x)-a
/b)*a+x*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))+1/32/c^4/b^2*(4*x^3*b*c
^3+4*(c^2*x^2+1)^(1/2)*x^2*b*c^2+3*arcsinh(c*x)*exp(-3*a/b)*Ei(1,-3*arcsinh
(c*x)-3*a/b)*b+3*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)*a+3*x*b*c+(c^2*x^2
+1)^(1/2)*b)/(a+b*arcsinh(c*x))-1/32/c^4/b^2*(16*x^5*b*c^5+16*(c^2*x^2+1)^(
1/2)*x^4*b*c^4+20*x^3*b*c^3+12*(c^2*x^2+1)^(1/2)*x^2*b*c^2+5*arcsinh(c*x)*e
xp(-5*a/b)*Ei(1,-5*arcsinh(c*x)-5*a/b)*b+5*exp(-5*a/b)*Ei(1,-5*arcsinh(c*x)
-5*a/b)*a+5*x*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(c^2x^5+x^3\right)\left(c^2x^2+1\right)+\left(c^3x^6+cx^4\right)\sqrt{c^2x^2+1}}{abc^3x^2+\sqrt{c^2x^2+1}\,abc^2x+abc+\left(b^2c^3x^2+\sqrt{c^2x^2+1}\,b^2c^2x+b^2c\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int\frac{}{abc^5x^4+\left(c^2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^5 + x^3)*(c^2*x^2 + 1) + (c^3*x^6 + c*x^4)*sqrt(c^2*x^2 + 1))/(a*b
*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^
2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((5*c^3
*x^5 + 2*c*x^3)*(c^2*x^2 + 1)^(3/2) + (10*c^4*x^6 + 11*c^2*x^4 + 3*x^2)*(c^
2*x^2 + 1) + (5*c^5*x^7 + 9*c^3*x^5 + 4*c*x^3)*sqrt(c^2*x^2 + 1))/(a*b*c^5*
x^4 + (c^2*x^2 + 1)*a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c
^2*x^2 + 1)*b^2*c^3*x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*
x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c
^2*x)*sqrt(c^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\sqrt{c^2\,x^2+1}}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x))^2,x)
```

```
[Out] int((x^3*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \sqrt{c^{2} x^{2} + 1}}{\left(a + b \operatorname{asinh}{\left(c x \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x**3*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x))**2, x)
```

3.412 $\int \frac{x^2\sqrt{1+c^2x^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=93

$$-\frac{\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{2b^2c^3}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{2b^2c^3}-\frac{x^2\left(c^2x^2+1\right)}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

```
[Out] -x^2*(c^2*x^2+1)/b/c/(a+b*arcsinh(c*x))+1/2*cosh(4*a/b)*Shi(4*(a+b*arcsinh(
c*x))/b)/b^2/c^3-1/2*Chi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b^2/c^3
```

Rubi [A] time = 0.56, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {5777, 5669, 5448, 12, 3303, 3298, 3301}

$$-\frac{\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{2b^2c^3}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{2b^2c^3}-\frac{x^2\left(c^2x^2+1\right)}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((x^2*(1 + c^2*x^2))/(b*c*(a + b*ArcSinh[c*x]))) - (CoshIntegral[(4*a)/b +
 4*ArcSinh[c*x]]*Sinh[(4*a)/b])/(2*b^2*c^3) + (Cosh[(4*a)/b]*SinhIntegral[(
4*a)/b + 4*ArcSinh[c*x]])/(2*b^2*c^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^
p*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-Dist[(f*m*d^IntPart[p
]*(d + e*x^2)^FracPart[p])/(b*c*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*
x)^(m - 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - D
ist[(c*(m + 2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(b*f*(n + 1)*(1
+ c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*A
rcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[2*p, 0]
```

Rubi steps

$$\int \frac{x^2\sqrt{1+c^2x^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{x^2\left(1+c^2x^2\right)}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{2\int \frac{x}{a+b\sinh^{-1}(cx)}\,dx}{bc}+\frac{(4c)\int \frac{x^3}{a+b\sinh^{-1}(cx)}\,dx}{b}$$

$$= -\frac{x^2\left(1+c^2x^2\right)}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{2\,\mathrm{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{bc^3}+\frac{4\,\mathrm{Subst}\left(\int \frac{\cos\ldots}{\ldots}\right.}{\ldots}$$

$$= -\frac{x^2\left(1+c^2x^2\right)}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{2\,\mathrm{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)}\,dx,x,\sinh^{-1}(cx)\right)}{bc^3}+\frac{4\,\mathrm{Subst}\left(\int\left(-\frac{\sinh(2\ldots}{4(a+b\ldots}\right.\right.}{\ldots}$$

$$= -\frac{x^2\left(1+c^2x^2\right)}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\mathrm{Subst}\left(\int \frac{\sinh(4x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{2bc^3}$$

$$= -\frac{x^2\left(1+c^2x^2\right)}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\cosh\left(\frac{4a}{b}\right)\mathrm{Subst}\left(\int \frac{\sinh\left(\frac{4a}{b}+4x\right)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{2bc^3}-\frac{\sinh\left(\ldots\right.}{\ldots}$$

$$= -\frac{x^2\left(1+c^2x^2\right)}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{\mathrm{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)\sinh\left(\frac{4a}{b}\right)}{2b^2c^3}+\frac{\cosh\left(\frac{4a}{b}\right)\mathrm{Shi}\left(\frac{4a}{b}+4\ldots\right.}{2b^2c^3}$$

Mathematica [A] time = 0.32, size = 82, normalized size = 0.88

$$\frac{-\frac{2bc^2x^2\left(c^2x^2+1\right)}{a+b\sinh^{-1}(cx)}-\sinh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+\cosh\left(\frac{4a}{b}\right)\mathrm{Shi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{2b^2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] ((-2*b*c^2*x^2*(1 + c^2*x^2))/(a + b*ArcSinh[c*x]) - CoshIntegral[4*(a/b +
ArcSinh[c*x])]*Sinh[(4*a)/b] + Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[
c*x])])/(2*b^2*c^3)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}\,x^2}{b^2\,\text{arsinh}\,(cx)^2+2\,ab\,\text{arsinh}\,(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^2/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a
^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}\,x^2}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)*x^2/(b*arcsinh(c*x) + a)^2, x)
```

maple [B] time = 0.28, size = 248, normalized size = 2.67

$$\frac{1}{8c^3\,(a+b\,\text{arcsinh}\,(cx))\,b}-\frac{8c^4x^4-8c^3x^3\sqrt{c^2x^2+1}+8c^2x^2-4cx\sqrt{c^2x^2+1}+1}{16c^3\,(a+b\,\text{arcsinh}\,(cx))\,b}+\frac{e^{\frac{4a}{b}}\,\text{Ei}\left(1,4\,\text{arcsinh}\,(cx)+\frac{4}{l}\right.}{4c^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/8/c^3/(a+b*arcsinh(c*x))/b-1/16*(8*c^4*x^4-8*c^3*x^3*(c^2*x^2+1)^(1/2)+8*
c^2*x^2-4*c*x*(c^2*x^2+1)^(1/2)+1)/c^3/(a+b*arcsinh(c*x))/b+1/4/c^3/b^2*exp
(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)-1/16/c^3/b^2*(8*x^4*b*c^4+8*(c^2*x^2+1)^
(1/2)*x^3*b*c^3+8*x^2*b*c^2+4*b*c*(c^2*x^2+1)^(1/2)*x+4*arcsinh(c*x)*Ei(1,-
4*arcsinh(c*x)-4*a/b)*exp(-4*a/b)*b+4*Ei(1,-4*arcsinh(c*x)-4*a/b)*exp(-4*a/
b)*a+b)/(a+b*arcsinh(c*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(c^2x^4+x^2\right)\left(c^2x^2+1\right)+\left(c^3x^5+cx^3\right)\sqrt{c^2x^2+1}}{abc^3x^2+\sqrt{c^2x^2+1}\,abc^2x+abc+\left(b^2c^3x^2+\sqrt{c^2x^2+1}\,b^2c^2x+b^2c\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int\frac{}{abc^5x^4+\left(c^2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^4 + x^2)*(c^2*x^2 + 1) + (c^3*x^5 + c*x^3)*sqrt(c^2*x^2 + 1))/(a*b
*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^
2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((4*c^3
*x^4 + c*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(4*c^4*x^5 + 4*c^2*x^3 + x)*(c^2*x^2
+ 1) + (4*c^5*x^6 + 7*c^3*x^4 + 3*c*x^2)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 +
(c^2*x^2 + 1)*a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2
 + 1)*b^2*c^3*x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqr
t(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*
sqrt(c^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{c^2\,x^2+1}}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x))^2,x)
```

```
[Out] int((x^2*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}\sqrt{c^{2}x^{2}+1}}{\left(a+b\operatorname{asinh}\left(cx\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x**2*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x))**2, x)
```

3.413 $\int \frac{x\sqrt{1+c^2x^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=149

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4b^2c^2}+\frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{4b^2c^2}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4b^2c^2}-\frac{3\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\right.}{4b^2}$$

```
[Out] -x*(c^2*x^2+1)/b/c/(a+b*arcsinh(c*x))+1/4*Chi((a+b*arcsinh(c*x))/b)*cosh(a/
b)/b^2/c^2+3/4*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b^2/c^2-1/4*Shi((a+b
*arcsinh(c*x))/b)*sinh(a/b)/b^2/c^2-3/4*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*
a/b)/b^2/c^2
```

Rubi [A] time = 0.41, antiderivative size = 198, normalized size of antiderivative = 1.33, number of steps used = 14, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {5777, 5657, 3303, 3298, 3301, 5669, 5448}

$$-\frac{3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4b^2c^2}+\frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{4b^2c^2}+\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c^2}+\frac{3\sinh}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((x*(1 + c^2*x^2))/(b*c*(a + b*ArcSinh[c*x]))) - (3*Cosh[a/b]*CoshIntegral
[a/b + ArcSinh[c*x]])/(4*b^2*c^2) + (3*Cosh[(3*a)/b]*CoshIntegral[(3*a)/b +
 3*ArcSinh[c*x]])/(4*b^2*c^2) + (Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*x]
)/b])/(b^2*c^2) + (3*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(4*b^2*c^2
) - (3*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(4*b^2*c^2) -
(Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(b^2*c^2)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^
p*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-Dist[(f*m*d^IntPart[p
]*(d + e*x^2)^FracPart[p])/(b*c*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*
x)^(m - 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - D
ist[(c*(m + 2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(b*f*(n + 1)*(1
+ c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*A
rcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[2*p, 0]
```

Rubi steps

$$\int \frac{x\sqrt{1+c^2x^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{x\left(1+c^2x^2\right)}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\int \frac{1}{a+b\sinh^{-1}(cx)}\,dx}{bc}+\frac{(3c)\int \frac{x^2}{a+b\sinh^{-1}(cx)}\,dx}{b}$$

$$= -\frac{x\left(1+c^2x^2\right)}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\sinh^{-1}(cx)\right)}{b^2c^2}+\frac{3\operatorname{Subst}\left(\int \frac{c}{}\right.}{}$$

$$= -\frac{x\left(1+c^2x^2\right)}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{3\operatorname{Subst}\left(\int \left(-\frac{\cosh(x)}{4(a+bx)}+\frac{\cosh(3x)}{4(a+bx)}\right)dx,x,\sinh^{-1}(cx)\right)}{bc^2}+\frac{\cosh}{}$$

$$= -\frac{x\left(1+c^2x^2\right)}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c^2}-\frac{\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c^2}$$

$$= -\frac{x\left(1+c^2x^2\right)}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c^2}-\frac{\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c^2}$$

$$= -\frac{x\left(1+c^2x^2\right)}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{3\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4b^2c^2}+\frac{3\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3a}{b}+3\right.}{4b^2c^2}$$

Mathematica [A] time = 0.38, size = 126, normalized size = 0.85

$$-\frac{\frac{4bc^3x^3}{a+b\sinh^{-1}(cx)}-\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-3\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a}{b}+\sinh^{-1}\right.}{4b^2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -1/4*((4*b*c*x)/(a + b*ArcSinh[c*x]) + (4*b*c^3*x^3)/(a + b*ArcSinh[c*x]) -
 Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - 3*Cosh[(3*a)/b]*CoshIntegral[
3*(a/b + ArcSinh[c*x])] + Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 3*Si
nh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])])/(b^2*c^2)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}\,x}{b^2\,\text{arsinh}\,(cx)^2+2\,ab\,\text{arsinh}\,(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}\,x}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)*x/(b*arcsinh(c*x) + a)^2, x)
```

maple [B] time = 0.27, size = 364, normalized size = 2.44

$$-\frac{4c^3x^3-4c^2x^2\sqrt{c^2x^2+1}+3cx-\sqrt{c^2x^2+1}}{8c^2b\,(a+b\,\text{arcsinh}\,(cx))}-\frac{3\,\mathrm{e}^{\frac{3a}{b}}\,\text{Ei}\left(1,3\,\text{arcsinh}\,(cx)+\frac{3a}{b}\right)}{8c^2b^2}-\frac{cx-\sqrt{c^2x^2+1}}{8c^2b\,(a+b\,\text{arcsinh}\,(cx))}-\frac{\mathrm{e}^{\frac{a}{b}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] -1/8*(4*c^3*x^3-4*c^2*x^2*(c^2*x^2+1)^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))/c^2/b/
(a+b*arcsinh(c*x))-3/8/c^2/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-1/8*(c
*x-(c^2*x^2+1)^(1/2))/c^2/b/(a+b*arcsinh(c*x))-1/8/c^2/b^2*exp(a/b)*Ei(1,ar
csinh(c*x)+a/b)-1/8/c^2/b^2*(arcsinh(c*x)*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)
*b+exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*a+x*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcs
inh(c*x))-1/8/c^2/b^2*(4*x^3*b*c^3+4*(c^2*x^2+1)^(1/2)*x^2*b*c^2+3*arcsinh(
c*x)*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)*b+3*exp(-3*a/b)*Ei(1,-3*arcsin
h(c*x)-3*a/b)*a+3*x*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(c^2x^3+x\right)\left(c^2x^2+1\right)+\left(c^3x^4+cx^2\right)\sqrt{c^2x^2+1}}{abc^3x^2+\sqrt{c^2x^2+1}\,abc^2x+abc+\left(b^2c^3x^2+\sqrt{c^2x^2+1}\,b^2c^2x+b^2c\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int\frac{}{abc^5x^4+\left(c^2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^3 + x)*(c^2*x^2 + 1) + (c^3*x^4 + c*x^2)*sqrt(c^2*x^2 + 1))/(a*b*c
^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2
+ 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate((3*(c^2*x
```

```
^2 + 1)^(3/2)*c^3*x^3 + (6*c^4*x^4 + 5*c^2*x^2 + 1)*(c^2*x^2 + 1) + (3*c^5*
x^5 + 5*c^3*x^3 + 2*c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c^2*x^2 + 1)*a*
b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 + 1)*b^2*c^3*x^
2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(c^2*x^2 + 1))*
log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sqrt(c^2*x^2 + 1
)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{c^2\,x^2+1}}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x))^2,x)
```

```
[Out] int((x*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{c^2x^2+1}}{\left(a+b\operatorname{asinh}\left(cx\right)\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x))**2, x)
```

3.414 $\int \frac{\sqrt{1+c^2x^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=85

$$-\frac{\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{b^2c}+\frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{b^2c}-\frac{c^2x^2+1}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

```
[Out] (-c^2*x^2-1)/b/c/(a+b*arcsinh(c*x))+cosh(2*a/b)*Shi(2*(a+b*arcsinh(c*x))/b)
/b^2/c-Chi(2*(a+b*arcsinh(c*x))/b)*sinh(2*a/b)/b^2/c
```

Rubi [A] time = 0.18, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {5696, 5669, 5448, 12, 3303, 3298, 3301}

$$-\frac{\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{b^2c}+\frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{b^2c}-\frac{c^2x^2+1}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + c^2*x^2]/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((1 + c^2*x^2)/(b*c*(a + b*ArcSinh[c*x]))) - (CoshIntegral[(2*a)/b + 2*Arc
Sinh[c*x]]*Sinh[(2*a)/b])/(b^2*c) + (Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2
*ArcSinh[c*x]])/(b^2*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5696

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(Sqrt[1 + c^2*x^2]*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^(n +
 1))/(b*c*(n + 1)), x] - Dist[(c*(2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(b*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ
[e, c^2*d] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+c^2x^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx &= -\frac{1+c^2x^2}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{(2c)\int \frac{x}{a+b\sinh^{-1}(cx)}\,dx}{b}\\
&= -\frac{1+c^2x^2}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{2\,\mathrm{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{bc}\\
&= -\frac{1+c^2x^2}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{2\,\mathrm{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)}\,dx,x,\sinh^{-1}(cx)\right)}{bc}\\
&= -\frac{1+c^2x^2}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\mathrm{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{bc}\\
&= -\frac{1+c^2x^2}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\cosh\left(\frac{2a}{b}\right)\mathrm{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{bc}-\frac{\sinh\left(\right.}{}\\
&= -\frac{1+c^2x^2}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{\mathrm{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{b^2c}+\frac{\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2a}{b}+2\right.}{b^2c}\end{aligned}$$

Mathematica [A] time = 0.17, size = 73, normalized size = 0.86

$$\frac{-\frac{bc^2x^2+b}{a+b\sinh^{-1}(cx)}-\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{b^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + c^2*x^2]/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-((b + b*c^2*x^2)/(a + b*ArcSinh[c*x])) - CoshIntegral[2*(a/b + ArcSinh[c*
x])]*Sinh[(2*a)/b] + Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])])/(b
^2*c)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{c^2x^2+1}}{b^2\,\mathrm{arsinh}\,(cx)^2+2\,ab\,\mathrm{arsinh}\,(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{(b \operatorname{arsinh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a)^2, x)
```

maple [B] time = 0.26, size = 192, normalized size = 2.26

$$-\frac{1}{2bc\,(a+b \operatorname{arcsinh}(cx))}-\frac{2c^2x^2-2cx\sqrt{c^2x^2+1}+1}{4c\,(a+b \operatorname{arcsinh}(cx))\,b}+\frac{e^{\frac{2a}{b}}\operatorname{Ei}\left(1,2 \operatorname{arcsinh}(cx)+\frac{2a}{b}\right)}{2c\,b^2}-\frac{2x^2b\,c^2+2bc\sqrt{c^2x^2+1}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] -1/2/b/c/(a+b*arcsinh(c*x))-1/4*(2*c^2*x^2-2*c*x*(c^2*x^2+1)^(1/2)+1)/c/(a+
b*arcsinh(c*x))/b+1/2/c/b^2*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)-1/4/c/b^2
*(2*x^2*b*c^2+2*b*c*(c^2*x^2+1)^(1/2)*x+2*arcsinh(c*x)*exp(-2*a/b)*Ei(1,-2*
arcsinh(c*x)-2*a/b)*b+2*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*a+b)/(a+b*a
rcsinh(c*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(c^2x^2+1\right)^2+\left(c^3x^3+cx\right)\sqrt{c^2x^2+1}}{abc^3x^2+\sqrt{c^2x^2+1}\,abc^2x+abc+\left(b^2c^3x^2+\sqrt{c^2x^2+1}\,b^2c^2x+b^2c\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int \frac{}{abc^4x^4+\left(c^2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^2 + 1)^2 + (c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(
c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x
 + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((2*c^2*x^2 - 1)*(c^2*x
^2 + 1)^(3/2) + 2*(2*c^3*x^3 + c*x)*(c^2*x^2 + 1) + (2*c^4*x^4 + 3*c^2*x^2
+ 1)*sqrt(c^2*x^2 + 1))/(a*b*c^4*x^4 + (c^2*x^2 + 1)*a*b*c^2*x^2 + 2*a*b*c^
2*x^2 + a*b + (b^2*c^4*x^4 + (c^2*x^2 + 1)*b^2*c^2*x^2 + 2*b^2*c^2*x^2 + b^
2 + 2*(b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1
)) + 2*(a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c^2\,x^2+1}}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(1/2)/(a + b*asinh(c*x))^2,x)
```

```
[Out] int((c^2*x^2 + 1)^(1/2)/(a + b*asinh(c*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{(a+b\,\mathrm{asinh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(1/2)/(a+b*asinh(c*x))**2,x)
[Out] Integral(sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x))**2, x)
```

3.415 $\int \frac{\sqrt{1+c^2x^2}}{x\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=103

$$-\frac{\text{Int}\left(\frac{1}{x^2\left(a+b\sinh^{-1}(cx)\right)},x\right)}{bc}+\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2}-\frac{c^2x^2+1}{bcx\left(a+b\sinh^{-1}(cx)\right)}$$

```
[Out] (-c^2*x^2-1)/b/c/x/(a+b*arcsinh(c*x))+Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b
^2-Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b^2-Unintegrable(1/x^2/(a+b*arcsinh(
c*x)),x)/b/c
```

Rubi [A] time = 0.23, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1+c^2x^2}}{x\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[1 + c^2*x^2]/(x*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -((1 + c^2*x^2)/(b*c*x*(a + b*ArcSinh[c*x]))) + (Cosh[a/b]*CoshIntegral[(a
+ b*ArcSinh[c*x])/b])/b^2 - (Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b]
)/b^2 - Defer[Int][1/(x^2*(a + b*ArcSinh[c*x])), x]/(b*c)
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+c^2x^2}}{x\left(a+b\sinh^{-1}(cx)\right)^2}\,dx &= -\frac{1+c^2x^2}{bcx\left(a+b\sinh^{-1}(cx)\right)}-\frac{\int \frac{1}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx}{bc}+\frac{c\int \frac{1}{a+b\sinh^{-1}(cx)}\,dx}{b}\\
&= -\frac{1+c^2x^2}{bcx\left(a+b\sinh^{-1}(cx)\right)}+\frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\sinh^{-1}(cx)\right)}{b^2}-\frac{\int \frac{}{x^2(a+}}{}\\
&= -\frac{1+c^2x^2}{bcx\left(a+b\sinh^{-1}(cx)\right)}-\frac{\int \frac{1}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx}{bc}+\frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx,\right.}{b^2}\\
&= -\frac{1+c^2x^2}{bcx\left(a+b\sinh^{-1}(cx)\right)}+\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}}{b}\right.}{b^2}
\end{aligned}$$

Mathematica [A] time = 10.87, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+c^2x^2}}{x\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[1 + c^2*x^2]/(x*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[Sqrt[1 + c^2*x^2]/(x*(a + b*ArcSinh[c*x])^2), x]
```

fricas [A] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}}{b^2x\,\text{arsinh}\,(cx)^2+2\,abx\,\text{arsinh}\,(cx)+a^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(b^2*x*arcsinh(c*x)^2 + 2*a*b*x*arcsinh(c*x) + a
^2*x), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{x\,(a+b\,\text{arcsinh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(c^2x^2+1\right)^2+\left(c^3x^3+cx\right)\sqrt{c^2x^2+1}}{abc^3x^3+\sqrt{c^2x^2+1}\,abc^2x^2+abcx+\left(b^2c^3x^3+\sqrt{c^2x^2+1}\,b^2c^2x^2+b^2cx\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int \frac{}{abc^5x^6+\left(c^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^2 + 1)^2 + (c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^3 + sqrt(
c^2*x^2 + 1)*a*b*c^2*x^2 + a*b*c*x + (b^2*c^3*x^3 + sqrt(c^2*x^2 + 1)*b^2*c
^2*x^2 + b^2*c*x)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((c^3*x^3 - 2*c
*x)*(c^2*x^2 + 1)^(3/2) + (2*c^4*x^4 - c^2*x^2 - 1)*(c^2*x^2 + 1) + (c^5*x^
5 + c^3*x^3)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^6 + (c^2*x^2 + 1)*a*b*c^3*x^4 +
2*a*b*c^3*x^4 + a*b*c*x^2 + (b^2*c^5*x^6 + (c^2*x^2 + 1)*b^2*c^3*x^4 + 2*b^
2*c^3*x^4 + b^2*c*x^2 + 2*(b^2*c^4*x^5 + b^2*c^2*x^3)*sqrt(c^2*x^2 + 1))*lo
g(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^5 + a*b*c^2*x^3)*sqrt(c^2*x^2 + 1
)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c^2\,x^2+1}}{x\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(1/2)/(x*(a + b*asinh(c*x))^2),x)
[Out] int((c^2*x^2 + 1)^(1/2)/(x*(a + b*asinh(c*x))^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{x\left(a+b\,\mathrm{asinh}\,(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(1/2)/x/(a+b*asinh(c*x))**2,x)
[Out] Integral(sqrt(c**2*x**2 + 1)/(x*(a + b*asinh(c*x))**2), x)
```

3.416 $\int \frac{\sqrt{1+c^2x^2}}{x^2\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=56

$$-\frac{2\text{Int}\left(\frac{1}{x^3\left(a+b\sinh^{-1}(cx)\right)},x\right)}{bc}-\frac{c^2x^2+1}{bcx^2\left(a+b\sinh^{-1}(cx)\right)}$$

```
[Out] (-c^2*x^2-1)/b/c/x^2/(a+b*arcsinh(c*x))-2*Unintegrable(1/x^3/(a+b*arcsinh(c
*x)),x)/b/c
```

Rubi [A] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1+c^2x^2}}{x^2\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[1 + c^2*x^2]/(x^2*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -((1 + c^2*x^2)/(b*c*x^2*(a + b*ArcSinh[c*x]))) - (2*Defer[Int][1/(x^3*(a +
 b*ArcSinh[c*x])), x])/(b*c)
```

Rubi steps

$$\int \frac{\sqrt{1+c^2x^2}}{x^2\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=-\frac{1+c^2x^2}{bcx^2\left(a+b\sinh^{-1}(cx)\right)}-\frac{2\int \frac{1}{x^3\left(a+b\sinh^{-1}(cx)\right)}\,dx}{bc}$$

Mathematica [A] time = 2.83, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+c^2x^2}}{x^2\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[1 + c^2*x^2]/(x^2*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[Sqrt[1 + c^2*x^2]/(x^2*(a + b*ArcSinh[c*x])^2), x]
```

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}}{b^2x^2\,\text{arsinh}(cx)^2+2\,abx^2\,\text{arsinh}(cx)+a^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*x)
 + a^2*x^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{(b\,\mathrm{arsinh}\,(cx)+a)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)^2*x^2), x)
```

maple [A] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{x^2\,(a+b\,\mathrm{arcsinh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x))^2,x)
[Out] int((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(c^2x^2+1\right)^2+\left(c^3x^3+cx\right)\sqrt{c^2x^2+1}}{abc^3x^4+\sqrt{c^2x^2+1}\,abc^2x^3+abcx^2+\left(b^2c^3x^4+\sqrt{c^2x^2+1}\,b^2c^2x^3+b^2cx^2\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}-\int\frac{}{abc^5x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -((c^2*x^2 + 1)^2 + (c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^4 + sqrt(
c^2*x^2 + 1)*a*b*c^2*x^3 + a*b*c*x^2 + (b^2*c^3*x^4 + sqrt(c^2*x^2 + 1)*b^2
*c^2*x^3 + b^2*c*x^2)*log(c*x + sqrt(c^2*x^2 + 1))) - integrate((3*(c^2*x^2
 + 1)^(3/2)*c*x + 2*(2*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^3*x^3 + c*x)*sqrt(c^
2*x^2 + 1))/(a*b*c^5*x^7 + (c^2*x^2 + 1)*a*b*c^3*x^5 + 2*a*b*c^3*x^5 + a*b*
c*x^3 + (b^2*c^5*x^7 + (c^2*x^2 + 1)*b^2*c^3*x^5 + 2*b^2*c^3*x^5 + b^2*c*x^
3 + 2*(b^2*c^4*x^6 + b^2*c^2*x^4)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2
 + 1)) + 2*(a*b*c^4*x^6 + a*b*c^2*x^4)*sqrt(c^2*x^2 + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c^2\,x^2+1}}{x^2\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(1/2)/(x^2*(a + b*asinh(c*x))^2),x)
[Out] int((c^2*x^2 + 1)^(1/2)/(x^2*(a + b*asinh(c*x))^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{x^2\,(a+b\,\mathrm{asinh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(1/2)/x**2/(a+b*asinh(c*x))**2,x)
[Out] Integral(sqrt(c**2*x**2 + 1)/(x**2*(a + b*asinh(c*x))**2), x)
```

3.417 $\int \frac{\sqrt{1+c^2x^2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{\sqrt{c^2x^2+1}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

[Out] Unintegrable((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x))^2,x)

Rubi [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1+c^2x^2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

[In] Int[Sqrt[1 + c^2*x^2]/(x^3*(a + b*ArcSinh[c*x])^2),x]

[Out] Defer[Int][Sqrt[1 + c^2*x^2]/(x^3*(a + b*ArcSinh[c*x])^2), x]

Rubi steps

$$\int \frac{\sqrt{1+c^2x^2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{\sqrt{1+c^2x^2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 19.09, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+c^2x^2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[Sqrt[1 + c^2*x^2]/(x^3*(a + b*ArcSinh[c*x])^2),x]

[Out] Integrate[Sqrt[1 + c^2*x^2]/(x^3*(a + b*ArcSinh[c*x])^2), x]

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}}{b^2x^3\operatorname{arsinh}(cx)^2+2\,abx^3\operatorname{arsinh}(cx)+a^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")

[Out] integral(sqrt(c^2*x^2 + 1)/(b^2*x^3*arcsinh(c*x)^2 + 2*a*b*x^3*arcsinh(c*x) + a^2*x^3), x)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.75, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{x^3\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(c^2x^2+1\right)^2+\left(c^3x^3+cx\right)\sqrt{c^2x^2+1}}{abc^3x^5+\sqrt{c^2x^2+1}\,abc^2x^4+abcx^3+\left(b^2c^3x^5+\sqrt{c^2x^2+1}\,b^2c^2x^4+b^2cx^3\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}-\int \frac{}{abc^5x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^2 + 1)^2 + (c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^5 + sqrt(
c^2*x^2 + 1)*a*b*c^2*x^4 + a*b*c*x^3 + (b^2*c^3*x^5 + sqrt(c^2*x^2 + 1)*b^2
*c^2*x^4 + b^2*c*x^3)*log(c*x + sqrt(c^2*x^2 + 1))) - integrate(((c^3*x^3 +
 4*c*x)*(c^2*x^2 + 1)^(3/2) + (2*c^4*x^4 + 7*c^2*x^2 + 3)*(c^2*x^2 + 1) + (
c^5*x^5 + 3*c^3*x^3 + 2*c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^8 + (c^2*x^2 + 1
)*a*b*c^3*x^6 + 2*a*b*c^3*x^6 + a*b*c*x^4 + (b^2*c^5*x^8 + (c^2*x^2 + 1)*b^
2*c^3*x^6 + 2*b^2*c^3*x^6 + b^2*c*x^4 + 2*(b^2*c^4*x^7 + b^2*c^2*x^5)*sqrt(
c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^7 + a*b*c^2*x^5)*
sqrt(c^2*x^2 + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{c^2\,x^2+1}}{x^3\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(1/2)/(x^3*(a + b*asinh(c*x))^2),x)
```

```
[Out] int((c^2*x^2 + 1)^(1/2)/(x^3*(a + b*asinh(c*x))^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{x^3\left(a+b\,\mathrm{asinh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(1/2)/x**3/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(sqrt(c**2*x**2 + 1)/(x**3*(a + b*asinh(c*x))**2), x)
```

3.418 $\int \frac{\sqrt{1+c^2x^2}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=30

$$\mathrm{Int}\left(\frac{\sqrt{c^2x^2+1}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1+c^2x^2}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[1 + c^2*x^2]/(x^4*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][Sqrt[1 + c^2*x^2]/(x^4*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{\sqrt{1+c^2x^2}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{\sqrt{1+c^2x^2}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 3.68, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+c^2x^2}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[1 + c^2*x^2]/(x^4*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[Sqrt[1 + c^2*x^2]/(x^4*(a + b*ArcSinh[c*x])^2), x]
```

fricas [A] time = 0.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{c^2x^2+1}}{b^2x^4\,\mathrm{arsinh}\,(cx)^2+2\,abx^4\,\mathrm{arsinh}\,(cx)+a^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)/(b^2*x^4*arcsinh(c*x)^2 + 2*a*b*x^4*arcsinh(c*x)
 + a^2*x^4), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{\left(b\,\mathrm{arsinh}\,(cx)+a\right)^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)^2*x^4), x)
```

maple [A] time = 0.85, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{x^4 (a + b \operatorname{arcsinh}(cx))^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x))^2,x)
[Out] int((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(c^2x^2+1\right)^2 + \left(c^3x^3+cx\right)\sqrt{c^2x^2+1}}{abc^3x^6 + \sqrt{c^2x^2+1}\,abc^2x^5 + abcx^4 + \left(b^2c^3x^6 + \sqrt{c^2x^2+1}\,b^2c^2x^5 + b^2cx^4\right)\log\left(cx+\sqrt{c^2x^2+1}\right)} - \int \frac{}{abc^5x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -((c^2*x^2 + 1)^2 + (c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^6 + sqrt(
c^2*x^2 + 1)*a*b*c^2*x^5 + a*b*c*x^4 + (b^2*c^3*x^6 + sqrt(c^2*x^2 + 1)*b^2
*c^2*x^5 + b^2*c*x^4)*log(c*x + sqrt(c^2*x^2 + 1))) - integrate(((2*c^3*x^3
 + 5*c*x)*(c^2*x^2 + 1)^(3/2) + 2*(2*c^4*x^4 + 5*c^2*x^2 + 2)*(c^2*x^2 + 1)
 + (2*c^5*x^5 + 5*c^3*x^3 + 3*c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^9 + (c^2*x
^2 + 1)*a*b*c^3*x^7 + 2*a*b*c^3*x^7 + a*b*c*x^5 + (b^2*c^5*x^9 + (c^2*x^2 +
 1)*b^2*c^3*x^7 + 2*b^2*c^3*x^7 + b^2*c*x^5 + 2*(b^2*c^4*x^8 + b^2*c^2*x^6)
*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^8 + a*b*c^2
*x^6)*sqrt(c^2*x^2 + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{c^2\,x^2+1}}{x^4\,(a + b\,\mathrm{asinh}(c\,x))^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(1/2)/(x^4*(a + b*asinh(c*x))^2),x)
[Out] int((c^2*x^2 + 1)^(1/2)/(x^4*(a + b*asinh(c*x))^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^2x^2+1}}{x^4 (a + b \operatorname{asinh}(cx))^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(1/2)/x**4/(a+b*asinh(c*x))**2,x)
[Out] Integral(sqrt(c**2*x**2 + 1)/(x**4*(a + b*asinh(c*x))**2), x)
```

3.419 $\int \frac{x^3(1+c^2x^2)^{3/2}}{(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=277

$$-\frac{3\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{64b^2c^4}-\frac{9\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{64b^2c^4}+\frac{5\cosh\left(\frac{5a}{b}\right)\operatorname{Chi}\left(\frac{5(a+b\sinh^{-1}(cx))}{b}\right)}{64b^2c^4}+\frac{7\cosh\left(\frac{7a}{b}\right.}{}$$

```
[Out] -x^3*(c^2*x^2+1)^2/b/c/(a+b*arcsinh(c*x))-3/64*Chi((a+b*arcsinh(c*x))/b)*co
sh(a/b)/b^2/c^4-9/64*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b^2/c^4+5/64*C
hi(5*(a+b*arcsinh(c*x))/b)*cosh(5*a/b)/b^2/c^4+7/64*Chi(7*(a+b*arcsinh(c*x)
)/b)*cosh(7*a/b)/b^2/c^4+3/64*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b^2/c^4+9
/64*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^4-5/64*Shi(5*(a+b*arcsinh
(c*x))/b)*sinh(5*a/b)/b^2/c^4-7/64*Shi(7*(a+b*arcsinh(c*x))/b)*sinh(7*a/b)/
b^2/c^4
```

Rubi [A] time = 1.01, antiderivative size = 273, normalized size of antiderivative = 0.99, number of steps used = 28, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5777, 5779, 5448, 3303, 3298, 3301}

$$-\frac{3\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{64b^2c^4}-\frac{9\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{64b^2c^4}+\frac{5\cosh\left(\frac{5a}{b}\right)\operatorname{Chi}\left(\frac{5a}{b}+5\sinh^{-1}(cx)\right)}{64b^2c^4}+-\frac{7}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((x^3*(1 + c^2*x^2)^2)/(b*c*(a + b*ArcSinh[c*x]))) - (3*Cosh[a/b]*CoshInte
gral[a/b + ArcSinh[c*x]])/(64*b^2*c^4) - (9*Cosh[(3*a)/b]*CoshIntegral[(3*a
)/b + 3*ArcSinh[c*x]])/(64*b^2*c^4) + (5*Cosh[(5*a)/b]*CoshIntegral[(5*a)/b
 + 5*ArcSinh[c*x]])/(64*b^2*c^4) + (7*Cosh[(7*a)/b]*CoshIntegral[(7*a)/b +
7*ArcSinh[c*x]])/(64*b^2*c^4) + (3*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x
]])/(64*b^2*c^4) + (9*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])
/(64*b^2*c^4) - (5*Sinh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcSinh[c*x]])/(6
4*b^2*c^4) - (7*Sinh[(7*a)/b]*SinhIntegral[(7*a)/b + 7*ArcSinh[c*x]])/(64*b
^2*c^4)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^
p*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-Dist[(f*m*d^IntPart[p
]*(d + e*x^2)^FracPart[p])/(b*c*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*
x)^(m - 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - D
ist[(c*(m + 2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(b*f*(n + 1)*(1
+ c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*A
rcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[2*p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^3\left(1+c^2x^2\right)^{3/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{x^3\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{3\int \frac{x^2\left(1+c^2x^2\right)}{a+b\sinh^{-1}(cx)}\,dx}{bc} + \frac{(7c)\int \frac{x^4\left(1+c^2x^2\right)}{a+b\sinh^{-1}(cx)}\,dx}{b}$$

$$= -\frac{x^3\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{3\,\text{Subst}\left(\int \frac{\cosh^3(x)\sinh^2(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{bc^4} + \frac{7\,\text{Subst}\left(\right.}{}$$

$$= -\frac{x^3\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{3\,\text{Subst}\left(\int \left(-\frac{\cosh(x)}{8(a+bx)} + \frac{\cosh(3x)}{16(a+bx)} + \frac{\cosh(5x)}{16(a+bx)}\right) dx, x, \sinh^{-1}\right.}{bc^4}$$

$$= -\frac{x^3\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{7\,\text{Subst}\left(\int \frac{\cosh(5x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{64bc^4} + \frac{7\,\text{Subst}\left(\int \frac{\cosh(}{a+b}\right.}{}$$

$$= -\frac{x^3\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\left(21\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{64bc^4} - \frac{(3}{}$$

$$= -\frac{x^3\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{64b^2c^4} - \frac{9\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}\right.}{64b^2c}$$

Mathematica [A] time = 0.82, size = 399, normalized size = 1.44

$$\frac{-3\cosh\left(\frac{a}{b}\right)\left(a+b\sinh^{-1}(cx)\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right) - 9\cosh\left(\frac{3a}{b}\right)\left(a+b\sinh^{-1}(cx)\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-64*b*c^3*x^3 - 128*b*c^5*x^5 - 64*b*c^7*x^7 - 3*(a + b*ArcSinh[c*x])*Cosh
[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - 9*(a + b*ArcSinh[c*x])*Cosh[(3*a)/
b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] + 5*a*Cosh[(5*a)/b]*CoshIntegral[5*
(a/b + ArcSinh[c*x])] + 5*b*ArcSinh[c*x]*Cosh[(5*a)/b]*CoshIntegral[5*(a/b
+ ArcSinh[c*x])] + 7*a*Cosh[(7*a)/b]*CoshIntegral[7*(a/b + ArcSinh[c*x])] +
 7*b*ArcSinh[c*x]*Cosh[(7*a)/b]*CoshIntegral[7*(a/b + ArcSinh[c*x])] + 3*a*
Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 3*b*ArcSinh[c*x]*Sinh[a/b]*Sin
hIntegral[a/b + ArcSinh[c*x]] + 9*a*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + Arc
Sinh[c*x])] + 9*b*ArcSinh[c*x]*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[
c*x])] - 5*a*Sinh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[c*x])] - 5*b*ArcSi
nh[c*x]*Sinh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[c*x])] - 7*a*Sinh[(7*a)
/b]*SinhIntegral[7*(a/b + ArcSinh[c*x])] - 7*b*ArcSinh[c*x]*Sinh[(7*a)/b]*S
inhIntegral[7*(a/b + ArcSinh[c*x])])/(64*b^2*c^4*(a + b*ArcSinh[c*x]))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(c^2x^5+x^3\right)\sqrt{c^2x^2+1}}{b^2\operatorname{arsinh}\left(cx\right)^2+2\,ab\operatorname{arsinh}\left(cx\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcs
inh(c*x) + a^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [B] time = 0.59, size = 958, normalized size = 3.46

$$-\frac{64c^7x^7-64c^6x^6\sqrt{c^2x^2+1}+112c^5x^5-80c^4x^4\sqrt{c^2x^2+1}+56c^3x^3-24c^2x^2\sqrt{c^2x^2+1}+7cx-\sqrt{c^2x^2+1}}{128c^4\left(a+b\operatorname{arcsinh}\left(cx\right)\right)b}-\frac{7\,\mathrm{e}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] -1/128*(64*c^7*x^7-64*c^6*x^6*(c^2*x^2+1)^(1/2)+112*c^5*x^5-80*c^4*x^4*(c^2
*x^2+1)^(1/2)+56*c^3*x^3-24*c^2*x^2*(c^2*x^2+1)^(1/2)+7*c*x-(c^2*x^2+1)^(1/
2))/c^4/(a+b*arcsinh(c*x))/b-7/128/c^4/b^2*exp(7*a/b)*Ei(1,7*arcsinh(c*x)+7
*a/b)-1/128*(16*c^5*x^5-16*c^4*x^4*(c^2*x^2+1)^(1/2)+20*c^3*x^3-12*c^2*x^2*
(c^2*x^2+1)^(1/2)+5*c*x-(c^2*x^2+1)^(1/2))/c^4/b/(a+b*arcsinh(c*x))-5/128/c
^4/b^2*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)+3/128*(4*c^3*x^3-4*c^2*x^2*(c^
2*x^2+1)^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))/c^4/b/(a+b*arcsinh(c*x))+9/128/c^4/
b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)+3/128*(c*x-(c^2*x^2+1)^(1/2))/c^4
/b/(a+b*arcsinh(c*x))+3/128/c^4/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)+3/128/c
^4/b^2*(arcsinh(c*x)*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*b+exp(-a/b)*Ei(1,-ar
csinh(c*x)-a/b)*a+x*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))+3/128/c^4/b
^2*(4*x^3*b*c^3+4*(c^2*x^2+1)^(1/2)*x^2*b*c^2+3*arcsinh(c*x)*exp(-3*a/b)*Ei
```

```
(1,-3*arcsinh(c*x)-3*a/b)*b+3*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)*a+3*x
*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-1/128/c^4/b^2*(16*x^5*b*c^5+16
*(c^2*x^2+1)^(1/2)*x^4*b*c^4+20*x^3*b*c^3+12*(c^2*x^2+1)^(1/2)*x^2*b*c^2+5*
arcsinh(c*x)*exp(-5*a/b)*Ei(1,-5*arcsinh(c*x)-5*a/b)*b+5*exp(-5*a/b)*Ei(1,-
5*arcsinh(c*x)-5*a/b)*a+5*x*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-1/1
28/c^4/b^2*(64*x^7*b*c^7+64*(c^2*x^2+1)^(1/2)*x^6*b*c^6+112*x^5*b*c^5+80*(c
^2*x^2+1)^(1/2)*x^4*b*c^4+56*x^3*b*c^3+24*(c^2*x^2+1)^(1/2)*x^2*b*c^2+7*arc
sinh(c*x)*Ei(1,-7*arcsinh(c*x)-7*a/b)*exp(-7*a/b)*b+7*Ei(1,-7*arcsinh(c*x)-
7*a/b)*exp(-7*a/b)*a+7*x*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(c^4x^7+2\,c^2x^5+x^3\right)\left(c^2x^2+1\right)+\left(c^5x^8+2\,c^3x^6+cx^4\right)\sqrt{c^2x^2+1}}{abc^3x^2+\sqrt{c^2x^2+1}\,abc^2x+abc+\left(b^2c^3x^2+\sqrt{c^2x^2+1}\,b^2c^2x+b^2c\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int\frac{}{abc^5x^4+\left(c^2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^4*x^7 + 2*c^2*x^5 + x^3)*(c^2*x^2 + 1) + (c^5*x^8 + 2*c^3*x^6 + c*x^4)
*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b
^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 +
1))) + integrate(((7*c^5*x^7 + 9*c^3*x^5 + 2*c*x^3)*(c^2*x^2 + 1)^(3/2) + (
14*c^6*x^8 + 27*c^4*x^6 + 16*c^2*x^4 + 3*x^2)*(c^2*x^2 + 1) + (7*c^7*x^9 +
18*c^5*x^7 + 15*c^3*x^5 + 4*c*x^3)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c^2*x
^2 + 1)*a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 + 1)*
b^2*c^3*x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(c^2*
x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sqrt(c
^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(c^2\,x^2+1\right)}^{3/2}}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x))^2,x)
```

```
[Out] int((x^3*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}}{\left(a+b\operatorname{asinh}{\left(cx\right)}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x**3*(c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x))**2, x)
```

3.420 $\int \frac{x^2(1+c^2x^2)^{3/2}}{(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=219

$$\frac{\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{16b^2c^3}-\frac{\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\sinh^{-1}(cx))}{b}\right)}{4b^2c^3}-\frac{3\sinh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\sinh^{-1}(cx))}{b}\right)}{16b^2c^3}-\frac{\cosh\left(\frac{2a}{b}\right)\text{Sh}}{1}$$

```
[Out] -x^2*(c^2*x^2+1)^2/b/c/(a+b*arcsinh(c*x))-1/16*cosh(2*a/b)*Shi(2*(a+b*arcsi
nh(c*x))/b)/b^2/c^3+1/4*cosh(4*a/b)*Shi(4*(a+b*arcsinh(c*x))/b)/b^2/c^3+3/1
6*cosh(6*a/b)*Shi(6*(a+b*arcsinh(c*x))/b)/b^2/c^3+1/16*Chi(2*(a+b*arcsinh(c
*x))/b)*sinh(2*a/b)/b^2/c^3-1/4*Chi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b^2
/c^3-3/16*Chi(6*(a+b*arcsinh(c*x))/b)*sinh(6*a/b)/b^2/c^3
```

Rubi [A] time = 0.72, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5777, 5779, 5448, 3303, 3298, 3301}

$$\frac{\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{16b^2c^3}-\frac{\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{4b^2c^3}-\frac{3\sinh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6a}{b}+6\sinh^{-1}(cx)\right)}{16b^2c^3}-\text{cos}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((x^2*(1 + c^2*x^2)^2)/(b*c*(a + b*ArcSinh[c*x]))) + (CoshIntegral[(2*a)/b
 + 2*ArcSinh[c*x]]*Sinh[(2*a)/b])/(16*b^2*c^3) - (CoshIntegral[(4*a)/b + 4*
ArcSinh[c*x]]*Sinh[(4*a)/b])/(4*b^2*c^3) - (3*CoshIntegral[(6*a)/b + 6*ArcS
inh[c*x]]*Sinh[(6*a)/b])/(16*b^2*c^3) - (Cosh[(2*a)/b]*SinhIntegral[(2*a)/b
 + 2*ArcSinh[c*x]])/(16*b^2*c^3) + (Cosh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*
ArcSinh[c*x]])/(4*b^2*c^3) + (3*Cosh[(6*a)/b]*SinhIntegral[(6*a)/b + 6*ArcS
inh[c*x]])/(16*b^2*c^3)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
```

```
& IGtQ[p, 0]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^
p*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-Dist[(f*m*d^IntPart[p
]*(d + e*x^2)^FracPart[p])/(b*c*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*
x)^(m - 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - D
ist[(c*(m + 2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(b*f*(n + 1)*(1
+ c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*A
rcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[2*p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2\left(1+c^2x^2\right)^{3/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{x^2\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{2\int \frac{x\left(1+c^2x^2\right)}{a+b\sinh^{-1}(cx)}\,dx}{bc} + \frac{(6c)\int \frac{x^3\left(1+c^2x^2\right)}{a+b\sinh^{-1}(cx)}\,dx}{b}$$

$$= -\frac{x^2\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{2\,\text{Subst}\left(\int \frac{\cosh^3(x)\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{bc^3} + \frac{6\,\text{Subst}\left(\right.}{}$$

$$= -\frac{x^2\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{2\,\text{Subst}\left(\int \left(\frac{\sinh(2x)}{4(a+bx)}+\frac{\sinh(4x)}{8(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{bc^3} + \frac{6\,\text{Su}}{}$$

$$= -\frac{x^2\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{3\,\text{Subst}\left(\int \frac{\sinh(6x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{16bc^3} + \frac{\text{Subst}\left(\int \frac{\sinh(4x}{a+bx}\right.}{4}$$

$$= -\frac{x^2\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{2bc^3} - \frac{\left(9\,\text{c}\right.}{}$$

$$= -\frac{x^2\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{16b^2c^3} - \frac{\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}\right.}{4b^2c}$$

Mathematica [A] time = 0.74, size = 306, normalized size = 1.40

$$-\frac{-\sinh\left(\frac{2a}{b}\right)\left(a+b\sinh^{-1}(cx)\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+4\sinh\left(\frac{4a}{b}\right)\left(a+b\sinh^{-1}(cx)\right)\text{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -1/16*(16*b*c^2*x^2 + 32*b*c^4*x^4 + 16*b*c^6*x^6 - (a + b*ArcSinh[c*x])*Co
shIntegral[2*(a/b + ArcSinh[c*x])]*Sinh[(2*a)/b] + 4*(a + b*ArcSinh[c*x])*C
oshIntegral[4*(a/b + ArcSinh[c*x])]*Sinh[(4*a)/b] + 3*a*CoshIntegral[6*(a/b
 + ArcSinh[c*x])]*Sinh[(6*a)/b] + 3*b*ArcSinh[c*x]*CoshIntegral[6*(a/b + Ar
cSinh[c*x])]*Sinh[(6*a)/b] + a*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[
c*x])] + b*ArcSinh[c*x]*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])]
- 4*a*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])] - 4*b*ArcSinh[c*x]
*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])] - 3*a*Cosh[(6*a)/b]*Sin
hIntegral[6*(a/b + ArcSinh[c*x])] - 3*b*ArcSinh[c*x]*Cosh[(6*a)/b]*SinhInte
gral[6*(a/b + ArcSinh[c*x])])/(b^2*c^3*(a + b*ArcSinh[c*x]))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(c^{2} x^{4}+x^{2}\right) \sqrt{c^{2} x^{2}+1}}{b^{2} \operatorname{arsinh}\left(c x\right)^{2}+2 \, a b \operatorname{arsinh}\left(c x\right)+a^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + x^2)*sqrt(c^2*x^2 + 1)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcs
inh(c*x) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^{2} x^{2}+1\right)^{\frac{3}{2}} x^{2}}{\left(b \operatorname{arsinh}\left(c x\right)+a\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(3/2)*x^2/(b*arcsinh(c*x) + a)^2, x)
```

maple [B] time = 0.60, size = 704, normalized size = 3.21

$$\frac{1}{16 c^{3} \left(a+b \operatorname{arcsinh}\left(c x\right)\right) b}-\frac{32 c^{6} x^{6}-32 c^{5} x^{5} \sqrt{c^{2} x^{2}+1}+48 c^{4} x^{4}-32 c^{3} x^{3} \sqrt{c^{2} x^{2}+1}+18 c^{2} x^{2}-6 c x \sqrt{c^{2} x^{2}+1}+}{64 c^{3} \left(a+b \operatorname{arcsinh}\left(c x\right)\right) b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/16/c^3/(a+b*arcsinh(c*x))/b-1/64*(32*c^6*x^6-32*c^5*x^5*(c^2*x^2+1)^(1/2)
+48*c^4*x^4-32*c^3*x^3*(c^2*x^2+1)^(1/2)+18*c^2*x^2-6*c*x*(c^2*x^2+1)^(1/2)
+1)/c^3/(a+b*arcsinh(c*x))/b+3/32/c^3/b^2*exp(6*a/b)*Ei(1,6*arcsinh(c*x)+6*
a/b)-1/32*(8*c^4*x^4-8*c^3*x^3*(c^2*x^2+1)^(1/2)+8*c^2*x^2-4*c*x*(c^2*x^2+1
)^(1/2)+1)/c^3/(a+b*arcsinh(c*x))/b+1/8/c^3/b^2*exp(4*a/b)*Ei(1,4*arcsinh(c
*x)+4*a/b)+1/64*(2*c^2*x^2-2*c*x*(c^2*x^2+1)^(1/2)+1)/c^3/(a+b*arcsinh(c*x)
)/b-1/32/c^3/b^2*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)+1/64/c^3/b^2*(2*x^2*
b*c^2+2*b*c*(c^2*x^2+1)^(1/2)*x+2*arcsinh(c*x)*exp(-2*a/b)*Ei(1,-2*arcsinh(
c*x)-2*a/b)*b+2*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*a+b)/(a+b*arcsinh(c
*x))-1/32/c^3/b^2*(8*x^4*b*c^4+8*(c^2*x^2+1)^(1/2)*x^3*b*c^3+8*x^2*b*c^2+4*
b*c*(c^2*x^2+1)^(1/2)*x+4*arcsinh(c*x)*Ei(1,-4*arcsinh(c*x)-4*a/b)*exp(-4*a
/b)*b+4*Ei(1,-4*arcsinh(c*x)-4*a/b)*exp(-4*a/b)*a+b)/(a+b*arcsinh(c*x))-1/6
4/c^3/b^2*(32*x^6*b*c^6+32*(c^2*x^2+1)^(1/2)*x^5*b*c^5+48*x^4*b*c^4+32*(c^2
*x^2+1)^(1/2)*x^3*b*c^3+18*x^2*b*c^2+6*b*c*(c^2*x^2+1)^(1/2)*x+6*arcsinh(c*
x)*Ei(1,-6*arcsinh(c*x)-6*a/b)*exp(-6*a/b)*b+6*Ei(1,-6*arcsinh(c*x)-6*a/b)*
exp(-6*a/b)*a+b)/(a+b*arcsinh(c*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(c^{4}x^{6}+2\,c^{2}x^{4}+x^{2}\right)\left(c^{2}x^{2}+1\right)+\left(c^{5}x^{7}+2\,c^{3}x^{5}+cx^{3}\right)\sqrt{c^{2}x^{2}+1}}{abc^{3}x^{2}+\sqrt{c^{2}x^{2}+1}\,abc^{2}x+abc+\left(b^{2}c^{3}x^{2}+\sqrt{c^{2}x^{2}+1}\,b^{2}c^{2}x+b^{2}c\right)\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)}+\int\frac{}{abc^{5}x^{4}+\left(c^{2}x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^4*x^6 + 2*c^2*x^4 + x^2)*(c^2*x^2 + 1) + (c^5*x^7 + 2*c^3*x^5 + c*x^3)
*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b
^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 +
1))) + integrate(((6*c^5*x^6 + 7*c^3*x^4 + c*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(
6*c^6*x^7 + 11*c^4*x^5 + 6*c^2*x^3 + x)*(c^2*x^2 + 1) + 3*(2*c^7*x^8 + 5*c^
5*x^6 + 4*c^3*x^4 + c*x^2)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c^2*x^2 + 1)*
a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 + 1)*b^2*c^3*
x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(c^2*x^2 + 1)
)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sqrt(c^2*x^2 +
 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x^{2}\left(c^{2}\,x^{2}+1\right)^{3/2}}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x))^2,x)
```

```
[Out] int((x^2*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{2}\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}}{\left(a+b\,\mathrm{asinh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x**2*(c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x))**2, x)
```

3.421 $\int \frac{x(1+c^2x^2)^{3/2}}{(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=213

$$\frac{\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8b^2c^2}+\frac{9\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{16b^2c^2}+\frac{5\cosh\left(\frac{5a}{b}\right)\mathrm{Chi}\left(\frac{5(a+b\sinh^{-1}(cx))}{b}\right)}{16b^2c^2}-\frac{\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\right.}{8b^2}$$

```
[Out] -x*(c^2*x^2+1)^2/b/c/(a+b*arcsinh(c*x))+1/8*Chi((a+b*arcsinh(c*x))/b)*cosh(
a/b)/b^2/c^2+9/16*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b^2/c^2+5/16*Chi(
5*(a+b*arcsinh(c*x))/b)*cosh(5*a/b)/b^2/c^2-1/8*Shi((a+b*arcsinh(c*x))/b)*s
inh(a/b)/b^2/c^2-9/16*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^2-5/16*
Shi(5*(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b^2/c^2
```

Rubi [A] time = 0.73, antiderivative size = 209, normalized size of antiderivative = 0.98, number of steps used = 22, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {5777, 5699, 3312, 3303, 3298, 3301, 5779, 5448}

$$\frac{\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{8b^2c^2}+\frac{9\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{16b^2c^2}+\frac{5\cosh\left(\frac{5a}{b}\right)\mathrm{Chi}\left(\frac{5a}{b}+5\sinh^{-1}(cx)\right)}{16b^2c^2}-\mathrm{sinh}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((x*(1 + c^2*x^2)^2)/(b*c*(a + b*ArcSinh[c*x]))) + (Cosh[a/b]*CoshIntegral
[a/b + ArcSinh[c*x]])/(8*b^2*c^2) + (9*Cosh[(3*a)/b]*CoshIntegral[(3*a)/b +
 3*ArcSinh[c*x]])/(16*b^2*c^2) + (5*Cosh[(5*a)/b]*CoshIntegral[(5*a)/b + 5*
ArcSinh[c*x]])/(16*b^2*c^2) - (Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/
(8*b^2*c^2) - (9*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(16*
b^2*c^2) - (5*Sinh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcSinh[c*x]])/(16*b^2
*c^2)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^
p*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-Dist[(f*m*d^IntPart[p
]*(d + e*x^2)^FracPart[p])/(b*c*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*
x)^(m - 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - D
ist[(c*(m + 2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(b*f*(n + 1)*(1
+ c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*A
rcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[2*p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x\left(1+c^2x^2\right)^{3/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{x\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\int \frac{1+c^2x^2}{a+b\sinh^{-1}(cx)}\,dx}{bc}+\frac{(5c)\int \frac{x^2\left(1+c^2x^2\right)}{a+b\sinh^{-1}(cx)}\,dx}{b}$$

$$= -\frac{x\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\text{Subst}\left(\int \frac{\cosh^3(x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{bc^2}+\frac{5\,\text{Subst}\left(\int \frac{\cosh^3(x)\sin}{a+bx}\right.}{}$$

$$= -\frac{x\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\text{Subst}\left(\int \left(\frac{3\cosh(x)}{4(a+bx)}+\frac{\cosh(3x)}{4(a+bx)}\right)dx,x,\sinh^{-1}(cx)\right)}{bc^2}+\frac{5\,\text{Subst}}{}$$

$$= -\frac{x\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\text{Subst}\left(\int \frac{\cosh(3x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{4bc^2}+\frac{5\,\text{Subst}\left(\int \frac{\cosh(3x)}{a+bx}\,d\right.}{16bc}$$

$$= -\frac{x\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{\left(5\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{8bc^2}+\frac{\left(3\cos\right.}{}$$

$$= -\frac{x\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{8b^2c^2}+\frac{9\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\,\text{s}\right.}{16b^2c^2}$$

Mathematica [A] time = 0.61, size = 295, normalized size = 1.38

$$-\frac{-2\cosh\left(\frac{a}{b}\right)\left(a+b\sinh^{-1}(cx)\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-9\cosh\left(\frac{3a}{b}\right)\left(a+b\sinh^{-1}(cx)\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -1/16*(16*b*c*x + 32*b*c^3*x^3 + 16*b*c^5*x^5 - 2*(a + b*ArcSinh[c*x])*Cosh
[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - 9*(a + b*ArcSinh[c*x])*Cosh[(3*a)/
b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] - 5*a*Cosh[(5*a)/b]*CoshIntegral[5*
(a/b + ArcSinh[c*x])] - 5*b*ArcSinh[c*x]*Cosh[(5*a)/b]*CoshIntegral[5*(a/b
+ ArcSinh[c*x])] + 2*a*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 2*b*Arc
Sinh[c*x]*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 9*a*Sinh[(3*a)/b]*Si
nhIntegral[3*(a/b + ArcSinh[c*x])] + 9*b*ArcSinh[c*x]*Sinh[(3*a)/b]*SinhInt
egral[3*(a/b + ArcSinh[c*x])] + 5*a*Sinh[(5*a)/b]*SinhIntegral[5*(a/b + Arc
Sinh[c*x])] + 5*b*ArcSinh[c*x]*Sinh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[
c*x])])/(b^2*c^2*(a + b*ArcSinh[c*x]))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^2x^3+x\right)\sqrt{c^2x^2+1}}{b^2\,\text{arsinh}\left(cx\right)^2+2\,ab\,\text{arsinh}\left(cx\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^2*x^3 + x)*sqrt(c^2*x^2 + 1)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsin
h(c*x) + a^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [B] time = 0.40, size = 633, normalized size = 2.97

$$-\frac{16c^5x^5-16c^4x^4\sqrt{c^2x^2+1}+20c^3x^3-12c^2x^2\sqrt{c^2x^2+1}+5cx-\sqrt{c^2x^2+1}}{32c^2b\left(a+b\operatorname{arcsinh}(cx)\right)}-\frac{5\,\mathrm{e}^{\frac{5a}{b}}\operatorname{Ei}\left(1,5\operatorname{arcsinh}(cx)+\frac{5a}{b}\right)}{32c^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] -1/32*(16*c^5*x^5-16*c^4*x^4*(c^2*x^2+1)^(1/2)+20*c^3*x^3-12*c^2*x^2*(c^2*x
^2+1)^(1/2)+5*c*x-(c^2*x^2+1)^(1/2))/c^2/b/(a+b*arcsinh(c*x))-5/32/c^2/b^2*
exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)-3/32*(4*c^3*x^3-4*c^2*x^2*(c^2*x^2+1)
^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))/c^2/b/(a+b*arcsinh(c*x))-9/32/c^2/b^2*exp(3
*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-1/16*(c*x-(c^2*x^2+1)^(1/2))/c^2/b/(a+b*ar
csinh(c*x))-1/16/c^2/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/16/c^2/b^2*(arcs
inh(c*x)*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*b+exp(-a/b)*Ei(1,-arcsinh(c*x)-a
/b)*a+x*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-3/32/c^2/b^2*(4*x^3*b*c
^3+4*(c^2*x^2+1)^(1/2)*x^2*b*c^2+3*arcsinh(c*x)*exp(-3*a/b)*Ei(1,-3*arcsinh
(c*x)-3*a/b)*b+3*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)*a+3*x*b*c+(c^2*x^2
+1)^(1/2)*b)/(a+b*arcsinh(c*x))-1/32/c^2/b^2*(16*x^5*b*c^5+16*(c^2*x^2+1)^(
1/2)*x^4*b*c^4+20*x^3*b*c^3+12*(c^2*x^2+1)^(1/2)*x^2*b*c^2+5*arcsinh(c*x)*e
xp(-5*a/b)*Ei(1,-5*arcsinh(c*x)-5*a/b)*b+5*exp(-5*a/b)*Ei(1,-5*arcsinh(c*x)
-5*a/b)*a+5*x*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(c^4x^5+2\,c^2x^3+x\right)\left(c^2x^2+1\right)+\left(c^5x^6+2\,c^3x^4+cx^2\right)\sqrt{c^2x^2+1}}{abc^3x^2+\sqrt{c^2x^2+1}\,abc^2x+abc+\left(b^2c^3x^2+\sqrt{c^2x^2+1}\,b^2c^2x+b^2c\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int\frac{}{abc^5x^4+\left(c^2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^4*x^5 + 2*c^2*x^3 + x)*(c^2*x^2 + 1) + (c^5*x^6 + 2*c^3*x^4 + c*x^2)*s
qrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2
*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1)
)) + integrate((5*(c^5*x^5 + c^3*x^3)*(c^2*x^2 + 1)^(3/2) + (10*c^6*x^6 + 1
7*c^4*x^4 + 8*c^2*x^2 + 1)*(c^2*x^2 + 1) + (5*c^7*x^7 + 12*c^5*x^5 + 9*c^3*
x^3 + 2*c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c^2*x^2 + 1)*a*b*c^3*x^2 +
2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 + 1)*b^2*c^3*x^2 + 2*b^2*c^
3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(c^2*x^2 + 1))*log(c*x + sq
rt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sqrt(c^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,{\left(c^2\,x^2+1\right)}^{3/2}}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x))^2,x)
```

```
[Out] int((x*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(c^2x^2+1\right)^{\frac{3}{2}}}{\left(a+b\operatorname{asinh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
[Out] Integral(x*(c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x))**2, x)
```

3.422 $\int \frac{\left(1+c^2x^2\right)^{3/2}}{\left(a+b\sinh^{-1}(cx)\right)^2} dx$

Optimal. Leaf size=149

$$-\frac{\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{b^2c}-\frac{\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{2b^2c}+\frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{b^2c}+\frac{\cosh\left(\frac{4a}{b}\right)}{}$$

```
[Out] -(c^2*x^2+1)^2/b/c/(a+b*arcsinh(c*x))+cosh(2*a/b)*Shi(2*(a+b*arcsinh(c*x))/
b)/b^2/c+1/2*cosh(4*a/b)*Shi(4*(a+b*arcsinh(c*x))/b)/b^2/c-Chi(2*(a+b*arcsi
nh(c*x))/b)*sinh(2*a/b)/b^2/c-1/2*Chi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b
^2/c
```

Rubi [A] time = 0.32, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5696, 5779, 5448, 3303, 3298, 3301}

$$-\frac{\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{b^2c}-\frac{\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{2b^2c}+\frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{b^2c}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(1 + c^2*x^2)^(3/2)/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((1 + c^2*x^2)^2/(b*c*(a + b*ArcSinh[c*x]))) - (CoshIntegral[(2*a)/b + 2*A
rcSinh[c*x]]*Sinh[(2*a)/b])/(b^2*c) - (CoshIntegral[(4*a)/b + 4*ArcSinh[c*x
]]*Sinh[(4*a)/b])/(2*b^2*c) + (Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSi
nh[c*x]])/(b^2*c) + (Cosh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcSinh[c*x]])/
(2*b^2*c)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5696

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(Sqrt[1 + c^2*x^2]*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^(n +
 1))/(b*c*(n + 1)), x] - Dist[(c*(2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(b*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ
[e, c^2*d] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{\left(1+c^2x^2\right)^{3/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{(4c)\int\frac{x\left(1+c^2x^2\right)}{a+b\sinh^{-1}(cx)}\,dx}{b}$$

$$= -\frac{\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{4\,\mathrm{Subst}\left(\int\frac{\cosh^3(x)\sinh(x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{bc}$$

$$= -\frac{\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{4\,\mathrm{Subst}\left(\int\left(\frac{\sinh(2x)}{4(a+bx)}+\frac{\sinh(4x)}{8(a+bx)}\right)dx,x,\sinh^{-1}(cx)\right)}{bc}$$

$$= -\frac{\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\mathrm{Subst}\left(\int\frac{\sinh(4x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{2bc}+\frac{\mathrm{Subst}\left(\int\frac{\sinh(2x)}{a+bx}\,dx,\right.}{bc}$$

$$= -\frac{\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\cosh\left(\frac{2a}{b}\right)\mathrm{Subst}\left(\int\frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{bc}+\frac{\cosh\left(\right.}{}$$

$$= -\frac{\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{\mathrm{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{b^2c}-\frac{\mathrm{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx\right.}{2b^2c}$$

Mathematica [A] time = 0.47, size = 122, normalized size = 0.82

$$\frac{-\frac{2b\left(c^2x^2+1\right)^2}{a+b\sinh^{-1}(cx)}-2\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-\sinh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+2\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(2\left(\right.\right.}{2b^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + c^2*x^2)^(3/2)/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] ((-2*b*(1 + c^2*x^2)^2)/(a + b*ArcSinh[c*x]) - 2*CoshIntegral[2*(a/b + ArcS
inh[c*x])]*Sinh[(2*a)/b] - CoshIntegral[4*(a/b + ArcSinh[c*x])]*Sinh[(4*a)/
b] + 2*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] + Cosh[(4*a)/b]*S
inhIntegral[4*(a/b + ArcSinh[c*x])])/(2*b^2*c)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{b^2\operatorname{arsinh}\left(cx\right)^2+2\,ab\operatorname{arsinh}\left(cx\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^2*x^2 + 1)^(3/2)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(3/2)/(b*arcsinh(c*x) + a)^2, x)
```

maple [B] time = 0.41, size = 420, normalized size = 2.82

$$-\frac{3}{8bc\left(a+b\operatorname{arcsinh}\left(cx\right)\right)}-\frac{8c^4x^4-8c^3x^3\sqrt{c^2x^2+1}+8c^2x^2-4cx\sqrt{c^2x^2+1}+1}{16c\left(a+b\operatorname{arcsinh}\left(cx\right)\right)b}+\frac{e^{\frac{4a}{b}}\operatorname{Ei}\left(1,4\operatorname{arcsinh}\left(cx\right)+\frac{4}{b}\right.}{4c\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] -3/8/b/c/(a+b*arcsinh(c*x))-1/16*(8*c^4*x^4-8*c^3*x^3*(c^2*x^2+1)^(1/2)+8*c
^2*x^2-4*c*x*(c^2*x^2+1)^(1/2)+1)/c/(a+b*arcsinh(c*x))/b+1/4/c/b^2*exp(4*a/
b)*Ei(1,4*arcsinh(c*x)+4*a/b)-1/4*(2*c^2*x^2-2*c*x*(c^2*x^2+1)^(1/2)+1)/c/(
a+b*arcsinh(c*x))/b+1/2/c/b^2*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)-1/4/c/b
^2*(2*x^2*b*c^2+2*b*c*(c^2*x^2+1)^(1/2)*x+2*arcsinh(c*x)*exp(-2*a/b)*Ei(1,-
2*arcsinh(c*x)-2*a/b)*b+2*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*a+b)/(a+b
*arcsinh(c*x))-1/16/c/b^2*(8*x^4*b*c^4+8*(c^2*x^2+1)^(1/2)*x^3*b*c^3+8*x^2*
b*c^2+4*b*c*(c^2*x^2+1)^(1/2)*x+4*arcsinh(c*x)*Ei(1,-4*arcsinh(c*x)-4*a/b)*
exp(-4*a/b)*b+4*Ei(1,-4*arcsinh(c*x)-4*a/b)*exp(-4*a/b)*a+b)/(a+b*arcsinh(c
*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(c^4x^4+2\,c^2x^2+1\right)\left(c^2x^2+1\right)+\left(c^5x^5+2\,c^3x^3+cx\right)\sqrt{c^2x^2+1}}{abc^3x^2+\sqrt{c^2x^2+1}\,abc^2x+abc+\left(b^2c^3x^2+\sqrt{c^2x^2+1}\,b^2c^2x+b^2c\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int\frac{}{abc^4x^4+\left(c^2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^4*x^4 + 2*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^5*x^5 + 2*c^3*x^3 + c*x)*sqr
t(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c
^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1)))
 + integrate(((4*c^4*x^4 + 3*c^2*x^2 - 1)*(c^2*x^2 + 1)^(3/2) + 4*(2*c^5*x^
5 + 3*c^3*x^3 + c*x)*(c^2*x^2 + 1) + (4*c^6*x^6 + 9*c^4*x^4 + 6*c^2*x^2 + 1
```

```
)*sqrt(c^2*x^2 + 1))/(a*b*c^4*x^4 + (c^2*x^2 + 1)*a*b*c^2*x^2 + 2*a*b*c^2*x
^2 + a*b + (b^2*c^4*x^4 + (c^2*x^2 + 1)*b^2*c^2*x^2 + 2*b^2*c^2*x^2 + b^2 +
 2*(b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))
+ 2*(a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c^2\,x^2+1\right)^{3/2}}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(3/2)/(a + b*asinh(c*x))^2,x)
```

```
[Out] int((c^2*x^2 + 1)^(3/2)/(a + b*asinh(c*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^{2} x^{2} + 1\right)^{\frac{3}{2}}}{\left(a + b \operatorname{asinh}{\left(c x \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x))**2, x)
```

3.423 $\int \frac{\left(1+c^2x^2\right)^{3/2}}{x\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=175

$$-\frac{\text{Int}\left(\frac{c^2x^2+1}{x^2\left(a+b\sinh^{-1}(cx)\right)},x\right)}{bc}+\frac{9\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4b^2}+\frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{4b^2}-\frac{9\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a-}{}\right.}{4b^2}$$

```
[Out] -(c^2*x^2+1)^2/b/c/x/(a+b*arcsinh(c*x))+9/4*Chi((a+b*arcsinh(c*x))/b)*cosh(
a/b)/b^2+3/4*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b^2-9/4*Shi((a+b*arcsi
nh(c*x))/b)*sinh(a/b)/b^2-3/4*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2-U
nintegrable((c^2*x^2+1)/x^2/(a+b*arcsinh(c*x)),x)/b/c
```

Rubi [A] time = 0.44, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1+c^2x^2\right)^{3/2}}{x\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(3/2)/(x*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -((1 + c^2*x^2)^2/(b*c*x*(a + b*ArcSinh[c*x]))) + (9*Cosh[a/b]*CoshIntegral
[a/b + ArcSinh[c*x]])/(4*b^2) + (3*Cosh[(3*a)/b]*CoshIntegral[(3*a)/b + 3*A
rcSinh[c*x]])/(4*b^2) - (9*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(4*b
^2) - (3*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(4*b^2) - De
fer[Int][(1 + c^2*x^2)/(x^2*(a + b*ArcSinh[c*x])), x]/(b*c)
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(1+c^2x^2\right)^{3/2}}{x\left(a+b\sinh^{-1}(cx)\right)^2}\,dx &= -\frac{\left(1+c^2x^2\right)^2}{bcx\left(a+b\sinh^{-1}(cx)\right)}-\frac{\int \frac{1+c^2x^2}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx}{bc}+\frac{(3c)\int \frac{1+c^2x^2}{a+b\sinh^{-1}(cx)}\,dx}{b}\\
&= -\frac{\left(1+c^2x^2\right)^2}{bcx\left(a+b\sinh^{-1}(cx)\right)}+\frac{3\,\text{Subst}\left(\int \frac{\cosh^3(x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{b}-\frac{\int \frac{1+c^2x^2}{x^2\left(a+b\sinh^{-1}\right.}}{bc}\\
&= -\frac{\left(1+c^2x^2\right)^2}{bcx\left(a+b\sinh^{-1}(cx)\right)}+\frac{3\,\text{Subst}\left(\int \left(\frac{3\cosh(x)}{4(a+bx)}+\frac{\cosh(3x)}{4(a+bx)}\right)dx,x,\sinh^{-1}(cx)\right)}{b}-\\
&= -\frac{\left(1+c^2x^2\right)^2}{bcx\left(a+b\sinh^{-1}(cx)\right)}+\frac{3\,\text{Subst}\left(\int \frac{\cosh(3x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{4b}+\frac{9\,\text{Subst}\left(\int \right.}{}\\
&= -\frac{\left(1+c^2x^2\right)^2}{bcx\left(a+b\sinh^{-1}(cx)\right)}-\frac{\int \frac{1+c^2x^2}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx}{bc}+\frac{\left(9\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+\right.}{a+bx}\right.}{4b}\\
&= -\frac{\left(1+c^2x^2\right)^2}{bcx\left(a+b\sinh^{-1}(cx)\right)}+\frac{9\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4b^2}+\frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\right.}{4}
\end{aligned}$$

Mathematica [A] time = 7.70, size = 0, normalized size = 0.00

$$\int \frac{\left(1+c^2x^2\right)^{3/2}}{x\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(3/2)/(x*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[(1 + c^2*x^2)^(3/2)/(x*(a + b*ArcSinh[c*x])^2), x]
```

fricas [A] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{b^2x\,\text{arsinh}\,(cx)^2+2\,abx\,\text{arsinh}\,(cx)+a^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral((c^2*x^2 + 1)^(3/2)/(b^2*x*arcsinh(c*x)^2 + 2*a*b*x*arcsinh(c*x) +
 a^2*x), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{x\left(a+b\,\text{arcsinh}\,(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x))^2,x)
[Out] int((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(c^4x^4+2\,c^2x^2+1\right)\left(c^2x^2+1\right)+\left(c^5x^5+2\,c^3x^3+cx\right)\sqrt{c^2x^2+1}}{abc^3x^3+\sqrt{c^2x^2+1}\,abc^2x^2+abcx+\left(b^2c^3x^3+\sqrt{c^2x^2+1}\,b^2c^2x^2+b^2cx\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int \frac{}{abc^5x^6+\left(c^{\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -((c^4*x^4 + 2*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^5*x^5 + 2*c^3*x^3 + c*x)*sqr
t(c^2*x^2 + 1))/(a*b*c^3*x^3 + sqrt(c^2*x^2 + 1)*a*b*c^2*x^2 + a*b*c*x + (b
^2*c^3*x^3 + sqrt(c^2*x^2 + 1)*b^2*c^2*x^2 + b^2*c*x)*log(c*x + sqrt(c^2*x^
```

```
2 + 1))) + integrate(((3*c^5*x^5 + c^3*x^3 - 2*c*x)*(c^2*x^2 + 1)^(3/2) + (
6*c^6*x^6 + 7*c^4*x^4 - 1)*(c^2*x^2 + 1) + 3*(c^7*x^7 + 2*c^5*x^5 + c^3*x^3
)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^6 + (c^2*x^2 + 1)*a*b*c^3*x^4 + 2*a*b*c^3*x
^4 + a*b*c*x^2 + (b^2*c^5*x^6 + (c^2*x^2 + 1)*b^2*c^3*x^4 + 2*b^2*c^3*x^4 +
 b^2*c*x^2 + 2*(b^2*c^4*x^5 + b^2*c^2*x^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqr
t(c^2*x^2 + 1)) + 2*(a*b*c^4*x^5 + a*b*c^2*x^3)*sqrt(c^2*x^2 + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c^2\,x^2+1\right)^{3/2}}{x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(3/2)/(x*(a + b*asinh(c*x))^2),x)
```

```
[Out] int((c^2*x^2 + 1)^(3/2)/(x*(a + b*asinh(c*x))^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}}{x\left(a+b\,\mathrm{asinh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(3/2)/x/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((c**2*x**2 + 1)**(3/2)/(x*(a + b*asinh(c*x))**2), x)
```

3.424 $\int \frac{\left(1+c^2x^2\right)^{3/2}}{x^2\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=99

$$\frac{2c\text{Int}\left(\frac{c^2x^2+1}{x\left(a+b\sinh^{-1}(cx)\right)},x\right)}{b}-\frac{2\text{Int}\left(\frac{c^2x^2+1}{x^3\left(a+b\sinh^{-1}(cx)\right)},x\right)}{bc}-\frac{\left(c^2x^2+1\right)^2}{bcx^2\left(a+b\sinh^{-1}(cx)\right)}$$

```
[Out] -(c^2*x^2+1)^2/b/c/x^2/(a+b*arcsinh(c*x))-2*Unintegrable((c^2*x^2+1)/x^3/(a
+b*arcsinh(c*x)),x)/b/c+2*c*Unintegrable((c^2*x^2+1)/x/(a+b*arcsinh(c*x)),x
)/b
```

Rubi [A] time = 0.25, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1+c^2x^2\right)^{3/2}}{x^2\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(3/2)/(x^2*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -((1 + c^2*x^2)^2/(b*c*x^2*(a + b*ArcSinh[c*x]))) - (2*Defer[Int][(1 + c^2*
x^2)/(x^3*(a + b*ArcSinh[c*x])), x])/(b*c) + (2*c*Defer[Int][(1 + c^2*x^2)/
(x*(a + b*ArcSinh[c*x])), x])/b
```

Rubi steps

$$\int \frac{\left(1+c^2x^2\right)^{3/2}}{x^2\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=-\frac{\left(1+c^2x^2\right)^2}{bcx^2\left(a+b\sinh^{-1}(cx)\right)}-\frac{2\int \frac{1+c^2x^2}{x^3\left(a+b\sinh^{-1}(cx)\right)}\,dx}{bc}+\frac{(2c)\int \frac{1+c^2x^2}{x\left(a+b\sinh^{-1}(cx)\right)}\,dx}{b}$$

Mathematica [A] time = 4.31, size = 0, normalized size = 0.00

$$\int \frac{\left(1+c^2x^2\right)^{3/2}}{x^2\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(3/2)/(x^2*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[(1 + c^2*x^2)^(3/2)/(x^2*(a + b*ArcSinh[c*x])^2), x]
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{b^2x^2\,\text{arsinh}\,(cx)^2+2\,abx^2\,\text{arsinh}\,(cx)+a^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

[Out] integral((c^2*x^2 + 1)^(3/2)/(b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*x) + a^2*x^2), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="giac")

[Out] integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)^2*x^2), x)

maple [A] time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{x^2\left(a+b\operatorname{arcsinh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x))^2,x)

[Out] int((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x))^2,x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(c^4x^4+2\,c^2x^2+1\right)\left(c^2x^2+1\right)+\left(c^5x^5+2\,c^3x^3+cx\right)\sqrt{c^2x^2+1}}{abc^3x^4+\sqrt{c^2x^2+1}\,abc^2x^3+abcx^2+\left(b^2c^3x^4+\sqrt{c^2x^2+1}\,b^2c^2x^3+b^2cx^2\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int \frac{}{abc^5x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")

[Out] -((c^4*x^4 + 2*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^5*x^5 + 2*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^4 + sqrt(c^2*x^2 + 1)*a*b*c^2*x^3 + a*b*c*x^2 + (b^2*c^3*x^4 + sqrt(c^2*x^2 + 1)*b^2*c^2*x^3 + b^2*c*x^2)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((2*c^5*x^5 - c^3*x^3 - 3*c*x)*(c^2*x^2 + 1)^(3/2) + 2*(2*c^6*x^6 + c^4*x^4 - 2*c^2*x^2 - 1)*(c^2*x^2 + 1) + (2*c^7*x^7 + 3*c^5*x^5 - c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^7 + (c^2*x^2 + 1)*a*b*c^3*x^5 + 2*a*b*c^3*x^5 + a*b*c*x^3 + (b^2*c^5*x^7 + (c^2*x^2 + 1)*b^2*c^3*x^5 + 2*b^2*c^3*x^5 + b^2*c*x^3 + 2*(b^2*c^4*x^6 + b^2*c^2*x^4)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^6 + a*b*c^2*x^4)*sqrt(c^2*x^2 + 1)), x)

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c^2\,x^2+1\right)^{3/2}}{x^2\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c^2*x^2 + 1)^(3/2)/(x^2*(a + b*asinh(c*x))^2),x)

[Out] int((c^2*x^2 + 1)^(3/2)/(x^2*(a + b*asinh(c*x))^2), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{x^2\left(a+b\operatorname{asinh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(3/2)/x**2/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((c**2*x**2 + 1)**(3/2)/(x**2*(a + b*asinh(c*x))**2), x)
```

3.425 $\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{(c^2x^2+1)^{3/2}}{x^3\,(a+b\sinh^{-1}(cx))^2},x\right)$$

```
[Out] Unintegrable((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3\,(a+b\sinh^{-1}(cx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(3/2)/(x^3*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][(1 + c^2*x^2)^(3/2)/(x^3*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3\,(a+b\sinh^{-1}(cx))^2}\,dx=\int \frac{(1+c^2x^2)^{3/2}}{x^3\,(a+b\sinh^{-1}(cx))^2}\,dx$$

Mathematica [A] time = 14.51, size = 0, normalized size = 0.00

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3\,(a+b\sinh^{-1}(cx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(3/2)/(x^3*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[(1 + c^2*x^2)^(3/2)/(x^3*(a + b*ArcSinh[c*x])^2), x]
```

fricas [A] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(c^2x^2+1)^{\frac{3}{2}}}{b^2x^3\,\text{arsinh}\,(cx)^2+2\,abx^3\,\text{arsinh}\,(cx)+a^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral((c^2*x^2 + 1)^(3/2)/(b^2*x^3*arcsinh(c*x)^2 + 2*a*b*x^3*arcsinh(c*
x) + a^2*x^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 1.29, size = 0, normalized size = 0.00

$$\int \frac{(c^2x^2+1)^{\frac{3}{2}}}{x^3 (a+b \operatorname{arcsinh}(cx))^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{(c^4x^4+2c^2x^2+1)(c^2x^2+1)+(c^5x^5+2c^3x^3+cx)\sqrt{c^2x^2+1}}{abc^3x^5+\sqrt{c^2x^2+1}\,abc^2x^4+abcx^3+\left(b^2c^3x^5+\sqrt{c^2x^2+1}\,b^2c^2x^4+b^2cx^3\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int \frac{\quad}{abc^5x^8+(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^4*x^4 + 2*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^5*x^5 + 2*c^3*x^3 + c*x)*sqr
t(c^2*x^2 + 1))/(a*b*c^3*x^5 + sqrt(c^2*x^2 + 1)*a*b*c^2*x^4 + a*b*c*x^3 +
(b^2*c^3*x^5 + sqrt(c^2*x^2 + 1)*b^2*c^2*x^4 + b^2*c*x^3)*log(c*x + sqrt(c^
2*x^2 + 1))) + integrate(((c^5*x^5 - 3*c^3*x^3 - 4*c*x)*(c^2*x^2 + 1)^(3/2)
 + (2*c^6*x^6 - 3*c^4*x^4 - 8*c^2*x^2 - 3)*(c^2*x^2 + 1) + (c^7*x^7 - 3*c^3
*x^3 - 2*c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^8 + (c^2*x^2 + 1)*a*b*c^3*x^6 +
 2*a*b*c^3*x^6 + a*b*c*x^4 + (b^2*c^5*x^8 + (c^2*x^2 + 1)*b^2*c^3*x^6 + 2*b
^2*c^3*x^6 + b^2*c*x^4 + 2*(b^2*c^4*x^7 + b^2*c^2*x^5)*sqrt(c^2*x^2 + 1))*l
og(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^7 + a*b*c^2*x^5)*sqrt(c^2*x^2 +
1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(c^2\,x^2+1)^{3/2}}{x^3\,(a+b\,\mathrm{asinh}(c\,x))^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(3/2)/(x^3*(a + b*asinh(c*x))^2),x)
```

```
[Out] int((c^2*x^2 + 1)^(3/2)/(x^3*(a + b*asinh(c*x))^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c^2x^2+1)^{\frac{3}{2}}}{x^3 (a+b \operatorname{asinh}(cx))^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(3/2)/x**3/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((c**2*x**2 + 1)**(3/2)/(x**3*(a + b*asinh(c*x))**2), x)
```

3.426 $\int \frac{\left(1+c^2x^2\right)^{3/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=67

$$-\frac{4\text{Int}\left(\frac{c^2x^2+1}{x^5\left(a+b\sinh^{-1}(cx)\right)},x\right)}{bc}-\frac{\left(c^2x^2+1\right)^2}{bcx^4\left(a+b\sinh^{-1}(cx)\right)}$$

```
[Out] -(c^2*x^2+1)^2/b/c/x^4/(a+b*arcsinh(c*x))-4*Unintegrable((c^2*x^2+1)/x^5/(a
+b*arcsinh(c*x)),x)/b/c
```

Rubi [A] time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1+c^2x^2\right)^{3/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(3/2)/(x^4*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -((1 + c^2*x^2)^2/(b*c*x^4*(a + b*ArcSinh[c*x]))) - (4*Defer[Int][(1 + c^2*
x^2)/(x^5*(a + b*ArcSinh[c*x])), x])/(b*c)
```

Rubi steps

$$\int \frac{\left(1+c^2x^2\right)^{3/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=-\frac{\left(1+c^2x^2\right)^2}{bcx^4\left(a+b\sinh^{-1}(cx)\right)}-\frac{4\int \frac{1+c^2x^2}{x^5\left(a+b\sinh^{-1}(cx)\right)}\,dx}{bc}$$

Mathematica [A] time = 2.78, size = 0, normalized size = 0.00

$$\int \frac{\left(1+c^2x^2\right)^{3/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(3/2)/(x^4*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[(1 + c^2*x^2)^(3/2)/(x^4*(a + b*ArcSinh[c*x])^2), x]
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{b^2x^4\,\text{arsinh}\left(cx\right)^2+2\,abx^4\,\text{arsinh}\left(cx\right)+a^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^2*x^2 + 1)^(3/2)/(b^2*x^4*arcsinh(c*x)^2 + 2*a*b*x^4*arcsinh(c*
x) + a^2*x^4), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)^2*x^4), x)
```

maple [A] time = 0.77, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{x^4\left(a+b\operatorname{arcsinh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(c^4x^4+2\,c^2x^2+1\right)\left(c^2x^2+1\right)+\left(c^5x^5+2\,c^3x^3+cx\right)\sqrt{c^2x^2+1}}{abc^3x^6+\sqrt{c^2x^2+1}\,abc^2x^5+abcx^4+\left(b^2c^3x^6+\sqrt{c^2x^2+1}\,b^2c^2x^5+b^2cx^4\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}-\int \frac{}{abc^5x^9+(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^4*x^4 + 2*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^5*x^5 + 2*c^3*x^3 + c*x)*sqr
t(c^2*x^2 + 1))/(a*b*c^3*x^6 + sqrt(c^2*x^2 + 1)*a*b*c^2*x^5 + a*b*c*x^4 +
(b^2*c^3*x^6 + sqrt(c^2*x^2 + 1)*b^2*c^2*x^5 + b^2*c*x^4)*log(c*x + sqrt(c^
2*x^2 + 1))) - integrate((5*(c^3*x^3 + c*x)*(c^2*x^2 + 1)^(3/2) + 4*(2*c^4*
x^4 + 3*c^2*x^2 + 1)*(c^2*x^2 + 1) + 3*(c^5*x^5 + 2*c^3*x^3 + c*x)*sqrt(c^2
*x^2 + 1))/(a*b*c^5*x^9 + (c^2*x^2 + 1)*a*b*c^3*x^7 + 2*a*b*c^3*x^7 + a*b*c
*x^5 + (b^2*c^5*x^9 + (c^2*x^2 + 1)*b^2*c^3*x^7 + 2*b^2*c^3*x^7 + b^2*c*x^5
 + 2*(b^2*c^4*x^8 + b^2*c^2*x^6)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2
+ 1)) + 2*(a*b*c^4*x^8 + a*b*c^2*x^6)*sqrt(c^2*x^2 + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c^2\,x^2+1\right)}^{3/2}}{x^4\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(3/2)/(x^4*(a + b*asinh(c*x))^2),x)
```

```
[Out] int((c^2*x^2 + 1)^(3/2)/(x^4*(a + b*asinh(c*x))^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{x^4\left(a+b\operatorname{asinh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(3/2)/x**4/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((c**2*x**2 + 1)**(3/2)/(x**4*(a + b*asinh(c*x))**2), x)
```

3.427 $\int \frac{x^3(1+c^2x^2)^{5/2}}{(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=277

$$-\frac{3\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{128b^2c^4}-\frac{3\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{32b^2c^4}+\frac{21\cosh\left(\frac{7a}{b}\right)\mathrm{Chi}\left(\frac{7\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{256b^2c^4}+\frac{9\cosh}{}$$

```
[Out] -x^3*(c^2*x^2+1)^3/b/c/(a+b*arcsinh(c*x))-3/128*Chi((a+b*arcsinh(c*x))/b)*c
osh(a/b)/b^2/c^4-3/32*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b^2/c^4+21/25
6*Chi(7*(a+b*arcsinh(c*x))/b)*cosh(7*a/b)/b^2/c^4+9/256*Chi(9*(a+b*arcsinh(
c*x))/b)*cosh(9*a/b)/b^2/c^4+3/128*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b^2/
c^4+3/32*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^4-21/256*Shi(7*(a+b*
arcsinh(c*x))/b)*sinh(7*a/b)/b^2/c^4-9/256*Shi(9*(a+b*arcsinh(c*x))/b)*sinh
(9*a/b)/b^2/c^4
```

Rubi [A] time = 1.22, antiderivative size = 273, normalized size of antiderivative = 0.99, number of steps used = 34, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5777, 5779, 5448, 3303, 3298, 3301}

$$-\frac{3\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{128b^2c^4}-\frac{3\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{32b^2c^4}+\frac{21\cosh\left(\frac{7a}{b}\right)\mathrm{Chi}\left(\frac{7a}{b}+7\sinh^{-1}(cx)\right)}{256b^2c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((x^3*(1 + c^2*x^2)^3)/(b*c*(a + b*ArcSinh[c*x]))) - (3*Cosh[a/b]*CoshInte
gral[a/b + ArcSinh[c*x]])/(128*b^2*c^4) - (3*Cosh[(3*a)/b]*CoshIntegral[(3*
a)/b + 3*ArcSinh[c*x]])/(32*b^2*c^4) + (21*Cosh[(7*a)/b]*CoshIntegral[(7*a)
/b + 7*ArcSinh[c*x]])/(256*b^2*c^4) + (9*Cosh[(9*a)/b]*CoshIntegral[(9*a)/b
 + 9*ArcSinh[c*x]])/(256*b^2*c^4) + (3*Sinh[a/b]*SinhIntegral[a/b + ArcSinh
[c*x]])/(128*b^2*c^4) + (3*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c
*x]])/(32*b^2*c^4) - (21*Sinh[(7*a)/b]*SinhIntegral[(7*a)/b + 7*ArcSinh[c*x
]])/(256*b^2*c^4) - (9*Sinh[(9*a)/b]*SinhIntegral[(9*a)/b + 9*ArcSinh[c*x]]
)/(256*b^2*c^4)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^
p*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-Dist[(f*m*d^IntPart[p
]*(d + e*x^2)^FracPart[p])/(b*c*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*
x)^(m - 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - D
ist[(c*(m + 2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(b*f*(n + 1)*(1
+ c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*A
rcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[2*p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^3\left(1+c^2x^2\right)^{5/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{x^3\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{3\int \frac{x^2\left(1+c^2x^2\right)^2}{a+b\sinh^{-1}(cx)}\,dx}{bc} + \frac{(9c)\int \frac{x^4\left(1+c^2x^2\right)^2}{a+b\sinh^{-1}(cx)}\,dx}{b}$$

$$= -\frac{x^3\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{3\,\text{Subst}\left(\int \frac{\cosh^5(x)\sinh^2(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{bc^4} + \frac{9\,\text{Subst}\left(\int \right.}{}$$

$$= -\frac{x^3\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{3\,\text{Subst}\left(\int \left(-\frac{5\cosh(x)}{64(a+bx)} + \frac{\cosh(3x)}{64(a+bx)} + \frac{3\cosh(5x)}{64(a+bx)} + \frac{\cosh(7x)}{64(a+bx)}\right) dx,\right.}{bc^4}$$

$$= -\frac{x^3\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{9\,\text{Subst}\left(\int \frac{\cosh(7x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{256bc^4} + \frac{9\,\text{Subst}\left(\int \frac{\cosh(9x)}{a+bx}\right.}{256}$$

$$= -\frac{x^3\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\left(27\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{128bc^4} - \frac{(15\,c}{}$$

$$= -\frac{x^3\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{128b^2c^4} - \frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\right.}{32b^2c^4}$$

Mathematica [A] time = 1.38, size = 408, normalized size = 1.47

$$-\frac{6\cosh\left(\frac{a}{b}\right)\left(a+b\sinh^{-1}(cx)\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right) + 24\cosh\left(\frac{3a}{b}\right)\left(a+b\sinh^{-1}(cx)\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -1/256*(256*b*c^3*x^3 + 768*b*c^5*x^5 + 768*b*c^7*x^7 + 256*b*c^9*x^9 + 6*(
a + b*ArcSinh[c*x])*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] + 24*(a + b*
ArcSinh[c*x])*Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] - 21*a*Cos
h[(7*a)/b]*CoshIntegral[7*(a/b + ArcSinh[c*x])] - 21*b*ArcSinh[c*x]*Cosh[(7
*a)/b]*CoshIntegral[7*(a/b + ArcSinh[c*x])] - 9*a*Cosh[(9*a)/b]*CoshIntegra
l[9*(a/b + ArcSinh[c*x])] - 9*b*ArcSinh[c*x]*Cosh[(9*a)/b]*CoshIntegral[9*(
a/b + ArcSinh[c*x])] - 6*a*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - 6*b
*ArcSinh[c*x]*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - 24*a*Sinh[(3*a)/
b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] - 24*b*ArcSinh[c*x]*Sinh[(3*a)/b]*S
inhIntegral[3*(a/b + ArcSinh[c*x])] + 21*a*Sinh[(7*a)/b]*SinhIntegral[7*(a/
b + ArcSinh[c*x])] + 21*b*ArcSinh[c*x]*Sinh[(7*a)/b]*SinhIntegral[7*(a/b +
ArcSinh[c*x])] + 9*a*Sinh[(9*a)/b]*SinhIntegral[9*(a/b + ArcSinh[c*x])] + 9
*b*ArcSinh[c*x]*Sinh[(9*a)/b]*SinhIntegral[9*(a/b + ArcSinh[c*x])])/(b^2*c^
4*(a + b*ArcSinh[c*x]))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^4x^7+2\,c^2x^5+x^3\right)\sqrt{c^2x^2+1}}{b^2\,\text{arsinh}\,(cx)^2+2\,ab\,\text{arsinh}\,(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^4*x^7 + 2*c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1)/(b^2*arcsinh(c*x)^2
+ 2*a*b*arcsinh(c*x) + a^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [B] time = 0.74, size = 1070, normalized size = 3.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] -1/512*(256*c^9*x^9-256*c^8*x^8*(c^2*x^2+1)^(1/2)+576*c^7*x^7-448*c^6*x^6*(
c^2*x^2+1)^(1/2)+432*c^5*x^5-240*c^4*x^4*(c^2*x^2+1)^(1/2)+120*c^3*x^3-40*c
^2*x^2*(c^2*x^2+1)^(1/2)+9*c*x-(c^2*x^2+1)^(1/2))/c^4/(a+b*arcsinh(c*x))/b-
9/512/c^4/b^2*exp(9*a/b)*Ei(1,9*arcsinh(c*x)+9*a/b)-3/512*(64*c^7*x^7-64*c^
6*x^6*(c^2*x^2+1)^(1/2)+112*c^5*x^5-80*c^4*x^4*(c^2*x^2+1)^(1/2)+56*c^3*x^3
-24*c^2*x^2*(c^2*x^2+1)^(1/2)+7*c*x-(c^2*x^2+1)^(1/2))/c^4/(a+b*arcsinh(c*x
))/b-21/512/c^4/b^2*exp(7*a/b)*Ei(1,7*arcsinh(c*x)+7*a/b)+1/64*(4*c^3*x^3-4
*c^2*x^2*(c^2*x^2+1)^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))/c^4/b/(a+b*arcsinh(c*x)
)+3/64/c^4/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)+3/256*(c*x-(c^2*x^2+1)
^(1/2))/c^4/b/(a+b*arcsinh(c*x))+3/256/c^4/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a
/b)+3/256/c^4/b^2*(arcsinh(c*x)*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*b+exp(-a/
b)*Ei(1,-arcsinh(c*x)-a/b)*a+x*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))+
1/64/c^4/b^2*(4*x^3*b*c^3+4*(c^2*x^2+1)^(1/2)*x^2*b*c^2+3*arcsinh(c*x)*exp(
```

```
-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)*b+3*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*
a/b)*a+3*x*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-3/512/c^4/b^2*(64*x^
7*b*c^7+64*(c^2*x^2+1)^(1/2)*x^6*b*c^6+112*x^5*b*c^5+80*(c^2*x^2+1)^(1/2)*x
^4*b*c^4+56*x^3*b*c^3+24*(c^2*x^2+1)^(1/2)*x^2*b*c^2+7*arcsinh(c*x)*Ei(1,-7
*arcsinh(c*x)-7*a/b)*exp(-7*a/b)*b+7*Ei(1,-7*arcsinh(c*x)-7*a/b)*exp(-7*a/b
)*a+7*x*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-1/512/c^4/b^2*(256*x^9*
b*c^9+256*(c^2*x^2+1)^(1/2)*x^8*b*c^8+576*x^7*b*c^7+448*(c^2*x^2+1)^(1/2)*x
^6*b*c^6+432*x^5*b*c^5+240*(c^2*x^2+1)^(1/2)*x^4*b*c^4+120*x^3*b*c^3+40*(c^
2*x^2+1)^(1/2)*x^2*b*c^2+9*arcsinh(c*x)*exp(-9*a/b)*Ei(1,-9*arcsinh(c*x)-9*
a/b)*b+9*exp(-9*a/b)*Ei(1,-9*arcsinh(c*x)-9*a/b)*a+9*x*b*c+(c^2*x^2+1)^(1/2
)*b)/(a+b*arcsinh(c*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(c^6x^9+3\,c^4x^7+3\,c^2x^5+x^3\right)\left(c^2x^2+1\right)+\left(c^7x^{10}+3\,c^5x^8+3\,c^3x^6+cx^4\right)\sqrt{c^2x^2+1}}{abc^3x^2+\sqrt{c^2x^2+1}\,abc^2x+abc+\left(b^2c^3x^2+\sqrt{c^2x^2+1}\,b^2c^2x+b^2c\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int\frac{\left(9\,c^7x^9+\right.}{abc^5x^4+\left(c^2x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^6*x^9 + 3*c^4*x^7 + 3*c^2*x^5 + x^3)*(c^2*x^2 + 1) + (c^7*x^10 + 3*c^5
*x^8 + 3*c^3*x^6 + c*x^4)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 +
1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*
log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((9*c^7*x^9 + 20*c^5*x^7 + 13*c^3
*x^5 + 2*c*x^3)*(c^2*x^2 + 1)^(3/2) + 3*(6*c^8*x^10 + 17*c^6*x^8 + 17*c^4*x
^6 + 7*c^2*x^4 + x^2)*(c^2*x^2 + 1) + (9*c^9*x^11 + 31*c^7*x^9 + 39*c^5*x^7
 + 21*c^3*x^5 + 4*c*x^3)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c^2*x^2 + 1)*a*
b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 + 1)*b^2*c^3*x^
2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(c^2*x^2 + 1))*
log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sqrt(c^2*x^2 + 1
)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(c^2\,x^2+1\right)}^{5/2}}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x))^2,x)
```

```
[Out] int((x^3*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \left(c^{2} x^{2} + 1\right)^{\frac{5}{2}}}{\left(a + b \operatorname{asinh}{\left(c x \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x**3*(c**2*x**2 + 1)**(5/2)/(a + b*asinh(c*x))**2, x)
```

3.428 $\int \frac{x^2\left(1+c^2x^2\right)^{5/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=281

$$\frac{\sinh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16b^2c^3}-\frac{\sinh\left(\frac{4a}{b}\right)\operatorname{Chi}\left(\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{8b^2c^3}-\frac{3\sinh\left(\frac{6a}{b}\right)\operatorname{Chi}\left(\frac{6\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16b^2c^3}-\frac{\sinh\left(\frac{8a}{b}\right)\cdots}{}$$

```
[Out] -x^2*(c^2*x^2+1)^3/b/c/(a+b*arcsinh(c*x))-1/16*cosh(2*a/b)*Shi(2*(a+b*arcsi
nh(c*x))/b)/b^2/c^3+1/8*cosh(4*a/b)*Shi(4*(a+b*arcsinh(c*x))/b)/b^2/c^3+3/1
6*cosh(6*a/b)*Shi(6*(a+b*arcsinh(c*x))/b)/b^2/c^3+1/16*cosh(8*a/b)*Shi(8*(a
+b*arcsinh(c*x))/b)/b^2/c^3+1/16*Chi(2*(a+b*arcsinh(c*x))/b)*sinh(2*a/b)/b^
2/c^3-1/8*Chi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b^2/c^3-3/16*Chi(6*(a+b*a
rcsinh(c*x))/b)*sinh(6*a/b)/b^2/c^3-1/16*Chi(8*(a+b*arcsinh(c*x))/b)*sinh(8
*a/b)/b^2/c^3
```

Rubi [A] time = 1.07, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5777, 5779, 5448, 3303, 3298, 3301}

$$\frac{\sinh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{16b^2c^3}-\frac{\sinh\left(\frac{4a}{b}\right)\operatorname{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{8b^2c^3}-\frac{3\sinh\left(\frac{6a}{b}\right)\operatorname{Chi}\left(\frac{6a}{b}+6\sinh^{-1}(cx)\right)}{16b^2c^3}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((x^2*(1 + c^2*x^2)^3)/(b*c*(a + b*ArcSinh[c*x]))) + (CoshIntegral[(2*a)/b
 + 2*ArcSinh[c*x]]*Sinh[(2*a)/b])/(16*b^2*c^3) - (CoshIntegral[(4*a)/b + 4*
ArcSinh[c*x]]*Sinh[(4*a)/b])/(8*b^2*c^3) - (3*CoshIntegral[(6*a)/b + 6*ArcS
inh[c*x]]*Sinh[(6*a)/b])/(16*b^2*c^3) - (CoshIntegral[(8*a)/b + 8*ArcSinh[c
*x]]*Sinh[(8*a)/b])/(16*b^2*c^3) - (Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*
ArcSinh[c*x]])/(16*b^2*c^3) + (Cosh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcSi
nh[c*x]])/(8*b^2*c^3) + (3*Cosh[(6*a)/b]*SinhIntegral[(6*a)/b + 6*ArcSinh[c
*x]])/(16*b^2*c^3) + (Cosh[(8*a)/b]*SinhIntegral[(8*a)/b + 8*ArcSinh[c*x]])
/(16*b^2*c^3)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^
p*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-Dist[(f*m*d^IntPart[p
]*(d + e*x^2)^FracPart[p])/(b*c*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*
x)^(m - 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - D
ist[(c*(m + 2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(b*f*(n + 1)*(1
+ c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*A
rcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[2*p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2\left(1+c^2x^2\right)^{5/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{x^2\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{2\int\frac{x\left(1+c^2x^2\right)^2}{a+b\sinh^{-1}(cx)}\,dx}{bc}+\frac{(8c)\int\frac{x^3\left(1+c^2x^2\right)^2}{a+b\sinh^{-1}(cx)}\,dx}{b}$$

$$= -\frac{x^2\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{2\,\mathrm{Subst}\left(\int\frac{\cosh^5(x)\sinh(x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{bc^3}+\frac{8\,\mathrm{Subst}\left(\int \mathrm{c}\right.}{}$$

$$= -\frac{x^2\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{2\,\mathrm{Subst}\left(\int\left(\frac{5\sinh(2x)}{32(a+bx)}+\frac{\sinh(4x)}{8(a+bx)}+\frac{\sinh(6x)}{32(a+bx)}\right)dx,x,\sinh^{-1}(cx\right.}{bc^3}$$

$$= -\frac{x^2\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\mathrm{Subst}\left(\int\frac{\sinh(6x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{16bc^3}+\frac{\mathrm{Subst}\left(\int\frac{\sinh(8x)}{a+bx}\,dx,\right.}{16bc^3}$$

$$= -\frac{x^2\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\left(5\cosh\left(\frac{2a}{b}\right)\right)\mathrm{Subst}\left(\int\frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{16bc^3}-\frac{(3}{}$$

$$= -\frac{x^2\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\mathrm{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{16b^2c^3}-\frac{\mathrm{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx\right.}{8b^2c^3}$$

Mathematica [A] time = 1.17, size = 413, normalized size = 1.47

$$-\frac{-\sinh\left(\frac{2a}{b}\right)\left(a+b\sinh^{-1}(cx)\right)\mathrm{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+2\sinh\left(\frac{4a}{b}\right)\left(a+b\sinh^{-1}(cx)\right)\mathrm{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -1/16*(16*b*c^2*x^2 + 48*b*c^4*x^4 + 48*b*c^6*x^6 + 16*b*c^8*x^8 - (a + b*A
rcSinh[c*x])*CoshIntegral[2*(a/b + ArcSinh[c*x])]*Sinh[(2*a)/b] + 2*(a + b*
ArcSinh[c*x])*CoshIntegral[4*(a/b + ArcSinh[c*x])]*Sinh[(4*a)/b] + 3*a*Cosh
Integral[6*(a/b + ArcSinh[c*x])]*Sinh[(6*a)/b] + 3*b*ArcSinh[c*x]*CoshInteg
ral[6*(a/b + ArcSinh[c*x])]*Sinh[(6*a)/b] + a*CoshIntegral[8*(a/b + ArcSinh
[c*x])]*Sinh[(8*a)/b] + b*ArcSinh[c*x]*CoshIntegral[8*(a/b + ArcSinh[c*x])]
*Sinh[(8*a)/b] + a*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] + b*A
rcSinh[c*x]*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] - 2*a*Cosh[(
4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])] - 2*b*ArcSinh[c*x]*Cosh[(4*a)/
b]*SinhIntegral[4*(a/b + ArcSinh[c*x])] - 3*a*Cosh[(6*a)/b]*SinhIntegral[6*
(a/b + ArcSinh[c*x])] - 3*b*ArcSinh[c*x]*Cosh[(6*a)/b]*SinhIntegral[6*(a/b
+ ArcSinh[c*x])] - a*Cosh[(8*a)/b]*SinhIntegral[8*(a/b + ArcSinh[c*x])] - b
*ArcSinh[c*x]*Cosh[(8*a)/b]*SinhIntegral[8*(a/b + ArcSinh[c*x])])/(b^2*c^3*
(a + b*ArcSinh[c*x]))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^4x^6+2\,c^2x^4+x^2\right)\sqrt{c^2x^2+1}}{b^2\,\text{arsinh}\,(cx)^2+2\,ab\,\text{arsinh}\,(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^4*x^6 + 2*c^2*x^4 + x^2)*sqrt(c^2*x^2 + 1)/(b^2*arcsinh(c*x)^2
+ 2*a*b*arcsinh(c*x) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(c^2x^2+1\right)^{\frac{5}{2}}x^2}{\left(b\,\text{arsinh}\,(cx)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)*x^2/(b*arcsinh(c*x) + a)^2, x)
```

maple [B] time = 0.89, size = 1044, normalized size = 3.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 5/128/c^3/(a+b*arcsinh(c*x))/b-1/256*(128*c^8*x^8-128*c^7*x^7*(c^2*x^2+1)^(
1/2)+256*c^6*x^6-192*c^5*x^5*(c^2*x^2+1)^(1/2)+160*c^4*x^4-80*c^3*x^3*(c^2*
x^2+1)^(1/2)+32*c^2*x^2-8*c*x*(c^2*x^2+1)^(1/2)+1)/c^3/(a+b*arcsinh(c*x))/b
+1/32/c^3/b^2*exp(8*a/b)*Ei(1,8*arcsinh(c*x)+8*a/b)-1/64*(32*c^6*x^6-32*c^5
*x^5*(c^2*x^2+1)^(1/2)+48*c^4*x^4-32*c^3*x^3*(c^2*x^2+1)^(1/2)+18*c^2*x^2-6
*c*x*(c^2*x^2+1)^(1/2)+1)/c^3/(a+b*arcsinh(c*x))/b+3/32/c^3/b^2*exp(6*a/b)*
Ei(1,6*arcsinh(c*x)+6*a/b)-1/64*(8*c^4*x^4-8*c^3*x^3*(c^2*x^2+1)^(1/2)+8*c^
2*x^2-4*c*x*(c^2*x^2+1)^(1/2)+1)/c^3/(a+b*arcsinh(c*x))/b+1/16/c^3/b^2*exp(
4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)+1/64*(2*c^2*x^2-2*c*x*(c^2*x^2+1)^(1/2)+1
)/c^3/(a+b*arcsinh(c*x))/b-1/32/c^3/b^2*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/
b)+1/64/c^3/b^2*(2*x^2*b*c^2+2*b*c*(c^2*x^2+1)^(1/2)*x+2*arcsinh(c*x)*exp(-
2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*b+2*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a
/b)*a+b)/(a+b*arcsinh(c*x))-1/64/c^3/b^2*(8*x^4*b*c^4+8*(c^2*x^2+1)^(1/2)*x
```

```
^3*b*c^3+8*x^2*b*c^2+4*b*c*(c^2*x^2+1)^(1/2)*x+4*arcsinh(c*x)*Ei(1,-4*arcsi
nh(c*x)-4*a/b)*exp(-4*a/b)*b+4*Ei(1,-4*arcsinh(c*x)-4*a/b)*exp(-4*a/b)*a+b)
/(a+b*arcsinh(c*x))-1/64/c^3/b^2*(32*x^6*b*c^6+32*(c^2*x^2+1)^(1/2)*x^5*b*c
^5+48*x^4*b*c^4+32*(c^2*x^2+1)^(1/2)*x^3*b*c^3+18*x^2*b*c^2+6*b*c*(c^2*x^2+
1)^(1/2)*x+6*arcsinh(c*x)*Ei(1,-6*arcsinh(c*x)-6*a/b)*exp(-6*a/b)*b+6*Ei(1,
-6*arcsinh(c*x)-6*a/b)*exp(-6*a/b)*a+b)/(a+b*arcsinh(c*x))-1/256/c^3/b^2*(1
28*x^8*b*c^8+128*(c^2*x^2+1)^(1/2)*x^7*b*c^7+256*x^6*b*c^6+192*(c^2*x^2+1)^
(1/2)*x^5*b*c^5+160*x^4*b*c^4+80*(c^2*x^2+1)^(1/2)*x^3*b*c^3+32*x^2*b*c^2+8
*arcsinh(c*x)*Ei(1,-8*arcsinh(c*x)-8*a/b)*exp(-8*a/b)*b+8*b*c*(c^2*x^2+1)^(
1/2)*x+8*Ei(1,-8*arcsinh(c*x)-8*a/b)*exp(-8*a/b)*a+b)/(a+b*arcsinh(c*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(c^{6} x^{8}+3 c^{4} x^{6}+3 c^{2} x^{4}+x^{2}\right)\left(c^{2} x^{2}+1\right)+\left(c^{7} x^{9}+3 c^{5} x^{7}+3 c^{3} x^{5}+c x^{3}\right) \sqrt{c^{2} x^{2}+1}}{a b c^{3} x^{2}+\sqrt{c^{2} x^{2}+1}\, a b c^{2} x+a b c+\left(b^{2} c^{3} x^{2}+\sqrt{c^{2} x^{2}+1}\, b^{2} c^{2} x+b^{2} c\right) \log \left(c x+\sqrt{c^{2} x^{2}+1}\right)}+\int \frac{\left(8 c^{7} x^{8} \right.}{a b c^{5} x^{4}+\left(c^{2} x^{2} \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^6*x^8 + 3*c^4*x^6 + 3*c^2*x^4 + x^2)*(c^2*x^2 + 1) + (c^7*x^9 + 3*c^5*
x^7 + 3*c^3*x^5 + c*x^3)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1
)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*l
og(c*x + sqrt(c^2*x^2 + 1))) + integrate(((8*c^7*x^8 + 17*c^5*x^6 + 10*c^3*
x^4 + c*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(8*c^8*x^9 + 22*c^6*x^7 + 21*c^4*x^5 +
 8*c^2*x^3 + x)*(c^2*x^2 + 1) + (8*c^9*x^10 + 27*c^7*x^8 + 33*c^5*x^6 + 17*
c^3*x^4 + 3*c*x^2)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c^2*x^2 + 1)*a*b*c^3*
x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 + 1)*b^2*c^3*x^2 + 2*
b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(c^2*x^2 + 1))*log(c*
x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sqrt(c^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(c^2\,x^2+1\right)}^{5/2}}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x))^2,x)
```

```
[Out] int((x^2*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(c^{2} x^{2} + 1\right)^{\frac{5}{2}}}{\left(a + b \operatorname{asinh}{\left(c x \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x**2*(c**2*x**2 + 1)**(5/2)/(a + b*asinh(c*x))**2, x)
```

3.429 $\int \frac{x(1+c^2x^2)^{5/2}}{(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=275

$$\frac{5\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{64b^2c^2}+\frac{27\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{64b^2c^2}+\frac{25\cosh\left(\frac{5a}{b}\right)\mathrm{Chi}\left(\frac{5(a+b\sinh^{-1}(cx))}{b}\right)}{64b^2c^2}+\frac{7\cosh}{}$$

```
[Out] -x*(c^2*x^2+1)^3/b/c/(a+b*arcsinh(c*x))+5/64*Chi((a+b*arcsinh(c*x))/b)*cosh
(a/b)/b^2/c^2+27/64*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b^2/c^2+25/64*C
hi(5*(a+b*arcsinh(c*x))/b)*cosh(5*a/b)/b^2/c^2+7/64*Chi(7*(a+b*arcsinh(c*x)
)/b)*cosh(7*a/b)/b^2/c^2-5/64*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b^2/c^2-2
7/64*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^2-25/64*Shi(5*(a+b*arcsi
nh(c*x))/b)*sinh(5*a/b)/b^2/c^2-7/64*Shi(7*(a+b*arcsinh(c*x))/b)*sinh(7*a/b
)/b^2/c^2
```

Rubi [A] time = 0.97, antiderivative size = 271, normalized size of antiderivative = 0.99, number of steps used = 28, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {5777, 5699, 3312, 3303, 3298, 3301, 5779, 5448}

$$\frac{5\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{64b^2c^2}+\frac{27\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{64b^2c^2}+\frac{25\cosh\left(\frac{5a}{b}\right)\mathrm{Chi}\left(\frac{5a}{b}+5\sinh^{-1}(cx)\right)}{64b^2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((x*(1 + c^2*x^2)^3)/(b*c*(a + b*ArcSinh[c*x]))) + (5*Cosh[a/b]*CoshIntegr
al[a/b + ArcSinh[c*x]])/(64*b^2*c^2) + (27*Cosh[(3*a)/b]*CoshIntegral[(3*a)
/b + 3*ArcSinh[c*x]])/(64*b^2*c^2) + (25*Cosh[(5*a)/b]*CoshIntegral[(5*a)/b
 + 5*ArcSinh[c*x]])/(64*b^2*c^2) + (7*Cosh[(7*a)/b]*CoshIntegral[(7*a)/b +
7*ArcSinh[c*x]])/(64*b^2*c^2) - (5*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x
]])/(64*b^2*c^2) - (27*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]]
)/(64*b^2*c^2) - (25*Sinh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcSinh[c*x]])/
(64*b^2*c^2) - (7*Sinh[(7*a)/b]*SinhIntegral[(7*a)/b + 7*ArcSinh[c*x]])/(64
*b^2*c^2)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^
p*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-Dist[(f*m*d^IntPart[p
]*(d + e*x^2)^FracPart[p])/(b*c*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*
x)^(m - 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - D
ist[(c*(m + 2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(b*f*(n + 1)*(1
+ c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*A
rcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[2*p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x\left(1+c^2x^2\right)^{5/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{x\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\int \frac{\left(1+c^2x^2\right)^2}{a+b\sinh^{-1}(cx)}\,dx}{bc} + \frac{(7c)\int \frac{x^2\left(1+c^2x^2\right)^2}{a+b\sinh^{-1}(cx)}\,dx}{b}$$

$$= -\frac{x\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\text{Subst}\left(\int \frac{\cosh^5(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{bc^2} + \frac{7\,\text{Subst}\left(\int \frac{\cosh^5(x}{a}\right.}{}$$

$$= -\frac{x\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\text{Subst}\left(\int \left(\frac{5\cosh(x)}{8(a+bx)} + \frac{5\cosh(3x)}{16(a+bx)} + \frac{\cosh(5x)}{16(a+bx)}\right)dx, x, \sinh^{-1}(c\right.}{bc^2}$$

$$= -\frac{x\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\text{Subst}\left(\int \frac{\cosh(5x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{16bc^2} + \frac{7\,\text{Subst}\left(\int \frac{\cosh(3x}{a+bx}\right.}{64}$$

$$= -\frac{x\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{\left(35\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{64bc^2} + \frac{(5}{}$$

$$= -\frac{x\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{5\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{64b^2c^2} + \frac{27\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}\right.}{64b^2}$$

Mathematica [A] time = 1.02, size = 404, normalized size = 1.47

$$-\frac{-5\cosh\left(\frac{a}{b}\right)\left(a+b\sinh^{-1}(cx)\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-27\cosh\left(\frac{3a}{b}\right)\left(a+b\sinh^{-1}(cx)\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -1/64*(64*b*c*x + 192*b*c^3*x^3 + 192*b*c^5*x^5 + 64*b*c^7*x^7 - 5*(a + b*A
rcSinh[c*x])*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - 27*(a + b*ArcSinh
[c*x])*Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] - 25*a*Cosh[(5*a)
/b]*CoshIntegral[5*(a/b + ArcSinh[c*x])] - 25*b*ArcSinh[c*x]*Cosh[(5*a)/b]*
CoshIntegral[5*(a/b + ArcSinh[c*x])] - 7*a*Cosh[(7*a)/b]*CoshIntegral[7*(a/
b + ArcSinh[c*x])] - 7*b*ArcSinh[c*x]*Cosh[(7*a)/b]*CoshIntegral[7*(a/b + A
rcSinh[c*x])] + 5*a*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 5*b*ArcSin
h[c*x]*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 27*a*Sinh[(3*a)/b]*Sinh
Integral[3*(a/b + ArcSinh[c*x])] + 27*b*ArcSinh[c*x]*Sinh[(3*a)/b]*SinhInte
gral[3*(a/b + ArcSinh[c*x])] + 25*a*Sinh[(5*a)/b]*SinhIntegral[5*(a/b + Arc
Sinh[c*x])] + 25*b*ArcSinh[c*x]*Sinh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh
[c*x])] + 7*a*Sinh[(7*a)/b]*SinhIntegral[7*(a/b + ArcSinh[c*x])] + 7*b*ArcS
inh[c*x]*Sinh[(7*a)/b]*SinhIntegral[7*(a/b + ArcSinh[c*x])])/(b^2*c^2*(a +
b*ArcSinh[c*x]))
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^4x^5+2\,c^2x^3+x\right)\sqrt{c^2x^2+1}}{b^2\,\text{arsinh}\,(cx)^2+2\,ab\,\text{arsinh}\,(cx)+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^4*x^5 + 2*c^2*x^3 + x)*sqrt(c^2*x^2 + 1)/(b^2*arcsinh(c*x)^2 +
2*a*b*arcsinh(c*x) + a^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [B] time = 0.62, size = 958, normalized size = 3.48

$$-\frac{64c^7x^7-64c^6x^6\sqrt{c^2x^2+1}+112c^5x^5-80c^4x^4\sqrt{c^2x^2+1}+56c^3x^3-24c^2x^2\sqrt{c^2x^2+1}+7cx-\sqrt{c^2x^2+1}}{128c^2\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)b}-\frac{7\,\mathrm{e}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] -1/128*(64*c^7*x^7-64*c^6*x^6*(c^2*x^2+1)^(1/2)+112*c^5*x^5-80*c^4*x^4*(c^2
*x^2+1)^(1/2)+56*c^3*x^3-24*c^2*x^2*(c^2*x^2+1)^(1/2)+7*c*x-(c^2*x^2+1)^(1/
2))/c^2/(a+b*arcsinh(c*x))/b-7/128/c^2/b^2*exp(7*a/b)*Ei(1,7*arcsinh(c*x)+7
*a/b)-5/128*(16*c^5*x^5-16*c^4*x^4*(c^2*x^2+1)^(1/2)+20*c^3*x^3-12*c^2*x^2*
(c^2*x^2+1)^(1/2)+5*c*x-(c^2*x^2+1)^(1/2))/c^2/b/(a+b*arcsinh(c*x))-25/128/
c^2/b^2*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)-9/128*(4*c^3*x^3-4*c^2*x^2*(c
^2*x^2+1)^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))/c^2/b/(a+b*arcsinh(c*x))-27/128/c^
2/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-5/128*(c*x-(c^2*x^2+1)^(1/2))/c
^2/b/(a+b*arcsinh(c*x))-5/128/c^2/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-5/128
/c^2/b^2*(arcsinh(c*x)*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*b+exp(-a/b)*Ei(1,-
arcsinh(c*x)-a/b)*a+x*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-9/128/c^2
/b^2*(4*x^3*b*c^3+4*(c^2*x^2+1)^(1/2)*x^2*b*c^2+3*arcsinh(c*x)*exp(-3*a/b)*
Ei(1,-3*arcsinh(c*x)-3*a/b)*b+3*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)*a+3
*x*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-5/128/c^2/b^2*(16*x^5*b*c^5+
16*(c^2*x^2+1)^(1/2)*x^4*b*c^4+20*x^3*b*c^3+12*(c^2*x^2+1)^(1/2)*x^2*b*c^2+
5*arcsinh(c*x)*exp(-5*a/b)*Ei(1,-5*arcsinh(c*x)-5*a/b)*b+5*exp(-5*a/b)*Ei(1
,-5*arcsinh(c*x)-5*a/b)*a+5*x*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-1
/128/c^2/b^2*(64*x^7*b*c^7+64*(c^2*x^2+1)^(1/2)*x^6*b*c^6+112*x^5*b*c^5+80*
(c^2*x^2+1)^(1/2)*x^4*b*c^4+56*x^3*b*c^3+24*(c^2*x^2+1)^(1/2)*x^2*b*c^2+7*a
rcsinh(c*x)*Ei(1,-7*arcsinh(c*x)-7*a/b)*exp(-7*a/b)*b+7*Ei(1,-7*arcsinh(c*x
)-7*a/b)*exp(-7*a/b)*a+7*x*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(c^6x^7+3\,c^4x^5+3\,c^2x^3+x\right)\left(c^2x^2+1\right)+\left(c^7x^8+3\,c^5x^6+3\,c^3x^4+cx^2\right)\sqrt{c^2x^2+1}}{abc^3x^2+\sqrt{c^2x^2+1}\,abc^2x+abc+\left(b^2c^3x^2+\sqrt{c^2x^2+1}\,b^2c^2x+b^2c\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int\frac{7(}{abc^5x^4+\left(c^2x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^6*x^7 + 3*c^4*x^5 + 3*c^2*x^3 + x)*(c^2*x^2 + 1) + (c^7*x^8 + 3*c^5*x^
6 + 3*c^3*x^4 + c*x^2)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*
a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log
(c*x + sqrt(c^2*x^2 + 1))) + integrate((7*(c^7*x^7 + 2*c^5*x^5 + c^3*x^3)*(
c^2*x^2 + 1)^(3/2) + (14*c^8*x^8 + 37*c^6*x^6 + 33*c^4*x^4 + 11*c^2*x^2 + 1
```

```
)*(c^2*x^2 + 1) + (7*c^9*x^9 + 23*c^7*x^7 + 27*c^5*x^5 + 13*c^3*x^3 + 2*c*x
)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c^2*x^2 + 1)*a*b*c^3*x^2 + 2*a*b*c^3*x
^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 + 1)*b^2*c^3*x^2 + 2*b^2*c^3*x^2 + b^2
*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2
+ 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sqrt(c^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\left(c^2x^2+1\right)^{5/2}}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x))^2,x)
[Out] int((x*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(c^2x^2+1\right)^{\frac{5}{2}}}{\left(a+b\,\mathrm{asinh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
[Out] Integral(x*(c**2*x**2 + 1)**(5/2)/(a + b*asinh(c*x))**2, x)
```

3.430 $\int \frac{(1+c^2x^2)^{5/2}}{(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=216

$$-\frac{15\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{16b^2c}-\frac{3\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\sinh^{-1}(cx))}{b}\right)}{4b^2c}-\frac{3\sinh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\sinh^{-1}(cx))}{b}\right)}{16b^2c}+\frac{15\cosh}{}$$

```
[Out] -(c^2*x^2+1)^3/b/c/(a+b*arcsinh(c*x))+15/16*cosh(2*a/b)*Shi(2*(a+b*arcsinh(
c*x))/b)/b^2/c+3/4*cosh(4*a/b)*Shi(4*(a+b*arcsinh(c*x))/b)/b^2/c+3/16*cosh(
6*a/b)*Shi(6*(a+b*arcsinh(c*x))/b)/b^2/c-15/16*Chi(2*(a+b*arcsinh(c*x))/b)*
sinh(2*a/b)/b^2/c-3/4*Chi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b^2/c-3/16*Ch
i(6*(a+b*arcsinh(c*x))/b)*sinh(6*a/b)/b^2/c
```

Rubi [A] time = 0.45, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5696, 5779, 5448, 3303, 3298, 3301}

$$-\frac{15\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{16b^2c}-\frac{3\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{4b^2c}-\frac{3\sinh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6a}{b}+6\sinh^{-1}(cx)\right)}{16b^2c}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + c^2*x^2)^(5/2)/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((1 + c^2*x^2)^3/(b*c*(a + b*ArcSinh[c*x]))) - (15*CoshIntegral[(2*a)/b +
2*ArcSinh[c*x]]*Sinh[(2*a)/b])/(16*b^2*c) - (3*CoshIntegral[(4*a)/b + 4*Arc
Sinh[c*x]]*Sinh[(4*a)/b])/(4*b^2*c) - (3*CoshIntegral[(6*a)/b + 6*ArcSinh[c
*x]]*Sinh[(6*a)/b])/(16*b^2*c) + (15*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2
*ArcSinh[c*x]])/(16*b^2*c) + (3*Cosh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcS
inh[c*x]])/(4*b^2*c) + (3*Cosh[(6*a)/b]*SinhIntegral[(6*a)/b + 6*ArcSinh[c*
x]])/(16*b^2*c)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
```

```
& IGtQ[p, 0]
```

Rule 5696

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(Sqrt[1 + c^2*x^2]*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^(n +
 1))/(b*c*(n + 1)), x] - Dist[(c*(2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(b*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ
[e, c^2*d] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{\left(1+c^2x^2\right)^{5/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{(6c)\int \frac{x\left(1+c^2x^2\right)^2}{a+b\sinh^{-1}(cx)}\,dx}{b}$$

$$= -\frac{\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{6\,\mathrm{Subst}\left(\int \frac{\cosh^5(x)\sinh(x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{bc}$$

$$= -\frac{\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{6\,\mathrm{Subst}\left(\int \left(\frac{5\sinh(2x)}{32(a+bx)}+\frac{\sinh(4x)}{8(a+bx)}+\frac{\sinh(6x)}{32(a+bx)}\right)dx,x,\sinh^{-1}(\right.}{bc}$$

$$= -\frac{\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{3\,\mathrm{Subst}\left(\int \frac{\sinh(6x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{16bc}+\frac{3\,\mathrm{Subst}\left(\int \frac{\sinh(}{a+b}\right.}{}$$

$$= -\frac{\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\left(15\cosh\left(\frac{2a}{b}\right)\right)\mathrm{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{16bc}+$$

$$= -\frac{\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{15\mathrm{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{16b^2c}-\frac{3\mathrm{Chi}\left(\frac{4a}{b}+4\sin\right.}{4}$$

Mathematica [A] time = 0.72, size = 311, normalized size = 1.44

$$-\frac{15\sinh\left(\frac{2a}{b}\right)\left(a+b\sinh^{-1}(cx)\right)\mathrm{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+12\sinh\left(\frac{4a}{b}\right)\left(a+b\sinh^{-1}(cx)\right)\mathrm{Chi}\left(4\left(\frac{a}{b}+\sinh^{-}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + c^2*x^2)^(5/2)/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -1/16*(16*b + 48*b*c^2*x^2 + 48*b*c^4*x^4 + 16*b*c^6*x^6 + 15*(a + b*ArcSin
h[c*x])*CoshIntegral[2*(a/b + ArcSinh[c*x])]*Sinh[(2*a)/b] + 12*(a + b*ArcS
inh[c*x])*CoshIntegral[4*(a/b + ArcSinh[c*x])]*Sinh[(4*a)/b] + 3*a*CoshInte
```

```
gral[6*(a/b + ArcSinh[c*x])]*Sinh[(6*a)/b] + 3*b*ArcSinh[c*x]*CoshIntegral[
6*(a/b + ArcSinh[c*x])]*Sinh[(6*a)/b] - 15*a*Cosh[(2*a)/b]*SinhIntegral[2*(
a/b + ArcSinh[c*x])] - 15*b*ArcSinh[c*x]*Cosh[(2*a)/b]*SinhIntegral[2*(a/b
+ ArcSinh[c*x])] - 12*a*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])]
- 12*b*ArcSinh[c*x]*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])] - 3*
a*Cosh[(6*a)/b]*SinhIntegral[6*(a/b + ArcSinh[c*x])] - 3*b*ArcSinh[c*x]*Cos
h[(6*a)/b]*SinhIntegral[6*(a/b + ArcSinh[c*x])])/(b^2*c*(a + b*ArcSinh[c*x]
))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^4x^4+2\,c^2x^2+1\right)\sqrt{c^2x^2+1}}{b^2\operatorname{arsinh}\left(cx\right)^2+2\,ab\operatorname{arsinh}\left(cx\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b^2*arcsinh(c*x)^2 +
2*a*b*arcsinh(c*x) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(c^2x^2+1\right)^{\frac{5}{2}}}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)/(b*arcsinh(c*x) + a)^2, x)
```

maple [B] time = 0.71, size = 704, normalized size = 3.26

$$-\frac{5}{16bc\left(a+b\operatorname{arcsinh}\left(cx\right)\right)}-\frac{32c^6x^6-32c^5x^5\sqrt{c^2x^2+1}+48c^4x^4-32c^3x^3\sqrt{c^2x^2+1}+18c^2x^2-6cx\sqrt{c^2x^2+1}+}{64c\left(a+b\operatorname{arcsinh}\left(cx\right)\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] -5/16/b/c/(a+b*arcsinh(c*x))-1/64*(32*c^6*x^6-32*c^5*x^5*(c^2*x^2+1)^(1/2)+
48*c^4*x^4-32*c^3*x^3*(c^2*x^2+1)^(1/2)+18*c^2*x^2-6*c*x*(c^2*x^2+1)^(1/2)+
1)/c/(a+b*arcsinh(c*x))/b+3/32/c/b^2*exp(6*a/b)*Ei(1,6*arcsinh(c*x)+6*a/b)-
3/32*(8*c^4*x^4-8*c^3*x^3*(c^2*x^2+1)^(1/2)+8*c^2*x^2-4*c*x*(c^2*x^2+1)^(1/
2)+1)/c/(a+b*arcsinh(c*x))/b+3/8/c/b^2*exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b
)-15/64*(2*c^2*x^2-2*c*x*(c^2*x^2+1)^(1/2)+1)/c/(a+b*arcsinh(c*x))/b+15/32/
c/b^2*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)-15/64/c/b^2*(2*x^2*b*c^2+2*b*c*
(c^2*x^2+1)^(1/2)*x+2*arcsinh(c*x)*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*
b+2*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*a+b)/(a+b*arcsinh(c*x))-3/32/c/
b^2*(8*x^4*b*c^4+8*(c^2*x^2+1)^(1/2)*x^3*b*c^3+8*x^2*b*c^2+4*b*c*(c^2*x^2+1
)^(1/2)*x+4*arcsinh(c*x)*Ei(1,-4*arcsinh(c*x)-4*a/b)*exp(-4*a/b)*b+4*Ei(1,-
4*arcsinh(c*x)-4*a/b)*exp(-4*a/b)*a+b)/(a+b*arcsinh(c*x))-1/64/c/b^2*(32*x^
6*b*c^6+32*(c^2*x^2+1)^(1/2)*x^5*b*c^5+48*x^4*b*c^4+32*(c^2*x^2+1)^(1/2)*x^
3*b*c^3+18*x^2*b*c^2+6*b*c*(c^2*x^2+1)^(1/2)*x+6*arcsinh(c*x)*Ei(1,-6*arcsi
nh(c*x)-6*a/b)*exp(-6*a/b)*b+6*Ei(1,-6*arcsinh(c*x)-6*a/b)*exp(-6*a/b)*a+b)
/(a+b*arcsinh(c*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(c^6x^6+3\,c^4x^4+3\,c^2x^2+1\right)\left(c^2x^2+1\right)+\left(c^7x^7+3\,c^5x^5+3\,c^3x^3+cx\right)\sqrt{c^2x^2+1}}{abc^3x^2+\sqrt{c^2x^2+1}\,abc^2x+abc+\left(b^2c^3x^2+\sqrt{c^2x^2+1}\,b^2c^2x+b^2c\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int\frac{}{abc^4x^4+\left(c^2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^6*x^6 + 3*c^4*x^4 + 3*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^7*x^7 + 3*c^5*x^
5 + 3*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*
b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c
*x + sqrt(c^2*x^2 + 1))) + integrate(((6*c^6*x^6 + 11*c^4*x^4 + 4*c^2*x^2 -
 1)*(c^2*x^2 + 1)^(3/2) + 6*(2*c^7*x^7 + 5*c^5*x^5 + 4*c^3*x^3 + c*x)*(c^2*
x^2 + 1) + (6*c^8*x^8 + 19*c^6*x^6 + 21*c^4*x^4 + 9*c^2*x^2 + 1)*sqrt(c^2*x
^2 + 1))/(a*b*c^4*x^4 + (c^2*x^2 + 1)*a*b*c^2*x^2 + 2*a*b*c^2*x^2 + a*b + (
b^2*c^4*x^4 + (c^2*x^2 + 1)*b^2*c^2*x^2 + 2*b^2*c^2*x^2 + b^2 + 2*(b^2*c^3*
x^3 + b^2*c*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^3
*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c^2\,x^2+1\right)^{5/2}}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(5/2)/(a + b*asinh(c*x))^2,x)
```

```
[Out] int((c^2*x^2 + 1)^(5/2)/(a + b*asinh(c*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2+1\right)^{\frac{5}{2}}}{\left(a+b\,\mathrm{asinh}\,(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((c**2*x**2 + 1)**(5/2)/(a + b*asinh(c*x))**2, x)
```

3.431 $\int \frac{\left(1+c^2x^2\right)^{5/2}}{x\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=233

$$-\frac{\text{Int}\left(\frac{\left(c^2x^2+1\right)^2}{x^2\left(a+b\sinh^{-1}(cx)\right)},x\right)}{bc}+\frac{25\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8b^2}+\frac{25\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16b^2}+\frac{5\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\right.}{16b}$$

```
[Out] -(c^2*x^2+1)^3/b/c/x/(a+b*arcsinh(c*x))+25/8*Chi((a+b*arcsinh(c*x))/b)*cosh
(a/b)/b^2+25/16*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b^2+5/16*Chi(5*(a+b
*arcsinh(c*x))/b)*cosh(5*a/b)/b^2-25/8*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)/
b^2-25/16*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2-5/16*Shi(5*(a+b*arcsi
nh(c*x))/b)*sinh(5*a/b)/b^2-Unintegrable((c^2*x^2+1)^2/x^2/(a+b*arcsinh(c*x
)),x)/b/c
```

Rubi [A] time = 0.57, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1+c^2x^2\right)^{5/2}}{x\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(5/2)/(x*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -((1 + c^2*x^2)^3/(b*c*x*(a + b*ArcSinh[c*x]))) + (25*Cosh[a/b]*CoshIntegra
l[a/b + ArcSinh[c*x]])/(8*b^2) + (25*Cosh[(3*a)/b]*CoshIntegral[(3*a)/b + 3
*ArcSinh[c*x]])/(16*b^2) + (5*Cosh[(5*a)/b]*CoshIntegral[(5*a)/b + 5*ArcSin
h[c*x]])/(16*b^2) - (25*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(8*b^2)
 - (25*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(16*b^2) - (5*
Sinh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcSinh[c*x]])/(16*b^2) - Defer[Int]
[(1 + c^2*x^2)^2/(x^2*(a + b*ArcSinh[c*x])), x]/(b*c)
```

Rubi steps

$$\int \frac{\left(1+c^2x^2\right)^{5/2}}{x\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{\left(1+c^2x^2\right)^3}{bcx\left(a+b\sinh^{-1}(cx)\right)} - \frac{\int \frac{\left(1+c^2x^2\right)^2}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx}{bc} + \frac{(5c)\int \frac{\left(1+c^2x^2\right)^2}{a+b\sinh^{-1}(cx)}\,dx}{b}$$

$$= -\frac{\left(1+c^2x^2\right)^3}{bcx\left(a+b\sinh^{-1}(cx)\right)} + \frac{5\,\mathrm{Subst}\left(\int \frac{\cosh^5(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{b} - \frac{\int \frac{\left(1+c^2x^2\right)}{x^2\left(a+b\sinh^{-1}\right.}}{bc}$$

$$= -\frac{\left(1+c^2x^2\right)^3}{bcx\left(a+b\sinh^{-1}(cx)\right)} + \frac{5\,\mathrm{Subst}\left(\int \left(\frac{5\cosh(x)}{8(a+bx)} + \frac{5\cosh(3x)}{16(a+bx)} + \frac{\cosh(5x)}{16(a+bx)}\right) dx, x, \mathrm{sinh}\right.}{b}$$

$$= -\frac{\left(1+c^2x^2\right)^3}{bcx\left(a+b\sinh^{-1}(cx)\right)} + \frac{5\,\mathrm{Subst}\left(\int \frac{\cosh(5x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{16b} + \frac{25\,\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{\left(1+c^2x^2\right)^3}{bcx\left(a+b\sinh^{-1}(cx)\right)} - \frac{\int \frac{\left(1+c^2x^2\right)^2}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx}{bc} + \frac{\left(25\cosh\left(\frac{a}{b}\right)\right)\mathrm{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}\right.}{a+b}\right.}{8b}$$

$$= -\frac{\left(1+c^2x^2\right)^3}{bcx\left(a+b\sinh^{-1}(cx)\right)} + \frac{25\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{8b^2} + \frac{25\cosh\left(\frac{3a}{b}\right)\mathrm{Ch}}{}$$

Mathematica [A] time = 9.52, size = 0, normalized size = 0.00

$$\int \frac{\left(1+c^2x^2\right)^{5/2}}{x\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(5/2)/(x*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[(1 + c^2*x^2)^(5/2)/(x*(a + b*ArcSinh[c*x])^2), x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(c^4x^4+2\,c^2x^2+1\right)\sqrt{c^2x^2+1}}{b^2x\,\mathrm{arsinh}\,(cx)^2+2\,abx\,\mathrm{arsinh}\,(cx)+a^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b^2*x*arcsinh(c*x)^2
+ 2*a*b*x*arcsinh(c*x) + a^2*x), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\left(c^{2} x^{2}+1\right)^{\frac{5}{2}}}{x \left(a +b \,\mathrm{arcsinh}\left(c x \right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(c^{6} x^{6}+3 c^{4} x^{4}+3 c^{2} x^{2}+1\right)\left(c^{2} x^{2}+1\right)+\left(c^{7} x^{7}+3 c^{5} x^{5}+3 c^{3} x^{3}+c x\right)\sqrt{c^{2} x^{2}+1}}{a b c^{3} x^{3}+\sqrt{c^{2} x^{2}+1}\, a b c^{2} x^{2}+a b c x+\left(b^{2} c^{3} x^{3}+\sqrt{c^{2} x^{2}+1}\, b^{2} c^{2} x^{2}+b^{2} c x\right)\log\left(c x+\sqrt{c^{2} x^{2}+1}\right)}+\int \frac{}{a b c^{5} x^{6}+\left(c^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^6*x^6 + 3*c^4*x^4 + 3*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^7*x^7 + 3*c^5*x^
5 + 3*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^3 + sqrt(c^2*x^2 + 1)*a*
b*c^2*x^2 + a*b*c*x + (b^2*c^3*x^3 + sqrt(c^2*x^2 + 1)*b^2*c^2*x^2 + b^2*c*
x)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((5*c^7*x^7 + 8*c^5*x^5 + c^3*
x^3 - 2*c*x)*(c^2*x^2 + 1)^(3/2) + (10*c^8*x^8 + 23*c^6*x^6 + 15*c^4*x^4 +
c^2*x^2 - 1)*(c^2*x^2 + 1) + 5*(c^9*x^9 + 3*c^7*x^7 + 3*c^5*x^5 + c^3*x^3)*
sqrt(c^2*x^2 + 1))/(a*b*c^5*x^6 + (c^2*x^2 + 1)*a*b*c^3*x^4 + 2*a*b*c^3*x^4
 + a*b*c*x^2 + (b^2*c^5*x^6 + (c^2*x^2 + 1)*b^2*c^3*x^4 + 2*b^2*c^3*x^4 + b
^2*c*x^2 + 2*(b^2*c^4*x^5 + b^2*c^2*x^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(
c^2*x^2 + 1)) + 2*(a*b*c^4*x^5 + a*b*c^2*x^3)*sqrt(c^2*x^2 + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c^2\,x^2+1\right)}^{5/2}}{x\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(5/2)/(x*(a + b*asinh(c*x))^2),x)
```

```
[Out] int((c^2*x^2 + 1)^(5/2)/(x*(a + b*asinh(c*x))^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^{2} x^{2} + 1\right)^{\frac{5}{2}}}{x \left(a + b \operatorname{asinh}{\left(c x \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(5/2)/x/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((c**2*x**2 + 1)**(5/2)/(x*(a + b*asinh(c*x))**2), x)
```

3.432 $\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^2\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=103

$$\frac{4c\text{Int}\left(\frac{\left(c^2x^2+1\right)^2}{x\left(a+b\sinh^{-1}(cx)\right)},x\right)}{b}-\frac{2\text{Int}\left(\frac{\left(c^2x^2+1\right)^2}{x^3\left(a+b\sinh^{-1}(cx)\right)},x\right)}{bc}-\frac{\left(c^2x^2+1\right)^3}{bcx^2\left(a+b\sinh^{-1}(cx)\right)}$$

```
[Out] -(c^2*x^2+1)^3/b/c/x^2/(a+b*arcsinh(c*x))-2*Unintegrable((c^2*x^2+1)^2/x^3/
(a+b*arcsinh(c*x)),x)/b/c+4*c*Unintegrable((c^2*x^2+1)^2/x/(a+b*arcsinh(c*x
)),x)/b
```

Rubi [A] time = 0.32, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^2\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(5/2)/(x^2*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -((1 + c^2*x^2)^3/(b*c*x^2*(a + b*ArcSinh[c*x]))) - (2*Defer[Int][(1 + c^2*
x^2)^2/(x^3*(a + b*ArcSinh[c*x])), x])/(b*c) + (4*c*Defer[Int][(1 + c^2*x^2
)^2/(x*(a + b*ArcSinh[c*x])), x])/b
```

Rubi steps

$$\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^2\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=-\frac{\left(1+c^2x^2\right)^3}{bcx^2\left(a+b\sinh^{-1}(cx)\right)}-\frac{2\int \frac{\left(1+c^2x^2\right)^2}{x^3\left(a+b\sinh^{-1}(cx)\right)}\,dx}{bc}+\frac{(4c)\int \frac{\left(1+c^2x^2\right)^2}{x\left(a+b\sinh^{-1}(cx)\right)}\,dx}{b}$$

Mathematica [A] time = 3.42, size = 0, normalized size = 0.00

$$\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^2\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(5/2)/(x^2*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[(1 + c^2*x^2)^(5/2)/(x^2*(a + b*ArcSinh[c*x])^2), x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^4x^4+2\,c^2x^2+1\right)\sqrt{c^2x^2+1}}{b^2x^2\,\text{arsinh}\,(cx)^2+2\,abx^2\,\text{arsinh}\,(cx)+a^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b^2*x^2*arcsinh(c*x)^
2 + 2*a*b*x^2*arcsinh(c*x) + a^2*x^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2+1\right)^{\frac{5}{2}}}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)^2*x^2), x)
```

maple [A] time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2+1\right)^{\frac{5}{2}}}{x^2\left(a+b\operatorname{arcsinh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(c^6x^6+3\,c^4x^4+3\,c^2x^2+1\right)\left(c^2x^2+1\right)+\left(c^7x^7+3\,c^5x^5+3\,c^3x^3+cx\right)\sqrt{c^2x^2+1}}{abc^3x^4+\sqrt{c^2x^2+1}\,abc^2x^3+abcx^2+\left(b^2c^3x^4+\sqrt{c^2x^2+1}\,b^2c^2x^3+b^2cx^2\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int \frac{}{abc^5x^7+(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^6*x^6 + 3*c^4*x^4 + 3*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^7*x^7 + 3*c^5*x^
5 + 3*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^4 + sqrt(c^2*x^2 + 1)*a*
b*c^2*x^3 + a*b*c*x^2 + (b^2*c^3*x^4 + sqrt(c^2*x^2 + 1)*b^2*c^2*x^3 + b^2*
c*x^2)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((4*c^7*x^7 + 5*c^5*x^5 -
2*c^3*x^3 - 3*c*x)*(c^2*x^2 + 1)^(3/2) + 2*(4*c^8*x^8 + 8*c^6*x^6 + 3*c^4*x
^4 - 2*c^2*x^2 - 1)*(c^2*x^2 + 1) + (4*c^9*x^9 + 11*c^7*x^7 + 9*c^5*x^5 + c
^3*x^3 - c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^7 + (c^2*x^2 + 1)*a*b*c^3*x^5 +
 2*a*b*c^3*x^5 + a*b*c*x^3 + (b^2*c^5*x^7 + (c^2*x^2 + 1)*b^2*c^3*x^5 + 2*b
^2*c^3*x^5 + b^2*c*x^3 + 2*(b^2*c^4*x^6 + b^2*c^2*x^4)*sqrt(c^2*x^2 + 1))*l
og(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^6 + a*b*c^2*x^4)*sqrt(c^2*x^2 +
1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c^2\,x^2+1\right)}^{5/2}}{x^2\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(5/2)/(x^2*(a + b*asinh(c*x))^2),x)
```

```
[Out] int((c^2*x^2 + 1)^(5/2)/(x^2*(a + b*asinh(c*x))^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2+1\right)^{\frac{5}{2}}}{x^2\left(a+b\,\mathrm{asinh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(5/2)/x**2/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((c**2*x**2 + 1)**(5/2)/(x**2*(a + b*asinh(c*x))**2), x)
```

3.433 $\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{\left(c^2x^2+1\right)^{5/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(5/2)/(x^3*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Defer[Int][(1 + c^2*x^2)^(5/2)/(x^3*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 14.57, size = 0, normalized size = 0.00

$$\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(5/2)/(x^3*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[(1 + c^2*x^2)^(5/2)/(x^3*(a + b*ArcSinh[c*x])^2), x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^4x^4+2\,c^2x^2+1\right)\sqrt{c^2x^2+1}}{b^2x^3\,\text{arsinh}\,(cx)^2+2\,abx^3\,\text{arsinh}\,(cx)+a^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b^2*x^3*arcsinh(c*x)^
2 + 2*a*b*x^3*arcsinh(c*x) + a^2*x^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 1.43, size = 0, normalized size = 0.00

$$\int \frac{\left(c^{2}x^{2}+1\right)^{\frac{5}{2}}}{x^{3}\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(c^{6}x^{6}+3\,c^{4}x^{4}+3\,c^{2}x^{2}+1\right)\left(c^{2}x^{2}+1\right)+\left(c^{7}x^{7}+3\,c^{5}x^{5}+3\,c^{3}x^{3}+cx\right)\sqrt{c^{2}x^{2}+1}}{abc^{3}x^{5}+\sqrt{c^{2}x^{2}+1}\,abc^{2}x^{4}+abcx^{3}+\left(b^{2}c^{3}x^{5}+\sqrt{c^{2}x^{2}+1}\,b^{2}c^{2}x^{4}+b^{2}cx^{3}\right)\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)}+\int \frac{}{abc^{5}x^{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^6*x^6 + 3*c^4*x^4 + 3*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^7*x^7 + 3*c^5*x^
5 + 3*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^5 + sqrt(c^2*x^2 + 1)*a*
b*c^2*x^4 + a*b*c*x^3 + (b^2*c^3*x^5 + sqrt(c^2*x^2 + 1)*b^2*c^2*x^4 + b^2*
c*x^3)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((3*c^7*x^7 + 2*c^5*x^5 -
5*c^3*x^3 - 4*c*x)*(c^2*x^2 + 1)^(3/2) + 3*(2*c^8*x^8 + 3*c^6*x^6 - c^4*x^4
 - 3*c^2*x^2 - 1)*(c^2*x^2 + 1) + (3*c^9*x^9 + 7*c^7*x^7 + 3*c^5*x^5 - 3*c^
3*x^3 - 2*c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^8 + (c^2*x^2 + 1)*a*b*c^3*x^6
+ 2*a*b*c^3*x^6 + a*b*c*x^4 + (b^2*c^5*x^8 + (c^2*x^2 + 1)*b^2*c^3*x^6 + 2*
b^2*c^3*x^6 + b^2*c*x^4 + 2*(b^2*c^4*x^7 + b^2*c^2*x^5)*sqrt(c^2*x^2 + 1))*
log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^7 + a*b*c^2*x^5)*sqrt(c^2*x^2 +
 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(c^2\,x^2+1\right)}^{5/2}}{x^3\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(5/2)/(x^3*(a + b*asinh(c*x))^2),x)
```

```
[Out] int((c^2*x^2 + 1)^(5/2)/(x^3*(a + b*asinh(c*x))^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^{2}x^{2}+1\right)^{\frac{5}{2}}}{x^{3}\left(a+b\,\mathrm{asinh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(5/2)/x**3/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((c**2*x**2 + 1)**(5/2)/(x**3*(a + b*asinh(c*x))**2), x)
```

3.434 $\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=30

$$\mathrm{Int}\left(\frac{\left(c^2x^2+1\right)^{5/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(5/2)/(x^4*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][(1 + c^2*x^2)^(5/2)/(x^4*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 3.08, size = 0, normalized size = 0.00

$$\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(5/2)/(x^4*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[(1 + c^2*x^2)^(5/2)/(x^4*(a + b*ArcSinh[c*x])^2), x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(c^4x^4+2\,c^2x^2+1\right)\sqrt{c^2x^2+1}}{b^2x^4\,\mathrm{arsinh}\left(cx\right)^2+2\,abx^4\,\mathrm{arsinh}\left(cx\right)+a^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b^2*x^4*arcsinh(c*x)^
2 + 2*a*b*x^4*arcsinh(c*x) + a^2*x^4), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2x^2+1\right)^{\frac{5}{2}}}{\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)^2*x^4), x)
```

maple [A] time = 1.24, size = 0, normalized size = 0.00

$$\int \frac{\left(c^{2} x^{2}+1\right)^{\frac{5}{2}}}{x^{4} \left(a+b \,\mathrm{arcsinh}\left(c x\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x))^2,x)
[Out] int((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(c^{6} x^{6}+3 c^{4} x^{4}+3 c^{2} x^{2}+1\right)\left(c^{2} x^{2}+1\right)+\left(c^{7} x^{7}+3 c^{5} x^{5}+3 c^{3} x^{3}+c x\right)\sqrt{c^{2} x^{2}+1}}{a b c^{3} x^{6}+\sqrt{c^{2} x^{2}+1}\, a b c^{2} x^{5}+a b c x^{4}+\left(b^{2} c^{3} x^{6}+\sqrt{c^{2} x^{2}+1}\, b^{2} c^{2} x^{5}+b^{2} c x^{4}\right)\log\left(c x+\sqrt{c^{2} x^{2}+1}\right)}+\int \frac{}{a b c^{5} x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -((c^6*x^6 + 3*c^4*x^4 + 3*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^7*x^7 + 3*c^5*x^
5 + 3*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^6 + sqrt(c^2*x^2 + 1)*a*
b*c^2*x^5 + a*b*c*x^4 + (b^2*c^3*x^6 + sqrt(c^2*x^2 + 1)*b^2*c^2*x^5 + b^2*
c*x^4)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((2*c^7*x^7 - c^5*x^5 - 8*
c^3*x^3 - 5*c*x)*(c^2*x^2 + 1)^(3/2) + 2*(2*c^8*x^8 + c^6*x^6 - 6*c^4*x^4 -
 7*c^2*x^2 - 2)*(c^2*x^2 + 1) + (2*c^9*x^9 + 3*c^7*x^7 - 3*c^5*x^5 - 7*c^3*
x^3 - 3*c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^9 + (c^2*x^2 + 1)*a*b*c^3*x^7 +
2*a*b*c^3*x^7 + a*b*c*x^5 + (b^2*c^5*x^9 + (c^2*x^2 + 1)*b^2*c^3*x^7 + 2*b^
2*c^3*x^7 + b^2*c*x^5 + 2*(b^2*c^4*x^8 + b^2*c^2*x^6)*sqrt(c^2*x^2 + 1))*lo
g(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^8 + a*b*c^2*x^6)*sqrt(c^2*x^2 + 1
)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(c^2\,x^2+1\right)}^{5/2}}{x^4\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(5/2)/(x^4*(a + b*asinh(c*x))^2),x)
[Out] int((c^2*x^2 + 1)^(5/2)/(x^4*(a + b*asinh(c*x))^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^{2} x^{2} + 1\right)^{\frac{5}{2}}}{x^{4} \left(a + b \operatorname{asinh}{\left(c x \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(5/2)/x**4/(a+b*asinh(c*x))**2,x)
[Out] Integral((c**2*x**2 + 1)**(5/2)/(x**4*(a + b*asinh(c*x))**2), x)
```

3.435 $\int \frac{x^5}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=204

$$\frac{5\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8b^2c^6}-\frac{15\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16b^2c^6}+\frac{5\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16b^2c^6}-\frac{5\sinh\left(\frac{a}{b}\right)\text{S}$$

```
[Out] -x^5/b/c/(a+b*arcsinh(c*x))+5/8*Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b^2/c^6
-15/16*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b^2/c^6+5/16*Chi(5*(a+b*arcs
inh(c*x))/b)*cosh(5*a/b)/b^2/c^6-5/8*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b^
2/c^6+15/16*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^6-5/16*Shi(5*(a+b
*arcsinh(c*x))/b)*sinh(5*a/b)/b^2/c^6
```

Rubi [A] time = 0.48, antiderivative size = 200, normalized size of antiderivative = 0.98, number of steps used = 13, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5774, 5669, 5448, 3303, 3298, 3301}

$$\frac{5\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{8b^2c^6}-\frac{15\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{16b^2c^6}+\frac{5\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5a}{b}+5\sinh^{-1}(cx)\right)}{16b^2c^6}-\frac{5}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(x^5/(b*c*(a + b*ArcSinh[c*x]))) + (5*Cosh[a/b]*CoshIntegral[a/b + ArcSinh
[c*x]])/(8*b^2*c^6) - (15*Cosh[(3*a)/b]*CoshIntegral[(3*a)/b + 3*ArcSinh[c*
x]])/(16*b^2*c^6) + (5*Cosh[(5*a)/b]*CoshIntegral[(5*a)/b + 5*ArcSinh[c*x]]
)/(16*b^2*c^6) - (5*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(8*b^2*c^6)
 + (15*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(16*b^2*c^6) -
 (5*Sinh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcSinh[c*x]])/(16*b^2*c^6)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{x^5}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{x^5}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{5\int \frac{x^4}{a+b\sinh^{-1}(cx)}\,dx}{bc}$$

$$= -\frac{x^5}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{5\,\mathrm{Subst}\left(\int \frac{\cosh(x)\sinh^4(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{bc^6}$$

$$= -\frac{x^5}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{5\,\mathrm{Subst}\left(\int \left(\frac{\cosh(x)}{8(a+bx)} - \frac{3\cosh(3x)}{16(a+bx)} + \frac{\cosh(5x)}{16(a+bx)}\right) dx$$

$$= -\frac{x^5}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{5\,\mathrm{Subst}\left(\int \frac{\cosh(5x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{16bc^6} + \frac{5\,\mathrm{Su}$$

$$= -\frac{x^5}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\left(5\cosh\left(\frac{a}{b}\right)\right)\mathrm{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}}{8bc^6}$$

$$= -\frac{x^5}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{5\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{8b^2c^6} - \frac{15\cosh\left(\right.}{}$$

Mathematica [A] time = 0.34, size = 158, normalized size = 0.77

$$\frac{5\left(2\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right) - 3\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) + \cosh\left(\frac{5a}{b}\right)\mathrm{Chi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx\right.\right.\right.}{1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(x^5/(b*c*(a + b*ArcSinh[c*x]))) + (5*(2*Cosh[a/b]*CoshIntegral[a/b + ArcS
inh[c*x]] - 3*Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] + Cosh[(5*
a)/b]*CoshIntegral[5*(a/b + ArcSinh[c*x])] - 2*Sinh[a/b]*SinhIntegral[a/b +
 ArcSinh[c*x]] + 3*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] - Sin
h[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[c*x])]))/(16*b^2*c^6)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{c^2x^2+1}\,x^5}{a^2c^2x^2 + \left(b^2c^2x^2+b^2\right)\mathrm{arsinh}(cx)^2 + a^2 + 2\left(abc^2x^2+ab\right)\mathrm{arsinh}(cx)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^5/(a^2*c^2*x^2 + (b^2*c^2*x^2 + b^2)*arcsinh(c
*x)^2 + a^2 + 2*(a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.42, size = 633, normalized size = 3.10

$$-\frac{16c^5x^5-16c^4x^4\sqrt{c^2x^2+1}+20c^3x^3-12c^2x^2\sqrt{c^2x^2+1}+5cx-\sqrt{c^2x^2+1}}{32c^6b\,(a+b\,\mathrm{arcsinh}\,(cx))}-\frac{5\,\mathrm{e}^{\frac{5a}{b}}\,\mathrm{Ei}\left(1,5\,\mathrm{arcsinh}\,(cx)+\frac{5a}{b}\right)}{32c^6b^2}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x)
```

```
[Out] -1/32*(16*c^5*x^5-16*c^4*x^4*(c^2*x^2+1)^(1/2)+20*c^3*x^3-12*c^2*x^2*(c^2*x
^2+1)^(1/2)+5*c*x-(c^2*x^2+1)^(1/2))/c^6/b/(a+b*arcsinh(c*x))-5/32/c^6/b^2*
exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)+5/32*(4*c^3*x^3-4*c^2*x^2*(c^2*x^2+1)
^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))/c^6/b/(a+b*arcsinh(c*x))+15/32/c^6/b^2*exp(
3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-5/16*(c*x-(c^2*x^2+1)^(1/2))/c^6/b/(a+b*a
rcsinh(c*x))-5/16/c^6/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-5/16/c^6/b^2*(arc
sinh(c*x)*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*b+exp(-a/b)*Ei(1,-arcsinh(c*x)-
a/b)*a+x*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))+5/32/c^6/b^2*(4*x^3*b*
c^3+4*(c^2*x^2+1)^(1/2)*x^2*b*c^2+3*arcsinh(c*x)*exp(-3*a/b)*Ei(1,-3*arcsin
h(c*x)-3*a/b)*b+3*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)*a+3*x*b*c+(c^2*x^
2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-1/32/c^6/b^2*(16*x^5*b*c^5+16*(c^2*x^2+1)^
(1/2)*x^4*b*c^4+20*x^3*b*c^3+12*(c^2*x^2+1)^(1/2)*x^2*b*c^2+5*arcsinh(c*x)*
exp(-5*a/b)*Ei(1,-5*arcsinh(c*x)-5*a/b)*b+5*exp(-5*a/b)*Ei(1,-5*arcsinh(c*x
)-5*a/b)*a+5*x*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^3x^8+cx^6+\left(c^2x^7+x^5\right)\sqrt{c^2x^2+1}}{\left(c^2x^2+1\right)abc^2x+\left(\left(c^2x^2+1\right)b^2c^2x+\left(b^2c^3x^2+b^2c\right)\sqrt{c^2x^2+1}\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(abc^3x^2+abc\right)\sqrt{c^2x^2+\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -(c^3*x^8 + c*x^6 + (c^2*x^7 + x^5)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)*a*b*c
^2*x + ((c^2*x^2 + 1)*b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*
log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + i
ntegrate((5*c^5*x^9 + 11*c^3*x^7 + 6*c*x^5 + (5*c^3*x^7 + 4*c*x^5)*(c^2*x^2
 + 1) + 5*(2*c^4*x^8 + 3*c^2*x^6 + x^4)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)^(
3/2)*a*b*c^3*x^2 + 2*(a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((c^2*x^2 +
1)^(3/2)*b^2*c^3*x^2 + 2*(b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5
```

```
*x^4 + 2*b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1
)) + (a*b*c^5*x^4 + 2*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{(a+b\operatorname{asinh}(c\,x))^2\,\sqrt{c^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)),x)
[Out] int(x^5/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a+b\operatorname{asinh}(cx))^2\,\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a+b*asinh(c*x))**2/(c**2*x**2+1)**(1/2),x)
[Out] Integral(x**5/((a + b*asinh(c*x))**2*sqrt(c**2*x**2 + 1)), x)
```

3.436 $\int \frac{x^4}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=141

$$\frac{\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{b^2c^5}-\frac{\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{2b^2c^5}-\frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{b^2c^5}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\cdots\right.}{2b\cdots}$$

```
[Out] -x^4/b/c/(a+b*arcsinh(c*x))-cosh(2*a/b)*Shi(2*(a+b*arcsinh(c*x))/b)/b^2/c^5
+1/2*cosh(4*a/b)*Shi(4*(a+b*arcsinh(c*x))/b)/b^2/c^5+Chi(2*(a+b*arcsinh(c*x
))/b)*sinh(2*a/b)/b^2/c^5-1/2*Chi(4*(a+b*arcsinh(c*x))/b)*sinh(4*a/b)/b^2/c
^5
```

Rubi [A] time = 0.40, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5774, 5669, 5448, 3303, 3298, 3301}

$$\frac{\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{b^2c^5}-\frac{\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{2b^2c^5}-\frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{b^2c^5}+\frac{\cosh\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(x^4/(b*c*(a + b*ArcSinh[c*x]))) + (CoshIntegral[(2*a)/b + 2*ArcSinh[c*x]]
*Sinh[(2*a)/b])/(b^2*c^5) - (CoshIntegral[(4*a)/b + 4*ArcSinh[c*x]]*Sinh[(4
*a)/b])/(2*b^2*c^5) - (Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]]
)/(b^2*c^5) + (Cosh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcSinh[c*x]])/(2*b^2
*c^5)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{x^4}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{4\int\frac{x^3}{a+b\sinh^{-1}(cx)}\,dx}{bc}$$

$$= -\frac{x^4}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{4\,\mathrm{Subst}\left(\int\frac{\cosh(x)\sinh^3(x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{bc^5}$$

$$= -\frac{x^4}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{4\,\mathrm{Subst}\left(\int\left(-\frac{\sinh(2x)}{4(a+bx)}+\frac{\sinh(4x)}{8(a+bx)}\right)dx,x,\sinh^{-1}\right.}{bc^5}$$

$$= -\frac{x^4}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\mathrm{Subst}\left(\int\frac{\sinh(4x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{2bc^5}-\frac{\mathrm{Subst}}{}$$

$$= -\frac{x^4}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{\cosh\left(\frac{2a}{b}\right)\mathrm{Subst}\left(\int\frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx,x,\sinh^{-1}(\right.}{bc^5}$$

$$= -\frac{x^4}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\mathrm{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{b^2c^5}-\frac{\mathrm{Chi}\left(\frac{4a}{b}\right.}{}$$

Mathematica [A] time = 0.29, size = 117, normalized size = 0.83

$$\frac{-\frac{2bc^4x^4}{a+b\sinh^{-1}(cx)}+2\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-\sinh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-2\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\right.}{2b^2c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] ((-2*b*c^4*x^4)/(a + b*ArcSinh[c*x]) + 2*CoshIntegral[2*(a/b + ArcSinh[c*x]
)]*Sinh[(2*a)/b] - CoshIntegral[4*(a/b + ArcSinh[c*x])]*Sinh[(4*a)/b] - 2*C
osh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] + Cosh[(4*a)/b]*SinhInteg
ral[4*(a/b + ArcSinh[c*x])])/(2*b^2*c^5)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{c^2x^2+1}\,x^4}{a^2c^2x^2+\left(b^2c^2x^2+b^2\right)\operatorname{arsinh}(cx)^2+a^2+2\left(abc^2x^2+ab\right)\operatorname{arsinh}(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^4/(a^2*c^2*x^2 + (b^2*c^2*x^2 + b^2)*arcsinh(c
*x)^2 + a^2 + 2*(a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{c^2x^2+1}\,(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)^2), x)
```

maple [B] time = 0.47, size = 420, normalized size = 2.98

$$-\frac{3}{8c^5(a+b\operatorname{arcsinh}(cx))\,b}-\frac{8c^4x^4-8c^3x^3\sqrt{c^2x^2+1}+8c^2x^2-4cx\sqrt{c^2x^2+1}+1}{16c^5(a+b\operatorname{arcsinh}(cx))\,b}+\frac{e^{\frac{4a}{b}}\operatorname{Ei}\left(1,4\operatorname{arcsinh}(cx)+\frac{4a}{b}\right)}{4c^5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x)
```

```
[Out] -3/8/c^5/(a+b*arcsinh(c*x))/b-1/16*(8*c^4*x^4-8*c^3*x^3*(c^2*x^2+1)^(1/2)+8
*c^2*x^2-4*c*x*(c^2*x^2+1)^(1/2)+1)/c^5/(a+b*arcsinh(c*x))/b+1/4/c^5/b^2*ex
p(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)+1/4*(2*c^2*x^2-2*c*x*(c^2*x^2+1)^(1/2)+
1)/c^5/(a+b*arcsinh(c*x))/b-1/2/c^5/b^2*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/
b)+1/4/c^5/b^2*(2*x^2*b*c^2+2*b*c*(c^2*x^2+1)^(1/2)*x+2*arcsinh(c*x)*exp(-2
*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*b+2*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/
b)*a+b)/(a+b*arcsinh(c*x))-1/16/c^5/b^2*(8*x^4*b*c^4+8*(c^2*x^2+1)^(1/2)*x^
3*b*c^3+8*x^2*b*c^2+4*b*c*(c^2*x^2+1)^(1/2)*x+4*arcsinh(c*x)*Ei(1,-4*arcsin
h(c*x)-4*a/b)*exp(-4*a/b)*b+4*Ei(1,-4*arcsinh(c*x)-4*a/b)*exp(-4*a/b)*a+b)/
(a+b*arcsinh(c*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^3x^7+cx^5+\left(c^2x^6+x^4\right)\sqrt{c^2x^2+1}}{\left(c^2x^2+1\right)abc^2x+\left(\left(c^2x^2+1\right)b^2c^2x+\left(b^2c^3x^2+b^2c\right)\sqrt{c^2x^2+1}\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(abc^3x^2+abc\right)\sqrt{c^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -(c^3*x^7 + c*x^5 + (c^2*x^6 + x^4)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)*a*b*c
^2*x + ((c^2*x^2 + 1)*b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*
log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + i
ntegrate((4*c^5*x^8 + 9*c^3*x^6 + 5*c*x^4 + (4*c^3*x^6 + 3*c*x^4)*(c^2*x^2
+ 1) + 4*(2*c^4*x^7 + 3*c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)^(3
/2)*a*b*c^3*x^2 + 2*(a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((c^2*x^2 + 1
)^(3/2)*b^2*c^3*x^2 + 2*(b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*
x^4 + 2*b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)
) + (a*b*c^5*x^4 + 2*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{(a+b\operatorname{asinh}(c\,x))^2\,\sqrt{c^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)),x)
[Out] int(x^4/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a + b \operatorname{asinh}(cx))^2 \sqrt{c^2x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a+b*asinh(c*x))**2/(c**2*x**2+1)**(1/2),x)
[Out] Integral(x**4/((a + b*asinh(c*x))**2*sqrt(c**2*x**2 + 1)), x)
```

3.437 $\int \frac{x^3}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=142

$$-\frac{3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4b^2c^4}+\frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{4b^2c^4}+\frac{3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4b^2c^4}-\frac{3\sinh\left(\frac{3a}{b}\right)\text{Shi}}{4}$$

```
[Out] -x^3/b/c/(a+b*arcsinh(c*x))-3/4*Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b^2/c^4
+3/4*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b^2/c^4+3/4*Shi((a+b*arcsinh(c
*x))/b)*sinh(a/b)/b^2/c^4-3/4*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c
^4
```

Rubi [A] time = 0.37, antiderivative size = 138, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5774, 5669, 5448, 3303, 3298, 3301}

$$-\frac{3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4b^2c^4}+\frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{4b^2c^4}+\frac{3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4b^2c^4}-3\sinh$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(x^3/(b*c*(a + b*ArcSinh[c*x]))) - (3*Cosh[a/b]*CoshIntegral[a/b + ArcSinh
[c*x]])/(4*b^2*c^4) + (3*Cosh[(3*a)/b]*CoshIntegral[(3*a)/b + 3*ArcSinh[c*x
]])/(4*b^2*c^4) + (3*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(4*b^2*c^4
) - (3*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(4*b^2*c^4)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}dx = -\frac{x^3}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{3\int\frac{x^2}{a+b\sinh^{-1}(cx)}dx}{bc}$$

$$= -\frac{x^3}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{3\,\text{Subst}\left(\int\frac{\cosh(x)\sinh^2(x)}{a+bx}dx,x,\sinh^{-1}(cx)\right)}{bc^4}$$

$$= -\frac{x^3}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{3\,\text{Subst}\left(\int\left(-\frac{\cosh(x)}{4(a+bx)}+\frac{\cosh(3x)}{4(a+bx)}\right)dx,x,\sinh^{-1}\right.}{bc^4}$$

$$= -\frac{x^3}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{3\,\text{Subst}\left(\int\frac{\cosh(x)}{a+bx}dx,x,\sinh^{-1}(cx)\right)}{4bc^4}+\frac{3\,\text{Sub}}{}$$

$$= -\frac{x^3}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{\left(3\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}dx,x,\sinh^{-1}\right.}{4bc^4}$$

$$= -\frac{x^3}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4b^2c^4}+\frac{3\cosh\left(\frac{3}{l}\right.}{}$$

Mathematica [A] time = 0.27, size = 113, normalized size = 0.80

$$\frac{3\left(-\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-\text{si}\right.}{4b^2c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(x^3/(b*c*(a + b*ArcSinh[c*x]))) + (3*(-(Cosh[a/b]*CoshIntegral[a/b + ArcS
inh[c*x]]) + Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] + Sinh[a/b]
*SinhIntegral[a/b + ArcSinh[c*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b + Arc
Sinh[c*x])]))/(4*b^2*c^4)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}\,x^3}{a^2c^2x^2+\left(b^2c^2x^2+b^2\right)\text{arsinh}\,(cx)^2+a^2+2\left(abc^2x^2+ab\right)\text{arsinh}\,(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^3/(a^2*c^2*x^2 + (b^2*c^2*x^2 + b^2)*arcsinh(c
*x)^2 + a^2 + 2*(a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.29, size = 364, normalized size = 2.56

$$-\frac{4c^3x^3-4c^2x^2\sqrt{c^2x^2+1}+3cx-\sqrt{c^2x^2+1}}{8c^4b\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)}-\frac{3\,\mathrm{e}^{\frac{3a}{b}}\,\mathrm{Ei}\left(1,3\,\mathrm{arcsinh}\left(cx\right)+\frac{3a}{b}\right)}{8c^4b^2}+\frac{\frac{3cx}{8}-\frac{3\sqrt{c^2x^2+1}}{8}}{c^4b\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)}+\frac{3\,\mathrm{e}^{\frac{a}{b}}\,\mathrm{Ei}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x)
```

```
[Out] -1/8*(4*c^3*x^3-4*c^2*x^2*(c^2*x^2+1)^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))/c^4/b/
(a+b*arcsinh(c*x))-3/8/c^4/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)+3/8*(c
*x-(c^2*x^2+1)^(1/2))/c^4/b/(a+b*arcsinh(c*x))+3/8/c^4/b^2*exp(a/b)*Ei(1,ar
csinh(c*x)+a/b)+3/8/c^4/b^2*(arcsinh(c*x)*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)
*b+exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*a+x*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcs
inh(c*x))-1/8/c^4/b^2*(4*x^3*b*c^3+4*(c^2*x^2+1)^(1/2)*x^2*b*c^2+3*arcsinh(
c*x)*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)*b+3*exp(-3*a/b)*Ei(1,-3*arcsin
h(c*x)-3*a/b)*a+3*x*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^3x^6+cx^4+\left(c^2x^5+x^3\right)\sqrt{c^2x^2+1}}{\left(c^2x^2+1\right)abc^2x+\left(\left(c^2x^2+1\right)b^2c^2x+\left(b^2c^3x^2+b^2c\right)\sqrt{c^2x^2+1}\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(abc^3x^2+abc\right)\sqrt{c^2x^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -(c^3*x^6 + c*x^4 + (c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)*a*b*c
^2*x + ((c^2*x^2 + 1)*b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*
log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + i
ntegrate((3*c^5*x^7 + 7*c^3*x^5 + 4*c*x^3 + (3*c^3*x^5 + 2*c*x^3)*(c^2*x^2
+ 1) + 3*(2*c^4*x^6 + 3*c^2*x^4 + x^2)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)^(3
/2)*a*b*c^3*x^2 + 2*(a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((c^2*x^2 + 1
)^(3/2)*b^2*c^3*x^2 + 2*(b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*
x^4 + 2*b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)
) + (a*b*c^5*x^4 + 2*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,\sqrt{c^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)),x)
[Out] int(x^3/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a + b\,\text{asinh}\,(cx))^2\,\sqrt{c^2x^2 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*asinh(c*x))**2/(c**2*x**2+1)**(1/2),x)
[Out] Integral(x**3/((a + b*asinh(c*x))**2*sqrt(c**2*x**2 + 1)), x)
```

3.438 $\int \frac{x^2}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=79

$$-\frac{\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{b^2c^3}+\frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{b^2c^3}-\frac{x^2}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

```
[Out] -x^2/b/c/(a+b*arcsinh(c*x))+cosh(2*a/b)*Shi(2*(a+b*arcsinh(c*x))/b)/b^2/c^3
-Chi(2*(a+b*arcsinh(c*x))/b)*sinh(2*a/b)/b^2/c^3
```

Rubi [A] time = 0.26, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {5774, 5669, 5448, 12, 3303, 3298, 3301}

$$-\frac{\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{b^2c^3}+\frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{b^2c^3}-\frac{x^2}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(x^2/(b*c*(a + b*ArcSinh[c*x]))) - (CoshIntegral[(2*a)/b + 2*ArcSinh[c*x]]
*Sinh[(2*a)/b])/(b^2*c^3) + (Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh
[c*x]])/(b^2*c^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx &= -\frac{x^2}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{2\int \frac{x}{a+b\sinh^{-1}(cx)}\,dx}{bc} \\
&= -\frac{x^2}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{2\,\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{bc^3} \\
&= -\frac{x^2}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{2\,\text{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)}\,dx,x,\sinh^{-1}(cx)\right)}{bc^3} \\
&= -\frac{x^2}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\text{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{bc^3} \\
&= -\frac{x^2}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx,x,\sinh^{-1}(\right.}{bc^3} \\
&= -\frac{x^2}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{b^2c^3}+\frac{\cosh\left(\frac{2}{}\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 70, normalized size = 0.89

$$\frac{-\frac{bc^2x^2}{a+b\sinh^{-1}(cx)}-\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{b^2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] (-((b*c^2*x^2)/(a + b*ArcSinh[c*x])) - CoshIntegral[2*(a/b + ArcSinh[c*x])]
*Sinh[(2*a)/b] + Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])])/(b^2*c
^3)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}\,x^2}{a^2c^2x^2+\left(b^2c^2x^2+b^2\right)\text{arsinh}\,(cx)^2+a^2+2\left(abc^2x^2+ab\right)\text{arsinh}\,(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^2/(a^2*c^2*x^2 + (b^2*c^2*x^2 + b^2)*arcsinh(c
*x)^2 + a^2 + 2*(a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{c^2x^2+1}\,(b\,\mathrm{arsinh}\,(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)^2), x)
```

maple [B] time = 0.22, size = 192, normalized size = 2.43

$$\frac{1}{2c^3\,(a+b\,\mathrm{arcsinh}\,(cx))\,b}-\frac{2c^2x^2-2cx\sqrt{c^2x^2+1}\,+1}{4c^3\,(a+b\,\mathrm{arcsinh}\,(cx))\,b}+\frac{\mathrm{e}^{\frac{2a}{b}}\,\mathrm{Ei}\left(1,2\,\mathrm{arcsinh}\,(cx)+\frac{2a}{b}\right)}{2c^3b^2}-\frac{2x^2b\,c^2+2bc\sqrt{c^2x^2+1}\,x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x)
```

```
[Out] 1/2/c^3/(a+b*arcsinh(c*x))/b-1/4*(2*c^2*x^2-2*c*x*(c^2*x^2+1)^(1/2)+1)/c^3/
(a+b*arcsinh(c*x))/b+1/2/c^3/b^2*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)-1/4/
c^3/b^2*(2*x^2*b*c^2+2*b*c*(c^2*x^2+1)^(1/2)*x+2*arcsinh(c*x)*exp(-2*a/b)*E
i(1,-2*arcsinh(c*x)-2*a/b)*b+2*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*a+b)
/(a+b*arcsinh(c*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^3x^5+cx^3+\left(c^2x^4+x^2\right)\sqrt{c^2x^2+1}}{\left(c^2x^2+1\right)abc^2x+\left(\left(c^2x^2+1\right)b^2c^2x+\left(b^2c^3x^2+b^2c\right)\sqrt{c^2x^2+1}\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(abc^3x^2+abc\right)\sqrt{c^2x^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -(c^3*x^5 + c*x^3 + (c^2*x^4 + x^2)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)*a*b*c
^2*x + ((c^2*x^2 + 1)*b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*
log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + i
ntegrate((2*c^5*x^6 + 5*c^3*x^4 + 3*c*x^2 + (2*c^3*x^4 + c*x^2)*(c^2*x^2 +
1) + 2*(2*c^4*x^5 + 3*c^2*x^3 + x)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)^(3/2)*
a*b*c^3*x^2 + 2*(a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((c^2*x^2 + 1)^(3
/2)*b^2*c^3*x^2 + 2*(b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*x^4
+ 2*b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) +
(a*b*c^5*x^4 + 2*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{(a+b\,\mathrm{asinh}\,(c\,x))^2\,\sqrt{c^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(x^2/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+b\,\mathrm{asinh}\,(cx)\right)^2\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*asinh(c*x))**2/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2/((a + b*asinh(c*x))**2*sqrt(c**2*x**2 + 1)), x)
```

3.439 $\int \frac{x}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=73

$$\frac{\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c^2}-\frac{\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c^2}-\frac{x}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

```
[Out] -x/b/c/(a+b*arcsinh(c*x))+Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b^2/c^2-Shi((
a+b*arcsinh(c*x))/b)*sinh(a/b)/b^2/c^2
```

Rubi [A] time = 0.16, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5774, 5657, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c^2}-\frac{\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c^2}-\frac{x}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(x/(b*c*(a + b*ArcSinh[c*x]))) + (Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*
x])/b])/(b^2*c^2) - (Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(b^2*c
^2)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{x}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\int \frac{1}{a+b\sinh^{-1}(cx)}\,dx}{bc}$$

$$= -\frac{x}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\sinh^{-1}(cx)\right)}{b^2c^2}$$

$$= -\frac{x}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx,x,a+b\sinh^{-1}\right.}{b^2c^2}$$

$$= -\frac{x}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c^2}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\right.}{b^2c}$$

Mathematica [A] time = 0.12, size = 60, normalized size = 0.82

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-\frac{bcx}{a+b\sinh^{-1}(cx)}}{b^2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] (-((b*c*x)/(a + b*ArcSinh[c*x])) + Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x
]] - Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(b^2*c^2)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}\,x}{a^2c^2x^2+\left(b^2c^2x^2+b^2\right)\text{arsinh}\left(cx\right)^2+a^2+2\left(abc^2x^2+ab\right)\text{arsinh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x/(a^2*c^2*x^2 + (b^2*c^2*x^2 + b^2)*arcsinh(c*x
)^2 + a^2 + 2*(a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{c^2x^2+1}\left(b\,\text{arsinh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)^2), x)
```

maple [B] time = 0.08, size = 151, normalized size = 2.07

$$-\frac{cx-\sqrt{c^2x^2+1}}{2c^2b\left(a+b\,\text{arcsinh}\left(cx\right)\right)}-\frac{e^{\frac{a}{b}}\,\text{Ei}\left(1,\text{arcsinh}\left(cx\right)+\frac{a}{b}\right)}{2c^2b^2}-\frac{\text{arcsinh}\left(cx\right)e^{-\frac{a}{b}}\,\text{Ei}\left(1,-\text{arcsinh}\left(cx\right)-\frac{a}{b}\right)b+e^{-\frac{a}{b}}\,\text{Ei}\left(\right.}{2c^2b^2\left(a+b\,\text{arcsi}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x)
```

```
[Out] -1/2*(c*x-(c^2*x^2+1)^(1/2))/c^2/b/(a+b*arcsinh(c*x))-1/2/c^2/b^2*exp(a/b)*
Ei(1,arcsinh(c*x)+a/b)-1/2/c^2/b^2*(arcsinh(c*x)*exp(-a/b)*Ei(1,-arcsinh(c*
x)-a/b)*b+exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*a+x*b*c+(c^2*x^2+1)^(1/2)*b)/(a
+b*arcsinh(c*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^{3}x^{4}+cx^{2}+\left(c^{2}x^{3}+x\right)\sqrt{c^{2}x^{2}+1}}{\left(c^{2}x^{2}+1\right)abc^{2}x+\left(\left(c^{2}x^{2}+1\right)b^{2}c^{2}x+\left(b^{2}c^{3}x^{2}+b^{2}c\right)\sqrt{c^{2}x^{2}+1}\right)\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)+\left(abc^{3}x^{2}+abc\right)\sqrt{c^{2}x^{2}+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -(c^3*x^4 + c*x^2 + (c^2*x^3 + x)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)*a*b*c^2
*x + ((c^2*x^2 + 1)*b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*lo
g(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + int
egrate((c^5*x^5 + (c^2*x^2 + 1)*c^3*x^3 + 3*c^3*x^3 + 2*c*x + (2*c^4*x^4 +
3*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)^(3/2)*a*b*c^3*x^2 + 2*(a*b
*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((c^2*x^2 + 1)^(3/2)*b^2*c^3*x^2 + 2*
(b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*x^4 + 2*b^2*c^3*x^2 + b^
2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^4 + 2*a*b
*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,\sqrt{c^2\,x^2+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(x/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a + b \operatorname{asinh}{\left(cx \right)}\right)^{2} \sqrt{c^{2}x^{2} + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*asinh(c*x))**2/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x/((a + b*asinh(c*x))**2*sqrt(c**2*x**2 + 1)), x)
```

3.440 $\int \frac{1}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=18

$$-\frac{1}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

```
[Out] -1/b/c/(a+b*arcsinh(c*x))
```

Rubi [A] time = 0.04, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {5675}

$$-\frac{1}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(1/(b*c*(a + b*ArcSinh[c*x])))
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{1}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$-\frac{1}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(1/(b*c*(a + b*ArcSinh[c*x])))
```

fricas [A] time = 0.54, size = 30, normalized size = 1.67

$$-\frac{1}{b^2c\log\left(cx+\sqrt{c^2x^2+1}\right)+abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/(b^2*c*log(c*x + sqrt(c^2*x^2 + 1)) + a*b*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c^2x^2+1}\,(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)^2), x)
```

maple [A] time = 0.01, size = 19, normalized size = 1.06

$$-\frac{1}{bc\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x)
```

```
[Out] -1/b/c/(a+b*arcsinh(c*x))
```

maxima [A] time = 0.36, size = 18, normalized size = 1.00

$$-\frac{1}{(b\,\mathrm{arsinh}\left(cx\right)+a)bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/((b*arcsinh(c*x) + a)*b*c)
```

mupad [B] time = 0.14, size = 18, normalized size = 1.00

$$-\frac{1}{c\,\mathrm{asinh}\left(c\,x\right)\,b^2+a\,c\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)),x)
```

```
[Out] -1/(b^2*c*asinh(c*x) + a*b*c)
```

sympy [A] time = 3.06, size = 36, normalized size = 2.00

$$\begin{cases} \frac{x}{a^2} & \text{for}\: b = 0 \wedge c = 0 \\ \frac{\mathrm{asinh}\left(cx\right)}{a^2c} & \text{for}\: b = 0 \\ \frac{x}{a^2} & \text{for}\: c = 0 \\ -\frac{1}{abc+b^2c\,\mathrm{asinh}\left(cx\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(c*x))**2/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((x/a**2, Eq(b, 0) & Eq(c, 0)), (asinh(c*x)/(a**2*c), Eq(b, 0)), (
x/a**2, Eq(c, 0)), (-1/(a*b*c + b**2*c*asinh(c*x)), True))
```

3.441 $\int \frac{1}{x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=47

$$-\frac{\mathrm{Int}\left(\frac{1}{x^2\left(a+b\sinh^{-1}(cx)\right)},x\right)}{bc}-\frac{1}{bcx\left(a+b\sinh^{-1}(cx)\right)}$$

```
[Out] -1/b/c/x/(a+b*arcsinh(c*x))-Unintegrable(1/x^2/(a+b*arcsinh(c*x)),x)/b/c
```

Rubi [A] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(1/(b*c*x*(a + b*ArcSinh[c*x]))) - Defer[Int][1/(x^2*(a + b*ArcSinh[c*x]))
, x]/(b*c)
```

Rubi steps

$$\int \frac{1}{x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=-\frac{1}{bcx\left(a+b\sinh^{-1}(cx)\right)}-\frac{\int \frac{1}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx}{bc}$$

Mathematica [A] time = 5.44, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2), x]
```

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{c^2x^2+1}}{a^2c^2x^3+a^2x+\left(b^2c^2x^3+b^2x\right)\operatorname{arsinh}(cx)^2+2\left(abc^2x^3+abx\right)\operatorname{arsinh}(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(a^2*c^2*x^3 + a^2*x + (b^2*c^2*x^3 + b^2*x)*arc
sinh(c*x)^2 + 2*(a*b*c^2*x^3 + a*b*x)*arcsinh(c*x)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^{2}\sqrt{c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x)
```

```
[Out] int(1/x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^{3}x^{3}+cx+\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}}{\left(c^{2}x^{2}+1\right)abc^{2}x^{2}+\left(\left(c^{2}x^{2}+1\right)b^{2}c^{2}x^{2}+\left(b^{2}c^{3}x^{3}+b^{2}cx\right)\sqrt{c^{2}x^{2}+1}\right)\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)+\left(abc^{3}x^{3}+abcx\right)\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/((c^2*x^2 + 1)*a*b*c^2*x^2 + ((c^2*x
^2 + 1)*b^2*c^2*x^2 + (b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2*x^2 + 1))*log(c*x +
sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)) - integrate
((c^5*x^5 + c^3*x^3 + (c^3*x^3 + 2*c*x)*(c^2*x^2 + 1) + (2*c^4*x^4 + 3*c^2*
x^2 + 1)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)^(3/2)*a*b*c^3*x^4 + 2*(a*b*c^4*x
^5 + a*b*c^2*x^3)*(c^2*x^2 + 1) + ((c^2*x^2 + 1)^(3/2)*b^2*c^3*x^4 + 2*(b^2
*c^4*x^5 + b^2*c^2*x^3)*(c^2*x^2 + 1) + (b^2*c^5*x^6 + 2*b^2*c^3*x^4 + b^2*
c*x^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^6 + 2*a
*b*c^3*x^4 + a*b*c*x^2)*sqrt(c^2*x^2 + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,\sqrt{c^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(1/(x*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\,\mathrm{asinh}\left(cx\right)\right)^{2}\sqrt{c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*asinh(c*x))**2/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(x*(a + b*asinh(c*x))**2*sqrt(c**2*x**2 + 1)), x)
```

3.442 $\int \frac{1}{x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=47

$$-\frac{2\text{Int}\left(\frac{1}{x^3\left(a+b\sinh^{-1}(cx)\right)},x\right)}{bc}-\frac{1}{bcx^2\left(a+b\sinh^{-1}(cx)\right)}$$

```
[Out] -1/b/c/x^2/(a+b*arcsinh(c*x))-2*Unintegrable(1/x^3/(a+b*arcsinh(c*x)),x)/b/
c
```

Rubi [A] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(1/(b*c*x^2*(a + b*ArcSinh[c*x]))) - (2*Defer[Int][1/(x^3*(a + b*ArcSinh[c
*x])), x])/(b*c)
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=-\frac{1}{bcx^2\left(a+b\sinh^{-1}(cx)\right)}-\frac{2\int \frac{1}{x^3\left(a+b\sinh^{-1}(cx)\right)}\,dx}{bc}$$

Mathematica [A] time = 1.25, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2), x]
```

fricas [A] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}}{a^2c^2x^4+a^2x^2+\left(b^2c^2x^4+b^2x^2\right)\operatorname{arsinh}(cx)^2+2\left(abc^2x^4+abx^2\right)\operatorname{arsinh}(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(a^2*c^2*x^4 + a^2*x^2 + (b^2*c^2*x^4 + b^2*x^2)
*arcsinh(c*x)^2 + 2*(a*b*c^2*x^4 + a*b*x^2)*arcsinh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c^2x^2+1}\,(b\operatorname{arsinh}(cx)+a)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)^2*x^2), x)
```

maple [A] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\,(a+b\operatorname{arcsinh}(cx))^2\,\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x)
[Out] int(1/x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^3x^3+cx+\left(c^2x^2+1\right)^{\frac{3}{2}}}{\left(c^2x^2+1\right)abc^2x^3+\left(\left(c^2x^2+1\right)b^2c^2x^3+\left(b^2c^3x^4+b^2cx^2\right)\sqrt{c^2x^2+1}\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(abc^3x^4+abcx^2\right)\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="maxima
")
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/((c^2*x^2 + 1)*a*b*c^2*x^3 + ((c^2*x
^2 + 1)*b^2*c^2*x^3 + (b^2*c^3*x^4 + b^2*c*x^2)*sqrt(c^2*x^2 + 1))*log(c*x
+ sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^4 + a*b*c*x^2)*sqrt(c^2*x^2 + 1)) - integ
rate((2*c^5*x^5 + 3*c^3*x^3 + (2*c^3*x^3 + 3*c*x)*(c^2*x^2 + 1) + c*x + 2*(
2*c^4*x^4 + 3*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)^(3/2)*a*b*c^3*
x^5 + 2*(a*b*c^4*x^6 + a*b*c^2*x^4)*(c^2*x^2 + 1) + ((c^2*x^2 + 1)^(3/2)*b^
2*c^3*x^5 + 2*(b^2*c^4*x^6 + b^2*c^2*x^4)*(c^2*x^2 + 1) + (b^2*c^5*x^7 + 2*
b^2*c^3*x^5 + b^2*c*x^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) +
(a*b*c^5*x^7 + 2*a*b*c^3*x^5 + a*b*c*x^3)*sqrt(c^2*x^2 + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,\sqrt{c^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)),x)
[Out] int(1/(x^2*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\,(a+b\operatorname{asinh}(cx))^2\,\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*asinh(c*x))**2/(c**2*x**2+1)**(1/2),x)
[Out] Integral(1/(x**2*(a + b*asinh(c*x))**2*sqrt(c**2*x**2 + 1)), x)
```

3.443 $\int \frac{x^3}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{x^3}{\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(x^3/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^3}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][x^3/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{x^3}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{x^3}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 8.10, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[x^3/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

fricas [A] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}\,x^3}{a^2c^4x^4+2\,a^2c^2x^2+\left(b^2c^4x^4+2\,b^2c^2x^2+b^2\right)\text{arsinh}\,(cx)^2+a^2+2\left(abc^4x^4+2\,abc^2x^2+ab\right)\text{arsinh}\,(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)*x^3/(a^2*c^4*x^4 + 2*a^2*c^2*x^2 + (b^2*c^4*x^4
+ 2*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 + 2*a*b*c^2*x^
2 + a*b)*arcsinh(c*x)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(c^2x^2+1\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^3/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{cx^4+\sqrt{c^2x^2+1}\,x^3}{\left(c^2x^2+1\right)abc^2x+\left(\left(c^2x^2+1\right)b^2c^2x+\left(b^2c^3x^2+b^2c\right)\sqrt{c^2x^2+1}\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(abc^3x^2+abc\right)\sqrt{c^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c*x^4 + sqrt(c^2*x^2 + 1)*x^3)/((c^2*x^2 + 1)*a*b*c^2*x + ((c^2*x^2 + 1)*
b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2
 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + integrate((c^5*x^7 + 5*
c^3*x^5 + 4*c*x^3 + (c^3*x^5 + 2*c*x^3)*(c^2*x^2 + 1) + (2*c^4*x^6 + 7*c^2*
x^4 + 3*x^2)*sqrt(c^2*x^2 + 1))/((a*b*c^5*x^4 + a*b*c^3*x^2)*(c^2*x^2 + 1)^
(3/2) + 2*(a*b*c^6*x^5 + 2*a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((b^2*c
^5*x^4 + b^2*c^3*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(b^2*c^6*x^5 + 2*b^2*c^4*x^3
+ b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^7*x^6 + 3*b^2*c^5*x^4 + 3*b^2*c^3*x^2 +
 b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^7*x^6 + 3*
a*b*c^5*x^4 + 3*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^3}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(c^2\,x^2+1\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)),x)
```

```
[Out] int(x^3/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a+b\,\mathrm{asinh}\left(cx\right)\right)^2\left(c^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x**3/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(3/2)), x)
```

3.444 $\int \frac{x^2}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=67

$$\frac{2\text{Int}\left(\frac{x}{\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)},x\right)}{bc}-\frac{x^2}{bc\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}$$

```
[Out] -x^2/b/c/(c^2*x^2+1)/(a+b*arcsinh(c*x))+2*Unintegrable(x/(c^2*x^2+1)^2/(a+b
*arcsinh(c*x)),x)/b/c
```

Rubi [A] time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(x^2/(b*c*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))) + (2*Defer[Int][x/((1 + c^2
*x^2)^2*(a + b*ArcSinh[c*x])), x])/(b*c)
```

Rubi steps

$$\int \frac{x^2}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=-\frac{x^2}{bc\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}+\frac{2\int \frac{x}{\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}\,dx}{bc}$$

Mathematica [A] time = 3.60, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[x^2/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}\,x^2}{a^2c^4x^4+2\,a^2c^2x^2+\left(b^2c^4x^4+2\,b^2c^2x^2+b^2\right)\text{arsinh}\,(cx)^2+a^2+2\left(abc^4x^4+2\,abc^2x^2+ab\right)\text{arsinh}\,(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^2/(a^2*c^4*x^4 + 2*a^2*c^2*x^2 + (b^2*c^4*x^4
+ 2*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 + 2*a*b*c^2*x^
2 + a*b)*arcsinh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(c^2x^2+1\right)^{\frac{3}{2}}(b\,\mathrm{arsinh}\,(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")

[Out] integrate(x^2/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)^2), x)

maple [A] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(c^2x^2+1\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arcsinh}\,(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)

[Out] int(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{cx^3+\sqrt{c^2x^2+1}\,x^2}{\left(c^2x^2+1\right)abc^2x+\left(\left(c^2x^2+1\right)b^2c^2x+\left(b^2c^3x^2+b^2c\right)\sqrt{c^2x^2+1}\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(abc^3x^2+abc\right)\sqrt{c^2x^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")

[Out] -(c*x^3 + sqrt(c^2*x^2 + 1)*x^2)/((c^2*x^2 + 1)*a*b*c^2*x + ((c^2*x^2 + 1)*b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + integrate((3*c^3*x^4 + (c^2*x^2 + 1)*c*x^2 + 3*c*x^2 + 2*(2*c^2*x^3 + x)*sqrt(c^2*x^2 + 1))/((a*b*c^5*x^4 + a*b*c^3*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(a*b*c^6*x^5 + 2*a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((b^2*c^5*x^4 + b^2*c^3*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(b^2*c^6*x^5 + 2*b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^7*x^6 + 3*b^2*c^5*x^4 + 3*b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^7*x^6 + 3*a*b*c^5*x^4 + 3*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(c^2\,x^2+1\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)),x)

[Out] int(x^2/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+b\,\mathrm{asinh}\,(cx)\right)^2\left(c^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x**2/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(3/2)), x)
```

3.445 $\int \frac{x}{(1+c^2x^2)^{3/2}(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{x}{(c^2x^2+1)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][x/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{x}{(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{x}{(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 5.36, size = 0, normalized size = 0.00

$$\int \frac{x}{(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[x/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}\,x}{a^2c^4x^4+2\,a^2c^2x^2+\left(b^2c^4x^4+2\,b^2c^2x^2+b^2\right)\text{arsinh}(cx)^2+a^2+2\left(abc^4x^4+2\,abc^2x^2+ab\right)\text{arsinh}(cx)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)*x/(a^2*c^4*x^4 + 2*a^2*c^2*x^2 + (b^2*c^4*x^4 +
2*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 + 2*a*b*c^2*x^2
+ a*b)*arcsinh(c*x)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(c^2x^2+1\right)^{\frac{3}{2}}\left(a+b\operatorname{arcsinh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{cx^2+\sqrt{c^2x^2+1}\,x}{\left(c^2x^2+1\right)abc^2x+\left(\left(c^2x^2+1\right)b^2c^2x+\left(b^2c^3x^2+b^2c\right)\sqrt{c^2x^2+1}\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(abc^3x^2+abc\right)\sqrt{c^2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c*x^2 + sqrt(c^2*x^2 + 1)*x)/((c^2*x^2 + 1)*a*b*c^2*x + ((c^2*x^2 + 1)*b^
2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 +
 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) - integrate((c^5*x^5 + (c^2
*x^2 + 1)*c^3*x^3 - c^3*x^3 - 2*c*x + (2*c^4*x^4 - c^2*x^2 - 1)*sqrt(c^2*x^
2 + 1))/((a*b*c^5*x^4 + a*b*c^3*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(a*b*c^6*x^5 +
 2*a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((b^2*c^5*x^4 + b^2*c^3*x^2)*(c
^2*x^2 + 1)^(3/2) + 2*(b^2*c^6*x^5 + 2*b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 +
1) + (b^2*c^7*x^6 + 3*b^2*c^5*x^4 + 3*b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1
))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^7*x^6 + 3*a*b*c^5*x^4 + 3*a*b*c^3*
x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2\,\left(c^2\,x^2+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)),x)
```

```
[Out] int(x/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+b\operatorname{asinh}\left(cx\right)\right)^2\left(c^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(3/2)), x)
```

3.446 $\int \frac{1}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=62

$$-\frac{2c\text{Int}\left(\frac{x}{\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)},x\right)}{b}-\frac{1}{bc\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}$$

```
[Out] -1/b/c/(c^2*x^2+1)/(a+b*arcsinh(c*x))-2*c*Unintegrable(x/(c^2*x^2+1)^2/(a+b
*arcsinh(c*x)),x)/b
```

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(1/(b*c*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))) - (2*c*Defer[Int][x/((1 + c^2
*x^2)^2*(a + b*ArcSinh[c*x])), x])/b
```

Rubi steps

$$\int \frac{1}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=-\frac{1}{bc\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}-\frac{(2c)\int \frac{x}{\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}\,dx}{b}$$

Mathematica [A] time = 2.44, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[1/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

fricas [A] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}}{a^2c^4x^4+2\,a^2c^2x^2+\left(b^2c^4x^4+2\,b^2c^2x^2+b^2\right)\text{arsinh}\,(cx)^2+a^2+2\left(abc^4x^4+2\,abc^2x^2+ab\right)\text{arsinh}\,(cx)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(a^2*c^4*x^4 + 2*a^2*c^2*x^2 + (b^2*c^4*x^4 + 2*
b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 + 2*a*b*c^2*x^2 +
a*b)*arcsinh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c^2x^2+1\right)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)^2), x)
```

maple [A] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c^2x^2+1\right)^{\frac{3}{2}}(a+b\operatorname{arcsinh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{cx+\sqrt{c^2x^2+1}}{\left(c^2x^2+1\right)abc^2x+\left(\left(c^2x^2+1\right)b^2c^2x+\left(b^2c^3x^2+b^2c\right)\sqrt{c^2x^2+1}\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(abc^3x^2+abc\right)\sqrt{c^2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c*x + sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)*a*b*c^2*x + ((c^2*x^2 + 1)*b^2*c^
2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))
 + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) - integrate((2*c^4*x^4 + c^2*x^
2 + (2*c^2*x^2 + 1)*(c^2*x^2 + 1) + 2*(2*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1) -
 1)/((a*b*c^4*x^4 + a*b*c^2*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(a*b*c^5*x^5 + 2*a
*b*c^3*x^3 + a*b*c*x)*(c^2*x^2 + 1) + ((b^2*c^4*x^4 + b^2*c^2*x^2)*(c^2*x^2
 + 1)^(3/2) + 2*(b^2*c^5*x^5 + 2*b^2*c^3*x^3 + b^2*c*x)*(c^2*x^2 + 1) + (b^
2*c^6*x^6 + 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 + b^2)*sqrt(c^2*x^2 + 1))*log(c*x
 + sqrt(c^2*x^2 + 1)) + (a*b*c^6*x^6 + 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 + a*b)
*sqrt(c^2*x^2 + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(a+b\operatorname{asinh}(c\,x))^2\,\left(c^2\,x^2+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)),x)
```

```
[Out] int(1/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\operatorname{asinh}(cx))^2\left(c^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(1/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(3/2)), x)
```

3.447 $\int \frac{1}{x(1+c^2x^2)^{3/2}(a+b\sinh^{-1}(cx))^2} dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{1}{x\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Defer[Int][1/(x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2} dx = \int \frac{1}{x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2} dx$$

Mathematica [A] time = 7.83, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[1/(x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

fricas [A] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}}{a^2c^4x^5+2\,a^2c^2x^3+a^2x+\left(b^2c^4x^5+2\,b^2c^2x^3+b^2x\right)\operatorname{arsinh}(cx)^2+2\left(abc^4x^5+2\,abc^2x^3+abx\right)\operatorname{arsinh}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(a^2*c^4*x^5 + 2*a^2*c^2*x^3 + a^2*x + (b^2*c^4*
x^5 + 2*b^2*c^2*x^3 + b^2*x)*arcsinh(c*x)^2 + 2*(a*b*c^4*x^5 + 2*a*b*c^2*x^
3 + a*b*x)*arcsinh(c*x)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{cx+\sqrt{c^{2}x^{2}+1}}{\left(c^{2}x^{2}+1\right)abc^{2}x^{2}+\left(\left(c^{2}x^{2}+1\right)b^{2}c^{2}x^{2}+\left(b^{2}c^{3}x^{3}+b^{2}cx\right)\sqrt{c^{2}x^{2}+1}\right)\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)+\left(abc^{3}x^{3}+abcx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c*x + sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)*a*b*c^2*x^2 + ((c^2*x^2 + 1)*b^2*
c^2*x^2 + (b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2
 + 1)) + (a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)) - integrate((3*c^5*x^5
+ 3*c^3*x^3 + (3*c^3*x^3 + 2*c*x)*(c^2*x^2 + 1) + (6*c^4*x^4 + 5*c^2*x^2 +
1)*sqrt(c^2*x^2 + 1))/((a*b*c^5*x^6 + a*b*c^3*x^4)*(c^2*x^2 + 1)^(3/2) + 2*
(a*b*c^6*x^7 + 2*a*b*c^4*x^5 + a*b*c^2*x^3)*(c^2*x^2 + 1) + ((b^2*c^5*x^6 +
 b^2*c^3*x^4)*(c^2*x^2 + 1)^(3/2) + 2*(b^2*c^6*x^7 + 2*b^2*c^4*x^5 + b^2*c^
2*x^3)*(c^2*x^2 + 1) + (b^2*c^7*x^8 + 3*b^2*c^5*x^6 + 3*b^2*c^3*x^4 + b^2*c
*x^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^7*x^8 + 3*a*
b*c^5*x^6 + 3*a*b*c^3*x^4 + a*b*c*x^2)*sqrt(c^2*x^2 + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(c^2\,x^2+1\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)),x)
```

```
[Out] int(1/(x*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\,\mathrm{asinh}\left(cx\right)\right)^{2}\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(1/(x*(a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(3/2)), x)
```

3.448 $\int \frac{1}{x^2(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2} dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{1}{x^2\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

[Out] Unintegrable(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)

Rubi [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]

[Out] Defer[Int][1/(x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]

Rubi steps

$$\int \frac{1}{x^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2} dx = \int \frac{1}{x^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2} dx$$

Mathematica [A] time = 20.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]

[Out] Integrate[1/(x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]

fricas [A] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}}{a^2c^4x^6+2\,a^2c^2x^4+a^2x^2+\left(b^2c^4x^6+2\,b^2c^2x^4+b^2x^2\right)\text{arsinh}\,(cx)^2+2\left(abc^4x^6+2\,abc^2x^4+abx^2\right)\text{arsi}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas
")

[Out] integral(sqrt(c^2*x^2 + 1)/(a^2*c^4*x^6 + 2*a^2*c^2*x^4 + a^2*x^2 + (b^2*c^
4*x^6 + 2*b^2*c^2*x^4 + b^2*x^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*x^6 + 2*a*b*c^
2*x^4 + a*b*x^2)*arcsinh(c*x)), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c^2x^2+1\right)^{\frac{3}{2}}\left(b\,\text{arsinh}\,(cx)+a\right)^2x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate(1/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)^2*x^2), x)
```

maple [A] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(c^{2} x^{2}+1\right)^{\frac{3}{2}} \left(a +b \,\mathrm{arcsinh}\left(c x \right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
[Out] int(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{cx+\sqrt{c^2x^2+1}}{\left(c^2x^2+1\right)abc^2x^3+\left(\left(c^2x^2+1\right)b^2c^2x^3+\left(b^2c^3x^4+b^2cx^2\right)\sqrt{c^2x^2+1}\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(abc^3x^4+abcx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima
")
[Out] -(c*x + sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)*a*b*c^2*x^3 + ((c^2*x^2 + 1)*b^2*
c^2*x^3 + (b^2*c^3*x^4 + b^2*c*x^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x
^2 + 1)) + (a*b*c^3*x^4 + a*b*c*x^2)*sqrt(c^2*x^2 + 1)) - integrate((4*c^5*
x^5 + 5*c^3*x^3 + (4*c^3*x^3 + 3*c*x)*(c^2*x^2 + 1) + c*x + 2*(4*c^4*x^4 +
4*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1))/((a*b*c^5*x^7 + a*b*c^3*x^5)*(c^2*x^2 + 1
)^(3/2) + 2*(a*b*c^6*x^8 + 2*a*b*c^4*x^6 + a*b*c^2*x^4)*(c^2*x^2 + 1) + ((b
^2*c^5*x^7 + b^2*c^3*x^5)*(c^2*x^2 + 1)^(3/2) + 2*(b^2*c^6*x^8 + 2*b^2*c^4*
x^6 + b^2*c^2*x^4)*(c^2*x^2 + 1) + (b^2*c^7*x^9 + 3*b^2*c^5*x^7 + 3*b^2*c^3
*x^5 + b^2*c*x^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^
7*x^9 + 3*a*b*c^5*x^7 + 3*a*b*c^3*x^5 + a*b*c*x^3)*sqrt(c^2*x^2 + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x^2\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(c^2\,x^2+1\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)),x)
[Out] int(1/(x^2*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(a + b \operatorname{asinh}{\left(c x \right)}\right)^{2} \left(c^{2} x^{2} + 1\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
[Out] Integral(1/(x**2*(a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(3/2)), x)
```

3.449 $\int \frac{x^3}{(1+c^2x^2)^{5/2}(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{x^3}{\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(x^3/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^3}{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][x^3/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{x^3}{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{x^3}{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 14.37, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[x^3/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}\,x^3}{a^2c^6x^6+3\,a^2c^4x^4+3\,a^2c^2x^2+\left(b^2c^6x^6+3\,b^2c^4x^4+3\,b^2c^2x^2+b^2\right)\text{arsinh}\,(cx)^2+a^2+2\left(abc^6x^6+3\,ab\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)*x^3/(a^2*c^6*x^6 + 3*a^2*c^4*x^4 + 3*a^2*c^2*x^2
 + (b^2*c^6*x^6 + 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2
 + 2*(a*b*c^6*x^6 + 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 1.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\left(c^{2}x^{2}+1\right)^{\frac{5}{2}}\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^3/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{cx^{4}+\sqrt{c^{2}x^{2}+1}\,x^{3}}{\left(abc^{4}x^{3}+abc^{2}x\right)\left(c^{2}x^{2}+1\right)+\left(\left(b^{2}c^{4}x^{3}+b^{2}c^{2}x\right)\left(c^{2}x^{2}+1\right)+\left(b^{2}c^{5}x^{4}+2\,b^{2}c^{3}x^{2}+b^{2}c\right)\sqrt{c^{2}x^{2}+1}\right)\log\left(cx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c*x^4 + sqrt(c^2*x^2 + 1)*x^3)/((a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) +
 ((b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*x^4 + 2*b^2*c^3*x^2 +
b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^4 + 2*a
*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) - integrate((c^5*x^7 - 3*c^3*x^5 - 4
*c*x^3 + (c^3*x^5 - 2*c*x^3)*(c^2*x^2 + 1) + (2*c^4*x^6 - 5*c^2*x^4 - 3*x^2
)*sqrt(c^2*x^2 + 1))/((a*b*c^7*x^6 + 2*a*b*c^5*x^4 + a*b*c^3*x^2)*(c^2*x^2
+ 1)^(3/2) + 2*(a*b*c^8*x^7 + 3*a*b*c^6*x^5 + 3*a*b*c^4*x^3 + a*b*c^2*x)*(c
^2*x^2 + 1) + ((b^2*c^7*x^6 + 2*b^2*c^5*x^4 + b^2*c^3*x^2)*(c^2*x^2 + 1)^(3
/2) + 2*(b^2*c^8*x^7 + 3*b^2*c^6*x^5 + 3*b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2
+ 1) + (b^2*c^9*x^8 + 4*b^2*c^7*x^6 + 6*b^2*c^5*x^4 + 4*b^2*c^3*x^2 + b^2*c
)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^9*x^8 + 4*a*b*c^
7*x^6 + 6*a*b*c^5*x^4 + 4*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^3}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(c^2\,x^2+1\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)),x)
```

```
[Out] int(x^3/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\left(a+b\,\mathrm{asinh}\left(cx\right)\right)^{2}\left(c^{2}x^{2}+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x**3/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(5/2)), x)
```

3.450 $\int \frac{x^2}{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2} dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{x^2}{\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2}{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][x^2/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{x^2}{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2} dx = \int \frac{x^2}{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2} dx$$

Mathematica [A] time = 5.87, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[x^2/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}\,x^2}{a^2c^6x^6+3\,a^2c^4x^4+3\,a^2c^2x^2+\left(b^2c^6x^6+3\,b^2c^4x^4+3\,b^2c^2x^2+b^2\right)\text{arsinh}\,(cx)^2+a^2+2\left(abc^6x^6+3\,ab\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)*x^2/(a^2*c^6*x^6 + 3*a^2*c^4*x^4 + 3*a^2*c^2*x^2
 + (b^2*c^6*x^6 + 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2
 + 2*(a*b*c^6*x^6 + 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(c^2x^2+1\right)^{\frac{5}{2}}\left(b\,\text{arsinh}\,(cx)+a\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate(x^2/((c^2*x^2 + 1)^(5/2)*(b*arcsinh(c*x) + a)^2), x)
```

maple [A] time = 0.83, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(c^2x^2+1\right)^{\frac{5}{2}}\left(a+b\operatorname{arcsinh}(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
[Out] int(x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{cx^3+\sqrt{c^2x^2+1}\,x^2}{\left(abc^4x^3+abc^2x\right)\left(c^2x^2+1\right)+\left(\left(b^2c^4x^3+b^2c^2x\right)\left(c^2x^2+1\right)+\left(b^2c^5x^4+2\,b^2c^3x^2+b^2c\right)\sqrt{c^2x^2+1}\right)\log\left(cx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -(c*x^3 + sqrt(c^2*x^2 + 1)*x^2)/((a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) +
 ((b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*x^4 + 2*b^2*c^3*x^2 +
b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^4 + 2*a
*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) - integrate((2*c^5*x^6 - c^3*x^4 - 3
*c*x^2 + (2*c^3*x^4 - c*x^2)*(c^2*x^2 + 1) + 2*(2*c^4*x^5 - c^2*x^3 - x)*sq
rt(c^2*x^2 + 1))/((a*b*c^7*x^6 + 2*a*b*c^5*x^4 + a*b*c^3*x^2)*(c^2*x^2 + 1)
^(3/2) + 2*(a*b*c^8*x^7 + 3*a*b*c^6*x^5 + 3*a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x
^2 + 1) + ((b^2*c^7*x^6 + 2*b^2*c^5*x^4 + b^2*c^3*x^2)*(c^2*x^2 + 1)^(3/2)
+ 2*(b^2*c^8*x^7 + 3*b^2*c^6*x^5 + 3*b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1)
 + (b^2*c^9*x^8 + 4*b^2*c^7*x^6 + 6*b^2*c^5*x^4 + 4*b^2*c^3*x^2 + b^2*c)*sq
rt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^9*x^8 + 4*a*b*c^7*x^
6 + 6*a*b*c^5*x^4 + 4*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(c^2\,x^2+1\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)),x)
[Out] int(x^2/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+b\operatorname{asinh}(cx)\right)^2\left(c^2x^2+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
[Out] Integral(x**2/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(5/2)), x)
```

3.451 $\int \frac{x}{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{x}{\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][x/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{x}{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{x}{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 11.06, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[x/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}\,x}{a^2c^6x^6+3\,a^2c^4x^4+3\,a^2c^2x^2+\left(b^2c^6x^6+3\,b^2c^4x^4+3\,b^2c^2x^2+b^2\right)\text{arsinh}\left(cx\right)^2+a^2+2\left(abc^6x^6+3\,ab\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)*x/(a^2*c^6*x^6 + 3*a^2*c^4*x^4 + 3*a^2*c^2*x^2 +
 (b^2*c^6*x^6 + 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2 +
 2*(a*b*c^6*x^6 + 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(c^{2}x^{2}+1\right)^{\frac{5}{2}}\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{cx^{2}+\sqrt{c^{2}x^{2}+1}\,x}{\left(abc^{4}x^{3}+abc^{2}x\right)\left(c^{2}x^{2}+1\right)+\left(\left(b^{2}c^{4}x^{3}+b^{2}c^{2}x\right)\left(c^{2}x^{2}+1\right)+\left(b^{2}c^{5}x^{4}+2\,b^{2}c^{3}x^{2}+b^{2}c\right)\sqrt{c^{2}x^{2}+1}\right)\log\left(cx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c*x^2 + sqrt(c^2*x^2 + 1)*x)/((a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + (
(b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*x^4 + 2*b^2*c^3*x^2 + b^
2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^4 + 2*a*b
*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) - integrate((3*c^5*x^5 + 3*(c^2*x^2 +
1)*c^3*x^3 + c^3*x^3 - 2*c*x + (6*c^4*x^4 + c^2*x^2 - 1)*sqrt(c^2*x^2 + 1))
/((a*b*c^7*x^6 + 2*a*b*c^5*x^4 + a*b*c^3*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(a*b*
c^8*x^7 + 3*a*b*c^6*x^5 + 3*a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((b^2*
c^7*x^6 + 2*b^2*c^5*x^4 + b^2*c^3*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(b^2*c^8*x^7
 + 3*b^2*c^6*x^5 + 3*b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^9*x^8
+ 4*b^2*c^7*x^6 + 6*b^2*c^5*x^4 + 4*b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))
*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^9*x^8 + 4*a*b*c^7*x^6 + 6*a*b*c^5*x^
4 + 4*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(c^2\,x^2+1\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)),x)
```

```
[Out] int(x/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+b\,\mathrm{asinh}\left(cx\right)\right)^{2}\left(c^{2}x^{2}+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(5/2)), x)
```

3.452 $\int \frac{1}{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=62

$$-\frac{4c\text{Int}\left(\frac{x}{\left(c^2x^2+1\right)^3\left(a+b\sinh^{-1}(cx)\right)},x\right)}{b}-\frac{1}{bc\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}$$

```
[Out] -1/b/c/(c^2*x^2+1)^2/(a+b*arcsinh(c*x))-4*c*Unintegrable(x/(c^2*x^2+1)^3/(a
+b*arcsinh(c*x)),x)/b
```

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(1/(b*c*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))) - (4*c*Defer[Int][x/((1 + c
^2*x^2)^3*(a + b*ArcSinh[c*x])), x])/b
```

Rubi steps

$$\int \frac{1}{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=-\frac{1}{bc\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}-\frac{(4c)\int \frac{x}{\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)}\,dx}{b}$$

Mathematica [A] time = 2.86, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[1/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

fricas [A] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}}{a^2c^6x^6+3\,a^2c^4x^4+3\,a^2c^2x^2+\left(b^2c^6x^6+3\,b^2c^4x^4+3\,b^2c^2x^2+b^2\right)\text{arsinh}\,(cx)^2+a^2+2\left(abc^6x^6+3\,ab\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(a^2*c^6*x^6 + 3*a^2*c^4*x^4 + 3*a^2*c^2*x^2 + (
b^2*c^6*x^6 + 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2 + 2
*(a*b*c^6*x^6 + 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c^2x^2+1\right)^{\frac{5}{2}}(b\,\mathrm{arsinh}\,(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((c^2*x^2 + 1)^(5/2)*(b*arcsinh(c*x) + a)^2), x)
```

maple [A] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c^2x^2+1\right)^{\frac{5}{2}}(a+b\,\mathrm{arcsinh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(1/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{cx+\sqrt{c^2x^2+1}}{\left(abc^4x^3+abc^2x\right)\left(c^2x^2+1\right)+\left(\left(b^2c^4x^3+b^2c^2x\right)\left(c^2x^2+1\right)+\left(b^2c^5x^4+2\,b^2c^3x^2+b^2c\right)\sqrt{c^2x^2+1}\right)\log\left(cx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c*x + sqrt(c^2*x^2 + 1))/((a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((b^2
*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*x^4 + 2*b^2*c^3*x^2 + b^2*c)
*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^4 + 2*a*b*c^3
*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) - integrate((4*c^4*x^4 + 3*c^2*x^2 + (4*c^
2*x^2 + 1)*(c^2*x^2 + 1) + 4*(2*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1) - 1)/((a*b
*c^6*x^6 + 2*a*b*c^4*x^4 + a*b*c^2*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(a*b*c^7*x^
7 + 3*a*b*c^5*x^5 + 3*a*b*c^3*x^3 + a*b*c*x)*(c^2*x^2 + 1) + ((b^2*c^6*x^6
+ 2*b^2*c^4*x^4 + b^2*c^2*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(b^2*c^7*x^7 + 3*b^2
*c^5*x^5 + 3*b^2*c^3*x^3 + b^2*c*x)*(c^2*x^2 + 1) + (b^2*c^8*x^8 + 4*b^2*c^
6*x^6 + 6*b^2*c^4*x^4 + 4*b^2*c^2*x^2 + b^2)*sqrt(c^2*x^2 + 1))*log(c*x + s
qrt(c^2*x^2 + 1)) + (a*b*c^8*x^8 + 4*a*b*c^6*x^6 + 6*a*b*c^4*x^4 + 4*a*b*c^
2*x^2 + a*b)*sqrt(c^2*x^2 + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(a+b\,\mathrm{asinh}\,(c\,x))^2\,\left(c^2\,x^2+1\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)),x)
```

```
[Out] int(1/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\,\mathrm{asinh}\,(cx))^2\left(c^2x^2+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(1/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(5/2)), x)
```

3.453 $\int \frac{1}{x\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{1}{x\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][1/(x*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{x\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{1}{x\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 12.78, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[1/(x*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}}{a^2c^6x^7+3\,a^2c^4x^5+3\,a^2c^2x^3+a^2x+\left(b^2c^6x^7+3\,b^2c^4x^5+3\,b^2c^2x^3+b^2x\right)\text{arsinh}\,(cx)^2+2\left(abc^6x^7\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)/(a^2*c^6*x^7 + 3*a^2*c^4*x^5 + 3*a^2*c^2*x^3 + a
^2*x + (b^2*c^6*x^7 + 3*b^2*c^4*x^5 + 3*b^2*c^2*x^3 + b^2*x)*arcsinh(c*x)^2
 + 2*(a*b*c^6*x^7 + 3*a*b*c^4*x^5 + 3*a*b*c^2*x^3 + a*b*x)*arcsinh(c*x)), x
)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 1.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c^2x^2+1\right)^{\frac{5}{2}}\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(1/x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{cx+\sqrt{c^2x^2+1}}{\left(abc^4x^4+abc^2x^2\right)\left(c^2x^2+1\right)+\left(\left(b^2c^4x^4+b^2c^2x^2\right)\left(c^2x^2+1\right)+\left(b^2c^5x^5+2\,b^2c^3x^3+b^2cx\right)\sqrt{c^2x^2+1}\right)\log\left(cx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c*x + sqrt(c^2*x^2 + 1))/((a*b*c^4*x^4 + a*b*c^2*x^2)*(c^2*x^2 + 1) + ((b
^2*c^4*x^4 + b^2*c^2*x^2)*(c^2*x^2 + 1) + (b^2*c^5*x^5 + 2*b^2*c^3*x^3 + b^
2*c*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^5 + 2*a
*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)) - integrate((5*c^5*x^5 + 5*c^3*x^3
 + (5*c^3*x^3 + 2*c*x)*(c^2*x^2 + 1) + (10*c^4*x^4 + 7*c^2*x^2 + 1)*sqrt(c^
2*x^2 + 1))/((a*b*c^7*x^8 + 2*a*b*c^5*x^6 + a*b*c^3*x^4)*(c^2*x^2 + 1)^(3/2
) + 2*(a*b*c^8*x^9 + 3*a*b*c^6*x^7 + 3*a*b*c^4*x^5 + a*b*c^2*x^3)*(c^2*x^2
+ 1) + ((b^2*c^7*x^8 + 2*b^2*c^5*x^6 + b^2*c^3*x^4)*(c^2*x^2 + 1)^(3/2) + 2
*(b^2*c^8*x^9 + 3*b^2*c^6*x^7 + 3*b^2*c^4*x^5 + b^2*c^2*x^3)*(c^2*x^2 + 1)
+ (b^2*c^9*x^10 + 4*b^2*c^7*x^8 + 6*b^2*c^5*x^6 + 4*b^2*c^3*x^4 + b^2*c*x^2
)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^9*x^10 + 4*a*b*c
^7*x^8 + 6*a*b*c^5*x^6 + 4*a*b*c^3*x^4 + a*b*c*x^2)*sqrt(c^2*x^2 + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(c^2\,x^2+1\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)),x)
```

```
[Out] int(1/(x*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\,\mathrm{asinh}\left(cx\right)\right)^{2}\left(c^{2}x^{2}+1\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(1/(x*(a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(5/2)), x)
```

3.454 $\int \frac{1}{x^2(1+c^2x^2)^{5/2}(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{1}{x^2\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

[Out] Unintegrable(1/x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)

Rubi [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

[In] Int[1/(x^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]

[Out] Defer[Int][1/(x^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]

Rubi steps

$$\int \frac{1}{x^2\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{1}{x^2\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 12.09, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]

[Out] Integrate[1/(x^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]

fricas [A] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}}{a^2c^6x^8+3\,a^2c^4x^6+3\,a^2c^2x^4+a^2x^2+\left(b^2c^6x^8+3\,b^2c^4x^6+3\,b^2c^2x^4+b^2x^2\right)\text{arsinh}\left(cx\right)^2+2\left(abc^6x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")

[Out] integral(sqrt(c^2*x^2 + 1)/(a^2*c^6*x^8 + 3*a^2*c^4*x^6 + 3*a^2*c^2*x^4 + a^2*x^2 + (b^2*c^6*x^8 + 3*b^2*c^4*x^6 + 3*b^2*c^2*x^4 + b^2*x^2)*arcsinh(c*x)^2 + 2*(a*b*c^6*x^8 + 3*a*b*c^4*x^6 + 3*a*b*c^2*x^4 + a*b*x^2)*arcsinh(c*x)), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c^2x^2+1\right)^{\frac{5}{2}}(b\,\mathrm{arsinh}\,(cx)+a)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((c^2*x^2 + 1)^(5/2)*(b*arcsinh(c*x) + a)^2*x^2), x)
```

maple [A] time = 1.19, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(c^2x^2+1\right)^{\frac{5}{2}}(a+b\,\mathrm{arcsinh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(1/x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{cx+\sqrt{c^2x^2+1}}{\left(abc^4x^5+abc^2x^3\right)\left(c^2x^2+1\right)+\left(\left(b^2c^4x^5+b^2c^2x^3\right)\left(c^2x^2+1\right)+\left(b^2c^5x^6+2\,b^2c^3x^4+b^2cx^2\right)\sqrt{c^2x^2+1}\right)\log\left(cx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima
")
```

```
[Out] -(c*x + sqrt(c^2*x^2 + 1))/((a*b*c^4*x^5 + a*b*c^2*x^3)*(c^2*x^2 + 1) + ((b
^2*c^4*x^5 + b^2*c^2*x^3)*(c^2*x^2 + 1) + (b^2*c^5*x^6 + 2*b^2*c^3*x^4 + b^
2*c*x^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^6 + 2
*a*b*c^3*x^4 + a*b*c*x^2)*sqrt(c^2*x^2 + 1)) - integrate((6*c^5*x^5 + 7*c^3
*x^3 + 3*(2*c^3*x^3 + c*x)*(c^2*x^2 + 1) + c*x + 2*(6*c^4*x^4 + 5*c^2*x^2 +
 1)*sqrt(c^2*x^2 + 1))/((a*b*c^7*x^9 + 2*a*b*c^5*x^7 + a*b*c^3*x^5)*(c^2*x^
2 + 1)^(3/2) + 2*(a*b*c^8*x^10 + 3*a*b*c^6*x^8 + 3*a*b*c^4*x^6 + a*b*c^2*x^
4)*(c^2*x^2 + 1) + ((b^2*c^7*x^9 + 2*b^2*c^5*x^7 + b^2*c^3*x^5)*(c^2*x^2 +
1)^(3/2) + 2*(b^2*c^8*x^10 + 3*b^2*c^6*x^8 + 3*b^2*c^4*x^6 + b^2*c^2*x^4)*(
c^2*x^2 + 1) + (b^2*c^9*x^11 + 4*b^2*c^7*x^9 + 6*b^2*c^5*x^7 + 4*b^2*c^3*x^
5 + b^2*c*x^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^9*x
^11 + 4*a*b*c^7*x^9 + 6*a*b*c^5*x^7 + 4*a*b*c^3*x^5 + a*b*c*x^3)*sqrt(c^2*x
^2 + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x^2\,(a+b\,\mathrm{asinh}\,(c\,x))^2\,\left(c^2\,x^2+1\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)),x)
```

```
[Out] int(1/(x^2*(a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)), x)
```

sympy [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(a + b \operatorname{asinh}\left(cx\right)\right)^2 \left(c^2x^2 + 1\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(1/(x**2*(a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(5/2)), x)
```

3.455 $\int \frac{x^m\left(1+c^2x^2\right)^{5/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{\left(c^2x^2+1\right)^{5/2}x^m}{\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m\left(1+c^2x^2\right)^{5/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] Defer[Int][(x^m*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2, x]
```

Rubi steps

$$\int \frac{x^m\left(1+c^2x^2\right)^{5/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{x^m\left(1+c^2x^2\right)^{5/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 1.03, size = 0, normalized size = 0.00

$$\int \frac{x^m\left(1+c^2x^2\right)^{5/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] Integrate[(x^m*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2, x]
```

fricas [A] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^4x^4+2\,c^2x^2+1\right)\sqrt{c^2x^2+1}\,x^m}{b^2\operatorname{arsinh}\left(cx\right)^2+2\,ab\operatorname{arsinh}\left(cx\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)*x^m/(b^2*arcsinh(c*x)^
2 + 2*a*b*arcsinh(c*x) + a^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [**F(-2)**] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \left(c^2 x^2+1\right)^{\frac{5}{2}}}{\left(a+b \operatorname{arcsinh}\left(c x\right)\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

maxima [**A**] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(c^6x^6+3\,c^4x^4+3\,c^2x^2+1\right)\left(c^2x^2+1\right)x^m+\left(c^7x^7+3\,c^5x^5+3\,c^3x^3+cx\right)\sqrt{c^2x^2+1}\,x^m}{abc^3x^2+\sqrt{c^2x^2+1}\,abc^2x+abc+\left(b^2c^3x^2+\sqrt{c^2x^2+1}\,b^2c^2x+b^2c\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int \frac{\left(c^7(m+6)x^7\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^6*x^6 + 3*c^4*x^4 + 3*c^2*x^2 + 1)*(c^2*x^2 + 1)*x^m + (c^7*x^7 + 3*c^
5*x^5 + 3*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1)*x^m)/(a*b*c^3*x^2 + sqrt(c^2*x^2
 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*
c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((c^7*(m + 6)*x^7 + c^5*(3*m +
 11)*x^5 + c^3*(3*m + 4)*x^3 + c*(m - 1)*x)*(c^2*x^2 + 1)^(3/2)*x^m + (2*c^
8*(m + 6)*x^8 + c^6*(7*m + 30)*x^6 + 3*c^4*(3*m + 8)*x^4 + c^2*(5*m + 6)*x^
2 + m)*(c^2*x^2 + 1)*x^m + (c^9*(m + 6)*x^9 + c^7*(4*m + 19)*x^7 + 3*c^5*(2
*m + 7)*x^5 + c^3*(4*m + 9)*x^3 + c*(m + 1)*x)*sqrt(c^2*x^2 + 1)*x^m)/(a*b*
c^5*x^5 + (c^2*x^2 + 1)*a*b*c^3*x^3 + 2*a*b*c^3*x^3 + a*b*c*x + (b^2*c^5*x^
5 + (c^2*x^2 + 1)*b^2*c^3*x^3 + 2*b^2*c^3*x^3 + b^2*c*x + 2*(b^2*c^4*x^4 +
b^2*c^2*x^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x
^4 + a*b*c^2*x^2)*sqrt(c^2*x^2 + 1)), x)
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m\,{\left(c^2\,x^2+1\right)}^{5/2}}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x))^2,x)
```

```
[Out] int((x^m*(c^2*x^2 + 1)^(5/2))/(a + b*asinh(c*x))^2, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

3.456 $\int \frac{x^m(1+c^2x^2)^{3/2}}{(a+b\sinh^{-1}(cx))^2} dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{(c^2x^2+1)^{3/2} x^m}{(a+b\sinh^{-1}(cx))^2}, x\right)$$

```
[Out] Unintegrable(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m (1+c^2x^2)^{3/2}}{(a+b\sinh^{-1}(cx))^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] Defer[Int][(x^m*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2, x]
```

Rubi steps

$$\int \frac{x^m (1+c^2x^2)^{3/2}}{(a+b\sinh^{-1}(cx))^2} dx = \int \frac{x^m (1+c^2x^2)^{3/2}}{(a+b\sinh^{-1}(cx))^2} dx$$

Mathematica [A] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{x^m (1+c^2x^2)^{3/2}}{(a+b\sinh^{-1}(cx))^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] Integrate[(x^m*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2, x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(c^2x^2+1)^{\frac{3}{2}} x^m}{b^2 \operatorname{arsinh}(cx)^2 + 2\,ab \operatorname{arsinh}(cx) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^2*x^2 + 1)^(3/2)*x^m/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) +
 a^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \left(c^2 x^2+1\right)^{\frac{3}{2}}}{\left(a+b \operatorname{arcsinh}\left(c x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(c^4x^4+2\,c^2x^2+1\right)\left(c^2x^2+1\right)x^m+\left(c^5x^5+2\,c^3x^3+cx\right)\sqrt{c^2x^2+1}\,x^m}{abc^3x^2+\sqrt{c^2x^2+1}\,abc^2x+abc+\left(b^2c^3x^2+\sqrt{c^2x^2+1}\,b^2c^2x+b^2c\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int\frac{\left(c^5(m+4)x^5\right.}{abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^4*x^4 + 2*c^2*x^2 + 1)*(c^2*x^2 + 1)*x^m + (c^5*x^5 + 2*c^3*x^3 + c*x)
*sqrt(c^2*x^2 + 1)*x^m)/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c
+ (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^
2 + 1))) + integrate(((c^5*(m + 4)*x^5 + c^3*(2*m + 3)*x^3 + c*(m - 1)*x)*(
c^2*x^2 + 1)^(3/2)*x^m + (2*c^6*(m + 4)*x^6 + c^4*(5*m + 12)*x^4 + 4*c^2*(m
 + 1)*x^2 + m)*(c^2*x^2 + 1)*x^m + (c^7*(m + 4)*x^7 + 3*c^5*(m + 3)*x^5 + 3
*c^3*(m + 2)*x^3 + c*(m + 1)*x)*sqrt(c^2*x^2 + 1)*x^m)/(a*b*c^5*x^5 + (c^2*
x^2 + 1)*a*b*c^3*x^3 + 2*a*b*c^3*x^3 + a*b*c*x + (b^2*c^5*x^5 + (c^2*x^2 +
1)*b^2*c^3*x^3 + 2*b^2*c^3*x^3 + b^2*c*x + 2*(b^2*c^4*x^4 + b^2*c^2*x^2)*sq
rt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^4 + a*b*c^2*x^
2)*sqrt(c^2*x^2 + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m\,{\left(c^2\,x^2+1\right)}^{3/2}}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x))^2,x)
```

```
[Out] int((x^m*(c^2*x^2 + 1)^(3/2))/(a + b*asinh(c*x))^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m} \left(c^{2} x^{2} + 1\right)^{\frac{3}{2}}}{\left(a + b \operatorname{asinh}{\left(c x \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x**m*(c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x))**2, x)
```

3.457 $\int \frac{x^m\sqrt{1+c^2x^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{\sqrt{c^2x^2+1}\,x^m}{\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m\sqrt{1+c^2x^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2,x]
[Out] Defer[Int][(x^m*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2, x]
```

Rubi steps

$$\int \frac{x^m\sqrt{1+c^2x^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{x^m\sqrt{1+c^2x^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{x^m\sqrt{1+c^2x^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2,x]
[Out] Integrate[(x^m*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2, x]
```

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}\,x^m}{b^2\,\text{arsinh}\,(cx)^2+2\,ab\,\text{arsinh}\,(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)*x^m/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a
^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \sqrt{c^2x^2+1}}{(a+b\,\mathrm{arcsinh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(c^2x^2+1\right)^2 x^m + \left(c^3x^3+cx\right)\sqrt{c^2x^2+1}\,x^m}{abc^3x^2+\sqrt{c^2x^2+1}\,abc^2x+abc+\left(b^2c^3x^2+\sqrt{c^2x^2+1}\,b^2c^2x+b^2c\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int \frac{}{abc^5x^5+\left(c^2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^2 + 1)^2*x^m + (c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1)*x^m)/(a*b*c^3*x^2
 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b
^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((c^3*(m + 2)*x
^3 + c*(m - 1)*x)*(c^2*x^2 + 1)^(3/2)*x^m + (2*c^4*(m + 2)*x^4 + c^2*(3*m +
 2)*x^2 + m)*(c^2*x^2 + 1)*x^m + (c^5*(m + 2)*x^5 + c^3*(2*m + 3)*x^3 + c*(
m + 1)*x)*sqrt(c^2*x^2 + 1)*x^m)/(a*b*c^5*x^5 + (c^2*x^2 + 1)*a*b*c^3*x^3 +
 2*a*b*c^3*x^3 + a*b*c*x + (b^2*c^5*x^5 + (c^2*x^2 + 1)*b^2*c^3*x^3 + 2*b^2
*c^3*x^3 + b^2*c*x + 2*(b^2*c^4*x^4 + b^2*c^2*x^2)*sqrt(c^2*x^2 + 1))*log(c
*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^4 + a*b*c^2*x^2)*sqrt(c^2*x^2 + 1)),
 x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m\,\sqrt{c^2\,x^2+1}}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x))^2,x)
```

```
[Out] int((x^m*(c^2*x^2 + 1)^(1/2))/(a + b*asinh(c*x))^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \sqrt{c^2x^2+1}}{(a+b\,\mathrm{asinh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x**m*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x))**2, x)
```

3.458 $\int \frac{x^m}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=49

$$\frac{m\text{Int}\left(\frac{x^{m-1}}{a+b\sinh^{-1}(cx)},x\right)}{bc}-\frac{x^m}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

```
[Out] -x^m/b/c/(a+b*arcsinh(c*x))+m*Unintegrable(x^(-1+m)/(a+b*arcsinh(c*x)),x)/b
/c
```

Rubi [A] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(x^m/(b*c*(a + b*ArcSinh[c*x]))) + (m*Defer[Int][x^(-1 + m)/(a + b*ArcSinh
[c*x]), x])/(b*c)
```

Rubi steps

$$\int \frac{x^m}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=-\frac{x^m}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{m\int \frac{x^{-1+m}}{a+b\sinh^{-1}(cx)}\,dx}{bc}$$

Mathematica [A] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[x^m/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2), x]
```

fricas [A] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}\,x^m}{a^2c^2x^2+\left(b^2c^2x^2+b^2\right)\operatorname{arsinh}(cx)^2+a^2+2\left(abc^2x^2+ab\right)\operatorname{arsinh}(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^m/(a^2*c^2*x^2 + (b^2*c^2*x^2 + b^2)*arcsinh(c
*x)^2 + a^2 + 2*(a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{c^2x^2+1}\,(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(x^m/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)^2), x)
```

maple [A] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{x^{m}}{\sqrt{c^{2}x^{2}+1}\,\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}x^{m}+\left(c^{3}x^{3}+cx\right)x^{m}}{\left(c^{2}x^{2}+1\right)abc^{2}x+\left(\left(c^{2}x^{2}+1\right)b^{2}c^{2}x+\left(b^{2}c^{3}x^{2}+b^{2}c\right)\sqrt{c^{2}x^{2}+1}\right)\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)+\left(abc^{3}x^{2}+abc\right)\sqrt{c^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^2 + 1)^(3/2)*x^m + (c^3*x^3 + c*x)*x^m)/((c^2*x^2 + 1)*a*b*c^2*x +
 ((c^2*x^2 + 1)*b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*
x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + integra
te(((c^3*m*x^3 + c*(m - 1)*x)*(c^2*x^2 + 1)*x^m + (2*c^4*m*x^4 + 3*c^2*m*x^
2 + m)*sqrt(c^2*x^2 + 1)*x^m + (c^5*m*x^5 + c^3*(2*m + 1)*x^3 + c*(m + 1)*x
)*x^m)/((c^2*x^2 + 1)^(3/2)*a*b*c^3*x^3 + 2*(a*b*c^4*x^4 + a*b*c^2*x^2)*(c^
2*x^2 + 1) + ((c^2*x^2 + 1)^(3/2)*b^2*c^3*x^3 + 2*(b^2*c^4*x^4 + b^2*c^2*x^
2)*(c^2*x^2 + 1) + (b^2*c^5*x^5 + 2*b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2*x^2 + 1
))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^5 + 2*a*b*c^3*x^3 + a*b*c*x)*s
qrt(c^2*x^2 + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{m}}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{2}\,\sqrt{c^{2}\,x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(x^m/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m}}{\left(a+b\,\mathrm{asinh}\left(cx\right)\right)^{2}\sqrt{c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x**m/((a + b*asinh(c*x))**2*sqrt(c**2*x**2 + 1)), x)
```

3.459 $\int \frac{x^m}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{x^m}{\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][x^m/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{x^m}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{x^m}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[x^m/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}\,x^m}{a^2c^4x^4+2\,a^2c^2x^2+\left(b^2c^4x^4+2\,b^2c^2x^2+b^2\right)\text{arsinh}\,(cx)^2+a^2+2\left(abc^4x^4+2\,abc^2x^2+ab\right)\text{arsinh}\,(cx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)*x^m/(a^2*c^4*x^4 + 2*a^2*c^2*x^2 + (b^2*c^4*x^4
+ 2*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 + 2*a*b*c^2*x^
2 + a*b)*arcsinh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(c^2x^2+1\right)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(x^m/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)^2), x)
```

maple [A] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(c^2x^2+1\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{cxx^m+\sqrt{c^2x^2+1}\,x^m}{\left(c^2x^2+1\right)abc^2x+\left(\left(c^2x^2+1\right)b^2c^2x+\left(b^2c^3x^2+b^2c\right)\sqrt{c^2x^2+1}\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(abc^3x^2+abc\right)\sqrt{c^2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c*x*x^m + sqrt(c^2*x^2 + 1)*x^m)/((c^2*x^2 + 1)*a*b*c^2*x + ((c^2*x^2 + 1
)*b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x
^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + integrate(((c^3*(m -
2)*x^3 + c*(m - 1)*x)*(c^2*x^2 + 1)*x^m + (2*c^4*(m - 2)*x^4 + c^2*(3*m - 2
)*x^2 + m)*sqrt(c^2*x^2 + 1)*x^m + (c^5*(m - 2)*x^5 + c^3*(2*m - 1)*x^3 + c
*(m + 1)*x)*x^m)/((a*b*c^5*x^5 + a*b*c^3*x^3)*(c^2*x^2 + 1)^(3/2) + 2*(a*b*
c^6*x^6 + 2*a*b*c^4*x^4 + a*b*c^2*x^2)*(c^2*x^2 + 1) + ((b^2*c^5*x^5 + b^2*
c^3*x^3)*(c^2*x^2 + 1)^(3/2) + 2*(b^2*c^6*x^6 + 2*b^2*c^4*x^4 + b^2*c^2*x^2
)*(c^2*x^2 + 1) + (b^2*c^7*x^7 + 3*b^2*c^5*x^5 + 3*b^2*c^3*x^3 + b^2*c*x)*s
qrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^7*x^7 + 3*a*b*c^5*x
^5 + 3*a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(c^2\,x^2+1\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)),x)
```

```
[Out] int(x^m/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(a+b\,\mathrm{asinh}\left(cx\right)\right)^2\left(c^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x**m/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(3/2)), x)
```

3.460 $\int \frac{x^m}{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{x^m}{\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(x^m/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m}{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][x^m/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{x^m}{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{x^m}{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 0.90, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[x^m/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

fricas [A] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}\,x^m}{a^2c^6x^6+3\,a^2c^4x^4+3\,a^2c^2x^2+\left(b^2c^6x^6+3\,b^2c^4x^4+3\,b^2c^2x^2+b^2\right)\text{arsinh}\left(cx\right)^2+a^2+2\left(abc^6x^6+3\,ab\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)*x^m/(a^2*c^6*x^6 + 3*a^2*c^4*x^4 + 3*a^2*c^2*x^2
 + (b^2*c^6*x^6 + 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2
 + 2*(a*b*c^6*x^6 + 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(c^2x^2+1\right)^{\frac{5}{2}}\left(b\,\text{arsinh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(x^m/((c^2*x^2 + 1)^(5/2)*(b*arcsinh(c*x) + a)^2), x)
```

maple [A] time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(c^2x^2+1\right)^{\frac{5}{2}} \left(a+b\operatorname{arcsinh}(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^m/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{cxx^m+\sqrt{c^2x^2+1}\,x^m}{\left(abc^4x^3+abc^2x\right)\left(c^2x^2+1\right)+\left(\left(b^2c^4x^3+b^2c^2x\right)\left(c^2x^2+1\right)+\left(b^2c^5x^4+2\,b^2c^3x^2+b^2c\right)\sqrt{c^2x^2+1}\right)\log\left(cx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c*x*x^m + sqrt(c^2*x^2 + 1)*x^m)/((a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1)
 + ((b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*x^4 + 2*b^2*c^3*x^2
+ b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^4 + 2
*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + integrate(((c^3*(m - 4)*x^3 + c*
(m - 1)*x)*(c^2*x^2 + 1)*x^m + (2*c^4*(m - 4)*x^4 + c^2*(3*m - 4)*x^2 + m)*
sqrt(c^2*x^2 + 1)*x^m + (c^5*(m - 4)*x^5 + c^3*(2*m - 3)*x^3 + c*(m + 1)*x)
*x^m)/((a*b*c^7*x^7 + 2*a*b*c^5*x^5 + a*b*c^3*x^3)*(c^2*x^2 + 1)^(3/2) + 2*
(a*b*c^8*x^8 + 3*a*b*c^6*x^6 + 3*a*b*c^4*x^4 + a*b*c^2*x^2)*(c^2*x^2 + 1) +
 ((b^2*c^7*x^7 + 2*b^2*c^5*x^5 + b^2*c^3*x^3)*(c^2*x^2 + 1)^(3/2) + 2*(b^2*
c^8*x^8 + 3*b^2*c^6*x^6 + 3*b^2*c^4*x^4 + b^2*c^2*x^2)*(c^2*x^2 + 1) + (b^2
*c^9*x^9 + 4*b^2*c^7*x^7 + 6*b^2*c^5*x^5 + 4*b^2*c^3*x^3 + b^2*c*x)*sqrt(c^
2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^9*x^9 + 4*a*b*c^7*x^7 + 6
*a*b*c^5*x^5 + 4*a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(c^2\,x^2+1\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)),x)
```

```
[Out] int(x^m/((a + b*asinh(c*x))^2*(c^2*x^2 + 1)^(5/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(a+b\operatorname{asinh}(cx)\right)^2 \left(c^2x^2+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x**m/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(5/2)), x)
```

3.461 $\int \frac{1}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{2a\sinh^{-1}(ax)^2}$$

```
[Out] -1/2/a/arcsinh(a*x)^2
```

Rubi [A] time = 0.04, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {5675}

$$-\frac{1}{2a\sinh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3),x]
```

```
[Out] -1/(2*a*ArcSinh[a*x]^2)
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}\,dx=-\frac{1}{2a\sinh^{-1}(ax)^2}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$-\frac{1}{2a\sinh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3),x]
```

```
[Out] -1/2*1/(a*ArcSinh[a*x]^2)
```

fricas [B] time = 0.42, size = 23, normalized size = 1.77

$$-\frac{1}{2\,a\log\left(ax+\sqrt{a^2x^2+1}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2/(a*log(a*x + sqrt(a^2*x^2 + 1))^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2x^2+1}\,\operatorname{arsinh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^3), x)
```

maple [A] time = 0.01, size = 12, normalized size = 0.92

$$-\frac{1}{2a \operatorname{arcsinh}(ax)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/2/a/arcsinh(a*x)^2
```

maxima [A] time = 0.64, size = 11, normalized size = 0.85

$$-\frac{1}{2 \, a \operatorname{arsinh}(ax)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2/(a*arcsinh(a*x)^2)
```

mupad [B] time = 0.09, size = 11, normalized size = 0.85

$$-\frac{1}{2 \, a \operatorname{asinh}(a \, x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(asinh(a*x)^3*(a^2*x^2 + 1)^(1/2)),x)
```

```
[Out] -1/(2*a*asinh(a*x)^2)
```

sympy [A] time = 0.99, size = 12, normalized size = 0.92

$$-\frac{1}{2a \operatorname{asinh}^2(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(a*x)**3/(a**2*x**2+1)**(1/2),x)
```

```
[Out] -1/(2*a*asinh(a*x)**2)
```

3.462 $\int \frac{x^3(d+c^2dx^2)}{(a+b\sinh^{-1}(cx))^{3/2}}\,dx$

Optimal. Leaf size=254

$$-\frac{3\sqrt{\frac{\pi}{2}}\,de^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}+\frac{\sqrt{\frac{3\pi}{2}}\,de^{\frac{6a}{b}}\operatorname{erf}\left(\frac{\sqrt{6}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}-\frac{3\sqrt{\frac{\pi}{2}}\,de^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}+\frac{\sqrt{\frac{3\pi}{2}}\,de^{-}}{}$$

```
[Out] -3/32*d*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi
^(1/2)/b^(3/2)/c^4-3/32*d*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^
(1/2)*Pi^(1/2)/b^(3/2)/c^4/exp(2*a/b)+1/32*d*exp(6*a/b)*erf(6^(1/2)*(a+b*ar
csinh(c*x))^(1/2)/b^(1/2))*6^(1/2)*Pi^(1/2)/b^(3/2)/c^4+1/32*d*erfi(6^(1/2)
*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*6^(1/2)*Pi^(1/2)/b^(3/2)/c^4/exp(6*a/b)-
2*d*x^3*(c^2*x^2+1)^(3/2)/b/c/(a+b*arcsinh(c*x))^(1/2)
```

Rubi [A] time = 1.31, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5777, 5779, 5448, 3307, 2180, 2204, 2205}

$$-\frac{3\sqrt{\frac{\pi}{2}}\,de^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}+\frac{\sqrt{\frac{3\pi}{2}}\,de^{\frac{6a}{b}}\operatorname{Erf}\left(\frac{\sqrt{6}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}-\frac{3\sqrt{\frac{\pi}{2}}\,de^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}+\frac{\sqrt{\frac{3\pi}{2}}\,de}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + c^2*d*x^2))/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-2*d*x^3*(1 + c^2*x^2)^(3/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (3*d*E^((2*
a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(16*b^(3/
2)*c^4) + (d*E^((6*a)/b)*Sqrt[(3*Pi)/2]*Erf[(Sqrt[6]*Sqrt[a + b*ArcSinh[c*x
]])/Sqrt[b]])/(16*b^(3/2)*c^4) - (3*d*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*A
rcSinh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^4*E^((2*a)/b)) + (d*Sqrt[(3*Pi)/2]*Er
fi[(Sqrt[6]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^4*E^((6*a)/b)
)
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
```

```
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^
p*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-Dist[(f*m*d^IntPart[p
]*(d + e*x^2)^FracPart[p])/(b*c*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*
x)^(m - 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - D
ist[(c*(m + 2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(b*f*(n + 1)*(1
+ c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*A
rcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[2*p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^3\left(d+c^2dx^2\right)}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2dx^3\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{(6d)\int\frac{x^2\sqrt{1+c^2x^2}}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{bc}+\frac{(12cd)\int\frac{x^4\sqrt{1+c^2x^2}}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{b}$$

$$= -\frac{2dx^3\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{(6d)\,\mathrm{Subst}\left(\int\frac{\cosh^2(x)\sinh^2(x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{bc^4}+\frac{(12d)\,\mathrm{Su}}{}$$

$$= -\frac{2dx^3\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{(6d)\,\mathrm{Subst}\left(\int\left(-\frac{1}{8\sqrt{a+bx}}+\frac{\cosh(4x)}{8\sqrt{a+bx}}\right)dx,x,\sinh^{-1}(cx)\right)}{bc^4}+$$

$$= -\frac{2dx^3\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}-\frac{(3d)\,\mathrm{Subst}\left(\int\frac{\cosh(2x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{8bc^4}+\frac{(3d)\,\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{2dx^3\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{(3d)\,\mathrm{Subst}\left(\int\frac{e^{-6x}}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{16bc^4}-\frac{(3d)\,\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{2dx^3\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{(3d)\,\mathrm{Subst}\left(\int e^{\frac{6a}{b}-\frac{6x^2}{b}}\,dx,x,\sqrt{a+b\sinh^{-1}(cx)}\right)}{8b^2c^4}-\frac{(3d)\,\mathrm{Su}}{}$$

$$= -\frac{2dx^3\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}-\frac{3de^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}+\frac{de^{\frac{6a}{b}}\sqrt{\frac{3\pi}{2}}\,\mathrm{erf}\left(\frac{\sqrt{6}\sqrt{a+b}}{}\right.}{16b^{3/2}c^4}$$

Mathematica [A] time = 0.92, size = 232, normalized size = 0.91

$$\frac{de^{-\frac{6a}{b}}\left(-8e^{\frac{6a}{b}}\sinh^3\left(2\sinh^{-1}(cx)\right)+\sqrt{6}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{6\left(a+b\sinh^{-1}(cx)\right)}{b}\right)-3\sqrt{2}\,e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\right.\right.}{32bc^4\sqrt{a+b}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(d + c^2*d*x^2))/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (d*(Sqrt[6]*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-6*(a + b*ArcSinh[c
*x]))/b] - 3*Sqrt[2]*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2,
 (-2*(a + b*ArcSinh[c*x]))/b] + 3*Sqrt[2]*E^((8*a)/b)*Sqrt[a/b + ArcSinh[c*
x]]*Gamma[1/2, (2*(a + b*ArcSinh[c*x]))/b] - Sqrt[6]*E^((12*a)/b)*Sqrt[a/b
+ ArcSinh[c*x]]*Gamma[1/2, (6*(a + b*ArcSinh[c*x]))/b] - 8*E^((6*a)/b)*Sinh
[2*ArcSinh[c*x]]^3))/(32*b*c^4*E^((6*a)/b)*Sqrt[a + b*ArcSinh[c*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2dx^2 + d\right)x^3}{\left(b \operatorname{arsinh}\left(cx\right) + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
[Out] integrate((c^2*d*x^2 + d)*x^3/(b*arcsinh(c*x) + a)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(c^2 d\, x^2 + d\right)}{\left(a + b \operatorname{arcsinh}\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
[Out] int(x^3*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2dx^2 + d\right)x^3}{\left(b \operatorname{arsinh}\left(cx\right) + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate((c^2*d*x^2 + d)*x^3/(b*arcsinh(c*x) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\left(d\,c^2\,x^2+d\right)}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + c^2*d*x^2))/(a + b*asinh(c*x))^(3/2),x)
[Out] int((x^3*(d + c^2*d*x^2))/(a + b*asinh(c*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d \left(\int \frac{x^{3}}{a \sqrt{a + b \operatorname{asinh}{\left(c x \right)}} + b \sqrt{a + b \operatorname{asinh}{\left(c x \right)}} \operatorname{asinh}{\left(c x \right)}}\, dx + \int \frac{c^{2} x^{5}}{a \sqrt{a + b \operatorname{asinh}{\left(c x \right)}} + b \sqrt{a + b \operatorname{asinh}{\left(c x \right)}} \operatorname{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*d*x**2+d)/(a+b*asinh(c*x))**(3/2),x)
[Out] d*(Integral(x**3/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c*x))*asinh
(c*x)), x) + Integral(c**2*x**5/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*as
inh(c*x))*asinh(c*x)), x))
```

3.463 $\int \frac{x^2(d+c^2dx^2)}{(a+b\sinh^{-1}(cx))^{3/2}}\,dx$

Optimal. Leaf size=335

$$\frac{\sqrt{\pi}\,de^{a/b}\mathrm{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c^3}-\frac{\sqrt{3\pi}\,de^{\frac{3a}{b}}\mathrm{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3}-\frac{\sqrt{5\pi}\,de^{\frac{5a}{b}}\mathrm{erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3}-\frac{\sqrt{\pi}\,de^{-\frac{a}{b}}\mathrm{erfi}\left(\frac{\sqrt{}}{}\right.}{8b^{3/2}}$$

```
[Out] 1/8*d*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^3-1
/8*d*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^3/exp(a/b)-1
/16*d*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(
1/2)/b^(3/2)/c^3+1/16*d*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1
/2)*Pi^(1/2)/b^(3/2)/c^3/exp(3*a/b)-1/16*d*exp(5*a/b)*erf(5^(1/2)*(a+b*arcs
inh(c*x))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/c^3+1/16*d*erfi(5^(1/2)*(
a+b*arcsinh(c*x))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/c^3/exp(5*a/b)-2*
d*x^2*(c^2*x^2+1)^(3/2)/b/c/(a+b*arcsinh(c*x))^(1/2)
```

Rubi [A] time = 1.38, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5777, 5779, 5448, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\,de^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c^3}-\frac{\sqrt{3\pi}\,de^{\frac{3a}{b}}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3}-\frac{\sqrt{5\pi}\,de^{\frac{5a}{b}}\mathrm{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3}-\frac{\sqrt{\pi}\,de^{-\frac{a}{b}}\mathrm{Erfi}\left(\right.}{8b^{3/}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + c^2*d*x^2))/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-2*d*x^2*(1 + c^2*x^2)^(3/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (d*E^(a/b)*
Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*b^(3/2)*c^3) - (d*E^((3*
a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(16*b^(3/
2)*c^3) - (d*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x]])/
Sqrt[b]])/(16*b^(3/2)*c^3) - (d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt
[b]])/(8*b^(3/2)*c^3*E^(a/b)) + (d*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcS
inh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^3*E^((3*a)/b)) + (d*Sqrt[5*Pi]*Erfi[(Sqr
t[5]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^3*E^((5*a)/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^
p*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-Dist[(f*m*d^IntPart[p
]*(d + e*x^2)^FracPart[p])/(b*c*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*
x)^(m - 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - D
ist[(c*(m + 2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(b*f*(n + 1)*(1
+ c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*A
rcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[2*p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2 \left(d + c^2 d x^2\right)}{\left(a + b \sinh^{-1}(cx)\right)^{3/2}} \, dx = -\frac{2dx^2 \left(1 + c^2x^2\right)^{3/2}}{bc\sqrt{a + b\sinh^{-1}(cx)}} + \frac{(4d) \int \frac{x\sqrt{1+c^2x^2}}{\sqrt{a+b\sinh^{-1}(cx)}} \, dx}{bc} + \frac{(10cd) \int \frac{x^3\sqrt{1+c^2x^2}}{\sqrt{a+b\sinh^{-1}(cx)}} \, dx}{b}$$

$$= -\frac{2dx^2 \left(1 + c^2x^2\right)^{3/2}}{bc\sqrt{a + b\sinh^{-1}(cx)}} + \frac{(4d)\operatorname{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{\sqrt{a+bx}} \, dx, x, \sinh^{-1}(cx)\right)}{bc^3} + \frac{(10d)\operatorname{Su}}{}$$

$$= -\frac{2dx^2 \left(1 + c^2x^2\right)^{3/2}}{bc\sqrt{a + b\sinh^{-1}(cx)}} + \frac{(4d)\operatorname{Subst}\left(\int \left(\frac{\sinh(x)}{4\sqrt{a+bx}} + \frac{\sinh(3x)}{4\sqrt{a+bx}}\right) dx, x, \sinh^{-1}(cx)\right)}{bc^3} + \frac{(1}{}$$

$$= -\frac{2dx^2 \left(1 + c^2x^2\right)^{3/2}}{bc\sqrt{a + b\sinh^{-1}(cx)}} - \frac{(5d)\operatorname{Subst}\left(\int \frac{\sinh(3x)}{\sqrt{a+bx}} \, dx, x, \sinh^{-1}(cx)\right)}{8bc^3} + \frac{(5d)\operatorname{Subst}\left(\int\right.}{}$$

$$= -\frac{2dx^2 \left(1 + c^2x^2\right)^{3/2}}{bc\sqrt{a + b\sinh^{-1}(cx)}} - \frac{(5d)\operatorname{Subst}\left(\int \frac{e^{-5x}}{\sqrt{a+bx}} \, dx, x, \sinh^{-1}(cx)\right)}{16bc^3} + \frac{(5d)\operatorname{Subst}\left(\int\right.}{}$$

$$= -\frac{2dx^2 \left(1 + c^2x^2\right)^{3/2}}{bc\sqrt{a + b\sinh^{-1}(cx)}} - \frac{(5d)\operatorname{Subst}\left(\int e^{\frac{5a}{b} - \frac{5x^2}{b}} \, dx, x, \sqrt{a + b\sinh^{-1}(cx)}\right)}{8b^2c^3} + \frac{(5d)\operatorname{S}}{}$$

$$= -\frac{2dx^2 \left(1 + c^2x^2\right)^{3/2}}{bc\sqrt{a + b\sinh^{-1}(cx)}} + \frac{de^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c^3} - \frac{de^{\frac{3a}{b}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}}}{\sqrt{b}}\right.}{16b^{3/2}c^3}$$

Mathematica [A] time = 0.75, size = 436, normalized size = 1.30

$$\frac{de^{-5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}\left(-e^{\frac{5a}{b}+2\sinh^{-1}(cx)} + 2e^{\frac{5a}{b}+4\sinh^{-1}(cx)} + 2e^{\frac{5a}{b}+6\sinh^{-1}(cx)} - e^{\frac{5a}{b}+8\sinh^{-1}(cx)} - e^{\frac{5a}{b}+10\sinh^{-1}(cx)} - 2e^{\frac{6a}{b}+5\sinh}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(d + c^2*d*x^2))/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (d*(-E^((5*a)/b) - E^((5*a)/b + 2*ArcSinh[c*x]) + 2*E^((5*a)/b + 4*ArcSinh[
c*x]) + 2*E^((5*a)/b + 6*ArcSinh[c*x]) - E^((5*a)/b + 8*ArcSinh[c*x]) - E^(
(5*a)/b + 10*ArcSinh[c*x]) - 2*E^((6*a)/b + 5*ArcSinh[c*x])*Sqrt[a/b + ArcS
inh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] + Sqrt[5]*E^(5*ArcSinh[c*x])*Sqrt[
-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-5*(a + b*ArcSinh[c*x]))/b] + Sqrt[3
]*E^((2*a)/b + 5*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (
-3*(a + b*ArcSinh[c*x]))/b] - 2*E^((4*a)/b + 5*ArcSinh[c*x])*Sqrt[-((a + b*
ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)] + Sqrt[3]*E^((8*a)/
b + 5*ArcSinh[c*x])*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c
*x]))/b] + Sqrt[5]*E^(5*((2*a)/b + ArcSinh[c*x]))*Sqrt[a/b + ArcSinh[c*x]]*
Gamma[1/2, (5*(a + b*ArcSinh[c*x]))/b]))/(16*b*c^3*E^(5*(a/b + ArcSinh[c*x]
))*Sqrt[a + b*ArcSinh[c*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2dx^2+d\right)x^2}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)*x^2/(b*arcsinh(c*x) + a)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(c^2d\,x^2+d\right)}{\left(a+b\operatorname{arcsinh}\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(x^2*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2dx^2+d\right)x^2}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 + d)*x^2/(b*arcsinh(c*x) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(d\,c^2\,x^2+d\right)}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + c^2*d*x^2))/(a + b*asinh(c*x))^(3/2),x)
```

```
[Out] int((x^2*(d + c^2*d*x^2))/(a + b*asinh(c*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int \frac{x^{2}}{a\sqrt{a+b\operatorname{asinh}{\left(cx \right)}}+b\sqrt{a+b\operatorname{asinh}{\left(cx \right)}}\operatorname{asinh}{\left(cx \right)}}\, dx + \int \frac{c^{2}x^{4}}{a\sqrt{a+b\operatorname{asinh}{\left(cx \right)}}+b\sqrt{a+b\operatorname{asinh}{\left(cx \right)}}\operatorname{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] d*(Integral(x**2/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c*x))*asinh
(c*x)), x) + Integral(c**2*x**4/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*as
inh(c*x))*asinh(c*x)), x))
```

3.464 $\int \frac{x(d+c^2dx^2)}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=236

$$\frac{\sqrt{\pi}\,de^{\frac{4a}{b}}\operatorname{erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}+\frac{\sqrt{\frac{\pi}{2}}\,de^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2b^{3/2}c^2}+\frac{\sqrt{\pi}\,de^{-\frac{4a}{b}}\operatorname{erfi}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}+\frac{\sqrt{\frac{\pi}{2}}\,de^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\ldots}{\ldots}\right)}{2b^{3/2}\ldots}$$

```
[Out] 1/4*d*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^(
1/2)/b^(3/2)/c^2+1/4*d*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/
2)*Pi^(1/2)/b^(3/2)/c^2/exp(2*a/b)+1/4*d*exp(4*a/b)*erf(2*(a+b*arcsinh(c*x)
)^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^2+1/4*d*erfi(2*(a+b*arcsinh(c*x))^(1/2)
/b^(1/2))*Pi^(1/2)/b^(3/2)/c^2/exp(4*a/b)-2*d*x*(c^2*x^2+1)^(3/2)/b/c/(a+b*
arcsinh(c*x))^(1/2)
```

Rubi [A] time = 0.77, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5777, 5699, 3312, 3307, 2180, 2204, 2205, 5779, 5448}

$$\frac{\sqrt{\pi}\,de^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}+\frac{\sqrt{\frac{\pi}{2}}\,de^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2b^{3/2}c^2}+\frac{\sqrt{\pi}\,de^{-\frac{4a}{b}}\operatorname{Erfi}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}+\frac{\sqrt{\frac{\pi}{2}}\,de^{-\frac{2a}{b}}\operatorname{Erfi}\left(\ldots\right)}{2b^{3}\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + c^2*d*x^2))/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-2*d*x*(1 + c^2*x^2)^(3/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (d*E^((4*a)/b
)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c^2) + (d*
E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(2*
b^(3/2)*c^2) + (d*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b
^(3/2)*c^2*E^((4*a)/b)) + (d*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*
x]])/Sqrt[b]])/(2*b^(3/2)*c^2*E^((2*a)/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
```

```
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^
p*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-Dist[(f*m*d^IntPart[p
]*(d + e*x^2)^FracPart[p])/(b*c*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*
x)^(m - 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - D
ist[(c*(m + 2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(b*f*(n + 1)*(1
+ c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*A
rcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[2*p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x\left(d+c^2dx^2\right)}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2dx\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(2d)\int\frac{\sqrt{1+c^2x^2}}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{bc} + \frac{(8cd)\int\frac{x^2\sqrt{1+c^2x^2}}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{b}$$

$$= -\frac{2dx\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(2d)\operatorname{Subst}\left(\int\frac{\cosh^2(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{bc^2} + \frac{(8d)\operatorname{Subst}\left(\int\right.}{}$$

$$= -\frac{2dx\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(2d)\operatorname{Subst}\left(\int\left(\frac{1}{2\sqrt{a+bx}}+\frac{\cosh(2x)}{2\sqrt{a+bx}}\right)dx, x, \sinh^{-1}(cx)\right)}{bc^2} + \frac{(8}{}$$

$$= -\frac{2dx\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d\operatorname{Subst}\left(\int\frac{\cosh(2x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{bc^2} + \frac{d\operatorname{Subst}\left(\int\frac{\cosh(4}{\sqrt{a+b}}\right.}{}$$

$$= -\frac{2dx\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d\operatorname{Subst}\left(\int\frac{e^{-4x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{2bc^2} + \frac{d\operatorname{Subst}\left(\int\frac{e^{-2x}}{\sqrt{a+bx}}\right.}{2b}$$

$$= -\frac{2dx\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d\operatorname{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{b^2c^2} + \frac{d\operatorname{Subst}\left(\right.}{}$$

$$= -\frac{2dx\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{de^{\frac{4a}{b}}\sqrt{\pi}\operatorname{erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2} + \frac{de^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-}}}{\sqrt{b}}\right.}{2b^{3/2}c^2}$$

Mathematica [A] time = 0.46, size = 227, normalized size = 0.96

$$\frac{de^{-\frac{4a}{b}}\left(\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2}, -\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)+\sqrt{2}\,e^{\frac{2a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2}, -\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)-e^{\frac{4a}{b}}\left(\sqrt{2}\,e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\right.\right.}{4bc^2\sqrt{}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(d + c^2*d*x^2))/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (d*(Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-4*(a + b*ArcSinh[c*x]))/b]
 + Sqrt[2]*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-2*(a +
b*ArcSinh[c*x]))/b] - E^((4*a)/b)*(Sqrt[2]*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c
*x]]*Gamma[1/2, (2*(a + b*ArcSinh[c*x]))/b] + E^((4*a)/b)*Sqrt[a/b + ArcSin
h[c*x]]*Gamma[1/2, (4*(a + b*ArcSinh[c*x]))/b] + 2*Sinh[2*ArcSinh[c*x]] + S
inh[4*ArcSinh[c*x]])))/(4*b*c^2*E^((4*a)/b)*Sqrt[a + b*ArcSinh[c*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c^2dx^2+d)x}{(b\,\mathrm{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
[Out] integrate((c^2*d*x^2 + d)*x/(b*arcsinh(c*x) + a)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(c^2d\,x^2+d\right)}{(a+b\,\mathrm{arcsinh}\,(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
[Out] int(x*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c^2dx^2+d)x}{(b\,\mathrm{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate((c^2*d*x^2 + d)*x/(b*arcsinh(c*x) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\left(d\,c^2\,x^2+d\right)}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + c^2*d*x^2))/(a + b*asinh(c*x))^(3/2),x)
[Out] int((x*(d + c^2*d*x^2))/(a + b*asinh(c*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int \frac{x}{a\sqrt{a+b\,\mathrm{asinh}\,(cx)}+b\sqrt{a+b\,\mathrm{asinh}\,(cx)}\,\mathrm{asinh}\,(cx)}\,dx+\int \frac{c^2x^3}{a\sqrt{a+b\,\mathrm{asinh}\,(cx)}+b\sqrt{a+b\,\mathrm{asinh}\,(cx)}\,\mathrm{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*d*x**2+d)/(a+b*asinh(c*x))**(3/2),x)
[Out] d*(Integral(x/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c*x))*asinh(c*
x)), x) + Integral(c**2*x**3/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh
(c*x))*asinh(c*x)), x))
```

3.465 $\int \frac{d+c^2dx^2}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=228

$$-\frac{3\sqrt{\pi}\,de^{a/b}\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}-\frac{\sqrt{3\pi}\,de^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}+\frac{3\sqrt{\pi}\,de^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}+\frac{\sqrt{3\pi}\,de^{-\frac{3a}{b}}\text{erfi}}{4}$$

```
[Out] -3/4*d*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c+3/
4*d*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c/exp(a/b)-1/4*
d*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)
/b^(3/2)/c+1/4*d*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^
(1/2)/b^(3/2)/c/exp(3*a/b)-2*d*(c^2*x^2+1)^(3/2)/b/c/(a+b*arcsinh(c*x))^(1/
2)
```

Rubi [A] time = 0.53, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5696, 5779, 5448, 3308, 2180, 2204, 2205}

$$-\frac{3\sqrt{\pi}\,de^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}-\frac{\sqrt{3\pi}\,de^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}+\frac{3\sqrt{\pi}\,de^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}+\frac{\sqrt{3\pi}\,de^{-\frac{3a}{b}}\text{Er}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c^2*d*x^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-2*d*(1 + c^2*x^2)^(3/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (3*d*E^(a/b)*Sq
rt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c) - (d*E^((3*a)/b
)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c)
 + (3*d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c*E^(a/
b)) + (d*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^
(3/2)*c*E^((3*a)/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
```

```
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5696

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(Sqrt[1 + c^2*x^2]*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^(n +
 1))/(b*c*(n + 1)), x] - Dist[(c*(2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(b*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ
[e, c^2*d] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{d+c^2dx^2}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2d\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{(6cd)\int \frac{x\sqrt{1+c^2x^2}}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{b}$$

$$= -\frac{2d\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{(6d)\,\text{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{bc}$$

$$= -\frac{2d\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{(6d)\,\text{Subst}\left(\int \left(\frac{\sinh(x)}{4\sqrt{a+bx}}+\frac{\sinh(3x)}{4\sqrt{a+bx}}\right)dx,x,\sinh^{-1}(cx)\right)}{bc}$$

$$= -\frac{2d\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{(3d)\,\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{2bc}+\frac{(3d)\,\text{Subst}\left(\right.}{}$$

$$= -\frac{2d\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}-\frac{(3d)\,\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{4bc}-\frac{(3d)\,\text{Subst}\left(\right.}{}$$

$$= -\frac{2d\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}-\frac{(3d)\,\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx,x,\sqrt{a+b\sinh^{-1}(cx)}\right)}{2b^2c}-\frac{(3d}{}$$

$$= -\frac{2d\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}-\frac{3de^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}-\frac{de^{\frac{3a}{b}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b}}{}\right.}{4b^{3/2}c}$$

Mathematica [A] time = 0.93, size = 295, normalized size = 1.29

$$\frac{de^{-3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}\left(-3e^{\frac{3a}{b}+2\sinh^{-1}(cx)}-3e^{\frac{3a}{b}+4\sinh^{-1}(cx)}-e^{\frac{3a}{b}+6\sinh^{-1}(cx)}+3e^{\frac{4a}{b}+3\sinh^{-1}(cx)}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+s\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + c^2*d*x^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (d*(-E^((3*a)/b) - 3*E^((3*a)/b + 2*ArcSinh[c*x]) - 3*E^((3*a)/b + 4*ArcSin
h[c*x]) - E^((3*a)/b + 6*ArcSinh[c*x]) + 3*E^((4*a)/b + 3*ArcSinh[c*x])*Sqr
t[a/b + ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] + Sqrt[3]*E^(3*ArcSinh
[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c*x]))
/b] + 3*E^((2*a)/b + 3*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[
1/2, -((a + b*ArcSinh[c*x])/b)] + Sqrt[3]*E^((6*a)/b + 3*ArcSinh[c*x])*Sqrt
[a/b + ArcSinh[c*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c*x]))/b]))/(4*b*c*E^(3*(
a/b + ArcSinh[c*x]))*Sqrt[a + b*ArcSinh[c*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c^2dx^2+d}{(b\operatorname{arsinh}(cx)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)/(b*arcsinh(c*x) + a)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{c^2d\,x^2+d}{(a+b\operatorname{arcsinh}(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int((c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c^2dx^2+d}{(b\operatorname{arsinh}(cx)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 + d)/(b*arcsinh(c*x) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{d\,c^2\,x^2+d}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + c^2*d*x^2)/(a + b*asinh(c*x))^(3/2),x)
```

```
[Out] int((d + c^2*d*x^2)/(a + b*asinh(c*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d\left(\int \frac{c^{2}x^{2}}{a\sqrt{a+b\operatorname{asinh}{\left(cx\right)}}+b\sqrt{a+b\operatorname{asinh}{\left(cx\right)}}\operatorname{asinh}{\left(cx\right)}}\,dx+\int \frac{1}{a\sqrt{a+b\operatorname{asinh}{\left(cx\right)}}+b\sqrt{a+b\operatorname{asinh}{\left(cx\right)}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] d*(Integral(c**2*x**2/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c*x))*
asinh(c*x)), x) + Integral(1/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh
(c*x))*asinh(c*x)), x))
```

3.466 $\int \frac{d+c^2dx^2}{x\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=175

$$-\frac{2d\text{Int}\left(\frac{1}{x^2\sqrt{c^2x^2+1}\sqrt{a+b\sinh^{-1}(cx)}},x\right)}{bc}+\frac{\sqrt{\frac{\pi}{2}}\,de^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\,de^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}}-\frac{2d\left(c^2\right.}{bcx\sqrt{a+}}$$

```
[Out] 1/2*d*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^(
1/2)/b^(3/2)+1/2*d*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*P
i^(1/2)/b^(3/2)/exp(2*a/b)-2*d*(c^2*x^2+1)^(3/2)/b/c/x/(a+b*arcsinh(c*x))^(
1/2)-2*d*Unintegrable(1/x^2/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^(1/2),x)/b
/c
```

Rubi [A] time = 0.80, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{d+c^2dx^2}{x\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d + c^2*d*x^2)/(x*(a + b*ArcSinh[c*x])^(3/2)),x]
```

```
[Out] (-2*d*(1 + c^2*x^2)^(3/2))/(b*c*x*Sqrt[a + b*ArcSinh[c*x]]) + (d*E^((2*a)/b
)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/b^(3/2) + (d*
Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(b^(3/2)*E^((2
*a)/b)) - (2*d*Defer[Int][1/(x^2*Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]]
), x])/(b*c)
```

Rubi steps

$$\int \frac{d+c^2dx^2}{x\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2d\left(1+c^2x^2\right)^{3/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} - \frac{(2d)\int \frac{\sqrt{1+c^2x^2}}{x^2\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{bc} + \frac{(4cd)\int \frac{\sqrt{1+c^2x^2}}{\sqrt{a+b\sinh^{-1}(cx)}}\,c}{b}$$

$$= -\frac{2d\left(1+c^2x^2\right)^{3/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(4d)\,\text{Subst}\left(\int \frac{\cosh^2(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{b} - \frac{(2d)\int\Big(}{}$$

$$= -\frac{2d\left(1+c^2x^2\right)^{3/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(4d)\,\text{Subst}\left(\int \left(\frac{1}{2\sqrt{a+bx}} + \frac{\cosh(2x)}{2\sqrt{a+bx}}\right) dx, x, \sinh^{-1}(cx)\right)}{b}$$

$$= -\frac{2d\left(1+c^2x^2\right)^{3/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(2d)\,\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{b} - \frac{(2d)\int -}{}$$

$$= -\frac{2d\left(1+c^2x^2\right)^{3/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d\,\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{b} + \frac{d\,\text{Subst}\Big(\int}{}$$

$$= -\frac{2d\left(1+c^2x^2\right)^{3/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(2d)\,\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{b^2} +$$

$$= -\frac{2d\left(1+c^2x^2\right)^{3/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} + \frac{de^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}} + \frac{de^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erfi}\Big(\sqrt{2}}{b^{3/}}$$

Mathematica [A] time = 4.43, size = 0, normalized size = 0.00

$$\int \frac{d+c^2dx^2}{x\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + c^2*d*x^2)/(x*(a + b*ArcSinh[c*x])^(3/2)),x]
[Out] Integrate[(d + c^2*d*x^2)/(x*(a + b*ArcSinh[c*x])^(3/2)), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)/x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)/x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{c^{2} d\,x^{2}+d}{x \left(a +b \,\mathrm{arcsinh}\left(c x \right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)/x/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int((c^2*d*x^2+d)/x/(a+b*arcsinh(c*x))^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c^{2} d x^{2} + d}{{\left(b \operatorname{arsinh}\left(c x\right) + a\right)}^{\frac{3}{2}} x}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)/x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 + d)/((b*arcsinh(c*x) + a)^(3/2)*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d\,c^2\,x^2+d}{x\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + c^2*d*x^2)/(x*(a + b*asinh(c*x))^(3/2)),x)
```

```
[Out] int((d + c^2*d*x^2)/(x*(a + b*asinh(c*x))^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$d \left(\int \frac{c^{2} x^{2}}{a x \sqrt{a + b \operatorname{asinh}{\left (c x \right)}} + b x \sqrt{a + b \operatorname{asinh}{\left (c x \right)}} \operatorname{asinh}{\left (c x \right)}}\, dx + \int \frac{1}{a x \sqrt{a + b \operatorname{asinh}{\left (c x \right)}} + b x \sqrt{a + b \operatorname{asinh}{\left (c x \right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)/x/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] d*(Integral(c**2*x**2/(a*x*sqrt(a + b*asinh(c*x)) + b*x*sqrt(a + b*asinh(c*
x))*asinh(c*x)), x) + Integral(1/(a*x*sqrt(a + b*asinh(c*x)) + b*x*sqrt(a +
 b*asinh(c*x))*asinh(c*x)), x))
```

3.467 $\int \frac{x^3\left(d+c^2dx^2\right)^2}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=474

$$-\frac{\sqrt{\pi}d^2e^{\frac{4a}{b}}\operatorname{erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}-\frac{3\sqrt{\frac{\pi}{2}}d^2e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}+\frac{\sqrt{\frac{\pi}{2}}d^2e^{\frac{8a}{b}}\operatorname{erf}\left(\frac{2\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}+\frac{\sqrt{\frac{3\pi}{2}}d^2}{}$$

```
[Out] -3/64*d^2*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*
Pi^(1/2)/b^(3/2)/c^4+1/64*d^2*exp(8*a/b)*erf(2*2^(1/2)*(a+b*arcsinh(c*x))^(
1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(3/2)/c^4-3/64*d^2*erfi(2^(1/2)*(a+b*arcsi
nh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(3/2)/c^4/exp(2*a/b)+1/64*d^2*er
fi(2*2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(3/2)/c^4
/exp(8*a/b)-1/32*d^2*exp(4*a/b)*erf(2*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^
(1/2)/b^(3/2)/c^4-1/32*d^2*erfi(2*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2
)/b^(3/2)/c^4/exp(4*a/b)+1/64*d^2*exp(6*a/b)*erf(6^(1/2)*(a+b*arcsinh(c*x))
^(1/2)/b^(1/2))*6^(1/2)*Pi^(1/2)/b^(3/2)/c^4+1/64*d^2*erfi(6^(1/2)*(a+b*arc
sinh(c*x))^(1/2)/b^(1/2))*6^(1/2)*Pi^(1/2)/b^(3/2)/c^4/exp(6*a/b)-2*d^2*x^3
*(c^2*x^2+1)^(5/2)/b/c/(a+b*arcsinh(c*x))^(1/2)
```

Rubi [A] time = 1.53, antiderivative size = 474, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5777, 5779, 5448, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}d^2e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}-\frac{3\sqrt{\frac{\pi}{2}}d^2e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}+\frac{\sqrt{\frac{\pi}{2}}d^2e^{\frac{8a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}+\frac{\sqrt{\frac{3\pi}{2}}d^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + c^2*d*x^2)^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-2*d^2*x^3*(1 + c^2*x^2)^(5/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (d^2*E^((
4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4
) - (3*d^2*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sq
rt[b]])/(32*b^(3/2)*c^4) + (d^2*E^((8*a)/b)*Sqrt[Pi/2]*Erf[(2*Sqrt[2]*Sqrt[
a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4) + (d^2*E^((6*a)/b)*Sqrt[(3*
Pi)/2]*Erf[(Sqrt[6]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4) -
(d^2*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4*E
^((4*a)/b)) - (3*d^2*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqr
t[b]])/(32*b^(3/2)*c^4*E^((2*a)/b)) + (d^2*Sqrt[Pi/2]*Erfi[(2*Sqrt[2]*Sqrt[
a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4*E^((8*a)/b)) + (d^2*Sqrt[(3*
Pi)/2]*Erfi[(Sqrt[6]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4*E^
((6*a)/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^
p*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-Dist[(f*m*d^IntPart[p
]*(d + e*x^2)^FracPart[p])/(b*c*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*
x)^(m - 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - D
ist[(c*(m + 2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(b*f*(n + 1)*(1
+ c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*A
rcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[2*p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^3\left(d+c^2dx^2\right)^2}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2d^2x^3\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{\left(6d^2\right)\int \frac{x^2\left(1+c^2x^2\right)^{3/2}}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{bc} + \frac{\left(16cd^2\right)\int \frac{x^4\left(1+c^2x^2\right)^{3/2}}{\sqrt{a+b\sinh^{-1}(cx)}}\,d}{b}$$

$$= -\frac{2d^2x^3\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{\left(6d^2\right)\text{Subst}\left(\int \frac{\cosh^4(x)\sinh^2(x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{bc^4} + (16$$

$$= -\frac{2d^2x^3\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{\left(6d^2\right)\text{Subst}\left(\int \left(-\frac{1}{16\sqrt{a+bx}} - \frac{\cosh(2x)}{32\sqrt{a+bx}} + \frac{\cosh(4x)}{16\sqrt{a+bx}} + \frac{\cosh}{32\sqrt{}}\right.\right.}{bc^4}$$

$$= -\frac{2d^2x^3\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d^2\,\text{Subst}\left(\int \frac{\cosh(8x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{8bc^4} - \frac{\left(3d^2\right)\text{Subst}\left(\right.}{}$$

$$= -\frac{2d^2x^3\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d^2\,\text{Subst}\left(\int \frac{e^{-8x}}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{16bc^4} + \frac{d^2\,\text{Subst}\left(\int \frac{}{\sqrt{}}\right.}{}$$

$$= -\frac{2d^2x^3\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d^2\,\text{Subst}\left(\int e^{\frac{8a}{b}-\frac{8x^2}{b}}\,dx,x,\sqrt{a+b\sinh^{-1}(cx)}\right)}{8b^2c^4} + \frac{d^2\,\text{Su}}{}$$

$$= -\frac{2d^2x^3\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4} - \frac{3d^2e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a-}}{}\right.}{32b^{3/2}c^4}$$

Mathematica [A] time = 1.01, size = 462, normalized size = 0.97

$$\frac{d^2e^{-\frac{8a}{b}}\left(6e^{\frac{8a}{b}}\sinh\left(2\sinh^{-1}(cx)\right) + 2e^{\frac{8a}{b}}\sinh\left(4\sinh^{-1}(cx)\right) - 2e^{\frac{8a}{b}}\sinh\left(6\sinh^{-1}(cx)\right) - e^{\frac{8a}{b}}\sinh\left(8\sinh^{-1}(cx\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(d + c^2*d*x^2)^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (d^2*(Sqrt[2]*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-8*(a + b*ArcSinh
[c*x]))/b] + Sqrt[6]*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2,
 (-6*(a + b*ArcSinh[c*x]))/b] - 2*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b
)]*Gamma[1/2, (-4*(a + b*ArcSinh[c*x]))/b] - 3*Sqrt[2]*E^((6*a)/b)*Sqrt[-((
a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-2*(a + b*ArcSinh[c*x]))/b] + 3*Sqrt[2]
*E^((10*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (2*(a + b*ArcSinh[c*x]))/
b] + 2*E^((12*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (4*(a + b*ArcSinh[c
*x]))/b] - Sqrt[6]*E^((14*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (6*(a +
 b*ArcSinh[c*x]))/b] - Sqrt[2]*E^((16*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[
1/2, (8*(a + b*ArcSinh[c*x]))/b] + 6*E^((8*a)/b)*Sinh[2*ArcSinh[c*x]] + 2*E
^((8*a)/b)*Sinh[4*ArcSinh[c*x]] - 2*E^((8*a)/b)*Sinh[6*ArcSinh[c*x]] - E^((
8*a)/b)*Sinh[8*ArcSinh[c*x]]))/(64*b*c^4*E^((8*a)/b)*Sqrt[a + b*ArcSinh[c*x
]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas
")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2dx^2+d\right)^2x^3}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^2*x^3/(b*arcsinh(c*x) + a)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(c^2d\,x^2+d\right)^2}{\left(a+b\operatorname{arcsinh}\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(x^3*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2dx^2+d\right)^2x^3}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate((c^2*d*x^2 + d)^2*x^3/(b*arcsinh(c*x) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\left(d\,c^2\,x^2+d\right)^2}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + c^2*d*x^2)^2)/(a + b*asinh(c*x))^(3/2),x)
```

```
[Out] int((x^3*(d + c^2*d*x^2)^2)/(a + b*asinh(c*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int \frac{x^3}{a\sqrt{a+b\operatorname{asinh}\left(cx\right)}+b\sqrt{a+b\operatorname{asinh}\left(cx\right)}\operatorname{asinh}\left(cx\right)}\,dx+\int \frac{2c^2x^5}{a\sqrt{a+b\operatorname{asinh}\left(cx\right)}+b\sqrt{a+b\operatorname{asinh}\left(cx\right)}\operatorname{asi}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*d*x**2+d)**2/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] d**2*(Integral(x**3/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c*x))*as
inh(c*x)), x) + Integral(2*c**2*x**5/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a +
 b*asinh(c*x))*asinh(c*x)), x) + Integral(c**4*x**7/(a*sqrt(a + b*asinh(c*x
)) + b*sqrt(a + b*asinh(c*x))*asinh(c*x)), x))
```

3.468 $\int \frac{x^2(d+c^2dx^2)^2}{(a+b\sinh^{-1}(cx))^{3/2}}\,dx$

Optimal. Leaf size=457

$$\frac{5\sqrt{\pi}\,d^2e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}-\frac{\sqrt{3\pi}\,d^2e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}-\frac{3\sqrt{5\pi}\,d^2e^{\frac{5a}{b}}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}-\frac{\sqrt{7\pi}\,d^2e^{\frac{7a}{b}}}{}$$

```
[Out] 5/64*d^2*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^
3-5/64*d^2*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^3/exp(
a/b)-1/64*d^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1
/2)*Pi^(1/2)/b^(3/2)/c^3+1/64*d^2*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(
1/2))*3^(1/2)*Pi^(1/2)/b^(3/2)/c^3/exp(3*a/b)-3/64*d^2*exp(5*a/b)*erf(5^(1/
2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/c^3+3/64*d^2*
erfi(5^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/c^3
/exp(5*a/b)-1/64*d^2*exp(7*a/b)*erf(7^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2
))*7^(1/2)*Pi^(1/2)/b^(3/2)/c^3+1/64*d^2*erfi(7^(1/2)*(a+b*arcsinh(c*x))^(1
/2)/b^(1/2))*7^(1/2)*Pi^(1/2)/b^(3/2)/c^3/exp(7*a/b)-2*d^2*x^2*(c^2*x^2+1)^
(5/2)/b/c/(a+b*arcsinh(c*x))^(1/2)
```

Rubi [A] time = 1.78, antiderivative size = 457, normalized size of antiderivative = 1.00, number of steps used = 42, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5777, 5779, 5448, 3308, 2180, 2204, 2205}

$$\frac{5\sqrt{\pi}\,d^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}-\frac{\sqrt{3\pi}\,d^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}-\frac{3\sqrt{5\pi}\,d^2e^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}-\frac{\sqrt{7\pi}\,d^2e^{\frac{7a}{b}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + c^2*d*x^2)^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-2*d^2*x^2*(1 + c^2*x^2)^(5/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (5*d^2*E^
(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(64*b^(3/2)*c^3) - (d
^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/
(64*b^(3/2)*c^3) - (3*d^2*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5]*Sqrt[a + b*Ar
cSinh[c*x]])/Sqrt[b]])/(64*b^(3/2)*c^3) - (d^2*E^((7*a)/b)*Sqrt[7*Pi]*Erf[(
Sqrt[7]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(64*b^(3/2)*c^3) - (5*d^2*Sqrt[
Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(64*b^(3/2)*c^3*E^(a/b)) + (d^2
*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(64*b^(3/2)*c
^3*E^((3*a)/b)) + (3*d^2*Sqrt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x]])
/Sqrt[b]])/(64*b^(3/2)*c^3*E^((5*a)/b)) + (d^2*Sqrt[7*Pi]*Erfi[(Sqrt[7]*Sqr
t[a + b*ArcSinh[c*x]])/Sqrt[b]])/(64*b^(3/2)*c^3*E^((7*a)/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^
p*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-Dist[(f*m*d^IntPart[p
]*(d + e*x^2)^FracPart[p])/(b*c*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*
x)^(m - 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - D
ist[(c*(m + 2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(b*f*(n + 1)*(1
+ c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*A
rcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[2*p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2\left(d+c^2dx^2\right)^2}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2d^2x^2\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{\left(4d^2\right)\int \frac{x\left(1+c^2x^2\right)^{3/2}}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{bc} + \frac{\left(14cd^2\right)\int \frac{x^3\left(1+c^2x^2\right)^{3/2}}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{b}$$

$$= -\frac{2d^2x^2\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{\left(4d^2\right)\text{Subst}\left(\int \frac{\cosh^4(x)\sinh(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{bc^3} + \frac{\left(14d^2\right)}{}$$

$$= -\frac{2d^2x^2\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{\left(4d^2\right)\text{Subst}\left(\int \left(\frac{\sinh(x)}{8\sqrt{a+bx}} + \frac{3\sinh(3x)}{16\sqrt{a+bx}} + \frac{\sinh(5x)}{16\sqrt{a+bx}}\right)dx, x, \sinh\right.}{bc^3}$$

$$= -\frac{2d^2x^2\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{\left(7d^2\right)\text{Subst}\left(\int \frac{\sinh(5x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{32bc^3} + \frac{\left(7d^2\right)\text{Subst}\left(\right.}{}$$

$$= -\frac{2d^2x^2\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{\left(7d^2\right)\text{Subst}\left(\int \frac{e^{-7x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{64bc^3} - \frac{\left(7d^2\right)\text{Subst}\left(\right.}{}$$

$$= -\frac{2d^2x^2\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{\left(7d^2\right)\text{Subst}\left(\int e^{\frac{7a}{b}-\frac{7x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{32b^2c^3} - \frac{\left(7d^2\right)}{}$$

$$= -\frac{2d^2x^2\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{5d^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3} - \frac{d^2e^{\frac{3a}{b}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\,\text{s}}}{\sqrt{}}\right.}{64b^{3/2}c^3}$$

<u>Mathematica</u> [A] time = 1.64, size = 577, normalized size = 1.26

$$-\frac{d^2e^{-7\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}\left(3e^{\frac{7a}{b}+2\sinh^{-1}(cx)} + e^{\frac{7a}{b}+4\sinh^{-1}(cx)} - 5e^{\frac{7a}{b}+6\sinh^{-1}(cx)} - 5e^{\frac{7a}{b}+8\sinh^{-1}(cx)} + e^{\frac{7a}{b}+10\sinh^{-1}(cx)} + 3e^{\frac{7a}{b}+12}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(d + c^2*d*x^2)^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] -1/64*(d^2*(E^((7*a)/b) + 3*E^((7*a)/b + 2*ArcSinh[c*x]) + E^((7*a)/b + 4*A
rcSinh[c*x]) - 5*E^((7*a)/b + 6*ArcSinh[c*x]) - 5*E^((7*a)/b + 8*ArcSinh[c*
x]) + E^((7*a)/b + 10*ArcSinh[c*x]) + 3*E^((7*a)/b + 12*ArcSinh[c*x]) + E^(
(7*a)/b + 14*ArcSinh[c*x]) + 5*E^((8*a)/b + 7*ArcSinh[c*x])*Sqrt[a/b + ArcS
inh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] - Sqrt[7]*E^(7*ArcSinh[c*x])*Sqrt[
-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-7*(a + b*ArcSinh[c*x]))/b] - 3*Sqrt
[5]*E^((2*a)/b + 7*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2,
 (-5*(a + b*ArcSinh[c*x]))/b] - Sqrt[3]*E^((4*a)/b + 7*ArcSinh[c*x])*Sqrt[-
((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c*x]))/b] + 5*E^((6
*a)/b + 7*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b
*ArcSinh[c*x])/b)] - Sqrt[3]*E^((10*a)/b + 7*ArcSinh[c*x])*Sqrt[a/b + ArcSi
nh[c*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c*x]))/b] - 3*Sqrt[5]*E^((12*a)/b + 7
*ArcSinh[c*x])*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (5*(a + b*ArcSinh[c*x]))
/b] - Sqrt[7]*E^(7*((2*a)/b + ArcSinh[c*x]))*Sqrt[a/b + ArcSinh[c*x]]*Gamma
[1/2, (7*(a + b*ArcSinh[c*x]))/b]))/(b*c^3*E^(7*(a/b + ArcSinh[c*x]))*Sqrt[
a + b*ArcSinh[c*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas
")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2dx^2+d\right)^2x^2}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^2*x^2/(b*arcsinh(c*x) + a)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(c^2d\,x^2+d\right)^2}{\left(a+b\operatorname{arcsinh}\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(x^2*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2dx^2+d\right)^2x^2}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate((c^2*d*x^2 + d)^2*x^2/(b*arcsinh(c*x) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\left(d\,c^2\,x^2+d\right)^2}{\left(a+b\operatorname{asinh}\left(c\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + c^2*d*x^2)^2)/(a + b*asinh(c*x))^(3/2),x)
```

```
[Out] int((x^2*(d + c^2*d*x^2)^2)/(a + b*asinh(c*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int \frac{x^2}{a\sqrt{a+b\operatorname{asinh}\left(cx\right)}+b\sqrt{a+b\operatorname{asinh}\left(cx\right)}\operatorname{asinh}\left(cx\right)}\,dx+\int \frac{2c^2x^4}{a\sqrt{a+b\operatorname{asinh}\left(cx\right)}+b\sqrt{a+b\operatorname{asinh}\left(cx\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)**2/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] d**2*(Integral(x**2/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c*x))*as
inh(c*x)), x) + Integral(2*c**2*x**4/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a +
 b*asinh(c*x))*asinh(c*x)), x) + Integral(c**4*x**6/(a*sqrt(a + b*asinh(c*x
)) + b*sqrt(a + b*asinh(c*x))*asinh(c*x)), x))
```

3.469 $\int \frac{x(d+c^2dx^2)^2}{(a+b\sinh^{-1}(cx))^{3/2}}\,dx$

Optimal. Leaf size=358

$$\frac{\sqrt{\pi}\,d^2e^{\frac{4a}{b}}\operatorname{erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}+\frac{5\sqrt{\frac{\pi}{2}}\,d^2e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2}+\frac{\sqrt{\frac{3\pi}{2}}\,d^2e^{\frac{6a}{b}}\operatorname{erf}\left(\frac{\sqrt{6}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2}+\frac{\sqrt{\pi}\,d^2e^{-}}{}$$

```
[Out] 5/32*d^2*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*P
i^(1/2)/b^(3/2)/c^2+5/32*d^2*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))
*2^(1/2)*Pi^(1/2)/b^(3/2)/c^2/exp(2*a/b)+1/4*d^2*exp(4*a/b)*erf(2*(a+b*arcs
inh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^2+1/4*d^2*erfi(2*(a+b*arcsinh(c
*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^2/exp(4*a/b)+1/32*d^2*exp(6*a/b)*erf
(6^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*6^(1/2)*Pi^(1/2)/b^(3/2)/c^2+1/3
2*d^2*erfi(6^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*6^(1/2)*Pi^(1/2)/b^(3/
2)/c^2/exp(6*a/b)-2*d^2*x*(c^2*x^2+1)^(5/2)/b/c/(a+b*arcsinh(c*x))^(1/2)
```

Rubi [A] time = 1.33, antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5777, 5699, 3312, 3307, 2180, 2204, 2205, 5779, 5448}

$$\frac{\sqrt{\pi}\,d^2e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}+\frac{5\sqrt{\frac{\pi}{2}}\,d^2e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2}+\frac{\sqrt{\frac{3\pi}{2}}\,d^2e^{\frac{6a}{b}}\operatorname{Erf}\left(\frac{\sqrt{6}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2}+\frac{\sqrt{\pi}\,d^2e}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + c^2*d*x^2)^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-2*d^2*x*(1 + c^2*x^2)^(5/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (d^2*E^((4*
a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c^2) +
 (5*d^2*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[
b]])/(16*b^(3/2)*c^2) + (d^2*E^((6*a)/b)*Sqrt[(3*Pi)/2]*Erf[(Sqrt[6]*Sqrt[a
 + b*ArcSinh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^2) + (d^2*Sqrt[Pi]*Erfi[(2*Sqrt
[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c^2*E^((4*a)/b)) + (5*d^2*Sqrt[P
i/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^2*E^((
2*a)/b)) + (d^2*Sqrt[(3*Pi)/2]*Erfi[(Sqrt[6]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt
[b]])/(16*b^(3/2)*c^2*E^((6*a)/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^
p*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-Dist[(f*m*d^IntPart[p
]*(d + e*x^2)^FracPart[p])/(b*c*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*
x)^(m - 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - D
ist[(c*(m + 2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(b*f*(n + 1)*(1
+ c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*A
rcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[2*p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x\left(d+c^2dx^2\right)^2}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}dx = -\frac{2d^2x\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{\left(2d^2\right)\int \frac{\left(1+c^2x^2\right)^{3/2}}{\sqrt{a+b\sinh^{-1}(cx)}}dx}{bc} + \frac{\left(12cd^2\right)\int \frac{x^2\left(1+c^2x^2\right)^{3/2}}{\sqrt{a+b\sinh^{-1}(cx)}}d}{b}$$

$$= -\frac{2d^2x\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{\left(2d^2\right)\text{Subst}\left(\int \frac{\cosh^4(x)}{\sqrt{a+bx}}dx, x, \sinh^{-1}(cx)\right)}{bc^2} + \frac{\left(12d^2\right)\text{Su}}{}$$

$$= -\frac{2d^2x\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{\left(2d^2\right)\text{Subst}\left(\int \left(\frac{3}{8\sqrt{a+bx}}+\frac{\cosh(2x)}{2\sqrt{a+bx}}+\frac{\cosh(4x)}{8\sqrt{a+bx}}\right)dx, x, \text{si}\right.}{bc^2}$$

$$= -\frac{2d^2x\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d^2\,\text{Subst}\left(\int \frac{\cosh(4x)}{\sqrt{a+bx}}dx, x, \sinh^{-1}(cx)\right)}{4bc^2} - \frac{\left(3d^2\right)\text{Subst}\left(\right.}{}$$

$$= -\frac{2d^2x\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d^2\,\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{a+bx}}dx, x, \sinh^{-1}(cx)\right)}{8bc^2} + \frac{d^2\,\text{Subst}\left(\int \frac{}{\sqrt{}}\right.}{}$$

$$= -\frac{2d^2x\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d^2\,\text{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{4b^2c^2} + \frac{d^2\,\text{Su}}{}$$

$$= -\frac{2d^2x\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2} + \frac{5d^2e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a}}{}\right.}{16b^{3/2}c^2}$$

Mathematica [A] time = 0.71, size = 351, normalized size = 0.98

$$-\frac{d^2e^{-\frac{6a}{b}}\left(10e^{\frac{6a}{b}}\sinh\left(2\sinh^{-1}(cx)\right)+8e^{\frac{6a}{b}}\sinh\left(4\sinh^{-1}(cx)\right)+2e^{\frac{6a}{b}}\sinh\left(6\sinh^{-1}(cx)\right)-\sqrt{6}\sqrt{-\frac{a+b\sinh^{-1}(cx}{b}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(d + c^2*d*x^2)^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] -1/32*(d^2*(-(Sqrt[6]*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-6*(a + b
*ArcSinh[c*x]))/b]) - 8*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1
/2, (-4*(a + b*ArcSinh[c*x]))/b] - 5*Sqrt[2]*E^((4*a)/b)*Sqrt[-((a + b*ArcS
inh[c*x])/b)]*Gamma[1/2, (-2*(a + b*ArcSinh[c*x]))/b] + 5*Sqrt[2]*E^((8*a)/
b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (2*(a + b*ArcSinh[c*x]))/b] + 8*E^((
10*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (4*(a + b*ArcSinh[c*x]))/b] +
Sqrt[6]*E^((12*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (6*(a + b*ArcSinh[
c*x]))/b] + 10*E^((6*a)/b)*Sinh[2*ArcSinh[c*x]] + 8*E^((6*a)/b)*Sinh[4*ArcS
inh[c*x]] + 2*E^((6*a)/b)*Sinh[6*ArcSinh[c*x]]))/(b*c^2*E^((6*a)/b)*Sqrt[a
+ b*ArcSinh[c*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2 d x^2 + d\right)^2 x}{\left(b \operatorname{arsinh}\left(c x\right) + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^2*x/(b*arcsinh(c*x) + a)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x \left(c^2 d\, x^2 + d\right)^2}{\left(a + b \operatorname{arcsinh}\left(c x\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(x*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2 d x^2 + d\right)^2 x}{\left(b \operatorname{arsinh}\left(c x\right) + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 + d)^2*x/(b*arcsinh(c*x) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x \left(d\, c^2\, x^2 + d\right)^2}{{\left(a + b \operatorname{asinh}\left(c\, x\right)\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + c^2*d*x^2)^2)/(a + b*asinh(c*x))^(3/2),x)
```

```
[Out] int((x*(d + c^2*d*x^2)^2)/(a + b*asinh(c*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^2 \left(\int \frac{x}{a \sqrt{a + b \operatorname{asinh}\left(c x\right)} + b \sqrt{a + b \operatorname{asinh}\left(c x\right)} \operatorname{asinh}\left(c x\right)}\, dx + \int \frac{2 c^2 x^3}{a \sqrt{a + b \operatorname{asinh}\left(c x\right)} + b \sqrt{a + b \operatorname{asinh}\left(c x\right)} \operatorname{asi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*d*x**2+d)**2/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] d**2*(Integral(x/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c*x))*asinh
(c*x)), x) + Integral(2*c**2*x**3/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*
asinh(c*x))*asinh(c*x)), x) + Integral(c**4*x**5/(a*sqrt(a + b*asinh(c*x))
+ b*sqrt(a + b*asinh(c*x))*asinh(c*x)), x))
```

3.470 $\int \frac{(d+c^2dx^2)^2}{(a+b\sinh^{-1}(cx))^{3/2}}\,dx$

Optimal. Leaf size=346

$$-\frac{5\sqrt{\pi}\,d^2e^{a/b}\mathrm{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c}-\frac{5\sqrt{3\pi}\,d^2e^{\frac{3a}{b}}\mathrm{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c}-\frac{\sqrt{5\pi}\,d^2e^{\frac{5a}{b}}\mathrm{erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c}+\frac{5\sqrt{\pi}\,\ldots}{}$$

```
[Out] -5/8*d^2*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c+
5/8*d^2*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c/exp(a/b)-
5/16*d^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*P
i^(1/2)/b^(3/2)/c+5/16*d^2*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3
^(1/2)*Pi^(1/2)/b^(3/2)/c/exp(3*a/b)-1/16*d^2*exp(5*a/b)*erf(5^(1/2)*(a+b*a
rcsinh(c*x))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/c+1/16*d^2*erfi(5^(1/2
)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/c/exp(5*a/b)-2
*d^2*(c^2*x^2+1)^(5/2)/b/c/(a+b*arcsinh(c*x))^(1/2)
```

Rubi [A] time = 0.73, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {5696, 5779, 5448, 3308, 2180, 2204, 2205}

$$-\frac{5\sqrt{\pi}\,d^2e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c}-\frac{5\sqrt{3\pi}\,d^2e^{\frac{3a}{b}}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c}-\frac{\sqrt{5\pi}\,d^2e^{\frac{5a}{b}}\mathrm{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c}+\frac{5\sqrt{\pi}\,\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c^2*d*x^2)^2/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-2*d^2*(1 + c^2*x^2)^(5/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (5*d^2*E^(a/b
)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*b^(3/2)*c) - (5*d^2*E^
((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(16*b
^(3/2)*c) - (d^2*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x
]])/Sqrt[b]])/(16*b^(3/2)*c) + (5*d^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]
]/Sqrt[b]])/(8*b^(3/2)*c*E^(a/b)) + (5*d^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a
+ b*ArcSinh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c*E^((3*a)/b)) + (d^2*Sqrt[5*Pi]*E
rfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c*E^((5*a)/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5696

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(Sqrt[1 + c^2*x^2]*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^(n +
 1))/(b*c*(n + 1)), x] - Dist[(c*(2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(b*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ
[e, c^2*d] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^2}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2d^2\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{\left(10cd^2\right)\int \frac{x\left(1+c^2x^2\right)^{3/2}}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{b}$$

$$= -\frac{2d^2\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{\left(10d^2\right)\text{Subst}\left(\int \frac{\cosh^4(x)\sinh(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{bc}$$

$$= -\frac{2d^2\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{\left(10d^2\right)\text{Subst}\left(\int \left(\frac{\sinh(x)}{8\sqrt{a+bx}} + \frac{3\sinh(3x)}{16\sqrt{a+bx}} + \frac{\sinh(5x)}{16\sqrt{a+bx}}\right)dx, x,}{bc}$$

$$= -\frac{2d^2\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{\left(5d^2\right)\text{Subst}\left(\int \frac{\sinh(5x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{8bc} + \frac{\left(5d^2\right)\text{Sub}}{}$$

$$= -\frac{2d^2\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{\left(5d^2\right)\text{Subst}\left(\int \frac{e^{-5x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{16bc} + \frac{\left(5d^2\right)\text{Subs}}{}$$

$$= -\frac{2d^2\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{\left(5d^2\right)\text{Subst}\left(\int e^{\frac{5a}{b}-\frac{5x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{8b^2c} + \frac{(5}{}$$

$$= -\frac{2d^2\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{5d^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c} - \frac{5d^2e^{\frac{3a}{b}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{}}{}\right.}{16b^{3/2}c}$$

Mathematica [A] time = 2.09, size = 440, normalized size = 1.27

$$\frac{d^2e^{-5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}\left(-5e^{\frac{5a}{b}+2\sinh^{-1}(cx)} - 10e^{\frac{5a}{b}+4\sinh^{-1}(cx)} - 10e^{\frac{5a}{b}+6\sinh^{-1}(cx)} - 5e^{\frac{5a}{b}+8\sinh^{-1}(cx)} - e^{\frac{5a}{b}+10\sinh^{-1}(cx)} + 1\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + c^2*d*x^2)^2/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (d^2*(-E^((5*a)/b) - 5*E^((5*a)/b + 2*ArcSinh[c*x]) - 10*E^((5*a)/b + 4*Arc
Sinh[c*x]) - 10*E^((5*a)/b + 6*ArcSinh[c*x]) - 5*E^((5*a)/b + 8*ArcSinh[c*x
]) - E^((5*a)/b + 10*ArcSinh[c*x]) + 10*E^((6*a)/b + 5*ArcSinh[c*x])*Sqrt[a
/b + ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] + Sqrt[5]*E^(5*ArcSinh[c*
x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-5*(a + b*ArcSinh[c*x]))/b]
 + 5*Sqrt[3]*E^((2*a)/b + 5*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*G
amma[1/2, (-3*(a + b*ArcSinh[c*x]))/b] + 10*E^((4*a)/b + 5*ArcSinh[c*x])*Sq
rt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)] + 5*Sqr
t[3]*E^((8*a)/b + 5*ArcSinh[c*x])*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (3*(a
 + b*ArcSinh[c*x]))/b] + Sqrt[5]*E^(5*((2*a)/b + ArcSinh[c*x]))*Sqrt[a/b +
ArcSinh[c*x]]*Gamma[1/2, (5*(a + b*ArcSinh[c*x]))/b]))/(16*b*c*E^(5*(a/b +
ArcSinh[c*x]))*Sqrt[a + b*ArcSinh[c*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2dx^2+d\right)^2}{(b\operatorname{arsinh}(cx)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^2/(b*arcsinh(c*x) + a)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2d\,x^2+d\right)^2}{(a+b\operatorname{arcsinh}(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int((c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2dx^2+d\right)^2}{(b\operatorname{arsinh}(cx)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 + d)^2/(b*arcsinh(c*x) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d\,c^2\,x^2+d\right)^2}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + c^2*d*x^2)^2/(a + b*asinh(c*x))^(3/2),x)
```

```
[Out] int((d + c^2*d*x^2)^2/(a + b*asinh(c*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int \frac{2c^2x^2}{a\sqrt{a+b\operatorname{asinh}(cx)}+b\sqrt{a+b\operatorname{asinh}(cx)}\operatorname{asinh}(cx)}\,dx+\int \frac{c^4x^4}{a\sqrt{a+b\operatorname{asinh}(cx)}+b\sqrt{a+b\operatorname{asinh}(cx)}\operatorname{asi}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**2/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] d**2*(Integral(2*c**2*x**2/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c
*x))*asinh(c*x)), x) + Integral(c**4*x**4/(a*sqrt(a + b*asinh(c*x)) + b*sqr
t(a + b*asinh(c*x))*asinh(c*x)), x) + Integral(1/(a*sqrt(a + b*asinh(c*x))
+ b*sqrt(a + b*asinh(c*x))*asinh(c*x)), x))
```

3.471 $\int \frac{\left(d+c^2dx^2\right)^2}{x\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=375

$$-\frac{2d^2\text{Int}\left(\frac{1}{x^2\sqrt{c^2x^2+1}\sqrt{a+b\sinh^{-1}(cx)}},x\right)}{bc}+\frac{\sqrt{\pi}d^2e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}}+\frac{\sqrt{2\pi}d^2e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}d}{}$$

```
[Out] 3/4*d^2*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi
^(1/2)/b^(3/2)+3/4*d^2*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/
2)*Pi^(1/2)/b^(3/2)/exp(2*a/b)+1/4*d^2*exp(4*a/b)*erf(2*(a+b*arcsinh(c*x))^
(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)+1/4*d^2*erfi(2*(a+b*arcsinh(c*x))^(1/2)/b^(
1/2))*Pi^(1/2)/b^(3/2)/exp(4*a/b)-2*d^2*(c^2*x^2+1)^(5/2)/b/c/x/(a+b*arcsin
h(c*x))^(1/2)-2*d^2*Unintegrable(1/x^2/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))
^(1/2),x)/b/c
```

Rubi [A] time = 1.44, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(d+c^2dx^2\right)^2}{x\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d + c^2*d*x^2)^2/(x*(a + b*ArcSinh[c*x])^(3/2)),x]
```

```
[Out] (-2*d^2*(1 + c^2*x^2)^(5/2))/(b*c*x*Sqrt[a + b*ArcSinh[c*x]]) + (d^2*E^((4*
a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)) - (d^
2*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(
2*b^(3/2)) + (d^2*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*
x]])/Sqrt[b]])/b^(3/2) + (d^2*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c*x]])/Sq
rt[b]])/(4*b^(3/2)*E^((4*a)/b)) - (d^2*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*
ArcSinh[c*x]])/Sqrt[b]])/(2*b^(3/2)*E^((2*a)/b)) + (d^2*Sqrt[2*Pi]*Erfi[(Sq
rt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(b^(3/2)*E^((2*a)/b)) - (2*d^2*De
fer[Int][1/(x^2*Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]]), x])/(b*c)
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^2}{x\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2d^2\left(1+c^2x^2\right)^{5/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} - \frac{\left(2d^2\right)\int \frac{\left(1+c^2x^2\right)^{3/2}}{x^2\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{bc} + \frac{\left(8cd^2\right)\int \frac{\left(1+c^2x^2\right)^{3/2}}{\sqrt{a+b\sinh^{-1}(cx)}}\,d}{b}$$

$$= -\frac{2d^2\left(1+c^2x^2\right)^{5/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} + \frac{\left(8d^2\right)\text{Subst}\left(\int \frac{\cosh^4(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{b} - \left(2d^2\right)\int\Big($$

$$= -\frac{2d^2\left(1+c^2x^2\right)^{5/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} + \frac{\left(8d^2\right)\text{Subst}\left(\int\left(\frac{3}{8\sqrt{a+bx}}+\frac{\cosh(2x)}{2\sqrt{a+bx}}+\frac{\cosh(4x)}{8\sqrt{a+bx}}\right)dx, x, \sin\right.}{b}$$

$$= -\frac{2d^2\left(1+c^2x^2\right)^{5/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} - \frac{2d^2\sqrt{a+b\sinh^{-1}(cx)}}{b^2} + \frac{d^2\,\text{Subst}\left(\int \frac{\cosh(4x)}{\sqrt{a+bx}}\,dx, x, \sin\right.}{b}$$

$$= -\frac{2d^2\left(1+c^2x^2\right)^{5/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} - \frac{2d^2\sqrt{a+b\sinh^{-1}(cx)}}{b^2} + \frac{d^2\,\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{a+bx}}\,dx, x, \sinh\right.}{2b}$$

$$= -\frac{2d^2\left(1+c^2x^2\right)^{5/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d^2\,\text{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{b^2} + d^2\,\text{Su}$$

$$= -\frac{2d^2\left(1+c^2x^2\right)^{5/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}} + \frac{d^2e^{\frac{2a}{b}}\sqrt{2\pi}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+}}{}\right.}{b^{3/2}}$$

$$= -\frac{2d^2\left(1+c^2x^2\right)^{5/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}} + \frac{d^2e^{\frac{2a}{b}}\sqrt{2\pi}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+}}{}\right.}{b^{3/2}}$$

$$= -\frac{2d^2\left(1+c^2x^2\right)^{5/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}} - \frac{d^2e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b}}{}\right.}{2b^{3/2}}$$

Mathematica [A] time = 3.31, size = 0, normalized size = 0.00

$$\int \frac{\left(d+c^2dx^2\right)^2}{x\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + c^2*d*x^2)^2/(x*(a + b*ArcSinh[c*x])^(3/2)),x]
[Out] Integrate[(d + c^2*d*x^2)^2/(x*(a + b*ArcSinh[c*x])^(3/2)), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2/x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2/x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^{2} d\,x^{2}+d\right)^{2}}{x \left(a +b \,\mathrm{arcsinh}\left(c x \right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^2/x/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int((c^2*d*x^2+d)^2/x/(a+b*arcsinh(c*x))^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{{\left(c^{2} d x^{2} + d\right)}^{2}}{{\left(b \operatorname{arsinh}\left(c x\right) + a\right)}^{\frac{3}{2}} x}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2/x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 + d)^2/((b*arcsinh(c*x) + a)^(3/2)*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d\,c^2\,x^2+d\right)}^2}{x\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + c^2*d*x^2)^2/(x*(a + b*asinh(c*x))^(3/2)),x)
```

```
[Out] int((d + c^2*d*x^2)^2/(x*(a + b*asinh(c*x))^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$d^{2} \left(\int \frac{2 c^{2} x^{2}}{a x \sqrt{a + b \operatorname{asinh}{\left(c x \right)}} + b x \sqrt{a + b \operatorname{asinh}{\left(c x \right)}} \operatorname{asinh}{\left(c x \right)}}\, dx + \int \frac{c^{4} x^{4}}{a x \sqrt{a + b \operatorname{asinh}{\left(c x \right)}} + b x \sqrt{a + b \operatorname{asinh}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**2/x/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] d**2*(Integral(2*c**2*x**2/(a*x*sqrt(a + b*asinh(c*x)) + b*x*sqrt(a + b*asi
nh(c*x))*asinh(c*x)), x) + Integral(c**4*x**4/(a*x*sqrt(a + b*asinh(c*x)) +
 b*x*sqrt(a + b*asinh(c*x))*asinh(c*x)), x) + Integral(1/(a*x*sqrt(a + b*as
inh(c*x)) + b*x*sqrt(a + b*asinh(c*x))*asinh(c*x)), x))
```

3.472 $\int \left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=319

$$\frac{\sqrt{\pi}\,c\sqrt{a^2cx^2+c}\,\mathrm{erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{256a\sqrt{a^2x^2+1}}+\frac{\sqrt{\frac{\pi}{2}}\,c\sqrt{a^2cx^2+c}\,\mathrm{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{16a\sqrt{a^2x^2+1}}-\frac{\sqrt{\pi}\,c\sqrt{a^2cx^2+c}\,\mathrm{erfi}\left(2\sqrt{\sinh^{-1}\right.}}{256a\sqrt{a^2x^2+1}}$$

```
[Out] 1/4*c*arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+1/32*c*erf
(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^
2+1)^(1/2)-1/32*c*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*
x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+1/256*c*erf(2*arcsinh(a*x)^(1/2))*Pi^(1/2)
*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-1/256*c*erfi(2*arcsinh(a*x)^(1/2))
*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+1/4*x*(a^2*c*x^2+c)^(3/2)
*arcsinh(a*x)^(1/2)+3/8*c*x*(a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {5684, 5682, 5675, 5669, 5448, 12, 3308, 2180, 2204, 2205, 5779}

$$\frac{\sqrt{\pi}\,c\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{256a\sqrt{a^2x^2+1}}+\frac{\sqrt{\frac{\pi}{2}}\,c\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{16a\sqrt{a^2x^2+1}}-\frac{\sqrt{\pi}\,c\sqrt{a^2cx^2+c}\,\mathrm{Erfi}\left(2\sqrt{\sinh^{-}\right.}}{256a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(3/2)*Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (3*c*x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/8 + (x*(c + a^2*c*x^2)^(3/2)
*Sqrt[ArcSinh[a*x]])/4 + (c*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2))/(4*a*Sq
rt[1 + a^2*x^2]) + (c*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erf[2*Sqrt[ArcSinh[a*x]]
])/(256*a*Sqrt[1 + a^2*x^2]) + (c*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2
]*Sqrt[ArcSinh[a*x]]])/(16*a*Sqrt[1 + a^2*x^2]) - (c*Sqrt[Pi]*Sqrt[c + a^2*
c*x^2]*Erfi[2*Sqrt[ArcSinh[a*x]]])/(256*a*Sqrt[1 + a^2*x^2]) - (c*Sqrt[Pi/2
]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(16*a*Sqrt[1 + a^2*
x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)}\,dx = \frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)}+\frac{1}{4}(3c)\int \sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}\,dx - \frac{\left(ac\right.}{}$$

$$= \frac{3}{8}cx\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}+\frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)}+\frac{\left(3c\sqrt{c+a^2\right.}{8}$$

$$= \frac{3}{8}cx\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}+\frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)}+\frac{c\sqrt{c+a^2cx}}{4a\sqrt{}}$$

$$= \frac{3}{8}cx\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}+\frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)}+\frac{c\sqrt{c+a^2cx}}{4a\sqrt{}}$$

$$= \frac{3}{8}cx\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}+\frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)}+\frac{c\sqrt{c+a^2cx}}{4a\sqrt{}}$$

$$= \frac{3}{8}cx\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}+\frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)}+\frac{c\sqrt{c+a^2cx}}{4a\sqrt{}}$$

$$= \frac{3}{8}cx\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}+\frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)}+\frac{c\sqrt{c+a^2cx}}{4a\sqrt{}}$$

$$= \frac{3}{8}cx\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}+\frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)}+\frac{c\sqrt{c+a^2cx}}{4a\sqrt{}}$$

Mathematica [A] time = 0.19, size = 142, normalized size = 0.45

$$\frac{c\sqrt{a^2cx^2+c}\left(-\sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\frac{3}{2},-4\sinh^{-1}(ax)\right)-8\sqrt{2}\sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\frac{3}{2},-2\sinh^{-1}(ax)\right)+\sqrt{\sinh^{-1}(ax)}\left(\right.\right.}{128a\sqrt{a^2x^2+1}\sqrt{\sinh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)*Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (c*Sqrt[c + a^2*c*x^2]*(-(Sqrt[-ArcSinh[a*x]]*Gamma[3/2, -4*ArcSinh[a*x]])
- 8*Sqrt[2]*Sqrt[-ArcSinh[a*x]]*Gamma[3/2, -2*ArcSinh[a*x]] + Sqrt[ArcSinh[
a*x]]*(32*ArcSinh[a*x]^(3/2) - 8*Sqrt[2]*Gamma[3/2, 2*ArcSinh[a*x]] - Gamma
[3/2, 4*ArcSinh[a*x]])))/(128*a*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \sqrt{\operatorname{arcsinh}\left(a x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^{2} c x^{2} + c\right)^{\frac{3}{2}} \sqrt{\operatorname{arsinh}\left(a x\right)}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*sqrt(arcsinh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\mathrm{asinh}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int(asinh(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c \left(a^{2} x^{2} + 1\right)\right)^{\frac{3}{2}} \sqrt{\operatorname{asinh}{\left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*asinh(a*x)**(1/2),x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*sqrt(asinh(a*x)), x)
```

3.473 $\int \sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=186

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\mathrm{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{16a\sqrt{a^2x^2+1}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\mathrm{erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{16a\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\,\sinh^{-1}(ax)^{3/2}}{3a\sqrt{a^2x^2+1}}+\frac{1}{2}x$$

```
[Out] 1/3*arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+1/32*erf(2^(
1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)
^(1/2)-1/32*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)
^(1/2)/a/(a^2*x^2+1)^(1/2)+1/2*x*(a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5682, 5675, 5669, 5448, 12, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{16a\sqrt{a^2x^2+1}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{16a\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\,\sinh^{-1}(ax)^{3/2}}{3a\sqrt{a^2x^2+1}}+\frac{1}{2}x$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/2 + (Sqrt[c + a^2*c*x^2]*ArcSinh
[a*x]^(3/2))/(3*a*Sqrt[1 + a^2*x^2]) + (Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[
Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(16*a*Sqrt[1 + a^2*x^2]) - (Sqrt[Pi/2]*Sqrt[c
+ a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(16*a*Sqrt[1 + a^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rubi steps

$$\int \sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}\,dx = \frac{1}{2}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} + \frac{\sqrt{c+a^2cx^2}\int\frac{\sqrt{\sinh^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx}{2\sqrt{1+a^2x^2}} - \frac{\Big(a\sqrt{c+a^2cx^2}}{4\sqrt{1}}$$

$$= \frac{1}{2}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} + \frac{\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}}{3a\sqrt{1+a^2x^2}} - \frac{\sqrt{c+a^2cx^2}\ \mathrm{Sub}}{}$$

$$= \frac{1}{2}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} + \frac{\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}}{3a\sqrt{1+a^2x^2}} - \frac{\sqrt{c+a^2cx^2}\ \mathrm{Sub}}{}$$

$$= \frac{1}{2}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} + \frac{\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}}{3a\sqrt{1+a^2x^2}} - \frac{\sqrt{c+a^2cx^2}\ \mathrm{Sub}}{}$$

$$= \frac{1}{2}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} + \frac{\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}}{3a\sqrt{1+a^2x^2}} + \frac{\sqrt{c+a^2cx^2}\ \mathrm{Sub}}{16}$$

$$= \frac{1}{2}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} + \frac{\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}}{3a\sqrt{1+a^2x^2}} + \frac{\sqrt{c+a^2cx^2}\ \mathrm{Sub}}{}$$

$$= \frac{1}{2}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} + \frac{\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}}{3a\sqrt{1+a^2x^2}} + \frac{\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}}{16a}$$

Mathematica [A] time = 0.10, size = 104, normalized size = 0.56

$$\frac{\sqrt{c\left(a^2x^2+1\right)}\left(16\sinh^{-1}(ax)^2-3\sqrt{2}\sqrt{\sinh^{-1}(ax)}\,\Gamma\left(\frac{3}{2},2\sinh^{-1}(ax)\right)-3\sqrt{2}\sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\frac{3}{2},-2\sinh^{-1}(ax)\right.\right.}{48a\sqrt{a^2x^2+1}\sqrt{\sinh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*(16*ArcSinh[a*x]^2 - 3*Sqrt[2]*Sqrt[-ArcSinh[a*x]]*G
amma[3/2, -2*ArcSinh[a*x]] - 3*Sqrt[2]*Sqrt[ArcSinh[a*x]]*Gamma[3/2, 2*ArcS
inh[a*x]]))/(48*a*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \sqrt{a^2c\,x^2+c}\,\sqrt{\mathrm{arcsinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^2cx^2+c}\,\sqrt{\mathrm{arsinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*sqrt(arcsinh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\mathrm{asinh}\,(a\,x)}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2),x)
[Out] int(asinh(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c\left(a^2x^2+1\right)}\sqrt{\operatorname{asinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)*asinh(a*x)**(1/2),x)
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*sqrt(asinh(a*x)), x)
```

3.474 $\int \frac{\sqrt{\sinh^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=42

$$\frac{2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^{3/2}}{3a\sqrt{a^2cx^2+c}}$$

```
[Out] 2/3*arcsinh(a*x)^(3/2)*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5677, 5675}

$$\frac{2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^{3/2}}{3a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcSinh[a*x]]/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))/(3*a*Sqrt[c + a^2*c*x^2])
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] && !GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\sinh^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx &= \frac{\sqrt{1+a^2x^2}\int \frac{\sqrt{\sinh^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}\\ &= \frac{2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3/2}}{3a\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 42, normalized size = 1.00

$$\frac{2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^{3/2}}{3a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcSinh[a*x]]/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))/(3*a*Sqrt[c + a^2*c*x^2])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arsinh}(ax)}}{\sqrt{a^2cx^2 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(arcsinh(a*x))/sqrt(a^2*c*x^2 + c), x)
```

maple [A] time = 0.06, size = 36, normalized size = 0.86

$$\frac{2 \operatorname{arcsinh}(ax)^{\frac{3}{2}} \sqrt{a^2x^2+1}}{3a\sqrt{c\left(a^2x^2+1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 2/3*arcsinh(a*x)^(3/2)/a/(c*(a^2*x^2+1))^(1/2)*(a^2*x^2+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arsinh}(ax)}}{\sqrt{a^2cx^2 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arcsinh(a*x))/sqrt(a^2*c*x^2 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\operatorname{asinh}(a\,x)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^(1/2)/(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(asinh(a*x)^(1/2)/(c + a^2*c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{asinh}(ax)}}{\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**(1/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(asinh(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

3.475 $\int \frac{\sqrt{\sinh^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=87

$$\frac{x\sqrt{\sinh^{-1}(ax)}}{c\sqrt{a^2cx^2+c}}-\frac{a\sqrt{a^2x^2+1}\,\mathrm{Int}\left(\frac{x}{(a^2x^2+1)\sqrt{\sinh^{-1}(ax)}},x\right)}{2c\sqrt{a^2cx^2+c}}$$

```
[Out] x*arcsinh(a*x)^(1/2)/c/(a^2*c*x^2+c)^(1/2)-1/2*a*(a^2*x^2+1)^(1/2)*Unintegr
able(x/(a^2*x^2+1)/arcsinh(a*x)^(1/2),x)/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{\sinh^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[ArcSinh[a*x]]/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (x*Sqrt[ArcSinh[a*x]])/(c*Sqrt[c + a^2*c*x^2]) - (a*Sqrt[1 + a^2*x^2]*Defer
[Int][x/((1 + a^2*x^2)*Sqrt[ArcSinh[a*x]]), x])/(2*c*Sqrt[c + a^2*c*x^2])
```

Rubi steps

$$\int \frac{\sqrt{\sinh^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx=\frac{x\sqrt{\sinh^{-1}(ax)}}{c\sqrt{c+a^2cx^2}}-\frac{\left(a\sqrt{1+a^2x^2}\right)\int \frac{x}{(1+a^2x^2)\sqrt{\sinh^{-1}(ax)}}\,dx}{2c\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sinh^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[ArcSinh[a*x]]/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] Integrate[Sqrt[ArcSinh[a*x]]/(c + a^2*c*x^2)^(3/2), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arsinh}(ax)}}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(arcsinh(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

maple [A] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
[Out] int(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arsinh}(ax)}}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(arcsinh(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\operatorname{asinh}(a\,x)}}{\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^(1/2)/(c + a^2*c*x^2)^(3/2),x)
[Out] int(asinh(a*x)^(1/2)/(c + a^2*c*x^2)^(3/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{asinh}(ax)}}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**(1/2)/(a**2*c*x**2+c)**(3/2),x)
[Out] Integral(sqrt(asinh(a*x))/(c*(a**2*x**2 + 1))**(3/2), x)
```

3.476 $\int \frac{\sqrt{\sinh^{-1}(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=179

$$-\frac{a\sqrt{a^2x^2+1}\,\text{Int}\left(\frac{x}{\left(a^2x^2+1\right)^2\sqrt{\sinh^{-1}(ax)}},x\right)}{6c^2\sqrt{a^2cx^2+c}}-\frac{a\sqrt{a^2x^2+1}\,\text{Int}\left(\frac{x}{\left(a^2x^2+1\right)\sqrt{\sinh^{-1}(ax)}},x\right)}{3c^2\sqrt{a^2cx^2+c}}+\frac{2x\sqrt{\sinh^{-1}(ax)}}{3c^2\sqrt{a^2cx^2+c}}+\frac{x\sqrt{\sinh^{-1}(a}}{3c\left(a^2cx^2+c\right.}$$

```
[Out] 1/3*x*arcsinh(a*x)^(1/2)/c/(a^2*c*x^2+c)^(3/2)+2/3*x*arcsinh(a*x)^(1/2)/c^2
/(a^2*c*x^2+c)^(1/2)-1/6*a*(a^2*x^2+1)^(1/2)*Unintegrable(x/(a^2*x^2+1)^2/a
rcsinh(a*x)^(1/2),x)/c^2/(a^2*c*x^2+c)^(1/2)-1/3*a*(a^2*x^2+1)^(1/2)*Uninte
grable(x/(a^2*x^2+1)/arcsinh(a*x)^(1/2),x)/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{\sinh^{-1}(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[ArcSinh[a*x]]/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (x*Sqrt[ArcSinh[a*x]])/(3*c*(c + a^2*c*x^2)^(3/2)) + (2*x*Sqrt[ArcSinh[a*x]
])/(3*c^2*Sqrt[c + a^2*c*x^2]) - (a*Sqrt[1 + a^2*x^2]*Defer[Int][x/((1 + a^
2*x^2)^2*Sqrt[ArcSinh[a*x]]), x])/(6*c^2*Sqrt[c + a^2*c*x^2]) - (a*Sqrt[1 +
 a^2*x^2]*Defer[Int][x/((1 + a^2*x^2)*Sqrt[ArcSinh[a*x]]), x])/(3*c^2*Sqrt[
c + a^2*c*x^2])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\sinh^{-1}(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx &= \frac{x\sqrt{\sinh^{-1}(ax)}}{3c\left(c+a^2cx^2\right)^{3/2}}+\frac{2\int \frac{\sqrt{\sinh^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{3c}-\frac{\left(a\sqrt{1+a^2x^2}\right)\int \frac{x}{\left(1+a^2x^2\right)^2\sqrt{\sinh^{-1}(ax)}}\,dx}{6c^2\sqrt{c+a^2cx^2}}\\ &= \frac{x\sqrt{\sinh^{-1}(ax)}}{3c\left(c+a^2cx^2\right)^{3/2}}+\frac{2x\sqrt{\sinh^{-1}(ax)}}{3c^2\sqrt{c+a^2cx^2}}-\frac{\left(a\sqrt{1+a^2x^2}\right)\int \frac{x}{\left(1+a^2x^2\right)^2\sqrt{\sinh^{-1}(ax)}}\,dx}{6c^2\sqrt{c+a^2cx^2}}-\frac{\left(a\sqrt{1+a^2\right.}}{}\end{aligned}$$

Mathematica [A] time = 1.42, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sinh^{-1}(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[ArcSinh[a*x]]/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] Integrate[Sqrt[ArcSinh[a*x]]/(c + a^2*c*x^2)^(5/2), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arsinh}(ax)}}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(arcsinh(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

maple [A] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{\left(a^2c\,x^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arsinh}(ax)}}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arcsinh(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\operatorname{asinh}(a\,x)}}{\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^(1/2)/(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] int(asinh(a*x)^(1/2)/(c + a^2*c*x^2)^(5/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{asinh}(ax)}}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**(1/2)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(sqrt(asinh(a*x))/(c*(a**2*x**2 + 1))**(5/2), x)
```

3.477 $\int \left(c+a^2cx^2\right)^{3/2} \sinh^{-1}(ax)^{3/2} \, dx$

Optimal. Leaf size=449

$$\frac{3\sqrt{\pi}\, c\sqrt{a^2cx^2+c}\,\text{erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{2048a\sqrt{a^2x^2+1}}+\frac{3\sqrt{\frac{\pi}{2}}\, c\sqrt{a^2cx^2+c}\,\text{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{64a\sqrt{a^2x^2+1}}+\frac{3\sqrt{\pi}\, c\sqrt{a^2cx^2+c}\,\text{erfi}\left(2\sqrt{\sin\right.}}{2048a\sqrt{a^2x^2+1}}$$

```
[Out] 1/4*x*(a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(3/2)+3/8*c*x*arcsinh(a*x)^(3/2)*(a^
2*c*x^2+c)^(1/2)+3/20*c*arcsinh(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1
)^(1/2)+3/128*c*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2
+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+3/128*c*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1
/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+3/2048*c*erf(2*arcsinh
(a*x)^(1/2))*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+3/2048*c*erfi
(2*arcsinh(a*x)^(1/2))*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-3/3
2*c*(a^2*x^2+1)^(3/2)*(a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2)/a-27/256*c*(a^
2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2)/a/(a^2*x^2+1)^(1/2)-9/32*a*c*x^2*(a^2*c
*x^2+c)^(1/2)*arcsinh(a*x)^(1/2)/(a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {5684, 5682, 5675, 5663, 5779, 3312, 3307, 2180, 2204, 2205, 5717, 5699}

$$\frac{3\sqrt{\pi}\, c\sqrt{a^2cx^2+c}\,\text{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{2048a\sqrt{a^2x^2+1}}+\frac{3\sqrt{\frac{\pi}{2}}\, c\sqrt{a^2cx^2+c}\,\text{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{64a\sqrt{a^2x^2+1}}+\frac{3\sqrt{\pi}\, c\sqrt{a^2cx^2+c}\,\text{Erfi}\left(2\sqrt{\text{si}\right.}}{2048a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-27*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/(256*a*Sqrt[1 + a^2*x^2]) -
(9*a*c*x^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/(32*Sqrt[1 + a^2*x^2]) -
 (3*c*(1 + a^2*x^2)^(3/2)*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/(32*a) +
(3*c*x*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2))/8 + (x*(c + a^2*c*x^2)^(3/2)
*ArcSinh[a*x]^(3/2))/4 + (3*c*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(5/2))/(20*a
*Sqrt[1 + a^2*x^2]) + (3*c*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erf[2*Sqrt[ArcSinh[
a*x]]])/(2048*a*Sqrt[1 + a^2*x^2]) + (3*c*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Er
f[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(64*a*Sqrt[1 + a^2*x^2]) + (3*c*Sqrt[Pi]*Sqr
t[c + a^2*c*x^2]*Erfi[2*Sqrt[ArcSinh[a*x]]])/(2048*a*Sqrt[1 + a^2*x^2]) + (
3*c*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(64*a*
Sqrt[1 + a^2*x^2])
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
```

```
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^{3/2}\,dx=\frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^{3/2}+\frac{1}{4}(3c)\int\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^{3/2}\,dx-\frac{(3c}{}$$

$$=-\frac{3c\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{32a}+\frac{3}{8}cx\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^{3/2}+$$

$$=-\frac{9acx^2\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{32\sqrt{1+a^2x^2}}-\frac{3c\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{32a}$$

$$=-\frac{9acx^2\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{32\sqrt{1+a^2x^2}}-\frac{3c\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{32a}$$

$$=\frac{9c\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{256a\sqrt{1+a^2x^2}}-\frac{9acx^2\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{32\sqrt{1+a^2x^2}}-\frac{3c\left(1+a^2x^2\right.}{}$$

$$=-\frac{27c\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{256a\sqrt{1+a^2x^2}}-\frac{9acx^2\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{32\sqrt{1+a^2x^2}}-\frac{3c\left(1+a^2\right.}{}$$

$$=-\frac{27c\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{256a\sqrt{1+a^2x^2}}-\frac{9acx^2\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{32\sqrt{1+a^2x^2}}-\frac{3c\left(1+a^2\right.}{}$$

$$=-\frac{27c\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{256a\sqrt{1+a^2x^2}}-\frac{9acx^2\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{32\sqrt{1+a^2x^2}}-\frac{3c\left(1+a^2\right.}{}$$

$$=-\frac{27c\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{256a\sqrt{1+a^2x^2}}-\frac{9acx^2\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}}{32\sqrt{1+a^2x^2}}-\frac{3c\left(1+a^2\right.}{}$$

Mathematica [A] time = 0.32, size = 186, normalized size = 0.41

$$\frac{c\sqrt{a^2cx^2+c}\left(60\sqrt{2\pi}\sqrt{\sinh^{-1}(ax)}\operatorname{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)+60\sqrt{2\pi}\sqrt{\sinh^{-1}(ax)}\operatorname{erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)+384\,\mathrm{s}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(3/2),x]
```

```
[Out] (c*Sqrt[c + a^2*c*x^2]*(384*ArcSinh[a*x]^3 - 480*ArcSinh[a*x]*Cosh[2*ArcSin
h[a*x]] + 60*Sqrt[2*Pi]*Sqrt[ArcSinh[a*x]]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]]
+ 60*Sqrt[2*Pi]*Sqrt[ArcSinh[a*x]]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]] + 5*Sqr
t[-ArcSinh[a*x]]*Gamma[5/2, -4*ArcSinh[a*x]] - 5*Sqrt[ArcSinh[a*x]]*Gamma[5
/2, 4*ArcSinh[a*x]] + 640*ArcSinh[a*x]^2*Sinh[2*ArcSinh[a*x]]))/(2560*a*Sqr
t[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \operatorname{arcsinh}\left(a x \right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int {\left(a^{2} c x^{2} + c\right)}^{\frac{3}{2}} \operatorname{arsinh}\left(a x\right)^{\frac{3}{2}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*arcsinh(a*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\mathrm{asinh}\left(a\,x\right)}^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int(asinh(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*asinh(a*x)**(3/2),x)
```

```
[Out] Timed out
```

3.478 $\int \sqrt{c+a^2cx^2} \sinh^{-1}(ax)^{3/2} dx$

Optimal. Leaf size=271

$$\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\text{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{64a\sqrt{a^2x^2+1}}+\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\text{erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{64a\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\sinh^{-1}(ax)^{5/2}}{5a\sqrt{a^2x^2+1}}$$

```
[Out] 1/2*x*arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)+1/5*arcsinh(a*x)^(5/2)*(a^2*c*
x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+3/128*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1
/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+3/128*erfi(2^(1/2)*arc
sinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-3
/16*(a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2)/a/(a^2*x^2+1)^(1/2)-3/8*a*x^2*(a
^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2)/(a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5682, 5675, 5663, 5779, 3312, 3307, 2180, 2204, 2205}

$$\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\text{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{64a\sqrt{a^2x^2+1}}+\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\text{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{64a\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\sinh^{-1}(ax)^{5/}}{5a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-3*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/(16*a*Sqrt[1 + a^2*x^2]) - (3*a
*x^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/(8*Sqrt[1 + a^2*x^2]) + (x*Sqr
t[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2))/2 + (Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^
(5/2))/(5*a*Sqrt[1 + a^2*x^2]) + (3*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt
[2]*Sqrt[ArcSinh[a*x]]])/(64*a*Sqrt[1 + a^2*x^2]) + (3*Sqrt[Pi/2]*Sqrt[c +
a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(64*a*Sqrt[1 + a^2*x^2])
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}\,dx = \frac{1}{2}x\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2} + \frac{\sqrt{c+a^2cx^2}\int\frac{\sinh^{-1}(ax)^{3/2}}{\sqrt{1+a^2x^2}}dx}{2\sqrt{1+a^2x^2}} - \frac{\left(3a\sqrt{c+a^2cx\right.}{4}$$

$$= -\frac{3ax^2\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2} + \frac{\sqrt{c+a^2cx}}{5a\sqrt{}}$$

$$= -\frac{3ax^2\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2} + \frac{\sqrt{c+a^2cx}}{5a\sqrt{}}$$

$$= -\frac{3ax^2\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2} + \frac{\sqrt{c+a^2cx}}{5a\sqrt{}}$$

$$= -\frac{3\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{16a\sqrt{1+a^2x^2}} - \frac{3ax^2\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2c}$$

$$= -\frac{3\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{16a\sqrt{1+a^2x^2}} - \frac{3ax^2\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2c}$$

$$= -\frac{3\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{16a\sqrt{1+a^2x^2}} - \frac{3ax^2\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2c}$$

$$= -\frac{3\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{16a\sqrt{1+a^2x^2}} - \frac{3ax^2\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2c}$$

Mathematica [A] time = 0.27, size = 126, normalized size = 0.46

$$\frac{\sqrt{c\left(a^2x^2+1\right)}\left(15\sqrt{2\pi}\operatorname{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)+15\sqrt{2\pi}\operatorname{erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)+8\sqrt{\sinh^{-1}(ax)}\left(4\sinh^{-1}(a\right.\right.}{640a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2),x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*(15*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]] + 15*
Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]] + 8*Sqrt[ArcSinh[a*x]]*(-15*Cos
h[2*ArcSinh[a*x]] + 4*ArcSinh[a*x]*(4*ArcSinh[a*x] + 5*Sinh[2*ArcSinh[a*x]]
))))/(640*a*Sqrt[1 + a^2*x^2])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \operatorname{arcsinh}(ax)^{\frac{3}{2}} \sqrt{a^2 c\, x^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^2 c x^2 + c}\, \operatorname{arsinh}(ax)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{asinh}(a\, x)^{3/2} \sqrt{c\, a^2\, x^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(asinh(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c\left(a^2 x^2 + 1\right)}\, \operatorname{asinh}^{\frac{3}{2}}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**(3/2)*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*asinh(a*x)**(3/2), x)
```

3.479 $\int \frac{\sinh^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=42

$$\frac{2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^{5/2}}{5a\sqrt{a^2cx^2+c}}$$

```
[Out] 2/5*arcsinh(a*x)^(5/2)*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5677, 5675}

$$\frac{2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^{5/2}}{5a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^(3/2)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(5/2))/(5*a*Sqrt[c + a^2*c*x^2])
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] && !GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx &= \frac{\sqrt{1+a^2x^2}\int \frac{\sinh^{-1}(ax)^{3/2}}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}\\ &= \frac{2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{5/2}}{5a\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 42, normalized size = 1.00

$$\frac{2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^{5/2}}{5a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^(3/2)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(5/2))/(5*a*Sqrt[c + a^2*c*x^2])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)^{\frac{3}{2}}}{\sqrt{a^2cx^2 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^(3/2)/sqrt(a^2*c*x^2 + c), x)
```

maple [A] time = 0.06, size = 36, normalized size = 0.86

$$\frac{2 \operatorname{arcsinh}(ax)^{\frac{5}{2}} \sqrt{a^2x^2+1}}{5a\sqrt{c\left(a^2x^2+1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 2/5*arcsinh(a*x)^(5/2)/a/(c*(a^2*x^2+1))^(1/2)*(a^2*x^2+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)^{\frac{3}{2}}}{\sqrt{a^2cx^2 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^(3/2)/sqrt(a^2*c*x^2 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{asinh}(a\,x)^{3/2}}{\sqrt{c\,a^2\,x^2 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^(3/2)/(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(asinh(a*x)^(3/2)/(c + a^2*c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^{\frac{3}{2}}(ax)}{\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(asinh(a*x)**(3/2)/sqrt(c*(a**2*x**2 + 1)), x)
```

3.480 $\int \frac{\sinh^{-1}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=87

$$\frac{x\sinh^{-1}(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}-\frac{3a\sqrt{a^2x^2+1}\,\text{Int}\left(\frac{x\sqrt{\sinh^{-1}(ax)}}{a^2x^2+1},x\right)}{2c\sqrt{a^2cx^2+c}}$$

```
[Out] x*arcsinh(a*x)^(3/2)/c/(a^2*c*x^2+c)^(1/2)-3/2*a*(a^2*x^2+1)^(1/2)*Unintegr
able(x*arcsinh(a*x)^(1/2)/(a^2*x^2+1),x)/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sinh^{-1}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcSinh[a*x]^(3/2)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (x*ArcSinh[a*x]^(3/2))/(c*Sqrt[c + a^2*c*x^2]) - (3*a*Sqrt[1 + a^2*x^2]*Def
er[Int][(x*Sqrt[ArcSinh[a*x]])/(1 + a^2*x^2), x])/(2*c*Sqrt[c + a^2*c*x^2])
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx=\frac{x\sinh^{-1}(ax)^{3/2}}{c\sqrt{c+a^2cx^2}}-\frac{\left(3a\sqrt{1+a^2x^2}\right)\int \frac{x\sqrt{\sinh^{-1}(ax)}}{1+a^2x^2}\,dx}{2c\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{-1}(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcSinh[a*x]^(3/2)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] Integrate[ArcSinh[a*x]^(3/2)/(c + a^2*c*x^2)^(3/2), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)^{\frac{3}{2}}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^(3/2)/(a^2*c*x^2 + c)^(3/2), x)
```

maple [A] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsinh}(ax)^{\frac{3}{2}}}{\left(a^{2} c\, x^{2}+c\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)^{\frac{3}{2}}}{\left(a^{2} c x^{2}+c\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^(3/2)/(a^2*c*x^2 + c)^(3/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}(a\, x)^{3/2}}{\left(c\, a^2\, x^2+c\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^(3/2)/(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int(asinh(a*x)^(3/2)/(c + a^2*c*x^2)^(3/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^{\frac{3}{2}}(ax)}{\left(c\left(a^{2} x^{2}+1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**(3/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(asinh(a*x)**(3/2)/(c*(a**2*x**2 + 1))**(3/2), x)
```

3.481 $\int \left(c+a^2cx^2\right)^{3/2} \sinh^{-1}(ax)^{5/2}\,dx$

Optimal. Leaf size=514

$$\frac{15\sqrt{\pi}\,c\sqrt{a^2cx^2+c}\,\mathrm{erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{16384a\sqrt{a^2x^2+1}}+\frac{15\sqrt{\frac{\pi}{2}}\,c\sqrt{a^2cx^2+c}\,\mathrm{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{256a\sqrt{a^2x^2+1}}-\frac{15\sqrt{\pi}\,c\sqrt{a^2cx^2+c}\,\mathrm{erfi}\left(\right.}{16384a\sqrt{a^2x^2}}$$

```
[Out] 1/4*x*(a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(5/2)-5/32*c*(a^2*x^2+1)^(3/2)*arcsi
nh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/a+3/8*c*x*arcsinh(a*x)^(5/2)*(a^2*c*x^2+c
)^(1/2)-45/256*c*arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)
-15/32*a*c*x^2*arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/(a^2*x^2+1)^(1/2)+3/2
8*c*arcsinh(a*x)^(7/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+15/512*c*erf
(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^
2+1)^(1/2)-15/512*c*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*
c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+15/16384*c*erf(2*arcsinh(a*x)^(1/2))*Pi^
(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-15/16384*c*erfi(2*arcsinh(a*x
)^(1/2))*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+225/512*c*x*(a^2*
c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2)+15/256*c*x*(a^2*x^2+1)*(a^2*c*x^2+c)^(1/2
)*arcsinh(a*x)^(1/2)
```

Rubi [A] time = 0.76, antiderivative size = 514, normalized size of antiderivative = 1.00, number of steps used = 39, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {5684, 5682, 5675, 5663, 5758, 5669, 5448, 12, 3308, 2180, 2204, 2205, 5717, 5779}

$$\frac{15\sqrt{\pi}\,c\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{16384a\sqrt{a^2x^2+1}}+\frac{15\sqrt{\frac{\pi}{2}}\,c\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{256a\sqrt{a^2x^2+1}}-\frac{15\sqrt{\pi}\,c\sqrt{a^2cx^2+c}\,\mathrm{Erfi}\left(\right.}{16384a\sqrt{a^2x}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(5/2),x]
```

```
[Out] (225*c*x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/512 + (15*c*x*(1 + a^2*x^2
)*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/256 - (45*c*Sqrt[c + a^2*c*x^2]*A
rcSinh[a*x]^(3/2))/(256*a*Sqrt[1 + a^2*x^2]) - (15*a*c*x^2*Sqrt[c + a^2*c*x
^2]*ArcSinh[a*x]^(3/2))/(32*Sqrt[1 + a^2*x^2]) - (5*c*(1 + a^2*x^2)^(3/2)*S
qrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2))/(32*a) + (3*c*x*Sqrt[c + a^2*c*x^2]*
ArcSinh[a*x]^(5/2))/8 + (x*(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(5/2))/4 + (3
*c*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(7/2))/(28*a*Sqrt[1 + a^2*x^2]) + (15*c
*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erf[2*Sqrt[ArcSinh[a*x]]])/(16384*a*Sqrt[1 +
a^2*x^2]) + (15*c*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a
*x]]])/(256*a*Sqrt[1 + a^2*x^2]) - (15*c*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erfi[
2*Sqrt[ArcSinh[a*x]]])/(16384*a*Sqrt[1 + a^2*x^2]) - (15*c*Sqrt[Pi/2]*Sqrt[
c + a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(256*a*Sqrt[1 + a^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
```

```
GtQ[p, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^{5/2}\,dx = \frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^{5/2}+\frac{1}{4}(3c)\int \sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^{5/2}\,dx - \frac{(5c}{}$$

$$= -\frac{5c\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^{3/2}}{32a}+\frac{3}{8}cx\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^{5/2}$$

$$= \frac{15}{256}cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)} - \frac{15acx^2\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^{3}}{32\sqrt{1+a^2x^2}}$$

$$= \frac{225}{512}cx\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}+\frac{15}{256}cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}$$

$$= \frac{225}{512}cx\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}+\frac{15}{256}cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}$$

$$= \frac{225}{512}cx\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}+\frac{15}{256}cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}$$

$$= \frac{225}{512}cx\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}+\frac{15}{256}cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}$$

$$= \frac{225}{512}cx\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}+\frac{15}{256}cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}$$

$$= \frac{225}{512}cx\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}+\frac{15}{256}cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}$$

$$= \frac{225}{512}cx\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}+\frac{15}{256}cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}\,\sqrt{\sinh^{-1}(ax)}$$

Mathematica [A] time = 0.35, size = 201, normalized size = 0.39

$$\frac{c\sqrt{a^2cx^2+c}\left(420\sqrt{2\pi}\sqrt{\sinh^{-1}(ax)}\operatorname{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)-420\sqrt{2\pi}\sqrt{\sinh^{-1}(ax)}\operatorname{erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)+15\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(5/2),x]
```

```
[Out] (c*Sqrt[c + a^2*c*x^2]*(1536*ArcSinh[a*x]^4 - 4480*ArcSinh[a*x]^2*Cosh[2*Ar
cSinh[a*x]] + 420*Sqrt[2*Pi]*Sqrt[ArcSinh[a*x]]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*
x]]] - 420*Sqrt[2*Pi]*Sqrt[ArcSinh[a*x]]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]] -
 7*Sqrt[-ArcSinh[a*x]]*Gamma[7/2, -4*ArcSinh[a*x]] - 7*Sqrt[ArcSinh[a*x]]*G
amma[7/2, 4*ArcSinh[a*x]] + 3360*ArcSinh[a*x]*Sinh[2*ArcSinh[a*x]] + 3584*A
rcSinh[a*x]^3*Sinh[2*ArcSinh[a*x]]))/(14336*a*Sqrt[1 + a^2*x^2]*Sqrt[ArcSin
h[a*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \operatorname{arcsinh}\left(a x \right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(5/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int {\left(a^{2} c x^{2} + c\right)}^{\frac{3}{2}} \operatorname{arsinh}\left(a x\right)^{\frac{5}{2}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*arcsinh(a*x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\mathrm{asinh}\left(a\,x\right)}^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*asinh(a*x)**(5/2),x)
```

```
[Out] Timed out
```

3.482 $\int \sqrt{c+a^2cx^2} \sinh^{-1}(ax)^{5/2}\,dx$

Optimal. Leaf size=298

$$\frac{15\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\mathrm{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{256a\sqrt{a^2x^2+1}}-\frac{15\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\mathrm{erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{256a\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\sinh^{-1}(ax)^{7/2}}{7a\sqrt{a^2x^2+1}}$$

```
[Out] 1/2*x*arcsinh(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2)-5/16*arcsinh(a*x)^(3/2)*(a^2*c
*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-5/8*a*x^2*arcsinh(a*x)^(3/2)*(a^2*c*x^2+c
)^(1/2)/(a^2*x^2+1)^(1/2)+1/7*arcsinh(a*x)^(7/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2
*x^2+1)^(1/2)+15/512*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*
c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-15/512*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*
2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+15/32*x*(a^2*c*x^2
+c)^(1/2)*arcsinh(a*x)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {5682, 5675, 5663, 5758, 5669, 5448, 12, 3308, 2180, 2204, 2205}

$$\frac{15\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{256a\sqrt{a^2x^2+1}}-\frac{15\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{256a\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\sinh^{-1}(ax)^{7/2}}{7a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(5/2),x]
```

```
[Out] (15*x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/32 - (5*Sqrt[c + a^2*c*x^2]*A
rcSinh[a*x]^(3/2))/(16*a*Sqrt[1 + a^2*x^2]) - (5*a*x^2*Sqrt[c + a^2*c*x^2]*
ArcSinh[a*x]^(3/2))/(8*Sqrt[1 + a^2*x^2]) + (x*Sqrt[c + a^2*c*x^2]*ArcSinh[
a*x]^(5/2))/2 + (Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(7/2))/(7*a*Sqrt[1 + a^2*
x^2]) + (15*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])
/(256*a*Sqrt[1 + a^2*x^2]) - (15*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[2
]*Sqrt[ArcSinh[a*x]]])/(256*a*Sqrt[1 + a^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int \sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^{5/2}\,dx = \frac{1}{2}x\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^{5/2} + \frac{\sqrt{c+a^2cx^2}\int\frac{\sinh^{-1}(ax)^{5/2}}{\sqrt{1+a^2x^2}}dx}{2\sqrt{1+a^2x^2}} - \frac{\left(5a\sqrt{c+a^2cx^2}\right)}{4\sqrt{1}}$$

$$= -\frac{5ax^2\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^{3/2}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^{5/2} + \frac{\sqrt{c+a^2cx^2}}{7a\sqrt{1}}$$

$$= \frac{15}{32}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} - \frac{5ax^2\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^{3/2}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2cx^2}$$

$$= \frac{15}{32}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} - \frac{5\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^{3/2}}{16a\sqrt{1+a^2x^2}} - \frac{5ax^2\sqrt{c+a^2cx^2}}{8\sqrt{1+a}}$$

$$= \frac{15}{32}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} - \frac{5\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^{3/2}}{16a\sqrt{1+a^2x^2}} - \frac{5ax^2\sqrt{c+a^2cx^2}}{8\sqrt{1+a}}$$

$$= \frac{15}{32}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} - \frac{5\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^{3/2}}{16a\sqrt{1+a^2x^2}} - \frac{5ax^2\sqrt{c+a^2cx^2}}{8\sqrt{1+a}}$$

$$= \frac{15}{32}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} - \frac{5\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^{3/2}}{16a\sqrt{1+a^2x^2}} - \frac{5ax^2\sqrt{c+a^2cx^2}}{8\sqrt{1+a}}$$

$$= \frac{15}{32}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} - \frac{5\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^{3/2}}{16a\sqrt{1+a^2x^2}} - \frac{5ax^2\sqrt{c+a^2cx^2}}{8\sqrt{1+a}}$$

$$= \frac{15}{32}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} - \frac{5\sqrt{c+a^2cx^2}\,\sinh^{-1}(ax)^{3/2}}{16a\sqrt{1+a^2x^2}} - \frac{5ax^2\sqrt{c+a^2cx^2}}{8\sqrt{1+a}}$$

Mathematica [A] time = 0.29, size = 135, normalized size = 0.45

$$\frac{\sqrt{c\left(a^2x^2+1\right)}\left(105\sqrt{2\pi}\,\mathrm{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)-105\sqrt{2\pi}\,\mathrm{erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)+8\sqrt{\sinh^{-1}(ax)}\left(64\sinh^{-1}(a\right.\right.}{3584a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(5/2),x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*(105*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]] - 10
5*Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]] + 8*Sqrt[ArcSinh[a*x]]*(64*Ar
cSinh[a*x]^3 - 140*ArcSinh[a*x]*Cosh[2*ArcSinh[a*x]] + 7*(15 + 16*ArcSinh[a
*x]^2)*Sinh[2*ArcSinh[a*x]])))/(3584*a*Sqrt[1 + a^2*x^2])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \operatorname{arcsinh}(ax)^{\frac{5}{2}} \sqrt{a^{2} c\, x^{2}+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(arcsinh(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^{2}cx^{2}+c}\, \operatorname{arsinh}(ax)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{asinh}(a\,x)^{5/2}\,\sqrt{c\,a^2\,x^2+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**(5/2)*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

3.483 $\int \frac{\sinh^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}} dx$

Optimal. Leaf size=42

$$\frac{2\sqrt{a^2x^2+1}\sinh^{-1}(ax)^{7/2}}{7a\sqrt{a^2cx^2+c}}$$

```
[Out] 2/7*arcsinh(a*x)^(7/2)*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5677, 5675}

$$\frac{2\sqrt{a^2x^2+1}\sinh^{-1}(ax)^{7/2}}{7a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^(5/2)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(7/2))/(7*a*Sqrt[c + a^2*c*x^2])
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] && !GtQ[d, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\sinh^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}} dx &= \frac{\sqrt{1+a^2x^2}\int \frac{\sinh^{-1}(ax)^{5/2}}{\sqrt{1+a^2x^2}} dx}{\sqrt{c+a^2cx^2}} \\ &= \frac{2\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{7/2}}{7a\sqrt{c+a^2cx^2}} \end{aligned}$$

Mathematica [A] time = 0.04, size = 42, normalized size = 1.00

$$\frac{2\sqrt{a^2x^2+1}\sinh^{-1}(ax)^{7/2}}{7a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^(5/2)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(7/2))/(7*a*Sqrt[c + a^2*c*x^2])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)^{\frac{5}{2}}}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^(5/2)/sqrt(a^2*c*x^2 + c), x)
```

maple [**A**] time = 0.06, size = 36, normalized size = 0.86

$$\frac{2\operatorname{arcsinh}(ax)^{\frac{7}{2}}\sqrt{a^2x^2+1}}{7a\sqrt{c\left(a^2x^2+1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 2/7*arcsinh(a*x)^(7/2)/a/(c*(a^2*x^2+1))^(1/2)*(a^2*x^2+1)^(1/2)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)^{\frac{5}{2}}}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^(5/2)/sqrt(a^2*c*x^2 + c), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{asinh}(a\,x)^{5/2}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^(5/2)/(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(asinh(a*x)^(5/2)/(c + a^2*c*x^2)^(1/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

3.484 $\int \frac{\sinh^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=87

$$\frac{x\sinh^{-1}(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}-\frac{5a\sqrt{a^2x^2+1}\,\mathrm{Int}\left(\frac{x\sinh^{-1}(ax)^{3/2}}{a^2x^2+1},x\right)}{2c\sqrt{a^2cx^2+c}}$$

```
[Out] x*arcsinh(a*x)^(5/2)/c/(a^2*c*x^2+c)^(1/2)-5/2*a*(a^2*x^2+1)^(1/2)*Unintegr
able(x*arcsinh(a*x)^(3/2)/(a^2*x^2+1),x)/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sinh^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcSinh[a*x]^(5/2)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (x*ArcSinh[a*x]^(5/2))/(c*Sqrt[c + a^2*c*x^2]) - (5*a*Sqrt[1 + a^2*x^2]*Def
er[Int][(x*ArcSinh[a*x]^(3/2))/(1 + a^2*x^2), x])/(2*c*Sqrt[c + a^2*c*x^2])
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx=\frac{x\sinh^{-1}(ax)^{5/2}}{c\sqrt{c+a^2cx^2}}-\frac{\left(5a\sqrt{1+a^2x^2}\right)\int \frac{x\sinh^{-1}(ax)^{3/2}}{1+a^2x^2}\,dx}{2c\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcSinh[a*x]^(5/2)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] Integrate[ArcSinh[a*x]^(5/2)/(c + a^2*c*x^2)^(3/2), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)^{\frac{5}{2}}}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2 + c)^(3/2), x)
```

maple [A] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsinh}(ax)^{\frac{5}{2}}}{\left(a^{2} c\, x^{2}+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)^{\frac{5}{2}}}{\left(a^{2} c x^{2}+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2 + c)^(3/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}(a\,x)^{5/2}}{\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^(5/2)/(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int(asinh(a*x)^(5/2)/(c + a^2*c*x^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**(5/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

3.485 $\int \left(a^2+x^2\right)^{3/2} \sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\, dx$

Optimal. Leaf size=309

$$\frac{3}{8}a^2x\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}+\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}+\frac{\sqrt{\pi}a^3\sqrt{a^2+x^2}\,\text{erf}\left(2\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{256\sqrt{\frac{x^2}{a^2}+1}}+\frac{\sqrt{\frac{\pi}{2}}a^3\sqrt{a^2+}$$

[Out] 1/4*a^3*arcsinh(x/a)^(3/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)+1/32*a^3*erf(2^(1/2)*arcsinh(x/a)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)-1/32*a^3*erfi(2^(1/2)*arcsinh(x/a)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)+1/256*a^3*erf(2*arcsinh(x/a)^(1/2))*Pi^(1/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)-1/256*a^3*erfi(2*arcsinh(x/a)^(1/2))*Pi^(1/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)+1/4*x*(a^2+x^2)^(3/2)*arcsinh(x/a)^(1/2)+3/8*a^2*x*(a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2)

Rubi [A] time = 0.36, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5684, 5682, 5675, 5669, 5448, 12, 3308, 2180, 2204, 2205, 5779}

$$\frac{\sqrt{\pi}a^3\sqrt{a^2+x^2}\,\text{Erf}\left(2\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{256\sqrt{\frac{x^2}{a^2}+1}}+\frac{\sqrt{\frac{\pi}{2}}a^3\sqrt{a^2+x^2}\,\text{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{16\sqrt{\frac{x^2}{a^2}+1}}-\frac{\sqrt{\pi}a^3\sqrt{a^2+x^2}\,\text{Erfi}\left(2\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{256\sqrt{\frac{x^2}{a^2}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + x^2)^(3/2)*Sqrt[ArcSinh[x/a]],x]
```

```
[Out] (3*a^2*x*Sqrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]])/8 + (x*(a^2 + x^2)^(3/2)*Sqrt[
ArcSinh[x/a]])/4 + (a^3*Sqrt[a^2 + x^2]*ArcSinh[x/a]^(3/2))/(4*Sqrt[1 + x^2
/a^2]) + (a^3*Sqrt[Pi]*Sqrt[a^2 + x^2]*Erf[2*Sqrt[ArcSinh[x/a]]])/(256*Sqrt
[1 + x^2/a^2]) + (a^3*Sqrt[Pi/2]*Sqrt[a^2 + x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[x
/a]]])/(16*Sqrt[1 + x^2/a^2]) - (a^3*Sqrt[Pi]*Sqrt[a^2 + x^2]*Erfi[2*Sqrt[A
rcSinh[x/a]]])/(256*Sqrt[1 + x^2/a^2]) - (a^3*Sqrt[Pi/2]*Sqrt[a^2 + x^2]*Er
fi[Sqrt[2]*Sqrt[ArcSinh[x/a]]])/(16*Sqrt[1 + x^2/a^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \left(a^2+x^2\right)^{3/2} \sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\, dx = \frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}\left(3a^2\right)\int \sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\, dx - \frac{\left(a\sqrt{a\right.}}{}$$

$$= \frac{3}{8}a^2x\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} - \frac{\left(3a\sqrt{a^2+x^2}\right)}{16\sqrt{}}$$

$$= \frac{3}{8}a^2x\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{a^3\sqrt{a^2+x^2}\ \mathrm{si}}{4\sqrt{1+}}$$

$$= \frac{3}{8}a^2x\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{a^3\sqrt{a^2+x^2}\ \mathrm{si}}{4\sqrt{1+}}$$

$$= \frac{3}{8}a^2x\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{a^3\sqrt{a^2+x^2}\ \mathrm{si}}{4\sqrt{1+}}$$

$$= \frac{3}{8}a^2x\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{a^3\sqrt{a^2+x^2}\ \mathrm{si}}{4\sqrt{1+}}$$

$$= \frac{3}{8}a^2x\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{a^3\sqrt{a^2+x^2}\ \mathrm{si}}{4\sqrt{1+}}$$

$$= \frac{3}{8}a^2x\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{a^3\sqrt{a^2+x^2}\ \mathrm{si}}{4\sqrt{1+}}$$

Mathematica [A] time = 0.18, size = 156, normalized size = 0.50

$$\frac{a^3\sqrt{a^2+x^2}\left(-\sqrt{-\sinh^{-1}\left(\frac{x}{a}\right)}\,\Gamma\left(\frac{3}{2},-4\sinh^{-1}\left(\frac{x}{a}\right)\right) - 8\sqrt{2}\sqrt{-\sinh^{-1}\left(\frac{x}{a}\right)}\,\Gamma\left(\frac{3}{2},-2\sinh^{-1}\left(\frac{x}{a}\right)\right) + \sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\left(32\right.\right.}{128\sqrt{\frac{x^2}{a^2}+1}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a^2 + x^2)^(3/2)*Sqrt[ArcSinh[x/a]],x]
```

```
[Out] (a^3*Sqrt[a^2 + x^2]*(-(Sqrt[-ArcSinh[x/a]]*Gamma[3/2, -4*ArcSinh[x/a]]) -
8*Sqrt[2]*Sqrt[-ArcSinh[x/a]]*Gamma[3/2, -2*ArcSinh[x/a]] + Sqrt[ArcSinh[x/
a]]*(32*ArcSinh[x/a]^(3/2) - 8*Sqrt[2]*Gamma[3/2, 2*ArcSinh[x/a]] - Gamma[3
/2, 4*ArcSinh[x/a]])))/(128*Sqrt[1 + x^2/a^2]*Sqrt[ArcSinh[x/a]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+x^2)^(3/2)*arcsinh(x/a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + x^2\right)^{\frac{3}{2}} \sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+x^2)^(3/2)*arcsinh(x/a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2 + x^2)^(3/2)*sqrt(arcsinh(x/a)), x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \left(a^2 + x^2\right)^{\frac{3}{2}} \sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+x^2)^(3/2)*arcsinh(x/a)^(1/2),x)
```

```
[Out] int((a^2+x^2)^(3/2)*arcsinh(x/a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + x^2\right)^{\frac{3}{2}} \sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+x^2)^(3/2)*arcsinh(x/a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a^2 + x^2)^(3/2)*sqrt(arcsinh(x/a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}\, \left(a^2 + x^2\right)^{3/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(x/a)^(1/2)*(a^2 + x^2)^(3/2),x)
```

```
[Out] int(asinh(x/a)^(1/2)*(a^2 + x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + x^2\right)^{\frac{3}{2}} \sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+x**2)**(3/2)*asinh(x/a)**(1/2),x)
```

```
[Out] Integral((a**2 + x**2)**(3/2)*sqrt(asinh(x/a)), x)
```

3.486 $\int \sqrt{a^2+x^2} \sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} \, dx$

Optimal. Leaf size=176

$$\frac{\sqrt{\frac{\pi}{2}}\, a\sqrt{a^2+x^2}\,\mathrm{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{16\sqrt{\frac{x^2}{a^2}+1}}-\frac{\sqrt{\frac{\pi}{2}}\, a\sqrt{a^2+x^2}\,\mathrm{erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{16\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\,\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2
$$

```
[Out] 1/3*a*arcsinh(x/a)^(3/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)+1/32*a*erf(2^(1/
2)*arcsinh(x/a)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)-1
/32*a*erfi(2^(1/2)*arcsinh(x/a)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2+x^2)^(1/2)/(1+
x^2/a^2)^(1/2)+1/2*x*(a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {5682, 5675, 5669, 5448, 12, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}}\, a\sqrt{a^2+x^2}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{16\sqrt{\frac{x^2}{a^2}+1}}-\frac{\sqrt{\frac{\pi}{2}}\, a\sqrt{a^2+x^2}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{16\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\,\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a
$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]],x]
```

```
[Out] (x*Sqrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]])/2 + (a*Sqrt[a^2 + x^2]*ArcSinh[x/a]^
(3/2))/(3*Sqrt[1 + x^2/a^2]) + (a*Sqrt[Pi/2]*Sqrt[a^2 + x^2]*Erf[Sqrt[2]*Sq
rt[ArcSinh[x/a]]])/(16*Sqrt[1 + x^2/a^2]) - (a*Sqrt[Pi/2]*Sqrt[a^2 + x^2]*E
rfi[Sqrt[2]*Sqrt[ArcSinh[x/a]]])/(16*Sqrt[1 + x^2/a^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rubi steps

$$\int \sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\,dx = \frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{\sqrt{a^2+x^2}\int\frac{\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{\sqrt{1+\frac{x^2}{a^2}}}\,dx}{2\sqrt{1+\frac{x^2}{a^2}}} - \frac{\sqrt{a^2+x^2}\int\frac{x}{\sqrt{\sinh^{-1}}}}{4a\sqrt{1+\frac{x^2}{a^2}}}$$

$$= \frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{a\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{1+\frac{x^2}{a^2}}} - \frac{\left(a\sqrt{a^2+x^2}\right)\text{Subst}(}{}$$

$$= \frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{a\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{1+\frac{x^2}{a^2}}} - \frac{\left(a\sqrt{a^2+x^2}\right)\text{Subst}(}{4}$$

$$= \frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{a\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{1+\frac{x^2}{a^2}}} - \frac{\left(a\sqrt{a^2+x^2}\right)\text{Subst}(}{8}$$

$$= \frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{a\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{1+\frac{x^2}{a^2}}} + \frac{\left(a\sqrt{a^2+x^2}\right)\text{Subst}(}{16\sqrt{}}$$

$$= \frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{a\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{1+\frac{x^2}{a^2}}} + \frac{\left(a\sqrt{a^2+x^2}\right)\text{Subst}(}{8}$$

$$= \frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{a\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{1+\frac{x^2}{a^2}}} + \frac{a\sqrt{\frac{\pi}{2}}\sqrt{a^2+x^2}\,\text{erf}(\sqrt{}}{16\sqrt{1+}}$$

Mathematica [A] time = 0.09, size = 110, normalized size = 0.62

$$\frac{a\sqrt{a^2+x^2}\left(16\sinh^{-1}\left(\frac{x}{a}\right)^2-3\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\,\Gamma\left(\frac{3}{2},2\sinh^{-1}\left(\frac{x}{a}\right)\right)-3\sqrt{2}\sqrt{-\sinh^{-1}\left(\frac{x}{a}\right)}\,\Gamma\left(\frac{3}{2},-2\sinh^{-1}\left(\frac{x}{a}\right)\right)\right)}{48\sqrt{\frac{x^2}{a^2}+1}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]],x]
```

```
[Out] (a*Sqrt[a^2 + x^2]*(16*ArcSinh[x/a]^2 - 3*Sqrt[2]*Sqrt[-ArcSinh[x/a]]*Gamma
[3/2, -2*ArcSinh[x/a]] - 3*Sqrt[2]*Sqrt[ArcSinh[x/a]]*Gamma[3/2, 2*ArcSinh[
x/a]]))/(48*Sqrt[1 + x^2/a^2]*Sqrt[ArcSinh[x/a]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^2+x^2}\sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2 + x^2)*sqrt(arcsinh(x/a)), x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \sqrt{a^2+x^2}\,\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2),x)
```

```
[Out] int((a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^2+x^2}\sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2 + x^2)*sqrt(arcsinh(x/a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}\,\sqrt{a^2+x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(x/a)^(1/2)*(a^2 + x^2)^(1/2),x)
```

```
[Out] int(asinh(x/a)^(1/2)*(a^2 + x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^2 + x^2} \sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+x**2)**(1/2)*asinh(x/a)**(1/2),x)
```

```
[Out] Integral(sqrt(a**2 + x**2)*sqrt(asinh(x/a)), x)
```

3.487 $\int \frac{\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx$

Optimal. Leaf size=39

$$\frac{2a\sqrt{\frac{x^2}{a^2}+1}\,\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{a^2+x^2}}$$

```
[Out] 2/3*a*arcsinh(x/a)^(3/2)*(1+x^2/a^2)^(1/2)/(a^2+x^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5677, 5675}

$$\frac{2a\sqrt{\frac{x^2}{a^2}+1}\,\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{a^2+x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcSinh[x/a]]/Sqrt[a^2 + x^2],x]
```

```
[Out] (2*a*Sqrt[1 + x^2/a^2]*ArcSinh[x/a]^(3/2))/(3*Sqrt[a^2 + x^2])
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] && !GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx &= \frac{\sqrt{1+\frac{x^2}{a^2}}\int \frac{\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{\sqrt{1+\frac{x^2}{a^2}}}\,dx}{\sqrt{a^2+x^2}}\\ &= \frac{2a\sqrt{1+\frac{x^2}{a^2}}\,\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{a^2+x^2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 39, normalized size = 1.00

$$\frac{2a\sqrt{\frac{x^2}{a^2}+1}\,\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{a^2+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcSinh[x/a]]/Sqrt[a^2 + x^2],x]
```

```
[Out] (2*a*Sqrt[1 + x^2/a^2]*ArcSinh[x/a]^(3/2))/(3*Sqrt[a^2 + x^2])
```

fricas [A] time = 0.44, size = 20, normalized size = 0.51

$$\frac{2}{3}\log\left(\frac{x+\sqrt{a^2+x^2}}{a}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(1/2),x, algorithm="fricas")
[Out] 2/3*log((x + sqrt(a^2 + x^2))/a)^(3/2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(arcsinh(x/a))/sqrt(a^2 + x^2), x)
```

maple [A] time = 0.06, size = 34, normalized size = 0.87

$$\frac{2\operatorname{arcsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}} a\sqrt{\frac{a^2+x^2}{a^2}}}{3\sqrt{a^2+x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(x/a)^(1/2)/(a^2+x^2)^(1/2),x)
[Out] 2/3*arcsinh(x/a)^(3/2)*a/(a^2+x^2)^(1/2)*((a^2+x^2)/a^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(arcsinh(x/a))/sqrt(a^2 + x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(x/a)^(1/2)/(a^2 + x^2)^(1/2),x)
[Out] int(asinh(x/a)^(1/2)/(a^2 + x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x/a)**(1/2)/(a**2+x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(asinh(x/a))/sqrt(a**2 + x**2), x)
```

3.488 $\int \frac{\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=84

$$\frac{x\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{a^2\sqrt{a^2+x^2}}-\frac{\sqrt{\frac{x^2}{a^2}+1}\,\text{Int}\left(\frac{x}{\left(\frac{x^2}{a^2}+1\right)\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}},x\right)}{2a^3\sqrt{a^2+x^2}}$$

```
[Out] x*arcsinh(x/a)^(1/2)/a^2/(a^2+x^2)^(1/2)-1/2*(1+x^2/a^2)^(1/2)*Unintegrable
(x/(1+x^2/a^2)/arcsinh(x/a)^(1/2),x)/a^3/(a^2+x^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[ArcSinh[x/a]]/(a^2 + x^2)^(3/2),x]
```

```
[Out] (x*Sqrt[ArcSinh[x/a]])/(a^2*Sqrt[a^2 + x^2]) - (Sqrt[1 + x^2/a^2]*Defer[Int
][x/((1 + x^2/a^2)*Sqrt[ArcSinh[x/a]]), x])/(2*a^3*Sqrt[a^2 + x^2])
```

Rubi steps

$$\int \frac{\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{3/2}}\,dx=\frac{x\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{a^2\sqrt{a^2+x^2}}-\frac{\sqrt{1+\frac{x^2}{a^2}}\int \frac{x}{\left(1+\frac{x^2}{a^2}\right)\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}\,dx}{2a^3\sqrt{a^2+x^2}}$$

Mathematica [A] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[ArcSinh[x/a]]/(a^2 + x^2)^(3/2),x]
```

```
[Out] Integrate[Sqrt[ArcSinh[x/a]]/(a^2 + x^2)^(3/2), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(arcsinh(x/a))/(a^2 + x^2)^(3/2), x)
```

maple [A] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(x/a)^(1/2)/(a^2+x^2)^(3/2),x)
[Out] int(arcsinh(x/a)^(1/2)/(a^2+x^2)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(arcsinh(x/a))/(a^2 + x^2)^(3/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(x/a)^(1/2)/(a^2 + x^2)^(3/2),x)
[Out] int(asinh(x/a)^(1/2)/(a^2 + x^2)^(3/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x/a)**(1/2)/(a**2+x**2)**(3/2),x)
[Out] Integral(sqrt(asinh(x/a))/(a**2 + x**2)**(3/2), x)
```

3.489 $\int \frac{\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=173

$$-\frac{\sqrt{\frac{x^2}{a^2}+1}\,\mathrm{Int}\left(\frac{x}{\left(\frac{x^2}{a^2}+1\right)^2\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}},x\right)}{6a^5\sqrt{a^2+x^2}}-\frac{\sqrt{\frac{x^2}{a^2}+1}\,\mathrm{Int}\left(\frac{x}{\left(\frac{x^2}{a^2}+1\right)\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}},x\right)}{3a^5\sqrt{a^2+x^2}}+\frac{x\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{3a^2\left(a^2+x^2\right)^{3/2}}+\frac{2x\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{3a^4\sqrt{a^2+x^2}}$$

```
[Out] 1/3*x*arcsinh(x/a)^(1/2)/a^2/(a^2+x^2)^(3/2)+2/3*x*arcsinh(x/a)^(1/2)/a^4/(
a^2+x^2)^(1/2)-1/6*(1+x^2/a^2)^(1/2)*Unintegrable(x/(1+x^2/a^2)^2/arcsinh(x
/a)^(1/2),x)/a^5/(a^2+x^2)^(1/2)-1/3*(1+x^2/a^2)^(1/2)*Unintegrable(x/(1+x^
2/a^2)/arcsinh(x/a)^(1/2),x)/a^5/(a^2+x^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[ArcSinh[x/a]]/(a^2 + x^2)^(5/2),x]
```

```
[Out] (x*Sqrt[ArcSinh[x/a]])/(3*a^2*(a^2 + x^2)^(3/2)) + (2*x*Sqrt[ArcSinh[x/a]])
/(3*a^4*Sqrt[a^2 + x^2]) - (Sqrt[1 + x^2/a^2]*Defer[Int][x/((1 + x^2/a^2)^2
*Sqrt[ArcSinh[x/a]]), x])/(6*a^5*Sqrt[a^2 + x^2]) - (Sqrt[1 + x^2/a^2]*Defe
r[Int][x/((1 + x^2/a^2)*Sqrt[ArcSinh[x/a]]), x])/(3*a^5*Sqrt[a^2 + x^2])
```

Rubi steps

$$\int \frac{\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{5/2}}\,dx=\frac{x\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{3a^2\left(a^2+x^2\right)^{3/2}}+\frac{2\int \frac{\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{3/2}}\,dx}{3a^2}-\frac{\sqrt{1+\frac{x^2}{a^2}}\int \frac{x}{\left(1+\frac{x^2}{a^2}\right)^2\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}\,dx}{6a^5\sqrt{a^2+x^2}}$$

$$=\frac{x\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{3a^2\left(a^2+x^2\right)^{3/2}}+\frac{2x\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{3a^4\sqrt{a^2+x^2}}-\frac{\sqrt{1+\frac{x^2}{a^2}}\int \frac{x}{\left(1+\frac{x^2}{a^2}\right)^2\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}\,dx}{6a^5\sqrt{a^2+x^2}}-\frac{\sqrt{1+\frac{x^2}{a^2}}\int \frac{x}{\left(1+\frac{x^2}{a^2}\right)\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}\,dx}{3a^5\sqrt{a^2+x^2}}$$

Mathematica [A] time = 0.93, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[ArcSinh[x/a]]/(a^2 + x^2)^(5/2),x]
```

```
[Out] Integrate[Sqrt[ArcSinh[x/a]]/(a^2 + x^2)^(5/2), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(arcsinh(x/a))/(a^2 + x^2)^(5/2), x)
```

maple [A] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(x/a)^(1/2)/(a^2+x^2)^(5/2),x)
```

```
[Out] int(arcsinh(x/a)^(1/2)/(a^2+x^2)^(5/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arcsinh(x/a))/(a^2 + x^2)^(5/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}}{{\left(a^2+x^2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(x/a)^(1/2)/(a^2 + x^2)^(5/2),x)
```

```
[Out] int(asinh(x/a)^(1/2)/(a^2 + x^2)^(5/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x/a)**(1/2)/(a**2+x**2)**(5/2),x)
```

```
[Out] Integral(sqrt(asinh(x/a))/(a**2 + x**2)**(5/2), x)
```

3.490 $\int \left(a^2+x^2\right)^{3/2} \sinh^{-1}\left(\frac{x}{a}\right)^{3/2} \, dx$

Optimal. Leaf size=433

$$\frac{3}{8}a^2x\sqrt{a^2+x^2}\,\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}-\frac{9ax^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}-\frac{3\left(a^2+x^2\right)^{5/2}\sqrt{\sinh^{-1}}}{32a\sqrt{\frac{x^2}{a^2}+1}}$$

```
[Out] 1/4*x*(a^2+x^2)^(3/2)*arcsinh(x/a)^(3/2)+3/8*a^2*x*arcsinh(x/a)^(3/2)*(a^2+
x^2)^(1/2)+3/20*a^3*arcsinh(x/a)^(5/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)+3/
128*a^3*erf(2^(1/2)*arcsinh(x/a)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2+x^2)^(1/2)/(1
+x^2/a^2)^(1/2)+3/128*a^3*erfi(2^(1/2)*arcsinh(x/a)^(1/2))*2^(1/2)*Pi^(1/2)
*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)+3/2048*a^3*erf(2*arcsinh(x/a)^(1/2))*Pi^
(1/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)+3/2048*a^3*erfi(2*arcsinh(x/a)^(1/2
))*Pi^(1/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)-3/32*(a^2+x^2)^(5/2)*arcsinh(
x/a)^(1/2)/a/(1+x^2/a^2)^(1/2)-27/256*a^3*(a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2
)/(1+x^2/a^2)^(1/2)-9/32*a*x^2*(a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2)/(1+x^2/a^
2)^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {5684, 5682, 5675, 5663, 5779, 3312, 3307, 2180, 2204, 2205, 5717, 5699}

$$\frac{3\sqrt{\pi}\,a^3\sqrt{a^2+x^2}\,\text{Erf}\left(2\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{2048\sqrt{\frac{x^2}{a^2}+1}}+\frac{3\sqrt{\frac{\pi}{2}}\,a^3\sqrt{a^2+x^2}\,\text{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{64\sqrt{\frac{x^2}{a^2}+1}}+\frac{3\sqrt{\pi}\,a^3\sqrt{a^2+x^2}\,\text{Erfi}\left(2\sqrt{\sinh}\right.}{2048\sqrt{\frac{x^2}{a^2}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + x^2)^(3/2)*ArcSinh[x/a]^(3/2),x]
```

```
[Out] (-27*a^3*Sqrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]])/(256*Sqrt[1 + x^2/a^2]) - (9*a
*x^2*Sqrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]])/(32*Sqrt[1 + x^2/a^2]) - (3*(a^2 +
 x^2)^(5/2)*Sqrt[ArcSinh[x/a]])/(32*a*Sqrt[1 + x^2/a^2]) + (3*a^2*x*Sqrt[a^
2 + x^2]*ArcSinh[x/a]^(3/2))/8 + (x*(a^2 + x^2)^(3/2)*ArcSinh[x/a]^(3/2))/4
 + (3*a^3*Sqrt[a^2 + x^2]*ArcSinh[x/a]^(5/2))/(20*Sqrt[1 + x^2/a^2]) + (3*a
^3*Sqrt[Pi]*Sqrt[a^2 + x^2]*Erf[2*Sqrt[ArcSinh[x/a]]])/(2048*Sqrt[1 + x^2/a
^2]) + (3*a^3*Sqrt[Pi/2]*Sqrt[a^2 + x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[x/a]]])/(
64*Sqrt[1 + x^2/a^2]) + (3*a^3*Sqrt[Pi]*Sqrt[a^2 + x^2]*Erfi[2*Sqrt[ArcSinh
[x/a]]])/(2048*Sqrt[1 + x^2/a^2]) + (3*a^3*Sqrt[Pi/2]*Sqrt[a^2 + x^2]*Erfi[
Sqrt[2]*Sqrt[ArcSinh[x/a]]])/(64*Sqrt[1 + x^2/a^2])
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
```

```
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \left(a^2+x^2\right)^{3/2} \sinh^{-1}\left(\frac{x}{a}\right)^{3/2} dx = \frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2} + \frac{1}{4}\left(3a^2\right)\int \sqrt{a^2+x^2}\,\sinh^{-1}\left(\frac{x}{a}\right)^{3/2} dx - \frac{\left(3a\sqrt{\right.}{}$$

$$= -\frac{3\left(a^2+x^2\right)^{5/2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{32a\sqrt{1+\frac{x^2}{a^2}}} + \frac{3}{8}a^2x\sqrt{a^2+x^2}\,\sinh^{-1}\left(\frac{x}{a}\right)^{3/2} + \frac{1}{4}x\left(a^2+x^2\right)^{3/2}$$

$$= -\frac{9ax^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{1+\frac{x^2}{a^2}}} - \frac{3\left(a^2+x^2\right)^{5/2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{32a\sqrt{1+\frac{x^2}{a^2}}} + \frac{3}{8}a^2x\sqrt{a^2+x^2}$$

$$= -\frac{9ax^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{1+\frac{x^2}{a^2}}} - \frac{3\left(a^2+x^2\right)^{5/2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{32a\sqrt{1+\frac{x^2}{a^2}}} + \frac{3}{8}a^2x\sqrt{a^2+x^2}$$

$$= \frac{9a^3\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{256\sqrt{1+\frac{x^2}{a^2}}} - \frac{9ax^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{1+\frac{x^2}{a^2}}} - \frac{3\left(a^2+x^2\right)^{5/2}}{32a\sqrt{1+}}$$

$$= -\frac{27a^3\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{256\sqrt{1+\frac{x^2}{a^2}}} - \frac{9ax^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{1+\frac{x^2}{a^2}}} - \frac{3\left(a^2+x^2\right)^{5/2}}{32a\sqrt{1}}$$

$$= -\frac{27a^3\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{256\sqrt{1+\frac{x^2}{a^2}}} - \frac{9ax^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{1+\frac{x^2}{a^2}}} - \frac{3\left(a^2+x^2\right)^{5/2}}{32a\sqrt{1}}$$

$$= -\frac{27a^3\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{256\sqrt{1+\frac{x^2}{a^2}}} - \frac{9ax^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{1+\frac{x^2}{a^2}}} - \frac{3\left(a^2+x^2\right)^{5/2}}{32a\sqrt{1}}$$

$$= -\frac{27a^3\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{256\sqrt{1+\frac{x^2}{a^2}}} - \frac{9ax^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{1+\frac{x^2}{a^2}}} - \frac{3\left(a^2+x^2\right)^{5/2}}{32a\sqrt{1}}$$

Mathematica [A] time = 0.30, size = 210, normalized size = 0.48

$$a^3\sqrt{a^2+x^2}\left(60\sqrt{2\pi}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\operatorname{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)+60\sqrt{2\pi}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\operatorname{erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)+384\sin\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a^2 + x^2)^(3/2)*ArcSinh[x/a]^(3/2),x]
```

```
[Out] (a^3*Sqrt[a^2 + x^2]*(384*ArcSinh[x/a]^3 - 480*ArcSinh[x/a]*Cosh[2*ArcSinh[
x/a]] + 60*Sqrt[2*Pi]*Sqrt[ArcSinh[x/a]]*Erf[Sqrt[2]*Sqrt[ArcSinh[x/a]]] +
60*Sqrt[2*Pi]*Sqrt[ArcSinh[x/a]]*Erfi[Sqrt[2]*Sqrt[ArcSinh[x/a]]] + 5*Sqrt[
-ArcSinh[x/a]]*Gamma[5/2, -4*ArcSinh[x/a]] - 5*Sqrt[ArcSinh[x/a]]*Gamma[5/2
, 4*ArcSinh[x/a]] + 640*ArcSinh[x/a]^2*Sinh[2*ArcSinh[x/a]]))/(2560*Sqrt[1
+ x^2/a^2]*Sqrt[ArcSinh[x/a]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+x^2)^(3/2)*arcsinh(x/a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + x^2\right)^{\frac{3}{2}} \operatorname{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+x^2)^(3/2)*arcsinh(x/a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2 + x^2)^(3/2)*arcsinh(x/a)^(3/2), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \left(a^2 + x^2\right)^{\frac{3}{2}} \operatorname{arcsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+x^2)^(3/2)*arcsinh(x/a)^(3/2),x)
```

```
[Out] int((a^2+x^2)^(3/2)*arcsinh(x/a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + x^2\right)^{\frac{3}{2}} \operatorname{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+x^2)^(3/2)*arcsinh(x/a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a^2 + x^2)^(3/2)*arcsinh(x/a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{asinh}\left(\frac{x}{a}\right)^{3/2}\, {\left(a^2+x^2\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(x/a)^(3/2)*(a^2 + x^2)^(3/2),x)
```

```
[Out] int(asinh(x/a)^(3/2)*(a^2 + x^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+x**2)**(3/2)*asinh(x/a)**(3/2),x)
```

```
[Out] Timed out
```

3.491 $\int \sqrt{a^2+x^2} \sinh^{-1}\left(\frac{x}{a}\right)^{3/2} dx$

Optimal. Leaf size=259

$$\frac{3\sqrt{\frac{\pi}{2}}a\sqrt{a^2+x^2}\,\mathrm{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{64\sqrt{\frac{x^2}{a^2}+1}}+\frac{3\sqrt{\frac{\pi}{2}}a\sqrt{a^2+x^2}\,\mathrm{erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{64\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}$$

```
[Out] 1/2*x*arcsinh(x/a)^(3/2)*(a^2+x^2)^(1/2)+1/5*a*arcsinh(x/a)^(5/2)*(a^2+x^2)
^(1/2)/(1+x^2/a^2)^(1/2)+3/128*a*erf(2^(1/2)*arcsinh(x/a)^(1/2))*2^(1/2)*Pi
^(1/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)+3/128*a*erfi(2^(1/2)*arcsinh(x/a)^
(1/2))*2^(1/2)*Pi^(1/2)*(a^2+x^2)^(1/2)/(1+x^2/a^2)^(1/2)-3/16*a*(a^2+x^2)^
(1/2)*arcsinh(x/a)^(1/2)/(1+x^2/a^2)^(1/2)-3/8*x^2*(a^2+x^2)^(1/2)*arcsinh(
x/a)^(1/2)/a/(1+x^2/a^2)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {5682, 5675, 5663, 5779, 3312, 3307, 2180, 2204, 2205}

$$\frac{3\sqrt{\frac{\pi}{2}}a\sqrt{a^2+x^2}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{64\sqrt{\frac{x^2}{a^2}+1}}+\frac{3\sqrt{\frac{\pi}{2}}a\sqrt{a^2+x^2}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{64\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{\frac{x^2}{a^2}+1}}+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + x^2]*ArcSinh[x/a]^(3/2),x]
```

```
[Out] (-3*a*Sqrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]])/(16*Sqrt[1 + x^2/a^2]) - (3*x^2*S
qrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]])/(8*a*Sqrt[1 + x^2/a^2]) + (x*Sqrt[a^2 +
x^2]*ArcSinh[x/a]^(3/2))/2 + (a*Sqrt[a^2 + x^2]*ArcSinh[x/a]^(5/2))/(5*Sqrt
[1 + x^2/a^2]) + (3*a*Sqrt[Pi/2]*Sqrt[a^2 + x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[x
/a]]])/(64*Sqrt[1 + x^2/a^2]) + (3*a*Sqrt[Pi/2]*Sqrt[a^2 + x^2]*Erfi[Sqrt[2
]*Sqrt[ArcSinh[x/a]]])/(64*Sqrt[1 + x^2/a^2])
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \sqrt{a^2+x^2} \sinh^{-1}\left(\frac{x}{a}\right)^{3/2} dx = \frac{1}{2}x\sqrt{a^2+x^2} \sinh^{-1}\left(\frac{x}{a}\right)^{3/2} + \frac{\sqrt{a^2+x^2} \int \frac{\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{1+\frac{x^2}{a^2}}} dx}{2\sqrt{1+\frac{x^2}{a^2}}} - \frac{\left(3\sqrt{a^2+x^2}\right)\int x\sqrt{}}{4a\sqrt{1+}}$$

$$= -\frac{3x^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{1+\frac{x^2}{a^2}}} + \frac{1}{2}x\sqrt{a^2+x^2} \sinh^{-1}\left(\frac{x}{a}\right)^{3/2} + \frac{a\sqrt{a^2+x^2} \sinh}{5\sqrt{1+\frac{x^2}{a^2}}}$$

$$= -\frac{3x^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{1+\frac{x^2}{a^2}}} + \frac{1}{2}x\sqrt{a^2+x^2} \sinh^{-1}\left(\frac{x}{a}\right)^{3/2} + \frac{a\sqrt{a^2+x^2} \sinh}{5\sqrt{1+\frac{x^2}{a^2}}}$$

$$= -\frac{3x^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{1+\frac{x^2}{a^2}}} + \frac{1}{2}x\sqrt{a^2+x^2} \sinh^{-1}\left(\frac{x}{a}\right)^{3/2} + \frac{a\sqrt{a^2+x^2} \sinh}{5\sqrt{1+\frac{x^2}{a^2}}}$$

$$= -\frac{3a\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{16\sqrt{1+\frac{x^2}{a^2}}} - \frac{3x^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{1+\frac{x^2}{a^2}}} + \frac{1}{2}x\sqrt{a^2+x^2} \sinh$$

$$= -\frac{3a\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{16\sqrt{1+\frac{x^2}{a^2}}} - \frac{3x^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{1+\frac{x^2}{a^2}}} + \frac{1}{2}x\sqrt{a^2+x^2} \sinh$$

$$= -\frac{3a\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{16\sqrt{1+\frac{x^2}{a^2}}} - \frac{3x^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{1+\frac{x^2}{a^2}}} + \frac{1}{2}x\sqrt{a^2+x^2} \sinh$$

$$= -\frac{3a\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{16\sqrt{1+\frac{x^2}{a^2}}} - \frac{3x^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{1+\frac{x^2}{a^2}}} + \frac{1}{2}x\sqrt{a^2+x^2} \sinh$$

Mathematica [A] time = 0.25, size = 133, normalized size = 0.51

$$\frac{a\sqrt{a^2+x^2}\left(15\sqrt{2\pi}\,\text{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)+15\sqrt{2\pi}\,\text{erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)+8\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\left(16\sinh^{-1}\left(\frac{x}{a}\right)^2\right.\right.}{640\sqrt{\frac{x^2}{a^2}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + x^2]*ArcSinh[x/a]^(3/2),x]
```

```
[Out] (a*Sqrt[a^2 + x^2]*(15*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ArcSinh[x/a]]] + 15*Sqrt
[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcSinh[x/a]]] + 8*Sqrt[ArcSinh[x/a]]*(16*ArcSinh[
x/a]^2 - 15*Cosh[2*ArcSinh[x/a]] + 20*ArcSinh[x/a]*Sinh[2*ArcSinh[x/a]])))/
(640*Sqrt[1 + x^2/a^2])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(3/2)*(a^2+x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^2 + x^2} \operatorname{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(3/2)*(a^2+x^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a^2 + x^2)*arcsinh(x/a)^(3/2), x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \operatorname{arcsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}} \sqrt{a^2 + x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(x/a)^(3/2)*(a^2+x^2)^(1/2),x)
[Out] int(arcsinh(x/a)^(3/2)*(a^2+x^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^2 + x^2} \operatorname{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(3/2)*(a^2+x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a^2 + x^2)*arcsinh(x/a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{asinh}\left(\frac{x}{a}\right)^{3/2} \sqrt{a^2 + x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(x/a)^(3/2)*(a^2 + x^2)^(1/2),x)
[Out] int(asinh(x/a)^(3/2)*(a^2 + x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^2 + x^2} \operatorname{asinh}^{\frac{3}{2}}\left(\frac{x}{a}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x/a)**(3/2)*(a**2+x**2)**(1/2),x)
[Out] Integral(sqrt(a**2 + x**2)*asinh(x/a)**(3/2), x)
```

3.492 $\int \frac{\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2+x^2}}\,dx$

Optimal. Leaf size=39

$$\frac{2a\sqrt{\frac{x^2}{a^2}+1}\,\sinh^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2+x^2}}$$

[Out] 2/5*a*arcsinh(x/a)^(5/2)*(1+x^2/a^2)^(1/2)/(a^2+x^2)^(1/2)

Rubi [A] time = 0.06, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5677, 5675}

$$\frac{2a\sqrt{\frac{x^2}{a^2}+1}\,\sinh^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2+x^2}}$$

Antiderivative was successfully verified.

[In] Int[ArcSinh[x/a]^(3/2)/Sqrt[a^2 + x^2],x]

[Out] (2*a*Sqrt[1 + x^2/a^2]*ArcSinh[x/a]^(5/2))/(5*Sqrt[a^2 + x^2])

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] &&  !GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2+x^2}}\,dx &= \frac{\sqrt{1+\frac{x^2}{a^2}}\int \frac{\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{1+\frac{x^2}{a^2}}}\,dx}{\sqrt{a^2+x^2}}\\ &= \frac{2a\sqrt{1+\frac{x^2}{a^2}}\,\sinh^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2+x^2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 39, normalized size = 1.00

$$\frac{2a\sqrt{\frac{x^2}{a^2}+1}\,\sinh^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2+x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[ArcSinh[x/a]^(3/2)/Sqrt[a^2 + x^2],x]

[Out] (2*a*Sqrt[1 + x^2/a^2]*ArcSinh[x/a]^(5/2))/(5*Sqrt[a^2 + x^2])

fricas [A] time = 0.48, size = 20, normalized size = 0.51

$$\frac{2}{5}\log\left(\frac{x+\sqrt{a^2+x^2}}{a}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(3/2)/(a^2+x^2)^(1/2),x, algorithm="fricas")
[Out] 2/5*log((x + sqrt(a^2 + x^2))/a)^(5/2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}}{\sqrt{a^2+x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(3/2)/(a^2+x^2)^(1/2),x, algorithm="giac")
[Out] integrate(arcsinh(x/a)^(3/2)/sqrt(a^2 + x^2), x)
```

maple [A] time = 0.06, size = 34, normalized size = 0.87

$$\frac{2\operatorname{arcsinh}\left(\frac{x}{a}\right)^{\frac{5}{2}} a\sqrt{\frac{a^2+x^2}{a^2}}}{5\sqrt{a^2+x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(x/a)^(3/2)/(a^2+x^2)^(1/2),x)
[Out] 2/5*arcsinh(x/a)^(5/2)*a/(a^2+x^2)^(1/2)*((a^2+x^2)/a^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}}{\sqrt{a^2+x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(3/2)/(a^2+x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(arcsinh(x/a)^(3/2)/sqrt(a^2 + x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{asinh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2+x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(x/a)^(3/2)/(a^2 + x^2)^(1/2),x)
[Out] int(asinh(x/a)^(3/2)/(a^2 + x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^{\frac{3}{2}}\left(\frac{x}{a}\right)}{\sqrt{a^2+x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x/a)**(3/2)/(a**2+x**2)**(1/2),x)
```

```
[Out] Integral(asinh(x/a)**(3/2)/sqrt(a**2 + x**2), x)
```

3.493 $\int \frac{\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2+x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=84

$$\frac{x\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{a^2\sqrt{a^2+x^2}}-\frac{3\sqrt{\frac{x^2}{a^2}+1}\,\mathrm{Int}\left(\frac{x\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{\frac{x^2}{a^2}+1},x\right)}{2a^3\sqrt{a^2+x^2}}$$

```
[Out] x*arcsinh(x/a)^(3/2)/a^2/(a^2+x^2)^(1/2)-3/2*(1+x^2/a^2)^(1/2)*Unintegrable
(x*arcsinh(x/a)^(1/2)/(1+x^2/a^2),x)/a^3/(a^2+x^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2+x^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcSinh[x/a]^(3/2)/(a^2 + x^2)^(3/2),x]
```

```
[Out] (x*ArcSinh[x/a]^(3/2))/(a^2*Sqrt[a^2 + x^2]) - (3*Sqrt[1 + x^2/a^2]*Defer[I
nt][(x*Sqrt[ArcSinh[x/a]])/(1 + x^2/a^2), x])/(2*a^3*Sqrt[a^2 + x^2])
```

Rubi steps

$$\int \frac{\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2+x^2\right)^{3/2}}\,dx=\frac{x\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{a^2\sqrt{a^2+x^2}}-\frac{\left(3\sqrt{1+\frac{x^2}{a^2}}\right)\int \frac{x\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{1+\frac{x^2}{a^2}}\,dx}{2a^3\sqrt{a^2+x^2}}$$

Mathematica [A] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2+x^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcSinh[x/a]^(3/2)/(a^2 + x^2)^(3/2),x]
```

```
[Out] Integrate[ArcSinh[x/a]^(3/2)/(a^2 + x^2)^(3/2), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(3/2)/(a^2+x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}}{\left(a^2+x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(3/2)/(a^2+x^2)^(3/2),x, algorithm="giac")
[Out] integrate(arcsinh(x/a)^(3/2)/(a^2 + x^2)^(3/2), x)
```

maple [A] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}}{\left(a^2+x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(x/a)^(3/2)/(a^2+x^2)^(3/2),x)
[Out] int(arcsinh(x/a)^(3/2)/(a^2+x^2)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}}{\left(a^2+x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(3/2)/(a^2+x^2)^(3/2),x, algorithm="maxima")
[Out] integrate(arcsinh(x/a)^(3/2)/(a^2 + x^2)^(3/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2+x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(x/a)^(3/2)/(a^2 + x^2)^(3/2),x)
[Out] int(asinh(x/a)^(3/2)/(a^2 + x^2)^(3/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^{\frac{3}{2}}\left(\frac{x}{a}\right)}{\left(a^2+x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x/a)**(3/2)/(a**2+x**2)**(3/2),x)
[Out] Integral(asinh(x/a)**(3/2)/(a**2 + x**2)**(3/2), x)
```

3.494 $\int \frac{x}{\sqrt{1+x^2}\sqrt{\sinh^{-1}(x)}}\,dx$

Optimal. Leaf size=33

$$\frac{1}{2}\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{\sinh^{-1}(x)}\right)-\frac{1}{2}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{\sinh^{-1}(x)}\right)$$

```
[Out] -1/2*erf(arcsinh(x)^(1/2))*Pi^(1/2)+1/2*erfi(arcsinh(x)^(1/2))*Pi^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {5779, 3308, 2180, 2204, 2205}

$$\frac{1}{2}\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\sinh^{-1}(x)}\right)-\frac{1}{2}\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\sinh^{-1}(x)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 + x^2]*Sqrt[ArcSinh[x]]),x]
```

```
[Out] -(Sqrt[Pi]*Erf[Sqrt[ArcSinh[x]]])/2 + (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[x]]])/2
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x}{\sqrt{1+x^2}\sqrt{\sinh^{-1}(x)}}\,dx = \text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(x)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(x)\right)\right) + \frac{1}{2}\text{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(x)\right)$$
$$= -\text{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\sinh^{-1}(x)}\right) + \text{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\sinh^{-1}(x)}\right)$$
$$= -\frac{1}{2}\sqrt{\pi}\,\text{erf}\left(\sqrt{\sinh^{-1}(x)}\right) + \frac{1}{2}\sqrt{\pi}\,\text{erfi}\left(\sqrt{\sinh^{-1}(x)}\right)$$

Mathematica [A] time = 0.06, size = 34, normalized size = 1.03

$$\frac{1}{2}\left(\frac{\sqrt{-\sinh^{-1}(x)}\,\Gamma\left(\frac{1}{2}, -\sinh^{-1}(x)\right)}{\sqrt{\sinh^{-1}(x)}} + \Gamma\left(\frac{1}{2}, \sinh^{-1}(x)\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[1 + x^2]*Sqrt[ArcSinh[x]]),x]
```

```
[Out] ((Sqrt[-ArcSinh[x]]*Gamma[1/2, -ArcSinh[x]])/Sqrt[ArcSinh[x]] + Gamma[1/2,
ArcSinh[x]])/2
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)^(1/2)/arcsinh(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^2+1}\sqrt{\text{arsinh}(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)^(1/2)/arcsinh(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(x^2 + 1)*sqrt(arcsinh(x))), x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^2+1}\,\sqrt{\text{arcsinh}(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+1)^(1/2)/arcsinh(x)^(1/2),x)
```

```
[Out] int(x/(x^2+1)^(1/2)/arcsinh(x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^2+1}\sqrt{\text{arsinh}(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)^(1/2)/arcsinh(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(x^2 + 1)*sqrt(arcsinh(x))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{\sqrt{\mathrm{asinh}(x)}\,\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(asinh(x)^(1/2)*(x^2 + 1)^(1/2)),x)
```

```
[Out] int(x/(asinh(x)^(1/2)*(x^2 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^2+1}\,\sqrt{\mathrm{asinh}\,(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+1)**(1/2)/asinh(x)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(x**2 + 1)*sqrt(asinh(x))), x)
```

3.495 $\int \frac{\left(c+a^2cx^2\right)^{5/2}}{\sqrt{\sinh^{-1}(ax)}}\,dx$

Optimal. Leaf size=396

$$\frac{3\sqrt{\pi}\,c^2\sqrt{a^2cx^2+c}\,\text{erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{64a\sqrt{a^2x^2+1}}+\frac{15\sqrt{\frac{\pi}{2}}\,c^2\sqrt{a^2cx^2+c}\,\text{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{64a\sqrt{a^2x^2+1}}+\frac{\sqrt{\frac{\pi}{6}}\,c^2\sqrt{a^2cx^2+c}\,\text{erf}\left(\right.}{64a\sqrt{a^2x^2}}$$

```
[Out] 1/384*c^2*erf(6^(1/2)*arcsinh(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1
/2)/a/(a^2*x^2+1)^(1/2)+1/384*c^2*erfi(6^(1/2)*arcsinh(a*x)^(1/2))*6^(1/2)*
Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+15/128*c^2*erf(2^(1/2)*arc
sinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+1
5/128*c^2*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(
1/2)/a/(a^2*x^2+1)^(1/2)+3/64*c^2*erf(2*arcsinh(a*x)^(1/2))*Pi^(1/2)*(a^2*c
*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+3/64*c^2*erfi(2*arcsinh(a*x)^(1/2))*Pi^(1
/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+5/8*c^2*(a^2*c*x^2+c)^(1/2)*arc
sinh(a*x)^(1/2)/a/(a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5702, 5699, 3312, 3307, 2180, 2204, 2205}

$$\frac{3\sqrt{\pi}\,c^2\sqrt{a^2cx^2+c}\,\text{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{64a\sqrt{a^2x^2+1}}+\frac{15\sqrt{\frac{\pi}{2}}\,c^2\sqrt{a^2cx^2+c}\,\text{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{64a\sqrt{a^2x^2+1}}+\frac{\sqrt{\frac{\pi}{6}}\,c^2\sqrt{a^2cx^2+c}\,\text{Erf}\left(\right.}{64a\sqrt{a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(5/2)/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (5*c^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/(8*a*Sqrt[1 + a^2*x^2]) + (3
*c^2*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erf[2*Sqrt[ArcSinh[a*x]]])/(64*a*Sqrt[1 +
 a^2*x^2]) + (15*c^2*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSin
h[a*x]]])/(64*a*Sqrt[1 + a^2*x^2]) + (c^2*Sqrt[Pi/6]*Sqrt[c + a^2*c*x^2]*Er
f[Sqrt[6]*Sqrt[ArcSinh[a*x]]])/(64*a*Sqrt[1 + a^2*x^2]) + (3*c^2*Sqrt[Pi]*S
qrt[c + a^2*c*x^2]*Erfi[2*Sqrt[ArcSinh[a*x]]])/(64*a*Sqrt[1 + a^2*x^2]) + (
15*c^2*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(64
*a*Sqrt[1 + a^2*x^2]) + (c^2*Sqrt[Pi/6]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[6]*Sq
rt[ArcSinh[a*x]]])/(64*a*Sqrt[1 + a^2*x^2])
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rule 5702

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(d^(p - 1/2)*Sqrt[d + e*x^2])/Sqrt[1 + c^2*x^2], Int[(1 +
 c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n}, x]
&& EqQ[e, c^2*d] && IGtQ[2*p, 0] &&  !(IntegerQ[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}}{\sqrt{\sinh^{-1}(ax)}}\,dx = \frac{\left(c^2\sqrt{c+a^2cx^2}\right)\int \frac{\left(1+a^2x^2\right)^{5/2}}{\sqrt{\sinh^{-1}(ax)}}\,dx}{\sqrt{1+a^2x^2}}$$

$$= \frac{\left(c^2\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh^6(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a\sqrt{1+a^2x^2}}$$

$$= \frac{\left(c^2\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \left(\frac{5}{16\sqrt{x}}+\frac{15\cosh(2x)}{32\sqrt{x}}+\frac{3\cosh(4x)}{16\sqrt{x}}+\frac{\cosh(6x)}{32\sqrt{x}}\right)dx, x, \sinh^{-1}(ax)\right)}{a\sqrt{1+a^2x^2}}$$

$$= \frac{5c^2\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{8a\sqrt{1+a^2x^2}} + \frac{\left(c^2\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh(6x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{32a\sqrt{1+a^2x^2}} + \frac{(3c}{}$$

$$= \frac{5c^2\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{8a\sqrt{1+a^2x^2}} + \frac{\left(c^2\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{e^{-6x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{64a\sqrt{1+a^2x^2}} + \frac{(c^2\sqrt{c}}{}$$

$$= \frac{5c^2\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{8a\sqrt{1+a^2x^2}} + \frac{\left(c^2\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int e^{-6x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{32a\sqrt{1+a^2x^2}} + \frac{(c^2}{}$$

$$= \frac{5c^2\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{8a\sqrt{1+a^2x^2}} + \frac{3c^2\sqrt{\pi}\sqrt{c+a^2cx^2}\,\text{erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{64a\sqrt{1+a^2x^2}} + \frac{15c^2\sqrt{\frac{\pi}{2}}\sqrt{c+a}}{6}$$

Mathematica [A] time = 0.31, size = 197, normalized size = 0.50

$$\frac{c^2\sqrt{a^2cx^2+c}\left(240\sinh^{-1}(ax)-45\sqrt{2}\sqrt{\sinh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},2\sinh^{-1}(ax)\right)-18\sqrt{\sinh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},4\sinh^{-1}(ax)\right)\cdots\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (c^2*Sqrt[c + a^2*c*x^2]*(240*ArcSinh[a*x] + Sqrt[6]*Sqrt[-ArcSinh[a*x]]*Ga
mma[1/2, -6*ArcSinh[a*x]] + 18*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -4*ArcSinh[a*
x]] + 45*Sqrt[2]*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -2*ArcSinh[a*x]] - 45*Sqrt[
2]*Sqrt[ArcSinh[a*x]]*Gamma[1/2, 2*ArcSinh[a*x]] - 18*Sqrt[ArcSinh[a*x]]*Ga
mma[1/2, 4*ArcSinh[a*x]] - Sqrt[6]*Sqrt[ArcSinh[a*x]]*Gamma[1/2, 6*ArcSinh[
a*x]]))/(384*a*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)/sqrt(arcsinh(a*x)), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2c\,x^2+c\right)^{\frac{5}{2}}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)/sqrt(arcsinh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,a^2\,x^2+c\right)^{5/2}}{\sqrt{\mathrm{asinh}\left(a\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(5/2)/asinh(a*x)^(1/2),x)
[Out] int((c + a^2*c*x^2)^(5/2)/asinh(a*x)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)/asinh(a*x)**(1/2),x)
[Out] Timed out
```

3.496 $\int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\sinh^{-1}(ax)}}\,dx$

Optimal. Leaf size=264

$$\frac{\sqrt{\pi}\,c\sqrt{a^2cx^2+c}\,\mathrm{erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{32a\sqrt{a^2x^2+1}}+\frac{\sqrt{\frac{\pi}{2}}\,c\sqrt{a^2cx^2+c}\,\mathrm{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{4a\sqrt{a^2x^2+1}}+\frac{\sqrt{\pi}\,c\sqrt{a^2cx^2+c}\,\mathrm{erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{32a\sqrt{a^2x^2+1}}$$

```
[Out] 1/8*c*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/
a/(a^2*x^2+1)^(1/2)+1/8*c*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)
*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+1/32*c*erf(2*arcsinh(a*x)^(1/2))*P
i^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+1/32*c*erfi(2*arcsinh(a*x)^
(1/2))*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+3/4*c*(a^2*c*x^2+c)
^(1/2)*arcsinh(a*x)^(1/2)/a/(a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5702, 5699, 3312, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\,c\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{32a\sqrt{a^2x^2+1}}+\frac{\sqrt{\frac{\pi}{2}}\,c\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{4a\sqrt{a^2x^2+1}}+\frac{\sqrt{\pi}\,c\sqrt{a^2cx^2+c}\,\mathrm{Erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{32a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(3/2)/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (3*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/(4*a*Sqrt[1 + a^2*x^2]) + (c*S
qrt[Pi]*Sqrt[c + a^2*c*x^2]*Erf[2*Sqrt[ArcSinh[a*x]]])/(32*a*Sqrt[1 + a^2*x
^2]) + (c*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(
4*a*Sqrt[1 + a^2*x^2]) + (c*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erfi[2*Sqrt[ArcSin
h[a*x]]])/(32*a*Sqrt[1 + a^2*x^2]) + (c*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erfi
[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(4*a*Sqrt[1 + a^2*x^2])
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
```

```
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rule 5702

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(d^(p - 1/2)*Sqrt[d + e*x^2])/Sqrt[1 + c^2*x^2], Int[(1 +
 c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n}, x]
&& EqQ[e, c^2*d] && IGtQ[2*p, 0] &&  !(IntegerQ[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}}{\sqrt{\sinh^{-1}(ax)}}\,dx = \frac{\left(c\sqrt{c+a^2cx^2}\right)\int \frac{\left(1+a^2x^2\right)^{3/2}}{\sqrt{\sinh^{-1}(ax)}}\,dx}{\sqrt{1+a^2x^2}}$$

$$= \frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh^4(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a\sqrt{1+a^2x^2}}$$

$$= \frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \left(\frac{3}{8\sqrt{x}}+\frac{\cosh(2x)}{2\sqrt{x}}+\frac{\cosh(4x)}{8\sqrt{x}}\right)dx, x, \sinh^{-1}(ax)\right)}{a\sqrt{1+a^2x^2}}$$

$$= \frac{3c\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{4a\sqrt{1+a^2x^2}} + \frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh(4x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{8a\sqrt{1+a^2x^2}} + \frac{\left(c\sqrt{c}\right.}{}$$

$$= \frac{3c\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{4a\sqrt{1+a^2x^2}} + \frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{16a\sqrt{1+a^2x^2}} + \frac{\left(c\sqrt{c+a\right.}{}$$

$$= \frac{3c\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{4a\sqrt{1+a^2x^2}} + \frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int e^{-4x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{8a\sqrt{1+a^2x^2}} + \frac{\left(c\sqrt{c}\right.}{}$$

$$= \frac{3c\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{4a\sqrt{1+a^2x^2}} + \frac{c\sqrt{\pi}\sqrt{c+a^2cx^2}\,\text{erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{32a\sqrt{1+a^2x^2}} + \frac{c\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\,\text{er}}{4a\sqrt{1}}$$

Mathematica [A] time = 0.18, size = 141, normalized size = 0.53

$$\frac{c\sqrt{a^2cx^2+c}\left(\sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},-4\sinh^{-1}(ax)\right)+4\sqrt{2}\sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},-2\sinh^{-1}(ax)\right)+\sqrt{\sinh^{-1}(ax)}\left(2\right.\right.}{32a\sqrt{a^2x^2+1}\sqrt{\sinh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (c*Sqrt[c + a^2*c*x^2]*(Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -4*ArcSinh[a*x]] + 4
*Sqrt[2]*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -2*ArcSinh[a*x]] + Sqrt[ArcSinh[a*x
]]*(24*Sqrt[ArcSinh[a*x]] - 4*Sqrt[2]*Gamma[1/2, 2*ArcSinh[a*x]] - Gamma[1/
2, 4*ArcSinh[a*x]])))/(32*a*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}}{\sqrt{\operatorname{arsinh}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)/sqrt(arcsinh(a*x)), x)
```

maple [**F**] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}}{\sqrt{\operatorname{arcsinh}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}}{\sqrt{\operatorname{arsinh}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)/sqrt(arcsinh(a*x)), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,a^2\,x^2+c\right)}^{3/2}}{\sqrt{\mathrm{asinh}\left(a\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(3/2)/asinh(a*x)^(1/2),x)
```

```
[Out] int((c + a^2*c*x^2)^(3/2)/asinh(a*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}{\sqrt{\operatorname{asinh}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)/asinh(a*x)**(1/2),x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)/sqrt(asinh(a*x)), x)
```

3.497 $\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\sinh^{-1}(ax)}}\,dx$

Optimal. Leaf size=156

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\mathrm{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{4a\sqrt{a^2x^2+1}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\mathrm{erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{4a\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\sqrt{\sinh^{-1}(ax)}}{a\sqrt{a^2x^2+1}}$$

```
[Out] 1/8*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/
(a^2*x^2+1)^(1/2)+1/8*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^
2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+(a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2)
/a/(a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5702, 5699, 3312, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{4a\sqrt{a^2x^2+1}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{4a\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\sqrt{\sinh^{-1}(ax)}}{a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + a^2*c*x^2]/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/(a*Sqrt[1 + a^2*x^2]) + (Sqrt[Pi/2
]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(4*a*Sqrt[1 + a^2*x^
2]) + (Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(4*
a*Sqrt[1 + a^2*x^2])
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rule 5702

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(d^(p - 1/2)*Sqrt[d + e*x^2])/Sqrt[1 + c^2*x^2], Int[(1 +
 c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n}, x]
&& EqQ[e, c^2*d] && IGtQ[2*p, 0] &&  !(IntegerQ[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\sinh^{-1}(ax)}}\,dx = \frac{\sqrt{c+a^2cx^2}\int \frac{\sqrt{1+a^2x^2}}{\sqrt{\sinh^{-1}(ax)}}\,dx}{\sqrt{1+a^2x^2}}$$

$$= \frac{\sqrt{c+a^2cx^2}\,\text{Subst}\left(\int \frac{\cosh^2(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a\sqrt{1+a^2x^2}}$$

$$= \frac{\sqrt{c+a^2cx^2}\,\text{Subst}\left(\int \left(\frac{1}{2\sqrt{x}}+\frac{\cosh(2x)}{2\sqrt{x}}\right)dx, x, \sinh^{-1}(ax)\right)}{a\sqrt{1+a^2x^2}}$$

$$= \frac{\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{a\sqrt{1+a^2x^2}} + \frac{\sqrt{c+a^2cx^2}\,\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{2a\sqrt{1+a^2x^2}}$$

$$= \frac{\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{a\sqrt{1+a^2x^2}} + \frac{\sqrt{c+a^2cx^2}\,\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{4a\sqrt{1+a^2x^2}} + \frac{\sqrt{c+a^2cx^2}\,\text{Su}}{}$$

$$= \frac{\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{a\sqrt{1+a^2x^2}} + \frac{\sqrt{c+a^2cx^2}\,\text{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{2a\sqrt{1+a^2x^2}} + \frac{\sqrt{c+a^2cx^2}}{}$$

$$= \frac{\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{a\sqrt{1+a^2x^2}} + \frac{\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\,\text{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{4a\sqrt{1+a^2x^2}} + \frac{\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\,\text{erfi}}{4a\sqrt{1}}$$

Mathematica [A] time = 0.10, size = 101, normalized size = 0.65

$$\frac{\sqrt{c\left(a^2x^2+1\right)}\left(8\sinh^{-1}(ax)-\sqrt{2}\sqrt{\sinh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},2\sinh^{-1}(ax)\right)+\sqrt{2}\sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},-2\sinh^{-1}(ax)\right)\right)}{8a\sqrt{a^2x^2+1}\sqrt{\sinh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*(8*ArcSinh[a*x] + Sqrt[2]*Sqrt[-ArcSinh[a*x]]*Gamma[
1/2, -2*ArcSinh[a*x]] - Sqrt[2]*Sqrt[ArcSinh[a*x]]*Gamma[1/2, 2*ArcSinh[a*x
]]))/(8*a*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2cx^2 + c}}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a^2*c*x^2 + c)/sqrt(arcsinh(a*x)), x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2c\,x^2 + c}}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x)
[Out] int((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2cx^2 + c}}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a^2*c*x^2 + c)/sqrt(arcsinh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,a^2\,x^2 + c}}{\sqrt{\operatorname{asinh}(a\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(1/2)/asinh(a*x)^(1/2),x)
[Out] int((c + a^2*c*x^2)^(1/2)/asinh(a*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\left(a^2x^2 + 1\right)}}{\sqrt{\operatorname{asinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)/asinh(a*x)**(1/2),x)
[Out] Integral(sqrt(c*(a**2*x**2 + 1))/sqrt(asinh(a*x)), x)
```

3.498 $\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}\,dx$

Optimal. Leaf size=40

$$\frac{2\sqrt{a^2x^2+1}\sqrt{\sinh^{-1}(ax)}}{a\sqrt{a^2cx^2+c}}$$

```
[Out] 2*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^(1/2)/a/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5677, 5675}

$$\frac{2\sqrt{a^2x^2+1}\sqrt{\sinh^{-1}(ax)}}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]]),x]
```

```
[Out] (2*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])/(a*Sqrt[c + a^2*c*x^2])
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] &&  !GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}\,dx &= \frac{\sqrt{1+a^2x^2}\int \frac{1}{\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}\,dx}{\sqrt{c+a^2cx^2}}\\ &= \frac{2\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{a\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 40, normalized size = 1.00

$$\frac{2\sqrt{a^2x^2+1}\sqrt{\sinh^{-1}(ax)}}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]]),x]
```

```
[Out] (2*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])/(a*Sqrt[c + a^2*c*x^2])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2cx^2 + c}\,\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*sqrt(arcsinh(a*x))), x)
```

maple [A] time = 0.06, size = 36, normalized size = 0.90

$$\frac{2\sqrt{\operatorname{arcsinh}(ax)}\,\sqrt{a^2x^2+1}}{a\sqrt{c\left(a^2x^2+1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x)
```

```
[Out] 2*arcsinh(a*x)^(1/2)/a/(c*(a^2*x^2+1))^(1/2)*(a^2*x^2+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2cx^2 + c}\,\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*sqrt(arcsinh(a*x))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\operatorname{asinh}(a\,x)}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(asinh(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2)),x)
```

```
[Out] int(1/(asinh(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\left(a^2x^2+1\right)}\,\sqrt{\operatorname{asinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(1/2)/asinh(a*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c*(a**2*x**2 + 1))*sqrt(asinh(a*x))), x)
```

3.499 $\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{1}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\sinh^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcSinh[a*x]]),x]
[Out] Defer[Int][1/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcSinh[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)}}\,dx=\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)}}\,dx$$

Mathematica [A] time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcSinh[a*x]]),x]
[Out] Integrate[1/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcSinh[a*x]]), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x, algorithm="giac")
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*sqrt(arcsinh(a*x))), x)
```

maple [A] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{2} c \,x^{2}+c\right)^{\frac{3}{2}} \sqrt{\operatorname{arcsinh}\left(a x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x)
[Out] int(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{2} c x^{2}+c\right)^{\frac{3}{2}} \sqrt{\operatorname{arsinh}\left(a x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*sqrt(arcsinh(a*x))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{\mathrm{asinh}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(asinh(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2)),x)
[Out] int(1/(asinh(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c \left(a^{2} x^{2} + 1\right)\right)^{\frac{3}{2}} \sqrt{\operatorname{asinh}{\left(a x \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(3/2)/asinh(a*x)**(1/2),x)
[Out] Integral(1/((c*(a**2*x**2 + 1))**(3/2)*sqrt(asinh(a*x))), x)
```

3.500 $\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\sqrt{\sinh^{-1}(ax)}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{1}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\sinh^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\sqrt{\sinh^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcSinh[a*x]]),x]
[Out] Defer[Int][1/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcSinh[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\sqrt{\sinh^{-1}(ax)}}\,dx=\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\sqrt{\sinh^{-1}(ax)}}\,dx$$

Mathematica [A] time = 1.66, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\sqrt{\sinh^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcSinh[a*x]]),x]
[Out] Integrate[1/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcSinh[a*x]]), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{\operatorname{arsinh}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x, algorithm="giac")
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*sqrt(arcsinh(a*x))), x)
```

maple [A] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{2} c\,x^{2}+c\right)^{\frac{5}{2}} \sqrt{\operatorname{arcsinh}\left(a x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x)
[Out] int(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{2} c x^{2}+c\right)^{\frac{5}{2}} \sqrt{\operatorname{arsinh}\left(a x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*sqrt(arcsinh(a*x))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{\operatorname{asinh}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(asinh(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)),x)
[Out] int(1/(asinh(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c \left(a^{2} x^{2} + 1\right)\right)^{\frac{5}{2}} \sqrt{\operatorname{asinh}{\left(a x \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(5/2)/asinh(a*x)**(1/2),x)
[Out] Integral(1/((c*(a**2*x**2 + 1))**(5/2)*sqrt(asinh(a*x))), x)
```

3.501 $\int \frac{\left(c+a^2cx^2\right)^{5/2}}{\sinh^{-1}(ax)^{3/2}} dx$

Optimal. Leaf size=391

$$-\frac{3\sqrt{\pi}c^2\sqrt{a^2cx^2+c}\,\mathrm{erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{8a\sqrt{a^2x^2+1}}-\frac{15\sqrt{\frac{\pi}{2}}c^2\sqrt{a^2cx^2+c}\,\mathrm{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{16a\sqrt{a^2x^2+1}}-\frac{\sqrt{\frac{3\pi}{2}}c^2\sqrt{a^2cx^2+c}\,\mathrm{erf}\left(\sqrt{\right.}}{16a\sqrt{a^2x^2+}}$$

```
[Out] -15/32*c^2*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(
1/2)/a/(a^2*x^2+1)^(1/2)+15/32*c^2*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)
*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-3/8*c^2*erf(2*arcsinh(a*x
)^(1/2))*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+3/8*c^2*erfi(2*ar
csinh(a*x)^(1/2))*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-1/32*c^2
*erf(6^(1/2)*arcsinh(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^
2*x^2+1)^(1/2)+1/32*c^2*erfi(6^(1/2)*arcsinh(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(
a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-2*(a^2*c*x^2+c)^(5/2)*(a^2*x^2+1)^(1
/2)/a/arcsinh(a*x)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5696, 5779, 5448, 3308, 2180, 2204, 2205}

$$-\frac{3\sqrt{\pi}c^2\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{8a\sqrt{a^2x^2+1}}-\frac{15\sqrt{\frac{\pi}{2}}c^2\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{16a\sqrt{a^2x^2+1}}-\frac{\sqrt{\frac{3\pi}{2}}c^2\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(\right.}{16a\sqrt{a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(5/2)/ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2]*(c + a^2*c*x^2)^(5/2))/(a*Sqrt[ArcSinh[a*x]]) - (3*c^
2*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erf[2*Sqrt[ArcSinh[a*x]]])/(8*a*Sqrt[1 + a^2
*x^2]) - (15*c^2*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*
x]]])/(16*a*Sqrt[1 + a^2*x^2]) - (c^2*Sqrt[(3*Pi)/2]*Sqrt[c + a^2*c*x^2]*Er
f[Sqrt[6]*Sqrt[ArcSinh[a*x]]])/(16*a*Sqrt[1 + a^2*x^2]) + (3*c^2*Sqrt[Pi]*S
qrt[c + a^2*c*x^2]*Erfi[2*Sqrt[ArcSinh[a*x]]])/(8*a*Sqrt[1 + a^2*x^2]) + (1
5*c^2*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(16*
a*Sqrt[1 + a^2*x^2]) + (c^2*Sqrt[(3*Pi)/2]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[6]
*Sqrt[ArcSinh[a*x]]])/(16*a*Sqrt[1 + a^2*x^2])
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5696

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(Sqrt[1 + c^2*x^2]*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^(n +
 1))/(b*c*(n + 1)), x] - Dist[(c*(2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(b*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ
[e, c^2*d] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}}{\sinh^{-1}(ax)^{3/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{5/2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\left(12ac^2\sqrt{c+a^2cx^2}\right)\int \frac{x\left(1+a^2x^2\right)^2}{\sqrt{\sinh^{-1}(ax)}}\,dx}{\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{5/2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\left(12c^2\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh^5(x)\sinh(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right.}{a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{5/2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\left(12c^2\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int\left(\frac{5\sinh(2x)}{32\sqrt{x}}+\frac{\sinh(4x)}{8\sqrt{x}}+\frac{\sinh(6x)}{32\sqrt{x}}\right)\right.}{a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{5/2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\left(3c^2\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\sinh(6x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{8a\sqrt{1+a^2x^2}} + \frac{(3c}{}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{5/2}}{a\sqrt{\sinh^{-1}(ax)}} - \frac{\left(3c^2\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{e^{-6x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{16a\sqrt{1+a^2x^2}} + \frac{(3c^2\sqrt{c}}{}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{5/2}}{a\sqrt{\sinh^{-1}(ax)}} - \frac{\left(3c^2\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int e^{-6x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{8a\sqrt{1+a^2x^2}} + \frac{(3c}{}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{5/2}}{a\sqrt{\sinh^{-1}(ax)}} - \frac{3c^2\sqrt{\pi}\sqrt{c+a^2cx^2}\,\text{erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{8a\sqrt{1+a^2x^2}} - \frac{15c^2\sqrt{\frac{\pi}{2}}\sqrt{c+a^2}}{16}$$

Mathematica [A] time = 1.07, size = 399, normalized size = 1.02

$$\frac{c^2\sqrt{a^2cx^2+c}\,e^{-6\sinh^{-1}(ax)}\left(-64a^2x^2e^{6\sinh^{-1}(ax)}-16\sqrt{2\pi}\,e^{6\sinh^{-1}(ax)}\sqrt{\sinh^{-1}(ax)}\,\text{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)+16\sqrt{2\pi}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)/ArcSinh[a*x]^(3/2),x]
```

```
[Out] (c^2*Sqrt[c + a^2*c*x^2]*(-1 - 6*E^(2*ArcSinh[a*x]) + E^(4*ArcSinh[a*x]) -
52*E^(6*ArcSinh[a*x]) + E^(8*ArcSinh[a*x]) - 6*E^(10*ArcSinh[a*x]) - E^(12*
ArcSinh[a*x]) - 64*a^2*E^(6*ArcSinh[a*x])*x^2 - 16*E^(6*ArcSinh[a*x])*Sqrt[
2*Pi]*Sqrt[ArcSinh[a*x]]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]] + 16*E^(6*ArcSinh[
a*x])*Sqrt[2*Pi]*Sqrt[ArcSinh[a*x]]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]] + Sqrt
[6]*E^(6*ArcSinh[a*x])*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -6*ArcSinh[a*x]] + 12
*E^(6*ArcSinh[a*x])*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -4*ArcSinh[a*x]] - Sqrt[
2]*E^(6*ArcSinh[a*x])*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -2*ArcSinh[a*x]] - Sqr
t[2]*E^(6*ArcSinh[a*x])*Sqrt[ArcSinh[a*x]]*Gamma[1/2, 2*ArcSinh[a*x]] + 12*
E^(6*ArcSinh[a*x])*Sqrt[ArcSinh[a*x]]*Gamma[1/2, 4*ArcSinh[a*x]] + Sqrt[6]*
E^(6*ArcSinh[a*x])*Sqrt[ArcSinh[a*x]]*Gamma[1/2, 6*ArcSinh[a*x]]))/(32*a*E^
(6*ArcSinh[a*x])*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}}{\operatorname{arsinh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)/arcsinh(a*x)^(3/2), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2c\,x^2+c\right)^{\frac{5}{2}}}{\operatorname{arcsinh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}}{\operatorname{arsinh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)/arcsinh(a*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,a^2\,x^2+c\right)}^{5/2}}{{\mathrm{asinh}\left(a\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(5/2)/asinh(a*x)^(3/2),x)
```

```
[Out] int((c + a^2*c*x^2)^(5/2)/asinh(a*x)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)/asinh(a*x)**(3/2),x)
```

```
[Out] Timed out
```

3.502 $\int \frac{\left(c+a^2cx^2\right)^{3/2}}{\sinh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=256

$$-\frac{\sqrt{\pi}\,c\sqrt{a^2cx^2+c}\,\mathrm{erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{4a\sqrt{a^2x^2+1}}-\frac{\sqrt{\frac{\pi}{2}}\,c\sqrt{a^2cx^2+c}\,\mathrm{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{a\sqrt{a^2x^2+1}}+\frac{\sqrt{\pi}\,c\sqrt{a^2cx^2+c}\,\mathrm{erfi}\left(2\sqrt{\sinh^{-1}\right.}{4a\sqrt{a^2x^2+1}}$$

```
[Out] -1/2*c*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)
/a/(a^2*x^2+1)^(1/2)+1/2*c*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2
)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-1/4*c*erf(2*arcsinh(a*x)^(1/2))*P
i^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+1/4*c*erfi(2*arcsinh(a*x)^(
1/2))*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-2*(a^2*c*x^2+c)^(3/2
)*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5696, 5779, 5448, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\,c\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{4a\sqrt{a^2x^2+1}}-\frac{\sqrt{\frac{\pi}{2}}\,c\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{a\sqrt{a^2x^2+1}}+\frac{\sqrt{\pi}\,c\sqrt{a^2cx^2+c}\,\mathrm{Erfi}\left(2\sqrt{\sinh\right.}{4a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(3/2)/ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2]*(c + a^2*c*x^2)^(3/2))/(a*Sqrt[ArcSinh[a*x]]) - (c*Sq
rt[Pi]*Sqrt[c + a^2*c*x^2]*Erf[2*Sqrt[ArcSinh[a*x]]])/(4*a*Sqrt[1 + a^2*x^2
]) - (c*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(a*
Sqrt[1 + a^2*x^2]) + (c*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erfi[2*Sqrt[ArcSinh[a*
x]]])/(4*a*Sqrt[1 + a^2*x^2]) + (c*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt
[2]*Sqrt[ArcSinh[a*x]]])/(a*Sqrt[1 + a^2*x^2])
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5696

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(Sqrt[1 + c^2*x^2]*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^(n +
 1))/(b*c*(n + 1)), x] - Dist[(c*(2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(b*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ
[e, c^2*d] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}}{\sinh^{-1}(ax)^{3/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{3/2}}{a\sqrt{\sinh^{-1}(ax)}}+\frac{\left(8ac\sqrt{c+a^2cx^2}\right)\int\frac{x\left(1+a^2x^2\right)}{\sqrt{\sinh^{-1}(ax)}}\,dx}{\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{3/2}}{a\sqrt{\sinh^{-1}(ax)}}+\frac{\left(8c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int\frac{\cosh^3(x)\sinh(x)}{\sqrt{x}}\,dx,x,\sinh^{-1}(ax\right)}{a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{3/2}}{a\sqrt{\sinh^{-1}(ax)}}+\frac{\left(8c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int\left(\frac{\sinh(2x)}{4\sqrt{x}}+\frac{\sinh(4x)}{8\sqrt{x}}\right)dx,x,\text{sinh}\right.}{a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{3/2}}{a\sqrt{\sinh^{-1}(ax)}}+\frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int\frac{\sinh(4x)}{\sqrt{x}}\,dx,x,\sinh^{-1}(ax)\right)}{a\sqrt{1+a^2x^2}}+\frac{(2}{}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{3/2}}{a\sqrt{\sinh^{-1}(ax)}}-\frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int\frac{e^{-4x}}{\sqrt{x}}\,dx,x,\sinh^{-1}(ax)\right)}{2a\sqrt{1+a^2x^2}}+\frac{\left(c\sqrt{c\right.}}{}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{3/2}}{a\sqrt{\sinh^{-1}(ax)}}-\frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int e^{-4x^2}\,dx,x,\sqrt{\sinh^{-1}(ax)}\right)}{a\sqrt{1+a^2x^2}}+\frac{(c}{}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{3/2}}{a\sqrt{\sinh^{-1}(ax)}}-\frac{c\sqrt{\pi}\sqrt{c+a^2cx^2}\,\text{erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{4a\sqrt{1+a^2x^2}}-\frac{c\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}}{a}$$

Mathematica [A] time = 0.37, size = 225, normalized size = 0.88

$$-\frac{c\sqrt{a^2cx^2+c}\,e^{-4\sinh^{-1}(ax)}\left(16a^2x^2e^{4\sinh^{-1}(ax)}+4\sqrt{2\pi}\,e^{4\sinh^{-1}(ax)}\sqrt{\sinh^{-1}(ax)}\,\text{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)-4\sqrt{2\pi}\,e^{4\sinh^{-1}(ax)}\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)/ArcSinh[a*x]^(3/2),x]
```

```
[Out] -1/8*(c*Sqrt[c + a^2*c*x^2]*(1 + 14*E^(4*ArcSinh[a*x]) + E^(8*ArcSinh[a*x])
 + 16*a^2*E^(4*ArcSinh[a*x])*x^2 + 4*E^(4*ArcSinh[a*x])*Sqrt[2*Pi]*Sqrt[Arc
Sinh[a*x]]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]] - 4*E^(4*ArcSinh[a*x])*Sqrt[2*Pi
]*Sqrt[ArcSinh[a*x]]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]] - 2*E^(4*ArcSinh[a*x]
)*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -4*ArcSinh[a*x]] - 2*E^(4*ArcSinh[a*x])*Sq
rt[ArcSinh[a*x]]*Gamma[1/2, 4*ArcSinh[a*x]]))/(a*E^(4*ArcSinh[a*x])*Sqrt[1
+ a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}}{\operatorname{arsinh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)/arcsinh(a*x)^(3/2), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}}{\operatorname{arcsinh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}}{\operatorname{arsinh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)/arcsinh(a*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,a^2\,x^2+c\right)^{3/2}}{\operatorname{asinh}\left(a\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(3/2)/asinh(a*x)^(3/2),x)
[Out] int((c + a^2*c*x^2)^(3/2)/asinh(a*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{3}{2}}}{\operatorname{asinh}^{\frac{3}{2}}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)/asinh(a*x)**(3/2),x)
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)/asinh(a*x)**(3/2), x)
```

3.503 $\int \frac{\sqrt{c+a^2cx^2}}{\sinh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=152

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\mathrm{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{a\sqrt{a^2x^2+1}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\mathrm{erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{a\sqrt{a^2x^2+1}}-\frac{2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}{a\sqrt{\sinh^{-1}(ax)}}$$

```
[Out] -1/2*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a
/(a^2*x^2+1)^(1/2)+1/2*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a
^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-2*(a^2*x^2+1)^(1/2)*(a^2*c*x^2+c)^(1/
2)/a/arcsinh(a*x)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5696, 5669, 5448, 12, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{a\sqrt{a^2x^2+1}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{a\sqrt{a^2x^2+1}}-\frac{2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}{a\sqrt{\sinh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + a^2*c*x^2]/ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2]*Sqrt[c + a^2*c*x^2])/(a*Sqrt[ArcSinh[a*x]]) - (Sqrt[P
i/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(a*Sqrt[1 + a^2*x
^2]) + (Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(a
*Sqrt[1 + a^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
```

```
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5696

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(Sqrt[1 + c^2*x^2]*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^(n +
 1))/(b*c*(n + 1)), x] - Dist[(c*(2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(b*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ
[e, c^2*d] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}}{\sinh^{-1}(ax)^{3/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}{a\sqrt{\sinh^{-1}(ax)}}+\frac{\left(4a\sqrt{c+a^2cx^2}\right)\int\frac{x}{\sqrt{\sinh^{-1}(ax)}}\,dx}{\sqrt{1+a^2x^2}}$$

$$=-\frac{2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}{a\sqrt{\sinh^{-1}(ax)}}+\frac{\left(4\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int\frac{\cosh(x)\sinh(x)}{\sqrt{x}}\,dx,x,\sinh^{-1}(ax)\right)}{a\sqrt{1+a^2x^2}}$$

$$=-\frac{2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}{a\sqrt{\sinh^{-1}(ax)}}+\frac{\left(4\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int\frac{\sinh(2x)}{2\sqrt{x}}\,dx,x,\sinh^{-1}(ax)\right)}{a\sqrt{1+a^2x^2}}$$

$$=-\frac{2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}{a\sqrt{\sinh^{-1}(ax)}}+\frac{\left(2\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int\frac{\sinh(2x)}{\sqrt{x}}\,dx,x,\sinh^{-1}(ax)\right)}{a\sqrt{1+a^2x^2}}$$

$$=-\frac{2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}{a\sqrt{\sinh^{-1}(ax)}}-\frac{\sqrt{c+a^2cx^2}\,\text{Subst}\left(\int\frac{e^{-2x}}{\sqrt{x}}\,dx,x,\sinh^{-1}(ax)\right)}{a\sqrt{1+a^2x^2}}+\frac{\sqrt{c+a^2cx^2}}{}$$

$$=-\frac{2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}{a\sqrt{\sinh^{-1}(ax)}}-\frac{\left(2\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int e^{-2x^2}\,dx,x,\sqrt{\sinh^{-1}(ax)}\right)}{a\sqrt{1+a^2x^2}}+\frac{\left(2\sqrt{c\right.}}{}$$

$$=-\frac{2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}{a\sqrt{\sinh^{-1}(ax)}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\,\text{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{a\sqrt{1+a^2x^2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\,\text{e}}{a\sqrt{}}$$

Mathematica [A] time = 0.19, size = 115, normalized size = 0.76

$$-\frac{\sqrt{a^2cx^2+c}\left(4a^2x^2+\sqrt{2\pi}\sqrt{\sinh^{-1}(ax)}\,\text{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)-\sqrt{2\pi}\sqrt{\sinh^{-1}(ax)}\,\text{erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)+4\right)}{2a\sqrt{a^2x^2+1}\sqrt{\sinh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/ArcSinh[a*x]^(3/2),x]
```

```
[Out] -1/2*(Sqrt[c + a^2*c*x^2]*(4 + 4*a^2*x^2 + Sqrt[2*Pi]*Sqrt[ArcSinh[a*x]]*Er
f[Sqrt[2]*Sqrt[ArcSinh[a*x]]] - Sqrt[2*Pi]*Sqrt[ArcSinh[a*x]]*Erfi[Sqrt[2]*
Sqrt[ArcSinh[a*x]]]))/(a*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2cx^2+c}}{\text{arsinh}\,(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)/arcsinh(a*x)^(3/2), x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2c\,x^2+c}}{\text{arcsinh}\,(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2cx^2+c}}{\text{arsinh}\,(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)/arcsinh(a*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,a^2\,x^2+c}}{{\text{asinh}\left(a\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(1/2)/asinh(a*x)^(3/2),x)
```

```
[Out] int((c + a^2*c*x^2)^(1/2)/asinh(a*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\left(a^{2}x^{2}+1\right)}}{\operatorname{asinh}^{\frac{3}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)/asinh(a*x)**(3/2),x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))/asinh(a*x)**(3/2), x)
```

3.504 $\int \frac{1}{\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=40

$$-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{a^2cx^2+c}\sqrt{\sinh^{-1}(ax)}}$$

```
[Out] -2*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5677, 5675}

$$-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{a^2cx^2+c}\sqrt{\sinh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(a*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] &&  !GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}}\,dx &= \frac{\sqrt{1+a^2x^2}\int \frac{1}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}\,dx}{\sqrt{c+a^2cx^2}}\\ &= -\frac{2\sqrt{1+a^2x^2}}{a\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 40, normalized size = 1.00

$$-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{a^2cx^2+c}\sqrt{\sinh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(a*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])
```

fricas [A] time = 0.43, size = 57, normalized size = 1.42

$$-\frac{2\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}}{(a^3cx^2+ac)\sqrt{\log\left(ax+\sqrt{a^2x^2+1}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)/((a^3*c*x^2 + a*c)*sqrt(log(a*x +
sqrt(a^2*x^2 + 1))))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2cx^2+c}\,\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^(3/2)), x)
```

maple [A] time = 0.06, size = 36, normalized size = 0.90

$$-\frac{2\sqrt{a^2x^2+1}}{\sqrt{\operatorname{arcsinh}(ax)}\,a\sqrt{c\left(a^2x^2+1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
[Out] -2/arcsinh(a*x)^(1/2)/a/(c*(a^2*x^2+1))^(1/2)*(a^2*x^2+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2cx^2+c}\,\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\operatorname{asinh}(a\,x)^{3/2}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(asinh(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(1/(asinh(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\left(a^2x^2+1\right)}\,\operatorname{asinh}^{\frac{3}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c*(a**2*x**2 + 1))*asinh(a*x)**(3/2)), x)
```

3.505 $\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=98

$$-\frac{4a\sqrt{a^2x^2+1}\,\text{Int}\left(\frac{x}{\left(a^2x^2+1\right)^2\sqrt{\sinh^{-1}(ax)}},x\right)}{c\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}}{a\left(a^2cx^2+c\right)^{3/2}\sqrt{\sinh^{-1}(ax)}}$$

```
[Out] -2*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2)-4*a*(a^2*x^2+
1)^(1/2)*Unintegrable(x/(a^2*x^2+1)^2/arcsinh(a*x)^(1/2),x)/c/(a^2*c*x^2+c)
^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(a*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcSinh[a*x]]) - (4*a*
Sqrt[1 + a^2*x^2]*Defer[Int][x/((1 + a^2*x^2)^2*Sqrt[ArcSinh[a*x]]), x])/(c
*Sqrt[c + a^2*c*x^2])
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^{3/2}}\,dx=-\frac{2\sqrt{1+a^2x^2}}{a\left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)}}-\frac{\left(4a\sqrt{1+a^2x^2}\right)\int \frac{x}{\left(1+a^2x^2\right)^2\sqrt{\sinh^{-1}(ax)}}\,dx}{c\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(3/2)),x]
```

```
[Out] Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(3/2)), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\operatorname{arsinh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x, algorithm="giac")
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*arcsinh(a*x)^(3/2)), x)
```

maple [A] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\operatorname{arcsinh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x)
[Out] int(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\operatorname{arsinh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*arcsinh(a*x)^(3/2)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\operatorname{asinh}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(asinh(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)),x)
[Out] int(1/(asinh(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{asinh}^{\frac{3}{2}}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(3/2)/asinh(a*x)**(3/2),x)
[Out] Integral(1/((c*(a**2*x**2 + 1))**(3/2)*asinh(a*x)**(3/2)), x)
```

3.506 $\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\sinh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=98

$$-\frac{8a\sqrt{a^2x^2+1}\,\mathrm{Int}\left(\frac{x}{\left(a^2x^2+1\right)^3\sqrt{\sinh^{-1}(ax)}},x\right)}{c^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}}{a\left(a^2cx^2+c\right)^{5/2}\sqrt{\sinh^{-1}(ax)}}$$

```
[Out] -2*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2)-8*a*(a^2*x^2+
1)^(1/2)*Unintegrable(x/(a^2*x^2+1)^3/arcsinh(a*x)^(1/2),x)/c^2/(a^2*c*x^2+
c)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\sinh^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(a*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcSinh[a*x]]) - (8*a*
Sqrt[1 + a^2*x^2]*Defer[Int][x/((1 + a^2*x^2)^3*Sqrt[ArcSinh[a*x]]), x])/(c
^2*Sqrt[c + a^2*c*x^2])
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\sinh^{-1}(ax)^{3/2}}\,dx=-\frac{2\sqrt{1+a^2x^2}}{a\left(c+a^2cx^2\right)^{5/2}\sqrt{\sinh^{-1}(ax)}}-\frac{\left(8a\sqrt{1+a^2x^2}\right)\int \frac{x}{\left(1+a^2x^2\right)^3\sqrt{\sinh^{-1}(ax)}}\,dx}{c^2\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 1.54, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\sinh^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^(3/2)),x]
```

```
[Out] Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^(3/2)), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\operatorname{arsinh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x, algorithm="giac")
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*arcsinh(a*x)^(3/2)), x)
```

maple [A] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\operatorname{arcsinh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x)
[Out] int(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\operatorname{arsinh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*arcsinh(a*x)^(3/2)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\operatorname{asinh}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(asinh(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)),x)
[Out] int(1/(asinh(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(5/2)/asinh(a*x)**(3/2),x)
[Out] Timed out
```

3.507 $\int \frac{(c+a^2cx^2)^{3/2}}{\sinh^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=296

$$\frac{2\sqrt{\pi}\,c\sqrt{a^2cx^2+c}\,\mathrm{erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{3a\sqrt{a^2x^2+1}}+\frac{2\sqrt{2\pi}\,c\sqrt{a^2cx^2+c}\,\mathrm{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{3a\sqrt{a^2x^2+1}}+\frac{2\sqrt{\pi}\,c\sqrt{a^2cx^2+c}\,\mathrm{erfi}\left(2\sqrt{\right.}}{3a\sqrt{a^2x^2+1}}$$

```
[Out] -2/3*(a^2*c*x^2+c)^(3/2)*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(3/2)+2/3*c*erf(2
*arcsinh(a*x)^(1/2))*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)+2/3*c
*erfi(2*arcsinh(a*x)^(1/2))*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2
)+2/3*c*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2
)/a/(a^2*x^2+1)^(1/2)+2/3*c*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/
2)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-16/3*c*x*(a^2*x^2+1)*(a^2*c*x^2+
c)^(1/2)/arcsinh(a*x)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {5696, 5777, 5699, 3312, 3307, 2180, 2204, 2205, 5779, 5448}

$$\frac{2\sqrt{\pi}\,c\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{3a\sqrt{a^2x^2+1}}+\frac{2\sqrt{2\pi}\,c\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{3a\sqrt{a^2x^2+1}}+\frac{2\sqrt{\pi}\,c\sqrt{a^2cx^2+c}\,\mathrm{Erfi}\left(2\right.}{3a\sqrt{a^2x^2+}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(3/2)/ArcSinh[a*x]^(5/2),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2]*(c + a^2*c*x^2)^(3/2))/(3*a*ArcSinh[a*x]^(3/2)) - (16
*c*x*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2])/(3*Sqrt[ArcSinh[a*x]]) + (2*c*Sqrt[
Pi]*Sqrt[c + a^2*c*x^2]*Erf[2*Sqrt[ArcSinh[a*x]]])/(3*a*Sqrt[1 + a^2*x^2])
+ (2*c*Sqrt[2*Pi]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(3*a
*Sqrt[1 + a^2*x^2]) + (2*c*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erfi[2*Sqrt[ArcSinh
[a*x]]])/(3*a*Sqrt[1 + a^2*x^2]) + (2*c*Sqrt[2*Pi]*Sqrt[c + a^2*c*x^2]*Erfi
[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(3*a*Sqrt[1 + a^2*x^2])
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5696

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(Sqrt[1 + c^2*x^2]*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^(n +
 1))/(b*c*(n + 1)), x] - Dist[(c*(2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(b*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ
[e, c^2*d] && LtQ[n, -1]
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^
p*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-Dist[(f*m*d^IntPart[p
]*(d + e*x^2)^FracPart[p])/(b*c*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*
x)^(m - 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - D
ist[(c*(m + 2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(b*f*(n + 1)*(1
+ c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*A
rcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[2*p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}}{\sinh^{-1}(ax)^{5/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{3/2}}{3a\sinh^{-1}(ax)^{3/2}} + \frac{\left(8ac\sqrt{c+a^2cx^2}\right)\int \frac{x(1+a^2x^2)}{\sinh^{-1}(ax)^{3/2}}\,dx}{3\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{3/2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{16cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}}{3\sqrt{\sinh^{-1}(ax)}} + \frac{\left(16c\sqrt{c+a^2cx^2}\right)\int \frac{\sqrt{1+}}{\sqrt{\sinh}}}{3\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{3/2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{16cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}}{3\sqrt{\sinh^{-1}(ax)}} + \frac{\left(16c\sqrt{c+a^2cx^2}\right)\mathrm{Subst}\left(\right.}{3a\sqrt{1}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{3/2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{16cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}}{3\sqrt{\sinh^{-1}(ax)}} + \frac{\left(16c\sqrt{c+a^2cx^2}\right)\mathrm{Subst}\left(\right.}{3}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{3/2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{16cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}}{3\sqrt{\sinh^{-1}(ax)}} + \frac{\left(8c\sqrt{c+a^2cx^2}\right)\mathrm{Subst}\left(\int\right.}{3a\sqrt{1}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{3/2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{16cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}}{3\sqrt{\sinh^{-1}(ax)}} + \frac{\left(4c\sqrt{c+a^2cx^2}\right)\mathrm{Subst}\left(\int\right.}{3a\sqrt{1+}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{3/2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{16cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}}{3\sqrt{\sinh^{-1}(ax)}} + \frac{\left(8c\sqrt{c+a^2cx^2}\right)\mathrm{Subst}\left(\int\right.}{3a\sqrt{1}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{3/2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{16cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}}{3\sqrt{\sinh^{-1}(ax)}} + \frac{2c\sqrt{\pi}\sqrt{c+a^2cx^2}\,\mathrm{erf}\left(2\sqrt{}\right.}{3a\sqrt{1+a^2x^2}}$$

Mathematica [A] time = 0.40, size = 262, normalized size = 0.89

$$-\frac{c\sqrt{a^2cx^2+c}\,e^{-4\sinh^{-1}(ax)}\left(16a^2x^2e^{4\sinh^{-1}(ax)} + 64ax\sqrt{a^2x^2+1}\,e^{4\sinh^{-1}(ax)}\sinh^{-1}(ax) + 14e^{4\sinh^{-1}(ax)} + e^{8\sinh^{-1}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)/ArcSinh[a*x]^(5/2),x]
```

```
[Out] -1/24*(c*Sqrt[c + a^2*c*x^2]*(1 + 14*E^(4*ArcSinh[a*x]) + E^(8*ArcSinh[a*x]
) + 16*a^2*E^(4*ArcSinh[a*x])*x^2 - 8*ArcSinh[a*x] + 8*E^(8*ArcSinh[a*x])*A
rcSinh[a*x] + 64*a*E^(4*ArcSinh[a*x])*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] + 16
*E^(4*ArcSinh[a*x])*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -4*ArcSinh[a*x]] + 16*
Sqrt[2]*E^(4*ArcSinh[a*x])*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -2*ArcSinh[a*x]
] + 16*Sqrt[2]*E^(4*ArcSinh[a*x])*ArcSinh[a*x]^(3/2)*Gamma[1/2, 2*ArcSinh[a
*x]] + 16*E^(4*ArcSinh[a*x])*ArcSinh[a*x]^(3/2)*Gamma[1/2, 4*ArcSinh[a*x]])
)/(a*E^(4*ArcSinh[a*x])*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}}{\operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x, algorithm="giac")
[Out] integrate((a^2*c*x^2 + c)^(3/2)/arcsinh(a*x)^(5/2), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}}{\operatorname{arcsinh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x)
[Out] int((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}}{\operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^(3/2)/arcsinh(a*x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,a^2\,x^2+c\right)}^{3/2}}{{\mathrm{asinh}\left(a\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(3/2)/asinh(a*x)^(5/2),x)
[Out] int((c + a^2*c*x^2)^(3/2)/asinh(a*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)/asinh(a*x)**(5/2),x)
[Out] Timed out
```

3.508 $\int \frac{\sqrt{c+a^2cx^2}}{\sinh^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=182

$$\frac{2\sqrt{2\pi}\sqrt{a^2cx^2+c}\,\mathrm{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{3a\sqrt{a^2x^2+1}}+\frac{2\sqrt{2\pi}\sqrt{a^2cx^2+c}\,\mathrm{erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{3a\sqrt{a^2x^2+1}}-\frac{8x\sqrt{a^2cx^2+c}}{3\sqrt{\sinh^{-1}(ax)}}-\frac{2\sqrt{a^2x}}{3a}$$

```
[Out] 2/3*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*c*x^2+c)^(1/2)/a/
(a^2*x^2+1)^(1/2)+2/3*erfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^
2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)-2/3*(a^2*x^2+1)^(1/2)*(a^2*c*x^2+c)^(1
/2)/a/arcsinh(a*x)^(3/2)-8/3*x*(a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5696, 5665, 3307, 2180, 2204, 2205}

$$\frac{2\sqrt{2\pi}\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{3a\sqrt{a^2x^2+1}}+\frac{2\sqrt{2\pi}\sqrt{a^2cx^2+c}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{3a\sqrt{a^2x^2+1}}-\frac{8x\sqrt{a^2cx^2+c}}{3\sqrt{\sinh^{-1}(ax)}}-\frac{2\sqrt{a^2x}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + a^2*c*x^2]/ArcSinh[a*x]^(5/2),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2]*Sqrt[c + a^2*c*x^2])/(3*a*ArcSinh[a*x]^(3/2)) - (8*x*
Sqrt[c + a^2*c*x^2])/(3*Sqrt[ArcSinh[a*x]]) + (2*Sqrt[2*Pi]*Sqrt[c + a^2*c*
x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(3*a*Sqrt[1 + a^2*x^2]) + (2*Sqrt[2*P
i]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(3*a*Sqrt[1 + a^2*
x^2])
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5696

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(Sqrt[1 + c^2*x^2]*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^(n +
 1))/(b*c*(n + 1)), x] - Dist[(c*(2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(b*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ
[e, c^2*d] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}}{\sinh^{-1}(ax)^{5/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}\,\sqrt{c+a^2cx^2}}{3a\sinh^{-1}(ax)^{3/2}} + \frac{\left(4a\sqrt{c+a^2cx^2}\right)\int \frac{x}{\sinh^{-1}(ax)^{3/2}}\,dx}{3\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\,\sqrt{c+a^2cx^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{8x\sqrt{c+a^2cx^2}}{3\sqrt{\sinh^{-1}(ax)}} + \frac{\left(8\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{x}}\,dx, x, \sinh^{-1}\right.}{3a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\,\sqrt{c+a^2cx^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{8x\sqrt{c+a^2cx^2}}{3\sqrt{\sinh^{-1}(ax)}} + \frac{\left(4\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(a\right.}{3a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\,\sqrt{c+a^2cx^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{8x\sqrt{c+a^2cx^2}}{3\sqrt{\sinh^{-1}(ax)}} + \frac{\left(8\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\sinh^{-1}}\right.}{3a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\,\sqrt{c+a^2cx^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{8x\sqrt{c+a^2cx^2}}{3\sqrt{\sinh^{-1}(ax)}} + \frac{2\sqrt{2\pi}\,\sqrt{c+a^2cx^2}\operatorname{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{3a\sqrt{1+a^2x^2}} +$$

Mathematica [A] time = 0.15, size = 122, normalized size = 0.67

$$-\frac{2\sqrt{a^2cx^2+c}\left(a^2x^2+4ax\sqrt{a^2x^2+1}\sinh^{-1}(ax)+\sqrt{2}\left(-\sinh^{-1}(ax)\right)^{3/2}\Gamma\left(\frac{1}{2},-2\sinh^{-1}(ax)\right)+\sqrt{2}\sinh^{-1}(ax)^{3/}\right.}{3a\sqrt{a^2x^2+1}\sinh^{-1}(ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/ArcSinh[a*x]^(5/2),x]
```

```
[Out] (-2*Sqrt[c + a^2*c*x^2]*(1 + a^2*x^2 + 4*a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]
 + Sqrt[2]*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -2*ArcSinh[a*x]] + Sqrt[2]*ArcS
inh[a*x]^(3/2)*Gamma[1/2, 2*ArcSinh[a*x]]))/(3*a*Sqrt[1 + a^2*x^2]*ArcSinh[
a*x]^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2cx^2+c}}{\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)/arcsinh(a*x)^(5/2), x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2c\,x^2+c}}{\operatorname{arcsinh}(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(5/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2cx^2+c}}{\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)/arcsinh(a*x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,a^2\,x^2+c}}{{\operatorname{asinh}\left(a\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + a^2*c*x^2)^(1/2)/asinh(a*x)^(5/2),x)
```

```
[Out] int((c + a^2*c*x^2)^(1/2)/asinh(a*x)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\left(a^2x^2+1\right)}}{\operatorname{asinh}^{\frac{5}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)/asinh(a*x)**(5/2),x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))/asinh(a*x)**(5/2), x)
```

3.509 $\int \frac{1}{\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=42

$$-\frac{2\sqrt{a^2x^2+1}}{3a\sqrt{a^2cx^2+c}\sinh^{-1}(ax)^{3/2}}$$

[Out] -2/3*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2)

Rubi [A] time = 0.07, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5677, 5675}

$$-\frac{2\sqrt{a^2x^2+1}}{3a\sqrt{a^2cx^2+c}\sinh^{-1}(ax)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(5/2)),x]

[Out] (-2*Sqrt[1 + a^2*x^2])/(3*a*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2))

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] && !GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{5/2}}\,dx &= \frac{\sqrt{1+a^2x^2}\int \frac{1}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{5/2}}\,dx}{\sqrt{c+a^2cx^2}}\\ &= -\frac{2\sqrt{1+a^2x^2}}{3a\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 42, normalized size = 1.00

$$-\frac{2\sqrt{a^2x^2+1}}{3a\sqrt{a^2cx^2+c}\sinh^{-1}(ax)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(5/2)),x]

[Out] (-2*Sqrt[1 + a^2*x^2])/(3*a*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2))

fricas [A] time = 0.57, size = 57, normalized size = 1.36

$$-\frac{2\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}}{3\left(a^3cx^2+ac\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)/((a^3*c*x^2 + a*c)*log(a*x + sqr
t(a^2*x^2 + 1))^(3/2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2cx^2+c}\,\operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^(5/2)), x)
```

maple [A] time = 0.06, size = 36, normalized size = 0.86

$$-\frac{2\sqrt{a^2x^2+1}}{3\operatorname{arcsinh}\left(ax\right)^{\frac{3}{2}}a\sqrt{c\left(a^2x^2+1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -2/3/arcsinh(a*x)^(3/2)/a/(c*(a^2*x^2+1))^(1/2)*(a^2*x^2+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2cx^2+c}\,\operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\operatorname{asinh}\left(a\,x\right)^{5/2}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2)),x)
```

```
[Out] int(1/(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\left(a^2x^2+1\right)}\,\operatorname{asinh}^{\frac{5}{2}}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c*(a**2*x**2 + 1))*asinh(a*x)**(5/2)), x)
```

3.510 $\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=102

$$-\frac{4a\sqrt{a^2x^2+1}\,\mathrm{Int}\left(\frac{x}{\left(a^2x^2+1\right)^2\sinh^{-1}(ax)^{3/2}},x\right)}{3c\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}}{3a\left(a^2cx^2+c\right)^{3/2}\sinh^{-1}(ax)^{3/2}}$$

```
[Out] -2/3*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2)-4/3*a*(a^2*
x^2+1)^(1/2)*Unintegrable(x/(a^2*x^2+1)^2/arcsinh(a*x)^(3/2),x)/c/(a^2*c*x^
2+c)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(5/2)),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(3*a*(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(3/2)) - (4*
a*Sqrt[1 + a^2*x^2]*Defer[Int][x/((1 + a^2*x^2)^2*ArcSinh[a*x]^(3/2)), x])/
(3*c*Sqrt[c + a^2*c*x^2])
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^{5/2}}\,dx=-\frac{2\sqrt{1+a^2x^2}}{3a\left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^{3/2}}-\frac{\left(4a\sqrt{1+a^2x^2}\right)\int \frac{x}{\left(1+a^2x^2\right)^2\sinh^{-1}(ax)^{3/2}}\,dx}{3c\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.73, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(5/2)), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x, algorithm="giac")
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*arcsinh(a*x)^(5/2)), x)
```

maple [**A**] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\operatorname{arcsinh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x)
[Out] int(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x)
```

maxima [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x, algorithm="maxima")
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*arcsinh(a*x)^(5/2)), x)
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\operatorname{asinh}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)),x)
[Out] int(1/(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(3/2)/asinh(a*x)**(5/2),x)
[Out] Timed out
```

3.511 $\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\sinh^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=102

$$-\frac{8a\sqrt{a^2x^2+1}\,\text{Int}\left(\frac{x}{\left(a^2x^2+1\right)^3\sinh^{-1}(ax)^{3/2}},x\right)}{3c^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}}{3a\left(a^2cx^2+c\right)^{5/2}\sinh^{-1}(ax)^{3/2}}$$

```
[Out] -2/3*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2)-8/3*a*(a^2*
x^2+1)^(1/2)*Unintegrable(x/(a^2*x^2+1)^3/arcsinh(a*x)^(3/2),x)/c^2/(a^2*c*
x^2+c)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\sinh^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^(5/2)),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(3*a*(c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^(3/2)) - (8*
a*Sqrt[1 + a^2*x^2]*Defer[Int][x/((1 + a^2*x^2)^3*ArcSinh[a*x]^(3/2)), x])/
(3*c^2*Sqrt[c + a^2*c*x^2])
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\sinh^{-1}(ax)^{5/2}}\,dx=-\frac{2\sqrt{1+a^2x^2}}{3a\left(c+a^2cx^2\right)^{5/2}\sinh^{-1}(ax)^{3/2}}-\frac{\left(8a\sqrt{1+a^2x^2}\right)\int \frac{x}{\left(1+a^2x^2\right)^3\sinh^{-1}(ax)^{3/2}}\,dx}{3c^2\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 1.55, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\sinh^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^(5/2)), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(5/2),x, algorithm="giac")
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*arcsinh(a*x)^(5/2)), x)
```

maple [A] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\operatorname{arcsinh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(5/2),x)
[Out] int(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(5/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(5/2),x, algorithm="maxima")
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*arcsinh(a*x)^(5/2)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\operatorname{asinh}\left(a\,x\right)^{5/2}\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)),x)
[Out] int(1/(asinh(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(5/2)/asinh(a*x)**(5/2),x)
[Out] Timed out
```

3.512 $\int x^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx$

Optimal. Leaf size=235

$$-\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^{n+1}}{8bc^3(n+1)\sqrt{c^2x^2+1}}+\frac{2^{-2(n+3)}e^{-\frac{4a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{4(a}{}\right.}{c^3\sqrt{c^2x^2+1}}$$

```
[Out] -1/8*(a+b*arcsinh(c*x))^(1+n)*(c^2*d*x^2+d)^(1/2)/b/c^3/(1+n)/(c^2*x^2+1)^(
1/2)+(a+b*arcsinh(c*x))^n*GAMMA(1+n,-4*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^
(1/2)/(2^(6+2*n))/c^3/exp(4*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1
/2)-exp(4*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,4*(a+b*arcsinh(c*x))/b)*(c^2*
d*x^2+d)^(1/2)/(2^(6+2*n))/c^3/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {5782, 5779, 5448, 3307, 2181}

$$\frac{2^{-2(n+3)}e^{-\frac{4a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{c^3\sqrt{c^2x^2+1}}-\frac{2^{-2(n+3)}e^{\frac{4a}{b}}\sqrt{c^2d}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] -(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^(1 + n))/(8*b*c^3*(1 + n)*Sqrt[1
 + c^2*x^2]) + (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-4
*(a + b*ArcSinh[c*x]))/b])/(2^(2*(3 + n))*c^3*E^((4*a)/b)*Sqrt[1 + c^2*x^2]
*(-((a + b*ArcSinh[c*x])/b))^n) - (E^((4*a)/b)*Sqrt[d + c^2*d*x^2]*(a + b*A
rcSinh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcSinh[c*x]))/b])/(2^(2*(3 + n))*c^3
*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
```

```
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5782

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPart[p])/(1 + c^2*
x^2)^FracPart[p], Int[x^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -
1] && IGtQ[m, 0] &&  !(IntegerQ[p] || GtQ[d, 0])
```

Rubi steps

$$\int x^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n dx = \frac{\sqrt{d+c^2dx^2}\int x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^n dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\sqrt{d+c^2dx^2}\operatorname{Subst}\left(\int (a+bx)^n\cosh^2(x)\sinh^2(x)\,dx,x,\sinh^{-1}(cx)\right)}{c^3\sqrt{1+c^2x^2}}$$

$$= \frac{\sqrt{d+c^2dx^2}\operatorname{Subst}\left(\int\left(-\frac{1}{8}(a+bx)^n+\frac{1}{8}(a+bx)^n\cosh(4x)\right)dx,x,s\ldots}{c^3\sqrt{1+c^2x^2}}$$

$$= -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{8bc^3(1+n)\sqrt{1+c^2x^2}} + \frac{\sqrt{d+c^2dx^2}\operatorname{Subst}\left(\int (a+b\ldots}{8c^3\sqrt{1\ldots}}$$

$$= -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{8bc^3(1+n)\sqrt{1+c^2x^2}} + \frac{\sqrt{d+c^2dx^2}\operatorname{Subst}\left(\int e^{-4x}(a\ldots}{16c^3\sqrt{1+\ldots}}$$

$$= -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{8bc^3(1+n)\sqrt{1+c^2x^2}} + \frac{4^{-3-n}e^{-\frac{4a}{b}}\sqrt{d+c^2dx^2}\left(a+b\ldots}{\ldots}$$

Mathematica [A] time = 0.99, size = 170, normalized size = 0.72

$$\frac{d\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^n\left(4^{-n}e^{-\frac{4a}{b}}\left(-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\left(\left(\frac{a}{b}+\sinh^{-1}(cx)\right)^n\Gamma\left(n+1,-\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)-\ldots}{64c^3\sqrt{d\left(c^2x^2+1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*((-8*(a + b*ArcSinh[c*x]))/(b +
 b*n) + ((a/b + ArcSinh[c*x])^n*Gamma[1 + n, (-4*(a + b*ArcSinh[c*x]))/b] -
 E^((8*a)/b)*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1 + n, (4*(a + b*ArcSinh[c
*x]))/b])/(4^n*E^((4*a)/b)*(-((a + b*ArcSinh[c*x])^2/b^2))^n)))/(64*c^3*Sqr
t[d*(1 + c^2*x^2)])
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\sqrt{c^2dx^2+d}\,(b\operatorname{arsinh}(cx)+a)^n x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n*x^2, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c^2dx^2 + d}\,(b\operatorname{arsinh}(cx) + a)^n x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n*x^2, x)

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int x^2\,(a + b\operatorname{arcsinh}(cx))^n\,\sqrt{c^2d\,x^2 + d}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x)

[Out] int(x^2*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c^2dx^2 + d}\,(b\operatorname{arsinh}(cx) + a)^n x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")

[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n*x^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,(a + b\operatorname{asinh}(c\,x))^n\,\sqrt{d\,c^2\,x^2 + d}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(1/2),x)

[Out] int(x^2*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\sqrt{d\left(c^{2}x^{2} + 1\right)}\,\left(a + b\operatorname{asinh}(cx)\right)^{n}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(a+b*asinh(c*x))**n*(c**2*d*x**2+d)**(1/2),x)

[Out] Integral(x**2*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**n, x)

3.513 $\int x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n dx$

Optimal. Leaf size=355

$$\frac{3^{-n-1}e^{-\frac{3a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{8c^2\sqrt{c^2x^2+1}}+\frac{e^{-\frac{a}{b}}\sqrt{c^2dx^2+d}\left(a+b\right.}{}$$

```
[Out] 1/8*3^(-1-n)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-3*(a+b*arcsinh(c*x))/b)*(c^2*d
*x^2+d)^(1/2)/c^2/exp(3*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)+
1/8*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(-a-b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/
2)/c^2/exp(a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)+1/8*exp(a/b)*
(a+b*arcsinh(c*x))^n*GAMMA(1+n,(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^
2/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)+1/8*3^(-1-n)*exp(3*a/b)*(a+b
*arcsinh(c*x))^n*GAMMA(1+n,3*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^2/
(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5782, 5779, 5448, 3308, 2181}

$$\frac{3^{-n-1}e^{-\frac{3a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{8c^2\sqrt{c^2x^2+1}}+\frac{e^{-\frac{a}{b}}\sqrt{c^2dx^2+d}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (3^(-1 - n)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-3*(a
+ b*ArcSinh[c*x]))/b])/(8*c^2*E^((3*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcSi
nh[c*x])/b))^n) + (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n,
-((a + b*ArcSinh[c*x])/b)])/(8*c^2*E^(a/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcS
inh[c*x])/b))^n) + (E^(a/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamm
a[1 + n, (a + b*ArcSinh[c*x])/b])/(8*c^2*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[
c*x])/b)^n) + (3^(-1 - n)*E^((3*a)/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x])^n*Gamma[1 + n, (3*(a + b*ArcSinh[c*x]))/b])/(8*c^2*Sqrt[1 + c^2*x^2]*((
a + b*ArcSinh[c*x])/b)^n)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5782

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPart[p])/(1 + c^2*
x^2)^FracPart[p], Int[x^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -
1] && IGtQ[m, 0] &&  !(IntegerQ[p] || GtQ[d, 0])
```

Rubi steps

$$\int x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n dx = \frac{\sqrt{d+c^2dx^2}\int x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^n dx}{\sqrt{1+c^2x^2}}$$
$$= \frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\int (a+bx)^n\cosh^2(x)\sinh(x)\,dx, x, \sinh^{-1}(cx)\right)}{c^2\sqrt{1+c^2x^2}}$$
$$= \frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\int\left(\frac{1}{4}(a+bx)^n\sinh(x)+\frac{1}{4}(a+bx)^n\sinh(3x)\right)dx, x\right.}{c^2\sqrt{1+c^2x^2}}$$
$$= \frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\int (a+bx)^n\sinh(x)\,dx, x, \sinh^{-1}(cx)\right)}{4c^2\sqrt{1+c^2x^2}} + \frac{\sqrt{d+c^2dx}}{}$$
$$= -\frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\int e^{-3x}(a+bx)^n\,dx, x, \sinh^{-1}(cx)\right)}{8c^2\sqrt{1+c^2x^2}} - \frac{\sqrt{d+c^2dx^2}\ \text{S}}{}$$
$$= \frac{3^{-1-n}e^{-\frac{3a}{b}}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\right.}{8c^2\sqrt{1+c^2x^2}}$$

Mathematica [A] time = 0.95, size = 229, normalized size = 0.65

$$\frac{de^{-\frac{3a}{b}}\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^n\left(\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\left(3^{-n}e^{\frac{6a}{b}}\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{2n}\left(-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\Gamma\left(n+1,\frac{3(}{}\right.\right.\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*((3*E^((4*a)/b)*Gamma[1 + n, a/
b + ArcSinh[c*x]])/(a/b + ArcSinh[c*x])^n + (Gamma[1 + n, (-3*(a + b*ArcSin
h[c*x]))/b]/3^n + 3*E^((2*a)/b)*Gamma[1 + n, -((a + b*ArcSinh[c*x])/b)] + (
E^((6*a)/b)*(-((a + b*ArcSinh[c*x])/b))^(2*n)*Gamma[1 + n, (3*(a + b*ArcSin
h[c*x]))/b])/(3^n*(-((a + b*ArcSinh[c*x])^2/b^2))^n))/(-((a + b*ArcSinh[c*x
])/b))^n))/(24*c^2*E^((3*a)/b)*Sqrt[d*(1 + c^2*x^2)])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{c^2dx^2+d}\,(b\,\text{arsinh}(cx)+a)^n x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n*x, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int x\left(a+b\operatorname{arcsinh}\left(cx\right)\right)^{n}\sqrt{c^{2}d\,x^{2}+d}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int(x*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c^{2}dx^{2}+d}\,(b\operatorname{arsinh}(cx)+a)^{n}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(a+b\operatorname{asinh}\left(c\,x\right)\right)^{n}\sqrt{d\,c^{2}\,x^{2}+d}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int(x*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{d\left(c^{2}x^{2}+1\right)}\,\left(a+b\operatorname{asinh}\left(cx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))**n*(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**n, x)
```

3.514 $\int \sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx$

Optimal. Leaf size=235

$$\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^{n+1}}{2bc(n+1)\sqrt{c^2x^2+1}}+\frac{2^{-n-3}e^{-\frac{2a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{c\sqrt{c^2x^2+1}}$$

```
[Out] 1/2*(a+b*arcsinh(c*x))^(1+n)*(c^2*d*x^2+d)^(1/2)/b/c/(1+n)/(c^2*x^2+1)^(1/2
)+2^(-3-n)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-2*(a+b*arcsinh(c*x))/b)*(c^2*d*x
^2+d)^(1/2)/c/exp(2*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)-2^(-
3-n)*exp(2*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,2*(a+b*arcsinh(c*x))/b)*(c^2
*d*x^2+d)^(1/2)/c/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5702, 5699, 3312, 3307, 2181}

$$\frac{2^{-n-3}e^{-\frac{2a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{c\sqrt{c^2x^2+1}}-\frac{2^{-n-3}e^{\frac{2a}{b}}\sqrt{c^2dx^2+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^(1 + n))/(2*b*c*(1 + n)*Sqrt[1 +
c^2*x^2]) + (2^(-3 - n)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1
+ n, (-2*(a + b*ArcSinh[c*x]))/b])/(c*E^((2*a)/b)*Sqrt[1 + c^2*x^2]*(-((a +
 b*ArcSinh[c*x])/b))^n) - (2^(-3 - n)*E^((2*a)/b)*Sqrt[d + c^2*d*x^2]*(a +
b*ArcSinh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcSinh[c*x]))/b])/(c*Sqrt[1 + c^2
*x^2]*((a + b*ArcSinh[c*x])/b)^n)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
```

```
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rule 5702

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(d^(p - 1/2)*Sqrt[d + e*x^2])/Sqrt[1 + c^2*x^2], Int[(1 +
 c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n}, x]
&& EqQ[e, c^2*d] && IGtQ[2*p, 0] &&  !(IntegerQ[p] || GtQ[d, 0])
```

Rubi steps

$$\int \sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n dx = \frac{\sqrt{d+c^2dx^2}\int \sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^n dx}{\sqrt{1+c^2x^2}}$$
$$= \frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\int (a+bx)^n\cosh^2(x)\,dx, x, \sinh^{-1}(cx)\right)}{c\sqrt{1+c^2x^2}}$$
$$= \frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\int \left(\frac{1}{2}(a+bx)^n+\frac{1}{2}(a+bx)^n\cosh(2x)\right)dx, x, \sinh\right.}{c\sqrt{1+c^2x^2}}$$
$$= \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{2bc(1+n)\sqrt{1+c^2x^2}} + \frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\int (a+bx)^n \right.}{2c\sqrt{1+c}}$$
$$= \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{2bc(1+n)\sqrt{1+c^2x^2}} + \frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\int e^{-2x}(a+b\right.}{4c\sqrt{1+c^2x}}$$
$$= \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{2bc(1+n)\sqrt{1+c^2x^2}} + \frac{2^{-3-n}e^{-\frac{2a}{b}}\sqrt{d+c^2dx^2}\left(a+b\sin\right.}{}$$

Mathematica [A] time = 0.60, size = 160, normalized size = 0.68

$$\frac{d\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^n\left(\frac{4a+4b\sinh^{-1}(cx)}{bn+b}-2^{-n}e^{\frac{2a}{b}}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)^{-n}\Gamma\left(n+1,\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)+2^{-n}e^{-\frac{2a}{b}}\right.}{8c\sqrt{d\left(c^2x^2+1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*((4*a + 4*b*ArcSinh[c*x])/(b +
b*n) + Gamma[1 + n, (-2*(a + b*ArcSinh[c*x]))/b]/(2^n*E^((2*a)/b)*(-((a + b
*ArcSinh[c*x])/b))^n) - (E^((2*a)/b)*Gamma[1 + n, (2*(a + b*ArcSinh[c*x]))/
b])/(2^n*(a/b + ArcSinh[c*x])^n)))/(8*c*Sqrt[d*(1 + c^2*x^2)])
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{c^2dx^2+d}\,(b\,\text{arsinh}\,(cx)+a)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int (a+b\,\mathrm{arcsinh}\,(cx))^n\,\sqrt{c^2d\,x^2+d}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c^2dx^2+d}\,(b\,\mathrm{arsinh}\,(cx)+a)^n\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{asinh}\,(c\,x))^n\,\sqrt{d\,c^2\,x^2+d}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\left(c^2x^2+1\right)}\,(a+b\,\mathrm{asinh}\,(cx))^n\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**n*(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**n, x)
```

3.515 $\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n}{x}\,dx$

Optimal. Leaf size=199

$$d\text{Int}\left(\frac{\left(a+b\sinh^{-1}(cx)\right)^n}{x\sqrt{c^2dx^2+d}},x\right)+\frac{de^{-\frac{a}{b}}\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{a+b\sinh^{-1}(cx)}{b}\right)}{2\sqrt{c^2dx^2+d}}+$$

```
[Out] 1/2*d*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(-a-b*arcsinh(c*x))/b)*(c^2*x^2+1)^(1/
2)/exp(a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+1/2*d*exp(a/b)*
(a+b*arcsinh(c*x))^n*GAMMA(1+n,(a+b*arcsinh(c*x))/b)*(c^2*x^2+1)^(1/2)/(((a
+b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+d*Unintegrable((a+b*arcsinh(c*x)
)^n/x/(c^2*d*x^2+d)^(1/2),x)
```

Rubi [A] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n)/x,x]
```

```
[Out] Defer[Int][(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n)/x, x]
```

Rubi steps

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n}{x}\,dx=\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n}{x}\,dx$$

Mathematica [A] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n)/x,x]
```

```
[Out] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n)/x, x]
```

fricas [A] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\,(b\,\text{arsinh}\,(cx)+a)^n}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n/x, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^{n}\,\sqrt{c^{2}d\,x^{2}+d}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x,x)
```

```
[Out] int((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^{2}dx^{2}+d}\,(b\,\mathrm{arsinh}\,(cx)+a)^{n}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))^{n}\,\sqrt{d\,c^{2}\,x^{2}+d}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(1/2))/x,x)
```

```
[Out] int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(1/2))/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\left(c^{2}x^{2}+1\right)}\,(a+b\,\mathrm{asinh}\,(cx))^{n}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**n*(c**2*d*x**2+d)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**n/x, x)
```

3.516 $\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n}{x^2}\,dx$

Optimal. Leaf size=84

$$d\mathrm{Int}\left(\frac{\left(a+b\sinh^{-1}(cx)\right)^n}{x^2\sqrt{c^2dx^2+d}},x\right)+\frac{cd\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^{n+1}}{b(n+1)\sqrt{c^2dx^2+d}}$$

```
[Out] c*d*(a+b*arcsinh(c*x))^(1+n)*(c^2*x^2+1)^(1/2)/b/(1+n)/(c^2*d*x^2+d)^(1/2)+
d*Unintegrable((a+b*arcsinh(c*x))^n/x^2/(c^2*d*x^2+d)^(1/2),x)
```

Rubi [A] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n)/x^2,x]
```

```
[Out] Defer[Int][(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n)/x^2, x]
```

Rubi steps

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n}{x^2}\,dx=\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n}{x^2}\,dx$$

Mathematica [A] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n)/x^2,x]
```

```
[Out] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n)/x^2, x]
```

fricas [A] time = 0.51, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{c^2dx^2+d}\,(b\,\mathrm{arsinh}\,(cx)+a)^n}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n/x^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^{n}\sqrt{c^{2}d\,x^{2}+d}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x^2,x)
```

```
[Out] int((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c^{2}dx^{2}+d}\,(b\,\mathrm{arsinh}\,(cx)+a)^{n}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n/x^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))^{n}\sqrt{d\,c^{2}\,x^{2}+d}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(1/2))/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(1/2))/x^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\left(c^{2}x^{2}+1\right)}\,(a+b\,\mathrm{asinh}\,(cx))^{n}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**n*(c**2*d*x**2+d)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**n/x**2, x)
```

3.517 $\int x^2 \left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right)^n \, dx$

Optimal. Leaf size=616

$$-\frac{d\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^{n+1}}{16bc^3(n+1)\sqrt{c^2x^2+1}}+\frac{d2^{-n-7}3^{-n-1}e^{-\frac{6a}{b}}\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(n\right.}{c^3\sqrt{c^2x^2+1}}$$

```
[Out] -1/16*d*(a+b*arcsinh(c*x))^(1+n)*(c^2*d*x^2+d)^(1/2)/b/c^3/(1+n)/(c^2*x^2+1
)^(1/2)+2^(-7-n)*3^(-1-n)*d*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-6*(a+b*arcsinh(
c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/exp(6*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^
2*x^2+1)^(1/2)+2^(-7-2*n)*d*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-4*(a+b*arcsinh(
c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/exp(4*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^
2*x^2+1)^(1/2)-2^(-7-n)*d*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-2*(a+b*arcsinh(c*
x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/exp(2*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*
x^2+1)^(1/2)+2^(-7-n)*d*exp(2*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,2*(a+b*ar
csinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+
1)^(1/2)-2^(-7-2*n)*d*exp(4*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,4*(a+b*arcs
inh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)
^(1/2)-2^(-7-n)*3^(-1-n)*d*exp(6*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,6*(a+b
*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x
^2+1)^(1/2)
```

Rubi [A] time = 0.83, antiderivative size = 616, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {5782, 5779, 5448, 3307, 2181}

$$\frac{d2^{-n-7}3^{-n-1}e^{-\frac{6a}{b}}\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{6\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{c^3\sqrt{c^2x^2+1}}+\frac{d2^{-2n-7}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] -(d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^(1 + n))/(16*b*c^3*(1 + n)*Sqr
t[1 + c^2*x^2]) + (2^(-7 - n)*3^(-1 - n)*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSi
nh[c*x])^n*Gamma[1 + n, (-6*(a + b*ArcSinh[c*x]))/b])/(c^3*E^((6*a)/b)*Sqrt
[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (2^(-7 - 2*n)*d*Sqrt[d + c^2
*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-4*(a + b*ArcSinh[c*x]))/b])/(
c^3*E^((4*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) - (2^(-7 -
 n)*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-2*(a + b*Ar
cSinh[c*x]))/b])/(c^3*E^((2*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])
/b))^n) + (2^(-7 - n)*d*E^((2*a)/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]
)^n*Gamma[1 + n, (2*(a + b*ArcSinh[c*x]))/b])/(c^3*Sqrt[1 + c^2*x^2]*((a +
b*ArcSinh[c*x])/b)^n) - (2^(-7 - 2*n)*d*E^((4*a)/b)*Sqrt[d + c^2*d*x^2]*(a
+ b*ArcSinh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcSinh[c*x]))/b])/(c^3*Sqrt[1 +
 c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n) - (2^(-7 - n)*3^(-1 - n)*d*E^((6*a)/b
)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (6*(a + b*ArcSinh
[c*x]))/b])/(c^3*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5782

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPart[p])/(1 + c^2*
x^2)^FracPart[p], Int[x^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -
1] && IGtQ[m, 0] &&  !(IntegerQ[p] || GtQ[d, 0])
```

Rubi steps

$$\int x^2\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx=\frac{\left(d\sqrt{d+c^2dx^2}\right)\int x^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx}{\sqrt{1+c^2x^2}}$$

$$=\frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\cosh^4(x)\sinh^2(x)\,dx,x,\sinh^{-1}(c\right.}{c^3\sqrt{1+c^2x^2}}$$

$$=\frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\left(-\frac{1}{16}(a+bx)^n-\frac{1}{32}(a+bx)^n\cosh(2x)+\frac{1}{1}\right.\right.}{c^3\sqrt{1+}}$$

$$=-\frac{d\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{16bc^3(1+n)\sqrt{1+c^2x^2}}-\frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int(a\right.}{32c^3}$$

$$=-\frac{d\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{16bc^3(1+n)\sqrt{1+c^2x^2}}+\frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int e^{-}\right.}{64c^3\sqrt{}}$$

$$=-\frac{d\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{16bc^3(1+n)\sqrt{1+c^2x^2}}+\frac{2^{-7-n}3^{-1-n}de^{-\frac{6a}{b}}\sqrt{d+c^2dx^2}}{}$$

Mathematica [A] time = 3.19, size = 429, normalized size = 0.70

$$-\frac{d^2 2^{-2n-7}3^{-n-1}e^{-\frac{6a}{b}}\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\left(2^ne^{\frac{6a}{b}}\left(2^{n+3}3^{n+1}\left(a+b\sinh^{-1}(cx)\right)\left(-\frac{(a+}{}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] -((2^(-7 - 2*n)*3^(-1 - n)*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*(-(
2^n*b*(1 + n)*(a/b + ArcSinh[c*x])^n*Gamma[1 + n, (-6*(a + b*ArcSinh[c*x]))
/b]) - 3^(1 + n)*b*E^((2*a)/b)*(1 + n)*(a/b + ArcSinh[c*x])^n*Gamma[1 + n,
(-4*(a + b*ArcSinh[c*x]))/b] + 2^n*3^(1 + n)*b*E^((4*a)/b)*(1 + n)*(a/b + A
rcSinh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcSinh[c*x]))/b] - 2^n*3^(1 + n)*b*
E^((8*a)/b)*(1 + n)*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1 + n, (2*(a + b*Ar
cSinh[c*x]))/b] + 3^(1 + n)*b*E^((10*a)/b)*(1 + n)*(-((a + b*ArcSinh[c*x])/
b))^n*Gamma[1 + n, (4*(a + b*ArcSinh[c*x]))/b] + 2^n*E^((6*a)/b)*(2^(3 + n)
*3^(1 + n)*(a + b*ArcSinh[c*x])*(-((a + b*ArcSinh[c*x])^2/b^2))^n + b*E^((6
*a)/b)*(1 + n)*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1 + n, (6*(a + b*ArcSinh
[c*x]))/b])))/(b*c^3*E^((6*a)/b)*(1 + n)*Sqrt[d + c^2*d*x^2]*(-((a + b*ArcS
inh[c*x])^2/b^2))^n))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(c^2dx^4 + dx^2\right)\sqrt{c^2dx^2 + d}\,(b\,\text{arsinh}\,(cx) + a)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="fricas
")
```

```
[Out] integral((c^2*d*x^4 + d*x^2)*sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c^2dx^2 + d\right)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx) + a)^n x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n*x^2, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int x^2\left(c^2d\,x^2 + d\right)^{\frac{3}{2}}(a + b\,\text{arcsinh}\,(cx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x)
```

```
[Out] int(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c^2dx^2 + d\right)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx) + a)^n x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="maxima
")
```

```
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n*x^2, x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 (a + b\,\mathrm{asinh}(c\,x))^n \left(d\,c^2\,x^2 + d\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(3/2),x)
[Out] int(x^2*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(3/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**n,x)
[Out] Timed out
```

3.518 $\int x\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx$

Optimal. Leaf size=542

$$\frac{d5^{-n-1}e^{-\frac{5a}{b}}\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{5\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{32c^2\sqrt{c^2x^2+1}}+\frac{d3^{-n}e^{-\frac{3a}{b}}\sqrt{c^2dx^2+d}}{}$$

```
[Out] 1/32*5^(-1-n)*d*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-5*(a+b*arcsinh(c*x))/b)*(c^
2*d*x^2+d)^(1/2)/c^2/exp(5*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/
2)+1/32*d*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-3*(a+b*arcsinh(c*x))/b)*(c^2*d*x^
2+d)^(1/2)/(3^n)/c^2/exp(3*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/
2)+1/16*d*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(-a-b*arcsinh(c*x))/b)*(c^2*d*x^2+
d)^(1/2)/c^2/exp(a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)+1/16*d*
exp(a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)
^(1/2)/c^2/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)+1/32*d*exp(3*a/b)*(
a+b*arcsinh(c*x))^n*GAMMA(1+n,3*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/(
3^n)/c^2/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)+1/32*5^(-1-n)*d*exp(5
*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,5*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^
(1/2)/c^2/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.63, antiderivative size = 542, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5782, 5779, 5448, 3308, 2181}

$$\frac{d5^{-n-1}e^{-\frac{5a}{b}}\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{5\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{32c^2\sqrt{c^2x^2+1}}+\frac{d3^{-n}e^{-\frac{3a}{b}}\sqrt{c^2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (5^(-1 - n)*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-5*(
a + b*ArcSinh[c*x]))/b])/(32*c^2*E^((5*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b*Ar
cSinh[c*x])/b))^n) + (d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1
+ n, (-3*(a + b*ArcSinh[c*x]))/b])/(32*3^n*c^2*E^((3*a)/b)*Sqrt[1 + c^2*x^2
]*(-((a + b*ArcSinh[c*x])/b))^n) + (d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x])^n*Gamma[1 + n, -((a + b*ArcSinh[c*x])/b)])/(16*c^2*E^(a/b)*Sqrt[1 + c^2
*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (d*E^(a/b)*Sqrt[d + c^2*d*x^2]*(a +
b*ArcSinh[c*x])^n*Gamma[1 + n, (a + b*ArcSinh[c*x])/b])/(16*c^2*Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])/b)^n) + (d*E^((3*a)/b)*Sqrt[d + c^2*d*x^2]*(a
+ b*ArcSinh[c*x])^n*Gamma[1 + n, (3*(a + b*ArcSinh[c*x]))/b])/(32*3^n*c^2*S
qrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n) + (5^(-1 - n)*d*E^((5*a)/b)*Sq
rt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (5*(a + b*ArcSinh[c*x
]))/b])/(32*c^2*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
```

```
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5782

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPart[p])/(1 + c^2*
x^2)^FracPart[p], Int[x^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -
1] && IGtQ[m, 0] &&  !(IntegerQ[p] || GtQ[d, 0])
```

Rubi steps

$$\int x\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx=\frac{\left(d\sqrt{d+c^2dx^2}\right)\int x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx}{\sqrt{1+c^2x^2}}$$

$$=\frac{\left(d\sqrt{d+c^2dx^2}\right)\mathrm{Subst}\left(\int(a+bx)^n\cosh^4(x)\sinh(x)\,dx,x,\sinh^{-1}(cx)\right.}{c^2\sqrt{1+c^2x^2}}$$

$$=\frac{\left(d\sqrt{d+c^2dx^2}\right)\mathrm{Subst}\left(\int\left(\frac{1}{8}(a+bx)^n\sinh(x)+\frac{3}{16}(a+bx)^n\sinh(3x\right.\right.}{c^2\sqrt{1+c^2x^2}}$$

$$=\frac{\left(d\sqrt{d+c^2dx^2}\right)\mathrm{Subst}\left(\int(a+bx)^n\sinh(5x)\,dx,x,\sinh^{-1}(cx)\right)}{16c^2\sqrt{1+c^2x^2}}+\frac{(d\sqrt{}}{}$$

$$=-\frac{\left(d\sqrt{d+c^2dx^2}\right)\mathrm{Subst}\left(\int e^{-5x}(a+bx)^n\,dx,x,\sinh^{-1}(cx)\right)}{32c^2\sqrt{1+c^2x^2}}+\frac{\left(d\sqrt{d\right.}}{}$$

$$=\frac{5^{-1-n}de^{-\frac{5a}{b}}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+\right.}{32c^2\sqrt{1+c^2x^2}}$$

Mathematica [A] time = 1.79, size = 390, normalized size = 0.72

$$\frac{d^2 15^{-n-1}e^{-\frac{5a}{b}}\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{b^2}\right)^{-2n}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)^n\left(3^n\left(-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{b^2}\right)^n\Gamma\left(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (15^(-1 - n)*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*(2*15^(1 + n)*E^(
(6*a)/b)*(-((a + b*ArcSinh[c*x])/b))^n*(-((a + b*ArcSinh[c*x])^2/b^2))^n*Ga
mma[1 + n, a/b + ArcSinh[c*x]] + 3*(a/b + ArcSinh[c*x])^n*(3^n*(-((a + b*Ar
cSinh[c*x])^2/b^2))^n*Gamma[1 + n, (-5*(a + b*ArcSinh[c*x]))/b] + 5^(1 + n)
*E^((2*a)/b)*(-((a + b*ArcSinh[c*x])^2/b^2))^n*Gamma[1 + n, (-3*(a + b*ArcS
inh[c*x]))/b] + 2*3^n*5^(1 + n)*E^((4*a)/b)*(-((a + b*ArcSinh[c*x])^2/b^2))
^n*Gamma[1 + n, -((a + b*ArcSinh[c*x])/b)] + 5^(1 + n)*E^((8*a)/b)*(-((a +
b*ArcSinh[c*x])/b))^(2*n)*Gamma[1 + n, (3*(a + b*ArcSinh[c*x]))/b] + 3^n*E^
((10*a)/b)*(-((a + b*ArcSinh[c*x])/b))^(2*n)*Gamma[1 + n, (5*(a + b*ArcSinh
[c*x]))/b])))/(32*c^2*E^((5*a)/b)*Sqrt[d + c^2*d*x^2]*(-((a + b*ArcSinh[c*x
])^2/b^2))^(2*n))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(c^2dx^3+dx\right)\sqrt{c^2dx^2+d}\,(b\,\text{arsinh}\,(cx)+a)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="fricas")
```

```
[Out] integral((c^2*d*x^3 + d*x)*sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int x\left(c^2d\,x^2+d\right)^{\frac{3}{2}}(a+b\,\text{arcsinh}\,(cx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x)
```

```
[Out] int(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c^2dx^2+d\right)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^n x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,(a+b\,\text{asinh}\,(c\,x))^n\,\left(d\,c^2\,x^2+d\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int(x*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**n,x)
```

```
[Out] Timed out
```

3.519 $\int \left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right)^n \, dx$

Optimal. Leaf size=420

$$\frac{3d\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^{n+1}}{8bc(n+1)\sqrt{c^2x^2+1}}+\frac{d2^{-2(n+3)}e^{-\frac{4a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(n+1\right.}{c\sqrt{c^2x^2+1}}$$

```
[Out] 3/8*d*(a+b*arcsinh(c*x))^(1+n)*(c^2*d*x^2+d)^(1/2)/b/c/(1+n)/(c^2*x^2+1)^(1
/2)+d*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-4*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)
^(1/2)/(2^(6+2*n))/c/exp(4*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/
2)+2^(-3-n)*d*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-2*(a+b*arcsinh(c*x))/b)*(c^2*
d*x^2+d)^(1/2)/c/exp(2*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)-2
^(-3-n)*d*exp(2*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,2*(a+b*arcsinh(c*x))/b)
*(c^2*d*x^2+d)^(1/2)/c/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)-d*exp(4
*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,4*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^
(1/2)/(2^(6+2*n))/c/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 420, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5702, 5699, 3312, 3307, 2181}

$$\frac{d2^{-2(n+3)}e^{-\frac{4a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{c\sqrt{c^2x^2+1}}+\frac{d2^{-n-3}e^{-\frac{2a}{b}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (3*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^(1 + n))/(8*b*c*(1 + n)*Sqrt[
1 + c^2*x^2]) + (d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n,
(-4*(a + b*ArcSinh[c*x]))/b])/(2^(2*(3 + n))*c*E^((4*a)/b)*Sqrt[1 + c^2*x^2
]*(-((a + b*ArcSinh[c*x])/b))^n) + (2^(-3 - n)*d*Sqrt[d + c^2*d*x^2]*(a + b
*ArcSinh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcSinh[c*x]))/b])/(c*E^((2*a)/b)*
Sqrt[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) - (2^(-3 - n)*d*E^((2*a)/b
)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcSinh
[c*x]))/b])/(c*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n) - (d*E^((4*a)/
b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcSin
h[c*x]))/b])/(2^(2*(3 + n))*c*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rule 5702

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(d^(p - 1/2)*Sqrt[d + e*x^2])/Sqrt[1 + c^2*x^2], Int[(1 +
 c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n}, x]
&& EqQ[e, c^2*d] && IGtQ[2*p, 0] &&  !(IntegerQ[p] || GtQ[d, 0])
```

Rubi steps

$$\int \left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx=\frac{\left(d\sqrt{d+c^2dx^2}\right)\int\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx}{\sqrt{1+c^2x^2}}$$
$$=\frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\cosh^4(x)\,dx,x,\sinh^{-1}(cx)\right)}{c\sqrt{1+c^2x^2}}$$
$$=\frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\left(\frac{3}{8}(a+bx)^n+\frac{1}{2}(a+bx)^n\cosh(2x)+\frac{1}{8}(a+b\right.\right.}{c\sqrt{1+c^2x^2}}$$
$$=\frac{3d\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{8bc(1+n)\sqrt{1+c^2x^2}}+\frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int(a+\right.}{8c\sqrt{1}}$$
$$=\frac{3d\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{8bc(1+n)\sqrt{1+c^2x^2}}+\frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int e^{-4x}(\right.}{16c\sqrt{1+}}$$
$$=\frac{3d\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{8bc(1+n)\sqrt{1+c^2x^2}}+\frac{4^{-3-n}de^{-\frac{4a}{b}}\sqrt{d+c^2dx^2}\left(a+b\right.}{}$$

Mathematica [A] time = 1.43, size = 287, normalized size = 0.68

$$\frac{d^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^n\left(4^{-n}e^{-\frac{4a}{b}}\left(-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\left(\left(\frac{a}{b}+\sinh^{-1}(cx)\right)^n\Gamma\left(n+1,-\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)-e^{\frac{8a}{b}}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*((-8*(a + b*ArcSinh[c*x]))/(b
 + b*n) + 8*((4*a + 4*b*ArcSinh[c*x])/(b + b*n) + Gamma[1 + n, (-2*(a + b*A
rcSinh[c*x]))/b]/(2^n*E^((2*a)/b)*(-((a + b*ArcSinh[c*x])/b))^n) - (E^((2*a
)/b)*Gamma[1 + n, (2*(a + b*ArcSinh[c*x]))/b])/(2^n*(a/b + ArcSinh[c*x])^n)
) + ((a/b + ArcSinh[c*x])^n*Gamma[1 + n, (-4*(a + b*ArcSinh[c*x]))/b] - E^(
(8*a)/b)*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1 + n, (4*(a + b*ArcSinh[c*x])
)/b])/(4^n*E^((4*a)/b)*(-((a + b*ArcSinh[c*x])^2/b^2))^n)))/(64*c*Sqrt[d +
c^2*d*x^2])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(c^2dx^2+d\right)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="fricas")
[Out] integral((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \left(c^2d\,x^2+d\right)^{\frac{3}{2}}(a+b\,\text{arcsinh}\,(cx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x)
[Out] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c^2dx^2+d\right)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="maxima")
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\text{asinh}\,(c\,x))^n\,\left(d\,c^2\,x^2+d\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(3/2),x)
[Out] int((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**n,x)
[Out] Timed out
```

3.520 $\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x}\,dx$

Optimal. Leaf size=390

$$d^2\text{Int}\left(\frac{\left(a+b\sinh^{-1}(cx)\right)^n}{x\sqrt{c^2dx^2+d}},x\right)+\frac{d^2 3^{-n-1}e^{-\frac{3a}{b}}\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{8\sqrt{c^2dx^2+d}}$$

```
[Out] 1/8*3^(-1-n)*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-3*(a+b*arcsinh(c*x))/b)*(c
^2*x^2+1)^(1/2)/exp(3*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+
5/8*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(-a-b*arcsinh(c*x))/b)*(c^2*x^2+1)^(
1/2)/exp(a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+5/8*d^2*exp(a
/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(a+b*arcsinh(c*x))/b)*(c^2*x^2+1)^(1/2)/
(((a+b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+1/8*3^(-1-n)*d^2*exp(3*a/b)*
(a+b*arcsinh(c*x))^n*GAMMA(1+n,3*(a+b*arcsinh(c*x))/b)*(c^2*x^2+1)^(1/2)/((
(a+b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+d^2*Unintegrable((a+b*arcsinh(
c*x))^n/x/(c^2*d*x^2+d)^(1/2),x)
```

Rubi [A] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n)/x,x]
[Out] Defer[Int][((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n)/x, x]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x}\,dx=\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x}\,dx$$

Mathematica [A] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n)/x,x]
[Out] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n)/x, x]
```

fricas [A] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^2dx^2+d\right)^{\frac{3}{2}}\left(b\,\text{arsinh}\left(cx\right)+a\right)^n}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x,x, algorithm="fricas")
[Out] integral((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n/x, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{\left(c^{2} d\,x^{2}+d\right)^{\frac{3}{2}} \left(a +b \,\mathrm{arcsinh}\left(c x \right)\right)^{n}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x,x)
[Out] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{{\left(c^{2} d x^{2} + d\right)}^{\frac{3}{2}} {\left(b \operatorname{arsinh}\left(c x\right) + a\right)}^{n}}{x}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x,x, algorithm="maxima")
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^n\,{\left(d\,c^2\,x^2+d\right)}^{3/2}}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(3/2))/x,x)
[Out] int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(3/2))/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d \left(c^{2} x^{2} + 1\right)\right)^{\frac{3}{2}} \left(a + b \operatorname{asinh}{\left(c x \right)}\right)^{n}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**n/x,x)
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**n/x, x)
```

3.521 $\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x^2}\,dx$

Optimal. Leaf size=273

$$d^2\text{Int}\left(\frac{\left(a+b\sinh^{-1}(cx)\right)^n}{x^2\sqrt{c^2dx^2+d}},x\right)+\frac{3cd^2\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)^{n+1}}{2b(n+1)\sqrt{c^2dx^2+d}}+\frac{cd^2 2^{-n-3}e^{-\frac{2a}{b}}\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2}}$$

```
[Out] 3/2*c*d^2*(a+b*arcsinh(c*x))^(1+n)*(c^2*x^2+1)^(1/2)/b/(1+n)/(c^2*d*x^2+d)^
(1/2)+2^(-3-n)*c*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-2*(a+b*arcsinh(c*x))/b
)*(c^2*x^2+1)^(1/2)/exp(2*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1
/2)-2^(-3-n)*c*d^2*exp(2*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,2*(a+b*arcsinh
(c*x))/b)*(c^2*x^2+1)^(1/2)/(((a+b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+
d^2*Unintegrable((a+b*arcsinh(c*x))^n/x^2/(c^2*d*x^2+d)^(1/2),x)
```

Rubi [A] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n)/x^2,x]
```

```
[Out] Defer[Int][((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n)/x^2, x]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x^2}\,dx=\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x^2}\,dx$$

Mathematica [A] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n)/x^2,x]
```

```
[Out] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n)/x^2, x]
```

fricas [A] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^2dx^2+d\right)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^n}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x^2,x, algorithm="fricas
")
```

```
[Out] integral((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n/x^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{\left(c^{2} d \,x^{2}+d \right)^{\frac{3}{2}} \left(a +b \,\mathrm{arcsinh}\left(c x \right)\right)^{n}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x^2,x)
```

```
[Out] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{{\left(c^{2} d x^{2} + d\right)}^{\frac{3}{2}} {\left(b \operatorname{arsinh}\left(c x\right) + a\right)}^{n}}{x^{2}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x^2,x, algorithm="maxima
")
```

```
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n/x^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^n\,{\left(d\,c^2\,x^2+d\right)}^{3/2}}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(3/2))/x^2,x)
```

```
[Out] int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(3/2))/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**n/x**2,x)
```

```
[Out] Timed out
```

3.522 $\int x^2 \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right)^n \, dx$

Optimal. Leaf size=816

$$\frac{2^{-3n-11}d^2e^{-\frac{8a}{b}}\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^n\Gamma\left(n+1,-\frac{8\left(a+b\sinh^{-1}(cx)\right)}{b}\right)\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}}{c^3\sqrt{c^2x^2+1}}+\frac{2^{-n-7}3^{-n-1}d^2e^{-\frac{6a}{b}}\sqrt{c^2}}{}$$

```
[Out] -5/128*d^2*(a+b*arcsinh(c*x))^(1+n)*(c^2*d*x^2+d)^(1/2)/b/c^3/(1+n)/(c^2*x^
2+1)^(1/2)+2^(-11-3*n)*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-8*(a+b*arcsinh(c
*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/exp(8*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2
*x^2+1)^(1/2)+2^(-7-n)*3^(-1-n)*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-6*(a+b*
arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/exp(6*a/b)/(((-a-b*arcsinh(c*x))/b
)^n)/(c^2*x^2+1)^(1/2)+d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-4*(a+b*arcsinh(c
*x))/b)*(c^2*d*x^2+d)^(1/2)/(2^(8+2*n))/c^3/exp(4*a/b)/(((-a-b*arcsinh(c*x)
)/b)^n)/(c^2*x^2+1)^(1/2)-2^(-7-n)*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-2*(a
+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/exp(2*a/b)/(((-a-b*arcsinh(c*x)
)/b)^n)/(c^2*x^2+1)^(1/2)+2^(-7-n)*d^2*exp(2*a/b)*(a+b*arcsinh(c*x))^n*GAMM
A(1+n,2*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/(((a+b*arcsinh(c*x))/
b)^n)/(c^2*x^2+1)^(1/2)-d^2*exp(4*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,4*(a+
b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/(2^(8+2*n))/c^3/(((a+b*arcsinh(c*x))
/b)^n)/(c^2*x^2+1)^(1/2)-2^(-7-n)*3^(-1-n)*d^2*exp(6*a/b)*(a+b*arcsinh(c*x)
)^n*GAMMA(1+n,6*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/(((a+b*arcsin
h(c*x))/b)^n)/(c^2*x^2+1)^(1/2)-2^(-11-3*n)*d^2*exp(8*a/b)*(a+b*arcsinh(c*x
))^n*GAMMA(1+n,8*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^3/(((a+b*arcsi
nh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.97, antiderivative size = 816, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {5782, 5779, 5448, 3307, 2181}

$$\frac{2^{-3n-11}d^2e^{-\frac{8a}{b}}\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^n\text{Gamma}\left(n+1,-\frac{8\left(a+b\sinh^{-1}(cx)\right)}{b}\right)\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}}{c^3\sqrt{c^2x^2+1}}+\frac{2^{-n-7}3^{-n-1}d^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (-5*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^(1 + n))/(128*b*c^3*(1 + n
)*Sqrt[1 + c^2*x^2]) + (2^(-11 - 3*n)*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSin
h[c*x])^n*Gamma[1 + n, (-8*(a + b*ArcSinh[c*x]))/b])/(c^3*E^((8*a)/b)*Sqrt[
1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (2^(-7 - n)*3^(-1 - n)*d^2*Sq
rt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-6*(a + b*ArcSinh[c*
x]))/b])/(c^3*E^((6*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n)
+ (d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-4*(a + b*A
rcSinh[c*x]))/b])/(2^(2*(4 + n))*c^3*E^((4*a)/b)*Sqrt[1 + c^2*x^2]*(-((a +
b*ArcSinh[c*x])/b))^n) - (2^(-7 - n)*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh
[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcSinh[c*x]))/b])/(c^3*E^((2*a)/b)*Sqrt[1
 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (2^(-7 - n)*d^2*E^((2*a)/b)*Sq
rt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcSinh[c*x
]))/b])/(c^3*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n) - (d^2*E^((4*a)/
b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcSin
h[c*x]))/b])/(2^(2*(4 + n))*c^3*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^
n) - (2^(-7 - n)*3^(-1 - n)*d^2*E^((6*a)/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcS
inh[c*x])^n*Gamma[1 + n, (6*(a + b*ArcSinh[c*x]))/b])/(c^3*Sqrt[1 + c^2*x^2
]*((a + b*ArcSinh[c*x])/b)^n) - (2^(-11 - 3*n)*d^2*E^((8*a)/b)*Sqrt[d + c^2
*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (8*(a + b*ArcSinh[c*x]))/b])/(c
^3*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5782

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPart[p])/(1 + c^2*
x^2)^FracPart[p], Int[x^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -
1] && IGtQ[m, 0] &&  !(IntegerQ[p] || GtQ[d, 0])
```

Rubi steps

$$\int x^2\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx=\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\int x^2\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx}{\sqrt{1+c^2x^2}}$$

$$=\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\cosh^6(x)\sinh^2(x)\,dx,x,\sinh^{-1}(\right.}{c^3\sqrt{1+c^2x^2}}$$

$$=\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\left(-\frac{5}{128}(a+bx)^n-\frac{1}{32}(a+bx)^n\cosh(2x)+\right.\right.}{}$$

$$=-\frac{5d^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{128bc^3(1+n)\sqrt{1+c^2x^2}}+\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\right.}{12}$$

$$=-\frac{5d^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{128bc^3(1+n)\sqrt{1+c^2x^2}}+\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\right.}{256c}$$

$$=-\frac{5d^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{128bc^3(1+n)\sqrt{1+c^2x^2}}+\frac{2^{-11-3n}d^2e^{-\frac{8a}{b}}\sqrt{d+c^2dx^2}}{}$$

Mathematica [A] time = 6.94, size = 667, normalized size = 0.82

$$-\frac{d^3 2^{-3n-11}3^{-n-1}e^{-\frac{8a}{b}}\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\left(e^{\frac{8a}{b}}\left(5a2^{3n+4}3^{n+1}\left(-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{b^2}\right)^n+5\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] -((2^(-11 - 3*n)*3^(-1 - n)*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*(-
(3^(1 + n)*b*(1 + n)*(a/b + ArcSinh[c*x])^n*Gamma[1 + n, (-8*(a + b*ArcSinh
[c*x]))/b]) - 4^(2 + n)*b*E^((2*a)/b)*(1 + n)*(a/b + ArcSinh[c*x])^n*Gamma[
1 + n, (-6*(a + b*ArcSinh[c*x]))/b] - 2^(3 + n)*3^(1 + n)*b*E^((4*a)/b)*(1
+ n)*(a/b + ArcSinh[c*x])^n*Gamma[1 + n, (-4*(a + b*ArcSinh[c*x]))/b] + 3^(
1 + n)*4^(2 + n)*b*E^((6*a)/b)*(1 + n)*(a/b + ArcSinh[c*x])^n*Gamma[1 + n,
(-2*(a + b*ArcSinh[c*x]))/b] + E^((8*a)/b)*(5*2^(4 + 3*n)*3^(1 + n)*a*(-((a
 + b*ArcSinh[c*x])^2/b^2))^n + 5*2^(4 + 3*n)*3^(1 + n)*b*ArcSinh[c*x]*(-((a
 + b*ArcSinh[c*x])^2/b^2))^n - 3^(1 + n)*4^(2 + n)*b*E^((2*a)/b)*(1 + n)*(-
((a + b*ArcSinh[c*x])/b))^n*Gamma[1 + n, (2*(a + b*ArcSinh[c*x]))/b] + 2^(3
 + n)*3^(1 + n)*b*E^((4*a)/b)*(1 + n)*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1
 + n, (4*(a + b*ArcSinh[c*x]))/b] + 4^(2 + n)*b*E^((6*a)/b)*(-((a + b*ArcSi
nh[c*x])/b))^n*Gamma[1 + n, (6*(a + b*ArcSinh[c*x]))/b] + 4^(2 + n)*b*E^((6
*a)/b)*n*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1 + n, (6*(a + b*ArcSinh[c*x])
)/b] + 3^(1 + n)*b*E^((8*a)/b)*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1 + n, (
8*(a + b*ArcSinh[c*x]))/b] + 3^(1 + n)*b*E^((8*a)/b)*n*(-((a + b*ArcSinh[c*
x])/b))^n*Gamma[1 + n, (8*(a + b*ArcSinh[c*x]))/b])))/(b*c^3*E^((8*a)/b)*(1
 + n)*Sqrt[d + c^2*d*x^2]*(-((a + b*ArcSinh[c*x])^2/b^2))^n))
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(c^4d^2x^6+2\,c^2d^2x^4+d^2x^2\right)\sqrt{c^2dx^2+d}\,(b\,\text{arsinh}\,(cx)+a)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="fricas
")
```

```
[Out] integral((c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2)*sqrt(c^2*d*x^2 + d)*(b*arc
sinh(c*x) + a)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c^2dx^2+d\right)^{\frac{5}{2}}(b \operatorname{arsinh}(cx)+a)^n x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^n*x^2, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int x^2\left(c^2d\,x^2+d\right)^{\frac{5}{2}}(a+b \operatorname{arcsinh}(cx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x)
```

```
[Out] int(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c^2dx^2+d\right)^{\frac{5}{2}}(b \operatorname{arsinh}(cx)+a)^n x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="maxima
")
```

```
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^n*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,(a+b\,\mathrm{asinh}(c\,x))^n\,\left(d\,c^2\,x^2+d\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int(x^2*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**n,x)
```

```
[Out] Timed out
```

3.523 $\int x\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx$

Optimal. Leaf size=745

$$\frac{d^2 7^{-n-1}e^{-\frac{7a}{b}}\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{7\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{128c^2\sqrt{c^2x^2+1}}+\frac{d^2 5^{-n}e^{-\frac{5a}{b}}\sqrt{c^2dx^2+d}\,(}{}$$

```
[Out] 1/128*7^(-1-n)*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-7*(a+b*arcsinh(c*x))/b)*
(c^2*d*x^2+d)^(1/2)/c^2/exp(7*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^
(1/2)+1/128*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-5*(a+b*arcsinh(c*x))/b)*(c^
2*d*x^2+d)^(1/2)/(5^n)/c^2/exp(5*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+
1)^(1/2)+1/128*3^(1-n)*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-3*(a+b*arcsinh(c
*x))/b)*(c^2*d*x^2+d)^(1/2)/c^2/exp(3*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2
*x^2+1)^(1/2)+5/128*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(-a-b*arcsinh(c*x))/
b)*(c^2*d*x^2+d)^(1/2)/c^2/exp(a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)
^(1/2)+5/128*d^2*exp(a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(a+b*arcsinh(c*x))
/b)*(c^2*d*x^2+d)^(1/2)/c^2/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)+1/
128*3^(1-n)*d^2*exp(3*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,3*(a+b*arcsinh(c*
x))/b)*(c^2*d*x^2+d)^(1/2)/c^2/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2)
+1/128*d^2*exp(5*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,5*(a+b*arcsinh(c*x))/b
)*(c^2*d*x^2+d)^(1/2)/(5^n)/c^2/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^(1/2
)+1/128*7^(-1-n)*d^2*exp(7*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,7*(a+b*arcsi
nh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c^2/(((a+b*arcsinh(c*x))/b)^n)/(c^2*x^2+1)^
(1/2)
```

Rubi [A] time = 0.79, antiderivative size = 745, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5782, 5779, 5448, 3308, 2181}

$$\frac{d^2 7^{-n-1}e^{-\frac{7a}{b}}\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{7\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{128c^2\sqrt{c^2x^2+1}}+\frac{d^2 5^{-n}e^{-\frac{5a}{b}}\sqrt{c^2d}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (7^(-1 - n)*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-7
*(a + b*ArcSinh[c*x]))/b])/(128*c^2*E^((7*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b
*ArcSinh[c*x])/b))^n) + (d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gam
ma[1 + n, (-5*(a + b*ArcSinh[c*x]))/b])/(128*5^n*c^2*E^((5*a)/b)*Sqrt[1 + c
^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (3^(1 - n)*d^2*Sqrt[d + c^2*d*x^2]
*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-3*(a + b*ArcSinh[c*x]))/b])/(128*c^2
*E^((3*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (5*d^2*Sqrt
[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, -((a + b*ArcSinh[c*x])/
b)])/(128*c^2*E^(a/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (5
*d^2*E^(a/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (a + b
*ArcSinh[c*x])/b])/(128*c^2*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n) +
 (3^(1 - n)*d^2*E^((3*a)/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamm
a[1 + n, (3*(a + b*ArcSinh[c*x]))/b])/(128*c^2*Sqrt[1 + c^2*x^2]*((a + b*Ar
cSinh[c*x])/b)^n) + (d^2*E^((5*a)/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x
])^n*Gamma[1 + n, (5*(a + b*ArcSinh[c*x]))/b])/(128*5^n*c^2*Sqrt[1 + c^2*x^
2]*((a + b*ArcSinh[c*x])/b)^n) + (7^(-1 - n)*d^2*E^((7*a)/b)*Sqrt[d + c^2*d
*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (7*(a + b*ArcSinh[c*x]))/b])/(128
*c^2*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5782

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPart[p])/(1 + c^2*
x^2)^FracPart[p], Int[x^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -
1] && IGtQ[m, 0] &&  !(IntegerQ[p] || GtQ[d, 0])
```

Rubi steps

$$\int x\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx=\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\int x\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx}{\sqrt{1+c^2x^2}}$$

$$=\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\cosh^6(x)\sinh(x)\,dx,x,\sinh^{-1}\right.}{c^2\sqrt{1+c^2x^2}}$$

$$=\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\left(\frac{5}{64}(a+bx)^n\sinh(x)+\frac{9}{64}(a+bx)^n\sin\right.\right.}{c^2}$$

$$=\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\sinh(7x)\,dx,x,\sinh^{-1}(cx)\right)}{64c^2\sqrt{1+c^2x^2}}+$$

$$=-\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int e^{-7x}(a+bx)^n\,dx,x,\sinh^{-1}(cx)\right)}{128c^2\sqrt{1+c^2x^2}}+\frac{\left(d^2\right.}{}$$

$$=\frac{7^{-1-n}d^2e^{-\frac{7a}{b}}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(\right.}{128c^2\sqrt{1+c^2x^2}}$$

Mathematica [A] time = 2.96, size = 685, normalized size = 0.92

$$d^3 105^{-n-1} e^{-\frac{7a}{b}} \sqrt{c^2x^2+1} \left(a+b\sinh^{-1}(cx)\right)^n \left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n} \left(-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{b^2}\right)^{-2n} \left(5^{n+2} 21^{n+1} e^{\frac{8a}{b}} \left(-\frac{a+b\sinh^{-1}(cx)}{b}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (105^(-1 - n)*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*(5^(2 + n)*21^(1
 + n)*E^((8*a)/b)*(-((a + b*ArcSinh[c*x])/b))^(2*n)*(-((a + b*ArcSinh[c*x])
^2/b^2))^n*Gamma[1 + n, a/b + ArcSinh[c*x]] + 15^(1 + n)*(-((a + b*ArcSinh[
c*x])^2/b^2))^(2*n)*Gamma[1 + n, (-7*(a + b*ArcSinh[c*x]))/b] + E^((2*a)/b)
*(5*21^(1 + n)*(-((a + b*ArcSinh[c*x])^2/b^2))^(2*n)*Gamma[1 + n, (-5*(a +
b*ArcSinh[c*x]))/b] + 9*35^(1 + n)*E^((2*a)/b)*(-((a + b*ArcSinh[c*x])^2/b^
2))^(2*n)*Gamma[1 + n, (-3*(a + b*ArcSinh[c*x]))/b] + 5^(2 + n)*21^(1 + n)*
E^((4*a)/b)*(-((a + b*ArcSinh[c*x])^2/b^2))^(2*n)*Gamma[1 + n, -((a + b*Arc
Sinh[c*x])/b)] + 35^(1 + n)*E^((8*a)/b)*(a/b + ArcSinh[c*x])^n*(-((a + b*Ar
cSinh[c*x])/b))^(3*n)*Gamma[1 + n, (3*(a + b*ArcSinh[c*x]))/b] + 8*35^(1 +
n)*E^((8*a)/b)*(-((a + b*ArcSinh[c*x])/b))^(2*n)*(-((a + b*ArcSinh[c*x])^2/
b^2))^n*Gamma[1 + n, (3*(a + b*ArcSinh[c*x]))/b] - 3^(2 + n)*7^(1 + n)*E^((
10*a)/b)*(a/b + ArcSinh[c*x])^n*(-((a + b*ArcSinh[c*x])/b))^(3*n)*Gamma[1 +
 n, (5*(a + b*ArcSinh[c*x]))/b] + 8*21^(1 + n)*E^((10*a)/b)*(-((a + b*ArcSi
nh[c*x])/b))^(2*n)*(-((a + b*ArcSinh[c*x])^2/b^2))^n*Gamma[1 + n, (5*(a + b
*ArcSinh[c*x]))/b] + 15^(1 + n)*E^((12*a)/b)*(a/b + ArcSinh[c*x])^n*(-((a +
 b*ArcSinh[c*x])/b))^(3*n)*Gamma[1 + n, (7*(a + b*ArcSinh[c*x]))/b])))/(128
*c^2*E^((7*a)/b)*Sqrt[d + c^2*d*x^2]*(-((a + b*ArcSinh[c*x])/b))^n*(-((a +
b*ArcSinh[c*x])^2/b^2))^(2*n))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(c^4d^2x^5 + 2\,c^2d^2x^3 + d^2x\right)\sqrt{c^2dx^2+d}\,(b\,\text{arsinh}\,(cx)+a)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="fricas")
```

```
[Out] integral((c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x)*sqrt(c^2*d*x^2 + d)*(b*arcsi
nh(c*x) + a)^n, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int x\left(c^2d\,x^2+d\right)^{\frac{5}{2}}(a+b\,\text{arcsinh}\,(cx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x)
```

```
[Out] int(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c^2dx^2 + d\right)^{\frac{5}{2}}(b\operatorname{arsinh}(cx) + a)^n x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="maxima")
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^n*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,(a + b\operatorname{asinh}(c\,x))^n\,\left(d\,c^2\,x^2 + d\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(5/2),x)
[Out] int(x*(a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**n,x)
[Out] Timed out
```

3.524 $\int \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right)^n \, dx$

Optimal. Leaf size=632

$$\frac{5d^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^{n+1}}{16bc(n+1)\sqrt{c^2x^2+1}}+\frac{d^2 2^{-n-7}3^{-n-1}e^{-\frac{6a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\Gamma\Big(n+}{c\sqrt{c^2x^2+1}}$$

```
[Out] 5/16*d^2*(a+b*arcsinh(c*x))^(1+n)*(c^2*d*x^2+d)^(1/2)/b/c/(1+n)/(c^2*x^2+1)
^(1/2)+2^(-7-n)*3^(-1-n)*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-6*(a+b*arcsinh
(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c/exp(6*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2
*x^2+1)^(1/2)+3*2^(-7-2*n)*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-4*(a+b*arcsi
nh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c/exp(4*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c
^2*x^2+1)^(1/2)+15*2^(-7-n)*d^2*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-2*(a+b*arcs
inh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c/exp(2*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(
c^2*x^2+1)^(1/2)-15*2^(-7-n)*d^2*exp(2*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,
2*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c/(((a+b*arcsinh(c*x))/b)^n)/(c
^2*x^2+1)^(1/2)-3*2^(-7-2*n)*d^2*exp(4*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,
4*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c/(((a+b*arcsinh(c*x))/b)^n)/(c
^2*x^2+1)^(1/2)-2^(-7-n)*3^(-1-n)*d^2*exp(6*a/b)*(a+b*arcsinh(c*x))^n*GAMMA
(1+n,6*(a+b*arcsinh(c*x))/b)*(c^2*d*x^2+d)^(1/2)/c/(((a+b*arcsinh(c*x))/b)^
n)/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.60, antiderivative size = 632, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5702, 5699, 3312, 3307, 2181}

$$\frac{d^2 2^{-n-7}3^{-n-1}e^{-\frac{6a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{6\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{c\sqrt{c^2x^2+1}}+\frac{3d^2 2^{-2n-7}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (5*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^(1 + n))/(16*b*c*(1 + n)*Sq
rt[1 + c^2*x^2]) + (2^(-7 - n)*3^(-1 - n)*d^2*Sqrt[d + c^2*d*x^2]*(a + b*Ar
cSinh[c*x])^n*Gamma[1 + n, (-6*(a + b*ArcSinh[c*x]))/b])/(c*E^((6*a)/b)*Sqr
t[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (3*2^(-7 - 2*n)*d^2*Sqrt[d
+ c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-4*(a + b*ArcSinh[c*x]))/
b])/(c*E^((4*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (15*2
^(-7 - n)*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-2*(
a + b*ArcSinh[c*x]))/b])/(c*E^((2*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcSinh
[c*x])/b))^n) - (15*2^(-7 - n)*d^2*E^((2*a)/b)*Sqrt[d + c^2*d*x^2]*(a + b*A
rcSinh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcSinh[c*x]))/b])/(c*Sqrt[1 + c^2*x^
2]*((a + b*ArcSinh[c*x])/b)^n) - (3*2^(-7 - 2*n)*d^2*E^((4*a)/b)*Sqrt[d + c
^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcSinh[c*x]))/b])/
(c*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n) - (2^(-7 - n)*3^(-1 - n)*d
^2*E^((6*a)/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (6*(
a + b*ArcSinh[c*x]))/b])/(c*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rule 5702

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(d^(p - 1/2)*Sqrt[d + e*x^2])/Sqrt[1 + c^2*x^2], Int[(1 +
 c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n}, x]
&& EqQ[e, c^2*d] && IGtQ[2*p, 0] && !(IntegerQ[p] || GtQ[d, 0])
```

Rubi steps

$$\int \left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx = \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\int\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\cosh^6(x)\,dx,x,\sinh^{-1}(cx)\right)}{c\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\left(\frac{5}{16}(a+bx)^n+\frac{15}{32}(a+bx)^n\cosh(2x)+\frac{3}{1}\right.\right.}{c\sqrt{1+c}}$$

$$= \frac{5d^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{16bc(1+n)\sqrt{1+c^2x^2}}+\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\right.}{32}$$

$$= \frac{5d^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{16bc(1+n)\sqrt{1+c^2x^2}}+\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\right.}{64c}$$

$$= \frac{5d^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{16bc(1+n)\sqrt{1+c^2x^2}}+\frac{2^{-7-n}3^{-1-n}d^2e^{-\frac{6a}{b}}\sqrt{d+c^2d}}{}$$

Mathematica [A] time = 5.37, size = 529, normalized size = 0.84

$$\frac{d^3 2^{-2n-7}3^{-n-1}e^{-\frac{6a}{b}}\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{b^2}\right)^{-2n}\left(5b2^n3^{n+2}(n+1)e^{\frac{4a}{b}}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)^n\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (2^(-7 - 2*n)*3^(-1 - n)*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*(2^n*
b*(1 + n)*(a/b + ArcSinh[c*x])^(2*n)*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1
+ n, (-6*(a + b*ArcSinh[c*x]))/b] + 3^(2 + n)*b*E^((2*a)/b)*(1 + n)*(a/b +
ArcSinh[c*x])^(2*n)*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1 + n, (-4*(a + b*A
rcSinh[c*x]))/b] + 5*2^n*3^(2 + n)*b*E^((4*a)/b)*(1 + n)*(a/b + ArcSinh[c*x
])^n*(-((a + b*ArcSinh[c*x])^2/b^2))^n*Gamma[1 + n, (-2*(a + b*ArcSinh[c*x]
))/b] - E^((6*a)/b)*(5*2^n*3^(2 + n)*b*E^((2*a)/b)*(1 + n)*(-((a + b*ArcSin
h[c*x])/b))^n*(-((a + b*ArcSinh[c*x])^2/b^2))^n*Gamma[1 + n, (2*(a + b*ArcS
inh[c*x]))/b] + 3^(2 + n)*b*E^((4*a)/b)*(1 + n)*(a/b + ArcSinh[c*x])^n*(-((
a + b*ArcSinh[c*x])/b))^(2*n)*Gamma[1 + n, (4*(a + b*ArcSinh[c*x]))/b] + 2^
n*(-5*2^(3 + n)*3^(1 + n)*(a + b*ArcSinh[c*x])*(-((a + b*ArcSinh[c*x])^2/b^
2))^(2*n) + b*E^((6*a)/b)*(1 + n)*(a/b + ArcSinh[c*x])^n*(-((a + b*ArcSinh[
c*x])/b))^(2*n)*Gamma[1 + n, (6*(a + b*ArcSinh[c*x]))/b]))))/(b*c*E^((6*a)/
b)*(1 + n)*Sqrt[d + c^2*d*x^2]*(-((a + b*ArcSinh[c*x])^2/b^2))^(2*n))
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(c^4d^2x^4 + 2\,c^2d^2x^2 + d^2\right)\sqrt{c^2dx^2 + d}\,(b\,\text{arsinh}\,(cx) + a)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="fricas")
```

```
[Out] integral((c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2)*sqrt(c^2*d*x^2 + d)*(b*arcsinh
(c*x) + a)^n, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \left(c^2d\,x^2 + d\right)^{\frac{5}{2}} (a + b\,\text{arcsinh}\,(cx))^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x)
```

```
[Out] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c^2dx^2 + d\right)^{\frac{5}{2}} (b\,\text{arsinh}\,(cx) + a)^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,\mathrm{asinh}\,(c\,x))^n \left(d\,c^2\,x^2 + d\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(5/2),x)
[Out] int((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**n,x)
[Out] Timed out
```

3.525 $\int \frac{(d+c^2dx^2)^{5/2}(a+b\sinh^{-1}(cx))^n}{x}\,dx$

Optimal. Leaf size=756

$$d^3\text{Int}\left(\frac{(a+b\sinh^{-1}(cx))^n}{x\sqrt{c^2dx^2+d}},x\right)+\frac{d^3 5^{-n-1}e^{-\frac{5a}{b}}\sqrt{c^2x^2+1}\,(a+b\sinh^{-1}(cx))^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{5(a+b\sinh^{-1}(cx))}{b}\right)}{32\sqrt{c^2dx^2+d}}$$

```
[Out] 1/32*5^(-1-n)*d^3*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-5*(a+b*arcsinh(c*x))/b)*(
c^2*x^2+1)^(1/2)/exp(5*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)
-5/32*3^(-1-n)*d^3*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-3*(a+b*arcsinh(c*x))/b)*
(c^2*x^2+1)^(1/2)/exp(3*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2
)+1/8*d^3*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-3*(a+b*arcsinh(c*x))/b)*(c^2*x^2+
1)^(1/2)/(3^n)/exp(3*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+1
1/16*d^3*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(-a-b*arcsinh(c*x))/b)*(c^2*x^2+1)^
(1/2)/exp(a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+11/16*d^3*ex
p(a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,(a+b*arcsinh(c*x))/b)*(c^2*x^2+1)^(1/
2)/(((a+b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)-5/32*3^(-1-n)*d^3*exp(3*a
/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,3*(a+b*arcsinh(c*x))/b)*(c^2*x^2+1)^(1/2
)/(((a+b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+1/8*d^3*exp(3*a/b)*(a+b*ar
csinh(c*x))^n*GAMMA(1+n,3*(a+b*arcsinh(c*x))/b)*(c^2*x^2+1)^(1/2)/(3^n)/(((
a+b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+1/32*5^(-1-n)*d^3*exp(5*a/b)*(a
+b*arcsinh(c*x))^n*GAMMA(1+n,5*(a+b*arcsinh(c*x))/b)*(c^2*x^2+1)^(1/2)/(((a
+b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)+d^3*Unintegrable((a+b*arcsinh(c*
x))^n/x/(c^2*d*x^2+d)^(1/2),x)
```

Rubi [A] time = 0.16, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\sinh^{-1}(cx))^n}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n)/x,x]
```

```
[Out] Defer[Int][((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n)/x, x]
```

Rubi steps

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\sinh^{-1}(cx))^n}{x}\,dx=\int \frac{(d+c^2dx^2)^{5/2}(a+b\sinh^{-1}(cx))^n}{x}\,dx$$

Mathematica [A] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\sinh^{-1}(cx))^n}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n)/x,x]
```

```
[Out] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n)/x, x]
```

fricas [A] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^4d^2x^4 + 2\,c^2d^2x^2 + d^2\right)\sqrt{c^2dx^2 + d}\,(b\,\text{arsinh}\,(cx) + a)^n}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x,x, algorithm="fricas")
```

```
[Out] integral((c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2)*sqrt(c^2*d*x^2 + d)*(b*arcsinh
(c*x) + a)^n/x, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2d\,x^2 + d\right)^{\frac{5}{2}} (a + b\,\text{arcsinh}\,(cx))^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x,x)
```

```
[Out] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2dx^2 + d\right)^{\frac{5}{2}} (b\,\text{arsinh}\,(cx) + a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x,x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^n/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\text{asinh}\,(c\,x))^n\,\left(d\,c^2\,x^2 + d\right)^{5/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(5/2))/x,x)
```

```
[Out] int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(5/2))/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**n/x,x)
```

```
[Out] Timed out
```

3.526 $\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x^2}\,dx$

Optimal. Leaf size=455

$$d^3\text{Int}\left(\frac{\left(a+b\sinh^{-1}(cx)\right)^n}{x^2\sqrt{c^2dx^2+d}},x\right)+\frac{15cd^3\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)^{n+1}}{8b(n+1)\sqrt{c^2dx^2+d}}+\frac{cd^32^{-2(n+3)}e^{-\frac{4a}{b}}\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(c\right.}{\sqrt{}}$$

```
[Out] 15/8*c*d^3*(a+b*arcsinh(c*x))^(1+n)*(c^2*x^2+1)^(1/2)/b/(1+n)/(c^2*d*x^2+d)
^(1/2)+c*d^3*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-4*(a+b*arcsinh(c*x))/b)*(c^2*x
^2+1)^(1/2)/(2^(6+2*n))/exp(4*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d
)^(1/2)+2^(-2-n)*c*d^3*(a+b*arcsinh(c*x))^n*GAMMA(1+n,-2*(a+b*arcsinh(c*x))
/b)*(c^2*x^2+1)^(1/2)/exp(2*a/b)/(((-a-b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^
(1/2)-2^(-2-n)*c*d^3*exp(2*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,2*(a+b*arcsi
nh(c*x))/b)*(c^2*x^2+1)^(1/2)/(((a+b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2
)-c*d^3*exp(4*a/b)*(a+b*arcsinh(c*x))^n*GAMMA(1+n,4*(a+b*arcsinh(c*x))/b)*(
c^2*x^2+1)^(1/2)/(2^(6+2*n))/(((a+b*arcsinh(c*x))/b)^n)/(c^2*d*x^2+d)^(1/2)
+d^3*Unintegrable((a+b*arcsinh(c*x))^n/x^2/(c^2*d*x^2+d)^(1/2),x)
```

Rubi [A] time = 0.16, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n)/x^2,x]
[Out] Defer[Int][((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n)/x^2, x]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x^2}\,dx=\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x^2}\,dx$$

Mathematica [A] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n)/x^2,x]
[Out] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n)/x^2, x]
```

fricas [A] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^4d^2x^4+2\,c^2d^2x^2+d^2\right)\sqrt{c^2dx^2+d}\,(b\,\text{arsinh}\,(cx)+a)^n}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x^2,x, algorithm="fricas")

[Out] integral((c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2)*sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n/x^2, x)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

maple [A] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{\left(c^{2} d\,x^{2}+d\right)^{\frac{5}{2}} \left(a +b \,\mathrm{arcsinh}\left(c x \right)\right)^{n}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x^2,x)

[Out] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x^2,x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^{2} d x^{2} + d\right)^{\frac{5}{2}} {\left(b \operatorname{arsinh}\left(c x\right) + a\right)}^{n}}{x^{2}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x^2,x, algorithm="maxima")

[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^n/x^2, x)

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^n\,{\left(d\,c^2\,x^2+d\right)}^{5/2}}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(5/2))/x^2,x)

[Out] int(((a + b*asinh(c*x))^n*(d + c^2*d*x^2)^(5/2))/x^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**n/x**2,x)

[Out] Timed out

3.527 $\int \frac{x^m \sinh^{-1}(ax)^n}{\sqrt{1+a^2x^2}} \, dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{x^m \sinh^{-1}(ax)^n}{\sqrt{a^2x^2+1}}, x\right)$$

```
[Out] Unintegrable(x^m*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m \sinh^{-1}(ax)^n}{\sqrt{1+a^2x^2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2],x]
[Out] Defer[Int][(x^m*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2], x]
```

Rubi steps

$$\int \frac{x^m \sinh^{-1}(ax)^n}{\sqrt{1+a^2x^2}} \, dx = \int \frac{x^m \sinh^{-1}(ax)^n}{\sqrt{1+a^2x^2}} \, dx$$

Mathematica [A] time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{x^m \sinh^{-1}(ax)^n}{\sqrt{1+a^2x^2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2],x]
[Out] Integrate[(x^m*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2], x]
```

fricas [A] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m \operatorname{arsinh}(ax)^n}{\sqrt{a^2x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(x^m*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \operatorname{arsinh}(ax)^n}{\sqrt{a^2x^2+1}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^m*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

maple [A] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{x^{m} \operatorname{arcsinh}\left(a x\right)^{n}}{\sqrt{a^{2} x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
[Out] int(x^m*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m} \operatorname{arsinh}\left(a x\right)^{n}}{\sqrt{a^{2} x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^m*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^{m} \operatorname{asinh}\left(a\, x\right)^{n}}{\sqrt{a^{2}\, x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*asinh(a*x)^n)/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x^m*asinh(a*x)^n)/(a^2*x^2 + 1)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m} \operatorname{asinh}^{n}\left(a x\right)}{\sqrt{a^{2} x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*asinh(a*x)**n/(a**2*x**2+1)**(1/2),x)
[Out] Integral(x**m*asinh(a*x)**n/sqrt(a**2*x**2 + 1), x)
```

3.528 $\int \frac{x^3 \sinh^{-1}(ax)^n}{\sqrt{1+a^2x^2}} \, dx$

Optimal. Leaf size=113

$$\frac{3^{-n-1} \sinh^{-1}(ax)^n \left(-\sinh^{-1}(ax)\right)^{-n} \Gamma\left(n+1,-3\sinh^{-1}(ax)\right)}{8a^4} - \frac{3\sinh^{-1}(ax)^n \left(-\sinh^{-1}(ax)\right)^{-n} \Gamma\left(n+1,-\sinh^{-1}(a\right.}{8a^4}$$

```
[Out] 1/8*3^(-1-n)*arcsinh(a*x)^n*GAMMA(1+n,-3*arcsinh(a*x))/a^4/((-arcsinh(a*x))
^n)-3/8*arcsinh(a*x)^n*GAMMA(1+n,-arcsinh(a*x))/a^4/((-arcsinh(a*x))^n)-3/8
*GAMMA(1+n,arcsinh(a*x))/a^4+1/8*3^(-1-n)*GAMMA(1+n,3*arcsinh(a*x))/a^4
```

Rubi [A] time = 0.25, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5779, 3312, 3308, 2181}

$$\frac{3^{-n-1} \sinh^{-1}(ax)^n \left(-\sinh^{-1}(ax)\right)^{-n} \text{Gamma}\left(n+1,-3\sinh^{-1}(ax)\right)}{8a^4} - \frac{3\sinh^{-1}(ax)^n \left(-\sinh^{-1}(ax)\right)^{-n} \text{Gamma}\left(n\right.}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (3^(-1 - n)*ArcSinh[a*x]^n*Gamma[1 + n, -3*ArcSinh[a*x]])/(8*a^4*(-ArcSinh[
a*x])^n) - (3*ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(8*a^4*(-ArcSinh[
a*x])^n) - (3*Gamma[1 + n, ArcSinh[a*x]])/(8*a^4) + (3^(-1 - n)*Gamma[1 + n
, 3*ArcSinh[a*x]])/(8*a^4)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^3 \sinh^{-1}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \frac{\text{Subst}\left(\int x^n \sinh^3(x)\,dx, x, \sinh^{-1}(ax)\right)}{a^4}$$
$$= \frac{i\,\text{Subst}\left(\int \left(\frac{3}{4}ix^n \sinh(x) - \frac{1}{4}ix^n \sinh(3x)\right)dx, x, \sinh^{-1}(ax)\right)}{a^4}$$
$$= \frac{\text{Subst}\left(\int x^n \sinh(3x)\,dx, x, \sinh^{-1}(ax)\right)}{4a^4} - \frac{3\,\text{Subst}\left(\int x^n \sinh(x)\,dx, x, \sinh^{-1}(ax)\right)}{4a^4}$$
$$= -\frac{\text{Subst}\left(\int e^{-3x}x^n\,dx, x, \sinh^{-1}(ax)\right)}{8a^4} + \frac{\text{Subst}\left(\int e^{3x}x^n\,dx, x, \sinh^{-1}(ax)\right)}{8a^4} + \frac{3\,\text{Subst}\left(\int\right.}{}$$
$$= \frac{3^{-1-n}\left(-\sinh^{-1}(ax)\right)^{-n}\sinh^{-1}(ax)^n\Gamma\left(1+n,-3\sinh^{-1}(ax)\right)}{8a^4} - \frac{3\left(-\sinh^{-1}(ax)\right)^{-n}\text{sinh}}{}$$

Mathematica [A] time = 0.20, size = 100, normalized size = 0.88

$$\frac{3^{-n-1}\left(-\sinh^{-1}(ax)\right)^{-n}\left(\left(-\sinh^{-1}(ax)\right)^n\left(\Gamma\left(n+1,3\sinh^{-1}(ax)\right)-3^{n+2}\Gamma\left(n+1,\sinh^{-1}(ax)\right)\right)+\sinh^{-1}(ax)^n\Gamma\right.}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (3^(-1 - n)*(ArcSinh[a*x]^n*Gamma[1 + n, -3*ArcSinh[a*x]] - 3^(2 + n)*ArcSi
nh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]] + (-ArcSinh[a*x])^n*(-(3^(2 + n)*Gamm
a[1 + n, ArcSinh[a*x]]) + Gamma[1 + n, 3*ArcSinh[a*x]])))/(8*a^4*(-ArcSinh[
a*x])^n)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3\,\text{arsinh}\,(ax)^n}{\sqrt{a^2x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^3*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\,\text{arcsinh}\,(ax)^n}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^3*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \operatorname{arsinh}(ax)^n}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \operatorname{asinh}(a\,x)^n}{\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*asinh(a*x)^n)/(a^2*x^2 + 1)^(1/2),x)
```

```
[Out] int((x^3*asinh(a*x)^n)/(a^2*x^2 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \operatorname{asinh}^n(ax)}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(a*x)**n/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3*asinh(a*x)**n/sqrt(a**2*x**2 + 1), x)
```

3.529 $\int \frac{x^2 \sinh^{-1}(ax)^n}{\sqrt{1+a^2x^2}} \, dx$

Optimal. Leaf size=80

$$-\frac{\sinh^{-1}(ax)^{n+1}}{2a^3(n+1)}+\frac{2^{-n-3}\sinh^{-1}(ax)^n\left(-\sinh^{-1}(ax)\right)^{-n}\Gamma\left(n+1,-2\sinh^{-1}(ax)\right)}{a^3}-\frac{2^{-n-3}\Gamma\left(n+1,2\sinh^{-1}(ax)\right)}{a^3}$$

```
[Out] -1/2*arcsinh(a*x)^(1+n)/a^3/(1+n)+2^(-3-n)*arcsinh(a*x)^n*GAMMA(1+n,-2*arcs
inh(a*x))/a^3/((-arcsinh(a*x))^n)-2^(-3-n)*GAMMA(1+n,2*arcsinh(a*x))/a^3
```

Rubi [A] time = 0.19, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5779, 3312, 3307, 2181}

$$\frac{2^{-n-3}\sinh^{-1}(ax)^n\left(-\sinh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1,-2\sinh^{-1}(ax)\right)}{a^3}-\frac{2^{-n-3}\text{Gamma}\left(n+1,2\sinh^{-1}(ax)\right)}{a^3}-\frac{\text{sir}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] -ArcSinh[a*x]^(1 + n)/(2*a^3*(1 + n)) + (2^(-3 - n)*ArcSinh[a*x]^n*Gamma[1
+ n, -2*ArcSinh[a*x]])/(a^3*(-ArcSinh[a*x])^n) - (2^(-3 - n)*Gamma[1 + n, 2
*ArcSinh[a*x]])/a^3
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2 \sinh^{-1}(ax)^n}{\sqrt{1+a^2x^2}}\,dx &= \frac{\text{Subst}\left(\int x^n \sinh^2(x)\,dx, x, \sinh^{-1}(ax)\right)}{a^3} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{x^n}{2}-\frac{1}{2}x^n\cosh(2x)\right)dx, x, \sinh^{-1}(ax)\right)}{a^3} \\
&= -\frac{\sinh^{-1}(ax)^{1+n}}{2a^3(1+n)}+\frac{\text{Subst}\left(\int x^n\cosh(2x)\,dx, x, \sinh^{-1}(ax)\right)}{2a^3} \\
&= -\frac{\sinh^{-1}(ax)^{1+n}}{2a^3(1+n)}+\frac{\text{Subst}\left(\int e^{-2x}x^n\,dx, x, \sinh^{-1}(ax)\right)}{4a^3}+\frac{\text{Subst}\left(\int e^{2x}x^n\,dx, x, \sinh^{-1}(ax)\right)}{4a^3} \\
&= -\frac{\sinh^{-1}(ax)^{1+n}}{2a^3(1+n)}+\frac{2^{-3-n}\left(-\sinh^{-1}(ax)\right)^{-n}\sinh^{-1}(ax)^n\Gamma\left(1+n,-2\sinh^{-1}(ax)\right)}{a^3}-\frac{2^{-3-n}\Gamma}{}
\end{aligned}
$$

Mathematica [A] time = 0.21, size = 86, normalized size = 1.08

$$
\frac{2^{-n-3}\left(-\sinh^{-1}(ax)\right)^{-n}\left((n+1)\sinh^{-1}(ax)^n\Gamma\left(n+1,-2\sinh^{-1}(ax)\right)-\left(-\sinh^{-1}(ax)\right)^n\left(2^{n+2}\sinh^{-1}(ax)^{n+1}+(n\right.\right.}{a^3(n+1)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (2^(-3 - n)*((1 + n)*ArcSinh[a*x]^n*Gamma[1 + n, -2*ArcSinh[a*x]] - (-ArcSi
nh[a*x])^n*(2^(2 + n)*ArcSinh[a*x]^(1 + n) + (1 + n)*Gamma[1 + n, 2*ArcSinh
[a*x]])))/(a^3*(1 + n)*(-ArcSinh[a*x])^n)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$
\text{integral}\left(\frac{x^2\,\text{arsinh}\,(ax)^n}{\sqrt{a^2x^2+1}}, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^2*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^2\,\text{arsinh}\,(ax)^n}{\sqrt{a^2x^2+1}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$
\int \frac{x^2\,\text{arcsinh}\,(ax)^n}{\sqrt{a^2x^2+1}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^2*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{arsinh}(ax)^n}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 \operatorname{asinh}(a\,x)^n}{\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*asinh(a*x)^n)/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x^2*asinh(a*x)^n)/(a^2*x^2 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{asinh}^n(ax)}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(a*x)**n/(a**2*x**2+1)**(1/2),x)
[Out] Integral(x**2*asinh(a*x)**n/sqrt(a**2*x**2 + 1), x)
```

3.530 $\int \frac{x \sinh^{-1}(ax)^n}{\sqrt{1+a^2x^2}}\, dx$

Optimal. Leaf size=49

$$\frac{\sinh^{-1}(ax)^n \left(-\sinh^{-1}(ax)\right)^{-n} \Gamma\left(n+1,-\sinh^{-1}(ax)\right)}{2a^2}+\frac{\Gamma\left(n+1,\sinh^{-1}(ax)\right)}{2a^2}$$

```
[Out] 1/2*arcsinh(a*x)^n*GAMMA(1+n,-arcsinh(a*x))/a^2/((-arcsinh(a*x))^n)+1/2*GAM
MA(1+n,arcsinh(a*x))/a^2
```

Rubi [A] time = 0.12, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5779, 3308, 2181}

$$\frac{\sinh^{-1}(ax)^n \left(-\sinh^{-1}(ax)\right)^{-n} \text{Gamma}\left(n+1,-\sinh^{-1}(ax)\right)}{2a^2}+\frac{\text{Gamma}\left(n+1,\sinh^{-1}(ax)\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(2*a^2*(-ArcSinh[a*x])^n) + Ga
mma[1 + n, ArcSinh[a*x]]/(2*a^2)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x \sinh^{-1}(ax)^n}{\sqrt{1+a^2x^2}}\, dx &= \frac{\text{Subst}\left(\int x^n \sinh(x)\, dx, x, \sinh^{-1}(ax)\right)}{a^2}\\
&= -\frac{\text{Subst}\left(\int e^{-x} x^n\, dx, x, \sinh^{-1}(ax)\right)}{2a^2}+\frac{\text{Subst}\left(\int e^{x} x^n\, dx, x, \sinh^{-1}(ax)\right)}{2a^2}\\
&= \frac{\left(-\sinh^{-1}(ax)\right)^{-n} \sinh^{-1}(ax)^n \Gamma\left(1+n,-\sinh^{-1}(ax)\right)}{2a^2}+\frac{\Gamma\left(1+n,\sinh^{-1}(ax)\right)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 43, normalized size = 0.88

$$\frac{\sinh^{-1}(ax)^n \left(-\sinh^{-1}(ax)\right)^{-n} \Gamma\left(n+1,-\sinh^{-1}(ax)\right)+\Gamma\left(n+1,\sinh^{-1}(ax)\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] ((ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(-ArcSinh[a*x])^n + Gamma[1 +
 n, ArcSinh[a*x]])/(2*a^2)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x \,\text{arsinh}\,(ax)^n}{\sqrt{a^2x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \,\text{arsinh}\,(ax)^n}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{x \,\text{arcsinh}\,(ax)^n}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \,\text{arsinh}\,(ax)^n}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x \,\text{asinh}\,(a\,x)^n}{\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*asinh(a*x)^n)/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x*asinh(a*x)^n)/(a^2*x^2 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \operatorname{asinh}^{n}(ax)}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(a*x)**n/(a**2*x**2+1)**(1/2),x)
[Out] Integral(x*asinh(a*x)**n/sqrt(a**2*x**2 + 1), x)
```

3.531 $\int \frac{\sinh^{-1}(ax)^n}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=17

$$\frac{\sinh^{-1}(ax)^{n+1}}{a(n+1)}$$

```
[Out] arcsinh(a*x)^(1+n)/a/(1+n)
```

Rubi [A] time = 0.04, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {5675}

$$\frac{\sinh^{-1}(ax)^{n+1}}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^n/Sqrt[1 + a^2*x^2],x]
```

```
[Out] ArcSinh[a*x]^(1 + n)/(a*(1 + n))
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \frac{\sinh^{-1}(ax)^{1+n}}{a(1+n)}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{\sinh^{-1}(ax)^{n+1}}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^n/Sqrt[1 + a^2*x^2],x]
```

```
[Out] ArcSinh[a*x]^(1 + n)/(a*(1 + n))
```

fricas [B] time = 0.46, size = 83, normalized size = 4.88

$$\frac{\cosh\left(n\log\left(\log\left(ax+\sqrt{a^2x^2+1}\right)\right)\right)\log\left(ax+\sqrt{a^2x^2+1}\right)+\log\left(ax+\sqrt{a^2x^2+1}\right)\sinh\left(n\log\left(\log\left(ax+\sqrt{a^2x^2+1}\right)\right)\right)}{an+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (cosh(n*log(log(a*x + sqrt(a^2*x^2 + 1))))*log(a*x + sqrt(a^2*x^2 + 1)) + l
og(a*x + sqrt(a^2*x^2 + 1))*sinh(n*log(log(a*x + sqrt(a^2*x^2 + 1)))))/(a*n
 + a)
```

giac [A] time = 0.88, size = 29, normalized size = 1.71

$$\frac{\log\left(ax + \sqrt{a^2x^2 + 1}\right)^{n+1}}{a(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] log(a*x + sqrt(a^2*x^2 + 1))^(n + 1)/(a*(n + 1))
```

maple [A] time = 0.01, size = 18, normalized size = 1.06

$$\frac{\operatorname{arcsinh}(ax)^{1+n}}{a(1+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
[Out] arcsinh(a*x)^(1+n)/a/(1+n)
```

maxima [A] time = 0.39, size = 17, normalized size = 1.00

$$\frac{\operatorname{arsinh}(ax)^{n+1}}{a(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] arcsinh(a*x)^(n + 1)/(a*(n + 1))
```

mupad [B] time = 0.28, size = 33, normalized size = 1.94

$$\begin{cases} \frac{\ln(\operatorname{asinh}(a\,x))}{a} & \text{if } n = -1 \\ \frac{\operatorname{asinh}(a\,x)^{n+1}}{a\,(n+1)} & \text{if } n \neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^n/(a^2*x^2 + 1)^(1/2),x)
[Out] piecewise(n == -1, log(asinh(a*x))/a, n ~= -1, asinh(a*x)^(n + 1)/(a*(n + 1
)))
```

sympy [A] time = 0.87, size = 34, normalized size = 2.00

$$\begin{cases} \tilde{\infty}x & \text{for } a = 0 \wedge n = -1 \\ 0^n x & \text{for } a = 0 \\ \frac{\log(\operatorname{asinh}(ax))}{a} & \text{for } n = -1 \\ \frac{\operatorname{asinh}(ax)\operatorname{asinh}^n(ax)}{an+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**n/(a**2*x**2+1)**(1/2),x)
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(n, -1)), (0**n*x, Eq(a, 0)), (log(asinh(a*x
))/a, Eq(n, -1)), (asinh(a*x)*asinh(a*x)**n/(a*n + a), True))
```

3.532 $\int \frac{\sinh^{-1}(ax)^n}{x\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{\sinh^{-1}(ax)^n}{x\sqrt{a^2x^2+1}},x\right)$$

```
[Out] Unintegrable(arcsinh(a*x)^n/x/(a^2*x^2+1)^(1/2),x)
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sinh^{-1}(ax)^n}{x\sqrt{1+a^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcSinh[a*x]^n/(x*Sqrt[1 + a^2*x^2]),x]
[Out] Defer[Int][ArcSinh[a*x]^n/(x*Sqrt[1 + a^2*x^2]), x]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^n}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\sinh^{-1}(ax)^n}{x\sqrt{1+a^2x^2}}\,dx$$

Mathematica [A] time = 5.93, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{-1}(ax)^n}{x\sqrt{1+a^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcSinh[a*x]^n/(x*Sqrt[1 + a^2*x^2]),x]
[Out] Integrate[ArcSinh[a*x]^n/(x*Sqrt[1 + a^2*x^2]), x]
```

fricas [A] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a^2x^2+1}\ \text{arsinh}\,(ax)^n}{a^2x^3+x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n/x/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^n/(a^2*x^3 + x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arsinh}\,(ax)^n}{\sqrt{a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n/x/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^n/(sqrt(a^2*x^2 + 1)*x), x)
```

maple [A] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^n/x/(a^2*x^2+1)^(1/2),x)
[Out] int(arcsinh(a*x)^n/x/(a^2*x^2+1)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)^n}{\sqrt{a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n/x/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(arcsinh(a*x)^n/(sqrt(a^2*x^2 + 1)*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{asinh}(a\,x)^n}{x\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^n/(x*(a^2*x^2 + 1)^(1/2)),x)
[Out] int(asinh(a*x)^n/(x*(a^2*x^2 + 1)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^n(ax)}{x\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**n/x/(a**2*x**2+1)**(1/2),x)
[Out] Integral(asinh(a*x)**n/(x*sqrt(a**2*x**2 + 1)), x)
```

3.533 $\int \frac{\sinh^{-1}(ax)^n}{x^2\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=26

$$\operatorname{Int}\left(\frac{\sinh^{-1}(ax)^n}{x^2\sqrt{a^2x^2+1}},x\right)$$

```
[Out] Unintegrable(arcsinh(a*x)^n/x^2/(a^2*x^2+1)^(1/2),x)
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sinh^{-1}(ax)^n}{x^2\sqrt{1+a^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcSinh[a*x]^n/(x^2*Sqrt[1 + a^2*x^2]),x]
[Out] Defer[Int][ArcSinh[a*x]^n/(x^2*Sqrt[1 + a^2*x^2]), x]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^n}{x^2\sqrt{1+a^2x^2}}\,dx=\int \frac{\sinh^{-1}(ax)^n}{x^2\sqrt{1+a^2x^2}}\,dx$$

Mathematica [A] time = 1.93, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{-1}(ax)^n}{x^2\sqrt{1+a^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcSinh[a*x]^n/(x^2*Sqrt[1 + a^2*x^2]),x]
[Out] Integrate[ArcSinh[a*x]^n/(x^2*Sqrt[1 + a^2*x^2]), x]
```

fricas [A] time = 0.51, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{a^2x^2+1}\,\operatorname{arsinh}(ax)^n}{a^2x^4+x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n/x^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^n/(a^2*x^4 + x^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)^n}{\sqrt{a^2x^2+1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n/x^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^n/(sqrt(a^2*x^2 + 1)*x^2), x)
```

maple [A] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x^2\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^n/x^2/(a^2*x^2+1)^(1/2),x)
[Out] int(arcsinh(a*x)^n/x^2/(a^2*x^2+1)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)^n}{\sqrt{a^2x^2+1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n/x^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(arcsinh(a*x)^n/(sqrt(a^2*x^2 + 1)*x^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{asinh}(a\,x)^n}{x^2\,\sqrt{a^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^n/(x^2*(a^2*x^2 + 1)^(1/2)),x)
[Out] int(asinh(a*x)^n/(x^2*(a^2*x^2 + 1)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^n(ax)}{x^2\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**n/x**2/(a**2*x**2+1)**(1/2),x)
[Out] Integral(asinh(a*x)**n/(x**2*sqrt(a**2*x**2 + 1)), x)
```

3.534 $\int (d+icdx)^{5/2}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)dx$

Optimal. Leaf size=416

$$-\frac{1}{4}c^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)+\frac{5d^2\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc\sqrt{c^2x^2+1}}+\frac{2id^2\left(c^2x^2+\right.}{}$$

```
[Out] 3/8*d^2*x*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-1/4*c^2*d^
2*x^3*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)+2/3*I*d^2*(c^2
*x^2+1)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c-2/3*I*b*d^
2*x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-3/16*b*c*d^2*x^2*
(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-2/9*I*b*c^2*d^2*x^3*(
d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/16*b*c^3*d^2*x^4*(d+
I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+5/16*d^2*(a+b*arcsinh(c*
x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.60, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {5712, 5821, 5682, 5675, 30, 5717, 5742, 5758}

$$-\frac{1}{4}c^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)+\frac{5d^2\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc\sqrt{c^2x^2+1}}+\frac{2id^2\left(c^2x^2+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (((-2*I)/3)*b*d^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 + c^2*x^2]
- (3*b*c*d^2*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(16*Sqrt[1 + c^2*x^2]
) - (((2*I)/9)*b*c^2*d^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 +
c^2*x^2] + (b*c^3*d^2*x^4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(16*Sqrt[1 +
 c^2*x^2]) + (3*d^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*
x]))/8 - (c^2*d^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*
x]))/4 + (((2*I)/3)*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(1 + c^2*x^2)*(
a + b*ArcSinh[c*x]))/c + (5*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*
ArcSinh[c*x])^2)/(16*b*c*Sqrt[1 + c^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)dx = \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int (d+icdx)^2\sqrt{1+c^2x^2}\left(a+b\sinh\right.}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \left(d^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\right.}{}$$

$$= \frac{\left(d^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}$$

$$= \frac{1}{2}d^2x\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{4}c^2d^2x^3\sqrt{d}$$

$$= -\frac{2ibd^2x\sqrt{d+icdx}\sqrt{f-icfx}}{3\sqrt{1+c^2x^2}}-\frac{bcd^2x^2\sqrt{d+icdx}\sqrt{f-icf}}{4\sqrt{1+c^2x^2}}$$

$$= -\frac{2ibd^2x\sqrt{d+icdx}\sqrt{f-icfx}}{3\sqrt{1+c^2x^2}}-\frac{3bcd^2x^2\sqrt{d+icdx}\sqrt{f-ic}}{16\sqrt{1+c^2x^2}}$$

Mathematica [A] time = 1.48, size = 361, normalized size = 0.87

$$\frac{720ad^{5/2}\sqrt{f}\sqrt{c^2x^2+1}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)+48ad^2\sqrt{c^2x^2+1}\left(-6c^3x^3+16ic^2x^2+9cx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (48*a*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]*(16*I + 9*c
*x + (16*I)*c^2*x^2 - 6*c^3*x^3) + 720*a*d^(5/2)*Sqrt[f]*Sqrt[1 + c^2*x^2]*
Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + 144*b*
d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(2*ArcSinh[c*x]^2 - Cosh[2*ArcSinh[
c*x]] + 2*ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]]) - (64*I)*b*d^2*Sqrt[d + I*c*d*
x]*Sqrt[f - I*c*f*x]*(9*c*x - 3*ArcSinh[c*x]*(3*Sqrt[1 + c^2*x^2] + Cosh[3*
ArcSinh[c*x]]) + Sinh[3*ArcSinh[c*x]]) + 9*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f -
 I*c*f*x]*(8*ArcSinh[c*x]^2 + Cosh[4*ArcSinh[c*x]] - 4*ArcSinh[c*x]*Sinh[4*
ArcSinh[c*x]]))/(1152*c*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\left(bc^2d^2x^2-2i\,bcd^2x-bd^2\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\log\left(cx+\sqrt{c^2x^2+1}\right)-\left(ac^2d^2x^2-2i\,acd^2x-a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(-(b*c^2*d^2*x^2 - 2*I*b*c*d^2*x - b*d^2)*sqrt(I*c*d*x + d)*sqrt(-I
*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) - (a*c^2*d^2*x^2 - 2*I*a*c*d^2*x -
 a*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int (icdx+d)^{\frac{5}{2}} (a+b\,\mathrm{arcsinh}\,(cx))\,\sqrt{-icfx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{asinh}\,(c\,x))\,\left(d+c\,d\,x\,1\mathrm{i}\right)^{5/2}\,\sqrt{f-c\,f\,x\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))*(f-I*c*f*x)**(1/2),x)
```

```
[Out] Timed out
```

3.535 $\int (d+icdx)^{3/2}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)dx$

Optimal. Leaf size=304

$$\frac{d\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{c^2x^2+1}}+\frac{id\left(c^2x^2+1\right)\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{3c}+\frac{1}{2}dx\sqrt{d+}$$

```
[Out] 1/2*d*x*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)+1/3*I*d*(c^2
*x^2+1)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c-1/3*I*b*d*
x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-1/4*b*c*d*x^2*(d+I*
c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-1/9*I*b*c^2*d*x^3*(d+I*c*d
*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/4*d*(a+b*arcsinh(c*x))^2*(d
+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {5712, 5821, 5682, 5675, 30, 5717}

$$\frac{d\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{c^2x^2+1}}+\frac{id\left(c^2x^2+1\right)\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{3c}+\frac{1}{2}dx\sqrt{d+}$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] ((-I/3)*b*d*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 + c^2*x^2] - (b*c
*d*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(4*Sqrt[1 + c^2*x^2]) - ((I/9)*
b*c^2*d*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 + c^2*x^2] + (d*x*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/2 + ((I/3)*d*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/c + (d*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[1 +
c^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
```

```
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)dx = \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int (d+icdx)\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(c\right.}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\left(d\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)+ic\right.}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)dx}{\sqrt{1+c^2x^2}}+$$

$$= \frac{1}{2}dx\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)+\frac{id\sqrt{d+icdx}\sqrt{j}}{}$$

$$= -\frac{ibdx\sqrt{d+icdx}\sqrt{f-icfx}}{3\sqrt{1+c^2x^2}}-\frac{bcdx^2\sqrt{d+icdx}\sqrt{f-icfx}}{4\sqrt{1+c^2x^2}}-\frac{ib}{}$$

Mathematica [A] time = 1.58, size = 273, normalized size = 0.90

$$\frac{12ad\left(2ic^2x^2+3cx+2i\right)\sqrt{d+icdx}\sqrt{f-icfx}+36ad^{3/2}\sqrt{f}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)+\frac{9bd\sqrt{d+i}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (12*a*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(2*I + 3*c*x + (2*I)*c^2*x^2) +
 36*a*d^(3/2)*Sqrt[f]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]] + (9*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(2*ArcSinh[c*x]^
2 - Cosh[2*ArcSinh[c*x]] + 2*ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]]))/Sqrt[1 + c
^2*x^2] - ((2*I)*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(9*c*x - 3*ArcSinh
[c*x]*(3*Sqrt[1 + c^2*x^2] + Cosh[3*ArcSinh[c*x]]) + Sinh[3*ArcSinh[c*x]]))
/Sqrt[1 + c^2*x^2])/(72*c)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left((i\,bcdx+bd)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\log\left(cx+\sqrt{c^2x^2+1}\right)+(i\,acdx+ad)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((I*b*c*d*x + b*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + s
qrt(c^2*x^2 + 1)) + (I*a*c*d*x + a*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f),
 x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument TypeError: Bad Argument
TypeError: Bad Argument TypeDone
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int (icdx + d)^{\frac{3}{2}} (a + b \operatorname{arcsinh}(cx)) \sqrt{-icfx + f}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b \operatorname{asinh}(c\, x))\, (d + c\, d\, x\, 1\mathrm{i})^{3/2} \sqrt{f - c\, f\, x\, 1\mathrm{i}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (id(cx - i))^{\frac{3}{2}} \sqrt{-if(cx + i)}\, (a + b \operatorname{asinh}(cx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))*(f-I*c*f*x)**(1/2),x)
```

```
[Out] Integral((I*d*(c*x - I))**(3/2)*sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x)), x)
```

3.536 $\int \sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)dx$

Optimal. Leaf size=147

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)-\frac{bcx^2\sqrt{d+icdx}\sqrt{f-icfx}}{4\sqrt{c^2x^2+1}}$$

```
[Out] 1/2*x*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-1/4*b*c*x^2*(d
+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/4*(a+b*arcsinh(c*x))^
2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {5712, 5682, 5675, 30}

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)-\frac{bcx^2\sqrt{d+icdx}\sqrt{f-icfx}}{4\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(b*c*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(4*Sqrt[1 + c^2*x^2]) + (x*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/2 + (Sqrt[d + I*c*
d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[1 + c^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rubi steps

$$\int \sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)dx = \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{1}{2}x\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)+\frac{\left(\sqrt{d+icdx}\sqrt{f\right.}$$

$$= -\frac{bcx^2\sqrt{d+icdx}\sqrt{f-icfx}}{4\sqrt{1+c^2x^2}}+\frac{1}{2}x\sqrt{d+icdx}\sqrt{f-icfx}\left(a+\right.$$

Mathematica [A] time = 0.60, size = 233, normalized size = 1.59

$$\frac{1}{2}ax\sqrt{id(cx-i)}\sqrt{-if(cx+i)}+\frac{a\sqrt{d}\sqrt{f}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{id(cx-i)}\sqrt{-if(cx+i)}\right)}{2c}-\frac{b\sqrt{i(cdx-id)}\sqrt{-i(cfx}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (a*x*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)])/2 + (a*Sqrt[d]*Sqrt[f]*Lo
g[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]])/(
2*c) - (b*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 +
 c^2*x^2))]*(Cosh[2*ArcSinh[c*x]] - 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*A
rcSinh[c*x]])))/(8*c*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x
^2])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{icdx+d}\sqrt{-icfx+f}\,b\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{icdx+d}\sqrt{-icfx+f}\,a,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2 + 1)
) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument TypeError: Bad Argument
TypeDone
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int (a+b\,\mathrm{arcsinh}(cx))\sqrt{icdx+d}\,\sqrt{-icfx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,\mathrm{asinh}\,(c\,x))\,\sqrt{d + c\,d\,x\,1\mathrm{i}}\,\sqrt{f - c\,f\,x\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{id\,(cx - i)}\,\sqrt{-if\,(cx + i)}\,\,(a + b\,\mathrm{asinh}\,(cx))\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(d+I*c*d*x)**(1/2)*(f-I*c*f*x)**(1/2),x)
```

```
[Out] Integral(sqrt(I*d*(c*x - I))*sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x)), x)
```

3.537 $\int \frac{\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+icdx}}\,dx$

Optimal. Leaf size=158

$$\frac{f\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{if\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{ibfx\sqrt{c^2x^2+1}}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

```
[Out] -I*f*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+I
*b*f*x*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/2*f*(a+b*arc
sinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5712, 5821, 5675, 5717, 8}

$$\frac{f\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{if\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{ibfx\sqrt{c^2x^2+1}}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/Sqrt[d + I*c*d*x],x]
```

```
[Out] (I*b*f*x*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (I*f*(1
 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) +
 (f*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*Sqrt[d + I*c*d*x]*Sqrt
[f - I*c*f*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
```

```
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int \frac{\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+icdx}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{(f-icfx)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\int \left(\frac{f\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}} - \frac{icfx\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\right)dx}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= \frac{\left(f\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\,\sqrt{f-icfx}} - \frac{\left(icf\sqrt{1+c^2x^2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= -\frac{if\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{f\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{(ib \cdots}{\sqrt{\cdots}}$$

$$= \frac{ibfx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\,\sqrt{f-icfx}} - \frac{if\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{f\sqrt{1+c^2x^2}\,(a \cdots}{2bc\sqrt{d+ic \cdots}}$$

Mathematica [A] time = 0.61, size = 227, normalized size = 1.44

$$\frac{2i\sqrt{d+icdx}\,\sqrt{f-icfx}\left(bcx-a\sqrt{c^2x^2+1}\right)+2a\sqrt{d}\,\sqrt{f}\,\sqrt{c^2x^2+1}\,\log\left(cdfx+\sqrt{d}\,\sqrt{f}\,\sqrt{d+icdx}\,\sqrt{f-icfx}\right)-2 \cdots}{2cd\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/Sqrt[d + I*c*d*x],x]
```

```
[Out] ((2*I)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(b*c*x - a*Sqrt[1 + c^2*x^2]) -
(2*I)*b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]
+ b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 + 2*a*Sqrt[d]*Sqrt[f
]*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x]])/(2*c*d*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{-i\sqrt{icdx+d}\,\sqrt{-icfx+f}\,b\log\left(cx+\sqrt{c^2x^2+1}\right)-i\sqrt{icdx+d}\,\sqrt{-icfx+f}\,a}{cdx-id}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((-I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2
+ 1)) - I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a)/(c*d*x - I*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-icfx+f}\,(b\,\text{arsinh}(cx)+a)}{\sqrt{icdx+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(sqrt(-I*c*f*x + f)*(b*arcsinh(c*x) + a)/sqrt(I*c*d*x + d), x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))\,\sqrt{-icfx+f}}{\sqrt{icdx+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\frac{f\,\mathrm{arsinh}\,(cx)}{cd\sqrt{\frac{f}{d}}}-\frac{i\sqrt{c^2dfx^2+df}}{cd}\right)+b\int\frac{\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)}{\sqrt{i\,cdx+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] a*(f*arcsinh(c*x)/(c*d*sqrt(f/d)) - I*sqrt(c^2*d*f*x^2 + d*f)/(c*d)) + b*in
tegrate(sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))/sqrt(I*c*d*x + d),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))\,\sqrt{f-c\,f\,x\,1\mathrm{i}}}{\sqrt{d+c\,d\,x\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(1/2),x)
```

```
[Out] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-if\,(cx+i)}\,(a+b\,\mathrm{asinh}\,(cx))}{\sqrt{id\,(cx-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(f-I*c*f*x)**(1/2)/(d+I*c*d*x)**(1/2),x)
```

```
[Out] Integral(sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x))/sqrt(I*d*(c*x - I)), x)
```

3.538 $\int \frac{\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{3/2}}\,dx$

Optimal. Leaf size=181

$$-\frac{f^2\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2if^2(1-icx)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2bf^2\left(c^2x^2+1\right)^{3/2}\log(-cx+i}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[Out] 2*I*f^2*(1-I*c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c
*f*x)^(3/2)-1/2*f^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/b/c/(d+I*c*d*x)^
(3/2)/(f-I*c*f*x)^(3/2)-2*b*f^2*(c^2*x^2+1)^(3/2)*ln(I-c*x)/c/(d+I*c*d*x)^(
3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A] time = 0.40, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {5712, 5833, 637, 5819, 12, 627, 31, 5675}

$$-\frac{f^2\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2if^2(1-icx)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2bf^2\left(c^2x^2+1\right)^{3/2}\log(-cx+i}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(3/2),x]
```

```
[Out] ((2*I)*f^2*(1 - I*c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)
^(3/2)*(f - I*c*f*x)^(3/2)) - (f^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])
^2)/(2*b*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (2*b*f^2*(1 + c^2*x^2
)^(3/2)*Log[I - c*x])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rule 5833

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{3/2}}\,dx = \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{(f-icfx)^2\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{3/2}\int \left(-\frac{2i\left(if^2+cf^2x\right)\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}-\frac{f^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{\left(2i\left(1+c^2x^2\right)^{3/2}\right)\int \frac{\left(if^2+cf^2x\right)\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(f^2\left(1+c^2x^2\right)^{3/2}\right)\int \frac{a-}{}}{(d+icdx)^{3/2}(f-}$$

$$= \frac{2if^2(1-icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{f^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sin\right.}{2bc(d+icdx)^{3/2}(f-ic}$$

$$= \frac{2if^2(1-icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{f^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sin\right.}{2bc(d+icdx)^{3/2}(f-ic}$$

$$= \frac{2if^2(1-icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{f^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sin\right.}{2bc(d+icdx)^{3/2}(f-ic}$$

$$= \frac{2if^2(1-icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{f^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sin\right.}{2bc(d+icdx)^{3/2}(f-ic}$$

Mathematica [A] time = 1.52, size = 283, normalized size = 1.56

$$-\frac{-\frac{4a\sqrt{d+icdx}\sqrt{f-icfx}}{cx-i}+2a\sqrt{d}\sqrt{f}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)+\frac{b\sqrt{d+icdx}\sqrt{f-icfx}\left(2\left(\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(3/2),x]
```

```
[Out] -1/2*((-4*a*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(-I + c*x) + 2*a*Sqrt[d]*S
qrt[f]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] +
 (b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(ArcSinh[c*x]*((-4*I)*Cosh[ArcSinh[
c*x]/2] - 4*Sinh[ArcSinh[c*x]/2]) + ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] +
I*Sinh[ArcSinh[c*x]/2]) + 2*((4*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + Log[1 + c
^2*x^2])*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/(Sqrt[1 + c^2*x^
2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/(c*d^2)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{icdx+d}\,\sqrt{-icfx+f}\,b\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{icdx+d}\,\sqrt{-icfx+f}\,a}{c^2d^2x^2-2icd^2x-d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral(-(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2 +
1)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a)/(c^2*d^2*x^2 - 2*I*c*d^2*x -
d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-icfx+f}\,(b\,\text{arsinh}\,(cx)+a)}{(icdx+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate(sqrt(-I*c*f*x + f)*(b*arcsinh(c*x) + a)/(I*c*d*x + d)^(3/2), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\text{arcsinh}\,(cx))\sqrt{-icfx+f}}{(icdx+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\frac{2i\sqrt{c^2dfx^2+df}}{ic^2d^2x+cd^2}-\frac{f\,\text{arsinh}\,(cx)}{cd^2\sqrt{\frac{f}{d}}}\right)+b\int \frac{\sqrt{-icfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)}{(icdx+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x, algorit
hm="maxima")
```

```
[Out] a*(2*I*sqrt(c^2*d*f*x^2 + d*f)/(I*c^2*d^2*x + c*d^2) - f*arcsinh(c*x)/(c*d^
2*sqrt(f/d))) + b*integrate(sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))
/(I*c*d*x + d)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{asinh}\,(c\,x))\,\sqrt{f - c\,f\,x\,1\mathrm{i}}}{(d + c\,d\,x\,1\mathrm{i})^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(3/2),x)
[Out] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-if\,(cx + i)}\,\left(a + b\,\mathrm{asinh}\,(cx)\right)}{\left(id\,(cx - i)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(f-I*c*f*x)**(1/2)/(d+I*c*d*x)**(3/2),x)
[Out] Integral(sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x))/(I*d*(c*x - I))**(3/2), x)
```

3.539 $\int \frac{\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{5/2}}\,dx$

Optimal. Leaf size=187

$$\frac{if^3(1-icx)^3\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2ibf^3\left(c^2x^2+1\right)^{5/2}}{3c(-cx+i)(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{bf^3\left(c^2x^2+1\right)^{5/2}\log(-cx+i)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

```
[Out] 2/3*I*b*f^3*(c^2*x^2+1)^(5/2)/c/(I-c*x)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
+1/3*I*f^3*(1-I*c*x)^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(
f-I*c*f*x)^(5/2)+1/3*b*f^3*(c^2*x^2+1)^(5/2)*ln(I-c*x)/c/(d+I*c*d*x)^(5/2)/
(f-I*c*f*x)^(5/2)
```

Rubi [A] time = 0.31, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {5712, 651, 5819, 12, 627, 43}

$$\frac{if^3(1-icx)^3\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2ibf^3\left(c^2x^2+1\right)^{5/2}}{3c(-cx+i)(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{bf^3\left(c^2x^2+1\right)^{5/2}\log(-cx+i)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(5/2),x]
```

```
[Out] (((2*I)/3)*b*f^3*(1 + c^2*x^2)^(5/2))/(c*(I - c*x)*(d + I*c*d*x)^(5/2)*(f -
 I*c*f*x)^(5/2)) + ((I/3)*f^3*(1 - I*c*x)^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*
x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (b*f^3*(1 + c^2*x^2)^(5/
2)*Log[I - c*x])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
```

```
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$\int \frac{\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{5/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(f-icfx)^3\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{if^3(1-icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(bc\left(1+c^2x^2\right)^{5/2}\right)\int \frac{if^3(1-i}{3c\left(1+c^2\right.}}{(d+icdx)^{5/2}(f-icfx)}$$

$$= \frac{if^3(1-icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(ibf^3\left(1+c^2x^2\right)^{5/2}\right)\int \frac{(1-i}{(1+c}}{3(d+icdx)^{5/2}(f-icfx}$$

$$= \frac{if^3(1-icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(ibf^3\left(1+c^2x^2\right)^{5/2}\right)\int \frac{1-i}{(1+i}}{3(d+icdx)^{5/2}(f-icfx)}$$

$$= \frac{if^3(1-icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(ibf^3\left(1+c^2x^2\right)^{5/2}\right)\int \left(-\frac{}{(}\right.}{3(d+icdx)^{5/2}(f-}$$

$$= \frac{2ibf^3\left(1+c^2x^2\right)^{5/2}}{3c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{if^3(1-icx)^3\left(1+c^2x^2\right)\left(a+b\sin\right.}{3c(d+icdx)^{5/2}(f-icfx)^{5}}$$

Mathematica [A] time = 0.50, size = 141, normalized size = 0.75

$$\frac{\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(-(cx+i)\left(a\sqrt{c^2x^2+1}+bcx-ib\right)-b(cx+i)\sqrt{c^2x^2+1}\,\sinh^{-1}(cx)+b(cx-i)^2\log(d+\right.}{3cd^3(cx-i)^2\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(5/2),x]
```

```
[Out] (Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-((I + c*x)*((-I)*b + b*c*x + a*Sqrt[
1 + c^2*x^2])) - b*(I + c*x)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + b*(-I + c*x)^
2*Log[d + I*c*d*x]))/(3*c*d^3*(-I + c*x)^2*Sqrt[1 + c^2*x^2])
```

fricas [B] time = 0.64, size = 565, normalized size = 3.02

$$-\frac{24\sqrt{c^2x^2+1}\,\sqrt{icdx+d}\,\sqrt{-icfx+f}\,bcx+3\left(4bc^2x^2+8ibcx-4b\right)\sqrt{icdx+d}\,\sqrt{-icfx+f}\,\log\left(cx+\sqrt{c^2x^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x, algorit
hm="fricas")
```

```
[Out] -1/3*(24*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x + 3*(
4*b*c^2*x^2 + 8*I*b*c*x - 4*b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x
 + sqrt(c^2*x^2 + 1)) + 2*(3*c^4*d^3*x^3 - 3*I*c^3*d^3*x^2 + 3*c^2*d^3*x -
3*I*c*d^3)*sqrt(b^2*f/(c^2*d^5))*log(1/3*(3*(-2*I*b*c^6*x^2 - 4*b*c^5*x + 4
*I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) + 2*(3*I*c
^9*d^3*x^4 + 6*c^8*d^3*x^3 + 3*I*c^7*d^3*x^2 + 6*c^6*d^3*x)*sqrt(b^2*f/(c^2
*d^5)))/(16*b*c^3*x^3 - 16*I*b*c^2*x^2 + 16*b*c*x - 16*I*b)) - 2*(3*c^4*d^3
*x^3 - 3*I*c^3*d^3*x^2 + 3*c^2*d^3*x - 3*I*c*d^3)*sqrt(b^2*f/(c^2*d^5))*log
(1/3*(3*(-2*I*b*c^6*x^2 - 4*b*c^5*x + 4*I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c
*d*x + d)*sqrt(-I*c*f*x + f) + 2*(-3*I*c^9*d^3*x^4 - 6*c^8*d^3*x^3 - 3*I*c^
7*d^3*x^2 - 6*c^6*d^3*x)*sqrt(b^2*f/(c^2*d^5)))/(16*b*c^3*x^3 - 16*I*b*c^2*
x^2 + 16*b*c*x - 16*I*b)) + 3*(4*a*c^2*x^2 + 8*I*a*c*x - 4*a)*sqrt(I*c*d*x
+ d)*sqrt(-I*c*f*x + f))/(12*c^4*d^3*x^3 - 12*I*c^3*d^3*x^2 + 12*c^2*d^3*x
- 12*I*c*d^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-i\,cfx+f}\,(b\operatorname{arsinh}(cx)+a)}{(i\,cdx+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate(sqrt(-I*c*f*x + f)*(b*arcsinh(c*x) + a)/(I*c*d*x + d)^(5/2), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{arcsinh}(cx))\sqrt{-icfx+f}}{(icdx+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x)
```

maxima [A] time = 0.60, size = 219, normalized size = 1.17

$$\frac{1}{3}\,bc\left(\frac{6\sqrt{f}}{3i\,c^3d^{\frac{5}{2}}x+3\,c^2d^{\frac{5}{2}}}+\frac{\sqrt{f}\,\log(cx-i)}{c^2d^{\frac{5}{2}}}\right)-\frac{1}{3}\,b\left(\frac{2i\sqrt{c^2dfx^2+df}}{c^3d^3x^2-2i\,c^2d^3x-cd^3}+\frac{3i\sqrt{c^2dfx^2+df}}{3i\,c^2d^3x+3\,cd^3}\right)\operatorname{arsinh}(cx)-\frac{1}{3}\,a\left(\frac{}{c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x, algorit
hm="maxima")
```

```
[Out] 1/3*b*c*(6*sqrt(f)/(3*I*c^3*d^(5/2)*x + 3*c^2*d^(5/2)) + sqrt(f)*log(c*x -
I)/(c^2*d^(5/2))) - 1/3*b*(2*I*sqrt(c^2*d*f*x^2 + d*f)/(c^3*d^3*x^2 - 2*I*c
^2*d^3*x - c*d^3) + 3*I*sqrt(c^2*d*f*x^2 + d*f)/(3*I*c^2*d^3*x + 3*c*d^3))*
arcsinh(c*x) - 1/3*a*(2*I*sqrt(c^2*d*f*x^2 + d*f)/(c^3*d^3*x^2 - 2*I*c^2*d^
3*x - c*d^3) + 3*I*sqrt(c^2*d*f*x^2 + d*f)/(3*I*c^2*d^3*x + 3*c*d^3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\sqrt{f-c\,f\,x\,1\mathrm{i}}}{(d+c\,d\,x\,1\mathrm{i})^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(5/2),x)
[Out] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(f-I*c*f*x)**(1/2)/(d+I*c*d*x)**(5/2),x)
[Out] Timed out
```

3.540 $\int (d+icdx)^{5/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=459

$$\frac{3dx(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{8\left(c^2x^2+1\right)}+\frac{3d(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc\left(c^2x^2+1\right)^{3/2}}+\frac{id\left(c^2x^2+1\right)(\ldots}{}$$

```
[Out] -1/5*I*b*d*x*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)-5/16*b*c
*d*x^2*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)-2/15*I*b*c^2*d
*x^3*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)-1/16*b*c^3*d*x^4
*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)-1/25*I*b*c^4*d*x^5*(
d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)+1/4*d*x*(d+I*c*d*x)^(3
/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))+3/8*d*x*(d+I*c*d*x)^(3/2)*(f-I*c*f
*x)^(3/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)+1/5*I*d*(d+I*c*d*x)^(3/2)*(f-I*c*f
*x)^(3/2)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c+3/16*d*(d+I*c*d*x)^(3/2)*(f-I*c*
f*x)^(3/2)*(a+b*arcsinh(c*x))^2/b/c/(c^2*x^2+1)^(3/2)
```

Rubi [A] time = 0.44, antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {5712, 5821, 5684, 5682, 5675, 30, 14, 5717, 194}

$$\frac{3dx(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{8\left(c^2x^2+1\right)}+\frac{3d(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc\left(c^2x^2+1\right)^{3/2}}+\frac{id\left(c^2x^2+1\right)(\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] ((-I/5)*b*d*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(1 + c^2*x^2)^(3/2)
- (5*b*c*d*x^2*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(16*(1 + c^2*x^2)^(
3/2)) - (((2*I)/15)*b*c^2*d*x^3*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(1
 + c^2*x^2)^(3/2) - (b*c^3*d*x^4*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(
16*(1 + c^2*x^2)^(3/2)) - ((I/25)*b*c^4*d*x^5*(d + I*c*d*x)^(3/2)*(f - I*c*
f*x)^(3/2))/(1 + c^2*x^2)^(3/2) + (d*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3
/2)*(a + b*ArcSinh[c*x]))/4 + (3*d*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2
)*(a + b*ArcSinh[c*x]))/(8*(1 + c^2*x^2)) + ((I/5)*d*(d + I*c*d*x)^(3/2)*(f
 - I*c*f*x)^(3/2)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/c + (3*d*(d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(16*b*c*(1 + c^2*x^2)^(
3/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx = \frac{\left((d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int (d+icdx)\left(1+c^2x^2\right)^{3/2}\left(a+b\,\mathrm{s}\right.}{\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{\left((d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int \left(d\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(c\right.\right.}{\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{\left(d(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int \left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx\right.}{\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{1}{4}dx(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{id(d+icd}{}$$

$$= \frac{1}{4}dx(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{3dx(d+i}{}$$

$$= -\frac{ibdx(d+icdx)^{3/2}(f-icfx)^{3/2}}{5\left(1+c^2x^2\right)^{3/2}}-\frac{5bcdx^2(d+icdx)^{3/2}(f-ic}{16\left(1+c^2x^2\right)^{3/2}}$$

Mathematica [A] time = 1.76, size = 683, normalized size = 1.49

$$\frac{3600ad^{5/2}f^{3/2}\sqrt{c^2x^2+1}\,\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)+3840iac^2d^2fx^2\sqrt{c^2x^2+1}\sqrt{d+icdx}\sqrt{f-i}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] ((-1200*I)*b*c*d^2*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (1920*I)*a*d^2
*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 6000*a*c*d^2*f*x
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (3840*I)*a*c^2*d^2
*f*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 2400*a*c^3*d
^2*f*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (1920*I)*a
*c^4*d^2*f*x^4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 1800
*b*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 - 1200*b*d^2*f*
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] - 75*b*d^2*f*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] + 3600*a*d^(5/2)*f^(3/2
)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x]] - (200*I)*b*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*Ar
cSinh[c*x]] + 60*b*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*(
(10*I)*Cosh[3*ArcSinh[c*x]] + (2*I)*Cosh[5*ArcSinh[c*x]] + 5*((4*I)*Sqrt[1
+ c^2*x^2] + 8*Sinh[2*ArcSinh[c*x]] + Sinh[4*ArcSinh[c*x]])) - (24*I)*b*d^2
*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[5*ArcSinh[c*x]])/(9600*c*Sqrt[1
 + c^2*x^2])
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(i\,bc^3d^2fx^3+bc^2d^2fx^2+i\,bcd^2fx+bd^2f\right)\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(i\,ac^3d^2fx^3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x, algorit
hm="fricas")
```

```
[Out] integral((I*b*c^3*d^2*f*x^3 + b*c^2*d^2*f*x^2 + I*b*c*d^2*f*x + b*d^2*f)*sq
rt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (I*a*c^3*
d^2*f*x^3 + a*c^2*d^2*f*x^2 + I*a*c*d^2*f*x + a*d^2*f)*sqrt(I*c*d*x + d)*sq
rt(-I*c*f*x + f), x)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument TypeError: Bad Argument
TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

maple [**F**] time = 0.28, size = 0, normalized size = 0.00

$$\int (icdx+d)^{\frac{5}{2}} \left(-icfx+f\right)^{\frac{3}{2}} (a+b\,\mathrm{arcsinh}\,(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{asinh}\,(c\,x))\,(d+c\,d\,x\,1\mathrm{i})^{5/2}\,\left(f-c\,f\,x\,1\mathrm{i}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(3/2),x)
```

```
[Out] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(3/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(5/2)*(f-I*c*f*x)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

3.541 $\int (d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)dx$

Optimal. Leaf size=247

$$\frac{3x(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{8\left(c^2x^2+1\right)}+\frac{3(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc\left(c^2x^2+1\right)^{3/2}}+\frac{1}{4}x(d+icdx)^{3/2}(f$$

```
[Out] -5/16*b*c*x^2*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)-1/16*b*
c^3*x^4*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)+1/4*x*(d+I*c*
d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))+3/8*x*(d+I*c*d*x)^(3/2)*(f-
I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)+3/16*(d+I*c*d*x)^(3/2)*(f-I*c
*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/b/c/(c^2*x^2+1)^(3/2)
```

Rubi [A] time = 0.25, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {5712, 5684, 5682, 5675, 30, 14}

$$\frac{3x(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{8\left(c^2x^2+1\right)}+\frac{3(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc\left(c^2x^2+1\right)^{3/2}}+\frac{1}{4}x(d+icdx)^{3/2}(f$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-5*b*c*x^2*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(16*(1 + c^2*x^2)^(3/2
)) - (b*c^3*x^4*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(16*(1 + c^2*x^2)^
(3/2)) + (x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/4
 + (3*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(8*(1
 + c^2*x^2)) + (3*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*
x])^2)/(16*b*c*(1 + c^2*x^2)^(3/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rubi steps

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)dx = \frac{\left((d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(c\right.}{\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{1}{4}x(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{\left(3(d+i\right.}{}$$

$$= \frac{1}{4}x(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{3x(d+i}{}$$

$$= -\frac{5bcx^2(d+icdx)^{3/2}(f-icfx)^{3/2}}{16\left(1+c^2x^2\right)^{3/2}}-\frac{bc^3x^4(d+icdx)^{3/2}(f-}{16\left(1+c^2x^2\right)^{3/}}$$

Mathematica [A] time = 0.90, size = 352, normalized size = 1.43

$$\frac{48ad^{3/2}f^{3/2}\sqrt{c^2x^2+1}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)+80acdfx\sqrt{c^2x^2+1}\sqrt{d+icdx}\sqrt{f-icfx}+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (80*a*c*d*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 32*a*
c^3*d*f*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 24*b*d*
f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 - 16*b*d*f*Sqrt[d + I*
c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] - b*d*f*Sqrt[d + I*c*d*x]*Sqr
t[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] + 48*a*d^(3/2)*f^(3/2)*Sqrt[1 + c^2*x^2
]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + 4*b*
d*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*(8*Sinh[2*ArcSinh[c*x]
] + Sinh[4*ArcSinh[c*x]]))/(128*c*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bc^2dfx^2+bdf\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(ac^2dfx^2+adf\right)\sqrt{i\,cdx+d}\sqrt{-i\,cf}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x, algorit
hm="fricas")
```

```
[Out] integral((b*c^2*d*f*x^2 + b*d*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c
*x + sqrt(c^2*x^2 + 1)) + (a*c^2*d*f*x^2 + a*d*f)*sqrt(I*c*d*x + d)*sqrt(-I
*c*f*x + f), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int (icdx+d)^{\frac{3}{2}} \left(-icfx+f\right)^{\frac{3}{2}} (a+b\,\mathrm{arcsinh}\,(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{asinh}\,(c\,x))\,(d+c\,d\,x\,1\mathrm{i})^{3/2}\,\left(f-c\,f\,x\,1\mathrm{i}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(3/2),x)
```

```
[Out] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(3/2)*(f-I*c*f*x)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

3.542 $\int \sqrt{d+icdx}\,(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)dx$

Optimal. Leaf size=304

$$\frac{f\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{c^2x^2+1}}-\frac{if\left(c^2x^2+1\right)\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{3c}+\frac{1}{2}fx\sqrt{d+}$$

```
[Out] 1/2*f*x*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-1/3*I*f*(c^2
*x^2+1)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c+1/3*I*b*f*
x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-1/4*b*c*f*x^2*(d+I*
c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/9*I*b*c^2*f*x^3*(d+I*c*d
*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/4*f*(a+b*arcsinh(c*x))^2*(d
+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {5712, 5821, 5682, 5675, 30, 5717}

$$\frac{f\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{c^2x^2+1}}-\frac{if\left(c^2x^2+1\right)\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{3c}+\frac{1}{2}fx\sqrt{d+}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] ((I/3)*b*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 + c^2*x^2] - (b*c*
f*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(4*Sqrt[1 + c^2*x^2]) + ((I/9)*b
*c^2*f*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 + c^2*x^2] + (f*x*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/2 - ((I/3)*f*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/c + (f*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[1 + c
^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
```

```
+ e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int \sqrt{d+icdx}\,(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)dx = \frac{\left(\sqrt{d+icdx}\,\sqrt{f-icfx}\right)\int(f-icfx)\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(c\right.}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(\sqrt{d+icdx}\,\sqrt{f-icfx}\right)\int\left(f\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)-ic\right.}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(f\sqrt{d+icdx}\,\sqrt{f-icfx}\right)\int\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{1}{2}fx\sqrt{d+icdx}\,\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)-\frac{if\sqrt{d+icdx}\,\sqrt{}}{}$$

$$= \frac{ibfx\sqrt{d+icdx}\,\sqrt{f-icfx}}{3\sqrt{1+c^2x^2}}-\frac{bcfx^2\sqrt{d+icdx}\,\sqrt{f-icfx}}{4\sqrt{1+c^2x^2}}+\frac{ib}{}$$

Mathematica [A] time = 1.59, size = 273, normalized size = 0.90

$$\frac{12af\left(-2ic^2x^2+3cx-2i\right)\sqrt{d+icdx}\,\sqrt{f-icfx}+36a\sqrt{d}\,f^{3/2}\log\left(cdfx+\sqrt{d}\,\sqrt{f}\,\sqrt{d+icdx}\,\sqrt{f-icfx}\right)+\frac{9bf\sqrt{d}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (12*a*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-2*I + 3*c*x - (2*I)*c^2*x^2)
+ 36*a*Sqrt[d]*f^(3/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt
[f - I*c*f*x]] + (9*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(2*ArcSinh[c*x]
^2 - Cosh[2*ArcSinh[c*x]] + 2*ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]]))/Sqrt[1 +
c^2*x^2] + ((2*I)*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(9*c*x - 3*ArcSin
h[c*x]*(3*Sqrt[1 + c^2*x^2] + Cosh[3*ArcSinh[c*x]]) + Sinh[3*ArcSinh[c*x]])
)/Sqrt[1 + c^2*x^2])/(72*c)
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-ibcfx+bf\right)\sqrt{icdx+d}\,\sqrt{-icfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(-iacfx+af\right)\sqrt{icdx+d}\,\sqrt{-icfx+f}\,,\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((-I*b*c*f*x + b*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
sqrt(c^2*x^2 + 1)) + (-I*a*c*f*x + a*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f
), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument TypeError: Bad Argument
TypeError: Bad Argument TypeDone
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \left(-icfx+f\right)^{\frac{3}{2}} \left(a+b\,\mathrm{arcsinh}\left(cx\right)\right) \sqrt{icdx+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\sqrt{d+c\,d\,x\,1{}\mathrm{i}}\,{\left(f-c\,f\,x\,1{}\mathrm{i}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(3/2),x)
```

```
[Out] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{id\left(cx-i\right)} \left(-if\left(cx+i\right)\right)^{\frac{3}{2}} \left(a+b\,\mathrm{asinh}\left(cx\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))*(d+I*c*d*x)**(1/2),x)
```

```
[Out] Integral(sqrt(I*d*(c*x - I))*(-I*f*(c*x + I))**(3/2)*(a + b*asinh(c*x)), x)
```

3.543 $\int \frac{(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+icdx}}\,dx$

Optimal. Leaf size=266

$$\frac{3f^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{f^2x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{2if^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{bcf}{4\sqrt{d+}}$$

```
[Out] -2*I*f^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/
2)-1/2*f^2*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(
1/2)+2*I*b*f^2*x*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/4*
b*c*f^2*x^2*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+3/4*f^2*(
a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/
2)
```

Rubi [A] time = 0.46, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5712, 5821, 5675, 5717, 8, 5758, 30}

$$\frac{3f^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{f^2x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{2if^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{bcf}{4\sqrt{d+}}$$

Antiderivative was successfully verified.

```
[In] Int[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/Sqrt[d + I*c*d*x],x]
```

```
[Out] ((2*I)*b*f^2*x*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (
b*c*f^2*x^2*Sqrt[1 + c^2*x^2])/(4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - ((
2*I)*f^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*Sqrt[d + I*c*d*x]*Sqrt[f -
I*c*f*x]) - (f^2*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(2*Sqrt[d + I*c*d*x]
*Sqrt[f - I*c*f*x]) + (3*f^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b
*c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int \frac{(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+icdx}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{(f-icfx)^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\int \left(\frac{f^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}} - \frac{2icf^2x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}} - \frac{c^2f^2x^2\left(a+b\sinh^{-1}(cx\right.}{\sqrt{1+c^2x^2}}\right.}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\left(f^2\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{\left(2icf^2\sqrt{1+c^2x^2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right.}{\sqrt{1+c^2x^2}}}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= -\frac{2if^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{f^2x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx\right.}{2\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{2ibf^2x\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{bcf^2x^2\sqrt{1+c^2x^2}}{4\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2if^2\left(1+c^2x^2\right)\left(a\right.}{c\sqrt{d+icdx}\sqrt{}}$$

Mathematica [A] time = 1.08, size = 344, normalized size = 1.29

$$12a\sqrt{d}f^{3/2}\sqrt{c^2x^2+1}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right) - 16iaf\sqrt{c^2x^2+1}\sqrt{d+icdx}\sqrt{f-icfx} - 4ac$$

Antiderivative was successfully verified.

```
[In] Integrate[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/Sqrt[d + I*c*d*x],x]
```

```
[Out] ((16*I)*b*c*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (16*I)*a*f*Sqrt[d + I
*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 4*a*c*f*x*Sqrt[d + I*c*d*x]*S
```

```
qrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 4*b*f*(4*I + c*x)*Sqrt[d + I*c*d*x]*Sq
rt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + 6*b*f*Sqrt[d + I*c*d*x]*Sq
rt[f - I*c*f*x]*ArcSinh[c*x]^2 + b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Co
sh[2*ArcSinh[c*x]] + 12*a*Sqrt[d]*f^(3/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + S
qrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]])/(8*c*d*Sqrt[1 + c^2*x^
2])
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(bcfx + ibf\right)\sqrt{icdx + d}\,\sqrt{-icfx + f}\,\log\left(cx + \sqrt{c^2x^2 + 1}\right) + \left(acfx + iaf\right)\sqrt{icdx + d}\,\sqrt{-icfx + f}}{cdx - id}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(-((b*c*f*x + I*b*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
 sqrt(c^2*x^2 + 1)) + (a*c*f*x + I*a*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f
))/(c*d*x - I*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-icfx + f\right)^{\frac{3}{2}}(b\,\mathrm{arsinh}\,(cx) + a)}{\sqrt{icdx + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((-I*c*f*x + f)^(3/2)*(b*arcsinh(c*x) + a)/sqrt(I*c*d*x + d), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{\left(-icfx + f\right)^{\frac{3}{2}}\,(a + b\,\mathrm{arcsinh}\,(cx))}{\sqrt{icdx + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{asinh}\,(c\,x))\,\left(f - c\,f\,x\,1\mathrm{i}\right)^{3/2}}{\sqrt{d + c\,d\,x\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(1/2),x)
```

```
[Out] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-if\left(cx+i\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{asinh}\left(cx\right)\right)}{\sqrt{id\left(cx-i\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))/(d+I*c*d*x)**(1/2),x)
```

```
[Out] Integral((-I*f*(c*x + I))**(3/2)*(a + b*asinh(c*x))/sqrt(I*d*(c*x - I)), x)
```

3.544 $\int \frac{(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{3/2}}\,dx$

Optimal. Leaf size=284

$$-\frac{3f^3\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{if^3\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{4if^3(1-icx)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}\right.}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[Out] -I*b*f^3*x*(c^2*x^2+1)^(3/2)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+4*I*f^3*(1
-I*c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2
)+I*f^3*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3
/2)-3/2*f^3*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/b/c/(d+I*c*d*x)^(3/2)/(f
-I*c*f*x)^(3/2)-4*b*f^3*(c^2*x^2+1)^(3/2)*ln(I-c*x)/c/(d+I*c*d*x)^(3/2)/(f-
I*c*f*x)^(3/2)
```

Rubi [A] time = 0.47, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5712, 5833, 637, 5819, 12, 627, 31, 5675, 5717, 8}

$$-\frac{3f^3\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{if^3\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{4if^3(1-icx)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}\right.}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(3/2),x]
```

```
[Out] ((-I)*b*f^3*x*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)
) + ((4*I)*f^3*(1 - I*c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*
d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (I*f^3*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x
]))/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (3*f^3*(1 + c^2*x^2)^(3/2
)*(a + b*ArcSinh[c*x])^2)/(2*b*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) -
 (4*b*f^3*(1 + c^2*x^2)^(3/2)*Log[I - c*x])/(c*(d + I*c*d*x)^(3/2)*(f - I*c
*f*x)^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rule 5833

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{3/2}}dx = \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{(f-icfx)^3\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{3/2}\int \left(-\frac{4i\left(if^3+cf^3x\right)\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}} - \frac{3f^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}} + \frac{icf^3x\left(a+b\sinh\ldots\right.}{\sqrt{1+c\ldots}}\right.}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{\left(4i\left(1+c^2x^2\right)^{3/2}\right)\int \frac{\left(if^3+cf^3x\right)\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(3f^3\left(1+c^2x^2\right)^{3/2}\right)\int \ldots}{(d+icdx)^{3/2}(f-\ldots}$$

$$= \frac{4if^3(1-icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{if^3\left(1+c^2x^2\right)^2\left(a+b\sinh\ldots\right.}{c(d+icdx)^{3/2}(f-icfx\ldots}$$

$$= -\frac{ibf^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4if^3(1-icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{ibf^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4if^3(1-icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{ibf^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4if^3(1-icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Mathematica [A] time = 2.38, size = 514, normalized size = 1.81

$$-\frac{6af^{3/2}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)}{d^{3/2}} + \frac{2af(5+icx)\sqrt{d+icdx}\sqrt{f-icfx}}{d^2(cx-i)} - \frac{bf\sqrt{d+icdx}\sqrt{f-icfx}\left(2\left(\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)+i\sinh\left(\frac{1}{2}\sinh^{-1}(cx\ldots\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(3/2),x]
```

```
[Out] ((2*a*f*(5 + I*c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(d^2*(-I + c*x)) -
 (6*a*f^(3/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*
f*x]])/d^(3/2) - (b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(ArcSinh[c*x]*((-
4*I)*Cosh[ArcSinh[c*x]/2] - 4*Sinh[ArcSinh[c*x]/2]) + ArcSinh[c*x]^2*(Cosh[
ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + 2*((4*I)*ArcTan[Tanh[ArcSinh[c*
x]/2]] + Log[1 + c^2*x^2])*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]))
)/(d^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) +
 (2*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-(ArcSinh[c*x]^2*(Cosh[ArcSinh
[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) + (c*x - 4*ArcTan[Coth[ArcSinh[c*x]/2]]
 - I*Log[1 + c^2*x^2])*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2]) +
 ArcSinh[c*x]*(I*(2 + Sqrt[1 + c^2*x^2])*Cosh[ArcSinh[c*x]/2] - (-2 + Sqrt[
1 + c^2*x^2])*Sinh[ArcSinh[c*x]/2])))/(d^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[
c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/(2*c)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ibcfx-bf\right)\sqrt{icdx+d}\sqrt{-icfx+f}\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(iacfx-af\right)\sqrt{icdx+d}\sqrt{-icfx+f}}{c^2d^2x^2-2icd^2x-d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral(((I*b*c*f*x - b*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
sqrt(c^2*x^2 + 1)) + (I*a*c*f*x - a*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)
)/(c^2*d^2*x^2 - 2*I*c*d^2*x - d^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{1544828329316635927383708931993249379254272 00,[3,10,0,10]%%%}+%%%{%%
{[-1043577678436828857913833475521211509427666944 0,0,%%%{104357767843682885
7913833475521211509427666944 0,[2,0,2]%%%}+%%%{-208715535687365771582766695 1
0424230188553338880,[0,0,0]%%%},15448283293166359273837089319932493792542 72
000]:[1,0,%%%{-2,[2,0,2]%%%},0,%%%{1,[4,0,4]%%%}+%%%{4,[2,0,2]%%%}+%%%{4,[0
,0,0]%%%}]%%},[3,8,0,8]%%%}+%%%{%%{[-8348621427494630863310667804169692075 4
213355520,0,%%%{834862142749463086331066780416969207542133 55520,[2,0,2]%%%}
+%%%{-1669724285498926172662133560833938415084267 11040,[0,0,0]%%%},%%%{5149
9447334972865995296164067863008297091072,[4,0,4]%%%}+%%%{205997789339891463
98118465627145203318836428 8,[2,0,2]%%%}+%%%{63853111066064351735160203842 44
449550205452288,[0,0,0]%%%}]:[1,0,%%%{-2,[2,0,2]%%%},0,%%%{1,[4,0,4]%%%}+%%
%{4,[2,0,2]%%%}+%%%{4,[0,0,0]%%%}]%%},[3,6,0,6]%%%}+%%%{%%{[%%%{-2964359278
91845317620371898030963111755776,[4,0,4]%%%}+%%%{-118574371156738127048148 7
592123852447023104,[2,0,2]%%%}+%%%{-2504598285685504932805905156126828861 15
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,6]%%%}+%%%{5928718557836906352407437960619262235115 52,[4,0,4]%%%}+%%%{2504
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[0,0,0]%%%}]:[1,0,%%%{-2,[2,0,2]%%%},0,%%%{1,[4,0,4]%%%}+%%%{4,[2,0,2]%%%}+
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92123852447023104,[4,0,4]%%%}+%%%{-474297484626952508192595036849540978809 2
416,[2,0,2]%%%}+%%%{-33394960007463150405750863811715617842664151449 6,[0,0,
0]%%%},0,%%%{11857437115673812704814875921238524470231 04,[6,0,6]%%%}+%%%{23
71487423134762540962975184247704894046208,[4,0,4]%%%}+%%%{33394011412493896
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9648,[8,0,8]%%%}+%%%{-668649109665903324659100725572731823718 4,[6,0,6]%%%}+
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0,4]%%%}+%%%{4,[2,0,2]%%%}+%%%{4,[0,0,0]%%%}]%%},[3,2,0,2]%%%}+%%%{%%{[%%%{
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451145463647436 8,[6,0,6]%%%}+%%%{411955459733202973762889766499369702467305
472,[4,0,4]%%%}+%%%{164792882279035843958350452211357044696101683 2,[2,0,2]%
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```

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```
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+%%%{4,[0,0,0]%%%}]%%},[0,10,0,10]%%%}+%%%{%%{[-2574743909706753632947823333
770872393994076160 0,0,%%%{25747439097067536329478233377087239399407616 00,[2
,0,2]%%%}+%%%{-51494878194135072658956466754174478798815232 00,[0,0,0]%%%},%
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19311407418705060946311178407 1168,[0,0,0]%%%}]:[1,0,%%%{-2,[2,0,2]%%%},0,%%
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40302089493165301050176962 56,[0,0,0]%%%},0,%%%{-5149294651542598694027110 03
79591413686337536,[6,0,6]%%%}+%%%{-1029858930308519738805422007591828273726
75072,[4,0,4]%%%}+%%%{105049474248887184795523777523532614145083965 44,[2,0,
2]%%%}+%%%{-20186007705530621168060417898633060210035392512,[0,0,0]%%%},%%%
{-166988690052249619030733320369677880506068238336,[4,0,4]%%%}+%%%{-6679547
60208998476122933281478711522024272953344,[2,0,2]%%%}+%%%{-4007288145931743
49905282001978743291631427282534 4,[0,0,0]%%%}]:[1,0,%%%{-2,[2,0,2]%%%},0,%%
%{1,[4,0,4]%%%}+%%%{4,[2,0,2]%%%}+%%%{4,[0,0,0]%%%}]%%},[0,6,0,6]%%%}+%%%{%
%{[%%%{30895767909255592164162660227754848211802521 6,[4,0,4]%%%}+%%%{123583
071637022368656650640911019392847210086 4,[2,0,2]%%%}+%%%{-19362120561283805
37701608029255959759105399193 6,[0,0,0]%%%},0,%%%{-308957679092555921641626 6
02277548482118025216,[6,0,6]%%%}+%%%{-617915358185111843283253204555096964 2
36050432,[4,0,4]%%%}+%%%{2183378199402425275014909311077998544799819366 4,[2
,0,2]%%%}+%%%{-387242411225676107540321605851191951821079838 72,[0,0,0]%%%},
%%%{12703611363707432963665219865954801732812 8,[8,0,8]%%%}+%%%{101628890909
659463709321758927638413862502 4,[6,0,6]%%%}+%%%{-500963021290021567308288 68
145626581236578883993 6,[4,0,4]%%%}+%%%{-200386021547135904199025147156577 74
60536264359936,[2,0,2]%%%}+%%%{-701286232663329476957435576546403810473954 1
417984,[0,0,0]%%%}]:[1,0,%%%{-2,[2,0,2]%%%},0,%%%{1,[4,0,4]%%%}+%%%{4,[2,0,
2]%%%}+%%%{4,[0,0,0]%%%}]%%},[0,4,0,4]%%%}+%%%{%%{[%%%{617915358185111843 28
325320455509696423605043 2,[4,0,4]%%%}+%%%{247166143274044737313301281822038
7856944201728,[2,0,2]%%%}+%%%{-181262898449135816904495738834494036625818 91
072,[0,0,0]%%%},0,%%%{-617915358185111843283253204555096964236050432,[6,0,6
]%%%}+%%%{-1235830716370223686566506409110193928472100864,[4,0,4]%%%}+%%%{2
306961271039447643671559951989017937647029452 8,[2,0,2]%%%}+%%%{-36252579689
82716338089914776689880732516378214 4,[0,0,0]%%%},%%%{5081444545482973185466
087946381920693125 12,[8,0,8]%%%}+%%%{406515563638637854837287035710553655 45
00096,[6,0,6]%%%}+%%%{-66794256474208931698728816286764020541460945305 6,[4,
0,4]%%%}+%%%{-26718027802134483589775396344334176659508738129 92,[2,0,2]%%%}
+%%%{-667901097339201515925050046614459755017198265958 4,[0,0,0]%%%}]:[1,0,%
%%{-2,[2,0,2]%%%},0,%%%{1,[4,0,4]%%%}+%%%{4,[2,0,2]%%%}+%%%{4,[0,0,0]%%%}]%
%},[0,2,0,2]%%%}+%%%{%%{[%%%{411943572123407895522168803036731309490700288,
[4,0,4]%%%}+%%%{164777428849363158208867521214692523796280115 2,[2,0,2]%%%}+
%%%{-659140622256798004334435946852099136984763596 8,[0,0,0]%%%},0,%%%{-4119
435721234078955221688030367313094907002 88,[6,0,6]%%%}+%%%{-823887144246815 7
910443376060734626189814005 76,[4,0,4]%%%}+%%%{988695479955524320752170989 28
14841845773238272,[2,0,2]%%%}+%%%{-131828124451359600866887189370419827396 9
5271936,[0,0,0]%%%},%%%{50814445454829731854660879463819206931251 2,[8,0,8]%
%%}+%%%{40651556363863785483728703571055365545000 96,[6,0,6]%%%}+%%%{-333965
184637590078925821522128284444024729763 84,[4,0,4]%%%}+%%%{-133589325979545
14067316730714759946219023279063 04,[2,0,2]%%%}+%%%{-2671634744395822188660 7
2451254019739069143685529 6,[0,0,0]%%%}]:[1,0,%%%{-2,[2,0,2]%%%},0,%%%{1,[4,
0,4]%%%}+%%%{4,[2,0,2]%%%}+%%%{4,[0,0,0]%%%}]%%},[0,0,0,0]%%%} / %%%{408989
1520512,[0,2,0,2]%%%}+%%%{8179783041024,[0,0,0,0]%%%} Error: Bad Argument V
alueWarning, replacing 0 by ` u`, a substitution variable should perhaps be
 purged.Evaluation time: 2.04sym2poly/r2sym(const gen & e,const index_m & i
,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{\left(-icfx+f\right)^{\frac{3}{2}} \left(a+b \operatorname{arcsinh}\left(cx\right)\right)}{\left(icdx+d\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x)
```

```
[Out] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\frac{i\left(c^2dfx^2+df\right)^{\frac{3}{2}}}{c^3d^3x^2-2i\,c^2d^3x-cd^3}+\frac{6i\sqrt{c^2dfx^2+df}\,f}{i\,c^2d^2x+cd^2}-\frac{3\,f^2\operatorname{arsinh}\left(cx\right)}{cd^2\sqrt{\frac{f}{d}}}\right)+b\int \frac{\left(-i\,cfx+f\right)^{\frac{3}{2}}\log\left(cx+\sqrt{c^2x^2+1}\right)}{\left(i\,cdx+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x, algorit
hm="maxima")
```

```
[Out] a*(I*(c^2*d*f*x^2 + d*f)^(3/2)/(c^3*d^3*x^2 - 2*I*c^2*d^3*x - c*d^3) + 6*I*
sqrt(c^2*d*f*x^2 + d*f)*f/(I*c^2*d^2*x + c*d^2) - 3*f^2*arcsinh(c*x)/(c*d^2
*sqrt(f/d))) + b*integrate((-I*c*f*x + f)^(3/2)*log(c*x + sqrt(c^2*x^2 + 1)
)/(I*c*d*x + d)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(f-c\,f\,x\,1{}\mathrm{i}\right)}^{3/2}}{{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(3/2),x)
```

```
[Out] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))/(d+I*c*d*x)**(3/2),x)
```

```
[Out] Timed out
```

3.545 $\int \frac{(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{5/2}}\,dx$

Optimal. Leaf size=364

$$-\frac{2if^4(1-icx)\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2if^4(1-icx)^3\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{f^4\left(c^2x^2+1\right)^{5/2}\text{si}}{c(d+ica}$$

```
[Out] 4/3*I*b*f^4*(c^2*x^2+1)^(5/2)/c/(I-c*x)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
-1/2*b*f^4*(c^2*x^2+1)^(5/2)*arcsinh(c*x)^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)
^(5/2)+2/3*I*f^4*(1-I*c*x)^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(
5/2)/(f-I*c*f*x)^(5/2)-2*I*f^4*(1-I*c*x)*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/c
/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+f^4*(c^2*x^2+1)^(5/2)*arcsinh(c*x)*(a+
b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+8/3*b*f^4*(c^2*x^2+1)
^(5/2)*ln(I-c*x)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A] time = 0.38, antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {5712, 669, 653, 215, 5819, 627, 43, 31, 5675}

$$-\frac{2if^4(1-icx)\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2if^4(1-icx)^3\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{f^4\left(c^2x^2+1\right)^{5/2}\text{si}}{c(d+ica}$$

Antiderivative was successfully verified.

```
[In] Int[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(5/2),x]
```

```
[Out] (((4*I)/3)*b*f^4*(1 + c^2*x^2)^(5/2))/(c*(I - c*x)*(d + I*c*d*x)^(5/2)*(f -
 I*c*f*x)^(5/2)) - (b*f^4*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x]^2)/(2*c*(d + I*c
*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((2*I)/3)*f^4*(1 - I*c*x)^3*(1 + c^2*x^
2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - ((2*
I)*f^4*(1 - I*c*x)*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(
5/2)*(f - I*c*f*x)^(5/2)) + (f^4*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x]*(a + b*Ar
cSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (8*b*f^4*(1 + c^
2*x^2)^(5/2)*Log[I - c*x])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$\int \frac{(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{5/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(f-icfx)^4\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{2if^4(1-icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2if^4(1-icx)\left(1+c^2x^2\right)}{c(d+icdx)^{5/2}(}$$

$$= \frac{2if^4(1-icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2if^4(1-icx)\left(1+c^2x^2\right)}{c(d+icdx)^{5/2}(}$$

$$= -\frac{bf^4\left(1+c^2x^2\right)^{5/2}\sinh^{-1}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2if^4(1-icx)^3\left(1+c^2x^2\right)\left(a+b\sinh\right.}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{bf^4\left(1+c^2x^2\right)^{5/2}\sinh^{-1}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2if^4(1-icx)^3\left(1+c^2x^2\right)\left(a+b\sinh\right.}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{4ibf^4\left(1+c^2x^2\right)^{5/2}}{3c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bf^4\left(1+c^2x^2\right)^{5/2}\sinh^{-1}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} +$$

Mathematica [A] time = 5.84, size = 706, normalized size = 1.94

$$\frac{\frac{12af^{3/2}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)}{d^{5/2}} - \frac{16af(2cx-i)\sqrt{d+icdx}\sqrt{f-icfx}}{d^3(cx-i)^2} - \frac{bf\sqrt{d+icdx}\sqrt{f-icfx}\left(\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)-i\sinh\left(\frac{1}{2}\sinh^{-1}(\right.\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(5/2),x]
```

```
[Out] ((-16*a*f*(-I + 2*c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(d^3*(-I + c*x)
^2) + (12*a*f^(3/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x]])/d^(5/2) - (b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(Cosh[ArcSi
nh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])*(Cosh[(3*ArcSinh[c*x])/2]*((-14 + (3*I
)*ArcSinh[c*x])*ArcSinh[c*x] - 28*ArcTan[Tanh[ArcSinh[c*x]/2]] + (7*I)*Log[
1 + c^2*x^2]) + Cosh[ArcSinh[c*x]/2]*(84*ArcTan[Tanh[ArcSinh[c*x]/2]] - I*(
8 - (6*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 + 21*Log[1 + c^2*x^2])) + 2*(4 -
(4*I)*ArcSinh[c*x] + 6*ArcSinh[c*x]^2 + (56*I)*ArcTan[Tanh[ArcSinh[c*x]/2]]
 + 14*Log[1 + c^2*x^2] + Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(-14*I + 3*ArcSinh
[c*x]) + (28*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 7*Log[1 + c^2*x^2]))*Sinh[Ar
cSinh[c*x]/2]))/(d^3*(I + c*x)*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/
2])^4) + ((2*I)*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(Cosh[ArcSinh[c*x]/
2] - I*Sinh[ArcSinh[c*x]/2])*((-I)*Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] -
 2*ArcTan[Coth[ArcSinh[c*x]/2]] - (I/2)*Log[1 + c^2*x^2]) + Cosh[ArcSinh[c*
x]/2]*(4 + (3*I)*ArcSinh[c*x] - (6*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + (3*Log
[1 + c^2*x^2])/2) + 2*((2 + Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + 2*(2 + Sqrt[1
 + c^2*x^2])*ArcTan[Coth[ArcSinh[c*x]/2]] + (I/2)*(4 + (2 + Sqrt[1 + c^2*x^
2])*Log[1 + c^2*x^2]))*Sinh[ArcSinh[c*x]/2]))/(d^3*(I + c*x)*(Cosh[ArcSinh[
c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^4))/(12*c)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bcfx+i\,bf\right)\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(acfx+i\,af\right)\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}}{c^3d^3x^3-3i\,c^2d^3x^2-3\,cd^3x+i\,d^3},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral(((b*c*f*x + I*b*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
sqrt(c^2*x^2 + 1)) + (a*c*f*x + I*a*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)
)/(c^3*d^3*x^3 - 3*I*c^2*d^3*x^2 - 3*c*d^3*x + I*d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-icfx + f\right)^{\frac{3}{2}}(b\operatorname{arsinh}(cx) + a)}{(icdx + d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((-I*c*f*x + f)^(3/2)*(b*arcsinh(c*x) + a)/(I*c*d*x + d)^(5/2), x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{\left(-icfx + f\right)^{\frac{3}{2}}(a + b\operatorname{arcsinh}(cx))}{(icdx + d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x)
```

```
[Out] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}a\left(-\frac{3i\left(c^2dfx^2 + df\right)^{\frac{3}{2}}}{-3ic^4d^4x^3 - 9c^3d^4x^2 + 9ic^2d^4x + 3cd^4} + \frac{2i\sqrt{c^2dfx^2 + df}\,f}{c^3d^3x^2 - 2ic^2d^3x - cd^3} + \frac{21i\sqrt{c^2dfx^2 + df}\,f}{3ic^2d^3x + 3cd^3} - \frac{3f^2\operatorname{arsinh}(c}{cd^3\sqrt{\frac{f}{d}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x, algorit
hm="maxima")
```

```
[Out] -1/3*a*(-3*I*(c^2*d*f*x^2 + d*f)^(3/2)/(-3*I*c^4*d^4*x^3 - 9*c^3*d^4*x^2 +
9*I*c^2*d^4*x + 3*c*d^4) + 2*I*sqrt(c^2*d*f*x^2 + d*f)*f/(c^3*d^3*x^2 - 2*I
*c^2*d^3*x - c*d^3) + 21*I*sqrt(c^2*d*f*x^2 + d*f)*f/(3*I*c^2*d^3*x + 3*c*d
^3) - 3*f^2*arcsinh(c*x)/(c*d^3*sqrt(f/d))) + b*integrate((-I*c*f*x + f)^(3
/2)*log(c*x + sqrt(c^2*x^2 + 1))/(I*c*d*x + d)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\operatorname{asinh}(c\,x))\left(f - c\,f\,x\,1\mathrm{i}\right)^{3/2}}{(d + c\,d\,x\,1\mathrm{i})^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(5/2),x)
```

```
[Out] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(5/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))/(d+I*c*d*x)**(5/2),x)
[Out] Timed out
```

3.546 $\int (d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=344

$$\frac{5x(d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{24\left(c^2x^2+1\right)}+\frac{5x(d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{16\left(c^2x^2+1\right)^2}+\frac{5(d+icdx)^{5/2}(f}{32}$$

```
[Out] -25/96*b*c*x^2*(d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)/(c^2*x^2+1)^(5/2)-5/96*b
*c^3*x^4*(d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)/(c^2*x^2+1)^(5/2)+1/6*x*(d+I*c
*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))+5/16*x*(d+I*c*d*x)^(5/2)*(
f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^2+5/24*x*(d+I*c*d*x)^(5/2)*
(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)+5/32*(d+I*c*d*x)^(5/2)*(f-
I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/b/c/(c^2*x^2+1)^(5/2)-1/36*b*(d+I*c*d*x
)^(5/2)*(f-I*c*f*x)^(5/2)*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] time = 0.30, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5712, 5684, 5682, 5675, 30, 14, 261}

$$\frac{5x(d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{24\left(c^2x^2+1\right)}+\frac{5x(d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{16\left(c^2x^2+1\right)^2}+\frac{5(d+icdx)^{5/2}(f}{32}$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-25*b*c*x^2*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))/(96*(1 + c^2*x^2)^(5/
2)) - (5*b*c^3*x^4*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))/(96*(1 + c^2*x^
2)^(5/2)) - (b*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*Sqrt[1 + c^2*x^2])/(
36*c) + (x*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/6
+ (5*x*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/(16*(1
 + c^2*x^2)^2) + (5*x*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSin
h[c*x]))/(24*(1 + c^2*x^2)) + (5*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a
 + b*ArcSinh[c*x])^2)/(32*b*c*(1 + c^2*x^2)^(5/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rubi steps

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)dx = \frac{\left((d+icdx)^{5/2}(f-icfx)^{5/2}\right)\int\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(c\right.}{\left(1+c^2x^2\right)^{5/2}}$$

$$= \frac{1}{6}x(d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{\left(5(d+i\right.}{}$$

$$= -\frac{b(d+icdx)^{5/2}(f-icfx)^{5/2}\sqrt{1+c^2x^2}}{36c}+\frac{1}{6}x(d+icdx)^{5/2}($$

$$= -\frac{b(d+icdx)^{5/2}(f-icfx)^{5/2}\sqrt{1+c^2x^2}}{36c}+\frac{1}{6}x(d+icdx)^{5/2}($$

$$= -\frac{25bcx^2(d+icdx)^{5/2}(f-icfx)^{5/2}}{96\left(1+c^2x^2\right)^{5/2}}-\frac{5bc^3x^4(d+icdx)^{5/2}(f}{96\left(1+c^2x^2\right)}$$

Mathematica [A] time = 1.18, size = 481, normalized size = 1.40

$$\frac{720ad^{5/2}f^{5/2}\sqrt{c^2x^2+1}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)+1584acd^2f^2x\sqrt{c^2x^2+1}\sqrt{d+icdx}\sqrt{f-ic}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (1584*a*c*d^2*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] +
 1248*a*c^3*d^2*f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^
```

```
2] + 384*a*c^5*d^2*f^2*x^5*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2
*x^2] + 360*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 -
270*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] - 27
*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] - 2*b*d
^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[6*ArcSinh[c*x]] + 720*a*d^(
5/2)*f^(5/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d
*x]*Sqrt[f - I*c*f*x]] + 12*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*A
rcSinh[c*x]*(45*Sinh[2*ArcSinh[c*x]] + 9*Sinh[4*ArcSinh[c*x]] + Sinh[6*ArcS
inh[c*x]]))/(2304*c*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(bc^4d^2f^2x^4+2\,bc^2d^2f^2x^2+bd^2f^2\right)\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(ac^4d^2f^2x^4+2\,ac^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x, algorit
hm="fricas")
```

```
[Out] integral((b*c^4*d^2*f^2*x^4 + 2*b*c^2*d^2*f^2*x^2 + b*d^2*f^2)*sqrt(I*c*d*x
 + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a*c^4*d^2*f^2*x^4
+ 2*a*c^2*d^2*f^2*x^2 + a*d^2*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int (icdx+d)^{\frac{5}{2}}\left(-icfx+f\right)^{\frac{5}{2}}(a+b\,\mathrm{arcsinh}\,(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{asinh}\,(c\,x))\,\left(d+c\,d\,x\,1\mathrm{i}\right)^{5/2}\,\left(f-c\,f\,x\,1\mathrm{i}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(5/2),x)
[Out] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(5/2)*(f-I*c*f*x)**(5/2)*(a+b*asinh(c*x)),x)
[Out] Timed out
```

3.547 $\int (d+icdx)^{3/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=459

$$\frac{3fx(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{8\left(c^2x^2+1\right)}+\frac{3f(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc\left(c^2x^2+1\right)^{3/2}}-\frac{if\left(c^2x^2+1\right)(}{}$$

```
[Out] 1/5*I*b*f*x*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)-5/16*b*c*
f*x^2*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)+2/15*I*b*c^2*f*
x^3*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)-1/16*b*c^3*f*x^4*
(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)+1/25*I*b*c^4*f*x^5*(d
+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)^(3/2)+1/4*f*x*(d+I*c*d*x)^(3/
2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))+3/8*f*x*(d+I*c*d*x)^(3/2)*(f-I*c*f*
x)^(3/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)-1/5*I*f*(d+I*c*d*x)^(3/2)*(f-I*c*f*
x)^(3/2)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c+3/16*f*(d+I*c*d*x)^(3/2)*(f-I*c*f
*x)^(3/2)*(a+b*arcsinh(c*x))^2/b/c/(c^2*x^2+1)^(3/2)
```

Rubi [A] time = 0.43, antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {5712, 5821, 5684, 5682, 5675, 30, 14, 5717, 194}

$$\frac{3fx(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{8\left(c^2x^2+1\right)}+\frac{3f(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc\left(c^2x^2+1\right)^{3/2}}-\frac{if\left(c^2x^2+1\right)(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] ((I/5)*b*f*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(1 + c^2*x^2)^(3/2) -
 (5*b*c*f*x^2*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(16*(1 + c^2*x^2)^(3
/2)) + (((2*I)/15)*b*c^2*f*x^3*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(1
+ c^2*x^2)^(3/2) - (b*c^3*f*x^4*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(1
6*(1 + c^2*x^2)^(3/2)) + ((I/25)*b*c^4*f*x^5*(d + I*c*d*x)^(3/2)*(f - I*c*f
*x)^(3/2))/(1 + c^2*x^2)^(3/2) + (f*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/
2)*(a + b*ArcSinh[c*x]))/4 + (3*f*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)
*(a + b*ArcSinh[c*x]))/(8*(1 + c^2*x^2)) - ((I/5)*f*(d + I*c*d*x)^(3/2)*(f
- I*c*f*x)^(3/2)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/c + (3*f*(d + I*c*d*x)
^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(16*b*c*(1 + c^2*x^2)^(3
/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)dx = \frac{\left((d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int (f-icfx)\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)dx}{\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{\left((d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int \left(f\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(c$$

$$= \frac{\left(f(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int \left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx$$

$$= \frac{1}{4}fx(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right) - \frac{if(d+icd$$

$$= \frac{1}{4}fx(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right) + \frac{3fx(d+i$$

$$= \frac{ibfx(d+icdx)^{3/2}(f-icfx)^{3/2}}{5\left(1+c^2x^2\right)^{3/2}} - \frac{5bcfx^2(d+icdx)^{3/2}(f-icf}{16\left(1+c^2x^2\right)^{3/2}}$$

Mathematica [A] time = 1.67, size = 683, normalized size = 1.49

$$\frac{3600ad^{3/2}f^{5/2}\sqrt{c^2x^2+1}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right) - 3840iac^2df^2x^2\sqrt{c^2x^2+1}\sqrt{d+icdx}\sqrt{f-ic}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] ((1200*I)*b*c*d*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (1920*I)*a*d*f^
2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 6000*a*c*d*f^2*x*
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (3840*I)*a*c^2*d*f^
2*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 2400*a*c^3*d*
f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (1920*I)*a*
c^4*d*f^2*x^4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 1800*
b*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 - 1200*b*d*f^2*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] - 75*b*d*f^2*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] + 3600*a*d^(3/2)*f^(5/2)
*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f -
 I*c*f*x]] + (200*I)*b*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*Arc
Sinh[c*x]] + 60*b*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*((
-10*I)*Cosh[3*ArcSinh[c*x]] - (2*I)*Cosh[5*ArcSinh[c*x]] + 5*((-4*I)*Sqrt[1
 + c^2*x^2] + 8*Sinh[2*ArcSinh[c*x]] + Sinh[4*ArcSinh[c*x]])) + (24*I)*b*d*
f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[5*ArcSinh[c*x]])/(9600*c*Sqrt[
1 + c^2*x^2])
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(-i\,bc^3df^2x^3+bc^2df^2x^2-i\,bcdf^2x+bdf^2\right)\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(-i\,ac^3df^2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x, algorit
hm="fricas")
```

```
[Out] integral((-I*b*c^3*d*f^2*x^3 + b*c^2*d*f^2*x^2 - I*b*c*d*f^2*x + b*d*f^2)*s
qrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (-I*a*c^
3*d*f^2*x^3 + a*c^2*d*f^2*x^2 - I*a*c*d*f^2*x + a*d*f^2)*sqrt(I*c*d*x + d)*
sqrt(-I*c*f*x + f), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument TypeError: Bad Argument
TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int (icdx+d)^{\frac{3}{2}} \left(-icfx+f\right)^{\frac{5}{2}} (a+b\,\mathrm{arcsinh}\,(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{asinh}\,(c\,x))\,(d+c\,d\,x\,1\mathrm{i})^{3/2}\,\left(f-c\,f\,x\,1\mathrm{i}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(5/2),x)
```

```
[Out] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(3/2)*(f-I*c*f*x)**(5/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

3.548 $\int \sqrt{d+icdx}\,(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=416

$$-\frac{1}{4}c^2f^2x^3\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)+\frac{5f^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)^2}{16bc\sqrt{c^2x^2+1}}-\frac{2if^2\left(c^2x^2+1\right)}{}$$

```
[Out] 3/8*f^2*x*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-1/4*c^2*f^
2*x^3*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-2/3*I*f^2*(c^2
*x^2+1)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c+2/3*I*b*f^
2*x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-3/16*b*c*f^2*x^2*
(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+2/9*I*b*c^2*f^2*x^3*(
d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/16*b*c^3*f^2*x^4*(d+
I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+5/16*f^2*(a+b*arcsinh(c*
x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.57, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {5712, 5821, 5682, 5675, 30, 5717, 5742, 5758}

$$-\frac{1}{4}c^2f^2x^3\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)+\frac{5f^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)^2}{16bc\sqrt{c^2x^2+1}}-\frac{2if^2\left(c^2x^2+1\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (((2*I)/3)*b*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 + c^2*x^2] -
 (3*b*c*f^2*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(16*Sqrt[1 + c^2*x^2])
 + (((2*I)/9)*b*c^2*f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 + c
^2*x^2] + (b*c^3*f^2*x^4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(16*Sqrt[1 +
c^2*x^2]) + (3*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x
]))/8 - (c^2*f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x
]))/4 - (((2*I)/3)*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(1 + c^2*x^2)*(a
 + b*ArcSinh[c*x]))/c + (5*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*A
rcSinh[c*x])^2)/(16*b*c*Sqrt[1 + c^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int \sqrt{d+icdx}\,(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)\,dx = \frac{\left(\sqrt{d+icdx}\,\sqrt{f-icfx}\right)\int (f-icfx)^2\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(\right.}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(\sqrt{d+icdx}\,\sqrt{f-icfx}\right)\int \left(f^2\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)-\right.}{}$$

$$= \frac{\left(f^2\sqrt{d+icdx}\,\sqrt{f-icfx}\right)\int \sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{1}{2}f^2x\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{4}c^2f^2x^3\sqrt{d+}$$

$$= \frac{2ibf^2x\sqrt{d+icdx}\,\sqrt{f-icfx}}{3\sqrt{1+c^2x^2}}-\frac{bcf^2x^2\sqrt{d+icdx}\,\sqrt{f-icfx}}{4\sqrt{1+c^2x^2}}+$$

$$= \frac{2ibf^2x\sqrt{d+icdx}\,\sqrt{f-icfx}}{3\sqrt{1+c^2x^2}}-\frac{3bcf^2x^2\sqrt{d+icdx}\,\sqrt{f-icfx}}{16\sqrt{1+c^2x^2}}$$

Mathematica [A] time = 1.23, size = 565, normalized size = 1.36

$$\frac{720a\sqrt{d}\,f^{5/2}\sqrt{c^2x^2+1}\,\log\left(cdfx+\sqrt{d}\,\sqrt{f}\,\sqrt{d+icdx}\,\sqrt{f-icfx}\right)-768iac^2f^2x^2\sqrt{c^2x^2+1}\,\sqrt{d+icdx}\,\sqrt{f-icfx}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] ((576*I)*b*c*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (768*I)*a*f^2*Sqrt
[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 432*a*c*f^2*x*Sqrt[d +
I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (768*I)*a*c^2*f^2*x^2*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 288*a*c^3*f^2*x^3*Sqrt[d
+ I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 360*b*f^2*Sqrt[d + I*c*d*x
]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 - 144*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I
*c*f*x]*Cosh[2*ArcSinh[c*x]] + 9*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*
Cosh[4*ArcSinh[c*x]] + 720*a*Sqrt[d]*f^(5/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x
+ Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + (64*I)*b*f^2*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]] + 12*b*f^2*Sqrt[d + I*c
*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*((-48*I)*Sqrt[1 + c^2*x^2] - (16*I)*Co
sh[3*ArcSinh[c*x]] + 24*Sinh[2*ArcSinh[c*x]] - 3*Sinh[4*ArcSinh[c*x]]))/(11
52*c*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(bc^2f^2x^2+2i\,bcf^2x-bf^2\right)\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)-\left(ac^2f^2x^2+2i\,acf^2x-af^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(-(b*c^2*f^2*x^2 + 2*I*b*c*f^2*x - b*f^2)*sqrt(I*c*d*x + d)*sqrt(-I
*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) - (a*c^2*f^2*x^2 + 2*I*a*c*f^2*x -
 a*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \left(-icfx+f\right)^{\frac{5}{2}} \left(a+b\,\mathrm{arcsinh}\left(cx\right)\right) \sqrt{icdx+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\sqrt{d+c\,d\,x\,1{}\mathrm{i}}\,{\left(f-c\,f\,x\,1{}\mathrm{i}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(5/2),x)
```

```
[Out] int((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))*(d+I*c*d*x)**(1/2),x)
```

```
[Out] Timed out
```

3.549 $\int \frac{(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+icdx}}\,dx$

Optimal. Leaf size=381

$$\frac{5f^3\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{icf^3x^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{3f^3x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\,\sqrt{f-icfx}}-11i$$

```
[Out] -11/3*I*f^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^
(1/2)-3/2*f^3*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x
)^(1/2)+1/3*I*c*f^3*x^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f
-I*c*f*x)^(1/2)+11/3*I*b*f^3*x*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f
*x)^(1/2)+3/4*b*c*f^3*x^2*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(
1/2)-1/9*I*b*c^2*f^3*x^3*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1
/2)+5/4*f^3*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f
-I*c*f*x)^(1/2)
```

Rubi [A] time = 0.63, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5712, 5821, 5675, 5717, 8, 5758, 30}

$$\frac{5f^3\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{icf^3x^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3\sqrt{d+icdx}\,\sqrt{f-icfx}}-\frac{3f^3x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\,\sqrt{f-icfx}}-11i$$

Antiderivative was successfully verified.

```
[In] Int[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/Sqrt[d + I*c*d*x],x]
```

```
[Out] (((11*I)/3)*b*f^3*x*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]
) + (3*b*c*f^3*x^2*Sqrt[1 + c^2*x^2])/(4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
]) - ((I/9)*b*c^2*f^3*x^3*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*
c*f*x]) - (((11*I)/3)*f^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*Sqrt[d + I
*c*d*x]*Sqrt[f - I*c*f*x]) - (3*f^3*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(
2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + ((I/3)*c*f^3*x^2*(1 + c^2*x^2)*(a
+ b*ArcSinh[c*x]))/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (5*f^3*Sqrt[1 +
c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]
)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
```

```
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int \frac{(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+icdx}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{(f-icfx)^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\int \left(\frac{f^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}} - \frac{3icf^3x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}} - \frac{3c^2f^3x^2\left(a+b\sinh^{-1}(c\right.}{\sqrt{1+c^2x^2}}\right.}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= \frac{\left(f^3\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\,\sqrt{f-icfx}} - \frac{\left(3icf^3\sqrt{1+c^2x^2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right.}{\sqrt{1+c^2x^2}}}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= -\frac{3if^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\,\sqrt{f-icfx}} - \frac{3f^3x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(c\right.}{2\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= \frac{3ibf^3x\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{3bcf^3x^2\sqrt{1+c^2x^2}}{4\sqrt{d+icdx}\,\sqrt{f-icfx}} - \frac{ibc^2f^3x^3\sqrt{1+c^2x}}{9\sqrt{d+icdx}\,\sqrt{f-i}}$$

$$= \frac{11ibf^3x\sqrt{1+c^2x^2}}{3\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{3bcf^3x^2\sqrt{1+c^2x^2}}{4\sqrt{d+icdx}\,\sqrt{f-icfx}} - \frac{ibc^2f^3x^3\sqrt{1+c^2}}{9\sqrt{d+icdx}\,\sqrt{f-}}$$

Mathematica [A] time = 1.71, size = 465, normalized size = 1.22

$$\frac{180a\sqrt{d}f^{5/2}\sqrt{c^2x^2+1}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)+24iac^2f^2x^2\sqrt{c^2x^2+1}\sqrt{d+icdx}\sqrt{f-icfx}-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/Sqrt[d + I*c*d*x],x]
```

```
[Out] ((264*I)*b*c*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (8*I)*b*c^3*f^2*x^
3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (264*I)*a*f^2*Sqrt[d + I*c*d*x]*Sqr
t[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 108*a*c*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f -
 I*c*f*x]*Sqrt[1 + c^2*x^2] + (24*I)*a*c^2*f^2*x^2*Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x]*Sqrt[1 + c^2*x^2] + 90*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
]*ArcSinh[c*x]^2 + 27*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcS
inh[c*x]] - 6*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*(9*(5*
I + 2*c*x)*Sqrt[1 + c^2*x^2] - I*Cosh[3*ArcSinh[c*x]]) + 180*a*Sqrt[d]*f^(5
/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]])/(72*c*d*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(ibc^2f^2x^2-2bcf^2x-ibf^2\right)\sqrt{icdx+d}\sqrt{-icfx+f}\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(iac^2f^2x^2-2acf^2x-ia\right.}{cdx-id}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(((I*b*c^2*f^2*x^2 - 2*b*c*f^2*x - I*b*f^2)*sqrt(I*c*d*x + d)*sqrt(
-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (I*a*c^2*f^2*x^2 - 2*a*c*f^2*x
 - I*a*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c*d*x - I*d), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{\left(-icfx+f\right)^{\frac{5}{2}}\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)}{\sqrt{icdx+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))\,\left(f-c\,f\,x\,1\mathrm{i}\right)^{5/2}}{\sqrt{d+c\,d\,x\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(1/2),x)
```

```
[Out] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))/(d+I*c*d*x)**(1/2),x)
```

```
[Out] Timed out
```

3.550 $\int \frac{(f-icfx)^{5/2}(a+b\sinh^{-1}(cx))}{(d+icdx)^{3/2}}\,dx$

Optimal. Leaf size=518

$$\frac{5if^4(1-icx)\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{15if^4\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2if^4(1-icx)^3\left(c^2x^2+1\right)(a+}{c(d+icdx)^{3/2}(f-icf}$$

```
[Out] -3/2*I*b*f^4*x*(c^2*x^2+1)^(3/2)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+b*c*f^
4*x^2*(c^2*x^2+1)^(3/2)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+5/4*b*f^4*(1-I*
c*x)^2*(c^2*x^2+1)^(3/2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+15/4*b*f^4*(
c^2*x^2+1)^(3/2)*arcsinh(c*x)^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+2*I*f
^4*(1-I*c*x)^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*
x)^(3/2)+15/2*I*f^4*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f
-I*c*f*x)^(3/2)+5/2*I*f^4*(1-I*c*x)*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/c/(d+I
*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-15/2*f^4*(c^2*x^2+1)^(3/2)*arcsinh(c*x)*(a+
b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-8*b*f^4*(c^2*x^2+1)^(
3/2)*ln(I-c*x)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A] time = 0.42, antiderivative size = 518, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {5712, 669, 671, 641, 215, 5819, 627, 43, 5675}

$$\frac{5if^4(1-icx)\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{15if^4\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2if^4(1-icx)^3\left(c^2x^2+1\right)(a+}{c(d+icdx)^{3/2}(f-icf}$$

Antiderivative was successfully verified.

```
[In] Int[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(3/2),x]
```

```
[Out] (((-3*I)/2)*b*f^4*x*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)
^(3/2)) + (b*c*f^4*x^2*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f
*x)^(3/2)) + (5*b*f^4*(1 - I*c*x)^2*(1 + c^2*x^2)^(3/2))/(4*c*(d + I*c*d*x)
^(3/2)*(f - I*c*f*x)^(3/2)) + (15*b*f^4*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]^2)
/(4*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + ((2*I)*f^4*(1 - I*c*x)^3*(
1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/
2)) + (((15*I)/2)*f^4*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2)) + (((5*I)/2)*f^4*(1 - I*c*x)*(1 + c^2*x^2)^2*(
a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (15*f^4*
(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*(a + b*ArcSinh[c*x]))/(2*c*(d + I*c*d*x)^(
3/2)*(f - I*c*f*x)^(3/2)) - (8*b*f^4*(1 + c^2*x^2)^(3/2)*Log[I - c*x])/(c*(
d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
```

```
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$\int \frac{(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{3/2}}\,dx = \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{(f-icfx)^4\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{2if^4(1-icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{15if^4\left(1+c^2x^2\right)^2\left(a+b\,\mathrm{si}\right.}{2c(d+icdx)^{3/2}(f-ic}$$

$$= -\frac{15ibf^4x\left(1+c^2x^2\right)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{5bf^4(1-icx)^2\left(1+c^2x^2\right)^{3/2}}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2if^4(1-ic}{c(}$$

$$= -\frac{15ibf^4x\left(1+c^2x^2\right)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{5bf^4(1-icx)^2\left(1+c^2x^2\right)^{3/2}}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{15bf^4\left(1+\right.}{4c(d+ic}$$

$$= -\frac{15ibf^4x\left(1+c^2x^2\right)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{5bf^4(1-icx)^2\left(1+c^2x^2\right)^{3/2}}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{15bf^4\left(1+\right.}{4c(d+ic}$$

$$= -\frac{3ibf^4x\left(1+c^2x^2\right)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{bcf^4x^2\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{5bf^4(1-icx)}{4c(d+icdx)^3}$$

Mathematica [A] time = 3.97, size = 779, normalized size = 1.50

$$\frac{\frac{4af^2\left(c^2x^2+7icx+24\right)\sqrt{d+icdx}\,\sqrt{f-icfx}}{d^2(cx-i)} - \frac{60af^{5/2}\log\left(cdfx+\sqrt{d}\,\sqrt{f}\,\sqrt{d+icdx}\,\sqrt{f-icfx}\right)}{d^{3/2}} + \frac{bf^2\sqrt{d+icdx}\,\sqrt{f-icfx}\left(2\sinh^{-1}(cx)\left(\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right.\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(3/2),x]
```

```
[Out] ((4*a*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(24 + (7*I)*c*x + c^2*x^2))/(
d^2*(-I + c*x)) - (60*a*f^(5/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*
d*x]*Sqrt[f - I*c*f*x]])/d^(3/2) - (4*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*
f*x]*(ArcSinh[c*x]*((-4*I)*Cosh[ArcSinh[c*x]/2] - 4*Sinh[ArcSinh[c*x]/2]) +
 ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + 2*((4*I)*
ArcTan[Tanh[ArcSinh[c*x]/2]] + Log[1 + c^2*x^2])*(Cosh[ArcSinh[c*x]/2] + I*
Sinh[ArcSinh[c*x]/2])))/(d^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Si
nh[ArcSinh[c*x]/2])) + (16*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-(Arc
Sinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) + (c*x - 4*Arc
Tan[Coth[ArcSinh[c*x]/2]] - I*Log[1 + c^2*x^2])*((-I)*Cosh[ArcSinh[c*x]/2]
+ Sinh[ArcSinh[c*x]/2]) + ArcSinh[c*x]*(I*(2 + Sqrt[1 + c^2*x^2])*Cosh[ArcS
inh[c*x]/2] - (-2 + Sqrt[1 + c^2*x^2])*Sinh[ArcSinh[c*x]/2])))/(d^2*Sqrt[1
+ c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) + (b*f^2*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*(-10*ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] + I
*Sinh[ArcSinh[c*x]/2]) - (Cosh[2*ArcSinh[c*x]] + 8*((2*I)*c*x + (4*I)*ArcTa
n[Tanh[ArcSinh[c*x]/2]] + Log[1 + c^2*x^2]))*(Cosh[ArcSinh[c*x]/2] + I*Sinh
[ArcSinh[c*x]/2]) + 2*ArcSinh[c*x]*(Sinh[ArcSinh[c*x]/2]*(8 - 8*Sqrt[1 + c^
2*x^2] + I*Sinh[2*ArcSinh[c*x]]) + Cosh[ArcSinh[c*x]/2]*((8*I)*(1 + Sqrt[1
+ c^2*x^2]) + Sinh[2*ArcSinh[c*x]]))))/(d^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh
[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/(8*c)
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(bc^2f^2x^2+2i\,bcf^2x-bf^2\right)\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(ac^2f^2x^2+2i\,acf^2x-af^2\right)}{c^2d^2x^2-2i\,cd^2x-d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral(((b*c^2*f^2*x^2 + 2*I*b*c*f^2*x - b*f^2)*sqrt(I*c*d*x + d)*sqrt(-I
*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a*c^2*f^2*x^2 + 2*I*a*c*f^2*x -
 a*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^2*d^2*x^2 - 2*I*c*d^2*x -
d^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{\left(-icfx+f\right)^{\frac{5}{2}} \left(a+b \operatorname{arcsinh}\left(cx\right)\right)}{\left(icdx+d\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x)
```

```
[Out] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(\frac{c^2f^3x^3}{\sqrt{c^2dfx^2+df}\,d}+\frac{8i\,cf^3x^2}{\sqrt{c^2dfx^2+df}\,d}+\frac{17\,f^3x}{\sqrt{c^2dfx^2+df}\,d}-\frac{15\,f^3\operatorname{arsinh}\left(cx\right)}{\sqrt{df}\,cd}+\frac{24i\,f^3}{\sqrt{c^2dfx^2+df}\,cd}\right)a+b\int \frac{(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x, algorit
hm="maxima")
```

```
[Out] 1/2*(c^2*f^3*x^3/(sqrt(c^2*d*f*x^2 + d*f)*d) + 8*I*c*f^3*x^2/(sqrt(c^2*d*f*
x^2 + d*f)*d) + 17*f^3*x/(sqrt(c^2*d*f*x^2 + d*f)*d) - 15*f^3*arcsinh(c*x)/
(sqrt(d*f)*c*d) + 24*I*f^3/(sqrt(c^2*d*f*x^2 + d*f)*c*d))*a + b*integrate((
-I*c*f*x + f)^(5/2)*log(c*x + sqrt(c^2*x^2 + 1))/(I*c*d*x + d)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(f-c\,f\,x\,1{}\mathrm{i}\right)}^{5/2}}{{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(3/2),x)
```

```
[Out] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(3/2), x)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))/(d+I*c*d*x)**(3/2),x)
```

```
[Out] Timed out
```

3.551 $\int \frac{(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{5/2}}\,dx$

Optimal. Leaf size=472

$$-\frac{5if^5\left(c^2x^2+1\right)^3\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{10if^5(1-icx)^2\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2if^5(1-icx)^4\left(c^2x^2+1\right)}{3c(d+icdx)^{5/2}(f}$$

```
[Out] I*b*f^5*x*(c^2*x^2+1)^(5/2)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+8/3*I*b*f^5
*(c^2*x^2+1)^(5/2)/c/(I-c*x)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-5/2*b*f^5*
(c^2*x^2+1)^(5/2)*arcsinh(c*x)^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+2/3*
I*f^5*(1-I*c*x)^4*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c
*f*x)^(5/2)-10/3*I*f^5*(1-I*c*x)^2*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/c/(d+I*
c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-5*I*f^5*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))/c/(
d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+5*f^5*(c^2*x^2+1)^(5/2)*arcsinh(c*x)*(a+
b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+28/3*b*f^5*(c^2*x^2+1
)^(5/2)*ln(I-c*x)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A] time = 0.44, antiderivative size = 472, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {5712, 669, 641, 215, 5819, 627, 43, 5675}

$$-\frac{5if^5\left(c^2x^2+1\right)^3\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{10if^5(1-icx)^2\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2if^5(1-icx)^4\left(c^2x^2+1\right)}{3c(d+icdx)^{5/2}(f}$$

Antiderivative was successfully verified.

```
[In] Int[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(5/2),x]
```

```
[Out] (I*b*f^5*x*(1 + c^2*x^2)^(5/2))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) +
 (((8*I)/3)*b*f^5*(1 + c^2*x^2)^(5/2))/(c*(I - c*x)*(d + I*c*d*x)^(5/2)*(f
- I*c*f*x)^(5/2)) - (5*b*f^5*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x]^2)/(2*c*(d +
I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((2*I)/3)*f^5*(1 - I*c*x)^4*(1 + c^2
*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (
((10*I)/3)*f^5*(1 - I*c*x)^2*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(c*(d +
I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - ((5*I)*f^5*(1 + c^2*x^2)^3*(a + b*Arc
Sinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (5*f^5*(1 + c^2*x
^2)^(5/2)*ArcSinh[c*x]*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*
c*f*x)^(5/2)) + (28*b*f^5*(1 + c^2*x^2)^(5/2)*Log[I - c*x])/(3*c*(d + I*c*d
*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$\int \frac{(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{5/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(f-icfx)^5\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{2if^5(1-icx)^4\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{10if^5(1-icx)^2\left(1+c^2x\right.}{3c(d+icdx)^{5/}}$$

$$= \frac{5ibf^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2if^5(1-icx)^4\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx\right.}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{5ibf^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{5bf^5\left(1+c^2x^2\right)^{5/2}\sinh^{-1}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2if^5(1-}{}$$

$$= \frac{5ibf^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{5bf^5\left(1+c^2x^2\right)^{5/2}\sinh^{-1}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2if^5(1-}{}$$

$$= \frac{ibf^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{8ibf^5\left(1+c^2x^2\right)^{5/2}}{3c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{5bf}{2c}$$

Mathematica [B] time = 7.43, size = 1005, normalized size = 2.13

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(5/2),x]
```

```
[Out] (((-4*I)*a*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-23 - (34*I)*c*x + 3*c^
2*x^2))/(d^3*(-I + c*x)^2) + (60*a*f^(5/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]])/d^(5/2) - (2*b*f^2*Sqrt[d + I*c*d*x]*Sq
rt[f - I*c*f*x]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])*(Cosh[(3*Ar
cSinh[c*x])/2]*((-14 + (3*I)*ArcSinh[c*x])*ArcSinh[c*x] - 28*ArcTan[Tanh[Ar
cSinh[c*x]/2]] + (7*I)*Log[1 + c^2*x^2]) + Cosh[ArcSinh[c*x]/2]*(84*ArcTan[
Tanh[ArcSinh[c*x]/2]] - I*(8 - (6*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 + 21*L
og[1 + c^2*x^2])) + 2*(4 - (4*I)*ArcSinh[c*x] + 6*ArcSinh[c*x]^2 + (56*I)*A
rcTan[Tanh[ArcSinh[c*x]/2]] + 14*Log[1 + c^2*x^2] + Sqrt[1 + c^2*x^2]*(ArcS
inh[c*x]*(-14*I + 3*ArcSinh[c*x]) + (28*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 7
*Log[1 + c^2*x^2]))*Sinh[ArcSinh[c*x]/2]))/(d^3*(I + c*x)*(Cosh[ArcSinh[c*x
]/2] + I*Sinh[ArcSinh[c*x]/2])^4) + ((2*I)*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f -
 I*c*f*x]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])*((-I)*Cosh[(3*Arc
Sinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]/2]] - (I/2)*Log[1
+ c^2*x^2]) + Cosh[ArcSinh[c*x]/2]*(4 + (3*I)*ArcSinh[c*x] - (6*I)*ArcTan[C
oth[ArcSinh[c*x]/2]] + (3*Log[1 + c^2*x^2])/2) + 2*((2 + Sqrt[1 + c^2*x^2])
*ArcSinh[c*x] + 2*(2 + Sqrt[1 + c^2*x^2])*ArcTan[Coth[ArcSinh[c*x]/2]] + (I
/2)*(4 + (2 + Sqrt[1 + c^2*x^2])*Log[1 + c^2*x^2]))*Sinh[ArcSinh[c*x]/2]))/
(d^3*(I + c*x)*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^4) + (b*f^2*
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(I*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[
c*x]/2])*(2*(4 + (6*I)*c*x - 6*c^2*x^2 + 52*(-I + c*x)*ArcTan[Coth[ArcSinh[
c*x]/2]] + 13*(1 + I*c*x)*Log[1 + c^2*x^2])*(Cosh[ArcSinh[c*x]/2] + I*Sinh[
ArcSinh[c*x]/2]) + 18*ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh
[c*x]/2])^3 + ArcSinh[c*x]*((-24*I)*Cosh[ArcSinh[c*x]/2] - (35*I)*Cosh[(3*A
rcSinh[c*x])/2] + (3*I)*Cosh[(5*ArcSinh[c*x])/2] - 24*Sinh[ArcSinh[c*x]/2]
+ 35*Sinh[(3*ArcSinh[c*x])/2] + 3*Sinh[(5*ArcSinh[c*x])/2])))/(d^3*(I + c*x
)*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^4))/(12*c)
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-i\,bc^2f^2x^2+2\,bcf^2x+i\,bf^2\right)\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(-i\,ac^2f^2x^2+2\,acf^2x+\right.}{c^3d^3x^3-3i\,c^2d^3x^2-3\,cd^3x+i\,d^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral(((-I*b*c^2*f^2*x^2 + 2*b*c*f^2*x + I*b*f^2)*sqrt(I*c*d*x + d)*sqrt
(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (-I*a*c^2*f^2*x^2 + 2*a*c*f^2
*x + I*a*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^3*d^3*x^3 - 3*I*c^2*
d^3*x^2 - 3*c*d^3*x + I*d^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{\left(-icfx+f\right)^{\frac{5}{2}}\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)}{\left(icdx+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x)
```

```
[Out] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}\left(\frac{3i\left(c^2dfx^2+df\right)^{\frac{5}{2}}}{c^5d^5x^4-4i\,c^4d^5x^3-6\,c^3d^5x^2+4i\,c^2d^5x+cd^5}-\frac{15i\left(c^2dfx^2+df\right)^{\frac{3}{2}}f}{-3i\,c^4d^4x^3-9\,c^3d^4x^2+9i\,c^2d^4x+3\,cd^4}+\frac{10i\sqrt{c^2dfx^2+}}{c^3d^3x^2-2i\,c^2d^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x, algorit
hm="maxima")
```

```
[Out] -1/3*(3*I*(c^2*d*f*x^2 + d*f)^(5/2)/(c^5*d^5*x^4 - 4*I*c^4*d^5*x^3 - 6*c^3*
d^5*x^2 + 4*I*c^2*d^5*x + c*d^5) - 15*I*(c^2*d*f*x^2 + d*f)^(3/2)*f/(-3*I*c
^4*d^4*x^3 - 9*c^3*d^4*x^2 + 9*I*c^2*d^4*x + 3*c*d^4) + 10*I*sqrt(c^2*d*f*x
^2 + d*f)*f^2/(c^3*d^3*x^2 - 2*I*c^2*d^3*x - c*d^3) + 105*I*sqrt(c^2*d*f*x^
2 + d*f)*f^2/(3*I*c^2*d^3*x + 3*c*d^3) - 15*f^3*arcsinh(c*x)/(c*d^3*sqrt(f/
d)))*a + b*integrate((-I*c*f*x + f)^(5/2)*log(c*x + sqrt(c^2*x^2 + 1))/(I*c
*d*x + d)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))\,\left(f-c\,f\,x\,1\mathrm{i}\right)^{5/2}}{(d+c\,d\,x\,1\mathrm{i})^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(5/2),x)
[Out] int(((a + b*asinh(c*x))*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))/(d+I*c*d*x)**(5/2),x)
[Out] Timed out
```

3.552 $\int \frac{(d+icdx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{f-icfx}}\,dx$

Optimal. Leaf size=381

$$\frac{5d^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{icd^3x^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{3d^3x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{11i\ldots}{\ldots}$$

```
[Out] 11/3*I*d^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(
1/2)-3/2*d^3*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)
^(1/2)-1/3*I*c*d^3*x^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-
I*c*f*x)^(1/2)-11/3*I*b*d^3*x*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*
x)^(1/2)+3/4*b*c*d^3*x^2*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1
/2)+1/9*I*b*c^2*d^3*x^3*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/
2)+5/4*d^3*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-
I*c*f*x)^(1/2)
```

Rubi [A] time = 0.63, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5712, 5821, 5675, 5717, 8, 5758, 30}

$$\frac{5d^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{icd^3x^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{3d^3x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{11i\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x]))/Sqrt[f - I*c*f*x],x]
```

```
[Out] (((-11*I)/3)*b*d^3*x*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
]) + (3*b*c*d^3*x^2*Sqrt[1 + c^2*x^2])/(4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
x]) + ((I/9)*b*c^2*d^3*x^3*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I
*c*f*x]) + (((11*I)/3)*d^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*Sqrt[d +
I*c*d*x]*Sqrt[f - I*c*f*x]) - (3*d^3*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/
(2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - ((I/3)*c*d^3*x^2*(1 + c^2*x^2)*(a
 + b*ArcSinh[c*x]))/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (5*d^3*Sqrt[1 +
 c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
```

```
+ e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int \frac{(d+icdx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{f-icfx}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{(d+icdx)^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\int \left(\frac{d^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}} + \frac{3icd^3x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}} - \frac{3c^2d^3x^2\left(a+b\sinh^{-1}(c\right.}{\sqrt{1+c^2x^2}}\right.}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= \frac{\left(d^3\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{\left(3icd^3\sqrt{1+c^2x^2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= \frac{3id^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\,\sqrt{f-icfx}} - \frac{3d^3x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= -\frac{3ibd^3x\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{3bcd^3x^2\sqrt{1+c^2x^2}}{4\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{ibc^2d^3x^3\sqrt{1+c^2}}{9\sqrt{d+icdx}\,\sqrt{f-}}$$

$$= -\frac{11ibd^3x\sqrt{1+c^2x^2}}{3\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{3bcd^3x^2\sqrt{1+c^2x^2}}{4\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{ibc^2d^3x^3\sqrt{1+}}{9\sqrt{d+icdx}\,\sqrt{f}}$$

Mathematica [A] time = 1.68, size = 465, normalized size = 1.22

$$180ad^{5/2}\sqrt{f}\sqrt{c^2x^2+1}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)-24iac^2d^2x^2\sqrt{c^2x^2+1}\sqrt{d+icdx}\sqrt{f-icfx}-$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x]))/Sqrt[f - I*c*f*x],x]
```

```
[Out] ((-264*I)*b*c*d^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (8*I)*b*c^3*d^2*x
^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (264*I)*a*d^2*Sqrt[d + I*c*d*x]*Sq
rt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 108*a*c*d^2*x*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x]*Sqrt[1 + c^2*x^2] - (24*I)*a*c^2*d^2*x^2*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 90*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
x]*ArcSinh[c*x]^2 + 27*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*Arc
Sinh[c*x]] - 6*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*(9*(-
5*I + 2*c*x)*Sqrt[1 + c^2*x^2] + I*Cosh[3*ArcSinh[c*x]]) + 180*a*d^(5/2)*Sq
rt[f]*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqr
t[f - I*c*f*x]])/(72*c*f*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-i\,bc^2d^2x^2-2\,bcd^2x+i\,bd^2\right)\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(-i\,ac^2d^2x^2-2\,acd^2x+i\right.}{cfx+i\,f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(((-I*b*c^2*d^2*x^2 - 2*b*c*d^2*x + I*b*d^2)*sqrt(I*c*d*x + d)*sqrt
(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (-I*a*c^2*d^2*x^2 - 2*a*c*d^2
*x + I*a*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c*f*x + I*f), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{(icdx+d)^{\frac{5}{2}}\,(a+b\,\mathrm{arcsinh}\,(cx))}{\sqrt{-icfx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{asinh}(c x)) (d + c d x 1\mathrm{i})^{5/2}}{\sqrt{f - c f x 1\mathrm{i}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(1/2),x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))/(f-I*c*f*x)**(1/2),x)
```

```
[Out] Timed out
```

3.553 $\int \frac{(d+icdx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{f-icfx}}\,dx$

Optimal. Leaf size=266

$$\frac{3d^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{d^2x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{2id^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{bcd}{4\sqrt{d+}}$$

```
[Out] 2*I*d^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2
)-1/2*d^2*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1
/2)-2*I*b*d^2*x*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/4*b
*c*d^2*x^2*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+3/4*d^2*(a
+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2
)
```

Rubi [A] time = 0.47, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5712, 5821, 5675, 5717, 8, 5758, 30}

$$\frac{3d^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{d^2x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{2id^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{bcd}{4\sqrt{d+}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x]))/Sqrt[f - I*c*f*x],x]
```

```
[Out] ((-2*I)*b*d^2*x*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) +
(b*c*d^2*x^2*Sqrt[1 + c^2*x^2])/(4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (
(2*I)*d^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*Sqrt[d + I*c*d*x]*Sqrt[f -
 I*c*f*x]) - (d^2*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(2*Sqrt[d + I*c*d*x
]*Sqrt[f - I*c*f*x]) + (3*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(4*
b*c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int \frac{(d+icdx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{f-icfx}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{(d+icdx)^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\int \left(\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}} + \frac{2icd^2x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}} - \frac{c^2d^2x^2\left(a+b\sinh^{-1}(cx\right.}{\sqrt{1+c^2x^2}}\right.}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\left(d^2\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{\left(2icd^2\sqrt{1+c^2x^2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right.}{\sqrt{1+c^2x^2}}}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{2id^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{d^2x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= -\frac{2ibd^2x\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{bcd^2x^2\sqrt{1+c^2x^2}}{4\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2id^2\left(1+c^2x^2\right)\left(a\right.}{c\sqrt{d+icdx}}$$

Mathematica [A] time = 1.03, size = 344, normalized size = 1.29

$$12ad^{3/2}\sqrt{f}\sqrt{c^2x^2+1}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)+16iad\sqrt{c^2x^2+1}\sqrt{d+icdx}\sqrt{f-icfx}-4a$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x]))/Sqrt[f - I*c*f*x],x]
```

```
[Out] ((-16*I)*b*c*d*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (16*I)*a*d*Sqrt[d +
I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 4*a*c*d*x*Sqrt[d + I*c*d*x]*
```

```
Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 4*b*d*(-4*I + c*x)*Sqrt[d + I*c*d*x]*
Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + 6*b*d*Sqrt[d + I*c*d*x]*
Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 + b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*
Cosh[2*ArcSinh[c*x]] + 12*a*d^(3/2)*Sqrt[f]*Sqrt[1 + c^2*x^2]*Log[c*d*f*x +
 Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]])/(8*c*f*Sqrt[1 + c^2*
x^2])
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(bcdx - i\,bd)\sqrt{i\,cdx + d}\,\sqrt{-i\,cfx + f}\,\log\left(cx + \sqrt{c^2x^2 + 1}\right) + (acdx - i\,ad)\sqrt{i\,cdx + d}\,\sqrt{-i\,cfx + f}}{cfx + i\,f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(-((b*c*d*x - I*b*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
 sqrt(c^2*x^2 + 1)) + (a*c*d*x - I*a*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f
))/(c*f*x + I*f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(i\,cdx + d)^{\frac{3}{2}}(b\,\mathrm{arsinh}\,(cx) + a)}{\sqrt{-i\,cfx + f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((I*c*d*x + d)^(3/2)*(b*arcsinh(c*x) + a)/sqrt(-I*c*f*x + f), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{(icdx + d)^{\frac{3}{2}}\,(a + b\,\mathrm{arcsinh}\,(cx))}{\sqrt{-icfx + f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{asinh}\,(c\,x))\,\left(d + c\,d\,x\,1\mathrm{i}\right)^{3/2}}{\sqrt{f - c\,f\,x\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(1/2),x)
[Out] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(id\,(cx-i))^{\frac{3}{2}}\,(a+b\operatorname{asinh}(cx))}{\sqrt{-if\,(cx+i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))/(f-I*c*f*x)**(1/2),x)
[Out] Integral((I*d*(c*x - I))**(3/2)*(a + b*asinh(c*x))/sqrt(-I*f*(c*x + I)), x)
```

3.554 $\int \frac{\sqrt{d+icdx}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{f-icfx}}\,dx$

Optimal. Leaf size=158

$$\frac{d\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{id\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{ibdx\sqrt{c^2x^2+1}}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

```
[Out] I*d*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-I*
b*d*x*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/2*d*(a+b*arcs
inh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5712, 5821, 5675, 5717, 8}

$$\frac{d\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{id\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{ibdx\sqrt{c^2x^2+1}}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x]))/Sqrt[f - I*c*f*x],x]
```

```
[Out] ((-I)*b*d*x*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (I*d
*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]
) + (d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*Sqrt[d + I*c*d*x]*S
qrt[f - I*c*f*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
```

```
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int \frac{\sqrt{d+icdx}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{f-icfx}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{(d+icdx)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\int \left(\frac{d\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}+\frac{icdx\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\right)dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\left(d\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{\left(icd\sqrt{1+c^2x^2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{id\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{d\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{(i}{\sqrt{}}$$

$$= -\frac{ibdx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{id\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{d\sqrt{1+c^2x}}{2bc\sqrt{d}}$$

Mathematica [A] time = 0.57, size = 227, normalized size = 1.44

$$\frac{-2i\sqrt{d+icdx}\sqrt{f-icfx}\left(bcx-a\sqrt{c^2x^2+1}\right)+2a\sqrt{d}\sqrt{f}\sqrt{c^2x^2+1}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right.}{2cf\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x]))/Sqrt[f - I*c*f*x],x]
```

```
[Out] ((-2*I)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(b*c*x - a*Sqrt[1 + c^2*x^2]) +
 (2*I)*b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]
 + b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 + 2*a*Sqrt[d]*Sqrt[
f]*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x]])/(2*c*f*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{i\sqrt{icdx+d}\sqrt{-icfx+f}\,b\log\left(cx+\sqrt{c^2x^2+1}\right)+i\sqrt{icdx+d}\sqrt{-icfx+f}\,a}{cfx+if},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2 +
 1)) + I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a)/(c*f*x + I*f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{icdx+d}\,(b\,\text{arsinh}\,(cx)+a)}{\sqrt{-icfx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(sqrt(I*c*d*x + d)*(b*arcsinh(c*x) + a)/sqrt(-I*c*f*x + f), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))\,\sqrt{icdx+d}}{\sqrt{-icfx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\frac{d\,\mathrm{arsinh}\,(cx)}{cf\sqrt{\frac{d}{f}}}+\frac{i\sqrt{c^2dfx^2+df}}{cf}\right)+b\int \frac{\sqrt{i\,cdx+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)}{\sqrt{-i\,cfx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] a*(d*arcsinh(c*x)/(c*f*sqrt(d/f)) + I*sqrt(c^2*d*f*x^2 + d*f)/(c*f)) + b*in
tegrate(sqrt(I*c*d*x + d)*log(c*x + sqrt(c^2*x^2 + 1))/sqrt(-I*c*f*x + f),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))\,\sqrt{d+c\,d\,x\,1\mathrm{i}}}{\sqrt{f-c\,f\,x\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(1/2),x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{id\,(cx-i)}\,\left(a+b\,\mathrm{asinh}\,(cx)\right)}{\sqrt{-if\,(cx+i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(1/2),x)
```

```
[Out] Integral(sqrt(I*d*(c*x - I))*(a + b*asinh(c*x))/sqrt(-I*f*(c*x + I)), x)
```

3.555 $\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx$

Optimal. Leaf size=59

$$\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

```
[Out] 1/2*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x
)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {5712, 5675}

$$\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x])
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx &= \frac{\sqrt{1+c^2x^2}\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}\\ &= \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}\end{aligned}$$

Mathematica [A] time = 0.45, size = 113, normalized size = 1.92

$$\frac{a\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)}{c\sqrt{d}\sqrt{f}}+\frac{b\sqrt{c^2x^2+1}\sinh^{-1}(cx)^2}{2c\sqrt{d+icdx}\sqrt{f-icfx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (b*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2)/(2*c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
x]) + (a*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]]
)/(c*Sqrt[d]*Sqrt[f])
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{icdx+d}\,\sqrt{-icfx+f}\,b\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{icdx+d}\,\sqrt{-icfx+f}\,a}{c^2dfx^2+df},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2 + 1
)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a)/(c^2*d*f*x^2 + d*f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{b\,\mathrm{arsinh}\,(cx)+a}{\sqrt{icdx+d}\,\sqrt{-icfx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int\frac{a+b\,\mathrm{arcsinh}\,(cx)}{\sqrt{icdx+d}\,\sqrt{-icfx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
```

maxima [A] time = 0.51, size = 32, normalized size = 0.54

$$\frac{b\,\mathrm{arsinh}\,(cx)^2}{2\,\sqrt{df}\,c}+\frac{a\,\mathrm{arsinh}\,(cx)}{\sqrt{df}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] 1/2*b*arcsinh(c*x)^2/(sqrt(d*f)*c) + a*arcsinh(c*x)/(sqrt(d*f)*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{a+b\,\mathrm{asinh}\,(c\,x)}{\sqrt{d+c\,d\,x\,1{}\mathrm{i}}\,\sqrt{f-c\,f\,x\,1{}\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(1/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asinh}(cx)}{\sqrt{id(cx - i)}\sqrt{-if(cx + i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(1/2),x)
[Out] Integral((a + b*asinh(c*x))/(sqrt(I*d*(c*x - I))*sqrt(-I*f*(c*x + I))), x)
```

3.556 $\int \frac{a+b\sinh^{-1}(cx)}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx$

Optimal. Leaf size=111

$$\frac{f(cx+i)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{bf\left(c^2x^2+1\right)^{3/2}\log(-cx+i)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[Out] f*(I+c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3
/2)-b*f*(c^2*x^2+1)^(3/2)*ln(I-c*x)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A] time = 0.24, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {5712, 637, 5819, 12, 627, 31}

$$\frac{f(cx+i)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{bf\left(c^2x^2+1\right)^{3/2}\log(-cx+i)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (f*(I + c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(3/2)*(f
- I*c*f*x)^(3/2)) - (b*f*(1 + c^2*x^2)^(3/2)*Log[I - c*x])/(c*(d + I*c*d*x)
^(3/2)*(f - I*c*f*x)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
```

```
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\sinh^{-1}(cx)}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx &= \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{(f-icfx)\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&= \frac{f(i+cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(bc\left(1+c^2x^2\right)^{3/2}\right)\int \frac{f(i+cx)}{c\left(1+c^2x^2\right)}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&= \frac{f(i+cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(bf\left(1+c^2x^2\right)^{3/2}\right)\int \frac{i+cx}{1+c^2x^2}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&= \frac{f(i+cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(bf\left(1+c^2x^2\right)^{3/2}\right)\int \frac{1}{-i+cx}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&= \frac{f(i+cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{bf\left(1+c^2x^2\right)^{3/2}\log(i-cx)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.38, size = 113, normalized size = 1.02

$$\frac{\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a\sqrt{c^2x^2+1}\,+b\sqrt{c^2x^2+1}\,\sinh^{-1}(cx)+b(-cx+i)\log(d+icdx)\right)}{cd^2f(cx-i)\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a*Sqrt[1 + c^2*x^2] + b*Sqrt[1 + c^2*
x^2]*ArcSinh[c*x] + b*(I - c*x)*Log[d + I*c*d*x]))/(c*d^2*f*(-I + c*x)*Sqrt
[1 + c^2*x^2])
```

fricas [B] time = 0.53, size = 445, normalized size = 4.01

$$2\sqrt{icdx+d}\,\sqrt{-icfx+f}\,b\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(c^2d^2fx-icd^2f\right)\sqrt{\frac{b^2}{c^2d^3f}}\,\log\left(\frac{\left(-2ibc^6x^2-4bc^5x+4ibc^4\right)\sqrt{c^2x^2+1}\,\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] 1/2*(2*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2 + 1))
+ (c^2*d^2*f*x - I*c*d^2*f)*sqrt(b^2/(c^2*d^3*f))*log(((-2*I*b*c^6*x^2 - 4*
b*c^5*x + 4*I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)
 + 2*(I*c^9*d^2*f*x^4 + 2*c^8*d^2*f*x^3 + I*c^7*d^2*f*x^2 + 2*c^6*d^2*f*x)*
sqrt(b^2/(c^2*d^3*f)))/(16*b*c^3*x^3 - 16*I*b*c^2*x^2 + 16*b*c*x - 16*I*b))
 - (c^2*d^2*f*x - I*c*d^2*f)*sqrt(b^2/(c^2*d^3*f))*log(((-2*I*b*c^6*x^2 - 4
*b*c^5*x + 4*I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f
```

```
) + 2*(-I*c^9*d^2*f*x^4 - 2*c^8*d^2*f*x^3 - I*c^7*d^2*f*x^2 - 2*c^6*d^2*f*x
)*sqrt(b^2/(c^2*d^3*f)))/(16*b*c^3*x^3 - 16*I*b*c^2*x^2 + 16*b*c*x - 16*I*b
)) + 2*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a)/(c^2*d^2*f*x - I*c*d^2*f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arsinh}(cx) + a}{(icdx + d)^{\frac{3}{2}} \sqrt{-icfx + f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((I*c*d*x + d)^(3/2)*sqrt(-I*c*f*x + f)), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arcsinh}(cx)}{(icdx + d)^{\frac{3}{2}} \sqrt{-icfx + f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x)
```

maxima [A] time = 0.74, size = 98, normalized size = 0.88

$$\frac{i\sqrt{c^2dfx^2 + df}\, b \operatorname{arsinh}(cx)}{ic^2d^2fx + cd^2f} + \frac{i\sqrt{c^2dfx^2 + df}\, a}{ic^2d^2fx + cd^2f} - \frac{b \log(icx + 1)}{cd^{\frac{3}{2}}\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] I*sqrt(c^2*d*f*x^2 + d*f)*b*arcsinh(c*x)/(I*c^2*d^2*f*x + c*d^2*f) + I*sqrt
(c^2*d*f*x^2 + d*f)*a/(I*c^2*d^2*f*x + c*d^2*f) - b*log(I*c*x + 1)/(c*d^(3/
2)*sqrt(f))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{asinh}(c\,x)}{(d + c\,d\,x\,1\mathrm{i})^{3/2}\,\sqrt{f - c\,f\,x\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(1/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asinh}(cx)}{(id(cx - i))^{\frac{3}{2}} \sqrt{-if(cx + i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(3/2)/(f-I*c*f*x)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/((I*d*(c*x - I))**(3/2)*sqrt(-I*f*(c*x + I))),
x)
```

3.557 $\int \frac{a+b\sinh^{-1}(cx)}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx$

Optimal. Leaf size=295

$$\frac{f^2x\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2if^2(1-icx)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{ibf^2\left(c^2x^2+1\right)^{5/2}}{3c(-cx+i)(d+icdx)^{5/2}(f-i}$$

```
[Out] 1/3*I*b*f^2*(c^2*x^2+1)^(5/2)/c/(I-c*x)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
+2/3*I*f^2*(1-I*c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-
I*c*f*x)^(5/2)+1/3*f^2*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)
/(f-I*c*f*x)^(5/2)-1/3*I*b*f^2*(c^2*x^2+1)^(5/2)*arctan(c*x)/c/(d+I*c*d*x)^
(5/2)/(f-I*c*f*x)^(5/2)-1/6*b*f^2*(c^2*x^2+1)^(5/2)*ln(c^2*x^2+1)/c/(d+I*c*
d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A] time = 0.34, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {5712, 653, 191, 5819, 627, 44, 203, 260}

$$\frac{f^2x\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2if^2(1-icx)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{ibf^2\left(c^2x^2+1\right)^{5/2}}{3c(-cx+i)(d+icdx)^{5/2}(f-i}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]),x]
```

```
[Out] ((I/3)*b*f^2*(1 + c^2*x^2)^(5/2))/(c*(I - c*x)*(d + I*c*d*x)^(5/2)*(f - I*c
*f*x)^(5/2)) + (((2*I)/3)*f^2*(1 - I*c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]
))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (f^2*x*(1 + c^2*x^2)^2*(a
+ b*ArcSinh[c*x]))/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - ((I/3)*b*f
^2*(1 + c^2*x^2)^(5/2)*ArcTan[c*x])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5
/2)) - (b*f^2*(1 + c^2*x^2)^(5/2)*Log[1 + c^2*x^2])/(6*c*(d + I*c*d*x)^(5/2
)*(f - I*c*f*x)^(5/2))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(f-icfx)^2\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{2if^2(1-icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^2x\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{2if^2(1-icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^2x\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{2if^2(1-icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^2x\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{2if^2(1-icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^2x\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{ibf^2\left(1+c^2x^2\right)^{5/2}}{3c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2if^2(1-icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{ibf^2\left(1+c^2x^2\right)^{5/2}}{3c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2if^2(1-icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Mathematica [A] time = 0.50, size = 143, normalized size = 0.48

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\left((cx-2i)\left(a\sqrt{c^2x^2+1}+bcx-ib\right)+b(cx-2i)\sqrt{c^2x^2+1}\sinh^{-1}(cx)-b(cx-i)^2\log(d+\right.}{3cd^3f(cx-i)^2\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((-2*I + c*x)*((-I)*b + b*c*x + a*Sqrt
[1 + c^2*x^2]) + b*(-2*I + c*x)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - b*(-I + c*
x)^2*Log[d + I*c*d*x]))/(3*c*d^3*f*(-I + c*x)^2*Sqrt[1 + c^2*x^2])
```

fricas [B] time = 0.75, size = 593, normalized size = 2.01

$$-\frac{12\sqrt{c^2x^2+1}\sqrt{icdx+d}\sqrt{-icfx+f}\,bcx-3\left(4bc^2x^2-4ibcx+8b\right)\sqrt{icdx+d}\sqrt{-icfx+f}\log\left(cx+\sqrt{c^2x^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] -1/3*(12*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x - 3*(
4*b*c^2*x^2 - 4*I*b*c*x + 8*b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x
 + sqrt(c^2*x^2 + 1)) - 2*(3*c^4*d^3*f*x^3 - 3*I*c^3*d^3*f*x^2 + 3*c^2*d^3*
f*x - 3*I*c*d^3*f)*sqrt(b^2/(c^2*d^5*f))*log(1/3*(3*(-2*I*b*c^6*x^2 - 4*b*c
^5*x + 4*I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) +
2*(3*I*c^9*d^3*f*x^4 + 6*c^8*d^3*f*x^3 + 3*I*c^7*d^3*f*x^2 + 6*c^6*d^3*f*x)
*sqrt(b^2/(c^2*d^5*f)))/(16*b*c^3*x^3 - 16*I*b*c^2*x^2 + 16*b*c*x - 16*I*b)
) + 2*(3*c^4*d^3*f*x^3 - 3*I*c^3*d^3*f*x^2 + 3*c^2*d^3*f*x - 3*I*c*d^3*f)*s
qrt(b^2/(c^2*d^5*f))*log(1/3*(3*(-2*I*b*c^6*x^2 - 4*b*c^5*x + 4*I*b*c^4)*sq
rt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) + 2*(-3*I*c^9*d^3*f*x^
4 - 6*c^8*d^3*f*x^3 - 3*I*c^7*d^3*f*x^2 - 6*c^6*d^3*f*x)*sqrt(b^2/(c^2*d^5*
f)))/(16*b*c^3*x^3 - 16*I*b*c^2*x^2 + 16*b*c*x - 16*I*b)) - 3*(4*a*c^2*x^2
- 4*I*a*c*x + 8*a)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(12*c^4*d^3*f*x^3
- 12*I*c^3*d^3*f*x^2 + 12*c^2*d^3*f*x - 12*I*c*d^3*f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\mathrm{arsinh}\,(cx)+a}{(icdx+d)^{\frac{5}{2}}\sqrt{-icfx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((I*c*d*x + d)^(5/2)*sqrt(-I*c*f*x + f)), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arcsinh}\,(cx)}{(icdx+d)^{\frac{5}{2}}\sqrt{-icfx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x)
```

maxima [A] time = 0.71, size = 233, normalized size = 0.79

$$\frac{1}{3}\,bc\left(\frac{3}{3i\,c^3d^{\frac{5}{2}}\sqrt{f}\,x+3\,c^2d^{\frac{5}{2}}\sqrt{f}}-\frac{\log\left(cx-i\right)}{c^2d^{\frac{5}{2}}\sqrt{f}}\right)+b\left(-\frac{i\,\sqrt{c^2dfx^2+df}}{3\,c^3d^3fx^2-6i\,c^2d^3fx-3\,cd^3f}+\frac{i\,\sqrt{c^2dfx^2+df}}{3i\,c^2d^3fx+3\,cd^3f}\right)\text{arsinh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] 1/3*b*c*(3/(3*I*c^3*d^(5/2)*sqrt(f)*x + 3*c^2*d^(5/2)*sqrt(f)) - log(c*x -
I)/(c^2*d^(5/2)*sqrt(f))) + b*(-I*sqrt(c^2*d*f*x^2 + d*f)/(3*c^3*d^3*f*x^2
- 6*I*c^2*d^3*f*x - 3*c*d^3*f) + I*sqrt(c^2*d*f*x^2 + d*f)/(3*I*c^2*d^3*f*x
 + 3*c*d^3*f))*arcsinh(c*x) + a*(-I*sqrt(c^2*d*f*x^2 + d*f)/(3*c^3*d^3*f*x^
2 - 6*I*c^2*d^3*f*x - 3*c*d^3*f) + I*sqrt(c^2*d*f*x^2 + d*f)/(3*I*c^2*d^3*f
*x + 3*c*d^3*f))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}\left(c\,x\right)}{{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^{5/2}\,\sqrt{f-c\,f\,x\,1{}\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(1/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{asinh}\left(cx\right)}{\left(id\left(cx-i\right)\right)^{\frac{5}{2}}\sqrt{-if\left(cx+i\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(5/2)/(f-I*c*f*x)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/((I*d*(c*x - I))**(5/2)*sqrt(-I*f*(c*x + I))),
x)
```

3.558 $\int \frac{(d+icdx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{(f-icfx)^{3/2}}\,dx$

Optimal. Leaf size=517

$$-\frac{5id^4(1+icx)\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{15id^4\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2id^4(1+icx)^3\left(c^2x^2+1\right)\left(\right.}{c(d+icdx)^{3/2}(f-\right.}$$

```
[Out] 3/2*I*b*d^4*x*(c^2*x^2+1)^(3/2)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+b*c*d^4
*x^2*(c^2*x^2+1)^(3/2)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+5/4*b*d^4*(1+I*c
*x)^2*(c^2*x^2+1)^(3/2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+15/4*b*d^4*(c
^2*x^2+1)^(3/2)*arcsinh(c*x)^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-2*I*d^
4*(1+I*c*x)^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x
)^(3/2)-15/2*I*d^4*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-
I*c*f*x)^(3/2)-5/2*I*d^4*(1+I*c*x)*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/c/(d+I*
c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-15/2*d^4*(c^2*x^2+1)^(3/2)*arcsinh(c*x)*(a+b
*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-8*b*d^4*(c^2*x^2+1)^(3
/2)*ln(I+c*x)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A] time = 0.42, antiderivative size = 517, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {5712, 669, 671, 641, 215, 5819, 627, 43, 5675}

$$-\frac{5id^4(1+icx)\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{15id^4\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2id^4(1+icx)^3\left(c^2x^2+1\right)\left(\right.}{c(d+icdx)^{3/2}(f-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(3/2),x]
```

```
[Out] (((3*I)/2)*b*d^4*x*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^
(3/2)) + (b*c*d^4*x^2*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*
x)^(3/2)) + (5*b*d^4*(1 + I*c*x)^2*(1 + c^2*x^2)^(3/2))/(4*c*(d + I*c*d*x)^
(3/2)*(f - I*c*f*x)^(3/2)) + (15*b*d^4*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]^2)/
(4*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - ((2*I)*d^4*(1 + I*c*x)^3*(1
 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2
)) - (((15*I)/2)*d^4*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)
^(3/2)*(f - I*c*f*x)^(3/2)) - (((5*I)/2)*d^4*(1 + I*c*x)*(1 + c^2*x^2)^2*(a
 + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (15*d^4*(
1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*(a + b*ArcSinh[c*x]))/(2*c*(d + I*c*d*x)^(3
/2)*(f - I*c*f*x)^(3/2)) - (8*b*d^4*(1 + c^2*x^2)^(3/2)*Log[I + c*x])/(c*(d
 + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
```

```
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$\int \frac{(d+icdx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{(f-icfx)^{3/2}}\,dx = \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{(d+icdx)^4\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{2id^4(1+icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{15id^4\left(1+c^2x^2\right)^2\left(a+\right.}{2c(d+icdx)^{3/2}(f}$$

$$= \frac{15ibd^4x\left(1+c^2x^2\right)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{5bd^4(1+icx)^2\left(1+c^2x^2\right)^{3/2}}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2id^4(1+i}{c}$$

$$= \frac{15ibd^4x\left(1+c^2x^2\right)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{5bd^4(1+icx)^2\left(1+c^2x^2\right)^{3/2}}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{15bd^4\left(1\right.}{4c(d+}$$

$$= \frac{15ibd^4x\left(1+c^2x^2\right)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{5bd^4(1+icx)^2\left(1+c^2x^2\right)^{3/2}}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{15bd^4\left(1\right.}{4c(d+}$$

$$= \frac{3ibd^4x\left(1+c^2x^2\right)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{bcd^4x^2\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{5bd^4(1+icx}{4c(d+icdx)}$$

Mathematica [A] time = 4.02, size = 781, normalized size = 1.51

$$\frac{\frac{4ad^2\left(c^2x^2-7icx+24\right)\sqrt{d+icdx}\sqrt{f-icfx}}{f^2(cx+i)} - \frac{60ad^{5/2}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)}{f^{3/2}} + \frac{4bd^2\sqrt{d+icdx}\sqrt{f-icfx}\left(2\left(\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)+i\cos\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(3/2),x]
```

```
[Out] ((4*a*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(24 - (7*I)*c*x + c^2*x^2))/(
f^2*(I + c*x)) - (60*a*d^(5/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d
*x]*Sqrt[f - I*c*f*x]])/f^(3/2) + (4*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f
*x]*(-(ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])) + 4*
ArcSinh[c*x]*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2]) + 2*(4*ArcT
an[Tanh[ArcSinh[c*x]/2]] + I*Log[1 + c^2*x^2])*(I*Cosh[ArcSinh[c*x]/2] + Si
nh[ArcSinh[c*x]/2])))/(f^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh
[ArcSinh[c*x]/2])) + (16*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-(ArcSi
nh[c*x]^2*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])) + (c*x - 4*ArcTa
n[Coth[ArcSinh[c*x]/2]] + I*Log[1 + c^2*x^2])*(I*Cosh[ArcSinh[c*x]/2] + Sin
h[ArcSinh[c*x]/2]) + ArcSinh[c*x]*((-I)*(2 + Sqrt[1 + c^2*x^2])*Cosh[ArcSin
h[c*x]/2] - (-2 + Sqrt[1 + c^2*x^2])*Sinh[ArcSinh[c*x]/2])))/(f^2*Sqrt[1 +
c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])) + (b*d^2*Sqrt[d +
 I*c*d*x]*Sqrt[f - I*c*f*x]*(-10*ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] - I*S
inh[ArcSinh[c*x]/2]) + (16*c*x + 32*ArcTan[Tanh[ArcSinh[c*x]/2]] + I*Cosh[2
*ArcSinh[c*x]] + (8*I)*Log[1 + c^2*x^2])*(I*Cosh[ArcSinh[c*x]/2] + Sinh[Arc
Sinh[c*x]/2]) + 2*ArcSinh[c*x]*(Sinh[ArcSinh[c*x]/2]*(8 - 8*Sqrt[1 + c^2*x^
2] - I*Sinh[2*ArcSinh[c*x]]) + Cosh[ArcSinh[c*x]/2]*((-8*I)*(1 + Sqrt[1 + c
^2*x^2]) + Sinh[2*ArcSinh[c*x]]))))/(f^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*
x]/2] - I*Sinh[ArcSinh[c*x]/2])))/(8*c)
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bc^2d^2x^2-2i\,bcd^2x-bd^2\right)\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(ac^2d^2x^2-2i\,acd^2x-ad\right.}{c^2f^2x^2+2i\,cf^2x-f^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral(((b*c^2*d^2*x^2 - 2*I*b*c*d^2*x - b*d^2)*sqrt(I*c*d*x + d)*sqrt(-I
*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a*c^2*d^2*x^2 - 2*I*a*c*d^2*x -
 a*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^2*f^2*x^2 + 2*I*c*f^2*x -
f^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{(icdx+d)^{\frac{5}{2}} (a+b\operatorname{arcsinh}(cx))}{\left(-icfx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(\frac{c^2d^3x^3}{\sqrt{c^2dfx^2+df}\,f}-\frac{8i\,cd^3x^2}{\sqrt{c^2dfx^2+df}\,f}+\frac{17\,d^3x}{\sqrt{c^2dfx^2+df}\,f}-\frac{15\,d^3\operatorname{arsinh}(cx)}{\sqrt{df}\,cf}-\frac{24i\,d^3}{\sqrt{c^2dfx^2+df}\,cf}\right)a+b\int\frac{(i\,cdx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x, algorit
hm="maxima")
```

```
[Out] 1/2*(c^2*d^3*x^3/(sqrt(c^2*d*f*x^2 + d*f)*f) - 8*I*c*d^3*x^2/(sqrt(c^2*d*f*
x^2 + d*f)*f) + 17*d^3*x/(sqrt(c^2*d*f*x^2 + d*f)*f) - 15*d^3*arcsinh(c*x)/
(sqrt(d*f)*c*f) - 24*I*d^3/(sqrt(c^2*d*f*x^2 + d*f)*c*f))*a + b*integrate((
I*c*d*x + d)^(5/2)*log(c*x + sqrt(c^2*x^2 + 1))/(-I*c*f*x + f)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\operatorname{asinh}(c\,x))\,{(d+c\,d\,x\,1\mathrm{i})}^{5/2}}{{\left(f-c\,f\,x\,1\mathrm{i}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(3/2),x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(3/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))/(f-I*c*f*x)**(3/2),x)
[Out] Timed out
```

3.559 $\int \frac{(d+icdx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{(f-icfx)^{3/2}}\,dx$

Optimal. Leaf size=283

$$-\frac{3d^3\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{id^3\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{4id^3(1+icx)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(\right.}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[Out] I*b*d^3*x*(c^2*x^2+1)^(3/2)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-4*I*d^3*(1+
I*c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
-I*d^3*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/
2)-3/2*d^3*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/b/c/(d+I*c*d*x)^(3/2)/(f-
I*c*f*x)^(3/2)-4*b*d^3*(c^2*x^2+1)^(3/2)*ln(I+c*x)/c/(d+I*c*d*x)^(3/2)/(f-I
*c*f*x)^(3/2)
```

Rubi [A] time = 0.48, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5712, 5833, 637, 5819, 12, 627, 31, 5675, 5717, 8}

$$-\frac{3d^3\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{id^3\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{4id^3(1+icx)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(\right.}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(3/2),x]
```

```
[Out] (I*b*d^3*x*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) -
 ((4*I)*d^3*(1 + I*c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2)) - (I*d^3*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))
/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (3*d^3*(1 + c^2*x^2)^(3/2)*(
a + b*ArcSinh[c*x])^2)/(2*b*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (4
*b*d^3*(1 + c^2*x^2)^(3/2)*Log[I + c*x])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*
x)^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rule 5833

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+icdx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{(f-icfx)^{3/2}}\,dx = \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{(d+icdx)^3\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{3/2}\int\left(-\frac{4i\left(id^3-cd^3x\right)\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}} - \frac{3d^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}} - \frac{icd^3x\left(a+b\sin\right.}{\sqrt{1+c^2x}}\right.}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{\left(4i\left(1+c^2x^2\right)^{3/2}\right)\int\frac{\left(id^3-cd^3x\right)\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(3d^3\left(1+c^2x^2\right)^{3/2}\right)\int\frac{a+}{}}{(d+icdx)^{3/2}(f-i}$$

$$= -\frac{4id^3(1+icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{id^3\left(1+c^2x^2\right)^2\left(a+b\sinh\right.}{c(d+icdx)^{3/2}(f-icf}$$

$$= \frac{ibd^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4id^3(1+icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} -$$

$$= \frac{ibd^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4id^3(1+icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} -$$

$$= \frac{ibd^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4id^3(1+icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} -$$

Mathematica [A] time = 2.43, size = 515, normalized size = 1.82

$$-\frac{6ad^{3/2}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)}{f^{3/2}} + \frac{2ad(5-icx)\sqrt{d+icdx}\sqrt{f-icfx}}{f^2(cx+i)} + \frac{bd\sqrt{d+icdx}\sqrt{f-icfx}\left(2\left(\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)+i\cosh\left(\frac{1}{2}\sinh^{-1}(cx\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(3/2),x]
```

```
[Out] ((2*a*d*(5 - I*c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(f^2*(I + c*x)) -
(6*a*d^(3/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f
*x]])/f^(3/2) + (b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-(ArcSinh[c*x]^2*
(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])) + 4*ArcSinh[c*x]*((-I)*Cos
h[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2]) + 2*(4*ArcTan[Tanh[ArcSinh[c*x]/2
]] + I*Log[1 + c^2*x^2])*(I*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2])))/
(f^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])) + (
2*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-(ArcSinh[c*x]^2*(Cosh[ArcSinh[c
*x]/2] - I*Sinh[ArcSinh[c*x]/2])) + (c*x - 4*ArcTan[Coth[ArcSinh[c*x]/2]] +
 I*Log[1 + c^2*x^2])*(I*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2]) + ArcS
inh[c*x]*((-I)*(2 + Sqrt[1 + c^2*x^2])*Cosh[ArcSinh[c*x]/2] - (-2 + Sqrt[1
+ c^2*x^2])*Sinh[ArcSinh[c*x]/2])))/(f^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*
x]/2] - I*Sinh[ArcSinh[c*x]/2])))/(2*c)
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(-i\,bcdx-bd)\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)+(-i\,acdx-ad)\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}}{c^2f^2x^2+2i\,cf^2x-f^2},x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral(((-I*b*c*d*x - b*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
 sqrt(c^2*x^2 + 1)) + (-I*a*c*d*x - a*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x +
f))/(c^2*f^2*x^2 + 2*I*c*f^2*x - f^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [c,d,t_nostep]=[30,75,38]Precision problem choosing root in common
_EXT, current precision 14Precision problem choosing root in common_EXT, cu
rrent precision 28Precision problem choosing root in common_EXT, current pr
ecision 56Precision problem choosing root in common_EXT, current precision
112Precision problem choosing root in common_EXT, current precision 224Prec
ision problem choosing root in common_EXT, current precision 448Precision p
roblem choosing root in common_EXT, current precision 896Unable to transpos
e Error: Bad Argument ValueWarning, replacing 0 by ` u`, a substitution var
iable should perhaps be purged.Evaluation time: 0.75sym2poly/r2sym(const ge
n & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{(icdx+d)^{\frac{3}{2}} (a+b\,\mathrm{arcsinh}\,(cx))}{\left(-icfx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x)
```

```
[Out] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(-\frac{i\left(c^2dfx^2+df\right)^{\frac{3}{2}}}{c^3f^3x^2+2i\,c^2f^3x-cf^3}-\frac{6i\sqrt{c^2dfx^2+df}\,d}{-i\,c^2f^2x+cf^2}-\frac{3\,d^2\,\mathrm{arsinh}\,(cx)}{cf^2\sqrt{\frac{d}{f}}}\right)+b\int\frac{(i\,cdx+d)^{\frac{3}{2}}\log\left(cx+\sqrt{c^2x^2+1}\right)}{\left(-i\,cfx+f\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x, algorit
hm="maxima")
```

```
[Out] a*(-I*(c^2*d*f*x^2 + d*f)^(3/2)/(c^3*f^3*x^2 + 2*I*c^2*f^3*x - c*f^3) - 6*I
*sqrt(c^2*d*f*x^2 + d*f)*d/(-I*c^2*f^2*x + c*f^2) - 3*d^2*arcsinh(c*x)/(c*f
^2*sqrt(d/f))) + b*integrate((I*c*d*x + d)^(3/2)*log(c*x + sqrt(c^2*x^2 + 1
))/(-I*c*f*x + f)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))\,\left(d+c\,d\,x\,1\mathrm{i}\right)^{3/2}}{\left(f-c\,f\,x\,1\mathrm{i}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(3/2),x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))/(f-I*c*f*x)**(3/2),x)
```

```
[Out] Timed out
```

3.560 $\int \frac{\sqrt{d+icdx}\left(a+b\sinh^{-1}(cx)\right)}{(f-icfx)^{3/2}}\,dx$

Optimal. Leaf size=180

$$-\frac{d^2\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2id^2(1+icx)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2bd^2\left(c^2x^2+1\right)^{3/2}\log(cx+}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[Out] -2*I*d^2*(1+I*c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*
c*f*x)^(3/2)-1/2*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/b/c/(d+I*c*d*x)
^(3/2)/(f-I*c*f*x)^(3/2)-2*b*d^2*(c^2*x^2+1)^(3/2)*ln(I+c*x)/c/(d+I*c*d*x)^
(3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A] time = 0.40, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {5712, 5833, 637, 5819, 12, 627, 31, 5675}

$$-\frac{d^2\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2id^2(1+icx)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2bd^2\left(c^2x^2+1\right)^{3/2}\log(cx+}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(3/2),x]
```

```
[Out] ((-2*I)*d^2*(1 + I*c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2)) - (d^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]
)^2)/(2*b*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (2*b*d^2*(1 + c^2*x^
2)^(3/2)*Log[I + c*x])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rule 5833

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+icdx}\left(a+b\sinh^{-1}(cx)\right)}{(f-icfx)^{3/2}}dx = \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{(d+icdx)^2\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{3/2}\int \left(-\frac{2i\left(id^2-cd^2x\right)\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}-\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{\left(2i\left(1+c^2x^2\right)^{3/2}\right)\int \frac{\left(id^2-cd^2x\right)\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(d^2\left(1+c^2x^2\right)^{3/2}\right)\int \frac{a+b\sin}{\sqrt{1+}}}{(d+icdx)^{3/2}(f-icf}$$

$$= -\frac{2id^2(1+icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh\right.}{2bc(d+icdx)^{3/2}(f-icf}$$

$$= -\frac{2id^2(1+icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh\right.}{2bc(d+icdx)^{3/2}(f-icf}$$

$$= -\frac{2id^2(1+icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh\right.}{2bc(d+icdx)^{3/2}(f-icf}$$

$$= -\frac{2id^2(1+icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh\right.}{2bc(d+icdx)^{3/2}(f-icf}$$

Mathematica [A] time = 1.28, size = 285, normalized size = 1.58

$$\frac{\frac{4a\sqrt{d+icdx}\sqrt{f-icfx}}{cx+i} - 2a\sqrt{d}\sqrt{f}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right) + \frac{b\sqrt{d+icdx}\sqrt{f-icfx}\left(2\left(\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)+i\cosh\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(3/2),x]
```

```
[Out] ((4*a*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(I + c*x) - 2*a*Sqrt[d]*Sqrt[f]*
Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + (b*Sqr
t[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-(ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] -
I*Sinh[ArcSinh[c*x]/2])) + 4*ArcSinh[c*x]*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh
[ArcSinh[c*x]/2]) + 2*(4*ArcTan[Tanh[ArcSinh[c*x]/2]] + I*Log[1 + c^2*x^2])
*(I*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2])))/(Sqrt[1 + c^2*x^2]*(Cosh
[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])))/(2*c*f^2)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{icdx+d}\,\sqrt{-icfx+f}\,b\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{icdx+d}\,\sqrt{-icfx+f}\,a}{c^2f^2x^2+2icf^2x-f^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral(-(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2 +
1)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a)/(c^2*f^2*x^2 + 2*I*c*f^2*x -
f^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{icdx+d}\,(b\,\text{arsinh}\,(cx)+a)}{\left(-icfx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate(sqrt(I*c*d*x + d)*(b*arcsinh(c*x) + a)/(-I*c*f*x + f)^(3/2), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\text{arcsinh}\,(cx))\,\sqrt{icdx+d}}{\left(-icfx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(-\frac{2i\sqrt{c^2dfx^2+df}}{-ic^2f^2x+cf^2}-\frac{d\,\text{arsinh}\,(cx)}{cf^2\sqrt{\frac{d}{f}}}\right)+b\int \frac{\sqrt{icdx+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)}{\left(-icfx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x, algorit
hm="maxima")
```

```
[Out] a*(-2*I*sqrt(c^2*d*f*x^2 + d*f)/(-I*c^2*f^2*x + c*f^2) - d*arcsinh(c*x)/(c*
f^2*sqrt(d/f))) + b*integrate(sqrt(I*c*d*x + d)*log(c*x + sqrt(c^2*x^2 + 1)
)/(-I*c*f*x + f)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{asinh}\,(c\,x))\,\sqrt{d + c\,d\,x\,1\mathrm{i}}}{\left(f - c\,f\,x\,1\mathrm{i}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(3/2),x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{id\,(cx - i)}\,(a + b\,\mathrm{asinh}\,(cx))}{\left(-if\,(cx + i)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(3/2),x)
```

```
[Out] Integral(sqrt(I*d*(c*x - I))*(a + b*asinh(c*x))/(-I*f*(c*x + I))**(3/2), x)
```

3.561 $\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+icdx}\,(f-icfx)^{3/2}}\,dx$

Optimal. Leaf size=112

$$-\frac{d(-cx+i)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{bd\left(c^2x^2+1\right)^{3/2}\log(cx+i)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[Out] -d*(I-c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(
3/2)-b*d*(c^2*x^2+1)^(3/2)*ln(I+c*x)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A] time = 0.25, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {5712, 637, 5819, 12, 627, 31}

$$-\frac{d(-cx+i)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{bd\left(c^2x^2+1\right)^{3/2}\log(cx+i)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)),x]
```

```
[Out] -((d*(I - c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(3/2)*(
f - I*c*f*x)^(3/2))) - (b*d*(1 + c^2*x^2)^(3/2)*Log[I + c*x])/(c*(d + I*c*d
*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
```

```
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+icdx}\,(f-icfx)^{3/2}}\,dx &= \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{(d+icdx)\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&= -\frac{d(i-cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{\left(bc\left(1+c^2x^2\right)^{3/2}\right)\int \frac{d(i-cx)}{c\left(1+c^2x^2\right)}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&= -\frac{d(i-cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{\left(bd\left(1+c^2x^2\right)^{3/2}\right)\int \frac{i-cx}{1+c^2x^2}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&= -\frac{d(i-cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{\left(bd\left(1+c^2x^2\right)^{3/2}\right)\int \frac{1}{-i-cx}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&= -\frac{d(i-cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{bd\left(1+c^2x^2\right)^{3/2}\log(i+cx)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.41, size = 94, normalized size = 0.84

$$\frac{\sqrt{f-icfx}\left(iacx+a-ib\sqrt{c^2x^2+1}\,\log(d(-1+icx))+(b+ibcx)\sinh^{-1}(cx)\right)}{cf^2(cx+i)\sqrt{d+icdx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)),x]
```

```
[Out] (Sqrt[f - I*c*f*x]*(a + I*a*c*x + (b + I*b*c*x)*ArcSinh[c*x] - I*b*Sqrt[1 +
 c^2*x^2]*Log[d*(-1 + I*c*x)]))/(c*f^2*(I + c*x)*Sqrt[d + I*c*d*x])
```

fricas [B] time = 0.65, size = 445, normalized size = 3.97

$$2\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,b\log\left(cx+\sqrt{c^2x^2+1}\right)-\left(c^2df^2x+i\,cdf^2\right)\sqrt{\frac{b^2}{c^2df^3}}\,\log\left(\frac{\left(2i\,bc^6x^2-4\,bc^5x-4i\,bc^4\right)\sqrt{c^2x^2+1}\,\sqrt{i\,cdx}}{16}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] 1/2*(2*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2 + 1))
- (c^2*d*f^2*x + I*c*d*f^2)*sqrt(b^2/(c^2*d*f^3))*log(((2*I*b*c^6*x^2 - 4*b
*c^5*x - 4*I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)
+ 2*(I*c^9*d*f^2*x^4 - 2*c^8*d*f^2*x^3 + I*c^7*d*f^2*x^2 - 2*c^6*d*f^2*x)*s
qrt(b^2/(c^2*d*f^3)))/(16*b*c^3*x^3 + 16*I*b*c^2*x^2 + 16*b*c*x + 16*I*b))
+ (c^2*d*f^2*x + I*c*d*f^2)*sqrt(b^2/(c^2*d*f^3))*log(((2*I*b*c^6*x^2 - 4*b
*c^5*x - 4*I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)
+ 2*(-I*c^9*d*f^2*x^4 + 2*c^8*d*f^2*x^3 - I*c^7*d*f^2*x^2 + 2*c^6*d*f^2*x)*
```

```
sqrt(b^2/(c^2*d*f^3)))/(16*b*c^3*x^3 + 16*I*b*c^2*x^2 + 16*b*c*x + 16*I*b))
 + 2*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a)/(c^2*d*f^2*x + I*c*d*f^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\sqrt{icdx + d}\left(-icfx + f\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(I*c*d*x + d)*(-I*c*f*x + f)^(3/2)), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arcsinh}(cx)}{\left(-icfx + f\right)^{\frac{3}{2}} \sqrt{icdx + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x)
```

maxima [A] time = 0.65, size = 98, normalized size = 0.88

$$-\frac{i\sqrt{c^2dfx^2 + df}\, b \operatorname{arsinh}(cx)}{-ic^2df^2x + cdf^2} - \frac{i\sqrt{c^2dfx^2 + df}\, a}{-ic^2df^2x + cdf^2} - \frac{b\log(icx - 1)}{c\sqrt{d}\, f^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] -I*sqrt(c^2*d*f*x^2 + d*f)*b*arcsinh(c*x)/(-I*c^2*d*f^2*x + c*d*f^2) - I*sq
rt(c^2*d*f*x^2 + d*f)*a/(-I*c^2*d*f^2*x + c*d*f^2) - b*log(I*c*x - 1)/(c*sq
rt(d)*f^(3/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{asinh}(c\,x)}{\sqrt{d + c\,d\,x\,1i}\,\left(f - c\,f\,x\,1i\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(3/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asinh}(cx)}{\sqrt{id(cx - i)}\,\left(-if(cx + i)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(f-I*c*f*x)**(3/2)/(d+I*c*d*x)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(sqrt(I*d*(c*x - I))*(-I*f*(c*x + I))**(3/2)),
x)
```

3.562 $\int \frac{a+b\sinh^{-1}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx$

Optimal. Leaf size=103

$$\frac{x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{b\left(c^2x^2+1\right)^{3/2}\log\left(c^2x^2+1\right)}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[Out] x*(c^2*x^2+1)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-1/2*b*
(c^2*x^2+1)^(3/2)*ln(c^2*x^2+1)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A] time = 0.21, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {5712, 5687, 260}

$$\frac{x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{b\left(c^2x^2+1\right)^{3/2}\log\left(c^2x^2+1\right)}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)),x]
```

```
[Out] (x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(
3/2)) - (b*(1 + c^2*x^2)^(3/2)*Log[1 + c^2*x^2])/(2*c*(d + I*c*d*x)^(3/2)*(
f - I*c*f*x)^(3/2))
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\sinh^{-1}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx &= \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{a+b\sinh^{-1}(cx)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&= \frac{x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{\left(bc\left(1+c^2x^2\right)^{3/2}\right)\int \frac{x}{1+c^2x^2}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&= \frac{x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{b\left(1+c^2x^2\right)^{3/2}\log\left(1+c^2x^2\right)}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.45, size = 118, normalized size = 1.15

$$\frac{i\sqrt{f-icfx}\left(2acx-b\sqrt{c^2x^2+1}\log(d(-1+icx))-b\sqrt{c^2x^2+1}\log(d+icdx)+2bcx\sinh^{-1}(cx)\right)}{2cdf^2(cx+i)\sqrt{d+icdx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)),x]
```

```
[Out] ((I/2)*Sqrt[f - I*c*f*x]*(2*a*c*x + 2*b*c*x*ArcSinh[c*x] - b*Sqrt[1 + c^2*x
^2]*Log[d*(-1 + I*c*x)] - b*Sqrt[1 + c^2*x^2]*Log[d + I*c*d*x]))/(c*d*f^2*(
I + c*x)*Sqrt[d + I*c*d*x])
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$4\sqrt{icdx+d}\sqrt{-icfx+f}\,bx\log\left(cx+\sqrt{c^2x^2+1}\right)+4\sqrt{icdx+d}\sqrt{-icfx+f}\,ax+\left(c^2d^2f^2x^2+d^2f^2\right)\sqrt{\frac{b^2}{c^2d^3f^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x, algorit
hm="fricas")
```

```
[Out] 1/4*(4*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*x*log(c*x + sqrt(c^2*x^2 + 1)
) + 4*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*x + (c^2*d^2*f^2*x^2 + d^2*f^2
)*sqrt(b^2/(c^2*d^3*f^3))*log((b*c^2*x^4 + sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x +
 d)*sqrt(-I*c*f*x + f)*c*d*f*x^2*sqrt(b^2/(c^2*d^3*f^3)) + b*x^2)/(b*c^4*x^
4 + 2*b*c^2*x^2 + b)) - (c^2*d^2*f^2*x^2 + d^2*f^2)*sqrt(b^2/(c^2*d^3*f^3))
*log((b*c^2*x^4 - sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*
d*f*x^2*sqrt(b^2/(c^2*d^3*f^3)) + b*x^2)/(b*c^4*x^4 + 2*b*c^2*x^2 + b)) - 2
*(c^2*d^2*f^2*x^2 + d^2*f^2)*sqrt(b^2/(c^2*d^3*f^3))*log((b*c^2*x^3 + sqrt(
c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d*f*x*sqrt(b^2/(c^2*d^3
*f^3)) + b*x)/(b*c^2*x^2 + b)) + 2*(c^2*d^2*f^2*x^2 + d^2*f^2)*sqrt(b^2/(c^
2*d^3*f^3))*log((b*c^2*x^3 - sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*
f*x + f)*c*d*f*x*sqrt(b^2/(c^2*d^3*f^3)) + b*x)/(b*c^2*x^2 + b)) + 4*(c^2*d
^2*f^2*x^2 + d^2*f^2)*integral(-sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I
*c*f*x + f)*b*c*x/(c^4*d^2*f^2*x^4 + 2*c^2*d^2*f^2*x^2 + d^2*f^2), x))/(c^2
*d^2*f^2*x^2 + d^2*f^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\mathrm{arsinh}\,(cx)+a}{(i\,cdx+d)^{\frac{3}{2}}\left(-i\,cfx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((I*c*d*x + d)^(3/2)*(-I*c*f*x + f)^(3/2)),
x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arcsinh}\,(cx)}{(icdx+d)^{\frac{3}{2}}\left(-icfx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x)
```

maxima [A] time = 0.58, size = 82, normalized size = 0.80

$$\frac{bx \operatorname{arsinh}(cx)}{\sqrt{c^2dfx^2 + df}\,df} + \frac{ax}{\sqrt{c^2dfx^2 + df}\,df} - \frac{b\sqrt{\frac{1}{df}}\log\left(x^2 + \frac{1}{c^2}\right)}{2\,cdf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x, algorit
hm="maxima")
```

```
[Out] b*x*arcsinh(c*x)/(sqrt(c^2*d*f*x^2 + d*f)*d*f) + a*x/(sqrt(c^2*d*f*x^2 + d*
f)*d*f) - 1/2*b*sqrt(1/(d*f))*log(x^2 + 1/c^2)/(c*d*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{asinh}(c\,x)}{(d + c\,d\,x\,1\mathrm{i})^{3/2}\left(f - c\,f\,x\,1\mathrm{i}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(3/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asinh}(cx)}{(id(cx - i))^{\frac{3}{2}}\left(-if(cx + i)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(3/2)/(f-I*c*f*x)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/((I*d*(c*x - I))**(3/2)*(-I*f*(c*x + I))**(3/2)
), x)
```

3.563 $\int \frac{a+b\sinh^{-1}(cx)}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx$

Optimal. Leaf size=282

$$\frac{2fx\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{f(cx+i)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{ibf\left(c^2x^2+1\right)^{5/2}}{6c(-cx+i)(d+icdx)^{5/2}(f-icfx)}$$

```
[Out] 1/6*I*b*f*(c^2*x^2+1)^(5/2)/c/(I-c*x)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1
/3*f*(I+c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)
^(5/2)+2/3*f*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*
x)^(5/2)-1/6*I*b*f*(c^2*x^2+1)^(5/2)*arctan(c*x)/c/(d+I*c*d*x)^(5/2)/(f-I*c
*f*x)^(5/2)-1/3*b*f*(c^2*x^2+1)^(5/2)*ln(c^2*x^2+1)/c/(d+I*c*d*x)^(5/2)/(f-
I*c*f*x)^(5/2)
```

Rubi [A] time = 0.31, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {5712, 639, 191, 5819, 627, 44, 203, 260}

$$\frac{2fx\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{f(cx+i)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{ibf\left(c^2x^2+1\right)^{5/2}}{6c(-cx+i)(d+icdx)^{5/2}(f-icfx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)),x]
```

```
[Out] ((I/6)*b*f*(1 + c^2*x^2)^(5/2))/(c*(I - c*x)*(d + I*c*d*x)^(5/2)*(f - I*c*f
*x)^(5/2)) + (f*(I + c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(3*c*(d + I*c
*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (2*f*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*
x]))/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - ((I/6)*b*f*(1 + c^2*x^2)
^(5/2)*ArcTan[c*x])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (b*f*(1 +
 c^2*x^2)^(5/2)*Log[1 + c^2*x^2])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5
/2))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(f-icfx)\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{f(i+cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2fx\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} -$$

$$= \frac{f(i+cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2fx\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} -$$

$$= \frac{f(i+cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2fx\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} -$$

$$= \frac{f(i+cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2fx\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} -$$

$$= \frac{ibf\left(1+c^2x^2\right)^{5/2}}{6c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f(i+cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} +$$

$$= \frac{ibf\left(1+c^2x^2\right)^{5/2}}{6c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f(i+cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} +$$

Mathematica [A] time = 0.64, size = 201, normalized size = 0.71

$$\frac{\sqrt{f-icfx}\left(8iac^2x^2+8acx+4ia-5ibcx\sqrt{c^2x^2+1}\log(d+icdx)+3b(-1-icx)\sqrt{c^2x^2+1}\log(d(-1+icx))-\right.}{12d^2f^2\left(c^3x^2+c\right)\sqrt{d+icdx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)),x]
```

```
[Out] (Sqrt[f - I*c*f*x]*((4*I)*a + 8*a*c*x + (8*I)*a*c^2*x^2 + 2*b*Sqrt[1 + c^2*
x^2] + 4*b*(I + 2*c*x + (2*I)*c^2*x^2)*ArcSinh[c*x] + 3*b*(-1 - I*c*x)*Sqrt
[1 + c^2*x^2]*Log[d*(-1 + I*c*x)] - 5*b*Sqrt[1 + c^2*x^2]*Log[d + I*c*d*x]
- (5*I)*b*c*x*Sqrt[1 + c^2*x^2]*Log[d + I*c*d*x]))/(12*d^2*f^2*Sqrt[d + I*c
*d*x]*(c + c^3*x^2))
```

fricas [F] time = 1.08, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x, algorit
hm="fricas")
```

```
[Out] -1/3*(48*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x - 3*(
64*b*c^2*x^2 - 64*I*b*c*x + 32*b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(
c*x + sqrt(c^2*x^2 + 1)) - 4*(9*c^4*d^3*f^2*x^3 - 9*I*c^3*d^3*f^2*x^2 + 9*c
^2*d^3*f^2*x - 9*I*c*d^3*f^2)*sqrt(b^2/(c^2*d^5*f^3))*log((4*I*sqrt(c^2*x^2
 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d^2*f*x*sqrt(b^2/(c^2*d^5*f^3)
) + 4*I*b*c^2*x^3 + 4*I*b*x)/(4*b*c^3*x^3 + 4*I*b*c^2*x^2 + 4*b*c*x + 4*I*b
)) + 4*(15*c^4*d^3*f^2*x^3 - 15*I*c^3*d^3*f^2*x^2 + 15*c^2*d^3*f^2*x - 15*I
*c*d^3*f^2)*sqrt(b^2/(c^2*d^5*f^3))*log((4*I*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x
 + d)*sqrt(-I*c*f*x + f)*c*d^2*f*x*sqrt(b^2/(c^2*d^5*f^3)) - 4*I*b*c^2*x^3
- 4*I*b*x)/(4*b*c^3*x^3 - 4*I*b*c^2*x^2 + 4*b*c*x - 4*I*b)) + 4*(9*c^4*d^3*
f^2*x^3 - 9*I*c^3*d^3*f^2*x^2 + 9*c^2*d^3*f^2*x - 9*I*c*d^3*f^2)*sqrt(b^2/(
c^2*d^5*f^3))*log((-4*I*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x +
 f)*c*d^2*f*x*sqrt(b^2/(c^2*d^5*f^3)) + 4*I*b*c^2*x^3 + 4*I*b*x)/(4*b*c^3*x
^3 + 4*I*b*c^2*x^2 + 4*b*c*x + 4*I*b)) - 4*(15*c^4*d^3*f^2*x^3 - 15*I*c^3*d
^3*f^2*x^2 + 15*c^2*d^3*f^2*x - 15*I*c*d^3*f^2)*sqrt(b^2/(c^2*d^5*f^3))*log
((-4*I*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d^2*f*x*sqr
t(b^2/(c^2*d^5*f^3)) - 4*I*b*c^2*x^3 - 4*I*b*x)/(4*b*c^3*x^3 - 4*I*b*c^2*x^
2 + 4*b*c*x - 4*I*b)) + 4*(24*c^4*d^3*f^2*x^3 - 24*I*c^3*d^3*f^2*x^2 + 24*c
^2*d^3*f^2*x - 24*I*c*d^3*f^2)*sqrt(b^2/(c^2*d^5*f^3))*log((sqrt(c^2*x^2 +
1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d^2*f*x*sqrt(b^2/(c^2*d^5*f^3)) +
 b*c^2*x^3 + b*x)/(b*c^2*x^2 + b)) - 4*(24*c^4*d^3*f^2*x^3 - 24*I*c^3*d^3*f
^2*x^2 + 24*c^2*d^3*f^2*x - 24*I*c*d^3*f^2)*sqrt(b^2/(c^2*d^5*f^3))*log(-(s
qrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d^2*f*x*sqrt(b^2/(c
^2*d^5*f^3)) - b*c^2*x^3 - b*x)/(b*c^2*x^2 + b)) - 3*(64*a*c^2*x^2 - 64*I*a
*c*x + 32*a)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) - 3*(96*c^4*d^3*f^2*x^3 -
 96*I*c^3*d^3*f^2*x^2 + 96*c^2*d^3*f^2*x - 96*I*c*d^3*f^2)*integral(-1/6*sq
rt(c^2*x^2 + 1)*(4*b*c*x + I*b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)/(c^4*d
^3*f^2*x^4 + 2*c^2*d^3*f^2*x^2 + d^3*f^2), x))/(96*c^4*d^3*f^2*x^3 - 96*I*c
^3*d^3*f^2*x^2 + 96*c^2*d^3*f^2*x - 96*I*c*d^3*f^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [c,d,t_nostep]=[-62,5,7]Warning, choosing root of [1,0,%%%{-8,[2,4
,2]%%%}+%%%{-6,[0,0,0]%%%},%%%{-32,[2,4,2]%%%}+%%%{-8,[0,0,0]%%%},%%%{16,[4
,8,4]%%%}+%%%{-24,[2,4,2]%%%}+%%%{-3,[0,0,0]%%%}] at parameters values [26,
-89,63]Warning, need to choose a branch for the root of a polynomial with p
arameters. This might be wrong.The choice was done assuming [c,d,t_nostep]=
[50,45,-24]Warning, choosing root of [1,0,%%%{-8,[2,4,2]%%%}+%%%{-6,[0,0,0]
%%%},%%%{-32,[2,4,2]%%%}+%%%{-8,[0,0,0]%%%},%%%{16,[4,8,4]%%%}+%%%{-24,[2,4
,2]%%%}+%%%{-3,[0,0,0]%%%}] at parameters values [-64,-30,70]Warning, need
to choose a branch for the root of a polynomial with parameters. This might
 be wrong.The choice was done assuming [c,d,t_nostep]=[-43,-99,-71]schur ro
w 3 -2.20626e-08Warning, need to choose a branch for the root of a polynomi
al with parameters. This might be wrong.The choice was done assuming [c,d,t
_nostep]=[29,40,-81]Warning, need to choose a branch for the root of a poly
nomial with parameters. This might be wrong.The choice was done assuming [c
,d,t_nostep]=[15,66,-69]schur row 1 3.08279e-07Francis algorithm not precis
e enough for[1.0,0.0,-1.62609692573e+14,-6.50438770291e+14,6.61047802965e+2
7]Warning, choosing root of [1,0,%%%{-8,[2,4,2]%%%}+%%%{-6,[0,0,0]%%%},%%%{
-32,[2,4,2]%%%}+%%%{-8,[0,0,0]%%%},%%%{16,[4,8,4]%%%}+%%%{-24,[2,4,2]%%%}+%
%%{-3,[0,0,0]%%%}] at parameters values [-80,-23,65]Warning, need to choose
 a branch for the root of a polynomial with parameters. This might be wrong
.The choice was done assuming [c,d,t_nostep]=[-63,70,-35]schur row 3 -1.418
72e-07Warning, choosing root of [1,0,%%%{-8,[2,4,2]%%%}+%%%{-6,[0,0,0]%%%},
%%%{-32,[2,4,2]%%%}+%%%{-8,[0,0,0]%%%},%%%{16,[4,8,4]%%%}+%%%{-24,[2,4,2]%%
%}+%%%{-3,[0,0,0]%%%}] at parameters values [-44,-22,93]Warning, need to ch
oose a branch for the root of a polynomial with parameters. This might be w
rong.The choice was done assuming [c,d,t_nostep]=[-72,80,-90]schur row 3 -2
.16313e-08Warning, need to choose a branch for the root of a polynomial wit
h parameters. This might be wrong.The choice was done assuming [c,d,t_noste
p]=[-90,-50,-96]ext_reduce Error: Bad Argument Typeext_reduce Error: Bad Ar
gument TypeEvaluation time: 2.09Done
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arcsinh}\,(cx)}{(icdx+d)^{\frac{5}{2}}\left(-icfx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x)
```

maxima [A] time = 0.52, size = 237, normalized size = 0.84

$$\frac{1}{12}bc\left(-\frac{2i\sqrt{d}\sqrt{f}}{c^3d^3f^2x-ic^2d^3f^2}-\frac{3\log(cx+i)}{c^2d^{\frac{5}{2}}f^{\frac{3}{2}}}-\frac{5\log(cx-i)}{c^2d^{\frac{5}{2}}f^{\frac{3}{2}}}\right)-\frac{1}{3}b\left(-\frac{3i}{3i\sqrt{c^2dfx^2+df}\,c^2d^2fx+3\sqrt{c^2dfx^2+df}\,c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x, algorit
hm="maxima")
```

```
[Out] 1/12*b*c*(-2*I*sqrt(d)*sqrt(f)/(c^3*d^3*f^2*x - I*c^2*d^3*f^2) - 3*log(c*x
+ I)/(c^2*d^(5/2)*f^(3/2)) - 5*log(c*x - I)/(c^2*d^(5/2)*f^(3/2))) - 1/3*b*
```

```
(-3*I/(3*I*sqrt(c^2*d*f*x^2 + d*f)*c^2*d^2*f*x + 3*sqrt(c^2*d*f*x^2 + d*f)*
c*d^2*f) - 2*x/(sqrt(c^2*d*f*x^2 + d*f)*d^2*f))*arcsinh(c*x) - 1/3*a*(-3*I/
(3*I*sqrt(c^2*d*f*x^2 + d*f)*c^2*d^2*f*x + 3*sqrt(c^2*d*f*x^2 + d*f)*c*d^2*
f) - 2*x/(sqrt(c^2*d*f*x^2 + d*f)*d^2*f))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}\left(c\,x\right)}{{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^{5/2}\,{\left(f-c\,f\,x\,1{}\mathrm{i}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(3/2)),x)
[Out] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(5/2)/(f-I*c*f*x)**(3/2),x)
[Out] Timed out
```

3.564 $\int \frac{(d+icdx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{(f-icfx)^{5/2}}\,dx$

Optimal. Leaf size=470

$$\frac{5id^5\left(c^2x^2+1\right)^3\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{10id^5(1+icx)^2\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2id^5(1+icx)^4\left(c^2x^2+1\right)\left(a+\right.}{3c(d+icdx)^{5/2}(f-ic}$$

```
[Out] -I*b*d^5*x*(c^2*x^2+1)^(5/2)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+8/3*I*b*d^
5*(c^2*x^2+1)^(5/2)/c/(I+c*x)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-5/2*b*d^5
*(c^2*x^2+1)^(5/2)*arcsinh(c*x)^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2/3
*I*d^5*(1+I*c*x)^4*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*
c*f*x)^(5/2)+10/3*I*d^5*(1+I*c*x)^2*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/c/(d+I
*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+5*I*d^5*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))/c/
(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+5*d^5*(c^2*x^2+1)^(5/2)*arcsinh(c*x)*(a
+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+28/3*b*d^5*(c^2*x^2+
1)^(5/2)*ln(I+c*x)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A] time = 0.44, antiderivative size = 470, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {5712, 669, 641, 215, 5819, 627, 43, 5675}

$$\frac{5id^5\left(c^2x^2+1\right)^3\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{10id^5(1+icx)^2\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2id^5(1+icx)^4\left(c^2x^2+1\right)\left(a+\right.}{3c(d+icdx)^{5/2}(f-ic}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(5/2),x]
```

```
[Out] ((-I)*b*d^5*x*(1 + c^2*x^2)^(5/2))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)
) + (((8*I)/3)*b*d^5*(1 + c^2*x^2)^(5/2))/(c*(I + c*x)*(d + I*c*d*x)^(5/2)*
(f - I*c*f*x)^(5/2)) - (5*b*d^5*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x]^2)/(2*c*(d
 + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((2*I)/3)*d^5*(1 + I*c*x)^4*(1 +
c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
+ (((10*I)/3)*d^5*(1 + I*c*x)^2*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(c*(d
 + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + ((5*I)*d^5*(1 + c^2*x^2)^3*(a + b*
ArcSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (5*d^5*(1 + c^
2*x^2)^(5/2)*ArcSinh[c*x]*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f -
 I*c*f*x)^(5/2)) + (28*b*d^5*(1 + c^2*x^2)^(5/2)*Log[I + c*x])/(3*c*(d + I*
c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$\int \frac{(d+icdx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{(f-icfx)^{5/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(d+icdx)^5\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{2id^5(1+icx)^4\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{10id^5(1+icx)^2\left(1+c^2x^2\right.}{3c(d+icdx)^{5/2}(}$$

$$= -\frac{5ibd^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2id^5(1+icx)^4\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{5ibd^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{5bd^5\left(1+c^2x^2\right)^{5/2}\sinh^{-1}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2id^5(1+i}{3c}$$

$$= -\frac{5ibd^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{5bd^5\left(1+c^2x^2\right)^{5/2}\sinh^{-1}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2id^5(1+i}{3c}$$

$$= -\frac{ibd^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{8ibd^5\left(1+c^2x^2\right)^{5/2}}{3c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{5bd^5}{2c(}$$

Mathematica [B] time = 8.75, size = 1331, normalized size = 2.83

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(5/2),x]
```

```
[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*((I*a*d^2)/f^3 + (((8*I)/3)*a*
d^2)/(f^3*(I + c*x)^2) - (28*a*d^2)/(3*f^3*(I + c*x))))/c + (5*a*d^(5/2)*Lo
g[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]])/(
c*f^(5/2)) - ((I/6)*b*d^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]
*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])
*(-(Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]/2]]
 + I*Log[Sqrt[1 + c^2*x^2]])) + Cosh[ArcSinh[c*x]/2]*(4*I + 3*ArcSinh[c*x]
- 6*ArcTan[Coth[ArcSinh[c*x]/2]] + (3*I)*Log[Sqrt[1 + c^2*x^2]]) + 2*(Sqrt[
1 + c^2*x^2]*(I*ArcSinh[c*x] + (2*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + Log[Sqr
t[1 + c^2*x^2]]) + 2*(1 + I*ArcSinh[c*x] + (2*I)*ArcTan[Coth[ArcSinh[c*x]/2
]] + Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/(c*f^3*(1 + I*c*x)*Sqr
t[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh
[c*x]/2])^4) + (b*d^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqr
t[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])*(Co
sh[(3*ArcSinh[c*x])/2]*((14*I - 3*ArcSinh[c*x])*ArcSinh[c*x] + (28*I)*ArcTa
n[Tanh[ArcSinh[c*x]/2]] - 14*Log[Sqrt[1 + c^2*x^2]]) + Cosh[ArcSinh[c*x]/2]
*(8 + (6*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 - (84*I)*ArcTan[Tanh[ArcSinh[c*
x]/2]] + 42*Log[Sqrt[1 + c^2*x^2]]) - (2*I)*(4 + (4*I)*ArcSinh[c*x] + 6*Arc
Sinh[c*x]^2 - (56*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 28*Log[Sqrt[1 + c^2*x^2
]] + Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(14*I + 3*ArcSinh[c*x]) - (28*I)*ArcTa
n[Tanh[ArcSinh[c*x]/2]] + 14*Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2])
)/(6*c*f^3*(1 + I*c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSin
h[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^4) - ((I/12)*b*d^2*Sqrt[I*((-I)*d + c*d
*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]
/2] + I*Sinh[ArcSinh[c*x]/2])*(-(Cosh[(3*ArcSinh[c*x])/2]*(9 - (35*I)*ArcSi
nh[c*x] + 9*ArcSinh[c*x]^2 + (52*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + 26*Log[S
qrt[1 + c^2*x^2]])) + Cosh[ArcSinh[c*x]/2]*(20 + (24*I)*ArcSinh[c*x] + 27*A
rcSinh[c*x]^2 + (156*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + 78*Log[Sqrt[1 + c^2*
```

```
x^2]]) - I*(3*(-I + ArcSinh[c*x])*Cosh[(5*ArcSinh[c*x])/2] + 2*(13 + (7*I)*
ArcSinh[c*x] + 18*ArcSinh[c*x]^2 + (104*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + (
3*I)*(I + ArcSinh[c*x])*Cosh[2*ArcSinh[c*x]] + 52*Log[Sqrt[1 + c^2*x^2]] +
Sqrt[1 + c^2*x^2]*(6 + (38*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 + (52*I)*ArcT
an[Coth[ArcSinh[c*x]/2]] + 26*Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]
)))/(c*f^3*(-I + c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh
[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^4)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(i\,bc^2d^2x^2+2\,bcd^2x-i\,bd^2\right)\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(i\,ac^2d^2x^2+2\,acd^2x-i\right.}{c^3f^3x^3+3i\,c^2f^3x^2-3\,cf^3x-i\,f^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral(((I*b*c^2*d^2*x^2 + 2*b*c*d^2*x - I*b*d^2)*sqrt(I*c*d*x + d)*sqrt(
-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (I*a*c^2*d^2*x^2 + 2*a*c*d^2*x
 - I*a*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^3*f^3*x^3 + 3*I*c^2*f^
3*x^2 - 3*c*f^3*x - I*f^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{(icdx+d)^{\frac{5}{2}}\,(a+b\,\mathrm{arcsinh}\,(cx))}{\left(-icfx+f\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(\frac{i\left(c^2dfx^2+df\right)^{\frac{5}{2}}}{c^5f^5x^4+4i\,c^4f^5x^3-6\,c^3f^5x^2-4i\,c^2f^5x+cf^5}-\frac{5i\left(c^2dfx^2+df\right)^{\frac{3}{2}}d}{3i\,c^4f^4x^3-9\,c^3f^4x^2-9i\,c^2f^4x+3\,cf^4}+\frac{10i\,\sqrt{c^2dfx^2}}{3\,c^3f^3x^2+6i\,c^2f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x, algorit
hm="maxima")
```

```
[Out] (I*(c^2*d*f*x^2 + d*f)^(5/2)/(c^5*f^5*x^4 + 4*I*c^4*f^5*x^3 - 6*c^3*f^5*x^2
 - 4*I*c^2*f^5*x + c*f^5) - 5*I*(c^2*d*f*x^2 + d*f)^(3/2)*d/(3*I*c^4*f^4*x^
3 - 9*c^3*f^4*x^2 - 9*I*c^2*f^4*x + 3*c*f^4) + 10*I*sqrt(c^2*d*f*x^2 + d*f)
*d^2/(3*c^3*f^3*x^2 + 6*I*c^2*f^3*x - 3*c*f^3) + 35*I*sqrt(c^2*d*f*x^2 + d*
f)*d^2/(-3*I*c^2*f^3*x + 3*c*f^3) + 5*d^3*arcsinh(c*x)/(c*f^3*sqrt(d/f)))*a
 + b*integrate((I*c*d*x + d)^(5/2)*log(c*x + sqrt(c^2*x^2 + 1))/(-I*c*f*x +
 f)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))\,\left(d+c\,d\,x\,1\mathrm{i}\right)^{5/2}}{\left(f-c\,f\,x\,1\mathrm{i}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(5/2),x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))/(f-I*c*f*x)**(5/2),x)
```

```
[Out] Timed out
```

3.565 $\int \frac{(d+icdx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{(f-icfx)^{5/2}}\,dx$

Optimal. Leaf size=362

$$\frac{2id^4(1+icx)\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2id^4(1+icx)^3\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{d^4\left(c^2x^2+1\right)^{5/2}\sinh}{c(d+icdx}$$

```
[Out] 4/3*I*b*d^4*(c^2*x^2+1)^(5/2)/c/(I+c*x)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
-1/2*b*d^4*(c^2*x^2+1)^(5/2)*arcsinh(c*x)^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)
^(5/2)-2/3*I*d^4*(1+I*c*x)^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(
5/2)/(f-I*c*f*x)^(5/2)+2*I*d^4*(1+I*c*x)*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/c
/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+d^4*(c^2*x^2+1)^(5/2)*arcsinh(c*x)*(a+
b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+8/3*b*d^4*(c^2*x^2+1)
^(5/2)*ln(I+c*x)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A] time = 0.39, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {5712, 669, 653, 215, 5819, 627, 43, 31, 5675}

$$\frac{2id^4(1+icx)\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2id^4(1+icx)^3\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{d^4\left(c^2x^2+1\right)^{5/2}\sinh}{c(d+icdx}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(5/2),x]
```

```
[Out] (((4*I)/3)*b*d^4*(1 + c^2*x^2)^(5/2))/(c*(I + c*x)*(d + I*c*d*x)^(5/2)*(f -
 I*c*f*x)^(5/2)) - (b*d^4*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x]^2)/(2*c*(d + I*c
*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((2*I)/3)*d^4*(1 + I*c*x)^3*(1 + c^2*x^
2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + ((2*
I)*d^4*(1 + I*c*x)*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(
5/2)*(f - I*c*f*x)^(5/2)) + (d^4*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x]*(a + b*Ar
cSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (8*b*d^4*(1 + c^
2*x^2)^(5/2)*Log[I + c*x])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$\int \frac{(d+icdx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{(f-icfx)^{5/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(d+icdx)^4\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{2id^4(1+icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2id^4(1+icx)\left(1+c^2x^2\right.}{c(d+icdx)^{5/2}}$$

$$= -\frac{2id^4(1+icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2id^4(1+icx)\left(1+c^2x^2\right.}{c(d+icdx)^{5/2}}$$

$$= -\frac{bd^4\left(1+c^2x^2\right)^{5/2}\sinh^{-1}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2id^4(1+icx)^3\left(1+c^2x^2\right)\left(a+b\sinh\right.}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{bd^4\left(1+c^2x^2\right)^{5/2}\sinh^{-1}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2id^4(1+icx)^3\left(1+c^2x^2\right)\left(a+b\sinh\right.}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{4ibd^4\left(1+c^2x^2\right)^{5/2}}{3c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bd^4\left(1+c^2x^2\right)^{5/2}\sinh^{-1}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} -$$

Mathematica [A] time = 5.64, size = 706, normalized size = 1.95

$$\frac{\frac{12ad^{3/2}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)}{f^{5/2}} - \frac{16ad(2cx+i)\sqrt{d+icdx}\sqrt{f-icfx}}{f^3(cx+i)^2} + \frac{bd\sqrt{d+icdx}\sqrt{f-icfx}\left(\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)+i\sinh\left(\frac{1}{2}\sinh^{-1}(\right.\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(5/2),x]
```

```
[Out] ((-16*a*d*(I + 2*c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(f^3*(I + c*x)^2
) + (12*a*d^(3/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f -
I*c*f*x]])/f^(5/2) - ((2*I)*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(Cosh[A
rcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])*(-(Cosh[(3*ArcSinh[c*x])/2]*(ArcSi
nh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]/2]] + (I/2)*Log[1 + c^2*x^2])) + Cosh[
ArcSinh[c*x]/2]*(4*I + 3*ArcSinh[c*x] - 6*ArcTan[Coth[ArcSinh[c*x]/2]] + ((
3*I)/2)*Log[1 + c^2*x^2]) + 2*(2 + (2*I)*ArcSinh[c*x] + (4*I)*ArcTan[Coth[A
rcSinh[c*x]/2]] + Log[1 + c^2*x^2] + (Sqrt[1 + c^2*x^2]*((2*I)*ArcSinh[c*x]
 + (4*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + Log[1 + c^2*x^2]))/2)*Sinh[ArcSinh[
c*x]/2]))/(f^3*(1 + I*c*x)*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^
4) + (b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(Cosh[ArcSinh[c*x]/2] + I*Sin
h[ArcSinh[c*x]/2])*(Cosh[(3*ArcSinh[c*x])/2]*((14*I - 3*ArcSinh[c*x])*ArcSi
nh[c*x] + (28*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] - 7*Log[1 + c^2*x^2]) + Cosh[
ArcSinh[c*x]/2]*(8 + (6*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 - (84*I)*ArcTan[
Tanh[ArcSinh[c*x]/2]] + 21*Log[1 + c^2*x^2]) - (2*I)*(4 + (4*I)*ArcSinh[c*x
] + 6*ArcSinh[c*x]^2 - (56*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 14*Log[1 + c^2
*x^2] + Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(14*I + 3*ArcSinh[c*x]) - (28*I)*Ar
cTan[Tanh[ArcSinh[c*x]/2]] + 7*Log[1 + c^2*x^2]))*Sinh[ArcSinh[c*x]/2]))/(f
^3*(1 + I*c*x)*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^4))/(12*c)
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(bcdx - i\,bd)\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right) + (acdx - i\,ad)\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}}{c^3f^3x^3 + 3i\,c^2f^3x^2 - 3\,cf^3x - i\,f^3}, x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral(((b*c*d*x - I*b*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
sqrt(c^2*x^2 + 1)) + (a*c*d*x - I*a*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)
)/(c^3*f^3*x^3 + 3*I*c^2*f^3*x^2 - 3*c*f^3*x - I*f^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(i\,cdx+d)^{\frac{3}{2}}(b\,\mathrm{arsinh}\,(cx)+a)}{\left(-i\,cfx+f\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((I*c*d*x + d)^(3/2)*(b*arcsinh(c*x) + a)/(-I*c*f*x + f)^(5/2), x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{(icdx+d)^{\frac{3}{2}}\,(a+b\,\mathrm{arcsinh}\,(cx))}{\left(-icfx+f\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x)
```

```
[Out] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(-\frac{i\left(c^2dfx^2+df\right)^{\frac{3}{2}}}{3i\,c^4f^4x^3-9\,c^3f^4x^2-9i\,c^2f^4x+3\,cf^4}+\frac{2i\,\sqrt{c^2dfx^2+df}\,d}{3\,c^3f^3x^2+6i\,c^2f^3x-3\,cf^3}+\frac{7i\,\sqrt{c^2dfx^2+df}\,d}{-3i\,c^2f^3x+3\,cf^3}+\frac{d^2\,\mathrm{arsinh}\,(cx)}{cf^3\sqrt{\frac{d}{f}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x, algorit
hm="maxima")
```

```
[Out] a*(-I*(c^2*d*f*x^2 + d*f)^(3/2)/(3*I*c^4*f^4*x^3 - 9*c^3*f^4*x^2 - 9*I*c^2*
f^4*x + 3*c*f^4) + 2*I*sqrt(c^2*d*f*x^2 + d*f)*d/(3*c^3*f^3*x^2 + 6*I*c^2*f
^3*x - 3*c*f^3) + 7*I*sqrt(c^2*d*f*x^2 + d*f)*d/(-3*I*c^2*f^3*x + 3*c*f^3)
+ d^2*arcsinh(c*x)/(c*f^3*sqrt(d/f))) + b*integrate((I*c*d*x + d)^(3/2)*log
(c*x + sqrt(c^2*x^2 + 1))/(-I*c*f*x + f)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))\,\left(d+c\,d\,x\,1{}\mathrm{i}\right)^{3/2}}{\left(f-c\,f\,x\,1{}\mathrm{i}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(5/2),x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(5/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))/(f-I*c*f*x)**(5/2),x)
[Out] Timed out
```

3.566 $\int \frac{\sqrt{d+icdx}\left(a+b\sinh^{-1}(cx)\right)}{(f-icfx)^{5/2}}\,dx$

Optimal. Leaf size=185

$$-\frac{id^3(1+icx)^3\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2ibd^3\left(c^2x^2+1\right)^{5/2}}{3c(cx+i)(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{bd^3\left(c^2x^2+1\right)^{5/2}\log(cx+i)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

```
[Out] 2/3*I*b*d^3*(c^2*x^2+1)^(5/2)/c/(I+c*x)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
-1/3*I*d^3*(1+I*c*x)^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(
f-I*c*f*x)^(5/2)+1/3*b*d^3*(c^2*x^2+1)^(5/2)*ln(I+c*x)/c/(d+I*c*d*x)^(5/2)/
(f-I*c*f*x)^(5/2)
```

Rubi [A] time = 0.31, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {5712, 651, 5819, 12, 627, 43}

$$-\frac{id^3(1+icx)^3\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2ibd^3\left(c^2x^2+1\right)^{5/2}}{3c(cx+i)(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{bd^3\left(c^2x^2+1\right)^{5/2}\log(cx+i)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(5/2),x]
```

```
[Out] (((2*I)/3)*b*d^3*(1 + c^2*x^2)^(5/2))/(c*(I + c*x)*(d + I*c*d*x)^(5/2)*(f -
 I*c*f*x)^(5/2)) - ((I/3)*d^3*(1 + I*c*x)^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*
x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (b*d^3*(1 + c^2*x^2)^(5/
2)*Log[I + c*x])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
```

```
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$\int \frac{\sqrt{d+icdx}\left(a+b\sinh^{-1}(cx)\right)}{(f-icfx)^{5/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(d+icdx)^3\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$
$$= -\frac{id^3(1+icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(bc\left(1+c^2x^2\right)^{5/2}\right)\int -\frac{id^3(}{3c(1}}{(d+icdx)^{5/2}(f-icf}$$
$$= -\frac{id^3(1+icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(ibd^3\left(1+c^2x^2\right)^{5/2}\right)\int \frac{(1}{(1+}}{3(d+icdx)^{5/2}(f-icf}$$
$$= -\frac{id^3(1+icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(ibd^3\left(1+c^2x^2\right)^{5/2}\right)\int \frac{1-}{(1-}}{3(d+icdx)^{5/2}(f-icf}$$
$$= -\frac{id^3(1+icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(ibd^3\left(1+c^2x^2\right)^{5/2}\right)\int \left(-\right.}{3(d+icdx)^{5/2}(f-}$$
$$= \frac{2ibd^3\left(1+c^2x^2\right)^{5/2}}{3c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{id^3(1+icx)^3\left(1+c^2x^2\right)\left(a+b\sin\right.}{3c(d+icdx)^{5/2}(f-icfx)^{5/}}$$

Mathematica [A] time = 0.43, size = 131, normalized size = 0.71

$$-\frac{id\sqrt{f-icfx}\left((cx-i)\left(acx-ia+b\sqrt{c^2x^2+1}\right)-b(cx+i)\sqrt{c^2x^2+1}\log(d(-1+icx))+b(cx-i)^2\sinh^{-1}(cx)\right.}{3cf^3(cx+i)^2\sqrt{d+icdx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(5/2),x]
```

```
[Out] ((-1/3*I)*d*Sqrt[f - I*c*f*x]*((-I + c*x)*((-I)*a + a*c*x + b*Sqrt[1 + c^2*
x^2]) + b*(-I + c*x)^2*ArcSinh[c*x] - b*(I + c*x)*Sqrt[1 + c^2*x^2]*Log[d*(
-1 + I*c*x)]))/(c*f^3*(I + c*x)^2*Sqrt[d + I*c*d*x])
```

fricas [B] time = 0.69, size = 565, normalized size = 3.05

$$-\frac{24\sqrt{c^2x^2+1}\sqrt{icdx+d}\sqrt{-icfx+f}\,bcx+3\left(4bc^2x^2-8ibcx-4b\right)\sqrt{icdx+d}\sqrt{-icfx+f}\log\left(cx+\sqrt{c^2x^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x, algorit
hm="fricas")
```

```
[Out] -1/3*(24*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x + 3*(
4*b*c^2*x^2 - 8*I*b*c*x - 4*b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x
 + sqrt(c^2*x^2 + 1)) - 2*(3*c^4*f^3*x^3 + 3*I*c^3*f^3*x^2 + 3*c^2*f^3*x +
3*I*c*f^3)*sqrt(b^2*d/(c^2*f^5))*log(1/3*(3*(2*I*b*c^6*x^2 - 4*b*c^5*x - 4*
I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) + 2*(3*I*c^
9*f^3*x^4 - 6*c^8*f^3*x^3 + 3*I*c^7*f^3*x^2 - 6*c^6*f^3*x)*sqrt(b^2*d/(c^2*
f^5)))/(16*b*c^3*x^3 + 16*I*b*c^2*x^2 + 16*b*c*x + 16*I*b)) + 2*(3*c^4*f^3*
x^3 + 3*I*c^3*f^3*x^2 + 3*c^2*f^3*x + 3*I*c*f^3)*sqrt(b^2*d/(c^2*f^5))*log(
1/3*(3*(2*I*b*c^6*x^2 - 4*b*c^5*x - 4*I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d
*x + d)*sqrt(-I*c*f*x + f) + 2*(-3*I*c^9*f^3*x^4 + 6*c^8*f^3*x^3 - 3*I*c^7*
f^3*x^2 + 6*c^6*f^3*x)*sqrt(b^2*d/(c^2*f^5)))/(16*b*c^3*x^3 + 16*I*b*c^2*x^
2 + 16*b*c*x + 16*I*b)) + 3*(4*a*c^2*x^2 - 8*I*a*c*x - 4*a)*sqrt(I*c*d*x +
d)*sqrt(-I*c*f*x + f))/(12*c^4*f^3*x^3 + 12*I*c^3*f^3*x^2 + 12*c^2*f^3*x +
12*I*c*f^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{i\,cdx+d}\,(b\,\mathrm{arsinh}\,(cx)+a)}{\left(-i\,cfx+f\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate(sqrt(I*c*d*x + d)*(b*arcsinh(c*x) + a)/(-I*c*f*x + f)^(5/2), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))\,\sqrt{icdx+d}}{\left(-icfx+f\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x)
```

maxima [A] time = 0.63, size = 220, normalized size = 1.19

$$-\frac{1}{3}\,bc\left(\frac{6\,\sqrt{d}}{3i\,c^3f^{\frac{5}{2}}x-3\,c^2f^{\frac{5}{2}}}-\frac{\sqrt{d}\,\log\,(cx+i)}{c^2f^{\frac{5}{2}}}\right)+b\left(\frac{2i\,\sqrt{c^2dfx^2+df}}{3\,c^3f^3x^2+6i\,c^2f^3x-3\,cf^3}+\frac{i\,\sqrt{c^2dfx^2+df}}{-3i\,c^2f^3x+3\,cf^3}\right)\mathrm{arsinh}\,(cx)+a\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x, algorit
hm="maxima")
```

```
[Out] -1/3*b*c*(6*sqrt(d)/(3*I*c^3*f^(5/2)*x - 3*c^2*f^(5/2)) - sqrt(d)*log(c*x +
 I)/(c^2*f^(5/2))) + b*(2*I*sqrt(c^2*d*f*x^2 + d*f)/(3*c^3*f^3*x^2 + 6*I*c^
2*f^3*x - 3*c*f^3) + I*sqrt(c^2*d*f*x^2 + d*f)/(-3*I*c^2*f^3*x + 3*c*f^3))*
arcsinh(c*x) + a*(2*I*sqrt(c^2*d*f*x^2 + d*f)/(3*c^3*f^3*x^2 + 6*I*c^2*f^3*
x - 3*c*f^3) + I*sqrt(c^2*d*f*x^2 + d*f)/(-3*I*c^2*f^3*x + 3*c*f^3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))\,\sqrt{d+c\,d\,x\,1\mathrm{i}}}{{\left(f-c\,f\,x\,1\mathrm{i}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(5/2),x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(5/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(5/2),x)
```

```
[Out] Timed out
```

3.567 $\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+icdx}\,(f-icfx)^{5/2}}\,dx$

Optimal. Leaf size=294

$$\frac{d^2x\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2id^2(1+icx)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{ibd^2\left(c^2x^2+1\right)^{5/2}}{3c(cx+i)(d+icdx)^{5/2}(f-icfx)^{5}}$$

```
[Out] 1/3*I*b*d^2*(c^2*x^2+1)^(5/2)/c/(I+c*x)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
-2/3*I*d^2*(1+I*c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-
I*c*f*x)^(5/2)+1/3*d^2*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)
/(f-I*c*f*x)^(5/2)+1/3*I*b*d^2*(c^2*x^2+1)^(5/2)*arctan(c*x)/c/(d+I*c*d*x)^
(5/2)/(f-I*c*f*x)^(5/2)-1/6*b*d^2*(c^2*x^2+1)^(5/2)*ln(c^2*x^2+1)/c/(d+I*c*
d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A] time = 0.33, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {5712, 653, 191, 5819, 627, 44, 203, 260}

$$\frac{d^2x\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2id^2(1+icx)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{ibd^2\left(c^2x^2+1\right)^{5/2}}{3c(cx+i)(d+icdx)^{5/2}(f-icfx)^{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)),x]
```

```
[Out] ((I/3)*b*d^2*(1 + c^2*x^2)^(5/2))/(c*(I + c*x)*(d + I*c*d*x)^(5/2)*(f - I*c
*f*x)^(5/2)) - (((2*I)/3)*d^2*(1 + I*c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]
))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (d^2*x*(1 + c^2*x^2)^2*(a
+ b*ArcSinh[c*x]))/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + ((I/3)*b*d
^2*(1 + c^2*x^2)^(5/2)*ArcTan[c*x])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5
/2)) - (b*d^2*(1 + c^2*x^2)^(5/2)*Log[1 + c^2*x^2])/(6*c*(d + I*c*d*x)^(5/2
)*(f - I*c*f*x)^(5/2))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+icdx}\,(f-icfx)^{5/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(d+icdx)^2\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{2id^2(1+icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^2x\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(c\right.}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{2id^2(1+icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^2x\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(c\right.}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{2id^2(1+icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^2x\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(c\right.}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{2id^2(1+icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^2x\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(c\right.}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{ibd^2\left(1+c^2x^2\right)^{5/2}}{3c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2id^2(1+icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx\right.}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{ibd^2\left(1+c^2x^2\right)^{5/2}}{3c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2id^2(1+icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx\right.}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Mathematica [A] time = 0.47, size = 139, normalized size = 0.47

$$\frac{\sqrt{f-icfx}\left((cx+2i)\left(iacx+a+ib\sqrt{c^2x^2+1}\right)+b(1-icx)\sqrt{c^2x^2+1}\log(d(-1+icx))+ib\left(c^2x^2+icx+2\right)\sinh^{-1}(cx)\right)}{3cf^3(cx+i)^2\sqrt{d+icdx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)),x]
```

```
[Out] (Sqrt[f - I*c*f*x]*((2*I + c*x)*(a + I*a*c*x + I*b*Sqrt[1 + c^2*x^2]) + I*b
*(2 + I*c*x + c^2*x^2)*ArcSinh[c*x] + b*(1 - I*c*x)*Sqrt[1 + c^2*x^2]*Log[d
*(-1 + I*c*x)]))/(3*c*f^3*(I + c*x)^2*Sqrt[d + I*c*d*x])
```

fricas [B] time = 0.79, size = 593, normalized size = 2.02

$$-\frac{12\sqrt{c^2x^2+1}\sqrt{icdx+d}\sqrt{-icfx+f}\,bcx-3\left(4bc^2x^2+4ibcx+8b\right)\sqrt{icdx+d}\sqrt{-icfx+f}\log\left(cx+\sqrt{c^2x^2+1}\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] -1/3*(12*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x - 3*(
4*b*c^2*x^2 + 4*I*b*c*x + 8*b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x
 + sqrt(c^2*x^2 + 1)) + 2*(3*c^4*d*f^3*x^3 + 3*I*c^3*d*f^3*x^2 + 3*c^2*d*f^
3*x + 3*I*c*d*f^3)*sqrt(b^2/(c^2*d*f^5))*log(1/3*(3*(2*I*b*c^6*x^2 - 4*b*c^
5*x - 4*I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) + 2
*(3*I*c^9*d*f^3*x^4 - 6*c^8*d*f^3*x^3 + 3*I*c^7*d*f^3*x^2 - 6*c^6*d*f^3*x)*
sqrt(b^2/(c^2*d*f^5)))/(16*b*c^3*x^3 + 16*I*b*c^2*x^2 + 16*b*c*x + 16*I*b))
 - 2*(3*c^4*d*f^3*x^3 + 3*I*c^3*d*f^3*x^2 + 3*c^2*d*f^3*x + 3*I*c*d*f^3)*sq
rt(b^2/(c^2*d*f^5))*log(1/3*(3*(2*I*b*c^6*x^2 - 4*b*c^5*x - 4*I*b*c^4)*sqrt
(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) + 2*(-3*I*c^9*d*f^3*x^4
+ 6*c^8*d*f^3*x^3 - 3*I*c^7*d*f^3*x^2 + 6*c^6*d*f^3*x)*sqrt(b^2/(c^2*d*f^5)
))/(16*b*c^3*x^3 + 16*I*b*c^2*x^2 + 16*b*c*x + 16*I*b)) - 3*(4*a*c^2*x^2 +
4*I*a*c*x + 8*a)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(12*c^4*d*f^3*x^3 +
12*I*c^3*d*f^3*x^2 + 12*c^2*d*f^3*x + 12*I*c*d*f^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\operatorname{arsinh}(cx)+a}{\sqrt{icdx+d}\left(-icfx+f\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(I*c*d*x + d)*(-I*c*f*x + f)^(5/2)), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{\left(-icfx+f\right)^{\frac{5}{2}}\sqrt{icdx+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x)
```

maxima [A] time = 0.64, size = 232, normalized size = 0.79

$$-\frac{1}{3}\,bc\left(\frac{3}{3i\,c^3\sqrt{d}\,f^{\frac{5}{2}}x-3\,c^2\sqrt{d}\,f^{\frac{5}{2}}}+\frac{\log\left(cx+i\right)}{c^2\sqrt{d}\,f^{\frac{5}{2}}}\right)+b\left(\frac{i\,\sqrt{c^2dfx^2+df}}{3\,c^3df^3x^2+6i\,c^2df^3x-3\,cdf^3}-\frac{i\,\sqrt{c^2dfx^2+df}}{-3i\,c^2df^3x+3\,cdf^3}\right)\text{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] -1/3*b*c*(3/(3*I*c^3*sqrt(d)*f^(5/2)*x - 3*c^2*sqrt(d)*f^(5/2)) + log(c*x +
 I)/(c^2*sqrt(d)*f^(5/2))) + b*(I*sqrt(c^2*d*f*x^2 + d*f)/(3*c^3*d*f^3*x^2
+ 6*I*c^2*d*f^3*x - 3*c*d*f^3) - I*sqrt(c^2*d*f*x^2 + d*f)/(-3*I*c^2*d*f^3*
x + 3*c*d*f^3))*arcsinh(c*x) + a*(I*sqrt(c^2*d*f*x^2 + d*f)/(3*c^3*d*f^3*x^
2 + 6*I*c^2*d*f^3*x - 3*c*d*f^3) - I*sqrt(c^2*d*f*x^2 + d*f)/(-3*I*c^2*d*f^
3*x + 3*c*d*f^3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}\left(c\,x\right)}{\sqrt{d+c\,d\,x\,1{}\mathrm{i}}\,{\left(f-c\,f\,x\,1{}\mathrm{i}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(5/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asinh}{\left(cx \right)}}{\sqrt{id \left(cx - i\right)} \left(- if \left(cx + i\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(f-I*c*f*x)**(5/2)/(d+I*c*d*x)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(sqrt(I*d*(c*x - I))*(-I*f*(c*x + I))**(5/2)),
x)
```

3.568 $\int \frac{a+b\sinh^{-1}(cx)}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx$

Optimal. Leaf size=282

$$\frac{2dx\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{d(-cx+i)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{ibd\left(c^2x^2+1\right)^{5/2}}{6c(cx+i)(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

```
[Out] 1/6*I*b*d*(c^2*x^2+1)^(5/2)/c/(I+c*x)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1
/3*d*(I-c*x)*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)
^(5/2)+2/3*d*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*
x)^(5/2)+1/6*I*b*d*(c^2*x^2+1)^(5/2)*arctan(c*x)/c/(d+I*c*d*x)^(5/2)/(f-I*c
*f*x)^(5/2)-1/3*b*d*(c^2*x^2+1)^(5/2)*ln(c^2*x^2+1)/c/(d+I*c*d*x)^(5/2)/(f-
I*c*f*x)^(5/2)
```

Rubi [A] time = 0.30, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {5712, 639, 191, 5819, 627, 44, 203, 260}

$$\frac{2dx\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{d(-cx+i)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{ibd\left(c^2x^2+1\right)^{5/2}}{6c(cx+i)(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)),x]
```

```
[Out] ((I/6)*b*d*(1 + c^2*x^2)^(5/2))/(c*(I + c*x)*(d + I*c*d*x)^(5/2)*(f - I*c*f
*x)^(5/2)) - (d*(I - c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(3*c*(d + I*c
*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (2*d*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*
x]))/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + ((I/6)*b*d*(1 + c^2*x^2)
^(5/2)*ArcTan[c*x])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (b*d*(1 +
 c^2*x^2)^(5/2)*Log[1 + c^2*x^2])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5
/2))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(d+icdx)\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{d(i-cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2dx\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx\right.}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{d(i-cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2dx\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx\right.}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{d(i-cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2dx\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx\right.}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{d(i-cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2dx\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx\right.}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{ibd\left(1+c^2x^2\right)^{5/2}}{6c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{d(i-cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{ibd\left(1+c^2x^2\right)^{5/2}}{6c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{d(i-cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Mathematica [A] time = 0.60, size = 202, normalized size = 0.72

$$\frac{\sqrt{f-icfx}\left(8iac^2x^2-8acx+4ia-3ibcx\sqrt{c^2x^2+1}\log(d+icdx)+5b(1-icx)\sqrt{c^2x^2+1}\log(d(-1+icx))+3b\sqrt{\right.}{12cdf^3(cx+i)^2\sqrt{d+icdx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)),x]
```

```
[Out] (Sqrt[f - I*c*f*x]*((4*I)*a - 8*a*c*x + (8*I)*a*c^2*x^2 - 2*b*Sqrt[1 + c^2*
x^2] + (4*I)*b*(1 + (2*I)*c*x + 2*c^2*x^2)*ArcSinh[c*x] + 5*b*(1 - I*c*x)*S
qrt[1 + c^2*x^2]*Log[d*(-1 + I*c*x)] + 3*b*Sqrt[1 + c^2*x^2]*Log[d + I*c*d*
x] - (3*I)*b*c*x*Sqrt[1 + c^2*x^2]*Log[d + I*c*d*x]))/(12*c*d*f^3*(I + c*x)
^2*Sqrt[d + I*c*d*x])
```

fricas [F] time = 1.11, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x, algorit
hm="fricas")
```

```
[Out] -1/3*(48*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x - 3*(
64*b*c^2*x^2 + 64*I*b*c*x + 32*b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(
c*x + sqrt(c^2*x^2 + 1)) - 4*(15*c^4*d^2*f^3*x^3 + 15*I*c^3*d^2*f^3*x^2 + 1
5*c^2*d^2*f^3*x + 15*I*c*d^2*f^3)*sqrt(b^2/(c^2*d^3*f^5))*log((4*I*sqrt(c^2
*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d*f^2*x*sqrt(b^2/(c^2*d^3*
f^5)) + 4*I*b*c^2*x^3 + 4*I*b*x)/(4*b*c^3*x^3 + 4*I*b*c^2*x^2 + 4*b*c*x + 4
*I*b)) + 4*(9*c^4*d^2*f^3*x^3 + 9*I*c^3*d^2*f^3*x^2 + 9*c^2*d^2*f^3*x + 9*I
*c*d^2*f^3)*sqrt(b^2/(c^2*d^3*f^5))*log((4*I*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x
 + d)*sqrt(-I*c*f*x + f)*c*d*f^2*x*sqrt(b^2/(c^2*d^3*f^5)) - 4*I*b*c^2*x^3
- 4*I*b*x)/(4*b*c^3*x^3 - 4*I*b*c^2*x^2 + 4*b*c*x - 4*I*b)) + 4*(15*c^4*d^2
*f^3*x^3 + 15*I*c^3*d^2*f^3*x^2 + 15*c^2*d^2*f^3*x + 15*I*c*d^2*f^3)*sqrt(b
^2/(c^2*d^3*f^5))*log((-4*I*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f
*x + f)*c*d*f^2*x*sqrt(b^2/(c^2*d^3*f^5)) + 4*I*b*c^2*x^3 + 4*I*b*x)/(4*b*c
^3*x^3 + 4*I*b*c^2*x^2 + 4*b*c*x + 4*I*b)) - 4*(9*c^4*d^2*f^3*x^3 + 9*I*c^3
*d^2*f^3*x^2 + 9*c^2*d^2*f^3*x + 9*I*c*d^2*f^3)*sqrt(b^2/(c^2*d^3*f^5))*log
((-4*I*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d*f^2*x*sqr
t(b^2/(c^2*d^3*f^5)) - 4*I*b*c^2*x^3 - 4*I*b*x)/(4*b*c^3*x^3 - 4*I*b*c^2*x^
2 + 4*b*c*x - 4*I*b)) + 4*(24*c^4*d^2*f^3*x^3 + 24*I*c^3*d^2*f^3*x^2 + 24*c
^2*d^2*f^3*x + 24*I*c*d^2*f^3)*sqrt(b^2/(c^2*d^3*f^5))*log((sqrt(c^2*x^2 +
1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d*f^2*x*sqrt(b^2/(c^2*d^3*f^5)) +
 b*c^2*x^3 + b*x)/(b*c^2*x^2 + b)) - 4*(24*c^4*d^2*f^3*x^3 + 24*I*c^3*d^2*f
^3*x^2 + 24*c^2*d^2*f^3*x + 24*I*c*d^2*f^3)*sqrt(b^2/(c^2*d^3*f^5))*log(-(s
qrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d*f^2*x*sqrt(b^2/(c
^2*d^3*f^5)) - b*c^2*x^3 - b*x)/(b*c^2*x^2 + b)) - 3*(64*a*c^2*x^2 + 64*I*a
*c*x + 32*a)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) - 3*(96*c^4*d^2*f^3*x^3 +
 96*I*c^3*d^2*f^3*x^2 + 96*c^2*d^2*f^3*x + 96*I*c*d^2*f^3)*integral(-1/6*sq
rt(c^2*x^2 + 1)*(4*b*c*x - I*b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)/(c^4*d
^2*f^3*x^4 + 2*c^2*d^2*f^3*x^2 + d^2*f^3), x))/(96*c^4*d^2*f^3*x^3 + 96*I*c
^3*d^2*f^3*x^2 + 96*c^2*d^2*f^3*x + 96*I*c*d^2*f^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [c,f,t_nostep]=[-96,-55,-34]schur row 3 -1.34095e-07schur row 3 1.
40652e-07Warning, choosing root of [1,0,%%%{-8,[2,4,2]%%%}+%%%{-6,[0,0,0]%%
%},%%%{-32,[2,4,2]%%%}+%%%{-8,[0,0,0]%%%},%%%{16,[4,8,4]%%%}+%%%{-24,[2,4,2
]%%%}+%%%{-3,[0,0,0]%%%}] at parameters values [26,-89,63]Warning, need to
choose a branch for the root of a polynomial with parameters. This might be
 wrong.The choice was done assuming [c,f,t_nostep]=[-64,-69,46]schur row 3
9.51285e-08schur row 3 -3.31158e-07Warning, choosing root of [1,0,%%%{-8,[2
,4,2]%%%}+%%%{-6,[0,0,0]%%%},%%%{-32,[2,4,2]%%%}+%%%{-8,[0,0,0]%%%},%%%{16,
[4,8,4]%%%}+%%%{-24,[2,4,2]%%%}+%%%{-3,[0,0,0]%%%}] at parameters values [-
64,-30,70]Warning, need to choose a branch for the root of a polynomial wit
h parameters. This might be wrong.The choice was done assuming [c,f,t_noste
p]=[93,80,-3]schur row 1 3.91518e-07Francis algorithm not precise enough fo
r[1.0,0.0,-2.550693888e+13,-5.101387776e+13,1.62650982757e+26]Warning, need
 to choose a branch for the root of a polynomial with parameters. This migh
t be wrong.The choice was done assuming [c,f,t_nostep]=[90,38,-42]schur row
 3 -1.59567e-07Warning, need to choose a branch for the root of a polynomia
l with parameters. This might be wrong.The choice was done assuming [c,f,t_
nostep]=[13,-42,-63]Warning, choosing root of [1,0,%%%{-8,[2,4,2]%%%}+%%%{-
6,[0,0,0]%%%},%%%{-32,[2,4,2]%%%}+%%%{-8,[0,0,0]%%%},%%%{16,[4,8,4]%%%}+%%%
{-24,[2,4,2]%%%}+%%%{-3,[0,0,0]%%%}] at parameters values [-80,-23,65]Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [c,f,t_nostep]=[-79,-6,-51]
schur row 3 9.78233e-07Warning, choosing root of [1,0,%%%{-8,[2,4,2]%%%}+%%
%{-6,[0,0,0]%%%},%%%{-32,[2,4,2]%%%}+%%%{-8,[0,0,0]%%%},%%%{16,[4,8,4]%%%}+
%%%{-24,[2,4,2]%%%}+%%%{-3,[0,0,0]%%%}] at parameters values [-44,-22,93]Wa
rning, need to choose a branch for the root of a polynomial with parameters
. This might be wrong.The choice was done assuming [c,f,t_nostep]=[-18,95,-
20]Warning, need to choose a branch for the root of a polynomial with param
eters. This might be wrong.The choice was done assuming [c,f,t_nostep]=[-67
,8,81]ext_reduce Error: Bad Argument Typeext_reduce Error: Bad Argument Typ
eEvaluation time: 2.3Done
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arcsinh}\,(cx)}{(icdx+d)^{\frac{3}{2}}\left(-icfx+f\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x)
```

maxima [A] time = 0.62, size = 237, normalized size = 0.84

$$\frac{1}{12}\,bc\left(\frac{2i\,\sqrt{d}\,\sqrt{f}}{c^3d^2f^3x+i\,c^2d^2f^3}-\frac{5\,\log\left(cx+i\right)}{c^2d^{\frac{3}{2}}f^{\frac{5}{2}}}-\frac{3\,\log\left(cx-i\right)}{c^2d^{\frac{3}{2}}f^{\frac{5}{2}}}\right)-\frac{1}{3}\,b\left(\frac{3i}{-3i\,\sqrt{c^2dfx^2+df}\,c^2df^2x+3\,\sqrt{c^2dfx^2+df}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x, algorit
hm="maxima")
```

```
[Out] 1/12*b*c*(2*I*sqrt(d)*sqrt(f)/(c^3*d^2*f^3*x + I*c^2*d^2*f^3) - 5*log(c*x +
 I)/(c^2*d^(3/2)*f^(5/2)) - 3*log(c*x - I)/(c^2*d^(3/2)*f^(5/2))) - 1/3*b*(
```

```
3*I/(-3*I*sqrt(c^2*d*f*x^2 + d*f)*c^2*d*f^2*x + 3*sqrt(c^2*d*f*x^2 + d*f)*c
*d*f^2) - 2*x/(sqrt(c^2*d*f*x^2 + d*f)*d*f^2))*arcsinh(c*x) - 1/3*a*(3*I/(-
3*I*sqrt(c^2*d*f*x^2 + d*f)*c^2*d*f^2*x + 3*sqrt(c^2*d*f*x^2 + d*f)*c*d*f^2
) - 2*x/(sqrt(c^2*d*f*x^2 + d*f)*d*f^2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}\left(c\,x\right)}{{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^{3/2}\,{\left(f-c\,f\,x\,1{}\mathrm{i}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(5/2)),x)
[Out] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(3/2)/(f-I*c*f*x)**(5/2),x)
[Out] Timed out
```

3.569 $\int \frac{a+b\sinh^{-1}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx$

Optimal. Leaf size=203

$$\frac{2x\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{b\left(c^2x^2+1\right)^{3/2}}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{b\left(c^2x^2+1\right)^{5/}}{3c(d+icdx)}$$

```
[Out] 1/6*b*(c^2*x^2+1)^(3/2)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*x*(c^2*x^
2+1)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+2/3*x*(c^2*x^2+
1)^2*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*b*(c^2*x^2+
1)^(5/2)*ln(c^2*x^2+1)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A] time = 0.24, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5712, 5690, 5687, 260, 261}

$$\frac{2x\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{b\left(c^2x^2+1\right)^{3/2}}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{b\left(c^2x^2+1\right)^{5/}}{3c(d+icdx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)),x]
```

```
[Out] (b*(1 + c^2*x^2)^(3/2))/(6*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (x*
(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5
/2)) + (2*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(3*(d + I*c*d*x)^(5/2)*(f
 - I*c*f*x)^(5/2)) - (b*(1 + c^2*x^2)^(5/2)*Log[1 + c^2*x^2])/(3*c*(d + I*c
*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{a+b\sinh^{-1}(cx)}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(2\left(1+c^2x^2\right)^{5/2}\right)\int \frac{a+b\sinh^{-1}(cx)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(bc\left(1+\right.\right.}{3(d+}$$

$$= \frac{b\left(1+c^2x^2\right)^{3/2}}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2x\left(1+c^2x^2\right)^2}{3(d+icdx)}$$

$$= \frac{b\left(1+c^2x^2\right)^{3/2}}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2x\left(1+c^2x^2\right)^2}{3(d+icdx)}$$

Mathematica [A] time = 0.57, size = 193, normalized size = 0.95

$$\frac{i\sqrt{f-icfx}\left(4ac^3x^3+6acx-2bc^2x^2\sqrt{c^2x^2+1}\log(d+icdx)-2b\left(c^2x^2+1\right)^{3/2}\log(d(-1+icx))-2b\sqrt{c^2x^2+1}\,\mathrm{l}\right.}{6cd^2f^3(cx-i)(cx+i)^2\sqrt{d+icdx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)),x]
```

```
[Out] ((I/6)*Sqrt[f - I*c*f*x]*(6*a*c*x + 4*a*c^3*x^3 + b*Sqrt[1 + c^2*x^2] + 2*b
*c*x*(3 + 2*c^2*x^2)*ArcSinh[c*x] - 2*b*(1 + c^2*x^2)^(3/2)*Log[d*(-1 + I*c
*x)] - 2*b*Sqrt[1 + c^2*x^2]*Log[d + I*c*d*x] - 2*b*c^2*x^2*Sqrt[1 + c^2*x^
2]*Log[d + I*c*d*x]))/(c*d^2*f^3*(-I + c*x)*(I + c*x)^2*Sqrt[d + I*c*d*x])
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$-\frac{\sqrt{c^2x^2+1}\,\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,bcx^2-2\left(2\,bc^2x^3+3\,bx\right)\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)-\left(c^4d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x, algorit
hm="fricas")
```

```
[Out] -1/6*(sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x^2 - 2*(2
*b*c^2*x^3 + 3*b*x)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2
*x^2 + 1)) - (c^4*d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)*sqrt(b^2/(c^2*
d^5*f^5))*log((sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d^2
*f^2*x^2*sqrt(b^2/(c^2*d^5*f^5)) + b*c^2*x^4 + b*x^2)/(b*c^4*x^4 + 2*b*c^2*
x^2 + b)) + (c^4*d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)*sqrt(b^2/(c^2*d
^5*f^5))*log(-(sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d^2
```

```
*f^2*x^2*sqrt(b^2/(c^2*d^5*f^5)) - b*c^2*x^4 - b*x^2)/(b*c^4*x^4 + 2*b*c^2*
x^2 + b)) + 2*(c^4*d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)*sqrt(b^2/(c^2
*d^5*f^5))*log((sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d^
2*f^2*x*sqrt(b^2/(c^2*d^5*f^5)) + b*c^2*x^3 + b*x)/(b*c^2*x^2 + b)) - 2*(c^
4*d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)*sqrt(b^2/(c^2*d^5*f^5))*log(-(
sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d^2*f^2*x*sqrt(b^2
/(c^2*d^5*f^5)) - b*c^2*x^3 - b*x)/(b*c^2*x^2 + b)) - 2*(2*a*c^2*x^3 + 3*a*
x)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) - 6*(c^4*d^3*f^3*x^4 + 2*c^2*d^3*f^
3*x^2 + d^3*f^3)*integral(-2/3*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*
c*f*x + f)*b*c*x/(c^4*d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3), x))/(c^4*
d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [c,d,t_nostep]=[-62,5,7]Warning, choosing root of [1,0,%%%{-8,[2,4
,2]%%%}+%%%{-6,[0,0,0]%%%},%%%{-32,[2,4,2]%%%}+%%%{-8,[0,0,0]%%%},%%%{16,[4
,8,4]%%%}+%%%{-24,[2,4,2]%%%}+%%%{-3,[0,0,0]%%%}] at parameters values [26,
-89,63]Warning, need to choose a branch for the root of a polynomial with p
arameters. This might be wrong.The choice was done assuming [c,d,t_nostep]=
[50,45,-24]Warning, choosing root of [1,0,%%%{-8,[2,4,2]%%%}+%%%{-6,[0,0,0]
%%%},%%%{-32,[2,4,2]%%%}+%%%{-8,[0,0,0]%%%},%%%{16,[4,8,4]%%%}+%%%{-24,[2,4
,2]%%%}+%%%{-3,[0,0,0]%%%}] at parameters values [-64,-30,70]Warning, need
to choose a branch for the root of a polynomial with parameters. This might
 be wrong.The choice was done assuming [c,d,t_nostep]=[-43,-99,-71]schur ro
w 3 -2.20626e-08Warning, need to choose a branch for the root of a polynomi
al with parameters. This might be wrong.The choice was done assuming [c,d,t
_nostep]=[29,40,-81]Warning, need to choose a branch for the root of a poly
nomial with parameters. This might be wrong.The choice was done assuming [c
,d,t_nostep]=[-69,45,-8]schur row 1 4.86814e-07Francis algorithm not precis
e enough for[1.0,0.0,-6.053520512e+13,-2.4214082048e+14,9.1612776473e+26]Wa
rning, choosing root of [1,0,%%%{-8,[2,4,2]%%%}+%%%{-6,[0,0,0]%%%},%%%{-32,
[2,4,2]%%%}+%%%{-8,[0,0,0]%%%},%%%{16,[4,8,4]%%%}+%%%{-24,[2,4,2]%%%}+%%%{-
3,[0,0,0]%%%}] at parameters values [-80,-23,65]Warning, need to choose a b
ranch for the root of a polynomial with parameters. This might be wrong.The
 choice was done assuming [c,d,t_nostep]=[-35,-31,-9]Warning, choosing root
 of [1,0,%%%{-8,[2,4,2]%%%}+%%%{-6,[0,0,0]%%%},%%%{-32,[2,4,2]%%%}+%%%{-8,[
0,0,0]%%%},%%%{16,[4,8,4]%%%}+%%%{-24,[2,4,2]%%%}+%%%{-3,[0,0,0]%%%}] at pa
rameters values [-44,-22,93]Warning, need to choose a branch for the root o
f a polynomial with parameters. This might be wrong.The choice was done ass
uming [c,d,t_nostep]=[-90,-5,-92]Warning, need to choose a branch for the r
oot of a polynomial with parameters. This might be wrong.The choice was don
e assuming [c,d,t_nostep]=[-96,-85,-8]schur row 1 1.20095e-07Francis algori
thm not precise enough for[1.0,0.0,-2.4631345152e+14,-4.9262690304e+14,1.51
675790999e+28]ext_reduce Error: Bad Argument Typeext_reduce Error: Bad Argu
ment TypeWarning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [c,f,t_nostep
]=[10,-24,-7]Warning, choosing root of [1,0,%%%{-8,[2,4,2]%%%}+%%%{-6,[0,0,
0]%%%},%%%{-32,[2,4,2]%%%}+%%%{-8,[0,0,0]%%%},%%%{16,[4,8,4]%%%}+%%%{-24,[2
,4,2]%%%}+%%%{-3,[0,0,0]%%%}] at parameters values [-24,-17,41]Warning, nee
d to choose a branch for the root of a polynomial with parameters. This mig
ht be wrong.The choice was done assuming [c,f,t_nostep]=[82,83,53]schur row
```

```
 3 2.05953e-07Warning, choosing root of [1,0,%%%{-8,[2,4,2]%%%}+%%%{-6,[0,0
,0]%%%},%%%{-32,[2,4,2]%%%}+%%%{-8,[0,0,0]%%%},%%%{16,[4,8,4]%%%}+%%%{-24,[
2,4,2]%%%}+%%%{-3,[0,0,0]%%%}] at parameters values [-51,-42,-65]Warning, n
eed to choose a branch for the root of a polynomial with parameters. This m
ight be wrong.The choice was done assuming [c,f,t_nostep]=[-93,50,64]schur
row 3 -1.05315e-07Warning, need to choose a branch for the root of a polyno
mial with parameters. This might be wrong.The choice was done assuming [c,f
,t_nostep]=[-26,-23,30]Warning, need to choose a branch for the root of a p
olynomial with parameters. This might be wrong.The choice was done assuming
 [c,f,t_nostep]=[90,62,-68]Warning, choosing root of [1,0,%%%{-8,[2,4,2]%%%
}+%%%{-6,[0,0,0]%%%},%%%{-32,[2,4,2]%%%}+%%%{-8,[0,0,0]%%%},%%%{16,[4,8,4]%
%%}+%%%{-24,[2,4,2]%%%}+%%%{-3,[0,0,0]%%%}] at parameters values [-7,81,82]
Warning, need to choose a branch for the root of a polynomial with paramete
rs. This might be wrong.The choice was done assuming [c,f,t_nostep]=[-11,3,
-26]Warning, choosing root of [1,0,%%%{-8,[2,4,2]%%%}+%%%{-6,[0,0,0]%%%},%%
%{-32,[2,4,2]%%%}+%%%{-8,[0,0,0]%%%},%%%{16,[4,8,4]%%%}+%%%{-24,[2,4,2]%%%}
+%%%{-3,[0,0,0]%%%}] at parameters values [-11,-50,-53]Warning, need to cho
ose a branch for the root of a polynomial with parameters. This might be wr
ong.The choice was done assuming [c,f,t_nostep]=[-78,84,2]schur row 3 -6.60
848e-07Warning, need to choose a branch for the root of a polynomial with p
arameters. This might be wrong.The choice was done assuming [c,f,t_nostep]=
[-59,-23,-54]schur row 1 5.11765e-07Francis algorithm not precise enough fo
r[1.0,0.0,-2.27244234819e+13,-4.54488469638e+13,1.29099855646e+26]ext_reduc
e Error: Bad Argument Typeext_reduce Error: Bad Argument TypeEvaluation tim
e: 4.24Done
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(icdx+d)^{\frac{5}{2}}\left(-icfx+f\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x)
```

maxima [A] time = 0.53, size = 159, normalized size = 0.78

$$\frac{1}{6}\,bc\left(\frac{1}{c^4d^{\frac{5}{2}}f^{\frac{5}{2}}x^2+c^2d^{\frac{5}{2}}f^{\frac{5}{2}}}-\frac{2\,\log\left(c^2x^2+1\right)}{c^2d^{\frac{5}{2}}f^{\frac{5}{2}}}\right)+\frac{1}{3}\,b\left(\frac{x}{\left(c^2dfx^2+df\right)^{\frac{3}{2}}df}+\frac{2\,x}{\sqrt{c^2dfx^2+df}\,d^2f^2}\right)\operatorname{arsinh}(cx)+\frac{1}{3}\,a\Bigg(\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x, algorit
hm="maxima")
```

```
[Out] 1/6*b*c*(1/(c^4*d^(5/2)*f^(5/2)*x^2 + c^2*d^(5/2)*f^(5/2)) - 2*log(c^2*x^2
+ 1)/(c^2*d^(5/2)*f^(5/2))) + 1/3*b*(x/((c^2*d*f*x^2 + d*f)^(3/2)*d*f) + 2*
x/(sqrt(c^2*d*f*x^2 + d*f)*d^2*f^2))*arcsinh(c*x) + 1/3*a*(x/((c^2*d*f*x^2
+ d*f)^(3/2)*d*f) + 2*x/(sqrt(c^2*d*f*x^2 + d*f)*d^2*f^2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}\left(c\,x\right)}{{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^{5/2}\,{\left(f-c\,f\,x\,1{}\mathrm{i}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(5/2)),x)
[Out] int((a + b*asinh(c*x))/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(5/2)), x)
```

sympy [**F(-1**)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(5/2)/(f-I*c*f*x)**(5/2),x)
[Out] Timed out
```

3.570 $\int (d+icdx)^{5/2}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=680

$$-\frac{1}{4}c^2d^2x^3\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)^2-\frac{3bcd^2x^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)}{8\sqrt{c^2x^2+1}}-\frac{4ibd^2x\sqrt{d}}{}$$

```
[Out] 8/9*I*b^2*d^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c+15/64*b^2*d^2*x*(d+I*c*
d*x)^(1/2)*(f-I*c*f*x)^(1/2)-1/32*b^2*c^2*d^2*x^3*(d+I*c*d*x)^(1/2)*(f-I*c*
f*x)^(1/2)+4/27*I*b^2*d^2*(c^2*x^2+1)*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c
+3/8*d^2*x*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-1/4*c^2
*d^2*x^3*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)+2/3*I*d^2
*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c-15/
64*b^2*d^2*arcsinh(c*x)*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c/(c^2*x^2+1)^(
1/2)-4/3*I*b*d^2*x*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(
c^2*x^2+1)^(1/2)-3/8*b*c*d^2*x^2*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*
c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-4/9*I*b*c^2*d^2*x^3*(a+b*arcsinh(c*x))*(d+I*
c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/8*b*c^3*d^2*x^4*(a+b*arc
sinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+5/24*d^2*(
a+b*arcsinh(c*x))^3*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/b/c/(c^2*x^2+1)^(1/
2)
```

Rubi [A] time = 1.11, antiderivative size = 680, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 13, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.351, Rules used = {5712, 5821, 5682, 5675, 5661, 321, 215, 5717, 5679, 444, 43, 5742, 5758}

$$\frac{bc^3d^2x^4\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)}{8\sqrt{c^2x^2+1}}-\frac{1}{4}c^2d^2x^3\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)^2-\frac{4ibc^2d^2x^3\sqrt{d}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (((8*I)/9)*b^2*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/c + (15*b^2*d^2*x*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/64 - (b^2*c^2*d^2*x^3*Sqrt[d + I*c*d*x]
*Sqrt[f - I*c*f*x])/32 + (((4*I)/27)*b^2*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c
*f*x]*(1 + c^2*x^2))/c - (15*b^2*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Ar
cSinh[c*x])/(64*c*Sqrt[1 + c^2*x^2]) - (((4*I)/3)*b*d^2*x*Sqrt[d + I*c*d*x]
*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2] - (3*b*c*d^2*x^2
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(8*Sqrt[1 + c^2*
x^2]) - (((4*I)/9)*b*c^2*d^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b
*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2] + (b*c^3*d^2*x^4*Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x]*(a + b*ArcSinh[c*x]))/(8*Sqrt[1 + c^2*x^2]) + (3*d^2*x*Sqrt[d +
 I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/8 - (c^2*d^2*x^3*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/4 + (((2*I)/3)*d^2*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/c +
 (5*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^3)/(24*b*c
*Sqrt[1 + c^2*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5679

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
```

```
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2\,dx = \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int (d+icdx)^2\sqrt{1+c^2x^2}\left(a+b\sinh\right.}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\left(d^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\right.}{}$$

$$= \frac{\left(d^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}$$

$$= \frac{1}{2}d^2x\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{1}{4}c^2d^2x^3\sqrt{}$$

$$= -\frac{4ibd^2x\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{3\sqrt{1+c^2x^2}}-\frac{bcd^2x^2}{}$$

$$= \frac{1}{4}b^2d^2x\sqrt{d+icdx}\sqrt{f-icfx}-\frac{1}{32}b^2c^2d^2x^3\sqrt{d+icdx}\sqrt{f-}$$

$$= \frac{15}{64}b^2d^2x\sqrt{d+icdx}\sqrt{f-icfx}-\frac{1}{32}b^2c^2d^2x^3\sqrt{d+icdx}\sqrt{f}$$

$$= \frac{8ib^2d^2\sqrt{d+icdx}\sqrt{f-icfx}}{9c}+\frac{15}{64}b^2d^2x\sqrt{d+icdx}\sqrt{f-ic}$$

Mathematica [A] time = 2.41, size = 890, normalized size = 1.31

$$\frac{-1728a^2c^3d^2\sqrt{icxd+d}\sqrt{f-icfx}\sqrt{c^2x^2+1}\,x^3+4608ia^2c^2d^2\sqrt{icxd+d}\sqrt{f-icfx}\sqrt{c^2x^2+1}\,x^2-6912iabcd^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] ((-6912*I)*a*b*c*d^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (4608*I)*a^2*d
^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (6912*I)*b^2*d^2
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 2592*a^2*c*d^2*x*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (4608*I)*a^2*c^2*d^2
*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 1728*a^2*c^3*d
^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 1440*b^2*d^2
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 - 1728*a*b*d^2*Sqrt[d +
 I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] + (256*I)*b^2*d^2*Sqrt[d +
 I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[3*ArcSinh[c*x]] + 108*a*b*d^2*Sqrt[d + I*c
*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] + 4320*a^2*d^(5/2)*Sqrt[f]*Sqr
t[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c
*f*x]] + 864*b^2*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x
]] - (768*I)*a*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x
]] - 27*b^2*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[4*ArcSinh[c*x]] +
12*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*((-576*I)*b*c*x +
 (576*I)*a*Sqrt[1 + c^2*x^2] - 144*b*Cosh[2*ArcSinh[c*x]] + (192*I)*a*Cosh[
3*ArcSinh[c*x]] + 9*b*Cosh[4*ArcSinh[c*x]] + 288*a*Sinh[2*ArcSinh[c*x]] - (
64*I)*b*Sinh[3*ArcSinh[c*x]] - 36*a*Sinh[4*ArcSinh[c*x]]) + 72*b*d^2*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(60*a + (48*I)*b*Sqrt[1 + c^2*
```

```
x^2] + (16*I)*b*Cosh[3*ArcSinh[c*x]] + 24*b*Sinh[2*ArcSinh[c*x]] - 3*b*Sinh
[4*ArcSinh[c*x]]))/(6912*c*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(b^2c^2d^2x^2 - 2i\,b^2cd^2x - b^2d^2\right)\sqrt{i\,cdx + d}\,\sqrt{-i\,cfx + f}\,\log\left(cx + \sqrt{c^2x^2 + 1}\right)^2 - \left(2\,abc^2d^2x^2 - 4i\,abcd^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(b^2*c^2*d^2*x^2 - 2*I*b^2*c*d^2*x - b^2*d^2)*sqrt(I*c*d*x + d)*s
qrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 - (2*a*b*c^2*d^2*x^2 - 4*I
*a*b*c*d^2*x - 2*a*b*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sq
rt(c^2*x^2 + 1)) - (a^2*c^2*d^2*x^2 - 2*I*a^2*c*d^2*x - a^2*d^2)*sqrt(I*c*d
*x + d)*sqrt(-I*c*f*x + f), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int (icdx + d)^{\frac{5}{2}} (a + b\,\mathrm{arcsinh}\,(cx))^2 \sqrt{-icfx + f}\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,\mathrm{asinh}\,(c\,x))^2\,(d + c\,d\,x\,1\mathrm{i})^{5/2}\,\sqrt{f - c\,f\,x\,1\mathrm{i}}\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(1/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))**2*(f-I*c*f*x)**(1/2),x)
[Out] Timed out
```

3.571 $\int (d+icdx)^{3/2}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=508

$$-\frac{bcdx^2\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{c^2x^2+1}}-\frac{2ibdx\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{3\sqrt{c^2x^2+1}}+\frac{d\sqrt{d+icdx}\sqrt{f}}{6}$$

```
[Out] 4/9*I*b^2*d*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c+1/4*b^2*d*x*(d+I*c*d*x)^(
1/2)*(f-I*c*f*x)^(1/2)+2/27*I*b^2*d*(c^2*x^2+1)*(d+I*c*d*x)^(1/2)*(f-I*c*f*
x)^(1/2)/c+1/2*d*x*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)
+1/3*I*d*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/
2)/c-1/4*b^2*d*arcsinh(c*x)*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c/(c^2*x^2+
1)^(1/2)-2/3*I*b*d*x*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)
/(c^2*x^2+1)^(1/2)-1/2*b*c*d*x^2*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*
c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)-2/9*I*b*c^2*d*x^3*(a+b*arcsinh(c*x))*(d+I*c*
d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/6*d*(a+b*arcsinh(c*x))^3*(
d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.63, antiderivative size = 508, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.297, Rules used = {5712, 5821, 5682, 5675, 5661, 321, 215, 5717, 5679, 444, 43}

$$-\frac{2ibc^2dx^3\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{9\sqrt{c^2x^2+1}}-\frac{bcdx^2\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{c^2x^2+1}}-\frac{2ibdx\sqrt{d+ic}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (((4*I)/9)*b^2*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/c + (b^2*d*x*Sqrt[d +
 I*c*d*x]*Sqrt[f - I*c*f*x])/4 + (((2*I)/27)*b^2*d*Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x]*(1 + c^2*x^2))/c - (b^2*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*A
rcSinh[c*x])/(4*c*Sqrt[1 + c^2*x^2]) - (((2*I)/3)*b*d*x*Sqrt[d + I*c*d*x]*S
qrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2] - (b*c*d*x^2*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(2*Sqrt[1 + c^2*x^2])
- (((2*I)/9)*b*c^2*d*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh
[c*x]))/Sqrt[1 + c^2*x^2] + (d*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b
*ArcSinh[c*x])^2)/2 + ((I/3)*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(1 + c^2
*x^2)*(a + b*ArcSinh[c*x])^2)/c + (d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a
 + b*ArcSinh[c*x])^3)/(6*b*c*Sqrt[1 + c^2*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5679

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
```

```
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2dx = \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int (d+icdx)\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(c\right.}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\left(d\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2+\right.}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{1}{2}dx\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{id\sqrt{d+icdx}}{}$$

$$= -\frac{2ibdx\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{3\sqrt{1+c^2x^2}}-\frac{bcdx^2\sqrt{d+}}{}$$

$$= \frac{1}{4}b^2dx\sqrt{d+icdx}\sqrt{f-icfx}-\frac{2ibdx\sqrt{d+icdx}\sqrt{f-icfx}\left(a\right.}{3\sqrt{1+c^2x^2}}$$

$$= \frac{1}{4}b^2dx\sqrt{d+icdx}\sqrt{f-icfx}-\frac{b^2d\sqrt{d+icdx}\sqrt{f-icfx}\sinh^{}}{4c\sqrt{1+c^2x^2}}$$

$$= \frac{4ib^2d\sqrt{d+icdx}\sqrt{f-icfx}}{9c}+\frac{1}{4}b^2dx\sqrt{d+icdx}\sqrt{f-icfx}+$$

Mathematica [A] time = 1.84, size = 705, normalized size = 1.39

$$\frac{108a^2d^{3/2}\sqrt{f}\sqrt{c^2x^2+1}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)+72ia^2c^2dx^2\sqrt{c^2x^2+1}\sqrt{d+icdx}\sqrt{f-icfx}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] ((-108*I)*a*b*c*d*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (72*I)*a^2*d*Sqrt
[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (108*I)*b^2*d*Sqrt[d +
I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 108*a^2*c*d*x*Sqrt[d + I*c*d
*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (72*I)*a^2*c^2*d*x^2*Sqrt[d + I*c
*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 36*b^2*d*Sqrt[d + I*c*d*x]*Sqrt
[f - I*c*f*x]*ArcSinh[c*x]^3 - 54*a*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]
*Cosh[2*ArcSinh[c*x]] + (4*I)*b^2*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cos
h[3*ArcSinh[c*x]] + 108*a^2*d^(3/2)*Sqrt[f]*Sqrt[1 + c^2*x^2]*Log[c*d*f*x +
 Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + 27*b^2*d*Sqrt[d + I
*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] + 18*b*d*Sqrt[d + I*c*d*x]*S
qrt[f - I*c*f*x]*ArcSinh[c*x]^2*(6*a + (3*I)*b*Sqrt[1 + c^2*x^2] + I*b*Cosh
```

```
[3*ArcSinh[c*x]] + 3*b*Sinh[2*ArcSinh[c*x]]) - (12*I)*a*b*d*Sqrt[d + I*c*d*
x]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]] + 6*b*d*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x]*ArcSinh[c*x]*(-9*b*Cosh[2*ArcSinh[c*x]] + 2*((-9*I)*b*c*x + (9*I
)*a*Sqrt[1 + c^2*x^2] + (3*I)*a*Cosh[3*ArcSinh[c*x]] + 9*a*Sinh[2*ArcSinh[c
*x]] - I*b*Sinh[3*ArcSinh[c*x]])))/(216*c*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(i\,b^2cdx + b^2d\right)\sqrt{i\,cdx + d}\,\sqrt{-i\,cfx + f}\,\log\left(cx + \sqrt{c^2x^2 + 1}\right)^2 + (2i\,abcdx + 2\,abd)\sqrt{i\,cdx + d}\,\sqrt{-i\,cf$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((I*b^2*c*d*x + b^2*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x
 + sqrt(c^2*x^2 + 1))^2 + (2*I*a*b*c*d*x + 2*a*b*d)*sqrt(I*c*d*x + d)*sqrt(
-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (I*a^2*c*d*x + a^2*d)*sqrt(I*c
*d*x + d)*sqrt(-I*c*f*x + f), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument TypeError: Bad Argument
TypeError: Bad Argument TypeDone
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int (icdx + d)^{\frac{3}{2}} (a + b\,\text{arcsinh}\,(cx))^2 \sqrt{-icfx + f}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,\text{asinh}\,(c\,x))^2\,(d + c\,d\,x\,1\text{i})^{3/2}\,\sqrt{f - c\,f\,x\,1\text{i}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (id (cx - i))^{\frac{3}{2}} \sqrt{-if (cx + i)} (a + b \operatorname{asinh}(cx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))**2*(f-I*c*f*x)**(1/2),x)
```

```
[Out] Integral((I*d*(c*x - I))**(3/2)*sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x))**2,
 x)
```

3.572 $\int \sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=244

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^3}{6bc\sqrt{c^2x^2+1}}-\frac{bcx^2\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{d+icdx}\sqrt{f-i}$$

```
[Out] 1/4*b^2*x*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)+1/2*x*(a+b*arcsinh(c*x))^2*(d
+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-1/4*b^2*arcsinh(c*x)*(d+I*c*d*x)^(1/2)*(f
-I*c*f*x)^(1/2)/c/(c^2*x^2+1)^(1/2)-1/2*b*c*x^2*(a+b*arcsinh(c*x))*(d+I*c*d
*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/6*(a+b*arcsinh(c*x))^3*(d+I
*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {5712, 5682, 5675, 5661, 321, 215}

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^3}{6bc\sqrt{c^2x^2+1}}-\frac{bcx^2\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{d+icdx}\sqrt{f-i}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (b^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/4 - (b^2*Sqrt[d + I*c*d*x]*Sqrt
[f - I*c*f*x]*ArcSinh[c*x])/(4*c*Sqrt[1 + c^2*x^2]) - (b*c*x^2*Sqrt[d + I*c
*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(2*Sqrt[1 + c^2*x^2]) + (x*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/2 + (Sqrt[d + I*c
*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^3)/(6*b*c*Sqrt[1 + c^2*x^2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rubi steps

$$\int \sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{\left(\sqrt{d+icdx}\,\sqrt{f-icfx}\right)\int\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)^2\,dx}{\sqrt{1+c^2x^2}}$$

$$=\frac{1}{2}x\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)^2+\frac{\left(\sqrt{d+icdx}\,\sqrt{f\right.}$$

$$=-\frac{bcx^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{1+c^2x^2}}+\frac{1}{2}x\sqrt{d+icdx}$$

$$=\frac{1}{4}b^2x\sqrt{d+icdx}\,\sqrt{f-icfx}-\frac{bcx^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\,s\right.}{2\sqrt{1+c^2x^2}}$$

$$=\frac{1}{4}b^2x\sqrt{d+icdx}\,\sqrt{f-icfx}-\frac{b^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\sinh^{-1}(cx}{4c\sqrt{1+c^2x^2}}$$

Mathematica [A] time = 1.02, size = 352, normalized size = 1.44

$$12a^2cx\sqrt{c^2x^2+1}\,\sqrt{d+icdx}\,\sqrt{f-icfx}+12a^2\sqrt{d}\,\sqrt{f}\,\sqrt{c^2x^2+1}\,\log\left(cdfx+\sqrt{d}\,\sqrt{f}\,\sqrt{d+icdx}\,\sqrt{f-icfx}\right)+6b$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (12*a^2*c*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 4*b^2*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 - 6*a*b*Sqrt[d + I*c*d*x]
*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] + 12*a^2*Sqrt[d]*Sqrt[f]*Sqrt[1 + c
^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]]
+ 3*b^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] - 6*b*Sqrt
[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*(b*Cosh[2*ArcSinh[c*x]] - 2*a*
Sinh[2*ArcSinh[c*x]]) + 6*b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x
]^2*(2*a + b*Sinh[2*ArcSinh[c*x]]))/(24*c*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{icdx+d}\,\sqrt{-icfx+f}\,b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2+2\,\sqrt{icdx+d}\,\sqrt{-icfx+f}\,ab\log\left(cx+\sqrt{c^2x^2+1}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2 +
1))^2 + 2*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2*x^2 +
 1)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument TypeError: Bad Argument
TypeDone
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int (a + b\,\mathrm{arcsinh}\,(cx))^2 \sqrt{icdx + d}\,\sqrt{-icfx + f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,\mathrm{asinh}\,(c\,x))^2 \sqrt{d + c\,d\,x\,1\mathrm{i}}\,\sqrt{f - c\,f\,x\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{id\,(cx - i)}\,\sqrt{-if\,(cx + i)}\,(a + b\,\mathrm{asinh}\,(cx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*(d+I*c*d*x)**(1/2)*(f-I*c*f*x)**(1/2),x)
```

```
[Out] Integral(sqrt(I*d*(c*x - I))*sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x))**2, x)
```

3.573 $\int \frac{\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+icdx}}\,dx$

Optimal. Leaf size=259

$$\frac{2iabfx\sqrt{c^2x^2+1}}{\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{f\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{if\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{2ib^2f\left(c^2x^2+1\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}}$$

```
[Out] -2*I*b^2*f*(c^2*x^2+1)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-I*f*(c^2*x^2+1
)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+2*I*a*b*f*x*(c
^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+2*I*b^2*f*x*arcsinh(c*x
)*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/3*f*(a+b*arcsinh(
c*x))^3*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {5712, 5821, 5675, 5717, 5653, 261}

$$\frac{2iabfx\sqrt{c^2x^2+1}}{\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{f\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{if\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{2ib^2f\left(c^2x^2+1\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/Sqrt[d + I*c*d*x],x]
```

```
[Out] ((2*I)*a*b*f*x*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (
(2*I)*b^2*f*(1 + c^2*x^2))/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + ((2*I)
*b^2*f*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
x]) - (I*f*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]) + (f*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(3*b*c*Sqrt[d
+ I*c*d*x]*Sqrt[f - I*c*f*x])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int \frac{\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+icdx}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{(f-icfx)\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\int \left(\frac{f\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}} - \frac{icfx\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\right)dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\left(f\sqrt{1+c^2x^2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{\left(icf\sqrt{1+c^2x^2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= -\frac{if\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{f\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{2iabfx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{if\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{f\sqrt{1+c^2}}{3bc\sqrt{d}}$$

$$= \frac{2iabfx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2ib^2fx\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{if\left(1+c^2x^2\right)}{c\sqrt{d+ic}}$$

$$= \frac{2iabfx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2ib^2f\left(1+c^2x^2\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2ib^2fx\sqrt{1+c^2x^2}\,\mathrm{si}}{\sqrt{d+icdx}\sqrt{f-}}$$

Mathematica [A] time = 1.17, size = 315, normalized size = 1.22

$$\frac{-3i\sqrt{d+icdx}\sqrt{f-icfx}\left(a^2\sqrt{c^2x^2+1}-2abcx+2b^2\sqrt{c^2x^2+1}\right)+3a^2\sqrt{d}\sqrt{f}\sqrt{c^2x^2+1}\log\left(cdfx+\sqrt{d}\sqrt{f}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/Sqrt[d + I*c*d*x],x]
```

```
[Out] ((-3*I)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-2*a*b*c*x + a^2*Sqrt[1 + c^2*
x^2] + 2*b^2*Sqrt[1 + c^2*x^2]) + (6*I)*b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
```

```
x]*(b*c*x - a*Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + 3*b*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]*(a - I*b*Sqrt[1 + c^2*x^2])*ArcSinh[c*x]^2 + b^2*Sqrt[d + I*c*
d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 + 3*a^2*Sqrt[d]*Sqrt[f]*Sqrt[1 + c^2*
x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]])/(3
*c*d*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{-i\sqrt{icdx+d}\sqrt{-icfx+f}\,b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2-2i\sqrt{icdx+d}\sqrt{-icfx+f}\,ab\log\left(cx+\sqrt{c^2x^2+1}\right.}{cdx-id}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((-I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^
2 + 1))^2 - 2*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2
*x^2 + 1)) - I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2)/(c*d*x - I*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-icfx+f}\,(b\operatorname{arsinh}(cx)+a)^2}{\sqrt{icdx+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(-I*c*f*x + f)*(b*arcsinh(c*x) + a)^2/sqrt(I*c*d*x + d), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2\sqrt{-icfx+f}}{\sqrt{icdx+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\frac{f\operatorname{arsinh}(cx)}{cd\sqrt{\frac{f}{d}}}-\frac{i\sqrt{c^2dfx^2+df}}{cd}\right)+\int \frac{\sqrt{-icfx+f}\,b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{\sqrt{icdx+d}}+\frac{2\sqrt{-icfx+f}\,ab\log\left(cx+\sqrt{c^2\right.}}{\sqrt{icdx+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] a^2*(f*arcsinh(c*x)/(c*d*sqrt(f/d)) - I*sqrt(c^2*d*f*x^2 + d*f)/(c*d)) + in
tegrate(sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/sqrt(I*c*d*x
+ d) + 2*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2*x^2 + 1))/sqrt(I*c*d*x +
 d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))^2\,\sqrt{f-c\,f\,x\,1\mathrm{i}}}{\sqrt{d+c\,d\,x\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(1/2),x)
[Out] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-if\,(cx+i)}\,\left(a+b\,\mathrm{asinh}\,(cx)\right)^2}{\sqrt{id\,(cx-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*(f-I*c*f*x)**(1/2)/(d+I*c*d*x)**(1/2),x)
[Out] Integral(sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x))**2/sqrt(I*d*(c*x - I)), x)
```

3.574 $\int \frac{\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}}\,dx$

Optimal. Leaf size=544

$$-\frac{f^2\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2f^2\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2if^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[Out] 2*I*f^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3
/2)+2*f^2*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^
(3/2)+2*f^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I
*c*f*x)^(3/2)-1/3*f^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*x
)^(3/2)/(f-I*c*f*x)^(3/2)-8*I*b*f^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*ar
ctan(c*x+(c^2*x^2+1)^(1/2))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-4*b*f^2*(
c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*
c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-4*b^2*f^2*(c^2*x^2+1)^(3/2)*polylog(2,-I*(c*
x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+4*b^2*f^2*(c^2*
x^2+1)^(3/2)*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*
c*f*x)^(3/2)-2*b^2*f^2*(c^2*x^2+1)^(3/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))
^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A] time = 1.01, antiderivative size = 544, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 13, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.351, Rules used = {5712, 5833, 5821, 5687, 5714, 3718, 2190, 2279, 2391, 5717, 5693, 4180, 5675}

$$-\frac{4b^2f^2\left(c^2x^2+1\right)^{3/2}\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{4b^2f^2\left(c^2x^2+1\right)^{3/2}\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2b^2f^2\left(c^2x^2+1\right)^{3/2}}{c(d+icdx}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(3/2),x]
```

```
[Out] ((2*I)*f^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f
 - I*c*f*x)^(3/2)) + (2*f^2*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/((d + I*
c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (2*f^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSi
nh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (f^2*(1 + c^2*x^2
)^(3/2)*(a + b*ArcSinh[c*x])^3)/(3*b*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3
/2)) - ((8*I)*b*f^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSi
nh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (4*b*f^2*(1 + c^2*x
^2)^(3/2)*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2)) - (4*b^2*f^2*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-
I)*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (4*b^2*f^
2*(1 + c^2*x^2)^(3/2)*PolyLog[2, I*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*
(f - I*c*f*x)^(3/2)) - (2*b^2*f^2*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(2*ArcS
inh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5833

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}}dx = \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{(f-icfx)^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{3/2}\int \left(-\frac{2i\left(if^2+cf^2x\right)\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}} - \frac{f^2\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{\left(2i\left(1+c^2x^2\right)^{3/2}\right)\int \frac{\left(if^2+cf^2x\right)\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(f^2\left(1+c^2x^2\right)^{3/2}\right)\int}{(d+icdx)^{3/2}(}$$

$$= -\frac{f^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(2i\left(1+c^2x^2\right)^{3/2}\right)\int\left(\frac{if^2(a+b\sinh}{(1+c^2x}\right.}{(d+icdx)^{3/}}$$

$$= -\frac{f^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(2f^2\left(1+c^2x^2\right)^{3/2}\right)\int\frac{(a+b\sinh^{-}}{(1+c^2x^2}}{(d+icdx)^{3/2}(f-icfx)^{3}}$$

$$= \frac{2if^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2f^2x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx\right.}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{2if^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2f^2x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx\right.}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{2if^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2f^2x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx\right.}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{2if^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2f^2x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx\right.}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{2if^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2f^2x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx\right.}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{2if^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2f^2x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx\right.}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Mathematica [A] time = 4.24, size = 594, normalized size = 1.09

$$\frac{\frac{6a^2\sqrt{d+icdx}\sqrt{f-icfx}}{cx-i} - 3a^2\sqrt{d}\sqrt{f}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right) - \frac{3ab\sqrt{d+icdx}\sqrt{f-icfx}\left(2\left(\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right.\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(3/2),x]
```

```
[Out] ((6*a^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(-I + c*x) - 3*a^2*Sqrt[d]*Sqr
t[f]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] - (
3*a*b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(ArcSinh[c*x]*((-4*I)*Cosh[ArcSin
h[c*x]/2] - 4*Sinh[ArcSinh[c*x]/2]) + ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2]
+ I*Sinh[ArcSinh[c*x]/2]) + 2*((4*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + Log[1 +
 c^2*x^2])*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/(Sqrt[1 + c^2*
x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) + (b^2*Sqrt[d + I*c*d
```

```
*x]*Sqrt[f - I*c*f*x]*((-6 + 6*I)*ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] - Si
nh[ArcSinh[c*x]/2]) - ArcSinh[c*x]^3*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh
[c*x]/2]) + (12*I)*Pi*(Log[1 - I/E^ArcSinh[c*x]] + 2*Log[1 + E^ArcSinh[c*x]
] - 2*Log[Cosh[ArcSinh[c*x]/2]] - Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]])*(C
osh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + 24*PolyLog[2, I/E^ArcSinh[c
*x]]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + 6*ArcSinh[c*x]*(Pi -
 (4*I)*Log[1 - I/E^ArcSinh[c*x]])*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh
[c*x]/2])))/(Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/
2])))/(3*c*d^2)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2+2\,\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,ab\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^2d^2x^2-2i\,cd^2x-d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2
+ 1))^2 + 2*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2*x^2
 + 1)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2)/(c^2*d^2*x^2 - 2*I*c*d^2
*x - d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-i\,cfx+f}\,(b\,\text{arsinh}\,(cx)+a)^2}{(i\,cdx+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(-I*c*f*x + f)*(b*arcsinh(c*x) + a)^2/(I*c*d*x + d)^(3/2), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\text{arcsinh}\,(cx))^2\,\sqrt{-icfx+f}}{(icdx+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\frac{2i\,\sqrt{c^2dfx^2+df}}{i\,c^2d^2x+cd^2}-\frac{f\,\text{arsinh}\,(cx)}{cd^2\sqrt{\frac{f}{d}}}\right)+\int\frac{\sqrt{-i\,cfx+f}\,b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{(i\,cdx+d)^{\frac{3}{2}}}+\frac{2\,\sqrt{-i\,cfx+f}\,ab\log\left(cx+\sqrt{c\right.}}{(i\,cdx+d)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x, algor
ithm="maxima")
```

```
[Out] a^2*(2*I*sqrt(c^2*d*f*x^2 + d*f)/(I*c^2*d^2*x + c*d^2) - f*arcsinh(c*x)/(c*
d^2*sqrt(f/d))) + integrate(sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2 +
 1))^2/(I*c*d*x + d)^(3/2) + 2*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2*x^
2 + 1))/(I*c*d*x + d)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{asinh}\,(c\,x))^2\,\sqrt{f - c\,f\,x\,1\mathrm{i}}}{(d + c\,d\,x\,1\mathrm{i})^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(3/2),x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-if\,(cx + i)}\,\,(a + b\,\mathrm{asinh}\,(cx))^2}{(id\,(cx - i))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*(f-I*c*f*x)**(1/2)/(d+I*c*d*x)**(3/2),x)
```

```
[Out] Integral(sqrt(-I*f*(c*x + I))*(a + b*asinh(c*x))**2/(I*d*(c*x - I))**(3/2),
 x)
```

3.575 $\int \frac{\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{5/2}}\,dx$

Optimal. Leaf size=518

$$-\frac{f^3\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{4bf^3\left(c^2x^2+1\right)^{5/2}\log\left(1+ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{if^3\left(c^2x^2+1\right)^{5/}}{}$$

```
[Out] -1/3*f^3*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*
f*x)^(5/2)-4/3*I*b^2*f^3*(c^2*x^2+1)^(5/2)*cot(1/4*Pi+1/2*I*arcsinh(c*x))/c
/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*I*f^3*(c^2*x^2+1)^(5/2)*(a+b*arcsi
nh(c*x))^2*cot(1/4*Pi+1/2*I*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(
5/2)+2/3*b*f^3*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*csc(1/4*Pi+1/2*I*arcsin
h(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*I*f^3*(c^2*x^2+1)^(5/2)
*(a+b*arcsinh(c*x))^2*cot(1/4*Pi+1/2*I*arcsinh(c*x))*csc(1/4*Pi+1/2*I*arcsi
nh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+4/3*b*f^3*(c^2*x^2+1)^(5/2
)*(a+b*arcsinh(c*x))*ln(1+I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f
-I*c*f*x)^(5/2)+4/3*b^2*f^3*(c^2*x^2+1)^(5/2)*polylog(2,-I*(c*x+(c^2*x^2+1)
^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A] time = 1.15, antiderivative size = 518, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 12, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.324, Rules used = {5712, 5833, 5831, 3318, 4186, 3767, 8, 4184, 3716, 2190, 2279, 2391}

$$\frac{4b^2f^3\left(c^2x^2+1\right)^{5/2}\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{f^3\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{4bf^3\left(c^2x^2+1\right)^{5/2}\log\left(1+ie\right.}{3c(d+icdx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(5/2),x]
```

```
[Out] -(f^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(3*c*(d + I*c*d*x)^(5/2)*
(f - I*c*f*x)^(5/2)) - (((4*I)/3)*b^2*f^3*(1 + c^2*x^2)^(5/2)*Cot[Pi/4 + (I
/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - ((I/3)*f^3
*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*Cot[Pi/4 + (I/2)*ArcSinh[c*x]])
/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (2*b*f^3*(1 + c^2*x^2)^(5/2)
*(a + b*ArcSinh[c*x])*Csc[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/(3*c*(d + I*c*d*x)^
(5/2)*(f - I*c*f*x)^(5/2)) + ((I/3)*f^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[
c*x])^2*Cot[Pi/4 + (I/2)*ArcSinh[c*x]]*Csc[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/(c
*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (4*b*f^3*(1 + c^2*x^2)^(5/2)*(a
 + b*ArcSinh[c*x])*Log[1 + I*E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f -
 I*c*f*x)^(5/2)) + (4*b^2*f^3*(1 + c^2*x^2)^(5/2)*PolyLog[2, (-I)*E^ArcSinh
[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
```

```
c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5833

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{5/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(f-icfx)^3\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{5/2}\int \left(-\frac{2f^3\left(a+b\sinh^{-1}(cx)\right)^2}{(-i+cx)^2\sqrt{1+c^2x^2}}+\frac{if^3\left(a+b\sinh^{-1}(cx)\right)^2}{(-i+cx)\sqrt{1+c^2x^2}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(if^3\left(1+c^2x^2\right)^{5/2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(-i+cx)\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(2f^3\left(1+c^2x^2\right)^{5/2}\right)\int \frac{\left(a+b\sinh^{-1}(c\right.}{(-i+cx)^2\sqrt{1+}}}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(if^3\left(1+c^2x^2\right)^{5/2}\right)\text{Subst}\left(\int \frac{(a+bx)^2}{-ic+c\sinh(x)}\,dx, x, \sinh^{-1}(cx)\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(2cf^3\left(1+c^2\right.\right.}{}$$

$$= -\frac{\left(f^3\left(1+c^2x^2\right)^{5/2}\right)\text{Subst}\left(\int (a+bx)^2\csc^2\left(\frac{\pi}{4}+\frac{ix}{2}\right)dx, x, \sinh^{-1}(cx)\right)}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} +$$

$$= -\frac{if^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\sinh^{-1}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2bf^3(1}{}$$

$$= -\frac{f^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{if^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx\right.}{3c(d+icdx)^{5/2}(f}$$

$$= -\frac{f^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{4ib^2f^3\left(1+c^2x^2\right)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\,\mathrm{s}\right.}{3c(d+icdx)^{5/2}(f-icfx)}$$

$$= -\frac{f^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{4ib^2f^3\left(1+c^2x^2\right)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\,\mathrm{s}\right.}{3c(d+icdx)^{5/2}(f-icfx)}$$

$$= -\frac{f^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{4ib^2f^3\left(1+c^2x^2\right)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\,\mathrm{s}\right.}{3c(d+icdx)^{5/2}(f-icfx)}$$

$$= -\frac{f^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{4ib^2f^3\left(1+c^2x^2\right)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\,\mathrm{s}\right.}{3c(d+icdx)^{5/2}(f-icfx)}$$

Mathematica [A] time = 8.18, size = 783, normalized size = 1.51

$$\frac{\sqrt{id(cx-i)}\sqrt{-if(cx+i)}\left(-\frac{a^2}{3d^3(cx-i)}-\frac{2ia^2}{3d^3(cx-i)^2}\right)}{c} + \frac{iab\sqrt{i(cdx-id)}\sqrt{-i(cfx+if)}\sqrt{-df\left(c^2x^2+1\right)}\left(\cosh\left(\frac{1}{2}\sinh^{-1}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(5/2),x]
```

```
[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*((((-2*I)/3)*a^2)/(d^3*(-I + c
*x)^2) - a^2/(3*d^3*(-I + c*x))))/c + ((I/3)*a*b*Sqrt[I*((-I)*d + c*d*x)]*S
qrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] -
I*Sinh[ArcSinh[c*x]/2])*((-I)*Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*Ar
cTan[Coth[ArcSinh[c*x]/2]] - I*Log[Sqrt[1 + c^2*x^2]]) + Cosh[ArcSinh[c*x]/
2]*(4 + (3*I)*ArcSinh[c*x] - (6*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + 3*Log[Sqr
t[1 + c^2*x^2]]) + 2*(Sqrt[1 + c^2*x^2]*(ArcSinh[c*x] + 2*ArcTan[Coth[ArcSi
nh[c*x]/2]] + I*Log[Sqrt[1 + c^2*x^2]]) + 2*(I + ArcSinh[c*x] + 2*ArcTan[Co
th[ArcSinh[c*x]/2]] + I*Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/(c*
d^3*(I + c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2]
 + I*Sinh[ArcSinh[c*x]/2])^4) + ((I/3)*b^2*(I + c*x)*Sqrt[I*((-I)*d + c*d*x
)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*((-1 + I)*ArcSinh[c*
x]^2 - (2*ArcSinh[c*x]*(-2*I + ArcSinh[c*x]))/(-I + c*x) + (2*I)*(Pi + (2*I
)*ArcSinh[c*x])*Log[1 - I/E^ArcSinh[c*x]] - I*Pi*(ArcSinh[c*x] - 4*Log[1 +
E^ArcSinh[c*x]] + 4*Log[Cosh[ArcSinh[c*x]/2]] + 2*Log[Sin[(Pi + (2*I)*ArcSi
nh[c*x])/4]]) + 4*PolyLog[2, I/E^ArcSinh[c*x]] - (4*ArcSinh[c*x]^2*Sinh[Arc
Sinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^3 + (2*(4 + A
rcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh
[c*x]/2])))/(c*d^3*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2
]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^2)
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$-\frac{\left(b^2cx + i\,b^2\right)\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2 - \left(3\,c^3d^3x^2 - 6i\,c^2d^3x - 3\,cd^3\right)\mathrm{integral}\left(\frac{3i\,\sqrt{i\,cdx+ \cdots}}{\cdots}\right.}{3\,c^3d^3x^2 - 6i\,c^2d^3x - 3\,cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x, algor
ithm="fricas")
```

```
[Out] -((b^2*c*x + I*b^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2
*x^2 + 1))^2 - (3*c^3*d^3*x^2 - 6*I*c^2*d^3*x - 3*c*d^3)*integral((3*I*sqrt
(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2 + (2*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x +
d)*sqrt(-I*c*f*x + f)*b^2 + 6*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b)*l
og(c*x + sqrt(c^2*x^2 + 1)))/(3*c^3*d^3*x^3 - 9*I*c^2*d^3*x^2 - 9*c*d^3*x +
 3*I*d^3), x))/(3*c^3*d^3*x^2 - 6*I*c^2*d^3*x - 3*c*d^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-i\,cfx+f}\,(b\,\mathrm{arsinh}\,(cx)+a)^2}{(i\,cdx+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(-I*c*f*x + f)*(b*arcsinh(c*x) + a)^2/(I*c*d*x + d)^(5/2), x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2\,\sqrt{-icfx+f}}{(icdx+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))^2\,\sqrt{f-c\,f\,x\,1\mathrm{i}}}{(d+c\,d\,x\,1\mathrm{i})^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(5/2),x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(1/2))/(d + c*d*x*1i)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*(f-I*c*f*x)**(1/2)/(d+I*c*d*x)**(5/2),x)
```

```
[Out] Timed out
```

3.576 $\int (d+icdx)^{5/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=774

$$-\frac{3bcdx^2(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{8\left(c^2x^2+1\right)^{3/2}}+\frac{3dx(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{8\left(c^2x^2+1\right)}-\frac{2ibdx}{}$$

```
[Out] 8/225*I*b^2*d*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/c+1/32*b^2*d*x*(d+I*c*d*x
)^(3/2)*(f-I*c*f*x)^(3/2)+16/75*I*b^2*d*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)
/c/(c^2*x^2+1)+15/64*b^2*d*x*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1
)+2/125*I*b^2*d*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(c^2*x^2+1)/c-9/64*b^2*
d*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*arcsinh(c*x)/c/(c^2*x^2+1)^(3/2)-2/5*
I*b*d*x*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^
(3/2)-3/8*b*c*d*x^2*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/
(c^2*x^2+1)^(3/2)-4/15*I*b*c^2*d*x^3*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a
+b*arcsinh(c*x))/(c^2*x^2+1)^(3/2)-2/25*I*b*c^4*d*x^5*(d+I*c*d*x)^(3/2)*(f-
I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(3/2)+1/4*d*x*(d+I*c*d*x)^(3/
2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2+3/8*d*x*(d+I*c*d*x)^(3/2)*(f-I*c*
f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(c^2*x^2+1)+1/5*I*d*(d+I*c*d*x)^(3/2)*(f-I*
c*f*x)^(3/2)*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c+1/8*d*(d+I*c*d*x)^(3/2)*(f-
I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^3/b/c/(c^2*x^2+1)^(3/2)-1/8*b*d*(d+I*c*d*
x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] time = 0.86, antiderivative size = 774, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.405, Rules used = {5712, 5821, 5684, 5682, 5675, 5661, 321, 215, 5717, 195, 194, 5679, 12, 1247, 698}

$$-\frac{2ibc^4dx^5(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{25\left(c^2x^2+1\right)^{3/2}}-\frac{4ibc^2dx^3(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{15\left(c^2x^2+1\right)^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (((8*I)/225)*b^2*d*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/c + (b^2*d*x*(d
 + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/32 + (((16*I)/75)*b^2*d*(d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2))/(c*(1 + c^2*x^2)) + (15*b^2*d*x*(d + I*c*d*x)^
(3/2)*(f - I*c*f*x)^(3/2))/(64*(1 + c^2*x^2)) + (((2*I)/125)*b^2*d*(d + I*c
*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(1 + c^2*x^2))/c - (9*b^2*d*(d + I*c*d*x)^(
3/2)*(f - I*c*f*x)^(3/2)*ArcSinh[c*x])/(64*c*(1 + c^2*x^2)^(3/2)) - (((2*I)
/5)*b*d*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(1
+ c^2*x^2)^(3/2) - (3*b*c*d*x^2*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a
+ b*ArcSinh[c*x]))/(8*(1 + c^2*x^2)^(3/2)) - (((4*I)/15)*b*c^2*d*x^3*(d + I
*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(1 + c^2*x^2)^(3/2)
 - (((2*I)/25)*b*c^4*d*x^5*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*A
rcSinh[c*x]))/(1 + c^2*x^2)^(3/2) - (b*d*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^
(3/2)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(8*c) + (d*x*(d + I*c*d*x)^(3
/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/4 + (3*d*x*(d + I*c*d*x)^(3
/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(8*(1 + c^2*x^2)) + ((I/5)*
d*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]
)^2)/c + (d*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^3)
/(8*b*c*(1 + c^2*x^2)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5679

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2 dx = \frac{\left((d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int (d+icdx)\left(1+c^2x^2\right)^{3/2}\left(a+b\right.}{\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{\left((d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int \left(d\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(\right.\right.}{\left(1+c^2x^2\right)^{3}}$$

$$= \frac{\left(d(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int \left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(c\right.}{\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{1}{4}dx(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2 + \frac{id(d+i}{}$$

$$= -\frac{2ibdx(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{5\left(1+c^2x^2\right)^{3/2}} - \frac{4ibc^2}{}$$

$$= \frac{1}{32}b^2dx(d+icdx)^{3/2}(f-icfx)^{3/2} - \frac{2ibdx(d+icdx)^{3/2}(f-ic}{5\left(1+\right.}$$

$$= \frac{1}{32}b^2dx(d+icdx)^{3/2}(f-icfx)^{3/2} + \frac{15b^2dx(d+icdx)^{3/2}(f-}{64\left(1+c^2x^2\right)}$$

$$= \frac{1}{32}b^2dx(d+icdx)^{3/2}(f-icfx)^{3/2} + \frac{15b^2dx(d+icdx)^{3/2}(f-}{64\left(1+c^2x^2\right)}$$

$$= \frac{8ib^2d(d+icdx)^{3/2}(f-icfx)^{3/2}}{225c} + \frac{1}{32}b^2dx(d+icdx)^{3/2}(f-i$$

Mathematica [A] time = 3.18, size = 1084, normalized size = 1.40

$$\frac{57600ia^2c^4d^2f\sqrt{icxd+d}\sqrt{f-icfx}\sqrt{c^2x^2+1}\,x^4 + 72000a^2c^3d^2f\sqrt{icxd+d}\sqrt{f-icfx}\sqrt{c^2x^2+1}\,x^3 + 115200ia^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] ((-72000*I)*a*b*c*d^2*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (57600*I)*a
^2*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (72000*I)*
b^2*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 180000*a^
2*c*d^2*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (115200
*I)*a^2*c^2*d^2*f*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]
 + 72000*a^2*c^3*d^2*f*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2
*x^2] + (57600*I)*a^2*c^4*d^2*f*x^4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqr
t[1 + c^2*x^2] + 36000*b^2*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSin
h[c*x]^3 - 72000*a*b*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSi
nh[c*x]] + (4000*I)*b^2*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[3*Ar
cSinh[c*x]] - 4500*a*b*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*Arc
Sinh[c*x]] + (288*I)*b^2*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[5*A
rcSinh[c*x]] + 108000*a^2*d^(5/2)*f^(3/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + S
qrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + 36000*b^2*d^2*f*Sqrt[
```

```
d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] - (12000*I)*a*b*d^2*f*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]] + 1125*b^2*d^2*f*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[4*ArcSinh[c*x]] + 1800*b*d^2*f*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(60*a + (20*I)*b*Sqrt[1 + c^2
*x^2] + (10*I)*b*Cosh[3*ArcSinh[c*x]] + (2*I)*b*Cosh[5*ArcSinh[c*x]] + 40*b
*Sinh[2*ArcSinh[c*x]] + 5*b*Sinh[4*ArcSinh[c*x]]) - (1440*I)*a*b*d^2*f*Sqrt
[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[5*ArcSinh[c*x]] + 60*b*d^2*f*Sqrt[d +
I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*((-1200*I)*b*c*x + (1200*I)*a*Sqrt[
1 + c^2*x^2] - 1200*b*Cosh[2*ArcSinh[c*x]] + (600*I)*a*Cosh[3*ArcSinh[c*x]]
 - 75*b*Cosh[4*ArcSinh[c*x]] + (120*I)*a*Cosh[5*ArcSinh[c*x]] + 2400*a*Sinh
[2*ArcSinh[c*x]] - (200*I)*b*Sinh[3*ArcSinh[c*x]] + 300*a*Sinh[4*ArcSinh[c*
x]] - (24*I)*b*Sinh[5*ArcSinh[c*x]]))/(288000*c*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(i\,b^2c^3d^2fx^3+b^2c^2d^2fx^2+i\,b^2cd^2fx+b^2d^2f\right)\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2+\left(2i\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="fricas")
```

```
[Out] integral((I*b^2*c^3*d^2*f*x^3 + b^2*c^2*d^2*f*x^2 + I*b^2*c*d^2*f*x + b^2*d
^2*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 +
 (2*I*a*b*c^3*d^2*f*x^3 + 2*a*b*c^2*d^2*f*x^2 + 2*I*a*b*c*d^2*f*x + 2*a*b*d
^2*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (
I*a^2*c^3*d^2*f*x^3 + a^2*c^2*d^2*f*x^2 + I*a^2*c*d^2*f*x + a^2*d^2*f)*sqrt
(I*c*d*x + d)*sqrt(-I*c*f*x + f), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument TypeError: Bad Argument
TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int (icdx+d)^{\frac{5}{2}}\left(-icfx+f\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{asinh}\,(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^{5/2}\,\left(f-c\,f\,x\,1\mathrm{i}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(3/2),x)
[Out] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(5/2)*(f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))**2,x)
[Out] Timed out
```

3.577 $\int (d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=396

$$\frac{(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^3}{8bc\left(c^2x^2+1\right)^{3/2}}+\frac{3x(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{8\left(c^2x^2+1\right)}-\frac{b\sqrt{c^2x^2+1}\,(d}{}$$

```
[Out] 1/32*b^2*x*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)+15/64*b^2*x*(d+I*c*d*x)^(3/2
)*(f-I*c*f*x)^(3/2)/(c^2*x^2+1)-9/64*b^2*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2
)*arcsinh(c*x)/c/(c^2*x^2+1)^(3/2)-3/8*b*c*x^2*(d+I*c*d*x)^(3/2)*(f-I*c*f*x
)^(3/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(3/2)+1/4*x*(d+I*c*d*x)^(3/2)*(f-I*c
*f*x)^(3/2)*(a+b*arcsinh(c*x))^2+3/8*x*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*
(a+b*arcsinh(c*x))^2/(c^2*x^2+1)+1/8*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a
+b*arcsinh(c*x))^3/b/c/(c^2*x^2+1)^(3/2)-1/8*b*(d+I*c*d*x)^(3/2)*(f-I*c*f*x
)^(3/2)*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] time = 0.48, antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {5712, 5684, 5682, 5675, 5661, 321, 215, 5717, 195}

$$\frac{(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^3}{8bc\left(c^2x^2+1\right)^{3/2}}+\frac{3x(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{8\left(c^2x^2+1\right)}-\frac{b\sqrt{c^2x^2+1}\,(d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (b^2*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/32 + (15*b^2*x*(d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2))/(64*(1 + c^2*x^2)) - (9*b^2*(d + I*c*d*x)^(3/2
)*(f - I*c*f*x)^(3/2)*ArcSinh[c*x])/(64*c*(1 + c^2*x^2)^(3/2)) - (3*b*c*x^2
*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(8*(1 + c^2*
x^2)^(3/2)) - (b*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*Sqrt[1 + c^2*x^2]*
(a + b*ArcSinh[c*x]))/(8*c) + (x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a
 + b*ArcSinh[c*x])^2)/4 + (3*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a +
 b*ArcSinh[c*x])^2)/(8*(1 + c^2*x^2)) + ((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^
(3/2)*(a + b*ArcSinh[c*x])^3)/(8*b*c*(1 + c^2*x^2)^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2 dx = \frac{\left((d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(\right.}{\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{1}{4}x(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2 + \frac{(3(d+}{}$$

$$= -\frac{b(d+icdx)^{3/2}(f-icfx)^{3/2}\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right.}{8c}$$

$$= \frac{1}{32}b^2x(d+icdx)^{3/2}(f-icfx)^{3/2} - \frac{3bcx^2(d+icdx)^{3/2}(f-}{8(1-}$$

$$= \frac{1}{32}b^2x(d+icdx)^{3/2}(f-icfx)^{3/2} + \frac{15b^2x(d+icdx)^{3/2}(f-}{64\left(1+c^2x^2\right)}$$

$$= \frac{1}{32}b^2x(d+icdx)^{3/2}(f-icfx)^{3/2} + \frac{15b^2x(d+icdx)^{3/2}(f-}{64\left(1+c^2x^2\right)}$$

Mathematica [A] time = 1.74, size = 524, normalized size = 1.32

$$\frac{96a^2d^{3/2}f^{3/2}\sqrt{c^2x^2+1}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)+160a^2cdfx\sqrt{c^2x^2+1}\sqrt{d+icdx}\sqrt{f-icf}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (160*a^2*c*d*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 64
*a^2*c^3*d*f*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 32
*b^2*d*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 - 64*a*b*d*f*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] - 4*a*b*d*f*Sqrt[d +
 I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] + 96*a^2*d^(3/2)*f^(3/2)*S
qrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I
*c*f*x]] + 32*b^2*d*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*
x]] + b^2*d*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[4*ArcSinh[c*x]] + 8*
b*d*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(12*a + 8*b*Sinh[2
*ArcSinh[c*x]] + b*Sinh[4*ArcSinh[c*x]]) - 4*b*d*f*Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x]*ArcSinh[c*x]*(16*b*Cosh[2*ArcSinh[c*x]] + b*Cosh[4*ArcSinh[c*x]
] - 4*a*(8*Sinh[2*ArcSinh[c*x]] + Sinh[4*ArcSinh[c*x]])))/(256*c*Sqrt[1 + c
^2*x^2])
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2c^2dfx^2+b^2df\right)\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2+2\left(abc^2dfx^2+abdf\right)\sqrt{i\,cdx+d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="fricas")
```

```
[Out] integral((b^2*c^2*d*f*x^2 + b^2*d*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*l
og(c*x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c^2*d*f*x^2 + a*b*d*f)*sqrt(I*c*d*x
```

```
+ d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a^2*c^2*d*f*x^2 + a
^2*d*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int (icdx + d)^{\frac{3}{2}} \left(-icfx + f\right)^{\frac{3}{2}} (a + b \operatorname{arcsinh}(cx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b \operatorname{asinh}(c\,x))^2 \,(d + c\,d\,x\,1\mathrm{i})^{3/2} \left(f - c\,f\,x\,1\mathrm{i}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(3/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(3/2)*(f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

3.578 $\int \sqrt{d+icdx}\,(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=508

$$-\frac{bcfx^2\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{c^2x^2+1}}+\frac{2ibfx\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{3\sqrt{c^2x^2+1}}+\frac{f\sqrt{d+icdx}}{}$$

```
[Out] -4/9*I*b^2*f*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c+1/4*b^2*f*x*(d+I*c*d*x)^
(1/2)*(f-I*c*f*x)^(1/2)-2/27*I*b^2*f*(c^2*x^2+1)*(d+I*c*d*x)^(1/2)*(f-I*c*f
*x)^(1/2)/c+1/2*f*x*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2
)-1/3*I*f*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1
/2)/c-1/4*b^2*f*arcsinh(c*x)*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c/(c^2*x^2
+1)^(1/2)+2/3*I*b*f*x*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2
)/(c^2*x^2+1)^(1/2)-1/2*b*c*f*x^2*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I
*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+2/9*I*b*c^2*f*x^3*(a+b*arcsinh(c*x))*(d+I*c
*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/6*f*(a+b*arcsinh(c*x))^3*
(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.65, antiderivative size = 508, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.297, Rules used = {5712, 5821, 5682, 5675, 5661, 321, 215, 5717, 5679, 444, 43}

$$\frac{2ibc^2fx^3\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{9\sqrt{c^2x^2+1}}-\frac{bcfx^2\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{c^2x^2+1}}+\frac{2ibfx\sqrt{d+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (((-4*I)/9)*b^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/c + (b^2*f*x*Sqrt[d
+ I*c*d*x]*Sqrt[f - I*c*f*x])/4 - (((2*I)/27)*b^2*f*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]*(1 + c^2*x^2))/c - (b^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*
ArcSinh[c*x])/(4*c*Sqrt[1 + c^2*x^2]) + (((2*I)/3)*b*f*x*Sqrt[d + I*c*d*x]*
Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2] - (b*c*f*x^2*Sqrt
[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(2*Sqrt[1 + c^2*x^2])
 + (((2*I)/9)*b*c^2*f*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSin
h[c*x]))/Sqrt[1 + c^2*x^2] + (f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a +
b*ArcSinh[c*x])^2)/2 - ((I/3)*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(1 + c^
2*x^2)*(a + b*ArcSinh[c*x])^2)/c + (f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(
a + b*ArcSinh[c*x])^3)/(6*b*c*Sqrt[1 + c^2*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5679

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
```

```
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int \sqrt{d+icdx}\,(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{\left(\sqrt{d+icdx}\,\sqrt{f-icfx}\right)\int(f-icfx)\sqrt{1+c^2x^2}\,\left(a+b\sinh\right.}{\sqrt{1+c^2x^2}}$$

$$=\frac{\left(\sqrt{d+icdx}\,\sqrt{f-icfx}\right)\int\left(f\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)\right.}{\sqrt{1+c^2x^2}}$$

$$=\frac{\left(f\sqrt{d+icdx}\,\sqrt{f-icfx}\right)\int\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}$$

$$=\frac{1}{2}fx\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)^2-\frac{if\sqrt{d+ic}}{}$$

$$=\frac{2ibfx\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)}{3\sqrt{1+c^2x^2}}-\frac{bcfx^2\sqrt{d}}{}$$

$$=\frac{1}{4}b^2fx\sqrt{d+icdx}\,\sqrt{f-icfx}+\frac{2ibfx\sqrt{d+icdx}\,\sqrt{f-icfx}}{3\sqrt{1+c^2x}}$$

$$=\frac{1}{4}b^2fx\sqrt{d+icdx}\,\sqrt{f-icfx}-\frac{b^2f\sqrt{d+icdx}\,\sqrt{f-icfx}\,\mathrm{si}}{4c\sqrt{1+c^2x^2}}$$

$$=-\frac{4ib^2f\sqrt{d+icdx}\,\sqrt{f-icfx}}{9c}+\frac{1}{4}b^2fx\sqrt{d+icdx}\,\sqrt{f-icf}$$

Mathematica [A] time = 1.83, size = 705, normalized size = 1.39

$$\frac{108a^2\sqrt{d}\,f^{3/2}\sqrt{c^2x^2+1}\,\log\left(cdfx+\sqrt{d}\,\sqrt{f}\,\sqrt{d+icdx}\,\sqrt{f-icfx}\right)-72ia^2c^2fx^2\sqrt{c^2x^2+1}\,\sqrt{d+icdx}\,\sqrt{f-icf}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] ((108*I)*a*b*c*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (72*I)*a^2*f*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (108*I)*b^2*f*Sqrt[d + I
*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 108*a^2*c*f*x*Sqrt[d + I*c*d*
x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (72*I)*a^2*c^2*f*x^2*Sqrt[d + I*c*
d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 36*b^2*f*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]*ArcSinh[c*x]^3 - 54*a*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*
Cosh[2*ArcSinh[c*x]] - (4*I)*b^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh
[3*ArcSinh[c*x]] + 108*a^2*Sqrt[d]*f^(3/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x +
Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + 27*b^2*f*Sqrt[d + I*
c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] + 18*b*f*Sqrt[d + I*c*d*x]*Sq
rt[f - I*c*f*x]*ArcSinh[c*x]^2*(6*a - (3*I)*b*Sqrt[1 + c^2*x^2] - I*b*Cosh[
```

```
3*ArcSinh[c*x]] + 3*b*Sinh[2*ArcSinh[c*x]]) + (12*I)*a*b*f*Sqrt[d + I*c*d*x
]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]] + 6*b*f*Sqrt[d + I*c*d*x]*Sqrt[f -
 I*c*f*x]*ArcSinh[c*x]*(-9*b*Cosh[2*ArcSinh[c*x]] + 2*((9*I)*b*c*x - (9*I)*
a*Sqrt[1 + c^2*x^2] - (3*I)*a*Cosh[3*ArcSinh[c*x]] + 9*a*Sinh[2*ArcSinh[c*x
]] + I*b*Sinh[3*ArcSinh[c*x]])))/(216*c*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-i\,b^2cfx + b^2f\right)\sqrt{i\,cdx + d}\,\sqrt{-i\,cfx + f}\,\log\left(cx + \sqrt{c^2x^2 + 1}\right)^2 + \left(-2i\,abcfx + 2\,abf\right)\sqrt{i\,cdx + d}\,\sqrt{-i\,c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((-I*b^2*c*f*x + b^2*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*
x + sqrt(c^2*x^2 + 1))^2 + (-2*I*a*b*c*f*x + 2*a*b*f)*sqrt(I*c*d*x + d)*sqr
t(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (-I*a^2*c*f*x + a^2*f)*sqrt(
I*c*d*x + d)*sqrt(-I*c*f*x + f), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument TypeError: Bad Argument
TypeError: Bad Argument TypeDone
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \left(-icfx + f\right)^{\frac{3}{2}} \left(a + b\,\mathrm{arcsinh}\left(cx\right)\right)^2 \sqrt{icdx + d}\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,\sqrt{d+c\,d\,x\,1{}\mathrm{i}}\,{\left(f-c\,f\,x\,1{}\mathrm{i}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(3/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{id\,(cx-i)}\,\left(-if\,(cx+i)\right)^{\frac{3}{2}}\,(a+b\operatorname{asinh}(cx))^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))**2*(d+I*c*d*x)**(1/2),x)
```

```
[Out] Integral(sqrt(I*d*(c*x - I))*(-I*f*(c*x + I))**(3/2)*(a + b*asinh(c*x))**2,
 x)
```

3.579 $\int \frac{(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+icdx}}\,dx$

Optimal. Leaf size=436

$$\frac{f^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{2if^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{f^2x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{2\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{bcf^2x}{}$$

```
[Out] -4*I*b^2*f^2*(c^2*x^2+1)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-1/4*b^2*f^2*
x*(c^2*x^2+1)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-2*I*f^2*(c^2*x^2+1)*(a+b*
arcsinh(c*x))^2/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-1/2*f^2*x*(c^2*x^2+1)
*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/4*b^2*f^2*arcsi
nh(c*x)*(c^2*x^2+1)^(1/2)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+4*I*b*f^2*x
*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1
/2*b*c*f^2*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*
c*f*x)^(1/2)+1/2*f^2*(a+b*arcsinh(c*x))^3*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)
^(1/2)/(f-I*c*f*x)^(1/2)
```

Rubi [A] time = 0.63, antiderivative size = 436, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {5712, 5831, 3317, 3296, 2638, 3311, 32, 2635, 8}

$$\frac{f^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{2if^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{f^2x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{2\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{bcf^2x}{}$$

Antiderivative was successfully verified.

```
[In] Int[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[d + I*c*d*x],x]
```

```
[Out] ((-4*I)*b^2*f^2*(1 + c^2*x^2))/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (b
^2*f^2*x*(1 + c^2*x^2))/(4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (b^2*f^2*
Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(4*c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) +
 ((4*I)*b*f^2*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(Sqrt[d + I*c*d*x]*
Sqrt[f - I*c*f*x]) + (b*c*f^2*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(
2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - ((2*I)*f^2*(1 + c^2*x^2)*(a + b*Ar
cSinh[c*x])^2)/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (f^2*x*(1 + c^2*x^
2)*(a + b*ArcSinh[c*x])^2)/(2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (f^2*S
qrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(2*b*c*Sqrt[d + I*c*d*x]*Sqrt[f -
I*c*f*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+icdx}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{(f-icfx)^2\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\,\text{Subst}\left(\int (a+bx)^2(cf-icf\sinh(x))^2\,dx, x, \sinh^{-1}(cx)\right)}{c^3\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\,\text{Subst}\left(\int \left(c^2f^2(a+bx)^2 - 2ic^2f^2(a+bx)^2\sinh(x) - c^2f^2(a\right.\right.}{c^3\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= \frac{f^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{d+icdx}\,\sqrt{f-icfx}} - \frac{\left(2if^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)^2\right.}{c\sqrt{d+icdx}\,\sqrt{f}}$$

$$= \frac{bcf^2x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\,\sqrt{f-icfx}} - \frac{2if^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx\right.}{c\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= -\frac{b^2f^2x\left(1+c^2x^2\right)}{4\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{4ibf^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{bcf^2}{}$$

$$= -\frac{4ib^2f^2\left(1+c^2x^2\right)}{c\sqrt{d+icdx}\,\sqrt{f-icfx}} - \frac{b^2f^2x\left(1+c^2x^2\right)}{4\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{b^2f^2\sqrt{1+c^2x^2}\,\text{sin}}{4c\sqrt{d+icdx}\,\sqrt{f}}$$

Mathematica [A] time = 2.26, size = 532, normalized size = 1.22

$$\frac{12a^2\sqrt{d}\,f^{3/2}\sqrt{c^2x^2+1}\,\log\left(cdfx+\sqrt{d}\,\sqrt{f}\,\sqrt{d+icdx}\,\sqrt{f-icfx}\right) - 16ia^2f\sqrt{c^2x^2+1}\,\sqrt{d+icdx}\,\sqrt{f-icfx} - 4a^2c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[d + I*c*d*x],x]
```

```
[Out] ((32*I)*a*b*c*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (16*I)*a^2*f*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (32*I)*b^2*f*Sqrt[d + I*c
*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 4*a^2*c*f*x*Sqrt[d + I*c*d*x]*S
qrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 4*b^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c
*f*x]*ArcSinh[c*x]^3 + 2*a*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*A
rcSinh[c*x]] + 2*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*((4*I
)*(4*b*c*x + a*(-4 + I*c*x)*Sqrt[1 + c^2*x^2]) + b*Cosh[2*ArcSinh[c*x]]) +
12*a^2*Sqrt[d]*f^(3/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt
[d + I*c*d*x]*Sqrt[f - I*c*f*x]] - b^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
]*Sinh[2*ArcSinh[c*x]] + 2*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[
c*x]^2*(6*a - (8*I)*b*Sqrt[1 + c^2*x^2] - b*Sinh[2*ArcSinh[c*x]]))/(8*c*d*S
qrt[1 + c^2*x^2])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(b^2cfx+ib^2f\right)\sqrt{icdx+d}\,\sqrt{-icfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2 + 2\left(abcfx+iabf\right)\sqrt{icdx+d}\,\sqrt{-icfx}}{cdx-id}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-((b^2*c*f*x + I*b^2*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c
*x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c*f*x + I*a*b*f)*sqrt(I*c*d*x + d)*sqrt(
-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a^2*c*f*x + I*a^2*f)*sqrt(I*c
*d*x + d)*sqrt(-I*c*f*x + f))/(c*d*x - I*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-i\,cfx + f\right)^{\frac{3}{2}}(b\,\operatorname{arsinh}\left(cx\right) + a)^{2}}{\sqrt{i\,cdx + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((-I*c*f*x + f)^(3/2)*(b*arcsinh(c*x) + a)^2/sqrt(I*c*d*x + d), x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{\left(-icfx + f\right)^{\frac{3}{2}}\left(a + b\,\operatorname{arcsinh}\left(cx\right)\right)^{2}}{\sqrt{icdx + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a + b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(f - c\,f\,x\,1{}\mathrm{i}\right)}^{3/2}}{\sqrt{d + c\,d\,x\,1{}\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(1/2),x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-if\left(cx + i\right)\right)^{\frac{3}{2}}\left(a + b\,\operatorname{asinh}\left(cx\right)\right)^{2}}{\sqrt{id\left(cx - i\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))**2/(d+I*c*d*x)**(1/2),x)
```

```
[Out] Integral((-I*f*(c*x + I))**(3/2)*(a + b*asinh(c*x))**2/sqrt(I*d*(c*x - I)),
 x)
```

3.580 $\int \frac{(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}}\,dx$

Optimal. Leaf size=752

$$-\frac{2iabf^3x\left(c^2x^2+1\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{f^3\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^3}{bc(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{if^3\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{4f^3\left(c^2x^2\cdots\right.}{c(d+\cdots}$$

```
[Out] -2*I*a*b*f^3*x*(c^2*x^2+1)^(3/2)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+2*I*b^
2*f^3*(c^2*x^2+1)^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-2*I*b^2*f^3*x*(c^
2*x^2+1)^(3/2)*arcsinh(c*x)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+4*I*f^3*(c^
2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+4*f^3*x
*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+4*f^3
*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/
2)+I*f^3*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)
^(3/2)-f^3*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*x)^(3/2)/(f-
I*c*f*x)^(3/2)-16*I*b*f^3*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*arctan(c*x+(
c^2*x^2+1)^(1/2))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-8*b*f^3*(c^2*x^2+1)
^(3/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/
2)/(f-I*c*f*x)^(3/2)-8*b^2*f^3*(c^2*x^2+1)^(3/2)*polylog(2,-I*(c*x+(c^2*x^2
+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+8*b^2*f^3*(c^2*x^2+1)^(3/
2)*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/
2)-4*b^2*f^3*(c^2*x^2+1)^(3/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I
*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A] time = 1.13, antiderivative size = 752, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 15, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.405, Rules used = {5712, 5833, 5821, 5687, 5714, 3718, 2190, 2279, 2391, 5717, 5693, 4180, 5675, 5653, 261}

$$-\frac{8b^2f^3\left(c^2x^2+1\right)^{3/2}\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{8b^2f^3\left(c^2x^2+1\right)^{3/2}\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{4b^2f^3\left(c^2x^2+1\right)^{3/2}\cdots}{c(d+icdx\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(3/2),x]
```

```
[Out] ((-2*I)*a*b*f^3*x*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(
3/2)) + ((2*I)*b^2*f^3*(1 + c^2*x^2)^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x
)^(3/2)) - ((2*I)*b^2*f^3*x*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x])/((d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2)) + ((4*I)*f^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]
)^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (4*f^3*x*(1 + c^2*x^2)*(
a + b*ArcSinh[c*x])^2)/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (4*f^3*(
1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*
f*x)^(3/2)) + (I*f^3*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*
x)^(3/2)*(f - I*c*f*x)^(3/2)) - (f^3*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x
])^3)/(b*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - ((16*I)*b*f^3*(1 + c^
2*x^2)^(3/2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^
(3/2)*(f - I*c*f*x)^(3/2)) - (8*b*f^3*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*
x])*Log[1 + E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)
) - (8*b^2*f^3*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*(d +
 I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (8*b^2*f^3*(1 + c^2*x^2)^(3/2)*PolyL
og[2, I*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (4*b
^2*f^3*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5833

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}}dx = \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{(f-icfx)^3\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{3/2}\int\left(-\frac{4i\left(if^3+cf^3x\right)\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}} - \frac{3f^3\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}} + \frac{icf^3x}{\cdots}\right.}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{\left(4i\left(1+c^2x^2\right)^{3/2}\right)\int \frac{\left(if^3+cf^3x\right)\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(3f^3\left(1+c^2x^2\right)^{3/2}\right.}{(d+icdx)^{3/}}$$

$$= \frac{if^3\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{f^3\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(\right.}{bc(d+icdx)^{3/2}(f-icfx)^{3/}}$$

$$= -\frac{2iabf^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{if^3\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \cdots$$

$$= -\frac{2iabf^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2ib^2f^3x\left(1+c^2x^2\right)^{3/2}\sinh^{-1}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4if}{\cdots}$$

$$= -\frac{2iabf^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2f^3\left(1+c^2x^2\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2ib^2f^3x\left(\right.}{(d+i}$$

$$= -\frac{2iabf^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2f^3\left(1+c^2x^2\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2ib^2f^3x\left(\right.}{(d+i}$$

$$= -\frac{2iabf^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2f^3\left(1+c^2x^2\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2ib^2f^3x\left(\right.}{(d+i}$$

$$= -\frac{2iabf^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2f^3\left(1+c^2x^2\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2ib^2f^3x\left(\right.}{(d+i}$$

$$= -\frac{2iabf^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2f^3\left(1+c^2x^2\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2ib^2f^3x\left(\right.}{(d+i}$$

Mathematica [B] time = 9.10, size = 1546, normalized size = 2.06

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(3/2),
x]

[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*((I*a^2*f)/d^2 + (4*a^2*f)/(d^
2*(-I + c*x))))/c - (3*a^2*f^(3/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[I*d*(
-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]])/(c*d^(3/2)) + ((2*I)*a*b*f*Sqrt[I*((-I)
*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcS
inh[c*x]/2]*(-(c*x) + 2*ArcSinh[c*x] + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + I*A
rcSinh[c*x]^2 + 4*ArcTan[Coth[ArcSinh[c*x]/2]] + (2*I)*Log[Sqrt[1 + c^2*x^2
]]) + I*(-(c*x) - 2*ArcSinh[c*x] + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + I*ArcSi
nh[c*x]^2 + 4*ArcTan[Coth[ArcSinh[c*x]/2]] + (2*I)*Log[Sqrt[1 + c^2*x^2]])*
Sinh[ArcSinh[c*x]/2]))/(c*d^2*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[
```

```
1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) - (a*b*f*Sqrt
[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(C
osh[ArcSinh[c*x]/2]*(ArcSinh[c*x]*(-4*I + ArcSinh[c*x]) + (8*I)*ArcTan[Tanh
[ArcSinh[c*x]/2]] + 4*Log[Sqrt[1 + c^2*x^2]]) + I*(ArcSinh[c*x]*(4*I + ArcS
inh[c*x]) + (8*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 4*Log[Sqrt[1 + c^2*x^2]])*
Sinh[ArcSinh[c*x]/2]))/(c*d^2*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[
1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) - (b^2*f*Sqrt
[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(C
osh[ArcSinh[c*x]/2]*((6*I)*Pi*ArcSinh[c*x] + (6 - 6*I)*ArcSinh[c*x]^2 + Arc
Sinh[c*x]^3 + 12*((-I)*Pi + 2*ArcSinh[c*x])*Log[1 - I/E^ArcSinh[c*x]] - (24
*I)*Pi*Log[1 + E^ArcSinh[c*x]] + (24*I)*Pi*Log[Cosh[ArcSinh[c*x]/2]] + (12*
I)*Pi*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]]) - 24*PolyLog[2, I/E^ArcSinh[c*
x]]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + (-6*Pi*ArcSinh[c*x] -
 (6 - 6*I)*ArcSinh[c*x]^2 + I*ArcSinh[c*x]^3 + 12*(Pi + (2*I)*ArcSinh[c*x])
*Log[1 - I/E^ArcSinh[c*x]] + 24*Pi*Log[1 + E^ArcSinh[c*x]] - 24*Pi*Log[Cosh
[ArcSinh[c*x]/2]] - 12*Pi*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]])*Sinh[ArcSi
nh[c*x]/2]))/(3*c*d^2*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*
x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) + ((I/3)*b^2*f*Sqrt[I
*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cos
h[ArcSinh[c*x]/2]*(-6*Pi*ArcSinh[c*x] - 6*c*x*ArcSinh[c*x] + (6 + 6*I)*ArcS
inh[c*x]^2 + (2*I)*ArcSinh[c*x]^3 + 3*Sqrt[1 + c^2*x^2]*(2 + ArcSinh[c*x]^2
) + 12*Pi*Log[1 - I/E^ArcSinh[c*x]] + (24*I)*ArcSinh[c*x]*Log[1 - I/E^ArcSi
nh[c*x]] + 24*Pi*Log[1 + E^ArcSinh[c*x]] - 24*Pi*Log[Cosh[ArcSinh[c*x]/2]]
- 12*Pi*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]]) + I*(-6*Pi*ArcSinh[c*x] - 6*
c*x*ArcSinh[c*x] - (6 - 6*I)*ArcSinh[c*x]^2 + (2*I)*ArcSinh[c*x]^3 + 3*Sqrt
[1 + c^2*x^2]*(2 + ArcSinh[c*x]^2) + 12*Pi*Log[1 - I/E^ArcSinh[c*x]] + (24*
I)*ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]] + 24*Pi*Log[1 + E^ArcSinh[c*x]] -
 24*Pi*Log[Cosh[ArcSinh[c*x]/2]] - 12*Pi*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/
4]])*Sinh[ArcSinh[c*x]/2] + 24*PolyLog[2, I/E^ArcSinh[c*x]]*((-I)*Cosh[ArcS
inh[c*x]/2] + Sinh[ArcSinh[c*x]/2])))/(c*d^2*Sqrt[-(((-I)*d + c*d*x)*(I*f +
 c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])
)
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(i b^2 c f x - b^2 f\right)\sqrt{i c d x + d}\,\sqrt{-i c f x + f}\,\log\left(cx + \sqrt{c^2x^2+1}\right)^2 + \left(2i\,abcfx - 2\,abf\right)\sqrt{i c d x + d}\,\sqrt{-i c f x}}{c^2d^2x^2 - 2i\,cd^2x - d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral(((I*b^2*c*f*x - b^2*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*
x + sqrt(c^2*x^2 + 1))^2 + (2*I*a*b*c*f*x - 2*a*b*f)*sqrt(I*c*d*x + d)*sqrt
(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (I*a^2*c*f*x - a^2*f)*sqrt(I*
c*d*x + d)*sqrt(-I*c*f*x + f))/(c^2*d^2*x^2 - 2*I*c*d^2*x - d^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{\left(-icfx+f\right)^{\frac{3}{2}} \left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^{2}}{\left(icdx+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x)
```

```
[Out] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\frac{i\left(c^{2}dfx^{2}+df\right)^{\frac{3}{2}}}{c^{3}d^{3}x^{2}-2i\,c^{2}d^{3}x-cd^{3}}+\frac{6i\,\sqrt{c^{2}dfx^{2}+df}\,f}{i\,c^{2}d^{2}x+cd^{2}}-\frac{3\,f^{2}\,\mathrm{arsinh}\left(cx\right)}{cd^{2}\sqrt{\frac{f}{d}}}\right)+\int\frac{\left(-i\,cfx+f\right)^{\frac{3}{2}}b^{2}\log\left(cx+\sqrt{c^{2}x^{2}+1}\right.}{\left(i\,cdx+d\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x, algor
ithm="maxima")
```

```
[Out] a^2*(I*(c^2*d*f*x^2 + d*f)^(3/2)/(c^3*d^3*x^2 - 2*I*c^2*d^3*x - c*d^3) + 6*
I*sqrt(c^2*d*f*x^2 + d*f)*f/(I*c^2*d^2*x + c*d^2) - 3*f^2*arcsinh(c*x)/(c*d
^2*sqrt(f/d))) + integrate((-I*c*f*x + f)^(3/2)*b^2*log(c*x + sqrt(c^2*x^2
+ 1))^2/(I*c*d*x + d)^(3/2) + 2*(-I*c*f*x + f)^(3/2)*a*b*log(c*x + sqrt(c^2
*x^2 + 1))/(I*c*d*x + d)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2\,{\left(f-c\,f\,x\,1{}\mathrm{i}\right)}^{3/2}}{{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(3/2),x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))**2/(d+I*c*d*x)**(3/2),x)
```

```
[Out] Timed out
```

3.581 $\int \frac{(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{5/2}}\,dx$

Optimal. Leaf size=580

$$\frac{f^4\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{8f^4\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{32bf^4\left(c^2x^2+1\right)^{5/2}\log\left(1+ie^{\sinh^{-1}(c}\right.}{3c(d+icdx)^{5/2}(f-ic}$$

```
[Out] -8/3*f^4*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*
f*x)^(5/2)+1/3*f^4*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*x)^(
5/2)/(f-I*c*f*x)^(5/2)-8/3*I*b^2*f^4*(c^2*x^2+1)^(5/2)*cot(1/4*Pi+1/2*I*arc
sinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-8/3*I*f^4*(c^2*x^2+1)^(5/2
)*(a+b*arcsinh(c*x))^2*cot(1/4*Pi+1/2*I*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(
f-I*c*f*x)^(5/2)+4/3*b*f^4*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*csc(1/4*Pi+
1/2*I*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+2/3*I*f^4*(c^2*
x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2*cot(1/4*Pi+1/2*I*arcsinh(c*x))*csc(1/4*Pi
+1/2*I*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+32/3*b*f^4*(c^
2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*ln(1+I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*
d*x)^(5/2)/(f-I*c*f*x)^(5/2)+32/3*b^2*f^4*(c^2*x^2+1)^(5/2)*polylog(2,-I*(c
*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A] time = 1.20, antiderivative size = 580, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.351, Rules used = {5712, 5833, 5675, 5831, 3318, 4186, 3767, 8, 4184, 3716, 2190, 2279, 2391}

$$\frac{32b^2f^4\left(c^2x^2+1\right)^{5/2}\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{f^4\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{8f^4\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh\right.}{3c(d+icdx)^{5/2}(f-icf}$$

Antiderivative was successfully verified.

```
[In] Int[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(5/2),x]
```

```
[Out] (-8*f^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(3*c*(d + I*c*d*x)^(5/2
)*(f - I*c*f*x)^(5/2)) + (f^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^3)/(
3*b*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((8*I)/3)*b^2*f^4*(1 + c^
2*x^2)^(5/2)*Cot[Pi/4 + (I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*
c*f*x)^(5/2)) - (((8*I)/3)*f^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*C
ot[Pi/4 + (I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
 + (4*b*f^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*Csc[Pi/4 + (I/2)*ArcSin
h[c*x]]^2)/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((2*I)/3)*f^4*(
1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*Cot[Pi/4 + (I/2)*ArcSinh[c*x]]*Cs
c[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
 + (32*b*f^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*Log[1 + I*E^ArcSinh[c
*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (32*b^2*f^4*(1 + c^2*
x^2)^(5/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I
*c*f*x)^(5/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5833

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{5/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(f-icfx)^4\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{5/2}\int\left(\frac{f^4\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}} - \frac{4f^4\left(a+b\sinh^{-1}(cx)\right)^2}{(-i+cx)^2\sqrt{1+c^2x^2}} + \frac{4if^4\left(a+b\sinh^{-1}(c\right.}{(-i+cx)\sqrt{1+c^2x}}\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(4if^4\left(1+c^2x^2\right)^{5/2}\right)\int\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(-i+cx)\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(f^4\left(1+c^2x^2\right)^{5/2}\right)\int\frac{(a+b\,\mathrm{si}}{\sqrt{1}}}{(d+icdx)^{5/2}(f-ic}$$

$$= \frac{f^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(4if^4\left(1+c^2x^2\right)^{5/2}\right)\mathrm{Subst}\left(\int\right.}{(d+icdx)^{5/2}(}$$

$$= \frac{f^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(f^4\left(1+c^2x^2\right)^{5/2}\right)\mathrm{Subst}\left(\int(a\right.}{c(d+ic}$$

$$= \frac{f^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{4if^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-}\right.}{c(d+icdx)^{5/}}$$

$$= -\frac{4f^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh\right.}{3bc(d+icdx)^{5/2}(f-icf}$$

$$= -\frac{8f^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh\right.}{3bc(d+icdx)^{5/2}(f-icf}$$

$$= -\frac{8f^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh\right.}{3bc(d+icdx)^{5/2}(f-icf}$$

$$= -\frac{8f^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh\right.}{3bc(d+icdx)^{5/2}(f-icf}$$

$$= -\frac{8f^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh\right.}{3bc(d+icdx)^{5/2}(f-icf}$$

Mathematica [B] time = 10.24, size = 1609, normalized size = 2.77

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(5/2),
x]
```

```
[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*((((-4*I)/3)*a^2*f)/(d^3*(-I +
 c*x)^2) - (8*a^2*f)/(3*d^3*(-I + c*x))))/c + (a^2*f^(3/2)*Log[c*d*f*x + Sq
rt[d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]])/(c*d^(5/2)) + (
(I/3)*a*b*f*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1
 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])*((-I)*Cosh[(3
*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]/2]] - I*Log[Sq
rt[1 + c^2*x^2]]) + Cosh[ArcSinh[c*x]/2]*(4 + (3*I)*ArcSinh[c*x] - (6*I)*Ar
cTan[Coth[ArcSinh[c*x]/2]] + 3*Log[Sqrt[1 + c^2*x^2]]) + 2*(Sqrt[1 + c^2*x^
2]*(ArcSinh[c*x] + 2*ArcTan[Coth[ArcSinh[c*x]/2]] + I*Log[Sqrt[1 + c^2*x^2]
]) + 2*(I + ArcSinh[c*x] + 2*ArcTan[Coth[ArcSinh[c*x]/2]] + I*Log[Sqrt[1 +
c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/(c*d^3*(I + c*x)*Sqrt[-(((-I)*d + c*d*x)
```

```
*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^4) - (a*b*
f*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2
))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])*(Cosh[(3*ArcSinh[c*x])/
2]*((-14 + (3*I)*ArcSinh[c*x])*ArcSinh[c*x] - 28*ArcTan[Tanh[ArcSinh[c*x]/2
]] + (14*I)*Log[Sqrt[1 + c^2*x^2]]) + Cosh[ArcSinh[c*x]/2]*(84*ArcTan[Tanh[
ArcSinh[c*x]/2]] - I*(8 - (6*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 + 42*Log[Sq
rt[1 + c^2*x^2]])) + 2*(4 - (4*I)*ArcSinh[c*x] + 6*ArcSinh[c*x]^2 + (56*I)*
ArcTan[Tanh[ArcSinh[c*x]/2]] + 28*Log[Sqrt[1 + c^2*x^2]] + Sqrt[1 + c^2*x^2
]*(ArcSinh[c*x]*(-14*I + 3*ArcSinh[c*x]) + (28*I)*ArcTan[Tanh[ArcSinh[c*x]/
2]] + 14*Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/(6*c*d^3*(I + c*x)
*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[Arc
Sinh[c*x]/2])^4) + ((I/3)*b^2*f*(I + c*x)*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I
)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*((-1 + I)*ArcSinh[c*x]^2 - (2*A
rcSinh[c*x]*(-2*I + ArcSinh[c*x]))/(-I + c*x) + (2*I)*(Pi + (2*I)*ArcSinh[c
*x])*Log[1 - I/E^ArcSinh[c*x]] - I*Pi*(ArcSinh[c*x] - 4*Log[1 + E^ArcSinh[c
*x]] + 4*Log[Cosh[ArcSinh[c*x]/2]] + 2*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]
]) + 4*PolyLog[2, I/E^ArcSinh[c*x]] - (4*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2
])/(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^3 + (2*(4 + ArcSinh[c*x]
^2)*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))
/(c*d^3*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[Arc
Sinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^2) + (b^2*f*(I + c*x)*Sqrt[I*((-I)*d
 + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(7*Pi*ArcSin
h[c*x] - (7 + 7*I)*ArcSinh[c*x]^2 - I*ArcSinh[c*x]^3 + (2*ArcSinh[c*x]*(-2*
I + ArcSinh[c*x]))/(1 + I*c*x) - 14*(Pi + (2*I)*ArcSinh[c*x])*Log[1 - I/E^A
rcSinh[c*x]] - 28*Pi*Log[1 + E^ArcSinh[c*x]] + 28*Pi*Log[Cosh[ArcSinh[c*x]/
2]] + 14*Pi*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]] + (28*I)*PolyLog[2, I/E^A
rcSinh[c*x]] - ((4*I)*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*
x]/2] + I*Sinh[ArcSinh[c*x]/2])^3 + (2*(4 + 7*ArcSinh[c*x]^2)*Sinh[ArcSinh[
c*x]/2])/((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2])))/(3*c*d^3*Sqrt
[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2]
 - I*Sinh[ArcSinh[c*x]/2])^2)
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2cfx + i\,b^2f\right)\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2 + 2\left(abcfx + i\,abf\right)\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx}}{c^3d^3x^3 - 3i\,c^2d^3x^2 - 3\,cd^3x + i\,d^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral(((b^2*c*f*x + I*b^2*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*
x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c*f*x + I*a*b*f)*sqrt(I*c*d*x + d)*sqrt(-
I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a^2*c*f*x + I*a^2*f)*sqrt(I*c*
d*x + d)*sqrt(-I*c*f*x + f))/(c^3*d^3*x^3 - 3*I*c^2*d^3*x^2 - 3*c*d^3*x + I
*d^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
```

```
assuming [c,d,t_nostep]=[21,-43,-38]schur row 3 -2.83202e-07Warning, choosi
ng root of [1,0,%%%{-8,[2,4,2]%%%}+%%%{-6,[0,0,0]%%%},%%%{-32,[2,4,2]%%%}+%
%%{-8,[0,0,0]%%%},%%%{16,[4,8,4]%%%}+%%%{-24,[2,4,2]%%%}+%%%{-3,[0,0,0]%%%}
] at parameters values [-89,63,-49]Warning, need to choose a branch for the
 root of a polynomial with parameters. This might be wrong.The choice was d
one assuming [c,d,t_nostep]=[40,82,-30]schur row 1 4.21049e-07Francis algor
ithm not precise enough for[1.0,0.0,-8.3670048e+13,-3.34680192e+14,1.750169
23308e+27]Warning, choosing root of [1,0,%%%{-8,[2,4,2]%%%}+%%%{-6,[0,0,0]%
%%},%%%{-32,[2,4,2]%%%}+%%%{-8,[0,0,0]%%%},%%%{16,[4,8,4]%%%}+%%%{-24,[2,4,
2]%%%}+%%%{-3,[0,0,0]%%%}] at parameters values [-30,70,22]Warning, need to
 choose a branch for the root of a polynomial with parameters. This might b
e wrong.The choice was done assuming [c,d,t_nostep]=[2,-99,37]schur row 3 -
9.03227e-07Warning, need to choose a branch for the root of a polynomial wi
th parameters. This might be wrong.The choice was done assuming [c,d,t_nost
ep]=[80,4,51]Warning, need to choose a branch for the root of a polynomial
with parameters. This might be wrong.The choice was done assuming [c,d,t_no
step]=[15,-14,48]schur row 3 -7.71736e-07Warning, choosing root of [1,0,%%%
{-8,[2,4,2]%%%}+%%%{-6,[0,0,0]%%%},%%%{-32,[2,4,2]%%%}+%%%{-8,[0,0,0]%%%},%
%%{16,[4,8,4]%%%}+%%%{-24,[2,4,2]%%%}+%%%{-3,[0,0,0]%%%}] at parameters val
ues [-85,28,-44]Warning, need to choose a branch for the root of a polynomi
al with parameters. This might be wrong.The choice was done assuming [c,d,t
_nostep]=[-45,-95,-43]schur row 3 -1.82298e-07Warning, choosing root of [1,
0,%%%{-8,[2,4,2]%%%}+%%%{-6,[0,0,0]%%%},%%%{-32,[2,4,2]%%%}+%%%{-8,[0,0,0]%
%%},%%%{16,[4,8,4]%%%}+%%%{-24,[2,4,2]%%%}+%%%{-3,[0,0,0]%%%}] at parameter
s values [91,31,-21]Warning, need to choose a branch for the root of a poly
nomial with parameters. This might be wrong.The choice was done assuming [c
,d,t_nostep]=[88,-89,-82]Warning, need to choose a branch for the root of a
 polynomial with parameters. This might be wrong.The choice was done assumi
ng [c,d,t_nostep]=[40,-67,-59]Evaluation time: 7.35sym2poly/r2sym(const gen
 & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\left(-icfx+f\right)^{\frac{3}{2}} \left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^{2}}{\left(icdx+d\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x)
[Out] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x, algor
ithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(f-c\,f\,x\,1{}\mathrm{i}\right)}^{3/2}}{{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(5/2),x)
[Out] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(3/2))/(d + c*d*x*1i)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))**2/(d+I*c*d*x)**(5/2),x)
[Out] Timed out
```

3.582 $\int (d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=548

$$\frac{5(d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{48bc\left(c^2x^2+1\right)^{5/2}}+\frac{5x(d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{24\left(c^2x^2+1\right)}+\frac{5x(d+icdx)}{}$$

```
[Out] 1/108*b^2*x*(d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)+245/1152*b^2*x*(d+I*c*d*x)^
(5/2)*(f-I*c*f*x)^(5/2)/(c^2*x^2+1)^2+65/1728*b^2*x*(d+I*c*d*x)^(5/2)*(f-I*
c*f*x)^(5/2)/(c^2*x^2+1)-115/1152*b^2*(d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*a
rcsinh(c*x)/c/(c^2*x^2+1)^(5/2)-5/16*b*c*x^2*(d+I*c*d*x)^(5/2)*(f-I*c*f*x)^
(5/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(5/2)+1/6*x*(d+I*c*d*x)^(5/2)*(f-I*c*f
*x)^(5/2)*(a+b*arcsinh(c*x))^2+5/16*x*(d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(
a+b*arcsinh(c*x))^2/(c^2*x^2+1)^2+5/24*x*(d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2
)*(a+b*arcsinh(c*x))^2/(c^2*x^2+1)+5/48*(d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)
*(a+b*arcsinh(c*x))^3/b/c/(c^2*x^2+1)^(5/2)-5/48*b*(d+I*c*d*x)^(5/2)*(f-I*c
*f*x)^(5/2)*(a+b*arcsinh(c*x))/c/(c^2*x^2+1)^(1/2)-1/18*b*(d+I*c*d*x)^(5/2)
*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] time = 0.61, antiderivative size = 548, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {5712, 5684, 5682, 5675, 5661, 321, 215, 5717, 195}

$$\frac{5(d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{48bc\left(c^2x^2+1\right)^{5/2}}+\frac{5x(d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{24\left(c^2x^2+1\right)}+\frac{5x(d+icdx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (b^2*x*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))/108 + (245*b^2*x*(d + I*c*d
*x)^(5/2)*(f - I*c*f*x)^(5/2))/(1152*(1 + c^2*x^2)^2) + (65*b^2*x*(d + I*c*
d*x)^(5/2)*(f - I*c*f*x)^(5/2))/(1728*(1 + c^2*x^2)) - (115*b^2*(d + I*c*d*
x)^(5/2)*(f - I*c*f*x)^(5/2)*ArcSinh[c*x])/(1152*c*(1 + c^2*x^2)^(5/2)) - (
5*b*c*x^2*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/(16
*(1 + c^2*x^2)^(5/2)) - (5*b*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b
*ArcSinh[c*x]))/(48*c*Sqrt[1 + c^2*x^2]) - (b*(d + I*c*d*x)^(5/2)*(f - I*c*
f*x)^(5/2)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(18*c) + (x*(d + I*c*d*x
)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/6 + (5*x*(d + I*c*d*x)^
(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(16*(1 + c^2*x^2)^2) + (5
*x*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(24*(1 +
 c^2*x^2)) + (5*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]
)^3)/(48*b*c*(1 + c^2*x^2)^(5/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx = \frac{\left((d+icdx)^{5/2}(f-icfx)^{5/2}\right)\int\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}\right.}{\left(1+c^2x^2\right)^{5/2}}$$

$$= \frac{1}{6}x(d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2 + \frac{(5(d+}{}$$

$$= -\frac{b(d+icdx)^{5/2}(f-icfx)^{5/2}\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right.}{18c}$$

$$= \frac{1}{108}b^2x(d+icdx)^{5/2}(f-icfx)^{5/2} - \frac{5b(d+icdx)^{5/2}(f-ic}{48c\sqrt{}}$$

$$= \frac{1}{108}b^2x(d+icdx)^{5/2}(f-icfx)^{5/2} + \frac{65b^2x(d+icdx)^{5/2}(f}{1728\left(1+c^2x\right.}$$

$$= \frac{1}{108}b^2x(d+icdx)^{5/2}(f-icfx)^{5/2} + \frac{245b^2x(d+icdx)^{5/2}(}{1152\left(1+c^2x\right.}$$

$$= \frac{1}{108}b^2x(d+icdx)^{5/2}(f-icfx)^{5/2} + \frac{245b^2x(d+icdx)^{5/2}(}{1152\left(1+c^2x\right.}$$

Mathematica [A] time = 2.34, size = 735, normalized size = 1.34

$$\frac{4320a^2d^{5/2}f^{5/2}\sqrt{c^2x^2+1}\,\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)+9504a^2cd^2f^2x\sqrt{c^2x^2+1}\sqrt{d+icdx}\sqrt{f}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (9504*a^2*c*d^2*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]
 + 7488*a^2*c^3*d^2*f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^
2*x^2] + 2304*a^2*c^5*d^2*f^2*x^5*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[
1 + c^2*x^2] + 1440*b^2*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh
[c*x]^3 - 3240*a*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSi
nh[c*x]] - 324*a*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcSi
nh[c*x]] - 24*a*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[6*ArcSin
h[c*x]] + 4320*a^2*d^(5/2)*f^(5/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*
Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + 1620*b^2*d^2*f^2*Sqrt[d + I*
c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] + 81*b^2*d^2*f^2*Sqrt[d + I*c
*d*x]*Sqrt[f - I*c*f*x]*Sinh[4*ArcSinh[c*x]] + 4*b^2*d^2*f^2*Sqrt[d + I*c*d
*x]*Sqrt[f - I*c*f*x]*Sinh[6*ArcSinh[c*x]] - 12*b*d^2*f^2*Sqrt[d + I*c*d*x]
*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*(270*b*Cosh[2*ArcSinh[c*x]] + 27*b*Cosh[4*A
rcSinh[c*x]] + 2*b*Cosh[6*ArcSinh[c*x]] - 540*a*Sinh[2*ArcSinh[c*x]] - 108*
a*Sinh[4*ArcSinh[c*x]] - 12*a*Sinh[6*ArcSinh[c*x]]) + 72*b*d^2*f^2*Sqrt[d +
 I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(60*a + 45*b*Sinh[2*ArcSinh[c*x]
] + 9*b*Sinh[4*ArcSinh[c*x]] + b*Sinh[6*ArcSinh[c*x]]))/(13824*c*Sqrt[1 + c
^2*x^2])
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(b^2c^4d^2f^2x^4+2\,b^2c^2d^2f^2x^2+b^2d^2f^2\right)\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2+2\left(abc^4d^2f^2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="fricas")
```

```
[Out] integral((b^2*c^4*d^2*f^2*x^4 + 2*b^2*c^2*d^2*f^2*x^2 + b^2*d^2*f^2)*sqrt(I
*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c^4*
d^2*f^2*x^4 + 2*a*b*c^2*d^2*f^2*x^2 + a*b*d^2*f^2)*sqrt(I*c*d*x + d)*sqrt(-
I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a^2*c^4*d^2*f^2*x^4 + 2*a^2*c^
2*d^2*f^2*x^2 + a^2*d^2*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int (icdx+d)^{\frac{5}{2}} \left(-icfx+f\right)^{\frac{5}{2}} (a+b\,\mathrm{arcsinh}\,(cx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{asinh}\,(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^{5/2}\,\left(f-c\,f\,x\,1\mathrm{i}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(5/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(5/2)*(f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

3.583 $\int (d+icdx)^{3/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=774

$$-\frac{3bcfx^2(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{8\left(c^2x^2+1\right)^{3/2}}+\frac{3fx(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{8\left(c^2x^2+1\right)}+\frac{2ibf}{}$$

```
[Out] -8/225*I*b^2*f*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/c+1/32*b^2*f*x*(d+I*c*d*
x)^(3/2)*(f-I*c*f*x)^(3/2)-16/75*I*b^2*f*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2
)/c/(c^2*x^2+1)+15/64*b^2*f*x*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)/(c^2*x^2+
1)-2/125*I*b^2*f*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(c^2*x^2+1)/c-9/64*b^2
*f*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*arcsinh(c*x)/c/(c^2*x^2+1)^(3/2)+2/5
*I*b*f*x*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)
^(3/2)-3/8*b*c*f*x^2*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))
/(c^2*x^2+1)^(3/2)+4/15*I*b*c^2*f*x^3*(d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(
a+b*arcsinh(c*x))/(c^2*x^2+1)^(3/2)+2/25*I*b*c^4*f*x^5*(d+I*c*d*x)^(3/2)*(f
-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(3/2)+1/4*f*x*(d+I*c*d*x)^(3
/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2+3/8*f*x*(d+I*c*d*x)^(3/2)*(f-I*c
*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(c^2*x^2+1)-1/5*I*f*(d+I*c*d*x)^(3/2)*(f-I
*c*f*x)^(3/2)*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c+1/8*f*(d+I*c*d*x)^(3/2)*(f
-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^3/b/c/(c^2*x^2+1)^(3/2)-1/8*b*f*(d+I*c*d
*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] time = 0.83, antiderivative size = 774, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.405, Rules used = {5712, 5821, 5684, 5682, 5675, 5661, 321, 215, 5717, 195, 194, 5679, 12, 1247, 698}

$$\frac{2ibc^4fx^5(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{25\left(c^2x^2+1\right)^{3/2}}+\frac{4ibc^2fx^3(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{15\left(c^2x^2+1\right)^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (((-8*I)/225)*b^2*f*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/c + (b^2*f*x*(
d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/32 - (((16*I)/75)*b^2*f*(d + I*c*d*
x)^(3/2)*(f - I*c*f*x)^(3/2))/(c*(1 + c^2*x^2)) + (15*b^2*f*x*(d + I*c*d*x)
^(3/2)*(f - I*c*f*x)^(3/2))/(64*(1 + c^2*x^2)) - (((2*I)/125)*b^2*f*(d + I*
c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(1 + c^2*x^2))/c - (9*b^2*f*(d + I*c*d*x)^
(3/2)*(f - I*c*f*x)^(3/2)*ArcSinh[c*x])/(64*c*(1 + c^2*x^2)^(3/2)) + (((2*I
)/5)*b*f*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(1
 + c^2*x^2)^(3/2) - (3*b*c*f*x^2*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a
 + b*ArcSinh[c*x]))/(8*(1 + c^2*x^2)^(3/2)) + (((4*I)/15)*b*c^2*f*x^3*(d +
I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(1 + c^2*x^2)^(3/2
) + (((2*I)/25)*b*c^4*f*x^5*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*
ArcSinh[c*x]))/(1 + c^2*x^2)^(3/2) - (b*f*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)
^(3/2)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(8*c) + (f*x*(d + I*c*d*x)^(
3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/4 + (3*f*x*(d + I*c*d*x)^(
3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(8*(1 + c^2*x^2)) - ((I/5)
*f*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x
])^2)/c + (f*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^3
)/(8*b*c*(1 + c^2*x^2)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5679

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx = \frac{\left((d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int (f-icfx)\left(1+c^2x^2\right)^{3/2}\left(a+b\right.}{\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{\left((d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int \left(f\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(\right.\right.}{\left(1+c^2x^2\right)^{3}}$$

$$= \frac{\left(f(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int \left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(c\right.}{\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{1}{4}fx(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2 - \frac{if(d+i}{}$$

$$= \frac{2ibfx(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{5\left(1+c^2x^2\right)^{3/2}} + \frac{4ibc^2f}{}$$

$$= \frac{1}{32}b^2fx(d+icdx)^{3/2}(f-icfx)^{3/2} + \frac{2ibfx(d+icdx)^{3/2}(f-i}{5\left(1+\right.}$$

$$= \frac{1}{32}b^2fx(d+icdx)^{3/2}(f-icfx)^{3/2} + \frac{15b^2fx(d+icdx)^{3/2}(f-}{64\left(1+c^2x^2\right)}$$

$$= \frac{1}{32}b^2fx(d+icdx)^{3/2}(f-icfx)^{3/2} + \frac{15b^2fx(d+icdx)^{3/2}(f-}{64\left(1+c^2x^2\right)}$$

$$= -\frac{8ib^2f(d+icdx)^{3/2}(f-icfx)^{3/2}}{225c} + \frac{1}{32}b^2fx(d+icdx)^{3/2}(f-$$

Mathematica [**A**] time = 3.19, size = 1084, normalized size = 1.40

$$\frac{-57600ia^2c^4df^2\sqrt{icxd+d}\sqrt{f-icfx}\sqrt{c^2x^2+1}\,x^4+72000a^2c^3df^2\sqrt{icxd+d}\sqrt{f-icfx}\sqrt{c^2x^2+1}\,x^3-115200ia}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] ((72000*I)*a*b*c*d*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (57600*I)*a^
2*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (72000*I)*b
^2*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 180000*a^2
*c*d*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (115200*
I)*a^2*c^2*d*f^2*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]
+ 72000*a^2*c^3*d*f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*
x^2] - (57600*I)*a^2*c^4*d*f^2*x^4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt
[1 + c^2*x^2] + 36000*b^2*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh
[c*x]^3 - 72000*a*b*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSin
h[c*x]] - (4000*I)*b^2*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[3*Arc
Sinh[c*x]] - 4500*a*b*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcS
inh[c*x]] - (288*I)*b^2*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[5*Ar
cSinh[c*x]] + 108000*a^2*d^(3/2)*f^(5/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sq
rt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + 36000*b^2*d*f^2*Sqrt[d
```

```
+ I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] + (12000*I)*a*b*d*f^2*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]] + 1125*b^2*d*f^2*Sqr
t[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[4*ArcSinh[c*x]] + 1800*b*d*f^2*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(60*a - (20*I)*b*Sqrt[1 + c^2*
x^2] - (10*I)*b*Cosh[3*ArcSinh[c*x]] - (2*I)*b*Cosh[5*ArcSinh[c*x]] + 40*b*
Sinh[2*ArcSinh[c*x]] + 5*b*Sinh[4*ArcSinh[c*x]]) + (1440*I)*a*b*d*f^2*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[5*ArcSinh[c*x]] + 60*b*d*f^2*Sqrt[d + I
*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*((1200*I)*b*c*x - (1200*I)*a*Sqrt[1
+ c^2*x^2] - 1200*b*Cosh[2*ArcSinh[c*x]] - (600*I)*a*Cosh[3*ArcSinh[c*x]] -
 75*b*Cosh[4*ArcSinh[c*x]] - (120*I)*a*Cosh[5*ArcSinh[c*x]] + 2400*a*Sinh[2
*ArcSinh[c*x]] + (200*I)*b*Sinh[3*ArcSinh[c*x]] + 300*a*Sinh[4*ArcSinh[c*x]
] + (24*I)*b*Sinh[5*ArcSinh[c*x]]))/(288000*c*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-i\,b^2c^3df^2x^3+b^2c^2df^2x^2-i\,b^2cdf^2x+b^2df^2\right)\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2+\left(-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="fricas")
```

```
[Out] integral((-I*b^2*c^3*d*f^2*x^3 + b^2*c^2*d*f^2*x^2 - I*b^2*c*d*f^2*x + b^2*
d*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2
+ (-2*I*a*b*c^3*d*f^2*x^3 + 2*a*b*c^2*d*f^2*x^2 - 2*I*a*b*c*d*f^2*x + 2*a*b
*d*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) +
 (-I*a^2*c^3*d*f^2*x^3 + a^2*c^2*d*f^2*x^2 - I*a^2*c*d*f^2*x + a^2*d*f^2)*s
qrt(I*c*d*x + d)*sqrt(-I*c*f*x + f), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument TypeError: Bad Argument
TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int (icdx+d)^{\frac{3}{2}}\left(-icfx+f\right)^{\frac{5}{2}}(a+b\,\mathrm{arcsinh}\,(cx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\operatorname{asinh}(c\,x))^2\,(d + c\,d\,x\,1\mathrm{i})^{3/2}\,\left(f - c\,f\,x\,1\mathrm{i}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(5/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(3/2)*(f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

3.584 $\int \sqrt{d+icdx}\,(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=680

$$-\frac{1}{4}c^2f^2x^3\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)^2-\frac{3bcf^2x^2\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)}{8\sqrt{c^2x^2+1}}+\frac{4ibf^2x}{}$$

```
[Out] -8/9*I*b^2*f^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c+15/64*b^2*f^2*x*(d+I*c
*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-1/32*b^2*c^2*f^2*x^3*(d+I*c*d*x)^(1/2)*(f-I*c
*f*x)^(1/2)-4/27*I*b^2*f^2*(c^2*x^2+1)*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/
c+3/8*f^2*x*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-1/4*c^
2*f^2*x^3*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)-2/3*I*f^
2*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c-15
/64*b^2*f^2*arcsinh(c*x)*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/c/(c^2*x^2+1)^
(1/2)+4/3*I*b*f^2*x*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/
(c^2*x^2+1)^(1/2)-3/8*b*c*f^2*x^2*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I
*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+4/9*I*b*c^2*f^2*x^3*(a+b*arcsinh(c*x))*(d+I
*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+1/8*b*c^3*f^2*x^4*(a+b*ar
csinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/(c^2*x^2+1)^(1/2)+5/24*f^2*
(a+b*arcsinh(c*x))^3*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2)/b/c/(c^2*x^2+1)^(1
/2)
```

Rubi [A] time = 1.06, antiderivative size = 680, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 13, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.351$, Rules used = {5712, 5821, 5682, 5675, 5661, 321, 215, 5717, 5679, 444, 43, 5742, 5758}

$$\frac{bc^3f^2x^4\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)}{8\sqrt{c^2x^2+1}}-\frac{1}{4}c^2f^2x^3\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)^2+\frac{4ibc^2f^2x}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (((-8*I)/9)*b^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/c + (15*b^2*f^2*x*
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/64 - (b^2*c^2*f^2*x^3*Sqrt[d + I*c*d*x
]*Sqrt[f - I*c*f*x])/32 - (((4*I)/27)*b^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*
c*f*x]*(1 + c^2*x^2))/c - (15*b^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*A
rcSinh[c*x])/(64*c*Sqrt[1 + c^2*x^2]) + (((4*I)/3)*b*f^2*x*Sqrt[d + I*c*d*x
]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2] - (3*b*c*f^2*x^
2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(8*Sqrt[1 + c^2
*x^2]) + (((4*I)/9)*b*c^2*f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a +
b*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2] + (b*c^3*f^2*x^4*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(8*Sqrt[1 + c^2*x^2]) + (3*f^2*x*Sqrt[d
+ I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/8 - (c^2*f^2*x^3*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/4 - (((2*I)/3)*f^2*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/c
+ (5*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^3)/(24*b*
c*Sqrt[1 + c^2*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5679

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
```

```
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int \sqrt{d+icdx}\,(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx = \frac{\left(\sqrt{d+icdx}\,\sqrt{f-icfx}\right)\int(f-icfx)^2\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}\right.}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(\sqrt{d+icdx}\,\sqrt{f-icfx}\right)\int\left(f^2\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)^2\right.}{}$$

$$= \frac{\left(f^2\sqrt{d+icdx}\,\sqrt{f-icfx}\right)\int\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)^2\,d}{\sqrt{1+c^2x^2}}$$

$$= \frac{1}{2}f^2x\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)^2-\frac{1}{4}c^2f^2x^3\sqrt{d}$$

$$= \frac{4ibf^2x\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(a+b\sinh^{-1}(cx)\right)}{3\sqrt{1+c^2x^2}}-\frac{bcf^2x^2\sqrt{d}}{}$$

$$= \frac{1}{4}b^2f^2x\sqrt{d+icdx}\,\sqrt{f-icfx}-\frac{1}{32}b^2c^2f^2x^3\sqrt{d+icdx}\,\sqrt{f-i}$$

$$= \frac{15}{64}b^2f^2x\sqrt{d+icdx}\,\sqrt{f-icfx}-\frac{1}{32}b^2c^2f^2x^3\sqrt{d+icdx}\,\sqrt{f-}$$

$$= -\frac{8ib^2f^2\sqrt{d+icdx}\,\sqrt{f-icfx}}{9c}+\frac{15}{64}b^2f^2x\sqrt{d+icdx}\,\sqrt{f-icf}$$

Mathematica [**A**] time = 2.43, size = 890, normalized size = 1.31

$$-1728a^2c^3f^2\sqrt{icxd+d}\,\sqrt{f-icfx}\,\sqrt{c^2x^2+1}\,x^3-4608ia^2c^2f^2\sqrt{icxd+d}\,\sqrt{f-icfx}\,\sqrt{c^2x^2+1}\,x^2+6912iabcf^2\sqrt{i}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] ((6912*I)*a*b*c*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (4608*I)*a^2*f^
2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (6912*I)*b^2*f^2*
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 2592*a^2*c*f^2*x*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (4608*I)*a^2*c^2*f^2*
x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 1728*a^2*c^3*f^
2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 1440*b^2*f^2*
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 - 1728*a*b*f^2*Sqrt[d +
I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] - (256*I)*b^2*f^2*Sqrt[d +
I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[3*ArcSinh[c*x]] + 108*a*b*f^2*Sqrt[d + I*c*
d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] + 4320*a^2*Sqrt[d]*f^(5/2)*Sqrt
[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*
f*x]] + 864*b^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]
] + (768*I)*a*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]
] - 27*b^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[4*ArcSinh[c*x]] + 1
2*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*((576*I)*b*c*x - (
576*I)*a*Sqrt[1 + c^2*x^2] - 144*b*Cosh[2*ArcSinh[c*x]] - (192*I)*a*Cosh[3*
ArcSinh[c*x]] + 9*b*Cosh[4*ArcSinh[c*x]] + 288*a*Sinh[2*ArcSinh[c*x]] + (64
*I)*b*Sinh[3*ArcSinh[c*x]] - 36*a*Sinh[4*ArcSinh[c*x]]) + 72*b*f^2*Sqrt[d +
 I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(60*a - (48*I)*b*Sqrt[1 + c^2*x^
```

```
2] - (16*I)*b*Cosh[3*ArcSinh[c*x]] + 24*b*Sinh[2*ArcSinh[c*x]] - 3*b*Sinh[4
*ArcSinh[c*x]]))/(6912*c*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(b^2c^2f^2x^2 + 2i\,b^2cf^2x - b^2f^2\right)\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2 - \left(2\,abc^2f^2x^2 + 4i\,ab\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(b^2*c^2*f^2*x^2 + 2*I*b^2*c*f^2*x - b^2*f^2)*sqrt(I*c*d*x + d)*s
qrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 - (2*a*b*c^2*f^2*x^2 + 4*I
*a*b*c*f^2*x - 2*a*b*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sq
rt(c^2*x^2 + 1)) - (a^2*c^2*f^2*x^2 + 2*I*a^2*c*f^2*x - a^2*f^2)*sqrt(I*c*d
*x + d)*sqrt(-I*c*f*x + f), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \left(-icfx+f\right)^{\frac{5}{2}} \left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^2 \sqrt{icdx+d}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,\sqrt{d+c\,d\,x\,1{}\mathrm{i}}\,{\left(f-c\,f\,x\,1{}\mathrm{i}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(5/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))**2*(d+I*c*d*x)**(1/2),x)
[Out] Timed out
```

3.585 $\int \frac{(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+icdx}}\,dx$

Optimal. Leaf size=615

$$\frac{5f^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{6bc\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{icf^3x^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{3f^3x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{2\sqrt{d+icdx}\sqrt{f-icfx}}$$

```
[Out] -68/9*I*b^2*f^3*(c^2*x^2+1)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-3/4*b^2*f
^3*x*(c^2*x^2+1)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+2/27*I*b^2*f^3*(c^2*x^
2+1)^2/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-11/3*I*f^3*(c^2*x^2+1)*(a+b*ar
csinh(c*x))^2/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-3/2*f^3*x*(c^2*x^2+1)*(
a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/3*I*c*f^3*x^2*(c^
2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+3/4*b^2*f
^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+22/
3*I*b*f^3*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f
*x)^(1/2)+3/2*b*c*f^3*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^
(1/2)/(f-I*c*f*x)^(1/2)-2/9*I*b*c^2*f^3*x^3*(a+b*arcsinh(c*x))*(c^2*x^2+1)^
(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+5/6*f^3*(a+b*arcsinh(c*x))^3*(c^2
*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

Rubi [A] time = 0.78, antiderivative size = 615, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.270, Rules used = {5712, 5831, 3317, 3296, 2638, 3311, 32, 2635, 8, 2633}

$$\frac{5f^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{6bc\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{icf^3x^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{3f^3x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{2\sqrt{d+icdx}\sqrt{f-icfx}}$$

Antiderivative was successfully verified.

```
[In] Int[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[d + I*c*d*x],x]
```

```
[Out] (((-68*I)/9)*b^2*f^3*(1 + c^2*x^2))/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])
 - (3*b^2*f^3*x*(1 + c^2*x^2))/(4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + ((
(2*I)/27)*b^2*f^3*(1 + c^2*x^2)^2)/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])
+ (3*b^2*f^3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(4*c*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x]) + (((22*I)/3)*b*f^3*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (3*b*c*f^3*x^2*Sqrt[1 + c^2*x^2]*(a
+ b*ArcSinh[c*x]))/(2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (((2*I)/9)*b*c
^2*f^3*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]) - (((11*I)/3)*f^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (3*f^3*x*(1 + c^2*x^2)*(a + b*ArcSinh[
c*x])^2)/(2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + ((I/3)*c*f^3*x^2*(1 + c^
2*x^2)*(a + b*ArcSinh[c*x])^2)/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (5*f
^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(6*b*c*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+icdx}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{(f-icfx)^3\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\,\text{Subst}\left(\int (a+bx)^2(cf-icf\sinh(x))^3\,dx, x, \sinh^{-1}(cx)\right)}{c^4\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\,\text{Subst}\left(\int \left(c^3f^3(a+bx)^2-3ic^3f^3(a+bx)^2\sinh(x)-3c^3f\right.\right.}{c^4\sqrt{d+icdx}}$$

$$= \frac{f^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{\left(if^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx\right.}{c\sqrt{d+icdx}}$$

$$= \frac{3bcf^3x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\,\sqrt{f-icfx}} - \frac{2ibc^2f^3x^3\sqrt{1+c^2x^2}\left(a+b\right.}{9\sqrt{d+icdx}\,\sqrt{f-ic}}$$

$$= -\frac{3b^2f^3x\left(1+c^2x^2\right)}{4\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{6ibf^3x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{3}{}$$

$$= -\frac{56ib^2f^3\left(1+c^2x^2\right)}{9c\sqrt{d+icdx}\,\sqrt{f-icfx}} - \frac{3b^2f^3x\left(1+c^2x^2\right)}{4\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{2ib^2f^3\left(1+\right.}{27c\sqrt{d+icdx}}$$

$$= -\frac{68ib^2f^3\left(1+c^2x^2\right)}{9c\sqrt{d+icdx}\,\sqrt{f-icfx}} - \frac{3b^2f^3x\left(1+c^2x^2\right)}{4\sqrt{d+icdx}\,\sqrt{f-icfx}} + \frac{2ib^2f^3\left(1+\right.}{27c\sqrt{d+icdx}}$$

Mathematica [A] time = 3.62, size = 723, normalized size = 1.18

$$540a^2\sqrt{d}\,f^{5/2}\sqrt{c^2x^2+1}\,\log\left(cdfx+\sqrt{d}\,\sqrt{f}\,\sqrt{d+icdx}\,\sqrt{f-icfx}\right)-792ia^2f^2\sqrt{c^2x^2+1}\,\sqrt{d+icdx}\,\sqrt{f-icfx}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[d + I*c*d*x],x]
```

```
[Out] ((1620*I)*a*b*c*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (792*I)*a^2*f^2
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (1620*I)*b^2*f^2*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 324*a^2*c*f^2*x*Sqrt
[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (72*I)*a^2*c^2*f^2*x^2*
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 180*b^2*f^2*Sqrt[d
+ I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 + 162*a*b*f^2*Sqrt[d + I*c*d*x]
*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] + (4*I)*b^2*f^2*Sqrt[d + I*c*d*x]*S
qrt[f - I*c*f*x]*Cosh[3*ArcSinh[c*x]] + 6*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f -
I*c*f*x]*ArcSinh[c*x]*(27*b*Cosh[2*ArcSinh[c*x]] + (2*I)*(-4*b*c*x*(-33 + c
^2*x^2) + 27*a*(-5 + (2*I)*c*x)*Sqrt[1 + c^2*x^2] + 3*a*Cosh[3*ArcSinh[c*x]
])) + 540*a^2*Sqrt[d]*f^(5/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[
f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] - 81*b^2*f^2*Sqrt[d + I*c*d*x]*Sqrt
[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] + 18*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*
c*f*x]*ArcSinh[c*x]^2*(30*a - (45*I)*b*Sqrt[1 + c^2*x^2] + I*b*Cosh[3*ArcSi
nh[c*x]] - 9*b*Sinh[2*ArcSinh[c*x]]) - (12*I)*a*b*f^2*Sqrt[d + I*c*d*x]*Sqr
t[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]])/(216*c*d*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(i\,b^2c^2f^2x^2-2\,b^2cf^2x-i\,b^2f^2\right)\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2+\left(2i\,abc^2f^2x^2-4\,ab\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(((I*b^2*c^2*f^2*x^2 - 2*b^2*c*f^2*x - I*b^2*f^2)*sqrt(I*c*d*x + d)
*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 + (2*I*a*b*c^2*f^2*x^2 -
 4*a*b*c*f^2*x - 2*I*a*b*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x
+ sqrt(c^2*x^2 + 1)) + (I*a^2*c^2*f^2*x^2 - 2*a^2*c*f^2*x - I*a^2*f^2)*sqrt
(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c*d*x - I*d), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{\left(-icfx+f\right)^{\frac{5}{2}} \left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^{2}}{\sqrt{icdx+d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(f-c\,f\,x\,1{}\mathrm{i}\right)}^{5/2}}{\sqrt{d+c\,d\,x\,1{}\mathrm{i}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(1/2),x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))**2/(d+I*c*d*x)**(1/2),x)
```

```
[Out] Timed out
```

3.586 $\int \frac{(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}}\,dx$

Optimal. Leaf size=972

$$-\frac{5\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^3f^4}{2bc(icxd+d)^{3/2}(f-icfx)^{3/2}}+\frac{b^2x\left(c^2x^2+1\right)^2f^4}{4(icxd+d)^{3/2}(f-icfx)^{3/2}}+\frac{8ib^2\left(c^2x^2+1\right)^2f^4}{c(icxd+d)^{3/2}(f-icfx)^{3/2}}+\frac{x\left(c^2x^2+1\right)^2}{2(icxd+}$$

```
[Out] -32*I*b*f^4*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/
2))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-8*I*a*b*f^4*x*(c^2*x^2+1)^(3/2)/(
d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+1/4*b^2*f^4*x*(c^2*x^2+1)^2/(d+I*c*d*x)^
(3/2)/(f-I*c*f*x)^(3/2)-1/4*b^2*f^4*(c^2*x^2+1)^(3/2)*arcsinh(c*x)/c/(d+I*c
*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+4*I*f^4*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/c/
(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-1/2*b*c*f^4*x^2*(c^2*x^2+1)^(3/2)*(a+b*
arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+8*I*b^2*f^4*(c^2*x^2+1)^2
/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+8*f^4*x*(c^2*x^2+1)*(a+b*arcsinh(c*x
))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+8*f^4*(c^2*x^2+1)^(3/2)*(a+b*arcsi
nh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-8*I*b^2*f^4*x*(c^2*x^2+1)^
(3/2)*arcsinh(c*x)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+1/2*f^4*x*(c^2*x^2+1
)^2*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-5/2*f^4*(c^2*x
^2+1)^(3/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+8*
I*f^4*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2
)-16*b*f^4*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2)
)^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-16*b^2*f^4*(c^2*x^2+1)^(3/2)*pol
ylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+16
*b^2*f^4*(c^2*x^2+1)^(3/2)*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*
x)^(3/2)/(f-I*c*f*x)^(3/2)-8*b^2*f^4*(c^2*x^2+1)^(3/2)*polylog(2,-(c*x+(c^2
*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A] time = 1.36, antiderivative size = 972, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 19, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.514, Rules used = {5712, 5833, 5821, 5687, 5714, 3718, 2190, 2279, 2391, 5717, 5693, 4180, 5675, 5653, 261, 5758, 5661, 321, 215}

$$-\frac{5\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^3f^4}{2bc(icxd+d)^{3/2}(f-icfx)^{3/2}}+\frac{b^2x\left(c^2x^2+1\right)^2f^4}{4(icxd+d)^{3/2}(f-icfx)^{3/2}}+\frac{8ib^2\left(c^2x^2+1\right)^2f^4}{c(icxd+d)^{3/2}(f-icfx)^{3/2}}+\frac{x\left(c^2x^2+1\right)^2}{2(icxd+}$$

Antiderivative was successfully verified.

```
[In] Int[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(3/2),x]
```

```
[Out] ((-8*I)*a*b*f^4*x*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(
3/2)) + ((8*I)*b^2*f^4*(1 + c^2*x^2)^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x
)^(3/2)) + (b^2*f^4*x*(1 + c^2*x^2)^2)/(4*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)
^(3/2)) - (b^2*f^4*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x])/(4*c*(d + I*c*d*x)^(3/
2)*(f - I*c*f*x)^(3/2)) - ((8*I)*b^2*f^4*x*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]
)/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (b*c*f^4*x^2*(1 + c^2*x^2)^(3
/2)*(a + b*ArcSinh[c*x]))/(2*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + ((8
*I)*f^4*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I
*c*f*x)^(3/2)) + (8*f^4*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/((d + I*c*d
*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (8*f^4*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[
c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + ((4*I)*f^4*(1 + c^2*
x^2)^2*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
 + (f^4*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(2*(d + I*c*d*x)^(3/2)*(f
 - I*c*f*x)^(3/2)) - (5*f^4*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^3)/(2*b
*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - ((32*I)*b*f^4*(1 + c^2*x^2)^(
3/2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f
 - I*c*f*x)^(3/2)) - (16*b*f^4*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])*Log
```

```
[1 + E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (16
*b^2*f^4*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*(d + I*c*d
*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (16*b^2*f^4*(1 + c^2*x^2)^(3/2)*PolyLog[2,
 I*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (8*b^2*f^
4*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/
2)*(f - I*c*f*x)^(3/2))
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
```

```
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5833

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}}\,dx = \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{(f-icfx)^4\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{3/2}\int\left(-\frac{8i\left(if^4+cf^4x\right)\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}} - \frac{7f^4\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}} + \frac{4icf^4 \cdots}{\cdots}\right.}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{\left(8i\left(1+c^2x^2\right)^{3/2}\right)\int \frac{\left(if^4+cf^4x\right)\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(7f^4\left(1+c^2x^2\right)^{3/2}\cdots\right.}{(d+icdx)^{3/\cdots}}$$

$$= \frac{4if^4\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{f^4x\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}\cdots\right.}{2(d+icdx)^{3/2}(f-icfx)^{3\cdots}}$$

$$= -\frac{8iabf^4x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{bcf^4x^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\cdots}{2(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{8iabf^4x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{b^2f^4x\left(1+c^2x^2\right)^2}{4(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8ib^2f^4x\left(\cdots\right.}{(d+i\cdots}$$

$$= -\frac{8iabf^4x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8ib^2f^4\left(1+c^2x^2\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{b^2f^4\cdots}{4(d+icd\cdots}$$

$$= -\frac{8iabf^4x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8ib^2f^4\left(1+c^2x^2\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{b^2f^4\cdots}{4(d+icd\cdots}$$

$$= -\frac{8iabf^4x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8ib^2f^4\left(1+c^2x^2\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{b^2f^4\cdots}{4(d+icd\cdots}$$

$$= -\frac{8iabf^4x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8ib^2f^4\left(1+c^2x^2\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{b^2f^4\cdots}{4(d+icd\cdots}$$

$$= -\frac{8iabf^4x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8ib^2f^4\left(1+c^2x^2\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{b^2f^4\cdots}{4(d+icd\cdots}$$

Mathematica [B] time = 11.83, size = 2492, normalized size = 2.56

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(3/2),
x]
```

```
[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*(((4*I)*a^2*f^2)/d^2 + (a^2*c*
f^2*x)/(2*d^2) + (8*a^2*f^2)/(d^2*(-I + c*x))))/c - (15*a^2*f^(5/2)*Log[c*d
*f*x + Sqrt[d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]])/(2*c*d
^(3/2)) + ((4*I)*a*b*f^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*
Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2]*(-(c*x) + 2*ArcSinh[c*x] +
 Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + I*ArcSinh[c*x]^2 + 4*ArcTan[Coth[ArcSinh[
c*x]/2]] + (2*I)*Log[Sqrt[1 + c^2*x^2]]) + I*(-(c*x) - 2*ArcSinh[c*x] + Sqr
t[1 + c^2*x^2]*ArcSinh[c*x] + I*ArcSinh[c*x]^2 + 4*ArcTan[Coth[ArcSinh[c*x]
/2]] + (2*I)*Log[Sqrt[1 + c^2*x^2]])*Sinh[ArcSinh[c*x]/2]))/(c*d^2*Sqrt[-((
```

```
(-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I
*Sinh[ArcSinh[c*x]/2])) - (a*b*f^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f
 + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2]*(ArcSinh[c*x]*(-
4*I + ArcSinh[c*x]) + (8*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 4*Log[Sqrt[1 + c
^2*x^2]]) + I*(ArcSinh[c*x]*(4*I + ArcSinh[c*x]) + (8*I)*ArcTan[Tanh[ArcSin
h[c*x]/2]] + 4*Log[Sqrt[1 + c^2*x^2]])*Sinh[ArcSinh[c*x]/2]))/(c*d^2*Sqrt[-
(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] +
 I*Sinh[ArcSinh[c*x]/2])) - (b^2*f^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*
f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2]*((6*I)*Pi*ArcS
inh[c*x] + (6 - 6*I)*ArcSinh[c*x]^2 + ArcSinh[c*x]^3 + 12*((-I)*Pi + 2*ArcS
inh[c*x])*Log[1 - I/E^ArcSinh[c*x]] - (24*I)*Pi*Log[1 + E^ArcSinh[c*x]] + (
24*I)*Pi*Log[Cosh[ArcSinh[c*x]/2]] + (12*I)*Pi*Log[Sin[(Pi + (2*I)*ArcSinh[
c*x])/4]]) - 24*PolyLog[2, I/E^ArcSinh[c*x]]*(Cosh[ArcSinh[c*x]/2] + I*Sinh
[ArcSinh[c*x]/2]) + (-6*Pi*ArcSinh[c*x] - (6 - 6*I)*ArcSinh[c*x]^2 + I*ArcS
inh[c*x]^3 + 12*(Pi + (2*I)*ArcSinh[c*x])*Log[1 - I/E^ArcSinh[c*x]] + 24*Pi
*Log[1 + E^ArcSinh[c*x]] - 24*Pi*Log[Cosh[ArcSinh[c*x]/2]] - 12*Pi*Log[Sin[
(Pi + (2*I)*ArcSinh[c*x])/4]])*Sinh[ArcSinh[c*x]/2]))/(3*c*d^2*Sqrt[-(((-I)
*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sin
h[ArcSinh[c*x]/2])) + (((2*I)/3)*b^2*f^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)
*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2]*(-6*Pi*Arc
Sinh[c*x] - 6*c*x*ArcSinh[c*x] + (6 + 6*I)*ArcSinh[c*x]^2 + (2*I)*ArcSinh[c
*x]^3 + 3*Sqrt[1 + c^2*x^2]*(2 + ArcSinh[c*x]^2) + 12*Pi*Log[1 - I/E^ArcSin
h[c*x]] + (24*I)*ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]] + 24*Pi*Log[1 + E^A
rcSinh[c*x]] - 24*Pi*Log[Cosh[ArcSinh[c*x]/2]] - 12*Pi*Log[Sin[(Pi + (2*I)*
ArcSinh[c*x])/4]]) + I*(-6*Pi*ArcSinh[c*x] - 6*c*x*ArcSinh[c*x] - (6 - 6*I)
*ArcSinh[c*x]^2 + (2*I)*ArcSinh[c*x]^3 + 3*Sqrt[1 + c^2*x^2]*(2 + ArcSinh[c
*x]^2) + 12*Pi*Log[1 - I/E^ArcSinh[c*x]] + (24*I)*ArcSinh[c*x]*Log[1 - I/E^
ArcSinh[c*x]] + 24*Pi*Log[1 + E^ArcSinh[c*x]] - 24*Pi*Log[Cosh[ArcSinh[c*x]
/2]] - 12*Pi*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]])*Sinh[ArcSinh[c*x]/2] +
24*PolyLog[2, I/E^ArcSinh[c*x]]*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c
*x]/2])))/(c*d^2*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*
(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) + (b^2*f^2*Sqrt[I*((-I)*d
+ c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(96*PolyLog[2
, I/E^ArcSinh[c*x]]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + Sinh[
ArcSinh[c*x]/2]*(24*Pi*ArcSinh[c*x] + 48*c*x*ArcSinh[c*x] + (24 - 24*I)*Arc
Sinh[c*x]^2 - (10*I)*ArcSinh[c*x]^3 + (3*I)*Sqrt[1 + c^2*x^2]*(c*x + (8*I)*
(2 + ArcSinh[c*x]^2)) - (3*I)*ArcSinh[c*x]*Cosh[2*ArcSinh[c*x]] - 48*Pi*Log
[1 - I/E^ArcSinh[c*x]] - (96*I)*ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]] - 96
*Pi*Log[1 + E^ArcSinh[c*x]] + 96*Pi*Log[Cosh[ArcSinh[c*x]/2]] + 48*Pi*Log[S
in[(Pi + (2*I)*ArcSinh[c*x])/4]] + (3*I)*ArcSinh[c*x]^2*Sinh[2*ArcSinh[c*x]
]) + Cosh[ArcSinh[c*x]/2]*(3*Sqrt[1 + c^2*x^2]*(c*x + (8*I)*(2 + ArcSinh[c*
x]^2)) - 3*ArcSinh[c*x]*Cosh[2*ArcSinh[c*x]] - I*(24*Pi*ArcSinh[c*x] + 48*c
*x*ArcSinh[c*x] - (24 + 24*I)*ArcSinh[c*x]^2 - (10*I)*ArcSinh[c*x]^3 - 48*P
i*Log[1 - I/E^ArcSinh[c*x]] - (96*I)*ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]]
 - 96*Pi*Log[1 + E^ArcSinh[c*x]] + 96*Pi*Log[Cosh[ArcSinh[c*x]/2]] + 48*Pi*
Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]] + (3*I)*ArcSinh[c*x]^2*Sinh[2*ArcSinh
[c*x]]))))/(12*c*d^2*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x
^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) + (a*b*f^2*Sqrt[I*((-I
)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(-(Sinh[A
rcSinh[c*x]/2]*((-16*I)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + Cosh[2*ArcSinh[c*x
]] + 2*((8*I)*c*x + (8*I)*ArcSinh[c*x] + 5*ArcSinh[c*x]^2 + (16*I)*ArcTan[T
anh[ArcSinh[c*x]/2]] + 8*Log[Sqrt[1 + c^2*x^2]] - ArcSinh[c*x]*Sinh[2*ArcSi
nh[c*x]]))) + Cosh[ArcSinh[c*x]/2]*(16*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + I*(
Cosh[2*ArcSinh[c*x]] + 2*((8*I)*c*x - (8*I)*ArcSinh[c*x] + 5*ArcSinh[c*x]^2
 + (16*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 8*Log[Sqrt[1 + c^2*x^2]] - ArcSinh
[c*x]*Sinh[2*ArcSinh[c*x]])))))/(4*c*d^2*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f
*x))]*Sqrt[1 + c^2*x^2]*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2]))
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2c^2f^2x^2+2i\,b^2cf^2x-b^2f^2\right)\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2+\left(2\,abc^2f^2x^2+4i\,abc\right.}{c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral(((b^2*c^2*f^2*x^2 + 2*I*b^2*c*f^2*x - b^2*f^2)*sqrt(I*c*d*x + d)*s
qrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 + (2*a*b*c^2*f^2*x^2 + 4*I
*a*b*c*f^2*x - 2*a*b*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sq
rt(c^2*x^2 + 1)) + (a^2*c^2*f^2*x^2 + 2*I*a^2*c*f^2*x - a^2*f^2)*sqrt(I*c*d
*x + d)*sqrt(-I*c*f*x + f))/(c^2*d^2*x^2 - 2*I*c*d^2*x - d^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\left(-icfx+f\right)^{\frac{5}{2}}\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^2}{\left(icdx+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x)
```

```
[Out] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(\frac{c^2f^3x^3}{\sqrt{c^2dfx^2+df}\,d}+\frac{8i\,cf^3x^2}{\sqrt{c^2dfx^2+df}\,d}+\frac{17\,f^3x}{\sqrt{c^2dfx^2+df}\,d}-\frac{15\,f^3\,\mathrm{arsinh}\left(cx\right)}{\sqrt{df}\,cd}+\frac{24i\,f^3}{\sqrt{c^2dfx^2+df}\,cd}\right)a^2+\int\frac{\left(-i\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/2*(c^2*f^3*x^3/(sqrt(c^2*d*f*x^2 + d*f)*d) + 8*I*c*f^3*x^2/(sqrt(c^2*d*f*
x^2 + d*f)*d) + 17*f^3*x/(sqrt(c^2*d*f*x^2 + d*f)*d) - 15*f^3*arcsinh(c*x)/
(sqrt(d*f)*c*d) + 24*I*f^3/(sqrt(c^2*d*f*x^2 + d*f)*c*d))*a^2 + integrate((
-I*c*f*x + f)^(5/2)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(I*c*d*x + d)^(3/2)
+ 2*(-I*c*f*x + f)^(5/2)*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(I*c*d*x + d)^(3/
2), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))^2\,\left(f-c\,f\,x\,1\mathrm{i}\right)^{5/2}}{(d+c\,d\,x\,1\mathrm{i})^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(3/2),x)
[Out] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(3/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))**2/(d+I*c*d*x)**(3/2),x)
[Out] Timed out
```

3.587 $\int \frac{(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{5/2}}\,dx$

Optimal. Leaf size=790

$$\frac{2iabf^5x\left(c^2x^2+1\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{5f^5\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{if^5\left(c^2x^2+1\right)^3\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{28f^5\left(c^2\right.}{3c}$$

```
[Out] 2*I*a*b*f^5*x*(c^2*x^2+1)^(5/2)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2*I*b^2
*f^5*(c^2*x^2+1)^3/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+2*I*b^2*f^5*x*(c^2
*x^2+1)^(5/2)*arcsinh(c*x)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-28/3*f^5*(c^
2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-I
*f^5*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/
2)+5/3*f^5*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*x)^(5/2)/(f-
I*c*f*x)^(5/2)-16/3*I*b^2*f^5*(c^2*x^2+1)^(5/2)*cot(1/4*Pi+1/2*I*arcsinh(c*
x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-28/3*I*f^5*(c^2*x^2+1)^(5/2)*(a+b
*arcsinh(c*x))^2*cot(1/4*Pi+1/2*I*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*
f*x)^(5/2)+8/3*b*f^5*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*csc(1/4*Pi+1/2*I*
arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+4/3*I*f^5*(c^2*x^2+1)
^(5/2)*(a+b*arcsinh(c*x))^2*cot(1/4*Pi+1/2*I*arcsinh(c*x))*csc(1/4*Pi+1/2*I
*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+112/3*b*f^5*(c^2*x^2
+1)^(5/2)*(a+b*arcsinh(c*x))*ln(1+I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^
(5/2)/(f-I*c*f*x)^(5/2)+112/3*b^2*f^5*(c^2*x^2+1)^(5/2)*polylog(2,-I*(c*x+(
c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A] time = 1.36, antiderivative size = 790, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 16, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.432, Rules used = {5712, 5833, 5675, 5717, 5653, 261, 5831, 3318, 4186, 3767, 8, 4184, 3716, 2190, 2279, 2391}

$$\frac{112b^2f^5\left(c^2x^2+1\right)^{5/2}\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2iabf^5x\left(c^2x^2+1\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{5f^5\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx\right.}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(5/2),x]
```

```
[Out] ((2*I)*a*b*f^5*x*(1 + c^2*x^2)^(5/2))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5
/2)) - ((2*I)*b^2*f^5*(1 + c^2*x^2)^3)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)
^(5/2)) + ((2*I)*b^2*f^5*x*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x])/((d + I*c*d*x)
^(5/2)*(f - I*c*f*x)^(5/2)) - (28*f^5*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*
x])^2)/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (I*f^5*(1 + c^2*x^2)
^3*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (5
*f^5*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^3)/(3*b*c*(d + I*c*d*x)^(5/2)
*(f - I*c*f*x)^(5/2)) - (((16*I)/3)*b^2*f^5*(1 + c^2*x^2)^(5/2)*Cot[Pi/4 +
(I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((28*I)
/3)*f^5*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*Cot[Pi/4 + (I/2)*ArcSinh
[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (8*b*f^5*(1 + c^2*x^2
)^(5/2)*(a + b*ArcSinh[c*x])*Csc[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/(3*c*(d + I*
c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((4*I)/3)*f^5*(1 + c^2*x^2)^(5/2)*(a +
 b*ArcSinh[c*x])^2*Cot[Pi/4 + (I/2)*ArcSinh[c*x]]*Csc[Pi/4 + (I/2)*ArcSinh[
c*x]]^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (112*b*f^5*(1 + c^2*
x^2)^(5/2)*(a + b*ArcSinh[c*x])*Log[1 + I*E^ArcSinh[c*x]])/(3*c*(d + I*c*d*
x)^(5/2)*(f - I*c*f*x)^(5/2)) + (112*b^2*f^5*(1 + c^2*x^2)^(5/2)*PolyLog[2,
 (-I)*E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
```

```
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5833

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{5/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(f-icfx)^5\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{5/2}\int\left(\frac{5f^5\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}} - \frac{icf^5x\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}} - \frac{8f^5\left(a+b\sinh^{-1}(cx\right.}{(-i+cx)^2\sqrt{1+c^2x^2}}\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(12if^5\left(1+c^2x^2\right)^{5/2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(-i+cx)\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(5f^5\left(1+c^2x^2\right)^{5/2}\right)\int \frac{\left(a+b\sin\right.}{\sqrt{1}}}{(d+icdx)^{5/2}(f-icf}$$

$$= -\frac{if^5\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{5f^5\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(c\right.}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{2iabf^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{if^5\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{5f^5\left(\right.}{3}$$

$$= \frac{2iabf^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ib^2f^5x\left(1+c^2x^2\right)^{5/2}\sinh^{-1}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{if^5\left(1+\right.}{c(d}$$

$$= \frac{2iabf^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ib^2f^5\left(1+c^2x^2\right)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ib^2f^5x\left(1+c\right.}{(d+icdx)^5}$$

$$= \frac{2iabf^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ib^2f^5\left(1+c^2x^2\right)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ib^2f^5x\left(1+c\right.}{(d+icdx)^5}$$

$$= \frac{2iabf^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ib^2f^5\left(1+c^2x^2\right)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ib^2f^5x\left(1+c\right.}{(d+icdx)^5}$$

$$= \frac{2iabf^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ib^2f^5\left(1+c^2x^2\right)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ib^2f^5x\left(1+c\right.}{(d+icdx)^5}$$

$$= \frac{2iabf^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ib^2f^5\left(1+c^2x^2\right)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ib^2f^5x\left(1+c\right.}{(d+icdx)^5}$$

Mathematica [B] time = 13.52, size = 2622, normalized size = 3.32

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(5/2),
x]
```

```
[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*(((-I)*a^2*f^2)/d^3 - (((8*I)/
3)*a^2*f^2)/(d^3*(-I + c*x)^2) - (28*a^2*f^2)/(3*d^3*(-I + c*x))))/c + (5*a
^2*f^(5/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(
I + c*x)]])/(c*d^(5/2)) + ((I/3)*a*b*f^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)
*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[A
rcSinh[c*x]/2])*((-I)*Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Cot
h[ArcSinh[c*x]/2]] - I*Log[Sqrt[1 + c^2*x^2]]) + Cosh[ArcSinh[c*x]/2]*(4 +
(3*I)*ArcSinh[c*x] - (6*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + 3*Log[Sqrt[1 + c^
2*x^2]]) + 2*(Sqrt[1 + c^2*x^2]*(ArcSinh[c*x] + 2*ArcTan[Coth[ArcSinh[c*x]/
2]] + I*Log[Sqrt[1 + c^2*x^2]]) + 2*(I + ArcSinh[c*x] + 2*ArcTan[Coth[ArcSi
nh[c*x]/2]] + I*Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/(c*d^3*(I +
```

```
c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] + I*Sin
h[ArcSinh[c*x]/2])^4) - (a*b*f^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f +
c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c
*x]/2])*(Cosh[(3*ArcSinh[c*x])/2]*((-14 + (3*I)*ArcSinh[c*x])*ArcSinh[c*x]
- 28*ArcTan[Tanh[ArcSinh[c*x]/2]] + (14*I)*Log[Sqrt[1 + c^2*x^2]]) + Cosh[A
rcSinh[c*x]/2]*(84*ArcTan[Tanh[ArcSinh[c*x]/2]] - I*(8 - (6*I)*ArcSinh[c*x]
 + 9*ArcSinh[c*x]^2 + 42*Log[Sqrt[1 + c^2*x^2]])) + 2*(4 - (4*I)*ArcSinh[c*
x] + 6*ArcSinh[c*x]^2 + (56*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 28*Log[Sqrt[1
 + c^2*x^2]] + Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(-14*I + 3*ArcSinh[c*x]) + (
28*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 14*Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSi
nh[c*x]/2]))/(3*c*d^3*(I + c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Co
sh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^4) + ((I/3)*b^2*f^2*(I + c*x)*
Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))
]*((-1 + I)*ArcSinh[c*x]^2 - (2*ArcSinh[c*x]*(-2*I + ArcSinh[c*x]))/(-I + c
*x) + (2*I)*(Pi + (2*I)*ArcSinh[c*x])*Log[1 - I/E^ArcSinh[c*x]] - I*Pi*(Arc
Sinh[c*x] - 4*Log[1 + E^ArcSinh[c*x]] + 4*Log[Cosh[ArcSinh[c*x]/2]] + 2*Log
[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]]) + 4*PolyLog[2, I/E^ArcSinh[c*x]] - (4*A
rcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[
c*x]/2])^3 + (2*(4 + ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*
x]/2] + I*Sinh[ArcSinh[c*x]/2])))/(c*d^3*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f
*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^2)
- ((I/3)*b^2*f^2*(I + c*x)*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)
]*Sqrt[-(d*f*(1 + c^2*x^2))]*(((6*I)*c*x*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] +
((13 - 13*I)*ArcSinh[c*x]^2)/Sqrt[1 + c^2*x^2] + (3*ArcSinh[c*x]^3)/Sqrt[1
+ c^2*x^2] + (2*ArcSinh[c*x]*(-2*I + ArcSinh[c*x]))/((-I + c*x)*Sqrt[1 + c^
2*x^2]) - (3*I)*(2 + ArcSinh[c*x]^2) + ((13*I)*(-2*(Pi + (2*I)*ArcSinh[c*x]
)*Log[1 - I/E^ArcSinh[c*x]] + Pi*(ArcSinh[c*x] - 4*Log[1 + E^ArcSinh[c*x]]
+ 4*Log[Cosh[ArcSinh[c*x]/2]] + 2*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]]) +
(4*I)*PolyLog[2, I/E^ArcSinh[c*x]]))/Sqrt[1 + c^2*x^2] + (4*ArcSinh[c*x]^2*
Sinh[ArcSinh[c*x]/2])/(Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[Arc
Sinh[c*x]/2])^3) - (2*(4 + 13*ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/(Sqrt[1
 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]))))/(c*d^3*Sqrt[
-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c
*x]/2])^2) + (2*b^2*f^2*(I + c*x)*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f +
 c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(7*Pi*ArcSinh[c*x] - (7 + 7*I)*ArcSinh[
c*x]^2 - I*ArcSinh[c*x]^3 + (2*ArcSinh[c*x]*(-2*I + ArcSinh[c*x]))/(1 + I*c
*x) - 14*(Pi + (2*I)*ArcSinh[c*x])*Log[1 - I/E^ArcSinh[c*x]] - 28*Pi*Log[1
+ E^ArcSinh[c*x]] + 28*Pi*Log[Cosh[ArcSinh[c*x]/2]] + 14*Pi*Log[Sin[(Pi + (
2*I)*ArcSinh[c*x])/4]] + (28*I)*PolyLog[2, I/E^ArcSinh[c*x]] - ((4*I)*ArcSi
nh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]
/2])^3 + (2*(4 + 7*ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/((-I)*Cosh[ArcSinh
[c*x]/2] + Sinh[ArcSinh[c*x]/2])))/(3*c*d^3*Sqrt[-(((-I)*d + c*d*x)*(I*f +
c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^
2) + ((I/6)*a*b*f^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[
-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])*(-3*C
osh[(5*ArcSinh[c*x])/2] + (3*I)*ArcSinh[c*x]*Cosh[(5*ArcSinh[c*x])/2] - Cos
h[(3*ArcSinh[c*x])/2]*(9 + (35*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 - (52*I)*
ArcTan[Coth[ArcSinh[c*x]/2]] + 26*Log[Sqrt[1 + c^2*x^2]]) + Cosh[ArcSinh[c*
x]/2]*(20 - (24*I)*ArcSinh[c*x] + 27*ArcSinh[c*x]^2 - (156*I)*ArcTan[Coth[A
rcSinh[c*x]/2]] + 78*Log[Sqrt[1 + c^2*x^2]]) + (20*I)*Sinh[ArcSinh[c*x]/2]
- 24*ArcSinh[c*x]*Sinh[ArcSinh[c*x]/2] + (27*I)*ArcSinh[c*x]^2*Sinh[ArcSinh
[c*x]/2] + 156*ArcTan[Coth[ArcSinh[c*x]/2]]*Sinh[ArcSinh[c*x]/2] + (78*I)*L
og[Sqrt[1 + c^2*x^2]]*Sinh[ArcSinh[c*x]/2] + (9*I)*Sinh[(3*ArcSinh[c*x])/2]
 + 35*ArcSinh[c*x]*Sinh[(3*ArcSinh[c*x])/2] + (9*I)*ArcSinh[c*x]^2*Sinh[(3*
ArcSinh[c*x])/2] + 52*ArcTan[Coth[ArcSinh[c*x]/2]]*Sinh[(3*ArcSinh[c*x])/2]
 + (26*I)*Log[Sqrt[1 + c^2*x^2]]*Sinh[(3*ArcSinh[c*x])/2] - (3*I)*Sinh[(5*A
rcSinh[c*x])/2] + 3*ArcSinh[c*x]*Sinh[(5*ArcSinh[c*x])/2]))/(c*d^3*(I + c*x
)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[Ar
cSinh[c*x]/2])^4)
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-i\,b^2c^2f^2x^2+2\,b^2cf^2x+i\,b^2f^2\right)\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2+\left(-2i\,abc^2f^2x^2+4\,a\right.}{c^3d^3x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral(((-I*b^2*c^2*f^2*x^2 + 2*b^2*c*f^2*x + I*b^2*f^2)*sqrt(I*c*d*x + d
)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 + (-2*I*a*b*c^2*f^2*x^2
 + 4*a*b*c*f^2*x + 2*I*a*b*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*
x + sqrt(c^2*x^2 + 1)) + (-I*a^2*c^2*f^2*x^2 + 2*a^2*c*f^2*x + I*a^2*f^2)*s
qrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^3*d^3*x^3 - 3*I*c^2*d^3*x^2 - 3*c*d
^3*x + I*d^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\left(-icfx+f\right)^{\frac{5}{2}}\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^{2}}{\left(icdx+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x)
```

```
[Out] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(f-c\,f\,x\,1{}\mathrm{i}\right)}^{5/2}}{{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(5/2),x)
[Out] int(((a + b*asinh(c*x))^2*(f - c*f*x*1i)^(5/2))/(d + c*d*x*1i)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))**2/(d+I*c*d*x)**(5/2),x)
[Out] Timed out
```

3.588 $\int \frac{(d+icdx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{f-icfx}}\,dx$

Optimal. Leaf size=615

$$\frac{5d^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{6bc\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{icd^3x^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{3d^3x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{2\sqrt{d+icdx}\sqrt{f-icfx}}+\cdots$$

```
[Out] 68/9*I*b^2*d^3*(c^2*x^2+1)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-3/4*b^2*d^
3*x*(c^2*x^2+1)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-2/27*I*b^2*d^3*(c^2*x^2
+1)^2/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+11/3*I*d^3*(c^2*x^2+1)*(a+b*arc
sinh(c*x))^2/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-3/2*d^3*x*(c^2*x^2+1)*(a
+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-1/3*I*c*d^3*x^2*(c^2
*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+3/4*b^2*d^
3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-22/3
*I*b*d^3*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*
x)^(1/2)+3/2*b*c*d^3*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(
1/2)/(f-I*c*f*x)^(1/2)+2/9*I*b*c^2*d^3*x^3*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(
1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+5/6*d^3*(a+b*arcsinh(c*x))^3*(c^2*
x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

Rubi [A] time = 0.79, antiderivative size = 615, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.270, Rules used = {5712, 5831, 3317, 3296, 2638, 3311, 32, 2635, 8, 2633}

$$\frac{5d^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{6bc\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{icd^3x^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{3d^3x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{2\sqrt{d+icdx}\sqrt{f-icfx}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[f - I*c*f*x],x]
```

```
[Out] (((68*I)/9)*b^2*d^3*(1 + c^2*x^2))/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])
- (3*b^2*d^3*x*(1 + c^2*x^2))/(4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (((
2*I)/27)*b^2*d^3*(1 + c^2*x^2)^2)/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) +
 (3*b^2*d^3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(4*c*Sqrt[d + I*c*d*x]*Sqrt[f -
 I*c*f*x]) - (((22*I)/3)*b*d^3*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (3*b*c*d^3*x^2*Sqrt[1 + c^2*x^2]*(a +
 b*ArcSinh[c*x]))/(2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (((2*I)/9)*b*c^
2*d^3*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x]) + (((11*I)/3)*d^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*Sqr
t[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (3*d^3*x*(1 + c^2*x^2)*(a + b*ArcSinh[c
*x])^2)/(2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - ((I/3)*c*d^3*x^2*(1 + c^2
*x^2)*(a + b*ArcSinh[c*x])^2)/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (5*d^
3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(6*b*c*Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{(d+icdx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{f-icfx}}dx = \frac{\sqrt{1+c^2x^2}\int \frac{(d+icdx)^3\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\,\text{Subst}\left(\int (a+bx)^2(cd+icd\sinh(x))^3\,dx, x, \sinh^{-1}(cx)\right)}{c^4\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\,\text{Subst}\left(\int \left(c^3d^3(a+bx)^2 + 3ic^3d^3(a+bx)^2\sinh(x) - 3c^3d^3(a\right.\right.}{c^4\sqrt{d+icdx}\sqrt{f\ -}}$$

$$= \frac{d^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{\left(id^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)^2\ \text{s}\right.}{c\sqrt{d+icdx}\sqrt{f\ -}}$$

$$= \frac{3bcd^3x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2ibc^2d^3x^3\sqrt{1+c^2x^2}\left(a+b\,\text{sinl}\right.}{9\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= -\frac{3b^2d^3x\left(1+c^2x^2\right)}{4\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{6ibd^3x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{3bcd}{}$$

$$= \frac{56ib^2d^3\left(1+c^2x^2\right)}{9c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{3b^2d^3x\left(1+c^2x^2\right)}{4\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2ib^2d^3\left(1+c^2x^2\right.}{27c\sqrt{d+icdx}\sqrt{f\ -}}$$

$$= \frac{68ib^2d^3\left(1+c^2x^2\right)}{9c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{3b^2d^3x\left(1+c^2x^2\right)}{4\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2ib^2d^3\left(1+c^2x^2\right.}{27c\sqrt{d+icdx}\sqrt{f\ -}}$$

Mathematica [A] time = 3.48, size = 723, normalized size = 1.18

$$\frac{540a^2d^{5/2}\sqrt{f}\sqrt{c^2x^2+1}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)+792ia^2d^2\sqrt{c^2x^2+1}\sqrt{d+icdx}\sqrt{f-icfx}\ -}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[f - I*c*f*x],x]
```

```
[Out] ((-1620*I)*a*b*c*d^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (792*I)*a^2*d^
2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (1620*I)*b^2*d^2*
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 324*a^2*c*d^2*x*Sqr
t[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (72*I)*a^2*c^2*d^2*x^2
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 180*b^2*d^2*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 + 162*a*b*d^2*Sqrt[d + I*c*d*x
]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] - (4*I)*b^2*d^2*Sqrt[d + I*c*d*x]*
Sqrt[f - I*c*f*x]*Cosh[3*ArcSinh[c*x]] + 6*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f -
 I*c*f*x]*ArcSinh[c*x]*(27*b*Cosh[2*ArcSinh[c*x]] + (2*I)*(4*b*c*x*(-33 + c
^2*x^2) + 27*a*(5 + (2*I)*c*x)*Sqrt[1 + c^2*x^2] - 3*a*Cosh[3*ArcSinh[c*x]]
)) + 540*a^2*d^(5/2)*Sqrt[f]*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f
]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] - 81*b^2*d^2*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] + 18*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c
*f*x]*ArcSinh[c*x]^2*(30*a + (45*I)*b*Sqrt[1 + c^2*x^2] - I*b*Cosh[3*ArcSin
h[c*x]] - 9*b*Sinh[2*ArcSinh[c*x]]) + (12*I)*a*b*d^2*Sqrt[d + I*c*d*x]*Sqrt
[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]])/(216*c*f*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-i\,b^2c^2d^2x^2 - 2\,b^2cd^2x + i\,b^2d^2\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2 + \left(-2i\,abc^2d^2x^2 - 4\,ab\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(((-I*b^2*c^2*d^2*x^2 - 2*b^2*c*d^2*x + I*b^2*d^2)*sqrt(I*c*d*x + d
)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 + (-2*I*a*b*c^2*d^2*x^2
 - 4*a*b*c*d^2*x + 2*I*a*b*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*
x + sqrt(c^2*x^2 + 1)) + (-I*a^2*c^2*d^2*x^2 - 2*a^2*c*d^2*x + I*a^2*d^2)*s
qrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c*f*x + I*f), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{(icdx+d)^{\frac{5}{2}} (a+b\operatorname{arcsinh}(cx))^2}{\sqrt{-icfx+f}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^{5/2}}{\sqrt{f-c\,f\,x\,1\mathrm{i}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(1/2),x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))**2/(f-I*c*f*x)**(1/2),x)
```

```
[Out] Timed out
```

3.589 $\int \frac{(d+icdx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{f-icfx}}\,dx$

Optimal. Leaf size=436

$$\frac{d^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{2id^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{d^2x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{2\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{bcd^2x}{}$$

```
[Out] 4*I*b^2*d^2*(c^2*x^2+1)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-1/4*b^2*d^2*x
*(c^2*x^2+1)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+2*I*d^2*(c^2*x^2+1)*(a+b*a
rcsinh(c*x))^2/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-1/2*d^2*x*(c^2*x^2+1)*
(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/4*b^2*d^2*arcsin
h(c*x)*(c^2*x^2+1)^(1/2)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-4*I*b*d^2*x*
(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/
2*b*c*d^2*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c
*f*x)^(1/2)+1/2*d^2*(a+b*arcsinh(c*x))^3*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^
(1/2)/(f-I*c*f*x)^(1/2)
```

Rubi [A] time = 0.67, antiderivative size = 436, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {5712, 5831, 3317, 3296, 2638, 3311, 32, 2635, 8}

$$\frac{d^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{2id^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{d^2x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{2\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{bcd^2x}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[f - I*c*f*x],x]
```

```
[Out] ((4*I)*b^2*d^2*(1 + c^2*x^2))/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (b^
2*d^2*x*(1 + c^2*x^2))/(4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (b^2*d^2*S
qrt[1 + c^2*x^2]*ArcSinh[c*x])/(4*c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) -
((4*I)*b*d^2*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(Sqrt[d + I*c*d*x]*S
qrt[f - I*c*f*x]) + (b*c*d^2*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + ((2*I)*d^2*(1 + c^2*x^2)*(a + b*Arc
Sinh[c*x])^2)/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (d^2*x*(1 + c^2*x^2
)*(a + b*ArcSinh[c*x])^2)/(2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (d^2*Sq
rt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(2*b*c*Sqrt[d + I*c*d*x]*Sqrt[f - I
*c*f*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{(d+icdx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{f-icfx}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{(d+icdx)^2\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$
$$= \frac{\sqrt{1+c^2x^2}\ \mathrm{Subst}\left(\int (a+bx)^2(cd+icd\sinh(x))^2\,dx, x, \sinh^{-1}(cx)\right)}{c^3\sqrt{d+icdx}\sqrt{f-icfx}}$$
$$= \frac{\sqrt{1+c^2x^2}\ \mathrm{Subst}\left(\int \left(c^2d^2(a+bx)^2 + 2ic^2d^2(a+bx)^2\sinh(x) - c^2d^2(a+\right.\right.}{c^3\sqrt{d+icdx}\sqrt{f-icfx}}$$
$$= \frac{d^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{\left(2id^2\sqrt{1+c^2x^2}\right)\mathrm{Subst}\left(\int (a+bx)^2\right.}{c\sqrt{d+icdx}\sqrt{f-}}$$
$$= \frac{bcd^2x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2id^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx\right.}{c\sqrt{d+icdx}\sqrt{f-icfx}}$$
$$= -\frac{b^2d^2x\left(1+c^2x^2\right)}{4\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{4ibd^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{bcd^2}{}$$
$$= \frac{4ib^2d^2\left(1+c^2x^2\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{b^2d^2x\left(1+c^2x^2\right)}{4\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{b^2d^2\sqrt{1+c^2x^2}\ \sinh}{4c\sqrt{d+icdx}\sqrt{f-}}$$

Mathematica [A] time = 2.21, size = 529, normalized size = 1.21

$$\frac{12a^2d^{3/2}\sqrt{f}\sqrt{c^2x^2+1}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)+16ia^2d\sqrt{c^2x^2+1}\sqrt{d+icdx}\sqrt{f-icfx}-4a^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[f - I*c*f*x],x]
```

```
[Out] ((-32*I)*a*b*c*d*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (16*I)*a^2*d*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (32*I)*b^2*d*Sqrt[d + I*
c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 4*a^2*c*d*x*Sqrt[d + I*c*d*x]*
Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 4*b^2*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*
c*f*x]*ArcSinh[c*x]^3 + 2*a*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*
ArcSinh[c*x]] + 2*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*((-1
6*I)*b*c*x - 4*a*(-4*I + c*x)*Sqrt[1 + c^2*x^2] + b*Cosh[2*ArcSinh[c*x]]) +
 12*a^2*d^(3/2)*Sqrt[f]*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqr
t[d + I*c*d*x]*Sqrt[f - I*c*f*x]] - b^2*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
x]*Sinh[2*ArcSinh[c*x]] + 2*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh
[c*x]^2*(6*a + (8*I)*b*Sqrt[1 + c^2*x^2] - b*Sinh[2*ArcSinh[c*x]]))/(8*c*f*
Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(b^2cdx-ib^2d\right)\sqrt{icdx+d}\sqrt{-icfx+f}\log\left(cx+\sqrt{c^2x^2+1}\right)^2+2\left(abcdx-iabd\right)\sqrt{icdx+d}\sqrt{-icfx}}{cfx+if}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-((b^2*c*d*x - I*b^2*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c
*x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c*d*x - I*a*b*d)*sqrt(I*c*d*x + d)*sqrt(
-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a^2*c*d*x - I*a^2*d)*sqrt(I*c
*d*x + d)*sqrt(-I*c*f*x + f))/(c*f*x + I*f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(i\,cdx+d)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)^2}{\sqrt{-i\,cfx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((I*c*d*x + d)^(3/2)*(b*arcsinh(c*x) + a)^2/sqrt(-I*c*f*x + f), x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{(icdx+d)^{\frac{3}{2}}\,(a+b\operatorname{arcsinh}(cx))^2}{\sqrt{-icfx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\operatorname{asinh}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^{3/2}}{\sqrt{f-c\,f\,x\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(1/2),x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(id\,(cx-i))^{\frac{3}{2}}\,(a+b\operatorname{asinh}(cx))^2}{\sqrt{-if\,(cx+i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))**2/(f-I*c*f*x)**(1/2),x)
```

```
[Out] Integral((I*d*(c*x - I))**(3/2)*(a + b*asinh(c*x))**2/sqrt(-I*f*(c*x + I)),
 x)
```

3.590 $\int \frac{\sqrt{d+icdx}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{f-icfx}}\,dx$

Optimal. Leaf size=259

$$-\frac{2iabdx\sqrt{c^2x^2+1}}{\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{d\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{id\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{2ib^2d\left(c^2x^2+1\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}}$$

```
[Out] 2*I*b^2*d*(c^2*x^2+1)/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+I*d*(c^2*x^2+1)
*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-2*I*a*b*d*x*(c^
2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)-2*I*b^2*d*x*arcsinh(c*x)
*(c^2*x^2+1)^(1/2)/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)+1/3*d*(a+b*arcsinh(c
*x))^3*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {5712, 5821, 5675, 5717, 5653, 261}

$$-\frac{2iabdx\sqrt{c^2x^2+1}}{\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{d\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{id\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{2ib^2d\left(c^2x^2+1\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x])^2)/Sqrt[f - I*c*f*x],x]
```

```
[Out] ((-2*I)*a*b*d*x*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) +
((2*I)*b^2*d*(1 + c^2*x^2))/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - ((2*I
)*b^2*d*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f
*x]) + (I*d*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*Sqrt[d + I*c*d*x]*Sqrt
[f - I*c*f*x]) + (d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(3*b*c*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int \frac{\sqrt{d+icdx}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{f-icfx}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{(d+icdx)\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\int \left(\frac{d\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}+\frac{icdx\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\right)dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\left(d\sqrt{1+c^2x^2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{\left(icd\sqrt{1+c^2x^2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{id\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{d\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}} -$$

$$= -\frac{2iabdx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{id\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{d\sqrt{1+c^2}}{3bc\sqrt{}}$$

$$= -\frac{2iabdx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2ib^2dx\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{id\left(1+c^2x^2\right)}{c\sqrt{d+i}}$$

$$= -\frac{2iabdx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2ib^2d\left(1+c^2x^2\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2ib^2dx\sqrt{1+c^2x^2}\,\mathrm{s}}{\sqrt{d+icdx}\sqrt{f}}$$

Mathematica [A] time = 1.15, size = 315, normalized size = 1.22

$$\frac{3i\sqrt{d+icdx}\sqrt{f-icfx}\left(a^2\sqrt{c^2x^2+1}-2abcx+2b^2\sqrt{c^2x^2+1}\right)+3a^2\sqrt{d}\sqrt{f}\sqrt{c^2x^2+1}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x])^2)/Sqrt[f - I*c*f*x],x]
```

```
[Out] ((3*I)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-2*a*b*c*x + a^2*Sqrt[1 + c^2*x
^2] + 2*b^2*Sqrt[1 + c^2*x^2]) - (6*I)*b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
```

```
]*(b*c*x - a*Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + 3*b*Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x]*(a + I*b*Sqrt[1 + c^2*x^2])*ArcSinh[c*x]^2 + b^2*Sqrt[d + I*c*d
*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 + 3*a^2*Sqrt[d]*Sqrt[f]*Sqrt[1 + c^2*x
^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]])/(3*
c*f*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{i\sqrt{icdx+d}\sqrt{-icfx+f}\,b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2+2i\sqrt{icdx+d}\sqrt{-icfx+f}\,ab\log\left(cx+\sqrt{c^2x^2+1}\right)}{cfx+if}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2
 + 1))^2 + 2*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2*
x^2 + 1)) + I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2)/(c*f*x + I*f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{icdx+d}\,(b\operatorname{arsinh}(cx)+a)^2}{\sqrt{-icfx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(I*c*d*x + d)*(b*arcsinh(c*x) + a)^2/sqrt(-I*c*f*x + f), x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2\sqrt{icdx+d}}{\sqrt{-icfx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\frac{d\operatorname{arsinh}(cx)}{cf\sqrt{\frac{d}{f}}}+\frac{i\sqrt{c^2dfx^2+df}}{cf}\right)+\int\frac{\sqrt{icdx+d}\,b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{\sqrt{-icfx+f}}+\frac{2\sqrt{icdx+d}\,ab\log\left(cx+\sqrt{c^2x^2+\right.}}{\sqrt{-icfx+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] a^2*(d*arcsinh(c*x)/(c*f*sqrt(d/f)) + I*sqrt(c^2*d*f*x^2 + d*f)/(c*f)) + in
tegrate(sqrt(I*c*d*x + d)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/sqrt(-I*c*f*x
+ f) + 2*sqrt(I*c*d*x + d)*a*b*log(c*x + sqrt(c^2*x^2 + 1))/sqrt(-I*c*f*x +
 f), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))^2\,\sqrt{d+c\,d\,x\,1\mathrm{i}}}{\sqrt{f-c\,f\,x\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(1/2),x)
[Out] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{id\,(cx-i)}\,\,(a+b\,\mathrm{asinh}\,(cx))^2}{\sqrt{-if\,(cx+i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(1/2),x)
[Out] Integral(sqrt(I*d*(c*x - I))*(a + b*asinh(c*x))**2/sqrt(-I*f*(c*x + I)), x)
```

3.591 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+icdx}\,\sqrt{f-icfx}}\,dx$

Optimal. Leaf size=59

$$\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

```
[Out] 1/3*(a+b*arcsinh(c*x))^3*(c^2*x^2+1)^(1/2)/b/c/(d+I*c*d*x)^(1/2)/(f-I*c*f*x
)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {5712, 5675}

$$\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(3*b*c*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x])
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+icdx}\,\sqrt{f-icfx}}\,dx &= \frac{\sqrt{1+c^2x^2}\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\,\sqrt{f-icfx}}\\ &= \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{d+icdx}\,\sqrt{f-icfx}}\end{aligned}$$

Mathematica [B] time = 0.85, size = 168, normalized size = 2.85

$$\frac{a^2\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\,\sqrt{f-icfx}\right)}{c\sqrt{d}\sqrt{f}}+\frac{ab\sqrt{c^2x^2+1}\,\sinh^{-1}(cx)^2}{c\sqrt{d+icdx}\,\sqrt{f-icfx}}+\frac{b^2\sqrt{c^2x^2+1}\,\sinh^{-1}(cx)^3}{3c\sqrt{d+icdx}\,\sqrt{f-icfx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (a*b*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2)/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
x]) + (b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^3)/(3*c*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x]) + (a^2*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f -
 I*c*f*x]])/(c*Sqrt[d]*Sqrt[f])
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{icdx+d}\sqrt{-icfx+f}\,b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2+2\sqrt{icdx+d}\sqrt{-icfx+f}\,ab\log\left(cx+\sqrt{c^2x^2+1}\right.}{c^2dfx^2+df}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2 +
 1))^2 + 2*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2*x^2
+ 1)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2)/(c^2*d*f*x^2 + d*f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{icdx+d}\sqrt{-icfx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\text{arcsinh}\,(cx))^2}{\sqrt{icdx+d}\,\sqrt{-icfx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
```

maxima [A] time = 0.59, size = 53, normalized size = 0.90

$$\frac{b^2\,\text{arsinh}\,(cx)^3}{3\sqrt{df}\,c}+\frac{ab\,\text{arsinh}\,(cx)^2}{\sqrt{df}\,c}+\frac{a^2\,\text{arsinh}\,(cx)}{\sqrt{df}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/3*b^2*arcsinh(c*x)^3/(sqrt(d*f)*c) + a*b*arcsinh(c*x)^2/(sqrt(d*f)*c) + a
^2*arcsinh(c*x)/(sqrt(d*f)*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,\text{asinh}\,(c\,x))^2}{\sqrt{d+c\,d\,x\,1\text{i}}\,\sqrt{f-c\,f\,x\,1\text{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(1/2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{asinh}(cx))^2}{\sqrt{id(cx - i)}\sqrt{-if(cx + i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(sqrt(I*d*(c*x - I))*sqrt(-I*f*(c*x + I))),
x)
```

3.592 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx$

Optimal. Leaf size=464

$$\frac{f\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{if\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{fx\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2bf}{}$$

```
[Out] I*f*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+
f*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+f*
(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2
)-4*I*b*f*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2)
)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-2*b*f*(c^2*x^2+1)^(3/2)*(a+b*arcsin
h(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/
2)-2*b^2*f*(c^2*x^2+1)^(3/2)*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c
*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+2*b^2*f*(c^2*x^2+1)^(3/2)*polylog(2,I*(c*x+(c
^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-b^2*f*(c^2*x^2+1)^(
3/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^
(3/2)
```

Rubi [A] time = 0.75, antiderivative size = 464, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.297, Rules used = {5712, 5821, 5687, 5714, 3718, 2190, 2279, 2391, 5717, 5693, 4180}

$$-\frac{2b^2f\left(c^2x^2+1\right)^{3/2}\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2b^2f\left(c^2x^2+1\right)^{3/2}\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{b^2f\left(c^2x^2+1\right)^{3/2}\text{P}}{c(d+icdx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (I*f*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*
f*x)^(3/2)) + (f*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/((d + I*c*d*x)^(3/
2)*(f - I*c*f*x)^(3/2)) + (f*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/(c
*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - ((4*I)*b*f*(1 + c^2*x^2)^(3/2)*
(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*
c*f*x)^(3/2)) - (2*b*f*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])*Log[1 + E^(
2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (2*b^2*f*(1
 + c^2*x^2)^(3/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(
f - I*c*f*x)^(3/2)) + (2*b^2*f*(1 + c^2*x^2)^(3/2)*PolyLog[2, I*E^ArcSinh[c
*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (b^2*f*(1 + c^2*x^2)^(3
/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(
3/2))
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx = \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{(f-icfx)\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{3/2}\int \left(\frac{f\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}} - \frac{icfx\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{\left(f\left(1+c^2x^2\right)^{3/2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(icf\left(1+c^2x^2\right)^{3/2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{if\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{fx\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{(2ibf}{}$$

$$= \frac{if\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{fx\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{(2ibf}{}$$

$$= \frac{if\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{fx\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{f(1+}{c}$$

$$= \frac{if\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{fx\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{f(1+}{c}$$

$$= \frac{if\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{fx\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{f(1+}{c}$$

$$= \frac{if\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{fx\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{f(1+}{c}$$

Mathematica [A] time = 2.26, size = 508, normalized size = 1.09

$$\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(\left(\cosh\left(\tfrac{1}{2}\sinh^{-1}(cx)\right)+i\sinh\left(\tfrac{1}{2}\sinh^{-1}(cx)\right)\right)\left(a^2cx+ia^2-ab\sqrt{c^2x^2+1}\,\log\left(c^2x^2+1\right)\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((-1 + I)*b^2*Sqrt[1 + c^2*x^2]*ArcSin
h[c*x]^2*(Cosh[ArcSinh[c*x]/2] - Sinh[ArcSinh[c*x]/2]) + (I*a^2 + a^2*c*x -
 (4*I)*a*b*Sqrt[1 + c^2*x^2]*ArcTan[Tanh[ArcSinh[c*x]/2]] + (2*I)*b^2*Pi*Sq
rt[1 + c^2*x^2]*Log[1 - I/E^ArcSinh[c*x]] + (4*I)*b^2*Pi*Sqrt[1 + c^2*x^2]*
Log[1 + E^ArcSinh[c*x]] - a*b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2] - (4*I)*b^
2*Pi*Sqrt[1 + c^2*x^2]*Log[Cosh[ArcSinh[c*x]/2]] - (2*I)*b^2*Pi*Sqrt[1 + c^
```

```
2*x^2]*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]])*(Cosh[ArcSinh[c*x]/2] + I*Sin
h[ArcSinh[c*x]/2]) + 4*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I/E^ArcSinh[c*x]]*(
Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + b*Sqrt[1 + c^2*x^2]*ArcSin
h[c*x]*(I*Cosh[ArcSinh[c*x]/2]*(2*a - b*Pi + (4*I)*b*Log[1 - I/E^ArcSinh[c*
x]]) + (2*a + b*Pi - (4*I)*b*Log[1 - I/E^ArcSinh[c*x]])*Sinh[ArcSinh[c*x]/2
])))/(c*d^2*f*(-I + c*x)*(I + c*x)*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c
*x]/2]))
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\frac{\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2+\left(c^2d^2fx-i\,cd^2f\right)\mathrm{integral}\left(\frac{-i\,\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,a^2-\left(2\,\sqrt{c^2x^2+1}\,\sqrt{i\,c\ldots}\right.}{c^3d^2\ldots}\right.}{c^2d^2fx-i\,cd^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] (sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2 +
(c^2*d^2*f*x - I*c*d^2*f)*integral((-I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)
*a^2 - (2*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2 + 2*I*
sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b)*log(c*x + sqrt(c^2*x^2 + 1)))/(c^
3*d^2*f*x^3 - I*c^2*d^2*f*x^2 + c*d^2*f*x - I*d^2*f), x))/(c^2*d^2*f*x - I*
c*d^2*f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^2}{(i\,cdx+d)^{\frac{3}{2}}\sqrt{-i\,cfx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((I*c*d*x + d)^(3/2)*sqrt(-I*c*f*x + f)),
x)
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2}{(icdx+d)^{\frac{3}{2}}\sqrt{-icfx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^2\int \frac{\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{(i\,cdx+d)^{\frac{3}{2}}\sqrt{-i\,cfx+f}}\,dx+\frac{2i\,\sqrt{c^2dfx^2+df}\,ab\,\mathrm{arsinh}\,(cx)}{i\,c^2d^2fx+cd^2f}+\frac{i\,\sqrt{c^2dfx^2+df}\,a^2}{i\,c^2d^2fx+cd^2f}-\frac{2\,ab\log\,(i\,cx+1)}{cd^{\frac{3}{2}}\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/((I*c*d*x + d)^(3/2)*sqrt(-I*c
*f*x + f)), x) + 2*I*sqrt(c^2*d*f*x^2 + d*f)*a*b*arcsinh(c*x)/(I*c^2*d^2*f*
x + c*d^2*f) + I*sqrt(c^2*d*f*x^2 + d*f)*a^2/(I*c^2*d^2*f*x + c*d^2*f) - 2*
a*b*log(I*c*x + 1)/(c*d^(3/2)*sqrt(f))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))^2}{(d+c\,d\,x\,1\mathrm{i})^{3/2}\,\sqrt{f-c\,f\,x\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(1/2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(cx))^2}{(id\,(cx-i))^{\frac{3}{2}}\,\sqrt{-if\,(cx+i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(3/2)/(f-I*c*f*x)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/((I*d*(c*x - I))**(3/2)*sqrt(-I*f*(c*x + I))
), x)
```

3.593 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx$

Optimal. Leaf size=942

$$-\frac{c^2f^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2x^3}{3(icxd+d)^{5/2}(f-icfx)^{5/2}}-\frac{bcf^2\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)x^2}{3(icxd+d)^{5/2}(f-icfx)^{5/2}}-\frac{2b^2f^2\left(c^2x^2+1\right)^2x}{3(icxd+d)^{5/2}(f-icfx)^{5/2}}+\frac{2f}{}$$

```
[Out] 2/3*I*f^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^
(5/2)-2/3*b^2*f^2*x*(c^2*x^2+1)^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*b
^2*f^2*(c^2*x^2+1)^(5/2)*arcsinh(c*x)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
+1/3*b*f^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*
f*x)^(5/2)-4/3*I*b*f^2*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*arctan(c*x+(c^2
*x^2+1)^(1/2))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*b*c*f^2*x^2*(c^2*x
^2+1)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2/3*I*b*
f^2*x*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5
/2)+1/3*f^2*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x
)^(5/2)-1/3*c^2*f^2*x^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/
(f-I*c*f*x)^(5/2)+2/3*f^2*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^
(5/2)/(f-I*c*f*x)^(5/2)+1/3*f^2*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d
+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2/3*I*b^2*f^2*(c^2*x^2+1)^2/c/(d+I*c*d*x)
^(5/2)/(f-I*c*f*x)^(5/2)-2/3*b*f^2*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*ln(
1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2/3*b^2*
f^2*(c^2*x^2+1)^(5/2)*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(
5/2)/(f-I*c*f*x)^(5/2)+2/3*b^2*f^2*(c^2*x^2+1)^(5/2)*polylog(2,I*(c*x+(c^2*
x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*b^2*f^2*(c^2*x^2+1
)^(5/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*
x)^(5/2)
```

Rubi [A] time = 1.33, antiderivative size = 942, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 18, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.486, Rules used = {5712, 5821, 5690, 5687, 5714, 3718, 2190, 2279, 2391, 5717, 191, 5693, 4180, 261, 5723, 5751, 288, 215}

$$-\frac{c^2f^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2x^3}{3(icxd+d)^{5/2}(f-icfx)^{5/2}}-\frac{bcf^2\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)x^2}{3(icxd+d)^{5/2}(f-icfx)^{5/2}}-\frac{2b^2f^2\left(c^2x^2+1\right)^2x}{3(icxd+d)^{5/2}(f-icfx)^{5/2}}+\frac{2f}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (((-2*I)/3)*b^2*f^2*(1 + c^2*x^2)^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(
5/2)) - (2*b^2*f^2*x*(1 + c^2*x^2)^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^
(5/2)) + (b^2*f^2*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x])/(3*c*(d + I*c*d*x)^(5/2
)*(f - I*c*f*x)^(5/2)) + (b*f^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(
3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((2*I)/3)*b*f^2*x*(1 + c^2*
x^2)^(3/2)*(a + b*ArcSinh[c*x]))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
 - (b*c*f^2*x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*(d + I*c*d*x)^(
5/2)*(f - I*c*f*x)^(5/2)) + (((2*I)/3)*f^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x
])^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (f^2*x*(1 + c^2*x^2)*(a
 + b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (c^2*f^
2*x^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c
*f*x)^(5/2)) + (2*f^2*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c
*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (f^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[
c*x])^2)/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((4*I)/3)*b*f^2*(
1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c
*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (2*b*f^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcS
inh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*
```

```
x)^(5/2)) - (2*b^2*f^2*(1 + c^2*x^2)^(5/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])
/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (2*b^2*f^2*(1 + c^2*x^2)^(
5/2)*PolyLog[2, I*E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(
5/2)) - (b^2*f^2*(1 + c^2*x^2)^(5/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*c*
(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
```

```
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
```

```
d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
NeQ[m, -1]
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(f-icfx)^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{5/2}\int \left(\frac{f^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}} - \frac{2icf^2x\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}} - \frac{c^2f^2x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(f^2\left(1+c^2x^2\right)^{5/2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(2icf^2\left(1+c^2x^2\right)^{5/2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}d}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{2if^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^2x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{c^2f^2}{}$$

$$= \frac{bf^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ibf^2x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} -$$

$$= -\frac{2ib^2f^2\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2f^2x\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bf^2\left(1+c^2x^2\right)^{3/2}(a}{3c(d+icdx)^{5/2}}$$

$$= -\frac{2ib^2f^2\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2f^2x\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b^2f^2\left(1+c^2x^2\right)^{5/2}s}{3c(d+icdx)^{5/2}(f-}$$

$$= -\frac{2ib^2f^2\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2f^2x\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b^2f^2\left(1+c^2x^2\right)^{5/2}s}{3c(d+icdx)^{5/2}(f-}$$

$$= -\frac{2ib^2f^2\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2f^2x\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b^2f^2\left(1+c^2x^2\right)^{5/2}s}{3c(d+icdx)^{5/2}(f-}$$

$$= -\frac{2ib^2f^2\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2f^2x\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b^2f^2\left(1+c^2x^2\right)^{5/2}s}{3c(d+icdx)^{5/2}(f-}$$

Mathematica [A] time = 7.14, size = 524, normalized size = 0.56

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\left(\frac{a^2(cx-2i)}{(cx-i)^2} - \frac{ab\left(2\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\left(\frac{1}{2}i\left(\left(\sqrt{c^2x^2+1}+2\right)\log\left(c^2x^2+1\right)-2\right)+\left(\sqrt{c^2x^2+1}-1\right)\sinh^{-1}(cx)+2\left(\sqrt{c^2x^2+1}+2\right)t\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((a^2*(-2*I + c*x))/(-I + c*x)^2 - (a*
b*((-I)*Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]
/2]] - (I/2)*Log[1 + c^2*x^2]) + Cosh[ArcSinh[c*x]/2]*(-2 - (3*I)*ArcSinh[c
*x] - (6*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + (3*Log[1 + c^2*x^2])/2) + 2*((-1
 + Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + 2*(2 + Sqrt[1 + c^2*x^2])*ArcTan[Coth[
ArcSinh[c*x]/2]] + (I/2)*(-2 + (2 + Sqrt[1 + c^2*x^2])*Log[1 + c^2*x^2]))*S
inh[ArcSinh[c*x]/2]))/(Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[Arc
Sinh[c*x]/2])^3) - (b^2*((1 - I)*ArcSinh[c*x]^2 - (ArcSinh[c*x]*(-2*I + Arc
```

```
Sinh[c*x]))/(-I + c*x) + 2*((-I)*Pi + 2*ArcSinh[c*x])*Log[1 - I/E^ArcSinh[c
*x]] + I*Pi*(ArcSinh[c*x] - 4*Log[1 + E^ArcSinh[c*x]] + 4*Log[Cosh[ArcSinh[
c*x]/2]] + 2*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]]) - 4*PolyLog[2, I/E^ArcS
inh[c*x]] - (2*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] +
 I*Sinh[ArcSinh[c*x]/2])^3 - (2*(-2 + ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])
/(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/Sqrt[1 + c^2*x^2]))/(3*c
*d^3*f)
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\frac{\left(b^2cx - 2i\,b^2\right)\sqrt{i\,cdx + d}\,\sqrt{-i\,cfx + f}\,\log\left(cx + \sqrt{c^2x^2 + 1}\right)^2 + \left(3\,c^3d^3fx^2 - 6i\,c^2d^3fx - 3\,cd^3f\right)\mathrm{integral}\left(-\frac{3\,\sqrt{}}{}\right.}{3\,c^3d^3fx^2 - 6i\,c^2d^3fx - 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] ((b^2*c*x - 2*I*b^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^
2*x^2 + 1))^2 + (3*c^3*d^3*f*x^2 - 6*I*c^2*d^3*f*x - 3*c*d^3*f)*integral(-(
3*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2 + (6*sqrt(I*c*d*x + d)*sqrt(-I*c
*f*x + f)*a*b + (2*b^2*c*x - 4*I*b^2)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*s
qrt(-I*c*f*x + f))*log(c*x + sqrt(c^2*x^2 + 1)))/(3*c^4*d^3*f*x^4 - 6*I*c^3
*d^3*f*x^3 - 6*I*c*d^3*f*x - 3*d^3*f), x))/(3*c^3*d^3*f*x^2 - 6*I*c^2*d^3*f
*x - 3*c*d^3*f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2}{(i\,cdx + d)^{\frac{5}{2}}\sqrt{-i\,cfx + f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((I*c*d*x + d)^(5/2)*sqrt(-I*c*f*x + f)),
x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arcsinh}(cx))^2}{(icdx + d)^{\frac{5}{2}}\sqrt{-icfx + f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{asinh}\,(c\,x))^2}{(d + c\,d\,x\,1\mathrm{i})^{5/2}\,\sqrt{f - c\,f\,x\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(1/2)),x)
[Out] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(5/2)/(f-I*c*f*x)**(1/2),x)
[Out] Timed out
```

3.594 $\int \frac{(d+icdx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(f-icfx)^{3/2}}\,dx$

Optimal. Leaf size=972

$$-\frac{5\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^3d^4}{2bc(icxd+d)^{3/2}(f-icfx)^{3/2}}+\frac{b^2x\left(c^2x^2+1\right)^2d^4}{4(icxd+d)^{3/2}(f-icfx)^{3/2}}-\frac{8ib^2\left(c^2x^2+1\right)^2d^4}{c(icxd+d)^{3/2}(f-icfx)^{3/2}}+\frac{x\left(c^2x^2+1\right)^2}{2(icxd+}$$

```
[Out] -8*I*b^2*d^4*(c^2*x^2+1)^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+8*I*a*b*d^
4*x*(c^2*x^2+1)^(3/2)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+1/4*b^2*d^4*x*(c^
2*x^2+1)^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-1/4*b^2*d^4*(c^2*x^2+1)^(3/2
)*arcsinh(c*x)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+32*I*b*d^4*(c^2*x^2+1)
^(3/2)*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2))/c/(d+I*c*d*x)^(3/2)
/(f-I*c*f*x)^(3/2)-1/2*b*c*d^4*x^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/(d+
I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-8*I*d^4*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c
/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+8*d^4*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))
^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+8*d^4*(c^2*x^2+1)^(3/2)*(a+b*arcsinh
(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-4*I*d^4*(c^2*x^2+1)^2*(a+b*a
rcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+1/2*d^4*x*(c^2*x^2+1)^
2*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-5/2*d^4*(c^2*x^2
+1)^(3/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+8*I*
b^2*d^4*x*(c^2*x^2+1)^(3/2)*arcsinh(c*x)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2
)-16*b*d^4*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2)
)^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+16*b^2*d^4*(c^2*x^2+1)^(3/2)*pol
ylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-16
*b^2*d^4*(c^2*x^2+1)^(3/2)*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*
x)^(3/2)/(f-I*c*f*x)^(3/2)-8*b^2*d^4*(c^2*x^2+1)^(3/2)*polylog(2,-(c*x+(c^2
*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A] time = 1.37, antiderivative size = 972, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 19, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.514, Rules used = {5712, 5833, 5821, 5687, 5714, 3718, 2190, 2279, 2391, 5717, 5693, 4180, 5675, 5653, 261, 5758, 5661, 321, 215}

$$-\frac{5\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^3d^4}{2bc(icxd+d)^{3/2}(f-icfx)^{3/2}}+\frac{b^2x\left(c^2x^2+1\right)^2d^4}{4(icxd+d)^{3/2}(f-icfx)^{3/2}}-\frac{8ib^2\left(c^2x^2+1\right)^2d^4}{c(icxd+d)^{3/2}(f-icfx)^{3/2}}+\frac{x\left(c^2x^2+1\right)^2}{2(icxd+}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(3/2),x]
```

```
[Out] ((8*I)*a*b*d^4*x*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3
/2)) - ((8*I)*b^2*d^4*(1 + c^2*x^2)^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)
^(3/2)) + (b^2*d^4*x*(1 + c^2*x^2)^2)/(4*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^
(3/2)) - (b^2*d^4*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x])/(4*c*(d + I*c*d*x)^(3/2
)*(f - I*c*f*x)^(3/2)) + ((8*I)*b^2*d^4*x*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x])
/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (b*c*d^4*x^2*(1 + c^2*x^2)^(3/
2)*(a + b*ArcSinh[c*x]))/(2*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - ((8*
I)*d^4*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*
c*f*x)^(3/2)) + (8*d^4*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/((d + I*c*d*
x)^(3/2)*(f - I*c*f*x)^(3/2)) + (8*d^4*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c
*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - ((4*I)*d^4*(1 + c^2*x
^2)^2*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) +
 (d^4*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(2*(d + I*c*d*x)^(3/2)*(f -
 I*c*f*x)^(3/2)) - (5*d^4*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^3)/(2*b*
c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + ((32*I)*b*d^4*(1 + c^2*x^2)^(3
/2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f
- I*c*f*x)^(3/2)) - (16*b*d^4*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])*Log[
```

```
1 + E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (16*
b^2*d^4*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*(d + I*c*d*
x)^(3/2)*(f - I*c*f*x)^(3/2)) - (16*b^2*d^4*(1 + c^2*x^2)^(3/2)*PolyLog[2,
I*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (8*b^2*d^4
*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2
)*(f - I*c*f*x)^(3/2))
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
```

```
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5833

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+icdx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(f-icfx)^{3/2}}\,dx = \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{(d+icdx)^4\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{3/2}\int \left(-\frac{8i\left(id^4-cd^4x\right)\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}} - \frac{7d^4\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}} - \frac{4icd^4x}{\quad}\right.}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{\left(8i\left(1+c^2x^2\right)^{3/2}\right)\int \frac{\left(id^4-cd^4x\right)\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(7d^4\left(1+c^2x^2\right)^{3/2}\right)}{(d+icdx)^{3/2}}$$

$$= -\frac{4id^4\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{d^4x\left(1+c^2x^2\right)^2\left(a+b\sinh^{-}\right.}{2(d+icdx)^{3/2}(f-icfx)}$$

$$= \frac{8iabd^4x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{bcd^4x^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} -$$

$$= \frac{8iabd^4x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{b^2d^4x\left(1+c^2x^2\right)^2}{4(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8ib^2d^4x\,(1}{(d+icd}$$

$$= \frac{8iabd^4x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8ib^2d^4\left(1+c^2x^2\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{b^2d^4x}{4(d+icdx)}$$

$$= \frac{8iabd^4x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8ib^2d^4\left(1+c^2x^2\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{b^2d^4x\,(}{4(d+icdx)}$$

$$= \frac{8iabd^4x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8ib^2d^4\left(1+c^2x^2\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{b^2d^4x\,(}{4(d+icdx)}$$

$$= \frac{8iabd^4x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8ib^2d^4\left(1+c^2x^2\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{b^2d^4x\,(}{4(d+icdx)}$$

$$= \frac{8iabd^4x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8ib^2d^4\left(1+c^2x^2\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{b^2d^4x\,(}{4(d+icdx)}$$

Mathematica [B] time = 14.70, size = 2143, normalized size = 2.20

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(3/2),
x]
```

```
[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*(((-4*I)*a^2*d^2)/f^2 + (a^2*c
*d^2*x)/(2*f^2) + (8*a^2*d^2)/(f^2*(I + c*x))))/c - (15*a^2*d^(5/2)*Log[c*d
*f*x + Sqrt[d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]])/(2*c*f
^(3/2)) - ((4*I)*a*b*d^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*
Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2]*(-(c*x) + 2*ArcSinh[c*x] +
 Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - I*ArcSinh[c*x]^2 + 4*ArcTan[Coth[ArcSinh[
c*x]/2]] - (2*I)*Log[Sqrt[1 + c^2*x^2]]) - ((-I)*c*x - (2*I)*ArcSinh[c*x] +
 I*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + ArcSinh[c*x]^2 + (4*I)*ArcTan[Coth[ArcS
inh[c*x]/2]] + 2*Log[Sqrt[1 + c^2*x^2]])*Sinh[ArcSinh[c*x]/2]))/(c*f^2*Sqrt
```

```
[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2]
 - I*Sinh[ArcSinh[c*x]/2])) - (a*b*d^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(
I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2]*(8*ArcTan[Ta
nh[ArcSinh[c*x]/2]] + I*(ArcSinh[c*x]*(4*I + ArcSinh[c*x]) + 4*Log[Sqrt[1 +
 c^2*x^2]])) + (ArcSinh[c*x]*(-4*I + ArcSinh[c*x]) - (8*I)*ArcTan[Tanh[ArcS
inh[c*x]/2]] + 4*Log[Sqrt[1 + c^2*x^2]])*Sinh[ArcSinh[c*x]/2]))/(c*f^2*Sqrt
[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(I*Cosh[ArcSinh[c*x]/
2] + Sinh[ArcSinh[c*x]/2])) - (b^2*d^2*(-I + c*x)*Sqrt[I*((-I)*d + c*d*x)]*
Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(-18*Pi*ArcSinh[c*x] -
(6 - 6*I)*ArcSinh[c*x]^2 + I*ArcSinh[c*x]^3 - 12*(Pi - (2*I)*ArcSinh[c*x])*
Log[1 + I/E^ArcSinh[c*x]] + 24*Pi*Log[1 + E^ArcSinh[c*x]] + 12*Pi*Log[-Cos[
(Pi + (2*I)*ArcSinh[c*x])/4]] - 24*Pi*Log[Cosh[ArcSinh[c*x]/2]] - (24*I)*Po
lyLog[2, (-I)/E^ArcSinh[c*x]] - ((12*I)*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2]
)/(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])))/(3*c*f^2*Sqrt[-(((-I)*d
 + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[
ArcSinh[c*x]/2])^2) - (((2*I)/3)*b^2*d^2*(-I + c*x)*Sqrt[I*((-I)*d + c*d*x)
]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(((-6*I)*c*x*ArcSinh[
c*x])/Sqrt[1 + c^2*x^2] + ((6 + 6*I)*ArcSinh[c*x]^2)/Sqrt[1 + c^2*x^2] + (2
*ArcSinh[c*x]^3)/Sqrt[1 + c^2*x^2] + (3*I)*(2 + ArcSinh[c*x]^2) + ((6*I)*(2
*(Pi - (2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] + Pi*(3*ArcSinh[c*x] -
 4*Log[1 + E^ArcSinh[c*x]] - 2*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] + 4*L
og[Cosh[ArcSinh[c*x]/2]]) + (4*I)*PolyLog[2, (-I)/E^ArcSinh[c*x]]))/Sqrt[1
+ c^2*x^2] - (12*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Sqrt[1 + c^2*x^2]*(C
osh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2]))))/(c*f^2*Sqrt[-(((-I)*d + c*
d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^2) + (
b^2*d^2*(-I + c*x)*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-
(d*f*(1 + c^2*x^2))]*((-96*c*x*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] + ((48 - 48*
I)*ArcSinh[c*x]^2)/Sqrt[1 + c^2*x^2] - ((20*I)*ArcSinh[c*x]^3)/Sqrt[1 + c^2
*x^2] + 48*(2 + ArcSinh[c*x]^2) + (6*I)*c*x*(1 + 2*ArcSinh[c*x]^2) - ((6*I)
*ArcSinh[c*x]*Cosh[2*ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (48*(2*(Pi - (2*I)*
ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] + Pi*(3*ArcSinh[c*x] - 4*Log[1 + E^
ArcSinh[c*x]] - 2*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] + 4*Log[Cosh[ArcSi
nh[c*x]/2]]) + (4*I)*PolyLog[2, (-I)/E^ArcSinh[c*x]]))/Sqrt[1 + c^2*x^2] +
((96*I)*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Sqrt[1 + c^2*x^2]*(Cosh[ArcSi
nh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2]))))/(24*c*f^2*Sqrt[-(((-I)*d + c*d*x)*(
I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^2) + (a*b*d^
2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2
))]*(Sinh[ArcSinh[c*x]/2]*(-16*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + I*Cosh[2*Ar
cSinh[c*x]] + 2*(8*c*x + 8*ArcSinh[c*x] + (5*I)*ArcSinh[c*x]^2 + 16*ArcTan[
Tanh[ArcSinh[c*x]/2]] + (8*I)*Log[Sqrt[1 + c^2*x^2]] - I*ArcSinh[c*x]*Sinh[
2*ArcSinh[c*x]])) - Cosh[ArcSinh[c*x]/2]*((16*I)*Sqrt[1 + c^2*x^2]*ArcSinh[
c*x] + Cosh[2*ArcSinh[c*x]] - 2*((8*I)*c*x - (8*I)*ArcSinh[c*x] - 5*ArcSinh
[c*x]^2 + (16*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] - 8*Log[Sqrt[1 + c^2*x^2]] +
ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]]))))/(4*c*f^2*Sqrt[-(((-I)*d + c*d*x)*(I*f
 + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2
]))
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2c^2d^2x^2 - 2i\,b^2cd^2x - b^2d^2\right)\sqrt{i\,cdx + d}\,\sqrt{-i\,cfx + f}\,\log\left(cx + \sqrt{c^2x^2 + 1}\right)^2 + \left(2\,abc^2d^2x^2 - 4i\,abcd^2x\right.}{c^2f^2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral(((b^2*c^2*d^2*x^2 - 2*I*b^2*c*d^2*x - b^2*d^2)*sqrt(I*c*d*x + d)*s
qrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 + (2*a*b*c^2*d^2*x^2 - 4*I
```

```
*a*b*c*d^2*x - 2*a*b*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sq
rt(c^2*x^2 + 1)) + (a^2*c^2*d^2*x^2 - 2*I*a^2*c*d^2*x - a^2*d^2)*sqrt(I*c*d
*x + d)*sqrt(-I*c*f*x + f))/(c^2*f^2*x^2 + 2*I*c*f^2*x - f^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{(icdx+d)^{\frac{5}{2}}(a+b\,\mathrm{arcsinh}\,(cx))^2}{\left(-icfx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(\frac{c^2d^3x^3}{\sqrt{c^2dfx^2+df}\,f}-\frac{8i\,cd^3x^2}{\sqrt{c^2dfx^2+df}\,f}+\frac{17\,d^3x}{\sqrt{c^2dfx^2+df}\,f}-\frac{15\,d^3\,\mathrm{arsinh}\,(cx)}{\sqrt{df}\,cf}-\frac{24i\,d^3}{\sqrt{c^2dfx^2+df}\,cf}\right)a^2+\int \frac{(i\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/2*(c^2*d^3*x^3/(sqrt(c^2*d*f*x^2 + d*f)*f) - 8*I*c*d^3*x^2/(sqrt(c^2*d*f*
x^2 + d*f)*f) + 17*d^3*x/(sqrt(c^2*d*f*x^2 + d*f)*f) - 15*d^3*arcsinh(c*x)/
(sqrt(d*f)*c*f) - 24*I*d^3/(sqrt(c^2*d*f*x^2 + d*f)*c*f))*a^2 + integrate((
I*c*d*x + d)^(5/2)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(-I*c*f*x + f)^(3/2)
+ 2*(I*c*d*x + d)^(5/2)*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(-I*c*f*x + f)^(3/
2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^{5/2}}{\left(f-c\,f\,x\,1\mathrm{i}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(3/2),x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))**2/(f-I*c*f*x)**(3/2),x)
```

```
[Out] Timed out
```

3.595 $\int \frac{(d+icdx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(f-icfx)^{3/2}}\,dx$

Optimal. Leaf size=752

$$\frac{2iabd^3x\left(c^2x^2+1\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{d^3\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^3}{bc(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{id^3\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{4d^3\left(c^2x^2\right.}{c(d\ldots}$$

```
[Out] 2*I*a*b*d^3*x*(c^2*x^2+1)^(3/2)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-2*I*b^2
*d^3*(c^2*x^2+1)^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+2*I*b^2*d^3*x*(c^2
*x^2+1)^(3/2)*arcsinh(c*x)/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-4*I*d^3*(c^2
*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+4*d^3*x*
(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+4*d^3*
(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2
)-I*d^3*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^
(3/2)-d^3*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*x)^(3/2)/(f-I
*c*f*x)^(3/2)+16*I*b*d^3*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*arctan(c*x+(c
^2*x^2+1)^(1/2))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-8*b*d^3*(c^2*x^2+1)^
(3/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/2
)/(f-I*c*f*x)^(3/2)+8*b^2*d^3*(c^2*x^2+1)^(3/2)*polylog(2,-I*(c*x+(c^2*x^2+
1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-8*b^2*d^3*(c^2*x^2+1)^(3/2
)*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2
)-4*b^2*d^3*(c^2*x^2+1)^(3/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*
c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A] time = 1.13, antiderivative size = 752, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 15, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.405, Rules used = {5712, 5833, 5821, 5687, 5714, 3718, 2190, 2279, 2391, 5717, 5693, 4180, 5675, 5653, 261}

$$\frac{8b^2d^3\left(c^2x^2+1\right)^{3/2}\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{8b^2d^3\left(c^2x^2+1\right)^{3/2}\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{4b^2d^3\left(c^2x^2+1\right)^{3/2}}{c(d+icd\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(3/2),x]
```

```
[Out] ((2*I)*a*b*d^3*x*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3
/2)) - ((2*I)*b^2*d^3*(1 + c^2*x^2)^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)
^(3/2)) + ((2*I)*b^2*d^3*x*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x])/((d + I*c*d*x)
^(3/2)*(f - I*c*f*x)^(3/2)) - ((4*I)*d^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])
^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (4*d^3*x*(1 + c^2*x^2)*(a
 + b*ArcSinh[c*x])^2)/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (4*d^3*(1
 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f
*x)^(3/2)) - (I*d^3*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2)) - (d^3*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]
)^3)/(b*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + ((16*I)*b*d^3*(1 + c^2
*x^2)^(3/2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(
3/2)*(f - I*c*f*x)^(3/2)) - (8*b*d^3*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x
])*Log[1 + E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
 + (8*b^2*d^3*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*(d +
I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (8*b^2*d^3*(1 + c^2*x^2)^(3/2)*PolyLo
g[2, I*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (4*b^
2*d^3*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)
^(3/2)*(f - I*c*f*x)^(3/2))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5833

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+icdx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(f-icfx)^{3/2}}\,dx = \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{(d+icdx)^3\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{3/2}\int\left(-\frac{4i\left(id^3-cd^3x\right)\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}} - \frac{3d^3\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}} - \frac{icd^3x(a+b}{\sqrt{1\cdot}}\right.}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{\left(4i\left(1+c^2x^2\right)^{3/2}\right)\int \frac{\left(id^3-cd^3x\right)\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(3d^3\left(1+c^2x^2\right)^{3/2}\right)\int}{(d+icdx)^{3/2}(f}$$

$$= -\frac{id^3\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^3\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx\right.}{bc(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{2iabd^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{id^3\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^3(1}{bc}$$

$$= \frac{2iabd^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2d^3x\left(1+c^2x^2\right)^{3/2}\sinh^{-1}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4id^3(1+}{c(d}$$

$$= \frac{2iabd^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2ib^2d^3\left(1+c^2x^2\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2d^3x(1+c}{(d+icdx)^3}$$

$$= \frac{2iabd^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2ib^2d^3\left(1+c^2x^2\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2d^3x(1+c}{(d+icdx)^3}$$

$$= \frac{2iabd^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2ib^2d^3\left(1+c^2x^2\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2d^3x(1+c}{(d+icdx)^3}$$

$$= \frac{2iabd^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2ib^2d^3\left(1+c^2x^2\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2d^3x(1+c}{(d+icdx)^3}$$

$$= \frac{2iabd^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2ib^2d^3\left(1+c^2x^2\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2d^3x(1+c}{(d+icdx)^3}$$

Mathematica [A] time = 11.19, size = 1346, normalized size = 1.79

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(3/2),
x]
```

```
[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*(((-I)*a^2*d)/f^2 + (4*a^2*d)/
(f^2*(I + c*x))))/c - (3*a^2*d^(3/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[I*d
*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]])/(c*f^(3/2)) - ((2*I)*a*b*d*Sqrt[I*((-
I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[Ar
cSinh[c*x]/2]*(-(c*x) + 2*ArcSinh[c*x] + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - I
*ArcSinh[c*x]^2 + 4*ArcTan[Coth[ArcSinh[c*x]/2]] - (2*I)*Log[Sqrt[1 + c^2*x
^2]]) - ((-I)*c*x - (2*I)*ArcSinh[c*x] + I*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] +
 ArcSinh[c*x]^2 + (4*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + 2*Log[Sqrt[1 + c^2*x
^2]])*Sinh[ArcSinh[c*x]/2]))/(c*f^2*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]
```

```
*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])) - (a*b*
d*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2
))]*(Cosh[ArcSinh[c*x]/2]*(8*ArcTan[Tanh[ArcSinh[c*x]/2]] + I*(ArcSinh[c*x]
*(4*I + ArcSinh[c*x]) + 4*Log[Sqrt[1 + c^2*x^2]])) + (ArcSinh[c*x]*(-4*I +
ArcSinh[c*x]) - (8*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 4*Log[Sqrt[1 + c^2*x^2
]])*Sinh[ArcSinh[c*x]/2]))/(c*f^2*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*S
qrt[1 + c^2*x^2]*(I*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2])) - (b^2*d*
(-I + c*x)*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1
+ c^2*x^2))]*(-18*Pi*ArcSinh[c*x] - (6 - 6*I)*ArcSinh[c*x]^2 + I*ArcSinh[c*
x]^3 - 12*(Pi - (2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] + 24*Pi*Log[1
 + E^ArcSinh[c*x]] + 12*Pi*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] - 24*Pi*L
og[Cosh[ArcSinh[c*x]/2]] - (24*I)*PolyLog[2, (-I)/E^ArcSinh[c*x]] - ((12*I)
*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSin
h[c*x]/2])))/(3*c*f^2*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*
x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^2) - ((I/3)*b^2*d*(-I
+ c*x)*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^
2*x^2))]*(((-6*I)*c*x*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] + ((6 + 6*I)*ArcSinh[
c*x]^2)/Sqrt[1 + c^2*x^2] + (2*ArcSinh[c*x]^3)/Sqrt[1 + c^2*x^2] + (3*I)*(2
 + ArcSinh[c*x]^2) + ((6*I)*(2*(Pi - (2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSin
h[c*x]] + Pi*(3*ArcSinh[c*x] - 4*Log[1 + E^ArcSinh[c*x]] - 2*Log[-Cos[(Pi +
 (2*I)*ArcSinh[c*x])/4]] + 4*Log[Cosh[ArcSinh[c*x]/2]]) + (4*I)*PolyLog[2,
(-I)/E^ArcSinh[c*x]]))/Sqrt[1 + c^2*x^2] - (12*ArcSinh[c*x]^2*Sinh[ArcSinh[
c*x]/2])/(Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])
)))/(c*f^2*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] +
I*Sinh[ArcSinh[c*x]/2])^2)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-i\,b^2cdx-b^2d\right)\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2+\left(-2i\,abcdx-2\,abd\right)\sqrt{i\,cdx+d}\,\sqrt{-}}{c^2f^2x^2+2i\,cf^2x-f^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral(((-I*b^2*c*d*x - b^2*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c
*x + sqrt(c^2*x^2 + 1))^2 + (-2*I*a*b*c*d*x - 2*a*b*d)*sqrt(I*c*d*x + d)*sq
rt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (-I*a^2*c*d*x - a^2*d)*sqrt
(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^2*f^2*x^2 + 2*I*c*f^2*x - f^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [c,d,t_nostep]=[50,8,69]Precision problem choosing root in common_
EXT, current precision 14Precision problem choosing root in common_EXT, cur
rent precision 28Precision problem choosing root in common_EXT, current pre
cision 56Precision problem choosing root in common_EXT, current precision 1
12Precision problem choosing root in common_EXT, current precision 224Preci
sion problem choosing root in common_EXT, current precision 448Precision pr
oblem choosing root in common_EXT, current precision 896Unable to transpose
```

```
Error: Bad Argument ValueWarning, replacing 0 by ` u`, a substitution vari
able should perhaps be purged.Warning, need to choose a branch for the root
 of a polynomial with parameters. This might be wrong.The choice was done a
ssuming [c,d,t_nostep]=[-85,-8,94]Precision problem choosing root in common
_EXT, current precision 14Precision problem choosing root in common_EXT, cu
rrent precision 28Precision problem choosing root in common_EXT, current pr
ecision 56Precision problem choosing root in common_EXT, current precision
112Precision problem choosing root in common_EXT, current precision 224Prec
ision problem choosing root in common_EXT, current precision 448Precision p
roblem choosing root in common_EXT, current precision 896Unable to transpos
e Error: Bad Argument ValueWarning, replacing 0 by ` u`, a substitution var
iable should perhaps be purged.Evaluation time: 1.71sym2poly/r2sym(const ge
n & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{(icdx+d)^{\frac{3}{2}} (a+b\,\mathrm{arcsinh}\,(cx))^2}{(-icfx+f)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x)
```

```
[Out] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(-\frac{i\left(c^2dfx^2+df\right)^{\frac{3}{2}}}{c^3f^3x^2+2i\,c^2f^3x-cf^3}-\frac{6i\sqrt{c^2dfx^2+df}\,d}{-i\,c^2f^2x+cf^2}-\frac{3\,d^2\,\mathrm{arsinh}\,(cx)}{cf^2\sqrt{\frac{d}{f}}}\right)+\int\frac{(i\,cdx+d)^{\frac{3}{2}}b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{(-i\,cfx+f)^{\frac{3}{2}}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x, algor
ithm="maxima")
```

```
[Out] a^2*(-I*(c^2*d*f*x^2 + d*f)^(3/2)/(c^3*f^3*x^2 + 2*I*c^2*f^3*x - c*f^3) - 6
*I*sqrt(c^2*d*f*x^2 + d*f)*d/(-I*c^2*f^2*x + c*f^2) - 3*d^2*arcsinh(c*x)/(c
*f^2*sqrt(d/f))) + integrate((I*c*d*x + d)^(3/2)*b^2*log(c*x + sqrt(c^2*x^2
 + 1))^2/(-I*c*f*x + f)^(3/2) + 2*(I*c*d*x + d)^(3/2)*a*b*log(c*x + sqrt(c^
2*x^2 + 1))/(-I*c*f*x + f)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^{3/2}}{(f-c\,f\,x\,1\mathrm{i})^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(3/2),x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))**2/(f-I*c*f*x)**(3/2),x)
```

```
[Out] Timed out
```

3.596 $\int \frac{\sqrt{d+icdx}\left(a+b\sinh^{-1}(cx)\right)^2}{(f-icfx)^{3/2}}\,dx$

Optimal. Leaf size=544

$$-\frac{d^2\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2d^2\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2id^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right.}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[Out] -2*I*d^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(
3/2)+2*d^2*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)
^(3/2)+2*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-
I*c*f*x)^(3/2)-1/3*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*
x)^(3/2)/(f-I*c*f*x)^(3/2)+8*I*b*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*a
rctan(c*x+(c^2*x^2+1)^(1/2))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-4*b*d^2*
(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I
*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+4*b^2*d^2*(c^2*x^2+1)^(3/2)*polylog(2,-I*(c
*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-4*b^2*d^2*(c^2
*x^2+1)^(3/2)*polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I
*c*f*x)^(3/2)-2*b^2*d^2*(c^2*x^2+1)^(3/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2)
)^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A] time = 0.99, antiderivative size = 544, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 13, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.351, Rules used = {5712, 5833, 5821, 5687, 5714, 3718, 2190, 2279, 2391, 5717, 5693, 4180, 5675}

$$\frac{4b^2d^2\left(c^2x^2+1\right)^{3/2}\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{4b^2d^2\left(c^2x^2+1\right)^{3/2}\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2b^2d^2\left(c^2x^2+1\right)^{3/2}}{c(d+icd}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(3/2),x]
```

```
[Out] ((-2*I)*d^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f
 - I*c*f*x)^(3/2)) + (2*d^2*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/((d + I
*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (2*d^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcS
inh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (d^2*(1 + c^2*x^
2)^(3/2)*(a + b*ArcSinh[c*x])^3)/(3*b*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(
3/2)) + ((8*I)*b*d^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcS
inh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (4*b*d^2*(1 + c^2*
x^2)^(3/2)*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*
x)^(3/2)*(f - I*c*f*x)^(3/2)) + (4*b^2*d^2*(1 + c^2*x^2)^(3/2)*PolyLog[2, (
-I)*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (4*b^2*d
^2*(1 + c^2*x^2)^(3/2)*PolyLog[2, I*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)
*(f - I*c*f*x)^(3/2)) - (2*b^2*d^2*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(2*Arc
Sinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5833

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+icdx}\left(a+b\sinh^{-1}(cx)\right)^2}{(f-icfx)^{3/2}}dx = \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{(d+icdx)^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{3/2}\int \left(-\frac{2i\left(id^2-cd^2x\right)\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}} - \frac{d^2\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{\left(2i\left(1+c^2x^2\right)^{3/2}\right)\int \frac{\left(id^2-cd^2x\right)\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(d^2\left(1+c^2x^2\right)^{3/2}\right)\int \frac{(a+b}{}}{(d+icdx)^{3/2}(f-}$$

$$= -\frac{d^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(2i\left(1+c^2x^2\right)^{3/2}\right)\int\left(\frac{id^2\left(a+b\sinh^{-1}(c\right.}{\left(1+c^2x^2\right)^{3/2}}\right.}{(d+icdx)^{3/2}(f}$$

$$= -\frac{d^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(2d^2\left(1+c^2x^2\right)^{3/2}\right)\int \frac{\left(a+b\sinh^{-1}(cx\right.}{\left(1+c^2x^2\right)^{3/2}}}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{2id^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2d^2x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{2id^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2d^2x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{2id^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2d^2x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{2id^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2d^2x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{2id^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2d^2x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{2id^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2d^2x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Mathematica [A] time = 5.53, size = 530, normalized size = 0.97

$$\frac{\frac{6a^2\sqrt{d+icdx}\sqrt{f-icfx}}{cx+i} - 3a^2\sqrt{d}\sqrt{f}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right) + \frac{3ab\sqrt{d+icdx}\sqrt{f-icfx}\left(2\left(\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)+i\right.\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(3/2),x]
```

```
[Out] ((6*a^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(I + c*x) - 3*a^2*Sqrt[d]*Sqrt
[f]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] - (b
^2*(-I + c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-18*Pi*ArcSinh[c*x] - (6
 - 6*I)*ArcSinh[c*x]^2 + I*ArcSinh[c*x]^3 - 12*(Pi - (2*I)*ArcSinh[c*x])*Lo
g[1 + I/E^ArcSinh[c*x]] + 24*Pi*Log[1 + E^ArcSinh[c*x]] + 12*Pi*Log[-Cos[(P
i + (2*I)*ArcSinh[c*x])/4]] - 24*Pi*Log[Cosh[ArcSinh[c*x]/2]] - (24*I)*Poly
```

```
Log[2, (-I)/E^ArcSinh[c*x]] - ((12*I)*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/
(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])))/(Sqrt[1 + c^2*x^2]*(Cosh[
ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^2) + (3*a*b*Sqrt[d + I*c*d*x]*Sqr
t[f - I*c*f*x]*(-(ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x
]/2])) + 4*ArcSinh[c*x]*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2])
+ 2*(4*ArcTan[Tanh[ArcSinh[c*x]/2]] + I*Log[1 + c^2*x^2])*(I*Cosh[ArcSinh[c
*x]/2] + Sinh[ArcSinh[c*x]/2])))/(Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] -
 I*Sinh[ArcSinh[c*x]/2])))/(3*c*f^2)
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2+2\,\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,ab\log\left(cx+\sqrt{c^2x^2+\right.}}{c^2f^2x^2+2i\,cf^2x-f^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2
+ 1))^2 + 2*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2*x^2
 + 1)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2)/(c^2*f^2*x^2 + 2*I*c*f^2
*x - f^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{i\,cdx+d}\,(b\,\text{arsinh}\,(cx)+a)^2}{\left(-i\,cfx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(I*c*d*x + d)*(b*arcsinh(c*x) + a)^2/(-I*c*f*x + f)^(3/2), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\text{arcsinh}\,(cx))^2\,\sqrt{icdx+d}}{\left(-icfx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(-\frac{2i\,\sqrt{c^2dfx^2+df}}{-i\,c^2f^2x+cf^2}-\frac{d\,\text{arsinh}\,(cx)}{cf^2\sqrt{\frac{d}{f}}}\right)+\int \frac{\sqrt{i\,cdx+d}\,b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{\left(-i\,cfx+f\right)^{\frac{3}{2}}}+\frac{2\,\sqrt{i\,cdx+d}\,ab\log\left(cx+\sqrt{\right.}}{\left(-i\,cfx+f\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x, algor
ithm="maxima")
```

```
[Out] a^2*(-2*I*sqrt(c^2*d*f*x^2 + d*f)/(-I*c^2*f^2*x + c*f^2) - d*arcsinh(c*x)/(
c*f^2*sqrt(d/f))) + integrate(sqrt(I*c*d*x + d)*b^2*log(c*x + sqrt(c^2*x^2
+ 1))^2/(-I*c*f*x + f)^(3/2) + 2*sqrt(I*c*d*x + d)*a*b*log(c*x + sqrt(c^2*x
^2 + 1))/(-I*c*f*x + f)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))^2\,\sqrt{d+c\,d\,x\,1\mathrm{i}}}{\left(f-c\,f\,x\,1\mathrm{i}\right)^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(3/2),x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{id\,(cx-i)}\,\left(a+b\,\mathrm{asinh}\,(cx)\right)^2}{\left(-if\,(cx+i)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(3/2),x)
```

```
[Out] Integral(sqrt(I*d*(c*x - I))*(a + b*asinh(c*x))**2/(-I*f*(c*x + I))**(3/2),
 x)
```

3.597 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+icdx}\,(f-icfx)^{3/2}}\,dx$

Optimal. Leaf size=464

$$\frac{d\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{id\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{dx\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2bd(}{}$$

```
[Out] -I*d*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
+d*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+d
*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/
2)+4*I*b*d*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/2
))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-2*b*d*(c^2*x^2+1)^(3/2)*(a+b*arcsi
nh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3
/2)+2*b^2*d*(c^2*x^2+1)^(3/2)*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*
c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-2*b^2*d*(c^2*x^2+1)^(3/2)*polylog(2,I*(c*x+(
c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-b^2*d*(c^2*x^2+1)^
(3/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)
^(3/2)
```

Rubi [A] time = 0.70, antiderivative size = 464, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.297, Rules used = {5712, 5821, 5687, 5714, 3718, 2190, 2279, 2391, 5717, 5693, 4180}

$$\frac{2b^2d\left(c^2x^2+1\right)^{3/2}\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2b^2d\left(c^2x^2+1\right)^{3/2}\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{b^2d\left(c^2x^2+1\right)^{3/2}\text{Pol}}{c(d+icdx)^{3/}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)),x]
```

```
[Out] ((-I)*d*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I
*c*f*x)^(3/2)) + (d*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/((d + I*c*d*x)^
(3/2)*(f - I*c*f*x)^(3/2)) + (d*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)
/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + ((4*I)*b*d*(1 + c^2*x^2)^(3/
2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f -
 I*c*f*x)^(3/2)) - (2*b*d*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])*Log[1 +
E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (2*b^2*d
*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2
)*(f - I*c*f*x)^(3/2)) - (2*b^2*d*(1 + c^2*x^2)^(3/2)*PolyLog[2, I*E^ArcSin
h[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (b^2*d*(1 + c^2*x^2)
^(3/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x
)^(3/2))
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+icdx}\,(f-icfx)^{3/2}}\,dx = \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{(d+icdx)\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{3/2}\int \left(\frac{d\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}+\frac{icdx\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{\left(d\left(1+c^2x^2\right)^{3/2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{\left(icd\left(1+c^2x^2\right)^{3/2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{id\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{dx\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{\left(2ib\right.}{}$$

$$= -\frac{id\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{dx\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{\left(2ib\right.}{}$$

$$= -\frac{id\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{dx\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{d\left(1\right.}{}$$

$$= -\frac{id\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{dx\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{d\left(1\right.}{}$$

$$= -\frac{id\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{dx\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{d\left(1\right.}{}$$

$$= -\frac{id\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{dx\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{d\left(1\right.}{}$$

Mathematica [A] time = 2.28, size = 511, normalized size = 1.10

$$\frac{\sqrt{d+icdx}\,\sqrt{f-icfx}\,\left(\left(\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)-i\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)\left(a^2cx-ia^2-ab\sqrt{c^2x^2+1}\,\log\left(c^2x^2+1\right)\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)),x]
```

```
[Out] (Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((-1 - I)*b^2*Sqrt[1 + c^2*x^2]*ArcSin
h[c*x]^2*(Cosh[ArcSinh[c*x]/2] - Sinh[ArcSinh[c*x]/2]) + ((-I)*a^2 + a^2*c*
x + (4*I)*a*b*Sqrt[1 + c^2*x^2]*ArcTan[Tanh[ArcSinh[c*x]/2]] - (2*I)*b^2*Pi
*Sqrt[1 + c^2*x^2]*Log[1 + I/E^ArcSinh[c*x]] + (4*I)*b^2*Pi*Sqrt[1 + c^2*x^
2]*Log[1 + E^ArcSinh[c*x]] - a*b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2] + (2*I)
*b^2*Pi*Sqrt[1 + c^2*x^2]*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] - (4*I)*b^
```

```
2*Pi*Sqrt[1 + c^2*x^2]*Log[Cosh[ArcSinh[c*x]/2]])*(Cosh[ArcSinh[c*x]/2] - I
*Sinh[ArcSinh[c*x]/2]) + 4*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)/E^ArcSinh[
c*x]]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2]) + b*Sqrt[1 + c^2*x^2]
*ArcSinh[c*x]*((-I)*Cosh[ArcSinh[c*x]/2]*(2*a + 3*b*Pi - (4*I)*b*Log[1 + I/
E^ArcSinh[c*x]]) + (2*a - 3*b*Pi + (4*I)*b*Log[1 + I/E^ArcSinh[c*x]])*Sinh[
ArcSinh[c*x]/2])))/(c*d*f^2*(-I + c*x)*(I + c*x)*(Cosh[ArcSinh[c*x]/2] - I*
Sinh[ArcSinh[c*x]/2]))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\frac{\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2+\left(c^2df^2x+i\,cdf^2\right)\text{integral}\left(\frac{i\,\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,a^2-\left(2\,\sqrt{c^2x^2+1}\,\sqrt{i\,cd}\right.}{c^3df^2}\right.}{c^2df^2x+i\,cdf^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] (sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2 +
(c^2*d*f^2*x + I*c*d*f^2)*integral((I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*
a^2 - (2*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2 - 2*I*s
qrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b)*log(c*x + sqrt(c^2*x^2 + 1)))/(c^3
*d*f^2*x^3 + I*c^2*d*f^2*x^2 + c*d*f^2*x + I*d*f^2), x))/(c^2*d*f^2*x + I*c
*d*f^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{i\,cdx+d}\,\left(-i\,cfx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(sqrt(I*c*d*x + d)*(-I*c*f*x + f)^(3/2)),
x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\text{arcsinh}\,(cx))^2}{\left(-icfx+f\right)^{\frac{3}{2}}\sqrt{icdx+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^2\int \frac{\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{\sqrt{i\,cdx+d}\,\left(-i\,cfx+f\right)^{\frac{3}{2}}}\,dx-\frac{2i\,\sqrt{c^2dfx^2+df}\,ab\,\text{arsinh}\,(cx)}{-i\,c^2df^2x+cdf^2}-\frac{i\,\sqrt{c^2dfx^2+df}\,a^2}{-i\,c^2df^2x+cdf^2}-\frac{2\,ab\log\left(i\,cx-1\right)}{c\sqrt{d}\,f^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/(sqrt(I*c*d*x + d)*(-I*c*f*x +
 f)^(3/2)), x) - 2*I*sqrt(c^2*d*f*x^2 + d*f)*a*b*arcsinh(c*x)/(-I*c^2*d*f^2
*x + c*d*f^2) - I*sqrt(c^2*d*f*x^2 + d*f)*a^2/(-I*c^2*d*f^2*x + c*d*f^2) -
2*a*b*log(I*c*x - 1)/(c*sqrt(d)*f^(3/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))^2}{\sqrt{d+c\,d\,x\,1\mathrm{i}}\,\left(f-c\,f\,x\,1\mathrm{i}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(3/2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(cx))^2}{\sqrt{id\,(cx-i)}\,\left(-if\,(cx+i)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(f-I*c*f*x)**(3/2)/(d+I*c*d*x)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(sqrt(I*d*(c*x - I))*(-I*f*(c*x + I))**(3/2)
), x)
```

3.598 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx$

Optimal. Leaf size=224

$$\frac{\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2b\left(c^2x^2+1\right)^{3/2}\log\left(e^{2\sinh^{-1}(cx)}+1\right)\left(a+b\,\mathrm{si}\right.}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[Out] x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)+(c^2
*x^2+1)^(3/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-2*
b*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d
+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)-b^2*(c^2*x^2+1)^(3/2)*polylog(2,-(c*x+(c^
2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2)
```

Rubi [A] time = 0.42, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {5712, 5687, 5714, 3718, 2190, 2279, 2391}

$$-\frac{b^2\left(c^2x^2+1\right)^{3/2}\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)),x]
```

```
[Out] (x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)
^(3/2)) + ((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/
2)*(f - I*c*f*x)^(3/2)) - (2*b*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])*Log
[1 + E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (b^
2*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/
2)*(f - I*c*f*x)^(3/2))
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx = \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(2bc\left(1+c^2x^2\right)^{3/2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{1+c^2x^2}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(2b\left(1+c^2x^2\right)^{3/2}\right)\mathrm{Subst}\left(\int (a+bx)\tanh\right.}{c(d+icdx)^{3/2}(f-icfx}$$

$$= \frac{x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(4b\right.}{}$$

$$= \frac{x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2b\,(1}{}$$

$$= \frac{x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2b\,(1}{}$$

$$= \frac{x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2b\,(1}{}$$

Mathematica [B] time = 1.36, size = 488, normalized size = 2.18

$$a^2cx - ab\sqrt{c^2x^2+1}\,\log\left(c^2x^2+1\right) + 2abcx\sinh^{-1}(cx) + 2b^2\sqrt{c^2x^2+1}\,\mathrm{Li}_2\left(-ie^{-\sinh^{-1}(cx)}\right) + 2b^2\sqrt{c^2x^2+1}\,\mathrm{Li}_2$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)),
x]
```

```
[Out] (a^2*c*x + 2*a*b*c*x*ArcSinh[c*x] - (2*I)*b^2*Pi*Sqrt[1 + c^2*x^2]*ArcSinh[
c*x] + b^2*c*x*ArcSinh[c*x]^2 - b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2 + I*b^
2*Pi*Sqrt[1 + c^2*x^2]*Log[1 - I/E^ArcSinh[c*x]] - 2*b^2*Sqrt[1 + c^2*x^2]*
ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]] - I*b^2*Pi*Sqrt[1 + c^2*x^2]*Log[1 +
 I/E^ArcSinh[c*x]] - 2*b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + I/E^ArcSi
nh[c*x]] + (4*I)*b^2*Pi*Sqrt[1 + c^2*x^2]*Log[1 + E^ArcSinh[c*x]] - a*b*Sqr
t[1 + c^2*x^2]*Log[1 + c^2*x^2] + I*b^2*Pi*Sqrt[1 + c^2*x^2]*Log[-Cos[(Pi +
 (2*I)*ArcSinh[c*x])/4]] - (4*I)*b^2*Pi*Sqrt[1 + c^2*x^2]*Log[Cosh[ArcSinh[
c*x]/2]] - I*b^2*Pi*Sqrt[1 + c^2*x^2]*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]]
 + 2*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)/E^ArcSinh[c*x]] + 2*b^2*Sqrt[1 +
 c^2*x^2]*PolyLog[2, I/E^ArcSinh[c*x]])/(c*d*f*Sqrt[d + I*c*d*x]*Sqrt[f - I
*c*f*x])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\frac{\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,b^2x\log\left(cx+\sqrt{c^2x^2+1}\right)^2+\left(c^2d^2f^2x^2+d^2f^2\right)\mathrm{integral}\left(\frac{\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,a^2-2\left(\sqrt{c^2x^2+1}\,\sqrt{i\,c}\right.}{c}\right.}{c^2d^2f^2x^2+d^2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x, algor
ithm="fricas")
```

```
[Out] (sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*x*log(c*x + sqrt(c^2*x^2 + 1))^2
+ (c^2*d^2*f^2*x^2 + d^2*f^2)*integral((sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f
)*a^2 - 2*(sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*c*x -
 sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b)*log(c*x + sqrt(c^2*x^2 + 1)))/(c
^4*d^2*f^2*x^4 + 2*c^2*d^2*f^2*x^2 + d^2*f^2), x))/(c^2*d^2*f^2*x^2 + d^2*f
^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^2}{(i\,cdx+d)^{\frac{3}{2}}\left(-i\,cfx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((I*c*d*x + d)^(3/2)*(-I*c*f*x + f)^(3/2))
, x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2}{(icdx+d)^{\frac{3}{2}}\left(-icfx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^2\int \frac{\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{(i\,cdx+d)^{\frac{3}{2}}\left(-i\,cfx+f\right)^{\frac{3}{2}}}\,dx+\frac{2\,abx\,\mathrm{arsinh}\,(cx)}{\sqrt{c^2dfx^2+df}\,df}+\frac{a^2x}{\sqrt{c^2dfx^2+df}\,df}-\frac{ab\sqrt{\frac{1}{df}}\,\log\left(x^2+\frac{1}{c^2}\right)}{cdf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x, algor
ithm="maxima")
```

```
[Out] b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/((I*c*d*x + d)^(3/2)*(-I*c*f*x
 + f)^(3/2)), x) + 2*a*b*x*arcsinh(c*x)/(sqrt(c^2*d*f*x^2 + d*f)*d*f) + a^2
*x/(sqrt(c^2*d*f*x^2 + d*f)*d*f) - a*b*sqrt(1/(d*f))*log(x^2 + 1/c^2)/(c*d*
f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))^2}{(d+c\,d\,x\,1\mathrm{i})^{3/2}\,\left(f-c\,f\,x\,1\mathrm{i}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(3/2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(cx))^2}{(id\,(cx-i))^{\frac{3}{2}}\left(-if\,(cx+i)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(3/2)/(f-I*c*f*x)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/((I*d*(c*x - I))**(3/2)*(-I*f*(c*x + I))**(3
/2)), x)
```

3.599 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx$

Optimal. Leaf size=743

$$\frac{2f\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2fx\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{bf\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{ib}{}$$

```
[Out] -1/3*I*b^2*f*(c^2*x^2+1)^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*b^2*f*
x*(c^2*x^2+1)^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*b*f*(c^2*x^2+1)^(3/
2)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*I*b*f*x*(c^
2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*I
*f*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1
/3*f*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
+2/3*f*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(
5/2)+2/3*f*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*
c*f*x)^(5/2)-2/3*I*b*f*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*arctan(c*x+(c^2
*x^2+1)^(1/2))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-4/3*b*f*(c^2*x^2+1)^(5
/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(5/2)/
(f-I*c*f*x)^(5/2)-1/3*b^2*f*(c^2*x^2+1)^(5/2)*polylog(2,-I*(c*x+(c^2*x^2+1)
^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*b^2*f*(c^2*x^2+1)^(5/2)*
polylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-
2/3*b^2*f*(c^2*x^2+1)^(5/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*
d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A] time = 0.91, antiderivative size = 743, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.378, Rules used = {5712, 5821, 5690, 5687, 5714, 3718, 2190, 2279, 2391, 5717, 191, 5693, 4180, 261}

$$-\frac{b^2f\left(c^2x^2+1\right)^{5/2}\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{b^2f\left(c^2x^2+1\right)^{5/2}\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2b^2f\left(c^2x^2+1\right)^{5/2}\text{PolyL}}{3c(d+icdx)^{5/2}(}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)),x]
```

```
[Out] ((-I/3)*b^2*f*(1 + c^2*x^2)^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
- (b^2*f*x*(1 + c^2*x^2)^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (
b*f*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c*(d + I*c*d*x)^(5/2)*(f -
 I*c*f*x)^(5/2)) - ((I/3)*b*f*x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(
(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + ((I/3)*f*(1 + c^2*x^2)*(a + b*Ar
cSinh[c*x])^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (f*x*(1 + c^2*
x^2)*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) +
(2*f*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/2)*(f -
I*c*f*x)^(5/2)) + (2*f*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(3*c*(d
+ I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((2*I)/3)*b*f*(1 + c^2*x^2)^(5/2)*
(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*
c*f*x)^(5/2)) - (4*b*f*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*Log[1 + E^(
2*ArcSinh[c*x])])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (b^2*f*(1
 + c^2*x^2)^(5/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)
*(f - I*c*f*x)^(5/2)) + (b^2*f*(1 + c^2*x^2)^(5/2)*PolyLog[2, I*E^ArcSinh[c
*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (2*b^2*f*(1 + c^2*x^2
)^(5/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*
f*x)^(5/2))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
```

```
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(f-icfx)\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{5/2}\int \left(\frac{f\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}} - \frac{icfx\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(f\left(1+c^2x^2\right)^{5/2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(icf\left(1+c^2x^2\right)^{5/2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{if\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{fx\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(2f\right.}{}$$

$$= \frac{bf\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{ibfx\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} +$$

$$= -\frac{ib^2f\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2fx\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bf\left(1+c^2x^2\right)^{3}}{3c(d+icd}$$

$$= -\frac{ib^2f\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2fx\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bf\left(1+c^2x^2\right)^{3}}{3c(d+icd}$$

$$= -\frac{ib^2f\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2fx\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bf\left(1+c^2x^2\right)^{3}}{3c(d+icd}$$

$$= -\frac{ib^2f\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2fx\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bf\left(1+c^2x^2\right)^{3}}{3c(d+icd}$$

$$= -\frac{ib^2f\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2fx\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bf\left(1+c^2x^2\right)^{3}}{3c(d+icd}$$

Mathematica [A] time = 8.34, size = 754, normalized size = 1.01

$$\frac{\sqrt{id(cx-i)}\sqrt{-if(cx+i)}\left(\frac{5a^2}{12d^3f^2(cx-i)} + \frac{a^2}{4d^3f^2(cx+i)} - \frac{ia^2}{6d^3f^2(cx-i)^2}\right)}{c} + \frac{iab\sqrt{i(cdx-id)}\sqrt{-i(cfx+if)}\left(\sqrt{c^2x^2+1}\right.\left(\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)),
x]
```

```
[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*(((-1/6*I)*a^2)/(d^3*f^2*(-I +
 c*x)^2) + (5*a^2)/(12*d^3*f^2*(-I + c*x)) + a^2/(4*d^3*f^2*(I + c*x))))/c
+ ((I/3)*a*b*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*(4*c*x*ArcSi
nh[c*x] + (2*I)*ArcSinh[c*x]*Cosh[2*ArcSinh[c*x]] + Sqrt[1 + c^2*x^2]*(1 -
(2*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 2*c*x*(ArcTan[Tanh[ArcSinh[c*x]/2]] -
(2*I)*Log[Sqrt[1 + c^2*x^2]]) - 4*Log[Sqrt[1 + c^2*x^2]])))/(c*d^2*f*(-I +
c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[-(d*f*(1 + c^2*x^2))]) +
((I/6)*b^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[1 + c^2*x
^2]*(7*Pi*ArcSinh[c*x] + ((2 + I*ArcSinh[c*x])*ArcSinh[c*x])/(-I + c*x) - (
```

```
1 + 4*I)*ArcSinh[c*x]^2 - 5*(Pi + (2*I)*ArcSinh[c*x])*Log[1 - I/E^ArcSinh[c
*x]] + 3*(Pi - (2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] - 16*Pi*Log[1
+ E^ArcSinh[c*x]] - 3*Pi*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] + 16*Pi*Log
[Cosh[ArcSinh[c*x]/2]] + 5*Pi*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]] + (6*I)
*PolyLog[2, (-I)/E^ArcSinh[c*x]] + (10*I)*PolyLog[2, I/E^ArcSinh[c*x]] + ((
3*I)*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[Ar
cSinh[c*x]/2]) + ((2*I)*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[
c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^3 + ((-4 + 5*ArcSinh[c*x]^2)*Sinh[ArcSinh
[c*x]/2])/((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2])))/(c*d^2*f*Sqr
t[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[-(d*f*(1 + c^2*x^2))])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\frac{\left(2\,b^{2}c^{2}x^{2}-2i\,b^{2}cx+b^{2}\right)\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)^{2}+\left(3\,c^{4}d^{3}f^{2}x^{3}-3i\,c^{3}d^{3}f^{2}x^{2}+3\,c^{2}d^{3}f^{2}x\right.}{3\,c^{4}d^{3}f^{2}x^{3}-3i\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x, algor
ithm="fricas")
```

```
[Out] ((2*b^2*c^2*x^2 - 2*I*b^2*c*x + b^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*l
og(c*x + sqrt(c^2*x^2 + 1))^2 + (3*c^4*d^3*f^2*x^3 - 3*I*c^3*d^3*f^2*x^2 +
3*c^2*d^3*f^2*x - 3*I*c*d^3*f^2)*integral((-3*I*sqrt(I*c*d*x + d)*sqrt(-I*c
*f*x + f)*a^2 + (-6*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b - (4*b^2*c^2
*x^2 - 4*I*b^2*c*x + 2*b^2)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f
*x + f))*log(c*x + sqrt(c^2*x^2 + 1)))/(3*c^5*d^3*f^2*x^5 - 3*I*c^4*d^3*f^2
*x^4 + 6*c^3*d^3*f^2*x^3 - 6*I*c^2*d^3*f^2*x^2 + 3*c*d^3*f^2*x - 3*I*d^3*f^
2), x))/(3*c^4*d^3*f^2*x^3 - 3*I*c^3*d^3*f^2*x^2 + 3*c^2*d^3*f^2*x - 3*I*c*
d^3*f^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [c,d,t_nostep]=[-33,71,-83]schur row 1 9.61551e-08Francis algorith
m not precise enough for[1.0,0.0,-1.5251320454e+15,-6.10052818161e+15,5.815
06938979e+29]schur row 1 1.96645e-07Francis algorithm not precise enough fo
r[1.0,0.0,-2.39675895199e+15,-9.58703580795e+15,1.43611336848e+30]Warning,
choosing root of [1,0,%%%{-8,[2,4,2]%%%}+%%%{-6,[0,0,0]%%%},%%%{-32,[2,4,2]
%%%}+%%%{-8,[0,0,0]%%%},%%%{16,[4,8,4]%%%}+%%%{-24,[2,4,2]%%%}+%%%{-3,[0,0,
0]%%%}] at parameters values [-89,63,-49]Warning, need to choose a branch f
or the root of a polynomial with parameters. This might be wrong.The choice
 was done assuming [c,d,t_nostep]=[-3,-93,97]Warning, choosing root of [1,0
,%%%{-8,[2,4,2]%%%}+%%%{-6,[0,0,0]%%%},%%%{-32,[2,4,2]%%%}+%%%{-8,[0,0,0]%%
%},%%%{16,[4,8,4]%%%}+%%%{-24,[2,4,2]%%%}+%%%{-3,[0,0,0]%%%}] at parameters
 values [-30,70,22]Warning, need to choose a branch for the root of a polyn
omial with parameters. This might be wrong.The choice was done assuming [c,
d,t_nostep]=[-27,64,92]Warning, need to choose a branch for the root of a p
olynomial with parameters. This might be wrong.The choice was done assuming
 [c,d,t_nostep]=[-48,-96,98]Warning, need to choose a branch for the root o
f a polynomial with parameters. This might be wrong.The choice was done ass
```

```
uming [c,d,t_nostep]=[-64,-23,17]schur row 1 1.0339e-07Francis algorithm no
t precise enough for[1.0,0.0,-1.383274802e+15,-5.533099208e+15,4.7836229446
2e+29]Warning, choosing root of [1,0,%%%{-8,[2,4,2]%%%}+%%%{-6,[0,0,0]%%%},
%%%{-32,[2,4,2]%%%}+%%%{-8,[0,0,0]%%%},%%%{16,[4,8,4]%%%}+%%%{-24,[2,4,2]%%
%}+%%%{-3,[0,0,0]%%%}] at parameters values [65,-85,28]Warning, need to cho
ose a branch for the root of a polynomial with parameters. This might be wr
ong.The choice was done assuming [c,d,t_nostep]=[-2,5,9]schur row 3 -3.4717
6e-07Warning, choosing root of [1,0,%%%{-8,[2,4,2]%%%}+%%%{-6,[0,0,0]%%%},%
%%{-32,[2,4,2]%%%}+%%%{-8,[0,0,0]%%%},%%%{16,[4,8,4]%%%}+%%%{-24,[2,4,2]%%%
}+%%%{-3,[0,0,0]%%%}] at parameters values [93,91,31]Warning, need to choos
e a branch for the root of a polynomial with parameters. This might be wron
g.The choice was done assuming [c,d,t_nostep]=[34,2,56]Warning, need to cho
ose a branch for the root of a polynomial with parameters. This might be wr
ong.The choice was done assuming [c,d,t_nostep]=[41,-70,47]schur row 1 4.95
324e-07Francis algorithm not precise enough for[1.0,0.0,-7.1325623432e+14,-
1.42651246864e+15,1.27183613949e+29]Evaluation time: 5.47sym2poly/r2sym(con
st gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [**F**] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(icdx+d)^{\frac{5}{2}}\left(-icfx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}\left(c\,x\right))^2}{(d+c\,d\,x\,1\mathrm{i})^{5/2}\,\left(f-c\,f\,x\,1\mathrm{i}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(3/2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(3/2)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(5/2)/(f-I*c*f*x)**(3/2),x)
```

```
[Out] Timed out
```

3.600 $\int \frac{(d+icdx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(f-icfx)^{5/2}} dx$

Optimal. Leaf size=794

$$-\frac{2iabd^5x\left(c^2x^2+1\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{5d^5\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{id^5\left(c^2x^2+1\right)^3\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{28d^5\left(c^2x\right.}{3c(d}$$

```
[Out] -2*I*a*b*d^5*x*(c^2*x^2+1)^(5/2)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+2*I*b^
2*d^5*(c^2*x^2+1)^3/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2*I*b^2*d^5*x*(c^
2*x^2+1)^(5/2)*arcsinh(c*x)/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+28/3*d^5*(c
^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+
I*d^5*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5
/2)+5/3*d^5*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*x)^(5/2)/(f
-I*c*f*x)^(5/2)+112/3*b*d^5*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*ln(1+I/(c*
x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-112/3*b^2*d^5*(
c^2*x^2+1)^(5/2)*polylog(2,-I/(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/
(f-I*c*f*x)^(5/2)+8/3*b*d^5*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*sec(1/4*Pi
+1/2*I*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+16/3*I*b^2*d^5
*(c^2*x^2+1)^(5/2)*tan(1/4*Pi+1/2*I*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*
c*f*x)^(5/2)+28/3*I*d^5*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2*tan(1/4*Pi+1
/2*I*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-4/3*I*d^5*(c^2*x^2
+1)^(5/2)*(a+b*arcsinh(c*x))^2*sec(1/4*Pi+1/2*I*arcsinh(c*x))^2*tan(1/4*Pi+
1/2*I*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A] time = 1.40, antiderivative size = 794, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 16, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.432, Rules used = {5712, 5833, 5675, 5717, 5653, 261, 5831, 3318, 4186, 3767, 8, 4184, 3716, 2190, 2279, 2391}

$$-\frac{112b^2d^5\left(c^2x^2+1\right)^{5/2}\text{PolyLog}\left(2,-ie^{-\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2iabd^5x\left(c^2x^2+1\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{5d^5\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right.}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(5/2),x]
```

```
[Out] ((-2*I)*a*b*d^5*x*(1 + c^2*x^2)^(5/2))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(
5/2)) + ((2*I)*b^2*d^5*(1 + c^2*x^2)^3)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x
)^(5/2)) - ((2*I)*b^2*d^5*x*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x])/((d + I*c*d*x
)^(5/2)*(f - I*c*f*x)^(5/2)) + (28*d^5*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c
*x])^2)/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (I*d^5*(1 + c^2*x^2
)^3*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (
5*d^5*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^3)/(3*b*c*(d + I*c*d*x)^(5/2
)*(f - I*c*f*x)^(5/2)) + (112*b*d^5*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]
)*Log[1 + I/E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
- (112*b^2*d^5*(1 + c^2*x^2)^(5/2)*PolyLog[2, (-I)/E^ArcSinh[c*x]])/(3*c*(d
 + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (8*b*d^5*(1 + c^2*x^2)^(5/2)*(a +
b*ArcSinh[c*x])*Sec[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/(3*c*(d + I*c*d*x)^(5/2)*
(f - I*c*f*x)^(5/2)) + (((16*I)/3)*b^2*d^5*(1 + c^2*x^2)^(5/2)*Tan[Pi/4 + (
I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((28*I)/
3)*d^5*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*Tan[Pi/4 + (I/2)*ArcSinh[
c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((4*I)/3)*d^5*(1 + c^
2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*Sec[Pi/4 + (I/2)*ArcSinh[c*x]]^2*Tan[Pi
/4 + (I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
```

```
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5833

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+icdx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(f-icfx)^{5/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(d+icdx)^5\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{5/2}\int\left(\frac{5d^5\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}+\frac{icd^5x\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}-\frac{8d^5\left(a+b\sinh^{-1}\right.}{(i+cx)^2\sqrt{1+c^2}}\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{\left(12id^5\left(1+c^2x^2\right)^{5/2}\right)\int\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(i+cx)\sqrt{1+c^2x^2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{\left(5d^5\left(1+c^2x^2\right)^{5/2}\right)\int\frac{(a}{}}{(d+icdx)^{5/2}(f-}$$

$$= \frac{id^5\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{5d^5\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}\right.}{3bc(d+icdx)^{5/2}(f-icfx)^{5}}$$

$$= -\frac{2iabd^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{id^5\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{5}{}$$

$$= -\frac{2iabd^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2ib^2d^5x\left(1+c^2x^2\right)^{5/2}\sinh^{-1}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{id^5}{}$$

$$= -\frac{2iabd^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2ib^2d^5\left(1+c^2x^2\right)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2ib^2d^5x\left(1\right.}{(d+ic}$$

$$= -\frac{2iabd^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2ib^2d^5\left(1+c^2x^2\right)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2ib^2d^5x\left(1\right.}{(d+ic}$$

$$= -\frac{2iabd^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2ib^2d^5\left(1+c^2x^2\right)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2ib^2d^5x\left(1\right.}{(d+ic}$$

$$= -\frac{2iabd^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2ib^2d^5\left(1+c^2x^2\right)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2ib^2d^5x\left(1\right.}{(d+ic}$$

$$= -\frac{2iabd^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2ib^2d^5\left(1+c^2x^2\right)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2ib^2d^5x\left(1\right.}{(d+ic}$$

Mathematica [B] time = 13.91, size = 2552, normalized size = 3.21

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(5/2),
x]
```

```
[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*((I*a^2*d^2)/f^3 + (((8*I)/3)*
a^2*d^2)/(f^3*(I + c*x)^2) - (28*a^2*d^2)/(3*f^3*(I + c*x))))/c + (5*a^2*d^
(5/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c
*x)]])/(c*f^(5/2)) - ((I/3)*a*b*d^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f
 + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSin
h[c*x]/2])*(-(Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[ArcSin
h[c*x]/2]] + I*Log[Sqrt[1 + c^2*x^2]])) + Cosh[ArcSinh[c*x]/2]*(4*I + 3*Arc
Sinh[c*x] - 6*ArcTan[Coth[ArcSinh[c*x]/2]] + (3*I)*Log[Sqrt[1 + c^2*x^2]])
+ 2*(Sqrt[1 + c^2*x^2]*(I*ArcSinh[c*x] + (2*I)*ArcTan[Coth[ArcSinh[c*x]/2]]
 + Log[Sqrt[1 + c^2*x^2]]) + 2*(1 + I*ArcSinh[c*x] + (2*I)*ArcTan[Coth[ArcS
inh[c*x]/2]] + Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/(c*f^3*(1 +
```

```
I*c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] - I*Si
nh[ArcSinh[c*x]/2])^4) + (a*b*d^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f +
 c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[
c*x]/2])*(Cosh[(3*ArcSinh[c*x])/2]*((14*I - 3*ArcSinh[c*x])*ArcSinh[c*x] +
(28*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] - 14*Log[Sqrt[1 + c^2*x^2]]) + Cosh[Arc
Sinh[c*x]/2]*(8 + (6*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 - (84*I)*ArcTan[Tan
h[ArcSinh[c*x]/2]] + 42*Log[Sqrt[1 + c^2*x^2]]) - (2*I)*(4 + (4*I)*ArcSinh[
c*x] + 6*ArcSinh[c*x]^2 - (56*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 28*Log[Sqrt
[1 + c^2*x^2]] + Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(14*I + 3*ArcSinh[c*x]) -
(28*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 14*Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcS
inh[c*x]/2]))/(3*c*f^3*(1 + I*c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*
(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^4) - ((I/3)*b^2*d^2*(-I + c
*x)*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x
^2))]*((-1 - I)*ArcSinh[c*x]^2 - (2*ArcSinh[c*x]*(2*I + ArcSinh[c*x]))/(I +
 c*x) - (2*I)*(Pi - (2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] - I*Pi*(3
*ArcSinh[c*x] - 4*Log[1 + E^ArcSinh[c*x]] - 2*Log[-Cos[(Pi + (2*I)*ArcSinh[
c*x])/4]] + 4*Log[Cosh[ArcSinh[c*x]/2]]) + 4*PolyLog[2, (-I)/E^ArcSinh[c*x]
] - (4*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[
ArcSinh[c*x]/2])^3 + (2*(4 + ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/(Cosh[Ar
cSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])))/(c*f^3*Sqrt[-(((-I)*d + c*d*x)*(I
*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]
/2])^2) + ((I/3)*b^2*d^2*(-I + c*x)*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f
 + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(((-6*I)*c*x*ArcSinh[c*x])/Sqrt[1 + c
^2*x^2] + ((13 + 13*I)*ArcSinh[c*x]^2)/Sqrt[1 + c^2*x^2] + (3*ArcSinh[c*x]^
3)/Sqrt[1 + c^2*x^2] + (2*ArcSinh[c*x]*(2*I + ArcSinh[c*x]))/((I + c*x)*Sqr
t[1 + c^2*x^2]) + (3*I)*(2 + ArcSinh[c*x]^2) + ((13*I)*(2*(Pi - (2*I)*ArcSi
nh[c*x])*Log[1 + I/E^ArcSinh[c*x]] + Pi*(3*ArcSinh[c*x] - 4*Log[1 + E^ArcSi
nh[c*x]] - 2*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] + 4*Log[Cosh[ArcSinh[c*
x]/2]]) + (4*I)*PolyLog[2, (-I)/E^ArcSinh[c*x]]))/Sqrt[1 + c^2*x^2] + (4*Ar
cSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2]
 - I*Sinh[ArcSinh[c*x]/2])^3) - (2*(4 + 13*ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x
]/2])/(Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2]))))
/(c*f^3*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] + I*S
inh[ArcSinh[c*x]/2])^2) + (2*b^2*d^2*(-I + c*x)*Sqrt[I*((-I)*d + c*d*x)]*Sq
rt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(-21*Pi*ArcSinh[c*x] - (7
 - 7*I)*ArcSinh[c*x]^2 + I*ArcSinh[c*x]^3 + ((2*I)*ArcSinh[c*x]*(2*I + ArcS
inh[c*x]))/(I + c*x) - 14*(Pi - (2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x
]] + 28*Pi*Log[1 + E^ArcSinh[c*x]] + 14*Pi*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x
])/4]] - 28*Pi*Log[Cosh[ArcSinh[c*x]/2]] - (28*I)*PolyLog[2, (-I)/E^ArcSinh
[c*x]] - ((2*I)*(4 + 7*ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[
c*x]/2] - I*Sinh[ArcSinh[c*x]/2]) + (4*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])
/(I*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2])^3))/(3*c*f^3*Sqrt[-(((-I)*
d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh
[ArcSinh[c*x]/2])^2) - ((I/6)*a*b*d^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I
*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcS
inh[c*x]/2])*(-(Cosh[(3*ArcSinh[c*x])/2]*(9 - (35*I)*ArcSinh[c*x] + 9*ArcSi
nh[c*x]^2 + (52*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + 26*Log[Sqrt[1 + c^2*x^2]]
)) + Cosh[ArcSinh[c*x]/2]*(20 + (24*I)*ArcSinh[c*x] + 27*ArcSinh[c*x]^2 + (
156*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + 78*Log[Sqrt[1 + c^2*x^2]]) - I*(3*(-I
 + ArcSinh[c*x])*Cosh[(5*ArcSinh[c*x])/2] + 2*(13 + (7*I)*ArcSinh[c*x] + 18
*ArcSinh[c*x]^2 + (104*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + (3*I)*(I + ArcSinh
[c*x])*Cosh[2*ArcSinh[c*x]] + 52*Log[Sqrt[1 + c^2*x^2]] + Sqrt[1 + c^2*x^2]
*(6 + (38*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 + (52*I)*ArcTan[Coth[ArcSinh[c
*x]/2]] + 26*Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2])))/(c*f^3*(-I +
c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh
[ArcSinh[c*x]/2])^4)
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(i\,b^2c^2d^2x^2+2\,b^2cd^2x-i\,b^2d^2\right)\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2+\left(2i\,abc^2d^2x^2+4\,ab\right.}{c^3f^3x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral(((I*b^2*c^2*d^2*x^2 + 2*b^2*c*d^2*x - I*b^2*d^2)*sqrt(I*c*d*x + d)
*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 + (2*I*a*b*c^2*d^2*x^2 +
 4*a*b*c*d^2*x - 2*I*a*b*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x
+ sqrt(c^2*x^2 + 1)) + (I*a^2*c^2*d^2*x^2 + 2*a^2*c*d^2*x - I*a^2*d^2)*sqrt
(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^3*f^3*x^3 + 3*I*c^2*f^3*x^2 - 3*c*f^3*
x - I*f^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{(icdx+d)^{\frac{5}{2}}\,(a+b\,\mathrm{arcsinh}\,(cx))^2}{\left(-icfx+f\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^{5/2}}{\left(f-c\,f\,x\,1\mathrm{i}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(5/2),x)
[Out] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(5/2))/(f - c*f*x*1i)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))**2/(f-I*c*f*x)**(5/2),x)
[Out] Timed out
```

3.601 $\int \frac{(d+icdx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(f-icfx)^{5/2}}\,dx$

Optimal. Leaf size=584

$$\frac{d^4\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{8d^4\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{32bd^4\left(c^2x^2+1\right)^{5/2}\log\left(1+ie^{-\sinh}\right.}{3c(d+icdx)^{5/2}(f-}$$

```
[Out] 8/3*d^4*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f
*x)^(5/2)+1/3*d^4*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^3/b/c/(d+I*c*d*x)^(5
/2)/(f-I*c*f*x)^(5/2)+32/3*b*d^4*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*ln(1+
I/(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-32/3*b^2*d
^4*(c^2*x^2+1)^(5/2)*polylog(2,-I/(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5
/2)/(f-I*c*f*x)^(5/2)+4/3*b*d^4*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*sec(1/
4*Pi+1/2*I*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+8/3*I*b^2*
d^4*(c^2*x^2+1)^(5/2)*tan(1/4*Pi+1/2*I*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f
-I*c*f*x)^(5/2)+8/3*I*d^4*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2*tan(1/4*Pi
+1/2*I*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2/3*I*d^4*(c^2*x
^2+1)^(5/2)*(a+b*arcsinh(c*x))^2*sec(1/4*Pi+1/2*I*arcsinh(c*x))^2*tan(1/4*P
i+1/2*I*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A] time = 1.21, antiderivative size = 584, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.351, Rules used = {5712, 5833, 5675, 5831, 3318, 4186, 3767, 8, 4184, 3716, 2190, 2279, 2391}

$$-\frac{32b^2d^4\left(c^2x^2+1\right)^{5/2}\text{PolyLog}\left(2,-ie^{-\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{d^4\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{8d^4\left(c^2x^2+1\right)^{5/2}\left(a+\right.}{3c(d+icdx)^{5/2}(f}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(5/2),x]
```

```
[Out] (8*d^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(3*c*(d + I*c*d*x)^(5/2)
*(f - I*c*f*x)^(5/2)) + (d^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^3)/(3
*b*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (32*b*d^4*(1 + c^2*x^2)^(5/
2)*(a + b*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)
*(f - I*c*f*x)^(5/2)) - (32*b^2*d^4*(1 + c^2*x^2)^(5/2)*PolyLog[2, (-I)/E^A
rcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (4*b*d^4*(1 +
 c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*Sec[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/(3*c
*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((8*I)/3)*b^2*d^4*(1 + c^2*x^2
)^(5/2)*Tan[Pi/4 + (I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x
)^(5/2)) + (((8*I)/3)*d^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*Tan[Pi
/4 + (I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((
2*I)/3)*d^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*Sec[Pi/4 + (I/2)*Arc
Sinh[c*x]]^2*Tan[Pi/4 + (I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*
c*f*x)^(5/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5833

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+icdx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(f-icfx)^{5/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(d+icdx)^4\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{5/2}\int\left(\frac{d^4\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}} - \frac{4d^4\left(a+b\sinh^{-1}(cx)\right)^2}{(i+cx)^2\sqrt{1+c^2x^2}} - \frac{4id^4\left(a+b\sinh^{-1}(cx)\right)^2}{(i+cx)\sqrt{1+c^2x^2}}\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{\left(4id^4\left(1+c^2x^2\right)^{5/2}\right)\int\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(i+cx)\sqrt{1+c^2x^2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(d^4\left(1+c^2x^2\right)^{5/2}\right)\int\frac{\left(a+b\sinh\right.}{\sqrt{1+c}}}{(d+icdx)^{5/2}(f-icfx}$$

$$= \frac{d^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(4id^4\left(1+c^2x^2\right)^{5/2}\right)\text{Subst}\left(\int\frac{(a}{ic+c}\right.}{(d+icdx)^{5/2}(f-}$$

$$= \frac{d^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(d^4\left(1+c^2x^2\right)^{5/2}\right)\text{Subst}\left(\int(a+b\right.}{c(d+icdx)^{5}}$$

$$= \frac{d^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{4bd^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(c\right.}{3c(d+icdx)^{5/2}(}$$

$$= \frac{4d^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx\right.}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{8d^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx\right.}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{8d^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx\right.}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{8d^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx\right.}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{8d^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx\right.}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

<u>Mathematica</u> [B] time = 10.22, size = 1617, normalized size = 2.77

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(5/2),
x]
```

```
[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*((((4*I)/3)*a^2*d)/(f^3*(I + c
*x)^2) - (8*a^2*d)/(3*f^3*(I + c*x))))/c + (a^2*d^(3/2)*Log[c*d*f*x + Sqrt[
d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]])/(c*f^(5/2)) - ((I/
3)*a*b*d*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 +
c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])*(-(Cosh[(3*ArcSi
nh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]/2]] + I*Log[Sqrt[1 +
 c^2*x^2]])) + Cosh[ArcSinh[c*x]/2]*(4*I + 3*ArcSinh[c*x] - 6*ArcTan[Coth[A
rcSinh[c*x]/2]] + (3*I)*Log[Sqrt[1 + c^2*x^2]]) + 2*(Sqrt[1 + c^2*x^2]*(I*A
rcSinh[c*x] + (2*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + Log[Sqrt[1 + c^2*x^2]])
+ 2*(1 + I*ArcSinh[c*x] + (2*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + Log[Sqrt[1 +
 c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/(c*f^3*(1 + I*c*x)*Sqrt[-(((-I)*d + c*d
```

```
*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^4) + (a
*b*d*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*
x^2))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])*(Cosh[(3*ArcSinh[c*x
])/2]*((14*I - 3*ArcSinh[c*x])*ArcSinh[c*x] + (28*I)*ArcTan[Tanh[ArcSinh[c*
x]/2]] - 14*Log[Sqrt[1 + c^2*x^2]]) + Cosh[ArcSinh[c*x]/2]*(8 + (6*I)*ArcSi
nh[c*x] + 9*ArcSinh[c*x]^2 - (84*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 42*Log[S
qrt[1 + c^2*x^2]]) - (2*I)*(4 + (4*I)*ArcSinh[c*x] + 6*ArcSinh[c*x]^2 - (56
*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 28*Log[Sqrt[1 + c^2*x^2]] + Sqrt[1 + c^2
*x^2]*(ArcSinh[c*x]*(14*I + 3*ArcSinh[c*x]) - (28*I)*ArcTan[Tanh[ArcSinh[c*
x]/2]] + 14*Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/(6*c*f^3*(1 + I
*c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] - I*Sin
h[ArcSinh[c*x]/2])^4) - ((I/3)*b^2*d*(-I + c*x)*Sqrt[I*((-I)*d + c*d*x)]*Sq
rt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*((-1 - I)*ArcSinh[c*x]^2
- (2*ArcSinh[c*x]*(2*I + ArcSinh[c*x]))/(I + c*x) - (2*I)*(Pi - (2*I)*ArcSi
nh[c*x])*Log[1 + I/E^ArcSinh[c*x]] - I*Pi*(3*ArcSinh[c*x] - 4*Log[1 + E^Arc
Sinh[c*x]] - 2*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] + 4*Log[Cosh[ArcSinh[
c*x]/2]]) + 4*PolyLog[2, (-I)/E^ArcSinh[c*x]] - (4*ArcSinh[c*x]^2*Sinh[ArcS
inh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^3 + (2*(4 + Ar
cSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[
c*x]/2])))/(c*f^3*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]
*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^2) + (b^2*d*(-I + c*x)*Sqr
t[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(
-21*Pi*ArcSinh[c*x] - (7 - 7*I)*ArcSinh[c*x]^2 + I*ArcSinh[c*x]^3 + ((2*I)*
ArcSinh[c*x]*(2*I + ArcSinh[c*x]))/(I + c*x) - 14*(Pi - (2*I)*ArcSinh[c*x])
*Log[1 + I/E^ArcSinh[c*x]] + 28*Pi*Log[1 + E^ArcSinh[c*x]] + 14*Pi*Log[-Cos
[(Pi + (2*I)*ArcSinh[c*x])/4]] - 28*Pi*Log[Cosh[ArcSinh[c*x]/2]] - (28*I)*P
olyLog[2, (-I)/E^ArcSinh[c*x]] - ((2*I)*(4 + 7*ArcSinh[c*x]^2)*Sinh[ArcSinh
[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2]) + (4*ArcSinh[c*x]
^2*Sinh[ArcSinh[c*x]/2])/(I*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2])^3)
)/(3*c*f^3*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[
ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^2)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2cdx - i\,b^2d\right)\sqrt{i\,cdx + d}\,\sqrt{-i\,cfx + f}\,\log\left(cx + \sqrt{c^2x^2 + 1}\right)^2 + 2\left(abcdx - i\,abd\right)\sqrt{i\,cdx + d}\,\sqrt{-i\,cfx}}{c^3f^3x^3 + 3i\,c^2f^3x^2 - 3\,cf^3x - i\,f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral(((b^2*c*d*x - I*b^2*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*
x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c*d*x - I*a*b*d)*sqrt(I*c*d*x + d)*sqrt(-
I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a^2*c*d*x - I*a^2*d)*sqrt(I*c*
d*x + d)*sqrt(-I*c*f*x + f))/(c^3*f^3*x^3 + 3*I*c^2*f^3*x^2 - 3*c*f^3*x - I
*f^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
```

```
assuming [c,f,t_nostep]=[-81,22,73]schur row 3 -2.04765e-07Warning, choosin
g root of [1,0,%%%{-8,[2,4,2]%%%}+%%%{-6,[0,0,0]%%%},%%%{-32,[2,4,2]%%%}+%%
%{-8,[0,0,0]%%%},%%%{16,[4,8,4]%%%}+%%%{-24,[2,4,2]%%%}+%%%{-3,[0,0,0]%%%}]
 at parameters values [-89,63,-49]Warning, need to choose a branch for the
root of a polynomial with parameters. This might be wrong.The choice was do
ne assuming [c,f,t_nostep]=[90,38,-42]schur row 3 -2.42082e-07Warning, choo
sing root of [1,0,%%%{-8,[2,4,2]%%%}+%%%{-6,[0,0,0]%%%},%%%{-32,[2,4,2]%%%}
+%%%{-8,[0,0,0]%%%},%%%{16,[4,8,4]%%%}+%%%{-24,[2,4,2]%%%}+%%%{-3,[0,0,0]%%
%}] at parameters values [-30,70,22]Warning, need to choose a branch for th
e root of a polynomial with parameters. This might be wrong.The choice was
done assuming [c,f,t_nostep]=[-47,-11,52]Warning, need to choose a branch f
or the root of a polynomial with parameters. This might be wrong.The choice
 was done assuming [c,f,t_nostep]=[-15,-55,-78]Warning, need to choose a br
anch for the root of a polynomial with parameters. This might be wrong.The
choice was done assuming [c,f,t_nostep]=[89,-43,35]Warning, choosing root o
f [1,0,%%%{-8,[2,4,2]%%%}+%%%{-6,[0,0,0]%%%},%%%{-32,[2,4,2]%%%}+%%%{-8,[0,
0,0]%%%},%%%{16,[4,8,4]%%%}+%%%{-24,[2,4,2]%%%}+%%%{-3,[0,0,0]%%%}] at para
meters values [-85,28,-44]Warning, need to choose a branch for the root of
a polynomial with parameters. This might be wrong.The choice was done assum
ing [c,f,t_nostep]=[-13,66,41]Warning, choosing root of [1,0,%%%{-8,[2,4,2]
%%%}+%%%{-6,[0,0,0]%%%},%%%{-32,[2,4,2]%%%}+%%%{-8,[0,0,0]%%%},%%%{16,[4,8,
4]%%%}+%%%{-24,[2,4,2]%%%}+%%%{-3,[0,0,0]%%%}] at parameters values [91,31,
-21]Warning, need to choose a branch for the root of a polynomial with para
meters. This might be wrong.The choice was done assuming [c,f,t_nostep]=[-3
,-92,-18]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [c,f,t_nostep
]=[-95,-42,-33]Evaluation time: 7.51sym2poly/r2sym(const gen & e,const inde
x_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{(icdx+d)^{\frac{3}{2}} (a+b \operatorname{arcsinh}(cx))^2}{\left(-icfx+f\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x)
```

```
[Out] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b \operatorname{asinh}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^{3/2}}{\left(f-c\,f\,x\,1\mathrm{i}\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(5/2),x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(3/2))/(f - c*f*x*1i)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))**2/(f-I*c*f*x)**(5/2),x)
[Out] Timed out
```

3.602 $\int \frac{\sqrt{d+icdx}\left(a+b\sinh^{-1}(cx)\right)^2}{(f-icfx)^{5/2}}\,dx$

Optimal. Leaf size=522

$$\frac{d^3\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{4bd^3\left(c^2x^2+1\right)^{5/2}\log\left(1+ie^{-\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2bd^3\left(c^2x^2+1\right)\right.$$

```
[Out] 1/3*d^3*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f
*x)^(5/2)+4/3*b*d^3*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*ln(1+I/(c*x+(c^2*x
^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-4/3*b^2*d^3*(c^2*x^2+1)
^(5/2)*polylog(2,-I/(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x
)^(5/2)+2/3*b*d^3*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*sec(1/4*Pi+1/2*I*arc
sinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+4/3*I*b^2*d^3*(c^2*x^2+1
)^(5/2)*tan(1/4*Pi+1/2*I*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2
)+1/3*I*d^3*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2*tan(1/4*Pi+1/2*I*arcsinh
(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*I*d^3*(c^2*x^2+1)^(5/2)*(a
+b*arcsinh(c*x))^2*sec(1/4*Pi+1/2*I*arcsinh(c*x))^2*tan(1/4*Pi+1/2*I*arcsin
h(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A] time = 1.15, antiderivative size = 522, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 12, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.324, Rules used = {5712, 5833, 5831, 3318, 4186, 3767, 8, 4184, 3716, 2190, 2279, 2391}

$$-\frac{4b^2d^3\left(c^2x^2+1\right)^{5/2}\text{PolyLog}\left(2,-ie^{-\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{d^3\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{4bd^3\left(c^2x^2+1\right)^{5/2}\log\left(1+\right.}{3c(d+icdx}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(5/2),x]
```

```
[Out] (d^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(3*c*(d + I*c*d*x)^(5/2)*(
f - I*c*f*x)^(5/2)) + (4*b*d^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*Log
[1 + I/E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (4*
b^2*d^3*(1 + c^2*x^2)^(5/2)*PolyLog[2, (-I)/E^ArcSinh[c*x]])/(3*c*(d + I*c*
d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (2*b*d^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSi
nh[c*x])*Sec[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/(3*c*(d + I*c*d*x)^(5/2)*(f - I*
c*f*x)^(5/2)) + (((4*I)/3)*b^2*d^3*(1 + c^2*x^2)^(5/2)*Tan[Pi/4 + (I/2)*Arc
Sinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + ((I/3)*d^3*(1 + c
^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*Tan[Pi/4 + (I/2)*ArcSinh[c*x]])/(c*(d
+ I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - ((I/3)*d^3*(1 + c^2*x^2)^(5/2)*(a +
 b*ArcSinh[c*x])^2*Sec[Pi/4 + (I/2)*ArcSinh[c*x]]^2*Tan[Pi/4 + (I/2)*ArcSin
h[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
```

```
c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5833

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+icdx}\left(a+b\sinh^{-1}(cx)\right)^2}{(f-icfx)^{5/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(d+icdx)^3\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{5/2}\int \left(-\frac{2d^3\left(a+b\sinh^{-1}(cx)\right)^2}{(i+cx)^2\sqrt{1+c^2x^2}}-\frac{id^3\left(a+b\sinh^{-1}(cx)\right)^2}{(i+cx)\sqrt{1+c^2x^2}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{\left(id^3\left(1+c^2x^2\right)^{5/2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(i+cx)\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(2d^3\left(1+c^2x^2\right)^{5/2}\right)\int \frac{\left(a+b\sinh^{-1}\right.}{(i+cx)^2\sqrt{1+}}}{(d+icdx)^{5/2}(f-icfx)^{5/}}$$

$$= -\frac{\left(id^3\left(1+c^2x^2\right)^{5/2}\right)\text{Subst}\left(\int \frac{(a+bx)^2}{ic+c\sinh(x)}\,dx,x,\sinh^{-1}(cx)\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(2cd^3\left(1+c^2\right.\right.}{}$$

$$= -\frac{\left(d^3\left(1+c^2x^2\right)^{5/2}\right)\text{Subst}\left(\int (a+bx)^2\csc^2\left(\frac{\pi}{4}-\frac{ix}{2}\right)dx,x,\sinh^{-1}(cx)\right)}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \cdots$$

$$= \frac{2bd^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{id^3\left(1+\right.}{}$$

$$= \frac{d^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2bd^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx\right.}{3c(d+icdx)^{5/2}(f}$$

$$= \frac{d^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{4ib^2d^3\left(1+c^2x^2\right)^{5/2}\cot\left(\frac{\pi}{4}-\frac{1}{2}i\sin\right.}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{d^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{4ib^2d^3\left(1+c^2x^2\right)^{5/2}\cot\left(\frac{\pi}{4}-\frac{1}{2}i\sin\right.}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{d^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{4ib^2d^3\left(1+c^2x^2\right)^{5/2}\cot\left(\frac{\pi}{4}-\frac{1}{2}i\sin\right.}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{d^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{4ib^2d^3\left(1+c^2x^2\right)^{5/2}\cot\left(\frac{\pi}{4}-\frac{1}{2}i\sin\right.}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Mathematica [A] time = 8.24, size = 788, normalized size = 1.51

$$\frac{\sqrt{id(cx-i)}\sqrt{-if(cx+i)}\left(\frac{2ia^2}{3f^3(cx+i)^2}-\frac{a^2}{3f^3(cx+i)}\right)}{c} - \frac{iab\sqrt{i(cdx-id)}\sqrt{-i(cfx+if)}\sqrt{-df\left(c^2x^2+1\right)}\left(\cosh\left(\frac{1}{2}\sinh^{-1}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(5/2),x]
```

```
[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*((((2*I)/3)*a^2)/(f^3*(I + c*x
)^2) - a^2/(3*f^3*(I + c*x))))/c - ((I/3)*a*b*Sqrt[I*((-I)*d + c*d*x)]*Sqrt
[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] + I*S
inh[ArcSinh[c*x]/2])*(-(Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[C
oth[ArcSinh[c*x]/2]] + I*Log[Sqrt[1 + c^2*x^2]])) + Cosh[ArcSinh[c*x]/2]*(4
*I + 3*ArcSinh[c*x] - 6*ArcTan[Coth[ArcSinh[c*x]/2]] + (3*I)*Log[Sqrt[1 + c
^2*x^2]]) + 2*(Sqrt[1 + c^2*x^2]*(I*ArcSinh[c*x] + (2*I)*ArcTan[Coth[ArcSin
h[c*x]/2]] + Log[Sqrt[1 + c^2*x^2]]) + 2*(1 + I*ArcSinh[c*x] + (2*I)*ArcTan
[Coth[ArcSinh[c*x]/2]] + Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/(c
*f^3*(1 + I*c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]
/2] - I*Sinh[ArcSinh[c*x]/2])^4) - ((I/3)*b^2*(-I + c*x)*Sqrt[I*((-I)*d + c
*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*((-1 - I)*ArcSin
h[c*x]^2 - (2*ArcSinh[c*x]*(2*I + ArcSinh[c*x]))/(I + c*x) - (2*I)*(Pi - (2
*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] - I*Pi*(3*ArcSinh[c*x] - 4*Log[
1 + E^ArcSinh[c*x]] - 2*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] + 4*Log[Cosh
[ArcSinh[c*x]/2]]) + 4*PolyLog[2, (-I)/E^ArcSinh[c*x]] - (4*ArcSinh[c*x]^2*
Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^3 + (
2*(4 + ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh
[ArcSinh[c*x]/2])))/(c*f^3*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 +
 c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^2)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$-\frac{\left(b^{2} c x - i\, b^{2}\right) \sqrt{i\, c d x + d} \sqrt{-i\, c f x + f} \log\left(c x + \sqrt{c^{2} x^{2} + 1}\right)^{2} - \left(3\, c^{3} f^{3} x^{2} + 6 i\, c^{2} f^{3} x - 3\, c f^{3}\right) {\rm integral}\left(\frac{-3 i \sqrt{i\, c d x + d}}{}\right)}{3\, c^{3} f^{3} x^{2} + 6 i\, c^{2} f^{3} x - 3\, c f^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x, algor
ithm="fricas")
```

```
[Out] -((b^2*c*x - I*b^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2
*x^2 + 1))^2 - (3*c^3*f^3*x^2 + 6*I*c^2*f^3*x - 3*c*f^3)*integral((-3*I*sqr
t(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2 + (2*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x +
 d)*sqrt(-I*c*f*x + f)*b^2 - 6*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b)*
log(c*x + sqrt(c^2*x^2 + 1)))/(3*c^3*f^3*x^3 + 9*I*c^2*f^3*x^2 - 9*c*f^3*x
- 3*I*f^3), x))/(3*c^3*f^3*x^2 + 6*I*c^2*f^3*x - 3*c*f^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{i\, c d x + d} {\left(b \operatorname{arsinh}\left(c x\right) + a\right)}^{2}}{{\left(-i\, c f x + f\right)}^{\frac{5}{2}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(I*c*d*x + d)*(b*arcsinh(c*x) + a)^2/(-I*c*f*x + f)^(5/2), x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{\left(a +b \arcsinh \left(c x \right)\right)^{2} \sqrt{i c d x +d}}{\left(-i c f x +f \right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))^2\,\sqrt{d+c\,d\,x\,1\mathrm{i}}}{\left(f-c\,f\,x\,1\mathrm{i}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(5/2),x)
```

```
[Out] int(((a + b*asinh(c*x))^2*(d + c*d*x*1i)^(1/2))/(f - c*f*x*1i)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(5/2),x)
```

```
[Out] Timed out
```

3.603 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+icdx}\,(f-icfx)^{5/2}}\,dx$

Optimal. Leaf size=942

$$-\frac{c^2d^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2x^3}{3(icxd+d)^{5/2}(f-icfx)^{5/2}}-\frac{bcd^2\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)x^2}{3(icxd+d)^{5/2}(f-icfx)^{5/2}}-\frac{2b^2d^2\left(c^2x^2+1\right)^2x}{3(icxd+d)^{5/2}(f-icfx)^{5/2}}+$$

```
[Out] 4/3*I*b*d^2*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*x^2+1)^(1/
2))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2/3*b^2*d^2*x*(c^2*x^2+1)^2/(d+I*
c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*b^2*d^2*(c^2*x^2+1)^(5/2)*arcsinh(c*x)/c
/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*b*d^2*(c^2*x^2+1)^(3/2)*(a+b*arcsi
nh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2/3*I*d^2*(c^2*x^2+1)*(a+b*a
rcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*b*c*d^2*x^2*(c^2*x
^2+1)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+2/3*I*b^
2*d^2*(c^2*x^2+1)^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*d^2*x*(c^2*x^
2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*c^2*d^2*x
^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+2/3
*d^2*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/
2)+1/3*d^2*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*
c*f*x)^(5/2)+2/3*I*b*d^2*x*(c^2*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)
^(5/2)/(f-I*c*f*x)^(5/2)-2/3*b*d^2*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*ln(
1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+2/3*b^2*
d^2*(c^2*x^2+1)^(5/2)*polylog(2,-I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(
5/2)/(f-I*c*f*x)^(5/2)-2/3*b^2*d^2*(c^2*x^2+1)^(5/2)*polylog(2,I*(c*x+(c^2*
x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*b^2*d^2*(c^2*x^2+1
)^(5/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*
x)^(5/2)
```

Rubi [A] time = 1.30, antiderivative size = 942, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 18, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.486, Rules used = {5712, 5821, 5690, 5687, 5714, 3718, 2190, 2279, 2391, 5717, 191, 5693, 4180, 261, 5723, 5751, 288, 215}

$$-\frac{c^2d^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2x^3}{3(icxd+d)^{5/2}(f-icfx)^{5/2}}-\frac{bcd^2\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)x^2}{3(icxd+d)^{5/2}(f-icfx)^{5/2}}-\frac{2b^2d^2\left(c^2x^2+1\right)^2x}{3(icxd+d)^{5/2}(f-icfx)^{5/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)),x]
```

```
[Out] (((2*I)/3)*b^2*d^2*(1 + c^2*x^2)^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5
/2)) - (2*b^2*d^2*x*(1 + c^2*x^2)^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(
5/2)) + (b^2*d^2*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x])/(3*c*(d + I*c*d*x)^(5/2)
*(f - I*c*f*x)^(5/2)) + (b*d^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3
*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((2*I)/3)*b*d^2*x*(1 + c^2*x
^2)^(3/2)*(a + b*ArcSinh[c*x]))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) -
 (b*c*d^2*x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*(d + I*c*d*x)^(5
/2)*(f - I*c*f*x)^(5/2)) - (((2*I)/3)*d^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]
)^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (d^2*x*(1 + c^2*x^2)*(a
+ b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (c^2*d^2
*x^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*
f*x)^(5/2)) + (2*d^2*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c*
d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (d^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c
*x])^2)/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((4*I)/3)*b*d^2*(1
 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*
d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (2*b*d^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSi
nh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x
```

```
)^(5/2)) + (2*b^2*d^2*(1 + c^2*x^2)^(5/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/
(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (2*b^2*d^2*(1 + c^2*x^2)^(5
/2)*PolyLog[2, I*E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5
/2)) - (b^2*d^2*(1 + c^2*x^2)^(5/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*c*(
d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
```

```
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
```

```
d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
NeQ[m, -1]
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+icdx}\,(f-icfx)^{5/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(d+icdx)^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{5/2}\int \left(\frac{d^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}} + \frac{2icd^2x\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}} - \frac{c^2d^2x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(d^2\left(1+c^2x^2\right)^{5/2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(2icd^2\left(1+c^2x^2\right)^{5/2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{5/2}}}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{2id^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^2x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{c}{}$$

$$= \frac{bd^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ibd^2x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{2ib^2d^2\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2d^2x\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bd^2\left(1+c^2x^2\right)^{3/2}}{3c(d+icdx)^{5/}}$$

$$= \frac{2ib^2d^2\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2d^2x\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b^2d^2\left(1+c^2x^2\right)^{5/2}}{3c(d+icdx)^{5/2}(f}$$

$$= \frac{2ib^2d^2\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2d^2x\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b^2d^2\left(1+c^2x^2\right)^{5/2}}{3c(d+icdx)^{5/2}(f}$$

$$= \frac{2ib^2d^2\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2d^2x\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b^2d^2\left(1+c^2x^2\right)^{5/2}}{3c(d+icdx)^{5/2}(f}$$

$$= \frac{2ib^2d^2\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2d^2x\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b^2d^2\left(1+c^2x^2\right)^{5/2}}{3c(d+icdx)^{5/2}(f}$$

Mathematica [A] time = 7.05, size = 528, normalized size = 0.56

$$\sqrt{d+icdx}\,\sqrt{f-icfx}\left(\frac{a^2(cx+2i)}{(cx+i)^2} - \frac{ab\left(2\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\left(-\frac{1}{2}i\left(\sqrt{c^2x^2+1}+2\right)\log\left(c^2x^2+1\right)+\left(\sqrt{c^2x^2+1}-1\right)\sinh^{-1}(cx)+2\left(\sqrt{c^2x^2+1}+2\right)\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)),x]
```

```
[Out] (Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((a^2*(2*I + c*x))/(I + c*x)^2 - (a*b*
(I*Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]/2]]
+ (I/2)*Log[1 + c^2*x^2]) + Cosh[ArcSinh[c*x]/2]*(-2 + (3*I)*ArcSinh[c*x] +
 (6*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + (3*Log[1 + c^2*x^2])/2) + 2*(I + (-1
+ Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + 2*(2 + Sqrt[1 + c^2*x^2])*ArcTan[Coth[A
rcSinh[c*x]/2]] - (I/2)*(2 + Sqrt[1 + c^2*x^2])*Log[1 + c^2*x^2])*Sinh[ArcS
inh[c*x]/2]))/(Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x
]/2])^3) - (b^2*((1 + I)*ArcSinh[c*x]^2 - (ArcSinh[c*x]*(2*I + ArcSinh[c*x]
```

```
))/(I + c*x) + 2*(I*Pi + 2*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] + I*Pi*(
3*ArcSinh[c*x] - 4*Log[1 + E^ArcSinh[c*x]] - 2*Log[-Cos[(Pi + (2*I)*ArcSinh
[c*x])/4]] + 4*Log[Cosh[ArcSinh[c*x]/2]]) - 4*PolyLog[2, (-I)/E^ArcSinh[c*x
]] - (2*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh
[ArcSinh[c*x]/2])^3 - (2*(-2 + ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/(Cosh[
ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])))/Sqrt[1 + c^2*x^2]))/(3*c*d*f^3)
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\frac{\left(b^{2} c x+2 i b^{2}\right) \sqrt{i c d x+d} \sqrt{-i c f x+f} \log\left(c x+\sqrt{c^{2} x^{2}+1}\right)^{2}+\left(3 c^{3} d f^{3} x^{2}+6 i c^{2} d f^{3} x-3 c d f^{3}\right) \text{integral}\left(-\frac{3 \sqrt{i c a}}{}\right.}{3 c^{3} d f^{3} x^{2}+6 i c^{2} d f^{3} x-3 c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] ((b^2*c*x + 2*I*b^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^
2*x^2 + 1))^2 + (3*c^3*d*f^3*x^2 + 6*I*c^2*d*f^3*x - 3*c*d*f^3)*integral(-(
3*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2 + (6*sqrt(I*c*d*x + d)*sqrt(-I*c
*f*x + f)*a*b + (2*b^2*c*x + 4*I*b^2)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*s
qrt(-I*c*f*x + f))*log(c*x + sqrt(c^2*x^2 + 1)))/(3*c^4*d*f^3*x^4 + 6*I*c^3
*d*f^3*x^3 + 6*I*c*d*f^3*x - 3*d*f^3), x))/(3*c^3*d*f^3*x^2 + 6*I*c^2*d*f^3
*x - 3*c*d*f^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(c x)+a)^{2}}{\sqrt{i c d x+d}\left(-i c f x+f\right)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(sqrt(I*c*d*x + d)*(-I*c*f*x + f)^(5/2)),
x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{(a+b \operatorname{arcsinh}(c x))^{2}}{\left(-i c f x+f\right)^{\frac{5}{2}} \sqrt{i c d x+d}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))^2}{\sqrt{d+c\,d\,x\,1\mathrm{i}}\,\left(f-c\,f\,x\,1\mathrm{i}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(5/2)),x)
[Out] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(1/2)*(f - c*f*x*1i)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(f-I*c*f*x)**(5/2)/(d+I*c*d*x)**(1/2),x)
[Out] Timed out
```

3.604 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx$

Optimal. Leaf size=743

$$\frac{2d\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2dx\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{bd\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+ib$$

```
[Out] 1/3*I*b^2*d*(c^2*x^2+1)^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*b^2*d*x
*(c^2*x^2+1)^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*b*d*(c^2*x^2+1)^(3/2
)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*I*b*d*x*(c^2
*x^2+1)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*I*
d*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/
3*d*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+
2/3*d*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5
/2)+2/3*d*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c
*f*x)^(5/2)+2/3*I*b*d*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*arctan(c*x+(c^2*
x^2+1)^(1/2))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-4/3*b*d*(c^2*x^2+1)^(5/
2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(5/2)/(
f-I*c*f*x)^(5/2)+1/3*b^2*d*(c^2*x^2+1)^(5/2)*polylog(2,-I*(c*x+(c^2*x^2+1)^
(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-1/3*b^2*d*(c^2*x^2+1)^(5/2)*p
olylog(2,I*(c*x+(c^2*x^2+1)^(1/2)))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2
/3*b^2*d*(c^2*x^2+1)^(5/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d
*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A] time = 0.90, antiderivative size = 743, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.378, Rules used = {5712, 5821, 5690, 5687, 5714, 3718, 2190, 2279, 2391, 5717, 191, 5693, 4180, 261}

$$\frac{b^2d\left(c^2x^2+1\right)^{5/2}\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{b^2d\left(c^2x^2+1\right)^{5/2}\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2b^2d\left(c^2x^2+1\right)^{5/2}\text{PolyLog}}{3c(d+icdx)^{5/2}(f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)),x]
```

```
[Out] ((I/3)*b^2*d*(1 + c^2*x^2)^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) -
 (b^2*d*x*(1 + c^2*x^2)^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (b
*d*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c*(d + I*c*d*x)^(5/2)*(f -
I*c*f*x)^(5/2)) + ((I/3)*b*d*x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/((
d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - ((I/3)*d*(1 + c^2*x^2)*(a + b*Arc
Sinh[c*x])^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (d*x*(1 + c^2*x
^2)*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (
2*d*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/2)*(f - I
*c*f*x)^(5/2)) + (2*d*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(3*c*(d +
 I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((2*I)/3)*b*d*(1 + c^2*x^2)^(5/2)*(
a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c
*f*x)^(5/2)) - (4*b*d*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*Log[1 + E^(2
*ArcSinh[c*x])])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (b^2*d*(1
+ c^2*x^2)^(5/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*
(f - I*c*f*x)^(5/2)) - (b^2*d*(1 + c^2*x^2)^(5/2)*PolyLog[2, I*E^ArcSinh[c*
x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (2*b^2*d*(1 + c^2*x^2)
^(5/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f
*x)^(5/2))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
```

```
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(d+icdx)\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{5/2}\int \left(\frac{d\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}+\frac{icdx\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(d\left(1+c^2x^2\right)^{5/2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(icd\left(1+c^2x^2\right)^{5/2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{id\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{dx\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{(2}{}$$

$$= \frac{bd\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{ibdx\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} -$$

$$= \frac{ib^2d\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2dx\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bd\left(1+c^2x^2\right)^{3/2}}{3c(d+icdx)}$$

$$= \frac{ib^2d\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2dx\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bd\left(1+c^2x^2\right)^{3/2}}{3c(d+icdx)}$$

$$= \frac{ib^2d\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2dx\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bd\left(1+c^2x^2\right)^{3/2}}{3c(d+icdx)}$$

$$= \frac{ib^2d\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2dx\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bd\left(1+c^2x^2\right)^{3/2}}{3c(d+icdx)}$$

$$= \frac{ib^2d\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2dx\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bd\left(1+c^2x^2\right)^{3/2}}{3c(d+icdx)}$$

Mathematica [A] time = 8.33, size = 757, normalized size = 1.02

$$\frac{\sqrt{id(cx-i)}\sqrt{-if(cx+i)}\left(\frac{a^2}{4d^2f^3(cx-i)}+\frac{5a^2}{12d^2f^3(cx+i)}+\frac{ia^2}{6d^2f^3(cx+i)^2}\right)}{c} - \frac{iab\sqrt{i(cdx-id)}\sqrt{-i(cfx+if)}\left(\sqrt{c^2x^2+1}\left(\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)),
x]
```

```
[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*(a^2/(4*d^2*f^3*(-I + c*x)) +
((I/6)*a^2)/(d^2*f^3*(I + c*x)^2) + (5*a^2)/(12*d^2*f^3*(I + c*x))))/c - ((
I/3)*a*b*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*(4*c*x*ArcSinh[c
*x] - (2*I)*ArcSinh[c*x]*Cosh[2*ArcSinh[c*x]] + Sqrt[1 + c^2*x^2]*(1 + (2*I
)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 2*c*x*(ArcTan[Tanh[ArcSinh[c*x]/2]] + (2*I
)*Log[Sqrt[1 + c^2*x^2]]) - 4*Log[Sqrt[1 + c^2*x^2]])))/(c*d*f^2*(I + c*x)*
Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[-(d*f*(1 + c^2*x^2))]) - ((I/6
)*b^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[1 + c^2*x^2]*(
-9*Pi*ArcSinh[c*x] + ((2 - I*ArcSinh[c*x])*ArcSinh[c*x])/(I + c*x) - (1 - 4
```

```
*I)*ArcSinh[c*x]^2 + 3*(Pi + (2*I)*ArcSinh[c*x])*Log[1 - I/E^ArcSinh[c*x]]
- 5*(Pi - (2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] + 16*Pi*Log[1 + E^A
rcSinh[c*x]] + 5*Pi*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] - 16*Pi*Log[Cosh
[ArcSinh[c*x]/2]] - 3*Pi*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]] - (10*I)*Pol
yLog[2, (-I)/E^ArcSinh[c*x]] - (6*I)*PolyLog[2, I/E^ArcSinh[c*x]] - ((2*I)*
ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh
[c*x]/2])^3 + (I*(4 - 5*ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh
[c*x]/2] - I*Sinh[ArcSinh[c*x]/2]) - ((3*I)*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x
]/2])/(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/(c*d*f^2*Sqrt[-(((-
I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[-(d*f*(1 + c^2*x^2))])
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\frac{\left(2\,b^2c^2x^2+2i\,b^2cx+b^2\right)\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2+\left(3\,c^4d^2f^3x^3+3i\,c^3d^2f^3x^2+3\,c^2d^2f^3x\right.}{3\,c^4d^2f^3x^3+3i\,c^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x, algor
ithm="fricas")
```

```
[Out] ((2*b^2*c^2*x^2 + 2*I*b^2*c*x + b^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*l
og(c*x + sqrt(c^2*x^2 + 1))^2 + (3*c^4*d^2*f^3*x^3 + 3*I*c^3*d^2*f^3*x^2 +
3*c^2*d^2*f^3*x + 3*I*c*d^2*f^3)*integral((3*I*sqrt(I*c*d*x + d)*sqrt(-I*c*
f*x + f)*a^2 + (6*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b - (4*b^2*c^2*x
^2 + 4*I*b^2*c*x + 2*b^2)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x
 + f))*log(c*x + sqrt(c^2*x^2 + 1)))/(3*c^5*d^2*f^3*x^5 + 3*I*c^4*d^2*f^3*x
^4 + 6*c^3*d^2*f^3*x^3 + 6*I*c^2*d^2*f^3*x^2 + 3*c*d^2*f^3*x + 3*I*d^2*f^3)
, x))/(3*c^4*d^2*f^3*x^3 + 3*I*c^3*d^2*f^3*x^2 + 3*c^2*d^2*f^3*x + 3*I*c*d^
2*f^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [c,f,t_nostep]=[-59,-77,-5]Warning, choosing root of [1,0,%%%{-8,[
2,4,2]%%%}+%%%{-6,[0,0,0]%%%},%%%{-32,[2,4,2]%%%}+%%%{-8,[0,0,0]%%%},%%%{16
,[4,8,4]%%%}+%%%{-24,[2,4,2]%%%}+%%%{-3,[0,0,0]%%%}] at parameters values [
-89,63,-49]Warning, need to choose a branch for the root of a polynomial wi
th parameters. This might be wrong.The choice was done assuming [c,f,t_nost
ep]=[85,33,-4]schur row 1 4.94474e-07Francis algorithm not precise enough f
or[1.0,0.0,-8.3670048e+13,-3.34680192e+14,1.75016923308e+27]Warning, choosi
ng root of [1,0,%%%{-8,[2,4,2]%%%}+%%%{-6,[0,0,0]%%%},%%%{-32,[2,4,2]%%%}+%
%%{-8,[0,0,0]%%%},%%%{16,[4,8,4]%%%}+%%%{-24,[2,4,2]%%%}+%%%{-3,[0,0,0]%%%}
] at parameters values [-30,70,22]Warning, need to choose a branch for the
root of a polynomial with parameters. This might be wrong.The choice was do
ne assuming [c,f,t_nostep]=[-58,38,-52]schur row 3 2.17351e-07Warning, need
 to choose a branch for the root of a polynomial with parameters. This migh
t be wrong.The choice was done assuming [c,f,t_nostep]=[-92,-95,41]schur ro
w 1 4.13061e-08Francis algorithm not precise enough for[1.0,0.0,-9.27102551
432e+15,-1.85420510286e+16,2.14879785218e+31]Warning, need to choose a bran
ch for the root of a polynomial with parameters. This might be wrong.The ch
```

```
oice was done assuming [c,f,t_nostep]=[-28,-28,68]schur row 1 1.30058e-07Fr
ancis algorithm not precise enough for[1.0,0.0,-1.383274802e+15,-5.53309920
8e+15,4.78362294462e+29]Warning, choosing root of [1,0,%%%{-8,[2,4,2]%%%}+%
%%{-6,[0,0,0]%%%},%%%{-32,[2,4,2]%%%}+%%%{-8,[0,0,0]%%%},%%%{16,[4,8,4]%%%}
+%%%{-24,[2,4,2]%%%}+%%%{-3,[0,0,0]%%%}] at parameters values [65,-85,28]Wa
rning, need to choose a branch for the root of a polynomial with parameters
. This might be wrong.The choice was done assuming [c,f,t_nostep]=[50,16,66
]schur row 3 1.3131e-07Warning, choosing root of [1,0,%%%{-8,[2,4,2]%%%}+%%
%{-6,[0,0,0]%%%},%%%{-32,[2,4,2]%%%}+%%%{-8,[0,0,0]%%%},%%%{16,[4,8,4]%%%}+
%%%{-24,[2,4,2]%%%}+%%%{-3,[0,0,0]%%%}] at parameters values [93,91,31]Warn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming [c,f,t_nostep]=[29,-15,-41
]Warning, need to choose a branch for the root of a polynomial with paramet
ers. This might be wrong.The choice was done assuming [c,f,t_nostep]=[57,27
,-59]Evaluation time: 5.99sym2poly/r2sym(const gen & e,const index_m & i,co
nst vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2}{(icdx+d)^{\frac{3}{2}}\left(-icfx+f\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))^2}{(d+c\,d\,x\,1\mathrm{i})^{3/2}\,\left(f-c\,f\,x\,1\mathrm{i}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(5/2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(3/2)*(f - c*f*x*1i)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(3/2)/(f-I*c*f*x)**(5/2),x)
```

```
[Out] Timed out
```

3.605 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx$

Optimal. Leaf size=386

$$\frac{2\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2x\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{b\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{x\left(c^2x\right.}{3(a}$$

```
[Out] -1/3*b^2*x*(c^2*x^2+1)^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*b*(c^2*x^2
+1)^(3/2)*(a+b*arcsinh(c*x))/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+1/3*x*(c
^2*x^2+1)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+2/3*x*(c
^2*x^2+1)^2*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)+2/3*(c
^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))^2/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-
4/3*b*(c^2*x^2+1)^(5/2)*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)/
c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)-2/3*b^2*(c^2*x^2+1)^(5/2)*polylog(2,-
(c*x+(c^2*x^2+1)^(1/2))^2)/c/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2)
```

Rubi [A] time = 0.53, antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.270, Rules used = {5712, 5690, 5687, 5714, 3718, 2190, 2279, 2391, 5717, 191}

$$-\frac{2b^2\left(c^2x^2+1\right)^{5/2}\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2x\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(c\right.}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)),x]
```

```
[Out] -(b^2*x*(1 + c^2*x^2)^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (b*(
1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*
f*x)^(5/2)) + (x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/
2)*(f - I*c*f*x)^(5/2)) + (2*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(3*(
d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (2*(1 + c^2*x^2)^(5/2)*(a + b*Arc
Sinh[c*x])^2)/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (4*b*(1 + c^2
*x^2)^(5/2)*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(3*c*(d + I*c
*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (2*b^2*(1 + c^2*x^2)^(5/2)*PolyLog[2, -E
^(2*ArcSinh[c*x])])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(2\left(1+c^2x^2\right)^{5/2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(2bc\right.}{}$$

$$= \frac{b\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2x\left(1+\right.}{3(d}$$

$$= -\frac{b^2x\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{x\left(1+c^2x^2\right.}{3(d+ica}$$

$$= -\frac{b^2x\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{x\left(1+c^2x^2\right.}{3(d+ica}$$

$$= -\frac{b^2x\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{x\left(1+c^2x^2\right.}{3(d+ica}$$

$$= -\frac{b^2x\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{x\left(1+c^2x^2\right.}{3(d+ica}$$

$$= -\frac{b^2x\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{x\left(1+c^2x^2\right.}{3(d+ica}$$

Mathematica [A] time = 8.16, size = 642, normalized size = 1.66

$$\frac{4a^2cx\left(2c^2x^2+3\right)+2ab\left(\sqrt{c^2x^2+1}\left(2-3\log\left(c^2x^2+1\right)\right)-\log\left(c^2x^2+1\right)\cosh\left(3\sinh^{-1}(cx)\right)+2\sinh^{-1}(cx)\left(3\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)),
x]
```

```
[Out] (4*a^2*c*x*(3 + 2*c^2*x^2) - b^2*(c*x - 6*c*x*ArcSinh[c*x]^2 + (4*I)*Pi*Arc
Sinh[c*x]*Cosh[3*ArcSinh[c*x]] + 2*ArcSinh[c*x]^2*Cosh[3*ArcSinh[c*x]] - (2
*I)*Pi*Cosh[3*ArcSinh[c*x]]*Log[1 - I/E^ArcSinh[c*x]] + 4*ArcSinh[c*x]*Cosh
[3*ArcSinh[c*x]]*Log[1 - I/E^ArcSinh[c*x]] + (2*I)*Pi*Cosh[3*ArcSinh[c*x]]*
Log[1 + I/E^ArcSinh[c*x]] + 4*ArcSinh[c*x]*Cosh[3*ArcSinh[c*x]]*Log[1 + I/E
^ArcSinh[c*x]] - (8*I)*Pi*Cosh[3*ArcSinh[c*x]]*Log[1 + E^ArcSinh[c*x]] - (2
*I)*Pi*Cosh[3*ArcSinh[c*x]]*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] + (8*I)*
Pi*Cosh[3*ArcSinh[c*x]]*Log[Cosh[ArcSinh[c*x]/2]] + (2*I)*Pi*Cosh[3*ArcSinh
[c*x]]*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]] + 2*Sqrt[1 + c^2*x^2]*(((-3*I)
*Pi + 6*ArcSinh[c*x])*Log[1 - I/E^ArcSinh[c*x]] + I*((2*I)*ArcSinh[c*x] + 6
*Pi*ArcSinh[c*x] - (3*I)*ArcSinh[c*x]^2 + 3*(Pi - (2*I)*ArcSinh[c*x])*Log[1
 + I/E^ArcSinh[c*x]] - 12*Pi*Log[1 + E^ArcSinh[c*x]] - 3*Pi*Log[-Cos[(Pi +
(2*I)*ArcSinh[c*x])/4]] + 12*Pi*Log[Cosh[ArcSinh[c*x]/2]] + 3*Pi*Log[Sin[(P
i + (2*I)*ArcSinh[c*x])/4]])) - 16*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-I)/E^Ar
cSinh[c*x]] - 16*(1 + c^2*x^2)^(3/2)*PolyLog[2, I/E^ArcSinh[c*x]] + Sinh[3*
ArcSinh[c*x]] - 2*ArcSinh[c*x]^2*Sinh[3*ArcSinh[c*x]]) + 2*a*b*(Sqrt[1 + c^
2*x^2]*(2 - 3*Log[1 + c^2*x^2]) - Cosh[3*ArcSinh[c*x]]*Log[1 + c^2*x^2] + 2
*ArcSinh[c*x]*(3*c*x + Sinh[3*ArcSinh[c*x]]))))/(12*d^2*f^2*Sqrt[d + I*c*d*x
]*Sqrt[f - I*c*f*x]*(c + c^3*x^2))
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\frac{\left(2\,b^2c^2x^3+3\,b^2x\right)\sqrt{i\,cdx+d}\,\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2+3\left(c^4d^3f^3x^4+2\,c^2d^3f^3x^2+d^3f^3\right)\mathrm{integral}}{3\left(c^4d^3f^3x^4+2\,c^2d^3f\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*((2*b^2*c^2*x^3 + 3*b^2*x)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x
 + sqrt(c^2*x^2 + 1))^2 + 3*(c^4*d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)
*integral(1/3*(3*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2 + 2*(3*sqrt(I*c*d
*x + d)*sqrt(-I*c*f*x + f)*a*b - (2*b^2*c^3*x^3 + 3*b^2*c*x)*sqrt(c^2*x^2 +
 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))*log(c*x + sqrt(c^2*x^2 + 1)))/(c^
6*d^3*f^3*x^6 + 3*c^4*d^3*f^3*x^4 + 3*c^2*d^3*f^3*x^2 + d^3*f^3), x))/(c^4*
d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [c,d,t_nostep]=[-33,71,-83]schur row 1 9.61551e-08Francis algorith
m not precise enough for[1.0,0.0,-1.5251320454e+15,-6.10052818161e+15,5.815
06938979e+29]schur row 1 1.96645e-07Francis algorithm not precise enough fo
r[1.0,0.0,-2.39675895199e+15,-9.58703580795e+15,1.43611336848e+30]Warning,
choosing root of [1,0,%%%{-8,[2,4,2]%%%}+%%%{-6,[0,0,0]%%%},%%%{-32,[2,4,2]
%%%}+%%%{-8,[0,0,0]%%%},%%%{16,[4,8,4]%%%}+%%%{-24,[2,4,2]%%%}+%%%{-3,[0,0,
0]%%%}] at parameters values [-89,63,-49]Warning, need to choose a branch f
or the root of a polynomial with parameters. This might be wrong.The choice
 was done assuming [c,d,t_nostep]=[-3,-93,97]Warning, choosing root of [1,0
,%%%{-8,[2,4,2]%%%}+%%%{-6,[0,0,0]%%%},%%%{-32,[2,4,2]%%%}+%%%{-8,[0,0,0]%%
%},%%%{16,[4,8,4]%%%}+%%%{-24,[2,4,2]%%%}+%%%{-3,[0,0,0]%%%}] at parameters
 values [-30,70,22]Warning, need to choose a branch for the root of a polyn
omial with parameters. This might be wrong.The choice was done assuming [c,
d,t_nostep]=[-27,64,92]Warning, need to choose a branch for the root of a p
olynomial with parameters. This might be wrong.The choice was done assuming
 [c,d,t_nostep]=[-48,-96,98]Warning, need to choose a branch for the root o
f a polynomial with parameters. This might be wrong.The choice was done ass
uming [c,d,t_nostep]=[-85,-13,-75]Warning, choosing root of [1,0,%%%{-8,[2,
4,2]%%%}+%%%{-6,[0,0,0]%%%},%%%{-32,[2,4,2]%%%}+%%%{-8,[0,0,0]%%%},%%%{16,[
4,8,4]%%%}+%%%{-24,[2,4,2]%%%}+%%%{-3,[0,0,0]%%%}] at parameters values [65
,-85,28]Warning, need to choose a branch for the root of a polynomial with
parameters. This might be wrong.The choice was done assuming [c,d,t_nostep]
=[-51,55,-32]Warning, choosing root of [1,0,%%%{-8,[2,4,2]%%%}+%%%{-6,[0,0,
0]%%%},%%%{-32,[2,4,2]%%%}+%%%{-8,[0,0,0]%%%},%%%{16,[4,8,4]%%%}+%%%{-24,[2
,4,2]%%%}+%%%{-3,[0,0,0]%%%}] at parameters values [93,91,31]Warning, need
to choose a branch for the root of a polynomial with parameters. This might
 be wrong.The choice was done assuming [c,d,t_nostep]=[1,-81,89]Warning, ne
ed to choose a branch for the root of a polynomial with parameters. This mi
ght be wrong.The choice was done assuming [c,d,t_nostep]=[71,12,-14]Evaluat
```

```
ion time: 5.7sym2poly/r2sym(const gen & e,const index_m & i,const vecteur &
 l) Error: Bad Argument Value
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{arcsinh}(cx))^2}{(icdx + d)^{\frac{5}{2}} \left(-icfx + f\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\, abc\left(\frac{1}{c^4 d^{\frac{5}{2}} f^{\frac{5}{2}} x^2 + c^2 d^{\frac{5}{2}} f^{\frac{5}{2}}} - \frac{2\,\log\left(c^2x^2+1\right)}{c^2 d^{\frac{5}{2}} f^{\frac{5}{2}}}\right) + \frac{2}{3}\, ab\left(\frac{x}{\left(c^2dfx^2+df\right)^{\frac{3}{2}} df} + \frac{2\,x}{\sqrt{c^2dfx^2+df}\, d^2 f^2}\right) \operatorname{arsinh}(cx) + \frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/3*a*b*c*(1/(c^4*d^(5/2)*f^(5/2)*x^2 + c^2*d^(5/2)*f^(5/2)) - 2*log(c^2*x^
2 + 1)/(c^2*d^(5/2)*f^(5/2))) + 2/3*a*b*(x/((c^2*d*f*x^2 + d*f)^(3/2)*d*f)
+ 2*x/(sqrt(c^2*d*f*x^2 + d*f)*d^2*f^2))*arcsinh(c*x) + 1/3*a^2*(x/((c^2*d*
f*x^2 + d*f)^(3/2)*d*f) + 2*x/(sqrt(c^2*d*f*x^2 + d*f)*d^2*f^2)) + b^2*inte
grate(log(c*x + sqrt(c^2*x^2 + 1))^2/((I*c*d*x + d)^(5/2)*(-I*c*f*x + f)^(5
/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\operatorname{asinh}(c\,x))^2}{(d + c\,d\,x\,1\mathrm{i})^{5/2} \left(f - c\,f\,x\,1\mathrm{i}\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(5/2)),x)
```

```
[Out] int((a + b*asinh(c*x))^2/((d + c*d*x*1i)^(5/2)*(f - c*f*x*1i)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(5/2)/(f-I*c*f*x)**(5/2),x)
```

```
[Out] Timed out
```

3.606 $\int \left(d+ex^2\right)^4 \left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=312

$$d^4x\left(a+b\sinh^{-1}(cx)\right)+\frac{4}{3}d^3ex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{6}{5}d^2e^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{4}{7}de^3x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{9}$$

```
[Out] -4/945*b*e*(105*c^6*d^3-189*c^4*d^2*e+135*c^2*d*e^2-35*e^3)*(c^2*x^2+1)^(3/
2)/c^9-2/525*b*e^2*(63*c^4*d^2-90*c^2*d*e+35*e^2)*(c^2*x^2+1)^(5/2)/c^9-4/4
41*b*(9*c^2*d-7*e)*e^3*(c^2*x^2+1)^(7/2)/c^9-1/81*b*e^4*(c^2*x^2+1)^(9/2)/c
^9+d^4*x*(a+b*arcsinh(c*x))+4/3*d^3*e*x^3*(a+b*arcsinh(c*x))+6/5*d^2*e^2*x^
5*(a+b*arcsinh(c*x))+4/7*d*e^3*x^7*(a+b*arcsinh(c*x))+1/9*e^4*x^9*(a+b*arcs
inh(c*x))-1/315*b*(315*c^8*d^4-420*c^6*d^3*e+378*c^4*d^2*e^2-180*c^2*d*e^3+
35*e^4)*(c^2*x^2+1)^(1/2)/c^9
```

Rubi [A] time = 0.35, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {194, 5704, 12, 1799, 1850}

$$\frac{6}{5}d^2e^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{4}{3}d^3ex^3\left(a+b\sinh^{-1}(cx)\right)+d^4x\left(a+b\sinh^{-1}(cx)\right)+\frac{4}{7}de^3x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^4*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(b*(315*c^8*d^4 - 420*c^6*d^3*e + 378*c^4*d^2*e^2 - 180*c^2*d*e^3 + 35*e^4
)*Sqrt[1 + c^2*x^2])/(315*c^9) - (4*b*e*(105*c^6*d^3 - 189*c^4*d^2*e + 135*
c^2*d*e^2 - 35*e^3)*(1 + c^2*x^2)^(3/2))/(945*c^9) - (2*b*e^2*(63*c^4*d^2 -
 90*c^2*d*e + 35*e^2)*(1 + c^2*x^2)^(5/2))/(525*c^9) - (4*b*(9*c^2*d - 7*e)
*e^3*(1 + c^2*x^2)^(7/2))/(441*c^9) - (b*e^4*(1 + c^2*x^2)^(9/2))/(81*c^9)
+ d^4*x*(a + b*ArcSinh[c*x]) + (4*d^3*e*x^3*(a + b*ArcSinh[c*x]))/3 + (6*d^
2*e^2*x^5*(a + b*ArcSinh[c*x]))/5 + (4*d*e^3*x^7*(a + b*ArcSinh[c*x]))/7 +
(e^4*x^9*(a + b*ArcSinh[c*x]))/9
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rule 5704

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0])
```

Rubi steps

$$\int \left(d+ex^2\right)^4 \left(a+b\sinh^{-1}(cx)\right)\,dx = d^4x\left(a+b\sinh^{-1}(cx)\right)+\frac{4}{3}d^3ex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{6}{5}d^2e^2x^5\left(a+b\sinh^{-1}\right.$$

$$= d^4x\left(a+b\sinh^{-1}(cx)\right)+\frac{4}{3}d^3ex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{6}{5}d^2e^2x^5\left(a+b\sinh^{-1}\right.$$

$$= d^4x\left(a+b\sinh^{-1}(cx)\right)+\frac{4}{3}d^3ex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{6}{5}d^2e^2x^5\left(a+b\sinh^{-1}\right.$$

$$= d^4x\left(a+b\sinh^{-1}(cx)\right)+\frac{4}{3}d^3ex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{6}{5}d^2e^2x^5\left(a+b\sinh^{-1}\right.$$

$$= -\frac{b\left(315c^8d^4-420c^6d^3e+378c^4d^2e^2-180c^2de^3+35e^4\right)\sqrt{1+c^2x^2}}{315c^9} - \frac{4be\left(\right.}{}$$

Mathematica [A] time = 0.36, size = 260, normalized size = 0.83

$$\frac{315ax\left(315d^4+420d^3ex^2+378d^2e^2x^4+180de^3x^6+35e^4x^8\right)-\frac{b\sqrt{c^2x^2+1}\left(c^8\left(99225d^4+44100d^3ex^2+23814d^2e^2x^4+8100de^3x^6+1\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^4*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (315*a*x*(315*d^4 + 420*d^3*e*x^2 + 378*d^2*e^2*x^4 + 180*d*e^3*x^6 + 35*e^
4*x^8) - (b*Sqrt[1 + c^2*x^2]*(4480*e^4 - 320*c^2*e^3*(81*d + 7*e*x^2) + 48
*c^4*e^2*(1323*d^2 + 270*d*e*x^2 + 35*e^2*x^4) - 8*c^6*e*(11025*d^3 + 3969*
d^2*e*x^2 + 1215*d*e^2*x^4 + 175*e^3*x^6) + c^8*(99225*d^4 + 44100*d^3*e*x^
2 + 23814*d^2*e^2*x^4 + 8100*d*e^3*x^6 + 1225*e^4*x^8)))/c^9 + 315*b*x*(315
*d^4 + 420*d^3*e*x^2 + 378*d^2*e^2*x^4 + 180*d*e^3*x^6 + 35*e^4*x^8)*ArcSin
h[c*x])/99225
```

fricas [A] time = 0.61, size = 333, normalized size = 1.07

$$\frac{11025\,ac^9e^4x^9+56700\,ac^9de^3x^7+119070\,ac^9d^2e^2x^5+132300\,ac^9d^3ex^3+99225\,ac^9d^4x+315\left(35\,bc^9e^4x^9+180\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/99225*(11025*a*c^9*e^4*x^9 + 56700*a*c^9*d*e^3*x^7 + 119070*a*c^9*d^2*e^2
*x^5 + 132300*a*c^9*d^3*e*x^3 + 99225*a*c^9*d^4*x + 315*(35*b*c^9*e^4*x^9 +
 180*b*c^9*d*e^3*x^7 + 378*b*c^9*d^2*e^2*x^5 + 420*b*c^9*d^3*e*x^3 + 315*b*
c^9*d^4*x)*log(c*x + sqrt(c^2*x^2 + 1)) - (1225*b*c^8*e^4*x^8 + 99225*b*c^8
*d^4 - 88200*b*c^6*d^3*e + 63504*b*c^4*d^2*e^2 - 25920*b*c^2*d*e^3 + 100*(8
1*b*c^8*d*e^3 - 14*b*c^6*e^4)*x^6 + 4480*b*e^4 + 6*(3969*b*c^8*d^2*e^2 - 16
20*b*c^6*d*e^3 + 280*b*c^4*e^4)*x^4 + 4*(11025*b*c^8*d^3*e - 7938*b*c^6*d^2
*e^2 + 3240*b*c^4*d*e^3 - 560*b*c^2*e^4)*x^2)*sqrt(c^2*x^2 + 1))/c^9
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.02, size = 451, normalized size = 1.45

$$\frac{\frac{a\left(\frac{1}{9}e^{4}c^{9}x^{9}+\frac{4}{7}c^{9}d\,e^{3}x^{7}+\frac{6}{5}c^{9}d^{2}e^{2}x^{5}+\frac{4}{3}c^{9}d^{3}e\,x^{3}+c^{9}d^{4}x\right)}{c^{8}}+\frac{b\left(\frac{\operatorname{arcsinh}(cx)e^{4}c^{9}x^{9}}{9}+\frac{4\operatorname{arcsinh}(cx)c^{9}d\,e^{3}x^{7}}{7}+\frac{6\operatorname{arcsinh}(cx)c^{9}d^{2}e^{2}x^{5}}{5}+\frac{4\operatorname{arcsinh}(cx)c^{9}d^{3}e\,x^{3}}{3}+\operatorname{arcsin}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^4*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c*(a/c^8*(1/9*e^4*c^9*x^9+4/7*c^9*d*e^3*x^7+6/5*c^9*d^2*e^2*x^5+4/3*c^9*d
^3*e*x^3+c^9*d^4*x)+b/c^8*(1/9*arcsinh(c*x)*e^4*c^9*x^9+4/7*arcsinh(c*x)*c^
9*d*e^3*x^7+6/5*arcsinh(c*x)*c^9*d^2*e^2*x^5+4/3*arcsinh(c*x)*c^9*d^3*e*x^3
+arcsinh(c*x)*c^9*d^4*x-1/9*e^4*(1/9*c^8*x^8*(c^2*x^2+1)^(1/2)-8/63*c^6*x^6
*(c^2*x^2+1)^(1/2)+16/105*c^4*x^4*(c^2*x^2+1)^(1/2)-64/315*c^2*x^2*(c^2*x^2
+1)^(1/2)+128/315*(c^2*x^2+1)^(1/2))-4/7*c^2*d*e^3*(1/7*c^6*x^6*(c^2*x^2+1)
^(1/2)-6/35*c^4*x^4*(c^2*x^2+1)^(1/2)+8/35*c^2*x^2*(c^2*x^2+1)^(1/2)-16/35*
(c^2*x^2+1)^(1/2))-6/5*c^4*d^2*e^2*(1/5*c^4*x^4*(c^2*x^2+1)^(1/2)-4/15*c^2*
x^2*(c^2*x^2+1)^(1/2)+8/15*(c^2*x^2+1)^(1/2))-4/3*c^6*d^3*e*(1/3*c^2*x^2*(c
^2*x^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/2))-c^8*d^4*(c^2*x^2+1)^(1/2)))
```

maxima [A] time = 0.41, size = 415, normalized size = 1.33

$$\frac{1}{9}\,ae^{4}x^{9}+\frac{4}{7}\,ade^{3}x^{7}+\frac{6}{5}\,ad^{2}e^{2}x^{5}+\frac{4}{3}\,ad^{3}ex^{3}+\frac{4}{9}\left(3\,x^{3}\operatorname{arsinh}\left(cx\right)-c\left(\frac{\sqrt{c^{2}x^{2}+1}\,x^{2}}{c^{2}}-\frac{2\,\sqrt{c^{2}x^{2}+1}}{c^{4}}\right)\right)bd^{3}e+\frac{2}{25}\left(15\,x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/9*a*e^4*x^9 + 4/7*a*d*e^3*x^7 + 6/5*a*d^2*e^2*x^5 + 4/3*a*d^3*e*x^3 + 4/9
*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c
^4))*b*d^3*e + 2/25*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4
*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*b*d^2*e^2 + 4/245*
(35*x^7*arcsinh(c*x) - (5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x
^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*b*d*e^3
 + 1/2835*(315*x^9*arcsinh(c*x) - (35*sqrt(c^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c
^2*x^2 + 1)*x^6/c^4 + 48*sqrt(c^2*x^2 + 1)*x^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x
^2/c^8 + 128*sqrt(c^2*x^2 + 1)/c^10)*c)*b*e^4 + a*d^4*x + (c*x*arcsinh(c*x)
 - sqrt(c^2*x^2 + 1))*b*d^4/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\operatorname{asinh}(c\,x))\,\left(e\,x^{2}+d\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + e*x^2)^4,x)
```

```
[Out] int((a + b*asinh(c*x))*(d + e*x^2)^4, x)
```

sympy [A] time = 18.00, size = 593, normalized size = 1.90

$$\begin{cases} ad^4x + \frac{4ad^3ex^3}{3} + \frac{6ad^2e^2x^5}{5} + \frac{4ade^3x^7}{7} + \frac{ae^4x^9}{9} + bd^4x\operatorname{asinh}(cx) + \frac{4bd^3ex^3\operatorname{asinh}(cx)}{3} + \frac{6bd^2e^2x^5\operatorname{asinh}(cx)}{5} + \frac{4bde^3x^7\operatorname{asinh}(cx)}{7} \\ a\left(d^4x + \frac{4d^3ex^3}{3} + \frac{6d^2e^2x^5}{5} + \frac{4de^3x^7}{7} + \frac{e^4x^9}{9}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**4*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*d**4*x + 4*a*d**3*e*x**3/3 + 6*a*d**2*e**2*x**5/5 + 4*a*d*e**3
*x**7/7 + a*e**4*x**9/9 + b*d**4*x*asinh(c*x) + 4*b*d**3*e*x**3*asinh(c*x)/
3 + 6*b*d**2*e**2*x**5*asinh(c*x)/5 + 4*b*d*e**3*x**7*asinh(c*x)/7 + b*e**4
*x**9*asinh(c*x)/9 - b*d**4*sqrt(c**2*x**2 + 1)/c - 4*b*d**3*e*x**2*sqrt(c*
*2*x**2 + 1)/(9*c) - 6*b*d**2*e**2*x**4*sqrt(c**2*x**2 + 1)/(25*c) - 4*b*d*
e**3*x**6*sqrt(c**2*x**2 + 1)/(49*c) - b*e**4*x**8*sqrt(c**2*x**2 + 1)/(81*
c) + 8*b*d**3*e*sqrt(c**2*x**2 + 1)/(9*c**3) + 8*b*d**2*e**2*x**2*sqrt(c**2
*x**2 + 1)/(25*c**3) + 24*b*d*e**3*x**4*sqrt(c**2*x**2 + 1)/(245*c**3) + 8*
b*e**4*x**6*sqrt(c**2*x**2 + 1)/(567*c**3) - 16*b*d**2*e**2*sqrt(c**2*x**2
+ 1)/(25*c**5) - 32*b*d*e**3*x**2*sqrt(c**2*x**2 + 1)/(245*c**5) - 16*b*e**
4*x**4*sqrt(c**2*x**2 + 1)/(945*c**5) + 64*b*d*e**3*sqrt(c**2*x**2 + 1)/(24
5*c**7) + 64*b*e**4*x**2*sqrt(c**2*x**2 + 1)/(2835*c**7) - 128*b*e**4*sqrt(
c**2*x**2 + 1)/(2835*c**9), Ne(c, 0)), (a*(d**4*x + 4*d**3*e*x**3/3 + 6*d**
2*e**2*x**5/5 + 4*d*e**3*x**7/7 + e**4*x**9/9), True))
```

3.607 $\int \left(d+ex^2\right)^3 \left(a+b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=221

$$d^3x\left(a+b\sinh^{-1}(cx)\right)+d^2ex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{7}e^3x^7\left(a+b\sinh^{-1}(cx)\right)-\frac{3be^2(\cdots}{\cdots}$$

```
[Out] -1/105*b*e*(35*c^4*d^2-42*c^2*d*e+15*e^2)*(c^2*x^2+1)^(3/2)/c^7-3/175*b*(7*
c^2*d-5*e)*e^2*(c^2*x^2+1)^(5/2)/c^7-1/49*b*e^3*(c^2*x^2+1)^(7/2)/c^7+d^3*x
*(a+b*arcsinh(c*x))+d^2*e*x^3*(a+b*arcsinh(c*x))+3/5*d*e^2*x^5*(a+b*arcsinh
(c*x))+1/7*e^3*x^7*(a+b*arcsinh(c*x))-1/35*b*(35*c^6*d^3-35*c^4*d^2*e+21*c^
2*d*e^2-5*e^3)*(c^2*x^2+1)^(1/2)/c^7
```

Rubi [A] time = 0.26, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {194, 5704, 12, 1799, 1850}

$$d^2ex^3\left(a+b\sinh^{-1}(cx)\right)+d^3x\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{7}e^3x^7\left(a+b\sinh^{-1}(cx)\right)-\frac{be\left(c^2\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(b*(35*c^6*d^3 - 35*c^4*d^2*e + 21*c^2*d*e^2 - 5*e^3)*Sqrt[1 + c^2*x^2])/(
35*c^7) - (b*e*(35*c^4*d^2 - 42*c^2*d*e + 15*e^2)*(1 + c^2*x^2)^(3/2))/(105
*c^7) - (3*b*(7*c^2*d - 5*e)*e^2*(1 + c^2*x^2)^(5/2))/(175*c^7) - (b*e^3*(1
 + c^2*x^2)^(7/2))/(49*c^7) + d^3*x*(a + b*ArcSinh[c*x]) + d^2*e*x^3*(a + b
*ArcSinh[c*x]) + (3*d*e^2*x^5*(a + b*ArcSinh[c*x]))/5 + (e^3*x^7*(a + b*Arc
Sinh[c*x]))/7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rule 5704

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0])
```

Rubi steps

$$\int \left(d+ex^2\right)^3 \left(a+b\sinh^{-1}(cx)\right)\,dx = d^3x\left(a+b\sinh^{-1}(cx)\right)+d^2ex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\sinh^{-1}(c\right.$$

$$= d^3x\left(a+b\sinh^{-1}(cx)\right)+d^2ex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\sinh^{-1}(c\right.$$

$$= d^3x\left(a+b\sinh^{-1}(cx)\right)+d^2ex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\sinh^{-1}(c\right.$$

$$= d^3x\left(a+b\sinh^{-1}(cx)\right)+d^2ex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\sinh^{-1}(c\right.$$

$$= -\frac{b\left(35c^6d^3-35c^4d^2e+21c^2de^2-5e^3\right)\sqrt{1+c^2x^2}}{35c^7}-\frac{be\left(35c^4d^2-42c^2de+\right.}{105c}$$

Mathematica [A] time = 0.28, size = 187, normalized size = 0.85

$$a\left(d^3x+d^2ex^3+\frac{3}{5}de^2x^5+\frac{e^3x^7}{7}\right)-\frac{b\sqrt{c^2x^2+1}\left(c^6\left(3675d^3+1225d^2ex^2+441de^2x^4+75e^3x^6\right)-2c^4e\left(1225d^2+2\right.\right.}{3675c^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] a*(d^3*x + d^2*e*x^3 + (3*d*e^2*x^5)/5 + (e^3*x^7)/7) - (b*Sqrt[1 + c^2*x^2
]*(-240*e^3 + 24*c^2*e^2*(49*d + 5*e*x^2) - 2*c^4*e*(1225*d^2 + 294*d*e*x^2
 + 45*e^2*x^4) + c^6*(3675*d^3 + 1225*d^2*e*x^2 + 441*d*e^2*x^4 + 75*e^3*x^
6)))/(3675*c^7) + b*(d^3*x + d^2*e*x^3 + (3*d*e^2*x^5)/5 + (e^3*x^7)/7)*Arc
Sinh[c*x]
```

fricas [A] time = 0.53, size = 241, normalized size = 1.09

$$\frac{525\,ac^7e^3x^7+2205\,ac^7de^2x^5+3675\,ac^7d^2ex^3+3675\,ac^7d^3x+105\left(5\,bc^7e^3x^7+21\,bc^7de^2x^5+35\,bc^7d^2ex^3+35\,b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/3675*(525*a*c^7*e^3*x^7 + 2205*a*c^7*d*e^2*x^5 + 3675*a*c^7*d^2*e*x^3 + 3
675*a*c^7*d^3*x + 105*(5*b*c^7*e^3*x^7 + 21*b*c^7*d*e^2*x^5 + 35*b*c^7*d^2*
e*x^3 + 35*b*c^7*d^3*x)*log(c*x + sqrt(c^2*x^2 + 1)) - (75*b*c^6*e^3*x^6 +
3675*b*c^6*d^3 - 2450*b*c^4*d^2*e + 1176*b*c^2*d*e^2 + 9*(49*b*c^6*d*e^2 -
10*b*c^4*e^3)*x^4 - 240*b*e^3 + (1225*b*c^6*d^2*e - 588*b*c^4*d*e^2 + 120*b
*c^2*e^3)*x^2)*sqrt(c^2*x^2 + 1))/c^7
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 316, normalized size = 1.43

$$\frac{\frac{a\left(\frac{1}{7}e^{3}c^{7}x^{7}+\frac{3}{5}c^{7}d\,e^{2}x^{5}+c^{7}d^{2}e\,x^{3}+x\,c^{7}d^{3}\right)}{c^{6}}+\frac{b\left(\frac{\operatorname{arcsinh}(cx)e^{3}c^{7}x^{7}}{7}+\frac{3\operatorname{arcsinh}(cx)c^{7}d\,e^{2}x^{5}}{5}+\operatorname{arcsinh}(cx)c^{7}d^{2}e\,x^{3}+\operatorname{arcsinh}(cx)c^{7}x\,d^{3}-\frac{e^{3}\left(\frac{c^{6}x^{6}\sqrt{c^{2}x^{2}+1}}{7}-\frac{6c^{4}\cdots}{\cdots}\right)}{\cdots}\right.}{\cdots}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c*(a/c^6*(1/7*e^3*c^7*x^7+3/5*c^7*d*e^2*x^5+c^7*d^2*e*x^3+x*c^7*d^3)+b/c^
6*(1/7*arcsinh(c*x)*e^3*c^7*x^7+3/5*arcsinh(c*x)*c^7*d*e^2*x^5+arcsinh(c*x)
*c^7*d^2*e*x^3+arcsinh(c*x)*c^7*x*d^3-1/7*e^3*(1/7*c^6*x^6*(c^2*x^2+1)^(1/2
)-6/35*c^4*x^4*(c^2*x^2+1)^(1/2)+8/35*c^2*x^2*(c^2*x^2+1)^(1/2)-16/35*(c^2*
x^2+1)^(1/2))-3/5*c^2*d*e^2*(1/5*c^4*x^4*(c^2*x^2+1)^(1/2)-4/15*c^2*x^2*(c^
2*x^2+1)^(1/2)+8/15*(c^2*x^2+1)^(1/2))-c^4*d^2*e*(1/3*c^2*x^2*(c^2*x^2+1)^(
1/2)-2/3*(c^2*x^2+1)^(1/2))-d^3*c^6*(c^2*x^2+1)^(1/2)))
```

maxima [A] time = 0.36, size = 287, normalized size = 1.30

$$\frac{1}{7}\,ae^{3}x^{7}+\frac{3}{5}\,ade^{2}x^{5}+ad^{2}ex^{3}+\frac{1}{3}\left(3\,x^{3}\,\operatorname{arsinh}\left(cx\right)-c\left(\frac{\sqrt{c^{2}x^{2}+1}\,x^{2}}{c^{2}}-\frac{2\,\sqrt{c^{2}x^{2}+1}}{c^{4}}\right)\right)bd^{2}e+\frac{1}{25}\left(15\,x^{5}\,\operatorname{arsinh}\left(cx\right)\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*e^3*x^7 + 3/5*a*d*e^2*x^5 + a*d^2*e*x^3 + 1/3*(3*x^3*arcsinh(c*x) - c
*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*d^2*e + 1/25*(15*
x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c
^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*b*d*e^2 + 1/245*(35*x^7*arcsinh(c*x) - (5*
sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 +
1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*b*e^3 + a*d^3*x + (c*x*arcsinh(c*
x) - sqrt(c^2*x^2 + 1))*b*d^3/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{asinh}\left(c\,x\right))\,{\left(e\,x^2+d\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + e*x^2)^3,x)
```

```
[Out] int((a + b*asinh(c*x))*(d + e*x^2)^3, x)
```

sympy [A] time = 6.65, size = 389, normalized size = 1.76

$$\begin{cases} ad^{3}x + ad^{2}ex^{3} + \frac{3ade^{2}x^{5}}{5} + \frac{ae^{3}x^{7}}{7} + bd^{3}x\operatorname{asinh}{\left(cx \right)} + bd^{2}ex^{3}\operatorname{asinh}{\left(cx \right)} + \frac{3bde^{2}x^{5}\operatorname{asinh}{\left(cx \right)}}{5} + \frac{be^{3}x^{7}\operatorname{asinh}{\left(cx \right)}}{7} - \frac{bd^{3}\sqrt{\cdots}}{\cdots} \cdots \\ a\left(d^{3}x + d^{2}ex^{3} + \frac{3de^{2}x^{5}}{5} + \frac{e^{3}x^{7}}{7}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*d**3*x + a*d**2*e*x**3 + 3*a*d*e**2*x**5/5 + a*e**3*x**7/7 + b
*d**3*x*asinh(c*x) + b*d**2*e*x**3*asinh(c*x) + 3*b*d*e**2*x**5*asinh(c*x)/
```

```
5 + b*e**3*x**7*asinh(c*x)/7 - b*d**3*sqrt(c**2*x**2 + 1)/c - b*d**2*e*x**2
*sqrt(c**2*x**2 + 1)/(3*c) - 3*b*d*e**2*x**4*sqrt(c**2*x**2 + 1)/(25*c) - b
*e**3*x**6*sqrt(c**2*x**2 + 1)/(49*c) + 2*b*d**2*e*sqrt(c**2*x**2 + 1)/(3*c
**3) + 4*b*d*e**2*x**2*sqrt(c**2*x**2 + 1)/(25*c**3) + 6*b*e**3*x**4*sqrt(c
**2*x**2 + 1)/(245*c**3) - 8*b*d*e**2*sqrt(c**2*x**2 + 1)/(25*c**5) - 8*b*e
**3*x**2*sqrt(c**2*x**2 + 1)/(245*c**5) + 16*b*e**3*sqrt(c**2*x**2 + 1)/(24
5*c**7), Ne(c, 0)), (a*(d**3*x + d**2*e*x**3 + 3*d*e**2*x**5/5 + e**3*x**7/
7), True))
```

3.608 $\int \left(d+ex^2\right)^2 \left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=147

$$d^2x\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\sinh^{-1}(cx)\right)-\frac{2be\left(c^2x^2+1\right)^{3/2}\left(5c^2d-3e\right)}{45c^5}-\frac{be^2}{}$$

```
[Out] -2/45*b*(5*c^2*d-3*e)*e*(c^2*x^2+1)^(3/2)/c^5-1/25*b*e^2*(c^2*x^2+1)^(5/2)/
c^5+d^2*x*(a+b*arcsinh(c*x))+2/3*d*e*x^3*(a+b*arcsinh(c*x))+1/5*e^2*x^5*(a+
b*arcsinh(c*x))-1/15*b*(15*c^4*d^2-10*c^2*d*e+3*e^2)*(c^2*x^2+1)^(1/2)/c^5
```

Rubi [A] time = 0.14, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {194, 5704, 12, 1247, 698}

$$d^2x\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\sinh^{-1}(cx)\right)-\frac{b\sqrt{c^2x^2+1}\left(15c^4d^2-10c^2de+3\right.}{15c^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(b*(15*c^4*d^2 - 10*c^2*d*e + 3*e^2)*Sqrt[1 + c^2*x^2])/(15*c^5) - (2*b*(5
*c^2*d - 3*e)*e*(1 + c^2*x^2)^(3/2))/(45*c^5) - (b*e^2*(1 + c^2*x^2)^(5/2))
/(25*c^5) + d^2*x*(a + b*ArcSinh[c*x]) + (2*d*e*x^3*(a + b*ArcSinh[c*x]))/3
 + (e^2*x^5*(a + b*ArcSinh[c*x]))/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 5704

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0])
```

Rubi steps

$$\int \left(d+ex^2\right)^2 \left(a+b\sinh^{-1}(cx)\right) dx = d^2x\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\sinh^{-1}(c\right.$$

$$= d^2x\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\sinh^{-1}(c\right.$$

$$= d^2x\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\sinh^{-1}(c\right.$$

$$= d^2x\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\sinh^{-1}(c\right.$$

$$= -\frac{b\left(15c^4d^2-10c^2de+3e^2\right)\sqrt{1+c^2x^2}}{15c^5}-\frac{2b\left(5c^2d-3e\right)e\left(1+c^2x^2\right)^{3/2}}{45c^5}-\frac{be}{}$$

Mathematica [A] time = 0.17, size = 125, normalized size = 0.85

$$\frac{1}{225}\left(15ax\left(15d^2+10dex^2+3e^2x^4\right)-\frac{b\sqrt{c^2x^2+1}\left(c^4\left(225d^2+50dex^2+9e^2x^4\right)-4c^2e\left(25d+3ex^2\right)+24e^2\right)}{c^5}+15\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (15*a*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) - (b*Sqrt[1 + c^2*x^2]*(24*e^2 -
4*c^2*e*(25*d + 3*e*x^2) + c^4*(225*d^2 + 50*d*e*x^2 + 9*e^2*x^4)))/c^5 + 1
5*b*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4)*ArcSinh[c*x])/225
```

fricas [A] time = 0.63, size = 163, normalized size = 1.11

$$\frac{45\,ac^5e^2x^5+150\,ac^5dex^3+225\,ac^5d^2x+15\left(3\,bc^5e^2x^5+10\,bc^5dex^3+15\,bc^5d^2x\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-\left(9\,bc^4e^2\right.}{225\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/225*(45*a*c^5*e^2*x^5 + 150*a*c^5*d*e*x^3 + 225*a*c^5*d^2*x + 15*(3*b*c^5
*e^2*x^5 + 10*b*c^5*d*e*x^3 + 15*b*c^5*d^2*x)*log(c*x + sqrt(c^2*x^2 + 1))
- (9*b*c^4*e^2*x^4 + 225*b*c^4*d^2 - 100*b*c^2*d*e + 24*b*e^2 + 2*(25*b*c^4
*d*e - 6*b*c^2*e^2)*x^2)*sqrt(c^2*x^2 + 1))/c^5
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.00, size = 204, normalized size = 1.39

$$\frac{\frac{a\left(\frac{1}{5}e^2c^5x^5+\frac{2}{3}c^5de\,x^3+x\,c^5d^2\right)}{c^4}+\frac{b\left(\frac{\operatorname{arcsinh}(cx)e^2c^5x^5}{5}+\frac{2\operatorname{arcsinh}(cx)c^5de\,x^3}{3}+\operatorname{arcsinh}(cx)c^5x\,d^2-\frac{e^2\left(\frac{c^4x^4\sqrt{c^2x^2+1}}{5}-\frac{4c^2x^2\sqrt{c^2x^2+1}}{15}+\frac{8\sqrt{c^2x^2+1}}{15}\right)}{5}-\frac{2c^2de\left(\frac{c^2x^2\sqrt{c}}{3}\right.}{}\right.}{c^4}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c*(a/c^4*(1/5*e^2*c^5*x^5+2/3*c^5*d*e*x^3+x*c^5*d^2)+b/c^4*(1/5*arcsinh(c
*x)*e^2*c^5*x^5+2/3*arcsinh(c*x)*c^5*d*e*x^3+arcsinh(c*x)*c^5*x*d^2-1/5*e^2
*(1/5*c^4*x^4*(c^2*x^2+1)^(1/2)-4/15*c^2*x^2*(c^2*x^2+1)^(1/2)+8/15*(c^2*x^
2+1)^(1/2))-2/3*c^2*d*e*(1/3*c^2*x^2*(c^2*x^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/2
))-d^2*c^4*(c^2*x^2+1)^(1/2)))
```

maxima [A] time = 0.45, size = 180, normalized size = 1.22

$$\frac{1}{5}\,ae^2x^5+\frac{2}{3}\,adex^3+\frac{2}{9}\left(3\,x^3\,\mathrm{arsinh}\,(cx)-c\left(\frac{\sqrt{c^2x^2+1}\,x^2}{c^2}-\frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)bde+\frac{1}{75}\left(15\,x^5\,\mathrm{arsinh}\,(cx)-\left(\frac{3\,\sqrt{c^2x}}{c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*e^2*x^5 + 2/3*a*d*e*x^3 + 2/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 +
 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*d*e + 1/75*(15*x^5*arcsinh(c*x) -
 (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^
2 + 1)/c^6)*c)*b*e^2 + a*d^2*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*b*d
^2/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\mathrm{asinh}\,(c\,x))\,\left(e\,x^2+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + e*x^2)^2,x)
```

```
[Out] int((a + b*asinh(c*x))*(d + e*x^2)^2, x)
```

sympy [A] time = 2.32, size = 240, normalized size = 1.63

$$\begin{cases} ad^2x+\frac{2adex^3}{3}+\frac{ae^2x^5}{5}+bd^2x\,\mathrm{asinh}\,(cx)+\frac{2bdex^3\,\mathrm{asinh}\,(cx)}{3}+\frac{be^2x^5\,\mathrm{asinh}\,(cx)}{5}-\frac{bd^2\sqrt{c^2x^2+1}}{c}-\frac{2bdex^2\sqrt{c^2x^2+1}}{9c}-\frac{be^2x^4\sqrt{c^2}}{25c} \\ a\left(d^2x+\frac{2dex^3}{3}+\frac{e^2x^5}{5}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*d**2*x + 2*a*d*e*x**3/3 + a*e**2*x**5/5 + b*d**2*x*asinh(c*x)
+ 2*b*d*e*x**3*asinh(c*x)/3 + b*e**2*x**5*asinh(c*x)/5 - b*d**2*sqrt(c**2*x
**2 + 1)/c - 2*b*d*e*x**2*sqrt(c**2*x**2 + 1)/(9*c) - b*e**2*x**4*sqrt(c**2
*x**2 + 1)/(25*c) + 4*b*d*e*sqrt(c**2*x**2 + 1)/(9*c**3) + 4*b*e**2*x**2*sq
rt(c**2*x**2 + 1)/(75*c**3) - 8*b*e**2*sqrt(c**2*x**2 + 1)/(75*c**5), Ne(c,
 0)), (a*(d**2*x + 2*d*e*x**3/3 + e**2*x**5/5), True))
```

3.609 $\int (d+ex^2)\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=81

$$dx\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\sinh^{-1}(cx)\right)-\frac{b\sqrt{c^2x^2+1}\,\left(3c^2d-e\right)}{3c^3}-\frac{be\left(c^2x^2+1\right)^{3/2}}{9c^3}$$

```
[Out] -1/9*b*e*(c^2*x^2+1)^(3/2)/c^3+d*x*(a+b*arcsinh(c*x))+1/3*e*x^3*(a+b*arcsin
h(c*x))-1/3*b*(3*c^2*d-e)*(c^2*x^2+1)^(1/2)/c^3
```

Rubi [A] time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5704, 444, 43}

$$dx\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\sinh^{-1}(cx)\right)-\frac{b\sqrt{c^2x^2+1}\,\left(3c^2d-e\right)}{3c^3}-\frac{be\left(c^2x^2+1\right)^{3/2}}{9c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(b*(3*c^2*d - e)*Sqrt[1 + c^2*x^2])/(3*c^3) - (b*e*(1 + c^2*x^2)^(3/2))/(9
*c^3) + d*x*(a + b*ArcSinh[c*x]) + (e*x^3*(a + b*ArcSinh[c*x]))/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 5704

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0])
```

Rubi steps

$$\int \left(d+ex^2\right)\left(a+b\sinh^{-1}(cx)\right)\,dx=dx\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\sinh^{-1}(cx)\right)-(bc)\int\frac{x\left(d+\frac{ex^2}{3}\right)}{\sqrt{1+c^2x^2}}\,dx$$

$$=dx\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{2}(bc)\,\text{Subst}\left(\int\frac{d+\frac{ex}{3}}{\sqrt{1+c^2x}}\right.$$

$$=dx\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{2}(bc)\,\text{Subst}\left(\int\left(\frac{3c^2d}{3c^2\sqrt{1+}}\right.\right.$$

$$=-\frac{b\left(3c^2d-e\right)\sqrt{1+c^2x^2}}{3c^3}-\frac{be\left(1+c^2x^2\right)^{3/2}}{9c^3}+dx\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}ex^3\,($$

Mathematica [A] time = 0.07, size = 71, normalized size = 0.88

$$\frac{1}{9}\left(3ax\left(3d+ex^2\right)-\frac{b\sqrt{c^2x^2+1}\left(c^2\left(9d+ex^2\right)-2e\right)}{c^3}+3bx\sinh^{-1}(cx)\left(3d+ex^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (3*a*x*(3*d + e*x^2) - (b*Sqrt[1 + c^2*x^2]*(-2*e + c^2*(9*d + e*x^2)))/c^3
 + 3*b*x*(3*d + e*x^2)*ArcSinh[c*x])/9
```

fricas [A] time = 0.55, size = 94, normalized size = 1.16

$$\frac{3\,ac^3ex^3+9\,ac^3dx+3\left(bc^3ex^3+3\,bc^3dx\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-\left(bc^2ex^2+9\,bc^2d-2\,be\right)\sqrt{c^2x^2+1}}{9\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/9*(3*a*c^3*e*x^3 + 9*a*c^3*d*x + 3*(b*c^3*e*x^3 + 3*b*c^3*d*x)*log(c*x +
sqrt(c^2*x^2 + 1)) - (b*c^2*e*x^2 + 9*b*c^2*d - 2*b*e)*sqrt(c^2*x^2 + 1))/c
^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.00, size = 109, normalized size = 1.35

$$\frac{\frac{a\left(\frac{1}{3}c^3x^3e+c^3dx\right)}{c^2}+\frac{b\left(\frac{\operatorname{arcsinh}(cx)c^3x^3e}{3}+\operatorname{arcsinh}(cx)c^3dx-\frac{e\left(\frac{c^2x^2\sqrt{c^2x^2+1}}{3}-\frac{2\sqrt{c^2x^2+1}}{3}\right)}{3}-c^2d\sqrt{c^2x^2+1}\right)}{c^2}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c*(a/c^2*(1/3*c^3*x^3*e+c^3*d*x)+b/c^2*(1/3*arcsinh(c*x)*c^3*x^3*e+arcsin
h(c*x)*c^3*d*x-1/3*e*(1/3*c^2*x^2*(c^2*x^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/2))-
c^2*d*(c^2*x^2+1)^(1/2)))
```

maxima [A] time = 0.33, size = 91, normalized size = 1.12

$$\frac{1}{3}\,aex^3+\frac{1}{9}\left(3\,x^3\operatorname{arsinh}\left(cx\right)-c\left(\frac{\sqrt{c^2x^2+1}\,x^2}{c^2}-\frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)be+adx+\frac{\left(cx\operatorname{arsinh}\left(cx\right)-\sqrt{c^2x^2+1}\right)bd}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*e*x^3 + 1/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sq
rt(c^2*x^2 + 1)/c^4))*b*e + a*d*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*
b*d/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,\mathrm{asinh}\,(c\,x))\,\left(e\,x^2 + d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + e*x^2),x)
```

```
[Out] int((a + b*asinh(c*x))*(d + e*x^2), x)
```

sympy [A] time = 0.56, size = 109, normalized size = 1.35

$$\begin{cases} adx + \frac{aex^3}{3} + bdx\,\mathrm{asinh}\,(cx) + \frac{bex^3\,\mathrm{asinh}\,(cx)}{3} - \frac{bd\sqrt{c^2x^2+1}}{c} - \frac{bex^2\sqrt{c^2x^2+1}}{9c} + \frac{2be\sqrt{c^2x^2+1}}{9c^3} & \text{for } c \neq 0 \\ a\left(dx + \frac{ex^3}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*d*x + a*e*x**3/3 + b*d*x*asinh(c*x) + b*e*x**3*asinh(c*x)/3 -
b*d*sqrt(c**2*x**2 + 1)/c - b*e*x**2*sqrt(c**2*x**2 + 1)/(9*c) + 2*b*e*sqrt
(c**2*x**2 + 1)/(9*c**3), Ne(c, 0)), (a*(d*x + e*x**3/3), True))
```

3.610 $\int \left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=30

$$ax-\frac{b\sqrt{c^2x^2+1}}{c}+bx\sinh^{-1}(cx)$$

```
[Out] a*x+b*x*arcsinh(c*x)-b*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5653, 261}

$$ax-\frac{b\sqrt{c^2x^2+1}}{c}+bx\sinh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcSinh[c*x],x]
```

```
[Out] a*x - (b*Sqrt[1 + c^2*x^2])/c + b*x*ArcSinh[c*x]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sinh^{-1}(cx)\right)\,dx&=ax+b\int \sinh^{-1}(cx)\,dx\\&=ax+bx\sinh^{-1}(cx)-(bc)\int \frac{x}{\sqrt{1+c^2x^2}}\,dx\\&=ax-\frac{b\sqrt{1+c^2x^2}}{c}+bx\sinh^{-1}(cx)\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$ax-\frac{b\sqrt{c^2x^2+1}}{c}+bx\sinh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcSinh[c*x],x]
```

```
[Out] a*x - (b*Sqrt[1 + c^2*x^2])/c + b*x*ArcSinh[c*x]
```

fricas [A] time = 0.50, size = 43, normalized size = 1.43

$$\frac{bcx\log\left(cx+\sqrt{c^2x^2+1}\right)+acx-\sqrt{c^2x^2+1}\,b}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(c*x),x, algorithm="fricas")
```

```
[Out] (b*c*x*log(c*x + sqrt(c^2*x^2 + 1)) + a*c*x - sqrt(c^2*x^2 + 1)*b)/c
```

giac [A] time = 0.25, size = 41, normalized size = 1.37

$$\left(x\log\left(cx+\sqrt{c^2x^2+1}\right)-\frac{\sqrt{c^2x^2+1}}{c}\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(c*x),x, algorithm="giac")
```

```
[Out] (x*log(c*x + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*b + a*x
```

maple [A] time = 0.01, size = 31, normalized size = 1.03

$$ax+\frac{b\left(\operatorname{arcsinh}(cx)\,cx-\sqrt{c^2x^2+1}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arcsinh(c*x),x)
```

```
[Out] a*x+b/c*(arcsinh(c*x)*c*x-(c^2*x^2+1)^(1/2))
```

maxima [A] time = 0.32, size = 30, normalized size = 1.00

$$ax+\frac{\left(cx\operatorname{arsinh}(cx)-\sqrt{c^2x^2+1}\right)b}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(c*x),x, algorithm="maxima")
```

```
[Out] a*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*b/c
```

mupad [B] time = 0.24, size = 28, normalized size = 0.93

$$a\,x-\frac{b\,\sqrt{c^2\,x^2+1}}{c}+b\,x\,\operatorname{asinh}(c\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*asinh(c*x),x)
```

```
[Out] a*x - (b*(c^2*x^2 + 1)^(1/2))/c + b*x*asinh(c*x)
```

sympy [A] time = 0.14, size = 26, normalized size = 0.87

$$ax+b\left(\begin{cases} x\operatorname{asinh}(cx)-\frac{\sqrt{c^2x^2+1}}{c} & \text{for}\ c\neq 0\\ 0 & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*asinh(c*x),x)
```

```
[Out] a*x + b*Piecewise((x*asinh(c*x) - sqrt(c**2*x**2 + 1)/c, Ne(c, 0)), (0, Tru
e))
```

3.611 $\int \frac{a+b\sinh^{-1}(cx)}{d+ex^2}\,dx$

Optimal. Leaf size=485

$$\frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}+1\right)}{2\sqrt{-d}\,\sqrt{e}}+\frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(\cdots\right.}{2\sqrt{-d}\,\sqrt{\cdots}}$$

```
[Out] 1/2*(a+b*arcsinh(c*x))*ln(1-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(
-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*
x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+1
/2*(a+b*arcsinh(c*x))*ln(1-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-
c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x
^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/
2*b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d+e)^(1/
2)))/(-d)^(1/2)/e^(1/2)+1/2*b*polylog(2,(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*
(-d)^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*b*polylog(2,-(c*x+(c^2
*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+
1/2*b*polylog(2,(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d+e)^(1
/2)))/(-d)^(1/2)/e^(1/2)
```

Rubi [A] time = 0.83, antiderivative size = 485, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5706, 5799, 5561, 2190, 2279, 2391}

$$-\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\,\sqrt{e}}+\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{\sqrt{e-c^2d}+c\sqrt{-d}}\right)}{2\sqrt{-d}\,\sqrt{e}}+\frac{b\text{PolyLog}\left(2,\frac{\sqrt{\cdots}}{\sqrt{e}\cdots}\right.}{2\sqrt{-d}\,\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(d + e*x^2),x]
```

```
[Out] ((a + b*ArcSinh[c*x])*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-
(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcSinh[c*x])*Log[1 + (Sqrt[
e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e])
 + ((a + b*ArcSinh[c*x])*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqr
t[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcSinh[c*x])*Log[1 + (Sq
rt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[
e]) - (b*PolyLog[2, -((Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d)
+ e]))])/(2*Sqrt[-d]*Sqrt[e]) + (b*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*S
qrt[-d] - Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - (b*PolyLog[2, -((Sqr
t[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e]))])/(2*Sqrt[-d]*Sqrt[
e]) + (b*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) +
e])])/(2*Sqrt[-d]*Sqrt[e])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{d+ex^2}\,dx = \int \left(\frac{\sqrt{-d}\,\left(a+b\sinh^{-1}(cx)\right)}{2d\left(\sqrt{-d}-\sqrt{e}\,x\right)}+\frac{\sqrt{-d}\,\left(a+b\sinh^{-1}(cx)\right)}{2d\left(\sqrt{-d}+\sqrt{e}\,x\right)}\right)dx$$

$$= -\frac{\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{-d}-\sqrt{e}\,x}\,dx}{2\sqrt{-d}}-\frac{\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{-d}+\sqrt{e}\,x}\,dx}{2\sqrt{-d}}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+bx)\cosh(x)}{c\sqrt{-d}-\sqrt{e}\,\sinh(x)}\,dx,x,\sinh^{-1}(cx)\right)}{2\sqrt{-d}}-\frac{\text{Subst}\left(\int \frac{(a+bx)\cosh(x)}{c\sqrt{-d}+\sqrt{e}\,\sinh(x)}\,dx,x,\sinh^{-1}(cx\right.}{2\sqrt{-d}}$$

$$= -\frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{c\sqrt{-d}-\sqrt{-c^2d+e}-\sqrt{e}\,e^x}\,dx,x,\sinh^{-1}(cx)\right)}{2\sqrt{-d}}-\frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{c\sqrt{-d}+\sqrt{-c^2d+e}-\sqrt{e}\,e^x}\,dx,x,s\right.}{2\sqrt{-d}}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}}$$

Mathematica [A] time = 0.44, size = 434, normalized size = 0.89

$$2a\sqrt{-d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)+b\sqrt{d}\,\text{Li}_2\left(\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)-b\sqrt{d}\,\text{Li}_2\left(\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{\sqrt{e-c^2d}-c\sqrt{-d}}\right)-b\sqrt{d}\,\text{Li}_2\left(-\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{\sqrt{-d}c+\sqrt{e-c^2d}}\right)+b\sqrt{d}\,\text{Li}_2$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + e*x^2),x]
```

```
[Out] (2*a*Sqrt[-d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - b*Sqrt[d]*ArcSinh[c*x]*Log[1 +
(Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])] + b*Sqrt[d]*Arc
Sinh[c*x]*Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) +
 e])] + b*Sqrt[d]*ArcSinh[c*x]*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d]
 + Sqrt[-(c^2*d) + e])] - b*Sqrt[d]*ArcSinh[c*x]*Log[1 + (Sqrt[e]*E^ArcSinh
[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])] + b*Sqrt[d]*PolyLog[2, (Sqrt[e]*E
^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])] - b*Sqrt[d]*PolyLog[2, (S
qrt[e]*E^ArcSinh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) + e])] - b*Sqrt[d]*Po
lyLog[2, -((Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e]))] + b
*Sqrt[d]*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) +
e])])/(2*Sqrt[-d^2]*Sqrt[e])
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arsinh}\,(cx)+a}{ex^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(e*x^2 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{b\,\text{arsinh}\,(cx)+a}{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(e*x^2 + d), x)
```

maple [C] time = 0.67, size = 224, normalized size = 0.46

$$\frac{a\arctan\left(\frac{xe}{\sqrt{de}}\right)}{\sqrt{de}}+\frac{cb\left(\sum_{\_R1=\text{RootOf}\left(e\_Z^4+\left(4c^2d-2e\right)\_Z^2+e\right)}\frac{\text{arcsinh}(cx)\ln\left(\frac{\_R1-cx-\sqrt{c^2x^2+1}}{\_R1}\right)+\text{dilog}\left(\frac{\_R1-cx-\sqrt{c^2x^2+1}}{\_R1}\right)}{\_R1\left(\_R1^2e+2c^2d-e\right)}\right)}{2}+\frac{cb\left(\sum_{\_R1=\text{Root}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(e*x^2+d),x)
```

```
[Out] a/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))+1/2*c*b*sum(1/_R1/(_R1^2*e+2*c^2*d-e)
*(arcsinh(c*x)*ln((_R1-c*x-(c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-c*x-(c^2*x^2+
1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d-2*e)*_Z^2+e))+1/2*c*b*sum(_R1/(_
R1^2*e+2*c^2*d-e)*(arcsinh(c*x)*ln((_R1-c*x-(c^2*x^2+1)^(1/2))/_R1)+dilog((
_R1-c*x-(c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d-2*e)*_Z^2+e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\int \frac{\log\left(cx + \sqrt{c^2x^2 + 1}\right)}{ex^2 + d}\,dx + \frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(e*x^2 + d), x) + a*arctan(e*x/sqr
t(d*e))/sqrt(d*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\,\mathrm{asinh}\,(c\,x)}{e\,x^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(d + e*x^2),x)
```

```
[Out] int((a + b*asinh(c*x))/(d + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{asinh}\,(cx)}{d + ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(e*x**2+d),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(d + e*x**2), x)
```

3.612 $\int \frac{a+b\sinh^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=707

$$-\frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}+1\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{\left(a+b\sinh^{-1}(cx)\right)\log}{4(-d)^{3/2}}$$

```
[Out] -1/4*(a+b*arcsinh(c*x))*ln(1-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-
(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2
*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-
1/4*(a+b*arcsinh(c*x))*ln(1-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(
-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*
x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1
/4*b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d+e)^(1
/2)))/(-d)^(3/2)/e^(1/2)-1/4*b*polylog(2,(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c
*(-d)^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*b*polylog(2,-(c*x+(c^
2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)
-1/4*b*polylog(2,(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d+e)^(
1/2)))/(-d)^(3/2)/e^(1/2)-1/4*b*c*arctan((-c^2*x*(-d)^(1/2)+e^(1/2))/(c^2*d
-e)^(1/2)/(c^2*x^2+1)^(1/2))/d/(c^2*d-e)^(1/2)/e^(1/2)-1/4*b*c*arctan((c^2*
x*(-d)^(1/2)+e^(1/2))/(c^2*d-e)^(1/2)/(c^2*x^2+1)^(1/2))/d/(c^2*d-e)^(1/2)/
e^(1/2)+1/4*(-a-b*arcsinh(c*x))/d/e^(1/2)/((-d)^(1/2)-x*e^(1/2))+1/4*(a+b*a
rcsinh(c*x))/d/e^(1/2)/((-d)^(1/2)+x*e^(1/2))
```

Rubi [A] time = 1.07, antiderivative size = 707, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5706, 5801, 725, 204, 5799, 5561, 2190, 2279, 2391}

$$\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{\sqrt{e-c^2d}+c\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e}{\sqrt{e-c}}\right)}{4(-d)^{3/2}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(d + e*x^2)^2,x]
```

```
[Out] -(a + b*ArcSinh[c*x])/(4*d*Sqrt[e]*(Sqrt[-d] - Sqrt[e]*x)) + (a + b*ArcSinh
[c*x])/(4*d*Sqrt[e]*(Sqrt[-d] + Sqrt[e]*x)) - (b*c*ArcTan[(Sqrt[e] - c^2*Sq
rt[-d]*x)/(Sqrt[c^2*d - e]*Sqrt[1 + c^2*x^2])])/(4*d*Sqrt[c^2*d - e]*Sqrt[e
]) - (b*c*ArcTan[(Sqrt[e] + c^2*Sqrt[-d]*x)/(Sqrt[c^2*d - e]*Sqrt[1 + c^2*x
^2])])/(4*d*Sqrt[c^2*d - e]*Sqrt[e]) - ((a + b*ArcSinh[c*x])*Log[1 - (Sqrt[
e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])])/(4*(-d)^(3/2)*Sqrt[e
]) + ((a + b*ArcSinh[c*x])*Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - S
qrt[-(c^2*d) + e])])/(4*(-d)^(3/2)*Sqrt[e]) - ((a + b*ArcSinh[c*x])*Log[1 -
 (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])])/(4*(-d)^(3/2)
*Sqrt[e]) + ((a + b*ArcSinh[c*x])*Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[
-d] + Sqrt[-(c^2*d) + e])])/(4*(-d)^(3/2)*Sqrt[e]) + (b*PolyLog[2, -((Sqrt[
e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e]))])/(4*(-d)^(3/2)*Sqrt[
e]) - (b*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) +
e])])/(4*(-d)^(3/2)*Sqrt[e]) + (b*PolyLog[2, -((Sqrt[e]*E^ArcSinh[c*x])/(c*
Sqrt[-d] + Sqrt[-(c^2*d) + e]))])/(4*(-d)^(3/2)*Sqrt[e]) - (b*PolyLog[2, (S
qrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])])/(4*(-d)^(3/2)*Sq
rt[e])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{\left(d+ex^2\right)^2}dx = \int \left(-\frac{e\left(a+b\sinh^{-1}(cx)\right)}{4d\left(\sqrt{-d}\sqrt{e}-ex\right)^2} - \frac{e\left(a+b\sinh^{-1}(cx)\right)}{4d\left(\sqrt{-d}\sqrt{e}+ex\right)^2} - \frac{e\left(a+b\sinh^{-1}(cx)\right)}{2d\left(-de-e^2x^2\right)}\right)dx$$

$$= -\frac{e\int \frac{a+b\sinh^{-1}(cx)}{\left(\sqrt{-d}\sqrt{e}-ex\right)^2}dx}{4d} - \frac{e\int \frac{a+b\sinh^{-1}(cx)}{\left(\sqrt{-d}\sqrt{e}+ex\right)^2}dx}{4d} - \frac{e\int \frac{a+b\sinh^{-1}(cx)}{-de-e^2x^2}dx}{2d}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)} + \frac{(bc)\int \frac{1}{\left(\sqrt{-d}\sqrt{e}-ex\right)\sqrt{1+c^2x^2}}dx}{4d} - \frac{(bc)}{}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)} + \frac{\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{-d}-\sqrt{e}x}dx}{4(-d)^{3/2}} + \frac{\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{-d}+\sqrt{e}x}d}{4(-d)^{3/2}}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)} - \frac{bc\tan^{-1}\left(\frac{\sqrt{e}-c^2\sqrt{-d}x}{\sqrt{c^2d-e}\sqrt{1+c^2x^2}}\right)}{4d\sqrt{c^2d-e}\sqrt{e}} - \frac{bc\tan^{-}}{4d}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)} - \frac{bc\tan^{-1}\left(\frac{\sqrt{e}-c^2\sqrt{-d}x}{\sqrt{c^2d-e}\sqrt{1+c^2x^2}}\right)}{4d\sqrt{c^2d-e}\sqrt{e}} - \frac{bc\tan^{-}}{4d}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)} - \frac{bc\tan^{-1}\left(\frac{\sqrt{e}-c^2\sqrt{-d}x}{\sqrt{c^2d-e}\sqrt{1+c^2x^2}}\right)}{4d\sqrt{c^2d-e}\sqrt{e}} - \frac{bc\tan^{-}}{4d}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)} - \frac{bc\tan^{-1}\left(\frac{\sqrt{e}-c^2\sqrt{-d}x}{\sqrt{c^2d-e}\sqrt{1+c^2x^2}}\right)}{4d\sqrt{c^2d-e}\sqrt{e}} - \frac{bc\tan^{-}}{4d}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)} - \frac{bc\tan^{-1}\left(\frac{\sqrt{e}-c^2\sqrt{-d}x}{\sqrt{c^2d-e}\sqrt{1+c^2x^2}}\right)}{4d\sqrt{c^2d-e}\sqrt{e}} - \frac{bc\tan^{-}}{4d}$$

Mathematica [C] time = 1.72, size = 622, normalized size = 0.88

$$\frac{1}{2}\left(\frac{a\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\sqrt{e}} + \frac{ax}{d^2+dex^2} + \frac{b\left(i\left(2\text{Li}_2\left(\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{\sqrt{e-c^2d}-ic\sqrt{d}}\right)+2\text{Li}_2\left(-\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{i\sqrt{d}c+\sqrt{e-c^2d}}\right)+\sinh^{-1}(cx)\left(-\sinh^{-1}(cx)+\right.\right.\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + e*x^2)^2,x]
```

```
[Out] ((a*x)/(d^2 + d*e*x^2) + (a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*Sqrt[e])
+ (b*(-2*Sqrt[d]*(-(ArcSinh[c*x]/(I*Sqrt[d] + Sqrt[e]*x)) + (c*ArcTan[(Sqrt
[e] - I*c^2*Sqrt[d]*x)/(Sqrt[c^2*d - e]*Sqrt[1 + c^2*x^2])])/Sqrt[c^2*d - e
]) + (2*I)*Sqrt[d]*(ArcSinh[c*x]/(Sqrt[d] + I*Sqrt[e]*x) + (c*ArcTanh[(I*Sq
rt[e] - c^2*Sqrt[d]*x)/(Sqrt[c^2*d - e]*Sqrt[1 + c^2*x^2])])/Sqrt[c^2*d - e
]) + I*(ArcSinh[c*x]*(-ArcSinh[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/(
I*c*Sqrt[d] - Sqrt[-(c^2*d) + e])] + Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/(I*c*
Sqrt[d] + Sqrt[-(c^2*d) + e])])) + 2*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/((
```

```
-I)*c*Sqrt[d] + Sqrt[-(c^2*d) + e])] + 2*PolyLog[2, -((Sqrt[e]*E^ArcSinh[c*
x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e]))]) - I*(ArcSinh[c*x]*(-ArcSinh[c*x]
+ 2*(Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) + e])
] + Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e])]))
+ 2*PolyLog[2, -((Sqrt[e]*E^ArcSinh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) +
 e]))] + 2*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d)
 + e])])))/(4*d^(3/2)*Sqrt[e]))/2
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\operatorname{arsinh}(cx)+a}{e^2x^4+2\,dex^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\operatorname{arsinh}(cx)+a}{\left(ex^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(e*x^2 + d)^2, x)
```

maple [C] time = 1.16, size = 1745, normalized size = 2.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(e*x^2+d)^2,x)
```

```
[Out] 1/2*c^2*a*x/d/(c^2*e*x^2+c^2*d)+1/2*a/d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))
+1/2*c^2*b*arcsinh(c*x)*x/d/(c^2*e*x^2+c^2*d)+1/4*c*b/d*sum(1/_R1/(_R1^2*e+
2*c^2*d-e)*(arcsinh(c*x)*ln((_R1-c*x-(c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-c*x
-(c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d-2*e)*_Z^2+e))+1/4*c*b/
d*sum(_R1/(_R1^2*e+2*c^2*d-e)*(arcsinh(c*x)*ln((_R1-c*x-(c^2*x^2+1)^(1/2))/
_R1)+dilog((_R1-c*x-(c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d-2*e
)*_Z^2+e))+c^5*b*(-(2*c^2*d-2*(d*c^2*(c^2*d-e))^(1/2)-e)*e)^(1/2)*arctanh((
c*x+(c^2*x^2+1)^(1/2))*e/((-2*c^2*d+2*(d*c^2*(c^2*d-e))^(1/2)+e)*e)^(1/2))*
d/(c^2*d-e)/e^3+c^3*b*(-(2*c^2*d-2*(d*c^2*(c^2*d-e))^(1/2)-e)*e)^(1/2)*arct
anh((c*x+(c^2*x^2+1)^(1/2))*e/((-2*c^2*d+2*(d*c^2*(c^2*d-e))^(1/2)+e)*e)^(1
/2))/(c^2*d-e)/e^3*(d*c^2*(c^2*d-e))^(1/2)-c^3*b*(-(2*c^2*d-2*(d*c^2*(c^2*d
-e))^(1/2)-e)*e)^(1/2)*arctanh((c*x+(c^2*x^2+1)^(1/2))*e/((-2*c^2*d+2*(d*c^
2*(c^2*d-e))^(1/2)+e)*e)^(1/2))/(c^2*d-e)/e^2-1/2*c*b*(-(2*c^2*d-2*(d*c^2*(
c^2*d-e))^(1/2)-e)*e)^(1/2)*arctanh((c*x+(c^2*x^2+1)^(1/2))*e/((-2*c^2*d+2*
(d*c^2*(c^2*d-e))^(1/2)+e)*e)^(1/2))/d/(c^2*d-e)/e^2*(d*c^2*(c^2*d-e))^(1/2
)-c^3*b*(-(2*c^2*d-2*(d*c^2*(c^2*d-e))^(1/2)-e)*e)^(1/2)*arctanh((c*x+(c^2*
x^2+1)^(1/2))*e/((-2*c^2*d+2*(d*c^2*(c^2*d-e))^(1/2)+e)*e)^(1/2))/e^3-c*b*(
-(2*c^2*d-2*(d*c^2*(c^2*d-e))^(1/2)-e)*e)^(1/2)*arctanh((c*x+(c^2*x^2+1)^(1
/2))*e/((-2*c^2*d+2*(d*c^2*(c^2*d-e))^(1/2)+e)*e)^(1/2))/d/e^3*(d*c^2*(c^2*
d-e))^(1/2)+1/2*c*b*(-(2*c^2*d-2*(d*c^2*(c^2*d-e))^(1/2)-e)*e)^(1/2)*arctan
h((c*x+(c^2*x^2+1)^(1/2))*e/((-2*c^2*d+2*(d*c^2*(c^2*d-e))^(1/2)+e)*e)^(1/2
))/d/e^2+c^5*b*((2*c^2*d+2*(d*c^2*(c^2*d-e))^(1/2)-e)*e)^(1/2)*arctan((c*x+
(c^2*x^2+1)^(1/2))*e/((2*c^2*d+2*(d*c^2*(c^2*d-e))^(1/2)-e)*e)^(1/2))*d/(c^
```

```
2*d-e)/e^3-c^3*b*((2*c^2*d+2*(d*c^2*(c^2*d-e))^(1/2)-e)*e)^(1/2)*arctan((c*
x+(c^2*x^2+1)^(1/2))*e/((2*c^2*d+2*(d*c^2*(c^2*d-e))^(1/2)-e)*e)^(1/2))/(c^
2*d-e)/e^3*(d*c^2*(c^2*d-e))^(1/2)-c^3*b*((2*c^2*d+2*(d*c^2*(c^2*d-e))^(1/2
)-e)*e)^(1/2)*arctan((c*x+(c^2*x^2+1)^(1/2))*e/((2*c^2*d+2*(d*c^2*(c^2*d-e)
)^(1/2)-e)*e)^(1/2))/(c^2*d-e)/e^2+1/2*c*b*((2*c^2*d+2*(d*c^2*(c^2*d-e))^(1
/2)-e)*e)^(1/2)*arctan((c*x+(c^2*x^2+1)^(1/2))*e/((2*c^2*d+2*(d*c^2*(c^2*d-
e))^(1/2)-e)*e)^(1/2))/d/(c^2*d-e)/e^2*(d*c^2*(c^2*d-e))^(1/2)-c^3*b*((2*c^
2*d+2*(d*c^2*(c^2*d-e))^(1/2)-e)*e)^(1/2)*arctan((c*x+(c^2*x^2+1)^(1/2))*e/
((2*c^2*d+2*(d*c^2*(c^2*d-e))^(1/2)-e)*e)^(1/2))/e^3+c*b*((2*c^2*d+2*(d*c^2
*(c^2*d-e))^(1/2)-e)*e)^(1/2)*arctan((c*x+(c^2*x^2+1)^(1/2))*e/((2*c^2*d+2*
(d*c^2*(c^2*d-e))^(1/2)-e)*e)^(1/2))/d/e^3*(d*c^2*(c^2*d-e))^(1/2)+1/2*c*b*
((2*c^2*d+2*(d*c^2*(c^2*d-e))^(1/2)-e)*e)^(1/2)*arctan((c*x+(c^2*x^2+1)^(1/
2))*e/((2*c^2*d+2*(d*c^2*(c^2*d-e))^(1/2)-e)*e)^(1/2))/d/e^2
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e-c^2*d>0)', see `assume?` for more
 details)Is e-c^2*d positive or negative?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}\left(c\,x\right)}{{\left(e\,x^2+d\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(d + e*x^2)^2,x)
```

```
[Out] int((a + b*asinh(c*x))/(d + e*x^2)^2, x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\operatorname{asinh}(cx)}{\left(d+ex^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Integral((a + b*asinh(c*x))/(d + e*x**2)**2, x)
```

3.613 $\int \left(d+ex^2\right)^3 \left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=559

$$-\frac{2bd^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c}-\frac{2bd^2ex^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{3c}-\frac{6bde^2x^4\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{25c}$$

```
[Out] 2*b^2*d^3*x-4/3*b^2*d^2*e*x/c^2+16/25*b^2*d*e^2*x/c^4-32/245*b^2*e^3*x/c^6+
2/9*b^2*d^2*e*x^3-8/75*b^2*d*e^2*x^3/c^2+16/735*b^2*e^3*x^3/c^4+6/125*b^2*d
*e^2*x^5-12/1225*b^2*e^3*x^5/c^2+2/343*b^2*e^3*x^7+d^3*x*(a+b*arcsinh(c*x))
^2+d^2*e*x^3*(a+b*arcsinh(c*x))^2+3/5*d*e^2*x^5*(a+b*arcsinh(c*x))^2+1/7*e^
3*x^7*(a+b*arcsinh(c*x))^2-2*b*d^3*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c+4
/3*b*d^2*e*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-16/25*b*d*e^2*(a+b*arcs
inh(c*x))*(c^2*x^2+1)^(1/2)/c^5+32/245*b*e^3*(a+b*arcsinh(c*x))*(c^2*x^2+1)
^(1/2)/c^7-2/3*b*d^2*e*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c+8/25*b*d*
e^2*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-16/245*b*e^3*x^2*(a+b*arcs
inh(c*x))*(c^2*x^2+1)^(1/2)/c^5-6/25*b*d*e^2*x^4*(a+b*arcsinh(c*x))*(c^2*x^
2+1)^(1/2)/c+12/245*b*e^3*x^4*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-2/49
*b*e^3*x^6*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] time = 0.97, antiderivative size = 559, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {5706, 5653, 5717, 8, 5661, 5758, 30}

$$-\frac{2bd^2ex^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{3c}+\frac{4bd^2e\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{3c^3}-\frac{2bd^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c}-$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] 2*b^2*d^3*x - (4*b^2*d^2*e*x)/(3*c^2) + (16*b^2*d*e^2*x)/(25*c^4) - (32*b^2
*e^3*x)/(245*c^6) + (2*b^2*d^2*e*x^3)/9 - (8*b^2*d*e^2*x^3)/(75*c^2) + (16*
b^2*e^3*x^3)/(735*c^4) + (6*b^2*d*e^2*x^5)/125 - (12*b^2*e^3*x^5)/(1225*c^2
) + (2*b^2*e^3*x^7)/343 - (2*b*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/
c + (4*b*d^2*e*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(3*c^3) - (16*b*d*e^
2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(25*c^5) + (32*b*e^3*Sqrt[1 + c^2
*x^2]*(a + b*ArcSinh[c*x]))/(245*c^7) - (2*b*d^2*e*x^2*Sqrt[1 + c^2*x^2]*(a
 + b*ArcSinh[c*x]))/(3*c) + (8*b*d*e^2*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh
[c*x]))/(25*c^3) - (16*b*e^3*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2
45*c^5) - (6*b*d*e^2*x^4*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(25*c) + (
12*b*e^3*x^4*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(245*c^3) - (2*b*e^3*x
^6*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(49*c) + d^3*x*(a + b*ArcSinh[c*
x])^2 + d^2*e*x^3*(a + b*ArcSinh[c*x])^2 + (3*d*e^2*x^5*(a + b*ArcSinh[c*x]
)^2)/5 + (e^3*x^7*(a + b*ArcSinh[c*x])^2)/7
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
```

```
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int \left(d+ex^2\right)^3 \left(a+b\sinh^{-1}(cx)\right)^2 \, dx = \int \left(d^3 \left(a+b\sinh^{-1}(cx)\right)^2 + 3d^2ex^2 \left(a+b\sinh^{-1}(cx)\right)^2 + 3de^2x^4 \left(a+b\sinh^{-1}\right.\right.$$

$$= d^3 \int \left(a+b\sinh^{-1}(cx)\right)^2 dx + \left(3d^2e\right) \int x^2 \left(a+b\sinh^{-1}(cx)\right)^2 dx + \left(3de^2\right.$$

$$= d^3x \left(a+b\sinh^{-1}(cx)\right)^2 + d^2ex^3 \left(a+b\sinh^{-1}(cx)\right)^2 + \frac{3}{5}de^2x^5 \left(a+b\sinh^{-1}\right.$$

$$= -\frac{2bd^3\sqrt{1+c^2x^2} \left(a+b\sinh^{-1}(cx)\right)}{c} - \frac{2bd^2ex^2\sqrt{1+c^2x^2} \left(a+b\sinh^{-1}(cx\right.}{3c}$$

$$= 2b^2d^3x + \frac{2}{9}b^2d^2ex^3 + \frac{6}{125}b^2de^2x^5 + \frac{2}{343}b^2e^3x^7 - \frac{2bd^3\sqrt{1+c^2x^2} \left(a+b\sin\right.}{c}$$

$$= 2b^2d^3x - \frac{4b^2d^2ex}{3c^2} + \frac{2}{9}b^2d^2ex^3 - \frac{8b^2de^2x^3}{75c^2} + \frac{6}{125}b^2de^2x^5 - \frac{12b^2e^3x^5}{1225c^2} + \frac{2}{34}$$

$$= 2b^2d^3x - \frac{4b^2d^2ex}{3c^2} + \frac{16b^2de^2x}{25c^4} + \frac{2}{9}b^2d^2ex^3 - \frac{8b^2de^2x^3}{75c^2} + \frac{16b^2e^3x^3}{735c^4} + \frac{6}{125}$$

$$= 2b^2d^3x - \frac{4b^2d^2ex}{3c^2} + \frac{16b^2de^2x}{25c^4} - \frac{32b^2e^3x}{245c^6} + \frac{2}{9}b^2d^2ex^3 - \frac{8b^2de^2x^3}{75c^2} + \frac{16b^2e}{735}$$

Mathematica [A] time = 0.61, size = 443, normalized size = 0.79

$$\frac{11025a^2c^7x\left(35d^3+35d^2ex^2+21de^2x^4+5e^3x^6\right) - 210ab\sqrt{c^2x^2+1}\left(c^6\left(3675d^3+1225d^2ex^2+441de^2x^4+75e^3x\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (11025*a^2*c^7*x*(35*d^3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 + 5*e^3*x^6) - 210*a
*b*Sqrt[1 + c^2*x^2]*(-240*e^3 + 24*c^2*e^2*(49*d + 5*e*x^2) - 2*c^4*e*(122
5*d^2 + 294*d*e*x^2 + 45*e^2*x^4) + c^6*(3675*d^3 + 1225*d^2*e*x^2 + 441*d*
e^2*x^4 + 75*e^3*x^6)) + 2*b^2*c*x*(-25200*e^3 + 840*c^2*e^2*(147*d + 5*e*x
^2) - 210*c^4*e*(1225*d^2 + 98*d*e*x^2 + 9*e^2*x^4) + c^6*(385875*d^3 + 428
75*d^2*e*x^2 + 9261*d*e^2*x^4 + 1125*e^3*x^6)) - 210*b*(-105*a*c^7*x*(35*d^
3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 + 5*e^3*x^6) + b*Sqrt[1 + c^2*x^2]*(-240*e^
3 + 24*c^2*e^2*(49*d + 5*e*x^2) - 2*c^4*e*(1225*d^2 + 294*d*e*x^2 + 45*e^2*
x^4) + c^6*(3675*d^3 + 1225*d^2*e*x^2 + 441*d*e^2*x^4 + 75*e^3*x^6)))*ArcSi
nh[c*x] + 11025*b^2*c^7*x*(35*d^3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 + 5*e^3*x^6
)*ArcSinh[c*x]^2)/(385875*c^7)
```

fricas [A] time = 0.57, size = 586, normalized size = 1.05

$$\frac{1125\left(49a^2+2b^2\right)c^7e^3x^7+189\left(49\left(25a^2+2b^2\right)c^7de^2-20b^2c^5e^3\right)x^5+35\left(1225\left(9a^2+2b^2\right)c^7d^2e-1176b^2c^5d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/385875*(1125*(49*a^2 + 2*b^2)*c^7*e^3*x^7 + 189*(49*(25*a^2 + 2*b^2)*c^7*
d*e^2 - 20*b^2*c^5*e^3)*x^5 + 35*(1225*(9*a^2 + 2*b^2)*c^7*d^2*e - 1176*b^2
*c^5*d*e^2 + 240*b^2*c^3*e^3)*x^3 + 11025*(5*b^2*c^7*e^3*x^7 + 21*b^2*c^7*d
*e^2*x^5 + 35*b^2*c^7*d^2*e*x^3 + 35*b^2*c^7*d^3*x)*log(c*x + sqrt(c^2*x^2
+ 1))^2 + 105*(3675*(a^2 + 2*b^2)*c^7*d^3 - 4900*b^2*c^5*d^2*e + 2352*b^2*c
```

```
^3*d*e^2 - 480*b^2*c*e^3)*x + 210*(525*a*b*c^7*e^3*x^7 + 2205*a*b*c^7*d*e^2
*x^5 + 3675*a*b*c^7*d^2*e*x^3 + 3675*a*b*c^7*d^3*x - (75*b^2*c^6*e^3*x^6 +
3675*b^2*c^6*d^3 - 2450*b^2*c^4*d^2*e + 1176*b^2*c^2*d*e^2 - 240*b^2*e^3 +
9*(49*b^2*c^6*d*e^2 - 10*b^2*c^4*e^3)*x^4 + (1225*b^2*c^6*d^2*e - 588*b^2*c
^4*d*e^2 + 120*b^2*c^2*e^3)*x^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2
+ 1)) - 210*(75*a*b*c^6*e^3*x^6 + 3675*a*b*c^6*d^3 - 2450*a*b*c^4*d^2*e + 1
176*a*b*c^2*d*e^2 - 240*a*b*e^3 + 9*(49*a*b*c^6*d*e^2 - 10*a*b*c^4*e^3)*x^4
 + (1225*a*b*c^6*d^2*e - 588*a*b*c^4*d*e^2 + 120*a*b*c^2*e^3)*x^2)*sqrt(c^2
*x^2 + 1))/c^7
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [B] time = 0.16, size = 1166, normalized size = 2.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c*(a^2/c^6*(1/7*e^3*c^7*x^7+3/5*c^7*d*e^2*x^5+c^7*d^2*e*x^3+x*c^7*d^3)+b^
2/c^6*(d^3*c^6*(arcsinh(c*x)^2*c*x-2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x)+
1/9*c^4*d^2*e*(9*arcsinh(c*x)^2*c^3*x^3-6*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^
2*x^2+27*arcsinh(c*x)^2*c*x+2*c^3*x^3-42*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+42*
c*x)-3*c^4*d^2*e*(arcsinh(c*x)^2*c*x-2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x
)+1/1125*d*e^2*c^2*(675*arcsinh(c*x)^2*c^5*x^5-270*arcsinh(c*x)*(c^2*x^2+1)
^(1/2)*c^4*x^4+2250*arcsinh(c*x)^2*c^3*x^3+54*c^5*x^5-1140*arcsinh(c*x)*(c^
2*x^2+1)^(1/2)*c^2*x^2+3375*arcsinh(c*x)^2*c*x+380*c^3*x^3-4470*arcsinh(c*x
)*(c^2*x^2+1)^(1/2)+4470*c*x)-2/9*d*e^2*c^2*(9*arcsinh(c*x)^2*c^3*x^3-6*arc
sinh(c*x)*(c^2*x^2+1)^(1/2)*c^2*x^2+27*arcsinh(c*x)^2*c*x+2*c^3*x^3-42*arcs
inh(c*x)*(c^2*x^2+1)^(1/2)+42*c*x)+3*d*e^2*c^2*(arcsinh(c*x)^2*c*x-2*arcsin
h(c*x)*(c^2*x^2+1)^(1/2)+2*c*x)+1/385875*e^3*(55125*arcsinh(c*x)^2*c^7*x^7-
15750*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^6*x^6+231525*arcsinh(c*x)^2*c^5*x^5+
2250*c^7*x^7-73710*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^4*x^4+385875*arcsinh(c*
x)^2*c^3*x^3+14742*c^5*x^5-158970*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^2*x^2+38
5875*arcsinh(c*x)^2*c*x+52990*c^3*x^3-453810*arcsinh(c*x)*(c^2*x^2+1)^(1/2)
+453810*c*x)-1/1125*e^3*(675*arcsinh(c*x)^2*c^5*x^5-270*arcsinh(c*x)*(c^2*x
^2+1)^(1/2)*c^4*x^4+2250*arcsinh(c*x)^2*c^3*x^3+54*c^5*x^5-1140*arcsinh(c*x
)*(c^2*x^2+1)^(1/2)*c^2*x^2+3375*arcsinh(c*x)^2*c*x+380*c^3*x^3-4470*arcsin
h(c*x)*(c^2*x^2+1)^(1/2)+4470*c*x)+1/9*e^3*(9*arcsinh(c*x)^2*c^3*x^3-6*arcs
inh(c*x)*(c^2*x^2+1)^(1/2)*c^2*x^2+27*arcsinh(c*x)^2*c*x+2*c^3*x^3-42*arcsi
nh(c*x)*(c^2*x^2+1)^(1/2)+42*c*x)-e^3*(arcsinh(c*x)^2*c*x-2*arcsinh(c*x)*(c
^2*x^2+1)^(1/2)+2*c*x))+2*a*b/c^6*(1/7*arcsinh(c*x)*e^3*c^7*x^7+3/5*arcsinh
(c*x)*c^7*d*e^2*x^5+arcsinh(c*x)*c^7*d^2*e*x^3+arcsinh(c*x)*c^7*x*d^3-1/7*e
^3*(1/7*c^6*x^6*(c^2*x^2+1)^(1/2)-6/35*c^4*x^4*(c^2*x^2+1)^(1/2)+8/35*c^2*x
^2*(c^2*x^2+1)^(1/2)-16/35*(c^2*x^2+1)^(1/2))-3/5*c^2*d*e^2*(1/5*c^4*x^4*(c
^2*x^2+1)^(1/2)-4/15*c^2*x^2*(c^2*x^2+1)^(1/2)+8/15*(c^2*x^2+1)^(1/2))-c^4*
d^2*e*(1/3*c^2*x^2*(c^2*x^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/2))-d^3*c^6*(c^2*x^
2+1)^(1/2)))
```

maxima [A] time = 0.53, size = 684, normalized size = 1.22

$$\frac{1}{7}b^2e^3x^7\operatorname{arsinh}(cx)^2+\frac{1}{7}a^2e^3x^7+\frac{3}{5}b^2de^2x^5\operatorname{arsinh}(cx)^2+\frac{3}{5}a^2de^2x^5+b^2d^2ex^3\operatorname{arsinh}(cx)^2+a^2d^2ex^3+b^2d^3x\operatorname{arsinh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*e^3*x^7*arcsinh(c*x)^2 + 1/7*a^2*e^3*x^7 + 3/5*b^2*d*e^2*x^5*arcsin
h(c*x)^2 + 3/5*a^2*d*e^2*x^5 + b^2*d^2*e*x^3*arcsinh(c*x)^2 + a^2*d^2*e*x^3
 + b^2*d^3*x*arcsinh(c*x)^2 + 2/3*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1
)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*d^2*e - 2/9*(3*c*(sqrt(c^2*x^2 +
1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3 - 6*x)/c^2)*b
^2*d^2*e + 2/25*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqr
t(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*d*e^2 - 2/375*(15*
(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2
 + 1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c^4)*b^2*d*e^2
 + 2/245*(35*x^7*arcsinh(c*x) - (5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x
^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c
)*a*b*e^3 - 2/25725*(105*(5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)
*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c*arcsin
h(c*x) - (75*c^6*x^7 - 126*c^4*x^5 + 280*c^2*x^3 - 1680*x)/c^6)*b^2*e^3 + 2
*b^2*d^3*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*d^3*x + 2*(c*x*arcsin
h(c*x) - sqrt(c^2*x^2 + 1))*a*b*d^3/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\operatorname{asinh}(c\,x))^2\left(e\,x^2+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + e*x^2)^3,x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + e*x^2)^3, x)
```

sympy [A] time = 13.83, size = 989, normalized size = 1.77

$$\begin{cases} a^2d^3x+a^2d^2ex^3+\frac{3a^2de^2x^5}{5}+\frac{a^2e^3x^7}{7}+2abd^3x\operatorname{asinh}(cx)+2abd^2ex^3\operatorname{asinh}(cx)+\frac{6abde^2x^5\operatorname{asinh}(cx)}{5}+\frac{2abe^3x^7\operatorname{asinh}(cx}{7} \\ a^2\left(d^3x+d^2ex^3+\frac{3de^2x^5}{5}+\frac{e^3x^7}{7}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d**3*x + a**2*d**2*e*x**3 + 3*a**2*d*e**2*x**5/5 + a**2*e**
3*x**7/7 + 2*a*b*d**3*x*asinh(c*x) + 2*a*b*d**2*e*x**3*asinh(c*x) + 6*a*b*d
*e**2*x**5*asinh(c*x)/5 + 2*a*b*e**3*x**7*asinh(c*x)/7 - 2*a*b*d**3*sqrt(c*
*2*x**2 + 1)/c - 2*a*b*d**2*e*x**2*sqrt(c**2*x**2 + 1)/(3*c) - 6*a*b*d*e**2
*x**4*sqrt(c**2*x**2 + 1)/(25*c) - 2*a*b*e**3*x**6*sqrt(c**2*x**2 + 1)/(49*
c) + 4*a*b*d**2*e*sqrt(c**2*x**2 + 1)/(3*c**3) + 8*a*b*d*e**2*x**2*sqrt(c**
2*x**2 + 1)/(25*c**3) + 12*a*b*e**3*x**4*sqrt(c**2*x**2 + 1)/(245*c**3) - 1
6*a*b*d*e**2*sqrt(c**2*x**2 + 1)/(25*c**5) - 16*a*b*e**3*x**2*sqrt(c**2*x**
2 + 1)/(245*c**5) + 32*a*b*e**3*sqrt(c**2*x**2 + 1)/(245*c**7) + b**2*d**3*
x*asinh(c*x)**2 + 2*b**2*d**3*x + b**2*d**2*e*x**3*asinh(c*x)**2 + 2*b**2*d
**2*e*x**3/9 + 3*b**2*d*e**2*x**5*asinh(c*x)**2/5 + 6*b**2*d*e**2*x**5/125
+ b**2*e**3*x**7*asinh(c*x)**2/7 + 2*b**2*e**3*x**7/343 - 2*b**2*d**3*sqrt(
```

```
c**2*x**2 + 1)*asinh(c*x)/c - 2*b**2*d**2*e*x**2*sqrt(c**2*x**2 + 1)*asinh(
c*x)/(3*c) - 6*b**2*d*e**2*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/(25*c) - 2*b
**2*e**3*x**6*sqrt(c**2*x**2 + 1)*asinh(c*x)/(49*c) - 4*b**2*d**2*e*x/(3*c*
*2) - 8*b**2*d*e**2*x**3/(75*c**2) - 12*b**2*e**3*x**5/(1225*c**2) + 4*b**2
*d**2*e*sqrt(c**2*x**2 + 1)*asinh(c*x)/(3*c**3) + 8*b**2*d*e**2*x**2*sqrt(c
**2*x**2 + 1)*asinh(c*x)/(25*c**3) + 12*b**2*e**3*x**4*sqrt(c**2*x**2 + 1)*
asinh(c*x)/(245*c**3) + 16*b**2*d*e**2*x/(25*c**4) + 16*b**2*e**3*x**3/(735
*c**4) - 16*b**2*d*e**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(25*c**5) - 16*b**2*
e**3*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(245*c**5) - 32*b**2*e**3*x/(245*c
**6) + 32*b**2*e**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/(245*c**7), Ne(c, 0)), (
a**2*(d**3*x + d**2*e*x**3 + 3*d*e**2*x**5/5 + e**3*x**7/7), True))
```

3.614 $\int \left(d+ex^2\right)^2 \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=329

$$-\frac{2bd^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c}-\frac{4bdex^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{9c}-\frac{2be^2x^4\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{25c}-$$

```
[Out] 2*b^2*d^2*x-8/9*b^2*d*e*x/c^2+16/75*b^2*e^2*x/c^4+4/27*b^2*d*e*x^3-8/225*b^
2*e^2*x^3/c^2+2/125*b^2*e^2*x^5+d^2*x*(a+b*arcsinh(c*x))^2+2/3*d*e*x^3*(a+b
*arcsinh(c*x))^2+1/5*e^2*x^5*(a+b*arcsinh(c*x))^2-2*b*d^2*(a+b*arcsinh(c*x)
)*(c^2*x^2+1)^(1/2)/c+8/9*b*d*e*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-16
/75*b*e^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^5-4/9*b*d*e*x^2*(a+b*arcsi
nh(c*x))*(c^2*x^2+1)^(1/2)/c+8/75*b*e^2*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^
(1/2)/c^3-2/25*b*e^2*x^4*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] time = 0.58, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {5706, 5653, 5717, 8, 5661, 5758, 30}

$$-\frac{2bd^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c}-\frac{4bdex^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{9c}+\frac{8bde\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{9c^3}-2-$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] 2*b^2*d^2*x - (8*b^2*d*e*x)/(9*c^2) + (16*b^2*e^2*x)/(75*c^4) + (4*b^2*d*e*
x^3)/27 - (8*b^2*e^2*x^3)/(225*c^2) + (2*b^2*e^2*x^5)/125 - (2*b*d^2*Sqrt[1
 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c + (8*b*d*e*Sqrt[1 + c^2*x^2]*(a + b*Arc
Sinh[c*x]))/(9*c^3) - (16*b*e^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(75
*c^5) - (4*b*d*e*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(9*c) + (8*b*e
^2*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(75*c^3) - (2*b*e^2*x^4*Sqrt
[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(25*c) + d^2*x*(a + b*ArcSinh[c*x])^2 +
 (2*d*e*x^3*(a + b*ArcSinh[c*x])^2)/3 + (e^2*x^5*(a + b*ArcSinh[c*x])^2)/5
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int \left(d+ex^2\right)^2 \left(a+b\sinh^{-1}(cx)\right)^2 \, dx = \int \left(d^2 \left(a+b\sinh^{-1}(cx)\right)^2 + 2dex^2 \left(a+b\sinh^{-1}(cx)\right)^2 + e^2x^4 \left(a+b\sin\right.\right.$$

$$= d^2 \int \left(a+b\sinh^{-1}(cx)\right)^2 \, dx + (2de) \int x^2 \left(a+b\sinh^{-1}(cx)\right)^2 \, dx + e^2$$

$$= d^2x \left(a+b\sinh^{-1}(cx)\right)^2 + \frac{2}{3}dex^3 \left(a+b\sinh^{-1}(cx)\right)^2 + \frac{1}{5}e^2x^5 \left(a+b\sin\right.$$

$$= -\frac{2bd^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c} - \frac{4bdex^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(\right.}{9c}$$

$$= 2b^2d^2x + \frac{4}{27}b^2dex^3 + \frac{2}{125}b^2e^2x^5 - \frac{2bd^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c}$$

$$= 2b^2d^2x - \frac{8b^2dex}{9c^2} + \frac{4}{27}b^2dex^3 - \frac{8b^2e^2x^3}{225c^2} + \frac{2}{125}b^2e^2x^5 - \frac{2bd^2\sqrt{1+c^2x^2}}{}$$

$$= 2b^2d^2x - \frac{8b^2dex}{9c^2} + \frac{16b^2e^2x}{75c^4} + \frac{4}{27}b^2dex^3 - \frac{8b^2e^2x^3}{225c^2} + \frac{2}{125}b^2e^2x^5 - \frac{2bd}{}$$

Mathematica [A] time = 0.43, size = 289, normalized size = 0.88

$$\frac{225a^2c^5x\left(15d^2+10dex^2+3e^2x^4\right) - 30ab\sqrt{c^2x^2+1}\left(c^4\left(225d^2+50dex^2+9e^2x^4\right) - 4c^2e\left(25d+3ex^2\right) + 24e^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (225*a^2*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) - 30*a*b*Sqrt[1 + c^2*x^2]
*(24*e^2 - 4*c^2*e*(25*d + 3*e*x^2) + c^4*(225*d^2 + 50*d*e*x^2 + 9*e^2*x^4
)) + 2*b^2*c*x*(360*e^2 - 60*c^2*e*(25*d + e*x^2) + c^4*(3375*d^2 + 250*d*e
```

```
*x^2 + 27*e^2*x^4)) - 30*b*(-15*a*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) +
 b*Sqrt[1 + c^2*x^2]*(24*e^2 - 4*c^2*e*(25*d + 3*e*x^2) + c^4*(225*d^2 + 50
*d*e*x^2 + 9*e^2*x^4)))*ArcSinh[c*x] + 225*b^2*c^5*x*(15*d^2 + 10*d*e*x^2 +
 3*e^2*x^4)*ArcSinh[c*x]^2)/(3375*c^5)
```

fricas [A] time = 1.02, size = 380, normalized size = 1.16

$$\frac{27\left(25\,a^2+2\,b^2\right)c^5e^2x^5+10\left(25\left(9\,a^2+2\,b^2\right)c^5de-12\,b^2c^3e^2\right)x^3+225\left(3\,b^2c^5e^2x^5+10\,b^2c^5dex^3+15\,b^2c^5d^2x\right)\mathrm{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/3375*(27*(25*a^2 + 2*b^2)*c^5*e^2*x^5 + 10*(25*(9*a^2 + 2*b^2)*c^5*d*e -
12*b^2*c^3*e^2)*x^3 + 225*(3*b^2*c^5*e^2*x^5 + 10*b^2*c^5*d*e*x^3 + 15*b^2*
c^5*d^2*x)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 15*(225*(a^2 + 2*b^2)*c^5*d^2 -
 200*b^2*c^3*d*e + 48*b^2*c*e^2)*x + 30*(45*a*b*c^5*e^2*x^5 + 150*a*b*c^5*d
*e*x^3 + 225*a*b*c^5*d^2*x - (9*b^2*c^4*e^2*x^4 + 225*b^2*c^4*d^2 - 100*b^2
*c^2*d*e + 24*b^2*e^2 + 2*(25*b^2*c^4*d*e - 6*b^2*c^2*e^2)*x^2)*sqrt(c^2*x^
2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 30*(9*a*b*c^4*e^2*x^4 + 225*a*b*c^4*
d^2 - 100*a*b*c^2*d*e + 24*a*b*e^2 + 2*(25*a*b*c^4*d*e - 6*a*b*c^2*e^2)*x^2
)*sqrt(c^2*x^2 + 1))/c^5
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [B] time = 0.10, size = 620, normalized size = 1.88

$$\frac{\frac{a^2\left(\frac{1}{5}e^2c^5x^5+\frac{2}{3}c^5de\,x^3+x\,c^5d^2\right)}{c^4}+\frac{b^2\left(d^2c^4\left(\operatorname{arcsinh}(cx)^2cx-2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}+2cx\right)+\frac{2c^2de\left(9\operatorname{arcsinh}(cx)^2c^3x^3-6\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,c^2x^2+27\operatorname{arcsinh}(cx\right.}{27}}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c*(a^2/c^4*(1/5*e^2*c^5*x^5+2/3*c^5*d*e*x^3+x*c^5*d^2)+b^2/c^4*(d^2*c^4*(
arcsinh(c*x)^2*c*x-2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x)+2/27*c^2*d*e*(9*
arcsinh(c*x)^2*c^3*x^3-6*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^2*x^2+27*arcsinh(
c*x)^2*c*x+2*c^3*x^3-42*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+42*c*x)-2*c^2*d*e*(a
rcsinh(c*x)^2*c*x-2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x)+1/3375*e^2*(675*a
rcsinh(c*x)^2*c^5*x^5-270*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^4*x^4+2250*arcsi
nh(c*x)^2*c^3*x^3+54*c^5*x^5-1140*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^2*x^2+33
75*arcsinh(c*x)^2*c*x+380*c^3*x^3-4470*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+4470*
c*x)-2/27*e^2*(9*arcsinh(c*x)^2*c^3*x^3-6*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^
2*x^2+27*arcsinh(c*x)^2*c*x+2*c^3*x^3-42*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+42*
c*x)+e^2*(arcsinh(c*x)^2*c*x-2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x))+2*a*b
/c^4*(1/5*arcsinh(c*x)*e^2*c^5*x^5+2/3*arcsinh(c*x)*c^5*d*e*x^3+arcsinh(c*x
)*c^5*x*d^2-1/5*e^2*(1/5*c^4*x^4*(c^2*x^2+1)^(1/2)-4/15*c^2*x^2*(c^2*x^2+1)
```

```
^(1/2)+8/15*(c^2*x^2+1)^(1/2))-2/3*c^2*d*e*(1/3*c^2*x^2*(c^2*x^2+1)^(1/2)-2
/3*(c^2*x^2+1)^(1/2))-d^2*c^4*(c^2*x^2+1)^(1/2)))
```

maxima [A] time = 0.50, size = 429, normalized size = 1.30

$$\frac{1}{5}b^2e^2x^5\operatorname{arsinh}(cx)^2+\frac{1}{5}a^2e^2x^5+\frac{2}{3}b^2dex^3\operatorname{arsinh}(cx)^2+\frac{2}{3}a^2dex^3+b^2d^2x\operatorname{arsinh}(cx)^2+\frac{4}{9}\left(3x^3\operatorname{arsinh}(cx)-c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*e^2*x^5*arcsinh(c*x)^2 + 1/5*a^2*e^2*x^5 + 2/3*b^2*d*e*x^3*arcsinh(
c*x)^2 + 2/3*a^2*d*e*x^3 + b^2*d^2*x*arcsinh(c*x)^2 + 4/9*(3*x^3*arcsinh(c*
x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*d*e - 4/2
7*(3*c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) -
 (c^2*x^3 - 6*x)/c^2)*b^2*d*e + 2/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2
 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a
*b*e^2 - 2/1125*(15*(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/
c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 1
20*x)/c^4)*b^2*e^2 + 2*b^2*d^2*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2
*d^2*x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*d^2/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\operatorname{asinh}(c\,x))^2\,\left(e\,x^2+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + e*x^2)^2,x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + e*x^2)^2, x)
```

sympy [A] time = 4.92, size = 595, normalized size = 1.81

$$\begin{cases} a^2d^2x+\frac{2a^2dex^3}{3}+\frac{a^2e^2x^5}{5}+2abd^2x\operatorname{asinh}(cx)+\frac{4abdex^3\operatorname{asinh}(cx)}{3}+\frac{2abe^2x^5\operatorname{asinh}(cx)}{5}-\frac{2abd^2\sqrt{c^2x^2+1}}{c}-\frac{4abdex^2\sqrt{c^2x^2+1}}{9c} \\ a^2\left(d^2x+\frac{2dex^3}{3}+\frac{e^2x^5}{5}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d**2*x + 2*a**2*d*e*x**3/3 + a**2*e**2*x**5/5 + 2*a*b*d**2*
x*asinh(c*x) + 4*a*b*d*e*x**3*asinh(c*x)/3 + 2*a*b*e**2*x**5*asinh(c*x)/5 -
 2*a*b*d**2*sqrt(c**2*x**2 + 1)/c - 4*a*b*d*e*x**2*sqrt(c**2*x**2 + 1)/(9*c
) - 2*a*b*e**2*x**4*sqrt(c**2*x**2 + 1)/(25*c) + 8*a*b*d*e*sqrt(c**2*x**2 +
 1)/(9*c**3) + 8*a*b*e**2*x**2*sqrt(c**2*x**2 + 1)/(75*c**3) - 16*a*b*e**2*
sqrt(c**2*x**2 + 1)/(75*c**5) + b**2*d**2*x*asinh(c*x)**2 + 2*b**2*d**2*x +
 2*b**2*d*e*x**3*asinh(c*x)**2/3 + 4*b**2*d*e*x**3/27 + b**2*e**2*x**5*asin
h(c*x)**2/5 + 2*b**2*e**2*x**5/125 - 2*b**2*d**2*sqrt(c**2*x**2 + 1)*asinh(
c*x)/c - 4*b**2*d*e*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(9*c) - 2*b**2*e**2
*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/(25*c) - 8*b**2*d*e*x/(9*c**2) - 8*b**
2*e**2*x**3/(225*c**2) + 8*b**2*d*e*sqrt(c**2*x**2 + 1)*asinh(c*x)/(9*c**3)
 + 8*b**2*e**2*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(75*c**3) + 16*b**2*e**2
*x/(75*c**4) - 16*b**2*e**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(75*c**5), Ne(c,
 0)), (a**2*(d**2*x + 2*d*e*x**3/3 + e**2*x**5/5), True))
```

3.615 $\int \left(d+ex^2\right)\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=153

$$-\frac{2bd\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)}{c}-\frac{2bex^2\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)}{9c}+\frac{4be\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)}{9c^3}+dx\,(a$$

```
[Out] 2*b^2*d*x-4/9*b^2*e*x/c^2+2/27*b^2*e*x^3+d*x*(a+b*arcsinh(c*x))^2+1/3*e*x^3
*(a+b*arcsinh(c*x))^2-2*b*d*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c+4/9*b*e*
(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-2/9*b*e*x^2*(a+b*arcsinh(c*x))*(c^
2*x^2+1)^(1/2)/c
```

Rubi [A] time = 0.28, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5706, 5653, 5717, 8, 5661, 5758, 30}

$$-\frac{2bd\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)}{c}-\frac{2bex^2\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)}{9c}+\frac{4be\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)}{9c^3}+dx\,(a$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] 2*b^2*d*x - (4*b^2*e*x)/(9*c^2) + (2*b^2*e*x^3)/27 - (2*b*d*Sqrt[1 + c^2*x^
2]*(a + b*ArcSinh[c*x]))/c + (4*b*e*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))
/(9*c^3) - (2*b*e*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(9*c) + d*x*(
a + b*ArcSinh[c*x])^2 + (e*x^3*(a + b*ArcSinh[c*x])^2)/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^2\right)\left(a+b\sinh^{-1}(cx)\right)^2\,dx &= \int \left(d\left(a+b\sinh^{-1}(cx)\right)^2+ex^2\left(a+b\sinh^{-1}(cx)\right)^2\right)dx\\
&= d\int \left(a+b\sinh^{-1}(cx)\right)^2\,dx+e\int x^2\left(a+b\sinh^{-1}(cx)\right)^2\,dx\\
&= dx\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{3}ex^3\left(a+b\sinh^{-1}(cx)\right)^2-(2bcd)\int \frac{x\left(a+b\mathrm{s}\right.}{\sqrt{1+}}\\
&= -\frac{2bd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c}-\frac{2bex^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right.}{9c}\\
&= 2b^2dx+\frac{2}{27}b^2ex^3-\frac{2bd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c}+\frac{4be\sqrt{1+c^2x^2}\left(\right.}{9}\\
&= 2b^2dx-\frac{4b^2ex}{9c^2}+\frac{2}{27}b^2ex^3-\frac{2bd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c}+\frac{4be\sqrt{1}}{}\end{aligned}$$

Mathematica [A] time = 0.22, size = 164, normalized size = 1.07

$$\frac{9a^2c^3x\left(3d+ex^2\right)-6ab\sqrt{c^2x^2+1}\left(c^2\left(9d+ex^2\right)-2e\right)-6b\sinh^{-1}(cx)\left(b\sqrt{c^2x^2+1}\left(c^2\left(9d+ex^2\right)-2e\right)-3ac\right.}{27c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (9*a^2*c^3*x*(3*d + e*x^2) - 6*a*b*Sqrt[1 + c^2*x^2]*(-2*e + c^2*(9*d + e*x
^2)) + 2*b^2*c*x*(-6*e + c^2*(27*d + e*x^2)) - 6*b*(-3*a*c^3*x*(3*d + e*x^2
) + b*Sqrt[1 + c^2*x^2]*(-2*e + c^2*(9*d + e*x^2)))*ArcSinh[c*x] + 9*b^2*c^
3*x*(3*d + e*x^2)*ArcSinh[c*x]^2)/(27*c^3)
```

fricas [A] time = 0.97, size = 209, normalized size = 1.37

$$\frac{\left(9a^2+2b^2\right)c^3ex^3+9\left(b^2c^3ex^3+3b^2c^3dx\right)\log\left(cx+\sqrt{c^2x^2+1}\right)^2+3\left(9\left(a^2+2b^2\right)c^3d-4b^2ce\right)x+6\left(3abc^3e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/27*((9*a^2 + 2*b^2)*c^3*e*x^3 + 9*(b^2*c^3*e*x^3 + 3*b^2*c^3*d*x)*log(c*x
 + sqrt(c^2*x^2 + 1))^2 + 3*(9*(a^2 + 2*b^2)*c^3*d - 4*b^2*c*e)*x + 6*(3*a*
b*c^3*e*x^3 + 9*a*b*c^3*d*x - (b^2*c^2*e*x^2 + 9*b^2*c^2*d - 2*b^2*e)*sqrt(
c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 6*(a*b*c^2*e*x^2 + 9*a*b*c^2*d
 - 2*a*b*e)*sqrt(c^2*x^2 + 1))/c^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.09, size = 271, normalized size = 1.77

$$\frac{\frac{a^2\left(\frac{1}{3}c^3x^3e+c^3dx\right)}{c^2}+\frac{b^2\left(c^2d\left(\operatorname{arcsinh}(cx)^2cx-2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}+2cx\right)+\frac{e\left(9\operatorname{arcsinh}(cx)^2c^3x^3-6\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,c^2x^2+27\operatorname{arcsinh}(cx)^2cx+2c^3x^3-42\operatorname{arcsir}\right.}{27}}{c^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c*(a^2/c^2*(1/3*c^3*x^3*e+c^3*d*x)+b^2/c^2*(c^2*d*(arcsinh(c*x)^2*c*x-2*a
rcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x)+1/27*e*(9*arcsinh(c*x)^2*c^3*x^3-6*arc
sinh(c*x)*(c^2*x^2+1)^(1/2)*c^2*x^2+27*arcsinh(c*x)^2*c*x+2*c^3*x^3-42*arcs
inh(c*x)*(c^2*x^2+1)^(1/2)+42*c*x)-e*(arcsinh(c*x)^2*c*x-2*arcsinh(c*x)*(c^
2*x^2+1)^(1/2)+2*c*x))+2*a*b/c^2*(1/3*arcsinh(c*x)*c^3*x^3*e+arcsinh(c*x)*c
^3*d*x-1/3*e*(1/3*c^2*x^2*(c^2*x^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/2))-c^2*d*(c
^2*x^2+1)^(1/2)))
```

maxima [A] time = 0.44, size = 218, normalized size = 1.42

$$\frac{1}{3}b^2ex^3\operatorname{arsinh}(cx)^2+\frac{1}{3}a^2ex^3+b^2dx\operatorname{arsinh}(cx)^2+\frac{2}{9}\left(3x^3\operatorname{arsinh}(cx)-c\left(\frac{\sqrt{c^2x^2+1}\,x^2}{c^2}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)abe-\frac{2}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*e*x^3*arcsinh(c*x)^2 + 1/3*a^2*e*x^3 + b^2*d*x*arcsinh(c*x)^2 + 2/9
*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c
^4))*a*b*e - 2/27*(3*c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4
)*arcsinh(c*x) - (c^2*x^3 - 6*x)/c^2)*b^2*e + 2*b^2*d*(x - sqrt(c^2*x^2 + 1
)*arcsinh(c*x)/c) + a^2*d*x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*
d/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\operatorname{asinh}(c\,x))^2\,\left(e\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + e*x^2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + e*x^2), x)
```

sympy [A] time = 1.37, size = 279, normalized size = 1.82

$$\begin{cases} a^2dx + \frac{a^2ex^3}{3} + 2abdx\operatorname{asinh}(cx) + \frac{2abex^3\operatorname{asinh}(cx)}{3} - \frac{2abd\sqrt{c^2x^2+1}}{c} - \frac{2abex^2\sqrt{c^2x^2+1}}{9c} + \frac{4abe\sqrt{c^2x^2+1}}{9c^3} + b^2dx\operatorname{asinh}^2(c \\ a^2\left(dx + \frac{ex^3}{3}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d*x + a**2*e*x**3/3 + 2*a*b*d*x*asinh(c*x) + 2*a*b*e*x**3*a
sinh(c*x)/3 - 2*a*b*d*sqrt(c**2*x**2 + 1)/c - 2*a*b*e*x**2*sqrt(c**2*x**2 +
 1)/(9*c) + 4*a*b*e*sqrt(c**2*x**2 + 1)/(9*c**3) + b**2*d*x*asinh(c*x)**2 +
 2*b**2*d*x + b**2*e*x**3*asinh(c*x)**2/3 + 2*b**2*e*x**3/27 - 2*b**2*d*sqr
t(c**2*x**2 + 1)*asinh(c*x)/c - 2*b**2*e*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x
)/(9*c) - 4*b**2*e*x/(9*c**2) + 4*b**2*e*sqrt(c**2*x**2 + 1)*asinh(c*x)/(9*
c**3), Ne(c, 0)), (a**2*(d*x + e*x**3/3), True))
```

3.616 $\int \left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=46

$$-\frac{2b\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c}+x\left(a+b\sinh^{-1}(cx)\right)^2+2b^2x$$

```
[Out] 2*b^2*x+x*(a+b*arcsinh(c*x))^2-2*b*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5653, 5717, 8}

$$-\frac{2b\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c}+x\left(a+b\sinh^{-1}(cx)\right)^2+2b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] 2*b^2*x - (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c + x*(a + b*ArcSinh
[c*x])^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sinh^{-1}(cx)\right)^2\,dx &= x\left(a+b\sinh^{-1}(cx)\right)^2-(2bc)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx\\ &= -\frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c}+x\left(a+b\sinh^{-1}(cx)\right)^2+\left(2b^2\right)\int 1\,dx\\ &= 2b^2x-\frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c}+x\left(a+b\sinh^{-1}(cx)\right)^2\end{aligned}$$

Mathematica [A] time = 0.06, size = 74, normalized size = 1.61

$$x\left(a^2+2b^2\right)-\frac{2ab\sqrt{c^2x^2+1}}{c}+\frac{2b\sinh^{-1}(cx)\left(acx-b\sqrt{c^2x^2+1}\right)}{c}+b^2x\sinh^{-1}(cx)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (a^2 + 2*b^2)*x - (2*a*b*Sqrt[1 + c^2*x^2])/c + (2*b*(a*c*x - b*Sqrt[1 + c^
2*x^2])*ArcSinh[c*x])/c + b^2*x*ArcSinh[c*x]^2
```

fricas [B] time = 0.61, size = 96, normalized size = 2.09

$$\frac{b^2cx\log\left(cx+\sqrt{c^2x^2+1}\right)^2+\left(a^2+2\,b^2\right)cx-2\,\sqrt{c^2x^2+1}\,ab+2\left(abcx-\sqrt{c^2x^2+1}\,b^2\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] (b^2*c*x*log(c*x + sqrt(c^2*x^2 + 1))^2 + (a^2 + 2*b^2)*c*x - 2*sqrt(c^2*x^
2 + 1)*a*b + 2*(a*b*c*x - sqrt(c^2*x^2 + 1)*b^2)*log(c*x + sqrt(c^2*x^2 + 1
)))/c
```

giac [B] time = 0.52, size = 111, normalized size = 2.41

$$2\left(x\log\left(cx+\sqrt{c^2x^2+1}\right)-\frac{\sqrt{c^2x^2+1}}{c}\right)ab+\left(x\log\left(cx+\sqrt{c^2x^2+1}\right)^2+2\,c\left(\frac{x}{c}-\frac{\sqrt{c^2x^2+1}\,\log\left(cx+\sqrt{c^2x^2+\right.}}{c^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] 2*(x*log(c*x + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*a*b + (x*log(c*x +
 sqrt(c^2*x^2 + 1))^2 + 2*c*(x/c - sqrt(c^2*x^2 + 1)*log(c*x + sqrt(c^2*x^2
 + 1))/c^2))*b^2 + a^2*x
```

maple [A] time = 0.05, size = 72, normalized size = 1.57

$$\frac{a^2cx+b^2\left(\operatorname{arcsinh}(cx)^2\,cx-2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,+2cx\right)+2ab\left(\operatorname{arcsinh}(cx)\,cx-\sqrt{c^2x^2+1}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c*(a^2*c*x+b^2*(arcsinh(c*x)^2*c*x-2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x
)+2*a*b*(arcsinh(c*x)*c*x-(c^2*x^2+1)^(1/2)))
```

maxima [A] time = 0.61, size = 72, normalized size = 1.57

$$b^2x\operatorname{arsinh}(cx)^2+2\,b^2\left(x-\frac{\sqrt{c^2x^2+1}\,\operatorname{arsinh}(cx)}{c}\right)+a^2x+\frac{2\left(cx\operatorname{arsinh}(cx)-\sqrt{c^2x^2+1}\right)ab}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] b^2*x*arcsinh(c*x)^2 + 2*b^2*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*x
 + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2,x)
```

```
[Out] int((a + b*asinh(c*x))^2, x)
```

sympy [A] time = 0.27, size = 82, normalized size = 1.78

$$\begin{cases} a^2x + 2abx\operatorname{asinh}(cx) - \frac{2ab\sqrt{c^2x^2+1}}{c} + b^2x\operatorname{asinh}^2(cx) + 2b^2x - \frac{2b^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{c} & \text{for } c \neq 0 \\ a^2x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*x*asinh(c*x) - 2*a*b*sqrt(c**2*x**2 + 1)/c + b**2
*x*asinh(c*x)**2 + 2*b**2*x - 2*b**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/c, Ne(c
, 0)), (a**2*x, True))
```

3.617 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d+ex^2}\,dx$

Optimal. Leaf size=739

$$-\frac{b\left(a+b\sinh^{-1}(cx)\right)\text{Li}_2\left(-\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{\sqrt{-d}\sqrt{e}}+\frac{b\left(a+b\sinh^{-1}(cx)\right)\text{Li}_2\left(\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{\sqrt{-d}\sqrt{e}}-\frac{b\left(a+b\sinh^{-1}(cx)\right)\text{Li}_2\left(-\right.}{\sqrt{-d}\sqrt{e}}$$

```
[Out] 1/2*(a+b*arcsinh(c*x))^2*ln(1-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)
-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arcsinh(c*x))^2*ln(1+(c*x+(
c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/
2)+1/2*(a+b*arcsinh(c*x))^2*ln(1-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1
/2)+(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arcsinh(c*x))^2*ln(1+(c*
x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^
(1/2)-b*(a+b*arcsinh(c*x))*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-
d)^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+b*(a+b*arcsinh(c*x))*polylog
(2,(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(1
/2)/e^(1/2)-b*(a+b*arcsinh(c*x))*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)
/(c*(-d)^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+b*(a+b*arcsinh(c*x))*p
olylog(2,(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d+e)^(1/2)))/(
-d)^(1/2)/e^(1/2)+b^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1
/2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-b^2*polylog(3,(c*x+(c^2*x^2+1)^(1
/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+b^2*polylo
g(3,-(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^
(1/2)/e^(1/2)-b^2*polylog(3,(c*x+(c^2*x^2+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(
-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)
```

Rubi [A] time = 1.32, antiderivative size = 739, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {5706, 5799, 5561, 2190, 2531, 2282, 6589}

$$-\frac{b\left(a+b\sinh^{-1}(cx)\right)\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{\sqrt{-d}\sqrt{e}}+\frac{b\left(a+b\sinh^{-1}(cx)\right)\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{\sqrt{-d}\sqrt{e}}-\frac{b\left(a+b\,\text{si}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + e*x^2),x]
```

```
[Out] ((a + b*ArcSinh[c*x])^2*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt
[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcSinh[c*x])^2*Log[1 + (S
qrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt
[e]) + ((a + b*ArcSinh[c*x])^2*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d]
 + Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcSinh[c*x])^2*Log
[1 + (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])])/(2*Sqrt[-
d]*Sqrt[e]) - (b*(a + b*ArcSinh[c*x])*PolyLog[2, -((Sqrt[e]*E^ArcSinh[c*x])
/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e]))])/(Sqrt[-d]*Sqrt[e]) + (b*(a + b*ArcSin
h[c*x])*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e
])])/(Sqrt[-d]*Sqrt[e]) - (b*(a + b*ArcSinh[c*x])*PolyLog[2, -((Sqrt[e]*E^A
rcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e]))])/(Sqrt[-d]*Sqrt[e]) + (b*(
a + b*ArcSinh[c*x])*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[
-(c^2*d) + e])])/(Sqrt[-d]*Sqrt[e]) + (b^2*PolyLog[3, -((Sqrt[e]*E^ArcSinh[
c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e]))])/(Sqrt[-d]*Sqrt[e]) - (b^2*PolyLo
g[3, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])])/(Sqrt[-d]
*Sqrt[e]) + (b^2*PolyLog[3, -((Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-
(c^2*d) + e]))])/(Sqrt[-d]*Sqrt[e]) - (b^2*PolyLog[3, (Sqrt[e]*E^ArcSinh[c*
x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])])/(Sqrt[-d]*Sqrt[e])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d+ex^2}\,dx = \int \left(\frac{\sqrt{-d}\left(a+b\sinh^{-1}(cx)\right)^2}{2d\left(\sqrt{-d}-\sqrt{e}\,x\right)}+\frac{\sqrt{-d}\left(a+b\sinh^{-1}(cx)\right)^2}{2d\left(\sqrt{-d}+\sqrt{e}\,x\right)}\right)dx$$

$$= -\frac{\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{-d}-\sqrt{e}\,x}\,dx}{2\sqrt{-d}}-\frac{\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{-d}+\sqrt{e}\,x}\,dx}{2\sqrt{-d}}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+bx)^2\cosh(x)}{c\sqrt{-d}-\sqrt{e}\,\sinh(x)}\,dx, x, \sinh^{-1}(cx)\right)}{2\sqrt{-d}}-\frac{\text{Subst}\left(\int \frac{(a+bx)^2\cosh(x)}{c\sqrt{-d}+\sqrt{e}\,\sinh(x)}\,dx, x, \sinh^{-1}(cx)\right.}{2\sqrt{-d}}$$

$$= -\frac{\text{Subst}\left(\int \frac{e^x(a+bx)^2}{c\sqrt{-d}-\sqrt{-c^2d+e}-\sqrt{e}\,e^x}\,dx, x, \sinh^{-1}(cx)\right)}{2\sqrt{-d}}-\frac{\text{Subst}\left(\int \frac{e^x(a+bx)^2}{c\sqrt{-d}+\sqrt{-c^2d+e}-\sqrt{e}\,e^x}\,d\right.}{2\sqrt{-d}}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1-\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+\frac{\sqrt{e}\,e^{\sin}}{c\sqrt{-d}-}\right.}{2\sqrt{-d}\,\sqrt{e}}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1-\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+\frac{\sqrt{e}\,e^{\sin}}{c\sqrt{-d}-}\right.}{2\sqrt{-d}\,\sqrt{e}}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1-\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+\frac{\sqrt{e}\,e^{\sin}}{c\sqrt{-d}-}\right.}{2\sqrt{-d}\,\sqrt{e}}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1-\frac{\sqrt{e}\,e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+\frac{\sqrt{e}\,e^{\sin}}{c\sqrt{-d}-}\right.}{2\sqrt{-d}\,\sqrt{e}}$$

Mathematica [A] time = 0.69, size = 985, normalized size = 1.33

$$\frac{2\sqrt{-d}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)a^2-2b\sqrt{d}\,\sinh^{-1}(cx)\log\left(\frac{e^{\sinh^{-1}(cx)}\sqrt{e}}{c\sqrt{-d}-\sqrt{e-c^2d}}+1\right)a+2b\sqrt{d}\,\sinh^{-1}(cx)\log\left(\frac{e^{\sinh^{-1}(cx)}\sqrt{e}}{\sqrt{e-c^2d}-c\sqrt{-d}}+1\right)a}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x^2),x]
```

```
[Out] (2*a^2*Sqrt[-d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - 2*a*b*Sqrt[d]*ArcSinh[c*x]*Lo
g[1 + (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])] - b^2*Sqr
t[d]*ArcSinh[c*x]^2*Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c
^2*d) + e])] + 2*a*b*Sqrt[d]*ArcSinh[c*x]*Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/
(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) + e])] + b^2*Sqrt[d]*ArcSinh[c*x]^2*Log[1 +
(Sqrt[e]*E^ArcSinh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) + e])] + 2*a*b*Sqrt
[d]*ArcSinh[c*x]*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*
d) + e])] + b^2*Sqrt[d]*ArcSinh[c*x]^2*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*
Sqrt[-d] + Sqrt[-(c^2*d) + e])] - 2*a*b*Sqrt[d]*ArcSinh[c*x]*Log[1 + (Sqrt[
e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])] - b^2*Sqrt[d]*ArcSinh
[c*x]^2*Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])]
 + 2*b*Sqrt[d]*(a + b*ArcSinh[c*x])*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*
Sqrt[-d] - Sqrt[-(c^2*d) + e])] - 2*b*Sqrt[d]*(a + b*ArcSinh[c*x])*PolyLog[
2, (Sqrt[e]*E^ArcSinh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) + e])] - 2*a*b*S
qrt[d]*PolyLog[2, -((Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) +
e]))] - 2*b^2*Sqrt[d]*ArcSinh[c*x]*PolyLog[2, -((Sqrt[e]*E^ArcSinh[c*x])/(c
```

```
*Sqrt[-d] + Sqrt[-(c^2*d) + e]))] + 2*a*b*Sqrt[d]*PolyLog[2, (Sqrt[e]*E^Arc
Sinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])] + 2*b^2*Sqrt[d]*ArcSinh[c*x]*
PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])] - 2*
b^2*Sqrt[d]*PolyLog[3, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d)
 + e])] + 2*b^2*Sqrt[d]*PolyLog[3, (Sqrt[e]*E^ArcSinh[c*x])/(-(c*Sqrt[-d])
+ Sqrt[-(c^2*d) + e])] + 2*b^2*Sqrt[d]*PolyLog[3, -((Sqrt[e]*E^ArcSinh[c*x]
)/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e]))] - 2*b^2*Sqrt[d]*PolyLog[3, (Sqrt[e]*E
^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d^2]*Sqrt[e])
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\operatorname{arsinh}(cx)^2 + 2\,ab\operatorname{arsinh}(cx) + a^2}{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d),x, algorithm="fricas")
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(e*x^2 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arsinh}(cx) + a)^2}{ex^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/(e*x^2 + d), x)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{arcsinh}(cx))^2}{e\,x^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(e*x^2+d),x)
[Out] int((a+b*arcsinh(c*x))^2/(e*x^2+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}} + \int \frac{b^2\log\left(cx + \sqrt{c^2x^2 + 1}\right)^2}{ex^2 + d} + \frac{2\,ab\log\left(cx + \sqrt{c^2x^2 + 1}\right)}{ex^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d),x, algorithm="maxima")
[Out] a^2*arctan(e*x/sqrt(d*e))/sqrt(d*e) + integrate(b^2*log(c*x + sqrt(c^2*x^2
+ 1))^2/(e*x^2 + d) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(e*x^2 + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\operatorname{asinh}(c\,x))^2}{e\,x^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(d + e*x^2),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(d + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{asinh}\,(cx))^2}{d + ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(e*x**2+d),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(d + e*x**2), x)
```

3.618 $\int \frac{(d+ex^2)^3}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=670

$$-\frac{5e^3\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{64bc^7}+\frac{9e^3\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{64bc^7}-\frac{5e^3\cosh\left(\frac{5a}{b}\right)\mathrm{Chi}\left(\frac{5\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{64bc^7}+\frac{e^3\,\mathrm{co}}{}$$

```
[Out] d^3*Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b/c-3/4*d^2*e*Chi((a+b*arcsinh(c*x)
)/b)*cosh(a/b)/b/c^3+3/8*d*e^2*Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b/c^5-5/
64*e^3*Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b/c^7+3/4*d^2*e*Chi(3*(a+b*arcsi
nh(c*x))/b)*cosh(3*a/b)/b/c^3-9/16*d*e^2*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3
*a/b)/b/c^5+9/64*e^3*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b/c^7+3/16*d*e
^2*Chi(5*(a+b*arcsinh(c*x))/b)*cosh(5*a/b)/b/c^5-5/64*e^3*Chi(5*(a+b*arcsin
h(c*x))/b)*cosh(5*a/b)/b/c^7+1/64*e^3*Chi(7*(a+b*arcsinh(c*x))/b)*cosh(7*a/
b)/b/c^7-d^3*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b/c+3/4*d^2*e*Shi((a+b*arc
sinh(c*x))/b)*sinh(a/b)/b/c^3-3/8*d*e^2*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)
/b/c^5+5/64*e^3*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b/c^7-3/4*d^2*e*Shi(3*(
a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b/c^3+9/16*d*e^2*Shi(3*(a+b*arcsinh(c*x))/
b)*sinh(3*a/b)/b/c^5-9/64*e^3*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b/c^7
-3/16*d*e^2*Shi(5*(a+b*arcsinh(c*x))/b)*sinh(5*a/b)/b/c^5+5/64*e^3*Shi(5*(a
+b*arcsinh(c*x))/b)*sinh(5*a/b)/b/c^7-1/64*e^3*Shi(7*(a+b*arcsinh(c*x))/b)*
sinh(7*a/b)/b/c^7
```

Rubi [A] time = 1.34, antiderivative size = 658, normalized size of antiderivative = 0.98, number of steps used = 42, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {5706, 5657, 3303, 3298, 3301, 5669, 5448}

$$-\frac{3d^2e\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4bc^3}+\frac{3d^2e\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{4bc^3}+\frac{3de^2\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(c\right.}{8bc^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-3*d^2*e*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]])/(4*b*c^3) + (3*d*e^2*
Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]])/(8*b*c^5) - (5*e^3*Cosh[a/b]*Co
shIntegral[a/b + ArcSinh[c*x]])/(64*b*c^7) + (3*d^2*e*Cosh[(3*a)/b]*CoshInt
egral[(3*a)/b + 3*ArcSinh[c*x]])/(4*b*c^3) - (9*d*e^2*Cosh[(3*a)/b]*CoshInt
egral[(3*a)/b + 3*ArcSinh[c*x]])/(16*b*c^5) + (9*e^3*Cosh[(3*a)/b]*CoshInte
gral[(3*a)/b + 3*ArcSinh[c*x]])/(64*b*c^7) + (3*d*e^2*Cosh[(5*a)/b]*CoshInt
egral[(5*a)/b + 5*ArcSinh[c*x]])/(16*b*c^5) - (5*e^3*Cosh[(5*a)/b]*CoshInte
gral[(5*a)/b + 5*ArcSinh[c*x]])/(64*b*c^7) + (e^3*Cosh[(7*a)/b]*CoshIntegra
l[(7*a)/b + 7*ArcSinh[c*x]])/(64*b*c^7) + (d^3*Cosh[a/b]*CoshIntegral[(a +
b*ArcSinh[c*x])/b])/(b*c) + (3*d^2*e*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c
*x]])/(4*b*c^3) - (3*d*e^2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(8*b
*c^5) + (5*e^3*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(64*b*c^7) - (3*
d^2*e*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(4*b*c^3) + (9*
d*e^2*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(16*b*c^5) - (9
*e^3*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(64*b*c^7) - (3*
d*e^2*Sinh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcSinh[c*x]])/(16*b*c^5) + (5
*e^3*Sinh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcSinh[c*x]])/(64*b*c^7) - (e^
3*Sinh[(7*a)/b]*SinhIntegral[(7*a)/b + 7*ArcSinh[c*x]])/(64*b*c^7) - (d^3*S
inh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(b*c)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^3}{a+b\sinh^{-1}(cx)}\,dx = \int \left(\frac{d^3}{a+b\sinh^{-1}(cx)}+\frac{3d^2ex^2}{a+b\sinh^{-1}(cx)}+\frac{3de^2x^4}{a+b\sinh^{-1}(cx)}+\frac{e^3x^6}{a+b\sinh^{-1}(cx)}\right)dx$$

$$= d^3\int \frac{1}{a+b\sinh^{-1}(cx)}\,dx+\left(3d^2e\right)\int \frac{x^2}{a+b\sinh^{-1}(cx)}\,dx+\left(3de^2\right)\int \frac{x^4}{a+b\sinh^{-1}(cx)}$$

$$= \frac{d^3\,\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\sinh^{-1}(cx)\right)}{bc}+\frac{\left(3d^2e\right)\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{a+bx}\,dx,x,\sin\right.}{c^3}$$

$$= \frac{\left(3d^2e\right)\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4(a+bx)}+\frac{\cosh(3x)}{4(a+bx)}\right)dx,x,\sinh^{-1}(cx)\right)}{c^3}+\frac{\left(3de^2\right)\text{Subst}\left(\int \left(\frac{\cosh(x)}{8(a+bx)}-\frac{3\,c}{16}\right.\right.}{}$$

$$= \frac{d^3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}-\frac{d^3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}-\frac{\left(3d^2e\right)\text{Subst}\left(\int \frac{\cosh(}{a+bx}\right.}{4c}$$

$$= \frac{d^3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}-\frac{d^3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}-\frac{\left(3d^2e\cosh\left(\frac{a}{b}\right)\right)\text{Subs}}{}$$

$$= -\frac{3d^2e\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4bc^3}+\frac{3de^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{8bc^5}-\frac{5e^3\cosh}{}$$

Mathematica [A] time = 1.03, size = 444, normalized size = 0.66

$$\frac{-64c^6d^3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+48c^4d^2e\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-48c^4d^2e\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+s\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3/(a + b*ArcSinh[c*x]),x]
```

```
[Out] ((64*c^6*d^3 - 48*c^4*d^2*e + 24*c^2*d*e^2 - 5*e^3)*Cosh[a/b]*CoshIntegral[
a/b + ArcSinh[c*x]] + 3*e*(16*c^4*d^2 - 12*c^2*d*e + 3*e^2)*Cosh[(3*a)/b]*C
oshIntegral[3*(a/b + ArcSinh[c*x])] + 12*c^2*d*e^2*Cosh[(5*a)/b]*CoshIntegr
al[5*(a/b + ArcSinh[c*x])] - 5*e^3*Cosh[(5*a)/b]*CoshIntegral[5*(a/b + ArcS
inh[c*x])] + e^3*Cosh[(7*a)/b]*CoshIntegral[7*(a/b + ArcSinh[c*x])] - 64*c^
6*d^3*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 48*c^4*d^2*e*Sinh[a/b]*S
inhIntegral[a/b + ArcSinh[c*x]] - 24*c^2*d*e^2*Sinh[a/b]*SinhIntegral[a/b +
 ArcSinh[c*x]] + 5*e^3*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - 48*c^4*
d^2*e*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] + 36*c^2*d*e^2*Sin
h[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] - 9*e^3*Sinh[(3*a)/b]*SinhI
ntegral[3*(a/b + ArcSinh[c*x])] - 12*c^2*d*e^2*Sinh[(5*a)/b]*SinhIntegral[5
*(a/b + ArcSinh[c*x])] + 5*e^3*Sinh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[
c*x])] - e^3*Sinh[(7*a)/b]*SinhIntegral[7*(a/b + ArcSinh[c*x])])/(64*b*c^7)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3}{b\,\text{arsinh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3)/(b*arcsinh(c*x) + a),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^3}{b\,\text{arsinh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^3/(b*arcsinh(c*x) + a), x)
```

maple [A] time = 0.47, size = 654, normalized size = 0.98

$$-\frac{e^3 e^{-\frac{7a}{b}} \text{Ei}\left(1,-7\,\text{arcsinh}(cx)-\frac{7a}{b}\right)}{128c^6 b}-\frac{e^3 e^{\frac{7a}{b}} \text{Ei}\left(1,7\,\text{arcsinh}(cx)+\frac{7a}{b}\right)}{128c^6 b}-\frac{e^{\frac{a}{b}} \text{Ei}\left(1,\text{arcsinh}(cx)+\frac{a}{b}\right)d^3}{2b}+\frac{3\,e^{\frac{a}{b}} \text{Ei}\left(1,\text{arcsinh}(cx)+\frac{a}{b}\right)d^2 e}{8c^2 b}-\frac{3\,e^{\frac{a}{b}} \text{Ei}(1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3/(a+b*arcsinh(c*x)),x)
[Out] 1/c*(-1/128/c^6*e^3/b*exp(-7*a/b)*Ei(1,-7*arcsinh(c*x)-7*a/b)-1/128/c^6*e^3
/b*exp(7*a/b)*Ei(1,7*arcsinh(c*x)+7*a/b)-1/2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a
/b)*d^3+3/8/c^2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*d^2*e-3/16/c^4/b*exp(a/b)
*Ei(1,arcsinh(c*x)+a/b)*d*e^2+5/128/c^6/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*e
^3-1/2/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*d^3+3/8/c^2/b*exp(-a/b)*Ei(1,-ar
csinh(c*x)-a/b)*d^2*e-3/16/c^4/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*d*e^2+5/
128/c^6/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*e^3-3/8/c^2*e/b*exp(3*a/b)*Ei(1
,3*arcsinh(c*x)+3*a/b)*d^2+9/32/c^4*e^2/b*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*
a/b)*d-9/128/c^6*e^3/b*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-3/8/c^2*e/b*ex
p(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)*d^2+9/32/c^4*e^2/b*exp(-3*a/b)*Ei(1,-
3*arcsinh(c*x)-3*a/b)*d-9/128/c^6*e^3/b*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*
a/b)-3/32/c^4*e^2/b*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)*d+5/128/c^6*e^3/b
*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)-3/32/c^4*e^2/b*exp(-5*a/b)*Ei(1,-5*a
rcsinh(c*x)-5*a/b)*d+5/128/c^6*e^3/b*exp(-5*a/b)*Ei(1,-5*arcsinh(c*x)-5*a/b
))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^3}{b\,\text{arsinh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^3/(b*arcsinh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)^3}{a+b\,\text{asinh}\,(c\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^3/(a + b*asinh(c*x)),x)
[Out] int((d + e*x^2)^3/(a + b*asinh(c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex^2\right)^3}{a+b\,\text{asinh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3/(a+b*asinh(c*x)),x)
```

```
[Out] Integral((d + e*x**2)**3/(a + b*asinh(c*x)), x)
```

3.619 $\int \frac{(d+ex^2)^2}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=388

$$\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8bc^5}-\frac{3e^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16bc^5}+\frac{e^2\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16bc^5}-\frac{e^2\sin}{}$$

```
[Out] d^2*Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b/c-1/2*d*e*Chi((a+b*arcsinh(c*x))/
b)*cosh(a/b)/b/c^3+1/8*e^2*Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b/c^5+1/2*d*
e*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b/c^3-3/16*e^2*Chi(3*(a+b*arcsinh
(c*x))/b)*cosh(3*a/b)/b/c^5+1/16*e^2*Chi(5*(a+b*arcsinh(c*x))/b)*cosh(5*a/b
)/b/c^5-d^2*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b/c+1/2*d*e*Shi((a+b*arcsin
h(c*x))/b)*sinh(a/b)/b/c^3-1/8*e^2*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b/c^
5-1/2*d*e*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b/c^3+3/16*e^2*Shi(3*(a+b
*arcsinh(c*x))/b)*sinh(3*a/b)/b/c^5-1/16*e^2*Shi(5*(a+b*arcsinh(c*x))/b)*si
nh(5*a/b)/b/c^5
```

Rubi [A] time = 0.79, antiderivative size = 380, normalized size of antiderivative = 0.98, number of steps used = 27, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {5706, 5657, 3303, 3298, 3301, 5669, 5448}

$$-\frac{de\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{2bc^3}+\frac{de\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{2bc^3}+\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{8bc^5}-$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(d*e*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]])/(2*b*c^3) + (e^2*Cosh[a/b
]*CoshIntegral[a/b + ArcSinh[c*x]])/(8*b*c^5) + (d*e*Cosh[(3*a)/b]*CoshInte
gral[(3*a)/b + 3*ArcSinh[c*x]])/(2*b*c^3) - (3*e^2*Cosh[(3*a)/b]*CoshIntegr
al[(3*a)/b + 3*ArcSinh[c*x]])/(16*b*c^5) + (e^2*Cosh[(5*a)/b]*CoshIntegral[
(5*a)/b + 5*ArcSinh[c*x]])/(16*b*c^5) + (d^2*Cosh[a/b]*CoshIntegral[(a + b*
ArcSinh[c*x])/b])/(b*c) + (d*e*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/
(2*b*c^3) - (e^2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(8*b*c^5) - (d
*e*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(2*b*c^3) + (3*e^2
*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(16*b*c^5) - (e^2*Si
nh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcSinh[c*x]])/(16*b*c^5) - (d^2*Sinh[
a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(b*c)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2}{a+b\sinh^{-1}(cx)}\,dx = \int \left(\frac{d^2}{a+b\sinh^{-1}(cx)}+\frac{2dex^2}{a+b\sinh^{-1}(cx)}+\frac{e^2x^4}{a+b\sinh^{-1}(cx)}\right)dx$$

$$= d^2\int \frac{1}{a+b\sinh^{-1}(cx)}\,dx+(2de)\int \frac{x^2}{a+b\sinh^{-1}(cx)}\,dx+e^2\int \frac{x^4}{a+b\sinh^{-1}(cx)}\,dx$$

$$= \frac{d^2\,\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\sinh^{-1}(cx)\right)}{bc}+\frac{(2de)\,\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{a+bx}\,dx,x,\sinh\right.}{c^3}$$

$$= \frac{(2de)\,\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4(a+bx)}+\frac{\cosh(3x)}{4(a+bx)}\right)dx,x,\sinh^{-1}(cx)\right)}{c^3}+\frac{e^2\,\text{Subst}\left(\int \left(\frac{\cosh(x)}{8(a+bx)}-\frac{3\cosh(3x)}{16(a+bx)}\right.\right.}{c}$$

$$= \frac{d^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}-\frac{d^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}-\frac{(de)\,\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\right.}{2c^3}$$

$$= \frac{d^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}-\frac{d^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}-\frac{\left(de\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\right.}{}$$

$$= -\frac{de\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{2bc^3}+\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{8bc^5}+\frac{de\cosh\left(\frac{3a}{b}\right)}{}$$

Mathematica [A] time = 0.55, size = 253, normalized size = 0.65

$$-16c^4d^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+e\cosh\left(\frac{3a}{b}\right)\left(8c^2d-3e\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+8c^2de\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (2*(8*c^4*d^2 - 4*c^2*d*e + e^2)*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]]
 + (8*c^2*d - 3*e)*e*Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] + e
^2*Cosh[(5*a)/b]*CoshIntegral[5*(a/b + ArcSinh[c*x])] - 16*c^4*d^2*Sinh[a/b
]*SinhIntegral[a/b + ArcSinh[c*x]] + 8*c^2*d*e*Sinh[a/b]*SinhIntegral[a/b +
 ArcSinh[c*x]] - 2*e^2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - 8*c^2*d
*e*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] + 3*e^2*Sinh[(3*a)/b]
*SinhIntegral[3*(a/b + ArcSinh[c*x])] - e^2*Sinh[(5*a)/b]*SinhIntegral[5*(a
/b + ArcSinh[c*x])])/(16*b*c^5)
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{e^2x^4 + 2\,dex^2 + d^2}{b\,\mathrm{arsinh}\,(cx) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^4 + 2*d*e*x^2 + d^2)/(b*arcsinh(c*x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2 + d\right)^2}{b\,\mathrm{arsinh}\,(cx) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2/(b*arcsinh(c*x) + a), x)
```

maple [A] time = 0.29, size = 380, normalized size = 0.98

$$-\frac{e^2 \mathrm{e}^{-\frac{5a}{b}}\,\mathrm{Ei}\left(1,-5\,\mathrm{arcsinh}(cx)-\frac{5a}{b}\right)}{32c^4b}-\frac{e^2 \mathrm{e}^{\frac{5a}{b}}\,\mathrm{Ei}\left(1,5\,\mathrm{arcsinh}(cx)+\frac{5a}{b}\right)}{32c^4b}-\frac{\mathrm{e}^{\frac{a}{b}}\,\mathrm{Ei}\left(1,\mathrm{arcsinh}(cx)+\frac{a}{b}\right)d^2}{2b}+\frac{\mathrm{e}^{\frac{a}{b}}\,\mathrm{Ei}\left(1,\mathrm{arcsinh}(cx)+\frac{a}{b}\right)de}{4c^2b}-\frac{\mathrm{e}^{\frac{a}{b}}\,\mathrm{Ei}\left(1,\mathrm{arcs}\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c*(-1/32/c^4*e^2/b*exp(-5*a/b)*Ei(1,-5*arcsinh(c*x)-5*a/b)-1/32/c^4*e^2/b
*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)-1/2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b
)*d^2+1/4/c^2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*d*e-1/16/c^4/b*exp(a/b)*Ei(
1,arcsinh(c*x)+a/b)*e^2-1/2/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*d^2+1/4/c^2
/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*d*e-1/16/c^4/b*exp(-a/b)*Ei(1,-arcsinh
(c*x)-a/b)*e^2-1/4/c^2*e/b*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)*d+3/32/c^4
*e^2/b*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-1/4/c^2*e/b*exp(-3*a/b)*Ei(1,-
3*arcsinh(c*x)-3*a/b)*d+3/32/c^4*e^2/b*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a
/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2 + d\right)^2}{b\,\mathrm{arsinh}\,(cx) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^2/(b*arcsinh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)^2}{a+b\,\mathrm{asinh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2/(a + b*asinh(c*x)),x)
```

```
[Out] int((d + e*x^2)^2/(a + b*asinh(c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex^2\right)^2}{a+b\,\mathrm{asinh}\left(cx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(a+b*asinh(c*x)),x)
```

```
[Out] Integral((d + e*x**2)**2/(a + b*asinh(c*x)), x)
```

3.620 $\int \frac{d+ex^2}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=180

$$-\frac{e\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4bc^3}+\frac{e\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{4bc^3}+\frac{e\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4bc^3}-\frac{e\sinh\left(\frac{3a}{b}\right)\text{S}}{}$$

```
[Out] d*Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b/c-1/4*e*Chi((a+b*arcsinh(c*x))/b)*c
osh(a/b)/b/c^3+1/4*e*Chi(3*(a+b*arcsinh(c*x))/b)*cosh(3*a/b)/b/c^3-d*Shi((a
+b*arcsinh(c*x))/b)*sinh(a/b)/b/c+1/4*e*Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)
/b/c^3-1/4*e*Shi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b/c^3
```

Rubi [A] time = 0.37, antiderivative size = 176, normalized size of antiderivative = 0.98, number of steps used = 15, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {5706, 5657, 3303, 3298, 3301, 5669, 5448}

$$-\frac{e\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4bc^3}+\frac{e\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{4bc^3}+\frac{e\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4bc^3}-\frac{e\,\text{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(e*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]])/(4*b*c^3) + (e*Cosh[(3*a)/b
]*CoshIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(4*b*c^3) + (d*Cosh[a/b]*CoshInte
gral[(a + b*ArcSinh[c*x])/b])/(b*c) + (e*Sinh[a/b]*SinhIntegral[a/b + ArcSi
nh[c*x]])/(4*b*c^3) - (e*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x
]])/(4*b*c^3) - (d*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(b*c)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
```

```
c, n}, x]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{d+ex^2}{a+b\sinh^{-1}(cx)}\,dx = \int \left(\frac{d}{a+b\sinh^{-1}(cx)}+\frac{ex^2}{a+b\sinh^{-1}(cx)}\right)dx$$

$$= d\int \frac{1}{a+b\sinh^{-1}(cx)}\,dx + e\int \frac{x^2}{a+b\sinh^{-1}(cx)}\,dx$$

$$= \frac{d\,\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\sinh^{-1}(cx)\right)}{bc} + \frac{e\,\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{a+bx}\,dx, x, \sinh^{-1}(c$$

$$= \frac{e\,\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4(a+bx)}+\frac{\cosh(3x)}{4(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{c^3} + \frac{\left(d\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x$$

$$= \frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc} - \frac{d\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc} - \frac{e\,\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x,\right.}{4c^3}$$

$$= \frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc} - \frac{d\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc} - \frac{\left(e\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \right.}{}$$

$$= -\frac{e\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4bc^3} + \frac{e\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{4bc^3} + \frac{d\cosh\left(\frac{a}{b}\right)\text{C}}{}$$

Mathematica [A] time = 0.25, size = 126, normalized size = 0.70

$$\frac{\cosh\left(\frac{a}{b}\right)\left(4c^2d-e\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-4c^2d\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+e\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(\right.\right.}{4bc^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(a + b*ArcSinh[c*x]),x]
```

```
[Out] ((4*c^2*d - e)*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] + e*Cosh[(3*a)/b]
*CoshIntegral[3*(a/b + ArcSinh[c*x])] - 4*c^2*d*Sinh[a/b]*SinhIntegral[a/b
+ ArcSinh[c*x]] + e*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - e*Sinh[(3*
a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])])/(4*b*c^3)
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ex^2+d}{b\,\text{arsinh}\,(cx)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((e*x^2 + d)/(b*arcsinh(c*x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2 + d}{b \operatorname{arsinh}(cx) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)/(b*arcsinh(c*x) + a), x)
```

maple [A] time = 0.20, size = 178, normalized size = 0.99

$$\frac{-\frac{e\,e^{-\frac{3a}{b}}\operatorname{Ei}\left(1,-3\operatorname{arcsinh}(cx)-\frac{3a}{b}\right)}{8c^2b}-\frac{e\,e^{\frac{3a}{b}}\operatorname{Ei}\left(1,3\operatorname{arcsinh}(cx)+\frac{3a}{b}\right)}{8c^2b}-\frac{e^{\frac{a}{b}}\operatorname{Ei}\left(1,\operatorname{arcsinh}(cx)+\frac{a}{b}\right)d}{2b}+\frac{e^{\frac{a}{b}}\operatorname{Ei}\left(1,\operatorname{arcsinh}(cx)+\frac{a}{b}\right)e}{8c^2b}-\frac{e^{-\frac{a}{b}}\operatorname{Ei}\left(1,-\operatorname{arcs}\right.}{2b}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(a+b*arcsinh(c*x)),x)
[Out] 1/c*(-1/8/c^2*e/b*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)-1/8/c^2*e/b*exp(3
*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-1/2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*d+1/
8/c^2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*e-1/2/b*exp(-a/b)*Ei(1,-arcsinh(c*x
)-a/b)*d+1/8/c^2/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2 + d}{b \operatorname{arsinh}(cx) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)/(b*arcsinh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e\,x^2 + d}{a + b\,\operatorname{asinh}(c\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(a + b*asinh(c*x)),x)
[Out] int((d + e*x^2)/(a + b*asinh(c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2}{a + b \operatorname{asinh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(a+b*asinh(c*x)),x)
[Out] Integral((d + e*x**2)/(a + b*asinh(c*x)), x)
```

3.621 $\int \frac{1}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=54

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}$$

```
[Out] Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b/c-Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)
/b/c
```

Rubi [A] time = 0.07, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5657, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^(-1),x]
```

```
[Out] (Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*x])/b])/(b*c) - (Sinh[a/b]*SinhInt
egral[(a + b*ArcSinh[c*x])/b])/(b*c)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rubi steps

$$\int \frac{1}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\sinh^{-1}(cx)\right)}{bc}$$
$$= \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx,x,a+b\sinh^{-1}(cx)\right)}{bc} - \frac{\sinh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx,x,\right.}{bc}$$
$$= \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc} - \frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 0.83

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^(-1),x]
```

```
[Out] (Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - Sinh[a/b]*SinhIntegral[a/b +
ArcSinh[c*x]])/(b*c)
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b\,\text{arsinh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*arcsinh(c*x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\,\text{arsinh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/(b*arcsinh(c*x) + a), x)
```

maple [A] time = 0.03, size = 56, normalized size = 1.04

$$\frac{-\frac{e^{\frac{a}{b}}\,\text{Ei}\left(1,\text{arcsinh}(cx)+\frac{a}{b}\right)}{2b}-\frac{e^{-\frac{a}{b}}\,\text{Ei}\left(1,-\text{arcsinh}(cx)-\frac{a}{b}\right)}{2b}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c*(-1/2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/2/b*exp(-a/b)*Ei(1,-arcsinh(c
*x)-a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\,\text{arsinh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*arcsinh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{a + b\,\mathrm{asinh}\,(c\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(c*x)),x)
```

```
[Out] int(1/(a + b*asinh(c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b\,\mathrm{asinh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(1/(a + b*asinh(c*x)), x)
```

3.622 $\int \frac{1}{(d+ex^2)(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=23

$$\operatorname{Int}\left(\frac{1}{(d+ex^2)\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)/(a+b*arcsinh(c*x)),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(d+ex^2)\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][1/((d + e*x^2)*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{1}{(d+ex^2)\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[1/((d + e*x^2)*(a + b*ArcSinh[c*x])), x]
```

fricas [A] time = 1.04, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{1}{aex^2+ad+\left(bex^2+bd\right)\operatorname{arsinh}(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(1/(a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsinh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex^2+d)(b\operatorname{arsinh}(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)*(b*arcsinh(c*x) + a)), x)
```

maple [A] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\,x^{2}+d\right)\left(a+b\,\mathrm{arcsinh}\left(c x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(a+b*arcsinh(c*x)),x)
[Out] int(1/(e*x^2+d)/(a+b*arcsinh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex^{2}+d\right)\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(1/((e*x^2 + d)*(b*arcsinh(c*x) + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\left(e\,x^{2}+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))*(d + e*x^2)),x)
[Out] int(1/((a + b*asinh(c*x))*(d + e*x^2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(cx\right)\right)\left(d+ex^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(a+b*asinh(c*x)),x)
[Out] Integral(1/((a + b*asinh(c*x))*(d + e*x**2)), x)
```

3.623 $\int \frac{1}{(d+ex^2)^2(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(d+ex^2)^2\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)^2/(a+b*arcsinh(c*x)),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(d+ex^2)^2\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^2\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{1}{(d+ex^2)^2\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 3.48, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)^2\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])), x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{ae^2x^4+2\,adex^2+ad^2+\left(be^2x^4+2\,bdex^2+bd^2\right)\text{arsinh}\,(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(1/(a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*
d^2)*arcsinh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex^2+d)^2(b\,\text{arsinh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate(1/((e*x^2 + d)^2*(b*arcsinh(c*x) + a)), x)
```

maple [**A**] time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\,x^{2}+d\right)^{2}\left(a+b\,\mathrm{arcsinh}\left(c x\right)\right)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(a+b*arcsinh(c*x)),x)
[Out] int(1/(e*x^2+d)^2/(a+b*arcsinh(c*x)),x)
```

maxima [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex^{2}+d\right)^{2}\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(1/((e*x^2 + d)^2*(b*arcsinh(c*x) + a)), x)
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(e\,x^2+d\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))*(d + e*x^2)^2),x)
[Out] int(1/((a + b*asinh(c*x))*(d + e*x^2)^2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(a+b*asinh(c*x)),x)
[Out] Timed out
```

3.624 $\int \frac{(d+ex^2)^2}{(a+b\sinh^{-1}(cx))^2} dx$

Optimal. Leaf size=495

$$-\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8b^2c^5}+\frac{9e^2\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{16b^2c^5}-\frac{5e^2\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\sinh^{-1}(cx))}{b}\right)}{16b^2c^5}+\frac{e^2 c}{}$$

```
[Out] d^2*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b^2/c-1/2*d*e*cosh(a/b)*Shi((a+b*ar
csinh(c*x))/b)/b^2/c^3+1/8*e^2*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b^2/c^5+
3/2*d*e*cosh(3*a/b)*Shi(3*(a+b*arcsinh(c*x))/b)/b^2/c^3-9/16*e^2*cosh(3*a/b
)*Shi(3*(a+b*arcsinh(c*x))/b)/b^2/c^5+5/16*e^2*cosh(5*a/b)*Shi(5*(a+b*arcsi
nh(c*x))/b)/b^2/c^5-d^2*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b^2/c+1/2*d*e*C
hi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b^2/c^3-1/8*e^2*Chi((a+b*arcsinh(c*x))/b
)*sinh(a/b)/b^2/c^5-3/2*d*e*Chi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^3
+9/16*e^2*Chi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^5-5/16*e^2*Chi(5*(a
+b*arcsinh(c*x))/b)*sinh(5*a/b)/b^2/c^5-d^2*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcs
inh(c*x))-2*d*e*x^2*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))-e^2*x^4*(c^2*x
^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))
```

Rubi [A] time = 0.87, antiderivative size = 483, normalized size of antiderivative = 0.98, number of steps used = 26, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {5706, 5655, 5779, 3303, 3298, 3301, 5665}

$$\frac{de\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{2b^2c^3}-\frac{3de\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{2b^2c^3}-\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{8b^2c^5}+$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((d^2*Sqrt[1 + c^2*x^2])/(b*c*(a + b*ArcSinh[c*x]))) - (2*d*e*x^2*Sqrt[1 +
 c^2*x^2])/(b*c*(a + b*ArcSinh[c*x])) - (e^2*x^4*Sqrt[1 + c^2*x^2])/(b*c*(a
 + b*ArcSinh[c*x])) - (d^2*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b])/(b^2
*c) + (d*e*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b])/(2*b^2*c^3) - (e^2*C
oshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b])/(8*b^2*c^5) - (3*d*e*CoshIntegra
l[(3*a)/b + 3*ArcSinh[c*x]]*Sinh[(3*a)/b])/(2*b^2*c^3) + (9*e^2*CoshIntegra
l[(3*a)/b + 3*ArcSinh[c*x]]*Sinh[(3*a)/b])/(16*b^2*c^5) - (5*e^2*CoshIntegr
al[(5*a)/b + 5*ArcSinh[c*x]]*Sinh[(5*a)/b])/(16*b^2*c^5) + (d^2*Cosh[a/b]*S
inhIntegral[a/b + ArcSinh[c*x]])/(b^2*c) - (d*e*Cosh[a/b]*SinhIntegral[a/b
+ ArcSinh[c*x]])/(2*b^2*c^3) + (e^2*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*
x]])/(8*b^2*c^5) + (3*d*e*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*
x]])/(2*b^2*c^3) - (9*e^2*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*
x]])/(16*b^2*c^5) + (5*e^2*Cosh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcSinh[c
*x]])/(16*b^2*c^5)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{(d+ex^2)^2}{(a+b\sinh^{-1}(cx))^2}\,dx = \int \left(\frac{d^2}{(a+b\sinh^{-1}(cx))^2} + \frac{2dex^2}{(a+b\sinh^{-1}(cx))^2} + \frac{e^2x^4}{(a+b\sinh^{-1}(cx))^2}\right)dx$$

$$= d^2\int \frac{1}{(a+b\sinh^{-1}(cx))^2}\,dx + (2de)\int \frac{x^2}{(a+b\sinh^{-1}(cx))^2}\,dx + e^2\int \frac{x^4}{(a+b\sinh$$

$$= -\frac{d^2\sqrt{1+c^2x^2}}{bc(a+b\sinh^{-1}(cx))} - \frac{2dex^2\sqrt{1+c^2x^2}}{bc(a+b\sinh^{-1}(cx))} - \frac{e^2x^4\sqrt{1+c^2x^2}}{bc(a+b\sinh^{-1}(cx))} + \frac{(cd^2)\int}{}$$

$$= -\frac{d^2\sqrt{1+c^2x^2}}{bc(a+b\sinh^{-1}(cx))} - \frac{2dex^2\sqrt{1+c^2x^2}}{bc(a+b\sinh^{-1}(cx))} - \frac{e^2x^4\sqrt{1+c^2x^2}}{bc(a+b\sinh^{-1}(cx))} + \frac{d^2\,\text{Subs}}{}$$

$$= -\frac{d^2\sqrt{1+c^2x^2}}{bc(a+b\sinh^{-1}(cx))} - \frac{2dex^2\sqrt{1+c^2x^2}}{bc(a+b\sinh^{-1}(cx))} - \frac{e^2x^4\sqrt{1+c^2x^2}}{bc(a+b\sinh^{-1}(cx))} + \frac{(d^2\cos}{}$$

$$= -\frac{d^2\sqrt{1+c^2x^2}}{bc(a+b\sinh^{-1}(cx))} - \frac{2dex^2\sqrt{1+c^2x^2}}{bc(a+b\sinh^{-1}(cx))} - \frac{e^2x^4\sqrt{1+c^2x^2}}{bc(a+b\sinh^{-1}(cx))} - \frac{d^2\text{Chi}(}{}$$

Mathematica [A] time = 2.08, size = 356, normalized size = 0.72

$$-\frac{-16c^4d^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+3e\sinh\left(\frac{3a}{b}\right)\left(8c^2d-3e\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+8c^2de\cosh\left(\frac{a}{b}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -1/16*((16*b*c^4*d^2*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (32*b*c^4*d*
e*x^2*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (16*b*c^4*e^2*x^4*Sqrt[1 +
c^2*x^2])/(a + b*ArcSinh[c*x]) + 2*(8*c^4*d^2 - 4*c^2*d*e + e^2)*CoshIntegr
al[a/b + ArcSinh[c*x]]*Sinh[a/b] + 3*(8*c^2*d - 3*e)*e*CoshIntegral[3*(a/b
+ ArcSinh[c*x])]*Sinh[(3*a)/b] + 5*e^2*CoshIntegral[5*(a/b + ArcSinh[c*x])]
*Sinh[(5*a)/b] - 16*c^4*d^2*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 8*
c^2*d*e*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - 2*e^2*Cosh[a/b]*SinhIn
tegral[a/b + ArcSinh[c*x]] - 24*c^2*d*e*Cosh[(3*a)/b]*SinhIntegral[3*(a/b +
 ArcSinh[c*x])] + 9*e^2*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])]
- 5*e^2*Cosh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[c*x])]])/(b^2*c^5)
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{e^2x^4+2\,dex^2+d^2}{b^2\,\text{arsinh}\,(cx)^2+2\,ab\,\text{arsinh}\,(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^4 + 2*d*e*x^2 + d^2)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*
x) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2+d)^2}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2/(b*arcsinh(c*x) + a)^2, x)
```

maple [B] time = 0.40, size = 1036, normalized size = 2.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c*(1/32*(16*c^5*x^5-16*c^4*x^4*(c^2*x^2+1)^(1/2)+20*c^3*x^3-12*c^2*x^2*(c
^2*x^2+1)^(1/2)+5*c*x-(c^2*x^2+1)^(1/2))*e^2/c^4/b/(a+b*arcsinh(c*x))+5/32*
e^2/c^4/b^2*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)-1/32/b*e^2/c^4*(16*c^5*x^
5+20*c^3*x^3+16*c^4*x^4*(c^2*x^2+1)^(1/2)+5*c*x+12*c^2*x^2*(c^2*x^2+1)^(1/2
)+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))-5/32/b^2*e^2/c^4*exp(-5*a/b)*Ei(1,-
5*arcsinh(c*x)-5*a/b)+1/2*(c*x-(c^2*x^2+1)^(1/2))*d^2/b/(a+b*arcsinh(c*x))+
1/2*d^2/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/4*(c*x-(c^2*x^2+1)^(1/2))*d*e
/c^2/b/(a+b*arcsinh(c*x))-1/4/c^2*d*e/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)+1
/16*(c*x-(c^2*x^2+1)^(1/2))*e^2/c^4/b/(a+b*arcsinh(c*x))+1/16/c^4*e^2/b^2*e
xp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/2/b*d^2*(c*x+(c^2*x^2+1)^(1/2))/(a+b*arcsi
nh(c*x))-1/2/b^2*d^2*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)+1/4/c^2/b*d*e*(c*x+(
c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))+1/4/c^2/b^2*d*e*exp(-a/b)*Ei(1,-arcsin
h(c*x)-a/b)-1/16/c^4/b*e^2*(c*x+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))-1/16/
c^4/b^2*e^2*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)+1/4*(4*c^3*x^3-4*c^2*x^2*(c^2
*x^2+1)^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))*d*e/c^2/b/(a+b*arcsinh(c*x))-3/32*(4
*c^3*x^3-4*c^2*x^2*(c^2*x^2+1)^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))*e^2/c^4/b/(a+
b*arcsinh(c*x))+3/4*e/c^2/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)*d-9/32*
e^2/c^4/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-1/4/c^2*e/b*(4*c^3*x^3+3*
c*x+4*c^2*x^2*(c^2*x^2+1)^(1/2)+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))*d+3/3
2/c^4*e^2/b*(4*c^3*x^3+3*c*x+4*c^2*x^2*(c^2*x^2+1)^(1/2)+(c^2*x^2+1)^(1/2))
/(a+b*arcsinh(c*x))-3/4/c^2*e/b^2*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)*d
+9/32/c^4*e^2/b^2*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^3e^2x^7 + \left(2\,c^3de + ce^2\right)x^5 + cd^2x + \left(c^3d^2 + 2\,cde\right)x^3 + \left(c^2e^2x^6 + \left(2\,c^2de + e^2\right)x^4 + \left(c^2d^2 + 2\,de\right)x^2 + d^2\right)\sqrt{c^2x^2 +}}{abc^3x^2 + \sqrt{c^2x^2+1}\,abc^2x + abc + \left(b^2c^3x^2 + \sqrt{c^2x^2+1}\,b^2c^2x + b^2c\right)\log\left(cx + \sqrt{c^2x^2+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*e^2*x^7 + (2*c^3*d*e + c*e^2)*x^5 + c*d^2*x + (c^3*d^2 + 2*c*d*e)*x^3
 + (c^2*e^2*x^6 + (2*c^2*d*e + e^2)*x^4 + (c^2*d^2 + 2*d*e)*x^2 + d^2)*sqrt
(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^
3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1)))
+ integrate((5*c^5*e^2*x^8 + 2*(3*c^5*d*e + 5*c^3*e^2)*x^6 + (c^5*d^2 + 12*
c^3*d*e + 5*c*e^2)*x^4 + c*d^2 + 2*(c^3*d^2 + 3*c*d*e)*x^2 + (5*c^3*e^2*x^6
 + 3*(2*c^3*d*e + c*e^2)*x^4 - c*d^2 + (c^3*d^2 + 2*c*d*e)*x^2)*(c^2*x^2 +
1) + (10*c^4*e^2*x^7 + (12*c^4*d*e + 13*c^2*e^2)*x^5 + 2*(c^4*d^2 + 7*c^2*d
*e + 2*e^2)*x^3 + (c^2*d^2 + 4*d*e)*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c
^2*x^2 + 1)*a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 +
 1)*b^2*c^3*x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(
c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sq
rt(c^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)^2}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2/(a + b*asinh(c*x))^2,x)
```

```
[Out] int((d + e*x^2)^2/(a + b*asinh(c*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex^2\right)^2}{\left(a+b\,\mathrm{asinh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((d + e*x**2)**2/(a + b*asinh(c*x))**2, x)
```

3.625 $\int \frac{d+ex^2}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=247

$$\frac{e\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4b^2c^3}-\frac{3e\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{4b^2c^3}-\frac{e\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4b^2c^3}+\frac{3e\cosh\left(\frac{3a}{b}\right)\text{Shi}}{4b}$$

```
[Out] d*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b^2/c-1/4*e*cosh(a/b)*Shi((a+b*arcsin
h(c*x))/b)/b^2/c^3+3/4*e*cosh(3*a/b)*Shi(3*(a+b*arcsinh(c*x))/b)/b^2/c^3-d*
Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b^2/c+1/4*e*Chi((a+b*arcsinh(c*x))/b)*s
inh(a/b)/b^2/c^3-3/4*e*Chi(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^3-d*(c
^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))-e*x^2*(c^2*x^2+1)^(1/2)/b/c/(a+b*arc
sinh(c*x))
```

Rubi [A] time = 0.48, antiderivative size = 239, normalized size of antiderivative = 0.97, number of steps used = 15, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5706, 5655, 5779, 3303, 3298, 3301, 5665}

$$\frac{e\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4b^2c^3}-\frac{3e\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{4b^2c^3}-\frac{e\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4b^2c^3}+\frac{3e\cosh}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((d*Sqrt[1 + c^2*x^2])/(b*c*(a + b*ArcSinh[c*x]))) - (e*x^2*Sqrt[1 + c^2*x
^2])/(b*c*(a + b*ArcSinh[c*x])) - (d*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[
a/b])/(b^2*c) + (e*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b])/(4*b^2*c^3)
- (3*e*CoshIntegral[(3*a)/b + 3*ArcSinh[c*x]]*Sinh[(3*a)/b])/(4*b^2*c^3) +
(d*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(b^2*c) - (e*Cosh[a/b]*SinhI
ntegral[a/b + ArcSinh[c*x]])/(4*b^2*c^3) + (3*e*Cosh[(3*a)/b]*SinhIntegral[
(3*a)/b + 3*ArcSinh[c*x]])/(4*b^2*c^3)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
```

```
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{d+ex^2}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = \int \left(\frac{d}{\left(a+b\sinh^{-1}(cx)\right)^2}+\frac{ex^2}{\left(a+b\sinh^{-1}(cx)\right)^2}\right)dx$$

$$= d\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx + e\int \frac{x^2}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

$$= -\frac{d\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{ex^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{(cd)\int \frac{x}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx}{b} -$$

$$= -\frac{d\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{ex^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{d\,\mathrm{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(\right.}{bc}$$

$$= -\frac{d\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{ex^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\left(d\cosh\left(\frac{a}{b}\right)\right)\mathrm{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\right.}{bc}$$

$$= -\frac{d\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{ex^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{d\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{b^2c}$$

Mathematica [A] time = 0.91, size = 190, normalized size = 0.77

$$-\frac{\sinh\left(\frac{a}{b}\right)\left(4c^2d-e\right)\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-4c^2d\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+\frac{4bc^2d\sqrt{c^2x^2+1}}{a+b\sinh^{-1}(cx)}+\frac{4bc^2ex^2\sqrt{c^2x^2+1}}{a+b\sinh^{-1}(cx)}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -1/4*((4*b*c^2*d*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (4*b*c^2*e*x^2*S
qrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (4*c^2*d - e)*CoshIntegral[a/b + A
rcSinh[c*x]]*Sinh[a/b] + 3*e*CoshIntegral[3*(a/b + ArcSinh[c*x])]*Sinh[(3*a
)/b] - 4*c^2*d*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + e*Cosh[a/b]*Sin
hIntegral[a/b + ArcSinh[c*x]] - 3*e*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + Arc
Sinh[c*x])])/(b^2*c^3)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{ex^2+d}{b^2\operatorname{arsinh}(cx)^2+2\,ab\operatorname{arsinh}(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{ex^2+d}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/(b*arcsinh(c*x) + a)^2, x)
```

maple [A] time = 0.25, size = 438, normalized size = 1.77

$$\frac{\frac{\left(4c^3x^3-4c^2x^2\sqrt{c^2x^2+1}+3cx-\sqrt{c^2x^2+1}\right)e}{8c^2b(a+b\operatorname{arcsinh}(cx))}+\frac{3e\,\mathrm{e}^{\frac{3a}{b}}\,\mathrm{Ei}\left(1,3\operatorname{arcsinh}(cx)+\frac{3a}{b}\right)}{8c^2b^2}-\frac{e\left(4c^3x^3+3cx+4c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}\right)}{8b\,c^2(a+b\operatorname{arcsinh}(cx))}-\frac{3e\,\mathrm{e}^{-\frac{3a}{b}}\,\mathrm{Ei}\left(1,-3\operatorname{arcsinh}\right.}{8b^2c^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c*(1/8*(4*c^3*x^3-4*c^2*x^2*(c^2*x^2+1)^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))*e/
c^2/b/(a+b*arcsinh(c*x))+3/8*e/c^2/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b
)-1/8/b*e/c^2*(4*c^3*x^3+3*c*x+4*c^2*x^2*(c^2*x^2+1)^(1/2)+(c^2*x^2+1)^(1/2
))/(a+b*arcsinh(c*x))-3/8/b^2*e/c^2*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)
+1/2*(c*x-(c^2*x^2+1)^(1/2))*d/b/(a+b*arcsinh(c*x))+1/2*d/b^2*exp(a/b)*Ei(1
,arcsinh(c*x)+a/b)-1/8*(c*x-(c^2*x^2+1)^(1/2))*e/c^2/b/(a+b*arcsinh(c*x))-1
/8/c^2*e/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/2/b*d*(c*x+(c^2*x^2+1)^(1/2)
)/(a+b*arcsinh(c*x))-1/2/b^2*d*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)+1/8/c^2/b*
e*(c*x+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))+1/8/c^2/b^2*e*exp(-a/b)*Ei(1,-
arcsinh(c*x)-a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^3ex^5+\left(c^3d+ce\right)x^3+cdx+\left(c^2ex^4+\left(c^2d+e\right)x^2+d\right)\sqrt{c^2x^2+1}}{abc^3x^2+\sqrt{c^2x^2+1}\,abc^2x+abc+\left(b^2c^3x^2+\sqrt{c^2x^2+1}\,b^2c^2x+b^2c\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int\frac{3}{abc^5x^4+\left(c^2x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*e*x^5 + (c^3*d + c*e)*x^3 + c*d*x + (c^2*e*x^4 + (c^2*d + e)*x^2 + d)
*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b
^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 +
1))) + integrate((3*c^5*e*x^6 + (c^5*d + 6*c^3*e)*x^4 + (2*c^3*d + 3*c*e)*x
^2 + (3*c^3*e*x^4 + (c^3*d + c*e)*x^2 - c*d)*(c^2*x^2 + 1) + c*d + (6*c^4*e
*x^5 + (2*c^4*d + 7*c^2*e)*x^3 + (c^2*d + 2*e)*x)*sqrt(c^2*x^2 + 1))/(a*b*c
^5*x^4 + (c^2*x^2 + 1)*a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 +
 (c^2*x^2 + 1)*b^2*c^3*x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c
^2*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*
b*c^2*x)*sqrt(c^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e\,x^2+d}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(a + b*asinh(c*x))^2,x)
```

```
[Out] int((d + e*x^2)/(a + b*asinh(c*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + e x^{2}}{\left(a + b \operatorname{asinh}{\left(c x \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((d + e*x**2)/(a + b*asinh(c*x))**2, x)
```

3.626 $\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=85

$$-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c}-\frac{\sqrt{c^2x^2+1}}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

```
[Out] cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b^2/c-Chi((a+b*arcsinh(c*x))/b)*sinh(a/
b)/b^2/c-(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))
```

Rubi [A] time = 0.19, antiderivative size = 81, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5655, 5779, 3303, 3298, 3301}

$$-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{b^2c}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{b^2c}-\frac{\sqrt{c^2x^2+1}}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^(-2),x]
```

```
[Out] -(Sqrt[1 + c^2*x^2]/(b*c*(a + b*ArcSinh[c*x]))) - (CoshIntegral[a/b + ArcSi
nh[c*x]]*Sinh[a/b])/(b^2*c) + (Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/
(b^2*c)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
```

```
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{c\int \frac{x}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx}{b}$$

$$= -\frac{\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{bc}$$

$$= -\frac{\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{bc}-\frac{\sinh\left(\right.}{}$$

$$= -\frac{\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{b^2c}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh\right.}{b^2c}$$

Mathematica [A] time = 0.15, size = 71, normalized size = 0.84

$$\frac{-\frac{b\sqrt{c^2x^2+1}}{a+b\sinh^{-1}(cx)}-\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{b^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^(-2),x]
[Out] (-((b*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])) - CoshIntegral[a/b + ArcSinh
[c*x]]*Sinh[a/b] + Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(b^2*c)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b^2\,\text{arsinh}\,(cx)^2+2\,ab\,\text{arsinh}\,(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^(-2), x)
```

maple [A] time = 0.05, size = 118, normalized size = 1.39

$$\frac{\frac{cx-\sqrt{c^2x^2+1}}{2b(a+b\,\text{arcsinh}(cx))}+\frac{e^{\frac{a}{b}}\,\text{Ei}\left(1,\text{arcsinh}(cx)+\frac{a}{b}\right)}{2b^2}-\frac{cx+\sqrt{c^2x^2+1}}{2b(a+b\,\text{arcsinh}(cx))}-\frac{e^{-\frac{a}{b}}\,\text{Ei}\left(1,-\text{arcsinh}(cx)-\frac{a}{b}\right)}{2b^2}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c*(1/2*(c*x-(c^2*x^2+1)^(1/2))/b/(a+b*arcsinh(c*x))+1/2/b^2*exp(a/b)*Ei(1
,arcsinh(c*x)+a/b)-1/2/b*(c*x+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))-1/2/b^2
*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^3x^3 + cx + \left(c^2x^2+1\right)^{\frac{3}{2}}}{abc^3x^2 + \sqrt{c^2x^2+1}\,abc^2x + abc + \left(b^2c^3x^2 + \sqrt{c^2x^2+1}\,b^2c^2x + b^2c\right)\log\left(cx + \sqrt{c^2x^2+1}\right)} + \int \frac{}{abc^4x^4 + \left(c^2x^2 + \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b
*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*
x + sqrt(c^2*x^2 + 1))) + integrate((c^4*x^4 + 2*c^2*x^2 + (c^2*x^2 + 1)*(c
^2*x^2 - 1) + (2*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1) + 1)/(a*b*c^4*x^4 + (c^2*
x^2 + 1)*a*b*c^2*x^2 + 2*a*b*c^2*x^2 + a*b + (b^2*c^4*x^4 + (c^2*x^2 + 1)*b
^2*c^2*x^2 + 2*b^2*c^2*x^2 + b^2 + 2*(b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2*x^2 +
 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2
+ 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(c*x))^2,x)
```

```
[Out] int(1/(a + b*asinh(c*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\operatorname{asinh}{\left(cx \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((a + b*asinh(c*x))**(-2), x)
```

3.627 $\int \frac{1}{(d+ex^2)(a+b\sinh^{-1}(cx))^2} dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(d+ex^2)(a+b\sinh^{-1}(cx))^2}, x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(d+ex^2)(a+b\sinh^{-1}(cx))^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][1/((d + e*x^2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)(a+b\sinh^{-1}(cx))^2} dx = \int \frac{1}{(d+ex^2)(a+b\sinh^{-1}(cx))^2} dx$$

Mathematica [A] time = 11.89, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)(a+b\sinh^{-1}(cx))^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[1/((d + e*x^2)*(a + b*ArcSinh[c*x])^2), x]
```

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{a^2ex^2 + a^2d + (b^2ex^2 + b^2d)\,\text{arsinh}(cx)^2 + 2(abex^2 + abd)\,\text{arsinh}(cx)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(1/(a^2*e*x^2 + a^2*d + (b^2*e*x^2 + b^2*d)*arcsinh(c*x)^2 + 2*(a*b
*e*x^2 + a*b*d)*arcsinh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex^2+d)(b\,\text{arsinh}(cx)+a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)*(b*arcsinh(c*x) + a)^2), x)
```

maple [A] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\,x^{2}+d\right)\left(a+b\,\mathrm{arcsinh}\left(c x\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(1/(e*x^2+d)/(a+b*arcsinh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^3x^3+cx+\left(c^2x^2+1\right)^{\frac{3}{2}}}{abc^3ex^4+\left(c^3d+ce\right)abx^2+abcd+\left(b^2c^3ex^4+\left(c^3d+ce\right)b^2x^2+b^2cd+\left(b^2c^2ex^3+b^2c^2dx\right)\sqrt{c^2x^2+1}\right)\log\left(cx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(a*b*c^3*e*x^4 + (c^3*d + c*e)*a*b*x
^2 + a*b*c*d + (b^2*c^3*e*x^4 + (c^3*d + c*e)*b^2*x^2 + b^2*c*d + (b^2*c^2*
e*x^3 + b^2*c^2*d*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b
*c^2*e*x^3 + a*b*c^2*d*x)*sqrt(c^2*x^2 + 1)) - integrate((c^5*e*x^6 - (c^5*
d - 2*c^3*e)*x^4 - (2*c^3*d - c*e)*x^2 + (c^3*e*x^4 - (c^3*d - 3*c*e)*x^2 +
 c*d)*(c^2*x^2 + 1) - c*d + (2*c^4*e*x^5 - (2*c^4*d - 5*c^2*e)*x^3 - (c^2*d
 - 2*e)*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*e^2*x^8 + 2*(c^5*d*e + c^3*e^2)*a*b*
x^6 + (c^5*d^2 + 4*c^3*d*e + c*e^2)*a*b*x^4 + a*b*c*d^2 + 2*(c^3*d^2 + c*d*
e)*a*b*x^2 + (a*b*c^3*e^2*x^6 + 2*a*b*c^3*d*e*x^4 + a*b*c^3*d^2*x^2)*(c^2*x
^2 + 1) + (b^2*c^5*e^2*x^8 + 2*(c^5*d*e + c^3*e^2)*b^2*x^6 + (c^5*d^2 + 4*c
^3*d*e + c*e^2)*b^2*x^4 + b^2*c*d^2 + 2*(c^3*d^2 + c*d*e)*b^2*x^2 + (b^2*c^
3*e^2*x^6 + 2*b^2*c^3*d*e*x^4 + b^2*c^3*d^2*x^2)*(c^2*x^2 + 1) + 2*(b^2*c^4
*e^2*x^7 + (2*c^4*d*e + c^2*e^2)*b^2*x^5 + b^2*c^2*d^2*x + (c^4*d^2 + 2*c^2
*d*e)*b^2*x^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4
*e^2*x^7 + (2*c^4*d*e + c^2*e^2)*a*b*x^5 + a*b*c^2*d^2*x + (c^4*d^2 + 2*c^2
*d*e)*a*b*x^3)*sqrt(c^2*x^2 + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,\left(e\,x^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))^2*(d + e*x^2)),x)
```

```
[Out] int(1/((a + b*asinh(c*x))^2*(d + e*x^2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(cx\right)\right)^{2}\left(d+ex^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(1/((a + b*asinh(c*x))**2*(d + e*x**2)), x)
```

3.628 $\int \frac{1}{(d+ex^2)^2 (a+b\sinh^{-1}(cx))^2} dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(d+ex^2)^2 (a+b\sinh^{-1}(cx))^2}, x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(d+ex^2)^2 (a+b\sinh^{-1}(cx))^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^2 (a+b\sinh^{-1}(cx))^2} dx = \int \frac{1}{(d+ex^2)^2 (a+b\sinh^{-1}(cx))^2} dx$$

Mathematica [A] time = 28.65, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)^2 (a+b\sinh^{-1}(cx))^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^2), x]
```

fricas [A] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{a^2e^2x^4 + 2a^2dex^2 + a^2d^2 + (b^2e^2x^4 + 2b^2dex^2 + b^2d^2)\,\text{arsinh}(cx)^2 + 2(abe^2x^4 + 2abdex^2 + abd^2)\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(1/(a^2*e^2*x^4 + 2*a^2*d*e*x^2 + a^2*d^2 + (b^2*e^2*x^4 + 2*b^2*d*
e*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*e^2*x^4 + 2*a*b*d*e*x^2 + a*b*d^2)
*arcsinh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex^2+d)^2 (b\,\text{arsinh}(cx)+a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)^2*(b*arcsinh(c*x) + a)^2), x)
```

maple [A] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\,x^{2}+d\right)^{2} \left(a +b \,\mathrm{arcsinh}\left(c x \right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^{3}x}{abc^{3}e^{2}x^{6} + \left(2\,c^{3}de + ce^{2}\right)abx^{4} + abcd^{2} + \left(c^{3}d^{2} + 2\,cde\right)abx^{2} + \left(b^{2}c^{3}e^{2}x^{6} + \left(2\,c^{3}de + ce^{2}\right)b^{2}x^{4} + b^{2}cd^{2} + \left(c^{3}d^{2} + \right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(a*b*c^3*e^2*x^6 + (2*c^3*d*e + c*e^
2)*a*b*x^4 + a*b*c*d^2 + (c^3*d^2 + 2*c*d*e)*a*b*x^2 + (b^2*c^3*e^2*x^6 + (
2*c^3*d*e + c*e^2)*b^2*x^4 + b^2*c*d^2 + (c^3*d^2 + 2*c*d*e)*b^2*x^2 + (b^2
*c^2*e^2*x^5 + 2*b^2*c^2*d*e*x^3 + b^2*c^2*d^2*x)*sqrt(c^2*x^2 + 1))*log(c*
x + sqrt(c^2*x^2 + 1)) + (a*b*c^2*e^2*x^5 + 2*a*b*c^2*d*e*x^3 + a*b*c^2*d^2
*x)*sqrt(c^2*x^2 + 1)) - integrate((3*c^5*e*x^6 - (c^5*d - 6*c^3*e)*x^4 - (
2*c^3*d - 3*c*e)*x^2 + (3*c^3*e*x^4 - (c^3*d - 5*c*e)*x^2 + c*d)*(c^2*x^2 +
 1) - c*d + (6*c^4*e*x^5 - (2*c^4*d - 11*c^2*e)*x^3 - (c^2*d - 4*e)*x)*sqrt
(c^2*x^2 + 1))/(a*b*c^5*e^3*x^10 + (3*c^5*d*e^2 + 2*c^3*e^3)*a*b*x^8 + (3*c
^5*d^2*e + 6*c^3*d*e^2 + c*e^3)*a*b*x^6 + (c^5*d^3 + 6*c^3*d^2*e + 3*c*d*e^
2)*a*b*x^4 + a*b*c*d^3 + (2*c^3*d^3 + 3*c*d^2*e)*a*b*x^2 + (a*b*c^3*e^3*x^8
 + 3*a*b*c^3*d*e^2*x^6 + 3*a*b*c^3*d^2*e*x^4 + a*b*c^3*d^3*x^2)*(c^2*x^2 +
1) + (b^2*c^5*e^3*x^10 + (3*c^5*d*e^2 + 2*c^3*e^3)*b^2*x^8 + (3*c^5*d^2*e +
 6*c^3*d*e^2 + c*e^3)*b^2*x^6 + (c^5*d^3 + 6*c^3*d^2*e + 3*c*d*e^2)*b^2*x^4
 + b^2*c*d^3 + (2*c^3*d^3 + 3*c*d^2*e)*b^2*x^2 + (b^2*c^3*e^3*x^8 + 3*b^2*c
^3*d*e^2*x^6 + 3*b^2*c^3*d^2*e*x^4 + b^2*c^3*d^3*x^2)*(c^2*x^2 + 1) + 2*(b^
2*c^4*e^3*x^9 + (3*c^4*d*e^2 + c^2*e^3)*b^2*x^7 + b^2*c^2*d^3*x + 3*(c^4*d^
2*e + c^2*d*e^2)*b^2*x^5 + (c^4*d^3 + 3*c^2*d^2*e)*b^2*x^3)*sqrt(c^2*x^2 +
1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*e^3*x^9 + (3*c^4*d*e^2 + c^2*
e^3)*a*b*x^7 + a*b*c^2*d^3*x + 3*(c^4*d^2*e + c^2*d*e^2)*a*b*x^5 + (c^4*d^3
 + 3*c^2*d^2*e)*a*b*x^3)*sqrt(c^2*x^2 + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(e\,x^2+d\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))^2*(d + e*x^2)^2),x)
```

```
[Out] int(1/((a + b*asinh(c*x))^2*(d + e*x^2)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

3.629 $\int \left(d+ex^2\right)^2 \sqrt{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=672

$$\frac{\sqrt{\pi}\sqrt{b}\,e^2e^{a/b}\mathrm{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32c^5}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}\,e^2e^{\frac{3a}{b}}\mathrm{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64c^5}+\frac{\sqrt{\frac{\pi}{5}}\sqrt{b}\,e^2e^{\frac{5a}{b}}\mathrm{erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{320c^5}-\sqrt{\pi}\ldots$$

```
[Out] 1/1600*e^2*exp(5*a/b)*erf(5^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)
*5^(1/2)*Pi^(1/2)/c^5-1/1600*e^2*erfi(5^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1
/2))*b^(1/2)*5^(1/2)*Pi^(1/2)/c^5/exp(5*a/b)+1/72*d*e*exp(3*a/b)*erf(3^(1/2
)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*3^(1/2)*Pi^(1/2)/c^3-1/192*e^2*
exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*3^(1/2)*Pi
^(1/2)/c^5-1/72*d*e*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*
3^(1/2)*Pi^(1/2)/c^3/exp(3*a/b)+1/192*e^2*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(
1/2)/b^(1/2))*b^(1/2)*3^(1/2)*Pi^(1/2)/c^5/exp(3*a/b)+1/4*d^2*exp(a/b)*erf(
(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c-1/8*d*e*exp(a/b)*erf((
a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c^3+1/32*e^2*exp(a/b)*erf
((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c^5-1/4*d^2*erfi((a+b*a
rcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c/exp(a/b)+1/8*d*e*erfi((a+b*a
rcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c^3/exp(a/b)-1/32*e^2*erfi((a+
b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c^5/exp(a/b)+d^2*x*(a+b*arc
sinh(c*x))^(1/2)+2/3*d*e*x^3*(a+b*arcsinh(c*x))^(1/2)+1/5*e^2*x^5*(a+b*arcs
inh(c*x))^(1/2)
```

Rubi [A] time = 1.88, antiderivative size = 672, normalized size of antiderivative = 1.00, number of steps used = 42, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {5706, 5653, 5779, 3308, 2180, 2204, 2205, 5663, 3312}

$$-\frac{\sqrt{\pi}\sqrt{b}\,dee^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8c^3}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}\,dee^{\frac{3a}{b}}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{24c^3}+\frac{\sqrt{\pi}\sqrt{b}\,dee^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8c^3}-\sqrt{\frac{\pi}{3}}\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] d^2*x*Sqrt[a + b*ArcSinh[c*x]] + (2*d*e*x^3*Sqrt[a + b*ArcSinh[c*x]])/3 + (
e^2*x^5*Sqrt[a + b*ArcSinh[c*x]])/5 + (Sqrt[b]*d^2*E^(a/b)*Sqrt[Pi]*Erf[Sqr
t[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*c) - (Sqrt[b]*d*e*E^(a/b)*Sqrt[Pi]*Erf[S
qrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*c^3) + (Sqrt[b]*e^2*E^(a/b)*Sqrt[Pi]*E
rf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(32*c^5) + (Sqrt[b]*d*e*E^((3*a)/b)*S
qrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(24*c^3) - (Sqrt
[b]*e^2*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[
b]])/(64*c^5) + (Sqrt[b]*e^2*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b
*ArcSinh[c*x]])/Sqrt[b]])/(320*c^5) - (Sqrt[b]*d^2*Sqrt[Pi]*Erfi[Sqrt[a + b
*ArcSinh[c*x]]/Sqrt[b]])/(4*c*E^(a/b)) + (Sqrt[b]*d*e*Sqrt[Pi]*Erfi[Sqrt[a
+ b*ArcSinh[c*x]]/Sqrt[b]])/(8*c^3*E^(a/b)) - (Sqrt[b]*e^2*Sqrt[Pi]*Erfi[Sq
rt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(32*c^5*E^(a/b)) - (Sqrt[b]*d*e*Sqrt[Pi/3]
*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(24*c^3*E^((3*a)/b)) + (
Sqrt[b]*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(6
4*c^5*E^((3*a)/b)) - (Sqrt[b]*e^2*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSi
nh[c*x]])/Sqrt[b]])/(320*c^5*E^((5*a)/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule steps

Rubi steps

$$\int \left(d+ex^2\right)^2 \sqrt{a+b\sinh^{-1}(cx)}\,dx = \int \left(d^2\sqrt{a+b\sinh^{-1}(cx)} + 2dex^2\sqrt{a+b\sinh^{-1}(cx)} + e^2x^4\sqrt{a+b\sinh^{-1}(\right.$$

$$= d^2\int \sqrt{a+b\sinh^{-1}(cx)}\,dx + (2de)\int x^2\sqrt{a+b\sinh^{-1}(cx)}\,dx + e^2\int x^4\sqrt{}$$

$$= d^2x\sqrt{a+b\sinh^{-1}(cx)} + \frac{2}{3}dex^3\sqrt{a+b\sinh^{-1}(cx)} + \frac{1}{5}e^2x^5\sqrt{a+b\sinh^{-1}(}$$

$$= d^2x\sqrt{a+b\sinh^{-1}(cx)} + \frac{2}{3}dex^3\sqrt{a+b\sinh^{-1}(cx)} + \frac{1}{5}e^2x^5\sqrt{a+b\sinh^{-1}(}$$

$$= d^2x\sqrt{a+b\sinh^{-1}(cx)} + \frac{2}{3}dex^3\sqrt{a+b\sinh^{-1}(cx)} + \frac{1}{5}e^2x^5\sqrt{a+b\sinh^{-1}(}$$

$$= d^2x\sqrt{a+b\sinh^{-1}(cx)} + \frac{2}{3}dex^3\sqrt{a+b\sinh^{-1}(cx)} + \frac{1}{5}e^2x^5\sqrt{a+b\sinh^{-1}(}$$

$$= d^2x\sqrt{a+b\sinh^{-1}(cx)} + \frac{2}{3}dex^3\sqrt{a+b\sinh^{-1}(cx)} + \frac{1}{5}e^2x^5\sqrt{a+b\sinh^{-1}(}$$

$$= d^2x\sqrt{a+b\sinh^{-1}(cx)} + \frac{2}{3}dex^3\sqrt{a+b\sinh^{-1}(cx)} + \frac{1}{5}e^2x^5\sqrt{a+b\sinh^{-1}(}$$

$$= d^2x\sqrt{a+b\sinh^{-1}(cx)} + \frac{2}{3}dex^3\sqrt{a+b\sinh^{-1}(cx)} + \frac{1}{5}e^2x^5\sqrt{a+b\sinh^{-1}(}$$

Mathematica [**A**] time = 6.40, size = 535, normalized size = 0.80

$$-\frac{be^{-\frac{5a}{b}}\left(25\sqrt{3}\,bee^{\frac{2a}{b}}\left(8c^2d-3e\right)\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\sqrt{-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{b^2}}\,\Gamma\left(\frac{3}{2},-\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)-bee^{\frac{8a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)^2*Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] -1/7200*(b*(450*E^((6*a)/b)*(8*a*c^4*d^2*Sqrt[a/b + ArcSinh[c*x]] + 8*b*c^4
*d^2*ArcSinh[c*x]*Sqrt[a/b + ArcSinh[c*x]] + b*(4*c^2*d - e)*e*Sqrt[-((a +
b*ArcSinh[c*x])/b)]*Sqrt[-((a + b*ArcSinh[c*x])^2/b^2)])*Gamma[3/2, a/b + A
rcSinh[c*x]] + 9*Sqrt[5]*b*e^2*Sqrt[a/b + ArcSinh[c*x]]*Sqrt[-((a + b*ArcSi
nh[c*x])^2/b^2)]*Gamma[3/2, (-5*(a + b*ArcSinh[c*x]))/b] + 25*Sqrt[3]*b*(8*
c^2*d - 3*e)*e*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Sqrt[-((a + b*ArcSinh[c
*x])^2/b^2)]*Gamma[3/2, (-3*(a + b*ArcSinh[c*x]))/b] + 450*E^((4*a)/b)*(8*a
*c^4*d^2*Sqrt[-((a + b*ArcSinh[c*x])/b)] + 8*b*c^4*d^2*ArcSinh[c*x]*Sqrt[-(
(a + b*ArcSinh[c*x])/b)] + b*e*(-4*c^2*d + e)*Sqrt[a/b + ArcSinh[c*x]]*Sqrt
[-((a + b*ArcSinh[c*x])^2/b^2)])*Gamma[3/2, -((a + b*ArcSinh[c*x])/b)] - b*
e*E^((8*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Sqrt[-((a + b*ArcSinh[c*x])^2
/b^2)]*(25*Sqrt[3]*(8*c^2*d - 3*e)*Gamma[3/2, (3*(a + b*ArcSinh[c*x]))/b] +
 9*Sqrt[5]*e*E^((2*a)/b)*Gamma[3/2, (5*(a + b*ArcSinh[c*x]))/b])))/(c^5*E^(
(5*a)/b)*(a + b*ArcSinh[c*x])^(3/2))
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^2+d\right)^2 \sqrt{b \operatorname{arsinh}(cx)+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*sqrt(b*arcsinh(c*x) + a), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \left(e\,x^2+d\right)^2 \sqrt{a+b \operatorname{arcsinh}(cx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arcsinh(c*x))^(1/2),x)
```

```
[Out] int((e*x^2+d)^2*(a+b*arcsinh(c*x))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^2+d\right)^2 \sqrt{b \operatorname{arsinh}(cx)+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^2*sqrt(b*arcsinh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+b \operatorname{asinh}(c\,x)}\, \left(e\,x^2+d\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^(1/2)*(d + e*x^2)^2,x)
```

```
[Out] int((a + b*asinh(c*x))^(1/2)*(d + e*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b \operatorname{asinh}(cx)}\, \left(d+ex^2\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*asinh(c*x))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*asinh(c*x))*(d + e*x**2)**2, x)
```

3.630 $\int (d + ex^2) \sqrt{a + b \sinh^{-1}(cx)}\, dx$

Optimal. Leaf size=322

$$-\frac{\sqrt{\pi}\sqrt{b}\,ee^{a/b}\mathrm{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}\,ee^{\frac{3a}{b}}\mathrm{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{48c^3}+\frac{\sqrt{\pi}\sqrt{b}\,ee^{-\frac{a}{b}}\mathrm{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}\,e\ldots}{}$$

```
[Out] 1/144*e*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*3^
(1/2)*Pi^(1/2)/c^3-1/144*e*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b
^(1/2)*3^(1/2)*Pi^(1/2)/c^3/exp(3*a/b)+1/4*d*exp(a/b)*erf((a+b*arcsinh(c*x)
)^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c-1/16*e*exp(a/b)*erf((a+b*arcsinh(c*x))^
(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c^3-1/4*d*erfi((a+b*arcsinh(c*x))^(1/2)/b^(
1/2))*b^(1/2)*Pi^(1/2)/c/exp(a/b)+1/16*e*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1
/2))*b^(1/2)*Pi^(1/2)/c^3/exp(a/b)+d*x*(a+b*arcsinh(c*x))^(1/2)+1/3*e*x^3*(
a+b*arcsinh(c*x))^(1/2)
```

Rubi [A] time = 0.93, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {5706, 5653, 5779, 3308, 2180, 2204, 2205, 5663, 3312}

$$-\frac{\sqrt{\pi}\sqrt{b}\,ee^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}\,ee^{\frac{3a}{b}}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{48c^3}+\frac{\sqrt{\pi}\sqrt{b}\,ee^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] d*x*Sqrt[a + b*ArcSinh[c*x]] + (e*x^3*Sqrt[a + b*ArcSinh[c*x]])/3 + (Sqrt[b
]*d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*c) - (Sqrt[b
]*e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(16*c^3) + (Sqr
t[b]*e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b
]])/(48*c^3) - (Sqrt[b]*d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/
(4*c*E^(a/b)) + (Sqrt[b]*e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])
/(16*c^3*E^(a/b)) - (Sqrt[b]*e*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[
c*x]])/Sqrt[b]])/(48*c^3*E^((3*a)/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \left(d+ex^2\right)\sqrt{a+b\sinh^{-1}(cx)}\,dx = \int \left(d\sqrt{a+b\sinh^{-1}(cx)}+ex^2\sqrt{a+b\sinh^{-1}(cx)}\right)dx$$

$$= d\int \sqrt{a+b\sinh^{-1}(cx)}\,dx + e\int x^2\sqrt{a+b\sinh^{-1}(cx)}\,dx$$

$$= dx\sqrt{a+b\sinh^{-1}(cx)} + \frac{1}{3}ex^3\sqrt{a+b\sinh^{-1}(cx)} - \frac{1}{2}(bcd)\int \frac{}{\sqrt{1+c^2x^2}\sqrt{a}}$$

$$= dx\sqrt{a+b\sinh^{-1}(cx)} + \frac{1}{3}ex^3\sqrt{a+b\sinh^{-1}(cx)} - \frac{(bd)\,\mathrm{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx,\right.}{2c}$$

$$= dx\sqrt{a+b\sinh^{-1}(cx)} + \frac{1}{3}ex^3\sqrt{a+b\sinh^{-1}(cx)} + \frac{(bd)\,\mathrm{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx,\right.}{4c}$$

$$= dx\sqrt{a+b\sinh^{-1}(cx)} + \frac{1}{3}ex^3\sqrt{a+b\sinh^{-1}(cx)} + \frac{d\,\mathrm{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\right.}{2c}$$

$$= dx\sqrt{a+b\sinh^{-1}(cx)} + \frac{1}{3}ex^3\sqrt{a+b\sinh^{-1}(cx)} + \frac{\sqrt{b}\,de^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\sinh}}{\sqrt{b}}\right.}{4c}$$

$$= dx\sqrt{a+b\sinh^{-1}(cx)} + \frac{1}{3}ex^3\sqrt{a+b\sinh^{-1}(cx)} + \frac{\sqrt{b}\,de^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\sinh}}{\sqrt{b}}\right.}{4c}$$

$$= dx\sqrt{a+b\sinh^{-1}(cx)} + \frac{1}{3}ex^3\sqrt{a+b\sinh^{-1}(cx)} + \frac{\sqrt{b}\,de^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\sinh}}{\sqrt{b}}\right.}{4c}$$

Mathematica [A] time = 2.90, size = 319, normalized size = 0.99

$$\frac{ee^{-\frac{3a}{b}}\sqrt{a+b\sinh^{-1}(cx)}\left(9e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{3}{2},-\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)\right.}{72c^3\sqrt{-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{b^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)*Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (d*Sqrt[a + b*ArcSinh[c*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c*x]])
/Sqrt[a/b + ArcSinh[c*x]]) + Gamma[3/2, -((a + b*ArcSinh[c*x])/b)]/Sqrt[-((
a + b*ArcSinh[c*x])/b)]))/(2*c*E^(a/b)) + (e*Sqrt[a + b*ArcSinh[c*x]]*(9*E^
((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[3/2, a/b + ArcSinh[c*x]] +
Sqrt[3]*Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2, (-3*(a + b*ArcSinh[c*x]))/b] -
9*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2, -((a + b*ArcSinh[c*x])/b)
] - Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[3/2, (3*(a +
b*ArcSinh[c*x]))/b]))/(72*c^3*E^((3*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])^2/b^2
)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^2 + d\right)\sqrt{b \operatorname{arsinh}(cx) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*sqrt(b*arcsinh(c*x) + a), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \left(e\,x^2 + d\right)\sqrt{a + b \operatorname{arcsinh}(cx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arcsinh(c*x))^(1/2),x)
```

```
[Out] int((e*x^2+d)*(a+b*arcsinh(c*x))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^2 + d\right)\sqrt{b \operatorname{arsinh}(cx) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)*sqrt(b*arcsinh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a + b \operatorname{asinh}(c\,x)}\,\left(e\,x^2 + d\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^(1/2)*(d + e*x^2),x)
```

```
[Out] int((a + b*asinh(c*x))^(1/2)*(d + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \operatorname{asinh}(cx)}\,\left(d + ex^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*asinh(c*x))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*asinh(c*x))*(d + e*x**2), x)
```

3.631 $\int \sqrt{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=102

$$\frac{\sqrt{\pi}\sqrt{b}e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}-\frac{\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}+x\sqrt{a+b\sinh^{-1}(cx)}$$

```
[Out] 1/4*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c-1/4*e
rfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c/exp(a/b)+x*(a+b*ar
csinh(c*x))^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5653, 5779, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\sqrt{b}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}-\frac{\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}+x\sqrt{a+b\sinh^{-1}(cx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] x*Sqrt[a + b*ArcSinh[c*x]] + (Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSi
nh[c*x]]/Sqrt[b]])/(4*c) - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/
Sqrt[b]])/(4*c*E^(a/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \sqrt{a+b\sinh^{-1}(cx)}\,dx = x\sqrt{a+b\sinh^{-1}(cx)}-\frac{1}{2}(bc)\int \frac{x}{\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}\,dx$$
$$= x\sqrt{a+b\sinh^{-1}(cx)}-\frac{b\,\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{2c}$$
$$= x\sqrt{a+b\sinh^{-1}(cx)}+\frac{b\,\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{4c}-\frac{b\,\text{Subst}\left(\int \frac{e^{x}}{\sqrt{a+bx}}\,dx\right.}{4c}$$
$$= x\sqrt{a+b\sinh^{-1}(cx)}+\frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\sinh^{-1}(cx)}\right)}{2c}-\frac{\text{Subst}\left(\int e^{-\frac{a}{b}+}\right.}{}$$
$$= x\sqrt{a+b\sinh^{-1}(cx)}+\frac{\sqrt{b}\,e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}-\frac{\sqrt{b}\,e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}}}{\sqrt{b}}\right.}{4c}$$

Mathematica [A] time = 0.20, size = 101, normalized size = 0.99

$$\frac{e^{-\frac{a}{b}}\sqrt{a+b\sinh^{-1}(cx)}\left(\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\sinh^{-1}(cx)}{b}\right)}{\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}}-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)}{\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}}\right)}{2c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (Sqrt[a + b*ArcSinh[c*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c*x]])/S
qrt[a/b + ArcSinh[c*x]]) + Gamma[3/2, -((a + b*ArcSinh[c*x])/b)]/Sqrt[-((a
+ b*ArcSinh[c*x])/b)]))/(2*c*E^(a/b))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\,\text{arsinh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arcsinh(c*x) + a), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \operatorname{arcsinh}(cx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^(1/2),x)
[Out] int((a+b*arcsinh(c*x))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \operatorname{arsinh}(cx) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*arcsinh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a + b \operatorname{asinh}(c\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^(1/2),x)
[Out] int((a + b*asinh(c*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \operatorname{asinh}(cx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**(1/2),x)
[Out] Integral(sqrt(a + b*asinh(c*x)), x)
```

3.632 $\int \frac{\sqrt{a+b\sinh^{-1}(cx)}}{d+ex^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{d+ex^2},x\right)$$

```
[Out] Unintegrable((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{a+b\sinh^{-1}(cx)}}{d+ex^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2),x]
[Out] Defer[Int][Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2), x]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sinh^{-1}(cx)}}{d+ex^2}\,dx = \int \frac{\sqrt{a+b\sinh^{-1}(cx)}}{d+ex^2}\,dx$$

Mathematica [A] time = 7.55, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sinh^{-1}(cx)}}{d+ex^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2),x]
[Out] Integrate[Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\,\text{arsinh}(cx)+a}}{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arcsinh(c*x) + a)/(e*x^2 + d), x)
```

maple [A] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b \operatorname{arcsinh}(cx)}}{e\,x^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d),x)
[Out] int((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b \operatorname{arsinh}(cx) + a}}{ex^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d),x, algorithm="maxima")
[Out] integrate(sqrt(b*arcsinh(c*x) + a)/(e*x^2 + d), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{a + b \operatorname{asinh}(c\,x)}}{e\,x^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^(1/2)/(d + e*x^2),x)
[Out] int((a + b*asinh(c*x))^(1/2)/(d + e*x^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b \operatorname{asinh}(cx)}}{d + ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**(1/2)/(e*x**2+d),x)
[Out] Integral(sqrt(a + b*asinh(c*x))/(d + e*x**2), x)
```

3.633 $\int \frac{\sqrt{a+b\sinh^{-1}(cx)}}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{(d+ex^2)^2},x\right)$$

```
[Out] Unintegrable((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d)^2,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{a+b\sinh^{-1}(cx)}}{(d+ex^2)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2)^2,x]
[Out] Defer[Int][Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2)^2, x]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sinh^{-1}(cx)}}{(d+ex^2)^2}\,dx=\int \frac{\sqrt{a+b\sinh^{-1}(cx)}}{(d+ex^2)^2}\,dx$$

Mathematica [A] time = 16.83, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sinh^{-1}(cx)}}{(d+ex^2)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2)^2,x]
[Out] Integrate[Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2)^2, x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d)^2,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\,\operatorname{arsinh}(cx)+a}}{(ex^2+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d)^2,x, algorithm="giac")
[Out] integrate(sqrt(b*arcsinh(c*x) + a)/(e*x^2 + d)^2, x)
```

maple [A] time = 0.93, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\,\mathrm{arcsinh}\left(cx\right)}}{\left(e\,x^{2}+d\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d)^2,x)
[Out] int((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\,\mathrm{arsinh}\left(cx\right)+a}}{\left(ex^{2}+d\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d)^2,x, algorithm="maxima")
[Out] integrate(sqrt(b*arcsinh(c*x) + a)/(e*x^2 + d)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{a+b\,\mathrm{asinh}\left(c\,x\right)}}{\left(e\,x^{2}+d\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^(1/2)/(d + e*x^2)^2,x)
[Out] int((a + b*asinh(c*x))^(1/2)/(d + e*x^2)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\,\mathrm{asinh}\left(cx\right)}}{\left(d+ex^{2}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**(1/2)/(e*x**2+d)**2,x)
[Out] Integral(sqrt(a + b*asinh(c*x))/(d + e*x**2)**2, x)
```

3.634 $\int \left(d+ex^2\right)\left(a+b\sinh^{-1}(cx)\right)^{3/2}\,dx$

Optimal. Leaf size=427

$$-\frac{3\sqrt{\pi}\,b^{3/2}ee^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32c^3}+\frac{\sqrt{\frac{\pi}{3}}\,b^{3/2}ee^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{96c^3}-\frac{3\sqrt{\pi}\,b^{3/2}ee^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32c^3}+\ldots$$

```
[Out] d*x*(a+b*arcsinh(c*x))^(3/2)+1/3*e*x^3*(a+b*arcsinh(c*x))^(3/2)+1/288*b^(3/
2)*e*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1
/2)/c^3+1/288*b^(3/2)*e*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1
/2)*Pi^(1/2)/c^3/exp(3*a/b)+3/8*b^(3/2)*d*exp(a/b)*erf((a+b*arcsinh(c*x))^(
1/2)/b^(1/2))*Pi^(1/2)/c-3/32*b^(3/2)*e*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/
2)/b^(1/2))*Pi^(1/2)/c^3+3/8*b^(3/2)*d*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2
))*Pi^(1/2)/c/exp(a/b)-3/32*b^(3/2)*e*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2)
)*Pi^(1/2)/c^3/exp(a/b)-3/2*b*d*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^(1/2)/
c+1/3*b*e*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^(1/2)/c^3-1/6*b*e*x^2*(c^2*x
^2+1)^(1/2)*(a+b*arcsinh(c*x))^(1/2)/c
```

Rubi [A] time = 1.26, antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5706, 5653, 5717, 5657, 3307, 2180, 2205, 2204, 5663, 5758, 5669, 5448}

$$-\frac{3\sqrt{\pi}\,b^{3/2}ee^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32c^3}+\frac{\sqrt{\frac{\pi}{3}}\,b^{3/2}ee^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{96c^3}-\frac{3\sqrt{\pi}\,b^{3/2}ee^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32c^3}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-3*b*d*Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]])/(2*c) + (b*e*Sqrt[1 + c
^2*x^2]*Sqrt[a + b*ArcSinh[c*x]])/(3*c^3) - (b*e*x^2*Sqrt[1 + c^2*x^2]*Sqrt
[a + b*ArcSinh[c*x]])/(6*c) + d*x*(a + b*ArcSinh[c*x])^(3/2) + (e*x^3*(a +
b*ArcSinh[c*x])^(3/2))/3 + (3*b^(3/2)*d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*Arc
Sinh[c*x]]/Sqrt[b]])/(8*c) - (3*b^(3/2)*e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*A
rcSinh[c*x]]/Sqrt[b]])/(32*c^3) + (b^(3/2)*e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sq
rt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(96*c^3) + (3*b^(3/2)*d*Sqrt[Pi]*
Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*c*E^(a/b)) - (3*b^(3/2)*e*Sqrt[P
i]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(32*c^3*E^(a/b)) + (b^(3/2)*e*Sq
rt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(96*c^3*E^((3*a)
/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int (d+ex^2)\left(a+b\sinh^{-1}(cx)\right)^{3/2}\,dx = \int \left(d\left(a+b\sinh^{-1}(cx)\right)^{3/2}+ex^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}\right)dx$$

$$= d\int \left(a+b\sinh^{-1}(cx)\right)^{3/2}dx+e\int x^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}dx$$

$$= dx\left(a+b\sinh^{-1}(cx)\right)^{3/2}+\frac{1}{3}ex^3\left(a+b\sinh^{-1}(cx)\right)^{3/2}-\frac{1}{2}(3bcd)\int \frac{x\sqrt{}}{}$$

$$= -\frac{3bd\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{2c}-\frac{bex^2\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx}}{6c}$$

$$= -\frac{3bd\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{2c}+\frac{be\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{3c^3}$$

$$= -\frac{3bd\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{2c}+\frac{be\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{3c^3}$$

$$= -\frac{3bd\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{2c}+\frac{be\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{3c^3}$$

$$= -\frac{3bd\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{2c}+\frac{be\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{3c^3}$$

$$= -\frac{3bd\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{2c}+\frac{be\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{3c^3}$$

$$= -\frac{3bd\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{2c}+\frac{be\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{3c^3}$$

Mathematica [A] time = 4.84, size = 770, normalized size = 1.80

$$\frac{aee^{-\frac{3a}{b}}\sqrt{a+b\sinh^{-1}(cx)}\left(9e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{3}{2},-\frac{3\left(a+b\sinh^{-1}(\right.}{b}\right.\right.}{72c^3\sqrt{-\frac{\left(a+b\sinh^{-1}(\right.}{b^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)*(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (a*d*Sqrt[a + b*ArcSinh[c*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c*x]
])/Sqrt[a/b + ArcSinh[c*x]]) + Gamma[3/2, -((a + b*ArcSinh[c*x])/b)]/Sqrt[-
((a + b*ArcSinh[c*x])/b)]))/(2*c*E^(a/b)) + (a*e*Sqrt[a + b*ArcSinh[c*x]]*(
9*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[3/2, a/b + ArcSinh[c*x]
] + Sqrt[3]*Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2, (-3*(a + b*ArcSinh[c*x]))/b
] - 9*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2, -((a + b*ArcSinh[c*x]
)/b)] - Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[3/2, (3*(
a + b*ArcSinh[c*x]))/b]))/(72*c^3*E^((3*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])^2
/b^2)]) + (Sqrt[b]*d*(4*Sqrt[b]*Sqrt[a + b*ArcSinh[c*x]]*(-3*Sqrt[1 + c^2*x
^2] + 2*c*x*ArcSinh[c*x]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*
x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b
*ArcSinh[c*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])))/(8*c) + (Sqrt[b]*e*(-9*(4
*Sqrt[b]*Sqrt[a + b*ArcSinh[c*x]]*(-3*Sqrt[1 + c^2*x^2] + 2*c*x*ArcSinh[c*x
]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]*(Cosh[a/b]
 - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]
*(Cosh[a/b] + Sinh[a/b])) + (2*a + b)*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*A
rcSinh[c*x]])/Sqrt[b]]*(Cosh[(3*a)/b] - Sinh[(3*a)/b]) + (-2*a + b)*Sqrt[3*
Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]]*(Cosh[(3*a)/b] + Sinh[(
3*a)/b]) + 12*Sqrt[b]*Sqrt[a + b*ArcSinh[c*x]]*(-Cosh[3*ArcSinh[c*x]] + 2*A
rcSinh[c*x]*Sinh[3*ArcSinh[c*x]])))/(288*c^3)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Simplification assuming c near 0Simplification assuming c
 near 0Simplification assuming c near 0Simplification assuming c near 0Simp
lification assuming a near 0Simplification assuming c near 0Simplification
assuming c near 0Simplification assuming c near 0Simplification assuming c
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ssuming c near 0Simplification assuming a near 0Simplification assuming c n
ear 0Simplification assuming c near 0Simplification assuming c near 0Simpli
fication assuming c near 0Simplification assuming a near 0Simplification as
suming c near 0Simplification assuming a near 0Simplification assuming c ne
ar 0Simplification assuming c near 0Simplification assuming c near 0Simplif
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uming c near 0Simplification assuming c near 0Simplification assuming c nea
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ation assuming c near 0Simplification assuming c near 0Simplification assum
ing c near 0Simplification assuming c near 0Simplification assuming c near
0Simplification assuming c near 0Simplification assuming a near 0Evaluation
```

```
time: 0.82sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l
) Error: Bad Argument Value
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \left(e\,x^{2}+d\right)\left(a+b\,\mathrm{arcsinh}\left(c x\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arcsinh(c*x))^(3/2),x)
[Out] int((e*x^2+d)*(a+b*arcsinh(c*x))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^{2}+d\right)\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)*(b*arcsinh(c*x) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{3/2}\,\left(e\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^(3/2)*(d + e*x^2),x)
[Out] int((a + b*asinh(c*x))^(3/2)*(d + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{asinh}\left(cx\right)\right)^{\frac{3}{2}}\left(d+ex^{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*asinh(c*x))**(3/2),x)
[Out] Integral((a + b*asinh(c*x))**(3/2)*(d + e*x**2), x)
```

3.635 $\int \left(a+b\sinh^{-1}(cx)\right)^{3/2}\,dx$

Optimal. Leaf size=135

$$\frac{3\sqrt{\pi}\,b^{3/2}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8c}+\frac{3\sqrt{\pi}\,b^{3/2}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8c}-\frac{3b\sqrt{c^2x^2+1}\sqrt{a+b\sinh^{-1}(cx)}}{2c}+x\left(a+b\sin\right.$$

```
[Out] x*(a+b*arcsinh(c*x))^(3/2)+3/8*b^(3/2)*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2
)/b^(1/2))*Pi^(1/2)/c+3/8*b^(3/2)*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi
^(1/2)/c/exp(a/b)-3/2*b*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^(1/2)/c
```

Rubi [A] time = 0.25, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5653, 5717, 5657, 3307, 2180, 2205, 2204}

$$\frac{3\sqrt{\pi}\,b^{3/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8c}+\frac{3\sqrt{\pi}\,b^{3/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8c}-\frac{3b\sqrt{c^2x^2+1}\sqrt{a+b\sinh^{-1}(cx)}}{2c}+x\left(a+b\sin\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-3*b*Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]])/(2*c) + x*(a + b*ArcSinh[
c*x])^(3/2) + (3*b^(3/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt
[b]])/(8*c) + (3*b^(3/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(
8*c*E^(a/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\sinh^{-1}(cx)\right)^{3/2}\,dx = x\left(a+b\sinh^{-1}(cx)\right)^{3/2} - \frac{1}{2}(3bc)\int \frac{x\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{1+c^2x^2}}\,dx$$

$$= -\frac{3b\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{2c} + x\left(a+b\sinh^{-1}(cx)\right)^{3/2} + \frac{1}{4}\left(3b^2\right)\int \frac{\cdots}{\sqrt{a+b\cdots}}$$

$$= -\frac{3b\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{2c} + x\left(a+b\sinh^{-1}(cx)\right)^{3/2} + \frac{(3b)\,\mathrm{Subst}\left(\int \frac{\cosh\cdots}{\cdots}\right.}{\cdots}$$

$$= -\frac{3b\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{2c} + x\left(a+b\sinh^{-1}(cx)\right)^{3/2} + \frac{(3b)\,\mathrm{Subst}\left(\int \frac{e^{-i\left(\cdots\right)}}{\cdots}\right.}{\cdots}$$

$$= -\frac{3b\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{2c} + x\left(a+b\sinh^{-1}(cx)\right)^{3/2} + \frac{(3b)\,\mathrm{Subst}\left(\int e^{\frac{a}{b}-\cdots}\right.}{\cdots}$$

$$= -\frac{3b\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{2c} + x\left(a+b\sinh^{-1}(cx)\right)^{3/2} + \frac{3b^{3/2}e^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\cdots\right.}{8c}$$

Mathematica [A] time = 0.64, size = 251, normalized size = 1.86

$$\frac{\sqrt{b}\left(4\sqrt{b}\left(2cx\sinh^{-1}(cx)-3\sqrt{c^2x^2+1}\right)\sqrt{a+b\sinh^{-1}(cx)}+\sqrt{\pi}\,(3b-2a)\left(\sinh\left(\frac{a}{b}\right)+\cosh\left(\frac{a}{b}\right)\right)\mathrm{erf}\left(\frac{\sqrt{a+b\,\mathrm{s}\cdots}}{\cdots}\right.\cdots}{8c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (a*Sqrt[a + b*ArcSinh[c*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c*x]])
/Sqrt[a/b + ArcSinh[c*x]]) + Gamma[3/2, -((a + b*ArcSinh[c*x])/b)]/Sqrt[-((
a + b*ArcSinh[c*x])/b)]))/(2*c*E^(a/b)) + (Sqrt[b]*(4*Sqrt[b]*Sqrt[a + b*Ar
cSinh[c*x]]*(-3*Sqrt[1 + c^2*x^2] + 2*c*x*ArcSinh[c*x]) + (2*a + 3*b)*Sqrt[
Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a
+ 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b
])))/(8*c)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arsinh}(cx) + a)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(3/2), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{arcsinh}(cx))^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arsinh}(cx) + a)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \operatorname{asinh}(c\,x))^{3/2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^(3/2),x)
```

```
[Out] int((a + b*asinh(c*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{asinh}(cx))^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**(3/2), x)
```

3.636 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{d+ex^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{d+ex^2},x\right)$$

```
[Out] Unintegrable((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d),x)
```

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{d+ex^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2),x]
[Out] Defer[Int][(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2), x]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{d+ex^2}\,dx=\int \frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{d+ex^2}\,dx$$

Mathematica [A] time = 2.89, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{d+ex^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2),x]
[Out] Integrate[(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}}{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(3/2)/(e*x^2 + d), x)
```

maple [A] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{(a + b \,\mathrm{arcsinh}\,(cx))^{\frac{3}{2}}}{e\,x^{2} + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d),x)
[Out] int((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \,\mathrm{arsinh}\,(cx) + a)^{\frac{3}{2}}}{ex^{2} + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d),x, algorithm="maxima")
[Out] integrate((b*arcsinh(c*x) + a)^(3/2)/(e*x^2 + d), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{3/2}}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^(3/2)/(d + e*x^2),x)
[Out] int((a + b*asinh(c*x))^(3/2)/(d + e*x^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \,\mathrm{asinh}\,(cx))^{\frac{3}{2}}}{d + ex^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**(3/2)/(e*x**2+d),x)
[Out] Integral((a + b*asinh(c*x))**(3/2)/(d + e*x**2), x)
```

3.637 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{\left(d+ex^2\right)^2},x\right)$$

```
[Out] Unintegrable((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d)^2,x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{\left(d+ex^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2)^2,x]
[Out] Defer[Int][(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2)^2, x]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{\left(d+ex^2\right)^2}\,dx=\int \frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{\left(d+ex^2\right)^2}\,dx$$

Mathematica [A] time = 9.45, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{\left(d+ex^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2)^2,x]
[Out] Integrate[(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2)^2, x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d)^2,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\text{arsinh}\left(cx\right)+a\right)^{\frac{3}{2}}}{\left(ex^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^(3/2)/(e*x^2 + d)^2, x)
```

maple [A] time = 0.95, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^{\frac{3}{2}}}{\left(e\,x^{2}+d\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d)^2,x)
[Out] int((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^{\frac{3}{2}}}{\left(ex^{2}+d\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d)^2,x, algorithm="maxima")
[Out] integrate((b*arcsinh(c*x) + a)^(3/2)/(e*x^2 + d)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))^{3/2}}{\left(e\,x^{2}+d\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^(3/2)/(d + e*x^2)^2,x)
[Out] int((a + b*asinh(c*x))^(3/2)/(d + e*x^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**(3/2)/(e*x**2+d)**2,x)
[Out] Timed out
```

3.638 $\int \frac{(d+ex^2)^2}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx$

Optimal. Leaf size=608

$$\frac{\sqrt{\pi}\,e^2e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16\sqrt{b}\,c^5}-\frac{\sqrt{3\pi}\,e^2e^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}\,c^5}+\frac{\sqrt{\frac{\pi}{5}}\,e^2e^{\frac{5a}{b}}\operatorname{erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}\,c^5}+\frac{\sqrt{\pi}\,e^2e^{-\frac{a}{b}}\operatorname{er}}{1}$$

```
[Out] 1/160*e^2*exp(5*a/b)*erf(5^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*5^(1/2)*
Pi^(1/2)/c^5/b^(1/2)+1/160*e^2*erfi(5^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2
))*5^(1/2)*Pi^(1/2)/c^5/exp(5*a/b)/b^(1/2)+1/12*d*e*exp(3*a/b)*erf(3^(1/2)*
(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/c^3/b^(1/2)+1/12*d*e*erf
i(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/c^3/exp(3*a/b)
/b^(1/2)+1/2*d^2*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c/
b^(1/2)-1/4*d*e*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c^3
/b^(1/2)+1/16*e^2*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c
^5/b^(1/2)+1/2*d^2*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c/exp(a/
b)/b^(1/2)-1/4*d*e*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c^3/exp(
a/b)/b^(1/2)+1/16*e^2*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c^5/e
xp(a/b)/b^(1/2)-1/32*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^
(1/2))*3^(1/2)*Pi^(1/2)/c^5/b^(1/2)-1/32*e^2*erfi(3^(1/2)*(a+b*arcsinh(c*x)
)^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/c^5/exp(3*a/b)/b^(1/2)
```

Rubi [A] time = 1.20, antiderivative size = 608, normalized size of antiderivative = 1.00, number of steps used = 39, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5706, 5657, 3307, 2180, 2205, 2204, 5669, 5448}

$$-\frac{\sqrt{\pi}\,dee^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,c^3}+\frac{\sqrt{\frac{\pi}{3}}\,dee^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,c^3}-\frac{\sqrt{\pi}\,dee^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,c^3}+\frac{\sqrt{\frac{\pi}{3}}\,dee^{-\frac{3a}{b}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (d^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c)
 - (d*e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*Sqrt[b]*c
^3) + (e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(16*Sqrt
[b]*c^5) + (d*e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]
])/Sqrt[b]])/(4*Sqrt[b]*c^3) - (e^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqr
t[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5) + (e^2*E^((5*a)/b)*Sqrt[P
i/5]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5) + (d
^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c*E^(a/b)) -
 (d*e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*Sqrt[b]*c^3*E^(a/
b)) + (e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(16*Sqrt[b]*c^5
*E^(a/b)) + (d*e*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]
])/(4*Sqrt[b]*c^3*E^((3*a)/b)) - (e^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*A
rcSinh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5*E^((3*a)/b)) + (e^2*Sqrt[Pi/5]*Erfi
[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5*E^((5*a)/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx = \int \left(\frac{d^2}{\sqrt{a+b\sinh^{-1}(cx)}}+\frac{2dex^2}{\sqrt{a+b\sinh^{-1}(cx)}}+\frac{e^2x^4}{\sqrt{a+b\sinh^{-1}(cx)}}\right)dx$$

$$= d^2\int \frac{1}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx+(2de)\int \frac{x^2}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx+e^2\int \frac{x^4}{\sqrt{a+b\sinh}}$$

$$= \frac{d^2\,\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\sinh^{-1}(cx)\right)}{bc}+\frac{(2de)\,\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{\sqrt{a+bx}}\,dx,x\right.}{c^3}$$

$$= \frac{d^2\,\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx,x,a+b\sinh^{-1}(cx)\right)}{2bc}+\frac{d^2\,\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx,x,a+b\sin\right.}{2bc}$$

$$= \frac{d^2\,\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\sinh^{-1}(cx)}\right)}{bc}+\frac{d^2\,\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\,s}\right.}{bc}$$

$$= \frac{d^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}+\frac{d^2e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}+\frac{(de)\,\text{Subst}\left(\int \frac{e}{\sqrt{a}}\right.}{}$$

$$= \frac{d^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}+\frac{d^2e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}+\frac{(de)\,\text{Subst}\left(\int e^{\frac{3a}{b}}\right.}{}$$

$$= \frac{d^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}-\frac{dee^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,c^3}+\frac{e^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\sqrt{a+}\right.}{16\sqrt{b}\,c^5}$$

Mathematica [A] time = 1.23, size = 530, normalized size = 0.87

$$e^{-\frac{5a}{b}}\left(240c^4d^2e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(cx)}{b}\right)+40\sqrt{3}\,c^2dee^{\frac{2a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)-\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)^2/Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (-30*(8*c^4*d^2 - 4*c^2*d*e + e^2)*E^((6*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gam
ma[1/2, a/b + ArcSinh[c*x]] + 3*Sqrt[5]*e^2*Sqrt[-((a + b*ArcSinh[c*x])/b)]
*Gamma[1/2, (-5*(a + b*ArcSinh[c*x]))/b] + 40*Sqrt[3]*c^2*d*e*E^((2*a)/b)*S
qrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c*x]))/b] - 15
*Sqrt[3]*e^2*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-3*(a
+ b*ArcSinh[c*x]))/b] + 240*c^4*d^2*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])
/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)] - 120*c^2*d*e*E^((4*a)/b)*Sqrt[-
((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)] + 30*e^2*E^
((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])
/b)] - 40*Sqrt[3]*c^2*d*e*E^((8*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (
3*(a + b*ArcSinh[c*x]))/b] + 15*Sqrt[3]*e^2*E^((8*a)/b)*Sqrt[a/b + ArcSinh[
c*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c*x]))/b] - 3*Sqrt[5]*e^2*E^((10*a)/b)*S
qrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (5*(a + b*ArcSinh[c*x]))/b])/(480*c^5*E^
((5*a)/b)*Sqrt[a + b*ArcSinh[c*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^2}{\sqrt{b\operatorname{arsinh}\left(cx\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(b*arcsinh(c*x) + a), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(e\,x^2+d\right)^2}{\sqrt{a+b\operatorname{arcsinh}\left(cx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x)
```

```
[Out] int((e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^2}{\sqrt{b\operatorname{arsinh}\left(cx\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(b*arcsinh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)^2}{\sqrt{a+b\,\mathrm{asinh}\left(c\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2/(a + b*asinh(c*x))^(1/2),x)
```

```
[Out] int((d + e*x^2)^2/(a + b*asinh(c*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex^2\right)^2}{\sqrt{a+b\operatorname{asinh}\left(cx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(a+b*asinh(c*x))**(1/2),x)
```

```
[Out] Integral((d + e*x**2)**2/sqrt(a + b*asinh(c*x)), x)
```

3.639 $\int \frac{d+ex^2}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx$

Optimal. Leaf size=287

$$-\frac{\sqrt{\pi}\,ee^{a/b}\mathrm{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,c^3}+\frac{\sqrt{\frac{\pi}{3}}\,ee^{\frac{3a}{b}}\mathrm{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,c^3}-\frac{\sqrt{\pi}\,ee^{-\frac{a}{b}}\mathrm{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,c^3}+\frac{\sqrt{\frac{\pi}{3}}\,ee^{-\frac{3a}{b}}\mathrm{erfi}\left(\cdots\right)}{8\sqrt{\cdots}}$$

```
[Out] 1/24*e*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^
(1/2)/c^3/b^(1/2)+1/24*e*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(
1/2)*Pi^(1/2)/c^3/exp(3*a/b)/b^(1/2)+1/2*d*exp(a/b)*erf((a+b*arcsinh(c*x))^
(1/2)/b^(1/2))*Pi^(1/2)/c/b^(1/2)-1/8*e*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/
2)/b^(1/2))*Pi^(1/2)/c^3/b^(1/2)+1/2*d*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2
))*Pi^(1/2)/c/exp(a/b)/b^(1/2)-1/8*e*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))
*Pi^(1/2)/c^3/exp(a/b)/b^(1/2)
```

Rubi [A] time = 0.57, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5706, 5657, 3307, 2180, 2205, 2204, 5669, 5448}

$$-\frac{\sqrt{\pi}\,ee^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,c^3}+\frac{\sqrt{\frac{\pi}{3}}\,ee^{\frac{3a}{b}}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,c^3}-\frac{\sqrt{\pi}\,ee^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,c^3}+\frac{\sqrt{\frac{\pi}{3}}\,ee^{-\frac{3a}{b}}\mathrm{Erfi}\left(\cdots\right)}{8\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c) -
(e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*Sqrt[b]*c^3)
+ (e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]]
)/(8*Sqrt[b]*c^3) + (d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*
Sqrt[b]*c*E^(a/b)) - (e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8
*Sqrt[b]*c^3*E^(a/b)) + (e*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]
])/Sqrt[b]])/(8*Sqrt[b]*c^3*E^((3*a)/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
```

```
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{d+ex^2}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx = \int \left(\frac{d}{\sqrt{a+b\sinh^{-1}(cx)}}+\frac{ex^2}{\sqrt{a+b\sinh^{-1}(cx)}}\right)dx$$
$$= d\int \frac{1}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx + e\int \frac{x^2}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx$$
$$= \frac{d\,\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\sinh^{-1}(cx)\right)}{bc} + \frac{e\,\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{\sqrt{a+bx}}\,dx, x, \sinh\right.}{c^3}$$
$$= \frac{d\,\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\sinh^{-1}(cx)\right)}{2bc} + \frac{d\,\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\sinh\right.}{2bc}$$
$$= \frac{d\,\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{bc} + \frac{d\,\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sin}\right.}{bc}$$
$$= \frac{de^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c} + \frac{de^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c} + \frac{e\,\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{a+bx}}\,d\right.}{8c}$$
$$= \frac{de^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c} + \frac{de^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c} + \frac{e\,\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\right.}{}$$
$$= \frac{de^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c} - \frac{ee^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,c^3} + \frac{ee^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+}\right.}{8\sqrt{b}\,c^3}$$

Mathematica [A] time = 0.69, size = 218, normalized size = 0.76

$$\frac{e^{-\frac{3a}{b}}\left(-3e^{\frac{4a}{b}}\left(4c^2d-e\right)\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)+3e^{\frac{2a}{b}}\left(4c^2d-e\right)\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\sinh\right.\right.}{24c^3\sqrt{a+b\sinh}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)/Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (-3*(4*c^2*d - e)*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, a/b + Arc
Sinh[c*x]] + Sqrt[3]*e*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-3*(a +
b*ArcSinh[c*x]))/b] + 3*(4*c^2*d - e)*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c*x
])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)] - Sqrt[3]*e*E^((6*a)/b)*Sqrt[a
/b + ArcSinh[c*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c*x]))/b])/(24*c^3*E^((3*a)
/b)*Sqrt[a + b*ArcSinh[c*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2 + d}{\sqrt{b \operatorname{arsinh}(cx) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)/sqrt(b*arcsinh(c*x) + a), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{e\,x^2 + d}{\sqrt{a + b \operatorname{arcsinh}(cx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x)
[Out] int((e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2 + d}{\sqrt{b \operatorname{arsinh}(cx) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)/sqrt(b*arcsinh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e\,x^2 + d}{\sqrt{a + b \operatorname{asinh}(c\,x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(a + b*asinh(c*x))^(1/2),x)
[Out] int((d + e*x^2)/(a + b*asinh(c*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2}{\sqrt{a + b \operatorname{asinh}(cx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(a+b*asinh(c*x))**(1/2),x)
[Out] Integral((d + e*x**2)/sqrt(a + b*asinh(c*x)), x)
```

3.640 $\int \frac{1}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx$

Optimal. Leaf size=88

$$\frac{\sqrt{\pi}\,e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}+\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}$$

```
[Out] 1/2*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c/b^(1/2)+1/2*e
rfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c/exp(a/b)/b^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5657, 3307, 2180, 2205, 2204}

$$\frac{\sqrt{\pi}\,e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}+\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c) + (S
qrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c*E^(a/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\sinh^{-1}(cx)\right)}{bc}$$
$$= \frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\sinh^{-1}(cx)\right)}{2bc} + \frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\sinh^{-1}(cx)\right)}{2bc}$$
$$= \frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{bc} + \frac{\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{bc}$$
$$= \frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}$$

Mathematica [A] time = 0.10, size = 101, normalized size = 1.15

$$\frac{e^{-\frac{a}{b}}\left(\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(cx)}{b}\right)-e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{2c\sqrt{a+b\sinh^{-1}(cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (-(E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]]) + S
qrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)])/(2*c*
E^(a/b)*Sqrt[a + b*ArcSinh[c*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\,\text{arsinh}\,(cx)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*arcsinh(c*x) + a), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\,\text{arcsinh}\,(cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(c*x))^(1/2),x)
```

```
[Out] int(1/(a+b*arcsinh(c*x))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\operatorname{arsinh}(cx)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*arcsinh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+b\operatorname{asinh}(c\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(c*x))^(1/2),x)
```

```
[Out] int(1/(a + b*asinh(c*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\operatorname{asinh}(cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(c*x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*asinh(c*x)), x)
```

3.641 $\int \frac{1}{(d+ex^2)\sqrt{a+b\sinh^{-1}(cx)}}\,dx$

Optimal. Leaf size=25

$$\mathrm{Int}\left(\frac{1}{(d+ex^2)\sqrt{a+b\sinh^{-1}(cx)}},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\sinh^{-1}(cx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)*Sqrt[a + b*ArcSinh[c*x]]),x]
```

```
[Out] Defer[Int][1/((d + e*x^2)*Sqrt[a + b*ArcSinh[c*x]]), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\sinh^{-1}(cx)}}\,dx = \int \frac{1}{(d+ex^2)\sqrt{a+b\sinh^{-1}(cx)}}\,dx$$

Mathematica [A] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\sinh^{-1}(cx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)*Sqrt[a + b*ArcSinh[c*x]]),x]
```

```
[Out] Integrate[1/((d + e*x^2)*Sqrt[a + b*ArcSinh[c*x]]), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex^2+d)\sqrt{b\,\mathrm{arsinh}\,(cx)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
[Out] integrate(1/((e*x^2 + d)*sqrt(b*arcsinh(c*x) + a)), x)
```

maple [A] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\,x^{2}+d\right)\sqrt{a+b\,\mathrm{arcsinh}\left(c x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x)
[Out] int(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex^{2}+d\right)\sqrt{b\,\mathrm{arsinh}\left(cx\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
[Out] integrate(1/((e*x^2 + d)*sqrt(b*arcsinh(c*x) + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{a+b\,\mathrm{asinh}\left(c\,x\right)}\,\left(e\,x^{2}+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))^(1/2)*(d + e*x^2)),x)
[Out] int(1/((a + b*asinh(c*x))^(1/2)*(d + e*x^2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\,\mathrm{asinh}\left(cx\right)}\,\left(d+ex^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(a+b*asinh(c*x))**(1/2),x)
[Out] Integral(1/(sqrt(a + b*asinh(c*x))*(d + e*x**2)), x)
```

3.642 $\int \frac{1}{\left(d+ex^2\right)^2 \sqrt{a+b\sinh^{-1}(cx)}}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{\left(d+ex^2\right)^2 \sqrt{a+b\sinh^{-1}(cx)}},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(d+ex^2\right)^2 \sqrt{a+b\sinh^{-1}(cx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)^2*Sqrt[a + b*ArcSinh[c*x]]),x]
[Out] Defer[Int][1/((d + e*x^2)^2*Sqrt[a + b*ArcSinh[c*x]]), x]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^2 \sqrt{a+b\sinh^{-1}(cx)}}\,dx = \int \frac{1}{\left(d+ex^2\right)^2 \sqrt{a+b\sinh^{-1}(cx)}}\,dx$$

Mathematica [A] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d+ex^2\right)^2 \sqrt{a+b\sinh^{-1}(cx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^2*Sqrt[a + b*ArcSinh[c*x]]),x]
[Out] Integrate[1/((d + e*x^2)^2*Sqrt[a + b*ArcSinh[c*x]]), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex^2+d\right)^2 \sqrt{b\,\text{arsinh}\left(cx\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
[Out] integrate(1/((e*x^2 + d)^2*sqrt(b*arcsinh(c*x) + a)), x)
```

maple [A] time = 0.91, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\,x^{2}+d\right)^{2} \sqrt{a +b \,\mathrm{arcsinh}\left(c x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x)
[Out] int(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{{\left(e x^{2} + d\right)}^{2} \sqrt{b \operatorname{arsinh}\left(c x\right) + a}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
[Out] integrate(1/((e*x^2 + d)^2*sqrt(b*arcsinh(c*x) + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{a+b\,\mathrm{asinh}\left(c\,x\right)}\,{\left(e\,x^2+d\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))^(1/2)*(d + e*x^2)^2),x)
[Out] int(1/((a + b*asinh(c*x))^(1/2)*(d + e*x^2)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b \operatorname{asinh}{\left(c x \right)}} \left(d + e x^{2}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(a+b*asinh(c*x))**(1/2),x)
[Out] Integral(1/(sqrt(a + b*asinh(c*x))*(d + e*x**2)**2), x)
```

3.643 $\int \frac{d+ex^2}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=349

$$\frac{\sqrt{\pi}\, e e^{a/b} \operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}-\frac{\sqrt{3\pi}\, e e^{\frac{3a}{b}} \operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}-\frac{\sqrt{\pi}\, e e^{-\frac{a}{b}} \operatorname{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}+\frac{\sqrt{3\pi}\, e e^{-\frac{3a}{b}} \operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{\cdots}}{\cdots}\right)}{4b^{3/2}c^3}$$

```
[Out] -d*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c+1/4*e*
exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^3+d*erfi(
(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c/exp(a/b)-1/4*e*erfi((a
+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^3/exp(a/b)-1/4*e*exp(3*a
/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/b^(3/2)/
c^3+1/4*e*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/b
^(3/2)/c^3/exp(3*a/b)-2*d*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))^(1/2)-2*
e*x^2*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))^(1/2)
```

Rubi [A] time = 0.69, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5706, 5655, 5779, 3308, 2180, 2204, 2205, 5665}

$$\frac{\sqrt{\pi}\, e e^{a/b} \operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}-\frac{\sqrt{3\pi}\, e e^{\frac{3a}{b}} \operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}-\frac{\sqrt{\pi}\, e e^{-\frac{a}{b}} \operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}+\frac{\sqrt{3\pi}\, e e^{-\frac{3a}{b}} \operatorname{Erfi}\left(\frac{\sqrt{3}\cdots}{\cdots}\right)}{4b^{3/2}c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-2*d*Sqrt[1 + c^2*x^2])/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (2*e*x^2*Sqrt[1 +
 c^2*x^2])/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a
+ b*ArcSinh[c*x]]/Sqrt[b]])/(b^(3/2)*c) + (e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a +
b*ArcSinh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c^3) - (e*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(
Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c^3) + (d*Sqrt[Pi]*E
rfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(b^(3/2)*c*E^(a/b)) - (e*Sqrt[Pi]*Er
fi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c^3*E^(a/b)) + (e*Sqrt[3*P
i]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c^3*E^((3*a
)/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{d+ex^2}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx = \int \left(\frac{d}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}} + \frac{ex^2}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\right)dx$$

$$= d\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx + e\int \frac{x^2}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$$

$$= -\frac{2d\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{2ex^2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(2cd)\int \frac{x}{\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{b}$$

$$= -\frac{2d\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{2ex^2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(2d)\,\mathrm{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-}\right.}{bc}$$

$$= -\frac{2d\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{2ex^2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{d\,\mathrm{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c\right.}{bc}$$

$$= -\frac{2d\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{2ex^2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{(2d)\,\mathrm{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\right.}{b^2c}$$

$$= -\frac{2d\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{2ex^2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{de^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} + \frac{ee}{}$$

Mathematica [A] time = 1.52, size = 303, normalized size = 0.87

$$\frac{e^{-3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}\left(\left(4c^2d-e\right)e^{\frac{4a}{b}+3\sinh^{-1}(cx)}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)+\left(4c^2d-e\right)e^{\frac{2a}{b}+3\sinh^{-1}(cx)}\sqrt{-\frac{a+b}{}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] ((4*c^2*d - e)*E^((4*a)/b + 3*ArcSinh[c*x])*Sqrt[a/b + ArcSinh[c*x]]*Gamma[
1/2, a/b + ArcSinh[c*x]] + Sqrt[3]*e*E^(3*ArcSinh[c*x])*Sqrt[-((a + b*ArcSi
nh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c*x]))/b] + (4*c^2*d - e)*E^((2*
a)/b + 3*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*
ArcSinh[c*x])/b)] + E^((3*a)/b)*(-((1 + E^(2*ArcSinh[c*x]))*(4*c^2*d*E^(2*A
rcSinh[c*x]) + e*(-1 + E^(2*ArcSinh[c*x]))^2)) + Sqrt[3]*e*E^(3*(a/b + ArcS
inh[c*x]))*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c*x]))/b])
)/(4*b*c^3*E^(3*(a/b + ArcSinh[c*x]))*Sqrt[a + b*ArcSinh[c*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2 + d}{(b \operatorname{arsinh}(cx) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)/(b*arcsinh(c*x) + a)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{e\,x^2 + d}{(a + b \operatorname{arcsinh}(cx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
[Out] int((e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2 + d}{(b \operatorname{arsinh}(cx) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)/(b*arcsinh(c*x) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e\,x^2 + d}{(a + b \operatorname{asinh}(c\,x))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(a + b*asinh(c*x))^(3/2),x)
[Out] int((d + e*x^2)/(a + b*asinh(c*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2}{(a + b \operatorname{asinh}(cx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(a+b*asinh(c*x))**(3/2),x)
[Out] Integral((d + e*x**2)/(a + b*asinh(c*x))**(3/2), x)
```

3.644 $\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=116

$$-\frac{\sqrt{\pi}\,e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}+\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\sinh^{-1}(cx)}}$$

```
[Out] -exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c+erfi((a+
b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c/exp(a/b)-2*(c^2*x^2+1)^(1
/2)/b/c/(a+b*arcsinh(c*x))^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5655, 5779, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\,e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}+\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\sinh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^(-3/2),x]
```

```
[Out] (-2*Sqrt[1 + c^2*x^2])/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (E^(a/b)*Sqrt[Pi]*E
rf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(b^(3/2)*c) + (Sqrt[Pi]*Erfi[Sqrt[a +
 b*ArcSinh[c*x]]/Sqrt[b]])/(b^(3/2)*c*E^(a/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
```

```
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{(2c)\int \frac{x}{\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{b}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{2\,\mathrm{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{bc}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}-\frac{\mathrm{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{bc}+\frac{\mathrm{Subst}\left(\int \frac{e^{x}}{\sqrt{a+bx}}\,d\right.}{b}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}-\frac{2\,\mathrm{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\sinh^{-1}(cx)}\right)}{b^2c}+\frac{2\,\mathrm{Subst}}{}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}-\frac{e^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}+\frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c}}{\sqrt{b}}\right.}{b^{3/2}c}$$

Mathematica [A] time = 0.23, size = 137, normalized size = 1.18

$$\frac{e^{-\frac{a+b\sinh^{-1}(cx)}{b}}\left(-e^{a/b}\left(e^{2\sinh^{-1}(cx)}+1\right)+e^{\frac{2a}{b}+\sinh^{-1}(cx)}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)+e^{\sinh^{-1}(cx)}\sqrt{-\frac{a+b\,\mathrm{si}}{}}\right.}{bc\sqrt{a+b\sinh^{-1}(cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^(-3/2),x]
```

```
[Out] (-(E^(a/b)*(1 + E^(2*ArcSinh[c*x]))) + E^((2*a)/b + ArcSinh[c*x])*Sqrt[a/b
+ ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] + E^ArcSinh[c*x]*Sqrt[-((a +
 b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)])/(b*c*E^((a + b*
ArcSinh[c*x])/b)*Sqrt[a + b*ArcSinh[c*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{arsinh}(cx) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(-3/2), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \operatorname{arcsinh}(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(1/(a+b*arcsinh(c*x))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{arsinh}(cx) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + b \operatorname{asinh}(c\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(c*x))^(3/2),x)
```

```
[Out] int(1/(a + b*asinh(c*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \operatorname{asinh}(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**(-3/2), x)
```

3.645 $\int \frac{1}{(d+ex^2)(a+b\sinh^{-1}(cx))^{3/2}}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{(d+ex^2)\left(a+b\sinh^{-1}(cx)\right)^{3/2}},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(d+ex^2)\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)*(a + b*ArcSinh[c*x])^(3/2)),x]
[Out] Defer[Int][1/((d + e*x^2)*(a + b*ArcSinh[c*x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx=\int \frac{1}{(d+ex^2)\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$$

Mathematica [A] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)*(a + b*ArcSinh[c*x])^(3/2)),x]
[Out] Integrate[1/((d + e*x^2)*(a + b*ArcSinh[c*x])^(3/2)), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex^2+d)(b\,\text{arsinh}(cx)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
[Out] integrate(1/((e*x^2 + d)*(b*arcsinh(c*x) + a)^(3/2)), x)
```

maple [A] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\,x^{2}+d\right)\left(a+b\,\mathrm{arcsinh}\left(c x\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
[Out] int(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex^{2}+d\right)\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((e*x^2 + d)*(b*arcsinh(c*x) + a)^(3/2)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{3/2}\,\left(e\,x^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))^(3/2)*(d + e*x^2)),x)
[Out] int(1/((a + b*asinh(c*x))^(3/2)*(d + e*x^2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(cx\right)\right)^{\frac{3}{2}}\left(d+ex^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(a+b*asinh(c*x))**(3/2),x)
[Out] Integral(1/((a + b*asinh(c*x))**(3/2)*(d + e*x**2)), x)
```

3.646 $\int \frac{1}{(d+ex^2)^2 (a+b\sinh^{-1}(cx))^{3/2}} dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{(d+ex^2)^2 (a+b\sinh^{-1}(cx))^{3/2}}, x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(d+ex^2)^2 (a+b\sinh^{-1}(cx))^{3/2}} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^(3/2)),x]
```

```
[Out] Defer[Int][1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^2 (a+b\sinh^{-1}(cx))^{3/2}} dx = \int \frac{1}{(d+ex^2)^2 (a+b\sinh^{-1}(cx))^{3/2}} dx$$

Mathematica [A] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)^2 (a+b\sinh^{-1}(cx))^{3/2}} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^(3/2)),x]
```

```
[Out] Integrate[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^(3/2)), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex^2+d)^2 (b\,\text{arsinh}(cx)+a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)^2*(b*arcsinh(c*x) + a)^(3/2)), x)
```

maple [A] time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\,x^{2}+d\right)^{2} \left(a +b \,\mathrm{arcsinh}\left(c x \right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{{\left(e x^{2} + d\right)}^{2} {\left(b \operatorname{arsinh}\left(c x\right) + a\right)}^{\frac{3}{2}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*x^2 + d)^2*(b*arcsinh(c*x) + a)^(3/2)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{3/2}\,{\left(e\,x^2+d\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))^(3/2)*(d + e*x^2)^2),x)
```

```
[Out] int(1/((a + b*asinh(c*x))^(3/2)*(d + e*x^2)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] Timed out
```

3.647 $\int \sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)dx$

Optimal. Leaf size=23

$$\operatorname{Int}\left(\sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable((a+b*arcsinh(c*x))*(e*x^2+d)^(1/2),x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x]),x]
[Out] Defer[Int][Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x]), x]
```

Rubi steps

$$\int \sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)dx=\int \sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)dx$$

Mathematica [A] time = 5.26, size = 0, normalized size = 0.00

$$\int \sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x]),x]
[Out] Integrate[Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x]), x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\sqrt{ex^2+d}\,(b\operatorname{arsinh}(cx)+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arcsinh(c*x) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex^2+d}\,(b\operatorname{arsinh}(cx)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsinh(c*x) + a), x)
```

maple [A] time = 0.46, size = 0, normalized size = 0.00

$$\int (a+b\operatorname{arcsinh}(cx))\sqrt{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))*(e*x^2+d)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e-c^2*d>0)', see `assume?` for more
 details)Is e-c^2*d zero or nonzero?
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int (a+b\,\mathrm{asinh}\,(c\,x))\,\sqrt{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + e*x^2)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))*(d + e*x^2)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\,\mathrm{asinh}\,(cx))\,\sqrt{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))*sqrt(d + e*x**2), x)
```

3.648 $\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{a+b\sinh^{-1}(cx)}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable((a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c*x])/Sqrt[d + e*x^2],x]
[Out] Defer[Int][(a + b*ArcSinh[c*x])/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

Mathematica [A] time = 3.92, size = 0, normalized size = 0.00

$$\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c*x])/Sqrt[d + e*x^2],x]
[Out] Integrate[(a + b*ArcSinh[c*x])/Sqrt[d + e*x^2], x]
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{ex^2+d}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral((b*arcsinh(c*x) + a)/sqrt(e*x^2 + d), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{ex^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/sqrt(e*x^2 + d), x)
```

maple [A] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arcsinh}\left(cx\right)}{\sqrt{e\,x^{2}+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$b\int \frac{\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)}{\sqrt{ex^{2}+d}}\,dx+\frac{a\,\mathrm{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/sqrt(e*x^2 + d), x) + a*arcsinh(e*
x/sqrt(d*e))/sqrt(e)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a+b\,\mathrm{asinh}\left(c\,x\right)}{\sqrt{e\,x^{2}+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(d + e*x^2)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))/(d + e*x^2)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{asinh}\left(cx\right)}{\sqrt{d+ex^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/sqrt(d + e*x**2), x)
```

3.649 $\int \frac{a+b\sinh^{-1}(cx)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=70

$$\frac{x\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+ex^2}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2+1}}{c\sqrt{d+ex^2}}\right)}{d\sqrt{e}}$$

```
[Out] -b*arctanh(e^(1/2)*(c^2*x^2+1)^(1/2)/c/(e*x^2+d)^(1/2))/d/e^(1/2)+x*(a+b*ar
csinh(c*x))/d/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {191, 5704, 12, 444, 63, 217, 206}

$$\frac{x\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+ex^2}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2+1}}{c\sqrt{d+ex^2}}\right)}{d\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(d + e*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcSinh[c*x]))/(d*Sqrt[d + e*x^2]) - (b*ArcTanh[(Sqrt[e]*Sqrt[1 +
 c^2*x^2])/(c*Sqrt[d + e*x^2])])/(d*Sqrt[e])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 5704

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0])
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{\left(d+ex^2\right)^{3/2}}\,dx = \frac{x\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+ex^2}} - (bc)\int \frac{x}{d\sqrt{1+c^2x^2}\,\sqrt{d+ex^2}}\,dx$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+ex^2}} - \frac{(bc)\int \frac{x}{\sqrt{1+c^2x^2}\,\sqrt{d+ex^2}}\,dx}{d}$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+ex^2}} - \frac{(bc)\,\text{Subst}\left(\int \frac{1}{\sqrt{1+c^2x}\,\sqrt{d+ex}}\,dx, x, x^2\right)}{2d}$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+ex^2}} - \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{d-\frac{e}{c^2}+\frac{ex^2}{c^2}}}\,dx, x, \sqrt{1+c^2x^2}\right)}{cd}$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+ex^2}} - \frac{b\,\text{Subst}\left(\int \frac{1}{1-\frac{ex^2}{c^2}}\,dx, x, \frac{\sqrt{1+c^2x^2}}{\sqrt{d+ex^2}}\right)}{cd}$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+ex^2}} - \frac{b\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{d\sqrt{e}}$$

Mathematica [C] time = 0.12, size = 75, normalized size = 1.07

$$\frac{x\left(2\left(a+b\sinh^{-1}(cx)\right)-bcx\sqrt{\frac{ex^2}{d}+1}\,F_1\left(1;\frac{1}{2},\frac{1}{2};2;-c^2x^2,-\frac{ex^2}{d}\right)\right)}{2d\sqrt{d+ex^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + e*x^2)^(3/2),x]
```

```
[Out] (x*(-(b*c*x*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, -(c^2*x^2), -((e*x
^2)/d)]) + 2*(a + b*ArcSinh[c*x])))/(2*d*Sqrt[d + e*x^2])
```

fricas [B] time = 0.54, size = 326, normalized size = 4.66

$$\left[\frac{4\sqrt{ex^2+d}\,bex\log\left(cx+\sqrt{c^2x^2+1}\right)+4\sqrt{ex^2+d}\,aex+\left(bex^2+bd\right)\sqrt{e}\,\log\left(8c^4e^2x^4+c^4d^2+6c^2de+8\left(c^4de\right.\right.}{4\left(de^2x^2+d^2e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(e*x^2 + d)*b*e*x*log(c*x + sqrt(c^2*x^2 + 1)) + 4*sqrt(e*x^2 +
 d)*a*e*x + (b*e*x^2 + b*d)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e
 + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d + c*e)*sqrt(c^2*x^2 +
 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2))/(d*e^2*x^2 + d^2*e), 1/2*(2*sqrt(e*x^2
+ d)*b*e*x*log(c*x + sqrt(c^2*x^2 + 1)) + 2*sqrt(e*x^2 + d)*a*e*x + (b*e*x^
2 + b*d)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d + e)*sqrt(c^2*x^2 + 1)*sq
rt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 + c*d*e + (c^3*d*e + c*e^2)*x^2)))/(d*e
^2*x^2 + d^2*e)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\left(ex^2 + d\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(e*x^2 + d)^(3/2), x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arcsinh}(cx)}{\left(e\,x^2 + d\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(e*x^2+d)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e-c^2*d>0)', see `assume?` for more
 details)Is e-c^2*d zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{asinh}(c\,x)}{\left(e\,x^2 + d\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(d + e*x^2)^(3/2),x)
```

```
[Out] int((a + b*asinh(c*x))/(d + e*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asinh}(cx)}{\left(d + ex^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(d + e*x**2)**(3/2), x)
```

3.650 $\int \frac{a+b\sinh^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx$

Optimal. Leaf size=146

$$\frac{2x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2+1}}{c\sqrt{d+ex^2}}\right)}{3d^2\sqrt{e}}-\frac{bc\sqrt{c^2x^2+1}}{3d\left(c^2d-e\right)\sqrt{d+ex^2}}$$

```
[Out] 1/3*x*(a+b*arcsinh(c*x))/d/(e*x^2+d)^(3/2)-2/3*b*arctanh(e^(1/2)*(c^2*x^2+1
)^(1/2)/c/(e*x^2+d)^(1/2))/d^2/e^(1/2)+2/3*x*(a+b*arcsinh(c*x))/d^2/(e*x^2+
d)^(1/2)-1/3*b*c*(c^2*x^2+1)^(1/2)/d/(c^2*d-e)/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {192, 191, 5704, 12, 571, 78, 63, 217, 206}

$$\frac{2x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2+1}}{c\sqrt{d+ex^2}}\right)}{3d^2\sqrt{e}}-\frac{bc\sqrt{c^2x^2+1}}{3d\left(c^2d-e\right)\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(d + e*x^2)^(5/2),x]
```

```
[Out] -(b*c*Sqrt[1 + c^2*x^2])/(3*d*(c^2*d - e)*Sqrt[d + e*x^2]) + (x*(a + b*ArcS
inh[c*x]))/(3*d*(d + e*x^2)^(3/2)) + (2*x*(a + b*ArcSinh[c*x]))/(3*d^2*Sqrt
[d + e*x^2]) - (2*b*ArcTanh[(Sqrt[e]*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + e*x^2])
])/(3*d^2*Sqrt[e])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 5704

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0])
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{\left(d+ex^2\right)^{5/2}}dx = \frac{x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} - (bc)\int \frac{x\left(3d+2ex^2\right)}{3d^2\sqrt{1+c^2x^2}\left(d+ex^2\right)^{3/2}}dx$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} - \frac{(bc)\int \frac{x\left(3d+2ex^2\right)}{\sqrt{1+c^2x^2}\left(d+ex^2\right)^{3/2}}dx}{3d^2}$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} - \frac{(bc)\,\text{Subst}\left(\int \frac{3d+2ex}{\sqrt{1+c^2x}(d+ex)^{3/2}}dx, x, x^2\right.}{6d^2}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{3d\left(c^2d-e\right)\sqrt{d+ex^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} - \frac{(bc)\,\text{Subst}}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{3d\left(c^2d-e\right)\sqrt{d+ex^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} - \frac{(2b)\,\text{Subs}}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{3d\left(c^2d-e\right)\sqrt{d+ex^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} - \frac{(2b)\,\text{Subs}}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{3d\left(c^2d-e\right)\sqrt{d+ex^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} - \frac{2b\tanh^{-1}}{3c}$$

Mathematica [C] time = 0.34, size = 139, normalized size = 0.95

$$\frac{ax\left(3d+2ex^2\right) - bcx^2\left(d+ex^2\right)\sqrt{\frac{ex^2}{d}+1}\,F_1\left(1;\frac{1}{2},\frac{1}{2};2;-c^2x^2,-\frac{ex^2}{d}\right) - \frac{bcd\sqrt{c^2x^2+1}\left(d+ex^2\right)}{c^2d-e} + bx\sinh^{-1}(cx)\left(3d+2\right.}{3d^2\left(d+ex^2\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + e*x^2)^(5/2),x]
```

```
[Out] (-((b*c*d*Sqrt[1 + c^2*x^2]*(d + e*x^2))/(c^2*d - e)) + a*x*(3*d + 2*e*x^2)
 - b*c*x^2*(d + e*x^2)*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, -(c^2*x
^2), -((e*x^2)/d)] + b*x*(3*d + 2*e*x^2)*ArcSinh[c*x])/(3*d^2*(d + e*x^2)^(
3/2))
```

fricas [B] time = 0.82, size = 738, normalized size = 5.05

$$\left[\frac{\left(bc^2d^3 + \left(bc^2de^2 - be^3\right)x^4 - bd^2e + 2\left(bc^2d^2e - bde^2\right)x^2\right)\sqrt{e}\,\log\left(8\,c^4e^2x^4 + c^4d^2 + 6\,c^2de + 8\left(c^4de + c^2e^2\right)x^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*((b*c^2*d^3 + (b*c^2*d*e^2 - b*e^3)*x^4 - b*d^2*e + 2*(b*c^2*d^2*e - b
*d*e^2)*x^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e +
```

```
c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d + c*e)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2
+ d)*sqrt(e) + e^2) + 2*(2*(b*c^2*d*e^2 - b*e^3)*x^3 + 3*(b*c^2*d^2*e - b*d
*e^2)*x)*sqrt(e*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(2*(a*c^2*d*e^2 -
 a*e^3)*x^3 + 3*(a*c^2*d^2*e - a*d*e^2)*x - (b*c*d*e^2*x^2 + b*c*d^2*e)*sqr
t(c^2*x^2 + 1))*sqrt(e*x^2 + d))/(c^2*d^5*e - d^4*e^2 + (c^2*d^3*e^3 - d^2*
e^4)*x^4 + 2*(c^2*d^4*e^2 - d^3*e^3)*x^2), 1/3*((b*c^2*d^3 + (b*c^2*d*e^2 -
 b*e^3)*x^4 - b*d^2*e + 2*(b*c^2*d^2*e - b*d*e^2)*x^2)*sqrt(-e)*arctan(1/2*
(2*c^2*e*x^2 + c^2*d + e)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e
^2*x^4 + c*d*e + (c^3*d*e + c*e^2)*x^2)) + (2*(b*c^2*d*e^2 - b*e^3)*x^3 + 3
*(b*c^2*d^2*e - b*d*e^2)*x)*sqrt(e*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) +
(2*(a*c^2*d*e^2 - a*e^3)*x^3 + 3*(a*c^2*d^2*e - a*d*e^2)*x - (b*c*d*e^2*x^2
 + b*c*d^2*e)*sqrt(c^2*x^2 + 1))*sqrt(e*x^2 + d))/(c^2*d^5*e - d^4*e^2 + (c
^2*d^3*e^3 - d^2*e^4)*x^4 + 2*(c^2*d^4*e^2 - d^3*e^3)*x^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\left(ex^2 + d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(e*x^2 + d)^(5/2), x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arcsinh}(cx)}{\left(e\,x^2 + d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(e*x^2+d)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\,a\left(\frac{2\,x}{\sqrt{ex^2 + d}\,d^2} + \frac{x}{\left(ex^2 + d\right)^{\frac{3}{2}}d}\right) + b\int \frac{\log\left(cx + \sqrt{c^2x^2 + 1}\right)}{\left(ex^2 + d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*(2*x/(sqrt(e*x^2 + d)*d^2) + x/((e*x^2 + d)^(3/2)*d)) + b*integrate(l
og(c*x + sqrt(c^2*x^2 + 1))/(e*x^2 + d)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{asinh}(c\,x)}{\left(e\,x^2 + d\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(d + e*x^2)^(5/2),x)
```

```
[Out] int((a + b*asinh(c*x))/(d + e*x^2)^(5/2), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{asinh}\,(cx)}{\left(d+e x^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(d + e*x**2)**(5/2), x)
```

3.651 $\int \frac{a+b\sinh^{-1}(cx)}{(d+ex^2)^{7/2}}\,dx$

Optimal. Leaf size=227

$$\frac{8x\left(a+b\sinh^{-1}(cx)\right)}{15d^3\sqrt{d+ex^2}}+\frac{4x\left(a+b\sinh^{-1}(cx)\right)}{15d^2\left(d+ex^2\right)^{3/2}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{5d\left(d+ex^2\right)^{5/2}}-\frac{8b\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2+1}}{c\sqrt{d+ex^2}}\right)}{15d^3\sqrt{e}}-\frac{2bc\sqrt{c^2x^2+1}\left(3c^2d\right.}{15d^2\left(c^2d-e\right)^2\sqrt{d}}$$

```
[Out] 1/5*x*(a+b*arcsinh(c*x))/d/(e*x^2+d)^(5/2)+4/15*x*(a+b*arcsinh(c*x))/d^2/(e
*x^2+d)^(3/2)-8/15*b*arctanh(e^(1/2)*(c^2*x^2+1)^(1/2)/c/(e*x^2+d)^(1/2))/d
^3/e^(1/2)-1/15*b*c*(c^2*x^2+1)^(1/2)/d/(c^2*d-e)/(e*x^2+d)^(3/2)+8/15*x*(a
+b*arcsinh(c*x))/d^3/(e*x^2+d)^(1/2)-2/15*b*c*(3*c^2*d-2*e)*(c^2*x^2+1)^(1/
2)/d^2/(c^2*d-e)^2/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.82, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {192, 191, 5704, 12, 6715, 949, 78, 63, 217, 206}

$$\frac{8x\left(a+b\sinh^{-1}(cx)\right)}{15d^3\sqrt{d+ex^2}}+\frac{4x\left(a+b\sinh^{-1}(cx)\right)}{15d^2\left(d+ex^2\right)^{3/2}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{5d\left(d+ex^2\right)^{5/2}}-\frac{2bc\sqrt{c^2x^2+1}\left(3c^2d-2e\right)}{15d^2\left(c^2d-e\right)^2\sqrt{d+ex^2}}-\frac{8b\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c}}{c\sqrt{d}}\right.}{15d^3\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(d + e*x^2)^(7/2),x]
```

```
[Out] -(b*c*Sqrt[1 + c^2*x^2])/(15*d*(c^2*d - e)*(d + e*x^2)^(3/2)) - (2*b*c*(3*c
^2*d - 2*e)*Sqrt[1 + c^2*x^2])/(15*d^2*(c^2*d - e)^2*Sqrt[d + e*x^2]) + (x*
(a + b*ArcSinh[c*x]))/(5*d*(d + e*x^2)^(5/2)) + (4*x*(a + b*ArcSinh[c*x]))/
(15*d^2*(d + e*x^2)^(3/2)) + (8*x*(a + b*ArcSinh[c*x]))/(15*d^3*Sqrt[d + e*
x^2]) - (8*b*ArcTanh[(Sqrt[e]*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(15*
d^3*Sqrt[e])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 5704

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0])
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{\left(d+ex^2\right)^{7/2}}\,dx = \frac{x\left(a+b\sinh^{-1}(cx)\right)}{5d\left(d+ex^2\right)^{5/2}}+\frac{4x\left(a+b\sinh^{-1}(cx)\right)}{15d^2\left(d+ex^2\right)^{3/2}}+\frac{8x\left(a+b\sinh^{-1}(cx)\right)}{15d^3\sqrt{d+ex^2}}-(bc)\int \frac{x\left(15d^2\right.}{15d^3\sqrt{1}}$$

$$=\frac{x\left(a+b\sinh^{-1}(cx)\right)}{5d\left(d+ex^2\right)^{5/2}}+\frac{4x\left(a+b\sinh^{-1}(cx)\right)}{15d^2\left(d+ex^2\right)^{3/2}}+\frac{8x\left(a+b\sinh^{-1}(cx)\right)}{15d^3\sqrt{d+ex^2}}-\frac{(bc)\int \frac{x\left(15d^2+20\right.}{\sqrt{1+c^2x^2}}}{15d}$$

$$=\frac{x\left(a+b\sinh^{-1}(cx)\right)}{5d\left(d+ex^2\right)^{5/2}}+\frac{4x\left(a+b\sinh^{-1}(cx)\right)}{15d^2\left(d+ex^2\right)^{3/2}}+\frac{8x\left(a+b\sinh^{-1}(cx)\right)}{15d^3\sqrt{d+ex^2}}-\frac{(bc)\,\text{Subst}\left(\int \frac{1}{\ }\right.}{}$$

$$=-\frac{bc\sqrt{1+c^2x^2}}{15d\left(c^2d-e\right)\left(d+ex^2\right)^{3/2}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{5d\left(d+ex^2\right)^{5/2}}+\frac{4x\left(a+b\sinh^{-1}(cx)\right)}{15d^2\left(d+ex^2\right)^{3/2}}+\frac{8x\left(a+b\right.}{15d^3\sqrt{}}$$

$$=-\frac{bc\sqrt{1+c^2x^2}}{15d\left(c^2d-e\right)\left(d+ex^2\right)^{3/2}}-\frac{2bc\left(3c^2d-2e\right)\sqrt{1+c^2x^2}}{15d^2\left(c^2d-e\right)^2\sqrt{d+ex^2}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{5d\left(d+ex^2\right)^{5/2}}+\frac{4x\left(a\right.}{15}$$

$$=-\frac{bc\sqrt{1+c^2x^2}}{15d\left(c^2d-e\right)\left(d+ex^2\right)^{3/2}}-\frac{2bc\left(3c^2d-2e\right)\sqrt{1+c^2x^2}}{15d^2\left(c^2d-e\right)^2\sqrt{d+ex^2}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{5d\left(d+ex^2\right)^{5/2}}+\frac{4x\left(a\right.}{15}$$

$$=-\frac{bc\sqrt{1+c^2x^2}}{15d\left(c^2d-e\right)\left(d+ex^2\right)^{3/2}}-\frac{2bc\left(3c^2d-2e\right)\sqrt{1+c^2x^2}}{15d^2\left(c^2d-e\right)^2\sqrt{d+ex^2}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{5d\left(d+ex^2\right)^{5/2}}+\frac{4x\left(a\right.}{15}$$

$$=-\frac{bc\sqrt{1+c^2x^2}}{15d\left(c^2d-e\right)\left(d+ex^2\right)^{3/2}}-\frac{2bc\left(3c^2d-2e\right)\sqrt{1+c^2x^2}}{15d^2\left(c^2d-e\right)^2\sqrt{d+ex^2}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{5d\left(d+ex^2\right)^{5/2}}+\frac{4x\left(a\right.}{15}$$

Mathematica [C] time = 0.42, size = 191, normalized size = 0.84

$$\frac{ax\left(15d^2+20dex^2+8e^2x^4\right)-4bcx^2\sqrt{\frac{ex^2}{d}+1}\left(d+ex^2\right)^2F_1\left(1;\frac{1}{2},\frac{1}{2};2;-c^2x^2,-\frac{ex^2}{d}\right)-\frac{bcd\sqrt{c^2x^2+1}\left(d+ex^2\right)\left(c^2d\left(7d+6ex^2\right)-\right.}{\left(e-c^2d\right)^2}}{15d^3\left(d+ex^2\right)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + e*x^2)^(7/2),x]
```

```
[Out] (a*x*(15*d^2 + 20*d*e*x^2 + 8*e^2*x^4) - (b*c*d*Sqrt[1 + c^2*x^2]*(d + e*x^
2)*(-(e*(5*d + 4*e*x^2)) + c^2*d*(7*d + 6*e*x^2)))/(-(c^2*d) + e)^2 - 4*b*c
*x^2*(d + e*x^2)^2*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, -(c^2*x^2),
 -((e*x^2)/d)] + b*x*(15*d^2 + 20*d*e*x^2 + 8*e^2*x^4)*ArcSinh[c*x])/(15*d^
3*(d + e*x^2)^(5/2))
```

fricas [B] time = 0.85, size = 1354, normalized size = 5.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/15*(2*(b*c^4*d^5 - 2*b*c^2*d^4*e + (b*c^4*d^2*e^3 - 2*b*c^2*d*e^4 + b*e^
5)*x^6 + b*d^3*e^2 + 3*(b*c^4*d^3*e^2 - 2*b*c^2*d^2*e^3 + b*d*e^4)*x^4 + 3*
(b*c^4*d^4*e - 2*b*c^2*d^3*e^2 + b*d^2*e^3)*x^2)*sqrt(e)*log(8*c^4*e^2*x^4
+ c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d
+ c*e)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + (8*(b*c^4*d^2*e^3
 - 2*b*c^2*d*e^4 + b*e^5)*x^5 + 20*(b*c^4*d^3*e^2 - 2*b*c^2*d^2*e^3 + b*d*e
^4)*x^3 + 15*(b*c^4*d^4*e - 2*b*c^2*d^3*e^2 + b*d^2*e^3)*x)*sqrt(e*x^2 + d)
*log(c*x + sqrt(c^2*x^2 + 1)) + (8*(a*c^4*d^2*e^3 - 2*a*c^2*d*e^4 + a*e^5)*
x^5 + 20*(a*c^4*d^3*e^2 - 2*a*c^2*d^2*e^3 + a*d*e^4)*x^3 + 15*(a*c^4*d^4*e
- 2*a*c^2*d^3*e^2 + a*d^2*e^3)*x - (7*b*c^3*d^4*e - 5*b*c*d^3*e^2 + 2*(3*b*
c^3*d^2*e^3 - 2*b*c*d*e^4)*x^4 + (13*b*c^3*d^3*e^2 - 9*b*c*d^2*e^3)*x^2)*sq
rt(c^2*x^2 + 1))*sqrt(e*x^2 + d))/(c^4*d^8*e - 2*c^2*d^7*e^2 + d^6*e^3 + (c
^4*d^5*e^4 - 2*c^2*d^4*e^5 + d^3*e^6)*x^6 + 3*(c^4*d^6*e^3 - 2*c^2*d^5*e^4
+ d^4*e^5)*x^4 + 3*(c^4*d^7*e^2 - 2*c^2*d^6*e^3 + d^5*e^4)*x^2), 1/15*(4*(b
*c^4*d^5 - 2*b*c^2*d^4*e + (b*c^4*d^2*e^3 - 2*b*c^2*d*e^4 + b*e^5)*x^6 + b*
d^3*e^2 + 3*(b*c^4*d^3*e^2 - 2*b*c^2*d^2*e^3 + b*d*e^4)*x^4 + 3*(b*c^4*d^4*
e - 2*b*c^2*d^3*e^2 + b*d^2*e^3)*x^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^
2*d + e)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 + c*d*e +
(c^3*d*e + c*e^2)*x^2)) + (8*(b*c^4*d^2*e^3 - 2*b*c^2*d*e^4 + b*e^5)*x^5 +
20*(b*c^4*d^3*e^2 - 2*b*c^2*d^2*e^3 + b*d*e^4)*x^3 + 15*(b*c^4*d^4*e - 2*b*
c^2*d^3*e^2 + b*d^2*e^3)*x)*sqrt(e*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) +
(8*(a*c^4*d^2*e^3 - 2*a*c^2*d*e^4 + a*e^5)*x^5 + 20*(a*c^4*d^3*e^2 - 2*a*c^
2*d^2*e^3 + a*d*e^4)*x^3 + 15*(a*c^4*d^4*e - 2*a*c^2*d^3*e^2 + a*d^2*e^3)*x
 - (7*b*c^3*d^4*e - 5*b*c*d^3*e^2 + 2*(3*b*c^3*d^2*e^3 - 2*b*c*d*e^4)*x^4 +
 (13*b*c^3*d^3*e^2 - 9*b*c*d^2*e^3)*x^2)*sqrt(c^2*x^2 + 1))*sqrt(e*x^2 + d)
)/(c^4*d^8*e - 2*c^2*d^7*e^2 + d^6*e^3 + (c^4*d^5*e^4 - 2*c^2*d^4*e^5 + d^3
*e^6)*x^6 + 3*(c^4*d^6*e^3 - 2*c^2*d^5*e^4 + d^4*e^5)*x^4 + 3*(c^4*d^7*e^2
- 2*c^2*d^6*e^3 + d^5*e^4)*x^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\left(ex^2 + d\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(e*x^2 + d)^(7/2), x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arcsinh}(cx)}{\left(e\,x^2 + d\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(e*x^2+d)^(7/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(e*x^2+d)^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{15}\, a\left(\frac{8\,x}{\sqrt{ex^2 + d}\,d^3} + \frac{4\,x}{\left(ex^2 + d\right)^{\frac{3}{2}} d^2} + \frac{3\,x}{\left(ex^2 + d\right)^{\frac{5}{2}} d}\right) + b\int \frac{\log\left(cx + \sqrt{c^2x^2 + 1}\right)}{\left(ex^2 + d\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*a*(8*x/(sqrt(e*x^2 + d)*d^3) + 4*x/((e*x^2 + d)^(3/2)*d^2) + 3*x/((e*x
^2 + d)^(5/2)*d)) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(e*x^2 + d)^(7
/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}\left(c\,x\right)}{{\left(e\,x^2+d\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(d + e*x^2)^(7/2),x)
```

```
[Out] int((a + b*asinh(c*x))/(d + e*x^2)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(e*x**2+d)**(7/2),x)
```

```
[Out] Timed out
```

3.652 $\int \sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=25

$$\text{Int}\left(\sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)^2,x\right)$$

```
[Out] Unintegrable((a+b*arcsinh(c*x))^2*(e*x^2+d)^(1/2),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2,x]
[Out] Defer[Int][Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2, x]
```

Rubi steps

$$\int \sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)^2 dx = \int \sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)^2 dx$$

Mathematica [A] time = 14.40, size = 0, normalized size = 0.00

$$\int \sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2,x]
[Out] Integrate[Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2, x]
```

fricas [A] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\,\text{arsinh}\,(cx)^2+2\,ab\,\text{arsinh}\,(cx)+a^2\right)\sqrt{ex^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)*sqrt(e*x^2 + d), x
)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex^2+d}\,(b\,\text{arsinh}\,(cx)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsinh(c*x) + a)^2, x)
```

maple [A] time = 0.26, size = 0, normalized size = 0.00

$$\int (a+b\,\text{arcsinh}\,(cx))^2\,\sqrt{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2*(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2*(e*x^2+d)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e-c^2*d>0)', see `assume?` for more
 details)Is e-c^2*d zero or nonzero?
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int (a+b\,\mathrm{asinh}\left(c\,x\right))^2\,\sqrt{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + e*x^2)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + e*x^2)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\,\mathrm{asinh}\,(cx))^2\,\sqrt{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2*sqrt(d + e*x**2), x)
```

3.653 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable((a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c*x])^2/Sqrt[d + e*x^2],x]
[Out] Defer[Int][(a + b*ArcSinh[c*x])^2/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+ex^2}}\,dx = \int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+ex^2}}\,dx$$

Mathematica [A] time = 11.41, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/Sqrt[d + e*x^2],x]
[Out] Integrate[(a + b*ArcSinh[c*x])^2/Sqrt[d + e*x^2], x]
```

fricas [A] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\,\text{arsinh}\,(cx)^2+2\,ab\,\text{arsinh}\,(cx)+a^2}{\sqrt{ex^2+d}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/sqrt(e*x^2 + d), x
)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\text{arsinh}\,(cx)+a\right)^2}{\sqrt{ex^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/sqrt(e*x^2 + d), x)
```

maple [A] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{arcsinh}\left(cx\right)\right)^{2}}{\sqrt{e\,x^{2} + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^{2} \operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{e}} + \int \frac{b^{2} \log\left(cx + \sqrt{c^{2}x^{2} + 1}\right)^{2}}{\sqrt{ex^{2} + d}} + \frac{2\,ab \log\left(cx + \sqrt{c^{2}x^{2} + 1}\right)}{\sqrt{ex^{2} + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] a^2*arcsinh(e*x/sqrt(d*e))/sqrt(e) + integrate(b^2*log(c*x + sqrt(c^2*x^2 +
 1))^2/sqrt(e*x^2 + d) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/sqrt(e*x^2 + d)
, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a + b \operatorname{asinh}\left(c\,x\right)\right)^{2}}{\sqrt{e\,x^{2} + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(d + e*x^2)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(d + e*x^2)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{asinh}\left(cx\right)\right)^{2}}{\sqrt{d + ex^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/sqrt(d + e*x**2), x)
```

3.654 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=25

$$\mathrm{Int}\left(\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable((a+b*arcsinh(c*x))^2/(e*x^2+d)^(3/2),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(3/2),x]
[Out] Defer[Int][(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{3/2}}\,dx$$

Mathematica [A] time = 3.13, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(3/2),x]
[Out] Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(3/2), x]
```

fricas [A] time = 0.60, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(b^2\operatorname{arsinh}(cx)^2+2\,ab\operatorname{arsinh}(cx)+a^2\right)\sqrt{ex^2+d}}{e^2x^4+2\,dex^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(3/2),x, algorithm="fricas")
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)*sqrt(e*x^2 + d)/(e
^2*x^4 + 2*d*e*x^2 + d^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\operatorname{arsinh}(cx)+a\right)^2}{\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(e*x^2 + d)^(3/2), x)
```

maple [A] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2}{\left(e\,x^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(e*x^2+d)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e-c^2*d>0)', see `assume?` for more
 details)Is e-c^2*d zero or nonzero?
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))^2}{\left(e\,x^2+d\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(d + e*x^2)^(3/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(d + e*x^2)^(3/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(cx))^2}{\left(d+ex^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(d + e*x**2)**(3/2), x)
```

3.655 $\int \frac{(a+b\sinh^{-1}(cx))^2}{(d+ex^2)^{5/2}}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{5/2}},x\right)$$

```
[Out] Unintegrable((a+b*arcsinh(c*x))^2/(e*x^2+d)^(5/2),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(5/2),x]
```

```
[Out] Defer[Int][(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(5/2), x]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{5/2}}\,dx$$

Mathematica [A] time = 5.90, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(5/2),x]
```

```
[Out] Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(5/2), x]
```

fricas [A] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2\operatorname{arsinh}(cx)^2+2\,ab\operatorname{arsinh}(cx)+a^2\right)\sqrt{ex^2+d}}{e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)*sqrt(e*x^2 + d)/(e
^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arsinh}(cx)+a)^2}{\left(ex^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(e*x^2 + d)^(5/2), x)
```

maple [A] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2}{\left(e\,x^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(e*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(e*x^2+d)^(5/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\,a^2\left(\frac{2\,x}{\sqrt{ex^2+d}\,d^2}+\frac{x}{\left(ex^2+d\right)^{\frac{3}{2}}d}\right)+\int\frac{b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{\left(ex^2+d\right)^{\frac{5}{2}}}+\frac{2\,ab\log\left(cx+\sqrt{c^2x^2+1}\right)}{\left(ex^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*a^2*(2*x/(sqrt(e*x^2 + d)*d^2) + x/((e*x^2 + d)^(3/2)*d)) + integrate(b
^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(e*x^2 + d)^(5/2) + 2*a*b*log(c*x + sqrt(
c^2*x^2 + 1))/(e*x^2 + d)^(5/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))^2}{\left(e\,x^2+d\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(d + e*x^2)^(5/2),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(d + e*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

3.656 $\int \frac{\sqrt{d+ex^2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\sqrt{d+ex^2}}{a+b\sinh^{-1}(cx)},x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{d+ex^2}}{a+b\sinh^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x]),x]
[Out] Defer[Int][Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x]), x]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{a+b\sinh^{-1}(cx)}\,dx = \int \frac{\sqrt{d+ex^2}}{a+b\sinh^{-1}(cx)}\,dx$$

Mathematica [A] time = 1.21, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2}}{a+b\sinh^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x]),x]
[Out] Integrate[Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x]), x]
```

fricas [A] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2+d}}{b\,\text{arsinh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)/(b*arcsinh(c*x) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2+d}}{b\,\text{arsinh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)/(b*arcsinh(c*x) + a), x)
```

maple [A] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\,x^{2}+d}}{a+b\,\mathrm{arcsinh}\left(cx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x)
[Out] int((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^{2}+d}}{b\,\mathrm{arsinh}\left(cx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(sqrt(e*x^2 + d)/(b*arcsinh(c*x) + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{e\,x^{2}+d}}{a+b\,\mathrm{asinh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(1/2)/(a + b*asinh(c*x)),x)
[Out] int((d + e*x^2)^(1/2)/(a + b*asinh(c*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^{2}}}{a+b\,\mathrm{asinh}\left(cx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/(a+b*asinh(c*x)),x)
[Out] Integral(sqrt(d + e*x**2)/(a + b*asinh(c*x)), x)
```

3.657 $\int \frac{1}{\sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{\sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/(a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{1}{\sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 1.08, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])), x]
```

fricas [A] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2+d}}{aex^2+ad+\left(bex^2+bd\right)\operatorname{arsinh}(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)/(a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsinh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ex^2+d}\,(b\operatorname{arsinh}(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(e*x^2 + d)*(b*arcsinh(c*x) + a)), x)
```

maple [A] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\,\mathrm{arcsinh}\,(cx))\,\sqrt{e\,x^{2}+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x)
[Out] int(1/(a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ex^{2}+d}\,(b\,\mathrm{arsinh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(e*x^2 + d)*(b*arcsinh(c*x) + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(a+b\,\mathrm{asinh}\,(c\,x))\,\sqrt{e\,x^{2}+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))*(d + e*x^2)^(1/2)),x)
[Out] int(1/((a + b*asinh(c*x))*(d + e*x^2)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\,\mathrm{asinh}\,(cx))\,\sqrt{d+ex^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(c*x))/(e*x**2+d)**(1/2),x)
[Out] Integral(1/((a + b*asinh(c*x))*sqrt(d + e*x**2)), x)
```

3.658 $\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 1.53, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

fricas [A] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2+d}}{ae^2x^4+2\,adex^2+ad^2+\left(be^2x^4+2\,bdex^2+bd^2\right)\text{arsinh}\,(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)/(a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*
b*d*e*x^2 + b*d^2)*arcsinh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex^2+d\right)^{\frac{3}{2}}\left(b\,\text{arsinh}\,(cx)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate(1/((e*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

maple [A] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\,x^{2}+d\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arcsinh}\left(c x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x)),x)
[Out] int(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex^{2}+d\right)^{\frac{3}{2}}\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(1/((e*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(e\,x^{2}+d\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))*(d + e*x^2)^(3/2)),x)
[Out] int(1/((a + b*asinh(c*x))*(d + e*x^2)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(cx\right)\right)\left(d+ex^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(3/2)/(a+b*asinh(c*x)),x)
[Out] Integral(1/((a + b*asinh(c*x))*(d + e*x**2)**(3/2)), x)
```

3.659 $\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{\left(d+ex^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

[Out] Unintegrable(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x)),x)

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

[In] Int[1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])),x]

[Out] Defer[Int][1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])), x]

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 3.72, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])),x]

[Out] Integrate[1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])), x]

fricas [A] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2+d}}{ae^3x^6+3\,ade^2x^4+3\,ad^2ex^2+ad^3+\left(be^3x^6+3\,bde^2x^4+3\,bd^2ex^2+bd^3\right)\text{arsinh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")

[Out] integral(sqrt(e*x^2 + d)/(a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 + 3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arcsinh(c*x)), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex^2+d\right)^{\frac{5}{2}}\left(b\,\text{arsinh}\left(cx\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)), x)
```

maple [A] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\,x^{2}+d\right)^{\frac{5}{2}}\left(a+b\,\mathrm{arcsinh}\left(c x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] int(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex^{2}+d\right)^{\frac{5}{2}}\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(e\,x^2+d\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))*(d + e*x^2)^(5/2)),x)
```

```
[Out] int(1/((a + b*asinh(c*x))*(d + e*x^2)^(5/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(cx\right)\right)\left(d+ex^{2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(5/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(1/((a + b*asinh(c*x))*(d + e*x**2)**(5/2)), x)
```

3.660 $\int \frac{\sqrt{d+ex^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\sqrt{d+ex^2}}{\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{d+ex^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x])^2,x]
[Out] Defer[Int][Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x])^2, x]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{\sqrt{d+ex^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 4.67, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x])^2,x]
[Out] Integrate[Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x])^2, x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2+d}}{b^2\operatorname{arsinh}(cx)^2+2\,ab\operatorname{arsinh}(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2), x
)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2+d}}{\left(b\operatorname{arsinh}(cx)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)/(b*arcsinh(c*x) + a)^2, x)
```

maple [A] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\,x^{2}+d}}{\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}\sqrt{ex^{2}+d}+\left(c^{3}x^{3}+cx\right)\sqrt{ex^{2}+d}}{abc^{3}x^{2}+\sqrt{c^{2}x^{2}+1}\,abc^{2}x+abc+\left(b^{2}c^{3}x^{2}+\sqrt{c^{2}x^{2}+1}\,b^{2}c^{2}x+b^{2}c\right)\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)}+\int\frac{}{abc^{5}ex^{6}+\left(c^{5}d+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^2 + 1)^(3/2)*sqrt(e*x^2 + d) + (c^3*x^3 + c*x)*sqrt(e*x^2 + d))/(a
*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*
x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((2*c
^3*e*x^4 + c^3*d*x^2 - c*d)*(c^2*x^2 + 1)*sqrt(e*x^2 + d) + (4*c^4*e*x^5 +
2*(c^4*d + 2*c^2*e)*x^3 + (c^2*d + e)*x)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d)
+ (2*c^5*e*x^6 + (c^5*d + 4*c^3*e)*x^4 + 2*(c^3*d + c*e)*x^2 + c*d)*sqrt(e*
x^2 + d))/(a*b*c^5*e*x^6 + (c^5*d + 2*c^3*e)*a*b*x^4 + (2*c^3*d + c*e)*a*b*
x^2 + a*b*c*d + (a*b*c^3*e*x^4 + a*b*c^3*d*x^2)*(c^2*x^2 + 1) + (b^2*c^5*e*
x^6 + (c^5*d + 2*c^3*e)*b^2*x^4 + (2*c^3*d + c*e)*b^2*x^2 + b^2*c*d + (b^2*
c^3*e*x^4 + b^2*c^3*d*x^2)*(c^2*x^2 + 1) + 2*(b^2*c^4*e*x^5 + b^2*c^2*d*x +
 (c^4*d + c^2*e)*b^2*x^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) +
 2*(a*b*c^4*e*x^5 + a*b*c^2*d*x + (c^4*d + c^2*e)*a*b*x^3)*sqrt(c^2*x^2 + 1
)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{e\,x^{2}+d}}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(1/2)/(a + b*asinh(c*x))^2,x)
```

```
[Out] int((d + e*x^2)^(1/2)/(a + b*asinh(c*x))^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^{2}}}{\left(a+b\,\mathrm{asinh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(sqrt(d + e*x**2)/(a + b*asinh(c*x))**2, x)
```

3.661 $\int \frac{1}{\sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{\sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Defer[Int][1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 6.45, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2), x]
```

fricas [A] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2+d}}{a^2ex^2+a^2d+\left(b^2ex^2+b^2d\right)\operatorname{arsinh}(cx)^2+2\left(abex^2+abd\right)\operatorname{arsinh}(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)/(a^2*e*x^2 + a^2*d + (b^2*e*x^2 + b^2*d)*arcsinh(c
*x)^2 + 2*(a*b*e*x^2 + a*b*d)*arcsinh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ex^2+d}\,(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(e*x^2 + d)*(b*arcsinh(c*x) + a)^2), x)
```

maple [A] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^{2}\sqrt{e\,x^{2}+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x)
```

```
[Out] int(1/(a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^{3}x^{3}+cx+\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}}{\sqrt{c^{2}x^{2}+1}\,\sqrt{ex^{2}+d}\,abc^{2}x+\left(\sqrt{c^{2}x^{2}+1}\,\sqrt{ex^{2}+d}\,b^{2}c^{2}x+\left(b^{2}c^{3}x^{2}+b^{2}c\right)\sqrt{ex^{2}+d}\right)\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)+\left(ab\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d)*a
*b*c^2*x + (sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d)*b^2*c^2*x + (b^2*c^3*x^2 + b^
2*c)*sqrt(e*x^2 + d))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*
sqrt(e*x^2 + d)) + integrate((c^5*d*x^4 + 2*c^3*d*x^2 + (c^2*x^2 + 1)*((c^3
*d - 2*c*e)*x^2 - c*d) + c*d + sqrt(c^2*x^2 + 1)*(2*(c^4*d - c^2*e)*x^3 + (
c^2*d - e)*x))/((a*b*c^3*e*x^4 + a*b*c^3*d*x^2)*(c^2*x^2 + 1)*sqrt(e*x^2 +
d) + 2*(a*b*c^4*e*x^5 + a*b*c^2*d*x + (c^4*d + c^2*e)*a*b*x^3)*sqrt(c^2*x^2
 + 1)*sqrt(e*x^2 + d) + ((b^2*c^3*e*x^4 + b^2*c^3*d*x^2)*(c^2*x^2 + 1)*sqrt
(e*x^2 + d) + 2*(b^2*c^4*e*x^5 + b^2*c^2*d*x + (c^4*d + c^2*e)*b^2*x^3)*sqr
t(c^2*x^2 + 1)*sqrt(e*x^2 + d) + (b^2*c^5*e*x^6 + (c^5*d + 2*c^3*e)*b^2*x^4
 + (2*c^3*d + c*e)*b^2*x^2 + b^2*c*d)*sqrt(e*x^2 + d))*log(c*x + sqrt(c^2*x
^2 + 1)) + (a*b*c^5*e*x^6 + (c^5*d + 2*c^3*e)*a*b*x^4 + (2*c^3*d + c*e)*a*b
*x^2 + a*b*c*d)*sqrt(e*x^2 + d)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))^2*(d + e*x^2)^(1/2)),x)
```

```
[Out] int(1/((a + b*asinh(c*x))^2*(d + e*x^2)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(cx\right)\right)^{2}\sqrt{d+ex^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(c*x))**2/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(1/((a + b*asinh(c*x))**2*sqrt(d + e*x**2)), x)
```

3.662 $\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 16.77, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

fricas [A] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2+d}}{a^2e^2x^4+2\,a^2dex^2+a^2d^2+\left(b^2e^2x^4+2\,b^2dex^2+b^2d^2\right)\text{arsinh}\left(cx\right)^2+2\left(abe^2x^4+2\,abdex^2+abd^2\right)a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)/(a^2*e^2*x^4 + 2*a^2*d*e*x^2 + a^2*d^2 + (b^2*e^2*
x^4 + 2*b^2*d*e*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*e^2*x^4 + 2*a*b*d*e*
x^2 + a*b*d^2)*arcsinh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex^2+d\right)^{\frac{3}{2}}\left(b\,\text{arsinh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^2), x)
```

maple [A] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\,x^2+d\right)^{\frac{3}{2}}\left(a+b\operatorname{arcsinh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^3x^3+cx+\left(c^2x^2+1\right)^{\frac{3}{2}}}{\left(abc^2ex^3+abc^2dx\right)\sqrt{c^2x^2+1}\,\sqrt{ex^2+d}+\left(\left(b^2c^2ex^3+b^2c^2dx\right)\sqrt{c^2x^2+1}\,\sqrt{ex^2+d}+\left(b^2c^3ex^4+\left(c^3d+ce\right)b^2x^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/((a*b*c^2*e*x^3 + a*b*c^2*d*x)*sqrt(
c^2*x^2 + 1)*sqrt(e*x^2 + d) + ((b^2*c^2*e*x^3 + b^2*c^2*d*x)*sqrt(c^2*x^2
+ 1)*sqrt(e*x^2 + d) + (b^2*c^3*e*x^4 + (c^3*d + c*e)*b^2*x^2 + b^2*c*d)*sq
rt(e*x^2 + d))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*e*x^4 + (c^3*d + c*e
)*a*b*x^2 + a*b*c*d)*sqrt(e*x^2 + d)) - integrate((2*c^5*e*x^6 - (c^5*d - 4
*c^3*e)*x^4 - 2*(c^3*d - c*e)*x^2 + (2*c^3*e*x^4 - (c^3*d - 4*c*e)*x^2 + c*
d)*(c^2*x^2 + 1) - c*d + (4*c^4*e*x^5 - 2*(c^4*d - 4*c^2*e)*x^3 - (c^2*d -
3*e)*x)*sqrt(c^2*x^2 + 1))/((a*b*c^3*e^2*x^6 + 2*a*b*c^3*d*e*x^4 + a*b*c^3*
d^2*x^2)*(c^2*x^2 + 1)*sqrt(e*x^2 + d) + 2*(a*b*c^4*e^2*x^7 + (2*c^4*d*e +
c^2*e^2)*a*b*x^5 + a*b*c^2*d^2*x + (c^4*d^2 + 2*c^2*d*e)*a*b*x^3)*sqrt(c^2*
x^2 + 1)*sqrt(e*x^2 + d) + ((b^2*c^3*e^2*x^6 + 2*b^2*c^3*d*e*x^4 + b^2*c^3*
d^2*x^2)*(c^2*x^2 + 1)*sqrt(e*x^2 + d) + 2*(b^2*c^4*e^2*x^7 + (2*c^4*d*e +
c^2*e^2)*b^2*x^5 + b^2*c^2*d^2*x + (c^4*d^2 + 2*c^2*d*e)*b^2*x^3)*sqrt(c^2*
x^2 + 1)*sqrt(e*x^2 + d) + (b^2*c^5*e^2*x^8 + 2*(c^5*d*e + c^3*e^2)*b^2*x^6
 + (c^5*d^2 + 4*c^3*d*e + c*e^2)*b^2*x^4 + b^2*c*d^2 + 2*(c^3*d^2 + c*d*e)*
b^2*x^2)*sqrt(e*x^2 + d))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*e^2*x^8 +
 2*(c^5*d*e + c^3*e^2)*a*b*x^6 + (c^5*d^2 + 4*c^3*d*e + c*e^2)*a*b*x^4 + a*
b*c*d^2 + 2*(c^3*d^2 + c*d*e)*a*b*x^2)*sqrt(e*x^2 + d)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(e\,x^2+d\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))^2*(d + e*x^2)^(3/2)),x)
```

```
[Out] int(1/((a + b*asinh(c*x))^2*(d + e*x^2)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\operatorname{asinh}\left(cx\right)\right)^2\left(d+ex^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(3/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(1/((a + b*asinh(c*x))**2*(d + e*x**2)**(3/2)), x)
```

3.663 $\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{\left(d+ex^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 29.13, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

fricas [A] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2+d}}{a^2e^3x^6+3\,a^2de^2x^4+3\,a^2d^2ex^2+a^2d^3+\left(b^2e^3x^6+3\,b^2de^2x^4+3\,b^2d^2ex^2+b^2d^3\right)\text{arsinh}\,(cx)^2+2\,\left(abe^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)/(a^2*e^3*x^6 + 3*a^2*d*e^2*x^4 + 3*a^2*d^2*e*x^2 +
 a^2*d^3 + (b^2*e^3*x^6 + 3*b^2*d*e^2*x^4 + 3*b^2*d^2*e*x^2 + b^2*d^3)*arcs
inh(c*x)^2 + 2*(a*b*e^3*x^6 + 3*a*b*d*e^2*x^4 + 3*a*b*d^2*e*x^2 + a*b*d^3)*
arcsinh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex^2+d\right)^{\frac{5}{2}}\left(b\,\text{arsinh}\,(cx)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^2), x)
```

maple [A] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\,x^{2}+d\right)^{\frac{5}{2}} \left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{}{\left(abc^{2}e^{2}x^{5}+2\,abc^{2}dex^{3}+abc^{2}d^{2}x\right)\sqrt{c^{2}x^{2}+1}\,\sqrt{ex^{2}+d}+\left(\left(b^{2}c^{2}e^{2}x^{5}+2\,b^{2}c^{2}dex^{3}+b^{2}c^{2}d^{2}x\right)\sqrt{c^{2}x^{2}+1}\,\sqrt{ex^{2}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/((a*b*c^2*e^2*x^5 + 2*a*b*c^2*d*e*x^
3 + a*b*c^2*d^2*x)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d) + ((b^2*c^2*e^2*x^5 +
2*b^2*c^2*d*e*x^3 + b^2*c^2*d^2*x)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d) + (b^2
*c^3*e^2*x^6 + (2*c^3*d*e + c*e^2)*b^2*x^4 + b^2*c*d^2 + (c^3*d^2 + 2*c*d*e
)*b^2*x^2)*sqrt(e*x^2 + d))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*e^2*x^6
 + (2*c^3*d*e + c*e^2)*a*b*x^4 + a*b*c*d^2 + (c^3*d^2 + 2*c*d*e)*a*b*x^2)*s
qrt(e*x^2 + d)) - integrate((4*c^5*e*x^6 - (c^5*d - 8*c^3*e)*x^4 - 2*(c^3*d
 - 2*c*e)*x^2 + (4*c^3*e*x^4 - (c^3*d - 6*c*e)*x^2 + c*d)*(c^2*x^2 + 1) - c
*d + (8*c^4*e*x^5 - 2*(c^4*d - 7*c^2*e)*x^3 - (c^2*d - 5*e)*x)*sqrt(c^2*x^2
 + 1))/((a*b*c^3*e^3*x^8 + 3*a*b*c^3*d*e^2*x^6 + 3*a*b*c^3*d^2*e*x^4 + a*b*
c^3*d^3*x^2)*(c^2*x^2 + 1)*sqrt(e*x^2 + d) + 2*(a*b*c^4*e^3*x^9 + (3*c^4*d*
e^2 + c^2*e^3)*a*b*x^7 + a*b*c^2*d^3*x + 3*(c^4*d^2*e + c^2*d*e^2)*a*b*x^5
+ (c^4*d^3 + 3*c^2*d^2*e)*a*b*x^3)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d) + ((b^
2*c^3*e^3*x^8 + 3*b^2*c^3*d*e^2*x^6 + 3*b^2*c^3*d^2*e*x^4 + b^2*c^3*d^3*x^2
)*(c^2*x^2 + 1)*sqrt(e*x^2 + d) + 2*(b^2*c^4*e^3*x^9 + (3*c^4*d*e^2 + c^2*e
^3)*b^2*x^7 + b^2*c^2*d^3*x + 3*(c^4*d^2*e + c^2*d*e^2)*b^2*x^5 + (c^4*d^3
+ 3*c^2*d^2*e)*b^2*x^3)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d) + (b^2*c^5*e^3*x^
10 + (3*c^5*d*e^2 + 2*c^3*e^3)*b^2*x^8 + (3*c^5*d^2*e + 6*c^3*d*e^2 + c*e^3
)*b^2*x^6 + (c^5*d^3 + 6*c^3*d^2*e + 3*c*d*e^2)*b^2*x^4 + b^2*c*d^3 + (2*c^
3*d^3 + 3*c*d^2*e)*b^2*x^2)*sqrt(e*x^2 + d))*log(c*x + sqrt(c^2*x^2 + 1)) +
 (a*b*c^5*e^3*x^10 + (3*c^5*d*e^2 + 2*c^3*e^3)*a*b*x^8 + (3*c^5*d^2*e + 6*c
^3*d*e^2 + c*e^3)*a*b*x^6 + (c^5*d^3 + 6*c^3*d^2*e + 3*c*d*e^2)*a*b*x^4 + a
*b*c*d^3 + (2*c^3*d^3 + 3*c*d^2*e)*a*b*x^2)*sqrt(e*x^2 + d)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(e\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))^2*(d + e*x^2)^(5/2)),x)
```

```
[Out] int(1/((a + b*asinh(c*x))^2*(d + e*x^2)^(5/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \operatorname{asinh}(cx))^2 \left(d + ex^2\right)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(5/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(1/((a + b*asinh(c*x))**2*(d + e*x**2)**(5/2)), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```